The Science Behind Beauty – Is Beauty Really Subjective?

Student Name : Odysseas Glenis - Konistis

Student number : 582822

Supervisor : Phyllis Wan

Second assessor : Michiel van Crombrugge

Programme : MSc Data Science and Marketing Analytics

Department : Business Economics

Date : 12 August 2021

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Abstract

In this paper, I aimed to find the key facial characteristics alongside with ratios that are mostly associated and influential to attractiveness. Furthermore, some psychological traits were tested. The task was performed on the Chicago Face Database with conjunction of the 10k US Adult Faces Database. Both databases had objective ratings that were the mean of various people rating each individual making it as unbiased as possible. Variables of interest were lengths and widths of the face with various ratios. The 10k US Adult Faces Database had only psychological variables while the Chicago Face Database consisted of both distances of features and psychological attributes. The research was specified in male individuals, to better distinguish and find the features that drive attractiveness in males, as usually when we talk about beauty and attractiveness females come in mind and thus are most researched. The methods used include a wide variety of supervised machine learning algorithms for regression and compares their predictive power. Relaxed Lasso was the winner and the variables it found influential were fit to the subsequent linear model to further examine significant features and their impact on attractiveness. Our final model found that Cheekbone height, (average mid-cheek to chin for right and left) divided by face length, second third of the face (eyebrow to end of nose) divided by face length and bi-zygomatic face width are the most influential variables positively associated with attractiveness. Lip thickness should be close to 1.6 times the philtrum, face width should be close to face length divided by 1.6, nose width, face width at cheek minus face width at mouth divided by face length (cheekbone prominence), lip to chin distance and the distance of the outer corner of the eyes should be close to three eye widths are most influential to changes in attractiveness. This shows the importance of the midface and lower third of the face. By making the chin bigger, acquiring more space from overall face length should raise someone’s attractiveness. Improvement in attractiveness also could be achieved by widening the cheeks. This could be done with a procedure that affect the width of the face for example to improve the fWHR ratio (face width to height) with cheek fillers and chin fillers or implants to improve the cheekbone height ratio. These insights help male individuals to understand what features and ratios influence attractiveness and guide them to enhance their looks if they want to.

Keywords: facial attractiveness, face characteristics, face ratios, golden ratios of the face, improving attractiveness

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1. Introduction

1.1 Motivation

It is said that beauty is subjective or beauty is on the eyes of the beholder. Our society however, seems to value beauty a lot, by preferring attractive people in general in all aspects of life making their life easier, known as the *halo effect*, according to Thorndike (1920) "A constant error in psychological ratings". This situation is also observed with the rise of social media where attractive people, males and females, have more followers which leads to more reach and engagement and this can be used for an additional income or even as a main job. As a coincidence, plastic surgeries have been heavily normalized, especially towards on young individuals, as they see what is happening and want the same privileges. While more and more people are undergoing procedures to change their characteristics in an attempt to improve their attractiveness, it is observed that most people persuade certain characteristics to improve their appearance, and that there is a similarity on the faces after the procedures. This suggests that there are some certain patterns and analogies on the features that "make of brake a face" and are internationally accepted. This paper will try to identify and rank them.

Knowing which specific features of the face drive beauty on men and their importance order could be used for marketing purposes as "everything being equal, good looks sell more." Ling Peng et al. (2020) "The Faces of Success: Beauty and Ugliness Premiums in Online Platforms", (*halo effect)*. Marketers know that by associating a product with something (or someone) attractive, they can raise the perceived value of the product as well. This happens because an attractive individual in an advertisement excites the areas of the brain that make the customer buy on impulse, bypassing the sections which control rational thought. This is a key factor of a successful influencer on social media. However marketers should be aware of *the vampire effect*, defined as "a decrease in brand recall for an advertising stimulus that features a celebrity endorser versus the same stimulus with an unknown but equally attractive endorser" according to Erfgen et al. (2015) "The vampire effect: When do celebrity endorsers harm brand recall?". So, careful selection of the attractive model and his name recall is crucial, as customers may forget the company and associate the product with the famous and attractive model. By having a model that takes as an input prospects faces, of applicants for a campaign, decision makers would ask the model to evaluate and pick the best looking ones for the task or selecting affiliates and influencers to work with**,** without having to worry about their possible preferences, the individual biases. This idea is already applied and can be further improved to the rising field of the cosmetic surgery too, by evaluating a face and propose the best alterations that would increase the attractiveness of each patient's individual face. The motivation of undergoing such procedures, besides the obvious reason that everyone wants to be more beautiful and thus more attractive, is to help him in every (business) aspect of life. For example, interestingly for anyone that works in the sales sector, researchers have found that "more attractive agents sell properties at higher prices than less-attractive agents", Sean P. Salter et al. (2012) "Broker beauty and boon: a study of physical attractiveness and its effect on real estate brokers' income and productivity".

1.2 Problem Description

Everyone would like to be more beautiful and attractive. The notion "What is beautiful is good" Berscheid et al. (1972), is an intuitive trait of humans known as the *halo effect* which suggests that more attractive people are more likely to get what they want in various aspects of life, e.g. first dates Riggio et al. (1984) The role of nonverbal cues and physical attractiveness in the selection of dating partners, Berscheid et al. (1971) Physical attractiveness and dating choice: A test of the matching hypothesis, jobs Jawahar et al. (2005) Sexism and Beautyism Effects in Selection as a Function of Self-Monitoring Level of Decision Maker, Cash et al. (1985) The Aye of the beholder: Susceptibility to sexism and beautyism in the evaluation of managerial applicants, Ruffle et al. (2014) Are Good-Looking People More Employable?, favored on crime judgement Sigal et al. (1975) Beautiful but dangerous: effects of offender attractiveness and nature of the crime on juridic judgment, higher salaries and in the end be more happy Hamermesh et al. (2013) Beauty is the promise of happiness?. Thus, knowing the specific features that make most of what humans perceive as beautiful and attractive would help to improve them, augmenting our chances for a successful career and persuading a happier life overall.

Overall, prior literature suggests that facial symmetry: Rhodes et al. (2001) Attractiveness of Facial Averageness and Symmetry in Non-Western Cultures: In Search of Biologically Based Standards of Beauty and facial proportion – averageness: Grammer et al. (1994) Human (Homo sapiens) Facial Attractiveness and Sexual Selection: The Role of Symmetry and Averageness, Langlois et al. (1990) Attractive Faces Are Only Average and facial expression, Hassin et al.

(2000) Facing faces: Studies on the cognitive aspects of physiognomy, Sutherland et al. (2017) Facial first impressions from another angle: How social judgements are influenced by changeable and invariant facial properties, are significant characteristics in the determination of facial beauty, regardless of traits such as race, age and sex. Supposing these assumptions are met, what characteristics specifically are more associated with attractiveness? This paper will try to answer this question and shed more light to what influence attractiveness by inspecting more on specific characteristics of the face and their ratios that are associated more with beauty on men and in order their importance. In the end, the goal is to objectively confirm or reject several hypotheses related to what people think drive beauty and attractiveness and point out that beauty is at least to some part objective.

The rest of the paper is structured as follows. Section 2 is the literature review, in section 3 data are explained, in section 4 the methods are described, section 5 are the results and section 6 the conclusion.

2. Literature Review

2.1 Pursuit of Attractiveness

As is previously said, there are some theoretical features that are supposed to drive beauty and attractiveness. Symmetry and averageness are two main factors that make a face attractive according to Langlois et al. (1990), Grammer et al. (1994), Rhodes et al. (1996) and Little et al. (2011). Symmetry and averageness are not restricted to Western cultures, found in the research of Rhodes et al. (2001). Masashi et al. (2009), found that "for both male and female faces, the faces that are further away from the average faces of the opposite sex (i.e., supernormal or extreme faces) were preferred and the faces that resemble the faces of the opposite sex had low attractiveness evaluations and more masculine faces" and that "more masculine faces were judged more attractive". Furthermore backing up this theory, Little et al. (2001), found that "there is a relatively increased preference for masculinity and an increased preference for symmetry for women who regard themselves as attractive" indicating a theory of a different way of measuring attractiveness of males depending on female self-portrayed attractiveness. In addition, Thompson et al. (2013), found that "women judged men with high facial masculinity to have had more previous romantic partners and to take longer to fall in love" indicating masculinity in general and masculine characteristics as strong predictive factors of above average sexual experiences in males. However, Holzleitner et al. (2017), in their work found that both high and low levels of masculinity are unattractive, meaning very high masculine characteristics on a male's face could have the opposite of the desired effect on females. Additional interesting findings gave the work of Little et al. (2002), when they showed that women value masculinity in men more when they want a partner for a short-term relationship. In contrast, they show higher preference for more feminine faced males when they seek a long-term relationship. This could be explained as i) evaluating high signs of good, healthy and strong genes desirable short-term, ii) preferring more feminine looking men because they assume a more feminine faced man will be less aggressive and more cooperative and caring for their offsprings. In an effort to further examine the effect of masculinity in attractiveness, Little et al. (2002), found that averageness and masculinity are antagonistic and masculinity is close related to distinctiveness, further supporting that attractiveness is related to masculinity while their findings showing a preference for feminine traits in male faces, indicating that one should either be more masculine or more feminine looking to be more attractive, supporting the *neoteny* theory in humans. *Neoteny* theory is the preference for partners with characteristics indicating youth and health, and is supported in the work of Jones et al. (1995), where they found that males from five different countries showed preference for females that have neotenous facial proportions, e.g., large eyes, small nose and full lips. The work of Perrett et al. (1998), confirmed the preference for feminized to average or masculinized shapes of a male face but enhancing masculine facial characteristics increased both perceived dominance and negative attributions relevant to relationships and paternal investment. These results indicate a selection pressure that limits sexual dimorphism and further encourages the *neoteny* theory in humans. Interesting is what Keating et al. (2003), found: "neotenous, submissive-looking facial characteristics cue social approach and elicit help while mature, dominant-looking facial traits cue avoidance", which is supported from the study of Berry et al. (1986), suggesting that adults with various childish facial qualities are perceived to be more submissive and honest than those with more mature faces. This means that: i) people see more baby-faced males as submissive and less dominant, a fact that women don't prefer when seeking for a male partner at least at first, and ii) males with more neotenous face receive more help from others. One thought comes immediately in mind: would more baby-faced yet attractive males had increased *halo effect* than attractive men in general? An interesting question for further future research.

Adding to the previously already mentioned papers of Ling Peng et al. (2020) "The Faces of Success: Beauty and Ugliness Premiums in Online Platforms", and Anne M. Brumbaugh (1993) *"*Physical Attractiveness and Personality in Advertising: More Than Just a Pretty Face?*"*, both showing that more attractive people sell more, the work of Salter et al. (2012), also confirms that claim and adds that "beauty augments more attractive agents' wages and that more attractive agents use beauty to supplement classic production-related characteristics such as effort, intelligence and organizational skills. Hence, it is implied that beauty can be an equal trait as character traits like effort and intelligence and that being attractive will get someone a higher salary. Interestingly, the research of Peng et al. (2020) adds that besides being on the high end of the attractiveness spectrum, one can be one the lower end (e.g. being unattractive) and still sell more than average looking competitors. This u-shaped relationship between facial attractiveness and sales is very interesting for e-commerce and marketing and in every field as high attractive faces have the advantages of the *halo effect* in life and are expected to sell more, however seriously unattractive faces can still be more profitable than average faces because of the better expertise customers think the unattractive individuals must have to overcome unattractiveness and compensate for their lack of attractiveness by working harder to achieve similar results, leading to a perception of greater competence. These findings question our decision making choices as we may think are logical and fact oriented thinkers but research shows facial attractiveness affects our choices without us even realizing it. From the neuro-marketing perspective, Cook et al. (2011), have found that "brain regions involved in decision-making and emotional processing were more active when individuals viewed ads that used logical persuasion than when they viewed ads that used non-rational influence." This basically means that ads showing attractive people as a way to drive sales and not logical facts about the product lead to less orthological thinking when it comes to decision making. These findings are in line with *evolutionary psychology* that basically says attractive faces signal good genes and health for reproduction, hence humans are imperceptibly attracted to that.

2.2 Halo effect

The *Halo effect* continue to influence other aspects of life. Some of them are meticulously examined: "call-backs to job interviews for attractive men are significantly higher than to men with no picture and to plain-looking men, nearly doubling the latter group" stated in the paper of Ruffle et al. (2014), more attractive people are more likely to win elections according to Jäckle et al. (2019) and Berggren et al. (2010), physically-attractive workers earn more Mobiuset al. (2006), people are more likely to accept unfair offers from attractive individuals compared with the unattractive ones, Ma et al. (2015), children prefer attractive teachers Hunsberger et al. (1988) and students attend classes of attractive instructors more frequently Wolbring et al. (2016). "Teachers hold negative expectancies towards children categorized with a deviancy label (some sort of unattractiveness) and maintain expectancies even when confronted with normal behavior, behavior inconsistent with the stated label", stated in the paper of Glen et al. (1976). Moreover, "physically attractive defendants were evaluated with less certainty of guilt, less severe recommended punishment and greater attraction than were unattractive defendants" Efran M. G. (1974) and Sigall et al. (1975), "hiring preferences were greater for attractive over unattractive applicants and males were favoured over females" Cash et al. (1985), "job applicants may encounter different employment opportunities as a function of their physical attractiveness" Jawahar et al. (2005). Adding to the long list of the *halo effect* is the assumption of a person to have a nice or at least the desired personality of the viewer, if the face is considered attractive, Little et al. (2006) and that beauty raises life's happiness, Hamermesh et al. (2013). The list could go on but I think these papers clearly show the reader how beauty and attractiveness affect life overall.

2.3 Face Width to Length Ratio (FWHR)

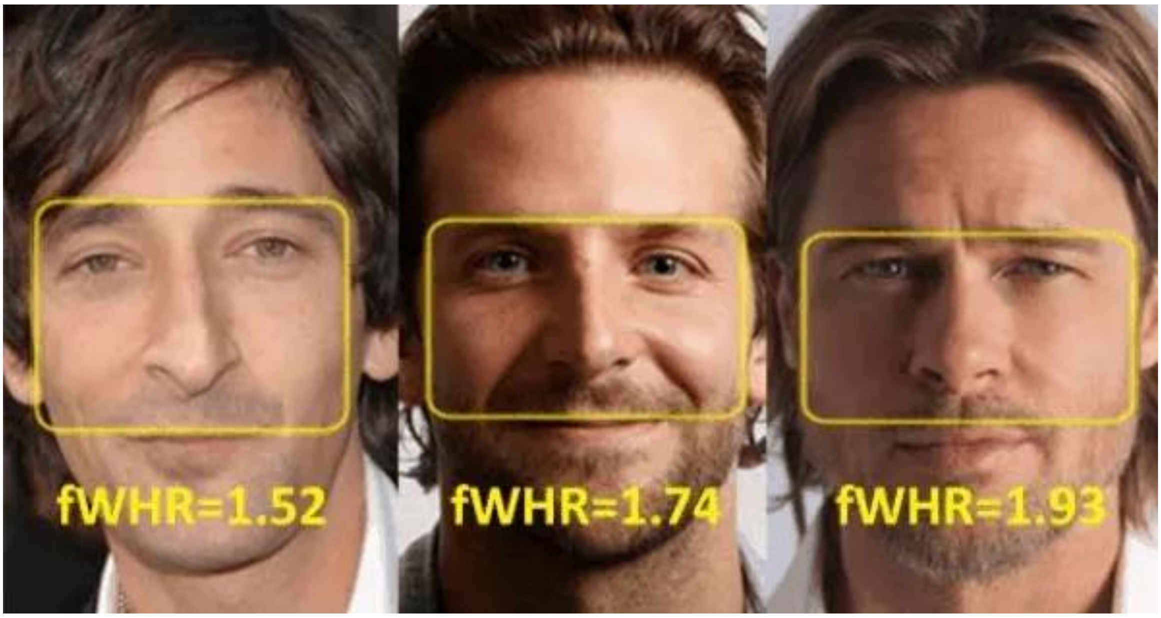
Special note deserves the fWHR ratio, which is the facial width to height ratio and is measured as the bi-zygomatic width (that is the distance between the cheekbones from one side to the other end) divided by the distance from the upper lip to the middle of the eyebrows. FWHR is associated with aggressiveness Wen et al. (2020). FWHR was positively associated with perceived dominance in males, likelihood of being chosen for a second date, and attractiveness to women for short-term, but not long-term, relationships which is in line with findings of masculinity, as a high fWHR ratio is a masculine characteristic. FWHR ratio is also shown to contribute to the perception of dominance and intensity, Merlhiot et al. (2021). High dominance emotion (anger, disgust, happiness) presented high fWHR and low dominance emotion (fear, sadness, surprise) presented low fWHR, Merlhiot et al. (2021). FWHR seems also to be associated with achievement-striving alongside associations with dominance and aggression as the study of Lewis et al. (2012) showed. All these studies mainly show correlation of high fWHR and aggression which is an anti-social trait. However, the study of Hahn T. et al (2017), proves that high fWHR is also "linked with high social rank in a more subtle fashion in both competitive as well as prosaically oriented settings" and Stirrat et al. (2010), found that "men with greater facial width were more likely to exploit the trust of others", somehow de-demonizating it.

2.4 Golden Ratio

Lastly, as this paper examines the importance of the existence of golden ratios in the human face, a description of golden ratio seems fit. The first definition we have goes back to Euclid, a Greek mathematician, and it is defined as "the ratio that divides a line by a point such that the ratio of the smallest part to the largest is equal to the ratio of the largest to the whole". The golden ratio analogy is a length to width proportion of 1 to 1.6. For example, if the eye is of y width or height, (y axis) then its length, (x axis) should be: x = 1.6*y. This concept of proportion was used in art and architecture creations like in the Parthenon of Athens, the Vitruvian Man by Leonardo da Vinci, etc. The golden ratio is also known as "*Phi*" from the Greek sculptor Phidias, most known for sculpting the Zeus's and Athena's statue at the temple of Olympia and Parthenon, respectively. Phi is also expressed by the Fibonacci sequence, in which, every subsequent number is the two predecessors added together. This sequence is also a geometric sequence as each number is the product of the previous number with the golden ratio 1.6. Furthermore, if we divide each number with its previous the outcome is always very close to 1.6, (1/1=1, 2/1=2, 3/2=1.5, 5/3=1.666, 8/5=1.6, 13/8=1.615, 21/13=1.625, 34/21=1.619, 55/34= 1.617, 89/55=1.618…). A well-known example of the Fibonacci sequence in nature is the "*golden spiral*" observed in shells, galaxies, the human ear, etc. A golden spiral grows geometrically by a factor of *phi* or "φ" in Greek for every quarter turn it makes. This is observed many times happening in nature, Borges R. F. (2004), with the most memorable example being the nautilus shell. The golden ratio is also observed in the way plants grow, in the human body, in the DNA, but also in human face as the nose and mouth both are located at the golden section of the distance between the eyes and the end of the chin. As a consequence, this idea of ideal proportion was believed to represent also the perfect beauty and it was widely used in the art of the Renaissance, as well as in architectures (page 55-57 in appendix).

Evidence in bibliography is shown by the work of Schmid et al. (2008), Rickets M.R. (1982). They found that "symmetry does not play as important a role in attractiveness as the proportions defined by the neoclassical canons and golden ratios” and that “the use of divine proportion in conjunction with the principles of maxillofacial surgery will lead to greater success for all concerned (including attractiveness), respectively. However, contradictive findings are that of the study of Rossetti et al. (2013) and Shell et al. (2004), found that “most of the facial ratios related to attractiveness were different from the golden ratio” and “the achievement of divine proportions seemed to have little, if any, influence on overall aesthetic outcomes” respectively. Furthermore, Kiekens et al. (2008) found that “few golden proportions have a significant relationship with facial esthetics in adolescents. The explained variance of these variables is too small to be of clinical importance” further diminishing the importance of golden ratios to facial attractiveness.

However, direct association of fWHR and attractiveness is only empirical and this study aims to find out if it holds real ground. In addition, evidence for the importance of golden ratios vary in bibliography and this paper will try to examine the importance of both of fWHR and golden ratios. Our ideal fWHR will be any value that falls in-between of the spectrum of 1.8 to 2 and this derives from empirical observations of attractive males. An example is below with three famous actors, supporting the 1.8 to 2 range.



3. Research Question and sub-questions

Thus, the research question is set: What are the most important features and ratios on the male face that make it attractive?

Sub-question: Do golden ratios presence in the face make it more attractive?

There are some more modern canons and ratios than of the golden ratio that claim to influence beauty and attractiveness. These are the rule of equal facial thirds (hairline to eyebrow – eyebrow to end of nose – end of nose to end of chin), thin nose, wide and full lips, wide and prominent cheeks, strong and wide jaw, positive canthal tilt of the eyes, little to no skin exposure of eyelid to name a few. However these canons are mostly empirical yet widely used in modern cosmetic surgery. Some of these claims formed our the beauty hypotheses for men:

1. The distance between hair-line to eyebrow – eyebrow to end of nose – end of nose to end of chin should be equal and is an important feature of attractiveness, rule of thirds.
2. Small, (narrow, thin and short) nose is an important feature of attractiveness.
3. A wide and full lip is an important feature of attractiveness.
4. Wide and prominent cheekbones (bi-zygomatic width), is an important feature of attractiveness.
5. A wide jaw is an important feature of attractiveness.
6. A broad and not too long face (high facial width-to-height ratio, fWHR) is perceived as a masculine feature and thus making the face more attractive, with the ideal considered in-between 1.8 to 2, is an important feature of attractiveness.
7. Wide palpebral fissure length and short palpebral fissure height are important features of attractiveness.
8. The outercanthal width is an important feature of attractiveness and should be equal to three eye widths.
9. The rule of fifths is an important feature of attractiveness. The face can be divided into five equal horizontal parts (fifths) and their distance should be equal to the width of the eye.
10. White skin is an important feature of attractiveness.

Anne M. Brumbaugh (1993) in her work "Physical Attractiveness and Personality in Advertising: More Than Just a Pretty Face?" suggests that "people's perception of an advertisement is affected not only by the spokesmodel's physical appearance, but also by personality inferences made by the viewer about the model." However, as said before, there is a correlation of how we see someone's personality by his/her looks. For this reason we can test the predictive power, explanation of attractiveness variance and importance of the physiological variables in a model separately from the objective features of the face and test if psychological traits such as happy, masculine, feminine, baby-faced and intelligent are significant factors and if they affect positively or negatively attractiveness. In addition, variables such as masculine or feminine probably are correlated with objective features and ratios of the face such as nose width or face roundness respectively, as wider nose is a masculine feature while a more rounded face is considered more feminine and baby-faced. We hereby see that more variables could be correlated especially as psychological ratings are influenced by objective features and ratios and vice versa. Interestingly, literature suggests that beautiful people are more intelligent, Kanazawa et al. (2004). However, this is an empirical study and no study has previously used real data to test this claim. Furthermore, there seems to be a trade-off between masculine and feminine characteristics in men's faces. Specifically, masculinity is associated with dominance and negative sentiments in general and women seem to prefer more masculine traits when seeking a short encounter while they prefer more feminine faced men when they search for a long-time partner (the father of their children), as they see the latter ones more caring and trustworthy, literature suggests. This finding supports the *neoteny theory* in humans, *(e.g., youthfulness)*, meaning people of both sexes in general prefer partners with youthful characteristics (baby-faced and feminine), as they show youth and health. However, studies show that this is important for males when looking for their female partner and has not been examined on them. Lastly, while there is literature that correlates attractiveness with facial expressions, most importantly smiling: Godinho et al. (2020), and skin colour: Fink et al. (2001). More precisely, dark skin, not light skin, was rated as most attractive, in contrast to many people beliefs. These debates add five more hypotheses to test (13 and 14 are antagonistic while 14 and 15 are probably highly correlated):

11. There is a significant and positive correlation between men with high ratings in intelligence variable and attractiveness.
12. There is a significant and positive rise in ratings of attractiveness when the subject is seemed happy or smiles, (happy) variable.

13. There is a significant and positive correlation between men with high ratings in masculine variable and attractiveness.
14. There is a significant and positive correlation between men with high ratings in feminine variable and attractiveness.
15. There is a significant and positive correlation between men with high ratings in baby-faced variable and attractiveness.

4. Data

For the analysis the Chicago Face Database and the 10k US Adult Faces Database were used.

4.1 Chicago Face Database description

All individuals were recruited from the University of Chicago, and were aged from 17 to 65 with various ethnicity backgrounds, making the dataset ideal for testing beauty preferences globally. However this research took into account only neutral images (for the convolutional neural network). All images were of same size, lighting and taken in same environmental conditions, making the dataset as objective as possible. Data include both physical variables (e.g., nose width and height) as well as subjective ratings (e.g., attractiveness, our dependent variable). The subjective ratings were made on the standardized neutral faces and consisted of equal proportion of males and females and also came from different racial backgrounds, making the dataset independent of race beauty standards, preferences and sex. Furthermore, the dataset consists ratings of various psychological traits (e.g., masculine, feminine, baby-faced, trustworthy and happy**)**. The ratings were conducted on a 1–7 Likert scale (1 = not at all, 7 = extremely). Each observation was rated by 15 random indivifuals and the actual values of the variables in the dataset is the mean of those fifteen ratings.

4.2 10k US Adult Faces Database description

“The 10k US Adult Faces Database is a natural and unbiased set of 10,168 face photographs of various angles and sizes, based on the 1990 US population that does not include people with glasses, unusual makeup, obvious deformities, celebrities were minimized” thus being highly objective. This dataset is supplementary to the research and will be used to inspect hypotheses that intelligent people are more attractive and validate more the findings of the analysis in the psychological variables of the previous dataset, as it is richer in those. More precisely, it consists of demographic attributes (e.g. age, attractive, friendly, gender) given from 12 subjects on 19 variables for 2,222 target images of the 10,168. These psychological attributes (e.g. boring, confident, intelligent, trustworthy, attractive, etc.) were rated on a scale of 1 (not at all) - 9 (extremely).

This paper uses the average scores of the symmetric variables, for example, distances of the hairline to right and left eyebrow (P030, P031). The notion behind the use of the average scores is that these individual (right and left) variables will be highly correlated to each other, (many are the same) and also the symmetry and averageness are considered as beauty traits "a prori" to this report, supported by the existing literature. In *Figure 1* one can see how the distances of the face are measured (a female face is shown but the procedure is exactly the same for males too). The makers of this database also provide some ratios (e.g., face roundness which is face width at mouth divided by face length, "heartshapeness" which stands for face width at cheeks divided by face width at mouth), and most appropriate for the research will be used too. However, as this research is interested in testing the importance of golden ratios of the face, new variables representing these ratios were made. This resulted in new eight variables representing the golden ratios, all following the 1 to 1.6 analogy: a variable for the ideal nose width ratio named "howclosetoidealnosewidth", a variable for the ideal mouth (height) to chin ratio named "howclosemouthtochin", a variable for the ideal face length named "howclosetoidealfacelength", a variable for the ideal lip height named "howclosetoideallipthickness", a variable for the ideal outercanthal width named "howclosetoidealoutercanthalwidth", a variable for the ideal face width named "howclosetoidealfacewidth, a variable for the philtrum distance (that is the distance from the end of the nose to the upper lip) and three dummy variables, the "facethirdsdummy", the "idealnoselengthdummy" and the "fWHRdummy", measuring if the rule of face thirds, the nose length being 0.67 of the second third of the face, Ding et al. (2020) and the face width to face length ratio being in-between 1.8 to 2, coded as 0 and 1, with 1 measuring if the golden ratio is met with the individual's face. All these variables are combined and summarized in Table 1.

These 24 variables alongside with the 9 added golden ratios and “Philtrumdistance” lead to 34 independent predictors and “Attractive” as the dependant variable. Variables made by me were calculated on excel based on every individual’s distances, as ideal golden ratios are different for everyone. By this point you may have noticed the “howcloseto” part of most of the variables for the golden ratios. This approximation of ideal-golden ratios was inevitable as for these variables no individual met the exact value of the golden ratio. Conveniently, the smaller this variable’s value the closer to ideal it is. For the calculation of these approximate golden ratios the *abs* function of excel was used so negative values won’t affect the comparisons, as someone may have bigger or smaller ratio value from the ideal. For example, for the variable “howclosetoidealnosewidth”, the ideal nose width is supposed to be the face width distance divided by five, so I calculated this value and then subtracted it from the real nose width. The difference (in absolute value) of them is the “howclosetoidealnosewidth” value. Same procedure was done to the other “howcloseto…” variables made. Only with the fWHR variable this wasn’t necessary as there were enough individuals to be within this ideal range.

Psychological variables of this dataset are:

Afraid, angry, baby-faced, feminine, masculine, trustworthy, unusual, surprised, sad and happy.

Psychological variables of the 10k US faces dataset are:

Calm, common, egotistic, interesting, intelligent, confident, introverted, kind, responsible, trustworthy, weird, aggressive, familiar, caring, emotional, friendly, happy, humble, memorable, normal, sociable and typical with the race variable of this dataset.

There were no missing values in any of the datasets. The variables of CFD dataset combined that are used for the main analysis are described in Table 1. Tables 2, 3 and 4 have the objective distances of features, the ratios given by the CFD dataset and my golden ratios respectively. Psychological variables of CFD dataset and all (psychological) variables of 10k US Faces dataset are presented in Table 5 and 6.

Variable	Description	Variable Type
Attractive	Attractiveness score (1 = not at all, 4 = neutral, 7 = extremely)	Numerical
LuminanceMedian	Median luminance for model's face only	Numerical
FaceLength	Distance between hairline and base of chin	Numerical
EyeHeightAvg	Average distance between upper and lower edge of visible eye within eyelids at centre of pupil for right and left eye	Numerical
EyeWidthAvg	Average distance between inner and outer corner of eye for right and left eye.	Numerical
FaceWidthMouth	Distance between outer edges of cheeks at level of mid-mouth.	Numerical
FaceWidthBZ	Maximum distance between left and right facial boundary	Numerical
BottomLipChin	Distance from bottom edge of lips to base of chin	Numerical
CheeksAvg	Average distance between mid-cheek and bottom of chin	Numerical
MidbrowHairlineAvg	Average distance between mid-brows to hairline: measured above the pupil of right and left eye in the middle of each eyebrow	Numerical
NoseShape	Nose width divided by nose length	Numerical
LipFullness	Lip thickness divided by face length	Numerical
EyeShape	Eye height divided by eye width	Numerical
EyeSize	Eye height divided by face length	Numerical
ForeheadHeight	Average mid-brow to hairline divided by face length	Numerical
CheekboneHeight	Average mid-cheek to chin divided by face length	Numerical
CheekboneProminence	Face width at cheek minus face width at mouth divided by face length	Numerical
FaceRoundness	Face width at mouth divided by face length	Numerical
eyebrow to end of nose	Second third of the face thirds	Numerical
forehead/facelength	First third of the face thirds divided by face length	Numerical
2ndthird/facelength	Second third of the face thirds divided by face length	Numerical
nosechin/facelength	Lower third of the face thirds divided by face length	Numerical
NoseWidth	Distance between outer edges of nostrils at widest point	Numerical
NoseLength	Distance between Forehead Bridge at level of visible upper eye edge to nose tip	Numerical
LipThicknes	Distance between uppermost and lowermost point of lips	Numerical
Howclosetoidealnosewidth	Actual nose width minus ideal nose width	

	(ideal nose width = 1/5 width of the face)	Numerical
howclosetoidealoutercanthalwidth	Actual distance of the outer corner of the eyes minus the ideal distance based on golden ratio (lip to chin distance = 1,6 lip height)	Numerical
howclosemouthtochin	Actual distance of end of lip to end of chin minus the ideal distance based on golden ratio (outercanthal width = three eyewidths)	Numerical
Howclosetoidealfacelength	Actual distance of length of the face minus the ideal ratio based on golden ratio (length of the face =1,6 width of the face)	Numerical
Howclosetoideallipthickness	Actual distance of lip height minus the ideal distance based on golden ratio (lip height = 1,6 philtrum distance)	Numerical
Howclosetoidealfacewidth	Actual distance width of the face minus the ideal ratio based on golden ratio (width of face = five eyewidths)	Numerical
Philtrumdistance	Distance from end of nose to beginning of the lip	Numerical
Facethirdsdummy	All face thirds are of equal distance	Binary
Idealnoselengthdummy	Nose length is 0.67(eyebrow to end of nose distance)*	Binary
fWHRdummy	Face width to length ratio is from 1.8 to 2	Binary

Table 1. Description of the variables in main analysis

* Ding A. & Zhang Y. (2020) What is the perfect nose? Lesson learnt from the literature

Variable	Description	Variable Type
Attractive	Attractiveness score (1 = not at all, 4 = neutral, 7 = extremely)	Numerical
LuminanceMedian	Median luminance for model's face only	Numerical
FaceLength	Distance between hairline and base of chin	Numerical
EyeHeightAvg	Average distance between upper and lower edge of visible eye within eyelids at centre of pupil for right and left eye	Numerical
EyeWidthAvg	Average distance between inner and outer corner of eye for right and left eye.	Numerical
FaceWidthMouth	Distance between outer edges of cheeks at level of mid-mouth.	Numerical
FaceWidthBZ	Maximum distance between left and right facial boundary	Numerical
BottomLipChin	Distance from bottom edge of lips to base of chin	Numerical
CheeksAvg	Average distance between mid-cheek and bottom of chin	Numerical
MidbrowHairlineAvg	Average distance between mid-brows to hairline: measured above the pupil of right and left eye in the middle of each eyebrow	Numerical
eyebrow to end of nose	Second third of the face thirds	Numerical
Philtrumdistance	Distance from end of nose to beginning of the lip	Numerical
NoseWidth	Distance between outer edges of nostrils at widest point	Numerical
NoseLength	Distance between Forehead Bridge at level of visible upper eye edge to nose tip	Numerical
LipThicknes	Distance between uppermost and lowermost point of lips	Numerical

Table 2. Description of the CFD variables used in analysis only with feature distances

Variable	Description	Variable Type
Attractive	Attractiveness score (1 = not at all, 4 = neutral, 7 = extremely)	Numerical
NoseShape	Nose width divided by nose length	Numerical
EyeShape	Eye height divided by eye width	Numerical

FaceRoundness	Face width at mouth divided by face length	Numerical
LipFullness	Lip thickness divided by face length	Numerical
EyeSize	Eye height divided by face length	Numerical
ForeheadHeight	Average mid-brow to hairline divided by face length	Numerical
CheekboneHeight	Average mid-cheek to chin divided by face length	Numerical
CheekboneProminence	Face width at cheek minus face width at mouth divided by face length	Numerical
ChinLength	Bottom of lip to chin divided by face length	Numerical
UpperHeadLength	Forehead divided by face length	Numerical
Heartshapeness	Face width at cheeks divided by face width at mouth	Numerical
fWHR2	Face width to height ratio: bi-zygomatic face width divided by the distance between the top of upper lip and the highest point of the eyelids.	Numerical
MidfaceLength	Average pupil to lip for right and left divided by face length	Numerical
FaceShape	Face width at cheeks divided by face length	Numerical

Table 3. Description of the variables used in analysis only with ratios given by the CFD dataset.

Variable	Description	Variable Type
Attractive	Attractiveness score (1 = not at all, 4 = neutral, 7 = extremely)	Numerical
Howclosetoidealnosewidth	Actual nose width minus ideal nose width (ideal nose width = 1/5 width of the face)	Numerical
howclosetoidealoutercanthalwidth	Actual distance of the outer corner of the eyes minus the ideal distance based on golden ratio (lip to chin distance = 1,6 lip height)	Numerical
howclosemouthtochin	Actual distance of end of lip to end of chin minus	

	the ideal distance based on golden ratio (outercanthal width = three eyewidths)	Numerical
Howclosetoidealfacelength	Actual distance of length of the face minus the ideal ratio based on golden ratio (length of the face =1,6 width of the face)	Numerical
Howclosetoideallipthickness	Actual distance of lip height minus the ideal distance based on golden ratio (lip height = 1,6 philtrum distance)	Numerical
Howclosetoidealfacewidth	Actual distance width of the face minus the ideal ratio based on golden ratio (width of face = five eyewidths)	Numerical
Facethirdsdummy	All face thirds are of equal distance	Binary
Idealnoselengthdummy	Nose length is 0.67(eyebrow to end of nose distance)	Binary
fWHRdummy	Face width to length ratio is from 1.8 to 2	Binary

Table 4. Description of the variables used in analysis only with golden ratios.

Variable	Description	Variable Type
Attractive	Attractiveness score (1 = not at all, 4 = neutral, 7 = extremely)	Numerical
Afraid	How afraid the individual looks (1 = not at all, 4 = neutral, 7 = extremely)	Numerical
Angry	How angry the individual looks (1 = not at all, 4 = neutral, 7 = extremely)	Numerical
Happy	How happy the individual looks (1 = not at all, 4 = neutral, 7 = extremely)	Numerical
Masculine	How masculine the individual looks (1 = not at all, 4 = neutral, 7 = extremely)	Numerical
Feminine	How feminine the individual looks (1 = not at all, 4 = neutral, 7 = extremely)	Numerical
Baby-faced	How baby-faced the individual looks (1 = not at all, 4 = neutral, 7 = extremely)	Numerical
Sad	How sad the individual looks (1 = not at all, 4 = neutral, 7 = extremely)	Numerical
Surprised	How sad the individual looks (1 = not at all, 4 = neutral, 7 = extremely)	Numerical
Unusual	How unusual the individual looks (1 = not at all, 4 = neutral, 7 = extremely)	Numerical
Trustworthy	How trustworthy the individual looks (1 = not at all, 4 = neutral, 7 = extremely)	Numerical

Table 5. Description of the CFD “psychological” variables.

Variable	Description	Variable Type
Attractive	Attractiveness score 1 (not at all) - 9 (extremely).	Numerical
Common	How common the individual looks 1 (not at all) - 9 (extremely).	Numerical
Calm	How calm the individual looks 1 (not at all) - 9 (extremely).	Numerical
Confident	How confident the individual looks 1 (not at all) - 9 (extremely).	Numerical
Egotistic	How egotistic the individual looks 1 (not at all) - 9 (extremely).	Numerical
Intelligent	How intelligent the individual looks 1 (not at all) - 9 (extremely).	Numerical
Introverted	How introverted the individual looks 1 (not at all) - 9 (extremely).	Numerical
Kind	How kind the individual looks 1 (not at all) - 9 (extremely).	Numerical
Responsible	How responsible the individual looks 1 (not at all) - 9 (extremely).	Numerical
Trustworthy	How trustworthy the individual looks 1 (not at all) - 9 (extremely).	Numerical
Weird	How weird the individual looks 1 (not at all) - 9 (extremely).	Numerical
Aggressive	How aggressive the individual looks 1 (not at all) - 9 (extremely).	Numerical
Caring	How caring the individual looks 1 (not at all) - 9 (extremely).	Numerical
Emotional	How emotional the individual looks 1 (not at all) - 9 (extremely).	Numerical
Familiar	How familiar the individual looks 1 (not at all) - 9 (extremely).	Numerical
Friendly	How friendly the individual looks 1 (not at all) - 9 (extremely).	Numerical
Happy	How happy the individual looks 1 (not at all) - 9 (extremely).	Numerical
Humble	How humble the individual looks 1 (not at all) - 9 (extremely).	Numerical
Interesting	How interesting the individual looks 1 (not at all) - 9 (extremely).	Numerical
Memorable	How memorable the individual is 1 (not at all) - 9 (extremely).	Numerical
Normal	How normal the individual looks 1 (not at all) - 9 (extremely).	Numerical
Sociable	How sociable the individual looks 1 (not at all) - 9 (extremely).	Numerical
Typical	How typical the individual looks 1 (not at all) - 9 (extremely).	Numerical
Race	Asian, Black, Hispanic (Latino), Caucasian (White), Middle Eastern and of mixed ancestry	Categorical

Table 6. Description of the 10k US Faces dataset variables (psychological).

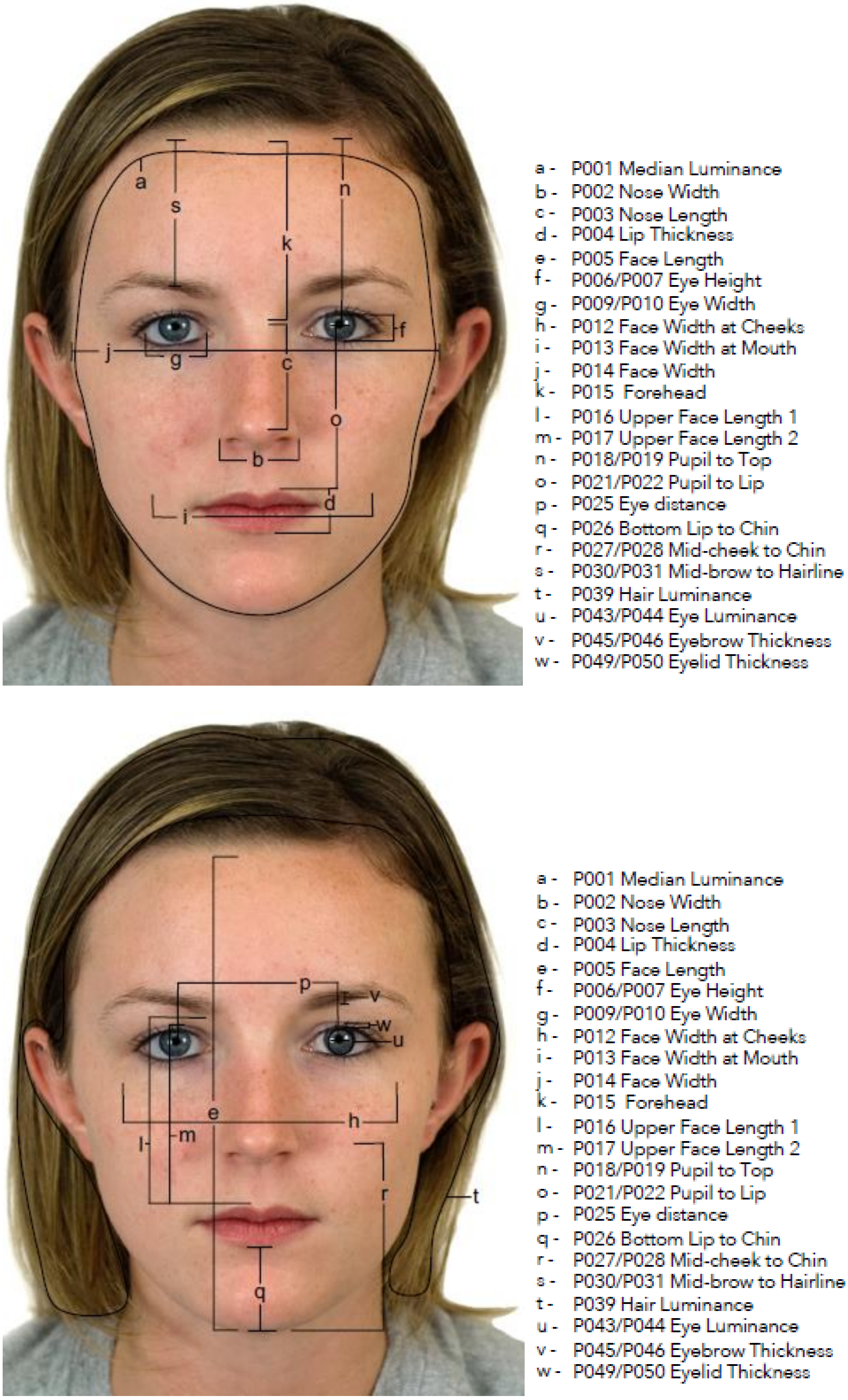


Figure 1: Measurement guide of the physical face attributes of the Chicago face database.

Note: Each image is named after the subject's self-reported ethnicity, gender, model and image ID, as well as expression (we use only neutral images in CNN).

Lastly, the database consisted of three independent datasets, the main consisting of images (and ratings) of 597 unique individuals, recruited in the United States, including self-identified Asian, Black, Latino, and White female and male models, of which 290 were males. The CFD-MR extension set including images of 88 unique individuals, who self-reported multiracial ancestry, of which 26 were males. Again, all models were recruited in the United States. The third dataset extension, CFD-INDIA, includes images of 142 unique individuals, recruited in Delhi, India of which 90 were males. All these three datasets were merged to one (by row in R) to acquire maximum observations for the analysis.

5. Research design

The analysis consists of multiple parts. It can be divided in two parts: examining the features and ratios of the face and examining the psychological traits that are most important for attractiveness.

In order to examine which features of the face alongside with their ratios, the variables of the CFD dataset, (Table 1) were separated in 3 parts and individual multiple linear regressions were performed on each subset of variables. The new subsets of variables are summarized in Tables 2, 3 and 4 having only the features of the face, only the ratios given by the developers of the CFD dataset and only my golden ratios, respectively. This separation was done to avoid multi-collinearity that is further explained in correlation analysis below. However, because I wanted to simultaneously examine the importance of features and ratios for attractiveness, the analysis continued with more sophisticated methods, described after the multiple linear regression in methods part, in order to select the most important variables of combined feature distances and ratios, (Table 1) that are not collinear with each other. This way, a comparison of the predictive power of these methods was possible and the metric used for the evaluation of the performance was RMSE. The best performing one (that with the lowest RMSE), was chosen for the variables applied in the final linear model consisting both distances of features and ratios.

To examine which psychological traits are most important for someone being attractive, two more multiple linear regressions were performed with variables of the CFD dataset, (Table 5) and variables from 10k US faces dataset (Table 6). This way we can inspect hypotheses stating that a more happy face is more attractive or if a person viewed as more intelligent is also more attractive. For this part of the analysis, removing correlated variables sufficiently was done by manually eliminating collinear variables after some variance inflator factor tests on the variables.

6. Methods

6.1 Multiple Linear Regression

Linear regression is one of the simplest approaches for supervised learning. Specifically, linear regression is the first tool someone would think to implement when the task is to predict a quantitative response. In this paper I use its extension, multiple linear regression, which as the name suggests, has multiple predictor variables, (instead of only one in the simple linear regression). The multiple linear regression of a model with p predictors takes the following form:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p + \varepsilon$$

Where:

Y represents the dependent variable (attractiveness),

$X_1, X_2, \ldots, X_p$ represent the predictors

β_0 is the intercept

$\beta_1, \beta_2, \ldots, \beta_p$ are the coefficients (slope terms) representing the linear relationship

ε is a mean-zero random error term

Interpretation of β_js: the average effect on the dependant variable Y of a one unit increase in X_j, holding all other predictors constant.

However, because of how the data were generated (ratios are fractions of 2 feature distances) the assumption of no or little multi-collinearity present in the linear regression was sure to break. Major consequence of multi-collinearity in the data is overfitting, and even though one can identify and remove collinear variables manually, (*vif* function in R), this way we may remove important information of the data and it is not always clear which variables to remove. Hence, methods that handle this problem are discussed and implemented for the analysis.

6.2 Regularization techniques

Regularized regression puts constraints on the weights of the coefficients and shrinks them towards zero. This constraint reduces the importance of some coefficients and as a result reduces the variance of the model.

6.2.1 Ridge regression

Ridge regression puts a penalty parameter λ to the objective function (L2). When λ = 0 no shrinkage is happening and our objective function equals the normal OLS regression. However, as the value of λ rises, heavier penalty is put on the coefficients shrinking their weight towards zero. Better understanding offers *Figure 2* where until log(λ)= -2 serious rise in variance for some variables is observed (e.g. red line quickly drops after when log(λ)= -2) which is an indicator of multi-collinearity. After log(λ)= 4 all coefficients are very close to zero. The dashed line represents the optimal λ value, (found by cross validation), within one standard error of the minimum MSE. However, this method does not perform any variable selection and all variables are kept.

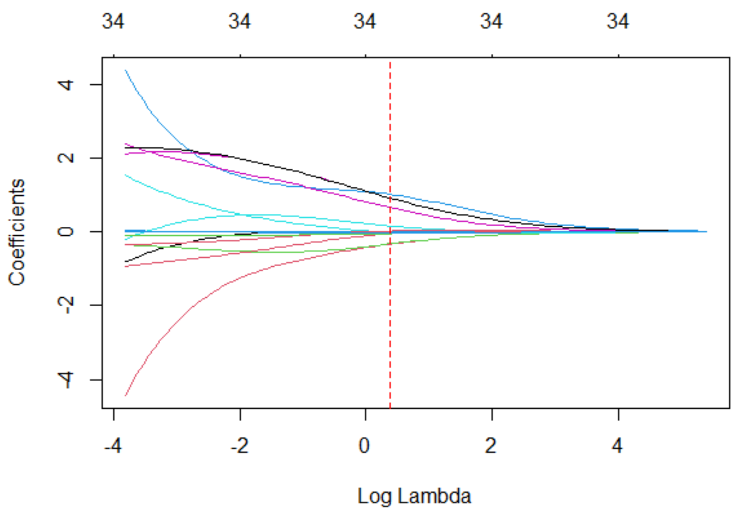


Figure 2: Ridge regression coefficients as λ grows from 0 and beyond.

6.2.2 Lasso regression

Lasso puts a slightly different penalty to the objective function (L1). Its use is to actually push some coefficients to exactly zero, thus performing variable selection too. Below in *Figure 3*, the drop of coefficients after log(λ) ~ -7 can be observed. Lasso performing variable selection results in selection of 14 from initially 34 variables with optimal λ found through cross validation for the λ that minimizes MSE and τhe equivalent of 1 standard error distance from minimum MSE.

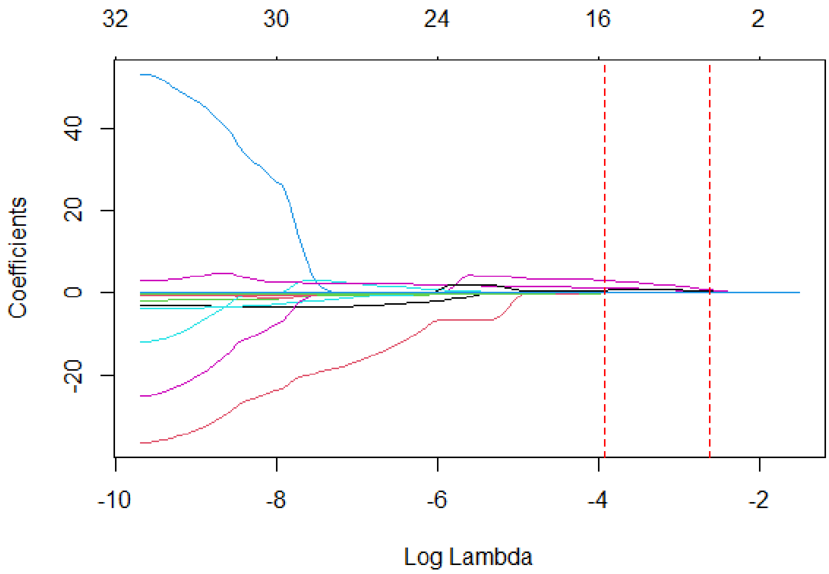


Figure 3: Lasso regression coefficients as λ grows from 0 and beyond.

6.2.3 Elastic Net regression

The elastic net regression combines both the penalty function of the ridge regression and that of the lasso regression. This way, elastic net was developed to benefit both from the regularization that ridge performs and reduces overfitting as well as doing feature selection and reduce the number of variables effectively tackling multi-collinearity. Below, λ_1 is the ridge penalty and λ_2 the lasso penalty.

$$minimize = \{ SSE + \lambda_1 \sum_{j=1}^{p} \beta_j^2 + \lambda_2 \sum_{j=1}^{p} |\beta_j| \}$$

Where:

P is the number of the variables,

λ_1 is the ridge penalty,

λ_2 is the lasso penalty,

β_j represent the coefficients

6.2.4 Relaxed Lasso

The motivation for the development of the "Relaxed Lasso" is that in Lasso many noise variables are selected if the estimator is chosen by cross-validation. Cross-validation is the technique used to split the data into k equal parts, train the model by iterating of in each of the k parts and then validate of the unseen k-1 part. Relaxed Lasso makes sure that the right parameter estimates for the variables are left in the model. This is achieved by eliminating noisy variables found in each iteration consequently ending with a less noisy variables than regular Lasso. Furthermore, relaxed lasso claims to give equal or better results than original Lasso Meinshausen N. (2017) and Hastie et al. (2017). For running relaxed lasso in R we set relax = TRUE, (FALSE is the default) and gamma = 0 in R. Relaxed Lasso found 12 from initially 34 variables to minimize MSE.

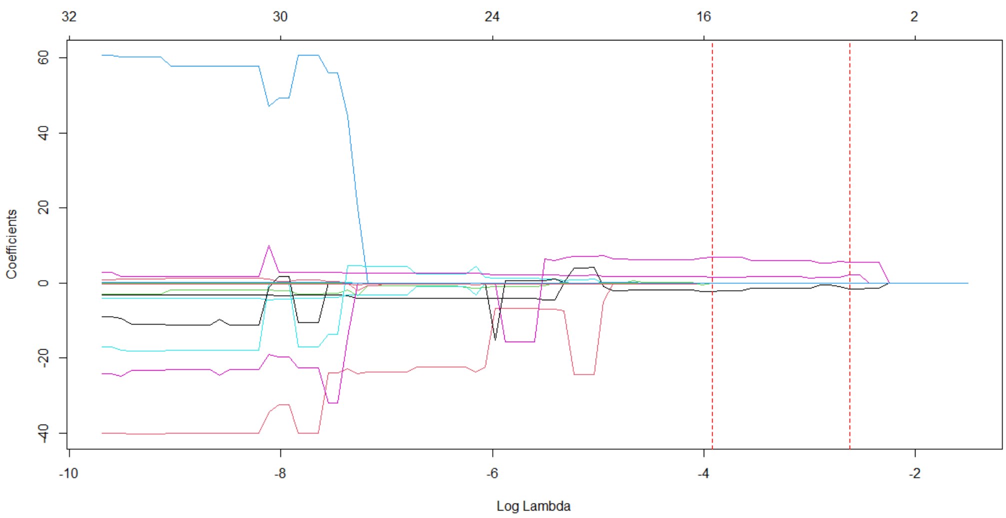


Figure 4: Relaxed lasso regression coefficients as λ grows from 0 and beyond with gamma = 0.

6.3 Decision Tree ensembles

Because Decision Trees are the foundation for the following machine learning algorithms (e.g. Random Forests and Boosted Decision Trees), a short explanation is provided about them. The basic idea is to represent data as a tree flowchart diagram, (hence its name). It consists of nodes (test of a condition), braches, (that represent the outcome of the test) and leaf nodes (terminal nodes) where there are the labels or the continuous output. At every decision node, the test of the condition is made with a variable containing the most information that is able to be gained. In the presence of two features which are highly correlated, the split will choose only one of them, reducing effectively tackling multi-collinearity. *Figure 5* shows a plain decision tree structure.

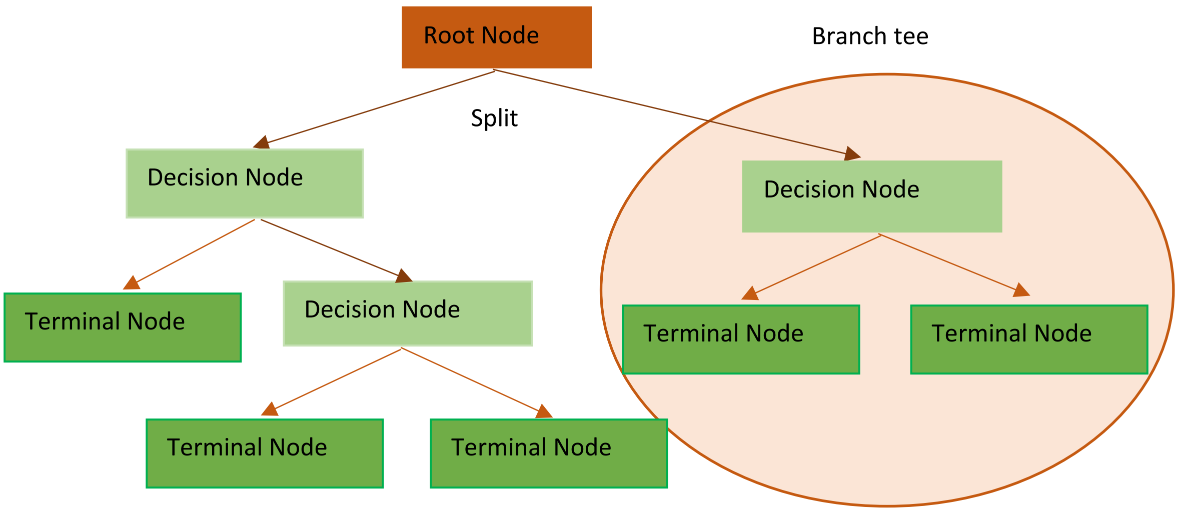


Figure 5: Graphical representation of a decision tree.

6.3.1 Random Forest

A random forest is a combination of many decision trees each of them containing a subsample of variables and a condition to grow a tree until a certain threshold (usually m=p/3). This way variables of each tree are de-correlated from variables of other trees further reducing multi-collinearity from plain decision trees. The final outcome is the majority vote of all trees. This way it reduces variance and predictive power is increased.

Parameters (tuning):

- ntree: number of trees, (to stabilize the error).
- mtry: number of random variables fitted in each split, (rule of thumb is 5), for de-correlation of trees.

- samplesize: number of samples to train on, (rule of thumb is a range in 60-80%).
- nodesize: number of samples within the terminal nodes, for pruning the trees, (small node size means deeper trees).
- maxnodes: number of terminal nodes. Another way to prune the trees (here more nodes mean deeper trees).

6.3.2 Gradient Boosting Machines

While Random Forest's ensemble structure is to use many independent trees and then take the majority vote, Gradient Boosting Machines introduce a sequence scheme of learning. GBMs build an ensemble of plain but successive trees, where each tree is made with information learned from the previous tree. This further improves the predictive power of the outcome (lower RMSE or higher accuracy). Parameters of GBMs:

- number of trees: same as Random Forest
- depth of trees: number of splits in each tree (equivalent of *nodesize* in Random Forest)
- Learning rate: (also called *shrinkage*) is the pace the algorithm learns to reach minimum MSE. Smaller values require more trees and more time to train.
- Subsampling (*bag_fraction*): the possibility to use part of the training observations, to reduce overfitting. Overfitting is the problem when the model performs very well on training set but has noticeably lower predictive power on test set. Equivalent to *samplesize* of Random Forest.

6.3.3 Extreme Gradient Boosting

Extreme Gradient Boosting is based on the same idea as gradient boosting machines but instead of using the loss function of the decision tree for minimizing the error of the final model, it takes the second order derivative as an approximation to do so. Also, it implements regularization (L1 & L2), like ridge and lasso. XGBOOST has proved numerous times best predictive power than GBMs and usually doing it with an average of ten times less time, making it probably the best machine learning algorithm today. Parameters to tune:

- learning rate (called *eta*), equivalent to GBM's *shrinkage*
- tree depth (*max_depth*),

- minimum node size (*min_child_weight*), equivalent to Random Forest's *nodesize*
- percent of training data to sample for each tree (*subsample*) which is equivalent to GBM's *bag.fraction* and Random Forest's *samplesize*
- colsample_bytrees: percent of columns to sample from for each tree

6.4 Dimensionality reduction techniques

6.4.1 Partial Least Squares

PLS is a supervised alternative to PCR. While principal components regression reduces dimensionality, it is an unsupervised method that does not take into account the dependent variable to determine the principal component directions. Thus, one cannot be sure that the predictors PCR found are associated with the outcome dependent variable as it was not taken into account to supervise the procedure in the training part. The first component of PLS explain most of the variability of the data and this procedure continues until a certain satisfying threshold of variance explained is found with components < p (number of variables), hence the dimension reduction. The principal components are linear combinations of all p variables with descending variance explained in every new component. Furthermore, every new component is the linear combination of all p variables that offer maximum variance explained and are uncorrelated to the previous component. Thus, PLS performs dimension reduction and produces fewer variables to fit into the model doing variable selection and eliminating multi-collinearity.

6.5 Deep Learning

6.5.1 Convolutional Neural Network (CNN)

A Convolutional Neural Network is a special case of combining Neural Networks and Kernel convolutions. Neural Networks consist of the input layer, the hidden layer(s) and the output layer. Convolutional Neural Networks take images as input and filters it with a convolution operation while repeating this process many times to learn interesting patterns about that image. It is mainly used for object detection and classification, but it can also be used for regression. CNN takes an image and converts it to a matrix of pixel values. Then a filter (or kernel) of our choice (usually the smaller the better, to capture more information, like 3*3) with multiple layers (each of them detecting different information of the image), is applied to the image to get the convolved feature, equivalent of a hidden layer of a normal neural network. This convolved feature is passed on to the next (hidden) layer to find combinations of the features detected before. By doing this many times and simultaneously shrink the image as we procced to the next layers, we end up having information about local patterns of the initial image. So, for example, the first layer extracts basic features, like edges, this output is passed on to the next layer which detects more complex features, and as we move deeper into the network it identifies even more complex features such as the nose, the eyes of the face, etc.. This way the convolutional neural network can learn which features of the face are important for the final output. *Figure 6* shows how this procedure looks like.

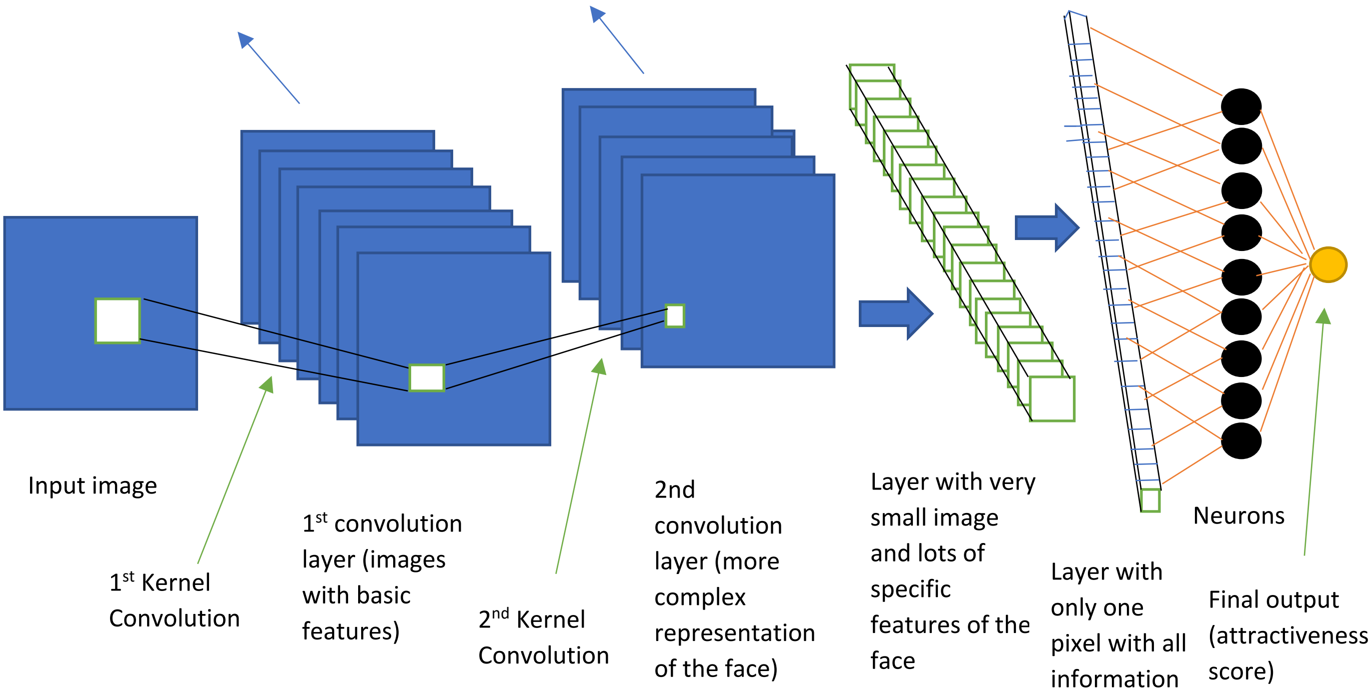


Figure 6: Structure of a Convolutional Neural Network.

7. Data Exploration

On CFD dataset, the dependent variable "attractive" has minimum value of 1.52, maximum value of 5.071, median and mean of 3.16 and 3.218 which is of normal distribution. On 10k US Adult Faces dataset dataset, the the dependent variable "attractive" has minimum value of 1.733, maximum value of 8.200, median and mean of 4.867 and 4.938 respectively, which is of normal distribution, *figure 6 and 7*. The ethnicity distribution of CFD dataset consists of 52 Asian males, 93 Black males, 90 Indian males, 52 Hispanic (Latino) males, 26 males of mixed ancestry and 93 Caucasian (White) males. This distribution offers in general a fair comparison, with multiracial males lacking the most. However the race distribution of 10k US Adult Faces dataset consists of 68 Asian males, 220 Black males, 72 Hispanic (Latino) males, only 2 males of mixed ancestry while having 1828 Caucasian (White) males alongside with 24 Middle Eastern males. A huge number of white males was observed that could interfere the validity of this variable**. *Figure 7 and 8* show graphically the distribution of the dependent "attractive" variable on both datasets and *figure 9 and 10* the distribution of CFD ethnicity and 10k US Adult Faces race respectively.

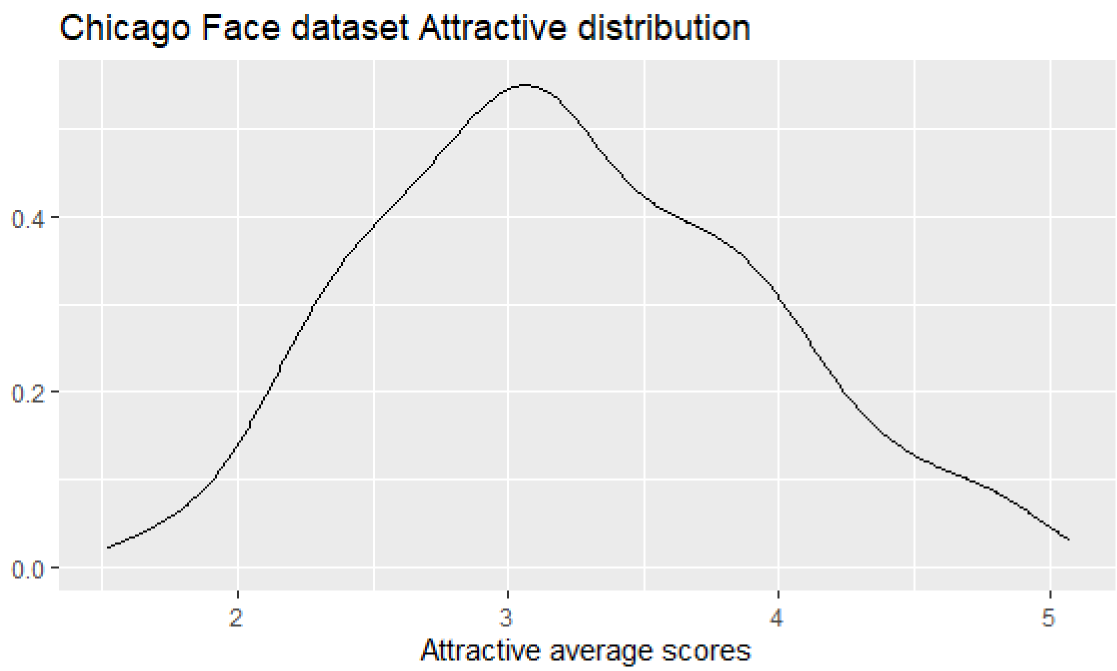


Figure 7: Distribution of attractiveness in males in the Chicago Face dataset.

*** Analysis showed that race wasn't significant so no further manipulation was done to balance this variable.*

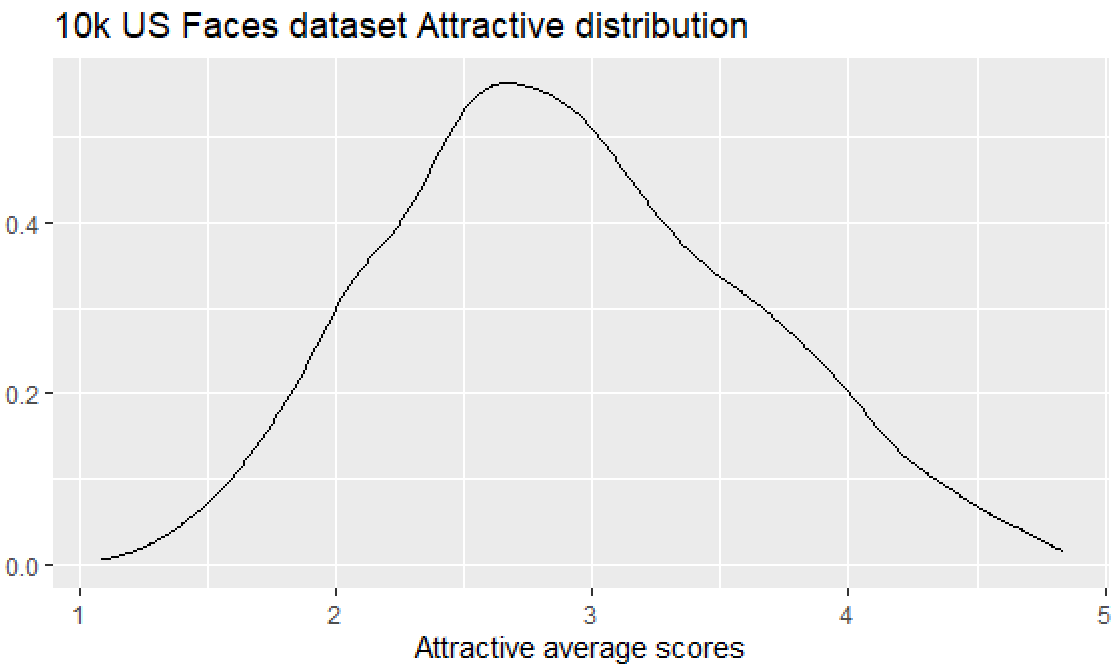


Figure 8: Distribution of attractiveness in males in the 10k US Faces dataset.

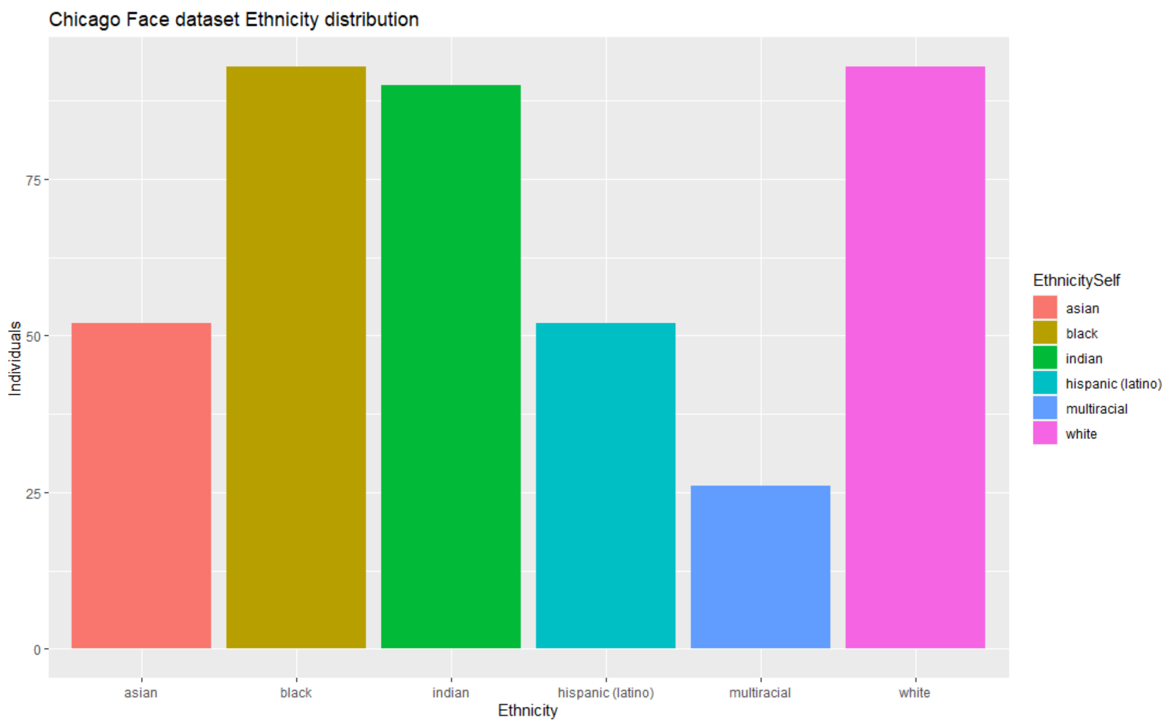


Figure 9: Distribution of ethnicities in males in the Chicago Face dataset.

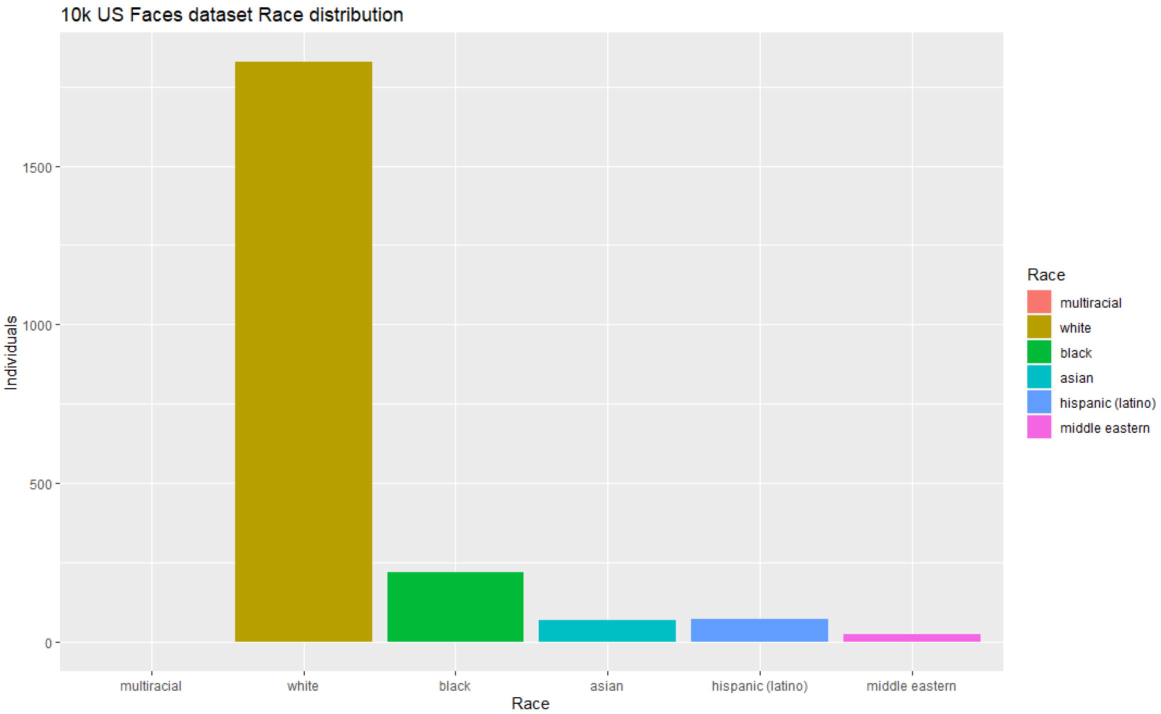


Figure 10: Distribution of races in males in the 10k US Faces dataset.

Lastly, important to note is that only 2 individuals matched the *facethirdsdummy*, 39 individuals matched the *idealnoselengthdummy* and 185 individuals matched the *fWHRdummy* from all 406 males of the CFD dataset. Meaning, only 2 had equal all three face thirds, only 39 had ideal nose length for their face and 185 were had equal or above 1.8 and equal or below to 2 face width to height ratio.

7.1 Correlation analysis

Intuitively, there was suspect for high correlations on the variables of Table 1 of CFD dataset. *Figure 11* shows the *correlogram* in which coefficients are coloured and sized according to the degree of correlation they have. For example, it can observed that lip thickness and lip fullness are very highly and positively correlated, taking a value of almost 1, meaning they are basically identical. In contrast, nose shape and nose length have a strong but negative correlation as both describe the nose but "*NoseShape*" variable is nose width divided by nose length. As the nose length is the denominator of nose shape they grow in opposite ways, whereas if we see nose shape correlation with nose width we observe a strong but positive correlation as nose width is the numerator of nose shape.

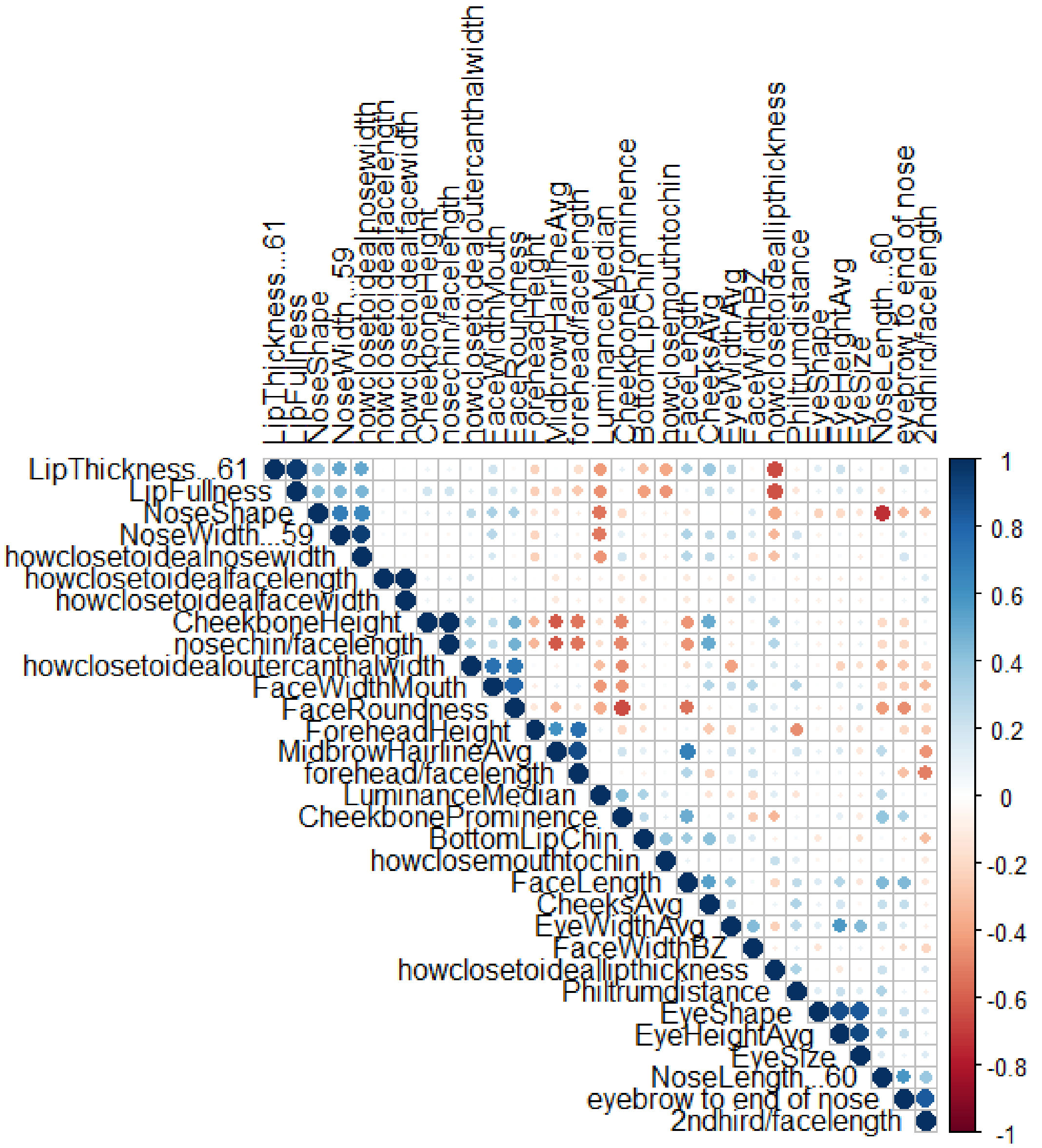


Figure 11: Correlations of variables and displayed based on colour intensity and circle size.

8. Results and Discussion

8.1 Measuring the quality of the models

For evaluation of the models Root Mean Square Error (RMSE) and Adjusted R Squared was used on test set. RMSE for finding the best model to choose our variables for the combined regression with variables of ratios and features and Adjusted R Squared for how well attractiveness is explained by our linear models.

Root Mean Square Error (RMSE) is preferred from Mean Squared Error (MSE) as its value is in the same unit as our dependent value. RMSE is the standard deviation of the prediction errors showing how close the predictions to actual observations are, on average. So, an RMSE of 0.6 means our model predicts wrongly the attractiveness of someone by 0.6, on average, and hence the smaller its value is, the better the model is.

Adjusted R Squared is a better way to see how explanatory power our model has than using the traditional R squared, James et al. (2014). That is because as we fit more variables in the model, R squared automatically increases, hence and because the higher values of R squared indicate a better fit, we do not know if the model is getting better or the R squared is inflated due to this phenomenon.

$$R^2 = \frac{TSS - RSS}{RSS} = 1 - \frac{RSS}{TSS}$$

where:

TSS is the total sum of squares, ($TSS = \sum(y_i - \bar{y})^2$), measuring the total variance in the response Y and,

RSS is the total sum of the residuals of the model, ($RSS = \sum_{i=1}^{n}(y_i - \hat{y_i})^2$).

With n being the observation pairs, y_i the actual value of attractiveness of the *ith* individual and $\hat{y}_i$ the prediction of attractiveness of the *ith* individual by the model.

$$R_{adj}^2 = 1 - (1 - R^2)\ \frac{n-1}{n-p-1}$$

where:

p is the total number of explanatory variables in the model and,

n is the sample size.

8.2 Psychological variables

To get quick insights about the importance and explanatory power of psychological traits of attractiveness, multiple linear regressions were performed on the CFD psychological variables (Table 5) and on the 10k US Faces dataset (Table 6). The multiple linear regressions on these analyses showed that nearly 60% of the variance in attractiveness can be explained by psychological variables alone. Notably, our variables of hypotheses 11, 13, 14 and 15, (intelligent, masculine, feminine and baby-faced) all were significant and important to attractiveness all were positively associated with an increase in attractiveness, except the variable happy. Happy was the equivalent of the hypothesis number 12, (smiling faces are seemed as more attractive). Happy wasn't significant on both analyses on both datasets so this hypothesis we conclude that a smiling face will not raise the attractiveness of an individual. In Table 11 is the multiple linear regression of CFD dataset only with psychological variables (Table 5) and Table 12 is the multiple linear regression of 10k US Faces dataset with psychological variables (Table 6):

Coefficients	Estimate	Standard Error	t-value	P-value	
(Intercept)	-1.11826	0.23274	-4.805	2.21e-06	***
Afraid	0.19150	0.10120	1.892	0.059172	.
Angry	-0.04140	0.05162	-0.802	0.422984	
Baby-faced	0.15239	0.04907	3.105	0.002037	**
Feminine	0.51690	0.09697	5.330	1.65e-07	***
Happy	0.06487	0.06221	1.043	0.297720	
Masculine	0.50596	0.05903	8.571	2.33e-16	***
Sad	-0.24987	0.06629	-3.769	0.000189	***
Surprised	-0.13287	0.11006	-1.207	0.228052	
Trustworthy	0.42068	0.07713	5.454	8.69e-08	***
Unusual	-0.18593	0.04738	-3.924	0.000103	***

Multiple R squared:	0.6077	Adjusted R squared:	0.5978
F statistic:	61.19	P-value:	< 2.2e-16

Table 11: Output of CFD psychological variables regression.

From the above table, one can see that the p-value of the F-statistic is < 2.2e-16, which is very close to zero and highly significant. This means that, at least one of the predictor variables is significantly related to the outcome variable (attractive). Angry, happy and surprised are not significant and can be excluded from the model. Doing so, the adjusted R squared is almost the same (0.5966) and all variables are significant *figure 17* in appendix. However, performing an anova test, a p-value of 0.2707 and an F-statistic of 1.2966 means that there isn't statistically significant difference between these 2 models. Variance inflation factor of all variables are below 5 thus the model does not suffer from multi-collinearity. It can be seen that, being more feminine by 1 rating, is associated with an increase of 0.51690 in attractiveness score, holding all other predictors fixed, while very close is masculine with a 0.50596 coefficient. These two antagonistic variables are significantly associated to changes in attractiveness while changes in happiness is not significantly associated with attractiveness. So, one should either be more masculine or more feminine looking from average to be more attractive, (having either more masculine characteristics like a broader jawline or more feminine characteristics like round face, and a good - not receding hairline). Being more attractive by having more feminine and baby-faced features supports the *neoteny* theory in humans stated in the literature review section.

Moving on to the 10k US Adult Faces dataset which has only psychological variables, we have the variable of interest "*intelligent*". In *Figure 18* in Appendix, one can observe that *calm, common, egotistic, intelligent, responsible, weird, familiar, friendly, humble, interesting, memorable, normal, sociable are* significant. However, some multicollinearity was observed through *vif* function and *caring*, *friendly happy* and *sociable* are collinear, so they are excluded from the final model of 10k US Faces dataset. Removing the non-significant variables resulted in *happy* not being significant at all anymore. By running the model again friendly resulted not being significant and having *vif* value above 10. Thus, for final model *friendly* variable was also removed and this is the final model with almost same Adjusted R squared and free of collinear variables. Now, variance inflation factor of all variables are below 5 thus this model too does not suffer from multi-collinearity. Table 12 below shows the final linear model on 10k US Faces dataset. Intelligent proved significant with a 1.365e-1 positive value (in decimal form is 0.1365), meaning a raise of 1 rating of an individual's perceived intelligence from the viewer, raises the attractiveness score by 0.1365, holding all other predictors fixed. In addition, it can be observed that most (positively) influential variable to attractiveness is *normal* and then *interesting*. Interestingly, the model suggests that someone being or perceived as *responsible* and/or *humble* affects negatively his attractiveness. Lastly, no particular race showed important association with attractiveness, hypothesis 10.

Coefficients	Estimate	Standard Error	t-value	P-value
(Intercept)	-3.485e-01	7.811e-01	-0.446	0.65553
calm	2.232e-01	4.134e-02	5.399	8.03e-08 ***
common	-1.071e-01	3.738e-02	-2.866	0.00423 **
egotistic	2.662e-01	3.598e-02	7.400	2.50e-13 ***
intelligent	1.365e-01	4.488e-02	3.041	0.00241 **
responsible	-4.360e-01	4.677e-02	-9.322	< 2e-16 ***
weird	-2.744e-01	3.492e-02	-7.858	8.42e-15 ***
familiar	1.732e-01	3.605e-02	4.805	1.74e-06 ***
humble	-1.195e-01	4.451e-02	-2.685	0.00735 **
interesting	2.838e-01	4.585e-02	6.191	8.11e-10 ***
memorable	1.641e-01	4.254e-02	3.857	0.00012 ***
normal	3.288e-01	4.598e-02	7.151	1.47e-12 ***
sociable	1.672e-01	5.271e-02	3.172	0.00155 **

Multiple R squared: 0.5881	Adjusted R squared: 0.5842
F statistic: 149.3	P-value: < 2.2e-16

Table 12: Multiple linear regression output of 10k US Faces dataset.

The analysis so far showed that hypotheses 12, 14, 15 and 16 are correct and support previous bibliography. The variable *happy* which was not significant on both dataset analyses and was the equivalent of the hypothesis 13 “smiling people are seemed more attractive” proved to not hold ground with actual data, confirmed on both uncorrelated datasets. Furthermore, normal appeared to be the most positive influential variable on 10k dataset and it could be interpreted the same as average and/or symmetrical, in the absent of these variables in the dataset, which is also supported from previous bibliography.

The analysis continues with multiple linear regressions on the objective distances, ratios given by the makers of the dataset CFD dataset and my golden ratios. Firstly, an analysis of only feature distances was done, *figure 20* on appendix. Some multicollinearity was inspected with the *pairs.panels* function, *figure 19* in appendix.

8.3 Features alone

First linear model with all distances showed only lip thickness, face length, eye width, face width at mouth as well as bi-zygomatic face width, lip to chin distance, distance of the lower third of the face (distance from end of nose to chin) – *CheeksAvg*, distance between mid-brow to hairline and eyebrow to end of nose distance to be significant, *figure 20* on appendix. Eliminating the non-significant variables made lip thickness not significant resulting in the final model of feature distances, *figure 21* on appendix. Again the anova test showed that there isn't statistically significant difference between these 2 models. These features can explain around 25% of attractiveness.

8.4 Ratios (conventional)

First linear model with all ratios given by the dataset showed only *LipFullness*, *EyeShape, EyeSize*, *ChinLength*, *CheekboneHeight CheekboneProminence* and *fWHR2* are significant, *figure 22* on appendix. Eliminating the non-significant variables the insignificant variables that are also collinear, resulted in only *LipFullness*, *EyeSize*, *ChinLength*, *CheekboneHeight* and *fWHR2* being significant, *figure 23* on appendix. Here the anova test showed that there is statistically significant difference between these 2 models with the latter being superior. These ratios can explain around 20% of attractiveness.

8.5 Golden ratios

After eliminating aliased as well as non-significant coefficients, the linear model with all golden ratios showed that only 4 (*howclosemouthtochin, howclosetoideallipthickness, howclosetoidealoutercanthalwidth* and *fWHRdummy)* out of 8 are significant and can only explain 0.07% of attractiveness, *figure 24* on appendix.

Separated analyses suggest that golden ratios are not significant in general and have very low explanatory power over attractiveness, below 10%. If we would choose some as important that would be *howclosemouthtochin*, *howclosetoideallipthickness*, *fWHRdummy1* and *howclosetoidealoutercanthalwidth*. This means the distance from bottom lip to end of chin is important to be close to 1.6 times the mouth height, same for lip height and philtrum distance, outercanthal width should be equal to 3 eye widths and facial width to height ratio should be in the 1.8 to 2 range. In addition, *LipFullness, EyeSize, ChinLength, CheekboneHeight* and *fWHR2* are other important ratios and lip thickness, face length, eye width, face width at mouth as well as bi-zygomatic face width, lip to chin distance, distance of the lower third of the face (distance from end of nose to chin) – *CheeksAvg* are important features associated the most with attractiveness. However, these findings are separated, meaning it wasn't examined that importance of features and ratios simultaneously. This was done on purpose, as one can clearly understand that many of the features distances are correlated with ratios, as correlation plot also showed before (*Figure 11)*. This raised the question of examining the most important features and their ratios that are mostly associated with attractiveness to get further insights in our understanding of face attractiveness. Hence, the analysis continued with distances, ratios given by the dataset and my golden ratios simultaneously. The variables of use were presented in Table 1. Methods used to handle multi-collinearity are described in methods section.

Data partition was 70% training and 30% testing for all methods used. The partition to training and testing was done in such way that no biases were introduced (every forth individual was assigned on the test set and all others to training set to be in line with the 70-30 scheme). Same seed were taken at all times and after the models were fully optimized with cross-validation and hyper-parameter tuning. The final results were acquired on test set and comparison of MSE and RMSE is presented in *Table 13*:

Models		MSE	RMSE
Ridge		0.3974787	0.6304591
Lasso		0.4000095	0.6324631
Relaxed Lasso		**0.3906833**	**0.6250466**
Elastic Net		0.3988026	0.6315082
Random Forest	OOB MSE: 0.3495669	0.3994698	
	OOB RMSE: 0.5912418		0.6320362
Gradient Boosting		0.4673332	0.6836178
Extreme Gradient Boosting		0.4250612	0.6519672
Partial Least Squares (with dummies)		0. 398559	0.6313153
Partial Least Squares (without dummies)		0. 3911241	0.6253992
Convolutional Neural Network		0.4552922	0.6747534

Table 13: Model comparison.

Hyperparameter tuning: results

- RF: the best Random Forest model found used *num.trees* = 1000, *mtry* = 24, terminal node size of 5 observations (*min.node.size*), and a sample size (*sample.fraction*) of 0.7 (70%).
- Gbm: *n.trees* = 398, *interaction.depth* = 3, *shrinkage* = 0.1, *n.minobsinnode* = 15, *bag.fraction* = 0.8 (80%).
- Xgboost: nrounds = 88, eta = 0.05, max_depth = 5, min_child_weight = 5, subsample = 0.65 (65%), colsample_bytree = 0.8 (80%).

We can see that all algorithms used 70, 80 and 65% of the available observations (*samplesize, bag.fraction, subsample*), Random Forest used considerably more trees to reach minimum MSE and there for RMSE, both GBM and XGBOOST used small learning rate (small steps seem to work best), Random Forest and XGBOOST used 5 observations in the terminal nodes while GBM 15.

Overall, all models were close but the best performing one was Relaxed Lasso with an RMSE of 0.6250 and very close was PLS (0.6253) without binary variables of *facethirdsdummy*, *idealnoselengthdummy* and *fWHRdummy*. Ridge had also a very competitive RMSE, however holding al the variables in. Elastic Net is next with an RMSE of 0.6315 very close to Ridge (0.6304) while doing feature selection. Quite surprisingly, considering the very small database of images, CNN outperformed GBM. In addition, Random Forest did a better job than GBM and XGBOOST. Random Forest’s usual case of underestimating OOB errors confirmed. Ridge’s lower RMSE than Lasso is normal as by eliminating some variables we sacrifice prediction power (for the sake of interpretability). Relaxed Lasso confirmed bibliography by outperforming standard Lasso and all other regularized models. Variables identified from Relaxed Lasso were selected to fit the final multiple linear regression, *Figure 25*. The first linear model had two aliased variables *Figure 26* (appendix). Table 14 shows the final model without these variables with a slightly improved Adjusted R squared and no collinear variables.

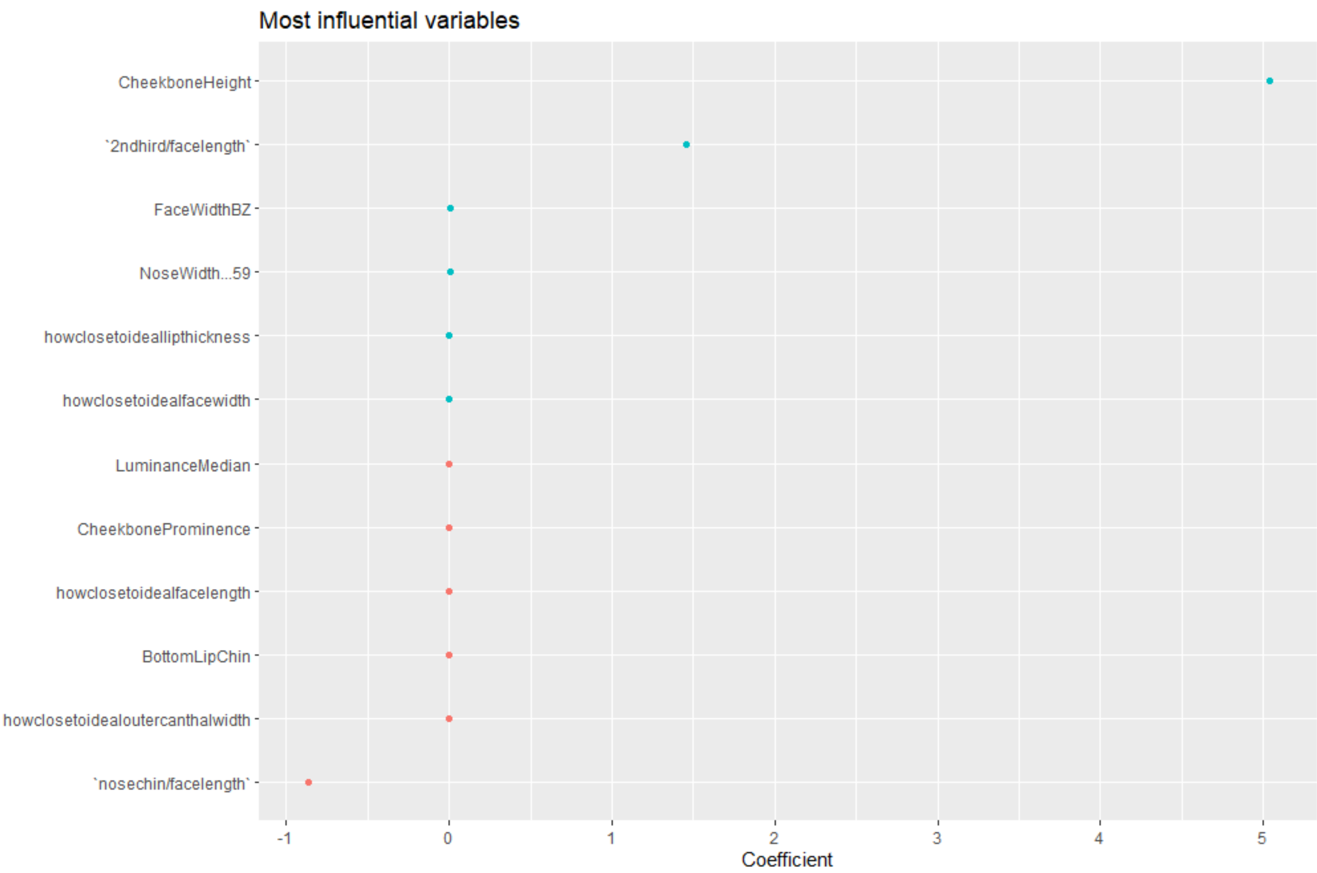


Figure 25: Most important variables selected by Relaxed Lasso.

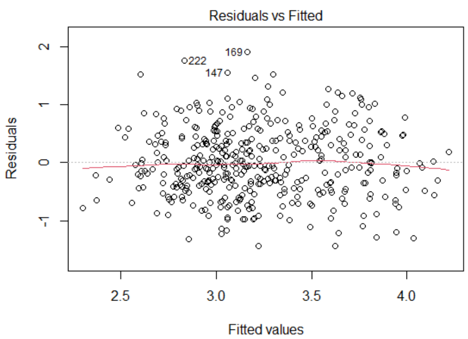
Coefficients	Estimate	Standard Error	t-value	P-value
(Intercept)	-3.2832254	1.0639092	-3.086	0.002171 **
CheekboneHeight	4.3378154	1.4777875	2.935	0.003526 **
FaceWidthBZ	0.0053074	0.0008712	6.092	2.65e-09 ***
howclosetoideallipthickness	0.0019326	0.0006679	2.893	0.004021 **
howclosetoidealfacewidth	-0.0020344	0.0009650	-2.108	0.035640 *
CheekboneProminence	-0.0026473	0.0007301	-3.626	0.000326 ***
BottomLipChin	-0.0029898	0.0012841	-2.328	0.020397 *
howclosetoidealoutercanthalwidth	-0.0037609	0.0008183	-4.596	5.80e-06 ***
secondthirdoffacedividedbyfacelength	1.9391416	1.1286483	1.718	0.086558 .
NoseWidth	0.0036424	0.0013473	2.704	0.007156 **

Multiple R squared:	0.2942	Adjusted R squared:	0.2781
F statistic:	18.34	P-value:	< 2.2e-16

Table 14: Final multiple linear regression output of variables selected by Relaxed Lasso.

8.6 Diagnostics

Table 14 shows an Adjusted R squared of 0.27, which is relatively small. We need to check if the model works well for the data. The plot in *Figure 27* shows that there aren't non-linear relationships and plot in *Figure 28* shows that the residuals are normally distributed. Residuals are spread equally along the ranges of predictors, (homoscedasticity), *Figure 29* and no outlier spotted outside Cook's distance that could alter the regression results, *Figure 30.*



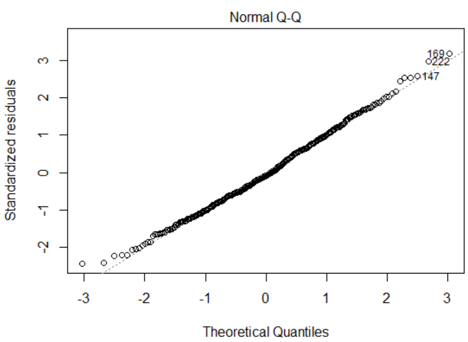


Figure 27 and 28. Linearity and normal distribution of the residuals.

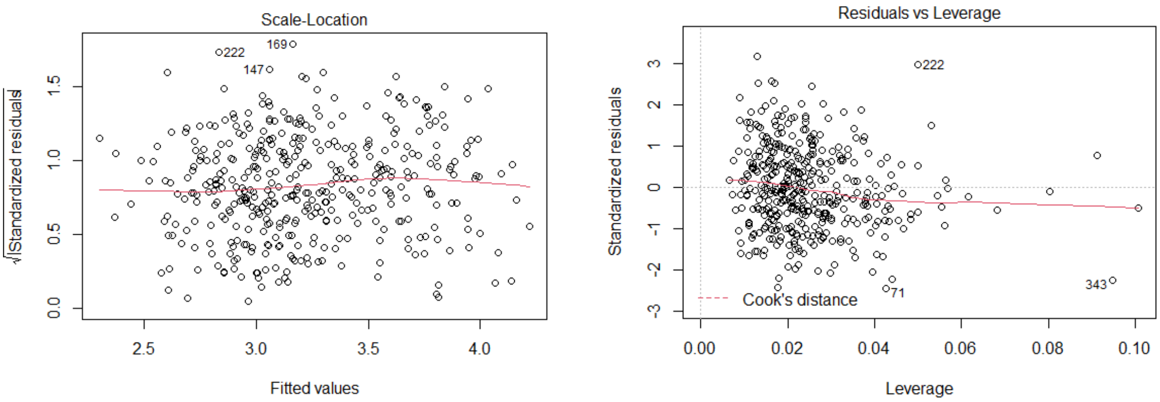


Figure 29 and 30. Homoscedasticity and no influential outliers in the model.

9. Conclusion

The 10k database few collinear variables that was easy to spot and remove from the linear model. However, the CFD dataset with features and ratios, had serious collinearity among variables resulting in a multicollinearity problem in the linear model. As I wanted to perform an analysis on objective distances and ratios simultaneously, I implemented a wide variety of machine learning supervised algorithms, so I can compare their predictive power, pick the best performing one and see the variable importance of that to select the most important features and ratios of the face that are most associated to attractiveness and then feed them on a new linear model, without multi-collinearity issues. Analysis of only the CFD psychological variables was made too, as well only with objective distances, ratios of the researchers and my golden ratios, separately. The multiple linear regressions on these analyses showed that nearly 60% of the variance of attractiveness can be explained with only psychological variables while only a 25% at best with features of the face. Our variables of hypotheses were significant and important, (feminine, baby-faced, masculine, intelligent) except the variable happy which was the equivalent of the hypothesis "smiling people are seemed more attractive". Happy wasn't significant on both analyses on both datasets so this hypothesis didn't hold ground.

The best performing method was Relaxed Lasso suggesting that Cheekbone height, (Average mid-cheek to chin for right and left) divided by face length, second third of the face (eyebrow to end of nose) divided by face length, bi-zygomatic face width are the most influential variables positively associated with attractiveness, followed by nose width, lip thickness should be close to 1.6 times the philtrum, face width should be close to face length divided by 1.6, luminance of the face, face width at cheek minus face width at mouth divided by face length, lip to chin distance, the distance of the outer corner of the eyes should be close to three eye widths and the lower third of the face (end of nose to chin) divided by face length are most influential to changes in attractiveness, *Figure 25*. The subsequent and final linear model with all these variables lead to the conclusion that Cheekbone height, (Average mid-cheek to chin for right and left) divided by face length, second third of the face (eyebrow to end of nose) divided by face length, bi-zygomatic face width are the most influential variables positively associated with attractiveness, lip thickness should be close to 1.6 times the philtrum, face width should be close to face length divided by 1.6, nose width, face width at cheek minus face width at mouth divided by face length (cheekbone prominence), lip to chin distance and the distance of the outer corner of the eyes should be close to three eye widths are most influential to changes in attractiveness, Table 14. Cheekbone height has the largest impact in attractiveness suggesting that by having larger lower third (meaning bigger chin) would make someone more attractive. That is because as cheekbone height is the mid-cheek to chin length divided by face length and mid cheek is under the eyes and somewhere is the center of the nose, the variable Cheekbone height is the ratio of the distance from the center of the nose to the end of the chin divided by face length. The center of the nose is very close to the center of the second third of the face thirds and should split the face in the center into two equal parts. It seems that a new ratio of attractiveness found significant and important, that consisting of half the second face third plus the third face third (lower third) divided by face length. From my way of seeing it with the golden ratios in mind, this ratio should be about 0.5 to be considered ideal, as it covers half of the face and based on the equal thirds of the face, if all three parts are equal, so should be these two half's of the face length. Here the model says that if the Cheekbone height ratio increases by 1, meaning this part of the face taking more space from the whole face length, the average attractiveness will increase by 4.33 rating, which is a significant change! Second most positive variable to attractiveness is the ratio of second third of the face (eyebrow to end of nose) divided by face length. An increase of 1 in this ratio will increase attractiveness rating by 1.94, on average, holding all the other variables constant. Bi-zygomatic face width (FaceWidthBZ), lip thickness being close to 1.6 times the philtrum and nose width are significant but with very small impact on attractiveness. Specifically, an increase of 1 in these variables will increase attractiveness rating by 0.005, 0.001 and 0.003 respectively. Negative coefficients have the *howclosetoidealfacewidth*, *CheekboneProminence*, *BottomLipChin* and *howclosetoidealoutercanthalwidth* suggesting that if these variables increase, the attractiveness score-rating will decrease. For example, as the distance of outer corners of the eyes deviates further from 3 eye widths the person would become more and more unattractive. However, the decrease in attractiveness by these variables are very small, 0.002 for the first three and 0.003 for the *howclosetoidealoutercanthalwidth.* The Adjusted R Squared is 0.278 meaning only the 27% of variance in attractiveness was able to be explained by the model. This normally would suggest a weak model and/or wrong pick of variables. However, an R Squared above 10% is generally accepted in social sciences as human behaviour. Furthermore, individual models and more precisely linear regression with only psychological variables gave an Adjusted R Squared of 0.58% which is twice that of the model with feature distances and ratios. This also suggests that attractiveness is better explained with psychological traits which can capture more proportion of variance of attractiveness. Last but not least, there is the possibility of not having picked the best variables to explain attractiveness or some important missing. It could be variables that had missing values or weren't measured for many of individuals, diminishing this research, such as the interpupilary distance or the eyebrow thickness. However, judging from the Adjusted R Squared of the models with distances and ratios, I doubt that any additional measures such as these would cause a substantial increase in the variance explained. Findings indicate that attractiveness is better explained by psychological variables. Features and their ratios can only explain around one fourth to one third of attractiveness. However, both psychological traits and features with ratios add up to around 85% explanation of attractiveness. In any case, these findings show the parts of the face that would increase the attractiveness most if altered.

9.1 Contributions to Knowledge – Summary – Key takeaways

In the literature review sector we discussed some factors that drive beauty in human face. These were mainly averageness, symmetry, masculine, feminine and baby-faced features. However, prior literature does not specifically say which features of the face are mostly associated and are indicators of attractiveness. This paper aimed to find the specific features that lead to beauty, order their importance and accept or reject the existing theories. Most researches were tailored either to white (Caucasian), female individuals or their combination. This research was tailored to males of all races (as I have seen males studied way less than females). This way we could see if there is a same global pattern of what is considered as a beauty indicator or if there are different perceptions in races of what leads to beauty, if a particular race proved significant, hypothesis 10, which didn't.

This paper's goal was to find objective trait in the human face that are highly associated with attractiveness proving that there is a pattern that undisputedly leads to beauty. 15 hypotheses were formed that included also psychological factors to further test assumptions of theoretical claims and see how well they can explain attractiveness compared to raw features and ratios. Findings support theoretical claims (hypotheses) of psychological traits to be significant and being able to explain a fair amount of attractiveness. Specifically, having more masculine (hypotheses 13) or feminine (hypothesis 14) and baby-faced (hypothesis 15) characteristics increase facial attractiveness. These findings imply that a broader face, a higher fWHR ratio, a bigger jaw (hypothesis 5) and chin would make someone more beautiful (from the masculine variable) or having a more round face, with big eyes, small nose and a good – not receding hairline would also make someone more beautiful as these are feminine and baby-faced characteristics. Furthermore, being seen as more intelligent is also associated with higher ratings in attractiveness, hence the claim more intelligent people are more attractive proved to be true, (hypothesis 11). In contrast, being seen more happy (hypothesis 12, smiling in photo) didn't prove significant in experiments with both datasets as well as race, hypothesis 11. Interesting is also the fact that someone being more humble and responsible negatively affects his attractiveness while being more egotistic was positively associated. Psychological traits on both datasets were able to explain nearly 60% of attractiveness. On the contrary, features and ratios were able to explain half the value of psychological traits did. Cheekbone height and second third of face divided by face length, described above, were the two most important ratios associated with the highest positive changes in attractiveness. By interpreting these two ratios can be inferred that the longer the distance from eye brows to end of nose is, the higher attractiveness gets and the same goes for the distance starting from the centre of the nose to the chin. Thus, brow lifts, canthoplasty or "fox eye" surgery as it is called, alongside with chin and/or lip height enlargement would have positive impact in attractiveness, as this way these areas will increase their distance in relation with whole face length. Face width enhancement may also increase attractiveness, as face width in bi-zygomatic area proved significant with positive association, (hypothesis 4). However, this is suggested to cause very little improvement to attractiveness as it has a 0.005 coefficient. The eight formed golden ratios didn't prove to be important and with substantial impact in improvement in attractiveness, if happen to exist in face. Even if *howclosetoideallipthickness*, *howclosetoidealfacewidth* and *howclosetoidealoutercanthalwidth* (hypothesis 8) golden ratios managed to be in the final model having a p-value less than 0.05, their impact in attractiveness was very small and wouldn't make a serious change in attractiveness. This paper's findings could be a reference for anyone trying to understand how attraction works, why and where he is lacking and how to improve his face aesthetics consequently having more confidence and improving quality of life and possibly be happier. From business perspective, almost in every working field better looks either sell more, augment your chances of getting a job, and in general positively affect all aspects of business and personal life.

9.2 Limitations and Future Work

Obvious limitation of this research was the low Adjusted R Squared of the models with features and ratios. The reason for that could be important variables missing from the model. This is either because some variables were not measured for the whole population, making them unavailable for the analysis (e.g. distance of the eye pupils or the colour of them, hair colour and eye lid exposure etc.). Other ratios not included in the model may be more important and with much more explanatory power over attractive variable. Furthermore, this paper examined attractiveness only from the front. Distances of the features of the (side) profile of the face are also very important for attractiveness and including them (thus examining the attractiveness from both front and side), would increase the proportion of variance explained, giving further insights of how attractiveness works. Lastly, another very important drawback of this research was the small data I had as a given. However, it was one of the largest available datasets fitted for this research. A dataset of the same quality but instead of having only 406 individuals (i.e. having 10 or 100 more times the observations) would yield more robust results, especially for the Convolutional Neural Network approach.

Appendix

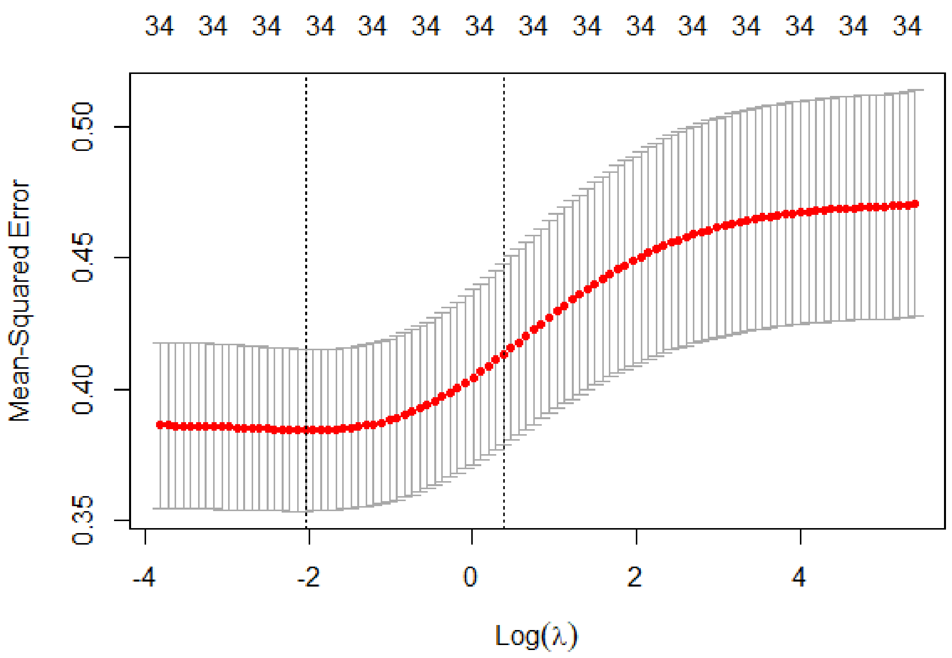


Figure 12: Optimal λ value of ridge regression found by cross-validation.

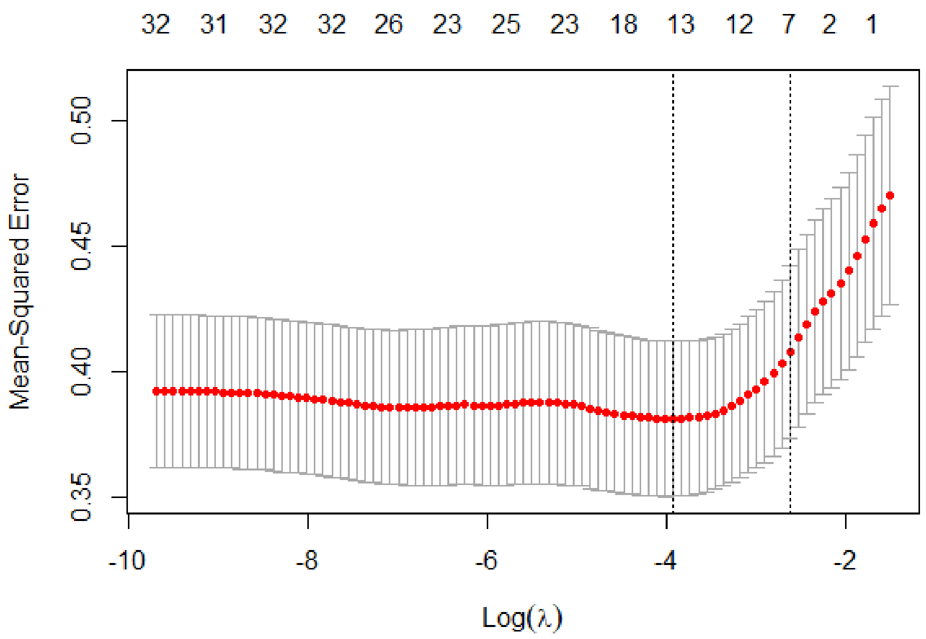


Figure 13: Optimal λ value of lasso regression found by cross-validation.

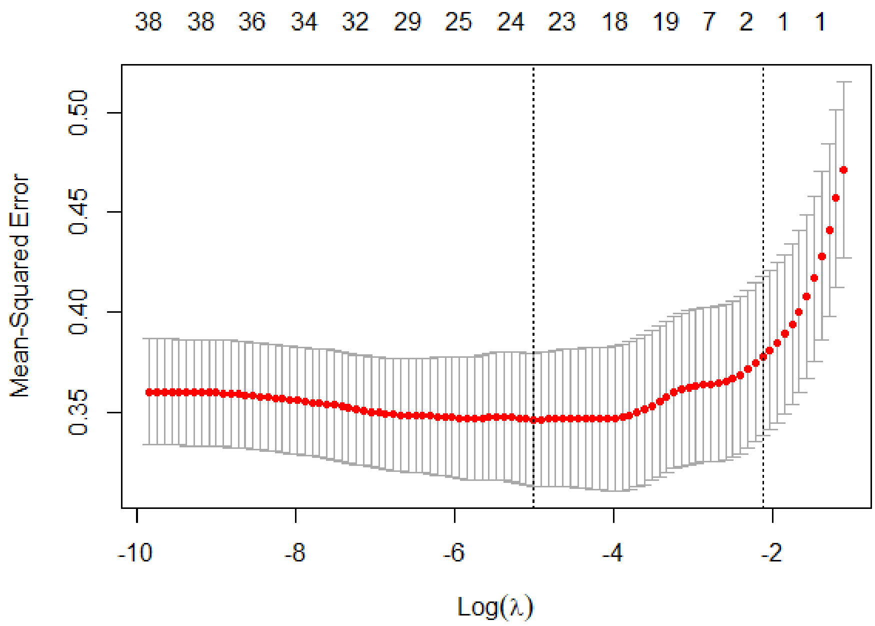


Figure 14: Optimal λ value of elastic net regression found by cross-validation.

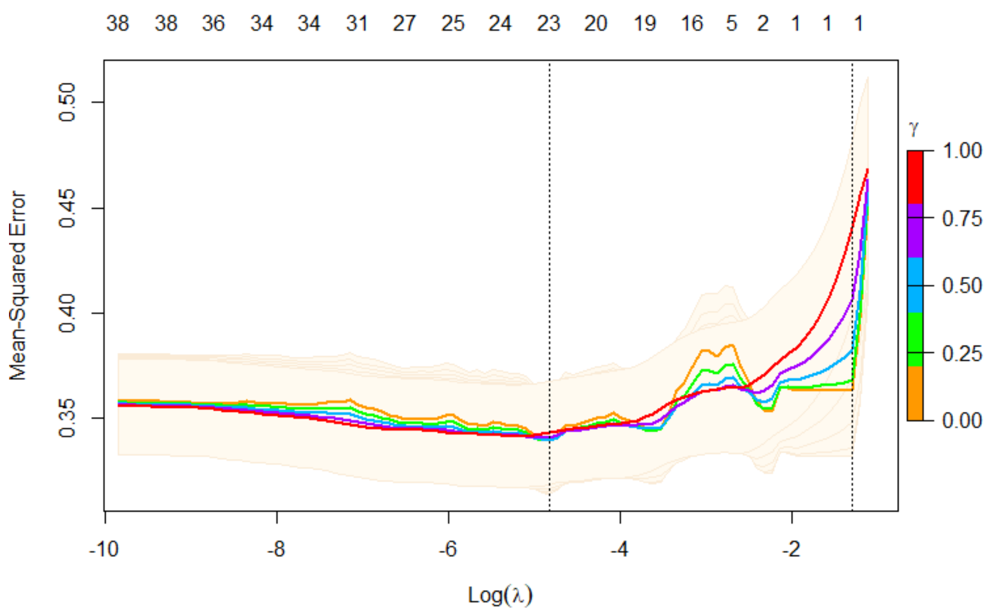


Figure 15: Optimal λ value of relaxed lasso regression found by cross-validation.

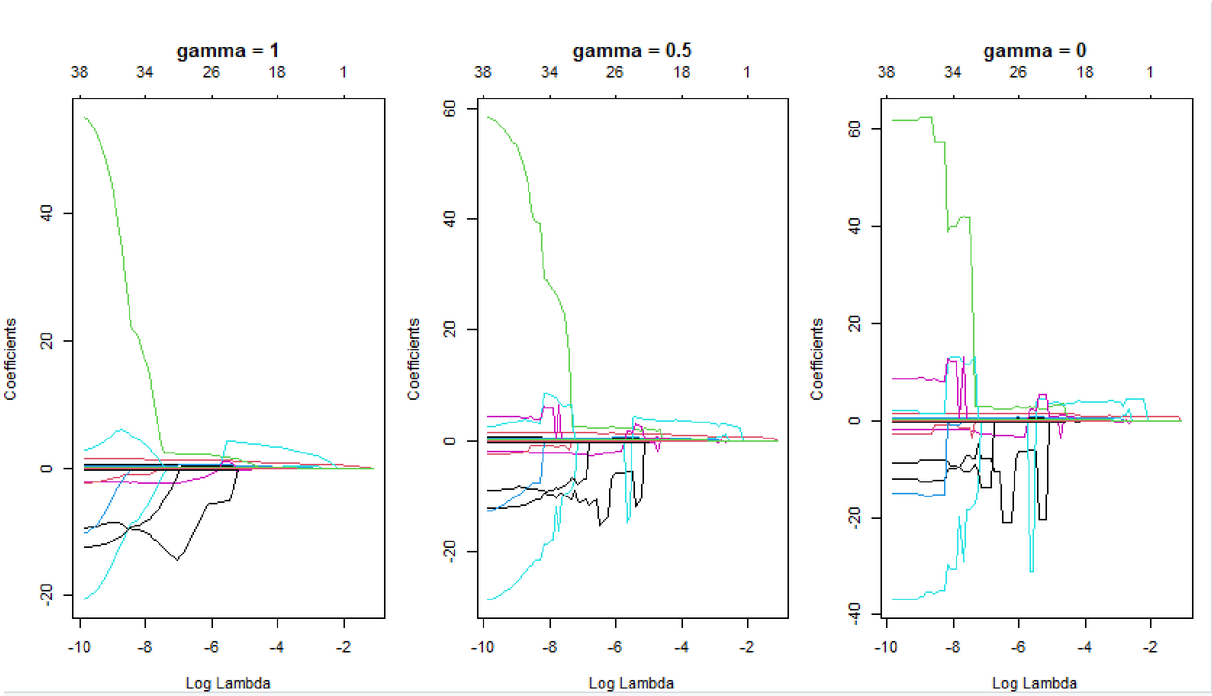


Figure 16: Relaxed lasso with gamma = 1 (regular lasso), gamma = 0 the unpenalized fit and gamma = 0.5 is a mixture of the two.

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) -1.11999    0.21614  -5.182 3.50e-07 ***
Babyfaced    0.15831    0.04838   3.272  0.00116 ** 
Feminine     0.51035    0.09237   5.525 5.96e-08 ***
Masculine    0.47906    0.04852   9.872  < 2e-16 ***
Sad         -0.21887    0.04347  -5.035 7.25e-07 ***
Trustworthy  0.49033    0.05876   8.345 1.18e-15 ***
Unusual     -0.18098    0.04531  -3.994 7.72e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.4505 on 399 degrees of freedom
Multiple R-squared:  0.6026,	Adjusted R-squared:  0.5966 
F-statistic: 100.8 on 6 and 399 DF,  p-value: < 2.2e-16
```

Figure 17: Final linear model of CFD database psychological variables.

```
Coefficients:
                        Estimate Std. Error t value Pr(>|t|)    
(Intercept)           -3.485e-01  7.811e-01  -0.446  0.65553    
calm                   2.232e-01  4.134e-02   5.399 8.03e-08 ***
common                -1.071e-01  3.738e-02  -2.866  0.00423 ** 
confident              6.462e-02  4.602e-02   1.404  0.16048    
egotistic              2.662e-01  3.598e-02   7.400 2.50e-13 ***
intelligent            1.365e-01  4.488e-02   3.041  0.00241 ** 
introverted            8.320e-03  3.808e-02   0.218  0.82709    
kind                   4.885e-02  4.889e-02   0.999  0.31786    
responsible           -4.360e-01  4.677e-02  -9.322  < 2e-16 ***
trustworthy            6.171e-03  5.397e-02   0.114  0.90899    
weird                 -2.744e-01  3.492e-02  -7.858 8.42e-15 ***
aggressive             4.777e-05  3.707e-02   0.001  0.99897    
caring                -1.160e-02  5.884e-02  -0.197  0.84379    
emotional              2.539e-02  3.698e-02   0.687  0.49248    
familiar               1.732e-01  3.605e-02   4.805 1.74e-06 ***
friendly               1.093e-01  6.124e-02   1.786  0.07441 .  
happy                 -8.684e-02  4.941e-02  -1.757  0.07909 .  
humble                -1.195e-01  4.451e-02  -2.685  0.00735 ** 
interesting            2.838e-01  4.585e-02   6.191 8.11e-10 ***
memorable              1.641e-01  4.254e-02   3.857  0.00012 ***
normal                 3.288e-01  4.598e-02   7.151 1.47e-12 ***
sociable               1.672e-01  5.271e-02   3.172  0.00155 ** 
typical               -9.423e-03  4.178e-02  -0.226  0.82162    
Racewhite             -3.486e-01  4.691e-01  -0.743  0.45751    
Raceblack             -4.925e-01  4.730e-01  -1.041  0.29800    
Raceasian             -3.948e-01  4.854e-01  -0.813  0.41611    
Racehispanic (latino) -3.966e-01  4.816e-01  -0.823  0.41039    
Racemiddle eastern    -1.654e-01  5.001e-01  -0.331  0.74086    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.6562 on 1240 degrees of freedom
Multiple R-squared:  0.5932,    Adjusted R-squared:  0.5843 
F-statistic: 66.97 on 27 and 1240 DF,  p-value: < 2.2e-16
```

Figure 18: First linear model of 10k US Faces database.

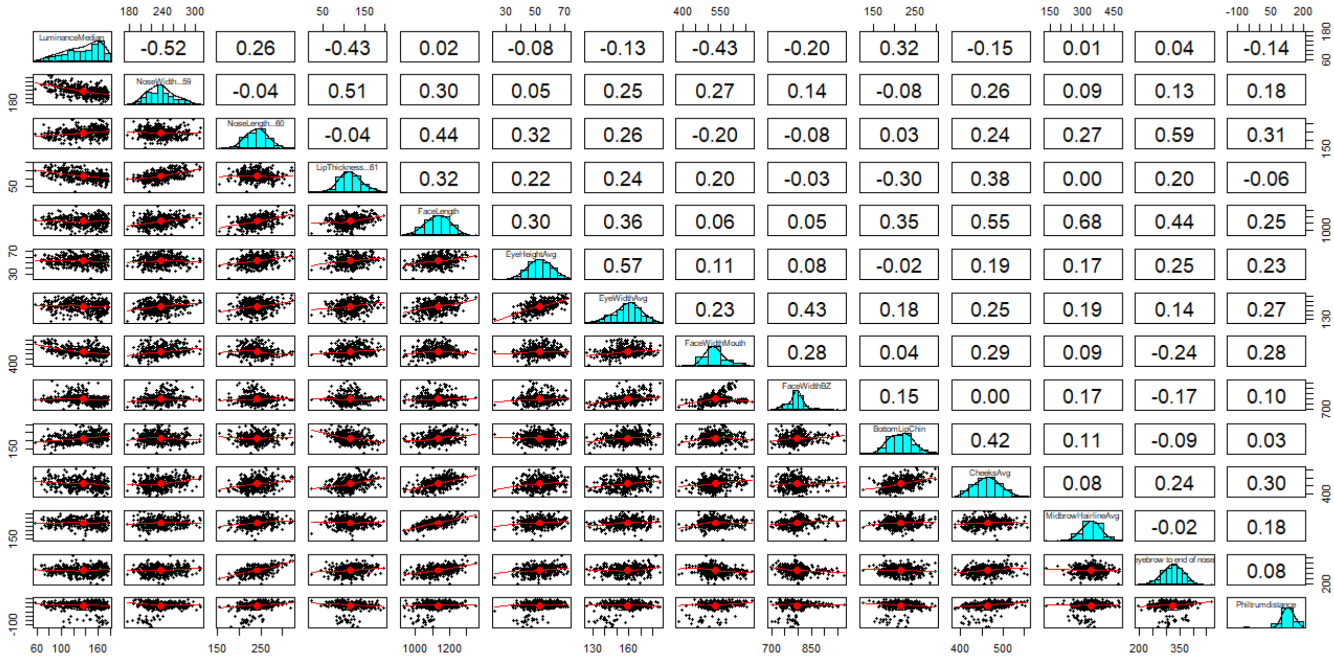


Figure 19: Collinear valiables by pairs.panels function.

```
Coefficients:
                         Estimate Std. Error t value Pr(>|t|)
(Intercept)            -0.3499598  1.0234380  -0.342 0.732576
LuminanceMedian        -0.0023522  0.0015701  -1.498 0.134916
NoseWidth...59          0.0023040  0.0017199   1.340 0.181148
NoseLength...60        -0.0010492  0.0018658  -0.562 0.574209
LipThickness...61      -0.0044504  0.0021691  -2.052 0.040860 *
FaceLength             -0.0057979  0.0014781  -3.923 0.000103 ***
EyeHeightAvg           -0.0003921  0.0050770  -0.077 0.938477
EyeWidthAvg             0.0069790  0.0041436   1.684 0.092927 .
FaceWidthMouth         -0.0030370  0.0008432  -3.602 0.000357 ***
FaceWidthBZ             0.0059084  0.0009241   6.394 4.62e-10 ***
BottomLipChin          -0.0053686  0.0020842  -2.576 0.010366 *
CheeksAvg               0.0109076  0.0017156   6.358 5.71e-10 ***
MidbrowHairlineAvg      0.0040133  0.0016089   2.494 0.013026 *
`eyebrow to end of nose` 0.0036164 0.0015751   2.296 0.022209 *
Philtrumdistance        0.0002265  0.0007630   0.297 0.766737
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.6131 on 391 degrees of freedom
Multiple R-squared:  0.2785,	Adjusted R-squared:  0.2527
F-statistic: 10.78 on 14 and 391 DF,  p-value: < 2.2e-16
```

```
Coefficients:
                          Estimate Std. Error t value Pr(>|t|)
(Intercept)             -0.8303816  0.8287035  -1.002 0.316942
FaceLength              -0.0067181  0.0011685  -5.750 1.79e-08 ***
EyeWidthAvg              0.0056463  0.0033376   1.692 0.091487 .
FaceWidthMouth          -0.0022685  0.0007840  -2.894 0.004020 **
FaceWidthBZ              0.0063469  0.0008956   7.087 6.32e-12 ***
BottomLipChin           -0.0041296  0.0013696  -3.015 0.002733 **
CheeksAvg                0.0103744  0.0015292   6.784 4.26e-11 ***
MidbrowHairlineAvg       0.0048143  0.0013150   3.661 0.000285 ***
`eyebrow to end of nose` 0.0040551  0.0012472   3.251 0.001247 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.6172 on 397 degrees of freedom
Multiple R-squared:  0.2576,	Adjusted R-squared:  0.2426
F-statistic: 17.22 on 8 and 397 DF,  p-value: < 2.2e-16
```

Figure 20 and 21: First and final model with feature’s distances.

```
Coefficients:
                     Estimate Std. Error t value Pr(>|t|)    
(Intercept)         9.039e+00  4.761e+00   1.898 0.058377 .  
FaceShape           5.433e+00  7.846e+00   0.692 0.489090    
Heartshapeness     -4.285e+00  3.591e+00  -1.193 0.233507    
NoseShape          -3.901e-01  3.036e-01  -1.285 0.199555    
LipFullness        -9.152e+00  3.098e+00  -2.954 0.003324 ** 
EyeShape           -7.221e+00  1.925e+00  -3.751 0.000203 ***
EyeSize             4.943e+01  1.173e+01   4.213 3.13e-05 ***
UpperHeadLength     4.708e-01  2.026e+00   0.232 0.816357    
MidfaceLength      -3.220e+00  3.166e+00  -1.017 0.309765    
ChinLength         -1.251e+01  3.027e+00  -4.134 4.37e-05 ***
ForeheadHeight     -6.394e-01  1.172e+00  -0.545 0.585760    
CheekboneHeight     1.301e+01  2.671e+00   4.873 1.60e-06 ***
CheekboneProminence -1.716e-03  9.683e-04  -1.772 0.077151 .  
FaceRoundness      -1.271e+01  8.950e+00  -1.420 0.156519    
fWHR2               9.561e-01  2.129e-01   4.491 9.33e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.6177 on 391 degrees of freedom
Multiple R-squared:  0.2676,	Adjusted R-squared:  0.2414 
F-statistic:  10.2 on 14 and 391 DF,  p-value: < 2.2e-16
```

```
Coefficients:
                Estimate Std. Error t value Pr(>|t|)    
(Intercept)      -0.6453     0.6160  -1.048   0.2954    
LipFullness      -8.2028     1.7958  -4.568 6.57e-06 ***
EyeSize           8.2792     4.7232   1.753   0.0804 .  
ChinLength      -11.1343     1.7991  -6.189 1.50e-09 ***
CheekboneHeight  10.4463     1.4642   7.134 4.60e-12 ***
fWHR2             1.0759     0.1453   7.407 7.75e-13 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.6392 on 400 degrees of freedom
Multiple R-squared:  0.1977,	Adjusted R-squared:  0.1877 
F-statistic: 19.71 on 5 and 400 DF,  p-value: < 2.2e-16
```

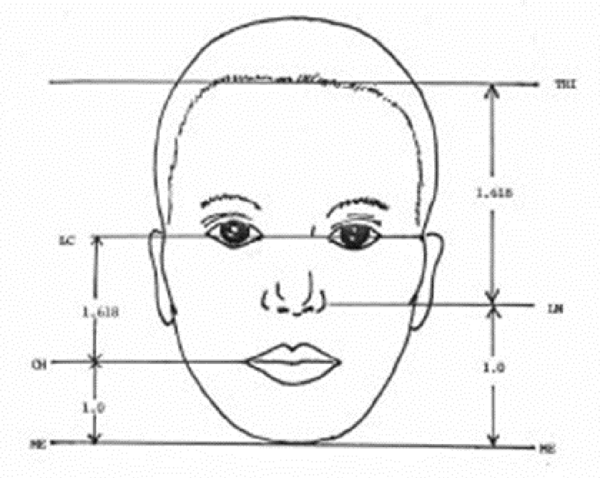
Figure 22 and 23: First and final model with ratios given from the dataset.

```
Coefficients:
                                 Estimate Std. Error t value Pr(>|t|)    
(Intercept)                     3.2330093  0.0905190  35.716  < 2e-16 ***
howclosemouthtochin            -0.0024531  0.0008800  -2.788  0.00556 ** 
howclosetoideallipthickness     0.0030530  0.0006851   4.456 1.08e-05 ***
howclosetoidealoutercanthalwidth -0.0017571  0.0007579  -2.318  0.02093 *  
fWHRdummy1                     -0.1924517  0.0684252  -2.813  0.00516 ** 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.6825 on 401 degrees of freedom
Multiple R-squared:  0.08307,	Adjusted R-squared:  0.07392 
F-statistic: 9.082 on 4 and 401 DF,  p-value: 4.959e-07
```

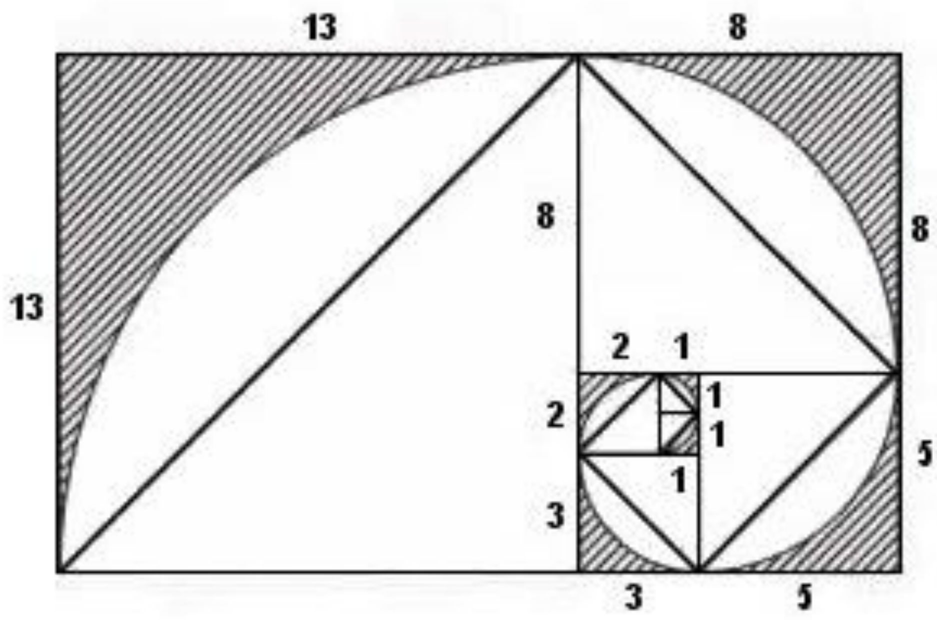
Figure 24: Final model only with significant golden ratios.

```
Coefficients: (2 not defined because of singularities)
                                     Estimate Std. Error t value Pr(>|t|)
(Intercept)                        -3.0444289  1.1330304  -2.687  0.00751 **
CheekboneHeight                     4.3232256  1.4791351   2.923  0.00367 **
FaceWidthBZ                         0.0052763  0.0008734   6.041 3.54e-09 ***
howclosetoideallipthickness         0.0019975  0.0006767   2.952  0.00335 **
howclosetoidealfacewidth           -0.0019995  0.0009674  -2.067  0.03940 *
CheekboneProminence                -0.0024667  0.0007873  -3.133  0.00186 **
LuminanceMedian                    -0.0009572  0.0015529  -0.616  0.53798
howclosetoidealfacelength                  NA         NA      NA       NA
BottomLipChin                      -0.0028136  0.0013166  -2.137  0.03321 *
howclosetoidealoutercanthalwidth   -0.0038127  0.0008232  -4.631 4.94e-06 ***
secondthirdoffacedividedbyfacelength 1.9403772  1.1295347   1.718  0.08661 .
NoseWidth                           0.0030801  0.0016279   1.892  0.05921 .
thirdthirdoffacedividedbyfacelength        NA         NA      NA       NA
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.6031 on 395 degrees of freedom
Multiple R-squared:  0.2949,    Adjusted R-squared:  0.277
F-statistic: 16.52 on 10 and 395 DF,  p-value: < 2.2e-16
```

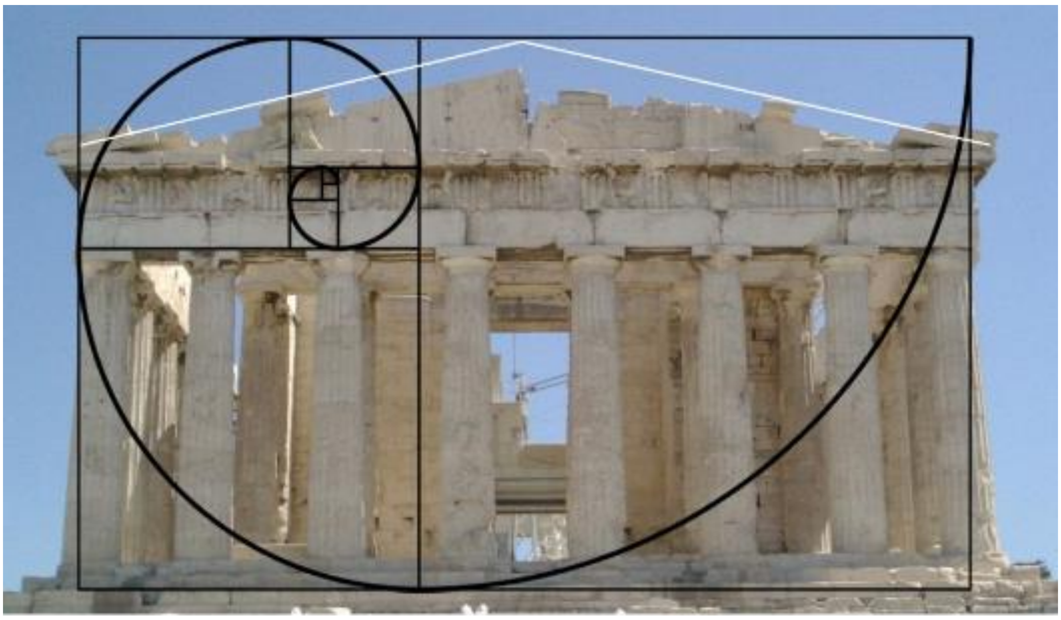
Figure 26: First linear model with variables selected from Relaxed Lasso.



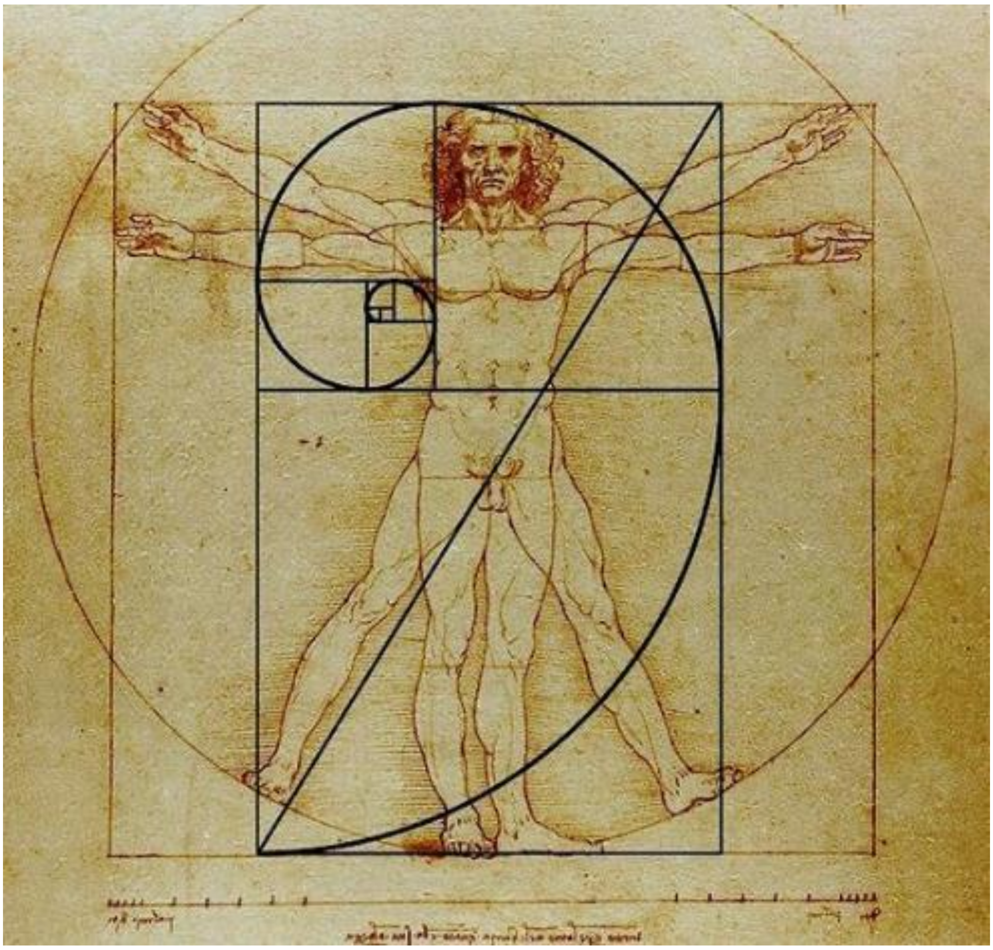
Golden ratios in human face



Phi sequence in nature (nautilus)



The golden ratio in the Parthenon (architecture)



The Vitruvian Man by Leonardo da Vinci (art)

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