ERASMUS UNIVERSITY ROTTERDAM
ERASMUS SCHOOL OF ECONOMICS
MSc Economics & Business
Master Specialisation Financial Economics

FACTORS INFLUENCE AND DETERMINE MAGNITUDE OF EXPECTED RETURNS AND VOLATILITY STRATEGIES APPLIED IN THE US MARKET

Name: Anh Tran

Student Number: 580527tt

Supervisor: Prof. P.J.P.M (Philippe) Versijp

ABSTRACT

Using the US stock observations from 1963 to 2020, this paper examines the combination of time window, which are three years, two years, one year, three months and one month, and data frequency, using daily, weekly, or monthly returns, to find the optimal chosen period to construct equal-weighted portfolios based on the low-volatility anomaly. Moreover, this study aims to determine the effect of macroeconomic factors and firm characteristics on the magnitude of the expected returns and the low-volatility strategy. Using the time window of one month with daily returns to determine the lowest volatile portfolio provides the highest portfolio performance, with the Sharpe ratio of around 0.8. The higher the least risky portfolios perform, the lower the highest volatile portfolios perform. Besides, the business cycle strongly impacts the portfolio's performance which is constructed by the one-month daily returns, the performance increases during the expansion periods and decreases in the recession period. This paper examines two macroeconomic variables which consist of inflation and interest rates, using the OLS regression model. It is surprising that although the inflation and interest rates exhibit opposite signs, both variables do not provide any impact on the relation between expected returns and the volatility anomalies. Lastly, the result indicates that firm characteristics, which contain the size, book-to-market, cashflow-to-price, and earnings-per-share ratio, do not influence the magnitude of expected returns and low-volatility strategy after controlling by the OLS regression model, while the dividend-to-price ratio does impact the existence of risk-adjusted returns and the low-volatility effect.

Date final version:

Contents

List of tables

List of graphs

I. Introduction

In 1970, Malkiel and Fama stated that in the Efficient Market Hypothesis (EMH), prices can reflect all the information about assets. Alhough this theory is a cornerstone of modern financial theory, the EMH is highly controversial in the empirical literature, and many recent researchers argue that many factors could also impact the changes in prices, as the market anomalies could produce abnormal returns (Schwert (2003)). One of the anomalies that have remained robust and can provide the outperformance strategy is the low-volatility effect. According to Blitz and Vliet in 2007, the low-volatility anomaly means that a low-volatility stock could yield higher returns than a market portfolio, while a high-volatility portfolio provides a lower return.

Even though the low-volatility strategy is one of the most common strategies applied in institutional investment decisions[1], this anomaly varies over time. In the period of 1857 and 1987, the stock market volatility fluctuated from two to twenty percent per month, according to Schwert (1989). In 2013, Blitz, Pang, and Vliet researched the effect of the low-volatility anomaly in emerging markets; the results provide negative returns, especially for the emerging markets related to the developed equity markets. Another study is introduced by Spiegel and Wang (2005), stating that a change of one unit in standard deviation could affect around 2.5 to 8 times the corresponding changes in liquidity on the expected returns. Some explanations of this fluctuation have been published. For example, Officer (1973) stated that the macroeconomics variables, such as money growth, inflation, are the reason for the varied volatilities, while Christie (1982) indicated that leverage can explain this fluctuation. Moreover, interest rates are believed to be the reason for this variation, according to Lauterbarch (1989). Thus, myriads of factors can significantly impact the performance of the low-volatility anomaly.

From these perspectives, this research examines the three factors that can mainly affect the performance of the low-volatility strategy. The main question that will be answered in this paper is: *"Which factors significantly influence, and determine the short-term magnitude of the expected return and the volatility strategy applied to the United States equity market?"*.

The first factor is the period of time in the past that was used to determine the volatility. In Blitz and Vliet's (2007) paper, they use weekly historical data over a period of three years to construct the decile portfolios. While Schwert (1989) and Ang et al. (2006) computed standard deviation using the daily returns on monthly data. Another study which was done by Engle and Patton (2007), used the one-year daily returns to identify the volatility. However, none of the researches mentioned above explains why the particular historical period used in their calculation was chosen. Moreover, Bali and Caki (2008) found that the frequency of data, which is monthly and daily data, significantly impacts the ex istence of the relation between idiosyncratic risk and expected returns. They, however, emphasized that their paper does not study the coefficient between the expected returns and the volatilities. The question is would the outcome of such calculation change when a different historical period was selected

[1] See MSCI report: MSCI Report (2016): Constructing low volatility strategies

instead. How would selecting a historical period affect the low-volatility strategy performance? Knowing the answer to this question would determine the optimal settings for the calculation of the volatility method.

Second, the macroeconomic variables are the other contributing factors to the volatility fluctuation over time (Officer (1973)). The macroeconomic factor comprises many elements such as inflation, interest rates, business cycle, unemployment rates. A study, which was published in 1996 by Hamilton and Lin, concluded that economic recessions primarily drive the volatility fluctuation on the stock market. During the market downturns period, the increase of global risk-aversion, which emphasizes the investors who like avoiding risks and want to minimize the risks, leads to the fall of expected returns and subsequently raises investment volatility. Besides, Chen et al. (2020) demonstrated that the macroeconomic factors and asset prices are linked through risk-aversion. Because of the sensitivity of global risk aversion in macroeconomy variables, recessions can strongly influence the negative correlation between returns and the volatility effect. When the risk-averse preferences increase during the recession periods, investors could accept losses to minimize the risks they could take by selling their assets at a lower price, leading to increased volatility on asset prices while decreasing its expected returns. However, in 2002, Li indicated that the unexpected inflation and interest rates significantly affected the stock and bond returns. Li (2002) also concluded that the business cycle does not have any impact on the performance of the stocks and bonds, although the US market has gone through many recessions and expansions, reporting by the National Bureau of Economic Research ('NBER')[2]. These papers raise another question, which macroeconomic factors mainly affect the relation between the stock returns and the volatility effect in today's market. The purpose of these findings is to determine which macroeconomic factors can impact the magnitude of the relation between the volatility and the expected returns. Knowing the importance of these factors would strengthen the predictability of asset price volatility and portfolio performance in different market trends.

Finally, the last factor which is considered in this research is firm characteristics. According to Kogan and Tian (2012), there are five firm characteristics related to the company valuation, which are market capitalization, book-to-market ratio, dividend-to-price ratio, earnings-to-price ratio, and cash flow-to-price ratio. These firm-specific variables are the proxies as the control variables of the dynamic link between the expected returns and the volatility effect. In 2013, Kogan and Papanikolaou showed that firm characteristics positively relate to the growth rate of the firm value and the stock returns. These firm characteristics are examined in this study to understand how much these characteristics could impact the magnitude and existence of the relation between the returns and volatility effect, thus, improving the performance of the volatility strategy even further.

---

[2] The business cycle of the US is obtained from MSCI Report: "The American Business Cycle: Continuity and Change" Historic Data Tables

The remainder of this paper is structured as follows. Section 2 gives an overview of the relevant research about the chosen factors. In Section 3, my hypothesis is presented. The data collection and cleaning process are presented in Section 4. Besides, Section 4 describes the methodology of constructing the portfolios between independent and dependent variables and the risk-factor models. Section 5 indicates and discusses the results. Lastly, in Section 6, a summary of the main points is provided. Any limitations of this paper and possible avenues for future research are discussed in this section.

II. Literature review

2.1 The low-volatility anomaly

The low volatility anomaly was first introduced in the mid-1970s; but only became a popular investment strategy after the financial crisis in 2008. The low-volatility investment strategy means that the portfolio with low-volatility asset prices would provide higher returns with a lower level of risk than the high-volatility portfolio. According to (Frazzini & Pedersen, 2014), the high exposure risks exhibit low expected returns, and the investors who go long in low-beta assets and short high-beta stocks receive significant positive risk-adjusted returns. Therefore, the investors who follow the low-volatility strategy would go long in the lowest-volatility portfolio to yield the highest expected returns with a low level of risk.

According to Blitz and Vliet (2007), most of the results show that the relation between historical volatility and returns is weak. However, in the risk-adjusted returns perspective, the low-volatility portfolios yield high average returns, whereas the high-volatility portfolio underperforms the market portfolio. Besides, Blitz and Vliet (2007) also found that the low-volatility effect performs strongly in a level of globalization, not only in the US market but also in the Japanese and European markets. According to the results, the high-volatility portfolios yield significant negative returns while the lowest volatile portfolio provides the highest positive profitability, so does the US performance. Possible explanations for the volatility effect are introduced by Blitz, Falkenstein, and Vliet (2014). One of the explanations relates to the absence of leverage constraints. They state that less leveraged-constrained investors would prefer low-beta stocks to have higher returns, resulting in underpricing the low-volatility stocks. Other explanations are that the perfect markets, or standard absolute utility preferences, etc. This paper finds it complicated to identify the critical explanation of the volatility effect. These explanations mainly involve other factors, as they gave a specific example about the winner's curse, where both overconfidence and short-selling constraints are considered.

2.2 Macroeconomic variables

According to Bender et al. (2013), some researchers who follow the EMH believe that systematic risks affect the stock prices, while others (Bender et al. (2013), Schwert (1989)) think that behavioral biases or time horizons are the main reason for stock prices' fluctuation. This statement contradicts to the EMH. Aside from market factors, Bender et al. (2013) stated that three main factors could strongly impact the volatilities of stock prices, which are macroeconomics, fundamental and statistical. Macroeconomics comprises those factors such as inflation, interest rates, gross national products, and vice versa. Fundamental factors include firm-specific variables, for example, firm valuation ratios, investment, technical indicators, and to name a few. Those firm characteristics that are considered most by researchers are value, growth, and size. Finally, statistical factors contain those factors using statistical techniques. These statistical factors, however, have not yet determined which one is best to debate.

One of the main categories of behavioral bias could be overconfidence. Investors who overestimate their ability to predict future stock prices are considered overconfident, leading to risky stocks being overpriced. Irrational investors can affect stock prices, which they react differently between the uptrend and downtrend market.

The data frequency has a crucial impact determining the existence and significance of the relation between the expected returns and idiosyncratic risks, according to the finding of Bali and Cakici (2008). Although using daily data to construct portfolios provides either significant or insignificant relation between the risks and expected returns, Bali and Cakici (2008) found no coefficient between the risk and expected returns when the idiosyncratic volatilities are calculated by the monthly data.

2.2.1 Business cycle

Other papers related to the research question are introduced by Schwert (1989) and Blitz and Vliet (2007). Schwert's paper (1989) examined the relation between market volatility and macroeconomic factors from 1857 to 1987. Schwert (1989) concluded that on the volatility puzzle, the business cycle significantly affects the stock market volatility, especially in the Great Depression, while other economic factors such as inflation, money growth, and interest rates are not closely related to the volatilities. Furthermore, Blitz and Vliet (2007) stated that the low volatility portfolios underperformed the market during the uptrend months and had a higher performance during the downtrend. Overall, the performance of the low volatility portfolios, which are 59% during up months and 41% during down months, can still offset between up and down months and exhibits positive returns. In contrast, the high-risk portfolios provide the opposite behavior, underperformed during the downtrend months and overperformed when the market recovers. However, overall, the bottom decile portfolios yielded a negative performance because the overperformance is not enough to offset the underperformance during the market downturn period. This finding is consistent and strengthens Schwert's finding in 1989.

2.2.2 Inflation rates and interest rates

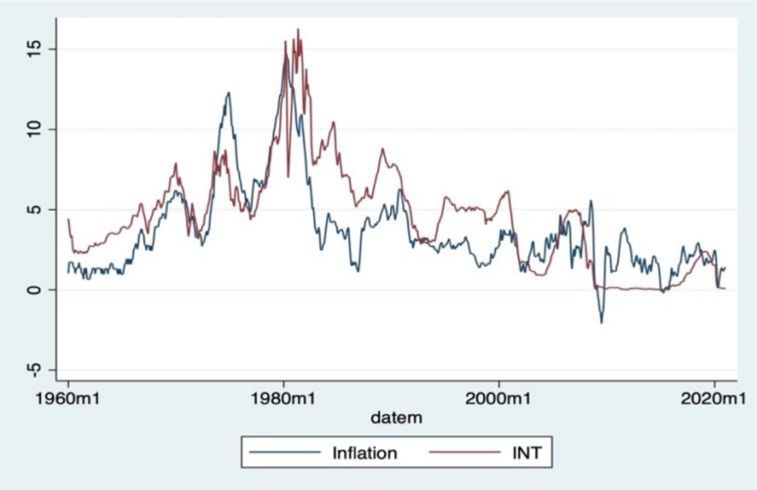
On the other hand, Li (2002) introduced the influence of macroeconomic factors on the correlation between bonds and stock returns. This paper concludes that unexpected inflation and interest rates drastically affect the performance of stocks and bonds. In comparison, the business cycle does not have any impact on the correlation of the stock-bond returns. There is an explanation for the contradicting results on empirical literature that is the risk-averse preferences. Investors who are risk-averse agents want to maximize their utility and only care about the mean and variance of the return. Blitz, Falkenstein, and van Vliet (2014) stated that risk-averse agents maximize the following objective function:

$$E(R_p) - \lambda\, E\left(R_p - E(R_p)\right)^2 \qquad (1)$$

Where R_p represents the absolute portfolio return and λ is a risk-aversion parameter that i`s greater than zero. Following the equation, if the risk-aversion increases, the mean-variance returns would reduce significantly. Empirically, Blitz, Falkenstein, and van Vliet (2014) show that the investors do not only care about the mean and variance of returns. Moreover, Brant and Wang (2003) explained the relationship among risk-aversion, asset prices, and inflation. In addition to the papers cited above, they also provided the positive coefficient between risk-aversion and inflation rates. While Campell and Barr in 1997 showed the negative coefficient between inflation rates and interest rates on the market in short horizons. However, they found that the correlation between interest rates and inflation is low in long horizons. Based on this literature, aside from the mean and variance of the return, the risk-averse agents aim to avoid risk in their portfolios when the economic market goes in a downward trend.

The inflation and interest rates of the US market from Jan 1963 to Dec 2020 are presented in Graph 1.

Graph 1. Inflation and interest rates data of the US market from Jan 1963 to Dec 2020.



Note: The graph displays the inflation, which is constructed by the Consumer Price Index ('CPI') data, and the interest rates. Both the data is obtained from FRED websites.

According to Graph 1, the inflation data and the interest rates of the US market fluctuated drastically through the business cycle. Both inflation and interest rates have the same pattern between 1963 and 2020 that increased during the recession period and decrease during the expansion period. In addition, both reached a peak of around 15% during 1980 and 1982, which is the contraction period. As shown in Graph 1, the US market's inflation rate was below 0 from March to October 2009, while the interest rate equals nearly 0 from 2008 to 2017, which was the lowest interest rate of the US market during the chosen period from 1963 to 2020. Moreover, the interest rates decrease drastically after the recession period ends, and the rates remain constants during the expansion periods. Besides, Barr and Campell (1997) indicated the opposite sign between inflation rates and the interest rates on the market, especially in short horizons. However, they found a low correlation between interest rates and inflation in longer horizons, which means following the increase of horizons, the positive relationship between interest rates and inflation also increases. Thus, there is the similar sign between inflation rates and the interest rates on the market.

2.2.3 Firm-specify variables

Finally, firm characteristics related to firm valuation are researched in the paper of Kogan and Tian (2012) and Kogan and Papanikolaou (2013). Kogan and Tian (2012) constructed 27 commonly used company characteristics to compute the asset pricing model. Those characteristics are divided into seven groups, which comprise the following groups: valuation, investment, prior returns, earnings, financial distress, external financing, and others. According to Kogan and Tian (2012), firm valuation yields statistically significant returns. While they stated that investment characteristics are used to predict future returns and have lower statistical significance, the return factors based on prior returns exhibit the opposite sign among each sub-factor. The other set of firm characteristics provide a negative coefficient between those firm characteristics and the excess returns of the portfolios. According to Bali and Cakici (2008), small-capitalization strongly drives the relation between expected returns and volatilities. Ang et al. (2006) indicated that there is a significant negative relation between the volatilities and expected returns, and other factors such as value, book-to-market, liquidity, and momentum cannot affect the relation between volatilities and expected returns. Bali and Cakici (2008) applied the same methodology as Ang et al. (2006) to test the robustness of their hypothesis. After removing the smallest firm valuation, the lowest stock prices, and the least liquidity, they found no relation between the volatilities and expected returns. This result rejects the finding of Ang et al. (2006) and shows that small-capitalization significantly impacts the fluctuation of stock prices. Moreover, Kogan, and Papanikolaou (2013) provide that profitability, valuation ratios, and volatilities are three out of the company characteristics related to the ratios of the growth and the firm value. Thus, the firm characteristics exhibit significant effects on the relationship between volatility and average excess returns.[3]

III. Hypotheses development

The purpose of this paper is to find which factors could influence the existence and determine the magnitude of the expected returns and volatilities.

Hypothesis 1: Using the historical data with the time window of one-year and the data frequency of weekly to identify volatility portfolios would provide the highest expected returns compared to other combination of the investment horizon and data frequency.

Following the recent empirical literature, investment horizons and frequency of the returns play significant roles to yield the expected returns. The combination between the alternative levels of data frequencies, which are high- (daily), medium- (weekly), and low- (monthly), and the different time horizons included long- (three years and two years), medium- (one year), and short- (three months and one month) could yield different portfolio's performance. This research examines the combination of three different frequency levels of data and five different time horizons to identify the optimal chosen data. Using the medium time window and medium frequency ranges would yield the highest return, which is the one-year weekly returns. Since either higher frequency data of illiquidity stocks (Ait-Sahalia and Yu (2008)) could create microstructure noise or monthly observations might not provide enough data to exhibit the optimal strategy, only weekly returns could solve these two problems and provide the highest returns. The reason is that the observations used for analysis and finding the optimal strategy could be more precise and reliable. The weekly returns capture the pattern of stock prices while avoiding the noise data, affecting the performance.

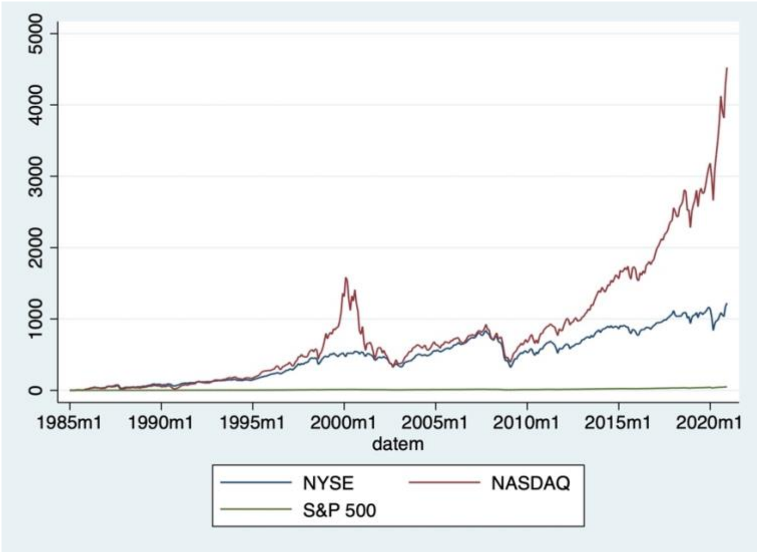
Not only the interval that matters, the investment horizon of the data also contributes to the results of the strategy. Using only one-month daily data to calculate could be affected by the noise from the daily data and both the systematic and unsystematic risks on a monthly basis. While weekly returns of one month do not give enough information on fundamental value. Moreover, using three-year returns would exhibit less robust results than the one-year historical data because, during examining either sub-period of the business cycle or shorter period, it faces the risk of lack of data. Thus, using the one-year historical data to compute the volatility effect can capture both the systematic and unsystematic markets' performance.

Each of the following hypotheses can interact with the choices for the historical period under investigation in Hypothesis 1.

[3] This paper restricts the firm valuation characteristics related to the stock information on fundamental values because these characteristics positively affect the portfolio's performance.

Graph 2 displays the value of $1 invested in S&P 500, NASDAQ, and NYSE from 1985 to 2020.

Graph 2. The value of $1 invested in S&P 500, NYSE, and NASDAQ from 1985 to 2020



Note: The graph shows the value of $1 invested in S&P 500, NYSE, and NASDAQ composite index from 1985 to 2020.

As can be seen in Graph 2, if the investors invest $1 in NASDAQ, NYSE, and S&P 500 at the beginning of 1985, they will get the value of approximately $4,500, more than $1,000, and nearly $100, respectively. From 1998 to 2001, which are in the expansion period, the value of $1 invested money grew significantly in the NASDAQ platform, while investors did not receive if investing in the NYSE and S&P 500 platforms. However, after the financial crisis between 2007 and 2009, the invested value in the NASDAQ and the NYSE platforms raised drastically from nearly $500 to nearly $4,500, and to more than $1,000, respectively. It is surprising that in the recession period of 2020, the stock price among the NYSE, NASDAQ and S&P 500 still grow significantly. According to Graph 2, the value of $1 invested increase subsequently during the expansion period and fall substantially during the contraction period.

Hypothesis 2: The business cycle significantly impacts the correlation between the expected returns and the volatility strategy.

A typical pattern that has been proved in much empirical research (Chen et al. (2020), Brandt and Wang (2003)) is that risk-aversion tends to be high during the recession, and when the economic market is in the expansion period, risk-averse preferences are low. Besides, when the risk-averse preferences increase, the investors could minimize their risks by selling their portfolios at lower prices, which leads to reducing the expected returns and raising the volatility of asset prices in the market. During the recession, inflation goes to the upward trend leading to the increase of risk aversion; thus, the performance of the risk-aversion preference portfolio is good and losing less. Moreover, Graph 2 indicates the positive relation between the value of $1 invested in the NYSE and NASDAQ platform and the business cycle. This pattern exhibits the positive correlation between the business cycle and the relation of the returns and the volatility strategy.

Hypothesis 3: Inflation does not impact the relation between expected returns and volatilities.

The aim of this paper is to identify whether there is a correlation between the change of inflation rates and the volatility in market trends. According to the recent empirical article, inflation rates positively correlate with risk-aversion, and the risk-averse preferences can explain the relation between inflation rates and asset prices (Brant and Wang (2003)). When the inflation rates increase during the recession, the aggregated risk-aversion goes in an upward trend, reducing investment and asset prices; whereas when the inflation rates go down, the risk-aversion falls and risk-seeking rises significantly, leading to the growth of asset prices and also volatility. However, according to Graph 1, inflation fluctuates drastically through the business cycle during the period 1963-2020, while Graph 2 shows that the stock prices only fluctuate significantly in the NASDAQ platform through the business cycle, especially in the expansion period 1998-2001 and from 2001 to 2020. Thus, there is no relation between inflation and the changes in stock prices during the period 1963-2020.

Hypothesis 4: There is no relation between the interest rates and the stock prices' volatility.

Besides, if the risk-averse preferences increase, the investors would raise their demand on the risk-free assets, yield lower interest rates, and the less risk-aversion, the higher the interest rates and volatility on asset prices and returns. The interest rate varies over the business cycle following the risk-averse behavior. Each market trend in the business cycle represents the difference between risk-averse preference and the varied interest rates. This trend explains why the risk-aversion investors usually lose less in bad times but gain less in the expansion period and emphasizes the impact of interest rates on the volatility of asset prices. On the other hand, although the interest rates in Graph 1 fluctuate significantly during 1963-2020, the value of $1 invested in the NYSE and S&P 500 shown in Graph 2 does not vary much through the business cycle. There is no clear evidence that shows the positive relation between the interest rates and the invested value in the NASDAQ platform. Therefore, the interest rate does not control the volatility of asset prices, and there is no relation between interest rates and the volatility effect.

Hypothesis 5: The firm characteristics on corporation valuation provide significant positive effects on the performance of the volatility strategy.

In this paper, the firm characteristics on corporation valuation include size, BM, DP, EP, and CP ratio as independent variables. Following the recent empirical literature, the firm characteristics provide a positive significant coefficient with the expected returns, whereas according to the low-volatility effect, the low-volatile portfolio yields higher returns than the performance of the high-volatile portfolio. Thus, those firm characteristics provide a significant positive influence on the performance of the volatility effect.

IV. Data and Research method:

4.1 Data

The dataset used for this paper comprises the daily stock prices in the US-listed companies from July 1963 to December 2020 on the CRSP database. The researcher choose this period because most of the Fama and French factors and macroeconomic data have been fully collected since 1963. Based on the asset pricing literature, similar to that of Fama & French (1993 and 2015), all equities listed on NYSE, AMEX, and NASDAQ are included. Non-financial firms and common shares shall be included in the dataset since the specific characteristics of financial firms could affect the research results. In addition, those companies which do not have the financial information between 1963 and 2020 are also eliminated throughout the research. The Fama-French 3-, 5- factors and the risk-free rate are obtained from the Kenneth R. French Data Library[4]. The excess returns are used in this paper to compute the standard deviation. The short-selling constraints and the transaction costs are ignored throughout my analysis.

Macroeconomic data originates from the FRED Economic data website ('FRED'). The monthly inflation rate is constructed by the Consumer Price Index ('CPI') data[5]. The interest rates are obtained via the FRED websites[6]. Besides, the US-business cycle information originates from the NBER website. The annual firm characteristics data, such as market capitalization, market-to-book, dividend, earnings, and cash flow, comes from the CRSP Compustat Merged.

4.2 Methodology

The methodology in this paper is inspired by the work of Blitz and Vliet in 2007. The volatility is calculated by using the standard deviation of historical returns of the assets in a particular period. The first step is to sort the stocks into ten-decile equally weighted portfolios introduced by constructing the three-year historical data with weekly returns to determine the standard deviation. In both Blitz and Vliet (2007)'s paper and this paper, the lowest volatile stocks are formed in the first decile portfolio, and the most volatile stocks are in the last portfolio.

4.2.1 The optimal historical information

This paper investigates the factors which can impact the performance of the low-volatility effect. In general, for constructing the low-volatility strategy as introduced by Blitz and Vliet in 2007, I sort the stocks into univariate decile portfolios. Decile one contains the smallest 10 percent of all stocks, while decile ten comprises the most volatile stocks. In this step, I calculate the volatility by using the short- (one month), mid- (three months and one year), and long- (two and three years) historical data. Moreover, Sahlgren (2016) stated that the model with the large data size combined with the medium to high-frequency ranges would perform better than the small-size model with long-frequency ranges.

[4] The FF 3-, 4-, and 5- factor models can be accessed via Kenneth R. French Library
[5] The CIP information can be accessed via FRED data: Consumer Price Index ('CPI')
[6] The interest rates data can be accessed via FRED data: Interest rates

However, obtaining the higher-frequency data could reduce the dataset quality (Zhu, Wu, and Chen (2003)). Thus, although daily observations are the most commonly used in the empirical literature, I expand the research by testing the different levels of the frequency of the data used to calculate the standard deviation, applying the daily, weekly, and monthly returns. Identifying the optimal testing period is to find the most effective strategy and the factors that affect the performance. Finally, the average monthly excess returns, standard deviation, and Sharpe ratios of each decile portfolio are calculated for performance evaluation. Since the Sharpe ratio introduced by William F. Sharpe presents the portfolio's risk-adjusted returns, the optimal portfolio performance would be the highest Sharpe ratio. The highest Sharpe ratio could provide the optimal combination of the investment horizon and the data frequency that give the answer for Hypothesis 1.

Once the returns are established, I then regress these results on the Capital Asset Pricing Model ('CAPM'), Fama, and French ('FF') 3-, 4- and 5-factor models. The purpose of this regression is to find out whether the strategy is explained by the existing factors. Another target is to test the anomaly of the volatility strategy discussed in this paper and strengthen Hypothesis 1. The tests under consideration are standard t-tests. The CAPM, which was first introduced by Sharpe (1964) and Lintner (1965), is the asset pricing model that measures the sensitivity of the assets to systematic risks and predicts the positive correlation between the risk and expected returns. The CAPM model is presented as the following equation:

$$R_{i,t} = \alpha_i + \beta_i^{MKT}\left(R_t^M - R_t^f\right) + \epsilon_{i,t} \tag{2}$$

Whereas $R_{i,t}$ denotes return on stocks i in time t in excess on the risk-free rate. $R_t^M - R_t^f$ represents the excess returns for the markets in time t. Besides, β_t is the estimated factor exposures, and $\epsilon_{i,t}$ is the residual return on stock i in time t. Gutierrez and Pirinsky (2007) argued that there is no qualitative change in their results when they use either standardize the residual returns or not. Therefore, I do not use the standardized residual returns throughout my analysis.

Moreover, in 1993, Fama and French built upon the CAPM by adding two additional factors: Small-minus-Big ('SMB') and High-minus-Low ('HML') factor. In their research, the FF 3-factor model explains the relationship between risks and the expected returns better than the CAPM. They indicated that the SMB and HML using the difference between firms capture a new dimension of systematic risk that is not explained by the market risk factor. The following expression describes the FF 3-factors model in detail:

$$R_{i,t} = \alpha_i + \beta_i^{MKT}\left(R_t^M - R_t^f\right) + \beta_i^{SMB}(SMB_t) + \beta_i^{HML}(HML_t) + \epsilon_{i,t} \tag{3}$$

Using the assumption and fundamental hypothesis of the FF 3-factors model, Blitz and van Vliet (2007) introduced the CAPM with controlling by four factors. The additional factor that they used is the (residual) momentum.

$$R_{i,t} = \alpha_i + \beta_i^{MKT}\left(R_t^M - R_t^f\right) + \beta_i^{SMB}(SMB_t) + \beta_i^{HML}(HML_t) + \beta_i^{UMD}(UMD_t) + \epsilon_{i,t} \quad (4)$$

In 2015, Fama and French expanded their FF 3-factor model with two other factors: profitability ('RMW') and investment ('CMA') factor. The RMW factor is the difference in the firms' returns between the high and low operating profitability portfolios, whereas the CMA indicates the difference in future returns between conservative investment and aggressive investment strategies. The following equation displays the FF 5-factor model.

$$\begin{aligned} R_{i,t} = \alpha_i + \beta_i^{MKT}\left(R_t^M - R_t^f\right) + \beta_i^{SMB}(SMB_t) + \beta_i^{HML}(HML_t) \\ + \beta_i^{RMW}(RMW_t) + \beta_i^{CMA}(CMA_t) + \epsilon_{i,t} \end{aligned} \quad (5)$$

4.2.2 Macroeconomic factors

Moving to the macroeconomic factors testing, I separate the testing into two subparts with alternative approaches for the three factors: one is for the business cycle, and another is for the inflation and the interest rates. Besides, I use the optimal historical information, which is found in previous part, to test whether the macroeconomic factors affect the performance of the sorted decile portfolios. The chosen dataset period can interact with the historical period found in the previous part.

First, for the testing of the business cycle's influence on the volatility effect, I divide the chosen dataset period from 1963 to 2020 into sub-periods which have two types, one is the expansion periods which is characterized by the upward trend of the markets, and the others are the recession periods when all the economic activities decline significantly. The details of the business cycle from 1963 to 2020 are presented in Table 1.

The next step is to calculate the average monthly excess returns, using the optimal historical information, on each sub-periods to see how the performance and the volatility of asset prices change over time. Finally, statistical regression of the CAPM, FF 3-, 4-, and 5-factor model is constructed to evaluate the relationship between the trends of the business cycle and the volatility. Either the significant or insignificant Beta results of these regressions would contribute to the conclusion of this research. If the value is less than the significant level of 0.01, the volatility under each market trend cannot be controlled by the CAPM, FF 3-, 4-, and 5-factor. The result of portfolio performance through the contraction periods and expansion periods could support Hypothesis 2, that the fluctuation of stock prices is affected by the business cycle.

Table 1. Business cycle from 1963 to 2020 obtained from Fred Institution

Beginning of period	Ending of period	Type	Duration (month)
Jan 1963	Nov 1969	*Expansion*	82
Dec 1969	Nov 1970	*Recession*	11
Dec 1970	Oct 1973	*Expansion*	34
Nov 1973	Mar 1975	*Recession*	16
Apr 1975	Dec 1979	*Expansion*	56
Jan 1980	Jul 1980	*Recession*	6
Aug 1980	Jun 1981	*Expansion*	10
Jul 1981	Nov 1982	*Recession*	16
Dec 1982	Jun 1990	*Expansion*	90
Jul 1990	Mar 1991	*Recession*	8
Apr 1991	Feb 2001	*Expansion*	118
Mar 2001	Nov 2001	*Recession*	8
Dec 2001	Nov 2007	*Expansion*	71
Dec 2007	Jun 2009	*Recession*	18
Jul 2009	Jan 2020	*Expansion*	126
Feb 2020	(Dec 2020) On going	*Recession*	10

Note: The table displays the details of business cycle from 1963 to 2020, which includes the starting, ending and duration of each period. The chosen dataset period is divided into two-type sub-periods, which include the expansion period and the recession period. The duration of each period is ranging from 6 to 126, which include the longest expansion period is from Jul 2009 to Jan 2020. The last contraction period in the table during the year 2020 keeps going on until now, however, in this paper, I only research on the dataset from Jan 1963 to Jan 2020.

Second, for the inflation and interest rates' impact, I use the Ordinary Least Squares ('OLS') regression model to test whether the inflation and interest rates affect the asset prices and the volatility of stock prices.

The period is divided into sub-periods which follow the business cycle. The rationale for dividing into sub-period is that the risk-averse preferences vary through each market trend, changing inflation and interest rates. Thus, inflation and interest rates control the risk-aversion that impacts asset price volatility in the market. In this methodology, inflation and interest rates are the explanatory variables, while the volatility of asset prices is the dependent variable. Thus, the dependent variable is expressed as the following equation:

Inflation rate:

$$\sigma_{i,t} = \alpha_i + \beta_t(INF) + \epsilon_{i,t} \tag{6}$$

Interest rate:

$$\sigma_{i,t} = \alpha_i + \beta_t(INT) + \epsilon_{i,t} \tag{7}$$

Where $\sigma_{i,t}$ is the standard deviation of stock i in time t calculated by the optimal historical period, which is found in Hypothesis 1. β_t denotes the regression coefficient between the volatility and each macroeconomic variable. Finally, $\epsilon_{i,t}$ represents the expected errors and assumes that have normal distribution.

After determining the dependent variables, the OLS regression model is used to estimate the coefficients between the volatility of asset prices and the macroeconomic variables. The p-value tests the null hypothesis that there is no correlation between the dependent variables and independent variables. This Beta results determines whether the relationship between the volatility of asset prices and the macroeconomic variables is significant with the benchmark is 0.01. Besides, if the value of Beta is higher than 0.01, the volatility can be controlled by inflation rates or interest rates. Aside from the Betas, R-squared is the determination coefficient representing the relation between the independent and dependent variables. If the figures close to one, there is a strong correlation between variables, but if R-squared equals zero when regression, there is no explanatory power between those variables. Thus, the null hypothesis can be rejected. These findings would support Hypothesis 3 and Hypothesis 4 that the inflation and interest rates do not impact the relation between the volatility effect and risk-adjusted returns.

4.2.3 Firm characteristics

Using a time-series regression framework, I want to find the statistical significance of each firm characteristic on the relation between the volatility effect and the returns. There is much empirical literature that has been constructed these firm characteristics. In 1993, Fama and French indicated two construction: stocks with low market capitalization provide higher expected returns, and stocks with high book-to-market ratios yield high returns. Following the recent empirical article, there is a positive correlation between the expected returns and the dividend-to-price ratio and the strong coefficient of the expected returns and the earning-to-price ratio. Finally, stocks with high cash flow-to-price exhibit high expected returns.

A similar methodology of the K. F. Liababry is applied to compute the firm-specific variables:

- Market capitalization is calculated by using the outstanding of share times the stock prices.
- The market-to-book is constructed by using the book-to-market ratio divided by the market capitalization. The book-to-market ratio uses total assets minus total liabilities, plus the deferred taxes on the balance sheet, then minus the redemption of preferred stocks.
- The earning per share is calculated by the total earnings excluding extraordinary items, divided by market capitalization.
- Using the total dividend paid during the year divided by the market capitalization computes the dividend-to-price.
- To identify the cashflows-to-price, I first calculate the equity's share using the market capitalization divided by total assets minus the book-to-market ratio plus market capitalization. The next step is to combine the total earning excluding extraordinary items plus the equity's share.

To determine the influence of those firm characteristics on the volatility of asset prices and the expected returns, my methodology uses the OLS regression model to test the relation between the volatility and firm variables. This methodology is inspired by Akdal's (2011) paper. Thus, the dependent variable is expressed as the following equation:

$$\sigma_{i,t} = \alpha_i + \beta_{1,t}(SIZE) + \beta_{2,t}(MB) + \beta_{3,t}(DP) + \beta_{4,t}(EP) + \beta_{5,t}(CP) + \epsilon_{i,t} \tag{8}$$

Where $\sigma_{i,t}$ is the standard deviation of stock i in time t calculated by the optimal historical period, which is found in Hypothesis 1. β_t denotes the regression coefficient between the volatility and each firm characteristic variable. Finally, $\epsilon_{i,t}$ represents the expected errors and assumes that have normal distribution.

After determining the dependent variables, the OLS model is used to estimate the coefficients between the volatility of asset prices and each firm characteristic variable. The Beta results are the indicators to determine whether the relationship between volatility and each firm characteristic variable is significant, using the significance level of 0.01 as a benchmark. Finally, the figures of R-squared are considered to determine the explanatory power between the firm-specific variables and the volatility effect. This finding would make more plausible Hypothesis 5 of the firm characteristics exhibit a significant positive relation between the volatility strategy and the expected returns.

V. Results

This section presents the research results. Subsection 5.1 indicates the different excess returns, standard deviation, and Sharpe ratio of each alternative chosen period. Besides, this subsection provides the alphas of the univariate-sorted portfolios obtained from the CAPM, FF 3-, 4-, and 5-factors models. The impact of the business cycle on the results is reported in subsection 5.2. Subsection 5.3 provides the influence of inflation and interest rates on the volatility strategy. Finally, subsection 5.4 demonstrates the impact of firm characteristics on the correlation between volatility and expected returns.

5.1 The optimal historical data setting

5.1.1 The optimal data settings

Before proceeding to the analysis, I first found out which time horizon and interval time generate the portfolios' optimal performance. The combination provides fifteen pairwise comparisons between long- (three years and two years), medium- (one year), short- (three months and one month) time horizon and high- (daily), medium- (weekly), low- (monthly) data frequencies. The best pair is the pair that exhibits the highest Sharpe ratio of its decile portfolio, especially of Decile 1, if the low-volatility effect exists.

Appendix A.1 indicates summary statistics for the univariate portfolios sorted into deciles on each frequency data range and investment horizon. In this paper, decile 1 contains the smallest 10 percent low-volatility portfolio, while decile 10 comprises the largest volatility portfolio, and D1-D10 is the top-minus-bottom portfolio. The results are divided into three separate tables for each frequency data of returns. The excess returns, standard deviation, and Sharpe ratio are presented on the tables. After that, based on the Share ratio, I determined the highest and lowest portfolios' performance. The details are given in Table 2.

Overall, according to Table 2, the bottom-decile portfolios with the highest volatility level yield higher expected returns than the top decile portfolio among investment horizons. However, all the standard deviation of the decile ten portfolios is the largest compared to other portfolios and is confident at the 1% level. Therefore, the bottom portfolios provide a worse-adjusted risk-adjusted performance relative to other portfolios, especially to the Decile one portfolio. This finding is consistent with the view of Blitz and van Vliet's research in 2007.

It is noted that using weekly returns to determine the decile portfolios provide the lowest excess returns among three data frequency. In contrast, the standard deviation of those portfolios using weekly historical returns are lower than those using monthly returns and higher than those using daily returns, leading to the portfolio's performance is better than those using data frequency of monthly returns. Overall, the portfolios constructed by daily historical data yield the highest performance, followed by those using weekly returns, and the worst performance among the three chosen data frequencies is monthly returns. Sub-section 5.1.2 would perform the robustness test of this finding by regressing the portfolios by the CAPM, FF3-, FF4-, and FF5- factor models.

Table 2. The highest and lowest performance portfolio among different time windows and the frequency of data.

Time window	Portfolio	Excess returns	Standard deviation	Sharpe ratio
Panel A: Using daily returns				
3 years	D1	0.0068***	0.0123***	0.552846
	D10	0.0175***	0.0622***	0.281350
2 years	D1	0.0066***	0.0120***	0.550000
	D10	0.0193***	0.0631***	0.305864
1 year	D1	0.0072***	0.0116***	0.620690
	D10	0.0181***	0.0643***	0.281493
3 months	D1	0.0071***	0.0104***	0.682692
	D10	0.0158***	0.0660***	0.239394
1 month	D1	0.0071***	0.0088***	0.806818
	D10	0.0134***	0.0678***	0.197640
Panel B: Using weekly returns				
3 years	D1	0.0064***	0.0261***	0.245211
	D10	0.0175***	0.1246***	0.140449
2 years	D1	0.0065***	0.0254***	0.255906
	D10	0.0191***	0.1271***	0.150275
1 year	D1	0.0067***	0.0241***	0.278008
	D10	0.0186***	0.1303***	0.142748
3 months	D1	0.0069***	0.0194***	0.355670
	D10	0.0167***	0.1371***	0.121809
1 month	D1	0.0068***	0.0111***	0.612613
	D10	0.0186***	0.1462***	0.127223
Panel C: Using monthly returns				
3 years	D1	0.0066***	0.0497***	0.132797
	D10	0.0158***	0.2599***	0.060793
2 years	D1	0.0065***	0.0477***	0.136268
	D10	0.0164***	0.2653***	0.061817
1 year	D1	0.0073***	0.0429***	0.170163
	D10	0.0163***	0.2733***	0.059641
3 months	D1	0.0091***	0.0183***	0.497268
	D10	0.0126***	0.2979***	0.042296
1 month	D2	0.0088***	0.0777***	0.113256
	D10	0.0153***	0.2544***	0.060142

*t-statistics in parentheses: * p<0.10, ** p<0.05, *** p<0.01*
Note: The table shows the highest and lowest performance among portfolios formed by the standard deviation calculated by five different time windows and three of the frequency of data. D1 contains the smallest 10 percent of all stocks, while D10 comprises the largest stocks. All portfolios are equal-weighted. The dataset used is the US stock observations from 1963 to 2020. The monthly expected excess returns, standard deviation, and Sharpe ratio is presented in the table. This table is based upon the tables that can be found in Appendix A.1.

Panel A on Table 2 shows that using one-month daily returns to construct the univariate sorted portfolios yields the optimal portfolio's performance, followed by 3-months daily returns, which have a Sharpe ratio of 0.81 and 0.68, respectively, in the lowest volatility portfolios. Computing the decile portfolio by two-years daily returns demonstrates the worst performance with a Sharpe ratio of 0.55, even though a statistic is higher than that on the different frequency data range. Nevertheless, the higher the Sharpe ratio on the top decile portfolios, the lower the performance on the bottom decile portfolios.

The bottom decile portfolio, which is constructed by one-month daily returns, provides the least Sharpe ratio of 0.197 compared to other investment horizons and other decile portfolios.

From Panel B on Table 2, it can be seen that using a one-month investment horizon demonstrates the highest performance on the top decile portfolio, which has a Sharpe ratio of 0.61, while the three-year investment horizon exhibits the lowest performance, which is 0.24 on the low-risk portfolio (D1). According to Table 2, forming the decile portfolio by three-month weekly returns provides the Sharpe ratio of 0.122, the lowest on the weekly frequency data range. Moreover, the result from Table 2 does not support Hypothesis 1, where 0.27 is the Sharpe ratio of the lowest volatile portfolio computed by one-year weekly returns, less than approximately three times the highest expected returns formed by one-month daily returns. The portfolio underperforms other investment horizons in the same frequency data range, which are a quarter and one month.

Finally, in Panel C on Table 2, it is not surprising that computing the standard deviation by monthly returns on a monthly basis provides the worst performance compared to other investment horizons. The reason is that the data is not sufficient, as using only one observation to construct the standard deviation, forming the univariate-sorted portfolios.

5.1.2 Testing the control of the CAPM, FF 3-, 4- and 5-factor models

Appendix A.2 presents the regression coefficient between the expected returns and the CAPM, the FF 3-, 4-, and 5- factor models on the ten decile portfolios. The excess returns and the market beta are included in Appendix A.2. Table 3 contains the figure detail of the highest and lowest performance of those portfolios after controlling by the CAPM, FF3-, FF4-, and FF5- factor models.

As can be seen in Table 2 and Table 3, the excess returns after regressing by the CAPM, FF3-, and FF5- factor models among different time windows and the data frequencies are slightly below than the results before regressing, except for those portfolios controlling by the FF4- factor model. A similar pattern with the results in Table 2 has been found that the performance of those portfolios constructed by weekly returns is lower than that of other frequencies. The portfolios using daily returns yield the optimal performance.

Table 3 indicates that all the time windows of the data and the frequency of data range provide significant positive Alphas among the CAPM, FF 3-, 4- and 5- factor models at the confident level of 1%. The alphas are provided in the results as the excess returns in the top decile portfolios are lower than that in the bottom decile portfolios. On the other hand, most of the market Beta in those time windows and the frequency of data generate positive and significant results, except for all the lowest volatility portfolios constructed by the three-year and two-year historical data. It is surprising that the second decile portfolio (D2), which is the optimal performance for the portfolios constructed by one-month monthly returns, exhibits insignificant market betas, indicating that the portfolios are well diversified in relation to market risk. Thus, according to the results, it can be concluded that most of the alphas of the low volatility effect do exist, and it succeeds in explaining the anomalies.

Table 3. The highest and lowest performance controlling by CAPM, FF3-, FF4-, and FF5- factor model

Time window	Portfolio	CAPM - α	CAPM - β	FF3 - α	FF3 - β	FF4 - α	FF4 - β	FF5 - α	FF5 - β
Panel A: Using daily returns									
3 years	D1	0.0066***	0.0391	0.0067***	0.0483	0.0071***	0.0371	0.0068***	0.0410
	D10	0.0152***	0.4259***	0.0156***	0.3476***	0.0162***	0.3323***	0.0164***	0.3238***
2 years	D1	0.0063***	0.0448	0.0063***	0.0546*	0.0068***	0.0434	0.0064***	0.0487
	D10	0.0167***	0.4805***	0.0169***	0.3967***	0.0174***	0.3856***	0.0179***	0.3681***
1 year	D1	0.0069***	0.0634**	0.0069***	0.0685**	0.0074***	0.0581*	0.0070***	0.0617*
	D10	0.0155***	0.4619***	0.0158***	0.3930***	0.0162***	0.3833***	0.0167***	0.3636***
3 months	D1	0.0064***	0.1161***	0.0064***	0.1170***	0.0069***	0.1046***	0.0065***	0.1108***
	D10	0.0136***	0.4060***	0.0140***	0.3331***	0.0145***	0.3227***	0.0149***	0.3109***
1 month	D1	0.0064***	0.1277***	0.0064***	0.1207***	0.0069***	0.1103***	0.0067***	0.1095***
	D10	0.0112***	0.3983***	0.0118***	0.3165***	0.0119***	0.3153***	0.0123***	0.3132***
Panel B: Using weekly returns									
3 years	D1	0.0063***	0.0290	0.0063***	0.0404	0.0067***	0.0309	0.0065***	0.0349
	D10	0.0151***	0.4415***	0.0157***	0.3609***	0.0162***	0.3472***	0.0166***	0.3348***
2 years	D1	0.0063***	0.0372	0.0063***	0.0487*	0.0068***	0.0383	0.0064***	0.0442
	D10	0.0165***	0.4853***	0.0169***	0.4066***	0.0174***	0.3945***	0.0179***	0.3775***
1 year	D1	0.0063***	0.0581**	0.0063***	0.0650**	0.0067***	0.0553*	0.0064***	0.0575*
	D10	0.0160***	0.4537***	0.0167***	0.3662***	0.0171***	0.3574***	0.0178***	0.3332***
3 months	D1	0.0063***	0.1179***	0.0063***	0.1174***	0.0067***	0.1076***	0.0064***	0.1110***
	D10	0.0145***	0.3878***	0.0153***	0.3174***	0.0159***	0.3022***	0.0163***	0.2916***
1 month	D1	0.0059***	0.1480***	0.0058***	0.1440***	0.0064***	0.1307***	0.0061***	0.1327***
	D10	0.0166***	0.3663***	0.0172***	0.3145***	0.0179***	0.2975***	0.0180***	0.2955***
Panel C: Using monthly returns									
3 years	D1	0.0064***	0.0338	0.0064***	0.0455	0.0067***	0.0378	0.0065***	0.0418
	D10	0.0137***	0.3874***	0.0142***	0.3244***	0.0145***	0.3166***	0.0152***	0.2969***
2 years	D1	0.0062***	0.0549*	0.0062***	0.0657**	0.0066***	0.0558*	0.0063***	0.0582*
	D10	0.0142***	0.4038***	0.0145***	0.3432***	0.0148***	0.3368***	0.0157***	0.3085***
1 year	D1	0.0068***	0.0864***	0.0067***	0.0906***	0.0071***	0.0811***	0.0070***	0.0789**
	D10	0.0144***	0.3459***	0.0151***	0.2816***	0.0154***	0.2753***	0.0158***	0.2618***
3 months	D1	0.0079***	0.2000***	0.0080***	0.1939***	0.0090***	0.1701***	0.0083***	0.1810***
	D10	0.0112***	0.2589***	0.0119***	0.1958**	0.0120***	0.1938**	0.0126***	0.1779**
1 month	D2	0.0085***	0.0547	0.0086***	0.0539	0.0091***	0.0424	0.0089***	0.0399
	D10	0.0127***	0.4684***	0.0135***	0.3636***	0.0144***	0.3427***	0.0142***	0.3510***

*t-statistics in parentheses: * p<0.10, ** p<0.05, *** p<0.01*

Note: The table displays the results of the highest and lowest performance portfolio of each strategy's returns regressed on the CAPM, FF 3-factor model, FF 4-factor, and FF 5-factor models. D1 contains the smallest 10 percent of all stocks, while D10 comprises the largest stocks. All portfolios are equal-weighted. The dataset used is the US stock observations from 1963 to 2020. The excess returns based on the CAPM, FF3-, FF4- , and FF5- factors are included on the table. Besides, the market Beta of CAPM, FF3-, FF4-, and FF5- factor models are presented on the table. This table is based upon the tables that can be found in Appendix A.2.

From Panel A in Table 3, it can be shown that the low volatility effect exists among five-time horizons at the significant level of 1% based on the Alpha's result, which contain the three years, two years, one year, three months and one month period. The significant market risk of the portfolio constructed by the investment horizon of two years still appears at the 90% confidence interval after correcting by the FF 3-factor model; however, it disappears when regressing by the CAPM, FF4-, and FF5- factor models. The top decile portfolios, constructed by the time window of three years, provide insignificant systematic risk related to the market. All the highest-risk portfolios generate both higher Alphas and Betas than the lowest volatility portfolios. It is not surprising that the Beta of those portfolios constructed by time horizons of one month shows the lowest results of Beta than that of other time horizons, suggesting that the portfolios using one month to sort have less exposure to market risks than others.

As shown in Panel B and Panel C in Table 3, the results among portfolios constructed using weekly and monthly returns after controlling by the CAPM, FF3-, FF4- and FF5- factor models are the same as those portfolios determined by the frequency of daily returns, except for the highest performance of portfolio using one-month monthly returns. The Beta of that portfolio exhibits insignificant results after controlling by the CAPM, FF3-, FF4- and FF5- factor models, suggesting that the low-volatility strategy is well-diversified in relation to market risks that risk-aversion investors are interested in.

Moreover, Appendix A.3 shows the detail of alpha and beta coefficients for the top decile portfolios and bottom decile portfolios among different time horizons and frequencies. From Appendix A.3, all factors of size, value, momentum, investment, and profitability cannot explain the lowest volatile portfolios as the results provide insignificant Betas for all factors. In contrast, after controlling by the FF3-, and FF4- factor models, the SMB factor generates significant results at the significance level of 5% for the portfolios constructed by daily returns. Aside from SMB factors, the bottom decile portfolios cannot be controlled by other factors. Thus, the size effect has little impact on the portfolios, which are determined by daily returns. However, it does not affect other data frequencies. The finding supports the research result of Ang et al. (2006) that factors such as size, momentum, and book-to-market cannot influence the fluctuation of stock prices.

Consequently, from the results presented in Table 2 and Table 3, it can be concluded that the one-month daily return is the optimal setting to compute the volatility strategy. The reason can be explained the finding is that using longer time windows of data would exhibit less robust results than the short-term historical data because, during examining either sub-period of the business cycle or shorter period, it faces the risk of lack of data. Besides, the frequency of weekly and monthly observations might not provide enough data to exhibit the optimal strategy, while the daily data could capture the pattern of asset prices. Therefore, using other investment horizons and frequency of data ranges could affect the portfolio's performance and provide lower expected returns.

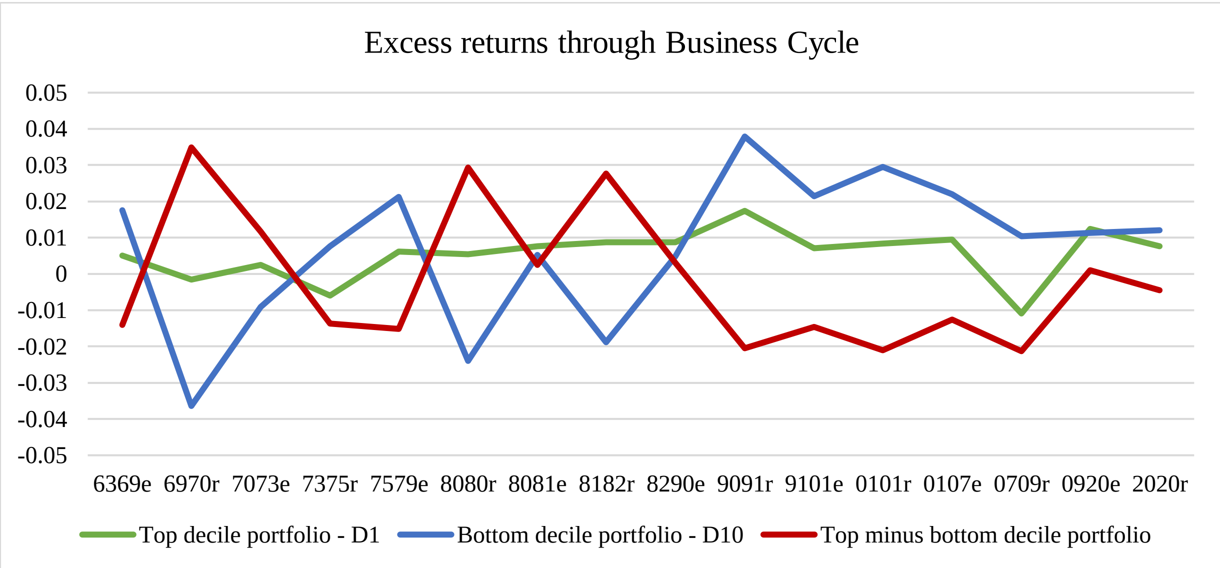
5.2 The portfolio's performance through business cycle

5.2.1 The portfolio's performance through each business cycle

This sub-section presents the portfolio's performance through the business cycle from January 1963 to December 2020 and the alphas and betas of the portfolios after regressing by the CAPM, the FF 3-, 4-, and 5-factor models. As mentioned in the Methodology section, to form the univariate-sorted portfolios based on the volatility strategy, I use the one-month daily returns to implement the standard deviation and divide the period into sub-periods of the business cycle which are the expansion and the contraction periods.

Graph 3 indicates the expected returns among the top decile portfolios, the bottom decile portfolio, and the top-minus-bottom decile portfolios through the expansion and recession periods from 1963 to 2020.

Graph 3. Excess returns through the business cycle

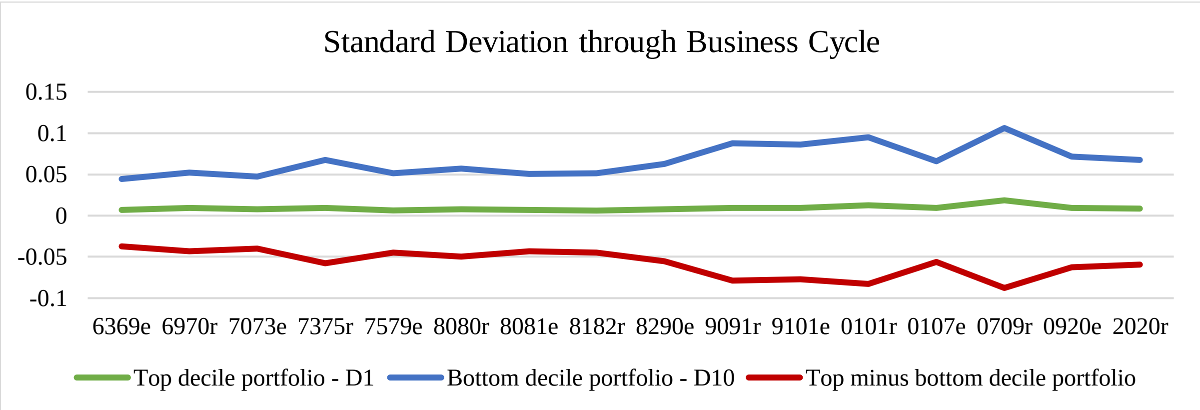


Note: The graph displays the monthly excess returns among the most-volatility portfolio, the lowest-volatility portfolio, and the top-minus-bottom portfolio from 1963 to 2020. The time series in this graph is divided into two types, the expansion periods and the recession periods. 6369e denotes the starting year of the expansion period is 1963 and the ending year of the expansion period is 1969. A similar approach is applied for other time horizons, where r represents the recession and e is the expansion.

As shown in Graph 3, the expected returns on the most volatile portfolio fluctuate significantly from 1963 to 2020, especially during the contraction periods. From the recession period 1990-1991, the bottom decile portfolio yields the highest expected returns. This finding is consistent with Blitz and Vliet's (2007)'s finding that relation between the low volatility anomaly and expected returns is weak. After the financial crisis, the expected returns have remained constant, with the results of excess returns around 0.01. However, the performance of the lowest volatile portfolio is more stable than the most volatile portfolio. The expected returns increase steadily in the expansion period and decrease slowly in the recession period from 1963 to 1979. In the next 30 years, an opposite pattern has been set up. As shown in Graph 3, the portfolio's performance constantly remained between expansion and contraction between 1979 and 1990, before decreasing in the expansion period and increasing in the contraction period between 1990 and 2007. The reason behind this trend could be explained by globalization and international risk sharing. Globalization emphasizes the international market integration where countries increase trading over the world. The second wave of international trading was expansion and developed after World War II, according to Ospina and Max Roser (2018). One of the benefits of globalization is risk diversification. In 2008, Artis and Hoffmann showed that country-specific factors are understated because of international risk sharing. Following data summarized by the NBER website[7], the recession periods from 1980 to the beginning of the 2000s are the USA's short recession and affect mainly the US market, while the global recession was reported four times over the past seven decades, which is in 1975, 1982, 1991, and 2009 (Kose et al. (2020)). Therefore, during the contraction periods between 1980 and 2001, the systematic risk of the US market was shared with other countries and did not affect much on the portfolio's performance, leading to the positive portfolio's performance. However, the Great Recession in 2009 was the most profound crisis, which led the USA's Gross Domestic Product ('GDP') to decline approximately 5.1%, from 4.5% at the end of 2007 to -0.81% in 2008, reported by Fred Websites[8]. That was the largest decline after the Great Depression in the period 1929-1930. That caused the worse performance in the contraction period 2007-2009 on the top decile portfolios.

Graph 4 demonstrates the volatilities of asset prices through the business cycle among the least volatile portfolio, the most volatile portfolio, and the top-minus-bottom portfolio.

Graph 4. Volatilities through the business cycle



Note: The graph displays the volatilities of asset prices among the most-volatility portfolio, the lowest-volatility portfolio, and the top-minus-bottom portfolio from 1963 to 2020. The time series in this graph is divided into two types, the expansion periods and the recession periods. 6369e denotes the starting year of the expansion period is 1963 and the ending year of the expansion period is 1969. A similar approach is applied for other time horizons, where r represents the recession and e is the expansion.

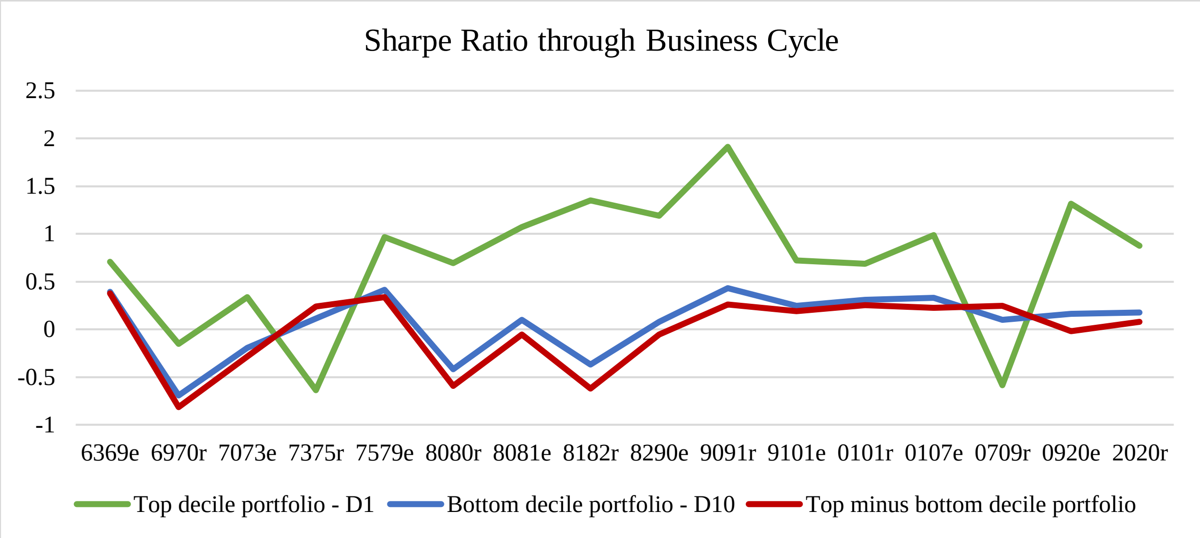
It can be seen in Graph 4 that the standard deviation of the least volatile portfolio - D1 is stable and nearly equals zero during the period, except for increasing slightly in the recession period 2007-2009. On the other hand, the volatilities of the D10 portfolio fluctuate between 1963 and 2020. The volatilities of the portfolios increase during the contraction periods compared to the expansion periods.

---

[7] The contraction period of the US market can be accessed via: List of recession period in the US
[8] The USA's GDP changed can be accessed via GDP of the US from 1963 to 2020

The portfolio's performance through the business cycle is exhibited in Graph 5.

Graph 5. Sharpe Ratio through the business cycle



Note: The graph displays the portfolio's performance of the most-volatility portfolio, the lowest-volatility portfolio, and the top-minus-bottom portfolio from 1963 to 2020. The time series in this graph is divided into two types, the expansion periods and the recession periods. 6369e denotes the starting year of the expansion period is 1963 and the ending year of the expansion period is 1969. A similar approach is applied for other time horizons, where r represents the recession and e is the expansion.

According to Graph 5, it can be concluded that the performance of the lowest volatile portfolios is better than that of the most volatile portfolios, except for two recession periods: 1973-1975 and 2007-2009 period, which are the periods that presented the highest volatility of stock prices on both portfolios. The top decile portfolio overperformed the bottom decile portfolio during the chosen period, with the Sharpe ratio ranging from approximately -0.5 to nearly 2. Moreover, the pattern of performance of the least volatile portfolio repeats among sub-periods, that decreases in the contraction periods and increases in the expansion periods, except for the recession period 1990-1991. The results on most of the sub-periods are higher than that of the period 1963-2020, with a Sharpe ratio of around 0.68 presented in Table 2. This result is in line with the finding of Blitz and Vliet (2007), which is that the performance of the low volatility portfolio provides significant positive returns because it can be offset the performance between the uptrend and the downtrend markets.

However, it is surprising that the performance of the bottom decile portfolio is better than expected, especially in the recession period 1990-1991. The Sharpe ratio of the highest volatility portfolio ranges between around -0.6 and 0.5, which yields a positive performance. Only four out of sixteen sub-period of the highest risk portfolios generate negative Sharpe ratio, all in the contraction periods. It is surprising that in the financial crisis between 2007 and 2009, the bottom decile portfolio provides a positive portfolio's performance, which is higher than the performance of the least volatile portfolios.

In conclusion, the top decile portfolio does not overperform the bottom decile portfolio in terms of excess returns but overperforms in term of portfolio's performance. The results indicate that the business cycle significantly impact the coefficient between the excess returns and the volatilities. The performance exhibit significant positive returns when the volatilities are low during the expansion period, and the performance yields statically significant negative returns when the volatilities are high in the contraction periods, leading to the fluctuation of the portfolio's performance during the business cycle. This finding makes more plausible Hypothesis 2 of the business cycle affect the magnitude of the expected returns and the volatility effect during the period 1963-2020.

5.2.1 Testing the control of the CAPM, FF 3-, 4- and 5-factor models

Appendix A.4 provides the alpha and beta coefficients of the decile portfolios after regressing by the CAPM, FF 3-, FF4-, and FF5-factor models through the business cycle for the period 1963-2020. The business cycle contains two types of periods: the expansion periods and the contraction period.

Table 4 contains the figure detail of the highest and lowest performance of those portfolios after controlling by the CAPM, FF3-, FF4-, and FF5- factor models through the business cycle.

As shown in Table 4, from 1963 to 1981, neither alphas nor beta coefficients of the top decile portfolios and bottom decile portfolios provide significant results. Therefore, the low-volatility anomaly does not appear in this period. Although market risks generate significant positive betas for the least volatile portfolio between 1981 and 1982 at a significant level of 1%, the volatility strategy provides insignificant alphas, indicating that the portfolio returns can be explained by the CAPM, FF3-, FF4-, and FF5- factor models. From the beginning of the 1990s, the top decile portfolios report significant risk-adjusted alphas at the confidence level of 1%. In the 2001-2007 and 2020-2020, the low-volatility effect strongly appears and can explain the anomalies since the beta and alpha coefficients provide significant positive results at a 99% confidence interval.

Table 4. The highest and lowest portfolio's performance through Business cycle controlling by the CAPM, FF3-, FF4-, and FF5- factor models

Sub-periods	Type	Portfolio	CAPM - α	CAPM - β	FF3 - α	FF3 - β	FF4 - α	FF4 - β	FF5 - α	FF5 - β
1963-1969	Expansion	D1	0.0048	0.0444	0.0042	0.0053	0.0040	0.0041	0.0037	0.0137
		D10	0.0150	0.5469*	0.0149	0.5058	0.0120	0.4836	0.0116	0.4677
1969-1970	Recession	D1	0.0023	0.2470	0.0061	0.1344	0.0101	0.2775	-0.0065	0.3369
		D10	-0.0263	0.6527	-0.0273	0.6363	-0.0256	0.6971	-0.0451	0.7563
1970-1973	Expansion	D1	0.0020	0.0987	0.0022	0.0943	0.0019	0.1076	-0.0003	0.1122
		D10	-0.0120	0.6026	-0.0120	0.6080	-0.0133	0.6610	-0.0147	0.6392
1973-1975	Recession	D1	-0.0037	0.1899	-0.0117	0.6100*	-0.0100	0.4800	-0.0326	0.8658
		D10	0.0086	0.0789	0.0002	0.8563	0.0031	0.6362	-0.0352	1.2628
1975-1979	Expansion	D1	0.0060	0.0197	0.0071	0.0195	0.0067	0.0099	0.0094*	0.0049
		D10	0.0194	0.3288	0.0198	0.3764	0.0182	0.3392	0.0194	0.3793
1980-1980	Recession	D1	0.0056	-0.0281	0.1020	-8.9380	0.0494	-2.8794	0.0075	0.9504
		D10	-0.0240	0.0108	0.2168	-23.8252	0.1464	-15.7132	0.1156	-12.3333
1980-1981	Expansion	D1	0.0083	-0.1077	0.0068	0.4381	0.0073	0.4860	0.0069	0.4445
		D10	0.0058	-0.0959	-0.0112	-0.2151	-0.0050	0.4784	-0.0076	-0.2405
1981-1982	Recession	D1	0.0105	0.5272***	0.0054	0.6900**	-0.0073	0.8125***	-0.0033	0.8728**
		D10	-0.0173	0.4720	-0.0299	0.8652*	-0.0499**	1.0588**	-0.0720**	1.5626***
1982-1990	Expansion	D1	0.0081**	0.0921	0.0096**	0.0340	0.0091**	0.0225	0.0124***	0.0133
		D10	0.0015	0.4306***	0.0035	0.3704**	0.0037	0.3757**	0.0072	0.3465*
1990-1991	Recession	D1	0.0159**	0.4344***	0.0206*	0.5051	0.0212	0.4478	0.0191	0.4535
		D10	0.0334	1.2507*	0.0554	1.6764	0.0630	0.9923	0.0157	0.6987
1991-2001	Expansion	D1	0.0069***	0.0095	0.0051**	0.0966	0.0059**	0.1037	0.0053**	0.0887
		D10	0.0200*	0.1598	0.0204*	0.1160	0.0160	0.0754	0.0170	0.3902
2001-2001	Recession	D1	0.0154	0.3510	0.0198	0.3874	0.0202	0.3481	0.0334**	-0.2569
		D10	0.0266	-0.1448	0.0399	-0.4550	0.0520	-1.8474	0.1296	-3.3116*
2001-2007	Expansion	D1	0.0081***	0.2518***	0.0077**	0.2765***	0.0080**	0.2445**	0.0075**	0.2899**
		D10	0.0188*	0.6218*	0.0226**	0.6453*	0.0230**	0.5989	0.0216*	0.7256
2007-2009	Recession	D1	-0.0030	0.3524*	0.0023	0.5138**	0.0029	0.5254*	-0.0031	0.5318
		D10	0.0364	1.1621**	0.0447	1.4819**	0.0334	1.2564	0.0453	1.0957
2009-2020	Expansion	D1	0.0141***	-0.1278*	0.0141***	-0.1277*	0.0144***	-0.1432*	0.0140***	-0.1246*
		D10	0.0115	-0.0061	0.0105	0.0902	0.0111	0.0598	0.0110	0.0812
2020-2020	Recession	D1	0.0068***	0.1440***	0.0069***	0.1365***	0.0073***	0.1256***	0.0072***	0.1240***
		D10	0.0100***	0.3621***	0.0106***	0.2957***	0.0111***	0.2837***	0.0108***	0.2967***

*t-statistics in parentheses: * p<0.10, ** p<0.05, *** p<0.01*

Note: The table displays the results of the highest and the lowest performance of each strategy's returns through the business cycle regressed on the CAPM, FF 3-factor model, FF 4-factor, and FF 5-factor models. D1 contains the least volatile portfolio and D10 is the most volatile portfolio. All portfolios are equal-weighted. The dataset used is the US stock observations from 1963 to 2020. The excess returns based on the CAPM, FF3-, FF4- , and FF5- factors are included on the table. Besides, the market Beta of CAPM, FF3-, FF4-, and FF5- factor models are presented on the table. The period is divided into sub-periods. There are two types of sub-periods, which include the expansion period and the recession period. All portfolios are zero-investment top-minus-bottom decile portfolios. The numbers above are monthly returns. RMRF presents the excess returns for the market. SMB denotes the Small-Minus-Big factor, while HML is the High-Minus-Low factor of the asset pricing model. Momentum factor is displayed by UMD. RMW and CMA present the profitability and investment factor respectively. This table is based upon the tables that can be found in Appendix A.4.

5.3 The impact of inflation and interest rates on the relation between expected returns and the low-volatility effect

As mentioned in the Methodology section, the next step in this paper is to check the relation between inflation and interest rates with the volatility effect in the chosen dataset period and the sub-periods following the business cycle.

5.3.1 Inflation

Table 5 represents the regression coefficients of inflation and the volatility effect from 1963 to 2020. Again, the period is divided into sub-periods through the business cycle.

During the period 1963-2020, either alphas or betas of both the expected returns and the volatility effect provide significant and positive results at the significance level of 1%, although the inflation constantly increases (as shown in Graph 1). Besides, the R squared in this period for regression on expected returns and the volatility effect is 0.0098% and 1.16%, respectively. That means neither the expected excess return nor the volatility effect is dependent on inflation.

As can be seen in Table 5, the alphas of the expected returns in the contraction period are less stable than those in the expansion periods. Five out of eight periods yield significant negative alphas, while the alphas coefficients generate significant positive results during the expansion periods. The highest alpha coefficients of the expansion period are in the 2001-2007 expansion period, which is higher around 18 times than Alphas of excess returns in the period 1963-2020. The results provide an insignificant positive alpha in the 1980-1981 expansion period and an insignificant negative beta in the same period. However, in the 2000s, the alphas of recession periods outperform those in the expansion periods, including the recession period 2001-2001 with the significant highest alphas of 19.14.

For the standard deviation of the portfolios, the low-volatility strategy yields significant alphas; two out of eight periods are negative results at a 99% confidence interval during the expansion periods. In the contraction periods, except for two periods that provide insignificant results, other recession periods exhibit significant alphas at the significance level of 1%

Indeed, the portfolios cannot be explained by the inflation data. The low-volatility strategy does exist and can explain its anomalies. This finding not only supports Hypothesis 3 but also is consistent with the view of Schwert in 1989 that inflation does not closely relate to the volatility of asset prices and business cycle influence on the volatility, especially in the recession periods.

Table 5. The alpha and beta results of OLS regression model between the inflation and the volatility effect, and the excess returns

					Expansion sub-periods				
	1963-2020	**1963-1969**	**1970-1973**	**1975-1979**	**1980-1981**	**1982-1990**	**1991-2001**	**2001-2007**	**2009-2020**
Panel A: Excess returns with controlling by inflation									
Intercept	0.0066***	0.0911***	0.1107***	0.0293***	0.1245	0.0254***	0.0394***	0.1237***	0.1279***
	(10.00)	(7.77)	(4.01)	(4.00)	(1.29)	(3.52)	(5.34)	(13.20)	(19.85)
Inflation	0.0001***	-0.0059***	-0.0063***	-0.0006**	-0.0030	-0.0003**	-0.0004***	-0.0014***	-0.0012***
	(8.18)	(-7.01)	(-4.11)	(-2.02)	(-1.17)	(-2.31)	(-3.71)	(-11.79)	(-18.29)
Panel B: Standard deviation with controlling by inflation									
Intercept	0.0208***	0.0007	-0.0211***	0.0269***	0.0722***	0.0091***	-0.0301***	0.1007***	0.0098***
	(205.10)	(0.45)	(-6.21)	(28.85)	(7.09)	(8.37)	(-27.08)	(77.97)	(9.64)
Inflation	0.0001***	0.0012***	0.0023***	-0.0003***	-0.0014***	0.0003***	0.0010***	-0.0009***	0.0002***
	(89.52)	(11.47)	(12.22)	(-7.20)	(-4.95)	(13.87)	(56.90)	(-57.35)	(17.74)
					Recession sub-periods				
		1969-1970	**1973-1975**	**1980-1980**	**1981-1982**	**1990-1991**	**2001-2001**	**2007-2009**	**2020-2020**
Panel C: Excess returns with controlling by inflation									
Intercept		-1.6705***	-0.6610***	-3.4183***	-1.6666***	-9.8642***	19.1431***	3.2113***	0.0066***
		(-10.12)	(-12.62)	(-23.40)	(-24.11)	(-26.89)	(26.29)	(26.50)	(10.01)
Inflation		0.1015***	0.0315***	0.1000***	0.0415***	0.1750***	-0.2555***	-0.0355***	0.0001***
		(10.06)	(12.68)	(23.45)	(24.18)	(26.97)	(-26.26)	(-26.54)	(8.18)
Panel D: Standard deviation with controlling by inflation									
Intercept		-0.0422**	-0.0207***	0.1185***	-0.0130	0.0218	1.0060***	0.8916***	0.0208***
		(-2.08)	(-3.33)	(5.95)	(-1.60)	(0.36)	(9.92)	(52.76)	(205.08)
Inflation		0.0040***	0.0023***	-0.0028***	0.0009***	0.0002	-0.0129***	-0.0093***	0.0001***
		(3.25)	(7.94)	(-4.77)	(4.25)	(0.18)	(-9.54)	(-49.99)	(89.54)
R^2 on the excess returns from 1963 to 2020				.0000983					
R^2 on the standard deviation from 1963 to 2020				.01162					

*t-statistics in parentheses: * p<0.10, ** p<0.05, *** p<0.01*

Note: The table displays coefficient results of the OLS regression model between the inflation and the expected returns, as well as the standard deviation of the US stock market from Jan 1963 to Dec 2020. The period is divided into sub-periods. There are two types of sub-periods, which include the expansion period and the recession period. The OLS regression equation is $\sigma_{i,t} = \alpha_i + \beta_t(INF) + \epsilon_{i,t}$.

Table 6. The alpha and beta results of OLS regression model between the interest rates and the volatility effect, and the excess returns

					Expansion sub-periods				
	1963-2020	**1963-1969**	**1970-1973**	**1975-1979**	**1980-1981**	**1982-1990**	**1991-2001**	**2001-2007**	**2009-2020**
Panel A: Excess returns with controlling by interest rates									
Intercept	0.0164***	0.0363***	-0.0066*	0.0259***	0.1310***	0.0275***	0.0260***	0.0251***	0.0145***
	(56.84)	(11.95)	(-1.71)	(9.65)	(5.92)	(8.86)	(8.97)	(25.96)	(35.69)
Interest rates	-0.0016***	-0.0059***	0.0007	-0.0017***	-0.0083***	-0.0025***	-0.0029***	-0.0042***	-0.0062***
	(-22.95)	(-9.23)	(1.07)	(-4.41)	(-5.39)	(-6.14)	(-4.88)	(-13.90)	(-16.50)
Panel B: Standard deviation with controlling by interest rates									
Intercept	0.0318***	0.0132***	0.0140***	0.0197***	0.0255***	0.0395***	0.0237***	0.0324***	0.0264***
	(716.34)	(33.99)	(29.37)	(57.64)	(10.83)	(85.89)	(53.56)	(241.90)	(415.30)
Interest rates	-0.0008***	0.0010***	0.0012***	0.0001	-0.0003	-0.0021***	0.0019***	-0.0020***	0.0018***
	(-76.08)	(12.35)	(14.09)	(1.62)	(-1.57)	(-34.31)	(21.32)	(-48.52)	(31.09)
					Recession sub-periods				
		1969-1970	**1973-1975**	**1980-1980**	**1981-1982**	**1990-1991**	**2001-2001**	**2007-2009**	**2020-2020**
Panel C: Excess returns with controlling by interest rates									
Intercept		0.1729***	0.4436***	0.2078***	0.2165***	0.6715***	0.0535***	0.0254***	0.0164***
		(7.76)	(28.86)	(24.89)	(31.67)	(25.25)	(5.86)	(11.14)	(56.84)
Interest rates		-0.0278***	-0.0590***	-0.0182***	-0.0181***	-0.0962***	-0.0108***	-0.0320***	-0.0016***
		(-8.26)	(-28.95)	(-25.15)	(-31.63)	(-24.17)	(-3.77)	(-16.75)	(-22.94)
Panel D: Standard deviation with controlling by interest rates									
Intercept		0.0241***	0.0477***	0.0174***	0.0275***	0.0374***	0.0456***	0.0626***	0.0318***
		(8.85)	(24.90)	(15.15)	(33.40)	(8.67)	(36.96)	(204.36)	(716.34)
Interest rates		-0.0001	-0.0026***	0.0005***	-0.0005***	-0.0007	-0.0022***	-0.0170***	-0.0008***
		(-0.14)	(-10.10)	(5.48)	(-7.39)	(-1.13)	(-5.65)	(-66.09)	(-76.10)
R^2 on the excess returns from 1963 to 2020				.0007726					
R^2 on the standard deviation from 1963 to 2020				.008424					

*t-statistics in parentheses: * p<0.10, ** p<0.05, *** p<0.01*

Note: The table displays coefficient results of the OLS regression model between the interest rates and the expected returns, as well as the standard deviation of the US stock market from Jan 1963 to Dec 2020. The period is divided into sub-periods. There are two types of sub-periods, which include the expansion period and the recession period. The OLS regression equation is $\sigma_{i,t} = \alpha_i + \beta_t(INT) + \epsilon_{i,t}$.

5.3.2 Interest rates

The regression results between the interest rates and the volatility effect from 1963 to 2020 are provided in Table 6. In this process, the impact on sub-periods is also examined since changing interest rates could influence the risk-averse preferences, impacting asset price volatilities in the market.

From Table 6, the performance of alphas after controlling by the interest rates is better than that controlling by inflation for the long-portfolio positions. The alphas coefficient for the portfolios' excess returns exhibit the figures of 0.0164 and 0.0066, respectively, and for the portfolio's volatilities are 0.0318 and 0.0208, respectively. Smart betas, however, perform worse than those betas controlling by inflation. The results provide significant negative alphas on both the expected returns and standard deviation with the confidence level of 1%. As the results of R squared are very low and nearly equal to zero in Table 6, the interest rates do not impact either expected returns or the standard deviation of asset prices during the period 1963-2020.

From Panel A and Panel C in Table 6, except for the excess returns after controlling by the interest rates in the expansion 1970-1973 period provide a significant negative alpha at a 90% confidence interval, the excess returns among other expansion periods and the contraction period yield significant and positive alphas with the significance level of 1%. The portfolio's excess returns after regressing by the interest rates among sub-periods range from -0.006 to 0.67, which is narrower than ranges of excess returns controlled by the inflation between -9.8 and 19.14.

For the portfolio's standard deviation controlled by the interest rates, even though the interest rates fluctuate drastically during the period 1963-2020, the results provide either significant alphas or betas coefficients, which are not significantly different from zero at a significant level of 1%, ranging from 0.01 to 0.04 among sub-periods.

Thus, the relation between the low volatility and the expected returns cannot be explained by the interest rates during 1963-2020. This finding is similar to the inflation's conclusion, although the interest rates and the inflation have an opposite sign. The result strengthens Hypothesis 4 and supports Schwert's view (1989), in which interest rates do not affect the volatility of asset prices.

5.4 The impact of firm characteristics on the low-volatility anomaly

Finally, firm characteristics are the last factors considered in this research. Table 7 shows the regression results between the firm-specific variables and the volatility effect from 1963 to 2020.

As shown in Panel A in Table 7, most of the alphas and betas coefficients are significant at a 99% confidence interval, except for the portfolio regressing by the dividend-to-price ratio. The alphas and betas coefficients for the portfolios' excess returns after controlling by the dividend-to-price ratio yield insignificant results. Moreover, the alphas of the portfolio's excess returns and the portfolio's standard deviation after regressing by the cashflow-to-price ratio are significantly negative at a 99% confidence interval, -4.3 and -3.5, respectively. Besides, the figure of R squared, ranging from 0.0004 to 0.01, exhibits the non-correlation between those firm characteristics and the expected returns.

Table 7. The alpha and beta results of OLS regression model between the firm characteristics and the volatility effect, and the excess returns

			1963-2020		
	Market cap	**Book-to-market**	**Dividend-to-Price**	**Earnings-to-price**	**Cashflows-to-price**
Panel A: Excess returns with controlling by firm characteristics					
Intercept	0.0292***	1.3554***	0.0033	-0.0000	-4.3905***
	(35.16)	(4.78)	(0.73)	(-0.93)	(-3.21)
Firm characteristics	-0.0000***	0.0256***	0.0214	0.0266***	0.0272***
	(-4.57)	(23.35)	(1.08)	(25.51)	(25.12)
R^2	.0004107	.0007526	.01004	.0000271	.0003414
Panel B: Standard deviation with controlling by firm characteristics					
Intercept	0.0299***	0.2631***	-0.0008	-0.0000***	-3.5010***
	(245.93)	(6.38)	(-1.41)	(-18.27)	(-17.73)
Firm characteristics	-0.0000***	0.0287***	0.0222***	0.0290***	0.0292***
	(-22.54)	(180.01)	(9.19)	(192.69)	(187.20)
R^2	.009878	.00134	.03622	.01039	.01032

*t-statistics in parentheses: * p<0.10, ** p<0.05, *** p<0.01*

Note: The table displays coefficient results of the OLS regression model between the firm characteristics and the expected returns, as well as the standard deviation of the US stock market from Jan 1963 to Dec 2020. The OLS regression equation applied is $\sigma_{i,t} = \alpha_i + \beta_{1,t}(SIZE) + \beta_{2,t}(MB) + \beta_{3,t}(DP) + \beta_{4,t}(EP) + \beta_{5,t}(CP) + \epsilon_{i,t}$.

It can be seen from Panel B in Table 7, after regression by the dividend-to-price ratio, the result of the volatility effect provides a significant negative alpha. At the same time, the beta coefficients are significant positive results with the confidence level of 1%, indicating that the dividend-to-price could impact the volatility of asset prices. There is a reason that could explain this finding. In this paper, the actual dividend paid is used to determine the relation between firm-specific variables and the volatility effect, while this figure directly influences investors' behavior, especially irrational investors.

In conclusion, the firm characteristics do not impact the magnitude of the volatility and expected returns, except for the dividend-to-price ratio. This finding is not similar to Hypothesis 5, but again, it supports partly the finding of Schwert in 1989 that only the business cycle could impact the relation between expected returns and the volatility strategy.

VI. Conclusion

In this paper, combining five investment horizons, which are long- (three years and two years), medium- (one year and three months), and short- (one month), with three different frequency data, consisted of high- (daily), medium- (weekly), and low- (monthly) frequencies, ranges provides the optimal data setting to construct the best portfolio's performance based on the low-volatility strategy. Furthermore, the portfolio's performance through the business cycle on the US market is considered in this research to determine the effect of expansion and contraction on the performance of the low-volatility effect. Afterward, the influence of macroeconomic factors, inflation and interest rates, and control variables, such as firm characteristics, on the magnitude between expected returns and the volatility effect is examined.

In line with Blitz and Vliet (2007), I find that the expected returns on the top decile portfolios could be lower than the bottom decile portfolios. Besides, the low-volatility effect is weak if only considering the expected returns. However, computing the Sharpe ratio of those decile portfolios provides the low-volatility effect's efficiency. The decile portfolios determined by the daily historical data, yield the highest portfolio performance on the top decile portfolios. In contrast, the most volatile portfolios constructed by the same frequency of data exhibit the lowest results compared to monthly and weekly historical data. Moreover, the shorter the investment horizons are, the better the performance is. The reason is that using the longer term of the data's time window could exhibit less robust results due to the lack of data in examining in shorter periods.

The business cycle strongly impacts the relation between expected returns and low-volatility anomaly in the period 1963-2020, especially in the recession period. The performance of the portfolios fluctuates drastically through the business cycle, which increases during the expansion periods and decreases during the contraction period. Globalization and international risk sharing could explain the changes in the expected returns of the low-volatility and the high-volatility portfolios during the expansion and contraction periods. On the other hand, the volatility on asset prices does not change much from Jan 1963 to Dec 2020 on the least volatile portfolio.

In addition, I find that macroeconomic variables, which are inflation and interest rates, and firm characteristics do not impact the magnitude of the expected returns and the volatility effect, except for the dividend-to-price ratio. During the chosen period, although the inflation and interest rates fluctuated through the business cycle, which increases during the recession periods and decreases during the expansion periods, the stock prices grew constantly. Therefore, those macroeconomic variables do not influence the stock prices' volatility. These findings are consistent with the view of Schwert in 1989 that only the business cycle could impact the volatility of asset prices.

With regards to the research, some limitations could limit the results of this paper. One of these issues is that I only use three macroeconomic variables and five firm characteristics. Other macroeconomic variables and firm characteristics could affect the expected returns and volatility strategies, such as the unemployment rate, gross domestic products, or gross national products. Besides, according to Kogan and Tian (2012), there are 27 commonly firm characteristics, as those variables could impact the conclusion of this research.

Another limitation is that I use the annual firm-specific variables data to examine the relation between the expected returns and volatility effect on a monthly basis. This process reduces the number of observations, which can influence the final results.

There are several avenues for further research that can improve my analysis. One is to consider the effect of globalization and international risk sharing on the relation between expected returns and the volatility effect through the business cycle, especially in the contraction period. This research can create an impact on the asset price volatility in different market trends. Lastly, following the above limitations, an expansion of the macroeconomic variables and firm-specific variables data should be considered to provide more robust results.

References:

Akdal, S. (2011). How do firm characteristics affect capital structure? Some UK evidence. *Some UK Evidence (March 3, 2011)*.

Aït-Sahalia, Y., & Yu, J. (2008). *High frequency market microstructure noise estimates and liquidity measures* (No. w13825). National Bureau of Economic Research.

Ang, A., Hodrick, R. J., Xing, Y., & Zhang, X. (2006). The cross‐section of volatility and expected returns. *The Journal of Finance*, *61*(1), 259-299.

Artis, M. J., & Hoffmann, M. (2008). Financial globalization, international business cycles and consumption risk sharing. *Scandinavian Journal of Economics*, *110*(3), 447-471.

Bali, T. G., & Cakici, N. (2008). Idiosyncratic volatility and the cross section of expected returns. *Journal of Financial and Quantitative Analysis*, *43*(1), 29-58.

Barr, D. G., & Campbell, J. Y. (1997). Inflation, real interest rates, and the bond market: A study of UK nominal and index-linked government bond prices. *Journal of Monetary Economics*, *39*(3), 361-383.

Bender, J., Briand, R., Melas, D., & Subramanian, R. A. (2013). Foundations of factor investing. *Available at SSRN 2543990*.

Blitz, D. C., & Van Vliet, P. (2007). The volatility effect. *The Journal of Portfolio Management*, *34*(1), 102-113.

Blitz, D., Huij, J., & Martens, M. (2011). Residual momentum. *Journal of Empirical Finance*, *18*(3), 506-521.

Blitz, D., Pang, J., & Van Vliet, P. (2013). The volatility effect in emerging markets. *Emerging Markets Review*, *16*, 31-45.

Blitz, D., Falkenstein, E., & Van Vliet, P. (2014). Explanations for the volatility effect: An overview based on the CAPM assumptions. *The Journal of Portfolio Management*, *40*(3), 61-76.

Brandt, M. W., & Wang, K. Q. (2003). Time-varying risk aversion and unexpected inflation. *Journal of Monetary Economics*, *50*(7), 1457-1498.

Chen, Z., Cooper, I., Ehling, P., & Xiouros, C. (2020). Risk aversion sensitive real business cycles. *Management Science*.

Christie, A. A. (1982). The stochastic behavior of common stock variances: Value, leverage and interest rate effects. *Journal of financial Economics*, *10*(4), 407-432.

Engle, R. F., & Patton, A. J. (2007). What good is a volatility model?. In *Forecasting volatility in the financial markets* (pp. 47-63). Butterworth-Heinemann.

Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of financial economics*, *33*(1), 3-56.

Fama, E. F., & French, K. R. (2015). A five-factor asset pricing model. *Journal of financial economics*, *116*(1), 1-22.

Frazzini, A., & Pedersen, L. H. (2014). Betting against beta. *Journal of Financial Economics*, *111*(1), 1-25.

Kose, M. A., Sugawara, N., & Terrones, M. E. (2020). Global recessions.

Gutierrez Jr, R. C., & Prinsky, C. A. (2007). Momentum, reversal, and the trading behaviors of institutions. *Journal of Financial Markets*, *10*(1), 48-75.

Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica: Journal of the econometric society*, 357-384.

Hamilton, J. D., & Lin, G. (1996). Stock market volatility and the business cycle. *Journal of applied econometrics*, *11*(5), 573-593.

Kogan, L., & Tian, M. H. (2012). Firm characteristics and empirical factor models: A data-mining experiment. *FRB International Finance discussion paper*, (1070).

Kogan, L., & Papanikolaou, D. (2013). Firm characteristics and stock returns: The role of investment-specific shocks. *The Review of Financial Studies*, *26*(11), 2718-2759.

Lauterbach, B. (1989). Consumption volatility, production volatility, spot-rate volatility, and the returns on treasury bills and bonds. *Journal of Financial Economics*, *24*(1), 155-179.

Li, L. (2002). Macroeconomic factors and the correlation of stock and bond returns. *Available at SSRN 363641*.

Lintner, J. (1965). Security prices, risk, and maximal gains from diversification. *The journal of finance*, *20*(4), 587-615.

Malkiel, B. G., & Fama, E. F. (1970). Efficient capital markets: A review of theory and empirical work. *The journal of Finance*, *25*(2), 383-417.

Officer, R. R. (1973). The variability of the market factor of the New York Stock Exchange. *the Journal of Business*, *46*(3), 434-453.

Ortiz-Ospina, E., Beltekian, D., & Roser, M. (2018). Trade and globalization. *Our World in Data*.

Sahlgren, M., & Lenci, A. (2016). The effects of data size and frequency range on distributional semantic models. *arXiv preprint arXiv:1609.08293*.

Schwert, G. W. (2003). Anomalies and market efficiency. *Handbook of the Economics of Finance*, *1*, 939-974.

Schwert, G. W. (1989). Why does stock market volatility change over time?. *The journal of finance*, *44*(5), 1115-1153.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The journal of finance*, *19*(3), 425-442.

Spiegel, M. I., & Wang, X. (2005). Cross-sectional variation in stock returns: Liquidity and idiosyncratic risk.

Zhu, X., Wu, X., & Chen, Q. (2003). Eliminating class noise in large datasets. In *Proceedings of the 20th International Conference on Machine Learning (ICML-03)* (pp. 920-927).

APPENDIX

APPENDIX A.1

Table A.1.1. Summary statistics for the decile portfolio using a daily basis

	Excess returns	**Standard deviation**	**Sharpe Ratio**
Panel A: Using 3-years daily returns			
D1	**0.0068*** **	**0.0123*** **	**0.552846**
D2	0.0081***	0.0156***	0.519231
D3	0.0085***	0.0179***	0.474860
D4	0.0085***	0.0200***	0.425000
D5	0.0095***	0.0223***	0.426009
D6	0.0106***	0.0251***	0.422311
D7	0.0105***	0.0288***	0.364583
D8	0.0129***	0.0337***	0.382789
D9	0.0135***	0.0410***	0.329268
D10	0.0175***	0.0622***	0.281350
D1-D10	-0.0107***	-0.0499***	0.214429
Panel B: Using 2-years daily returns			
D1	**0.0066*** **	**0.0120*** **	**0.550000**
D2	0.0078***	0.0154***	0.506494
D3	0.0084***	0.0177***	0.474576
D4	0.0090***	0.0199***	0.452261
D5	0.0092***	0.0222***	0.414414
D6	0.0107***	0.0251***	0.426295
D7	0.0113***	0.0288***	0.392361
D8	0.0129***	0.0337***	0.382789
D9	0.0139***	0.0411***	0.338200
D10	0.0193***	0.0631***	0.305864
D1-D10	-0.0127***	-0.0511***	0.248532
Panel C: Using 1-year daily returns			
D1	**0.0072*** **	**0.0116*** **	**0.620690**
D2	0.0076***	0.0150***	0.506667
D3	0.0090***	0.0174***	0.517241
D4	0.0091***	0.0196***	0.464286
D5	0.0099***	0.0221***	0.447964
D6	0.0112***	0.0250***	0.448000
D7	0.0114***	0.0287***	0.397213
D8	0.0130***	0.0337***	0.385757
D9	0.0137***	0.0412***	0.332524
D10	0.0181***	0.0643***	0.281493
D1-D10	-0.0108***	-0.0528***	0.204545

	Excess returns	**Standard deviation**	**Sharpe Ratio**
Panel D: Using 3-months daily returns			
D1	**0.0071*** **	**0.0104*** **	**0.682692**
D2	0.0083***	0.0140***	0.592857
D3	0.0089***	0.0165***	0.539394
D4	0.0094***	0.0188***	0.500000
D5	0.0109***	0.0214***	0.509346
D6	0.0119***	0.0243***	0.489712
D7	0.0134***	0.0280***	0.478571
D8	0.0112***	0.0330***	0.339394
D9	0.0131***	0.0407***	0.321867
D10	0.0158***	0.0660***	0.239394
D1-D10	-0.0088***	-0.0556***	0.158273
Panel E: Using 1-month daily returns			
D1	**0.0071*** **	**0.0088*** **	**0.806818**
D2	0.0079***	0.0127***	0.622047
D3	0.0097***	0.0152***	0.638158
D4	0.0097***	0.0176***	0.551136
D5	0.0104***	0.0203***	0.512315
D6	0.0120***	0.0232***	0.517241
D7	0.0127***	0.0270***	0.470370
D8	0.0125***	0.0321***	0.389408
D9	0.0115***	0.0404***	0.284653
D10	0.0134***	0.0678***	0.197640
D1-D10	-0.0064*	-0.0590***	0.108475

*t-statistics in parentheses: * p<0.10, ** p<0.05, *** p<0.01*

Note: The table displays the decile portfolios formed by the standard deviation calculated by a daily basis of the US stock, D1 contains the least volatile portfolio and D10 is the most volatile portfolio. D1-D10 is the top-minus-bottom decile portfolio. All portfolios are equal-weighted. The dataset used is the US stock observations from 1963 to 2020. The monthly expected excess returns, standard deviation and Sharpe ratio is presented on the table.

Table A.1.2. Summary statistics for the decile portfolio using a weekly basis

	Excess returns	Standard deviation	Sharpe Ratio
Panel A: Using 3-years weekly returns			
D1	**0.0064*** **	**0.0261*** **	**0.245211**
D2	0.0077***	0.0338***	0.227811
D3	0.0078***	0.0390***	0.200000
D4	0.0092***	0.0438***	0.210046
D5	0.0094***	0.0487***	0.193018
D6	0.0105***	0.0543***	0.193370
D7	0.0114***	0.0615***	0.185366
D8	0.0124***	0.0713***	0.173913
D9	0.0150***	0.0854***	0.175644
D10	0.0175***	0.1246***	0.140449
D1-D10	-0.0111***	-0.0985***	0.112690
Panel B: Using 2-years weekly returns			
D1	**0.0065*** **	**0.0254*** **	**0.255906**
D2	0.0075***	0.0332***	0.225904
D3	0.0082***	0.0385***	0.212987
D4	0.0094***	0.0434***	0.216590
D5	0.0094***	0.0484***	0.194215
D6	0.0110***	0.0542***	0.202952
D7	0.0122***	0.0615***	0.198374
D8	0.0120***	0.0714***	0.168067
D9	0.0148***	0.0858***	0.172494
D10	0.0191***	0.1271***	0.150275
D1-D10	-0.0126***	-0.1016***	0.124016
Panel C: Using 1-year weekly returns			
D1	**0.0067*** **	**0.0241*** **	**0.278008**
D2	0.0071***	0.0321***	0.221184
D3	0.0087***	0.0375***	0.232000
D4	0.0094***	0.0425***	0.221176
D5	0.0100***	0.0477***	0.209644
D6	0.0108***	0.0537***	0.201117
D7	0.0124***	0.0612***	0.202614
D8	0.0124***	0.0712***	0.174157
D9	0.0144***	0.0859***	0.167637
D10	0.0186***	0.1303***	0.142748
D1-D10	-0.0119***	-0.1062***	0.112053

	Excess returns	Standard deviation	Sharpe Ratio
Panel D: Using 3-months weekly returns			
D1	**0.0069*** **	**0.0194*** **	**0.355670**
D2	0.0076***	0.0279***	0.272401
D3	0.0084***	0.0337***	0.249258
D4	0.0094***	0.0390***	0.241026
D5	0.0106***	0.0446***	0.237668
D6	0.0113***	0.0509***	0.222004
D7	0.0122***	0.0586***	0.208191
D8	0.0133***	0.0689***	0.193033
D9	0.0134***	0.0847***	0.158205
D10	0.0167***	0.1371***	0.121809
D1-D10	-0.0098***	-0.1177***	0.083263
Panel E: Using 1-month weekly returns			
D1	**0.0068*** **	**0.0111*** **	**0.612613**
D2	0.0080***	0.0197***	0.406091
D3	0.0091***	0.0259***	0.351351
D4	0.0091***	0.0318***	0.286164
D5	0.0096***	0.0380***	0.252632
D6	0.0104***	0.0451***	0.230599
D7	0.0117***	0.0537***	0.217877
D8	0.0132***	0.0652***	0.202454
D9	0.0142***	0.0836***	0.169856
D10	0.0186***	0.1462***	0.127223
D1-D10	-0.0118***	-0.1351***	0.087343

*t-statistics in parentheses: * p<0.10, ** p<0.05, *** p<0.01*

Note: The table displays the decile portfolios formed by the standard deviation calculated by a weekly basis of the US stock, D1 contains the least volatile portfolio and D10 is the most volatile portfolio. D1-D10 is the top-minus-bottom decile portfolio. All portfolios are equal-weighted. The dataset used is the US stock observations from 1963 to 2020. The monthly expected excess returns, standard deviation and Sharpe ratio is presented on the table.

Table A.1.3. Summary statistics for the decile portfolio using a monthly basis

	Excess returns	Standard deviation	Sharpe Ratio
Panel A: Using 3 years returns on a monthly basis			
D1	**0.0066*** **	**0.0497*** **	**0.132797**
D2	0.0079***	0.0653***	0.120980
D3	0.0081***	0.0762***	0.106299
D4	0.0085***	0.0861***	0.098722
D5	0.0096***	0.0963***	0.099688
D6	0.0106***	0.1081***	0.098057
D7	0.0118***	0.1231***	0.095857
D8	0.0143***	0.1432***	0.099860
D9	0.0144***	0.1726***	0.083430
D10	0.0158***	0.2599***	0.060793
D1-D10	-0.0092***	-0.2102***	0.043768
Panel B: Using 2 years returns on a monthly basis			
D1	**0.0065*** **	**0.0477*** **	**0.136268**
D2	0.0081***	0.0636***	0.127358
D3	0.0084***	0.0745***	0.112752
D4	0.0086***	0.0848***	0.101415
D5	0.0100***	0.0954***	0.104822
D6	0.0110***	0.1075***	0.102326
D7	0.0125***	0.1224***	0.102124
D8	0.0134***	0.1425***	0.094035
D9	0.0152***	0.1726***	0.088065
D10	0.0164***	0.2653***	0.061817
D1-D10	-0.0099***	-0.2176***	0.045496
Panel C: Using 1 year returns on a monthly basis			
D1	**0.0073*** **	**0.0429*** **	**0.170163**
D2	0.0082***	0.0595***	0.137815
D3	0.0085***	0.0708***	0.120056
D4	0.0093***	0.0814***	0.114251
D5	0.0101***	0.0924***	0.109307
D6	0.0111***	0.1050***	0.105714
D7	0.0118***	0.1201***	0.098251
D8	0.0134***	0.1404***	0.095442
D9	0.0145***	0.1718***	0.084400
D10	0.0163***	0.2733***	0.059641
D1-D10	-0.0090***	-0.2305***	0.039046

	Excess returns	Standard deviation	Sharpe Ratio
Panel D: Using 3 months returns on a monthly basis			
D1	**0.0091*** **	**0.0183*** **	**0.497268**
D2	0.0089***	0.0358***	0.248603
D3	0.0101***	0.0491***	0.205703
D4	0.0100***	0.0617***	0.162075
D5	0.0113***	0.0750***	0.150667
D6	0.0112***	0.0898***	0.124722
D7	0.0119***	0.1078***	0.110390
D8	0.0116***	0.1315***	0.088213
D9	0.0131***	0.1694***	0.077332
D10	0.0126***	0.2979***	0.042296
D1-D10	-0.0036	-0.2796***	0.012876
Panel E: Using 1 month returns on a monthly basis			
D1	0.0065***	0.0608***	0.106908
D2	**0.0088*** **	**0.0777*** **	**0.113256**
D3	0.0099***	0.0893***	0.110862
D4	0.0115***	0.1001***	0.114885
D5	0.0111***	0.1114***	0.099641
D6	0.0120***	0.1236***	0.097087
D7	0.0128***	0.1379***	0.092821
D8	0.0115***	0.1564***	0.073529
D9	0.0116***	0.1827***	0.063492
D10	0.0153***	0.2544***	0.060142
D1-D10	-0.0088**	-0.1936***	0.045455

*t-statistics in parentheses: * p<0.10, ** p<0.05, *** p<0.01*

Note: The table displays the decile portfolios formed by the standard deviation calculated by a monthly basis of the US stock, D1 contains the least volatile portfolio and D10 is the most volatile portfolio. D1-D10 is the top-minus-bottom decile portfolio. All portfolios are equal-weighted. The dataset used is the US stock observations from 1963 to 2020. The monthly expected excess returns, standard deviation and Sharpe ratio is presented on the table.

APPENDIX A.2

Table A.2.1. The portfolio's performance on a daily basis controlling by the CAPM, FF 3-, 4-, and 5-factors

	CAPM - α	**CAPM - β**	**FF3 - α**	**FF3 - β**	**FF4 - α**	**FF4 - β**	**FF5 - α**	**FF5 - β**
Panel A: Using 3-years daily returns								
D1	0.0066***	0.0391	0.0067***	0.0483	0.0071***	0.0371	0.0068***	0.0410
D2	0.0077***	0.0767**	0.0077***	0.0767**	0.0082***	0.0635	0.0081***	0.0643
D3	0.0079***	0.1105***	0.0079***	0.1056***	0.0082***	0.0963**	0.0081***	0.0969**
D4	0.0079***	0.1210***	0.0078***	0.1175***	0.0084***	0.1031**	0.0084***	0.0994**
D5	0.0087***	0.1440***	0.0087***	0.1396***	0.0093***	0.1261**	0.0091***	0.1262**
D6	0.0098***	0.1460***	0.0099***	0.1332**	0.0106***	0.1156**	0.0106***	0.1120**
D7	0.0095***	0.1748***	0.0097***	0.1537***	0.0103***	0.1387**	0.0105***	0.1272**
D8	0.0117***	0.2236***	0.0118***	0.1974***	0.0124***	0.1837***	0.0124***	0.1850***
D9	0.0120***	0.2743***	0.0124***	0.2455***	0.0133***	0.2223***	0.0131***	0.2242***
D10	0.0152***	0.4259***	0.0156***	0.3476***	0.0162***	0.3323***	0.0164***	0.3238***
D1-D10	-0.0086**	-0.3868***	-0.0089***	-0.2994***	-0.0091***	-0.2952***	-0.0096***	-0.2828***
Panel B: Using 2-years daily returns								
D1	0.0063***	0.0448	0.0063***	0.0546*	0.0068***	0.0434	0.0064***	0.0487
D2	0.0073***	0.0775**	0.0073***	0.0794**	0.0079***	0.0652*	0.0077***	0.0681*
D3	0.0078***	0.1118***	0.0078***	0.1093***	0.0081***	0.1000**	0.0081***	0.0974**
D4	0.0082***	0.1400***	0.0082***	0.1348***	0.0086***	0.1248***	0.0086***	0.1211**
D5	0.0085***	0.1396***	0.0085***	0.1309***	0.0092***	0.1130**	0.0090***	0.1131**
D6	0.0098***	0.1577***	0.0098***	0.1475***	0.0107***	0.1266**	0.0104***	0.1307**
D7	0.0103***	0.1824***	0.0105***	0.1592***	0.0110***	0.1484**	0.0113***	0.1327**
D8	0.0117***	0.2122***	0.0118***	0.1875***	0.0123***	0.1757**	0.0126***	0.1688**
D9	0.0124***	0.2704***	0.0126***	0.2387***	0.0138***	0.2099***	0.0130***	0.2253***
D10	0.0167***	0.4805***	0.0169***	0.3967***	0.0174***	0.3856***	0.0179***	0.3681***
D1-D10	-0.0103***	-0.4356***	-0.0106***	-0.3421***	-0.0106***	-0.3422***	-0.0115***	-0.3194***

	CAPM - α	CAPM - β	FF3 - α	FF3 - β	FF4 - α	FF4 - β	FF5 - α	FF5 - β
Panel C: Using 1-year daily returns								
D1	0.0069***	0.0634**	0.0069***	0.0685**	0.0074***	0.0581*	0.0070***	0.0617*
D2	0.0070***	0.0951***	0.0070***	0.0962***	0.0076***	0.0830**	0.0074***	0.0828**
D3	0.0084***	0.1055***	0.0085***	0.0996**	0.0090***	0.0870**	0.0090***	0.0836*
D4	0.0083***	0.1404***	0.0082***	0.1352***	0.0086***	0.1266***	0.0086***	0.1208**
D5	0.0093***	0.1207***	0.0093***	0.1168**	0.0099***	0.1009**	0.0098***	0.0997*
D6	0.0103***	0.1620***	0.0104***	0.1534***	0.0112***	0.1331**	0.0111***	0.1315**
D7	0.0104***	0.1952***	0.0104***	0.1745***	0.0111***	0.1592***	0.0110***	0.1514**
D8	0.0119***	0.2073***	0.0121***	0.1824***	0.0126***	0.1726**	0.0128***	0.1632**
D9	0.0120***	0.3070***	0.0122***	0.2686***	0.0133***	0.2409***	0.0127***	0.2533***
D10	0.0155***	0.4619***	0.0158***	0.3930***	0.0162***	0.3833***	0.0167***	0.3636***
D1-D10	-0.0086**	-0.3985***	-0.0089***	-0.3246***	-0.0089**	-0.3252***	-0.0096***	-0.3019***
Panel D: Using 3-months daily returns								
D1	0.0064***	0.1161***	0.0064***	0.1170***	0.0069***	0.1046***	0.0065***	0.1108***
D2	0.0078***	0.0897***	0.0078***	0.0883**	0.0084***	0.0750**	0.0078***	0.0840**
D3	0.0082***	0.1205***	0.0082***	0.1102***	0.0087***	0.1003**	0.0089***	0.0885**
D4	0.0087***	0.1155***	0.0087***	0.1156**	0.0093***	0.1017**	0.0091***	0.1013**
D5	0.0099***	0.1670***	0.0099***	0.1539***	0.0107***	0.1370***	0.0106***	0.1323**
D6	0.0110***	0.1638***	0.0110***	0.1462***	0.0119***	0.1249**	0.0115***	0.1325**
D7	0.0121***	0.2331***	0.0122***	0.2094***	0.0130***	0.1916***	0.0131***	0.1800***
D8	0.0101***	0.2053***	0.0104***	0.1863***	0.0109***	0.1736**	0.0110***	0.1603**
D9	0.0114***	0.3084***	0.0116***	0.2715***	0.0126***	0.2489***	0.0121***	0.2550***
D10	0.0136***	0.4060***	0.0140***	0.3331***	0.0145***	0.3227***	0.0149***	0.3109***
D1-D10	-0.0071**	-0.2899***	-0.0076**	-0.2160***	-0.0075**	-0.2180***	-0.0084**	-0.2001**

	Excess returns	Standard deviation	Sharpe Ratio	CAPM - α	CAPM - β	FF3 - α	FF4 - α	FF5 - α
Panel E: Using 1-month daily returns								
D1	0.0064***	0.1277***	0.0064***	0.1207***	0.0069***	0.1103***	0.0067***	0.1095***
D2	0.0072***	0.1232***	0.0073***	0.1138***	0.0079***	0.0990**	0.0076***	0.1042***
D3	0.0088***	0.1549***	0.0090***	0.1416***	0.0096***	0.1287***	0.0096***	0.1169**
D4	0.0089***	0.1340***	0.0089***	0.1196***	0.0097***	0.1020**	0.0094***	0.1027**
D5	0.0095***	0.1562***	0.0095***	0.1491***	0.0101***	0.1351***	0.0099***	0.1339**
D6	0.0111***	0.1688***	0.0111***	0.1522***	0.0118***	0.1370**	0.0118***	0.1257**
D7	0.0117***	0.1791***	0.0119***	0.1512***	0.0125***	0.1375**	0.0125***	0.1363**
D8	0.0113***	0.2174***	0.0115***	0.2027***	0.0122***	0.1862***	0.0120***	0.1873***
D9	0.0099***	0.2894***	0.0101***	0.2576***	0.0111***	0.2341***	0.0114***	0.2135***
D10	0.0112***	0.3983***	0.0118***	0.3165***	0.0119***	0.3153***	0.0123***	0.3132***
D1-D10	-0.0050	-0.2597***	-0.0056*	-0.1843**	-0.0052	-0.1940**	-0.0059*	-0.1912**

*t-statistics in parentheses: * p<0.10, ** p<0.05, *** p<0.01*

Note: The table displays the results of each strategy's returns regressed on the CAPM, FF 3-factor model, FF 4-factor, and FF 5-factor models. D1 contains the least volatile portfolio and D10 is the most volatile portfolio. D1-D10 is the top-minus-bottom decile portfolio. All portfolios are equal-weighted. The dataset used is the US stock observations from 1963 to 2020. The excess returns based on the CAPM, FF3-, FF4- , and FF5- factors are included on the table. Besides, the market Beta of CAPM, FF3-, FF4-, and FF5- factor models are presented on the table.

Table A.2.2. The portfolio's performance on a weekly basis controlling by the CAPM, FF 3-, 4-, and 5-factors

	CAPM - α	**CAPM - β**	**FF3 - α**	**FF3 - β**	**FF4 - α**	**FF4 - β**	**FF5 - α**	**FF5 - β**
Panel A: Using 3-years weekly returns								
D1	0.0063***	0.0290	0.0063***	0.0404	0.0067***	0.0309	0.0065***	0.0349
D2	0.0073***	0.0666*	0.0074***	0.0687*	0.0079***	0.0564	0.0076***	0.0602
D3	0.0073***	0.0897**	0.0073***	0.0900**	0.0078***	0.0776*	0.0077***	0.0769*
D4	0.0085***	0.1181***	0.0084***	0.1191***	0.0091***	0.1024**	0.0088***	0.1068**
D5	0.0086***	0.1382***	0.0087***	0.1255***	0.0092***	0.1114**	0.0091***	0.1106**
D6	0.0095***	0.1824***	0.0095***	0.1697***	0.0103***	0.1502***	0.0101***	0.1497***
D7	0.0104***	0.1881***	0.0106***	0.1669***	0.0111***	0.1533***	0.0113***	0.1447**
D8	0.0112***	0.2044***	0.0114***	0.1771***	0.0119***	0.1650**	0.0122***	0.1621**
D9	0.0135***	0.2768***	0.0137***	0.2421***	0.0146***	0.2209***	0.0144***	0.2180***
D10	0.0151***	0.4415***	0.0157***	0.3609***	0.0162***	0.3472***	0.0166***	0.3348***
D1-D10	-0.0088**	-0.4125***	-0.0094***	-0.3205***	-0.0095***	-0.3163***	-0.0102***	-0.2999***
Panel B: Using 2-years weekly returns								
D1	0.0063***	0.0372	0.0063***	0.0487*	0.0068***	0.0383	0.0064***	0.0442
D2	0.0071***	0.0782**	0.0071***	0.0783**	0.0077***	0.0640*	0.0074***	0.0669*
D3	0.0077***	0.0904**	0.0076***	0.0935**	0.0081***	0.0833**	0.0080***	0.0802*
D4	0.0088***	0.1151***	0.0087***	0.1142**	0.0092***	0.1023**	0.0091***	0.1004**
D5	0.0085***	0.1575***	0.0085***	0.1408***	0.0092***	0.1240**	0.0090***	0.1232**
D6	0.0101***	0.1698***	0.0101***	0.1566***	0.0109***	0.1376**	0.0107***	0.1333**
D7	0.0110***	0.2098***	0.0112***	0.1934***	0.0120***	0.1750***	0.0118***	0.1788***
D8	0.0108***	0.2111***	0.0110***	0.1777***	0.0115***	0.1669**	0.0119***	0.1549**
D9	0.0134***	0.2556***	0.0137***	0.2165***	0.0144***	0.1982**	0.0142***	0.2010**
D10	0.0165***	0.4853***	0.0169***	0.4066***	0.0174***	0.3945***	0.0179***	0.3775***
D1-D10	-0.0101***	-0.4481***	-0.0106***	-0.3579***	-0.0106***	-0.3562***	-0.0115***	-0.3333***

	CAPM - α	CAPM - β	FF3 - α	FF3 - β	FF4 - α	FF4 - β	FF5 - α	FF5 - β
Panel C: Using 1-year weekly returns								
D1	0.0063***	0.0581**	0.0063***	0.0650**	0.0067***	0.0553*	0.0064***	0.0575*
D2	0.0066***	0.0974***	0.0066***	0.0954***	0.0071***	0.0817**	0.0069***	0.0828**
D3	0.0082***	0.1009***	0.0081***	0.1038**	0.0087***	0.0896**	0.0084***	0.0910**
D4	0.0088***	0.1176***	0.0087***	0.1173***	0.0093***	0.1044**	0.0091***	0.1019**
D5	0.0092***	0.1490***	0.0092***	0.1396***	0.0098***	0.1248**	0.0097***	0.1248**
D6	0.0098***	0.1799***	0.0098***	0.1596***	0.0105***	0.1418***	0.0105***	0.1378**
D7	0.0113***	0.1961***	0.0115***	0.1831***	0.0120***	0.1708***	0.0122***	0.1578**
D8	0.0113***	0.2057***	0.0114***	0.1733***	0.0119***	0.1625**	0.0121***	0.1561**
D9	0.0126***	0.3211***	0.0128***	0.2846***	0.0136***	0.2663***	0.0137***	0.2604***
D10	0.0160***	0.4537***	0.0167***	0.3662***	0.0171***	0.3574***	0.0178***	0.3332***
D1-D10	-0.0097***	-0.3956***	-0.0104***	-0.3012***	-0.0104***	-0.3022***	-0.0114***	-0.2757***
Panel D: Using 3-months weekly returns								
D1	0.0063***	0.1179***	0.0063***	0.1174***	0.0067***	0.1076***	0.0064***	0.1110***
D2	0.0070***	0.1054***	0.0070***	0.1023***	0.0075***	0.0886**	0.0073***	0.0891**
D3	0.0077***	0.1223***	0.0077***	0.1124***	0.0084***	0.0964**	0.0081***	0.0982**
D4	0.0085***	0.1469***	0.0084***	0.1431***	0.0090***	0.1276***	0.0087***	0.1268***
D5	0.0098***	0.1498***	0.0098***	0.1400***	0.0105***	0.1236**	0.0104***	0.1219**
D6	0.0101***	0.2037***	0.0102***	0.1858***	0.0109***	0.1696***	0.0111***	0.1592***
D7	0.0112***	0.1855***	0.0114***	0.1598***	0.0119***	0.1480**	0.0118***	0.1441**
D8	0.0119***	0.2509***	0.0123***	0.2175***	0.0128***	0.2061***	0.0129***	0.1939***
D9	0.0118***	0.2823***	0.0122***	0.2344***	0.0128***	0.2212***	0.0130***	0.2117***
D10	0.0145***	0.3878***	0.0153***	0.3174***	0.0159***	0.3022***	0.0163***	0.2916***
D1-D10	-0.0083***	-0.2699***	-0.0090***	-0.2000***	-0.0092***	-0.1946**	-0.0099***	-0.1806**

	CAPM - α	CAPM - β	FF3 - α	FF3 - β	FF4 - α	FF4 - β	FF5 - α	FF5 - β
Panel E: Using 1-month weekly returns								
D1	0.0059***	0.1480***	0.0058***	0.1440***	0.0064***	0.1307***	0.0061***	0.1327***
D2	0.0071***	0.1498***	0.0072***	0.1397***	0.0077***	0.1265***	0.0076***	0.1253***
D3	0.0081***	0.1634***	0.0082***	0.1388***	0.0087***	0.1279***	0.0087***	0.1224***
D4	0.0082***	0.1544***	0.0083***	0.1314***	0.0089***	0.1163**	0.0089***	0.1064**
D5	0.0086***	0.1791***	0.0086***	0.1688***	0.0092***	0.1549***	0.0091***	0.1528***
D6	0.0094***	0.1759***	0.0095***	0.1560***	0.0104***	0.1354**	0.0102***	0.1291**
D7	0.0106***	0.2033***	0.0107***	0.1948***	0.0112***	0.1823***	0.0113***	0.1748***
D8	0.0120***	0.2096***	0.0123***	0.1744***	0.0130***	0.1583**	0.0130***	0.1521**
D9	0.0129***	0.2260***	0.0131***	0.1894***	0.0135***	0.1799**	0.0133***	0.1810**
D10	0.0166***	0.3663***	0.0172***	0.3145***	0.0179***	0.2975***	0.0180***	0.2955***
D1-D10	-0.0106***	-0.2183***	-0.0113***	-0.1705***	-0.0115***	-0.1668**	-0.0119***	-0.1628**

*t-statistics in parentheses: * p<0.10, ** p<0.05, *** p<0.01*

Note: The table displays the results of each strategy's returns regressed on the CAPM, FF 3-factor model, FF 4-factor, and FF 5-factor models. D1 contains the least volatile portfolio and D10 is the most volatile portfolio. D1-D10 is the top-minus-bottom decile portfolio. All portfolios are equal-weighted. The dataset used is the US stock observations from 1963 to 2020. The excess returns based on the CAPM, FF3-, FF4- , and FF5- factors are included on the table. Besides, the market Beta of CAPM, FF3-, FF4-, and FF5- factor models are presented on the table.

Table A.2.3. The portfolio's performance on a monthly basis controlling by the CAPM, FF 3-, 4-, and 5-factors

	CAPM - α	**CAPM - β**	**FF3 - α**	**FF3 - β**	**FF4 - α**	**FF4 - β**	**FF5 - α**	**FF5 - β**
Panel A: Using 3-years monthly returns								
D1	0.0064***	0.0338	0.0064***	0.0455	0.0067***	0.0378	0.0065***	0.0418
D2	0.0076***	0.0570*	0.0076***	0.0565	0.0082***	0.0421	0.0079***	0.0451
D3	0.0074***	0.1149***	0.0074***	0.1128***	0.0079***	0.0996**	0.0079***	0.0948**
D4	0.0079***	0.1156***	0.0079***	0.1069**	0.0086***	0.0907**	0.0083***	0.0925*
D5	0.0088***	0.1395***	0.0088***	0.1308***	0.0093***	0.1176**	0.0092***	0.1177**
D6	0.0096***	0.1851***	0.0096***	0.1735***	0.0104***	0.1536***	0.0103***	0.1506***
D7	0.0107***	0.1932***	0.0107***	0.1776***	0.0115***	0.1594***	0.0114***	0.1582**
D8	0.0129***	0.2485***	0.0132***	0.2081***	0.0139***	0.1894***	0.0139***	0.1906***
D9	0.0129***	0.2616***	0.0133***	0.2253***	0.0140***	0.2093***	0.0138***	0.2135***
D10	0.0137***	0.3874***	0.0142***	0.3244***	0.0145***	0.3166***	0.0152***	0.2969***
D1-D10	-0.0073**	-0.3535***	-0.0077**	-0.2789***	-0.0077**	-0.2787***	-0.0087**	-0.2552***
Panel B: Using 2-years monthly returns								
D1	0.0062***	0.0549*	0.0062***	0.0657**	0.0066***	0.0558*	0.0063***	0.0582*
D2	0.0077***	0.0721**	0.0076***	0.0677*	0.0081***	0.0554	0.0079***	0.0570
D3	0.0078***	0.1113***	0.0078***	0.1111***	0.0084***	0.0982**	0.0082***	0.0954**
D4	0.0078***	0.1320***	0.0078***	0.1246***	0.0084***	0.1101**	0.0081***	0.1122**
D5	0.0091***	0.1504***	0.0091***	0.1426***	0.0099***	0.1230**	0.0098***	0.1194**
D6	0.0100***	0.1818***	0.0100***	0.1666***	0.0107***	0.1503***	0.0104***	0.1556***
D7	0.0115***	0.1877***	0.0115***	0.1623***	0.0125***	0.1398**	0.0124***	0.1318**
D8	0.0120***	0.2532***	0.0122***	0.2152***	0.0128***	0.2020***	0.0127***	0.2051***
D9	0.0137***	0.2689***	0.0141***	0.2336***	0.0148***	0.2189***	0.0147***	0.2207***
D10	0.0142***	0.4038***	0.0145***	0.3432***	0.0148***	0.3368***	0.0157***	0.3085***
D1-D10	-0.0080**	-0.3489***	-0.0084**	-0.2776***	-0.0082**	-0.2810***	-0.0094***	-0.2503***

	CAPM - α	CAPM - β	FF3 - α	FF3 - β	FF4 - α	FF4 - β	FF5 - α	FF5 - β
Panel C: Using 1-year monthly returns								
D1	0.0068***	0.0864***	0.0067***	0.0906***	0.0071***	0.0811***	0.0070***	0.0789**
D2	0.0076***	0.1092***	0.0076***	0.1082***	0.0081***	0.0960***	0.0077***	0.1007***
D3	0.0078***	0.1214***	0.0077***	0.1172***	0.0083***	0.1019**	0.0081***	0.1041**
D4	0.0085***	0.1428***	0.0086***	0.1316***	0.0092***	0.1162**	0.0090***	0.1142**
D5	0.0092***	0.1556***	0.0092***	0.1437***	0.0099***	0.1269**	0.0097***	0.1261**
D6	0.0101***	0.1761***	0.0100***	0.1654***	0.0110***	0.1429***	0.0106***	0.1464***
D7	0.0107***	0.2053***	0.0108***	0.1817***	0.0115***	0.1642***	0.0115***	0.1582***
D8	0.0120***	0.2462***	0.0122***	0.2201***	0.0128***	0.2071***	0.0132***	0.1901***
D9	0.0128***	0.2958***	0.0132***	0.2505***	0.0135***	0.2437***	0.0141***	0.2279***
D10	0.0144***	0.3459***	0.0151***	0.2816***	0.0154***	0.2753***	0.0158***	0.2618***
D1-D10	-0.0076**	-0.2595***	-0.0084***	-0.1910**	-0.0082**	-0.1942**	-0.0088***	-0.1829**
Panel D: Using 3-months monthly returns								
D1	0.0079***	0.2000***	0.0080***	0.1939***	0.0090***	0.1701***	0.0083***	0.1810***
D2	0.0079***	0.1776***	0.0078***	0.1688***	0.0085***	0.1525***	0.0081***	0.1538***
D3	0.0090***	0.1843***	0.0089***	0.1747***	0.0095***	0.1605***	0.0094***	0.1587***
D4	0.0091***	0.1694***	0.0092***	0.1484***	0.0098***	0.1325***	0.0097***	0.1292***
D5	0.0103***	0.1790***	0.0102***	0.1641***	0.0110***	0.1465***	0.0107***	0.1456***
D6	0.0102***	0.1907***	0.0101***	0.1805***	0.0108***	0.1640***	0.0108***	0.1611***
D7	0.0109***	0.1877***	0.0111***	0.1597***	0.0115***	0.1507***	0.0117***	0.1399**
D8	0.0106***	0.1910***	0.0107***	0.1692***	0.0112***	0.1577**	0.0113***	0.1535**
D9	0.0118***	0.2202***	0.0124***	0.1811***	0.0130***	0.1679**	0.0133***	0.1525**
D10	0.0112***	0.2589***	0.0119***	0.1958**	0.0120***	0.1938**	0.0126***	0.1779**
D1-D10	-0.0032	-0.0588	-0.0040	-0.0019	-0.0030	-0.0237	-0.0043*	0.0030

	CAPM - α	CAPM - β	FF3 - α	FF3 - β	FF4 - α	FF4 - β	FF5 - α	FF5 - β
Panel E: Using 1-month monthly returns								
D1	0.0066***	-0.0163	0.0067***	-0.0059	0.0070***	-0.0128	0.0069***	-0.0129
D2	0.0085***	0.0547	0.0086***	0.0539	0.0091***	0.0424	0.0089***	0.0399
D3	0.0092***	0.1142***	0.0093***	0.1049**	0.0101***	0.0852*	0.0097***	0.0865*
D4	0.0107***	0.1419***	0.0108***	0.1362***	0.0113***	0.1248***	0.0112***	0.1199**
D5	0.0102***	0.1611***	0.0101***	0.1574***	0.0107***	0.1422***	0.0104***	0.1438***
D6	0.0108***	0.2119***	0.0109***	0.1916***	0.0115***	0.1791***	0.0114***	0.1711***
D7	0.0114***	0.2487***	0.0115***	0.2251***	0.0123***	0.2061***	0.0123***	0.1985***
D8	0.0099***	0.2767***	0.0101***	0.2590***	0.0107***	0.2449***	0.0108***	0.2366***
D9	0.0098***	0.3065***	0.0100***	0.2579***	0.0104***	0.2479***	0.0107***	0.2272***
D10	0.0127***	0.4684***	0.0135***	0.3636***	0.0144***	0.3427***	0.0142***	0.3510***
D1-D10	-0.0060*	-0.4847***	-0.0068*	-0.3694***	-0.0074**	-0.3555***	-0.0073**	-0.3639***

*t-statistics in parentheses: * p<0.10, ** p<0.05, *** p<0.01*

Note: The table displays the results of each strategy's returns regressed on the CAPM, FF 3-factor model, FF 4-factor, and FF 5-factor models. D1 contains the least volatile portfolio and D10 is the most volatile portfolio. D1-D10 is the top-minus-bottom decile portfolio. All portfolios are equal-weighted. The dataset used is the US stock observations from 1963 to 2020. The excess returns based on the CAPM, FF3-, FF4- , and FF5- factors are included on the table. Besides, the market Beta of CAPM, FF3-, FF4-, and FF5- factor models are presented on the table.

APPENDIX A.3

Table A.3.1. The portfolio on a daily basis controlling by the CAPM, FF 3-, 4-, and 5-factors

Portfolio	Model	Alpha	RMRF	SMB	HML	UMD	RMW	CMA
Panel A: Using 3 years daily returns								
D1	CAPM	0.0066***	0.0391					
	FF3	0.0067***	0.0483	-0.0464	-0.0022			
	FF4	0.0071***	0.0371	-0.0478	-0.0231	-0.0547*		
	FF5	0.0068***	0.0410	-0.0495	0.0345		-0.0136	-0.0670
D10	CAPM	0.0152***	0.4259***					
	FF3	0.0156***	0.3476***	0.2509*	-0.1886			
	FF4	0.0162***	0.3323***	0.2491*	-0.2172	-0.0749		
	FF5	0.0164***	0.3238***	0.2262*	-0.1911		-0.1997	-0.0645
Panel B: Using 2 years daily returns								
D1	CAPM	0.0063***	0.0448					
	FF3	0.0063***	0.0546*	-0.0431	0.0063			
	FF4	0.0068***	0.0434	-0.0438	-0.0144	-0.0558*		
	FF5	0.0064***	0.0487	-0.0410	0.0401		0.0074	-0.0635
D10	CAPM	0.0167***	0.4805***					
	FF3	0.0169***	0.3967***	0.2901**	-0.1674			
	FF4	0.0174***	0.3856***	0.2894**	-0.1880	-0.0553		
	FF5	0.0179***	0.3681***	0.2598*	-0.1658		-0.2351	-0.0865
Panel C: Using 1 year daily returns								
D1	CAPM	0.0069***	0.0634**					
	FF3	0.0069***	0.0685**	-0.0250	-0.0008			
	FF4	0.0074***	0.0581*	-0.0257	-0.0198	-0.0515*		
	FF5	0.0070***	0.0617*	-0.0233	0.0324		0.0052	-0.0681
D10	CAPM	0.0155***	0.4619***					
	FF3	0.0158***	0.3930***	0.2322*	-0.1492			
	FF4	0.0162***	0.3833***	0.2316*	-0.1670	-0.0485		
	FF5	0.0167***	0.3636***	0.2130	-0.1339		-0.2084	-0.1014
Panel D: Using 3 months daily returns								
D1	CAPM	0.0064***	0.1161***					
	FF3	0.0064***	0.1170***	-0.0013	0.0050			
	FF4	0.0069***	0.1046***	-0.0021	-0.0180	-0.0624**		
	FF5	0.0065***	0.1108***	0.0013	0.0321		0.0094	-0.0627
D10	CAPM	0.0136***	0.4060***					
	FF3	0.0140***	0.3331***	0.2223*	-0.1954			
	FF4	0.0145***	0.3227***	0.2216*	-0.2146	-0.0523		
	FF5	0.0149***	0.3109***	0.1887	-0.2108		-0.2513	-0.0239
Panel E: Using 1 month daily returns								
D1	CAPM	0.0064***	0.1277***					
	FF3	0.0064***	0.1207***	0.0245	-0.0138			
	FF4	0.0069***	0.1103***	0.0237	-0.0331	-0.0525		
	FF5	0.0067***	0.1095***	0.0234	0.0205		-0.0218	-0.0857
D10	CAPM	0.0112***	0.3983***					
	FF3	0.0118***	0.3165***	0.2461*	-0.2271*			
	FF4	0.0119***	0.3153***	0.2460*	-0.2293	-0.0061		
	FF5	0.0123***	0.3132***	0.2115	-0.3177*		-0.2376	0.1402

*t-statistics in parentheses: * p<0.10, ** p<0.05, *** p<0.01*

Note: The table displays the results of each strategy's returns regressed on the CAPM, FF 3-factor model, FF 4-factor, and FF 5-factor models. D1 contains the least volatile portfolio and D10 is the most volatile portfolio. All portfolios are equal-weighted. The dataset used is the US stock observations from 1963 to 2020. The numbers are monthly returns. RMRF presents the excess returns for the market. SMB denotes the Small-Minus-Big factor, while HML is the High-Minus-Low factor of the asset pricing model. Momentum factor is displayed by UMD. RMW and CMA present the profitability and investment factor respectively.

Table A.3.2. The portfolio on a weekly basis controlling by the CAPM, FF 3-, 4-, and 5-factors

Portfolio	Model	Alpha	RMRF	SMB	HML	UMD	RMW	CMA
Panel A: Using 3 years weekly returns								
D1	CAPM	0.0063***	0.0290					
	FF3	0.0063***	0.0404	-0.0537	0.0025			
	FF4	0.0067***	0.0309	-0.0546	-0.0150	-0.0465		
	FF5	0.0065***	0.0349	-0.0555	0.0338		-0.0085	-0.0529
D10	CAPM	0.0151***	0.4415***					
	FF3	0.0157***	0.3609***	0.2285*	-0.2373*			
	FF4	0.0162***	0.3472***	0.2271*	-0.2626*	-0.0672		
	FF5	0.0166***	0.3348***	0.1956	-0.2389		-0.2503	-0.0565
Panel B: Using 2 years weekly returns								
D1	CAPM	0.0063***	0.0372					
	FF3	0.0063***	0.0487*	-0.0510	0.0061			
	FF4	0.0068***	0.0383	-0.0516	-0.0129	-0.0513*		
	FF5	0.0064***	0.0442	-0.0475	0.0352		0.0106	-0.0504
D10	CAPM	0.0165***	0.4853***					
	FF3	0.0169***	0.4066***	0.2338*	-0.2147			
	FF4	0.0174***	0.3945***	0.2331*	-0.2369*	-0.0599		
	FF5	0.0179***	0.3775***	0.2021	-0.2039		-0.2471	-0.0877
Panel C: Using 1 year weekly returns								
D1	CAPM	0.0063***	0.0581**					
	FF3	0.0063***	0.0650**	-0.0281	0.0083			
	FF4	0.0067***	0.0553*	-0.0287	-0.0096	-0.0488*		
	FF5	0.0064***	0.0575*	-0.0242	0.0458		0.0117	-0.0773
D10	CAPM	0.0160***	0.4537***					
	FF3	0.0167***	0.3662***	0.2351*	-0.2787**			
	FF4	0.0171***	0.3574***	0.2345*	-0.2948**	-0.0436		
	FF5	0.0178***	0.3332***	0.2046	-0.2541		-0.2476	-0.1208
Panel D: Using 3 months weekly returns								
D1	CAPM	0.0063***	0.1179***					
	FF3	0.0063***	0.1174***	0.0040	0.0027			
	FF4	0.0067***	0.1076***	0.0034	-0.0155	-0.0495		
	FF5	0.0064***	0.1110***	0.0054	0.0260		-0.0029	-0.0548
D10	CAPM	0.0145***	0.3878***					
	FF3	0.0153***	0.3174***	0.1673	-0.2586*			
	FF4	0.0159***	0.3022***	0.1663	-0.2868**	-0.0767		
	FF5	0.0163***	0.2916***	0.1217	-0.2511		-0.2862	-0.0550
Panel E: Using 1 month weekly returns								
D1	CAPM	0.0059***	0.1480***					
	FF3	0.0058***	0.1440***	0.0382	0.0279			
	FF4	0.0064***	0.1307***	0.0373	0.0033	-0.0669*		
	FF5	0.0061***	0.1327***	0.0347	0.0546		-0.0416	-0.0718
D10	CAPM	0.0166***	0.3663***					
	FF3	0.0172***	0.3145***	0.1115	-0.2077			
	FF4	0.0179***	0.2975***	0.1103	-0.2392*	-0.0856		
	FF5	0.0180***	0.2955***	0.0724	-0.2134		-0.2605	-0.0075

*t-statistics in parentheses: * p<0.10, ** p<0.05, *** p<0.01*

Note: The table displays the results of each strategy's returns regressed on the CAPM, FF 3-factor model, FF 4-factor, and FF 5-factor models. D1 contains the least volatile portfolio and D10 is the most volatile portfolio. All portfolios are equal-weighted. The dataset used is the US stock observations from 1963 to 2020. The numbers are monthly returns. RMRF presents the excess returns for the market. SMB denotes the Small-Minus-Big factor, while HML is the High-Minus-Low factor of the asset pricing model. Momentum factor is displayed by UMD. RMW and CMA present the profitability and investment factor respectively.

Table A.3.3. The portfolio on a monthly basis controlling by the CAPM, FF 3-, 4-, and 5-factors

Portfolio	Model	Alpha	RMRF	SMB	HML	UMD	RMW	CMA
Panel A: Using 3 years monthly returns								
D1	CAPM	0.0064***	0.0338					
	FF3	0.0064***	0.0455	-0.0505	0.0092			
	FF4	0.0067***	0.0378	-0.0512	-0.0050	-0.0375		
	FF5	0.0065***	0.0418	-0.0463	0.0350		0.0113	-0.0430
D10	CAPM	0.0137***	0.3874***					
	FF3	0.0142***	0.3244***	0.1761	-0.1896			
	FF4	0.0145***	0.3166***	0.1753	-0.2039	-0.0381		
	FF5	0.0152***	0.2969***	0.1413	-0.1683		-0.2354	-0.0919
Panel B: Using 2 years monthly returns								
D1	CAPM	0.0062***	0.0549*					
	FF3	0.0062***	0.0657**	-0.0385	0.0199			
	FF4	0.0066***	0.0558*	-0.0390	0.0017	-0.0491		
	FF5	0.0063***	0.0582*	-0.0327	0.0592		0.0173	-0.0786
D10	CAPM	0.0142***	0.4038***					
	FF3	0.0145***	0.3432***	0.1729	-0.1752			
	FF4	0.0148***	0.3368***	0.1726	-0.1871	-0.0322		
	FF5	0.0157***	0.3085***	0.1395	-0.1237		-0.2356	-0.1608
Panel C: Using 1 year monthly returns								
D1	CAPM	0.0068***	0.0864***					
	FF3	0.0067***	0.0906***	-0.0084	0.0176			
	FF4	0.0071***	0.0811***	-0.0090	0.0001	-0.0476		
	FF5	0.0070***	0.0789**	-0.0088	0.0656		-0.0064	-0.1069
D10	CAPM	0.0144***	0.3459***					
	FF3	0.0151***	0.2816***	0.1392	-0.2544*			
	FF4	0.0154***	0.2753***	0.1389	-0.2659*	-0.0313		
	FF5	0.0158***	0.2618***	0.1210	-0.2531		-0.1858	-0.0388
Panel D: Using 3 months daily returns								
D1	CAPM	0.0079***	0.2000***					
	FF3	0.0080***	0.1939***	0.0183	-0.0171			
	FF4	0.0090***	0.1701***	0.0167	-0.0611	-0.1198***		
	FF5	0.0083***	0.1810***	0.0177	0.0193		-0.0373	-0.0876
D10	CAPM	0.0112***	0.2589***					
	FF3	0.0119***	0.1958**	0.1314	-0.2593**			
	FF4	0.0120***	0.1938**	0.1313	-0.2630**	-0.0099		
	FF5	0.0126***	0.1779**	0.1095	-0.2494		-0.1548	-0.0573
Panel E: Using 1 month monthly returns								
D1	CAPM	0.0066***	-0.0163					
	FF3	0.0067***	-0.0059	-0.0603	-0.0145			
	FF4	0.0070***	-0.0128	-0.0608	-0.0274	-0.0350		
	FF5	0.0069***	-0.0129	-0.0658	0.0212		-0.0257	-0.0595
D10	CAPM	0.0127***	0.4684***					
	FF3	0.0135***	0.3636***	0.2906**	-0.3239**			
	FF4	0.0144***	0.3427***	0.2892**	-0.3626***	-0.1052		
	FF5	0.0142***	0.3510***	0.2517*	-0.3966**		-0.2768	0.0858

*t-statistics in parentheses: * p<0.10, ** p<0.05, *** p<0.01*

Note: The table displays the results of each strategy's returns regressed on the CAPM, FF 3-factor model, FF 4-factor, and FF 5-factor models. D1 contains the least volatile portfolio and D10 is the most volatile portfolio. All portfolios are equal-weighted. The dataset used is the US stock observations from 1963 to 2020. The numbers are monthly returns. RMRF presents the excess returns for the market. SMB denotes the Small-Minus-Big factor, while HML is the High-Minus-Low factor of the asset pricing model. Momentum factor is displayed by UMD. RMW and CMA present the profitability and investment factor respectively.

APPENDIX A.4

The portfolio's performance on sub-periods through the business cycle controlling by the CAPM, FF 3-, 4-, and 5-factors

	CAPM - α	**CAPM - β**	**FF3 - α**	**FF3 - β**	**FF4 - α**	**FF4 - β**	**FF5 - α**	**FF5 - β**
Panel A: 1963-1969 expansion period								
D1	**0.0048**	**0.0444**	**0.0042**	**0.0053**	**0.0040**	**0.0041**	**0.0037**	**0.0137**
D2	0.0050	-0.0404	0.0041	-0.0782	0.0033	-0.0842	0.0028	-0.0684
D3	0.0057	0.0326	0.0054	-0.0067	0.0045	-0.0135	0.0047	-0.0095
D4	0.0063	0.0076	0.0064	0.0097	0.0053	0.0034	0.0056	-0.0142
D5	0.0069	0.0667	0.0068	0.0322	0.0052	0.0193	0.0057	-0.0029
D6	0.0116**	0.0324	0.0104**	-0.0647	0.0090*	-0.0754	0.0094*	-0.0637
D7	0.0130**	0.0274	0.0132**	0.0404	0.0120*	0.0334	0.0120*	-0.0000
D8	0.0108*	0.0583	0.0110	0.0734	0.0089	0.0569	0.0095	0.0273
D9	0.0161*	0.0823	0.0181**	0.1714	0.0162*	0.1568	0.0158*	0.1047
D10	**0.0150**	**0.5469***	**0.0149**	**0.5058**	**0.0120**	**0.4836**	**0.0116**	**0.4677**
D1-D10	**-0.0116**	**-0.5085****	**-0.0117**	**-0.4914**	**-0.0091**	**-0.4713**	**-0.0093**	**-0.4104**
Panel B: 1969-1970 recession period								
D1	**0.0023**	**0.2470**	**0.0061**	**0.1344**	**0.0101**	**0.2775**	**-0.0065**	**0.3369**
D2	-0.0010	0.1650	0.0022	0.0271	0.0079	0.2326	-0.0153	0.3166
D3	-0.0032	0.1203	0.0013	-0.0272	0.0064	0.1565	-0.0168	0.2545
D4	0.0005	0.1524	0.0036	0.0094	0.0088	0.1978	-0.0133	0.2799
D5	0.0053	0.1893	0.0092	0.0540	0.0158	0.2922	-0.0159	0.4495
D6	-0.0054	0.3173	-0.0013	0.1337	0.0026	0.2721	-0.0188	0.3740
D7	-0.0055	0.2411	-0.0044	0.1317	-0.0009	0.2571	-0.0243	0.4191
D8	-0.0052	0.3414	-0.0067	0.2983	-0.0031	0.4267	-0.0268	0.5726
D9	-0.0222	0.3819	-0.0228	0.3335	-0.0190	0.4714	-0.0444	0.6065
D10	**-0.0263**	**0.6527**	**-0.0273**	**0.6363**	**-0.0256**	**0.6971**	**-0.0451**	**0.7563**
D1-D10	**0.0286**	**-0.4057**	**0.0334**	**-0.5018**	**0.0357**	**-0.4197**	**0.0386**	**-0.4194**

	CAPM - α	**CAPM - β**	**FF3 - α**	**FF3 - β**	**FF4 - α**	**FF4 - β**	**FF5 - α**	**FF5 - β**
Panel C: 1970-1973 expansion period								
D1	**0.0020**	**0.0987**	**0.0022**	**0.0943**	**0.0019**	**0.1076**	**-0.0003**	**0.1122**
D2	0.0018	-0.0344	0.0029	-0.1129	0.0037	-0.1451	0.0043	-0.0950
D3	0.0011	0.1989	0.0030	0.0339	0.0037	0.0050	0.0021	0.0589
D4	-0.0007	0.0596	0.0003	-0.0398	0.0005	-0.0473	0.0015	-0.0255
D5	0.0048	0.2727	0.0063	0.1414	0.0066	0.1310	0.0091	0.1609
D6	0.0082	0.2448	0.0091	0.1806	0.0084	0.2074	0.0094	0.1895
D7	0.0095	0.6563*	0.0122	0.4444	0.0099	0.5388	0.0123	0.4616
D8	0.0046	0.4125	0.0049	0.3881	0.0029	0.4732	0.0023	0.3994
D9	-0.0019	0.4457	0.0006	0.2108	-0.0007	0.2623	0.0018	0.2095
D10	**-0.0120**	**0.6026**	**-0.0120**	**0.6080**	**-0.0133**	**0.6610**	**-0.0147**	**0.6392**
D1-D10	**0.0140**	**-0.5039**	**0.0142**	**-0.5137**	**0.0152**	**-0.5534**	**0.0143**	**-0.5270**
Panel D: 1973-1975 recession period								
D1	**-0.0037**	**0.1899**	**-0.0117**	**0.6100***	**-0.0100**	**0.4800**	**-0.0326**	**0.8658**
D2	-0.0030	0.1216	-0.0084	0.4821	-0.0062	0.3093	-0.0324	0.8225
D3	0.0009	0.1273	-0.0059	0.4702	-0.0033	0.2748	-0.0318	0.8177
D4	0.0005	0.1071	-0.0058	0.4542	-0.0037	0.2975	-0.0183	0.5783
D5	-0.0026	0.0049	-0.0079	0.3004	-0.0060	0.1570	-0.0169	0.3502
D6	0.0039	0.1031	-0.0018	0.5014	0.0001	0.3553	-0.0281	0.8507
D7	0.0141	0.0642	0.0065	0.5365	0.0086	0.3779	-0.0051	0.6164
D8	0.0064	0.0109	0.0000	0.5151	0.0022	0.3534	-0.0248	0.7903
D9	0.0076	0.0577	-0.0030	0.6941	-0.0011	0.5507	-0.0273	0.9163
D10	**0.0086**	**0.0789**	**0.0002**	**0.8563**	**0.0031**	**0.6362**	**-0.0352**	**1.2628**
D1-D10	**-0.0124**	**0.1111**	**-0.0119**	**-0.2464**	**-0.0131**	**-0.1562**	**0.0027**	**-0.3970**

	CAPM - α	CAPM - β	FF3 - α	FF3 - β	FF4 - α	FF4 - β	FF5 - α	FF5 - β
Panel E 1975-1979 expansion period								
D1	**0.0060**	**0.0197**	**0.0071**	**0.0195**	**0.0067**	**0.0099**	**0.0094***	**0.0049**
D2	0.0091*	0.0488	0.0102*	0.0629	0.0106*	0.0725	0.0109*	0.0607
D3	0.0109*	-0.0329	0.0132**	-0.0198	0.0124*	-0.0388	0.0148**	-0.0277
D4	0.0129**	0.0927	0.0130*	0.1056	0.0128*	0.1008	0.0132*	0.1047
D5	0.0155**	-0.0218	0.0158**	-0.0093	0.0149*	-0.0295	0.0172**	-0.0146
D6	0.0176**	0.1619	0.0188**	0.2183	0.0186*	0.2140	0.0187*	0.2184
D7	0.0199**	-0.0561	0.0194**	-0.0800	0.0169*	-0.1360	0.0211**	-0.0931
D8	0.0252***	0.0819	0.0283***	0.1355	0.0267**	0.0982	0.0289***	0.1338
D9	0.0223**	0.0507	0.0234**	0.1139	0.0210*	0.0591	0.0241**	0.1107
D10	**0.0194**	**0.3288**	**0.0198**	**0.3764**	**0.0182**	**0.3392**	**0.0194**	**0.3793**
D1-D10	**-0.0134**	**-0.3091**	**-0.0127**	**-0.3570**	**-0.0115**	**-0.3293**	**-0.0100**	**-0.3744**
Panel F: 1980-1980 recession period								
D1	**0.0056**	**-0.0281**	**0.1020**	**-8.9380**	**0.0494**	**-2.8794**	**0.0075**	**0.9504**
D2	0.0077	-0.0658	0.0740	-5.4921	0.0153	1.2640	-0.0367	5.5633
D3	0.0081	0.0591	0.0826	-5.9168	0.0239	0.8425	-0.0275	5.1524
D4	0.0164	-0.0835	0.1094	-8.0340	0.0616	-2.5270	0.0272	0.4090
D5	0.0091	0.1713	0.1035	-7.6449	0.0608	-2.7359	0.0344	-0.6217
D6	0.0116	0.0623	0.1242	-9.9436	0.0655	-3.1860	0.0231	0.4687
D7	0.0161	0.1460	0.1567	-12.8398	0.1166	-8.2202	0.1022	-6.8509
D8	0.0082	0.1709	0.1854	-16.3993	0.1159	-8.3976	0.0698	-3.8745
D9	-0.0032	0.0983	0.2245	-21.2338	0.1677	-14.6935	0.1576	-13.7878
D10	**-0.0240**	**0.0108**	**0.2168**	**-23.8252**	**0.1464**	**-15.7132**	**0.1156**	**-12.3333**
D1-D10	**0.0296**	**-0.0389**	**-0.1148**	**14.8871**	**-0.0970**	**12.8338**	**-0.1080**	**13.2838**

	CAPM - α	CAPM - β	FF3 - α	FF3 - β	FF4 - α	FF4 - β	FF5 - α	FF5 - β
Panel G: 1980-1981 expansion period								
D1	**0.0083**	**-0.1077**	**0.0068**	**0.4381**	**0.0073**	**0.4860**	**0.0069**	**0.4445**
D2	0.0076	-0.1763	0.0094	-0.0416	0.0092	-0.0641	0.0077	0.1545
D3	0.0119	-0.1256	0.0138	-0.1342	0.0130	-0.2327	0.0123	0.0586
D4	0.0212	-0.0637	0.0145	0.1838	0.0128	-0.0078	0.0134	0.2813
D5	0.0104	0.0620	0.0051	0.0974	0.0043	-0.0003	0.0043	0.2229
D6	0.0073	-0.2223	-0.0033	0.0299	-0.0032	0.0414	-0.0034	0.1389
D7	0.0309	-0.1395	0.0149	0.4075	0.0176	0.7098	0.0169	0.4707
D8	0.0192	-0.2058	0.0056	-0.6395	0.0041	-0.8093	0.0031	-0.3234
D9	0.0066	-0.2409	-0.0034	-0.4545	-0.0018	-0.2807	-0.0046	-0.1850
D10	**0.0058**	**-0.0959**	**-0.0112**	**-0.2151**	**-0.0050**	**0.4784**	**-0.0076**	**-0.2405**
D1-D10	**0.0025**	**-0.0119**	**0.0181**	**0.6532**	**0.0123**	**0.0076**	**0.0145**	**0.6850**
Panel H: 1981-1982 recession period								
D1	**0.0105**	**0.5272*** **	**0.0054**	**0.6900** **	**-0.0073**	**0.8125*** **	**-0.0033**	**0.8728** **
D2	0.0126	0.4417**	0.0078	0.5860**	-0.0048	0.7083**	-0.0105	0.9627***
D3	0.0121	0.2920	0.0043	0.5408*	-0.0120	0.6982**	-0.0166	0.9491**
D4	0.0098	0.1879	0.0040	0.3675	-0.0109	0.5110	-0.0149	0.7575*
D5	0.0101	0.2503	0.0002	0.5638	-0.0167	0.7268**	-0.0258	1.1195**
D6	0.0099	0.2227	-0.0025	0.6194*	-0.0190	0.7796**	-0.0290	1.1603**
D7	0.0094	0.3225	0.0004	0.6103	-0.0228	0.8349*	-0.0430	1.4005**
D8	0.0031	0.4232	-0.0103	0.8410*	-0.0309	1.0407**	-0.0495*	1.5587**
D9	-0.0075	0.2929	-0.0146	0.4960	-0.0356	0.6990	-0.0553*	1.2399*
D10	**-0.0173**	**0.4720**	**-0.0299**	**0.8652***	**-0.0499** **	**1.0588** **	**-0.0720** **	**1.5626*** **
D1-D10	**0.0279** **	**0.0552**	**0.0353** **	**-0.1752**	**0.0427** **	**-0.2463**	**0.0687*** **	**-0.6898** **

	CAPM - α	CAPM - β	FF3 - α	FF3 - β	FF4 - α	FF4 - β	FF5 - α	FF5 - β
Panel I: 1982-1990 expansion period								
D1	**0.0081****	**0.0921**	**0.0096****	**0.0340**	**0.0091****	**0.0225**	**0.0124*****	**0.0133**
D2	0.0108**	0.0997	0.0117**	0.0733	0.0111**	0.0605	0.0157***	0.0448
D3	0.0086*	0.0770	0.0095*	0.0474	0.0092	0.0396	0.0144**	0.0121
D4	0.0104*	0.0420	0.0118**	0.0061	0.0115*	-0.0009	0.0170***	-0.0300
D5	0.0079	0.0812	0.0088	0.0525	0.0087	0.0494	0.0142**	0.0147
D6	0.0094	0.0878	0.0113*	0.0197	0.0109*	0.0099	0.0190***	-0.0347
D7	0.0100	0.0961	0.0109	0.0692	0.0110	0.0726	0.0172**	0.0251
D8	0.0103	0.1461	0.0110	0.1271	0.0113	0.1328	0.0180**	0.0779
D9	0.0040	0.2530*	0.0070	0.1378	0.0073	0.1449	0.0150*	0.0806
D10	**0.0015**	**0.4306*****	**0.0035**	**0.3704****	**0.0037**	**0.3757****	**0.0072**	**0.3465***
D1-D10	**0.0058**	**-0.3377*****	**0.0053**	**-0.3331****	**0.0045**	**-0.3505****	**0.0043**	**-0.3293****
Panel J: 1990-1991 recession period								
D1	**0.0159****	**0.4344*****	**0.0206***	**0.5051**	**0.0212**	**0.4478**	**0.0191**	**0.4535**
D2	0.0197	0.5445**	0.0206	0.8179	0.0210	0.7803	0.0204	0.6003
D3	0.0162	0.5511*	0.0118	0.8019	0.0143	0.5743	-0.0013	0.5054
D4	0.0222	0.6939**	0.0157	0.8338	0.0176	0.6632	0.0065	0.5164
D5	0.0307*	0.8881**	0.0298	0.9534	0.0325	0.7136	0.0176	0.5858
D6	0.0295	0.7802*	0.0288	1.2510	0.0332	0.8575	0.0084	0.6853
D7	0.0418*	1.1847**	0.0366	1.3497	0.0411	0.9449	0.0137	0.8566
D8	0.0268	1.1769**	0.0311	1.3947	0.0363	0.9273	0.0032	0.8982*
D9	0.0391	1.1876**	0.0359	1.2675	0.0397	0.9278	0.0203	0.6445
D10	**0.0334**	**1.2507***	**0.0554**	**1.6764**	**0.0630**	**0.9923**	**0.0157**	**0.6987**
D1-D10	**-0.0175**	**-0.8163**	**-0.0348**	**-1.1714**	**-0.0417**	**-0.5445**	**0.0035**	**-0.2452**

	CAPM - α	CAPM - β	FF3 - α	FF3 - β	FF4 - α	FF4 - β	FF5 - α	FF5 - β
Panel K: 1991-2001 expansion period								
D1	**0.0069*** **	**0.0095**	**0.0051** **	**0.0966**	**0.0059** **	**0.1037**	**0.0053** **	**0.0887**
D2	0.0093***	0.0067	0.0078**	0.0745	0.0087***	0.0829	0.0080**	0.0701
D3	0.0102***	0.0624	0.0089**	0.1244	0.0093**	0.1283	0.0094**	0.0934
D4	0.0112***	0.0066	0.0100***	0.0642	0.0105***	0.0694	0.0097**	0.0844
D5	0.0117***	-0.0076	0.0101**	0.0707	0.0097**	0.0668	0.0095**	0.1176
D6	0.0144***	-0.0406	0.0128**	0.0354	0.0134**	0.0408	0.0123**	0.0737
D7	0.0158***	0.1006	0.0152***	0.1204	0.0139**	0.1085	0.0137**	0.2344
D8	0.0132*	0.0871	0.0122*	0.1284	0.0108	0.1151	0.0102	0.2829
D9	0.0153*	0.1666	0.0154*	0.1472	0.0119	0.1148	0.0134	0.3060
D10	**0.0200***	**0.1598**	**0.0204***	**0.1160**	**0.0160**	**0.0754**	**0.0170**	**0.3902**
D1-D10	**-0.0131**	**-0.1503**	**-0.0153**	**-0.0194**	**-0.0101**	**0.0283**	**-0.0118**	**-0.3016**
Panel L: 2001-2001 recession period								
D1	**0.0154**	**0.3510**	**0.0198**	**0.3874**	**0.0202**	**0.3481**	**0.0334** **	**-0.2569**
D2	0.0152	0.2900	0.0187	0.2418	0.0229	-0.2408	0.0394*	-0.8325
D3	0.0219	0.3251	0.0269	0.2100	0.0303	-0.1733	0.0577*	-1.0492*
D4	0.0214	0.2388	0.0285	0.0262	0.0279	0.1005	0.0606*	-1.3693*
D5	0.0294	0.2939	0.0372	-0.0121	0.0371	-0.0047	0.0794*	-1.6300*
D6	0.0337	0.2543	0.0481	0.1666	0.0461	0.3964	0.0938*	-1.6300*
D7	0.0328	0.4512	0.0490	0.1987	0.0513	-0.0675	0.1079*	-1.8299*
D8	0.0301	0.1672	0.0439	-0.0091	0.0451	-0.1483	0.1081*	-2.2045
D9	0.0219	0.2656	0.0332	0.0807	0.0381	-0.4847	0.1123	-2.5021
D10	**0.0266**	**-0.1448**	**0.0399**	**-0.4550**	**0.0520**	**-1.8474**	**0.1296**	**-3.3116***
D1-D10	**-0.0112**	**0.4958**	**-0.0201**	**0.8424**	**-0.0318**	**2.1955**	**-0.0962**	**3.0547***

	CAPM - α	CAPM - β	FF3 - α	FF3 - β	FF4 - α	FF4 - β	FF5 - α	FF5 - β
Panel M: 2001-2007 expansion period								
D1	**0.0081*** **	**0.2518*** **	**0.0077** **	**0.2765*** **	**0.0080** **	**0.2445** **	**0.0075** **	**0.2899** **
D2	0.0086**	0.2776**	0.0086**	0.2965**	0.0087**	0.2762*	0.0085**	0.2990*
D3	0.0100**	0.3056**	0.0103**	0.3212**	0.0106**	0.2870*	0.0101**	0.3533**
D4	0.0095*	0.3195**	0.0100*	0.3403**	0.0104**	0.3029*	0.0099*	0.3584*
D5	0.0101*	0.3433**	0.0115**	0.3685**	0.0120**	0.3170	0.0111*	0.4025*
D6	0.0105*	0.4053**	0.0121*	0.4390**	0.0125*	0.3947*	0.0119*	0.4791*
D7	0.0088	0.4333**	0.0098	0.4562**	0.0102	0.4156*	0.0096	0.4882*
D8	0.0140*	0.4793**	0.0159*	0.4973**	0.0157*	0.5126*	0.0151*	0.5793*
D9	0.0152*	0.7002***	0.0183**	0.7003**	0.0181*	0.7206**	0.0173*	0.8382**
D10	**0.0188***	**0.6218***	**0.0226** **	**0.6453***	**0.0230** **	**0.5989**	**0.0216***	**0.7256**
D1-D10	**-0.0107**	**-0.3700**	**-0.0149**	**-0.3688**	**-0.0150**	**-0.3544**	**-0.0141**	**-0.4357**
Panel N: 2007-2009 recession period								
D1	**-0.0030**	**0.3524***	**0.0023**	**0.5138** **	**0.0029**	**0.5254***	**-0.0031**	**0.5318**
D2	0.0014	0.4460*	0.0116	0.7371**	0.0129	0.7630*	0.0053	0.7082
D3	-0.0002	0.4836	0.0106	0.8059**	0.0105	0.8044	0.0077	0.7836
D4	0.0020	0.5202*	0.0112	0.7959**	0.0086	0.7451	0.0053	0.7557
D5	0.0059	0.6350*	0.0172	1.0031**	0.0151	0.9611*	0.0255	0.9894
D6	0.0109	0.7287*	0.0245	1.1402**	0.0220	1.0919*	0.0309	1.0581
D7	0.0106	0.6704*	0.0230	1.0847**	0.0211	1.0459*	0.0257	1.1277
D8	0.0168	0.8829**	0.0317	1.3148**	0.0245	1.1708*	0.0368	1.1352
D9	0.0328	1.1590**	0.0412	1.4477**	0.0284	1.1916	0.0418	1.3192
D10	**0.0364**	**1.1621** **	**0.0447**	**1.4819** **	**0.0334**	**1.2564**	**0.0453**	**1.0957**
D1-D10	**-0.0394**	**-0.8097** **	**-0.0424**	**-0.9680***	**-0.0305**	**-0.7310**	**-0.0484**	**-0.5639**

	CAPM - α	**CAPM - β**	**FF3 - α**	**FF3 - β**	**FF4 - α**	**FF4 - β**	**FF5 - α**	**FF5 - β**
Panel N: 2009-2020 expansion period								
D1	**0.0141*****	**-0.1278***	**0.0141*****	**-0.1277***	**0.0144*****	**-0.1432***	**0.0140*****	**-0.1246***
D2	0.0137***	-0.1516*	0.0135***	-0.1246	0.0139***	-0.1448	0.0134***	-0.1201
D3	0.0141***	-0.1752*	0.0135***	-0.1260	0.0139***	-0.1462	0.0137***	-0.1287
D4	0.0140***	-0.1614	0.0136***	-0.1310	0.0141***	-0.1528	0.0137***	-0.1309
D5	0.0134***	-0.1043	0.0131***	-0.0669	0.0135***	-0.0883	0.0132***	-0.0674
D6	0.0127**	-0.1736	0.0121**	-0.1259	0.0126**	-0.1501	0.0122**	-0.1258
D7	0.0112**	-0.1103	0.0106*	-0.0623	0.0111*	-0.0860	0.0107*	-0.0616
D8	0.0127**	-0.0623	0.0123**	-0.0269	0.0129**	-0.0594	0.0120*	-0.0136
D9	0.0115*	0.0229	0.0112*	0.0675	0.0120*	0.0251	0.0118*	0.0525
D10	**0.0115**	**-0.0061**	**0.0105**	**0.0902**	**0.0111**	**0.0598**	**0.0110**	**0.0812**
D1-D10	**0.0026**	**-0.1217**	**0.0036**	**-0.2179**	**0.0033**	**-0.2030**	**0.0030**	**-0.2058**
Panel O: 2020-2020 recession period								
D1	**0.0068*****	**0.1440*****	**0.0069*****	**0.1365*****	**0.0073*****	**0.1256*****	**0.0072*****	**0.1240*****
D2	0.0077***	0.1230***	0.0078***	0.1128***	0.0084***	0.0989**	0.0080***	0.1070***
D3	0.0079***	0.1275***	0.0080***	0.1240***	0.0088***	0.1056**	0.0085***	0.1036**
D4	0.0089***	0.1242***	0.0089***	0.1075**	0.0096***	0.0909*	0.0094***	0.0883*
D5	0.0092***	0.1401***	0.0092***	0.1374***	0.0099***	0.1202**	0.0094***	0.1276**
D6	0.0108***	0.1577***	0.0108***	0.1457***	0.0116***	0.1269**	0.0112***	0.1278**
D7	0.0115***	0.1682***	0.0118***	0.1408**	0.0125***	0.1234**	0.0125***	0.1200*
D8	0.0128***	0.2294***	0.0131***	0.2018***	0.0137***	0.1882**	0.0136***	0.1900**
D9	0.0099***	0.3065***	0.0102***	0.2693***	0.0111***	0.2478***	0.0115***	0.2252***
D10	**0.0100*****	**0.3621*****	**0.0106*****	**0.2957*****	**0.0111*****	**0.2837*****	**0.0108*****	**0.2967*****
D1-D10	**-0.0033**	**-0.2186*****	**-0.0039**	**-0.1619****	**-0.0039**	**-0.1601****	**-0.0038**	**-0.1751****

*t-statistics in parentheses: * p<0.10, ** p<0.05, *** p<0.01*

Note: The table displays the results of each strategy's returns through the business cycle regressed on the CAPM, FF 3-factor model, FF 4-factor, and FF 5-factor models. D1 contains the least volatile portfolio and D10 is the most volatile portfolio. D1-D10 is the top-minus-bottom decile portfolio. All portfolios are equal-weighted. The dataset used is the US stock observations from 1963 to 2020. The excess returns based on the CAPM, FF3-, FF4- , and FF5- factors are included on the table. Besides, the market Beta of CAPM, FF3-, FF4-, and FF5- factor models are presented on the table. The period is divided into sub-periods. There are two types of sub-periods, which include the expansion period and the recession period. All portfolios are zero-investment top-minus-bottom decile portfolios. The numbers above are monthly returns. RMRF presents the excess returns for the market. SMB denotes the Small-Minus-Big factor, while HML is the High-Minus-Low factor of the asset pricing model. Momentum factor is displayed by UMD. RMW and CMA present the profitability and investment factor respectively.