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Cross-sectional pricing of volatility and a model-free volatility risk premium

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Abstract

In this research I find that the negative pricing of systematic and idiosyncratic volatility, as found by Ang, Hodrick, Xing and Zhang (2009), is still present in an updated 1990-2020 data sample. A volatility risk premium (*VRP*) measure, computed as the implied volatility minus the realized volatility is also negatively priced in the cross-section. The negative price of the *VRP* is in line with theory's predictions, if *VRP* is indeed a proxy for risk-aversion. The negative pricing of *VRP*, as well as the negative price on systematic volatility, in the cross-section is more pronounced in times of crisis, as identified by the NBER recession indicator.

Keywords: Systematic Volatility, Idiosyncratic Volatility, Volatility Risk Premium, Risk Premium, Risk-Aversion

Contents

1 Introduction

How volatility on individual assets and on the stock market as a whole impact asset prices is a widely researched topic. The intertemporal CAPM (ICAPM) of Merton (1973) predicts a time-series relationship between risk and expected returns, as well as a cross-sectional one. Previous work such as French, Schwert and Stambaugh (1987) and Campbell & Hentschel (1992) show a negative relation between unexpected market returns and expected volatility, especially in times of market crashes. Glosten, Jagannathan & Runkle (1993) also find a negative correlation between the conditional mean and conditional volatility of excess return on stocks, using a GARCH-M framework to model stochastic volatility of stock returns. In this paper I investigate whether this negative relation between expected market volatility and market returns is also present in the cross-section.

Ang, Hodrick, Xing and Zhang (2006) (henceforth AHXZ) extend the relation between stock returns and market volatility by analyzing the cross-section of U.S stock returns. In their paper they document a negative risk premium on systematic volatility using innovation in the *VIX*, an index of future expected volatility based on option prices, as their measure for systematic market volatility. Their interpretation for this negative coefficient is that risk-averse investors pay a premium to hedge against volatility increases. AHXZ also report significant negative returns for a portfolio that goes long stocks with a high sensitivity to systematic volatility, while going short in stocks with a low sensitivity to systematic volatility. They argue that investors want to hedge against volatility since high volatility is associated with downward market movements. Therefore stocks with high sensitivity to innovations in volatility have a higher price, and thus lower expected return. Apart from investigating the cross-sectional effect of the systematic volatility, AHXZ also test the effect of idiosyncratic volatility on the distribution of stock returns. By doing this the authors attempt to capture the price of un-systematic volatility risk that is not priced in the standard asset pricing models. They find that a zero-investment strategy with maximum exposure to idiosyncratic risk relative to the Fama & French (1993) three factor model shows a negative monthly return of 1,06%. This negative return is interpreted as puzzling, since behavioral (Barbaris & Huang, 2001) and imperfect-information (Merton, 1987) theory predicts that investors require higher returns for holding high idiosyncratic volatility stocks. The robust negative premium on systematic volatility and negative returns on the zero-investment strategy found by AHXZ are computed over U.S. stocks during 1986 to 2000. The first goal of this research is to test the robustness of their findings in an updated data sample, using the cross-section of U.S stocks over the period 1990-2020. This leads to the first hypothesis; *The negative price of systematic volatility and puzzling high (low) returns on low (high) idiosyncratic volatility stocks found by AHXZ are present in the 1990-2020 period.*

Apart from testing the robustness of AHXZ's results in newer data, I also employ an alternative way of measuring changes in market volatility, and idiosyncratic volatility. As stated by AHXZ their measure for systematic volatility (the *VIX* index) includes jump components and can not distinguish between the stochastic volatility and the volatility risk premium. They do not make any specification to address this for the sake of simplicity. I will instead follow the methodology of Bollerslev, Tauchen & Zhou (2009) (Henceforth BTZ) by distinguishing between implied and realized variance (volatility squared). In their paper BTZ find that the variance risk premium, defined as implied variance minus realized variance, explains more than fifteen percent of the market return ex-post. This research aims to answer whether a volatility premium measure (*VRP*), equal to the square root of BTZ's variance risk premium, explains cross-sectional differences in stock prices. This question result in the second hypothesis; *The VRP is a cross-sectionally priced factor.*

The implied volatility is defined as the ex-ante risk-neutral expected future volatility. It can be measured by the *VIX* since this index is based on option prices. The realized volatility on the other hand is based on observed return data, and therefore gives an accurate ex-post view of the return volatility. Then, subtracting the ex-post realized volatility from the ex-ante implied volatility, results in the *VRP* (BTZ, Drechsler & Yaron (2011)). Both of these volatility measures are so-called "model-free" volatilities, since they are not based on Black-Scholes based option pricing models.

Influential papers such as Fama, Schwert and Stambaugh (1987) and Campbell & Hentschel report the variability of stock market volatility. Especially in times of unexpected low market returns, volatility is often high. I take a detailed look of the cross-sectional effects of systematic and un-systematic volatility by comparing the result of these effects for the full sample and during crisis months, defined by the NBER recession indicator. In these crisis months, expected volatility, measured by the *VIX*, is often high. The *VRP* is often referred to as a measure for risk aversion in the market (BTZ), which is generally also high in times of crisis. Hence the third hypothesis reads; *The cross-sectional effects of systematic and un-systematic volatility, as well as the VRP, are more pronounced in crisis months.*

If the premium on systematic volatility risk, proxied by the *VIX*, is negative, stocks with high (low) sensitivity to the *VIX* should have low (high) returns. In the 1990-2020 data sample negative risk-adjusted returns are found for a difference portfolio that goes long in the 20% highest sensitivity to *VIX* and shorts the 20% lowest sensitivity stocks. Controlling for the FF-3 model, this portfolio results in an alpha of -0,38% per month (t-stat -2,08). The risk premium associated with a factor based on innovations in *VIX* found in the sample is -1,85% per month (t-stat -2,84). A similar difference portfolio is formed using idiosyncratic volatility as sorting variable, resulting in a negative FF-3 alpha of -0,98% per month (t-stat -4,17). These results indicate that AHXZ's findings in their 1986-2000 dataset are still present in a newer sample. Using the model-free measure *VRP* as sorting variable, I find similar return patterns on the difference portfolio. The cross-sectional risk premium found for the *VRP* is -2,35% per month with a t-stat of -3,27. Moreover, this risk factor is more robust to adding additional factors to the estimation model, compared to the systematic risk factor of AHXZ. Finally, when comparing these results in crisis months, the estimated risk premia for the *VIX* and *VRP* measures are more pronounced in times of crisis. The difference portfolios show negative returns that are statistically insignificant, which can be attributed to the low number of observations in the crisis subsample.

The next section described the theoretical derived expectations for the cross-sectional pricing of *VIX* and *VRP*. In the data section I describe the collection of data that is needed to perform the methods which are laid out in the methodology section. The result section provides an overview of the empirical results that result from this methodology, which are carefully interpreted. The final section concludes.

2 Theoretical motivation

2.1 Pricing of changes in systematic volatility

Influential papers of the previous century, such as Merton (1973) and Campbell (1993) have established theoretical models in which variables that contain information about future market returns command a risk premium. In these models investors are assumed to maximize their lifetime utility. These theories are based on the set of investment opportunities and aggregate consumption levels. Assets that covary positively (negatively) with increases in future return forecasts have higher (lower) returns, assuming risk averse representative agents. The underlaying intuition is that investors want to hedge against a low payout in a bad state of the economy, proxied by the return on the market portfolio. Thus, when an asset pays out when market returns are low, signaling a bad state of the economy, investors will pay a premium to hold this asset, leading to lower expected returns on said asset. Chen (2002) shows that investors do not only want to hedge against lower future returns, but also against higher future volatility. Chen argues that a decrease in future volatility allows investors to increase their consumption today, as they reduce their precautionary savings.

Chen theoretically derives an equilibrium pricing relation, using the Euler equation. This model assumes rational investors, and perfect capital markets without transaction costs, short-sales constraints or any other market imperfections. Also it assumes the presence of a riskless asset, and the law of one price to hold. For a detailed derivation see Chen (2002). The expected return is then given by;

$$E_t[r^e_{i,t+1}] + \frac{V_{ii,t}}{2} = \gamma V_{iw,t} + (\gamma - 1)V_{ih,t} - \tfrac{1}{2}(\gamma - 1)^2 V_{iv,t} \tag{1}$$

An asset's return therefore depends on its covariance with three factors; $V_{iw,t}$ (asset's covariance with the market), $V_{ih,t}$ (asset's covariance with changes in the forecasts of future market returns) and $V_{iv,t}$ (asset's covariance with changes in the forecast of future market volatility). The covariance of an asset with changes in forecasts of future volatility enters negatively in the return equation, since risk-averse investors tend to increase their precautionary savings in times of high volatility which lowers current consumption. Assets that positively covary with changes in the VIX index, which is based on investors expected future volatility, should then earn lower returns.

2.2 Pricing of changes in *VRP*

The *VRP* is computed as the difference between a measure of risk-neutral expectations of future volatility and an ex-post realized (physical) volatility measure. Because this measure displays the difference between investor expectation actual observed volatilities, the *VRP* is often interpreted as a measure of risk-aversion (BTZ). Bakshi & Madan (2006) formally show that the volatility spread, a similar measure to the *VRP,* defined as the relative difference of implied and physical volatilities, can be expressed as a non-linear function of risk-aversion. Assuming a quadratic pricing kernel and power utility, they show how the volatility is approximated by the following relationship;

$$\frac{\sigma^2_{rn}(t,\tau) - \sigma^2_p(t,\tau)}{\sigma^2_p(t,\tau)} \approx -\gamma[\sigma^2_p(t,\tau)]^{1/2} \times \theta_p(t,\tau) + \frac{\gamma^2}{2}[\sigma^2_p(t,\tau)] \times [\boldsymbol{\kappa_p}(\boldsymbol{t},\tau) - 3] \tag{2}$$

Here, the left hand expression is Bakshi & Madan's definition of volatility spread. The $\theta_p(t,\tau)$ and $\kappa_p(t,\tau)$ represent the physical skewness and kurtosis, respectively. The coefficient of relative risk aversion is denoted by γ. Therefore the divergence of risk-neutral 'implied' volatility and physical 'realized' volatility can be attributed to exposure to tail events, fatter left-tails of the physical distribution, and the risk-averse behavior of investors (Bakshi & Madan, 2006).

Analyzing the exact relation between a volatility measure such as the volatility spread or *VRP* and the investor risk-aversion requires careful modelling of the relationship. In this research I focus on the cross sectional pricing of the *VRP* measures, therefore it is sufficient to establish a positive relation between *VRP* and risk-aversion.

In order to justify a risk premium on risk-aversion, it must be allowed to vary over time. Instead of assuming a constant risk-aversion γ as in Chen's (2002) model, using habit persistent utility function as introduced by Campbell (1996), risk-aversion can vary over time. In Campbell's habit persistence model, investors maximize the following intertemporal utility function;

$$E\sum_{t=0}^{\infty}\delta_t\frac{(C_t - X_t)^{1-\alpha} - 1}{1-\alpha} \tag{3}$$

Here C_t and X_t are defined as consumption and level of habit, respectively. Now assume that the habit is external and the law of one price holds. Also, assume that a stochastic discount factor m_t prices the assets so that $E_t[m_{t+1}R_{t+1}] = 1$. Using the above utility function, the stochastic discount factor can be expressed as;

$$m_{t+1} = \delta(\frac{C_t}{C_{t+1}})^{\alpha} \times (\frac{\gamma_{t+1}}{\gamma_t})^{\alpha} \tag{4}$$

For a detailed derivation see Nyberg & Wilhelmsson (2010). Given the assumption that habit is exogenous, it is not affected by current consumption. As long as this habit level is positive, the relative risk-aversion γ correlates negatively with consumption, given Campbell's utility function. Consequently, the two bracketed terms in equation (4) are positively correlated. In the finance literature, it is well known that an asset's covariance with the stochastic discount factor plays a role in determining it's equilibrium asset price. An asset that correlates positively with the SDF has lower expected returns. Then, from equation (4) is follows that assets that covary positively (negatively) with changes in risk-aversion must have lower (higher) lower returns, since assets that covary positively with risk-aversion also covary positively with the SDF.

In finance literature, the SDF is described as a proxy for the state of the economy. In a bad state, a high SDF is found. Since the model predicts that risk-aversion positively influences the SDF, risk-aversion can be seen as a proxy for the state of the economy. Intuitively, a negative risk premium on risk-aversion then makes sense. If an asset's return covaries positively with changes in risk-aversion, it earns high returns in bad states of the economy (high SDF), when risk-averse investors value them most. As these assets are considered as a hedge against bad states of the economy, investors pay a premium to hold these assets, leading to a low return on these assets. Hence, the *VRP*, as a proxy for risk-aversion, is expected to be negatively priced in the cross section. This will be investigated in the empirical application of this research.

3 Data

The *VIX* index, obtained directly from the CBOE, is used as a measure for expected volatility. This index is based on S&P 500 index option bid/ask quotes, and measures the expected volatility of the S&P 500 index over the next 30 days. Since the *VIX* is available from 1990 onwards, data is collected from January 1990 to December 2020. To replicate AHXZ's methods to test the pricing of systematic volatility, The *VIX* index is converted to daily innovation in the *VIX* (denoted as $dVIX$), measured as the daily change of the *VIX* index. This transformation is done due to high autocorrelation in the *VIX* index, as shown by AHXZ.

In order to distinguish between stochastic aggregate volatility and the *VRP*, ex-post realized volatility is subtracted from the *VIX* index. BTZ compute the realized volatility as the sum of the squares of five-minute returns on the S&P 500 index. The intraday data on S&P 500 prices is not available publicly, therefore other measures are used in this research. Shu & Zhang (2003) investigate four different ways to capture realized volatility. The first measure is the traditional close-to-close estimator of volatility. Second, an extreme value estimator introduced by Parkinson (1980), using daily high and low prices is computed. Third, Yang & Zhang (2000)'s volatility measure, based on daily high and low as well as open and close prices, is tested. And last, an intraday volatility measure based on 5-minute returns is used, similar to BTZ. Considering availability of data, I will employ the first three realized volatility measures. In order to compute these measures daily closing, open high and low prices of the S&P 500 index are obtained from the CRSP database.

To test the effect of volatility risk on the cross-sectional pricing of U.S stocks I collect the prices of all AMEX, NYSE and NASDAQ listed common stocks from 1990 through 2020 from the CRSP database. Other asset classes such as ADRs, REITs, closed-end funds, and units of beneficial interests are excluded. The common Fama & French risk factors used in the analyses described in the next section are downloaded from the

Kenneth R. French library[1]. The liquidity factor *LIQ*, as defined by Pastor & Stambaugh (2003), is downloaded from Robert F. Stambaugh's website[2].

4 Methodology

4.1 Systematic volatility

To examine the pricing of systematic volatility in recent years I follow the methodology used by AHXZ. The cross section of U.S stocks are regressed on a market factor *Mkt* and innovation in the *VIX* index *dVIX*, as specified in equation (5). The market factor resembles the excess return on the CRSP value weighted market index.

$$r_t^i = \propto^i + \beta_{Mkt}^i Mkt + \beta_{dVIX}^i dVIX + \varepsilon_t^i \qquad (5)$$

The coefficient β_{dVIX} as specified in equation (5), resembles a stock's sensitivity to innovations in systematic volatility. This sensitivity is estimated over the past one month window, with a daily return frequency. This window allows for time-varying factor loadings while maintaining a sufficient number of observations. Only those stocks which have available price levels for at least 15 observations in the estimation month are included. The stocks are then sorted into five quintile portfolios based on their sensitivity β_{dVIX}, applying monthly rebalancing. The portfolios are value-weighted, using a stock's market equity relative to the portfolio's total market equity, at the beginning of the respective month. The market equity values are winsorized at the first and 99th percentile, in order to maintain reasonable stock weights. Quintile 1 resembles the portfolio with the lowest *dVIX* factor loadings, while quintile 5 has the highest loadings. According to previous research the return on a portfolio with high (low) sensitivity to the volatility measure should be low (high). Therefore the portfolio return difference of quintiles 5-1 is expected to be negative. As well as negative returns, a negative alpha with relation to various asset pricing model is expected.

A negative return on the 5-1 portfolio indicates the factor loadings during the pre-formation period have an effect on future returns. However, in order to establish a risk factor explanation, the relation between sensitivity to innovations in volatility and returns (alphas) must be shown in the same period. To estimate the premium associated with the volatility risk, AHXZ and other research such as Breeden et al. (1989) and Lamont (2001) construct a factor mimicking portfolio that aims to replicate the

[1] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html
[2] http://finance.wharton.upenn.edu/~stambaug/

innovation in *VIX*. Hereby the price of volatility risk can be estimated at any frequency by cumulating the daily returns on this portfolio. To construct the factor mimicking portfolio the following regression is used:

$$dVIX_t = c + b'X_t + \varepsilon_t \tag{6}$$

X_t represents the base assets at time t, and b the portfolio weights needed to mimic the innovations in *VIX*. $b'X_t$ equals the daily return on the factor mimicking portfolio, denoted as *FVIX*. The weights b are rebalanced monthly. The quintile portfolios based on sensitivity to *dVIX*, as described earlier, are used as the base assets X_t.

To assess the premium related to the systematic volatility factor, 25 test portfolios, comprised of the universe of U.S stocks listed on the AMEX, NYSE and NASDAQ, are regressed on multiple factors over the sample period (1990-2020). These 25 portfolios are the result of a 5x5 double sort first by market beta, using the CRSP value weighted index as market portfolio, and then by sensitivity to innovations in systematic volatility, β_{dVIX}. Using the Fama-Macbeth (1973) procedure it is possible to control for a set of different factors in determining the *FVIX* premium. I follow AHXZ using the FF3 model with the addition of a momentum factor *UMD* as constructed by Fama & French (2018), a liquidity factor *LIQ* as introduced by Pastor & Stambaugh (2003) and the *FVIX* factor. Naturally, the FF-3 factors are used as the FF-3 model was the commonly agreed upon model at the time of AHXZ's research. The addition of the *LIQ* and *UMD* factors allows controlling for momentum and liquidity effects, leading to a more accurate systematic volatility factor premium. Alternatively I also estimate the factor premia using the FF5 model, including a profitability factor *RMW*, and an investment factor *CMA*, as constructed by Fama & French (2015) as well as the *UMD, LIQ and FVIX* factors. This results in equation (7);

$$r_t^i = c + \beta_{Mkt}^i \lambda_{Mkt} + \beta_{SMB}^i \lambda_{SMB} + \beta_{HML}^i \lambda_{HML} + \beta_{RMW}^i \lambda_{RMW} + \beta_{CMA}^i \lambda_{CMA} + \beta_{UMD}^i \lambda_{UMD} + \beta_{LIQ}^i \lambda_{LIQ} + \beta_{FVIX}^i \lambda_{FVIX} + \varepsilon_t^i \tag{7}$$

Estimating the factor premia is done in two steps. First the betas used in equation (7) are estimated using cross-sectional multivariate regressions. These betas are then used to estimate the factor premia λ for each factor at each month t. the factor premia are averaged over time and checked for significance. To check the results for robustness of the model specifications, I estimate the factor premia using four models. The first model being the FF3 factors and the *FVIX* factor, after which the remaining factors;

UMD, *LIQ*, *RMW* and *CMA* are added to each subsequent model, to arrive at equation (7).

To asses whether a cross-sectional pricing effect is present for the *VRP*, I differentiate between implied and realized volatility using three different measures of realized volatility (*RV)* as defined by Shu & Zhang (2003). To be able to compare the implied and realized volatility, they should be based off of the same market index, namely the S&P500. The measures for *RV* are computed on a monthly basis using daily price data, and multiplied by 100 to compare them to the *VIX* index. The first measure RV_1 is based on close-to-close daily S&P 500 prices and computed as:

$$RV_1 = \sqrt{\frac{d}{n}\sum_{t=1}^{n} r_t^2} \times 100 \tag{8}$$

Where r_t is the return on day t calculated by the natural logarithm of the ratio of two consecutive closing prices, and d the number of trading days in the estimation month. RV_2 Resembles the Parkinson estimator based on high and low daily prices and is computed as:

$$\mathrm{RV}_2 = \sqrt{\frac{\mathrm{d}}{\mathrm{n}}\sum_{\mathrm{t}=1}^{\mathrm{n}} \frac{1}{4\ln 2}(\ln \mathrm{H_t} - \ln \mathrm{L_t})^2} \times 100 \tag{9}$$

Daily high and low prices of the S&P500 are denoted as H_t and L_t, respectively. Finally the Yang and Zhang estimator, RV_3, is computed as:

$$RV_3 = \sqrt{d} * \sqrt{V_o + kV_c + (1-k)V_{RS}} \times 100 \tag{10}$$

V_0, V_c and V_{RS} from equation (10) are defined as follows:

$$V_o = \frac{1}{n-1}\sum_{t=1}^{n}(o_t - o)^2, \qquad o_t = ln\, O_t - ln\, O_{t-1}, \qquad o = \frac{1}{n}\sum_{t=1}^{n} o_t\ ,$$

$$V_c = \frac{1}{n-1}\sum_{t=1}^{n}(c_t - c)^2, \qquad c_t = ln\, C_t - ln\, C_{t-1}, \qquad c = \frac{1}{n}\sum_{t=1}^{n} c_t\ ,$$

$$V_{RS} = \frac{1}{n}\sum_{t=1}^{n}[(ln\, H_t - ln\, O_t)(ln\, H_t - ln\, C_t) + (ln\, L_t - ln\, O_t)(ln\, L_t - ln\, C_t)].$$

Here O_t and C_t are the daily open and close prices of the S&P 500 on date t. k from equation (10) is a constant, chosen to minimize the variance of the estimator, computed by

$$k = \frac{0.34}{1.34 + \frac{n+1}{n-1}}\ .$$

Using the three different measures of realized volatility as described by Shu & Zhang (2003), three realized volatility factors are constructed. A set of factors is constructed by subtracting the realized volatility measures from the implied volatility, leading to the *VRP* as described in equation (11). This *VRP* time-series, often interpreted as a measure of risk-aversion, is then translated into a risk factor using the same methodology as used to construct *FVIX*. Note that the implied volatility IV_t is forward looking, based on option prices over the period *t* to *t+1*. To estimate the *VRP* over this time period, the realized variance RV_{t+1}, must be subtracted. BTZ subtract the past month realized variance instead, in favor of a forecasting perspective. In this research, however, I aim to discover the distribution of the *VRP* in the cross-section instead of discovering profitable investment strategies or forecasting the state of the stock market. Therefore the *VRP* is computed as;

$$VRP_t^x \equiv IV_t - RV_{t+1} \tag{11}$$

4.2 Idiosyncratic volatility

The volatility measures discussed in the previous section are based on the volatility of the S&P 500 market index, which reflects the systematic component of volatility. AHXZ also investigate the influence of idiosyncratic volatility on the expected returns, defined as residual volatility relative to the Fama & French 3-factor model. In order to isolate the idiosyncratic stock volatility more effectively, I will instead consider idiosyncratic volatility with relation to the Fama & French (2018) 6-factor, adding a profitability, investment and momentum factor. Therefore the equation estimating the residuals is as follows:

$$r_t^i = \propto^i + \beta_{Mkt}^i Mkt + \beta_{SMB}^i SMB + \beta_{HML}^i HML + \beta_{RMW}^i RMW + \beta_{CMA}^i CMA + \beta_{UMD}^i UMD + \varepsilon_t^i \tag{12}$$

The market factor *Mkt* is defined as the excess return on the CRSP value-weighted market index. The other factors are constructed as in Fama & French (2018). The idiosyncratic volatility is then computed as $\sqrt{var(\varepsilon^i)}$. I investigate the pricing of idiosyncratic volatility by setting up a trading strategy based on the residual volatility, similar to AHXZ. Equation (12) is used to estimate the residuals on a daily frequency for the entire universe of U.S stocks that trade on the NYSE, AMEX and NASDAQ stock markets. The idiosyncratic volatility is estimated on a monthly bases using the previous month's residuals. Five value-weighted portfolios are formed using monthly rebalancing, sorted on idiosyncratic volatility. Quintile 5 resembles the 20% of firms with the highest idiosyncratic volatility, whereas quintile 1 resembles the 20% of lowest idiosyncratic volatility firms.

4.3 Volatility and crisis

In order to test the premia on systematic and idiosyncratic volatility in times of crisis, I define a subsample of crisis months. The full sample consists of 372 months over the 1990-2020 period, 44 of which are listed as ‘crisis months’, following the NBER recession indicator. These 44 months are the result of the early 1990s recession, the internet bubble of 2000, the credit crisis of 2007 and the corona crisis. The time series of factor premia related to the systematic volatility measures, as well as the return on the trading strategy based on idiosyncratic volatility are investigated in this subsample.

5 Empirical findings

In this section, the empirical findings resulting from the methods described in the research design are laid out and discussed. First, the pricing of systematic volatility is investigated, here I will discuss the different cross-sectional effects of sensitivity to the *VIX* index and the *VRP*, as defined in the data section. Secondly the returns of portfolios sorted on idiosyncratic volatility relative to the Fama & French (2018) 6-factor model are analyzed. Finally, the effects of both systematic and idiosyncratic volatilities are compared in times of expansion and recession.

5.1 Pricing of systematic volatility

5.1.1 The *VIX* and *VRP* measures

Summary statistics of the *VIX* index and the three measures of the *VRP* as well as their respective first order differences are given in table (1). Both the *VIX* and *VRP* measures are highly autocorrelated time series. The *VIX* has a first-order autocorrelation of 0,98, that of the *VRP* measures varies between 0,96 and 0,97. Therefore first order differences are used to analyze the sensitivities to these measures. The time series of the first order differences each have low autocorrelation and a mean close to zero (between 0,001 and 0,002). The analyses performed in the rest of this research are based on the first-order difference time series of volatility measures. Table (1) also reports each measure’s Z-value, resulting from the dickey-fuller test for stationarity. The null-hypothesis of this test states that a unit root is present and is rejected in all cases, meaning that each measure is stationary.

Table 1: Summary statistics

The table shows summary statistics over for the daily time series over the period 1990-2020. *VIX* refers to the CBOE's daily closing price on their *VIX* index. *VRP1, VRP2* and *VRP3* are the result of subtracting *RV1, RV2* and *RV3*, respectively from the *VIX* index. *dVIX, dVRP1, dVRP2 and dVRP3* are each of the measures first-order differences. The reported Z-score is the result of a dickey-fuller test for stationarity.

	Nominal series				First-order-differenes			
	VIX	*VRP1*	*VRP2*	*VRP3*	*dVIX*	*dVRP1*	*dVRP2*	*dVRP3*
mean	19.472	15.037	15.735	13.825	0.001	0.001	0.001	0.002
St. dev	8.116	6.290	6.615	5.960	1.636	1.636	1.627	1.637
skewness	2.196	1.845	2.119	1.937	1.476	1.702	1.703	1.786
kurtosis	11.265	9.561	10.922	10.241	31.637	32.389	32.678	32.356
AR(1)	0.980	0.966	0.970	0.962	-0.134	-0.122	-0.135	-0.145
Z-score	-7.343	-9.805	-9.098	-10.167	-	-	-	-

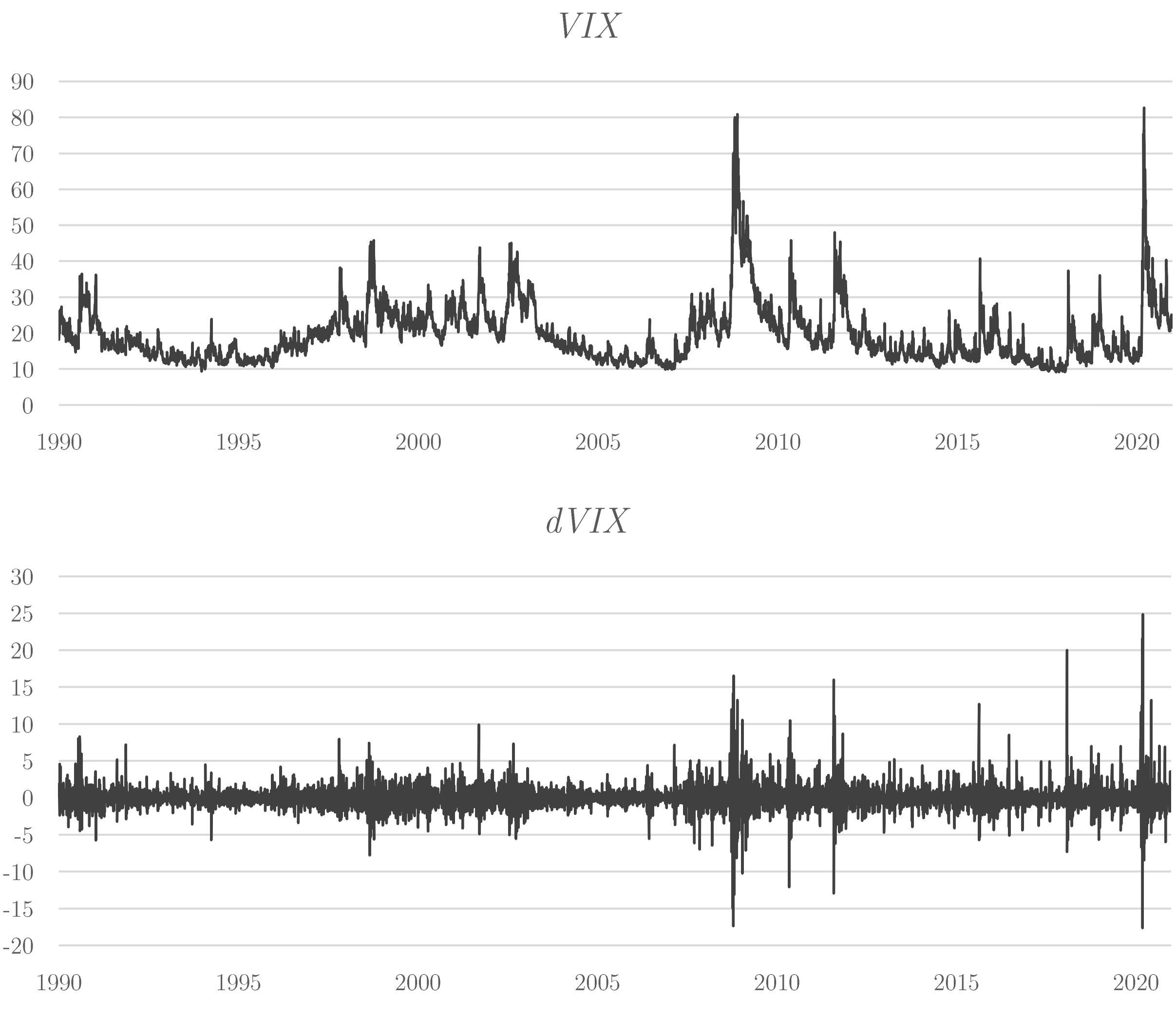

Figure 1: daily time-series of *VIX*, and daily innovations in *VIX*, *dVIX*.

Figure 1 shows the *VIX* and its first-order difference counterpart. The time-series of the three *VRP*s and their respective first-order differences show a similar pattern, displayed in appendix A. One can clearly identify the 2008 credit crisis and the initial shock of the corona crisis of march 2020, by the high levels of the *VIX* index.

5.1.2 *VIX* and *VRP* sorted portfolio returns

The full sample over the 1990-2020 period is sorted into five quintile portfolios based on their sensitivity to one of the four measures displayed in table (2). This process, described in the research design, result in the portfolios displayed in table (2). portfolio returns are decreasing in ranking on sensitivity to $dVIX$. For the *VIX* measure, the value weighted return on the difference portfolio 5-1 (return on the portfolio with the highest sensitivity to $dVIX$ minus the return on the portfolio with the lowest sensitivity to $dVIX$) averages -0,27% per month, which is statistically indifferent from zero (t-statistic of -1,24). The *VRP* based difference portfolios have similar negative average returns which are also statistically insignificant. Contrary to what AHXZ find in their 1986-2000 sample, no significant negative raw returns are found for a difference portfolio sorted on sensitivity to innovations in systematic volatility in the more recent 1990-2020 sample.

Table 2: β_{dVIX} and β_{dVRP} sorted portfolios

This table displays the raw and risk-adjusted returns for the quintile portfolios sorted on β_{dVIX}, β_{dVRP1}, β_{dVRP2} and β_{dVRP3}. The quintile portfolio 'Rank 1' is the portfolio with lowest respective sensitivity, while 'Rank 5' has the highest sensitivity. The difference portfolio '5-1' is the return on portfolio rank 5 minus portfolio rank 1. Risk-adjusted returns with relation to the CAPM and Fama & French 3-factor models are given by the CAPM and FF3 alpha. The t-statistics are in parenthesis.

Quintile portfolios sorted on β_{dVIX}				
Rank	Mean	Std. Dev.	CAPM alpha	FF3 alpha
1	1.103	5.821	0.252	0.260
			(1.96)	(2.13)
2	1.079	4.392	0.400	0.391
			(5.33)	(5.69)
3	0.975	4.157	0.326	0.309
			(5.56)	(7.99)
4	0.981	4.732	0.249	0.239
			(3.23)	(3.44)
5	0.831	6.747	-0.137	-0.122
			(-1.11)	(-1.18)
5-1	-0.272		-0.390	-0.382
	(-1.24)		(-2.11)	(-2.08)

(continued)

Quintile portfolios sorted on β_{dVRP1}

Rank	Mean	Std. Dev.	CAPM alpha	FF3 alpha
1	1.760	5.720	0.933	0.938
			(5.71)	(6.08)
2	1.413	4.401	0.735	0.725
			(6.75)	(7.62)
3	1.257	4.172	0.605	0.590
			(8.73)	(10.92)
4	1.243	4.658	0.517	0.507
			(7.69)	(8.43)
5	1.461	6.944	0.478	0.496
			(3.45)	(4.31)
5-1	-0.299		-0.455	-0.442
	(-1.33)		(-2.25)	(-2.21)

Quintile portfolios sorted on β_{dVRP2}

Rank	Mean	Std. Dev.	CAPM alpha	FF3 alpha
1	1.788	5.715	0.959	0.964
			(5.88)	(6.35)
2	1.355	4.380	0.679	0.669
			(7.18)	(8.05)
3	1.254	4.153	0.604	0.589
			(8.31)	(10.33)
4	1.304	4.706	0.574	0.566
			(7.48)	(7.94)
5	1.458	6.868	0.480	0.470
			(3.48)	(4.36)
5-1	-0.331		-0.479	-0.468
	(-1.52)		(-2.54)	(-2.47)

Quintile portfolios sorted on β_{dVRP3}

Rank	Mean	Std. Dev.	CAPM alpha	FF3 alpha
1	1.743	5.761	0.905	0.911
			(5.81)	(6.40)
2	1.384	4.349	0.712	0.704
			(7.93)	(8.58)
3	1.260	4.144	0.616	0.599
			(8.09)	(10.10)
4	1.324	4.732	0.592	0.585
			(7.73)	(7.99)
5	1.463	6.918	0.471	0.482
			(3.62)	(4.34)
5-1	-0.280		-0.434	-0.429
	(-1.35)		(-2.49)	(-2.40)

When the returns are risk-adjusted using the CAPM or Fama & French 3 factor model, results are in line with those in AHXZ. Table (2) reports alphas w.r.t both models for each of the four volatility measures. The β_{dVIX} sorted portfolios 1 through 4 each have significant positive alphas, while portfolio 5, the highest β_{dVIX} portfolio shows a negative, although insignificant, alpha. The difference portfolio has a significant negative alpha of -0,38 per month, compared to AHXZ's alpha of -0,83. The FF-3 alphas of the difference portfolios when sorting on β_{dVRP} vary between -0.43 and -0.48. An interesting observation concerning the sorted portfolio returns is the high risk-adjusted returns on value weighted portfolio with the lowest sensitivity to the VRP measures. The monthly FF3 alphas of the three VRP measures varies between 0.91 and 0.96. These alphas drive the negative alpha on the respective difference portfolio. On average, stocks that have low correlation with last month's VRP seem to earn higher risk-adjusted returns. When comparing the three VRP measures, the results appear to be similar. To summarize, AHXZ's result sorting portfolios based on sensitivity to systematic volatility is generally replicated in this newer sample, albeit less pronounced. Moreover, using the volatility risk premium VRP as sorting variables does not significantly change the results.

5.1.3 Factor mimicking portfolios and risk premia

Using the portfolios displayed in table (2) as base assets, a factor mimicking portfolio is constructed using equation (5). Using daily returns of the base assets, the regression based on equation (5) is performed for each month. The monthly portfolio weights b resulting from these regressions are then used to estimate the daily factor mimicking portfolio returns $b'X_t$. The factor mimicking changes in VIX is constructed using the five quintile portfolios based on sensitivity to VIX, whereas the factor mimicking changes in VRP are constructed using the respective quintile portfolios based on each of the sensitivities to the VRP measures. Correlations of the mimicking portfolios and their respective factors are shown in table (3), as well as the correlations between the factors used to estimate the risk premia on a monthly basis.

Table 3: Factor correlations

Panel A shows the daily correlations of each of the four factor mimicking portfolios with their respective volatility measure. Panel B shows the monthly correlations of the common risk factors with the factor mimicking portfolios. Here the factor mimicking portfolios are aggregated to monthly factor returns.

Panel A: daily factor mimicking portfolio correlations

	dVIX	*dVRP1*	*dVRP2*	*dVRP3*
FVIX	0.879			
FVRP1		0.864		
FVRP2			0.873	
FVRP3				0.869

Panel B: monthly factor correlations

	MKT	*SMB*	*HML*	*RMW*	*CMA*	*UMD*	*LIQ*
FVIX	-0.786	-0.194	0.057	0.250	0.247	0.203	-0.134
FVRP1	-0.819	-0.172	0.035	0.262	0.243	0.193	-0.170
FVRP2	-0.824	-0.197	0.049	0.270	0.231	0.207	-0.166
FVRP3	-0.783	-0.200	0.064	0.253	0.248	0.173	-0.135

The factor mimicking portfolios appear to be good proxies for their respective factor, judging by the high correlations in panel A of table (3) which lie between 0.87 and 0.88. The advantage of using factor mimicking portfolios is the interchangeable time dimension of these portfolios. The returns on the factor mimicking portfolios are accumulated by month. These monthly returns are then used to compute the monthly factor premia as described in equation (7). Correlations between the monthly factor mimicking portfolio returns and the risk factors used in equation (7) are displayed in panel B of table (3). The *VIX* and *VRP* factors show a high negative correlation with the market factor, consistent with previous literature, such as French, Schwert and Stambaugh (1987) and Campbell & Hentschel (1992), showing that high volatility is negatively related to market returns. The factor mimicking changes in *VIX* has a correlation with market returns of -0,79, while the lowest correlation with market returns is that between *VRP2* and the market with a value of -0.82. Correlations with other factors are weaker, he highest correlations being between *VRP2* and profitability factor *RMW* of 0.27, and *VRP3* and investment factor *CMA* of 0.25, both being relatively low.

Table 4: Fama-Macbeth (1973) factor premia

The estimated factor premia resulting from equation (7) are displayed. Models (1) through (5) are estimated using each of the four volatility factors, *FVIX, FVRP1, FVRP2* and *FVRP3*. The estimated risk premium λ is shown in the table, in parenthesis are the robust Newey-West (1987) t-statistics.

	FVIX					*FVRP1*				
	1	2	3	4	5	1	2	3	4	5
Cons	1.178	1.290	1.406	1.252	1.281	1.054	1.165	1.157	1.004	1.027
	(3.82)	(4.06)	(4.99)	(4.69)	(4.62)	(3.37)	(3.57)	(4.17)	(3.68)	(3.69)
MKT	-0.506	-0.491	-0.566	-0.583	-0.657	-0.196	-0.270	-0.264	-0.294	-0.341
	(-1.50)	(-1.47)	(-1.78)	(-1.81)	(-1.89)	(-0.60)	(-0.78)	(-0.81)	(-0.89)	(-0.98)
SMB	0.670	0.001	-0.036	0.344	0.339	0.467	0.044	0.046	0.440	0.429
	(1.56)	(0.00)	(-0.10)	(0.74)	(0.73)	(1.12)	(0.12)	(0.13)	(0.93)	(0.90)
HML	0.119	0.462	0.519	0.332	0.492	0.003	0.280	0.275	0.091	0.199
	(0.26)	(0.97)	(1.07)	(0.66)	(0.87)	(0.01)	(0.59)	(0.57)	(0.18)	(0.35)
FVIX/	-1.851	-0.901	-0.714	-0.588	-0.569	-2.354	-1.560	-1.577	-1.467	-1.422
FVRP	(-2.84)	(-1.42)	(-1.13)	(-0.89)	(-0.87)	(-3.27)	(-2.18)	(-2.30)	(-2.14)	(-2.11)
UMD		-1.371	-1.391	-1.154	-1.151		-0.754	-0.749	-0.507	-0.517
		(-2.26)	(-2.28)	(-1.96)	(-1.96)		(-1.30)	(-1.29)	(-0.87)	(-0.88)
LIQ			-1.390	-1.977	-1.539			-0.040	-0.598	-0.343
			(-1.08)	(-1.51)	(-1.17)			(-0.03)	(-0.51)	(-0.29)
RMW				0.362	0.323				0.277	0.256
				(1.39)	(1.23)				(1.14)	(1.04)
CMA					0.045					-0.020
					(0.16)					(-0.07)
adj.R^2	0.508	0.534	0.540	0.551	0.568	0.530	0.546	0.548	0.558	0.574

	FVRP2					*FVRP3*				
	1	2	3	4	5	1	2	3	4	5
Cons	1.342	1.386	1.457	1.258	1.306	1.377	1.412	1.473	1.281	1.344
	(4.24)	(4.31)	(4.99)	(4.61)	(4.60)	(4.34)	(4.40)	(5.11)	(4.66)	(4.68)
MKT	-0.416	-0.435	-0.492	-0.516	-0.634	-0.567	-0.543	-0.586	-0.603	-0.758
	(-1.25)	(-1.29)	(-1.52)	(-1.58)	(-1.82)	(-1.67)	(-1.63)	(-1.84)	(-1.87)	(-2.12)
SMB	0.427	-0.108	-0.126	0.352	0.356	0.639	0.028	0.007	0.458	0.476
	(1.02)	(-0.29)	(-0.33)	(0.75)	(0.76)	(1.52)	(0.08)	(0.02)	(0.96)	(1.00)
HML	-0.131	0.283	0.344	0.122	0.358	-0.064	0.316	0.361	0.147	0.430
	(-0.28)	(0.59)	(0.70)	(0.24)	(0.64)	(-0.14)	(0.66)	(0.74)	(0.28)	(0.76)
FVIX/	-1.588	-0.773	-0.611	-0.562	-0.632	-1.546	-0.859	-0.756	-0.760	-0.805
FVRP	(-2.83)	(-1.30)	(-1.06)	(-0.96)	(-1.07)	(-2.68)	(-1.50)	(-1.38)	(-1.39)	(-1.46)
UMD		-1.163	-1.207	-0.896	-0.803		-1.162	-1.186	-0.888	-0.816
		(-1.85)	(-1.89)	(-1.47)	(-1.34)		(-1.97)	(-1.99)	(-1.51)	(-1.40)
LIQ			-0.975	-1.595	-0.634			-0.951	-1.540	-0.538
			(-0.76)	(-1.23)	(-0.49)			(-0.77)	(-1.29)	(-0.44)
RMW				0.372	0.295				0.356	0.271
				(1.46)	(1.15)				(1.44)	(1.11)
CMA					-0.154					-0.181
					(-0.51)					(-0.57)
adj.R^2	0.511	0.536	0.542	0.553	0.571	0.511	0.535	0.540	0.550	0.570

Table (4) shows the result of the factor premia estimation resulting from equation (7), this equation is represented by model (5) in the table. The estimation is done based on 25 test portfolios double sorted on both market beta and the respective volatility measure. These test portfolios, by construction, have a sufficient distribution in sensitivity to the volatility measures to make cross-sectional inferences on the factor premia. The table also shows models (1) through (5), adding one of the factors, *UMD*, *LIQ*, *RMW* and *CMA* to the Fama-Macbeth procedure each time. Adding one factor at a time allows for a better analysis of the contribution of each separate factor. Looking at each of the four volatility measures, the adjusted R^2 increases only slightly when moving from model (1) to (2) through (5). The *FVIX* models' adjusted R^2 increases from 0,51 to 0,57 for models (1) and (5) respectively. The addition of the *UMD, LIQ, RMW* and *CMA* factors does not seem to add much to the explanatory power of the model for the 25 relative test portfolios.

The premium on the market factor appears to be negative over the full sample, the t-statistic lies between -1,50 and -2,00 in most of the estimated models. The exception being the models based on *VRP1*, where the t-stat is above -1,00 for each model specification, making the market factor premium insignificant. This negative premium on the market beta is consistent with the notion that high beta assets and/or portfolios earn lower risk adjusted returns and Sharpe ratios than assets and/or portfolios with low beta. Frazzini & Pedersen (2014) for example find positive significant risk-adjusted returns on a betting-against-beta factor that goes long in low-beta assets and shorts high-beta assets. The insignificant premia on Fama & French's size and value factors, *SMB* and *HML*, is consistent with the poor performance of these factors in the last few decades. The negative premium on the momentum factor, *UMD*, is significant in most of the specified models. This result may seem surprising given the results of papers such as Jegadeesh and Titman (1993) who find positive returns on momentum strategies. Blitz, Huij & Martens (2011) find returns on a total return momentum strategy, such as the *UMD* factor, of -8,54% from 2000 to 2009. The results from table (4) suggests these poor returns on a total momentum strategy continue into the 2010s, given the negative premium on the total 1990-2020 sample. Risk premia on the liquidity (*LIQ*), profitability (*RMW*) and investment (*CMA*) factors are insignificant in each specified model, similar to the size (*SMB*) and (*HML*) factors.

The *FVIX* factor premium estimated in models (1) through (5) are negative, consistent with the results from AHXZ. When only the *MKT, SMB, HML* and *FVIX* factors are used in the regression estimation this negative premium is -1,85% per month, significant at the 1% level. The statistical significance of the *FVIX* premium disappears when adding the *UMD, LIQ, RMW* and *CMA* factors, and the coefficient is lowered to -0,57%. The negative premium on innovations in systematic volatility found by AHXZ over their 1986-2000 sample is robust to using an updated data sample.

The addition of *UMD*, the momentum factor seems to have the biggest impact on the *FVIX* premium. It is possible that the systematic volatility premium actually captures some stock momentum effect. Given the negative price on the momentum factor, which is found in all of the specified models and significant in most, stocks with high momentum factor loadings generally have lower returns. The results shown in table (4) suggests that stocks with a high loading on the innovations in *VIX* volatility or *VRP* factors are often stocks that earned high returns in the past (months *t-2* to *t-12*). Then, due to the negative premium on momentum, these high volatility loading stocks have, on average, lower returns.

Comparing the results of *FVIX* to those of the three *FVRP* factors, similar patterns are observed. Especially looking at *FVRP2* and *FVRP3*, the factor premium of the volatility measure is absorbed by including the *UMD* and *LIQ* factors. Interestingly, the *FVRP1* factor premium stays negative and significant, at the 5% level, irrespective of the model used. The *UMD* and *LIQ* factors are less negative and not statistically significant, and do not seem to absorb the effect of the *FVRP1* factor. *FVRP1*, the measure for the *VRP*, computed using a realized volatility measure based on only end-to-end closing prices appears to be effective at capturing a volatility risk price story. This measure for *VRP* has a stronger negative premium of -2,35% per month compared to the *VIX* volatility premium of -1,85%. These results are consistent with the theoretical prediction that an asset's covariance with changes in risk-aversion lead to lower expected returns. This negative premium on risk-aversion, proxied by the *VRP*, is stronger and more robust than the negative premium on systematic volatility risk, as proxied by changes in the VIX index. Furthermore significance of the *VRP* factor premium does not disappear when adding common risk factors such as *UMD*, *LIQ*, *RMW* and *CMA*.

5.2 Pricing of idiosyncratic volatility

5.2.1 Portfolios sorted on idiosyncratic volatility

The previous section investigates the pricing of systematic volatility using the *VIX* and *VRP* measures, based on S&P500 prices. These price effects are based on sensitivity to systematic volatility risks. In this section I investigate the cross-sectional price effects of idiosyncratic volatility risk, using residual volatility with relation to the Fama & French (2018) 6-factor model. The residual volatility is computed as the square root of the variance of the error terms resulting from equation (12) over the last month.

Table 5: Portfolios sorted on idiosyncratic volatility

This table displays the raw and risk-adjusted returns for the quintile portfolios sorted on idiosyncratic volatility. The idiosyncratic volatility is computed as the square root of the variance of the residuals estimated by equation (12), over the past one month returns. The quintile portfolio 'Rank 1' is the portfolio with idiosyncratic volatility, while 'Rank 5' has the highest idiosyncratic volatility. The difference portfolio '5-1' is the return on portfolio rank 5 minus portfolio rank 1. Risk-adjusted returns with relation to the CAPM and Fama & French 3-factor models are given by the CAPM and FF3 alpha. The t-statistics are in parenthesis.

Rank	Mean	Std. Dev.	CAPM alpha	FF3 alpha
1	1.025	3.808	0.460	0.435
			(4.68)	(4.05)
2	1.044	4.850	0.291	0.277
			(3.32)	(3.93)
3	1.086	6.279	0.135	0.152
			(1.17)	(1.70)
4	0.935	7.884	-0.188	-0.151
			(-1.00)	(-1.12)
5	0.639	9.503	-0.593	-0.550
			(-2.14)	(-2.80)
5-1	-0.387		-1.053	-0.984
	(-1.24)		(-3.00)	(-4.17)

Table (5) shows the portfolio return and standard deviations for each quintile portfolios sorted on idiosyncratic volatility with relation to the FF6 model, as well as their respective CAPM and FF3 alphas. Portfolio 1 is the quintile portfolio with the lowest past month idiosyncratic volatility, while portfolio 5 is the quintile portfolio with the highest past month idiosyncratic volatility. The 5-1 portfolio is defined as the difference portfolio between portfolio 5 and 1. The 5-1 portfolio has negative monthly returns of -0,39%, although not statistically significant. The quintile portfolios show a clear pattern in risk-adjusted returns (alphas) with relation to the CAPM as well as the FF3 model. The portfolios with high idiosyncratic volatility loadings enjoy high risk-adjusted returns, whereas the portfolios with lower idiosyncratic volatility loading show lower risk-adjusted returns. Furthermore, a self-financing portfolio long leveraged in high idiosyncratic volatility stocks, financed by going short in low idiosyncratic volatility stocks, has a negative monthly alpha of -1,05. Hence, risk-adjusted returns appear to decrease with previous month's residual volatility with relation to the FF6 model. In unreported results, I employ the same methodology using FF3 residuals, yielding similar outcome.

AHXZ find a negative FF-3 alpha of -1,31 on a 5-1 idiosyncratic difference portfolio. They describe this negative risk-adjusted return as a puzzle, as it contradicts existing theory. Merton (1987) described how in informationally segmented markets, stocks with high firm-specific risks require higher return. Similarly behavioral theory as in Barbaris & Huang (2001) also predict a positive relation between idiosyncratic volatility and returns, resulting from investors requiring a higher return for holding the stock in their portfolio. Given the significant negative 5-1 alpha of -0,98 from table (5), the idiosyncratic volatility puzzle is still present in a modern sample.

5.2.2 Controlling for sensitivity to systematic volatility and *VRP*

In the previous section a negative risk premium was found for the factors based on systematic volatility (*FVIX*) and volatility risk premium (*FVRP*), using the 25 test portfolio returns. The negative risk-adjusted returns resulting from the idiosyncratic volatility trading strategy shown in table (5) could be attributed to the stocks' sensitivity to these risk factors. To investigate this possibility, I compute the idiosyncratic volatility sorted portfolio returns, while controlling for either systematic volatility, or the *VRP*. This is done by first sorting the cross section of stocks into five portfolios based on sensitivity to the volatility measure, and then sorting each of these portfolios into five portfolios based on idiosyncratic volatility. The average of the value-weighted returns of the five portfolios sorted on sensitivity to volatility measure is then taken across all five idiosyncratic sorted portfolios. Each of the five average value weighted portfolio returns, ranking from lowest idiosyncratic volatility (1) to highest volatility (5) should then have equal sensitivities to the volatility measure used. The resulting alphas w.r.t the FF-3 model are shown in table (6), as well as the mean average value weighted return of the difference (5-1) portfolio.

Using sensitivity to changes in VIX as control variable, the pattern of alphas remains the same; high idiosyncratic volatility portfolios earn low returns. When one of the *VRP* measures is used as control variable, this pattern changes. Alphas monotonically increase with idiosyncratic volatility, and the 5-1 portfolio has raw mean returns of 1.29% per month, significantly different from zero at 1% level. The idiosyncratic volatility puzzle disappears. This effect is similar for each of the three *VRP* measures. This result suggests that the negative returns on high idiosyncratic stocks, shown by AHXZ and in this research, is caused by a high sensitivity to the *VRP*. When the sensitivity to the *VRP* is controlled for, stocks with a higher past month idiosyncratic volatility relative to the FF-3 model, earn higher returns. This positive relation is logical, considering that investors require higher returns for holding more volatile stocks. Controlling for each of the three *VRP* measures all lead to similar results. These results suggest that the idiosyncratic volatility puzzle is actually the result of assets sensitivity to changes in risk-aversion.

One possible explanation is that high past-month idiosyncratic volatility assets earn low returns because they are a hedge against increases in risk-aversion, and thus bad states of the economy. To investigate this relation, however, is beyond the scope of this research.

Table 6: Idiosyncratic sorted portfolios controlled for VIX/VRP
This table shows the FF3-alphas of the quintile portfolios sorting on idiosyncratic volatility, controlling for one of the systematic volatility measures. The idiosyncratic volatility is computed as the square root of the variance of the residuals estimated by equation (12), over the past one month returns. Controlling for *VIX* or *VRP* is done by first sorting the cross section of stocks into five portfolios based on sensitivity to the volatility measure, and then sorting each of these portfolios into five portfolios based on idiosyncratic volatility. The average of the value-weighted returns of the five portfolios sorted on sensitivity to volatility measure is then taken across all five idiosyncratic sorted portfolios. The last column displays the difference '5-1' portfolio's raw returns. The t-statistics are displayed in parenthesis.

	FF3-alphas						mean
	Ranking on idiosyncratic volatility						return
	1	2	3	4	5	5-1	5-1
VIX controlled	0.355	0.193	0.075	-0.041	-0.386	-0.741	-0.270
	(6.02)	(3.29)	(0.87)	(-0.32)	(-2.13)	(-3.56)	(-0.76)
VRP1 controlled	0.579	0.635	0.695	0.941	1.378	0.799	1.293
	(8.21)	(8.31)	(7.09)	(5.86)	(5.56)	(3.08)	(3.36)
VRP2 controlled	0.588	0.638	0.691	0.980	1.365	0.777	1.269
	(8.37)	(8.29)	(6.77)	(5.86)	(5.89)	(3.25)	(3.30)
VRP3 controlled	0.552	0.653	0.707	0.991	1.390	0.838	1.346
	(8.15)	(8.77)	(7.15)	(6.14)	(5.77)	(3.29)	(3.47)

5.3 Volatility and crisis

The full sample of stock returns dating from 1990 to 2020 show negative risk-adjusted returns on trading strategies that go long in stocks with high sensitivity to one of the volatility measures and short in those stocks with a low sensitivity to the same volatility measure. Over this 30 year period, 44 months are defined as 'recession months' by the NBER. To investigate the relation between systematic volatility, the *VRP* and stock returns during crisis months, a subsample using only the recession months is compared to the full sample, shown in table (7). In this subsample, the trading strategy based on systematic volatility earn negative risk-adjusted returns. This alpha, however, is not statistically significant, due to the low number of observations in the NBER recessions subsample. The same pattern is found for the trading strategies based on the sensitivity to the *VRP* measures. The trading strategy based on previous month's idiosyncratic volatility has a positive average return, and a negative risk-adjusted return, both statistically insignificant. None of the trading strategies seem to result in statistically significant returns during crisis months, this is mainly due to the small size of this subsample.

Table 7: Difference '5-1' portfolios in different subsamples

In this table the Full sample is compared to the sample comprising of NBER recession months. The full sample consists of all months in the 1990-2020 sample, whereas the NBER recession subsample consists of the 44 crisis months defined by the NBER. The table displays the means and FF3-alphas of the difference '5-1' portfolios sorting on sensitivity to *dVIX, dVRP1, dVRP2* and *dVRP3*, and idiosyncratic volatility. The t-statistics are in parenthesis.

	d*VIX*		d*VRP1*		d*VRP2*		d*VRP3*		Idiosyn. Vol.	
	mean	FF3 alpha	mean	FF3 alpha	mean	FF3 alpha	mean	FF3 alpha	mean	FF3 alpha
Full sample	-0.272	-0.382	-0.299	-0.442	-0.331	-0.468	-0.280	-0.429	-0.387	-0.984
	(-1.24)	(-2.08)	(-1.33)	(-2.21)	(-1.52)	(-2.47)	(-1.35)	(-2.40)	(-1.24)	(-4.17)
NBER	-0.645	-0.918	-0.562	-0.964	-0.792	-0.970	-0.550	-0.643	0.440	-0.429
Recessions	(-0.69)	(-1.20)	(-0.54)	(-1.04)	(-0.79)	(-1.38)	(-0.55)	(-0.92)	(0.29)	(-0.35)

The factor premia of the factors computed in the previous section, *FVIX* and *FVRP*, are also investigated during the recession months. Table (8) shows the factor premia for the full sample as well as the NBER recession subsample. For each factor a lower premium is found in the recession subsample. *FVIX* has a negative premium of -3,16% per month during crisis months, compared to -0,57% in the full sample. *FVRP1*, the most robust out of the *VRP*s, has a factor premium of -3,82% in crisis months compared to -1,42%. All of the factor premia in the crisis subsample are statistically significant at a 1% level, despite the small sample size. The negative cross-sectional price on systematic volatility appears to be stronger during crisis months. Assuming that the *VRP* is a proxy for risk-aversion, as derived in the theoretical section, the higher risk-aversion in crisis months leads to a stronger negative cross-sectional premium on sensitivity to the changes in risk-aversion.

Table 8: Fama-Macbeth (1973) risk premia in different subsamples

The estimated factor premia resulting from equation (7) are displayed for the full sample and the NBER recession subsample. Model (1) as in table (4) is estimated using each of the four volatility factors, *FVIX, FVRP1, FVRP2* and *FVRP3*. The estimated risk premium λ is shown in the table, in parenthesis are the robust Newey-West (1987) t-statistics.

	Factor premium			
	FVIX	*FVRP1*	*FVRP2*	*FVRP3*
Full sample	-0.569	-1.422	-0.632	-0.805
	(-0.89)	(-2.11)	(-1.07)	(-1.46)
NBER recessions	-3.157	-3.816	-3.448	-3.492
	(-1.57)	(-2.22)	(-2.13)	(-2.76)

6 Conclusion

This paper aims to replicate AHXZ's results for the pricing of systematic volatility as well as idiosyncratic volatility in a newer data sample. The pricing of systematic volatility, proxied by changes in the *VIX* index, is analyzed first by forming portfolios sorted on sensitivity to changes in *VIX*. As in AHXZ's data sample, a difference portfolio going long in high sensitivity to changes in *VIX* assets, and short in assets with a low sensitivity, earns statistically negative risk-adjusted returns. By constructing a factor mimicking portfolio based on changes in *VIX*, a risk premium is estimated using Fama-Macbeth procedure. The resulting risk premium found in this research is negative and significant, similar to that found by AHXZ. To test the presence of a idiosyncratic volatility puzzle, as defined by AHXZ, in the newer data sample, five portfolios are formed in a similar fashion as those based on changes in systematic volatility. The resulting alpha of the difference portfolio using idiosyncratic volatility w.r.t the FF-6 model is negative and statistically significant, consistent with the idiosyncratic volatility puzzle. Following these results, the first hypothesis; *"The negative price of systematic volatility and puzzling high (low) returns on low (high) idiosyncratic volatility stocks found by AHXZ are present in the 1990-2020 period"*, is true.

In this research I define the volatility risk premium *VRP* as the difference between implied and realized volatility. This model-free measure is known to have explanatory power on future market return, and often interpreted as a proxy for risk-aversion. I show that changes in *VRP* is negatively priced in the cross-section, using three different methods to compute the *VRP*. The negative risk premium for *VRP*, found using a Fama-Macbeth procedure, is robust to including common risk factors. Furthermore. a difference portfolio based on *VRP* sorted quintile portfolios, similar to the difference portfolios in AHXZ, shows a significant negative FF-3 alpha. The second hypothesis; *"The VRP is a cross-sectionally priced factor"* is supported by the data. There is potential for a risk-aversion explanation of the cross-sectional effect of the *VRP*. Assuming Campbell's habit persistence, the existence of a stochastic discount factor and the law of one price to hold, risk-aversion is time varying. Then, if the *VRP* is indeed a proxy for risk-aversion, asset's sensitivity to *VRP* would hold information about an asset's payoff in a bad or good state of the economy which according to standard finance theory should impact the asset's expected return. A more detailed look into the relation between the *VRP* and risk-aversion, as well as risk-aversion and the state of the economy could be an interesting subject for future research.

The final hypothesis; *"The cross-sectional effects of systematic and un-systematic volatility, as well as the VRP, are more pronounced in crisis months"* is tested by comparing the full 1990-2020 sample to the results in a subsample of NBER recession months. Looking at the difference portfolio returns, similar results are found for the NBER recession subsample compared to the full sample. The risk factor premia resulting from the Fama-Macbeth procedure show a stronger negative coefficient for the recession months, with a monthly premium for the *VRP* of -3,82%. Given the mixed results, the third hypothesis is false, however the stronger coefficients for the *VIX* and *VRP* factor premia do indicate a stronger cross-sectional effect for systematic volatility during crisis months.

Another interesting finding is the disappearance of the idiosyncratic volatility puzzle when controlling for sensitivity to *VRP*. The negative returns on a difference portfolio, found by AHXZ as well as in this research, are inverted to a positive raw as well as risk-adjusted return after controlling for *VRP*. This positive return is in line with the predictions of standard asset pricing theory. Investigating the relation between the idiosyncratic volatility puzzle and the *VRP*, and the role that risk-aversion plays herein, is beyond the scope of this research.

As mentioned, previous research dealing with measures of *VRP* in a 'model-free' way, and therefor dealing with measure of realized volatility, make use of more precise data frequencies. BTZ, as well as Bollerslev, Gibson & Zhou (2011) and others stress the benefits of using 5-minute frequency return data to estimate the physical realized volatility. Because of the limited availability of intra-day price data for U.S. stocks, the measure is not used in this research. To test the robustness of the results of this research, a realized volatility measure based off these 5-minute returns would be an interesting addition.

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APPENDIX A

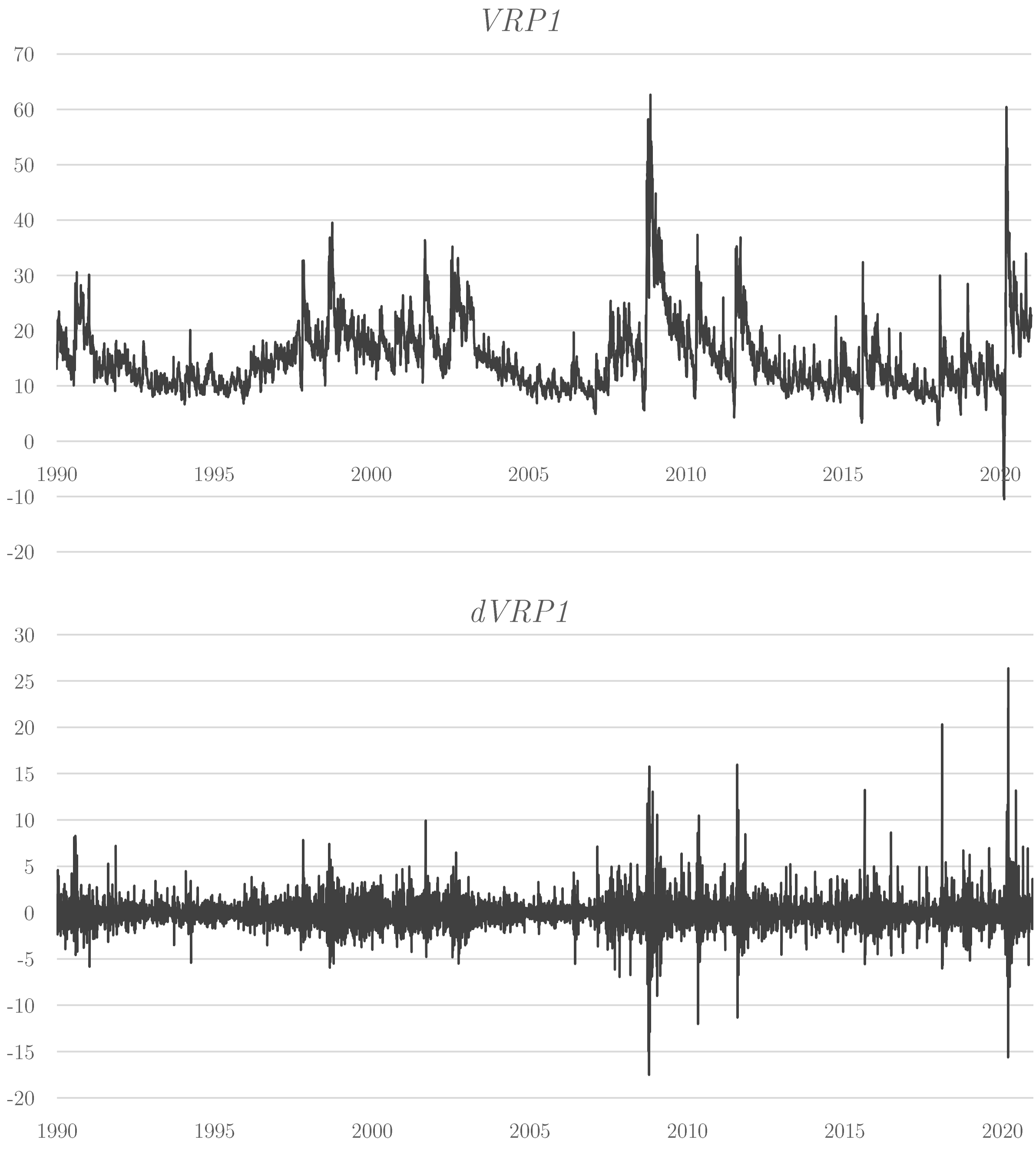

Figure 1: daily time-series of *VRP1* and its first order difference.

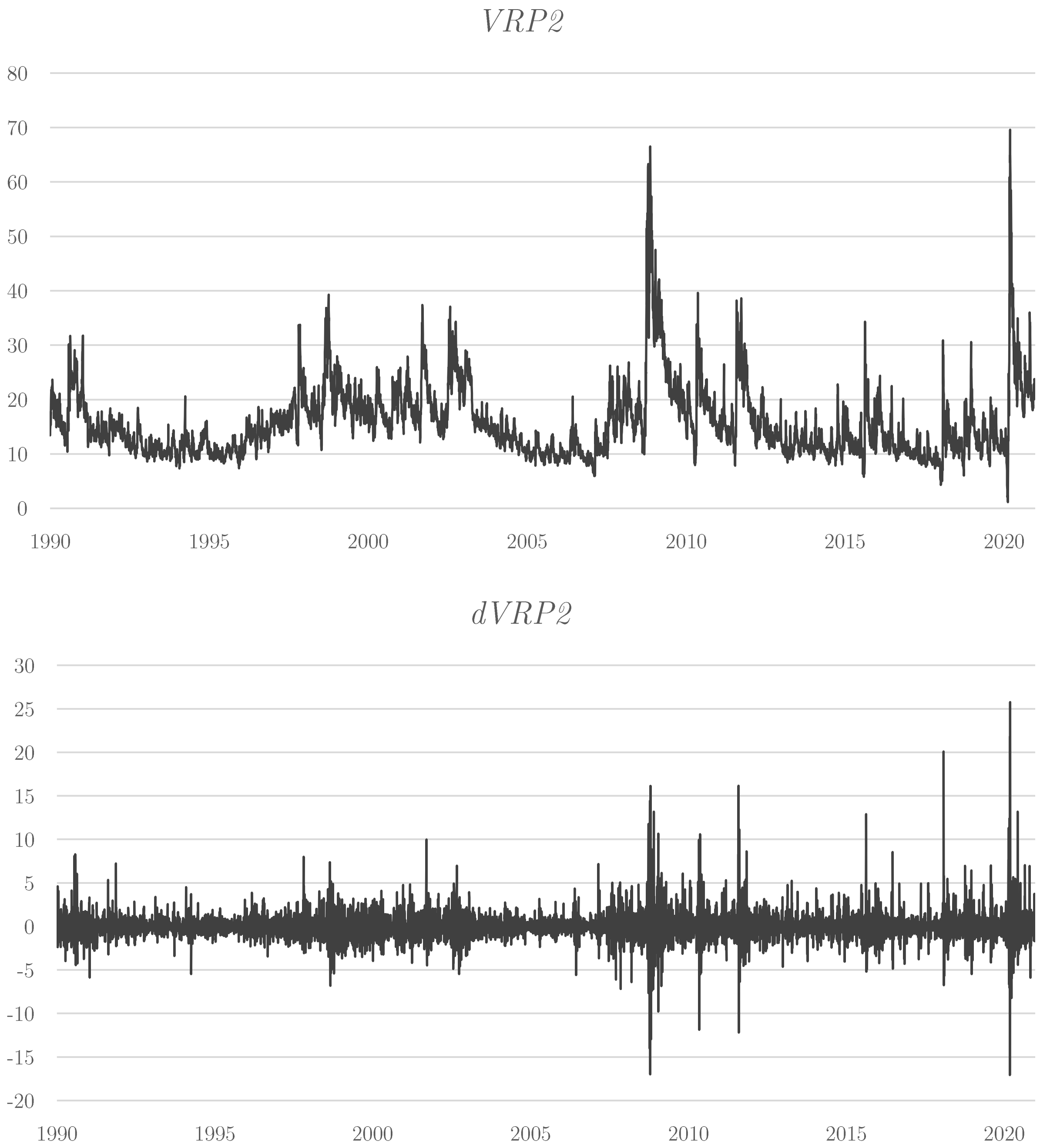

Figure 2: daily time-series of *VRP2* and its first order difference.

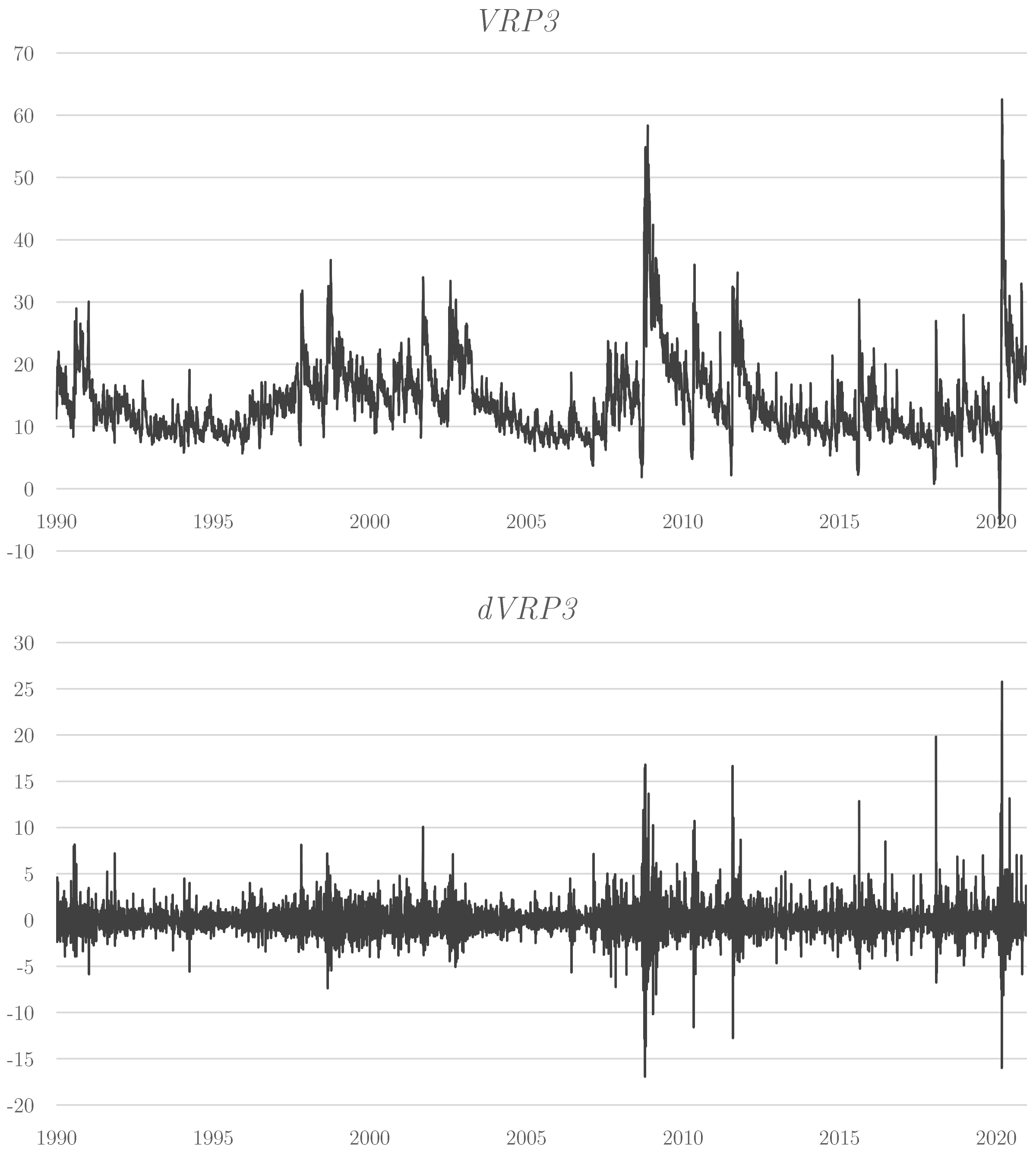

Figure 3: daily time-series of *VRP3* and its first order difference.