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Master thesis

Empirical Analysis on the Low Volatility Anomaly: The Impact of Business Cycles, Open Market Operations, and Shadow Rates

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Abstract

In this paper an empirical research is performed on possible drivers of the inversed CAPM model, also known as the low volatility anomaly. The inverse relationship is observed on financial markets. Which shows that low volatile or low beta stocks earn a higher return than that is expected, according to the CAPM. Possible explanations for this occurrence are researched, consisting of business cycles and quantitative easing. With the latter being divided into open market operations performed by the FED, and the difference on the shadow rate, which serves as a proxy for the FED rate. This research shows that different business cycles do change the behaviour of asset returns as predicted by the CAPM, but no conclusive evidence is found that business cycles are a driver for the low volatility anomaly. The same is true for quantitative easing, both measured in open market operations as in the difference on the shadow rate. However, both open market operations and the difference on the shadow rate do show a significant impact on the aggregate market, but this does not translate to individual portfolios.

Content

1. Introduction

According to traditional economics the price of an asset is based on supply and demand. But what if there is another way to calculate the price of the asset? A way to model the price is the CAPM, an asset pricing model introduced by Treynor (1961), Sharpe (1964) and Linter (1965) which continued the work on modern portfolio theory as described by Markowitz (1952). This model calculates the price, based on how much risk the underlying asset exhibits, the higher the exposure to systematic risk (measured in beta), the higher the expected return.

The CAPM model has been a staple in asset pricing and is still relevant to this day. However, in the last decades people started to question the empirical relevance of the CAPM. Gibbons, Ross and Shanken (1989) found that that high beta stock earn too little and low beta stocks earn too much. But the CAPM was still relevant, which was later used as the market factor in the Fama and French three factor model (1996). Major push-back didn't happen until the paper of Blitz and Van Vliet (2007). In which they suggested that the CAPM relation does not always hold. Blitz and Van Vliet showed that low volatility assets earn on average a higher return; also called the low volatility anomaly.

In a later paper Blitz, Falkenstein and Van Vliet (2014) argued that the CAPM is not a fundamental wrong model but relies on assumptions that do not hold. Therefore, the mathematical derivation that the CAPM is built upon, does not always describe an accurate reality. Blitz, Falkenstein and Van Vliet (2014) give multiple explanations on why the CAPM does not always hold and thus why the low volatility anomaly is observed. In this paper, factors that potentially explain the inversed CAPM relationship are researched.

One of the assumptions of the CAPM is that every investor is risk-averse, they maximise their expected utility, and only care about the mean-variance of the expected return (Blitz, Falkenstein, & Vliet, 2014). While, in reality, different investors have different utility preferences. For example, Baker, Bradley and Wurgler (2011) argue that fund managers only care about their outperforming their benchmark. They chase high beta stocks in bull markets and are not interested in the long-term results that are achieved with low volatility stocks. Assuming this is true, it seems that the state of the economy can explain why the CAPM assumptions do not always hold. Perhaps the state of the economy can play an important role in explaining the low volatility anomaly.

Another assumption of the CAPM is that markets are perfect and that the price is the result of an equilibrium. This again implies that investors are risk-averse, expectations are homogeneous because of free and instant information, investments are one-period, a riskless asset exists, and the market exhibits no frictions. A market that exhibits no frictions, is a market that doesn't have transaction costs and liquidity is always available. Within the perfect market assumption, this paper will focus on liquidity.

Liquidity can be divided into market liquidity and funding liquidity. Without market liquidity assets cannot be sold, and without funding liquidity investors are unable to secure funding for buying assets.

Without liquidity the CAPM assumption of a perfect market cannot be true. And as it happens, liquidity sometimes dries up. This usually happens when markets are in an economic downturn, which can lead to an economic collapse. To prevent this from happening and to prevent liquidity from drying up, central banks step in to provide liquidity in the form of quantitative easing (QE). QE are programs, ran by central banks, in which they try to inject liquidity into the market by either buying assets or by making funding available for investors. This artificially lowers the volatility in the market and can drive up prices, as explained by Veronesi (1999), and Villaneuva (2015) respectively. The presence of QE can be a factor for the CAPM not holding. Since liquidity is artificially injected into the market which can throw off the equilibrium between risk and return. Thus, making it a potential explanation for the low volatility anomaly.

In addition, one could argue that QE can also distort the homogeneous expectations between investors, which results into the perfect market assumption not holding. Although this is outside of the scope of this paper.

QE does come with a potential problem. Since QE programs are announced multiple periods before the central bank performs the QE, measuring the effect of the program can be problematic. The reason being, according to the Efficient Market Hypothesis, is that the expected impact of QE is incorporated into the equities market before the actual program starts, as shown by Blanchard (1981). This makes modelling the impact of QE harder. A second problem of using QE programs, as a measure of monetary policy, is that those programs are not continuous. They only happen on occasion, in contrast to the low volatility anomaly, which seems to be always present. To combat these problems QE is divided into two programs. The Open Market Operations (OMO) program of the FED in which they buy assets on the equity market. The second program is their discount rate monitoring. The discount rate is the interest rate that is charged to commercial banks and other depository institutions (FED, 2021). The rate is used as a continuous tool to have a direct link on the health of the economy. For this discount rate the FED has a specific target that is monitored periodically. This target is called the FED funds target rate and is used in this paper.

The interest rate can be seen as a fundamental rate that is intertwined with every financial market. They rate is influenced by macroeconomic events according to the Taylor rule, which was used by central banks to estimate the interest rate (FED, 2021). But one of the disadvantages of the Taylor rule is that it doesn’t allow for nominal interest rates to become negative, while this is certainly possible and has happened before (Black, 1995). Black explains why nominal rates should not become negative, but emphasizes that it has happened. The reason nominal rates should not become negative is because interest rates can be seen as an option. If the rate on your bank account is negative, you have the option to withdraw your money which means that your amount of money does not decrease. If you didn’t exercise this option the amount does decrease. So, the rate should not go lower than zero, but when the option is not exercised and the rate drops below zero, Black calls this the shadow rate.

The FED does run into the same problem as their FED funds target rate has never become negative before (FED, 2021). But when it should break through zero the rate becomes a shadow rate. Wu and Zhang introduced a new Keynesian model that models the shadow rate (2019) of the FED rate. The shadow rate is the estimated discount rate when the FED rate has reached the Zero Lower Bound (ZLB).

This paper will research the impact of QE on equity prices and the low volatility anomaly. The QE will be divided into two programs. First the Open Market Operations, these are programs ran by the FED in which they inject liquidity into the market. Second, the FED rate, but since the FED rate cannot become negative, the first difference of the shadow rate is used as proxy.

To tie this together, the CAPM assumption of investors being risk-averse, and the assumption that markets are perfect are researched. This is done by comparing the impact of business cycles, and QE on the low volatility. QE is divided into the OMO and the fluctuations on the shadow rate. The main question that is researched is the following: What is the impact of business cycles, quantitative easing, and the shadow rate on the low volatility anomaly, and thus, are these factors responsible for the inversed CAPM relationship?

In the following section, the literature regarding low volatility, QE, and shadow rates are discussed. This is then used to build the hypotheses. The research continues with which methods will be used to test the hypotheses, what data will be used and from where they're retrieved. Thereafter the results of the statistical tests are presented, which are interpreted and discussed. The paper concludes with further research suggestions and limitations.

2. Literature review

2.1. Low volatility

The CAPM argues that the risk versus reward is a linear sloping line in which assets with high volatility earn a higher risk. The CAPM tries to model expected returns of an equity, based on the exposure of said equity to the overall market. The higher the exposure (higher beta) the higher the expected return. But a high return doesn't tell the complete story. Since achieving a higher return can be relatively easy. Achieving a high return with low risk is what is desired according to the CAPM. To make this comparison, the returns need to be risk-adjusted. This can be done through the Sharpe ratio, which divides the return on the asset by the total volatility of that asset. Incorporating the Sharpe ratio into the CAPM, the CAPM creates a frontier of all possible asset combinations (portfolios) of which one is the most optimal. This portfolio contains a variety of different assets with certain weights, so that the Sharpe ratio of the portfolio cannot become any higher. The weights of the different assets in the portfolio are based on the correlation between the assets. By taking advantage of the correlation, a portfolio can be created which has a great diversification and thus has close to none unsystematic risk. The optimal portfolio achieves the highest Sharpe ratio and has the highest risk-adjusted return (Sharpe, 1964).

In this paper two ratios are used to risk-adjust the returns, the Sharpe ratio and the Treynor ratio. The first takes total risk of the portfolio into consideration, while the latter uses the exposure to the overall market. Two ratios are chosen to be able to better understand the performance of the portfolio.

Gibbons, Ross and Shanken (1989) challenge the CAPM. They construct mean-variance efficient portfolios and test those. They find that high beta stocks earn too little and low beta stocks earn too much, compared to the CAPM. The advantage of a mean-variance efficient portfolio is that it can contain all sorts of assets, but the specific combination of those assets minimizes the overall volatility of the portfolio. Meaning its mathematically impossible to earn a higher return with lower risk.

Additional research upon CAPM, and its efficient frontier is done by Clarke, De Silva and Thorley (2006). They show signs that a minimum-variance portfolio beats the market portfolio in risk-adjusted returns, in the CAPM-based framework. They note that the Fama and French (1996) size and value factor, and the Jegadeesh and Titman (1993) momentum are 'key market wide determinants' of portfolio returns. The return on their portfolio do exhibit a value and a small-size bias, but no consistent bias on momentum. They conclude that lowering variance by using mean-variance efficient portfolios seems to work in achieving competitive returns, which can, potentially, be attributed to the low volatility anomaly.

Blitz and Van Vliet (2007) introduce the low volatility anomaly. Which contains empirical evidence for something that has been assumed by previous researchers; that not only stocks with higher idiosyncratic volatility have lower returns (Ang, Hodrick, Xing, & Zhang, 2006), but that assets with lower volatility earn a higher risk-adjusted return.

Baker, Bradley, and Wurgler (2011) confirm the existence of the low volatility anomaly but add that simple methods for volatility have a stronger low volatility effect than complicated methods. This can be attributed to the fact investment managers measure their performance in tracking error. Baker, Bradley, and Wurgler argue that managers care more about outperforming during bull markets than underperforming during bear markets, which increases their demand for high beta stocks. Only adding to the low volatility anomaly.

In most papers about asset pricing anomalies the researchers make use of a self-financing portfolio. To be able to do this the investors must be able to create leverage. Leverage can be created in multiple ways, but the idea is that an investor borrows money, which he invests into an asset that creates a larger return than the cost of borrowing money. For example, Fama and French (1996) theorize that exposure to certain factors creates a return that is above than that of the market. They rank assets in how likely they exhibit said factor. The low-ranking assets are borrowed and sold (short position) and the money generated by selling those assets is used to invest (long position) into the high-ranking assets. The idea is that the high-ranking assets achieve a higher return than the low-ranking assets and thus, net a profit.

Baker, Bradley, and Wurgler note that leverage can be a problem for a low volatility, self-financing, trading strategy. First of all, not all investors have access to leverage. Secondly, for a self-financing low volatility portfolio to work, the investor needs to be able to short the portfolio which includes assets with the highest available volatility. Usually, these assets have a smaller market capitalization, making shorting those assets harder and perhaps more expensive than assets with a higher market capitalization. As of the time of writing this report, short-term interest rates are negative (OECD, 2021). This implies that leverage creates value in addition to the value created with the leverage-strategy. This can become an obstacle for the investor seeking leverage, since the counterparty loses value by lending. This phenomenon can create an entirely different discussion on the equilibrium and the transaction costs of leverage. To keep it simple, in this paper long-only portfolios are used.

2.2. Business cycles

Blitz, and Van Vliet (2007) note that low volatility underperforms in bull markets, but that the low volatility outclasses this underperformance with outperformance in the bear markets. Which Baker, Bradley, and Wurgler (2011) confirms. This would mean that in bull markets a rational investor should disinvest in low volatility assets and during bear markets the investor buys the low volatility assets. The business cycles seem to have an effect on the low volatility anomaly. Blitz, Falkenstein, and Van Vliet (2014) argue that perhaps the current incentive structure of asset managers is wrong, which contributes to the observation regarding the business cycle. Asset managers are rewarded for outperformance of the market, which can result in risk-seeking behaviour in times when the economy is booming. They hold high volatile stocks, since the changes that they outperform the market is higher than low volatile stocks. The downside is ignored for the chance of achieving abnormal returns (Baker & Haugen, 2012). When the economy takes a downturn a flight to quality can happen. When this happens, investors disinvest their equity portfolios and try to find safer investments such as bonds.

To incorporated business cycles into the research inspiration is drawn from Blitz, Huij and Martens. In a paper on residual momentum Blitz, Huij, and Martens (2011) find that incorporating dummy variables for up- and down-markets, and other things, improves the momentum trading strategy. This is based on the conclusion made by Chordia, and Shivakumar (2002). They conclude that "The profits to momentum strategies are explained by a parsimonious set of macroeconomic variables that are related to the business cycle.".

Concluding that perhaps business cycles can explain the low volatility anomaly, and contradict the CAPM. To test if this is the case the following hypothesis will be tested.

Hypothesis 1: *The low volatility anomaly has a greater performance in bull markets than in bear markets*.

2.3. Open Market Operations

One of the assumptions of the CAPM is that markets are perfect, this means that assets are valued correctly, information is perfect and the same for everyone, and that the price of the assets are set by the exposure to risk. But what if the market doesn't behave as assumed by the CAPM? For example, a central bank performing countercyclical policy. The role of a central bank is to influence factors in markets to reach economic goals. One of these factors is liquidity which has a link to the stock market. Perhaps the influence of central banks can distort the equilibrium in the market?

Some events on the stock market can lead to liquidity to drying up, for example economic downturns. To prevent this from happening central banks step in with quantitative easing. This is a countercyclical attempt to get the economy moving again. Veronesi (1999) creates a theoretical framework for how macroeconomic events influence market volatility. Concluding that investors require extra discounts when in anticipation of higher volatility. So, if central banks can reduce the anticipation of higher volatility with QE, the volatility could be less. This can attribute to the market not being perfect and a preference for the low volatility anomaly.

So, do the QE programs strengthen the low volatility anomaly? Before this question can be answered, quantitative easing needs to be elaborated. The FED has multiple tools for monetary policy, one of them being Open Market Operations (FED, 2021). This is a program in which the FED buys assets to inject liquidity into the market. Tan and Kohli (2011) found a significant negative relation between OMO and stock volatility, Steeley and Matyushkin (2015) found that QE neutralizes the increase of volatility during financial crisis. Although research points to a direct link, Villanueva (2015) argues that this link is more complicated, because of other macro-economic factors. One of these factors being the federal funds rate, which is discussed later in this paper.

To research the impact of OMO on the low volatility anomaly, the impact of OMO will be tested. To assess the impact, the difference between periods with OMO and periods without OMO are analysed. Creating the following hypothesis:

Hypothesis 2: *In periods without Open Market Operations the low volatility anomaly is less pronounced.*

The OMO has evolved considerable since the financial crisis in 2008 (FED, 2021). In this paper OMO are mentioned which all refer to programs ran by the FED after the crisis on 2008.

2.4. Shadow rates

Testing the impact of QE on the low volatility can be tricky, when the effects of QE on the market are happening before its implementation. The reason being is that QE programs are announced. The Efficient Market Hypothesis suggests that equity prices are influenced by available information. Meaning that the anticipation of QE can alter the price of assets (Shogbuyi & Steeley, 2017) before the programs is performed. In summary this means that perhaps the QE is indeed a driving factor of the low volatility anomaly, but empirically evidence can be hard to come by. This can happen when the effect happens before the implementation of the QE program and thus does not show up at the same time as the program.

A different and more immediate way to measure QE is the FED discount rate. In times of financial downturn central banks can lower the interest rate which in turn lowers the cost of liabilities. This lowers the threshold of a positive net present value of investment opportunities. So, a decrease of interest rate could negatively influence investments on the financial market and thus boost prices and lower volatility. Which in turn can be the driving factor of the low volatility anomaly. The demand for cheaper funding is observed after the Salomon brother scandal (A Review of Financial Market Events, 1998). During the economic collapse of 2008 (Central bank operations in response to the financial turmoil, 2008). During the crisis of 2013 (Asset encumbrance, financial reform and the demand for collateral assets, 2013). And after the 2020 COVID pandemic. (US dollar funding: an international perspective, 2020)[1]. However, literature regarding that the funding is used for projects that would otherwise been rejected, is not found.

While lowering the funding rate does improve liquidity, the FED rate cannot be lower indefinitely. At one point the ZLB is reached, which means the FED cannot lower the rate anymore. If the interest rate is used to test the impact on the low volatility anomaly it would run into the problem of the ZLB. To combat this problem a proxy is used; the shadow rate. The

[1] No report could be found on the dot.com bubble

shadow rate of the New Keynesian Model is used, which is introduced by Wu and Zhang (2019). The shadow rate can be negative and is used as proxy for the FED rate.

Hypothesis 3: *The changes in the shadow rate have a negative relationship with the return on the low volatility anomaly*.

3. Methodology & data

3.1. Methodology

3.1.1. Business cycle

To test the first hypothesis: *The low volatility anomaly has a greater performance in bull markets than in bear markets* The CRSP dataset will be used to calculate lognormal returns on the American equities. Thereafter, the different assets will be ranked into ten different categories based on their historical volatility, calculated with a rolling window standard deviation. The window has a length of three years, the same as used in the original paper of Blitz and Van Vliet (2007). This creates a time series dataset with a returns per month for ten portfolios ranked on volatility. Ranging from lowest volatility to the highest. For every month the business cycle will be established; a bear market or a bull market. The model is set as a bear market when the monthly return on the overall market, in the same month, is negative. Hereafter the Treynor and Sharpe Ratio, over the horizon for bear and bull market, are calculated and tested if the difference between them is significant. This is done by applying the Z-test developed by Jobson and Korkie (1981), with the addition of the correction developed by Memmel (2003). Although the addition of Memmel is focussed on the Sharpe Ratio it's assumed it also works for the Treynor Ratio.

$$Z = \frac{TR_1 - TR_2}{\sqrt{\frac{1}{T}\left[2\left(1-\rho_{1,2}\right)+\frac{1}{2}\left(TR_1^2+TR_2^2-TR_1TR_2\left(1+\rho_{1,2}^2\right)\right)\right]}} \tag{1}$$

Where TR_1 stands for the Treynor Ratio in bear markets and TR_2 for bull markets. T is the amount of monthly observations in the times series and ρ is the correlation between the bear and bull market portfolio return. For the Z-test for Sharpe ratio the Treynor ratio is switched for Sharpe ratio.

The difference in Treynor ratios is tested between bull and bear markets. This is done for all ten volatility portfolios. The first hypothesis will be rejected if the Z-score is not significant, meaning that the difference between Treynor Ratios in bear and bull markets does not differ.

3.1.2. Open Market Operations

To test the second hypothesis: *In periods without Open Market Operations the low volatility anomaly is less pronounced*. To test this hypothesis first a few terms need to be clarified. First in what periods did the FED ran their OMO program; March 2009 – October 2009, November 2010 – June 2011, September 2011 – June 2012, and September 2012 – October 2014 (FED, 2021).

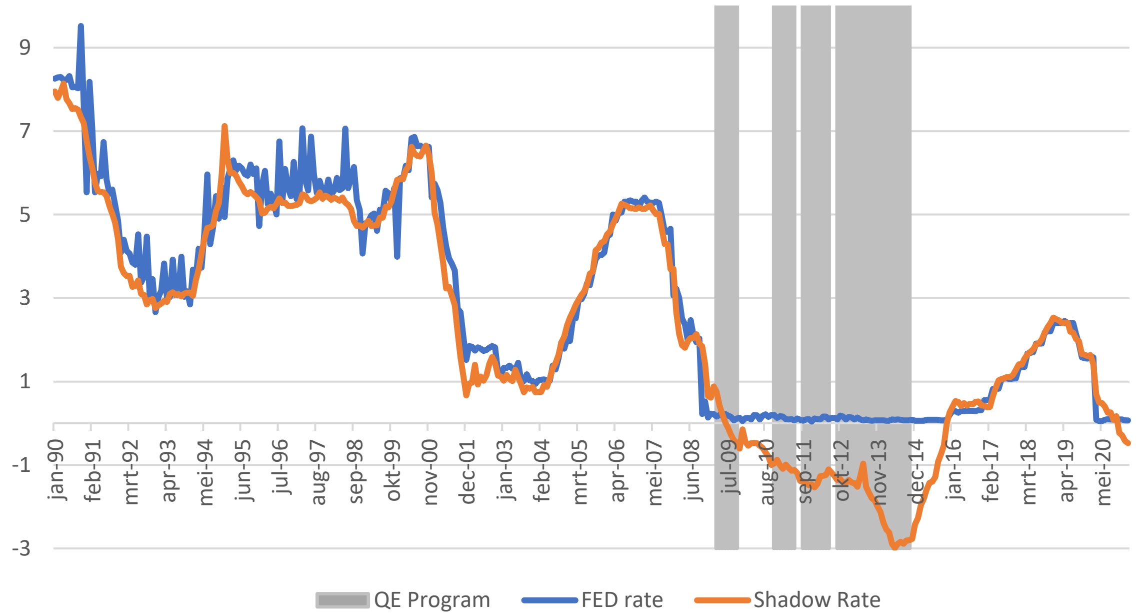

Figure 1 The course of the FED rate, Shadow rate as calculated by Wu and Zhang (2019), and the periods when the FED performed QE. Data retrieved from the FED website (Wu-Xia Shadow Federal Funds Rate, 2021)

Secondly, what qualifies as less pronounced? To test what part of the variation in returns on the constructed volatility portfolios is actually to result of it being low volatility and not because of intervention by the FED. This is tested by estimating the coefficient of a dummy variable of QE in an OLS regression. To make sure the results are not clouded by different factors, the regression will be correct for market, size, value, operating profitability and investment factor as explained in Fama and French (2015).

$$R_{p,t} = \alpha_p + \beta_{1,p}RMRF_t + \beta_{2,p}SMB_t + \beta_{3,p}HML_t + \beta_{4,p}RMW_t + \beta_{5,p}CMA_t + \beta_{6,p}QE_t + \varepsilon_{p,t} \quad (2)$$

The regression of equation will be estimated eleven times, once for every volatility portfolio and once for the market portfolio. QE_t is a dummy variable that takes on the value of one when the FED performs their OMO program, and zero otherwise. The coefficient β_6 says how much of the return variation is explained by the OMO program. If the OMO program affects low volatility assets in a different way than the market the estimated coefficient should be different between the regression coefficients β_6. To test if this difference is statically significant a Z-test will be performed. This Z-test is designed by Clogg, Petkova and Haritou (1995), see formula 3. Depending on the sign of the coefficient and the test statistic of the Z-test the second hypothesis can be rejected.

$$Z = \frac{\beta_1 - \beta_2}{\sqrt{(SE\beta_1)^2 + (SE\beta_2)^2}} \tag{3}$$

One thing to note is that the regression might exhibit multicollinearity since the dummy variable might be linearly related with one of the other factors.

3.1.3. Shadow Rate

Lastly the third hypothesis is tested: *The changes in the shadow rate have a negative relationship with the return on the low volatility anomaly*. If the lowering of the interest rate corresponds to intervention by the FED, the lowering of the rate would increase the prices on the equities market. This implies a negative correlation between the rate and the return. To test if this is true the same procedure and equation are used as in hypothesis 2. Except now instead of a dummy variable the first difference on the shadow rate is used as independent variable.

$$R_{p,t} = \alpha_p + \beta_{1,p}RMRF_t + \beta_{2,p}SMB_t + \beta_{3,p}HML_t + \beta_{4,p}RMW_t + \beta_{5,p}CMA_t + \beta_{6,p}\Delta SR_t + \varepsilon_{p,t} \tag{4}$$

Again, the regression is estimated eleven times, once for the volatility portfolios and once for the market portfolio. The estimated coefficients β_6 are Z-tested on statistically significant difference, with formula 3.

3.2. Data

To empirically test the hypothesis that are stated in the previous section the CRSP database is used to retrieve data on the American equities market. The research is limited to the New York, American, and The Nasdaq Stock exchange. Within these stock exchanges only the ordinary common shares are used. To eliminate small valued stocks from this dataset only equities which have a price larger than \$1 dollar are included. Of these prices the monthly lognormal returns are calculated. The dataset includes assets from January 1990 until the end of 2020. This allows to include multiple periods of high volatility.

The dataset that is used for the measuring monetary policy in terms of OMO will be retrieved from the website of the FED (Open market operations, 2021). Additionally, the shadow rates as computed in Wu and Zhang (2019) is retrieved from the FED website (Wu-Xia Shadow Federal Funds Rate, 2021). All observations will be monthly. To correct the return on the volatility portfolios for the five Fama & French factors, the website of Kenneth French will be used to retrieve the factor data (2021).

Table 1

Descriptive statistics of the ten value weighted volatility portfolios, volatility based on three-year historic standard deviation, divided by bear and bull markets. Returns on American equity markets for the period Jan 1993 - Dec 2020

Category	Obs.	Mean	St. Dev	Skewness	Kurtosis
Bear market					
Bottom 10%	151	-0.018	0.040	-1.078	5.875
10%-20%	151	-0.022	0.043	-0.770	4.980
20%-30%	151	-0.024	0.041	-1.146	5.783
30%-40%	151	-0.027	0.048	-1.679	8.922
40%-50%	151	-0.029	0.052	-0.654	4.703
50%-60%	151	-0.035	0.053	-1.383	5.371
60%-70%	151	-0.036	0.052	-0.758	3.935
70%-80%	151	-0.041	0.061	-1.152	4.796
80%-90%	151	-0.049	0.082	0.314	9.780
Top 10%	151	-0.045	0.078	-0.993	3.842
Bull market					
Bottom 10%	184	0.014	0.030	-0.961	5.586
10%-20%	184	0.026	0.032	0.133	5.778
20%-30%	184	0.030	0.042	-2.473	23.896
30%-40%	184	0.029	0.048	-4.470	47.370
40%-50%	184	0.035	0.037	0.253	5.042
50%-60%	184	0.038	0.038	0.534	4.710
60%-70%	184	0.044	0.051	-1.692	21.428
70%-80%	184	0.053	0.056	1.052	6.133
80%-90%	184	0.057	0.062	0.712	4.370
Top 10%	184	0.065	0.082	0.898	5.587

As observed in Table 1 the mean doesn’t follow the expected pattern, which is that lower volatility portfolios would earn a higher mean return. Although later in this paper some proof is provided for lower volatility portfolios earning a higher risk-adjusted return. The dataset contains a total of 335 observations. Of those 335 around 45% are from the bear market and 55% from the bull market.

4. Results

Because of the nature of the data, I wanted to research if there isn't a factor bias in the dataset. To do this, the returns of the volatility portfolios are regressed on the five Fama and French factor model.

Table 2

Regression results of $R_{p,t} = \alpha_p + \beta_{1,p}RMRF_t + \beta_{2,p}SMB_t + \beta_{3,p}HML_t + \beta_{4,p}RMW_t + \beta_{5,p}CMA_t + \varepsilon_{p,t}$ for the period Jan 1993 - Dec 2020 where portfolios are ranked on historic three-year volatility.

		Coefficients				
Portfolios	*Alpha*	*RMRF*	*SMB*	*HML*	*RMW*	*CMA*
Bottom 10%	-0.0076***	0.0077***	-0.0015***	-0.0001	0.0034***	0.0042***
	(0.0014)	(0.0004)	(0.0005)	(0.0006)	(0.0007)	(0.0009)
10%-20%	-0.0046***	0.0101***	-0.0008**	0.0020***	0.0034***	0.0021***
	(0.0010)	(0.0002)	(0.0003)	(0.0004)	(0.0004)	(0.0006)
20%-30%	-0.0016	0.0094***	0.0005	0.0031***	0.0011	0.0005
	(0.0016)	(0.0004)	(0.0006)	(0.0007)	(0.0008)	(0.0010)
30%-40%	-0.0050***	0.0106***	0.0010	0.0011	0.0028***	0.0015
	(0.0019)	(0.0005)	(0.0007)	(0.0008)	(0.0009)	(0.0012)
40%-50%	-0.0032**	0.0115***	0.0001	0.0017***	0.0027***	0.0009
	(0.0015)	(0.0004)	(0.0005)	(0.0006)	(0.0007)	(0.0009)
50%-60%	-0.0033**	0.0114***	0.0015***	-0.0008	-0.0011*	-0.0005
	(0.0013)	(0.0003)	(0.0005)	(0.0006)	(0.0006)	(0.0008)
60%-70%	0.0004	0.0113***	0.0016**	0.0008	-0.0017*	-0.0011
	(0.0022)	(0.0005)	(0.0008)	(0.0009)	(0.0010)	(0.0014)
70%-80%	0.0017	0.0133***	0.0035***	-0.0024***	-0.0028***	0.0000
	(0.0019)	(0.0005)	(0.0007)	(0.0008)	(0.0009)	(0.0012)
80%-90%	0.0006	0.0141***	0.0033***	-0.0018	-0.0050***	-0.0026
	(0.0027)	(0.0007)	(0.0010)	(0.0012)	(0.0013)	(0.0017)
Top 10%	0.0070**	0.0145***	0.0035***	-0.0024*	-0.0072***	-0.0026
	(0.0030)	(0.0008)	(0.0011)	(0.0013)	(0.0014)	(0.0019)

*Note: * denote significance; * p < 0.1; ** p < 0.05; and *** p < 0.01.*

Although not all the coefficients are significant, a pattern is observed. The first being the alpha, which shows a positive relation with the volatility portfolios. However, the fourth, fifth, and sixth portfolio do not follow this pattern. Two possible explanations can be given. On one hand, the coefficient for the third portfolio can be an outlier, since it lacks significance at, at least the 10% level. Meaning that the positive relationship is only broken by a lack of certainty. On the other hand, the disruption in the pattern can mean that portfolios in the middle have, on average less inherent return, in the absence of the other coefficients. That could mean that investors are more likely to pay a higher price for assets that are in the middle of the volatility spectrum. In behavioural economics this is called extremeness aversion (Simonson & Tversky, 1992).

The same positive relation is observed with the market factor. This is in line with the traditional CAPM theory. The size factor tells us that assets with a small market capitalization earn on average higher returns. The volatility portfolios exhibit a positive relation with the size factor. Concluding that the size factor plays a more prominent role for higher volatility portfolios than for lower ones. However, the coefficients for the lower volatility portfolios are not significant at even a 10% level.

Lastly, the value, the profitability, and the investment factor all show a negative relation with the volatility portfolios. Where the lower volatility portfolios have positive coefficients, while higher volatility portfolios have a negative coefficient. Concluding that those factors have a positive impact on lower volatility portfolios which this is the opposite for higher volatility portfolios. Although this also comes with a sidenote, not all coefficients are significant. This is especially the case for the investment factor. This factor only has significant coefficients for the lowest two volatility portfolios.

4.1. Value weighted portfolios

To test the first hypothesis, ten portfolios based on three-year historic volatility are constructed and their lognormal returns calculated. The first hypothesis researches if the low volatility anomaly has a greater performance in bear markets than in bull markets. This is tested by comparing return metrics, on statistically significant differences between bear and bull markets. This is done within the 10 different volatility portfolios. In the column *mean return* in Table 3, it is shown that the average return increases with the volatility levels. This seems to be the case for the bull market, while the opposite is true for the bear market. Which shows that low volatility portfolios do perform better in bear markets than higher volatility portfolios, but the mean return for bear markets is below zero.

The standard deviations for the portfolios are F-tested, the results for this test can be found in appendix I. The standard deviations that are near each other (e.g., one and two, two and three) are not significant at a 5% level, this is the case for both the bear and bull market. Portfolios that are further apart have standard deviations that are significantly different.

Concluding that the volatility levels do increase with the portfolios, which is to be expected, but the difference is not always significant.

Table 3

Summary statistics ten value weighted volatility portfolios divided between bear and bull markets over the period Jan 1993 – Dec 2020, the returns are monthly

Category	**Mean**	**St. Dev**	**CAPM Alpha**	**CAPM Beta**	**Treynor**	**Sharpe**	**Return >0**
Bear							
Bottom 10%	-0.0179	0.0395	0.0007	0.0997***	-0.180	-0.453	52
10%-20%	-0.0220	0.0427	0.0075**	0.1581***	-0.139	-0.515	43
20%-30%	-0.0244	0.0411	0.0063**	0.1643***	-0.148	-0.593	42
30%-40%	-0.0270	0.0476	0.0075**	0.1845***	-0.146	-0.567	42
40%-50%	-0.0285	0.0522	0.0046	0.1773***	-0.161	-0.547	36
50%-60%	-0.0353	0.0527	0.0046	0.2136***	-0.165	-0.670	35
60%-70%	-0.0356	0.0520	0.0010	0.1962***	-0.181	-0.685	37
70%-80%	-0.0406	0.0611	0.0018	0.2274***	-0.179	-0.665	35
80%-90%	-0.0489	0.0818	0.0003	0.2635***	-0.186	-0.598	47
Top 10%	-0.0455	0.0776	0.0022	0.2552***	-0.178	-0.586	41
Bull							
Bottom 10%	0.0140	0.0303	0.0049	0.0634***	0.221	0.464	159
10%-20%	0.0258	0.0324	0.0074**	0.1268***	0.203	0.795	157
20%-30%	0.0305	0.0424	0.0076*	0.1577***	0.193	0.718	161
30%-40%	0.0295	0.0476	0.0036	0.1788***	0.165	0.619	165
40%-50%	0.0346	0.0369	0.0087**	0.1786***	0.194	0.938	166
50%-60%	0.0377	0.0383	0.0072**	0.2107***	0.179	0.985	164
60%-70%	0.0443	0.0514	0.0084*	0.2480***	0.179	0.863	164
70%-80%	0.0535	0.0557	0.0055	0.3316***	0.161	0.959	157
80%-90%	0.0573	0.0616	0.0068	0.3488***	0.164	0.930	156
Top 10%	0.0654	0.0817	0.0012	0.4437***	0.147	0.801	131

*Note: * denote significance; * $p < 0.1$; ** $p < 0.05$; and *** $p < 0.01$.*

To put the results in perspective, the CAPM alpha and beta are estimated, see Table 3. Not all alphas are significant, meaning they could as well be zero. The CAPM betas are significant at the highest level for every portfolio and business cycle. The portfolios exhibit a positive linear relationship with the market, which is in line with the traditional CAPM theory. The betas in the lower volatility portfolios seems to be higher in bear markets than in bull markets, while the opposite is true for higher volatility portfolios. Furthermore, it seems that, on average, lower volatility portfolios exhibit more positive returns in bear markets, than higher volatility portfolios.

Table 4

Overview of statistically significant difference between bear and bull markets for every volatility portfolio. Performed for both the Treynor as the Sharpe ratio.

Category		Treynor	Z-statistic	p-value	Sharpe Ratio	Z-statistic	p-value
Bottom 10%	Bear market	-0.180	-5.087	0.000	-0.453	-10.973	0.000
	Bull market	0.221			0.464		
10%-20%	Bear market	-0.139	-4.319	0.000	-0.515	-14.486	0.000
	Bull market	0.203			0.795		
20%-30%	Bear market	-0.148	-4.355	0.000	-0.593	-14.678	0.000
	Bull market	0.193			0.718		
30%-40%	Bear market	-0.146	-3.733	0.000	-0.567	-12.801	0.000
	Bull market	0.165			0.619		
40%-50%	Bear market	-0.161	-4.712	0.000	-0.547	-16.106	0.000
	Bull market	0.194			0.938		
50%-60%	Bear market	-0.165	-4.34	0.000	-0.670	-17.371	0.000
	Bull market	0.179			0.985		
60%-70%	Bear market	-0.181	-4.536	0.000	-0.685	-16.628	0.000
	Bull market	0.179			0.863		
70%-80%	Bear market	-0.179	-4.048	0.000	-0.665	-17.166	0.000
	Bull market	0.161			0.959		
80%-90%	Bear market	-0.186	-4.32	0.000	-0.598	-11.549	0.000
	Bull market	0.164			0.930		
Top 10%	Bear market	-0.178	-3.897	0.000	-0.586	-15.367	0.000
	Bull market	0.147			0.801		

To start the analysis of the first hypothesis; *The low volatility anomaly has a greater performance in bull markets than in bear markets*, the Sharpe and Treynor ratios are analysed. The Sharpe and the Treynor ratio are compared within a volatility portfolio between the bear and bull market. Based on the corresponding Z-score the significance of the difference is established. In the current form, all Sharpe and Treynor ratios are negative in bear markets and positive in bull markets. Calculating the corresponding Z-statistics gives that every difference, within portfolios between markets, is significant at the highest level. The statistics are tabulated in Table 4.

It does seem that Treynor ratio is slightly decreasing for both the bear market and the bull market, this would indicate that a higher volatility gives less return per unit of volatility. This is not the case for the Sharpe ratio. The Sharpe ratio is slightly decreasing for bear markets. The Sharpe ratio in bull markets has a slight increase, but flattens when reaching higher volatility portfolios see Figure 2.

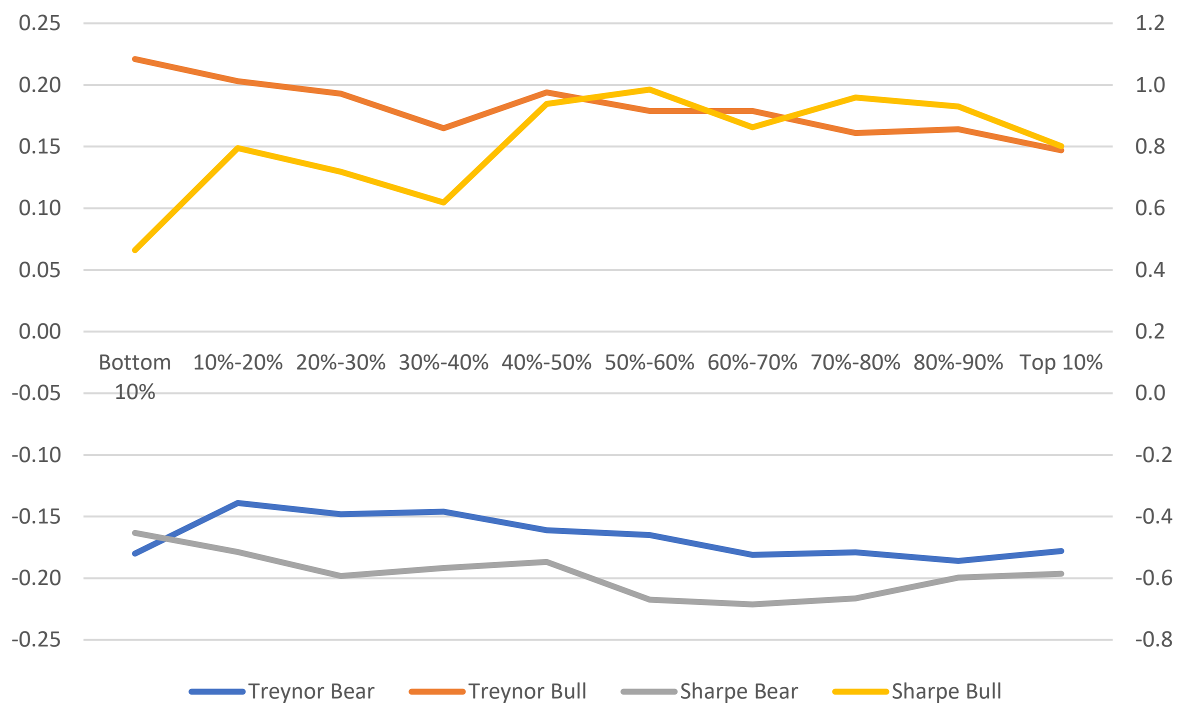

Figure 2 The ten value weighted volatility portfolios visualised over the time period 1990-2020 divided into bear and bull markets, based on the Treynor and Sharpe ratio (Treynor on the left axis, Sharpe on the right).

The Z-statistic being negative means that both the bear market Sharpe and Treynor ratio is lower than that of the bull market. None of the Z-statistics are insignificant, meaning that in all portfolios the bull markets Sharpe and Treynor ratio is higher and differs significantly from the bear market.

4.2. Open Market Operations

To test the effect of open market operations on the low volatility anomaly the Fama and French (2015) five factor regression is performed with the addition of a dummy variable for the periods that the FED ran their Open Market Operations in the US. The regression is run 11 times, once for every volatility portfolio, and once for the overall market. The beta coefficient for the dummy variable QE should differ if QE has a different impact on the different portfolios. To test if the difference between beta coefficient is significant the Z-test of Clogg, Petkova and Haritou (1995) is performed.

Table 5

Regression result of $R_{p,t} = \alpha_p + \beta_{1,p}RMRF_t + \beta_{2,p}SMB_t + \beta_{3,p}HML_t + \beta_{4,p}RMW_t + \beta_{5,p}CMA_t + \beta_{6,p}QE_t + \varepsilon_{p,t}$ for the period Jan 1993 – Dec 2020.

		Coefficients					
Portfolios	*Alpha*	*RMRF*	*SMB*	*HML*	*RMW*	*CMA*	*QE*
Bottom 10%	-0.0084***	0.0076***	-0.0015***	-0.0001	0.0035***	0.0042***	0.0054
	(0.0015)	(0.0004)	(0.0005)	(0.0006)	(0.0007)	(0.0009)	(0.0037)
10%-20%	-0.0048***	0.0101***	-0.0008**	0.0019***	0.0035***	0.0021***	0.0018
	-0.001	-0.0002	-0.0003	-0.0004	-0.0004	-0.0006	-0.0026
20%-30%	-0.0020	0.0094***	0.0006	0.0031***	0.0011	0.0005	0.0029
	(0.0018)	(0.0004)	(0.0006)	(0.0007)	(0.0008)	(0.0010)	(0.0044)
30%-40%	-0.0039*	0.0107***	0.001	0.0011	0.0028***	0.0015	-0.0075
	(0.0020)	(0.0005)	(0.0007)	(0.0008)	(0.0009)	(0.0012)	(0.0051)
40%-50%	-0.0037**	0.0114***	0.0001	0.0016**	0.0027***	0.0009	0.003
	(0.0016)	(0.0004)	(0.0005)	(0.0006)	(0.0007)	(0.0009)	(0.0040)
50%-60%	-0.0040***	0.0113***	0.0015***	-0.0009	-0.0011*	-0.0005	0.0049
	(0.0014)	(0.0003)	(0.0005)	(0.0006)	(0.0006)	(0.0008)	(0.0035)
60%-70%	-0.0003	0.0112***	0.0016**	0.0007	-0.0017*	-0.0011	0.0044
	(0.0023)	(0.0005)	(0.0008)	(0.0009)	(0.0010)	(0.0014)	(0.0057)
70%-80%	0.0018	0.0133***	0.0035***	-0.0024***	-0.0028***	0.0000	-0.0007
	(0.0020)	(0.0005)	(0.0007)	(0.0008)	(0.0009)	(0.0012)	(0.0050)
80%-90%	-0.0007	0.0140***	0.0033***	-0.0019	-0.0049***	-0.0027	0.0085
	(0.0029)	(0.0007)	(0.0010)	(0.0012)	(0.0013)	(0.0017)	(0.0072)
Top 10%	0.0064*	0.0145***	0.0035***	-0.0025*	-0.0072***	-0.0026	0.0039
	(0.0033)	(0.0008)	(0.0011)	(0.0013)	(0.0014)	(0.0019)	(0.0081)
Market	-0.0036		0.0307***	0.0288***	-0.0280***	-0.0404***	0.0623**
	(0.0102)		(0.0034)	(0.0040)	(0.0043)	(0.0057)	(0.0255)

*Note: * denote significance; * p < 0.1; ** p < 0.05; and *** p < 0.01. Standard errors within brackets.*

One of the first things to note in Table 5 is that the coefficients of the dummy variable QE is insignificant for all portfolios. The dummy variable is however significant at a 5% level for the overall market. The dummy variable having a low explanatory value for the return on the portfolios does make sense since the Fama and French five factors should be able to explain the return on a market. However, the combined effect of QE seems to have an effect on the overall market. This is also expected, since the whole point of QE is to lower volatility and kick-start the market.

Performing a Z-test on the dummy variable coefficient to test if the difference between portfolios is significant would not give the desired results. The coefficients are not significant at, at least a 10% level meaning they could as well be zero. If the test is performed anyway, the Z-statistic is significant at the highest level. Thus, concluding that each coefficient is statically different from one another.

The procedure is repeated for the Fama and French three factor model. However, the same results are observed; the QE coefficient only being statistically significant for the market and not for the individual portfolios.

4.3. Shadow Rates

One of the disadvantages of using Open Market Operations as a form of Quantitative Easing is that the FED does not actively perform OMO, but only in certain periods. However, the FED does monitor the economy continuously through interest rates. As discussed in the previous section, the FED rate should not be able to become negative. Therefore, in this research the estimated shadow rate is used as a proxy for continuously QE by the FED.

As the results in Table 6 shows, the coefficient for Delta Shadow rate is for most portfolios not significant, except for the portfolio with the highest volatility and for the overall market. However, every coefficient is negative, meaning a positive change in the Shadow rate corresponds with a negative return on the portfolio and the market, which is expected.

Table 6

Regression result of $R_{p,t} = \alpha_p + \beta_{1,p}RMRF_t + \beta_{2,p}SMB_t + \beta_{3,p}HML_t + \beta_{4,p}RMW_t + \beta_{5,p}CMA_t + \beta_{6,p}\Delta SR_t + \varepsilon_{p,t}$ for the period Jan 1993 – Dec 2020.

		Coefficients					
Portfolios	*Alpha*	*RMRF*	*SMB*	*HML*	*RMW*	*CMA*	*ΔSR*
Bottom 10%	-0.0076***	0.0077***	-0.0015***	0.0000	0.0034***	0.0042***	-0.0015
	(0.0014)	(0.0004)	(0.0005)	(0.0006)	(0.0007)	(0.0009)	(0.0029)
10%-20%	-0.0046***	0.0101***	-0.0008**	0.0020***	0.0034***	0.0021***	-0.0009
	-0.001	-0.0002	-0.0003	-0.0004	-0.0004	-0.0006	-0.002
20%-30%	-0.0016	0.0094***	0.0005	0.0031***	0.0011	0.0004	(0.0010)
	(0.0016)	(0.0004)	(0.0006)	(0.0007)	(0.0008)	(0.0011)	(0.0034)
30%-40%	-0.0050***	0.0106***	0.001	0.0011	0.0028***	0.0015	-0.0007
	(0.0019)	(0.0005)	(0.0007)	(0.0008)	(0.0009)	(0.0012)	(0.0039)
40%-50%	-0.0032**	0.0115***	0.0001	0.0017***	0.0027***	0.0008	-0.0014
	(0.0015)	(0.0004)	(0.0005)	(0.0006)	(0.0007)	(0.0010)	(0.0031)
50%-60%	-0.0032**	0.0113***	0.0014***	-0.0008	-0.0012*	-0.0005	-0.0015
	(0.0013)	(0.0003)	(0.0005)	(0.0006)	(0.0006)	(0.0008)	(0.0027)
60%-70%	0.0004	0.0111***	0.0015**	0.0008	-0.0018*	-0.0013	-0.0067
	(0.0021)	(0.0006)	(0.0008)	(0.0009)	(0.0010)	(0.0014)	(0.0044)
70%-80%	0.0017	0.0133***	0.0035***	-0.0024***	-0.0028***	0.0000	0.0000
	(0.0019)	(0.0005)	(0.0007)	(0.0008)	(0.0009)	(0.0012)	(0.0038)

80%-90%	0.0006	0.0141***	0.0033***	-0.0018	-0.0050***	-0.0027	-0.0006
	(0.0027)	(0.0007)	(0.0010)	(0.0012)	(0.0013)	(0.0017)	(0.0056)
Top 10%	0.0070**	0.0143***	0.0033***	-0.0024*	-0.0073***	-0.003	-0.0150**
	(0.0030)	(0.0008)	(0.0011)	(0.0013)	(0.0014)	(0.0019)	(0.0062)
Market	0.0058		0.0300***	0.0296***	-0.0288***	-0.0413***	-0.0337*
	(0.0093)		(0.0034)	(0.0040)	(0.0043)	(0.0058)	(0.0197)

*Note: * denote significance; * p < 0.1; ** p < 0.05; and *** p < 0.01. Standard errors within brackets.*

Because of the lack of significant coefficients, the proposed Z-test, to test the significance of the difference in coefficients, would not give the desired results. When performed anyway, all de Z-scores and corresponding p-values are significant at the highest level. Meaning that the difference between all the coefficients is statically significant if the coefficients are not equal to zero.

The same procedure is also performed with only the first three factors from the Fama & French model, but the results do not change.

4.4. Robustness

To research whether the performed tests and their results are robust, additional tests are performed with different variables. For the factor business cycles, value weighted portfolios are researched, and the length which is used to calculate historic volatility is changed.

4.4.1. Equally weighted portfolios

According to the literature of Blitz and Van Vliet (2007) the low volatility anomaly is observed within both value and equally weighted portfolios. To get a better understanding of the low volatility anomaly, the first hypothesis is also tested with equally weighted portfolios.

With equally weighted portfolios a few things change from that of value weighted portfolios. The Sharpe and Treynor ratio seem to have a negative relationship with volatility, when the business cycle is a bear market. This relationship is more pronounced than with the value weighted portfolio. The relationship between the Treynor ratio and volatility seems also negative in the bull market, while the Sharpe ratio shows a clear concave relationship. The highest Sharpe ratio is observed in the 40%-50% volatility portfolio, while the highest Treynor ratio is observed in the lowest volatility portfolio. The total returns above zero also differ from the value weighted portfolios. The returns above zero in bear markets is no longer linear, but a convex function, with the lowest amount of returns above zero being in the 40%-50% volatility portfolio.

Table 7

Summary statistics ten equally weighted volatility portfolios divided between bear and bull markets over the period Jan 1993 – Dec 2020, the returns are monthly.

Category	Mean	St. Dev	CAPM Alpha	CAPM Beta	Treynor	Sharpe	Return >0
Bear							
Bottom 10%	-0.0166	0.0318	0.0067***	0.0498***	-0.332	-0.521	35
10%-20%	-0.0248	0.0382	0.0064***	0.0667***	-0.372	-0.649	30
20%-30%	-0.0305	0.0423	0.0056***	0.0773***	-0.394	-0.719	23
30%-40%	-0.0352	0.0471	0.0062***	0.0888***	-0.397	-0.748	21
40%-50%	-0.0394	0.0479	0.0042***	0.0934***	-0.421	-0.821	18
50%-60%	-0.0456	0.0546	0.0051***	0.1086***	-0.420	-0.836	7
60%-70%	-0.0536	0.0576	0.0005	0.1159***	-0.463	-0.931	8
70%-80%	-0.0626	0.0576	-0.0036*	0.1264***	-0.495	-1.087	15
80%-90%	-0.0727	0.0714	-0.0115***	0.1310***	-0.555	-1.018	12
Top 10%	-0.0860	0.0859	-0.0196***	0.1422***	-0.605	-1.000	41
Bull							
Bottom 10%	0.0165	0.0175	0.0053***	0.0310***	0.532	0.939	169
10%-20%	0.0235	0.0220	0.0047***	0.0518***	0.453	1.069	167
20%-30%	0.0281	0.0249	0.0051***	0.0635***	0.443	1.131	171
30%-40%	0.0311	0.0265	0.0043***	0.0739***	0.421	1.173	176
40%-50%	0.0344	0.0274	0.0049***	0.0816***	0.421	1.254	180
50%-60%	0.0379	0.0316	0.0024**	0.0980***	0.387	1.199	176
60%-70%	0.0412	0.0358	0.0005	0.1125***	0.366	1.151	172
70%-80%	0.0460	0.0438	-0.0022	0.1331***	0.345	1.049	164
80%-90%	0.0520	0.0536	-0.0059**	0.1598***	0.325	0.970	138
Top 10%	0.0514	0.0714	-0.0191***	0.1948***	0.264	0.720	131

*Note: * denote significance; * p < 0.1; ** p < 0.05; and *** p < 0.01.*

The same test for significant differences between Treynor and Sharpe ratio, within portfolios, between business cycles is again performed. None of the differences are insignificant, so no change compared to value weighted portfolios.

The standard deviations are F-tested on significant difference between the portfolios. These results can be found in appendix I. For the bear market, most portfolios that are near each other are not significantly different, except for portfolios one and two and nine and ten. For the bull market, only the lower portfolios are statistically indifferent from each other. Again, as with equally weighted portfolios, a linear pattern is observed between portfolios and volatility.

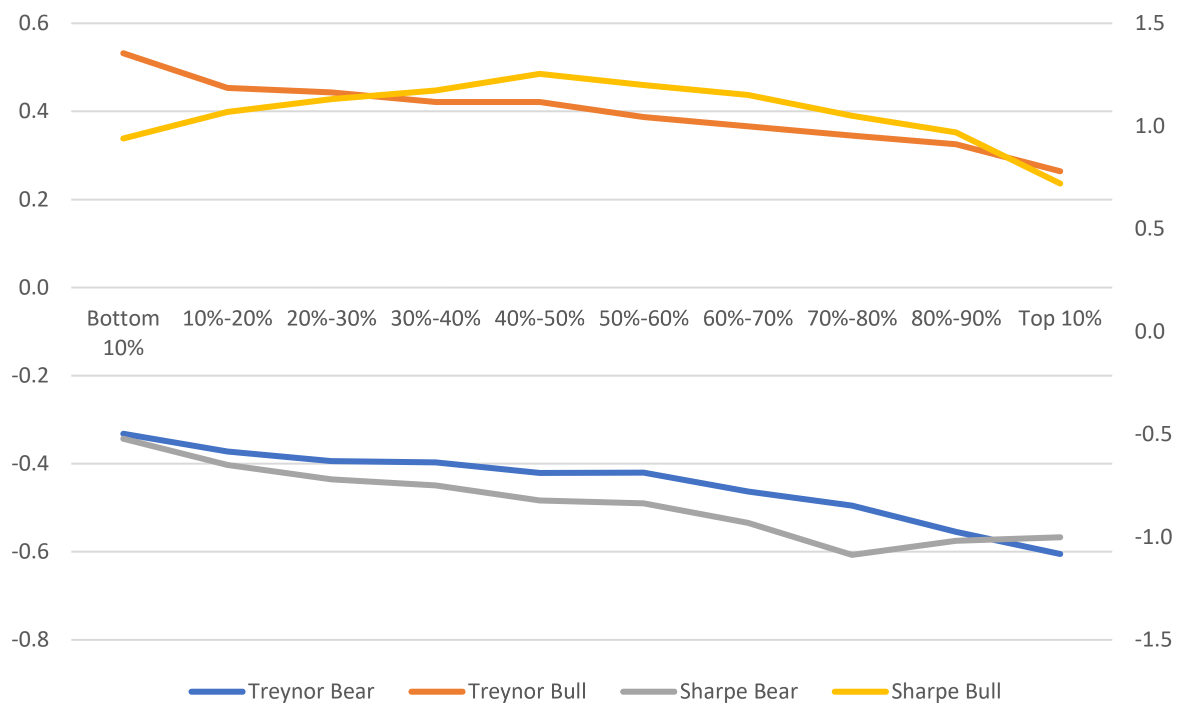

Figure 3 The ten equally weighted volatility portfolios visualised over the time period 1990-2020 divided into bear and bull markets, based on the Treynor and Sharpe ratio (Treynor on the left axis, Sharpe on the right).

4.4.2. Historic volatility window

The ten volatility portfolios, which are used in this paper, are constructed based on 36 months historic volatility. To test if the results change when the calculation window changes. The same analysis is performed, but now on portfolios which are constructed based on their 12 months prior volatility instead of 36 months. The statistics are tabulated below.

Table 8

Summary statistics ten value weighted volatility portfolios divided between bear and bull markets over the period Jan 1991 – Dec 2020, the returns are monthly, and the volatility is based on the past 12 months.

Category	Mean	St. Dev	CAPM Alpha	CAPM Beta	Treynor	Sharpe	Return >0
Bear							
Bottom 10%	-0.0138	0.0342	0.0021	0.3310***	-0.042	-0.404	59
10%-20%	-0.0209	0.0403	0.0024	0.4826***	-0.043	-0.517	57
20%-30%	-0.0249	0.0448	0.0062*	0.6452***	-0.039	-0.555	52
30%-40%	-0.0330	0.0460	-0.0022	0.6401***	-0.052	-0.717	40
40%-50%	-0.0321	0.0492	0.0054*	0.7782***	-0.041	-0.652	37
50%-60%	-0.0360	0.0521	0.0051	0.8539***	-0.042	-0.690	42
60%-70%	-0.0384	0.0578	0.0074**	0.9523***	-0.040	-0.665	44
70%-80%	-0.0350	0.0618	0.0105**	0.9457***	-0.037	-0.567	48

80%-90%	-0.0404	0.0701	0.0124***	1.0968***	-0.037	-0.576	47
Top 10%	-0.0437	0.0796	0.0094	1.1053***	-0.040	-0.549	54
Bull							
Bottom 10%	0.0174	0.0283	0.0113***	0.1602**	0.108	0.614	145
10%-20%	0.0231	0.0338	0.0107***	0.3273***	0.070	0.682	159
20%-30%	0.0310	0.0324	0.0128***	0.4817***	0.064	0.957	165
30%-40%	0.0344	0.0354	0.0103***	0.6301***	0.055	0.973	166
40%-50%	0.0384	0.0601	0.0066	0.8416***	0.046	0.639	172
50%-60%	0.0483	0.0414	0.0129***	0.9384***	0.051	1.167	174
60%-70%	0.0504	0.0518	0.0121***	1.0132***	0.050	0.974	172
70%-80%	0.0568	0.0482	0.0186***	1.0087***	0.056	1.178	178
80%-90%	0.0586	0.0619	0.0098*	1.2922***	0.045	0.947	170
Top 10%	0.0674	0.0740	0.0123*	1.4585***	0.046	0.911	172

*Note: * denote significance; * p < 0.1; ** p < 0.05; and *** p < 0.01.*

Using a rolling window of 12 months for historic volatility does not change the statistics by much. Obviously, more observations are present, increasing to 359, of which 47% are observed in the bear market. The Treynor and Sharpe ratio in the bear market seems flatter. While the Sharpe ratio in the bull market has a less of a pattern and jumps.

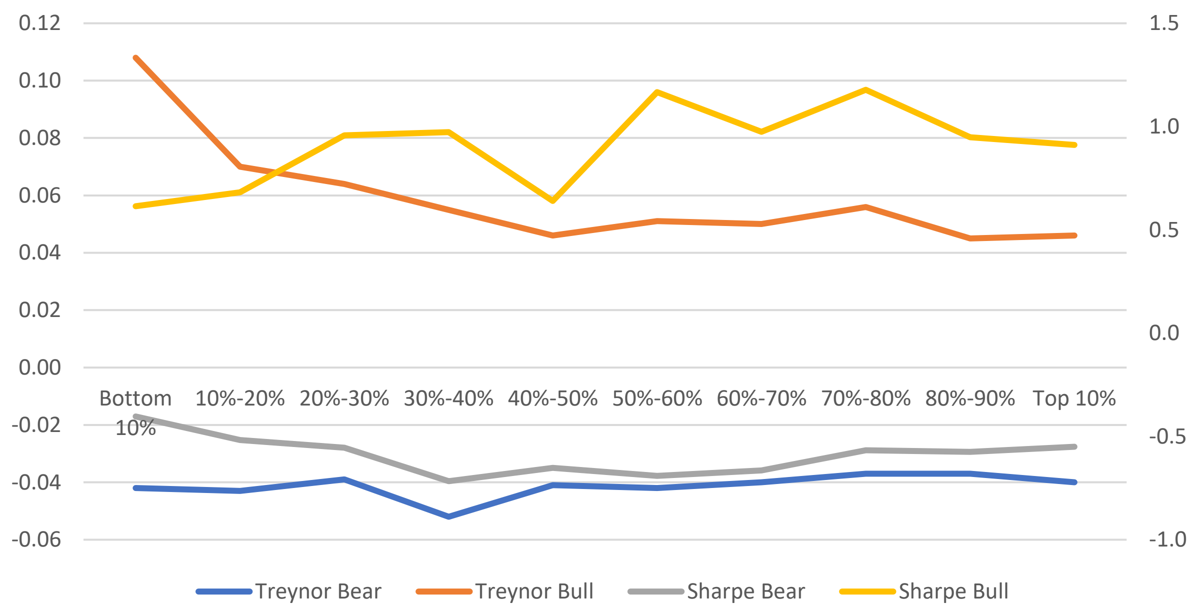

Figure 4 The course of Treynor and Sharpe ratios of the volatility portfolios, with the ranking on historic volatility based on only one year. Treynor on the left Sharpe on the right.

4.4.3. Amount of portfolios

One of the observed results from hypothesis two and three is that QE and ΔSR, seems to only have an effect on the overall market and not on the individual portfolios. To test the robustness of this result, the amount of portfolios is changed. From ten portfolios to only four.

Table 9

Regression result of $R_{p,t} = \alpha_p + \beta_{1,p}RMRF_t + \beta_{2,p}SMB_t + \beta_{3,p}HML_t + \beta_{4,p}RMW_t + \beta_{5,p}CMA_t + \beta_{6,p}QE_t + \varepsilon_{p,t}$ for the period Jan 1991 – Dec 2020, with one year historic volatility.

		Coefficients					
Portfolios	*Alpha*	*RMRF*	*SMB*	*HML*	*RMW*	*CMA*	*QE*
Bottom 25%	-0.0060***	0.0085***	-0.0011***	0.0011***	0.0030***	0.0030***	0.0032
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.003)
26-50%	-0.0034***	0.0109***	0.0001	0.0014***	0.0026***	0.0013*	-0.0023
	(0.001)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.003)
51-75%	-0.0015	0.0113***	0.0017***	-0.0005	-0.0015**	-0.0005	0.0037
	(0.001)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.003)
Top 25%	0.0017	0.0139***	0.0028***	-0.0024***	-0.0053***	-0.0027**	0.008
	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.005)
Market	-0.0036		0.0307***	0.0288***	-0.0280***	-0.0404***	0.0623***
	(0.005)		(0.002)	(0.002)	(0.002)	(0.003)	(0.013)

*Note: * denote significance; * p < 0.1; ** p < 0.05; and *** p < 0.01. Standard errors within brackets.*

Using four portfolios instead of ten does not change the significance of the coefficient of the QE variable. Meaning it still hasn't had a significant impact on the return on the volatility portfolios. However, the impact is still significant on the overall market.

Table 10

Regression result of $R_{p,t} = \alpha_p + \beta_{1,p}RMRF_t + \beta_{2,p}SMB_t + \beta_{3,p}HML_t + \beta_{4,p}RMW_t + \beta_{5,p}CMA_t + \beta_{6,p}\Delta SR_t + \varepsilon_{p,t}$ for the period Jan 1991 – Dec 2020, with one year historic volatility.

		Coefficients					
Portfolios	*Alpha*	*RMRF*	*SMB*	*HML*	*RMW*	*CMA*	*ΔSR*
Bottom 25%	-0.0056***	0.0085***	-0.0011***	0.0011***	0.0030***	0.0030***	-0.0011
	(0.0009)	(0.0002)	(0.0003)	(0.0004)	(0.0004)	(0.0006)	(0.0019)
26-50%	-0.0037***	0.0109***	0.0001	0.0014***	0.0026***	0.0012	-0.0012
	(0.0012)	(0.0003)	(0.0004)	(0.0005)	(0.0006)	(0.0008)	(0.0024)
51-75%	-0.0009	0.0113***	0.0017***	-0.0004	-0.0016***	-0.0006	-0.0022
	(0.0013)	(0.0003)	(0.0005)	(0.0006)	(0.0006)	(0.0008)	(0.0026)
Top 25%	0.0029	0.0140***	0.0027***	-0.0024***	-0.0054***	-0.0028**	-0.0030
	(0.0020)	(0.0005)	(0.0007)	(0.0009)	(0.0009)	(0.0013)	(0.0041)
Market	0.0058		0.0300***	0.0296***	-0.0288***	-0.0413***	-0.0337***
	(0.0046)		(0.0017)	(0.0020)	(0.0021)	(0.0029)	(0.0098)

*Note: * denote significance; * p < 0.1; ** p < 0.05; and *** p < 0.01. Standard errors within brackets.*

The impact of the shadow rate on the individual portfolios also doesn't change when four portfolios are used instead of ten. None of the dummy variable coefficients are significant, except for the overall market.

4.4.4. Break point

The last robustness test that is performed is by creating a break in the dataset. The breakpoint is placed after the end of 2007. This splits the data into two, with 179 observations before the split and 156 after the split. The financial crisis of 2008 is the reason this point was chosen.

To evaluate the difference, Table 3 is recreated twice, once with the period Jan 1993 – Dec 2007, and once Jan 2008 – Dec 2020. The statistics are found in appendix II. But do not differ by much, relative to the full period. The means are t-tested per portfolio, but no significant different means are found.

The Treynor and Sharpe ratios are also compared between the datasets. Figure 5 displays the graphs. The course of the ratios is not that different from Figure 2. Except for the before 2008 graph, here the Treynor ratio for the bull market deviates. But this can be the results of not enough data, which lowers the significance of the observed value. One thing that is noticeable between the graphs is that the ratios are, on average, higher for the before 2008 period.

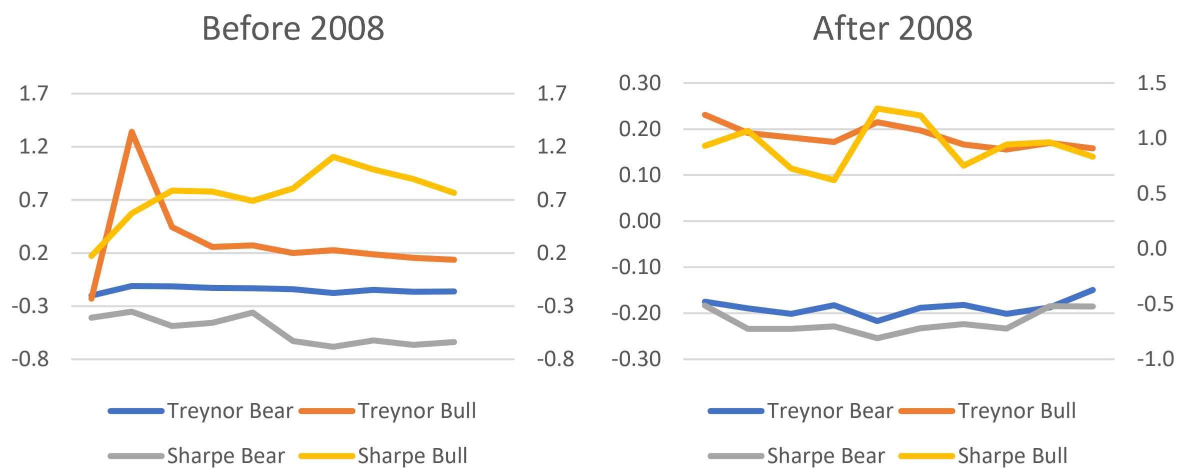

Figure 5 The ten value weighted volatility portfolios visualised before and after 2008. Based on the Treynor and Sharpe ratio (Treynor on the left axis, Sharpe on the right).

However, when formula one is performed to test the difference between bear and bull markets within the volatility portfolios, for both the Treynor and Sharpe ratio. Only one test isn't significant meaning that most ratios are similar. This isn't the case for the Treynor ratio in the first volatility decile, regarding observations before the break. But as discussed before, this can be the result of insufficient observations.

5. Conclusion

Since the invention of the CAPM model, researches have discussed the applicability of the theory in practice. The model is built on five underlying assumptions that need to hold for the equation to hold. In practice, observations are made that contradict the CAPM theory and suggestions are made for why that is the case. This paper researches three factors that may be responsible for the observations that contradict the theory. It tries to answer the main question: What is the impact of business cycles, quantitative easing, and the shadow rate on the low volatility anomaly, and thus, are these factors responsible for the inversed CAPM relationship?

5.1. Business cycle

To answer the main question, the question is divided into three hypotheses, one for each of the factors. The first hypothesis *The low volatility anomaly has a greater performance in bull markets than in bear markets*, is rejected, but comes with some implications. The results show that the low volatility anomaly is not as clearly observed in the American equity market for the period 1990-2020, as the literature suggests. The mean return on ten value weighted volatility portfolios have increasing returns when the volatility in the portfolio increases. When these returns are risk-adjusted the relationship seems to change in favour of the low volatility anomaly.

When looking at the Treynor ratio, a slight decrease is observed, this means that higher volatility portfolios exhibit a higher return, but to achieve this return a higher risk profile needs to be taken. Meaning that the return for lower volatility portfolios is higher equivalent to the risk that is taken. This is the case for both bear and bull markets, and suggests the presence of the low volatility anomaly, albeit small.

When looking at the Sharpe ratio, the conclusion differs between bear and bull markets. The Sharpe ratio is slightly decreasing for bear markets, while increasing for bull markets, but does flatten at higher volatility portfolios.

Based on the Treynor ratio the low volatility anomaly can be ruled in favour for, but no overwhelming evidence is found. Based on the Sharpe ratio, the low volatility anomaly does not seem to exist in bull markets, but is observed in bear markets. This seems to indicate that portfolios do exhibit a low volatility anomaly for portfolios which have systematic risk, but not when idiosyncratic risk is used as a risk measure.

According to the literature, the low volatility anomaly has a greater presence in bear markets than in bull markets. In bear markets, the return on lower volatile portfolios is less bad than on higher volatile portfolios, but on average still negative. However, the amount of returns above zero is greater than in higher volatility portfolios. Although, in bull markets the average return is always positive. Therefore, it cannot be concluded that the low volatility anomaly outperforms in bear markets compared to bull markets.

When the portfolios are constructed using equally weighted returns, the low volatility anomaly is more present than with value weighted returns. This contradicts Blitz and Van Vliet (2007), in which they say that the low volatility anomaly is observed for both ways of weighting assets within a portfolio.

This leaves us with an unsolved problem. The literature suggests the existence of a low volatility anomaly, that is even more strong in bear markets. While this research barely finds any evidence of the low volatility let alone positive returns in bear markets.

5.2. Quantitative Easing

The second hypothesis focusses on the influence of central banks on the low volatility anomaly. In this case the Open Market Operations of the FED on the American equity market after the financial crisis of 2008. The second hypothesis: *In periods without Open Market Operations the low volatility anomaly is less pronounced*. This hypothesis is answered by running a regression which uses a dummy variable for the month that the FED ran their OMO program. This is done to observes the differences in returns in periods with OMO and without OMO. The program ran in the following periods: March 2009 – October 2009, November 2010 – June 2011, September 2011 – June 2012, and September 2012 – October 2014 (FED, 2021). To make sure that the observed variance of the portfolio returns is not clouded by other factors that influence equity returns, the return on the volatility portfolios is corrected for the five Fama and French factors. Subsequently the beta coefficients of the dummy variable are Z-tested on significance difference between each other.

Unfortunately, none of the beta coefficient of the dummy variable are significant and can thus very well be zero, meaning no influence on the return of the portfolios. The only time the coefficient is significant is when regressed on the overall market. Intuitively, which makes sense. The idea of OMO is to boost the economy, so if it doesn't influence the overall market, one could argue that their actions are in vain.

To test the second hypothesis a Z-test between the OMO coefficients is performed to test the statistical difference between the coefficients. No evidence is found that OMO is more pronounced on portfolios that have low volatility than of that, that have a higher volatility. Namely the Z-test shows that the coefficients do differ from each other at the highest level, but this does not confirm the second hypothesis. Because of the lack of significance of the coefficients.

5.3. Shadow Rates

To test a different form of QE, performed by the FED, shadow rates are regressed on the portfolio returns. The advantage of using shadow rates instead of OMO is that shadow rates are continuous. The same procedure for the second hypothesis is used to test the third hypothesis: *The changes in the shadow rate have a negative relationship with the return on the low volatility anomaly*. The first difference of the shadow rate is regressed on the return on the volatility portfolios, while corrected for the five Fama and French factors. Thereafter, the coefficients of the delta shadow rate are Z-tested on statistically significant differences.

However, the estimated coefficients are not significant, meaning they could as well be zero. Only the coefficient for the highest volatility portfolio is significant at a 5% level, and for the overall market, significant at a 10% level. This makes testing the difference with a Z-test less useful, since the true value of the coefficient is not known. When the test is performed anyway, the p-value is significant at the highest level. Concluding that the coefficients differ from each other.

The main questions asks if the three factors are responsible for the inversed CAPM relationship observed on the market. During this research no overwhelming evidence could be found in favour of the low volatility anomaly, this makes answering the main question a bit more difficult. There's evidence that the business cycle has a different impact on the volatility portfolios. Namely, in a bear market the performance on the portfolios with low volatile assets is less bad than on the portfolio with higher volatile assets. If taken out of context this seems pretty obvious. The whole idea of a portfolio with lower volatility is that the returns are closer to the mean than that of a portfolio with a higher volatility. The only side note is that in some cases portfolios with lower volatility perform better in bear and bull markets, but saying that the business cycle has a causal relationship with the low volatility anomaly, is not proven.

Besides the business cycle, quantitative easing could also have an impact on the low volatility anomaly. In this paper QE is divided into OMO and shadow rates and their influences are researched. After correcting the returns on the volatility portfolios for the five factors of Fama and French. No influence of QE could be proven. Suggesting that both forms of QE do not significantly impact the low volatility anomaly. The only relationship that could be found is the aggregate relation on the overall market.

6. Further research and limitations

Since there is plenty of research on the low volatility anomaly and observations that contradict the CAPM. One could argue that it exists even though this paper only found marginal evidence of such an anomaly. This can be attributed that there are other factors in place that are responsible for the low volatility anomaly, other than business cycle, OMO, and shadow rates. Finding these factors could provide a better understanding of the low volatility anomaly.

This research is based on the American equity market. The low volatility anomaly was observed in multiple markets. It may be the case that results found within this paper are different in other markets. Shogbuyi and Steeley (2017). Found that QE has different effect in American markets opposed to European markets. It could be the case the OMO and shadow rates have an effect on the low volatility anomaly in different markets.

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8. Appendix I – Tables with F-test results on difference between standard deviations

Table 11

P-values of F-tested differences between standard deviations for the value weighted volatility portfolio, for the bear market. Values in red denote p-values higher than 5%.

	Bear 1	Bear 2	Bear 3	Bear 4	Bear 5	Bear 6	Bear 7	Bear 8	Bear 9	Bear 10
Bear 1		0.1715	0.3160	0.0116	0.0004	0.0002	0.0004	0.0000	0.0000	0.0000
Bear 2	0.1715		0.3193	0.0922	0.0076	0.0054	0.0085	0.0000	0.0000	0.0000
Bear 3	0.3160	0.3193		0.0363	0.0019	0.0013	0.0022	0.0000	0.0000	0.0000
Bear 4	0.0116	0.0922	0.0363		0.1338	0.1095	0.1433	0.0013	0.0000	0.0000
Bear 5	0.0004	0.0076	0.0019	0.1338		0.4520	0.4828	0.0272	0.0000	0.0000
Bear 6	0.0002	0.0054	0.0013	0.1095	0.4520		0.4350	0.0356	0.0000	0.0000
Bear 7	0.0004	0.0085	0.0022	0.1433	0.4828	0.4350		0.0246	0.0000	0.0000
Bear 8	0.0000	0.0000	0.0000	0.0013	0.0272	0.0356	0.0246		0.0002	0.0018
Bear 9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002		0.2566
Bear 10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0018	0.2566	

Table 12

P-values of F-tested differences between standard deviations for the value weighted volatility portfolio, for the bull market. Values in red denote p-values higher than 5%.

	Bull 1	Bull 2	Bull 3	Bull 4	Bull 5	Bull 6	Bull 7	Bull 8	Bull 9	Bull 10
Bull 1		0.3743	0.1061	0.0000	0.0237	0.0082	0.0006	0.0000	0.0000	0.0000
Bull 2	0.3743		0.1768	0.0000	0.0481	0.0187	0.0016	0.0000	0.0000	0.0000
Bull 3	0.1061	0.1768		0.0000	0.2303	0.1235	0.0214	0.0000	0.0000	0.0000
Bull 4	0.0000	0.0000	0.0000		0.0000	0.0002	0.0042	0.1577	0.0107	0.0000
Bull 5	0.0237	0.0481	0.2303	0.0000		0.3371	0.0983	0.0000	0.0000	0.0000
Bull 6	0.0082	0.0187	0.1235	0.0002	0.3371		0.1918	0.0000	0.0000	0.0000
Bull 7	0.0006	0.0016	0.0214	0.0042	0.0983	0.1918		0.0001	0.0000	0.0000
Bull 8	0.0000	0.0000	0.0000	0.1577	0.0000	0.0000	0.0001		0.0966	0.0000
Bull 9	0.0000	0.0000	0.0000	0.0107	0.0000	0.0000	0.0000	0.0966		0.0017
Bull 10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0017	

Table 13

P-values of F-tested differences between standard deviations for the equal weighted volatility portfolio, for the bear market. Values in red denote p-values higher than 5%.

	Bear 1	Bear 2	Bear 3	Bear 4	Bear 5	Bear 6	Bear 7	Bear 8	Bear 9	Bear 10
Bear 1		0.0123	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Bear 2	0.0123		0.1031	0.0054	0.0028	0.0000	0.0000	0.0000	0.0000	0.0000
Bear 3	0.0002	0.1031		0.0985	0.0649	0.0010	0.0001	0.0000	0.0000	0.0000
Bear 4	0.0000	0.0054	0.0985		0.4107	0.0355	0.0069	0.0001	0.0000	0.0000
Bear 5	0.0000	0.0028	0.0649	0.4107		0.0570	0.0126	0.0002	0.0000	0.0000
Bear 6	0.0000	0.0000	0.0010	0.0355	0.0570		0.2540	0.0252	0.0005	0.0000

Bear 7	0.0000	0.0000	0.0001	0.0069	0.0126	0.2540		0.0972	0.0045	0.0000
Bear 8	0.0000	0.0000	0.0000	0.0001	0.0002	0.0252	0.0972		0.0926	0.0002
Bear 9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0005	0.0045	0.0926		0.0119
Bear 10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0119	

Table 14

P-values of F-tested differences between standard deviations for the equal weighted volatility portfolio, for the bull market. Values in red denote p-values higher than 5%.

	Bull 1	Bull 2	Bull 3	Bull 4	Bull 5	Bull 6	Bull 7	Bull 8	Bull 9	Bull 10
Bull 1		0.0109	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Bull 2	0.0109		0.0818	0.0165	0.0072	0.0001	0.0000	0.0000	0.0000	0.0000
Bull 3	0.0001	0.0818		0.2292	0.1441	0.0093	0.0000	0.0000	0.0000	0.0000
Bull 4	0.0000	0.0165	0.2292		0.3741	0.0528	0.0003	0.0000	0.0000	0.0000
Bull 5	0.0000	0.0072	0.1441	0.3741		0.0971	0.0008	0.0000	0.0000	0.0000
Bull 6	0.0000	0.0001	0.0093	0.0528	0.0971		0.0310	0.0000	0.0000	0.0000
Bull 7	0.0000	0.0000	0.0000	0.0003	0.0008	0.0310		0.0049	0.0000	0.0000
Bull 8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0049		0.0042	0.0000
Bull 9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0042		0.0001
Bull 10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	

9. Appendix II – Tables containing results for break point robustness test

Table 15

Summary statistics ten value weighted volatility portfolios divided between bear and bull markets over the period Jan 1993 – Dec 2007, the returns are monthly

Category	Mean	St. Dev	CAPM Alpha	CAPM Beta	Treynor	Sharpe	Return >0
Bear							
Bottom 10%	-0.0169	0.0414	-0.0029	0.0842***	-0.201	-0.408	29
10%-20%	-0.0138	0.0393	0.0068	0.1248***	-0.111	-0.352	34
20%-30%	-0.0172	0.0352	0.0080*	0.1523***	-0.113	-0.488	27
30%-40%	-0.0196	0.0428	0.0053	0.1503***	-0.130	-0.458	27
40%-50%	-0.0183	0.0507	0.0047	0.1386***	-0.132	-0.361	27
50%-60%	-0.0327	0.0521	0.0059	0.2327***	-0.141	-0.628	22
60%-70%	-0.0351	0.0514	-0.0023	0.1978***	-0.177	-0.683	18
70%-80%	-0.0395	0.0635	0.0048	0.2675***	-0.148	-0.622	22
80%-90%	-0.0514	0.0773	0.0006	0.3136***	-0.164	-0.665	23
Top 10%	-0.0543	0.0853	0.0008	0.3327***	-0.163	-0.637	27
Bull							
Bottom 10%	0.0057	0.033	0.0089	-0.0247	-0.230	0.172	64
10%-20%	0.0180	0.0313	0.0162***	0.0134	1.340	0.574	78
20%-30%	0.0238	0.0303	0.0168***	0.0538*	0.443	0.787	77
30%-40%	0.0214	0.0275	0.0105**	0.0837***	0.256	0.778	77
40%-50%	0.0236	0.0341	0.0122**	0.0871**	0.271	0.691	79
50%-60%	0.0294	0.0364	0.0101*	0.1469***	0.200	0.807	80
60%-70%	0.0416	0.0377	0.0177***	0.1829***	0.228	1.105	82
70%-80%	0.0499	0.0506	0.0151**	0.2650***	0.188	0.985	82
80%-90%	0.0556	0.0621	0.0085	0.3594***	0.155	0.895	79
Top 10%	0.0608	0.0793	0.0024	0.4457***	0.137	0.767	79

*Note: * denote significance; * p < 0.1; ** p < 0.05; and *** p < 0.01.*

Table 16

Summary statistics ten value weighted volatility portfolios divided between bear and bull markets over the period Jan 2008 – Dec 2020, the returns are monthly

Category	Mean	St. Dev	CAPM Alpha	CAPM Beta	Treynor	Sharpe	Return >0
Bear							
Bottom 10%	-0.0192	0.0373	0.0043	0.1098***	-0.175	-0.514	26
10%-20%	-0.0325	0.0449	0.0041	0.1714***	-0.190	-0.724	18
20%-30%	-0.0336	0.0463	0.0020	0.1668***	-0.202	-0.726	16
30%-40%	-0.0365	0.0520	0.0062*	0.1999***	-0.183	-0.702	15
40%-50%	-0.0417	0.0515	-0.0007	0.1919***	-0.217	-0.810	15
50%-60%	-0.0386	0.0537	0.0052	0.2049***	-0.188	-0.719	14
60%-70%	-0.0362	0.0531	0.0064	0.1993***	-0.182	-0.682	17
70%-80%	-0.0421	0.0583	0.0025	0.2088***	-0.202	-0.722	15

80%-90%	-0.0458	0.0879	0.0064	0.2442***	-0.187	-0.521	12
Top 10%	-0.0340	0.0651	0.0146**	0.2274***	-0.150	-0.522	20
Bull							
Bottom 10%	0.0228	0.0244	0.0071**	0.0986***	0.231	0.932	77
10%-20%	0.0339	0.0317	0.0057*	0.1772***	0.191	1.069	81
20%-30%	0.0374	0.0515	0.0047	0.2056***	0.182	0.727	80
30%-40%	0.0379	0.0610	0.0028	0.2203***	0.172	0.621	84
40%-50%	0.0461	0.0363	0.0120***	0.2140***	0.215	1.269	86
50%-60%	0.0465	0.0385	0.0089***	0.2360***	0.197	1.207	86
60%-70%	0.0471	0.0627	0.0021	0.2831***	0.167	0.752	82
70%-80%	0.0572	0.0607	-0.0013	0.3679***	0.156	0.943	82
80%-90%	0.0591	0.0613	0.0038	0.3477***	0.170	0.963	78
Top 10%	0.0702	0.0843	-0.0006	0.4448***	0.158	0.833	77

*Note: * denote significance; * p < 0.1; ** p < 0.05; and *** p < 0.01.*

Table 17

Test statistics and corresponding p-values for testing the difference in mean on the volatility categories. Using data from Jan 1993 – Dec 2007 and Jan 2008 – Dec 2020, on the whole period from Jan 1993 – Dec 2020.

	Before 2008		**After 2008**	
Category	*Statistic*	*P-value*	*Statistic*	*P-value*
Bear				
Bottom 10%	0.0543	0.3984	-0.0723	0.3979
10%-20%	0.4377	0.3625	-0.5413	0.3446
20%-30%	0.4027	0.3679	-0.4724	0.3568
30%-40%	0.3786	0.3713	-0.4571	0.3594
40%-50%	0.4877	0.3542	-0.6259	0.3280
50%-60%	0.1226	0.3960	-0.1542	0.3942
60%-70%	0.0239	0.3988	-0.0298	0.3988
70%-80%	0.0488	0.3985	-0.0666	0.3981
80%-90%	-0.0957	0.3971	0.1159	0.3963
Top 10%	-0.3320	0.3775	0.4709	0.3571
Bull				
Bottom 10%	-0.5020	0.3517	0.5820	0.3368
10%-20%	-0.4758	0.3562	0.4902	0.3538
20%-30%	-0.3872	0.3701	0.3392	0.3766
30%-40%	-0.4691	0.3574	0.3809	0.3710
40%-50%	-0.6353	0.3260	0.6474	0.3235
50%-60%	-0.4674	0.3577	0.4839	0.3549
60%-70%	-0.1393	0.3951	0.1265	0.3958
70%-80%	-0.1719	0.3931	0.1668	0.3934
80%-90%	-0.0744	0.3978	0.0779	0.3977
Top 10%	-0.1740	0.3929	0.1786	0.3926

Table 18

Overview of statistically significant difference between bear and bull markets for every volatility portfolio. Performed for both the Treynor as the Sharpe ratio before 2008.

Categories		Treynor	Z-statistic	P-value	Sharpe	Z-statistic	P-value
Bottom 10%	Bear market	-0.201	0.376	0.646	-0.408	-7.237	0.000
	Bull market	-0.230			0.172		
10%-20%	Bear market	-0.111	-15.236	0.000	-0.352	-10.973	0.000
	Bull market	1.340			0.574		
20%-30%	Bear market	-0.113	-6.946	0.000	-0.488	-14.370	0.000
	Bull market	0.443			0.787		
30%-40%	Bear market	-0.130	-4.616	0.000	-0.458	-13.337	0.000
	Bull market	0.256			0.778		
40%-50%	Bear market	-0.132	-5.332	0.000	-0.361	-12.760	0.000
	Bull market	0.271			0.691		
50%-60%	Bear market	-0.141	-4.293	0.000	-0.628	-15.586	0.000
	Bull market	0.200			0.807		
60%-70%	Bear market	-0.177	-4.965	0.000	-0.683	-17.768	0.000
	Bull market	0.228			1.105		
70%-80%	Bear market	-0.148	-3.998	0.000	-0.622	-16.166	0.000
	Bull market	0.188			0.985		
80%-90%	Bear market	-0.164	-3.939	0.000	-0.665	-16.288	0.000
	Bull market	0.155			0.895		
Top 10%	Bear market	-0.163	-3.594	0.000	-0.637	-14.725	0.000
	Bull market	0.137			0.767		

Table 19

Overview of statistically significant difference between bear and bull markets for every volatility portfolio. Performed for both the Treynor as the Sharpe ratio After 2008.

Categories		Treynor	Z-statistic	P-value	Sharpe	Z-statistic	P-value
Bottom 10%	Bear market	-0.175	-5.147	0.000	-0.514	-15.745	0.000
	Bull market	0.231			0.932		
10%-20%	Bear market	-0.190	-4.797	0.000	-0.724	-18.117	0.000
	Bull market	0.191			1.069		
20%-30%	Bear market	-0.202	-4.875	0.000	-0.726	-15.868	0.000
	Bull market	0.182			0.727		
30%-40%	Bear market	-0.183	-4.244	0.000	-0.702	-14.092	0.000
	Bull market	0.172			0.621		
40%-50%	Bear market	-0.217	-5.715	0.000	-0.810	-20.346	0.000
	Bull market	0.215			1.269		
50%-60%	Bear market	-0.188	-4.845	0.000	-0.719	-18.884	0.000
	Bull market	0.197			1.207		
60%-70%	Bear market	-0.182	-4.285	0.000	-0.682	-15.305	0.000

	Bull market	0.167			0.752		
70%-80%	Bear market	-0.202	-4.250	0.000	-0.722	-16.597	0.000
	Bull market	0.156			0.943		
80%-90%	Bear market	-0.187	-4.409	0.000	-0.521	-15.669	0.000
	Bull market	0.170			0.963		
Top 10%	Bear market	-0.150	-3.683	0.000	-0.522	-14.313	0.000
	Bull market	0.158			0.833		