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The Evolution of Bitcoin as an Asset Class: Diversification, Hedging and Safe Haven Benefits of the Virtual Gold

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Preface and Acknowledgements

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Abstract

This thesis analyses Bitcoin's diversification, hedging and safe haven benefits, as well as its position in an efficiently diversified portfolio. Through the analysis of daily price and volume data of Bitcoin and other conventional assets for the period September 2011 to March 2021, this thesis shows that Bitcoin can serve as a diversifier but not as a hedge against conventional assets during regular market periods. Bitcoin is further shown to serve as a safe haven against American large cap, mid cap and small cap stocks as well as oil during the COVID-19 crisis. Furthermore, the Bitcoin market can be deemed inefficient due to the presence of autocorrelation in the returns. Bitcoin is additionally shown to be more liquid than gold in the period ranging from 2013 to 2021. Finally, this thesis shows that adding Bitcoin in an efficiently diversified portfolio increases the risk-return trade-off for multiple levels of risk aversion.

Keywords:

Bitcoin, Investment Decisions, Alternative Investments, Portfolio Optimisation, Information and Market Efficiency

JEL Classification:

G11, G14, B26, C12

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Chapter 1. Introduction

In recent years, cryptocurrencies have received increased and worldwide attention. The most famous, Bitcoin, has seen its price jump to high peaks several times in its lifetime, the first being in the second half of 2017 at roughly USD 19,000, and the second being in March 2021 at roughly USD 60,000 (Yahoo Finance, 2021). Contrasting opinions exist regarding the nature of the Bitcoin as an investment vehicle. One perspective on the Bitcoin is that it currently offers means of unregulated and anonymous trading, acting like a virtual currency, while others have described the Bitcoin as an asset instead of a currency due to its high volatility and other asset-like properties (Yermack, 2015). Due to these asset like properties, it is seen as a possible investment opportunity (Lee, Guo, & Wang, 2018). The Bitcoin has even been described as the “virtual gold”, drawing the comparison to the hedging and safe haven characteristics of gold and the Bitcoin (Dyhrberg, 2016). On the other side of the argument however, Kubat (2015) states that Bitcoin’s historical volatility is much higher than gold’s historical volatility, and the risk involved therefore indicates that Bitcoin does not have a similar store of value function. This casts doubt on the safe haven capabilities mentioned by Dyhrberg (2016). Furthermore, Urquhart (2016) argues that the Bitcoin market could be moving to maturity, giving cause to re-examine Bitcoin’s role in the market.

To address the existing controversy about the Bitcoin’s role in the current market, and to assess if the Bitcoin has any place in an investor’s portfolio through its diversification, hedging and safe haven capabilities to traditional assets, this thesis will have the following research question:

What are the diversification, hedging and safe haven benefits of including the Bitcoin as an investment vehicle in a portfolio of stocks, bonds and commodities, and how does it influence portfolio performance?

In 2009 an individual or a group of programmers using the pseudonym Satoshi Nakamoto introduced a new virtual currency called Bitcoin to the world. Bitcoins are created through a process called mining and offer means for peer-to-peer transactions without any form of regulation and interference of other parties (Nakamoto, 2009). Its anonymous nature has caused high criticism, mainly due to the high usage of Bitcoin for illegal transactions. From an investor’s perspective it has been looked down upon as nothing more than a speculative investment due to its extremely high volatility and uncertain nature. Bouoiyour and Selmi (2015) conclude from their research that Bitcoin shows high volatility in the period from December 2010 to June 2015, which can support the argument of a non-matured and under-developed market which causes the Bitcoin to behave more like a speculative investment. However, Urquhart (2016) states that the Bitcoin market could be moving towards efficiency, a market state in which past returns have no predictive power for future returns. Brière, Oosterlinck and Szafarz (2015) argue that while currencies are used often as means to diversify an investment portfolio, literature so far has overlooked to research these same characteristics in Bitcoin.

In order to answer the research question of this thesis, a description and evaluation of the research already executed with regard to this topic will be given to sketch an overview of the Bitcoin as an asset class. A more detailed analysis and problem statement for this research will follow from the literature review, as many different results and opinions exist regarding the Bitcoin, and many questions are still unanswered. Furthermore, with the Bitcoin's "birth" being in 2009, the increased attention over the past years has also caused the Bitcoin to mature. Additionally, as described by Bouoiyour and Selmi (2015), the volatility of the Bitcoin in its earliest life stages far exceeds the volatility that is observed in its more mature life stages, allowing for better conclusions to be drawn from research involving Bitcoin. Following the discussion of the existing literature surrounding the Bitcoin, the hypotheses needed to answer the research question will be developed. After this, the Data and methodology section will cover the statistical methodology to test the hypotheses, as well as describe the data used to perform these tests.

The results chapter of this thesis shows that Bitcoin can be effectively used as a diversifier against conventional assets, while hedging capabilities are not found. Bitcoin is further shown to serve as a safe haven against American large cap, mid cap and small cap stocks as well as oil during the COVID-19 crisis. Additionally, the Bitcoin market is shown to be inefficient due to autocorrelation being present in the returns. Moreover, this thesis shows through different measures that Bitcoin has a positive effect on the risk-return trade-off in an efficient portfolio, and that Bitcoin is investible under different levels of risk aversion. The thesis finally concludes by providing a discussion on the reported results and gives possible openings for future research.

Chapter 2. Literature review

This chapter will cover the existing literature surrounding the research question. This will be done by first providing a clear definition of a diversifier, hedge and safe haven based on the existing literature. Following this, the literature surrounding Bitcoin with regard to each of these three definitions will be covered. Once these three subjects have been covered, the possible move to an efficient Bitcoin market mentioned by Urquhart (2016) will be reviewed together with Bitcoin's liquidity.

2.1. Diversification, hedging and safe haven definitions

In order to answer the research question of this thesis, clear and testable definitions of a diversifier, hedge and safe haven are needed. Stensås et al. (2019) have shown that the first testable definitions can be derived from Baur and Lucey (2010). These characteristics can be defined as follows:

"A hedge is defined as an asset that is uncorrelated or negatively correlated with another asset or portfolio on average. A strict hedge is (strictly) negatively correlated with another asset or a portfolio on average.

A diversifier is defined as an asset that is positively (but not perfectly correlated) with another asset or portfolio on average.

A safe haven is defined as an asset that is uncorrelated or negatively correlated with another asset or portfolio in times of market stress or turmoil." (Baur & Lucey, 2010, p. 5)

These three definitions will be used throughout the thesis when referring to diversification, hedging and safe haven capabilities of Bitcoin.

2.2. Diversification and portfolio optimisation

It is commonly accepted throughout financial literature that investors will seek to optimise their portfolio in order to achieve the maximum risk-adjusted return for a certain level of risk. All assets are exposed to both systematic risks and idiosyncratic risks, of which systematic risks are commonly shared among all assets and are thus undiversifiable. This can be seen as the inherent risk of investing and participating in the stock market. The idiosyncratic risk however is linked to an individual asset, which therefore by definition allows investors to decide what idiosyncratic risk they carry in their portfolio by weighing and balancing their portfolio through diversification. This can be done by constructing the efficient frontier as described by Markowitz (1952), allowing an investor to construct a portfolio with the maximum expected return for a given level of risk that investor is willing to take.

Brière et al. (2015) use weekly data between 2010 and 2013 to analyse the diversification benefits Bitcoin may offer to a diversified portfolio. Over this period Bitcoin showed extremely high volatility and returns, while simultaneously showing a low and positive correlation with most other assets within the comparison. While including Bitcoin in the portfolio showed a significant improvement in the risk-return ratio of diversified investment portfolios, the authors do mention that these characteristics and results could be caused by the Bitcoin being a relatively young asset class, and that they may not hold up in the long run. The conclusion that Bitcoin can serve as an effective diversifier is supported by Bouri et al. (2017), who come to this conclusion while analysing the diversification capabilities in relation to major, worldwide stock indices.

Bitcoin being an alternative asset, as it does not fall within the traditional category of stocks and bonds, also brings a degree of risk compared to conventional assets. Platanakis et al. (2019) show that adding five different forms of alternative assets, namely real estate, commodities, hedge funds, emerging markets and private equity are harmful for investors over an 18-year period from 1997 to 2015. They attribute this mainly to estimation risk, which their empirical research shows to be higher for alternative assets than for equities and bonds. Estimation risk in asset pricing can be described as "investor uncertainty about the parameters of the return- or cashflow-generating process" (Shanken & Lewellen, 2000, p. 5). This estimation risk in turn leads to excessive portfolio weights for the alternative assets in order to reach the desired goal of diversification. One could argue that Bitcoin and cryptocurrencies as a whole are a relatively new market and therefore would be especially susceptible to estimation risk, as not much history is available to efficiently estimate returns over a longer period. This relation between market efficiency and estimation risk is analysed by Shanken and Lewellen (2000). They state that estimation risk helps explain volatility beyond what can be justified by changes in dividends, and further mention that estimation risk is an important characterization of market efficiency. These findings show that in order to effectively use Bitcoin as a diversifier without running the risk of excessively high portfolio weights, market efficiency is important.

Charfeddine et al. (2020) support the idea that the convergence of multiple studies regarding the low correlation between cryptocurrencies, specifically Bitcoin, and other assets, highlights the possible value of further examination of this topic. By examining this, they find that the correlation between cryptocurrencies and conventional assets changes over time but is generally low. This supports the diversification argument through impacting the idiosyncratic risk of an investment portfolio. Additionally, their results show that the optimal diversification within a portfolio can be reached by holding a relatively small part of cryptocurrencies, which is a counterargument to the possible estimation risks regarding alternative assets described by Platanakis et al. (2019).

Corbet et al. (2018) analyse the dynamic relationship between three popular cryptocurrencies (Bitcoin, Ripple and Litecoin) and other financial assets. They find that cryptocurrencies can serve as a diversifier against other financial assets, as they are relatively isolated from market shocks and price movements are mostly disassociated from popular financial assets. They do however state that this is not without risk, as the cryptocurrency markets contain idiosyncratic risks of their own which are difficult to hedge against.

2.3. Hedging and uncertainty

Other than constructing a diversified portfolio to reduce idiosyncratic risk, investors commonly hedge their investments to reduce the risk of adverse price movements. This can be done by for example purchasing a put-option of the same underlying asset as is held in the portfolio. That way a drop in price of the asset can be (partially) offset by the increase in value of the purchased put option. Other ways in which a hedge can be set up is by purchasing assets that show zero or an inverse correlation to the market – a common example of this is gold. Baur and Lucey (2010) studied the relations between U.S., UK and German stock and bond returns on the one hand, and gold returns on the other, and find that gold is a hedge against stocks on average.

Dyhrberg (2016) used a GARCH volatility analysis to determine that the Bitcoin can be compared to the USD in terms of the exchange means, while it also shares the hedging capabilities of gold. In further research, Dyhrberg (2016) again shows that the Bitcoin can serve as a hedge against stocks in the Financial Times Stock Exchange Index, as well as against the USD when evaluating a shorter horizon. Kristoufek's research (2013) can provide additional insights. He finds that Bitcoin can not only serve as a hedge against stock markets, but the price of Bitcoin is also affected by factors such as Wikipedia and Google search queries, which shows a contrast compared to traditional assets such as stocks or bonds. This relation can be an explanation as to why Bitcoin shows zero or negative correlation with traditional assets, leading to its possible hedging capabilities.

A possible counterargument for the hedging capabilities of the Bitcoin can be drawn from Bouri et al. (2017) using a Dynamic Conditional Correlation (DCC) model (Engle, 2002) for major stock indices around the world. Using daily and weekly data from July 2011 to December 2015, they find that the Bitcoin is a poor hedge in most cases and that it is more suitable as a diversification instrument. Cho and Parhizgari (2008) find that the DCC model does a better job capturing time-varying correlations with less computational problems. Engle (2002) argues that the process of estimating parameters in the correlation process independently from the number of series to be correlated leads to a more accurate way of estimating large correlation matrices, which is the basis for the development of the DCC model.

Charfeddine et al. (2020) find that in case of hedging, cryptocurrencies are a poor tool. They attribute this to their finding that the relationship between cryptocurrencies and conventional assets is sensitive to external shocks, possibly impacting the effectiveness of cryptocurrencies as hedging instruments.

Adebola et al. (2019) come to a similar conclusion. They examine the relationship of multiple cryptocurrencies, including Bitcoin, and gold prices. They find that the connection between the cryptocurrency market and the gold market is limited, and that it would be very difficult to predict movements in the gold market based on movements in the cryptocurrencies market and vice versa.

2.4. Safe haven during market turmoil

Baur and Lucey (2010) show that additionally to being an effective hedging instrument, gold is also a safe haven. They find that gold is a safe haven for stocks, and generally not for bonds. Another interesting aspect they find in the safe haven capabilities of gold is that the effect is relatively short-lived. This means that an investor that holds gold for longer than 15 days after a negative shock on average loses money while holding gold.

Bouri et al. (2017) use the DCC model proposed by Engle (2002). Despite finding that Bitcoin is a poor hedge, they also find indications that Bitcoin shows safe haven capabilities against weekly extreme down movements in Asian stocks. They argue that the high volatility in their dataset ranging from 2011 to 2015 could imply that these results vary over time, further highlighting the importance of examining the maturing of the Bitcoin market in relation to these findings.

Bouri et al. (2016) test for a difference between the return-volatility relation before and after the price crash of 2013. Here they find a significant inverse relation between past shocks and volatility before the price crash while finding no significant result after the price crash, a finding which is opposite to the pattern that is observed in traditional assets like stocks. They propose the safe haven effect (Baur, 2012) as an explanation for this finding, which relates to the safe haven property of gold where an increase in the price of gold is seen by investors as a sign of adverse future markets in other asset classes.

While using Bitcoin as a safe haven instrument similar to gold could be a valuable tool, it is important to also assess the risk related to this. A risk associated with Bitcoin that is often unforeseen is described by Ghabri et al. (2020). They state that most of the research regarding Bitcoin uses the Mean-Variance method, as is evident from earlier research described in this thesis. This approach only takes into account the volatility risk associated with Bitcoin and ignores another important factor – liquidity risk. Liquidity risk is especially important when using Bitcoin as a safe haven, as these investments often have a short horizon as described by Baur and Lucey (2010). Ghabri et al. (2020) find that adding Bitcoin to traditional assets can diversify liquidity risk, potentially increasing gains within the portfolio. Their results indicate that assigning a small weight in a portfolio to Bitcoin can reduce the liquidity risk and improve the Sharpe ratio (Sharpe, 1966). Possible reasons for this finding can be derived from research by Kim (2017). Kim, using daily Bitcoin quotes and prices in 16 different currencies, finds that transaction costs of Bitcoin are lower than retail foreign exchange markets, while also having a 2% narrower bid-ask spread on average. As a result, conversions between one currency and another can often be made using Bitcoin as intermediary to save on transaction costs.

Where most research covered until now shows promising signs for the safe haven capabilities of Bitcoin, Klein et al. (2018) argue that Bitcoin is not the new gold, as Bitcoin behaves the exact opposite of gold in bearish markets. Similar to stocks, they find that Bitcoin declines when markets are declining, meaning that the safe haven effect is not observed. However, they find nuance in this conclusion by noting that their sample includes a relatively small number of downturns, and that future maturity of the Bitcoin market with less extreme volatility is reason to re-examine this topic.

Junttila et al. (2018) use daily data from 1989 to 2016 to analyse the relation between gold and oil market futures and equity returns in the US market. They find that the correlations differ strongly during market crisis periods compared to normal market circumstances. They conclude that correlations between gold futures and US equities become negative during periods of market crisis, supporting the safe haven hypothesis. If Bitcoin behaves as the digital gold, then analysing the possible changing relation in the light of the current COVID-19 crisis is valuable.

2.5. The maturing of the Bitcoin market

Throughout its relatively short lifetime, different functions of Bitcoin have been examined. Arguments can be made for Bitcoin being a currency, a speculative investment, or an instrument used to increase the return of a diversified portfolio through further diversification, hedging or safe haven capabilities. The possible maturing of the Bitcoin market can be valuable for its diversification benefits through impacting the estimation risk, which Shanken and Lewellen (2000) argue is closely linked to market efficiency. Furthermore, the possible lower volatility in a more mature market allows for better estimations of the effectiveness of hedging and safe haven properties associated with Bitcoin as is mentioned in several papers previously discussed.

Yermack (2015) evaluates whether the Bitcoin complies with the general characteristics of a currency. He determines that the Bitcoin fails to satisfy the basic criteria of a currency, due to the fact it is not generally accepted everywhere as means of payment, and that its volatility is much higher than the volatility of commonly used currencies, meaning it does not properly serve as a unit of account. He concludes that the Bitcoin behaves more like a speculative investment than a currency. Corbet et al. (2018) examine whether the introduction of Bitcoin futures resolved these fallacies for Bitcoin as a currency. Their analysis shows that the volatility increased due to the announcement of Bitcoin futures, and that therefore the criteria as a unit of account (Yermack, 2015) is still not satisfied given the unpredictability of the Bitcoin price. Corbet et al. (2018) conclude that the Bitcoin should still be seen as a speculative asset rather than a currency, despite the introduction of futures trading. The combination of these findings together with the research of Glaser et al. (2015), who find that most individuals see digital currencies as an investment vehicle rather than a currency, forms the basis of evaluating Bitcoin as an investment vehicle in this thesis, rather than a currency.

Urquhart (2016) examines the efficiency of the Bitcoin market. Using a series of robustness tests, Urquhart determines that over the full sample, ranging from July 2010 to June 2016, the Bitcoin returns show non-randomness. He further indicates that in an efficient market, future prices should not be predictable, which leads to the conclusion of an inefficient Bitcoin market due to the non-randomness of the Bitcoin returns. Splitting the sample into two periods, Urquhart finds that according to some, but not all of the executed tests the Bitcoin market is efficient in the later period. This research implies that the Bitcoin as an asset class might be "maturing", leaving behind its period of extremely high volatility as described by Bouoiyour & Selmi (2015). The signs of the Bitcoin market moving to maturity are in contrast with research that was previously discussed, where the market was deemed nothing more than that of a speculative asset despite changing circumstances. The evolution of the Bitcoin as an asset class therefore means that research regarding the hedging and diversification capabilities of the Bitcoin can be of great value in the current field of research and has the possibility of finding nuance between contrasting opinions.

As a direct response to Urquhart (2016), Nadarajah and Chu (2017) further examine the efficiency of the Bitcoin market. They argue that by simply taking an odd integer power of the Bitcoin returns, the data is less variable, more peaked, more skewed, and less serially correlated. Furthermore, by taking an odd integer power of the returns, they argue that no information is lost, because returns of zero will remain zero and negative returns will remain negative. Using similar tests as Urquhart (2016) to determine efficiency, they find that using the transformed data Bitcoin shows weak market efficiency (Fama, 1970).

Dyhrberg et al. (2018) examine the investibility of Bitcoin. They find that trading costs are generally low, with both quoted and effective spreads often narrower than most equity exchanges around the world. They find that overall, Bitcoin is investible, especially for retail sized trades. However, they do not find the same investibility for institutional sized trades, mostly due to the market depth only being adequate for retail sized trades. This could indicate that large trading volumes at the institutional scale would impact Bitcoin prices too much, increasing liquidity risk for large trades. This in turn could cause differences in effectiveness of the diversification, hedging and safe haven capabilities of Bitcoin for retail investors compared to institutional investors.

The liquidity risk of Bitcoin is further examined by Marshall et al. (2019). They use intraday Bitcoin data across 14 exchanges and include Bitcoin prices against 13 currencies. In their research they find a spread ranging from 0.04% to 1.28% in Chinese Yuan and Canadian Dollars respectively. They further find that changes in Bitcoin liquidity are significantly dependent on changes in currency liquidity, but not the other way around. Bitcoin liquidity is however not strongly influenced by the VIX (CBOE.com, 2019), which does affect stock liquidity. This finding is especially important for the hedging and safe haven capabilities of Bitcoin – a negative effect on Bitcoin liquidity in market turmoil would increase liquidity risk after all, rendering the hedging and safe haven options less efficient, especially when considering the relatively short investor horizon for gold mentioned by Baur and Lucey (2010). Finally, Marshall et al. (2019) mention the important relation between the efficiency of the Bitcoin market and liquidity, stating that an illiquid market takes longer to remove pricing inefficiencies.

Chapter 3. Hypothesis development

This chapter will discuss the hypotheses that are used to answer the research question of this thesis. The below hypotheses will follow the definitions of a diversifier, hedge and safe haven proposed by Baur and Lucey (2010) and will help answer the research question of this thesis. The methodology used to test the hypotheses will be further elaborated on in the Data and methodology chapter.

For the Bitcoin to classify as a diversifier, the following hypothesis must be accepted:

(1) H_0: *The correlation between Bitcoin and conventional assets is larger than zero and smaller than one during periods of regular market behaviour.*

For the Bitcoin to classify as a hedging instrument, the following hypothesis must be accepted:

(2) H_0: *The correlation between Bitcoin and conventional assets is negative during periods of regular market behaviour.*

For the Bitcoin to classify as a safe haven, the following hypothesis must be accepted:

(3) H_0: *The correlation between Bitcoin and conventional assets is negative during periods of market downturn.*

Evaluating the above hypotheses will help draw conclusions on the diversification, hedging and safe haven benefits of Bitcoin as an investment vehicle. However, as described by Platanakis et al. (2019), estimation risk is often prevalent in alternative assets where price movements and volatility are harder to predict. The degree of estimation risk in turn is closely linked to market efficiency (Shanken & Lewellen, 2000), highlighting the importance of testing the following hypothesis:

(4) H_0: *The Bitcoin market shows efficiency in the weak form as described by Fama (1970).*

To determine if the Bitcoin market shows efficiency in the weak form, the methodology of Urquhart (2016) will be used, in which tests are executed to determine if autocorrelation exists within the daily Bitcoin returns. If no autocorrelation is present in the returns, then efficiency in the weak form can be assumed.

Besides the above hypotheses covering the volatility risk associated with Bitcoin, it is important to also assess the liquidity risk as mentioned by Ghabri et al. (2020) to have a more complete estimate of the risks associated with the inclusion of Bitcoin in an investment portfolio. The Amihud Illiquidity Ratio (Amihud, 2002) is one of the most widely used measures to assess liquidity of assets, and Brauneis et al. (2021) find that the Amihud Illiquidity Ratio outperforms most other measures when estimating liquidity levels of cryptocurrency markets. It is defined as follows:

$$ILLIQ_t^i = \frac{1}{D_t^i} \sum_{d=1}^{D_t^i} \frac{|R_d^i|}{V_d^i}$$

where D_t^i is the number of traded days in month *t* for asset *i*, R_d^i is the daily return of asset *i* on day *d*, and V_d^i is the daily dollar volume traded of asset *i* on day *d*. This ratio can be interpreted as the daily price response associated with one dollar of trading volume and illustrates the ability to execute large trades without causing the price to drop significantly as a result. As mentioned by Dyhrberg et al. (2018), the market depth for Bitcoin is only found to be adequate for retail sized trades. By comparing the Amihud Illiquidty Ratio of Bitcoin against the that of conventional assets, more insight can be gained regarding the liquidity risk associated with Bitcoin. The following hypothesis follows from this:

(5) H_0: *Bitcoin's Amihud illiquidity Ratio is lower than or equal to the Amihud illiquidity Ratio of conventional assets, especially gold.*

It is important to note that the illiquidity ratio of Bitcoin is expected to be much higher than for example the illiquidity ratio of the S&P 500. This is the case because these large indexes are globally traded on an extremely large scale. The resulting dollar volume of this is extremely high, and therefore causes the illiquidity ratio to approach zero. The most important comparison for Bitcoin is gold since similar properties are suggested in terms of hedging and safe haven capabilities.

Finally, the analysis of Bitcoin's place in an investment portfolio can be done by constructing the efficient frontier as described by Markowitz (1952). To determine if adding Bitcoin to an investment portfolio is valuable, the portfolio including Bitcoin should show higher risk-adjusted returns for various levels of risk-averseness, resulting in a more optimal efficient frontier. This leads to the following hypothesis:

(6) H_0: *Including Bitcoin in an investment portfolio results in a better efficient frontier and better portfolio performance.*

An important factor is, as described by Platanakis et al. (2019), that alternative assets are often assigned excessive portfolio weights due to a number of reasons, the most prevalent being estimation risk. To tackle this problem, an analysis on Bitcoin's presence in an efficient portfolio can be made under the assumption of various levels of risk aversion under the expected utility framework.

The final measure that will be used to determine if portfolio performance improves when including Bitcoin is the Jobson-Korkie-test of equal Sharpe Ratios (Jobson & Korkie, 1981) modified by Memmel (2003). By employing this method, the Sharpe Ratio (Sharpe, 1966) of the portfolio including Bitcoin can be compared to the Sharpe Ratio of the portfolio not including Bitcoin, allowing further conclusions to be drawn on the effectiveness of Bitcoin in an efficient portfolio.

Chapter 4. Data and methodology

This section of the thesis will cover the methodology used to test the hypotheses outlined in the literature review, as well as the relevant data that is used. This chapter will cover the methodology and data used per hypothesis to give a structured overview of each hypothesis that will be tested.

4.1. Diversification, hedging and safe haven

To test hypotheses (1), (2) and (3), an analysis will be made on the relation between Bitcoin and conventional assets. In order to do this, daily data is collected for major stock indices across the world for the period September 2011 to March 2021 from Yahoo finance (Yahoo Finance, 2021). To give a balanced view the of the relation between large-, mid-, and small cap stocks and Bitcoin, the S&P 500, S&P 400 and S&P 600 indices will be used. To further broaden the scope to beyond the United States, the Euro Stoxx 50 and Nikkei 225 indices are also added to the analysis. The iShares 7-10 Year Treasury Bond ETF, gold and crude oil will also be used in this comparison in order to effectively capture other assets one might find in a diversified portfolio. Daily Bitcoin data is collected from bitcoincharts.com and coinmarketcap.com for the period September 2011 to March 2021. Throughout this thesis the daily returns are derived by taking the natural logarithm of $Close_t$ divided by $Close_{t-1}$, which are the closing prices of an asset at time *t* and *t – 1* respectively.

Table 1 shows the descriptive statistics for the variables used to test hypothesis (1). As expected, the mean daily Bitcoin return is far higher than the rest of the assets included in the comparison. The standard deviation is also highest out of all assets included. This large difference in daily returns and standard deviation is in line with expectations and previous research. Additionally, Bitcoin shows little comparison to gold, as daily returns, standard deviation, skewness and kurtosis differ greatly. Kurtosis for all assets in general is very high, implying a relatively high number of extreme values within the daily returns distribution of the assets. The data in the research conducted by Brière et al. (2015) shows kurtosis ranging from three to nine for similar assets. This is however derived from weekly return data, which causes a lower volume of extreme values than daily data due to large positive and negative daily shocks cancelling each other out throughout the weekly returns.

Table 2 shows the correlation matrix of the same assets. Again, as expected, Bitcoin shows a relatively low correlation to other assets. The highest correlation that Bitcoin shows is with the S&P 500, however a correlation of 0.105 is still relatively low compared to the correlations the other assets have with each other. Similar to Bitcoin, gold also shows low correlations with other assets. It is striking however that the correlation between Bitcoin and gold is only 0.049, since one would expect to find a higher correlation if both assets are to be considered hedging instruments with similar characteristics. A possible explanation for this could be the large sample of daily returns, where in the beginning of its lifetime Bitcoin showed extremely high volatility and market inefficiency, which in turn could mean that any possible hedging capabilities had not yet fully settled. Based on the correlation matrix of Table 2, Bitcoin is a hedge for treasury bonds due to the negative correlation (albeit very close to zero), and a diversifier for the other asset classes due to the positive correlations found here. However, since this correlation matrix only displays the correlation between these assets on average, it is not possible to answer the first three hypotheses based on this.

To examine the first three hypotheses concerning the diversification, hedging and safe haven properties of Bitcoin, the approach from the research by Bouri et al. (2017) and Stensås et al. (2019) will be followed. This approach involves using the Dynamic Conditional Correlation-GARCH (DCC-GARCH) model introduced by Engle (2002), which allows the correlation to change over time. As can be seen in Figure 1, Bitcoin shows clear signs of time varying volatility between 2011 and 2021, implying that a model which captures these changes over time is the right fit and that correlations on average are insufficient for this purpose. In line with the research by Bouri et al. (2017), the DCC-GARCH model is estimated between pairs of returns. This provides the opportunity to assess Bitcoin's diversification, hedging and safe haven benefits in relation to each individual asset class, rather than in relation to the market as a whole.

Table 1: Descriptive statistics of Bitcoin and conventional assets

This table shows descriptive statistics (number of observations, mean, standard deviation, minimum and maximum returns, skewness and kurtosis) of daily returns of Bitcoin (in USD), global stock indices, 7-10 year U.S. treasury bonds, gold and crude oil for the period 2011-2021.

Variables	*N*	*Mean*	*SD*	*Min*	*Max*	*Skewness*	*Kurtosis*
Bitcoin	3,457	0.264%	4.927%	-66.395%	44.554%	-1.415	28.211
S&P 500	2,382	0.048%	1.075%	-12.765%	8.968%	-0.933	22.846
S&P 400	2,382	0.044%	1.242%	-14.788%	10.173%	-1.266	22.573
S&P 600	2,126	0.049%	1.355%	-14.282%	8.624%	-1.159	19.297
Euro Stoxx 50	2,364	0.021%	1.286%	-13.241%	8.834%	-0.716	12.185
Nikkei 225	2,303	0.055%	1.315%	-8.253%	7.731%	-0.252	7.443
T-bonds	2,382	0.011%	0.358%	-2.539%	2.607%	-0.028	7.170
Oil	2,335	0.014%	2.742%	-28.221%	31.963%	0.308	33.428
Gold	2,335	-0.004%	1.031%	-9.821%	5.778%	-0.613	10.436

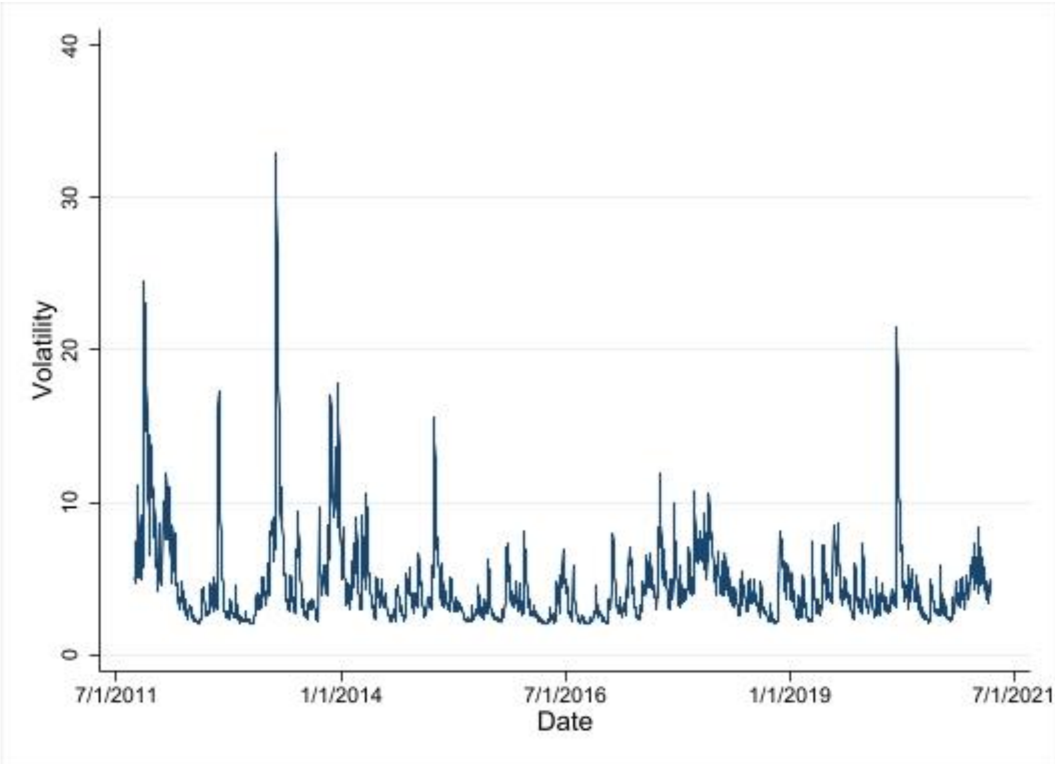
Table 2: Correlation matrix between Bitcoin and conventional assets

This table shows a correlation matrix between daily returns of Bitcoin (in USD), global stock indices, 7-10 year U.S. treasury bonds, gold and crude oil for the period 2011-2021.

Assets	*Bitcoin*	*S&P 500*	*S&P 400*	*S&P 600*	*Euro Stoxx 50*	*Nikkei 225*	*T-bonds*	*Oil*	*Gold*
Bitcoin	1.000	0.103	0.096	0.101	0.058	0.005	-0.002	0.025	0.049
S&P 500	0.103	1.000	0.925	0.867	0.613	0.195	-0.357	0.284	0.016
S&P 400	0.096	0.925	1.000	0.965	0.621	0.206	-0.340	0.294	0.021
S&P 600	0.101	0.867	0.965	1.000	0.595	0.186	-0.348	0.273	0.018
Euro Stoxx 50	0.058	0.613	0.621	0.595	1.000	0.332	-0.332	0.257	-0.105
Nikkei 225	0.005	0.195	0.206	0.186	0.332	1.000	-0.135	0.079	-0.054
T-bonds	-0.002	-0.357	-0.340	-0.348	-0.322	-0.135	1.000	-0.136	0.257
Oil	0.025	0.284	0.294	0.273	0.257	0.079	-0.136	1.000	0.085
Gold	0.049	0.016	0.021	0.018	-0.105	-0.054	0.257	0.085	1.000

Figure 1: Bitcoin volatility over time

Figure 1 shows the volatility of Bitcoin returns for the period 2011-2021. The volatility is derived by predicting the variance of a GARCH (1,1) model and taking the square root of the variance.



After estimating the DCC-GARCH model between Bitcoin and each of the asset classes and following the methodology of Bouri et al. (2017) and Stensås et al. (2019), the Dynamic Conditional Correlation is calculated by equation (1):

$$\rho_{ij,t} = \frac{q_{ij,t}}{(\sqrt{q_{ii,t}}\sqrt{q_{jj,t}})} \tag{1}$$

where $\rho_{ij,t}$ is the DCC between assets *i* and *j*, and q_t is the time-varying unconditional correlation matrix of the standardised residuals which are obtained from the GARCH estimation process. Following the computation of the DCC for the asset pair, it can then be used in the following regression:

$$DCC = \beta_0 + \beta_1 D_{q1} + \beta_2 D_{q5} + \beta_3 D_{q10} + \beta_4 D_{COVID} + \beta_5 D_{COVID} D_{q1} + \beta_6 D_{COVID} D_{q5} + \beta_7 D_{COVID} D_{q10} + \varepsilon_t \tag{2}$$

where the DCC obtained from equation (1) is regressed against dummy variables (D_{q1}, D_{q5} and D_{q10}) representing the lower first, fifth and tenth percentile of the return distribution. In this regression, D_q will be equal to one if the daily return of the asset falls into one or more of these percentiles. This methodology is chosen to capture the dynamic correlation between two assets in regular market periods, represented by the constant β_0, as well as the dynamic correlation in periods of market downturns, represented by β_1, β_2 and β_3. If β_0 is larger than zero and smaller than one, Bitcoin qualifies as a diversifier against the other asset. If β_0 is smaller than zero, Bitcoin qualifies as a hedge against the other asset. If any of β_1, β_2 and β_3 are negative, Bitcoin can be seen as a safe haven in times of market downturn against the other asset. Furthermore, depending on whether β_1, β_2 and β_3 are all negative or only one or two are negative, further conclusions can be made on which degree of market downturn is necessary for Bitcoin to qualify as a safe haven for a particular asset.

Additionally to the dummy variables for the lower first, fifth and tenth percentile of the return distribution, a dummy variable has also been added to signal the period of the COVID-19 crisis, represented by D_{COVID}. Since Junttila et al. (2018) find that correlations between gold and oil market futures and equity returns in the US market differ strongly during market crisis periods compared to normal market circumstances, it is valuable to test for the same phenomenon for Bitcoin in relation to the covid-19 crisis. As of the 12th of March 2020, the World Health Organisation announced that the COVID-19 outbreak was characterised as a pandemic, marking the official starting point for the COVID-19 crisis in this thesis (World Health Orginization, 2020).

Finally, interaction effects have been added between the COVID dummy variable and the dummy variables for the lower first, fifth and tenth percentile of the return distribution. These interaction terms capture situations where asset returns are within these low percentiles of the distribution while the COVID-19 crisis was active simultaneously. If β_4, β_5, β_6 and β_7 in equation (2) are significantly different from zero, this indicates that the COVID-19 pandemic has had an influence on the Dynamic Conditional Correlation between Bitcoin and alternative assets similarly to the findings of Junttila et al. (2018).

4.2. The Bitcoin market and liquidity

Hypothesis (4) covers the possible efficiency of the Bitcoin market. This can be reviewed by expanding on research conducted by Urquhart (2016). In this research, a series of tests are executed to determine the efficiency of the Bitcoin market. Specifically, efficiency in the weak form as described by Fama (1970) will be examined, defining a market in which past information cannot be used to predict future returns. This form of market efficiency implies that price changes are unpredictable and therefore follow a random walk. To evaluate this hypothesis, daily Bitcoin data from the period September 2011 to March 2021 will be used.

The first test that will be executed is the Ljung-Box test (Ljung & Box, 1978). The null hypothesis in this test is that there is no autocorrelation, which enables an analysis of whether past returns significantly influence future returns. Additionally, the Breusch-Godfrey Lagrange multiplier test will be used, where again the null hypothesis is no autocorrelation (Breusch, 1978) (Godfrey, 1978). The advantage of using this test is that the null hypothesis can be tested over a range of lags, meaning that different orders of lags can be tested for autocorrelation. If these tests show that there is no autocorrelation in the Bitcoin returns, then market efficiency in the weak form can be assumed which in turn allows for a degree of comfort in other conclusions that are drawn from this research.

Because the Bitcoin market is still relatively young, it is possible that using a sample that covers nearly the entire lifetime of the market causes the tests to show non-randomness while in the more recent years this was not the case. To be able to draw more accurate conclusions, a structural break test was executed to determine if there are any breaks in the Bitcoin returns in the period 2011-2021. The null hypothesis for the structural break test is that there is no structural break present. This null hypothesis is rejected with a p-value of 0.0002, with the estimated break taking place on the 7^{th} of May 2013. To account for this break, the autocorrelation tests will be executed for the sample as a whole, as well as for the period before and after the break. This way of testing the hypothesis allows for a more nuanced conclusion on whether or not the Bitcoin market shows efficiency in the weak form. To specify, results indicating there are no signs of market efficiency in the period 2011-2013 while in the period 2013-2021 the test results do show market efficiency, mean that there could be a move to maturity similar to the findings of Urquhart (2016).

Following the analysis on the weak form of market efficiency for Bitcoin, the Amihud Illiquidity Ratio (Amihud, 2002) is used to test hypothesis (5). It is described as the daily ratio of absolute stock returns to dollar volume, and can be interpreted as the ability to execute large trades without causing the price to drop significantly as a result. It is denoted as follows:

$$ILLIQ_t^i = \frac{1}{D_t^i} \sum_{d=1}^{D_t^i} \frac{|R_d^i|}{V_d^i} \qquad (3)$$

where D_t^i is the number of traded days in month *t* for asset *i*, R_d^i is the daily return of asset *i* on day *d*, and V_d^i is the daily dollar volume traded of asset *i* on day *d*. For analytical purposes, the ratio in this thesis is multiplied by a factor of 1,000,000 in order to allow for easier readability. It is shown that the expected market illiquidity over time positively affects stock returns, as stocks with higher illiquidity levels carry more risk and thus offer an illiquidity premium (Amihud, 2002). In order to analyse the illiquidity of Bitcoin compared to conventional assets, the monthly illiquidity ratio will be computed for each asset according to equation (3). The means of these illiquidity ratios can then be compared to each other by means of a two sample t-test between the illiquidity ratios of Bitcoin and the other asset. This allows conclusions to be made on whether or not Bitcoin's illiquidity is significantly different from other assets. However, as can be seen in Figure 2, Bitcoin's illiquidity ratio is far higher in the first months of the sample compared to the rest of the sample.

Similarly to the testing of the fourth hypothesis, a structural break test was executed for hypothesis (5) as well in order to verify if the variability of Bitcoin's illiquidity ratio over time displayed in Figure 2 can be statistically verified. The null hypothesis of no structural break was rejected with a p-value of 0.0000, with the estimated break taking place on the 3rd of March 2013. This means that the two sample t-tests will be executed for the period 2011-2021, as well as for the two periods on each side of the break, being 2011-2013 and 2013-2021 respectively. By applying this methodology, an analysis can be made on whether Bitcoin illiquidity as a whole differs from conventional assets, as well as for the more recent years where the period of extremely high volatility and illiquidity has been left behind.

Figure 2: Bitcoin illiquidity ratio over time

Figure 2 shows the Amihud Illiquidity Ratio (2002) of Bitcoin for the period 2011-2021.

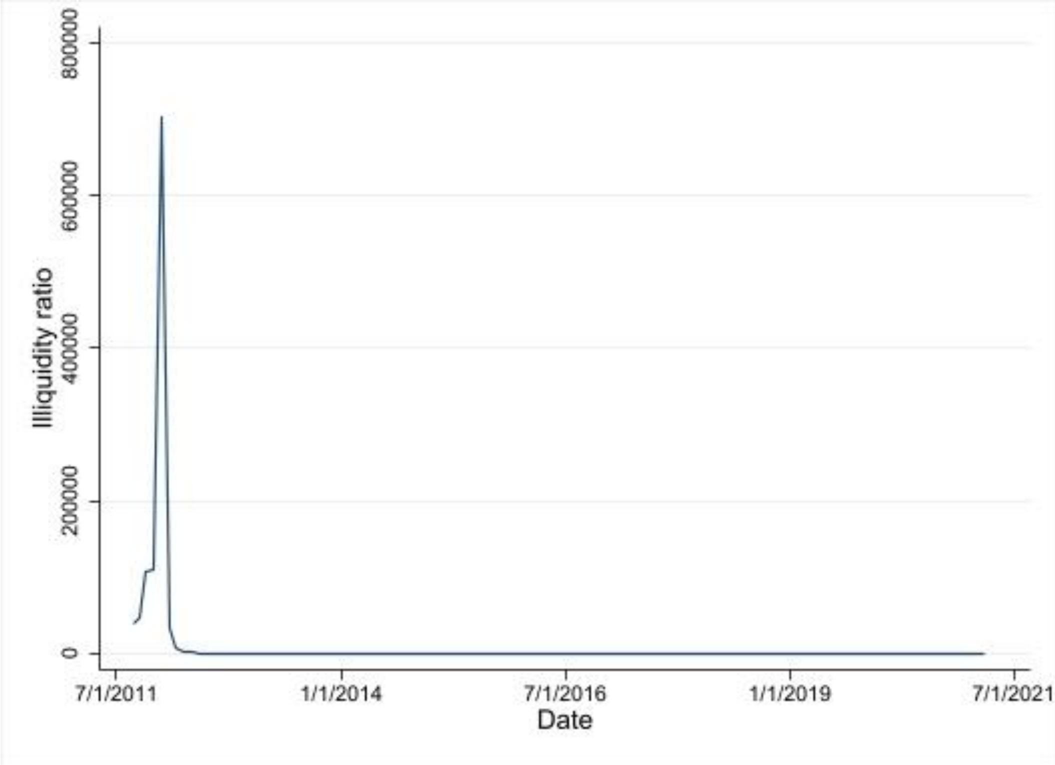


Table 3 shows the monthly Amihud illiquidity ratios for all assets that are used to test hypothesis (5). These are the same assets that are used to test hypotheses 1-3 (the S&P 500, S&P 400, S&P 600, the Euro Stoxx 50 and Nikkei 225, 7-10 year treasury bonds, gold and crude oil). The illiquidity ratio is displayed for the sample as a whole, as well as for the two sub samples. No volume data on the Euro Stoxx 50 was available on Yahoo finance for the first months of the sample.

The difference between mean illiquidity ratios for Bitcoin in the first sample compared to the second sample is confirmation of what was already visible in Figure 2 – the period 2011-2013 shows much higher illiquidity ratios for Bitcoin than the period 2013-2021. Furthermore, as expected, the illiquidity ratios of the large indices are extremely low and approach zero due to the enormous daily dollar volume traded. What is most striking is that the extreme decrease of Bitcoin's illiquidity ratio causes it to be lower than the illiquidity ratio of gold. The relatively high illiquidity ratio of gold compared to other assets in both sub samples could be signalling to the findings of Baur and Lucey (2010), who show that an investor that holds gold for longer than 15 days after a negative shock on average loses money while holding gold. This could partially be due to the relatively low liquidity, which causes losses when the market tries to close its long position in gold shortly after a negative shock.

Table 3: Amihud Illiquidity ratios for Bitcoin and conventional assets

Table 3 shows the Amihud Illiquidity Ratio (2002) of Bitcoin and conventional assets for the entire sample ranging from 2011 to 2021, as well as for two sub samples ranging from 2011 to 2013 and 2013 to 2021.

	Amihud Illiquidity Ratio					
	2011-2021		*2011-2013*		*2013-2021*	
Variables	*N*	*Mean*	*N*	*Mean*	*N*	*Mean*
Bitcoin	115	9,112	19	55,149	96	0.435
S&P 500	115	0.000	19	0.000	96	0.000
S&P 400	115	0.000	19	0.000	96	0.000
S&P 600	102	0.000	6	0.000	96	0.000
Euro Stoxx 50	96	0.000	-	-	96	0.000
Nikkei 225	115	0.000	19	0.000	96	0.000
T-bonds	115	0.002	19	0.004	96	0.001
Oil	115	0.090	19	0.051	96	0.097
Gold	115	15.500	19	12.548	96	16.081

As mentioned before, Bitcoin is not expected to show an illiquidity ratio that is similar to the ratios of the large indices, since the daily dollar volume for these assets is far larger than Bitcoin's as can be seen in Table 4. Bitcoin shows a remarkable increase in daily dollar volume from the first to the second sample, which is the main reason that Bitcoin's illiquidity ratio has improved.

Table 4: Daily dollar volume for Bitcoin and conventional assets

Table 4 shows the daily dollar volume of Bitcoin and conventional assets for the entire sample ranging from 2011 to 2021, as well as for two sub samples ranging from 2011 to 2013 and 2013 to 2021.

	Daily Dollar Volume					
	2011-2021		*2011-2013*		*2013-2021*	
Variables	*N*	*Mean*	*N*	*Mean*	*N*	*Mean*
Bitcoin	3,458	4.30×10^{7}	520	21,620	2,938	5.06×10^{7}
S&P 500	2,383	8.71×10^{12}	359	5.07×10^{12}	2,024	9.35×10^{12}
S&P 400	2,227	5.74×10^{11}	219	1.14×10^{11}	2,008	6.24×10^{11}
S&P 600	2,080	2.24×10^{11}	70	2.96×10^{10}	2,010	2.30×10^{11}
Euro Stoxx 50	1,910	1.55×10^{11}	-	-	1,910	1.55×10^{11}
Nikkei 225	2,320	2.04×10^{12}	353	1.35×10^{12}	1,967	2.16×10^{12}
T-bonds	2,383	2.86×10^{8}	359	9.45×10^{7}	2,024	3.20×10^{8}
Oil	2,361	2.65×10^{7}	355	2.46×10^{7}	2,006	2.68×10^{7}
Gold	2,360	7,828,964	355	9,486,902	2,005	7,525,414

4.3. Bitcoin and portfolio optimisation

The final hypothesis that will be tested is hypothesis (6), where an analysis is made on Bitcoin's place in an optimised portfolio. This will be done by constructing the efficient frontier as described by Markowitz (1952), which represents all levels of returns that can be obtained given a corresponding level of risk tolerance and given that the variance of the portfolio is minimised, which is represented by equation (4) in case of a portfolio consisting of n assets:

$$\sigma^2 = \Sigma_{i,j=1}^{n} \omega_i \omega_j \sigma_{ij} \quad (4)$$

where ω_i and ω_j are the portfolio weights of assets *i* and *j*, and σ_{ij} is the covariance between asset *i* and *j*. The expected return of the portfolio is represented by equation (5):

$$\mathrm{E}(r_p) = \Sigma_{i=1}^{n} \omega_i r_i \quad (5)$$

where ω_i again represents the portfolio weight of asset *i*, and r_i represents the average return for asset *i*. We further assume that the sum of all portfolio weights is equal to one, meaning that 100% of an investor's funds are invested and that no remaining assets are left unallocated, as is represented by equation (6).

$$\Sigma_{i=1}^{n} \omega_i = 1 \quad (6)$$

To analyse whether or not adding Bitcoin in a portfolio increases expected portfolio performance, two efficient frontiers will be constructed – one including Bitcoin, and one excluding Bitcoin. The other assets that are included in this analysis are the same assets used in the previous hypotheses (the S&P 500, S&P 400, S&P 600, the Euro Stoxx 50 and Nikkei 225, 7-10 year treasury bonds, gold and crude oil), with data ranging from September 2011 to March 2021. If Bitcoin does indeed provide benefits for the risk-return trade-off in an optimised portfolio, then the efficient frontiers should display that higher levels of returns can be obtained given a certain amount of risk when Bitcoin is included.

To continue the analysis on Bitcoin's place in an efficient portfolio, an analysis will be made on whether or not Bitcoin would be included in a portfolio given a certain amount of risk aversion. Expected utility associated with an investment portfolio is represented by equation (7):

$$\mathrm{E}(U_p) = \mathrm{E}(r_p) - \frac{\theta\sigma^2}{2} \quad (7)$$

where θ represents a coefficient for risk aversion and σ^2 represents the variance of the portfolio returns. Three portfolios will be constructed that maximise expected utility, denoted by equation (7), for three levels of risk aversion; $\theta = 2$ representing a slight risk aversion, $\theta = 3.5$ representing a moderate level of risk aversion, and $\theta = 5$ representing extreme risk aversion. This will yield three portfolio's that are present on the efficient frontier constructed by minimising the variance, denoted by equation (4). If Bitcoin's portfolio weight is larger than zero in one or more of these portfolios, it shows that including Bitcoin in an investment portfolio is beneficial for the risk-return trade-off given a set amount of risk aversion. Additionally, short-selling restrictions will be imposed on these portfolios to more accurately represent the possibilities of small institutional and retail investors that do not have access to short-selling. Since Dyhrberg et al. (2018) mentioned that the market depth for Bitcoin is only adequate for retail sized trades, the restriction of short-selling is a valuable addition to the analysis.

The final measure that will be used to determine portfolio performance is the Jobson-Korkie-test of equal Sharpe Ratios (Jobson & Korkie, 1981) modified by Memmel (2003). The Sharpe Ratio (Sharpe, 1966) is defined as follows:

$$Sh_p = \frac{R_p - R_f}{\sigma_p} \quad (8)$$

where R_p represents the return of the portfolio, R_f represents the risk-free rate and σ_p is the standard deviation of the portfolio. If the addition of Bitcoin to a portfolio is indeed valuable, then the portfolio including Bitcoin should outperform the portfolio that does not include Bitcoin. This can be denoted in the following null- and alternative hypothesis:

$$H_o : Sh_t - Sh_b \geq 0$$

$$H_1 : Sh_t - Sh_b < 0$$

where Sh_t represents the Sharpe Ratio of the traditional portfolio excluding Bitcoin, and Sh_b represents the Sharpe Ratio of the portfolio including Bitcoin. Memmel (2003) has shown that his modified version of the the Jobson-Korkie-test of equal Sharpe Ratios (Jobson & Korkie, 1981) does not lose any statistical properties or power. In his simplification the test statistic uses simple differences between Sharpe Ratios of the two portfolios, as demonstrated by the following equation:

$$\mathrm{z} = \frac{Sh_b - Sh_t}{\sqrt{V}} \tag{9}$$

where V is the asymptotic variance of the Sharpe Ratio difference:

$$V = \frac{1}{T}\left[2\sigma_t^2\sigma_b^2 - 2\sigma_t\sigma_b\sigma_{tb} + \frac{1}{2}\mu_t^2\sigma_b^2 + \frac{1}{2}\mu_b^2\sigma_t^2 - \frac{\mu_t\mu_b}{\sigma_t\sigma_b}\sigma_{tb}^2\right] \tag{10}$$

where T is the total number of observations of each portfolio, σ_t and σ_b are the standard deviations of portfolio t and portfolio b, σ_{tb}^2 is the covariance of the two portfolio returns and μ_t and μ_b are the mean returns of portfolio t and portfolio b. Since the test concerns a one-sided right-tailed test, the Z-score of equation (9) should be larger than 1.28, 1.645 or 2.33 if the null-hypothesis is to be rejected at the 10%, 5% and 1% level respectively.

Chapter 5. Results

This section will display and cover the results of the various tests that were performed to test the hypotheses of this thesis. As per the structure of the previous chapters, this chapter too will cover the results per hypothesis.

5.1. Diversification, hedging and safe haven

As mentioned in the Data and methodology chapter, the DCC-GARCH model (Engle, 2002) is used to test the first three hypotheses regarding the diversification, hedging and safe haven benefits of Bitcoin. The pairwise DCC is then calculated according to equation (1), after which it is used in the regression displayed in equation (2). These results are displayed in Table 5.

Table 5: Diversification, hedging and safe haven capabilities of Bitcoin

Table 5 shows the results of the regression in equation (2). The pairwise DCC between Bitcoin and conventional assets is regressed against three dummy variables representing the lower first, fifth and tenth percentile of the return distribution. ***, **, *, indicate statistical significance at the 1% level, 5% level and 10% level, respectively.

Variables	S&P 500	S&P 400	S&P 600	Euro Stoxx 50	Nikkei 225	T-bonds	Oil	Gold
β_0	0.053^{***}	0.045^{***}	0.056^{***}	0.064^{***}	0.011^{***}	0.001^{***}	0.029^{***}	0.055^{**}
β_1	0.021	-0.011	0.036^{***}	0.008	0.000	0.001	0.004	0.010
β_2	0.011	0.007	0.002	-0.000	-0.000	0.000	-0.001	0.006
β_3	-0.001	-0.002	0.005	-0.004	-0.000	0.000	0.004^{*}	-0.003
β_4	0.160^{***}	0.142^{***}	0.131^{***}	0.018^{***}	0.009^{***}	-0.001^{**}	0.050^{***}	0.064^{**}
β_5	-0.046^{*}	-0.019	-0.041^{**}	0.043	-0.000	-0.001	-0.025^{**}	0.008
β_6	-0.007	-0.022^{*}	-0.002	0.017	0.003^{*}	-0.000	-0.003	0.020
β_7	0.012	0.013	0.000	-0.017	0.001	-0.000	-0.004	-0.005
R-squared	0.535	0.555	0.539	0.027	0.292	0.074	0.227	0.290

As discussed in the Data and methodology chapter, the constant β_0 captures the dynamic correlation in periods of regular market behaviour. Following the definitions of Baur and Lucey (2010) discussed in the literature review, if β_0 is larger than zero and smaller than one, Bitcoin qualifies as a diversifier against the other asset. If β_0 is smaller than zero, Bitcoin can be seen as a hedge. As can be derived from Table 5, all constants of all regressions show values larger than zero and smaller than one with significance at the 1% level. This indicates that, based on the data used in this research, Bitcoin qualifies as a diversifier for all conventional assets, supporting the conclusion of Brière et al. (2015). This also means that Bitcoin is not a hedge for these assets, showing a contrast to research conducted by Dyhrberg (2016), but supporting the conclusions of Bouri et al. (2017) and Charfeddine et al. (2020) who respectively state that Bitcoin and cryptocurrencies in general are a poor hedging instrument.

To analyse the safe haven characteristics of Bitcoin, equation (2) also regresses the DCC against dummy variables (D_{q1}, D_{q5} and D_{q10}) that represent the lower first, fifth and tenth percentile of the return distribution. The variables β_1, β_2 and β_3 are coefficients that capture the dynamic correlation between Bitcoin and conventional assets in times of market downturns represented by these three dummy variables. A negative sign for these coefficients indicates a negative Dynamic Conditional Correlation between Bitcoin and a conventional asset within the lower first, fifth or tenth percentile of the return distribution, implying a safe haven effect. There are several negative signs for β_1, β_2 and β_3 in Table 5. While the negative signs might hint at a possible safe haven effect, they are not significant at the 10% level. Moreover, the coefficients are very close to zero, indicating that any hint at a possible safe haven effect should be investigated further before drawing any conclusions. This is largely in support of Bouri et al. (2017), who only find a safe haven effect for Bitcoin against extreme weekly down movements in Asian stocks and nowhere else. Klein et al. (2018) also report similar results and state that Bitcoin declines when the market declines, which is in line with the positive signs that most coefficients show in this thesis.

Additionally, the effect of the COVID-19 pandemic was analysed in the regression. β_4 is positive and significant at the 1% level in each regression, implying that the COVID-19 crisis has had a significant effect on the Dynamic Conditional Correlation between Bitcoin and conventional assets. The positive sign indicates that during the COVID-19 crisis Bitcoin and conventional assets have become more correlated with each other, again implying that no hedging benefits are present.

β_5, β_6 and β_7 show the possible safe haven effects during the COVID-19 crisis. In the pairwise regressions between Bitcoin on the one hand and the S&P 500, S&P 600 and oil on the other hand, β_5 is negative and significantly different from zero at either the 5% or 10% level. β_6 is negative and different from zero at the 10% level in the pairwise regression between Bitcoin and the S&P 400. These coefficients indicate that Bitcoin is a safe haven for the S&P 500, S&P 600 and oil in the lower first percentile of the return distribution and for the S&P 400 in the lower fifth percentile of the return distribution during the COVID-19 crisis. This does not only show promise for Bitcoin's use as a safe haven tool in times of crises, but also confirms Bitcoin's behaviour to be similar to that of gold as described by Junttila et al. (2018), who find that correlations between gold and oil market futures and equity returns in the US market differ strongly during market crisis periods compared to normal market circumstances.

Finally, the R-squared values for each regression model are also displayed in Table 5. Some R-squared values are relatively low, like in the regressions concerning the Euro Stoxx 50 and Treasury bonds, where the values are 0.027 and 0.074 respectively. The R-squared values for the other regressions range from 0.227 to 0.555, implying that in most cases, more than 50% of the variability in the data cannot be explained by the model. This is to be expected, since it is unlikely that dummies for the lower first, fifth and tenth percentile of the return distribution and the COVID-19 crisis are the only explanatory variables for the pairwise DCC between Bitcoin and conventional assets.

5.2. The Bitcoin market and liquidity

The fourth hypothesis that is being tested covers the market efficiency in the weak form (Fama, 1970). In order to do this, the Ljung-Box test (Ljung & Box, 1978) and the Breusch-Godfrey Lagrange multiplier test (Breusch, 1978) (Godfrey, 1978) are executed to test for autocorrelation within the returns. As discussed in the Data and methodology chapter, the two tests are executed for the sample as a whole (2011-2021), as well as for two sub samples ranging from 2011-2013 and 2013-2021. The corresponding P-values for the tests can be found in Table 6.

Table 6: Bitcoin market efficiency

Table 6 shows the P-values for the Ljung-Box test (Ljung & Box, 1978) and the Breusch-Godfrey Lagrange multiplier test (Breusch, 1978) (Godfrey, 1978) to test for weak form market efficiency in the Bitcoin market.

	2011-2021		*2011-2013*		*2013-2021*	
	Ljung-Box	*Breusch-Godfrey*	*Ljung-Box*	*Breusch-Godfrey*	*Ljung-Box*	*Breusch-Godfrey*
P-value	(0.000)	(0.013)	(0.003)	(0.440)	(0.000)	(0.009)

As can be derived from the P-values in Table 6, the Ljung-Box test rejects the null-hypothesis of no autocorrelation when tested for the period 2011-2021, as well as for the two periods 2011-2013 and 2013-2021. The P-values of the Breusch-Godfrey test show different results. For the period 2011-2021, the null-hypothesis of no autocorrelation is still rejected, albeit with slightly less statistical significance than the Ljung-Box test. What is striking however is that in the period 2011-2013, the Breusch-Godfrey test fails to reject the null-hypothesis, implying that no autocorrelation is present and therefore the market can be deemed efficient in the weak form. In the period 2013-2021 the null is rejected again, implying the existence of an inefficient Bitcoin market in this period. The contrast in the results between the Ljung-Box test and the Breusch-Godfrey test, especially in the first sample ranging from 2011 to 2013, calls for further investigation into this topic in future research. Furthermore, both different measures for testing autocorrelation in the Bitcoin returns show the existence of an inefficient market in the period 2011-2021 as well as the period 2013-2021, and there is no hint to a possible move to maturity as described by Urquhart (2016). The contrast in the results concerning the period 2011-2013 call for further examination of Bitcoin's market efficiency as well.

The fifth hypothesis that is tested in this thesis uses the Amihud Illiquidity Ratio (Amihud, 2002) to analyse the liquidity risk associated with Bitcoin. The ratio is calculated according to equation (3) for Bitcoin and the alternative assets. Table 3 shows the Amihud Illiquidity Ratios for each asset for the period 2011-2021, as well as for the two sub samples ranging from 2011-2013 and 2013-2021. As mentioned before, the most important comparison is the one between Bitcoin and gold, as Bitcoin is expected to show a much higher illiquidity ratio than indices like the S&P 500. This can be confirmed by the values displayed in Table 3 as well. Therefore, to keep this thesis as concise as possible, only the results of the two sample t-tests between Bitcoin and gold for each (sub)sample will be reported in the main body of text. The tests between Bitcoin and other conventional assets for the period 2011-2021 can be found in Appendix A. As expected, Bitcoin shows significantly higher illiquidity ratios than each conventional asset for this time period.

Because interest lies in whether or not Bitcoin's liquidity is better or worse than that of gold, a two-tailed test is not of much use, since this type of test is only able to display whether or not the difference between the samples is equal to zero or not. Therefore, the tests that are executed are left-tailed tests, since the goal is to find out if the illiquidity ratio of Bitcoin is lower than that of gold.

Table 7: Two sample t-test between Bitcoin and gold illiquidity (2011-2021)

Table 7 shows the results of the two sample t-test between the Amihud Illiquidity Ratios of Bitcoin and gold for sample period 2011-2021. The descriptive statistics for the two assets are displayed in the table, as well as the results of the one-tailed t-test with the corresponding t value, p value and degrees of freedom (DF).

Variables	*N*	*Mean*	*Std. Error*	*Std. Deviation*
Bitcoin	115	9,112	6,259	67,121
Gold	115	15.50	2.05	21.95
t-test for $\mu_{Bitcoin} - \mu_{gold} < 0$	t = 1.45	p = 0.926	DF = 114	

Table 8: Two sample t-test between Bitcoin and gold illiquidity (2011-2013)

Table 8 shows the results of the two sample t-test between the Amihud Illiquidity Ratios of Bitcoin and gold for sample period 2011-2013. The descriptive statistics for the two assets are displayed in the table, as well as the results of the one-tailed t-test with the corresponding t value, p value and degrees of freedom (DF).

Variables	*N*	*Mean*	*Std. Error*	*Std. Deviation*
Bitcoin	19	55,149	36,888	160,790
Gold	19	12.55	2.60	11.33
t-test for $\mu_{Bitcoin} - \mu_{gold} < 0$	t = 1.500	p = 0.924	DF = 18	

Table 9: Two sample t-test between Bitcoin and gold illiquidity (2013-2021)

Table 9 shows the results of the two sample t-test between the Amihud Illiquidity Ratios of Bitcoin and gold for sample period 2013-2021. The descriptive statistics for the two assets are displayed in the table, as well as the results of the one-tailed t-test with the corresponding t value, p value and degrees of freedom (DF).

Variables	*N*	*Mean*	*Std. Error*	*Std. Deviation*
Bitcoin	96	0.435	0.091	0.890
Gold	96	17.70	2.398	23.490
t-test for $\mu_{Bitcoin} - \mu_{gold} < 0$	t = -6.521	p = 0.000	DF = 95	

Tables 7, 8 and 9 show the results of the two sample t-test performed for the period 2011-2021 and the two periods 2011-2013 and 2013-2021 respectively. As can be derived from the p value of 0.926 from Table 7, there is no evidence to reject the null hypothesis of equal means for the entire sample, and therefore we cannot say that Bitcoin has a better (lower) illiquidity ratio than gold for the period 2011-2021. When looking at the results displayed in Table 8 and Table 9 however, the suspicion that was raised in the Data and methodology chapter due to the extreme drop of Bitcoin's illiquidity ratio is confirmed. We can see that for the period 2011-2013, the result is similar to that of the entire sample, meaning that Bitcoin does not have a better illiquidity ratio than gold. Table 9 shows that for the period 2013-2021, the t-test results in a p value of 0.000, meaning that the null hypothesis of equal means can be rejected and that the alternative hypothesis $\mu_{Bitcoin} - \mu_{gold} < 0$ is accepted. Since volume, efficiency and liquidity are often linked, the rising dollar volume that was observed in Table 4 shows promising signs for the future of the Bitcoin market. If this trend continues, one may possibly expect movements towards an efficient Bitcoin market and corresponding positive effects on liquidity.

5.3. Bitcoin and portfolio optimisation

The final hypothesis that will be tested is hypothesis 6, where an analysis is made on Bitcoin's place in an optimised portfolio. This analysis consists of three steps; firstly, an efficient frontier (Markowitz, Portfolio Selection, 1952) is constructed for a portfolio excluding and including Bitcoin. If the efficient frontier of the portfolio including Bitcoin shows a more efficient risk-return trade-off than the efficient frontier excluding Bitcoin, then this indicates that there is value in including Bitcoin in a diversified portfolio. Secondly, an analysis will be made on whether or not Bitcoin is included in a diversified portfolio given a certain degree of risk-averseness. Finally, the Jobson-Korkie-test of equal Sharpe Ratios (Jobson & Korkie, 1981) modified by Memmel (2003) is executed to test whether or not the portfolio including Bitcoin has a better Sharpe ratio than the portfolio without Bitcoin.

For step one of the analysis, as can be seen in Figure 3, the efficient frontier including Bitcoin does indeed show a more efficient risk-return trade-off, since for every level of risk the expected return is higher. Bitcoin itself does not feature on this graph visually because it is so far to the top right that it would make the rest of the graph difficult to read. For reference however, Bitcoin in this graph would be plotted on the point where risk equals 94% and return equals 57%. Despite the high individual risk, the overall risk-return trade-off improves if Bitcoin is included in the portfolio while minimising equation (4), simply because a more efficient asset allocation can be made.

Table 10 shows the portfolio weights for three portfolios maximising the expected utility represented by equation (7) for slight, moderate and extreme risk aversion. As can be seen, Bitcoin has been assigned a positive weight in each of the portfolios, indicating that under the three assumed levels of risk aversion, Bitcoin is still investible. As can be expected of an asset that carries relatively high risk, Bitcoin weights decrease as risk aversion increases. The portfolios that have short selling restrictions also include Bitcoin, which is an important finding considering Dyhrberg et al. (2018) mentioned that the market depth for Bitcoin might only be adequate for retail sized trades. Since retail investors often do not have access to short selling, this finding shows that including Bitcoin in your portfolio can still be of value as a retail investor, and combined with the possible move to maturity and the corresponding improvements in liquidity, retail investibility under current circumstances might point to institutional investibility in the future.

Figure 3: Efficient Frontier

Figure 3 shows the efficient frontiers for portfolio's including and excluding Bitcoin. Additionally, risk-return trade-offs for single conventional assets are also displayed. Lastly, the global mean variance portfolio (MVP) is displayed.

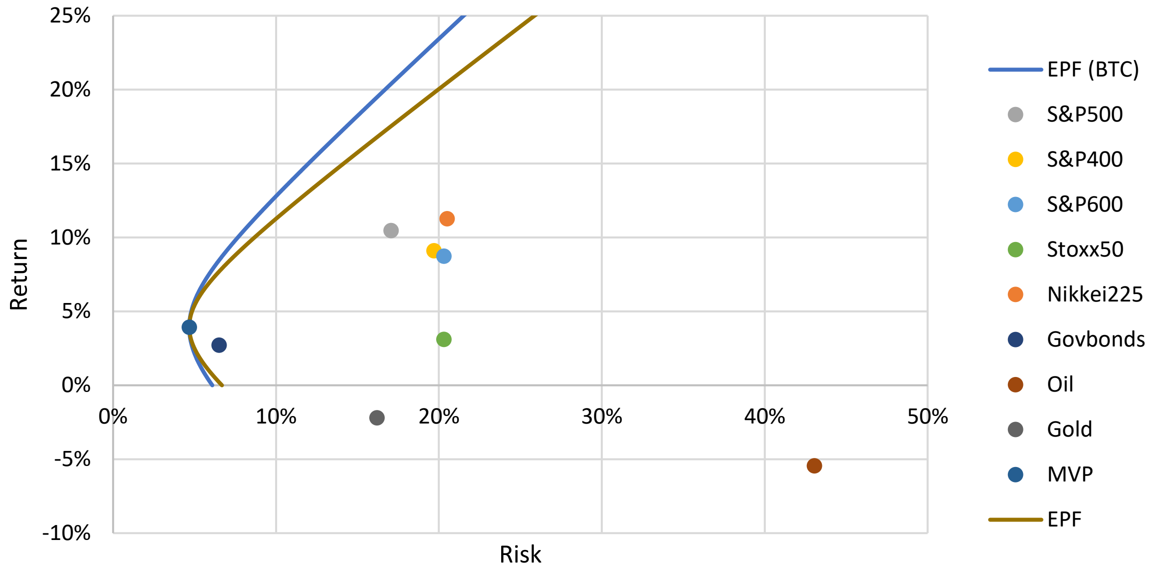


Table 10: Portfolio weights under the expected utility framework

Table 10 shows the portfolio weights for six portfolios maximising equation (7): $\mathrm{E}(U_p) = \mathrm{E}(r_p) - \frac{\theta\sigma^2}{2}$. Three levels of risk aversion are chosen to represent a slight risk aversion (θ = 2), a moderate risk aversion (θ = 3.5) and extreme risk aversion (θ = 5). The corresponding expected return and risk for each portfolio are also displayed. Portfolios marked with * are portfolios where short sale restrictions are imposed.

	$\theta = 2$		$\theta = 3.5$		$\theta = 5$	
Variables	ω	ω^*	ω	ω^*	ω	ω^*
Bitcoin	30%	28%	17%	16%	12%	11%
S&P 500	346%	32%	205%	44%	148%	38%
S&P 400	-122%	0%	-72%	0%	-52%	0%
S&P 600	-11%	0%	-6%	0%	-4%	0%
Euro Stoxx 50	-123%	0%	-67%	0%	-45%	0%
Nikkei 225	102%	41%	60%	40%	43%	31%
T-bonds	30%	0%	48%	0%	56%	19%
Oil	-31%	0%	-18%	0%	-12%	0%
Gold	-121%	0%	-68%	0%	-47%	0%
Return	54.29%	23.72%	32.70%	18.41%	24.07%	14.63%
Risk	50.41%	28.66%	29.06%	20.11%	20.61%	14.96%

The final measure that will be used to determine portfolio performance is the Jobson-Korkie-test of equal Sharpe Ratios (Jobson & Korkie, 1981) modified by Memmel (2003). To do this, three portfolios including Bitcoin and three portfolios excluding Bitcoin are constructed that maximise equation (7) under the three assumed levels of risk-averseness. No short selling restrictions are imposed to mimic the conditions under which institutional investors operate. For each of the six portfolios the Sharpe ratios are calculated according to equation (8), after which they are compared to each other according to equation (9) and (10). R_f has been set at 2.7%, as this is the average return of treasury bonds within the sample, representing the rate which investors can expect as returns without being exposed to risk.

Table 11: Portfolio weights and Sharpe ratios under the expected utility framework

Table 11 shows the portfolio weights for six portfolios maximising equation (7): $\mathrm{E}(U_p) = \mathrm{E}(r_p) - \frac{\theta\sigma^2}{2}$. Three levels of risk aversion are chosen to represent a slight risk aversion (θ = 2), a moderate risk aversion (θ = 3.5) and extreme risk aversion (θ = 5). The corresponding expected return and risk for each portfolio are also displayed, as well as the Sharpe ratios. Portfolios marked with * are portfolios where Bitcoin is excluded.

	$\theta = 2$		$\theta = 3.5$		$\theta = 5$	
Variables	ω	ω^*	ω	ω^*	ω	ω^*
Bitcoin	30%	0%	17%	0%	12%	0%
S&P 500	346%	358%	205%	212%	148%	153%
S&P 400	-122%	-127%	-72%	-75%	-52%	-54%
S&P 600	-11%	-5%	-6%	-2%	-4%	-1%
Euro Stoxx 50	-123%	-120%	-67%	-66%	-45%	-44%
Nikkei 225	102%	100%	60%	59%	43%	43%
T-bonds	30%	36%	48%	52%	56%	59%
Oil	-31%	-31%	-18%	-17%	-12%	-12%
Gold	-121%	112%	-68%	-63%	-47%	-43%
Return	54.29%	38.18%	32.70%	23.51%	24.07%	17.64%
Risk	50.41%	41.63%	29.06%	24.10%	20.61%	17.20%
Sharpe ratio	1.02	0.85	1.03	0.86	1.04	0.87

Table 11 shows the six constructed portfolios, the weights of each asset within the portfolio, along with the corresponding return, risk and Sharpe ratio. A clear pattern is visible where all portfolios that include Bitcoin have a Sharpe ratio over 1.00, while all portfolios excluding Bitcoin show Sharpe ratios of around 0.85. A portfolio including Bitcoin and a portfolio excluding Bitcoin are tested against each other for each level of risk-averseness, resulting in three Z-scores displayed in Table 12.

The comparison of two portfolios under the assumption of slight risk aversion resulted in a Z-score of 0.624, while the comparison of two portfolios under the assumptions of moderate and extreme risk aversion resulted in Z-scores of 2.932 and 5.683 respectively, meaning that the portfolios including Bitcoin outperform the portfolios excluding Bitcoin under moderate and extreme levels of risk aversion. These results imply that an investor that optimises its utility function benefits most from including Bitcoin in their portfolio when risk aversion is high, something that on the surface seems counterintuitive given the inherent risk that Bitcoin carries. However, the most likely reason for this result is the fact that Bitcoin exposure decreases in a portfolio when risk aversion increases, ultimately leading to a better risk-return trade-off overall.

Table 12: Jobson-Korkie-test of equal Sharpe Ratios

Table 12 shows the results of the Jobson-Korkie-test of equal Sharpe Ratios (Jobson & Korkie, 1981) modified by Memmel (2003). For each level of risk aversion the two corresponding portfolios displayed in Table 11 have been compared to each other according to equations (8), (9) and (10). The resulting Z-scores have been displayed in this table.

Test statistic	$\theta = 2$	$\theta = 3.5$	$\theta = 5$
$z = \frac{Sh_b - Sh_t}{\sqrt{V}}$	0.624	2.932	5.683

To summarise, incorporating Bitcoin in an optimised portfolio causes a better efficient frontier. Furthermore, it is shown that Bitcoin is investible under three levels of risk aversion, even when restrictions on short selling are imposed. Finally, portfolios including Bitcoin outperform portfolios excluding Bitcoin for two out of three risk aversion levels under the Jobson-Korkie Sharpe Ratio testing framework modified my Memmel (2003).

Chapter 6. Conclusion and discussion

This thesis aimed to cover a broad range of Bitcoin's investibility in order to answer the research question:

What are the diversification, hedging and safe haven benefits of including the Bitcoin as an investment vehicle in a portfolio of stocks, bonds and commodities, and how does it influence portfolio performance?

In order to answer the research question, an analysis was made on the diversification, hedging and safe haven benefits of Bitcoin by following the methodology of Bouri et al. (2017) and Stensås et al. (2019). This methodology involves the pairwise regression of the Dynamic Conditional Correlation obtained from the DCC-GARCH model estimation against dummy variables representing lower percentiles of the return distribution. From this analysis the conclusion can be drawn that Bitcoin serves as a diversifier and not a hedge against traditional assets, and that the overall results show that Bitcoin is more likely to move along with the market instead of against it. Moreover, the COVID-19 crisis is found to have an effect on the Dynamic Conditional Correlations between Bitcoin and conventional assets. In general, the COVID-19 crisis has caused Bitcoin and conventional assets to become more correlated with each other, supporting the conclusion of Klein et al. (2018), who state that Bitcoin is more inclined to move with the market rather than against it. In the cases of the S&P 500, S&P 400, S&P 600 and oil however, Bitcoin is shown to be a safe haven during the COVID-19 crisis. This shows promise for Bitcoin's use as a safe haven in times of a stock market crisis, and also confirms Bitcoin behaves similarly to gold described by Junttila et al. (2018), who find that correlations between gold market futures and equity returns in the US market differ strongly during market crisis periods compared to normal market circumstances.

Next, an analysis was made on efficiency and liquidity in the Bitcoin market in order to provide a broader view on the implications of the previous analysis. The analysis on market efficiency shows varying results and calls for further examination, but overall point to an inefficient Bitcoin market. It is possible that due to the extreme volatility and returns at the beginning of Bitcoin's life the tests are not as effective as expected, causing unexpected results. There are no signs of a possible move to market efficiency, which is in contrast with the conclusion of Urquhart (2016). This contrast between conclusions together with the contradicting results of the Ljung-Box test (Ljung & Box, 1978) and the Breusch-Godfrey test (Breusch, 1978) (Godfrey, 1978) calls for more future research to be conducted on Bitcoin market efficiency.

Bitcoin's liquidity has been shown to have improved dramatically in a 10-year period. In the years 2013-2021 Bitcoin shows a statistically significant difference in Amihud Illiquidity Ratio (Amihud, 2002) compared to gold. The fact that this result was obtained in a market that is not efficient shows promise for the future of Bitcoin liquidity, since a possible move towards efficiency in the future caused by more trading volume can lead to further improvements in liquidity.

Finally, this thesis assessed if the incorporation of Bitcoin in an efficient portfolio influences portfolio performance. It is shown that incorporating Bitcoin in an optimised portfolio causes a better efficient frontier and thus improves the risk-return trade-off. Additionally, Bitcoin remains investible even under extreme levels of risk aversion with and without imposing restrictions on short selling, implying that even highly risk averse retail investors could benefit from including Bitcoin in their portfolio. Furthermore, Sharpe Ratios of portfolios including Bitcoin are shown to be significantly better at the 1% level under the assumption of moderate and extreme risk aversion.

It is important to note that, during the writing of this thesis, Bitcoin has received a lot of media coverage due to the sharp price decrease since April, dropping from over USD 60,000 to about USD 30,000 in the timespan of about three months. This shows that while research has been covering Bitcoin increasingly over the past years, outside factors still have a massive influence on Bitcoin's price, creating more doubts on the matters of market efficiency, stability and liquidity. Examples of this are the influence of large public figures on the Bitcoin market, a topic which has started to gain traction[1], as well as increasing regulations imposed by governing bodies and banks on cryptocurrencies in general[2]. Future research can incorporate these outside factors by assessing their influence on possible diversification, hedging and safe haven benefits, as well as their implications on market efficiency.

[1] Related research includes: (Ante, 2021) and (Hobbs, 2021)
[2] Related research includes: (Hughes, 2017), (Xie, 2019) and (Nabilou & Prüm, 2019)

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Appendix A. Comparison of Amihud Illiquidity Ratios between Bitcoin and conventional assets

Table 13: Two sample t-test between Bitcoin and S&P 500 illiquidity (2011-2021)

Table 13 shows the results of the two sample t-test between the Amihud Illiquidity Ratios of Bitcoin and the S&P 500 for sample period 2011-2021. The descriptive statistics for the two assets are displayed in the table, as well as the results of the one-tailed t-test with the corresponding t value, p value and degrees of freedom (DF).

Variables	*N*	*Mean*	*Std. Error*	*Std. Deviation*
Bitcoin	115	9,112	6,259	67,121
S&P 500	115	0.000	0.000	0.000
t-test for $\mu_{Bitcoin} - \mu_{S\&P500} < 0$	t = 1.46	p = 0.926	DF = 114	

Table 14: Two sample t-test between Bitcoin and S&P 400 illiquidity (2011-2021)

Table 14 shows the results of the two sample t-test between the Amihud Illiquidity Ratios of Bitcoin and the S&P 400 for sample period 2011-2021. The descriptive statistics for the two assets are displayed in the table, as well as the results of the one-tailed t-test with the corresponding t value, p value and degrees of freedom (DF).

Variables	*N*	*Mean*	*Std. Error*	*Std. Deviation*
Bitcoin	115	9,112	6,259	67,121
S&P 400	115	0.000	0.000	0.000
t-test for $\mu_{Bitcoin} - \mu_{S\&P400} < 0$	t = 1.46	p = 0.926	DF = 114	

Table 15: Two sample t-test between Bitcoin and S&P 600 illiquidity (2011-2021)

Table 15 shows the results of the two sample t-test between the Amihud Illiquidity Ratios of Bitcoin and the S&P 600 for sample period 2011-2021. The descriptive statistics for the two assets are displayed in the table, as well as the results of the one-tailed t-test with the corresponding t value, p value and degrees of freedom (DF).

Variables	*N*	*Mean*	*Std. Error*	*Std. Deviation*
Bitcoin	115	9,112	6,259	67,121
S&P 600	115	0.000	0.000	0.000
t-test for $\mu_{Bitcoin} - \mu_{S\&P600} < 0$	t = 1.46	p = 0.926	DF = 114	

Table 16: Two sample t-test between Bitcoin and Euro Stoxx 50 illiquidity (2011-2021)

Table 16 shows the results of the two sample t-test between the Amihud Illiquidity Ratios of Bitcoin and the Euro Stoxx 50 for sample period 2011-2021. The descriptive statistics for the two assets are displayed in the table, as well as the results of the one-tailed t-test with the corresponding t value, p value and degrees of freedom (DF).

Variables	*N*	*Mean*	*Std. Error*	*Std. Deviation*
Bitcoin	115	9,112	6,259	67,121
Euro Stoxx 50	115	0.000	0.000	0.000
t-test for $\mu_{Bitcoin} - \mu_{Stoxx} < 0$	t = 1.46	p = 0.926	DF = 114	

Table 17: Two sample t-test between Bitcoin and Nikkei 225 illiquidity (2011-2021)

Table 17 shows the results of the two sample t-test between the Amihud Illiquidity Ratios of Bitcoin and the Nikkei 225 for sample period 2011-2021. The descriptive statistics for the two assets are displayed in the table, as well as the results of the one-tailed t-test with the corresponding t value, p value and degrees of freedom (DF).

Variables	*N*	*Mean*	*Std. Error*	*Std. Deviation*
Bitcoin	115	9,112	6,259	67,121
Nikkei 225	115	0.000	0.000	0.000
t-test for $\mu_{Bitcoin} - \mu_{Nikkei} < 0$	t = 1.46	p = 0.926	DF = 114	

Table 18: Two sample t-test between Bitcoin and Treasury bonds illiquidity (2011-2021)

Table 18 shows the results of the two sample t-test between the Amihud Illiquidity Ratios of Bitcoin and Treasury bonds for sample period 2011-2021. The descriptive statistics for the two assets are displayed in the table, as well as the results of the one-tailed t-test with the corresponding t value, p value and degrees of freedom (DF).

Variables	*N*	*Mean*	*Std. Error*	*Std. Deviation*
Bitcoin	115	9,112	6,259	67,121
T-bonds	115	0.000	0.000	0.000
t-test for $\mu_{Bitcoin} - \mu_{Tbonds} < 0$	t = 1.46	p = 0.926	DF = 114	

Table 19: Two sample t-test between Bitcoin and oil illiquidity (2011-2021)

Table 19 shows the results of the two sample t-test between the Amihud Illiquidity Ratios of Bitcoin and oil for sample period 2011-2021. The descriptive statistics for the two assets are displayed in the table, as well as the results of the one-tailed t-test with the corresponding t value, p value and degrees of freedom (DF).

Variables	*N*	*Mean*	*Std. Error*	*Std. Deviation*
Bitcoin	115	9,112	6,259	67,121
Oil	115	0.000	0.000	0.000
t-test for $\mu_{Bitcoin} - \mu_{Oil} < 0$	t = 1.46	p = 0.926	DF = 114	