Nash Tariffs: The US-China Trade War

R.J.J. van Tiggelen BSc

Supervisor: Prof. dr. Bosker, Maarten
Second Assessor: dr. Erbahar, Aksel

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ABSTRACT: Many scenario analyses using models by GTAP and CEPII have been performed to gauge the impact and implications of the US-China trade war. It is however frequently observed that both the US and China seem to lose in terms of welfare. So why did they engage in this trade war? Can the observed non-cooperative tariff setting be explained by looking at the strategic interaction between the US and China following the optimal tariff literature? This paper studies whether a comprehensive Applied General Equilibrium model, combining the trade models as put forth by Bernard, Redding and Schott (2007) and Caliendo et al. (2017) containing many features of contemporary trade models, amongst which vertical linkages, can give rise to the observed tariffs during the US-China trade war when considering Nash tariffs. We adopt the approach outlined by Ossa (2016) to compute Nash tariffs and treat the problem of finding these Nash tariffs as a Minimisation Problem under Equilibrium Constraints (MPEC). This permits us to also consider heterogeneous Nash tariffs across several sectors. We calibrate the model using WIOD and UNCTAD-TRAINS data and find that the model implies tariff changes which are consistent with the tariff changes implemented during the first round by the US. Given the actual reaction of China, the model implies that US would have had no further incentive to change its tariffs after the first round, which could be indicative of other political economic motives affecting US tariff setting during the second round. Tariff setting by China deviates strongly from implied optimal tariff setting as it seems to have a unilateral incentive to decrease its tariffs (in particular negative optimal tariffs in the case of the agricultural sector in China) rather than increase its tariffs in accordance with retaliation under WTO guidelines. This finding emphasises the difficulties faced by the WTO trading system, as full WTO retaliation might be feasible but it is not necessarily optimal nor desirable for the country which was initially subject to tariff increases to retaliate in this manner.

Keywords: *Trade Policy, Optimal Non-Cooperative Tariffs, Nash Tariffs, Applied General Equilibrium Analysis, US-China Trade War*

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The views stated in this thesis are those of the author and not necessarily those of the supervisor, second assessor, the Erasmus School of Economics or the Erasmus University Rotterdam. The supporting material, id est MATLAB code as well as processed data, are available upon request.

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1 Introduction and Summary

Since the emergence of the US-China trade war, an increased focus has been placed on the bilateral breakdown of trade relations between the US and China. As a result, this has generated a vast literature on the ramifications of such a trade war. Several authors, amongst whom Kutlina-Dimitrova and Lakatos (2017) and Bollen and Rojas-Romagosa (2018), have specifically emphasised the effects on welfare and trade stemming from the increase in tariffs across a wide variety of commodities by relying on (dynamic) Applied General Equilibrium (AGE) models developed by GTAP or CEPII. However, a substantial number these scenario analyses indicate that both the US and China lose in terms of welfare from such a trade dispute. This raises the question of how and whether modelling strategic interaction using similar models could give rise to the observed non-cooperative tariffs which have been imposed by the US and China during the US-China trade war. Only a couple of papers have tried to assess what tariffs would arise as a result of the strategic interaction between the US and China, most notably by Balistreri and Hillberry (2017) and Bouët and Laborde (2018). They build upon the traditional optimal non-cooperative tariff literature by assuming that both governments set their tariffs as to maximise their citizen's welfare while playing their best responses to the other's tariffs.

In order to compute these Nash tariffs for the US-China trade war, Balistreri and Hillberry (2017) and Bouët and Laborde (2018) compute the welfare changes, as implied by the underlying AGE model from either GTAP or CEPII, from various tariff change combinations. These welfare changes are then subsequently represented in the normal form corresponding to the tariff setting game to determine the Nash equilibrium tariff changes. To be able to represent it in normal form and retain a parsimonious and tractable analysis, however, they convert the continuous choice space of tariff changes into a discrete choice space (thus restricting the strategy space) and limit the analysis to a *uniform* tariff change across all sectors. Nevertheless, imposing homogeneous tariff changes across sectors is not inconsequential when assessing to what extent the observed tariff changes could have arisen through strategic interaction, especially when considering tariffs on the sectoral level. Real world tariff data during the US-China trade war indicates that there is considerable heterogeneity in tariff changes across sectors (see Li, 2018, Li, Balistreri and Zhang, 2020, or alternatively section 2.3), implying that heterogeneity in tariff changes across different sectors is at the heart of the strategic interaction between the US and China. In addition, it is unclear to what extent imposing homogeneous tariffs (in comprehensive AGE models) affects the magnitude of the implied Nash tariffs.

Building upon the work of Su and Judd (2012), Ossa (2016) proposes to treat the determination of Nash tariffs as a Minimisation Problem under Equilibrium Constraints (MPEC), which does not require homogeneous tariffs nor a discrete choice space to retain tractability. The governments are assumed to maximise the welfare of their citizens by imposing tariffs,[1] whilst satisfying the equilibrium conditions corresponding to the underlying AGE model. The governments then set their optimal non-cooperative tariffs in alternating fashion (first the US then China) which repeats until neither government would want to change its tariffs, thus achieving their Nash tariffs. This paper aims to complement the literature by implementing this method, outlined in Ossa (2016), to assess whether the strategic interaction between the US and China could explain the observed tariff changes during the US-China trade war. As emphasised earlier, this permits heterogeneous Nash tariff changes on the sectoral level (in this study limited to the following sectors: agriculture, mining, manufacturing and services) and is thus able to incorporate the heterogeneous tariff changes in the strategic interaction between the US and China. The underlying AGE model used to describe the implied welfare changes is, to maintain comparability with previous work on Nash tariffs for the US-China trade war as well as for the purpose of realism, similar to the AGE models used by Balistreri and Hillberry (2017) and Bouët and Laborde (2018).

[1] Please note that this is equivalent to minimising the *negative* of welfare subject to the general equilibrium conditions imposed by the underlying AGE model. In other words, the constrained maximisation problem can be solved using methods from constrained convex optimisation by taking the negative of the objective function. Hence, the methodology is called MPEC, where the letter M stands for minimisation rather than maximisation.

In particular, the AGE model used to study whether the strategic interaction between the US and China could rationalise the observed tariff changes features vertical linkages between sectors as well as multiple primary factors of production (heterogeneous resource endowments) and Melitz (2003) style competition similar to Balistreri and Hillberry (2017) and Bouët and Laborde (2018).[2] The specific features of the underlying AGE model are important for the implied Nash tariffs (and optimal trade policy in general) as these affect the trade-offs which a government faces when increasing or decreasing its tariffs. The argument in favour of non-zero Nash tariff changes stems from exploiting the terms of trade of a country (increasing exporting prices relative to importing prices) whilst balancing these with the distortions which a tariff creates. The magnitude of both effects, however, depend on the structure of the economy. In particular, several authors have emphasised accounting for several of these features, most notably Caliendo and Parro (2015) for including vertical linkages when devising optimal trade policy and Balistreri and Markusen (2009) for using appropriate market structures when computing Nash tariffs.[3] To that end, this paper merges the international trade models as proposed by Bernard, Redding and Schott (2007) and Caliendo et al. (2017), which accommodates vertical linkages between sectors as well as multiple primary factors of production (heterogeneous resource endowments) and Melitz (2003) style competition. The resulting model is referred to as the BRS-CFRT model.

After calibrating the BRS-CFRT model on the latest available IO table of the World Input-Ouput Database (WIOD) and the corresponding tariffs from the UNCTAD-TRAINS database, we can then derive the implied heterogeneous tariff changes for the US-China trade war. When comparing these with the actual increases in tariffs documented by Li (2018) we find that the first response (first three rounds) of the US seems to closely align with the implied tariffs by the BRS-CFRT model across all four sectors (agriculture, mining, manufacturing and services). The implied best response of China to the initial US tariffs however diverges from the tariffs which were actually observed, implying negative tariffs on most goods and services except for manufacturing. This would have several implications for WTO retaliation rules as it would indicate that full WTO retaliation is feasible but not desirable and as such does not guarantee stability in the trading system when parties are left to fight bilateral trade wars. The subsequent round mirrors to a large extent the first round of implied non-cooperative tariffs and thereafter neither government can favourably change its tariffs to attain a higher level of welfare. Given the large observed deviation from implied best response tariffs by China, we also computed the optimal response of the US to see what the optimal response would have been to the actual Chinese tariffs. It appears that the US had no further incentive to retaliate thereafter. To rationalise the second round of tariffs, one would probably have to account for political economic motives.

This paper is organised as follows. The first part of section 2 more formally defines what is meant by Nash tariffs and discusses what trade-offs are faced by governments when setting optimal non-cooperative tariffs as well as why these are non-zero. The implications of several features which have been included in the BRS-CFRT model on Nash tariffs will be highlighted in this section. The subsequent section introduces the BRS-CFRT model in levels, closely following Caliendo et al. (2017), and characterises the general equilibrium in the same manner as Ossa (2014) and Ossa (2016). In section 4, the model is reformulated from a model in levels, a CGE model, to an AGE model which can be calibrated on real world data using hat algebra by Dekle, Eaton and Kortum (2007) (also known more commonly as calibrated share form). Section 5 discusses how the Nash tariffs have been computed using the MPEC methodology outlined by Ossa (2016). This section is concluded by a discussion of how the BRS-CFRT model has been calibrated and which data has been used for this purpose. The subsequent section discusses the results and the conclusion provides a summary of the results as well as implications thereof, limitations and potential directions for future research.

[2]Please note that the market structure is not identical to those used by Balistreri and Hillberry (2017) and Bouët and Laborde (2018). In particular, the former used a Melitz-style market structure for manufacturing and business services (and an Armington market structure for the other sectors) for whilst the latter used a perfectly competitive market structure.

[3]A more elaborate discussion on the importance of several prominent features of the AGE model which has been used in this paper, the BRS-CFRT model, can be found in section 2.2.

2 Nash Tariffs and the US-China Trade War

In order to study optimal non-cooperative tariffs, in this specific case Nash tariffs, we first provide the readership with a definition of Nash tariffs. We subsequently try to build an understanding of how optimal non-cooperative tariff setting works by reviewing the optimal-non-cooperative tariff literature. Particular emphasis is given to how different modelling assumptions affect optimal non-cooperative tariffs, thereby providing a rationale for the AGE model used to study Nash tariffs in this paper: the BRS-CFRT model. In order to provide the readership with an idea of how tariffs have actually developed during the US-China trade war, we assess what non-cooperative tariffs have actually been observed during the US-China trade dispute, drawing from the work of Li (2018) and Li, Balistreri and Zhang (2020). The level of aggregation in the discussion of these non-cooperative tariffs matches the level of aggregation used in the results section. It is important to note that tariffs have been aggregated to the desired level of aggregation by computing trade-weighted average tariffs. These trade weighted average tariffs are used to compare and contrast implied and observed tariff changes in the results section. The last part of this section briefly discusses the uniform Nash tariff changes which have been documented by earlier work, such as Balistreri and Hillberry (2017), Bouët and Laborde (2018) and Li, He and Lin (2018), to which we will relate during the discussion of our findings.

2.1 Defining Nash Tariffs

In order to define optimal non-cooperative tariffs, and in particular Nash tariffs, we have to define what non-cooperative tariffs are and what is meant by *optimal* tariffs. Non-cooperative tariffs are tariffs which are imposed by a country during a strategic interaction with another country but without coordinating on said tariffs. In accordance with Ossa (2016), a set of non-cooperative tariffs which is imposed by a government is considered optimal if it maximises the objective function of said government. The objective function of the government could feature various motives, amongst which optimising the real income (or welfare) of its citizens as well as other political economic motives (Ossa, 2014). In this paper we abstract from including political economic motives and restrict the scope to governments which have as objective to maximise the welfare of its citizens. In the model used in this paper, this is equivalent to maximising real income. In other words, the governments of the US and China have been assumed to act benevolently towards their citizens. When modelling the trade war in the remainder of the paper, we assume that the US plays its best response to the current state of affairs and that China responds by playing its best response to said US tariff change. This subsequently invokes a new best response from the US. The US and China keep interacting until revising their tariffs would no longer benefit either country. At that point, both countries have played their best response to each-other's strategy and have thus reached a Nash equilibrium with its corresponding Nash tariffs.

However, why would the Nash equilibrium concept make sense to model the interaction between these agents? The first explanation which is consistent with this way of modelling the interaction between China and the US could be that the agents are considered myopic and thus take the strategy of the other player as given. In other words, they do not anticipate the reaction of the opponent. This might seem implausible as governments probably do consider the reaction which a potential tariff increase might elicit from one's opponent. The second explanation, which we prefer, could be that the US resorts to the folk theorems. The US is then assumed to try to achieve a Nash equilibrium which is sub-optimal for China (and potentially harmful to the US as well) in order to support a more favourable outcome for the US when negotiating. A logical reason why the US might then have initiated the trade war would be because the perceived bargaining weights changed in its favour, whilst China was content with the status quo. It is important to note that strategic interaction between countries might however not be solely responsible for shaping the tariff structure across sectors. Erbahar and Zi (2017) for instance find evidence that downstream firms actively petition protection when upstream firms receive protection, which would imply that internal rather than external bargaining shapes trade policy.

2.2 The Case for Non-Zero Nash Tariffs

The earliest use of a CGE approach in optimal non-cooperative tariff setting can be traced back to the seminal contribution of Johnson (1953), in which he addressed the circumstances under which a country - in a 2×2 exchange model, id est two countries and two commodities - can benefit by deviating to optimal non-cooperative tariffs from a free trade equilibrium, even if the opposing country retaliates. This notion stemmed from the classical Bickerdicke proposition and Kaldor's (1940) assessment that, contingent on the magnitude of the elasticity of import demand, the improvement in the terms of trade (ToT) could make a (sizeable) country better-off. Whilst the general equilibrium quantities which were consumed and traded could readily be computed, the combinations of the elasticities of import demand under which applying non-cooperative tariffs would be beneficial could not be derived analytically. As a result, Johnson (1953) implemented a CGE approach to find that if the ratio of import demand elasticities is sufficiently high for country I, then country I would stand to benefit from imposing its optimal non-cooperative tariff, even in the presence of retaliation.[4]

Subsequent extensions to multi-country, multi-industry models by Kuga (1973) and generalisations to different indifference maps by Gorman (1958) of these exchange models also drew on such numerical computations. Whilst instrumental in providing insights on optimal non-cooperative tariff setting in exchange economies, it was still distant to actual calibration of models based on actual trade data (i.e. it were CGEs, not AGEs). Hamilton and Whalley (1983) were amongst the first to calibrate Johnson's CGE model with two countries, extended with production and CES preferences, to real world data, drawing from trade elasticity data gathered by Stern, Francis and Schumacher (1976). This early AGE model by Hamilton and Whalley (1983) implies that trade wars might severely raise tariffs around the globe, with optimal non-cooperative (or Nash) tariffs for the US being 300% higher than those which where actually observed at the time. Perroni and Whalley (2000) utilise an Armington trade model with LES preferences and an increased number of countries to build a better understanding of optimal non-cooperative tariffs to rationalise why small countries make more concessions in trade agreements. The authors find that the optimal non-cooperative tariffs are again higher than those observed, varying between the 120% up to 500% depending on the elasticity of import demand.

These tariffs are relatively sizeable especially when compared to the tariffs which arose after the Smoot-Hawley tariffs during the Great Depression, raising the average tariffs by 17.4% over the first year (Irwin, 1996) despite being one of the most severe breakdowns in cooperative tariff setting to date.[5] Balistreri and Markusen (2009) argue that the reason for these high estimates of optimal non-cooperative tariffs in Armington models, with zero profit conditions enforced, can be traced back to the implicit market power which has been granted to perfectly competitive firms in a standard Armington model. Whilst the firms do not exert that market power, a social planner (or in this case the government) would exert said market power to increase welfare, which leads to higher relative prices of domestic goods to foreign goods through the implementation of tariffs. This emphasises the need to properly account for market structure in which firms internalise their market power. Examples of market structures which properly accommodate market power are Krugman-style or Melitz-style trade models (Melitz, 2003; Chaney, 2008), where the latter can also rationalise the observed productivity heterogeneity and self selection amongst exporting and non-exporting firms (i.e. large firms typically tend to export rather than small firms). As a result, the model used in this paper, the BRS-CFRT model, adopts a Melitz-style market structure for intermediate good producers across all sectors.

[4] If the elasticity of demand in country I is higher (to a sufficient degree) than that in country II, country I can more effectively change its ToT in its favour whilst country II will have little effect on the ToT. As such country I would stand to benefit from setting a non-cooperative tariffs. However, it Kaldor (1940) noted that such large asymmetries were "unusual".

[5] One would have expected these tariffs to come closer to actual optimal non-cooperative tariffs if the model accurately captured the economy and the government's objective function. For a sceptic of the AGE literature, this would point at two potential flaws: (1) the underlying model did not capture the economy sufficiently or (2) the government's objective function was wrongly specified (or both). Note that this particularly emphasises the need to use models which are more realistic and capture many aspects of international trade, similar to the BRS-CFRT model developed in section 3.

Demidova and Rodriguez-Clare (2009) and Felbermayr et al (2013) decompose the effects which drive optimal non-cooperative tariffs in the setting of the standard (one sector) Melitz (2003) model of international trade with free entry and heterogeneous productivity parameters across firms. Both papers distinguish three effects which effect the optimal non-cooperative tariff, namely: (1) the standard terms of trade effect; (2) the *mark-up distortion* effect; and (3) the *entry-distortion* effect. The mark-up distortion effect pertains to the fact that a benevolent social planner which maximizes the welfare of its people - and has no internal tax instruments available - will mitigate the distortion which market power has generated by imposing a tariff on foreign commodities of which the opportunity cost relative to that of the domestic varieties is too small.[6] In addition to this *second-best* motive for implementing non-zero non-cooperative tariffs, Felbermayr et al (2013) also note that the implementation of the tariffs distorts the consumer surplus which consumers derived from the foreign varieties, which is reduced due to tariff measures (only the more productive firms export if tariffs are raised). As such, a social planner would ideally subsidize imports which drives down the optimal tariffs which is referred to as the entry distortion effect. It is important to note that Felbermayr et al. (2013) remark that in their symmetric equilibrium, the Melitz-type selection channel of firms serves to reduce the optimal non-cooperative tariffs vis-a-vis the model without Melitz style selection channels.[7]

In addition, more contemporary models also feature vertical linkages when studying optimal trade policy. These comprehensive models (including trade in intermediaries and vertical linkages) do not easily lend themselves to a formal analytical exposition, but welfare gain computations from trade in these trade models have been conducted by for instance Costinot and Rodriguez-Clare (2014). Their computations reveal that the welfare gains for models with trade in intermediates and monopolistic competition have more sizeable gains from trade, on average an increase in welfare of 40.0%, compared to models without trade in intermediaries but with multiple sectors (on average 14%). In comparison to the simple one-sector Armington model with CES preferences, which is similar - but not equivalent - to the model used by Perroni and Whalley (2000), countries increase their welfare by only 4.4% on average. In addition, as the gains derived from trade are considerably more sizeable in the more realistic models with vertical linkages and trade in intermediates, it stands to reason that a trade war would also be more costly in those models compared to standard Armington models of trade and that optimal non-cooperative tariffs would be lower. Caliendo and Parro (2015) corroborate the notion that accounting for IO linkages is important when assessing the gains from tariff changes as these play an important role in their multi-sector, perfect competition model with input-output linkages and trade in intermediaries which is based on Eaton and Kortum (2002).

The analytical derivation of the welfare gains (or real income increases) from trade, which can be derived in the Ricardian-style models, by Caliendo and Parro (2015) reveals two predominant factors which contribute to the welfare gains in these type of trade models: (1) the standard terms of trade effect and (2) the *volume of trade* (VoT) effect (this trade-off features in nearly all models in international trade). Whilst these two effects are not remarkable en sich, the composition of these effects is interesting. The inclusion of vertical linkages and trade in intermediates gives rise to a weighted average of ToT effects across sectors which are not necessarily positive for all sectors, even if a country is sizeable. To illustrate this fact, an increase in tariffs on the input of a particular sector might turn the sector from a net exporter into a net importer and, assuming that the change in relative

[6] In other words, the prices charged by domestic firms are above their marginal costs and as such are "too expensive" relative too their foreign counterparts from a socially optimal perspective.

[7] Such a model would more closely resemble a typical Armington model with CES preferences as in this case the distribution would have attained maximal dispersion where a few (highly productive) firms produce the goods (if it were exactly one firm it would be an Armington model augmented by a productivity parameter). This follows from the fact that the Melitz-type selection channel is muted as the shape parameter of the pareto distribution tends to the elasticity of substitution minus unity (see Burstein and Vogel, 2011). Balistreri, Bohringer and Rutherford (2018) note that Melitz-type models rightfully allocate the market power which is inherent to the CES demand structure to firms (based on the critique on Armington models by Balistreri and Markusen (2009)) and thus serves to decrease the optimal non-cooperative tariffs set by a governments. This would indeed rationalise the finding by Felbermayr et al (2013).

import prices and export prices is sufficiently small, this would serve to negatively contribute to a countries ToT effect when vertical linkages exist of imposing a tariff on the ”input” sector. Naturally, if the country is sufficiently sizeable and the increase in tariffs changes the relative prices of the input sector in its favour (and the input sector is not a sizeable net importer), this serves to increase the ToT. Theoretically, it is not ex ante clear which effect dominates which. The VoT contributes positively to welfare as long as the *real* import volume increases as a result of the increase or decrease in the tariffs as discussed by Caliendo and Parro (2015).[8] Given the importance of vertical linkages when devising trade policy, the model used to study Nash tariff setting in this paper also incorporates IO linkages.

When augmenting said model by Melitz-style monopolistic competition whilst retaining the vertical linkages and the trade in intermediaries, similar to the model used in this paper, Caliendo et al. (2017) notes that this would also introduce the second best motives for imposing tariffs as encountered in the standard Melitz model, i.e. the mark-up and entry distortion effects (see: Demidova and Rodriguez-Clare (2009) and Felbermayr et al. (2013)). It is important to note that these effects might be more pronounced in models featuring vertical linkages as the model thus features double marginalisation due to the linkages between monopolistically competitive sectors (Caliendo et al., 2017). Few papers include multiple primary factors of production as well, all of which to our knowledge rely on the GTAP or CEPII models (see: Balistreri and Hillberry, 2017, Bouët and Laborde, 2018). We also include multiple factors of production, as introduced in Melitz-style models by Bernard, Redding and Schott (2007), which increases comparability with previous papers which study Nash tariffs in the US-China trade war context. In addition it might also affect trade policy through two different channels. Firstly, the US and China are different in their resource endowments (China has more labour relative to capital) and as a result changes in tariffs are likely to have heterogeneous effects on the income received from different primary factors of production, which might affect the effective country size (which affects the ToT) to a different degree than if the primary factors of production were to be considered homogeneous. Secondly, the primary factors of production are used in differing intensities across sectors and as a result a country might want to capitalise on this heterogeneity when setting its tariffs (changing imports will affect the marginal product of capital and labour as well given the fact that labour and capital are both used in production in addition to other composite goods).[9]

2.3 The US-China Trade War

We now turn to the documentation regarding the impact of the US-Sino trade dispute with respect to the implemented tariffs and the effects on international trade. For that purpose we draw upon the trade war tariff database by Li (2018), which contains at the GTAP product level all bilateral tariffs imposed by either China or the US during the trade war. Although the trade war is typically modelled as a one shot game, this is not consistent with the way in which the trade war unfolded. In specific, the first set of bilateral tariffs were imposed by the US on steel and aluminium from China on the 23rd of March, 2018. China responded to this increase in US tariffs on its exports by imposing tariffs of its own on the 2nd of April 2018. In total, these tariffs effected about 3 billion USD worth of US exports and were primarily aimed at agricultural and manufacturing products. Despite the fact that this effectively concluded the first round of bilateral tariffs, further increases followed on the 6th of July that same year. Again the US chose to impose tariffs mainly on the manufacturing industry. In contrast with the first round, the tariffs imposed in the second round were substantially bigger and covered about 50 billion USD worth of Chinese exports. Instead of targeting aliminium and steel exports, the tariff increases imposed on exports from China to the US were now imposed on machinery (12%), transport

[8] Note that real import volume refers to the value of imports which have been corrected for changing importer prices as in Caliendo and Parro (2015) to obtain import volume/quantity.

[9] It is also important to note that there have also been alterior efforts to study the impact of global value chains on optimal non-cooperative tariff setting with different utility functions or political economic motives which capture different effects on optimal non-cooperative tariffs beyond those which have been mentioned (see for instance: Blanchard, Bown and Johnson, 2015; Ossa, 2016).

equipment (8%) and motor vehicles (2%). China reacted to the newly imposed tariffs the same day by raising tariffs on 50 billion worth of US exports to China. Like the US, China shifted its focus as well and retaliated by levying tariffs on primarily agricultural products such as wheat (24%), paddy rice (25%) and plant based fibers (25%). This retaliation spurred yet another round of tariff increases on an additional 50 billion worth of trade flows between the two countries. In specific, the US increased its tariffs again on imported manufactured goods from China early August 2018, raising tariffs further on importing machinery (4%), transport equipment (5%) and petroleum and coal products (19%). In reaction thereto, China further diversified the type of commodities subject to tariffs. Moreover, tariffs were now levied on services (the distribution and transportation of gas, 25%), mining (coal, 25%) and manufactured goods (12% on motor vehicles and related parts, and 16% on non-ferrous metals).

As of the third round, the imposed tariffs covered nearly 10% of all US imports from China. Nevertheless, a further escalation of the trade dispute between China and the US occurred at the 24th of September, 2018. The fourth round of tariffs was, however, markedly different from any of the three rounds that preceded it, as the amount of trade subject to said tariffs expanded much more rapidly. In specific, the US decided to increase tariffs on 200 billion USD worth of imports from China. Product categories that had previously remained unaffected such as agricultural commodities, mined minerals and services (gass distribution and transportation) now experienced an increase weighted by trade flow of circa 8.3%, 9.9% and 2.3% respectively. In addition, tariffs on manufactured goods were further increased by 3.6% when weighting the tariff increases by trade flow. Although, China had previously matched the US in terms of the amount of trade that was subject to additional tariffs, it now retaliated on only 60 billion USD worth of imports from the US on the 24th of September, 2018. A second increase on 60 billion USD worth of imports from the US followed medio May, 2019. During those two increases tariffs increased little on agricultural goods (1.2%) and services (0.0%), while tariffs increased more strongly on mining (16.4%) and manufacturing (9.7%) on exports from the US to China. Despite efforts to prevent an additional rise in tariffs, higher tariffs were enacted on a larger variety of products at the 1st of September, 2019. Although this new set of extra tariffs effected 300 billion worth of Chinese exports to the US, the increases were across all commodities smaller.

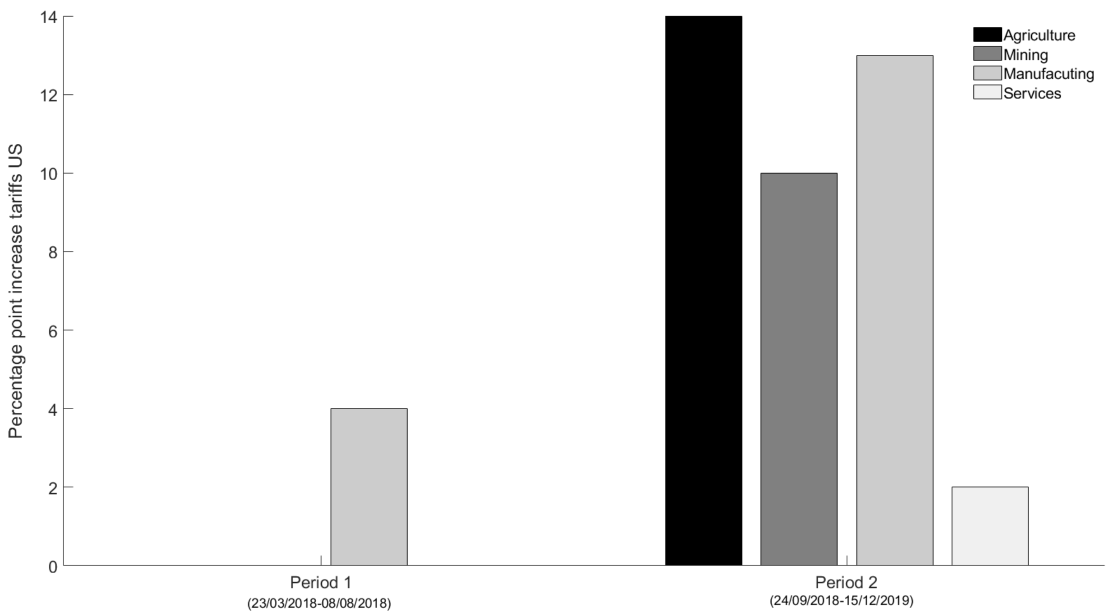

Figure 1: *Actual US tariffs during the first and second period of the US-CHN trade war. Changes in tariffs have been sourced from Li (2018) at the GTAP product level and have been weighted by GTAP product level trade flows from the UN COMTRADE to derive aggregate tariff change in a given sector.*

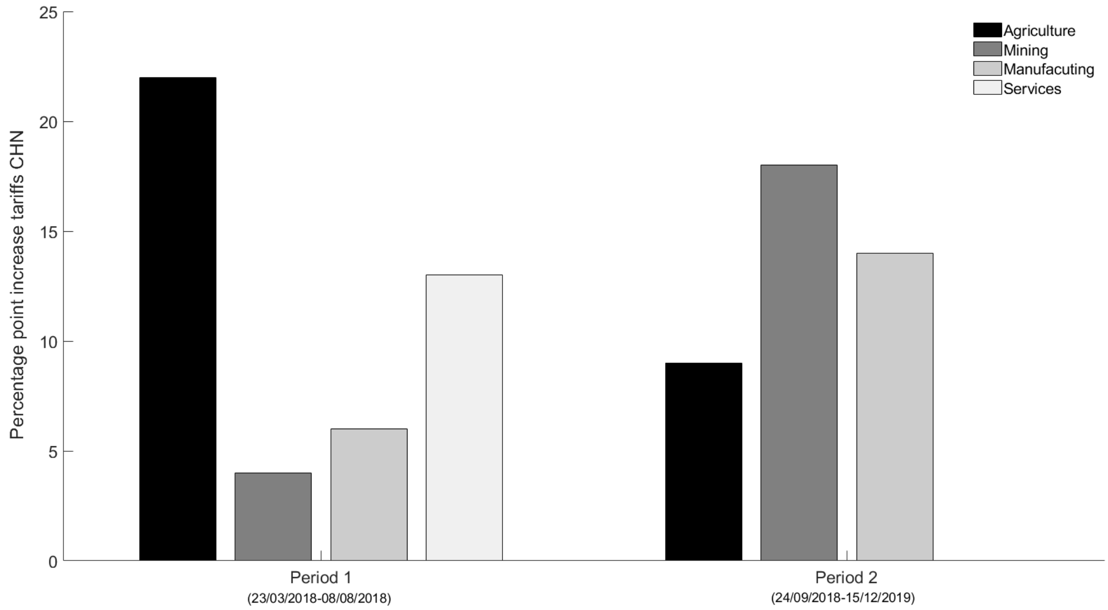

Figure 2: *Actual CHN tariffs during the first and second period of the US-CHN trade war. Changes in tariffs have been sourced from Li (2018) at the GTAP product level and have been weighted by GTAP product level trade flows from the UN COMTRADE to derive aggregate tariff change in a given sector.*

Specifically, the weighted tariff increases were relatively small as compared to the fourth round and amounted to an increase of 3.6% on agricultural goods, of 0.1% on mined minerals, of 3.0% on manufactured goods and of 0.0% on services. China's retaliation was implemented the same day, resulting in an increase in tariffs of about 6.5% on agricultural imports from the US. Furthermore, mining and manufactured goods imports from the US were also effected as tariffs increased thereon by 1.5% and 0.8%, respectively. Finally, the US threatened with a sixth round which would increase tariffs again on 300 billion worth of trade flows if no deal could be reached by the 15th of December 2019. Had this sixth round been enacted, tariffs would have further increased on agricultural goods by 1.6%) and on manufactured goods by 6.3%. In turn, China also threatened the US with additional tariffs on agricultural imports (1.7%), mined mineral imports (0.2%) and manufactured goods imports (3.2%). This last round was ultimately avoided as China agreed to a purchase list encompassed in the phase 1 deal. In line with the calibration and model presented in subsequent sections we divide the trade war into two periods: period 1 (round 1 till 3) and period 2 (round 4 till 6). The trade flow weighted increases in tariffs during both periods are depicted in Figure 1 for the US and Figure 2 for China. Furthermore, the tariff increases have weighted with trade flows from UN COMTRADE as to form sector tariff increases which is at the same sector level as the analysis in subsequent sections.

2.4 Recent Work on Nash Tariffs for the US-China Trade War

Contemporary analysis of the US trade dispute using AGEs is predominantly impacted by the models developed by the GTAP and CEPII communities which uses comprehensive AGE models to derive the optimal *uniform* import tariffs. Balistreri and Hillberry (2017) study the potential for a trade war between China and the US using the GTAP-in-GAMS model (the standard version as presented by Lanz and Rutherford (2016)), and find that optimal non-cooperative tariffs would lead the US to raise its tariffs by 11 percentage points on Chinese imports whilst China would only increase its tariffs by -5 percentage points on US imports. These findings are in line with the fact that China would have a unilateral incentive to lower tariffs as discussed by Balistreri and Rutherford (2013) and the authors speculate that the low optimal tariffs (in part) stem from the Melitz-type structure in comparison to the standard Armington models. However, it is important to note that this might be an artifact of allowing only one homogeneous tariff across sectors. An important additional point made by Balistreri and Hillberry (2017) is that retaliation according to WTO rules is only in line with the self interest of the country which was harmed due the implementation of non-cooperative tariffs by an opponent as long as the tariffs are not too sizeable. As such, bilateral retaliation might not be sufficient to enforce compliance of all countries in the trading system. Bouët and Laborde (2018) use the MIRAGRODEP model by CEPII to study optimal non-cooperative tariffs and report *uniform* increases in Nash tariffs of around the 7 percentage points for the US and 3 percentage points for China. In a similar fashion Li, He and Lin (2018) utilise an endogenous trade imbalance GE model (with Armington demand structure and zero profit conditions enforced) and determine with the aid of simulation the optimal non-cooperative, i.e. Nash, tariffs. They find that the increase towards Nash tariffs would see an increase in tariffs of around the 45 percentage points for the US and 0 percentage points for the China.

3 The BRS-CFRT Model in Standard Level Form

The model used to study optimal tariffs in this paper is a combination of two prominent trade models: the BRS and the CFRT model.[10] The general model discussed here features $|\mathcal{S}|$ sectors with each sector consisting out of two stages of production, intermediate and final good production. Intermediate good production in sector $s \in \mathcal{S}$ is organised in a similar fashion to production in NNT style models (see in particular: Melitz, 2003) and features monopolistic competition, heterogeneous firm productivity, scale economies and free entry and exit. In contrast to the typical NNT models, intermediate good production in this model is characterised by the fact that these intermediate good producers also use the final goods produced by other sectors to produce their intermediates, giving rise to the IO linkages in the CFRT model. In addition, intermediate good producers in this model also use two factors of production, labour and capital (which is characteristic for the BRS model), instead of only labour. Moreover, intermediate goods are the only traded goods between countries subject to iceberg trade costs $\vec{\tau}$ and tariffs $\vec{t}$ (i.e. final goods are non-traded). Final good producers compile the continuum of intermediate goods into the final good of sector s through a two stage CES aggregator, differentiating between domestic and foreign intermediate goods. These producers do not use any factors of production and exclusively sell their products to a representative consumer[11] and intermediate good producers in the same country at its marginal costs (i.e. perfectly competitive).

3.1 The Representative Consumer

The representative consumer in country $i \in \mathcal{N}$ (where $\mathcal{N} = \{1, 2, \ldots, N\}$ represents the set of all countries in this model) maximises its utility U_i which it derives from the consumption of a bundle of final goods produced in i, whilst spending no more on the consumption of final goods than that its income $\mathcal{I}_i$ permits. Assuming U_i can - as stated before - be characterised as a Cobb-Douglas utility function, the consumer is faced with the following problem:

$$\max_{\{Q_{i,s}\}} \quad U_i = \prod_{s=1}^{S} \left(Q_{i,s}^{\alpha_{i,s}}\right) \quad \text{subject to} \quad \sum_{s=1}^{S} (Q_{i,s} P_{i,s}) - \mathcal{I}_i = 0 \quad ; \quad \sum_{s=1}^{S} \alpha_i = 1 \tag{1}$$

[10]The CFRT (2015) model only used labour as an additional factor of production and as a result, the emphasis of the CFRT model was not laid on the typical comparative advantage effects encountered in HO models due to heterogeneous resource allocations of labour and capital. Nonetheless, these heterogeneous allocations of labour and capital also tend to give rise to different optimal trade policies. Given the fact that China and the US have markedly different resource endowments, it is only natural to permit for these differences in the model used to study the Sino-US trade war. As a result, the model in this paper is thus a synthesis of the BRS and the CFRT model.

[11]Alteratively one could think of the representative consumer as consisting out of a continuum of agents which satisfy Gorman aggregation conditions and yield an aggregate Cobb-Douglass utility function after aggregation.

where: $Q_{i,s}$ denotes the quantity consumed from the final good produced by county i's sector s; $P_{i,s}$ denotes the price per unit of the final good of sector s in i; and α_i represents the share of income which a representative consumer in country i spends on the final good of sector s. In optimum, such a representative consumer has the following indirect utility function V_i and demand functions $Q_{i,s}$ for all sectors $s \in \mathcal{S}$ (where $\mathcal{S} = \{1, 2, \ldots, S\}$ represents the set of all sectors in this model) and $i \in \mathcal{N}$:

$$V_i = \mathcal{I}_i \cdot \prod_{s=1}^{S} \left(\frac{P_{i,s}}{\alpha_{i,s}} \right)^{-\alpha_{i,s}} \quad ; \quad [Y_{i,s}]_{con} \equiv [Q_{i,s} P_{i,s}]_{\text{con}} = \alpha_{i,s} \mathcal{I}_i \tag{2}$$

here we have added $[\ \cdot\ ]_{\text{con}}$ to emphasise the fact that these expenditures, represented by $Y_{i,s}$, refer to *consumer* demand to prevent confusion later on when we refer to the demand for these final goods from IO linkages. To be more precise with regard to the income received by a representative consumer, $\mathcal{I}_i$ consists out of: (1) the total wage bill, denoted by $w_i L_i$ with L_i representing the labour supplied in country i and w_i representing the wage rate in i; (2) the total expenses on capital in i, denoted by $r_i K_i$ with K_i representing the capital supplied in country i and r_i representing the rental rate in i; (3) the tariff revenue received by the government of i, denoted by T_i[12], which is redistributed to the representative agent; and (4) the net transfers (deficits on the trade balance) of other countries as these cannot be rationalised in a static model as presented here. The indirect utility function has been provided here as a welfare optimising government of i would seek to maximise the indirect utility function V_i. The derivation of equation (2) can be found in B.1.

3.2 Intermediate and Final Good Producers

As mentioned before, intermediate good production in sector s is organised in a similar fashion to production in Melitz-style models, but augmented with final goods, labour and capital as factors of production. This means that firms operate in a monopolistically competitive environment with their unique variety ω and corresponing productivity parameter φ. The latter is obtained after entering the market, similar to Melitz (2003) and Bernard, Redding and Schott (2007), and is drawn from a Pareto distribution characterised by the PDF $g_s(\varphi)$, with scale parameter θ_s and unity as its minimum value. Analoguous to CFRT (2017), profit maximising intermediate good producers choose the bundle of inputs which minimises the cost per unit $c_{i,s}(\varphi)$, given their productivity parameter. Assuming the production function for producing $q_{i,s}(\varphi)$ units of the intermediate good in country i's sector s, with a productivity parameter of φ, can be characterised by a Cobb Douglas production function, the constrained minimisation problem faced by a intermediate good producer is as follows:

$$\min_{\left\{Q_{i,s',s}(\varphi), L_{i,s}(\varphi), K_{i,s}(\varphi)\right\}} \quad c_{i,s}(\varphi) = \sum_{s'=1}^{S} P_{i,s'} Q_{i,s',s}(\varphi) + w_i L_{i,s}(\varphi) + r_i K_{i,s}(\varphi) \tag{3}$$

Subject to producing (at least) one unit of the intermediate good (i.e. $q_{i,s} \geq 1$, which binds as $c_{i,s}(\varphi)$ and $q_{i,s}(\varphi)$ are strictly increasing in its inputs) and the share parameter restrictions on $\beta_{i,s',s}$:

$$q_{i,s}(\varphi) = \varphi \prod_{s'=1}^{S} \left(Q_{i,s',s}(\varphi)^{\beta_{i,s',s}} \right) L_{i,s}(\varphi)^{\beta_{i,S+1,s}} K_{i,s}(\varphi)^{\beta_{i,S+2,s}} = 1 \quad \text{and} \quad \sum\nolimits_{s'=1}^{S+2} \beta_{i,s',s} = 1 \tag{4}$$

Here $Q_{i,s',s}(\varphi)$ is the quantity demanded by an intermediate good producer in country i's sector s, with φ as its productivity parameter, from the final goods produced by country i's sector s'. $L_{i,s}(\varphi)$ and $K_{i,s}(\varphi)$ denote the labour and capital used by a firm in sector s in country i with productivity φ, respectively. The share parameter $\beta_{i,s',s}$, for all $s' \in \mathcal{S}$, represents the share of the total variable costs,

[12]Letting $X_{ij,s}$ denote the trade flow net of tariffs from country i to j in sector s and letting $t_{ji,s}$ denote the tariff on trade flows in sector s from j to i, we can write that tariff income accruing to country i as $T_i = \sum_{s \in \mathcal{S}} \sum_{j \in \mathcal{I}} t_{ji,s} X_{ji,s}$.

including the expenditures on labour and capital, which are spent on the output of sector s'. Given the constant returns to scale Cobb-Douglas structure of the production function, the optimal per unit (or marginal) costs are constant and thus independent of total production $q_{i,s}(\varphi)$. As a result, we can write the constant marginal costs $c_{i,s}(\varphi)$ for the intermediate goods sector s in country i as:

$$c_{i,s}(\varphi) = \frac{c_{i,s}}{\varphi}; \quad c_{i,s} = \prod_{s'=1}^{S} \left(\frac{P_{i,s'}}{\beta_{i,s',s}}\right)^{\beta_{i,s',s}} \left(\frac{w_i}{\beta_{i,S+1,s}}\right)^{\beta_{i,S+1,s}} \left(\frac{r_i}{\beta_{i,S+2,s}}\right)^{\beta_{i,S+2,s}} \tag{5}$$

of which the derivation can be found in appendix B.2. Moreover, analogous to NNT style models, intermediate good producers in i incur some fixed operating costs $f_{ij,s}$ per destination country $j \in \mathcal{N}$ if they choose to serve j's market. Analogous to the BRS model, we will assume that these fixed costs require capital and labour and are produced using a constant returns to scale Cobb Douglas production function.[13] Similar to the constant cost per unit of output $q_{i,s}(\varphi)$, the cost per unit of $f_{ij,s}$ using a cost minimising bundel of inputs is also constant and is denoted in this paper by $c_{i,s,f}$. When exporting to j, an intermediate good producer in i's sector s also incurs iceberg trade costs, implying for $q_{ij,s}(\varphi)$ units to arrive from i to j, $\tau_{ij,s}q_{ij,s}(\varphi)$ units need to be shipped and thus produced at the production location in i. As a result, the total cost incurred of exporting to a country j by an intermediate good producer in i's sector s with φ is given by $C_{ij,s}(\varphi) = c_{i,s}(\varphi)\tau_{ij,s}q_{ij,s}(\varphi) + c_{i,s,f}f_{ij,s}$. Tariffs in this model are *demand shifters* as defined by Felbermayr, Jung and Larch (2015), and serve to decrease the revenues received from selling in j - which is realistic - instead of increasing $C_{ij,s}$. In particular, the total revenues received by an intermediate good producer in i's sector s selling in j with productivity φ is equal to $R_{ij,s}(\varphi) = p_{ij,s}(\varphi)q_{ij,s}(\varphi)/(1 + t_{ij,s})$.[14] As a result, we can write the operating profit received by an intermediate good producer in i's sector s, with productivity parameter φ, as:

$$\pi_{i,s}(\varphi) = \sum_{j=1}^{N} R_{ij,s}(\varphi) + \sum_{j=1}^{N} C_{ij,s}(\varphi) = \sum_{j=1}^{N} \left(\frac{p_{ij,s}(\varphi)q_{ij,s}(\varphi)}{1 + t_{ij,s}} - \frac{c_{i,s}}{\varphi}\tau_{ij,s}q_{ij,s}(\varphi) - c_{i,s,f}f_{ij,s} \right) \tag{6}$$

Given that both intermediate and final good producers are profit maximising economic agents, the producers of intermediates maximise $\pi_{i,s}(\varphi)$ with respect to $p_{ij,s}(\varphi)$. The demand for a particular variety is naturally contingent on the price charged by the producer of said variety. In order to thus maximise equation (6), we require a explicit expression for the quantity $q_{ij,s}(\varphi)$ sold to country j from sector s in country i of a variety ω. Final good producers in the same sector s, for all $j \in \mathcal{N}$, have been assumed to be the only economic agents that demand the intermediates produced by the intermediate good producers in sector s. As final good producers operate in a perfectly competitive market, the price per unit of $Q_{i,s}$ at which the representative consumers and other industries can buy final goods of sector s in i is equal to the marginal cost of producing a single unit of the final good. As mentioned before, final good producers produce according to a two stage CES production function. Firstly, they compose a domestic ($Q_{ii,s}$) and a foreign composite good ($Q^F_{i,s}$) through a CES production function over all domestic and imported varieties of the intermediate good in sector s, respectively, with an elasticity of substitution σ_s. Subsequently, final good producers in i's sector s produce the final product $Q_{i,s}$ from $Q_{ii,s}$ and $Q^F_{i,s}$ using a second CES production function with a elasticity of substitution ς_s. In both stages, costs are minimised as a final good producer operates in a perfectly competitive market. Solving the three constrained optimisation problems for final good producers then yields the following demand functions for domestically produced and imported varieties:

$$q_{ii,s}(\varphi) = \left(\frac{p_{ii,s}(\varphi)}{P_{ii,s}}\right)^{-\sigma_s} \left(\frac{P_{ii,s}}{P_{i,s}}\right)^{-\varsigma_s} Q_{i,s}; \quad q_{ji,s}(\varphi) = \left(\frac{p_{ji,s}(\varphi)}{P^F_{i,s}}\right)^{-\sigma_s} \left(\frac{P^F_{i,s}}{P_{i,s}}\right)^{-\varsigma_s} Q_{i,s} \tag{7}$$

[13]In particular, we assume that $f_{ij,s} = L_{i,s}^{\gamma_{i,s}} K_{i,s}^{1-\gamma_{i,s}}$. Please note that exporting to j thus requires additional capital and labour in country i and not in j. Minimising the cost per unit of $f_{ij,s}$, denoted by $c_{i,s,f}$, one finds that $c_{i,s,f} = (w_i/\gamma_{i,s})^{\gamma_{i,s}}(r_i/[1-\gamma_{i,s}])^{1-\gamma_{i,s}}$. Its derivation is analogous to that of equation (5) (see appendix B.2).

[14]Here $p_{ij,s}$ represents the price charged by an intermediate good producer in sector s to a final good producer in sector s in country j.

where we have that: $P_{ii,s}$ is the price per unit of the domestic composite good; $P^F_{i,s}$ is the price per unit of the foreign composite good; $P_{i,s}$ is the price of the final good in country i, produced by sector s; $q_{ji,s}$ represents the quantity received in i from a producer in j (*id est* excluding the units sent to j but lost due to iceberg trade costs); and $Q_{i,s}$ is the total quantity demanded from country i's sector s final good producers (including demand by consumers and other sectors). To keep this exposition parsimonious, we have omitted an in-depth discussion of the minimisation problems faced by the final good producers as well as a discussion of the price indexes. For completeness, a formal derivation and discussion has been provided in appendix B.3.[15] Now we have obtained the demand for a specific variety of the intermediate good produced by an intermediate good producer in i's sector s in all countries $j \in \mathcal{N}$ (note that i and j can be readily interchanged in equation (10.2)) analoguous to CFRT (2017), we derive optimal prices $p_{ij,s}(\varphi)$ per export destination j, the quantity demanded as a result thereof in j (*id est* $q_{ij,s}(\varphi)$) and the corresponding operating profits per exporting location j and i itself (*id est* $\pi_{ij,s}(\varphi)$). Maximising equation (5) subject to equation (10) and letting the per destination operating profit be denoted by $\pi_{ij,s}(\varphi) = R_{ij,s}(\varphi) - C_{ij,s}(\varphi)$ and $Y_{j,s} = Q_{j,s}P_{j,s}$, one could readily obtain the following set of optimal per destination price and quantity pairs $(p_{ij,s}(\varphi),\, q_{ij,s}(\varphi))$ for an individual intermediate good producer in i's sector s, exporting to j with a productivity parameter φ:

$$p_{ij,s}(\varphi) = \left(\frac{\sigma_s}{\sigma_s - 1}\right)\left(\frac{(1+t_{ij,s})\tau_{ij,s}c_{i,s}}{\varphi}\right) \quad ; \quad q_{ij,s}(\varphi) = (p_{ij,s})^{-\sigma_s}\left(P^F_{j,s}\right)^{\sigma_s - 1}\left(\frac{P^F_{j,s}}{P_{j,s}}\right)^{1-\varsigma_s} Y_{j,s} \tag{8}$$

The corresponding operating profits for every destination $j \in \mathcal{N}$ can now also be obtained by combining the price quantity combinations and substituting those into the expression for $\pi_{ij,s}$ which yields:

$$\pi_{ij,s}(\varphi) = \left(\frac{1}{\sigma_s - 1}\right)\left(\frac{(1+t_{ij,s})\tau_{ij,s}c_{i,s}}{\varphi}\right)\frac{q_{ij,s}(\varphi)}{(1+t_{ij,s})} - c_{i,s,f}f_{ij,s} \tag{9}$$

Naturally, intermediate good producers in country i and sector s choose to produce for the market of j as long as $\pi_{ij,s}(\varphi) \geq 0$ which causes firms to self select into foreign markets (which is standard in Melitz-style models). Following BRS (2007) and CFRT (2017), we can equate equation (9) to zero which then subsequently allows one to obtain an expression for the lower bound productivity parameter $\varphi^*_{ij,s}$. Below $\varphi^*_{ij,s}$, intermediate good producers in country i in sector s chose to set the quantity supplied to the final good producers in j equal to zero as $\pi_{ij,s}(\varphi) \leq 0$ for any $\varphi < \varphi^*_{ij,s}$. Completely analogous to CFRT (2017), equating equation (9) to zero and substituting the optimal price expression into the optimal quantity supplied expression (i.e. for $q_{ij,s}(\varphi)$) then yields the following domestic and foreign market zero profit cut-off conditions after some tedious rewriting (see appendix B.4):

$$\varphi^*_{ii,s} = \left(\frac{\sigma_s}{\sigma_s - 1}\right)\left(\frac{\sigma_s(1+t_{ii,s})c_{i,s,f}f_{ii,s}}{Y_{i,s}}\right)^{\frac{1}{\sigma_s - 1}}(P_{ii,s})^{-1}\left(\frac{P_{ii,s}}{P_{i,s}}\right)^{\frac{1-\varsigma_s}{1-\sigma_s}}\Big((1+t_{ii,s})\tau_{ii,s}c_{i,s}\Big) \tag{10}$$

$$\varphi^*_{ij,s} = \left(\frac{\sigma_s}{\sigma_s - 1}\right)\left(\frac{\sigma_s(1+t_{ij,s})c_{i,s,f}f_{ij,s}}{Y_{j,s}}\right)^{\frac{1}{\sigma_s - 1}}\left(P^F_{j,s}\right)^{-1}\left(\frac{P^F_{j,s}}{P_{j,s}}\right)^{\frac{1-\varsigma_s}{1-\sigma_s}}\Big((1+t_{ij,s})\tau_{ij,s}c_{i,s}\Big) \tag{11}$$

In addition to the ZCP conditions, intermediate good producers only decide to enter (free entry) into sector s in i if the expected profits obtained from all possible export destinations and the home market yields an *expected* profit in excess of the entry costs $c_{i,s,f}f_{i,s,e}$. Here it has been assumed that the fixed operating costs $f_{ij,s}$ and the fixed entry costs $f_{i,s,e}$ both require the same production technology such that the cost per unit of $f_{ij,s}$ and $f_{i,s,e}$ are identical this will simplify the factor market clearing

[15]The fact that these price indexes are not discussed here does *not* imply these are not important. In later sections these price indexes will feature prominently, but would only serve to clutter the exposition here. The derivation will be presumed common knowledge in later sections, but a derivation - albeit not step-by-step - has been provided in appendix B.3.

conditions in section 3.3. Using the fact that before entry an intermediate good producer's φ is not known, similar to Melitz (2003), and after entry is drawn from a probability distribution with PDF $g_s(\varphi)$, which is the PDF of the Pareto distribution with scale parameter θ_s and minimum φ equal to unity, the free entry condition yields that (using equations 8-11; see appendix B.5 for the derivation):

$$\sum_{j=1}^{N} \mathbb{E}\left[\pi_{ij,s}(\varphi)\right] = \left(\frac{\sigma_s - 1}{\theta_s - (\sigma_s - 1)}\right) \sum_{j=1}^{N} c_{i,s,f} f_{ij,s} \left(\varphi_{ij,s}^{*}\right)^{-\theta_s} = c_{i,s,f} f_{i,s,e} \tag{12}$$

where $\mathbb{E}$ is the expectations operator and where we used the fact that the PDF of a Pareto distribution with minimum value of 1 and scale parameter θ_s is given by $g_s(\varphi) = \theta_s \varphi^{-(\theta+1)}$.

3.3 Trade Shares, Average Productivity and Gravity

In order to save on some of the cumbersome expressions in general equilibrium and to advance variable reduction in section 3.4, it is convenient to define several auxilliary expressions. The first auxilliary expression concerns the trade shares $\lambda_{ji,s}$ for all $j \in \mathcal{N}$, following CFRT (2017), and the average productivity parameter $\tilde{\varphi}_{ij,s}$. The trade shares $\lambda_{ji,s}$ measure the relative value of imports in intermediates goods from j to i in sector s to the total value of output of sector s in country i (in terms of final goods). As such, it captures the extent to which intermediates from j, within sector s, are used in the production of final goods in i, within sector s. Let $M_{j,s}$ denote the mass of entrants in country j's sector s (for intermediate good producers) and let, as a natural result thereof, $M_{j,s} g_s(\varphi)$ denote the mass of firms with a productivity parameter φ. We note that only intermediate good producers with $\varphi \geq \varphi_{ji,s}^{*}$ export to i. As such, we have that the share of aggregate production in stemming from j in sector s in country i's sector s is equal to (denoted by $\lambda_{ji,s}$):

$$\lambda_{ji,s} \equiv \Bigg(\underbrace{\int_{\varphi_{\min}}^{\varphi_{ji,s}^{*}} 0 \cdot M_{j,s} g_s(\varphi) d\varphi}_{(1)} + \underbrace{\int_{\varphi_{ji,s}^{*}}^{\infty} p_{ji,s}(\varphi) q_{ji,s}(\varphi) M_{j,s} g_s(\varphi) d\varphi}_{(2)} \Bigg) / Y_{i,s} \tag{13}$$

where (1) denotes the total value of products exported to country i by producers with a productivity lower than the ZCP productivity $\varphi_{ji,s}^{*}$ (which is zero as those firms opt out) and where (2) denotes the total value of products exported to country i by producers with a productivity higher than the ZCP productivity $\varphi_{ji,s}^{*}$. In order to solve for $\lambda_{ji,s}$ explicitly, we plug in the optimal price and quantity combinations governed by equation (8) and find that:

$$\lambda_{ji,s} = {\varphi_{ji,s}^{*}}^{-\theta_s} M_{j,s} \left(\frac{\sigma_s}{\sigma_s - 1} \frac{\tau_{ji,s}(1 + t_{ji,s}) c_{j,s}}{P_{i,s}^{F}}\right)^{1-\sigma_s} \left(\frac{P_{i,s}^{F}}{P_{i,s}}\right)^{1-\varsigma_s} \int_{\varphi_{ji,s}^{*}}^{\infty} \varphi^{\sigma_s - 1} \frac{g_s(\varphi)}{{\varphi_{ji,s}^{*}}^{-\theta_s}} d\varphi \tag{14}$$

In a completely analogous fashion we also obtain the share of output in sector s in country i which is paid to domestic intermediate good producers:

$$\lambda_{ii,s} = {\varphi_{ii,s}^{*}}^{-\theta_s} M_{i,s} \left(\frac{\sigma_s}{\sigma_s - 1} \frac{\tau_{ii,s}(1 + t_{ii,s}) c_{i,s}}{P_{ii,s}}\right)^{1-\sigma_s} \left(\frac{P_{ii,s}}{P_{i,s}}\right)^{1-\varsigma_s} \int_{\varphi_{ii,s}^{*}}^{\infty} \varphi^{\sigma_s - 1} \frac{g_s(\varphi)}{{\varphi_{ii,s}^{*}}^{-\theta_s}} d\varphi \tag{15}$$

where for $j = i$ the price index $P_{i,s}^{F}$ is replaced by $P_{ii,s}$in the expression for $\lambda_{ji,s}$. In addition, it is important to note that the last term reflects the average productivity of intermediate good producers that export to the market i from j in sector s, raised to the power $(\sigma_s - 1)$. Note that the fraction $g_s(\varphi)/(\varphi_{ij,s}^{*})^{-\theta_s}$ represents the conditional probability of an intermediate good producer's productivity *given* that this good producer exports to country j from country i (in sector s). To simplify the exposition in later sections, we will adopt the common notation $\tilde{\varphi}^*_{ji,s}$ to denote this average productivity. As such, we have that the average productivity is:

$$\tilde{\varphi}^{\sigma_s-1}_{ji,s} \equiv \int_{\varphi^*_{ji,s}}^{\infty} \varphi^{\sigma_s-1}\frac{g_s(\varphi)}{\left(\varphi^*_{ji,s}\right)^{-\theta_s}}d\varphi = \left(\frac{\theta_s}{\theta_s-(\sigma_s-1)}\right)\left(\varphi^*_{ji,s}\right)^{\sigma_s-1} \quad \text{for} \quad \theta_s-(\sigma_s-1)>0 \quad (16)$$

Following Ossa (2014, 2016), we will also define $X_{ij,s}$ as the net of tariff trade flow in intermediates from country i to country j, within sector s. As a result, these trade flows $X_{ij.s}$ represent the value of exports which is received by producers in i as tariffs are levied over the value of exports and thus accrue to the government of j. To obtain said net of tariff trade flows, we thus essentially divide by $(1+t_{ij,s})$ (i.e. the tariffs applied to a trade flow going from country i to country j). Given the fact that final good producers utilise a two stage CES production function, we obtain a gravity equation. Please note that, whilst this equation does not seem like the typical gravity equation due to the lack of origin specific factors governing said trade flow, the term $\lambda_{ij,s}$ has absorbed those terms. As a result, this equation represents a typical gravity equation (see: Head & Mayer, 2014; Ossa, 2014; Ossa, 2016). The gravity equation in this model is given by:

$$X_{ij,s} \equiv \frac{\lambda_{ij,s}}{(1+t_{ij,s})}Y_{j,s} = \left(\varphi^*_{ij,s}\right)^{-\theta_s} M_{i,s}\left(\frac{\sigma_s}{\sigma_s-1}\frac{\tau_{ij,s}(1+t_{ij,s})c_{i,s}}{\tilde{\varphi}_{ij,s}P^F_{j,s}}\right)^{1-\sigma_s}\left(\frac{P^F_{j,s}}{P_{j,s}}\right)^{1-\varsigma_s}\frac{Y_{j,s}}{(1+t_{ij,s})} \quad (17)$$

3.4 Simplifying Price Indexes and Trade Shares

From the above exposition it is evident that if we were to operationalise this model "as is", it would contain a vast amount of equations for, amongst others, average and threshold productivities (denoted by $\tilde{\varphi}_{ij,s},\ \varphi^*_{ij,s}$) as well as domestic and foreign price indexes (denoted by $P_{ii,s},\ P^F_{i,s}$). In order to minimise the number of variables that define the general equilibrium in section 3.5 (and thus the number of equations governing the general equilibrium), we set out here to define the price indexes $P_{i,s}$ and the trade shares $\lambda_{ij,s}$ in terms of (other) final good price indexes, sectoral output, the mass of entrants, the wage and the rental rates and the set of parameters $\mathcal{P}$.[16] The first set of variables that will be removed from both the price indexes are the ZCP and average productivities by substituting (10), (11) and (16) into the expressions for $P_{ii,s}$ and $P^F_{i,s}$ which have been provided in appendix B.3. Subsequently, we substitute the expressions we have found for $P_{ii,s}$ and $P^F_{i,s}$ (after substituting in (10), (11) and (16)) into the price index of final goods in optimum. These steps yield that the price index $P_{i,s}$ is characterised by the following expression (see appendix B.7 for a complete derivation):

$$P_{i,s} = \left(\left(\Lambda_{ii,s}\right)^{\xi_s} + \left(\sum_{j\neq i}\Lambda_{ji,s}\right)^{\xi_s}\right)^{-\frac{1}{\xi_s\theta_s}} \quad \text{where} \quad \xi_s \equiv \frac{(\sigma_s-1)(1-\varsigma_s)}{(\sigma_s-1)(1-\varsigma_s)-\theta_s(\sigma_s-\varsigma_s)} \quad (18)$$

with $\Lambda_{ii,s}$ and $\Lambda_{ji,s}$ defined as follows:

$$\Lambda_{ii,s} \equiv \left(\frac{\sigma_s}{\sigma_s-1}(1+t_{ii,s})\tau_{ii,s}c_{i,s}\right)^{-\theta_s}B_{ii,s} \quad ; \quad \Lambda_{ji,s} \equiv \left(\frac{\sigma_s}{\sigma_s-1}(1+t_{ji,s})\tau_{ji,s}c_{j,s}\right)^{-\theta_s}B_{ji,s} \quad (19)$$

with $B_{ji,s}$ defined as:

$$B_{ji,s} \equiv \left(\left(\frac{\theta_s}{\theta_s-(\sigma_s-1)}\right)M_{j,s}\right)\left(\frac{\sigma_s(1+t_{ji,s})c_{j,s,f}f_{ji,s}}{Y_{i,s}}\right)^{\frac{\sigma_s-1-\theta_s}{\sigma_s-1}} \quad (20)$$

[16]To summarize, the parameter space is relatively extensive and instead of continuously writing out all parameters we use $\mathcal{P}$ to denote the parameter space. To be more precise, $\mathcal{P} = \{\alpha_{i,s}, \beta_{i,s',s}, \gamma_{i,s}, \sigma_s, \varsigma_s, \tau_{ij,s}, \theta_s, f_{ij,s}, f_{i,s,e}\}$ for all combinations of counties $(i,j)\in\mathcal{N}\times\mathcal{N}$ and all combinations of sectors $(s,s')\in\mathcal{S}\times\mathcal{S}$.

From (18)-(20) it is apparent that the price index $P_{i,s}$ is now written in terms of only other final good price indexes (through the term $c_{i,s}$ and $c_{j,s}$), the mass of entrants ($M_{i,s}$ and $M_{j,s}$), sectoral outputs ($Y_{i,s}$ and $Y_{j,s}$) and the wage and rental rates (through the terms $c_{i,s}$, $c_{j,s}$, $c_{i,s,f}$ and $c_{j,s,f}$). In a similar fashion, we can also substitute the ZCP conditions in (14) and (15) into the definition for trade shares in (17) to eliminate the average and ZCP productivities. Using the definitions of $P_{ii,s}$ and $P^F_{i,s}$ in terms of $P_{i,s}$, $\Lambda_{ii,s}$ and $\Lambda_{ji,s}$, we obtain the following expression for the trade shares $\lambda_{ij,s}$:

$$\lambda_{ij,s} = \Lambda_{ij,s}\left(\sum\nolimits_{i\neq j}^{N}\Lambda_{ij,s}\right)^{\xi_s-1}\left((\Lambda_{jj,s})^{\xi_s}+\left(\sum\nolimits_{i\neq j}^{N}\Lambda_{ij,s}\right)^{\xi_s}\right)^{-1} \tag{21}$$

of which the formal derivation has been provided in appendix B.8. Here it is evident that the trade shares are also only a function of other final good price indexes, the mass of entrants, sectoral outputs, the wage rates, the rental rates and the set of parameters $\mathcal{P}$. Using the fact that the trade shares and the price indexes can be defined as in (18) and (21), we can now characterise the general equilibrium in section 3.5 using only the $N \times S$ price indexes, the $N \times S$ sectoral outputs, the $N \times S$ masses of entrants in every country and sector, N wage rates and N rental rates. As a result, we will require $N \times (2 + 3S)$ conditions to define the general equilibrium which arises in the BRS-CFRT model.

3.5 Characterising the General Equilibrium

Using the equations derived before (which characterize optimal consumer and firm behaviour), we have guaranteed that consumers have maximised their utility (section 3.1) and that intermediate and final good producing firms have maximised their profits (section 3.2-3.3) across all countries $i \in \mathcal{N}$ and sectors $s \in \mathcal{S}$. As such, we can readily define the conditions that govern the general equilibrium (GE) in the BRS-CFRT model we use to study the Sino-US trade war, both in terms of (1) what formally constitutes a general equilibrium in the type of economy described by the model and (2) what explicit equations govern the variables in general equilibrium. The conditions constituting a general equilibrium in the BRS-CFRT model is given by the following definition:[17]

Definition 1: the economy as described in the BRS-CFRT model is in general equilibrium if *all* of the following objectives and market clearing conditions have been met, given the parameters $\mathcal{P}$:

1) Consumers have maximised their utility given their aggregate income $\mathcal{I}_i$ and prices of final goods $P_{i,s}$ across all sectors $s \in \mathcal{S}$ *and* for all countries $i \in \mathcal{N}$;

2) Firms, whether intermediate *or* final good producers, have maximised their profits given input prices $P_{i,s}$, w_i and r_i across all sectors $s \in \mathcal{S}$, in all countries $i \in \mathcal{N}$;

3) All goods markets clear for all $s \in \mathcal{S}$ and all countries $i \in \mathcal{N}$ and all factor markets, i.e. the labour and capital market, clear for all countries $i \in \mathcal{N}$

Using the previously derived optimal consumer and firm behaviour, thus effectively guaranteeing conditions (1) and (2) are met, we now need to define the goods and factor market clearing conditions. The goods market clearing condition in the BRS-CFRT model is more involving than in conventional trade models, as demand does not only stem from consumer demand (derived in section 3.1), but also from intermediate good suppliers which use output of other sectors to produce their intermediate goods. The formal derivation is somewhat technical and has been omitted here, but it has been provided

[17] Throughout this paper we state definitions as these aids us when trying to compactly, yet comprehensively, write down optimisation problems as well as when referring to these conditions in general.

in appendix B.9 for the readership. The goods market clearing condition in the BRS-CFRT trade model, guaranteeing that goods market demand equals goods market supply, is given by:

$$Y_{i,s} = [Y_{i,s}]_{\text{con}} + [Y_{i,s}]_{\text{io}} = \alpha_{i,s}\mathcal{I}_i + \sum_{s'=1}^{S} \beta_{i,s,s'} \left(\frac{\sigma_{s'} - 1}{\sigma_{s'}} \right) \sum_{j=1}^{N} X_{ij,s'} \tag{22}$$

for all sectors $s \in \mathcal{S}$ and all countries $i \in \mathcal{N}$. To intuitively understand this condition, we first note that consumer expenditure in country i on sector s is a constant share of total income to consumers in i, as shown in section 3.1. Demand for sector s output stemming from a particular sector s' can be decomposed in the demand for sector s' output per destination, i.e. $X_{ij,s'}$. Intuitively, it the demand for sector s output in country i is a fixed share ($\beta_{i,s,s'}$) of the total variable costs incurred by intermediate good producers in i's sector s^* for exporting (net of tariffs) $X_{ij,s'}$ worth of intermediate goods to country j's sector s'. Note that $X_{ij,s'}$ is the total value of exports and that the total amount spent on variable costs is the value of exports minus the mark-up applied by the intermediate good producers, i.e. multiplying by inverse of the mark-up denoted by $(\sigma_{s'} - 1)/(\sigma_{s'})$. Given the fact that there are N countries and S sectors, the number of goods market clearing conditions determining the demand per sector, i.e. $Y_{i,s}$, is then $N \times S$. For labour and capital markets to clear, the demand for both factors stemming from the variable production costs (*id est* the labour and capital employed to produce a given quantity of the intermediate goods) and from fixed production costs needs to equal the supply of capital and labour. In this model, the capital and labour are provided completely inelastically (i.e. the supply elasticities of capital and labour are $\epsilon_{K_i,r_i} = 0$ and $\epsilon_{L_i,w_i} = 0$, respectively), implying there is a fixed capital and labour stock K_i and L_i for all countries $i \in \mathcal{N}$. Deriving these conditions is rather tedious and is analogous to the derivation of the goods market clearing conditions. A formal derivation has been provided for the factor market clearing conditions in appendix B.10. The factor market clearing conditions for all countries $i \in \mathcal{N}$ are given by:

$$w_i L_i = \sum_{s'=1}^{S} \gamma_{i,s'} M_{i,s'} \left(\frac{\theta_{s'}}{\sigma_{s'} - 1} \right) c_{i,s',f} f_{i,s',e} + \sum_{s'=1}^{S} \beta_{i,S+1,s'} \left(\frac{\sigma_{s'} - 1}{\sigma_{s'}} \right) \sum_{j=1}^{N} X_{ij,s'} \tag{23}$$

$$r_i K_i = \sum_{s'=1}^{S} (1 - \gamma_{i,s'}) M_{i,s'} \left(\frac{\theta_{s'}}{\sigma_{s'} - 1} \right) c_{i,s',f} f_{i,s',e} + \sum_{s'=1}^{S} \beta_{i,S+2,s'} \left(\frac{\sigma_{s'} - 1}{\sigma_{s'}} \right) \sum_{j=1}^{N} X_{ij,s'} \tag{24}$$

where we used the free entry condition to simplify the first summation in equations (23) and (24). Whilst we now have the N labour and N capital market clearing conditions, the $N \times S$ goods market equilibrium equations in (22) and the $N \times S$ price indexes in equation (18), we still need to define the mass of entrants to (represented by $N \times S$ variables and equations) be able to formally define the general equilibrium, given the dependence of other variables on the mass of entrants. Analogous to CFRT (2017), we obtain an expression for the mass of entrants in equilibrium by summing the gravity equation over j, isolating $M_{i,s}$ and using the free entry condition to simplify as far as possible[18]:

$$M_{i,s} = \sum_{j=1}^{N} X_{ij,s} \left(\left(\frac{\sigma_s \theta_s}{\sigma_s - 1} \right) (c_{i,s,f} f_{i,s,e}) \right)^{-1} \tag{25}$$

Now we have equations (18), (22) and (23)-(25), we can explicitly define what constitutes a general equilibrium (for a given set of tariffs) in terms of the set of the variables $\{Y_{i,s}, P_{i,s}, M_{i,s}, w_i, r_i\}$ and the set of parameters $\mathcal{P}$, given that the number of unknown variables is $N \times (3S + 2)$, which can be pinned down by the $N \times (3S + 2)$ equations we now have. The general equilibrium conditions in the BRS-CFRT model are given by the definition stated below in definition 2.

[18]It goes without saying that a formal derivation has also been provided for the equation governing the mass of entrants. For this derivation, the readership is referred to appendix B.11.

Definition 2: for a given set of tariffs $\vec{t}$, the economy described by the BRS-CFRT model is in general equilibrium if, given the set of parameters $\mathcal{P}$, the following equalities governing $\{Y_{i,s}, P_{i,s}, M_{i,s}, w_i, r_i\}$ have all been satisfied simultaneously, for all countries $i \in \mathcal{N}$ and all sectors $s \in \mathcal{S}$:

$$Y_{i,s} = \alpha_{i,s}\mathcal{I}_i + \sum_{s'=1}^{S} \beta_{i,s,s'} \left(\frac{\sigma_{s'} - 1}{\sigma_{s'}} \right) \sum_{j=1}^{N} X_{ij,s'} \tag{26}$$

$$P_{i,s} = \left((\Lambda_{ii,s})^{\xi_s} + \left(\sum_{j \neq i} \Lambda_{ji,s} \right)^{\xi_s} \right)^{-\frac{1}{\xi_s \theta_s}} \tag{27}$$

$$M_{i,s} = \sum_{j=1}^{N} X_{ij,s} \left(\left(\frac{\sigma_s \theta_s}{\sigma_s - 1} \right) (c_{i,s,f} f_{i,s,e}) \right)^{-1} \tag{28}$$

$$w_i L_i = \sum_{s'=1}^{S} \gamma_{i,s'} M_{i,s'} \left(\frac{\theta_{s'}}{\sigma_{s'} - 1} \right) c_{i,s',f} f_{i,s',e} + \sum_{s'=1}^{S} \beta_{i,S+1,s'} \left(\frac{\sigma_{s'} - 1}{\sigma_{s'}} \right) \sum_{j=1}^{N} X_{ij,s'} \tag{29}$$

$$r_i K_i = \sum_{s'=1}^{S} (1 - \gamma_{i,s'}) M_{i,s'} \left(\frac{\theta_{s'}}{\sigma_{s'} - 1} \right) c_{i,s',f} f_{i,s',e} + \sum_{s'=1}^{S} \beta_{i,S+2,s'} \left(\frac{\sigma_{s'} - 1}{\sigma_{s'}} \right) \sum_{j=1}^{N} X_{ij,s'} \tag{30}$$

where the other variables, $X_{ij,s}$, $\lambda_{ij,s}$ and ξ_s are governed by equations (17), (21) and (18), respectively (which are thus functions of the variables described in the set of equations above, and hence the system is fully identified). In addition, $\Lambda_{ij,s}$ is governed by equation (19).

There are two aspects about definition 2 which are convenient to note here, which also explains the Dekle, Eaton and Kortum (2008) hat algebra which follows. Whilst equations (26)-(30) constitutes a system of $N \times (2 + 3S)$ equations and $N \times (2 + 3S)$ unknowns which can thus be solved, the parameter space $\mathcal{P}$ still has some undesirable features. In particular, the parameter space contains parameters regarding the magnitude of the fixed costs of exporting and entry which are rarely, if ever, observed. To mitigate this problem, the system will be transformed using hat algebra, following CFRT (2017). In addition, this system is highly non-linear and can be solved with solvers in MATLAB or GAMS. Unlike in Ossa (2016), the IO linkages, the multiple factors of production and the free entry in this model prohibit us from further simplifying the model such that we only have 2 equations, one for output and one for labour, for every single country $i \in \mathcal{N}$. This also leads us to choose fewer countries and industries compared to Ossa (2016) to prevent us from running into computation constraints.

4 The BRS-CFRT Model in Calibrated Share Form

Now we have derived the equilibrium conditions, we want to proceed to calibrate the model on actual trade data. Unfortunately, the parameter space of the BRS-CFRT model contains parameters such as $f_{ij,s}$ and $f_{i,s,e}$ on which data is not readily available. To circumvent the need to estimate said parameters, we conduct the thought experiment analoguous to that in CFRT (2017) - following Dekle, Eaton and Kortum (2008) - of changing the policy variables $\vec{t}$ (*id est* tariffs). Given the fact that we are only interested in optimal tariff setting, without changing the actual scenario such as resource endowments and sectoral linkages, this permits us to define the model in changes relative to the current set of tariffs and trade flows. As a result, we can define the BRS-CFRT model in exact hat algebra and obtain the optimal non-cooperative tariff set for the US and China. In this exposition we will adhere to common notation within the field and define $\widehat{z} = z(\vec{t}')/z(\vec{t})$, where $\widehat{z}$ is the ratio of the variable z under the revised trade policy set $\vec{t'}$ and the value of z under the initial policy set $\vec{t}$. Note that for both the revised and current tariff set, equations (26)-(30) need to hold with equality, such that the economy is in a state of general equilibrium (in changes)[19]. This exposition is structured parallel to the exposition given in Ossa (2016) and uses the insights from CFRT (2017) to obtain an estimable model.

4.1 Indirect Utility in Hat Notation

The indirect utility to a representative consumer is its *real* income, and we can divide the real income under the revised tariff set by the real income under the actual tariff set to obtain $\widehat{V}_i$. Changing the tariff set changes the income received by the representative consumer as well as the prices indexes of final goods, but leaves the fundamental parameters such as $\alpha_{i,s}$ unchanged. Using the definition of V_i in equation (2) in section 3, we divide V_i' by V_i and obtain for all countries $i \in \mathcal{N}$:

$$\widehat{V}_i = \frac{\widehat{\mathcal{I}}_i}{\prod_{s=1}^{S} \left(\widehat{P}_{i,s}\right)^{\alpha_{i,s}}} \tag{31}$$

Given the fact that the indirect utility function is multiplicative, dividing V_i' by V_i yields that every single component under the revised tariff set, whether $\mathcal{I}_i'$ or $P_{i,s}'$, is divided by the same component under the current tariff set, *id est* $\mathcal{I}_i$ and $P_{i,s}$. As a result, the change in utility to a representative consumer is proportional to the change in its income and inversely related to the change in the price indexes for the final goods (i.e. higher prices serve to erode real income).

4.2 Trade Shares, Marginal Costs and Gravity in Hat Notation

Before performing Dekle, Eaton and Kortum's hat algebra on the system of equations governing the general equilibrium in the BRS-CFRT model as stated in definition 2, we first perform hat algebra on four auxiliary variables which appear in the equations governing the general equilibrium: national income $\mathcal{I}_i$; the value of exports to j from producers in country i's sector s (net of tariffs) $X_{ij,s}$; the price index for fixed costs $c_{i,s,f}$; and the marginal cost price index $c_{i,s}$. Unfortunately, not all expressions are multiplicative such that we can easily obtain hat algebraic expressions from the expressions we have found previously in levels. National income $\mathcal{I}_i$ is for instance a sum of several components, consisting out of primary factor market income, income from import tariffs and net international transfers Ω_i. In order to rewrite national income into hat algebra we use the fact that if $z = x + y$, then $\widehat{z} = x'/z + y'/z$ $= (x/z)\widehat{x} + (y/z)\widehat{y}$ (see: Ossa, 2016), where (x/z) and (y/z) are coefficients which can be calibrated on data before tariffs changed. When applied to national income we obtain the following equation:

$$\widehat{\mathcal{I}}_i = \delta_{\mathcal{I}1,(i)}\widehat{w}_i + \delta_{\mathcal{I}2,(i)}\widehat{r}_i + \delta_{\mathcal{I}3,(i)}\widehat{T}_i + \delta_{\mathcal{I}4,(i)}\widehat{\Omega}_i \tag{32}$$

in hat notation[20]. Here the coefficients $\delta_{\mathcal{I}1,(i)}$, $\delta_{\mathcal{I}2,(i)}$, $\delta_{\mathcal{I}3,(i)}$ and $\delta_{\mathcal{I}4,(i)}$ have been defined as:

$$\delta_{\mathcal{I}1,(i)} \equiv \left(\frac{w_i L_i}{\mathcal{I}_i}\right) \quad ; \quad \delta_{\mathcal{I}2,(i)} \equiv \left(\frac{r_i K_i}{\mathcal{I}_i}\right) \quad ; \quad \delta_{\mathcal{I}3,(i)} \equiv \left(\frac{T_i}{\mathcal{I}_i}\right) \quad ; \quad \delta_{\mathcal{I}4,(i)} \equiv \left(\frac{\Omega_i}{\mathcal{I}_i}\right) \tag{33}$$

These coefficients - or weights - on the changes in the components of the national income $\mathcal{I}_i$ are simply the shares which labour income, capital income, tariff income and international transfers made

[19]If tariffs remain unchanged, the entire system of equations in definition 2 in hat notation, which in hat notation represents the changes in the endogenous variables in general equilibrium due to a changes in tariffs $\vec{t}$, is reduces to zeroes.

[20]To obtain the income from tariffs in hat notation we divide T_i' by T_i. It can be readily shown that this yields:

$$\widehat{T}_i = \sum_{s=1}^{S}\sum_{j=1}^{N} \frac{X_{ji,s}}{T_i} \left((\widehat{1+t_{ji,s}})(1+t_{ji,s}) - 1\right) \widehat{X}_{ji,s} \;\text{ with }\; \delta_{T1,(ij,s)} = \frac{X_{ij,s}}{T_j} \;\;;\;\; \delta_{T2,(ij,s)} = (1+t_{ij,s})$$

up of national income under the initial tariff set $\vec{t}$. In order to write the general equilibrium conditions in terms of $\{\widehat{Y}_{i,s}, \widehat{P}_{i,s}, \widehat{M}_{i,s}, \widehat{w}_i, \widehat{r}_i\}$, we now also need to express the change in trade shares in terms of said set of variables. The formal derivation for the change in trade shares $\lambda_{ij,s}$ has been provided to the readership in appendix C.2. Using the expression for $\lambda_{ij,s}$ in section 3.4 we readily find that:

$$\widehat{\lambda}_{ij,s} = \widehat{\Lambda}_{ij,s} \left(\sum\nolimits_{i \neq j}^{N} \frac{\lambda_{ij,s}}{(1-\lambda_{jj,s})} \widehat{\Lambda}_{ij,s} \right)^{\xi_s - 1} \left(\widehat{P}_{j,s} \right)^{\theta_s \xi_s} \tag{34}$$

for $i \neq j$.[21] With $\widehat{\Lambda}_{ij,s}$ given by:

$$\widehat{\Lambda}_{ij,s} \equiv \left((\widehat{1+t_{ij,s}}) \widehat{\tau}_{ij,s} \widehat{c}_{i,s} \right)^{-\theta_s} \widehat{M}_{i,s} \left(\frac{(\widehat{1+t_{ij,s}}) \widehat{c}_{i,s,f}}{\widehat{Y}_{j,s}} \right)^{\frac{\sigma_s - 1 - \theta_s}{\sigma_s - 1}} \tag{35}$$

The only parameters which need to be estimated from current trade data for equation (34) are the trade shares $\lambda_{ij,s}$ (i.e. contemporary trade shares). In an analogous manner we proceed to derive an expression in hat notation for the variables which are not explicitly represented in the general equilibrium conditions: marginal costs indexes $c_{i,s,f}$ and $c_{i,s}$. Using the definitions of these marginal costs indexes, which can be found in footnote (12) and equation (5), we can readily (as these indexes are multiplicative) derive these equations in hat algebra as follows:

$$\widehat{c}_{i,s} = \prod_{s'=1}^{S} \left(\widehat{P}_{i,s'} \right)^{\beta_{i,s',s}} (\widehat{w}_i)^{\beta_{i,S+1,s}} (\widehat{r}_i)^{\beta_{i,S+2,s}} \quad ; \quad \widehat{c}_{i,s,f} = (\widehat{w}_i)^{\gamma_{i,s}} (\widehat{r}_i)^{1-\gamma_{i,s}}$$

Analogous to section 3.3 we can now also rewrite the gravity equation governing $X_{ij,s}$ in hat algebra, as a function of the change in trade shares $\widehat{\lambda}_{ij,s}$, the change in tariffs on trade flows from i to j and the change in aggregate output of sector s in country j, $\widehat{Y}_{j,s}$. Applying hat algebra to equation (17), we readily find the following expression for $\widehat{X}_{ij,s}$:

$$\widehat{X}_{ij,s} = \left(\widehat{\lambda}_{ij,s} / (\widehat{1+t_{ij,s}}) \right) \widehat{Y}_{j,s} \tag{36}$$

4.3 General Equilibrium Conditions in Hat Notation

The general equilibrium conditions, given by equations (26)-(30) in definition 2, can now also be converted into exact hat notation in a relatively straightforward manner. We will convert those equations for price indexes, goods and factor market clearance as well as the mass of entrants for all sectors $s \in \mathcal{S}$ and countries $i \in \mathcal{N}$ in the same order as they have been presented in definition 2. As such, we will start with equation (26) which governs goods market clearance. Using the fact that $z = x + y \implies \widehat{z} = (x/z)\widehat{x} + (y/z)\widehat{y}$, we can obtain the expression for $Y_{i,s}$ in hat notation:

$$\widehat{Y}_{i,s} = \alpha_i \delta_{\mathrm{SD1},(i,s)} \widehat{\mathcal{I}}_i + \sum_{s'=1}^{S} \beta_{i,s,s'} \left(\frac{\sigma_{s'} - 1}{\sigma_{s'}} \right) \sum_{j=1}^{N} \delta_{\mathrm{SD2},(ij,s,s')} \widehat{X}_{ij,s'} \tag{37}$$

where the coefficients, $\delta_{\mathrm{SD1},(i,s)}$ and $\delta_{\mathrm{SD2},(ij,s,s')}$, for the SD conditions (where SD represents Supply=Demand conditions, i.e. good or factor market clearing) are defined as follows:

$$\delta_{\mathrm{SD1},(i,s)} \equiv \left(\frac{\mathcal{I}_i}{Y_{i,s}} \right) \quad ; \quad \delta_{\mathrm{SD2},(ij,s,s')} \equiv \left(\frac{X_{ij,s'}}{Y_{i,s}} \right) \tag{38}$$

[21] Note that the equation in the case that $i = j$ then drastically simplifies to: $\widehat{\lambda}_{ii,s} = \widehat{\Lambda}_{ii,s}^{\xi_s} \left(\widehat{P}_{i,s} \right)^{\theta_s \xi_s}$

Note that the coefficients only pertain to the values which characterise the current general equilibrium and as such can be calibrated using real world data as long as we have data on income $\mathcal{I}_i$, sectoral trade flows $X_{ij,s}$ and sectoral output $Y_{i,s}$ for all $i,j \in \mathcal{N}$ and $s \in \mathcal{S}$. These values can be obtained from trade and production data using, in this paper, the WIOT tables. A discussion on the calibration exercise and the difficulties encountered when calibrating the model are discussed in section 5. In a slightly more involving manner, we can also rewrite the price indexes into hat notation. The formal derivation has been provided for the readership in appendix C.3. The expression for $\widehat{P}_{i,s}$ is given by:

$$\widehat{P}_{i,s} = \left(\lambda_{ii,s} \widehat{\Lambda}_{ii,s}^{\xi_s} + (1-\lambda_{ii,s}) \left(\sum_{j \neq i} \frac{\lambda_{ji,s}}{1-\lambda_{ii,s}} \widehat{\Lambda}_{ji,s} \right)^{\xi_s} \right)^{-\frac{1}{\xi_s \theta_s}} \tag{39}$$

where the only coefficients which need to be estimated are the trade shares $\lambda_{ji,s}$ for all countries $i,j \in \mathcal{N}$ and $s \in \mathcal{S}$, which can be calibrated as long as we have a complete dataset on $X_{ij,s}$ and tariffs. Converting the mass of entrants into hat notation is also relatively straightforward and can be obtained from equation (28) by dividing $M'_{i,s}$ by $M_{i,s}$. This division then yields the following expression for the change in the mass of entrants in i's sector s due to a change in tariffs or trade costs:

$$\widehat{M}_{i,s} = \sum_{j=1}^{N} \delta_{M1,(ij,s)} \widehat{X}_{ij,s} \left(\widehat{c}_{i,s,f} \right)^{-1} \quad \text{with} \quad \delta_{M1,(ij,s)} \equiv \left(\frac{X_{ij,s}}{\sum_{j=1}^{N} X_{ij,s}} \right) \tag{40}$$

where the coefficient $\delta_{M1,(ij,s)}$ can be calibrated with the net of tariff trade flows. Analogous to the goods market conditions, we can also repeat this procedure, i.e. converting the equation in definition 2 into exact hat algebra, to the factor market clearing conditions in equations (29) and (30). For the labour market clearing condition (for all $i \in \mathcal{N}$), we find the following expression for $\widehat{w}_i$:

$$\widehat{w}_i = \sum_{s'=1}^{S} \delta_{\text{SD3},(i,s')} \left(\widehat{c}_{i,s',f} \widehat{M}_{i,s'} \right) + \sum_{s'=1}^{S} \beta_{i,S+1,s'} \left(\frac{\sigma_{s'}-1}{\sigma_{s'}} \right) \sum_{j=1}^{N} \left(\delta_{\text{SD4},(ij,s')} \widehat{X}_{ij,s'} \right) \tag{41}$$

where the coefficients $\delta_{\text{SD3},(ij,s)}$ and $\delta_{\text{SD4},(ij,s)}$ are defined as follows:

$$\delta_{\text{SD3},(i,s)} \equiv \left(\frac{\gamma_{i,s} FC_{i,s}}{w_i L_i} \right) \quad ; \quad \delta_{\text{SD4},(ij,s)} \equiv \left(\frac{X_{ij,s}}{w_i L_i} \right) \quad ; \quad FC_{i,s} \equiv c_{i,s,f} \left(\frac{\theta_s}{\sigma_s - 1} \right) M_{i,s} f_{i,s,e} \tag{42}$$

Here, $\gamma_{i,s}$ represents the share of fixed costs in country i's sector s (denoted by $FC_{i,s}$) spent on labour.[22] As a result, the coefficient $\delta_{\text{SD3},(ij,s)}$ measures the share of the wage bill which stems from paying the fixed costs. Given the fact that the current wage bill $w_i L_i$ in country i (per sector) as well as net of tariff trade flows are observed and available in the WIOD database, we can also calibrate the labour market clearing conditions (*id est* the coefficients) using real world data. In an analogous fashion, we can obtain the capital market clearing conditions in hat notation for equation (30):

$$\widehat{r}_i = \sum_{s'=1}^{S} \delta_{\text{SD5},(i,s')} \left(\widehat{c}_{i,s',f} \widehat{M}_{i,s'} \right) + \sum_{s'=1}^{S} \beta_{i,S+2,s'} \left(\frac{\sigma_{s'}-1}{\sigma_{s'}} \right) \sum_{j=1}^{N} \left(\delta_{\text{SD6},(ij,s')} \widehat{X}_{ij,s'} \right) \tag{43}$$

[22] To see why the term $c_{i,s,f} \left(\theta_s / \sigma_s - 1 \right) M_{i,s} f_{i,s,e}$ represents the mark-up on trade flows in industry s in country i which are used to cover fixed costs, one can infer this from equation (9) or from substituting equation (28) into equation (29) and (30). Aggregate equation (9) over all countries j and φs and enforce zero aggregate profits (due to free entry) across all entrants to obtain the aforementioned value added term. This yields that all profits, the value added in this model, are paid to cover the fixed operating and entry costs which is what the term $c_{i,s,f} \left(\theta_s / \sigma_s - 1 \right) M_{i,s} f_{i,s,e}$ resembles. Here, $\theta_s f_{i,s,e}$ also contains the fixed operating costs by applying the free entry condition in (16).

where, completely analoguous to the coefficients arising in equation (37), the coefficients $\delta_{\text{SD5},(i,s)}$ and $\delta_{\text{SD6},(ij,s)}$ have been defined in the following manner:

$$\delta_{\text{SD5},(i,s)} \equiv \left(\frac{(1-\gamma_{i,s})FC_{i,s}}{r_i K_i}\right) \quad ; \quad \delta_{\text{SD6},(ij,s)} \equiv \left(\frac{X_{ij,s}}{r_i K_i}\right) \quad ; \quad FC_{i,s} \equiv c_{i,s,f}\left(\frac{\theta_s}{\sigma_s - 1}\right) M_{i,s} f_{i,s,e} \tag{44}$$

Here $(1-\gamma_{i,s})$ represents the share of fixed costs accruing to capital. As a result, the coefficient $\delta_{\text{SD3},(ij,s)}$ measures the share of the income accruing to capital which stems form fixed costs relative to total capital income, analogous to the coefficient found in the labour market clearing condition in hat notation, and can be calibrated in a similar fashion as the labour market clearing condition provided we have data on the current wage bill $w_i L_i$ in country i (per sector) as well as net of tariff trade flows. Now we have written all equilibrium and auxiliary conditions in hat notation from a change in $\vec{t}$ and $\vec{\tau}$, treating the parameter set $\widehat{\mathcal{P}}$ as given (where $\widehat{\mathcal{P}} = \mathcal{P}\backslash\{f_{ij,s}, f_{i,s,e}\}$), we can proceed to define the general equilibrium in hat notation which will be used to study non-cooperative tariffs.

4.4 Characterising the General Equilibrium in Hat Notation

Now we have derived the expressions for the general equilibrium in hat notation, we can use these equations (which are equations: (37), (39)-(40), (41) and (43)) as well as the auxiliary equations in section 4.2, to explicitly define what constitutes a general equilibrium (for a given set of tariffs) in terms of the set of the variables $\{\widehat{Y}_{i,s}, \widehat{P}_{i,s}, \widehat{M}_{i,s}, \widehat{w}_i, \widehat{r}_i\}$, and the set of parameters $\widehat{\mathcal{P}}$ (excluding tariffs and trade costs) and the set of tariff and trade cost changes $\{\widehat{(1-t_{ij,s})}, \widehat{\tau}_{ij,s}\}$. Given that the equilibrium is once again characterised by $N \times (3S+2)$ unknown variables, it is possible to compute the changes in these endogenous variables for a given change in tariffs or trade costs. Of course, if there is no change in tariffs or trade costs, the trivial solution, i.e. all changes in variables are zero or alternatively all endogenous hat variables take the value of unity, is indeed the only solution which solves the system of equations. As discussed before, none of the equations governing the general equilibrium or the auxiliary equations contain the parameters $f_{ij,s}$ and $f_{i,s,e}$. The general equilibrium conditions in the BRS-CFRT model in hat notation are summarised by the definition below for future reference.

Definition 3: for a given change in the set of tariffs $\vec{t}$, the economy described by the BRS-CFRT model is in equilibrium if, given the set of parameters $\widehat{\mathcal{P}}$, the following equalities governing $\{\widehat{Y}_{i,s}, \widehat{P}_{i,s}, \widehat{M}_{i,s}, \widehat{w}_i, \widehat{r}_i\}$ have all been satisfied simultaneously, for all countries $i \in \mathcal{N}$ and all sectors $s \in \mathcal{S}$:

$$\widehat{Y}_{i,s} = \alpha_{i,s}\delta_{\text{SD1},(i,s)}\widehat{\mathcal{I}}_i + \sum_{s'=1}^{S} \beta_{i,s,s'}\left(\frac{\sigma_{s'}-1}{\sigma_{s'}}\right)\sum_{j=1}^{N}\delta_{\text{SD2},(ij,s,s')}\widehat{X}_{ij,s'} \tag{45}$$

$$\widehat{P}_{i,s} = \left(\lambda_{ii,s}\widehat{\Lambda}_{ii,s}^{\xi_s} + (1-\lambda_{ii,s})\left(\sum_{j\neq i}\frac{\lambda_{ji,s}}{1-\lambda_{ii,s}}\widehat{\Lambda}_{ji,s}\right)^{\xi_s}\right)^{-\frac{1}{\xi_s\theta_s}} \tag{46}$$

$$\widehat{M}_{i,s} = \sum_{j=1}^{N}\delta_{M1,(ij,s)}\widehat{X}_{ij,s}\left(\widehat{c}_{i,s,f}\right)^{-1} \tag{47}$$

$$\widehat{w}_i = \sum_{s'=1}^{S}\delta_{\text{SD3},(i,s')}\left(\widehat{c}_{i,s',f}\widehat{M}_{i,s'}\right) + \sum_{s'=1}^{S}\beta_{i,S+1,s'}\left(\frac{\sigma_{s'}-1}{\sigma_{s'}}\right)\sum_{j=1}^{N}\left(\delta_{\text{SD4},(ij,s')}\widehat{X}_{ij,s'}\right) \tag{48}$$

$$\widehat{r}_i = \sum_{s'=1}^{S}\delta_{\text{SD5},(i,s')}\left(\widehat{c}_{i,s',f}\widehat{M}_{i,s'}\right) + \sum_{s'=1}^{S}\beta_{i,S+2,s'}\left(\frac{\sigma_{s'}-1}{\sigma_{s'}}\right)\sum_{j=1}^{N}\left(\delta_{\text{SD6},(ij,s')}\widehat{X}_{ij,s'}\right) \tag{49}$$

where: (1) the other variables such as the change in marginal costs, the change in the exports and the change in the trade shares are given by the auxiliary equations in section 4.2; and (2) the parameters δ are defined as described in section 4.3. It is important to note the the auxiliary equations express the aforementioned changes in terms of the variables $\{\widehat{Y}_{i,s}, \widehat{P}_{i,s}, \widehat{M}_{i,s}, \widehat{w}_i, \widehat{r}_i\}$.

The MATLAB program uses equations (45)-(49) for all country-sector pairs (i.e. for all $i \in \mathcal{N}$ and sectors $s \in \mathcal{S}$) to compute the implications on trade, production and income of increasing the tariffs on Chinese or US goods respectively. In addition to these $N \times (3S + 2)$, equations, we also explicitly include the auxiliary equations for trade flows (i.e. the gravity equations) as well as the equations governing $\Lambda_{ij,s}$ in order to keep the code parsimonious and readable.

5 Nash Tariff Computation and Model Calibration

In order to compute the implied optimal non-cooperative tariffs for the Sino-US trade war by the BRS-CFRT model as outlined in section 2.1, using the general equilibrium definition in changes as stated in definition 3, we require real world data to calibrate the model's parameters. This section provides both a discussion on how the model's parameters have been calibrated (in particular which data has been used for this calibration exercise) as well as a detailed account on how the non-cooperative tariffs have been computed using the numeric fmincon solver in MATLAB. To keep both the coding tractable and implementable using the numeric fmincon solver, we reduced the dimension of sectors and countries. The applied general equilibrium model contains three trade blocks, which are the US (US), China (CHN) and the Rest of the World (ROW) , and four sectors, which are agriculture (AGR), mining (MIN), manufacturing (MAN) and services (SER) (in other words $|\mathcal{N}| = 3$ and $|\mathcal{S}| = 4$).

5.1 Computing Model Consistent Nash Tariffs

In order to compute the implied non-cooperative tariffs, we need to formally define the structure imposed on the strategic interaction between the governments. It is assumed that the US initiates the trade war, as we have observed in reality (see Li, 2018), and that it implements the set of tariffs on trade flows originating from China such that it maximises its national welfare, taking into account the effects of the tariffs on trade flows, production and prices across the economies.[23] After the US has enacted its tariffs on imports from China and the world economy is in general equilibrium, the Chinese government is assumed to do the same as the US government but for the Chinese representative consumer and impose a new set of tariffs on imports from the US. Thereafter the US responds again and the cycle continues until neither party desires to change its tariffs (or changing its tariffs does not seem to be beneficial), reaching the final state of the trade war. As mentioned in section 2.1, the game theoretical equilibrium concept applied to model the strategic interaction between the US and China is thus a Nash equilibrium (see Ossa, 2016, who uses the NE to model the strategic interaction in the event of a world wide trade war). The ROW is presumed to act as the neutral party in the trade dispute, under the strict assumption that the US and China respect that neutrality.[24]

More formally, we have that the US government has the objective to maximise the change in the indirect utility of the US representative consumer as stated in equation (31) subject to the equilibrium conditions outlined in definition 3 for the BRS-CFRT model (or for alternative model specifications

[23] In other words, the government of the US needs to respect and incorporate the fact that the allocations and prices will adjust to changes which are compatible with the general equilibrium conditions.

[24] This might not be completely realistic as the former Trump administration also sought confrontation with other trade blocs, amongst which the EU, but otherwise the computations would become more elaborate and every iteration would take longer to run (more arguments of optimisation) as the RoW would have to react to the US and Chinese tariffs as well. This would then impose the rather stringent assumption that all other countries in the world except for the US and China would uniformly change their tariffs which is in and of itself also rather stringent (as this would require extreme coordination). Hence, we impose the simplifying assumption that the US and China are the primary belligerents in this trade dispute.

definitions 4-6). As such, the US plays its best response to the given state of the world economy by changing its tariff set denoted by $\widehat{1+t}_{\text{CHN, US}}$. In a completely analogous fashion, the Chinese government seeks to maximise the indirect utility of the Chinese representative consumer by changing its tariff set by $\widehat{1+t}_{\text{US, CHN}}$given the world economy must be in general equilibrium (and thus prices and production and consumption choices change accordingly). Now let the superscript h denote the value of the change in optimal tariffs at the h^{th} iteration. Then the US and Chinese governments are said to have achieved their optimal Nash tariffs if the change in tariffs at iteration h is sufficiently small (or the there is no improvement in indirect utility). In other words, if the vector of change in tariffs, $[\widehat{1+t}^{h}_{\text{CHN, US}} - \vec{1}, \widehat{1+t}^{h}_{\text{US, CHN}} - \vec{1}]$, at some iteration h is sufficiently close to $\vec{0}$ (with some deviation $\varepsilon > 0$), then the tariffs that have arisen after this iterative procedure are the NE tariffs.

Definition 4: the set of optimal non-cooperative tariffs is given by:

$$(1+t_{ij,s})^{\text{NE}} = \prod\nolimits_{h=0}^{H} \left(\widehat{1+t_{ij,s}}\right)^{h} [1+t_{ij,s}]_{h=0} \tag{50}$$

where H corresponds to the iteration for which $|| [\widehat{1+t}^{h}_{\text{CHN, US}} - \vec{1}, \widehat{1+t}^{h}_{\text{US, CHN}}] - \vec{1} || < \varepsilon$ for the first time (for some tolerance limit ε). The optimal tariff changes per iteration are then computed by solving the constrained optimisation problems using $\{\widehat{Y}_{i,s}, \widehat{P}_{i,s}, \widehat{M}_{i,s}, \widehat{w}_i, \widehat{r}_i\}$ and $(\widehat{1+t_{CHN,US}})$ or $(\widehat{1+t_{US,CHN}})$, for the first and second optimisation problem respectively, as the arguments of optimisation:

$$\min -\widehat{V}_{US} \quad s.t. \ \textbf{Definition 3} \text{ with parameter set } \widehat{\mathcal{P}}^{h}_{US} \tag{51}$$

$$\min -\widehat{V}_{CHN} \quad s.t. \ \textbf{Definition 3} \text{ with parameter set } \widehat{\mathcal{P}}^{h}_{CHN} \tag{52}$$

where $\widehat{\mathcal{P}}^{h}_{i}$ (incl. λ's) denotes the updated parameter set after the US has updated its tariffs at iteration h if $i =$ CHN or, if $i =$ US, the parameter set after CHN has updated its tariffs at $h-1$.

Note that after every optimisation the parameter set is re-calibrated to reflect the changed general equilibrium conditions due to the tariff change by the trading partner (i.e. the parameter set which includes the δ's changes at every iteration and as such the parameter vector $\widehat{\mathcal{P}}^{h}_{i}$ is also indexed by an h). As an example, the parameter $\delta_{\text{SD1}} = \mathcal{I}_{\rangle}/Y_{i,s}$is updated after every iteration by multiplying it by $\widehat{\mathcal{I}}_{\rangle}/\widehat{Y}_{i,s}$ such that it reflects the updated value of income relative to production in sector s.[25] The constrained minimisation problem faced by the US or China at every iteration as stated by equations (51) and (52) will be solved using the numeric solver fmincon in MATLAB. This effectively treats the problem as a Minimisation Problem under Equilibrium Constraints (MPEC) as outlined by Su and Judd (2012) and which has been advocated and clearly outlined by Ossa (2016).

5.2 Calibrating the Model in Calibrated Share Form

To calibrate the model we require data on several aspects regarding the economies of the US, China and ROW. Given the fact that recent data (ideally of 2017) was not readily available for all types of data required to calibrate the model, we calibrated the model on the most recent year for which all required data was available, which was 2014. The discussion is structured in such a fashion that we first discuss the data used to calibrate the parameters in $\widehat{\mathcal{P}}$ and subsequently discuss how the δ parameters have been calibrated (using the data we also required for $\widehat{\mathcal{P}}$). In order to calibrate the parameter vector, we require: (1) data on both the share of final consumption which is spent on each different sector (to calibrate $\alpha_{i,s}$); (2) the share of variable production costs spent on different sectors and primary inputs to calibrate the vertical linkages; (3) the relative share of labour to capital in the fixed costs to

[25] In other words, $(\mathcal{I}_{\rangle}/Y_{i,s})(\widehat{\mathcal{I}}_{\rangle}/\widehat{Y}_{i,s}) = \widehat{\mathcal{I}}_{\rangle}'/\widehat{Y}'_{i,s}$

calibrate $\gamma_{i,s}$; and (4) estimates of the elasticity of substitution parameters σ_s and ς_s and the Pareto shape parameter θ_s. The data on trade flows per sector as well as output and final consumption were obtained from the WIOD tables (Timmer et al., 2015) and the elasticity of substitution parameters and Pareto parameters were obtained from Caliendo et al. (2017).

In order to calibrate the model using the WIOD tables, several features of the WIOD tables which the model could not accommodate were reallocated first. In particular, the WIOD 2014 IO tables show that there are direct trade flows between country i and j from sector s to sector s', which the BRS-CFRT model cannot accommodate (i.e. intermediates can only be imported by the same sector's final good producers in the foreign country). As such, we add these trade flows to the trade between sector s' in i and sector s' in j and the internal trade flow from the sector s to s' in i is increased accordingly. Subsequently, the change in inventory and gross fixed capital formation in the usage columns were assigned proportionally to the trade flows stemming from said sector (i.e. inventory usage was proportional to the sales occurring between trade partners) and were reallocated to final demand, respectively. Thereafter, the WIOD table was reduced to the desired level of aggregation for both countries and sectors.[26] The total labour and capital income, the latter including also land (i.e. it is a residual), were obtained from the Socio-Economic Accounts (see: Timmer et al., 2015). To obtain total income, given the fact that $I_i = w_i L_i + r_i K_i + T_i + \Omega_i$, we also require data on tariffs (Ω_i absorbed any trade imbalances) to compute the tariff income received by consumers in country i.

The trade weighted average tariffs on various trade flows across agriculture, mining, manufacturing and services are obtained from the UNCTAD-TRAINS database using WITS. Caliendo et al. (2017) shows that trade weighted average tariffs are model consistent (for CFRT and thus for BRS-CFRT as well) when aggregating to a higher level of aggregation than individual product lines. These weighted tariffs are then multiplied with the trade flows in the reduced WIOT to obtain tariff revenue. Any remaining disparity between $\mathcal{I}_i$ and the sum of its components ($w_i L_i + r_i K_i + T_i + \Omega_i$) were absorbed by capital (capital income was increased per sector proportional to initial expenditure on capital) except in the case of the ROW where the disparity was reallocated to labour and capital in a similar fashion to how capital was adjusted for the US and China.[27] The elasticity of substitution σ_s as well as an estimate of the Pareto distribution's shape parameter θ_s were obtained from Caliendo et al. (2017) who used the method of triple differencing the gravity equation, as introduced by Caliendo and Parro (2015), to obtain elasticities for agriculture, mining, manufacturing and services.[28]

To obtain estimates of the value of production of final goods produced by a sector s in country i ($Y_{i,s}$) we note that this is equivalent to all sales of agriculture to domestic sectors (i.e. the IO linkages) plus the final consumption by domestic consumers from sector s. The trade flows to other countries $j \neq i$ is at basic prices and do not include any taxes (but do include the mark-up) and thus resemble the sales of intermediaries to a country j ($X_{ij,s}$). To obtain the domestic sale of intermediates ($X_{ii,s}$) we use the fact that $\sum_{j \in \mathcal{N}} \lambda_{ji} = 1$ and that $Y_{i,s}$ is known to compute $\lambda_{ii,s}$ and as a result $X_{ii,s}$. Subsequently we realise that the expenditure on fixed costs for sector s can be readily computed as $(1/\sigma_s) \sum_{j \in \mathcal{N}} X_{ij,s}$. These are then subtracted from the capital and labour expenditures in a particular sector which then permits the subsequent calibration of the $\beta_{i,s,s'}$ parameters. The remaining labour and capital expenditures are then reallocated to fixed and variable costs in the same proportion (i.e. $\gamma_{i,s} = \beta_{i,S+1,s} / (\beta_{i,S+1,s} + \beta_{i,S+2,s})$ as gamma could give rise to an arbitrary reallocation of capital

[26]In this case all countries that were not the US or China were aggregated into the ROW. All sectors in the WIOT which were marked with an A were defined as agricultural sectors, those marked with a B as mining sectors and those denoted by a C as the manufacturing sectors. The other categories were treated as service.

[27]Please note that the Socio Economic Accounts do not contain capital and labour income for the WIOT's rest of the world and hence it makes sense that the "missing income" is largely labour and capital.

[28]Note that only the elasticity of substitution for the manufacturing industry has been changed to 6 instead of 4.4, as with such a low elasticity the fixed costs exceeded the empirical payments to labour and capital. In contrast, setting sigma equal to 6 is consistent with the median estimated elasticity of substitution for the EU manufacturing industry (Chen and Novy, 2011).

and labour in the variable and fixed cost expenditures). Computing the fixed costs from the data in the manufacturing sector sometimes revealed that labour and capital income in a particular sector were not sizeable enough to cover the fixed costs implied by $(1/\sigma_s)\sum_{j\in\mathcal{N}} X_{ij,s}$ and as such the parameter value of σ_{MAN} was adjusted upward to remedy this problem.[29]

For proper calibration of the BRS-CFRT model, equations (26), (29) and (30) need to hold with equality, implying that all the economies should be in equilibrium. Given the fact that we have obtained the net of tariff trade flows $X_{ij,s}$ as well as income, we can compute the production $Y_{i,s}$, labour income ($w_i L_i$) and capital income ($r_i K_i$) which the model would imply. These can subsequently be compared to the data we have on $Y_{i,s}$, $w_i L_i$ and $r_i K_i$. However, there appear to be small deviations between observed and model consistent production, labour income and capital income which cause slight mismatches (deviations are typically less than 0.5% of observed the actually value). The only parameters, without changing the elasticity of substitution, which can remedy this mismatch are the $\beta_{i,s,s'}$ which were subsequently adjusted to remedy these deviations (average change in beta from data based estimates is smaller than 1e-6; can be inspected in the accompanying Excel sheet).[30] Using the data obtained from the WIOD, UNCTAD-TRAINS database and Caliendo et al. (2017), all δ (i.e. calibrated share form) parameters can be calibrated according to the definitions provided in section 4.

6 The BRS-CFRT Implied US-China Nash Tariffs

We now turn to the non-cooperative tariffs which would be implied by the BRS-CFRT model across agriculture, mining, manufacturing and services. It is important to note that MPEC studied in this paper appears to be well behaved (flat best response surfaces) such that MATLAB's numeric solver fmincon, at reasonable error tolerance levels and step size (smaller than 1e-4), quickly converges to the optimal non-cooperative tariffs (this would indicate that the analysis could be extended to lower levels of aggregation). In our case, no country can improve its welfare by changing its tariffs after two rounds and as a result we conclude that the tariffs found are the NE tariffs. This appears to be in accordance with the fact that the MPECs encountered by Ossa (2014) and Ossa (2016) are also sufficiently well behaved and converge relatively quickly to the Nash equilibrium tariffs (the best response functions are relatively flat). We compare the first round of implied retaliation to the first 3 observed rounds (defined as period $t = 1$) and the second round to the 3 last observed rounds (defined as period $t = 2$).[31]

6.1 Implied Tariffs for the US-China Trade War

During the first period of strategic interaction between the US and China we find that the implied change in tariffs across sectors in the US aligns very closely to the actual tariff increases witnessed during the first rounds of the trade war (see figure 3). In particular, the US's implied increase in tariffs on manufacturing goods originating from China would be a 4.32 percentage points increase in tariffs compared to the 4.29 percentage point increase in tariffs which was observed during the first three rounds of the US-China trade war. The implied change in tariffs across the agricultural, mining and service sectors seem to be slightly higher (around 0.5 percentage points) than those observed in reality, which did not change during the first period. Nonetheless, the implied and observed tariff profiles seem to display large similarities. However, when turning to China's reaction to the revised US tariffs there appears to be a mismatch between what was observed and what was implied (see figure 4). In specific,

[29]The estimate used corresponds to the median elasticity of substitution as used by Chen and Novy (2011).

[30]Please note that it is essential that the model consistent output and factor incomes align as otherwise the model's economy would be in disequilibrium. The BRS-CFRT model cannot accommodate this.

[31]Please note that there are multiple reasons why this division into two periods makes sense. Firstly, we require two periods to compare the two rounds of implied NE tariffs to. Secondly, both periods contain an equal amount of rounds and, as has been described in section 2.3, the behaviour is relatively similar within both periods. That is, the first period appears more passive whilst the second period is characterised by a much more aggressive attitude of the US. As such, the first 3 observed rounds are taken as period 1 and the last 3 observed rounds are taken as period 2.

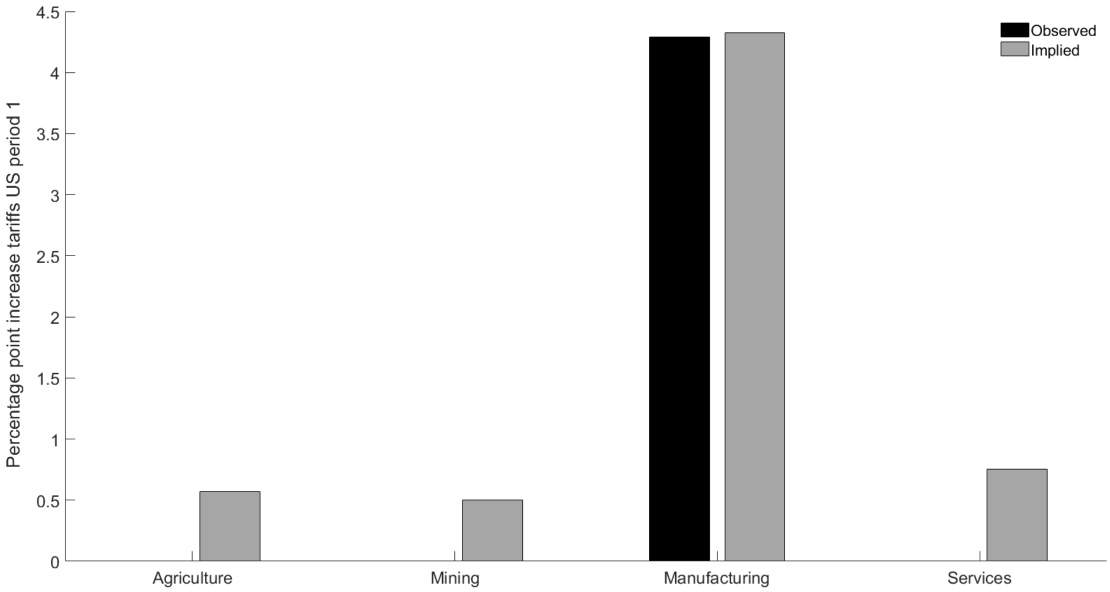

Figure 3: *Comparison of the observed and implied tariffs of the US during period 1, where the calibration is as described in section 5. Observed changes in tariffs have been sourced from Li (2018) at the GTAP product level and have been weighted by GTAP product level trade flows from the UN COMTRADE to derive aggregate tariff change in a given sector. Initial tariffs (2014) were obtained from the UNCTAD-TRAINS database.*

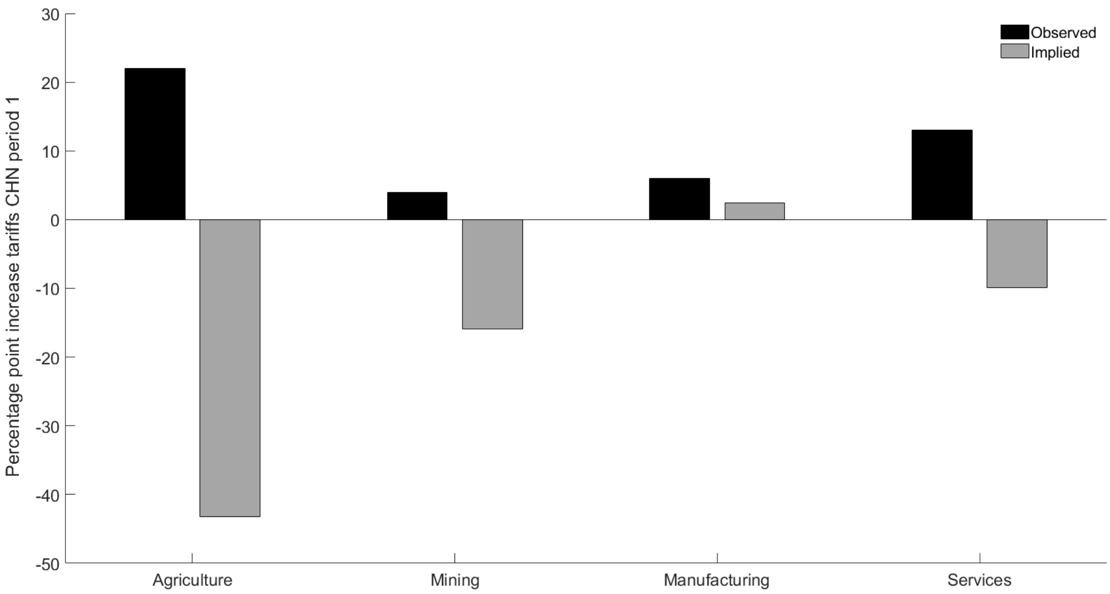

Figure 4: *Comparison of the observed and implied tariffs of China during period 1, where the calibration is as described in section 5. Observed changes in tariffs have been sourced from Li (2018) at the GTAP product level and have been weighted by GTAP product level trade flows from the UN COMTRADE to derive aggregate tariff change in a given sector. Initial tariffs (2014) were obtained from the UNCTAD-TRAINS database.*

the implied tariffs suggests that China would have had a unilateral incentive to lower its tariffs (providing import subsidies) on agriculture, mining and services whilst imposing only a slightly higher tariff on manufacturing goods from the US during the first period. The observed change in tariffs appears to however be the mirror opposite of these implied tariff changes, as tariffs have been mainly placed on agriculture and services (21.84 pp and 13.2 pp increase in tariffs respectively).

This raises the question whether we can rationalise the implied tariffs during the first period. It is important to note, however, that the observations made here are case specific and do not constitute general results regarding the importance of the different aspects of the model. In both the model as well as in reality, the Ramsey principle of optimal taxation seems to be at odds with the data. Broda, Limao and Weinstein (2008) argue that the sectors with the highest trade elasticities should be subject to the least amount of tariffs as these tariffs would be disproportionately distortive. This also appears to hold true for the model used by Ossa (2016). As a result, we would naturally expect that the tariff changes plus the initial tariff levels would yield a ranking in which import tariffs are highest on services, which is than followed by manufacturing, agriculture and mining. It is however apparent that during the first period, the US predominantly increases its tariffs on manufactured goods originating from China, rather than increasing tariffs on services from China. In addition, the observed tariff changes implemented by the Chinese government on agriculture also seems to be at odds with this rationale as agriculture would become the most heavily taxed commodity. In addition, the implied tariffs reveal that the best response of China would be to make agriculture the least taxed commodity.

The exact reason for these deviations requires a more in-depth analysis of the model and would require a paper in and of itself. However, there does seem to be a narrative which seems relatively plausible and could rationalise the implied and observed tariff changes by the US. The reason why services are taxed to a lesser extent could be due to the interdependence between the service sector and other sectors, which is mainly one-sided as services use very few inputs from other sectors whilst the other sectors extensively rely on the service sector (see for an overview of $\beta_{i,s,s'}$ appendix A). This would make a price distortion in the service sector much more costly than a price distortion in the manufacturing sector. Under the Melitz-style market structure, imposing a tariff would than serve to additionally increase the domestic price level as less efficient firms enter the market due to increased tariffs which would provide further incentives to the US government to make the increase of its optimal tariffs on services less substantial. Rationalising the optimal non-cooperative tariffs imposed in the first period by China is slightly more difficult to understand given the fact that negative import tariffs are not frequently encountered (with the notable exception of Balistreri and Hillberry, 2017). It is apparent that the traditional reason for increasing one's tariffs, which relies on raising tariff income and thus aggregate income to compensate for the distortions caused by the implementation of said tariff, is rendered infeasible as China effectively subsidizes its imports across nearly all sectors and obtains *negative* tariff income. So what could rationalise this behaviour?

There are two possible motives which could explain this finding. The first motive pertains to the fact that if all firms operate in a monopolistically competitive market, then input markets are inevitably also imperfectly competitive which results in double marginalisation. A welfare optimising government would want to remedy this double marginalisation and as a result set lower (possibly even negative) tariffs in optimum. This notion was first discussed by CFRT (2019) and posits a second best argument for negative tariffs. However, we find this explanation less plausible in light of the fact that in this case the Chinese manufacturing sector is the sector which most heavily relies on inputs from other sectors and as such should then have been corrected most heavily for this (manufacturing tariffs should have been much lower). The other argument would be that China's real income could also be raised in an alternative manner: by increasing labour or capital income. In fact, whilst labour income remains relatively stable (increases by 2%), capital income drastically increases (by 12%) as a result of the optimal non-cooperative tariff imposed by China during the first period. Intuitively, subsidizing the imports in the agricultural sector serves to reduce firm entry by (roughly) 50% in the Chinese agricultural sector which primarily uses labour. This labour is then reallocated to sectors which predominantly rely on capital such as manufacturing and services which serves to increase the marginal product of capital and thus increases capital income. In addition, this refocuses the US economy on agriculture and as a result eliminates competition from US manufacturing in the ROW.

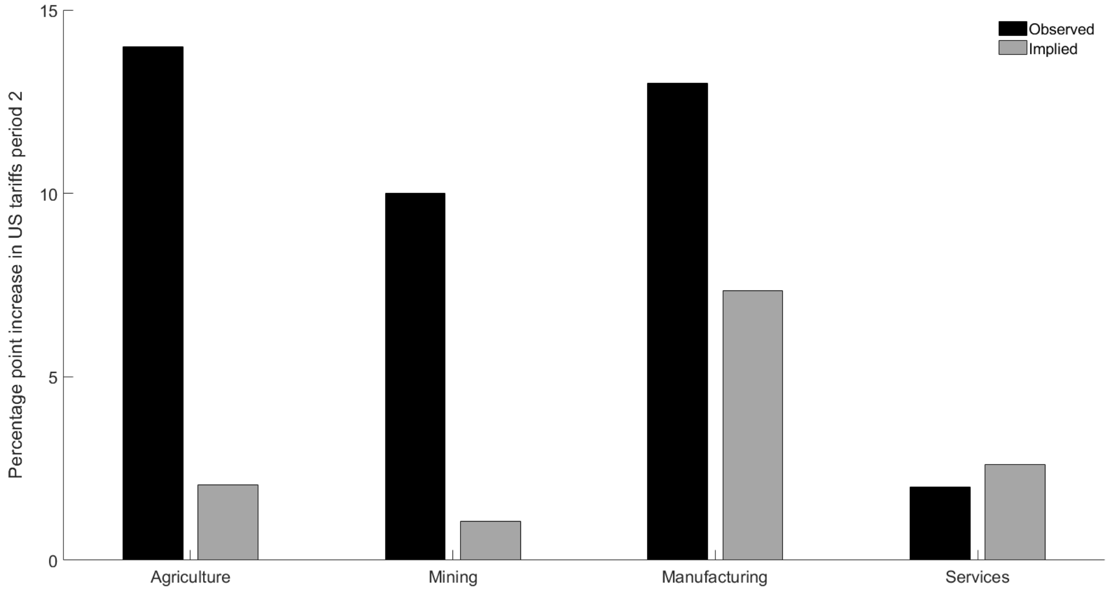

Figure 5: *Comparison of the observed and implied tariffs of the US during period 2, where the calibration is as described in section 5. Observed changes in tariffs have been sourced from Li (2018) at the GTAP product level and have been weighted by GTAP product level trade flows from the UN COMTRADE to derive aggregate tariff change in a given sector. Initial tariffs (2014) were obtained from the UNCTAD-TRAINS database.*

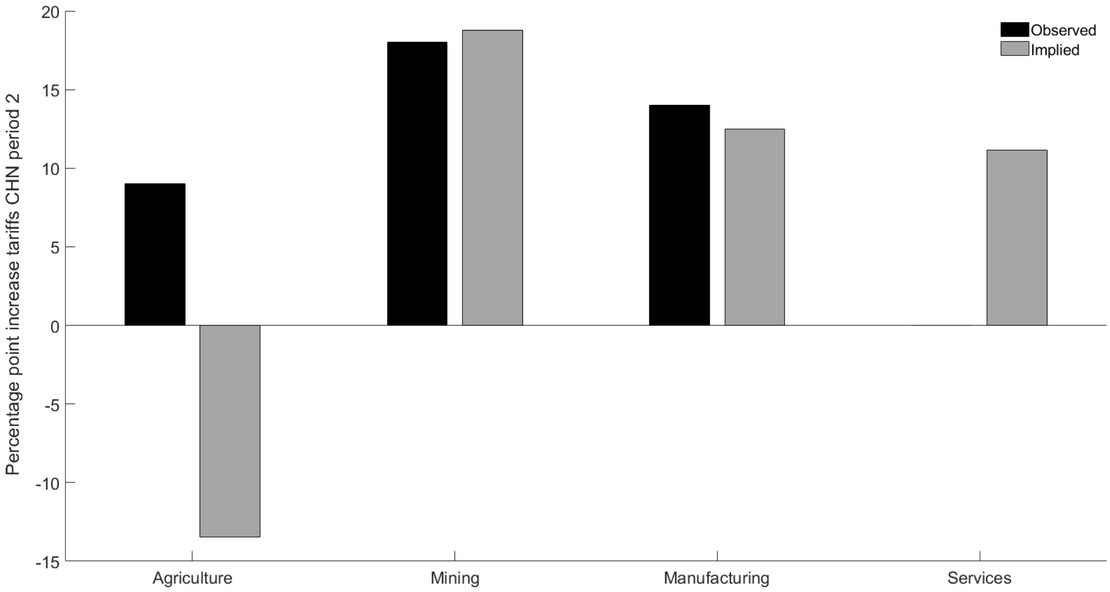

Figure 6: *Comparison of the observed and implied tariffs of China during period 2, where the calibration is as described in section 5. Observed changes in tariffs have been sourced from Li (2018) at the GTAP product level and have been weighted by GTAP product level trade flows from the UN COMTRADE to derive aggregate tariff change in a given sector. Initial tariffs (2014) were obtained from the UNCTAD-TRAINS database.*

During the second period, we observe that the US increases its tariffs across all commodities as has been observed, but are in magnitude rather different from those observed in the second period (see figure 5). The implied change in tariffs on Chinese goods in period 2 could be interpreted as follows. The best response of China to the initial tariff change by the US served to substantially increase manufacturing imports by US final good producers in the manufacturing sector and as a result the US's best response is to increase its tariffs to increase its ToT gains. However, the fact that observed tariffs and implied tariffs do not align could originate from the fact that China did not, in reality, decrease its tariffs in the first period but instead relied on full WTO retaliation. So what would the best response of the US have been if China imposed the tariffs which they did in reality? To that end we constrain the tariff changes to the actual tariff changes for China, but it appears that the US does not have an incentive to retaliate thereafter as the impact of the tariffs imposed by China was negligible and as such period 1 tariffs were optimal. In response to the best response of the US, China's best response, depicted in figure 6, is to once again use a similar tariff profile as used before and thus predominantly subsidizes the imports into the labour intensive agricultural sector whilst imposing positive tariffs on the other sectors which use relatively more capital compared to labour.

6.2 Placing the Implied Tariffs in Perspective

To facilitate the comparison of the aforementioned results to the optimal non-cooperative tariffs found in the literature, we generate trade weighted average tariff changes of the implied Nash equilibrium tariff adjustments, such that it resembles a uniform increase in tariffs. The implied increase in tariffs consistent with a Nash equilibrium in the BRS-CFRT model for the US over the entire trade war would be around the 10 percentage points whilst that of China would be close to zero percentage points. As a result, it appears that China would have had no (non-political) incentive to resort to full WTO retaliation. These results are reminiscent of the results obtained by Balistreri and Hillberry (2017), who estimate an 11 percentage point increase in tariffs by the US on Chinese goods and services during the trade war and a 5 percentage point decrease in Chinese tariffs. As such it is important to note that the decrease in tariffs due to optimal strategic interaction by China is probably not an artifact of permitting only one homogeneous tariff across sectors. It is important to note that we also encountered negative optimal tariffs during this numerical exercise, which likely arose from the IO structure and the complementarity with the primary factors of production rather than the Melitz-style structure. Bouët and Laborde (2018) use the MIRAGRODEP model by CEPII to study optimal non-cooperative tariffs and report *uniform* increases in Nash tariffs of around the 7 percentage points for the US and 3 percentage points for China. The estimates obtained in this paper are in between the estimates obtained by Bouët and Laborde (2018) and Balistreri and Hillberry (2017). We did not find any relatively large NE tariffs as documented by for instance Li, He and Lin (2018).

In addition, the implied welfare changes in the BRS-CFRT model do deviate quite remarkably from what has commonly been documented in the literature. Although the trade war between the US and China is generally perceived as harmful to both nations (See for instance Bollen and Rojas-Romagosa, 2018, Kutlina-Dimitrova and Lakatos, 2017), or to at least to China (Li, He and Lin, 2018), this does not appear to be the case had China played the implied best response. In fact, once China has imposed its optimal tariffs in the first period its welfare increases by about 7.56%, yielding a utility increase of about 7.09% at the end of period 1. In contrast, the tariffs imposed by the US would serve to increase its welfare only marginally (by about 0.02%) and after the implementation of China's optimal tariffs in period 1 the US actually ends period 1 with a loss of 0.04% in welfare. During the second period a similar pattern emerges albeit that the US now manages to improve its welfare through setting higher tariffs across more sectors. Hence, had both China and the US played optimally, the BRS-CFRT implied welfare gains would have been circa 0.15% for the US and 9.59% for China. That is, the trade war would have been a Pareto improvement for both the US and China. However, the BRS-CFRT model also indicates that the strategy chosen by China (full WTO retaliation) would actually have been welfare decreasing. Moreover, under such retaliation the US would have experienced a welfare loss of circa 0.04% and China would have reduced its welfare by about 0.18% by the end of the first period. Although the welfare decreases are small, this would seem to support the idea that the *observed* trade war during the first period has been detrimental to the US, as well as to China.

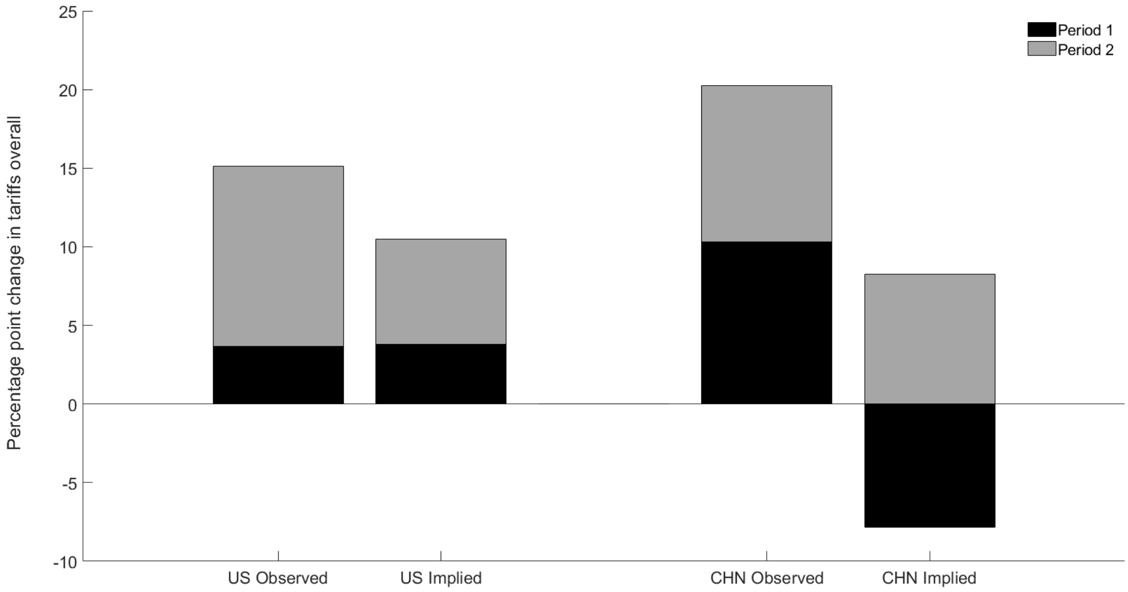

Figure 7: *Trade flow weighted average percentage point increase in implied and observed tariffs set by the US and China during the trade disputed. Breakdown is given by the respective periods in which these changes occurred.*

6.3 A Note on the Importance of HO Motives for Trade Policy

Few papers have actually included multiple primary factors of production in conjunction with vertical linkages and a Melitz-style market structure, all of which to our knowledge rely on the GTAP or CEPII models (see: Balistreri and Hillberry, 2017, Bouët and Laborde, 2018). However, none of these papers have analysed the effect of including capital as well as labour in a comprehensive trade model on optimal trade policy. In other words, would the results have changed drastically if we had not modelled the market clearance conditions for capital? To assess to what extent the analysis would have changed in the absence of capital income in the model, we rerun the calibration and computation exercise for the CFRT model whilst treating capital income as labour income. The optimal non-cooperative tariffs change slightly, increasing US tariffs on manufacturing trade flows originating from China by about 0.2 percentage points in period 1 and raising the tariffs imposed on mining trade flows stemming from the US by China by around 5 percentage points. These changes are thus not major, but also non-negligible.[32] In other words, including multiple primary factors of production does affect trade policy. In addition, whilst it does not seem like the most important feature of the model in this particular scenario, this combination of the BRS and CFRT models does permit some interesting extensions for future research, which will be discussed in the conclusion.

[32]Please note that this seems at odds with the second explanation which was given for the negative tariffs on agriculture, mining and services for China in period 1, which would ascribe a larger role to capital and labour (and thus multiple factors of production) than which is probably warranted. As a result, one could conclude that the motives which were described by Caliendo et al. (2017, 2019) are most likely the driving force behind the negative optimal non-cooperative tariffs.

7 Conclusion

Over the course of this paper we have developed a rich AGE model by combining the models as put forth by Bernard, Redding and Schott (2007) and Caliendo et al. (2017), which lends itself to estimating Nash tariffs using a MPEC approach as advocated by Ossa (2016). After calibrating the BRS-CFRT model on the latest available IO table of the World Input-Ouput Database (WIOD) and the corresponding tariffs from the UNCTAD-TRAINS database, we then derived the implied heterogeneous tariffs for the US-China trade war. When comparing these with the actual increases in tariffs documented by Li (2018) we find that the first response (first three rounds) of the US seems to closely align with the implied tariffs by the BRS-CFRT model across all four sectors (agriculture, mining, manufacturing and services). The implied best response of China to the initial US tariffs however diverges from the tariffs which were actually observed, implying negative tariffs on most goods and services except for manufacturing. The subsequent round mirrors to a large extent the first round of implied non-cooperative tariffs and thereafter neither government can favourably change its tariffs to attain a higher level of welfare. Given the large observed deviation from implied best response tariffs by China, we also computed the optimal response of the US to see what the optimal response would have been to the actual Chinese tariff changes. It appears that the US had no further incentive to retaliate thereafter. Thus, to rationalise the second round of tariffs, one would probably have to account for political economic motives to rationalise the tariff increases by the US during the last three rounds. Allowing for heterogeneous tariffs instead of homogeneous tariffs yielded trade weighted Nash tariffs which were smaller, in absolute magnitude, than the estimates by Balistreri and Hillberry (2017).

These findings can be interpreted as follows. When the trade war is modelled using Nash tariffs using the MPEC methodology as outlined by Ossa (2016), it appears that the US did implement its best response during the initial stages of the trade war with China. The fact that the US initiated the trade war and played its best response could be rationalised by the US resorting to the folk theorems, in which the US is assumed to try to achieve a Nash equilibrium which is sub-optimal for China (and potentially harmful to the US as well) in order to support a more favourable outcome for the US when negotiating. This would make sense if the (perceived) Nash bargaining weights had changed. The subsequent retaliation of China is however markedly different from the implied best response of China to the initial US tariff increases, which would have implied imposing import subsidies across nearly all sectors. This would support the notion that a trade war would have been ”easy” to win for the US, as China had no incentive to retaliate. Nonetheless, China retaliated according to full WTO retaliation. The best response of the US would have been to refrain from further tariff increases in this case, but the tariff increases in the last rounds would indicate other motives, most likely political economic motives, to have been the dominant motives during the second round (instead of a welfare optimising government). This analysis has an important policy implication which is similar to that offered by Balistreri and Hillberry (2017). The results indicate that China had the incentive to not retaliate (imposing import subsidies instead) which indicates that full WTO retaliation is feasible but not necessarily desirable. As a result, current WTO retaliation rules do not guarantee stability in the trading system when parties are left to fight bilateral trade wars.[33]

There are several additional aspects that were documented in the results section which deserve additional attention and might be of interest for the readership as well as policy makers. Our findings challenge the notion that trade wars would be necessarily harmful. In fact, had the US and China played the implied optimal non-cooperative tariffs, then the welfare change for both countries would have been positive. This positive effect stems from the fact that some countries appear to have high tariffs which would be reduced during a trade war when playing there best response to the tariffs

[33] An important question which is raised by this observation is whether some optimal contract could be designed which would resolve this instability, as a government anticipating this (see the US) could easily take advantage of smaller countries due to its larger ToT gains. One could perhaps entertain the idea of having other countries (partly) compensate the damages sustained by the party subject to this form of non-cooperative tariff setting such that it would become a collective problem.

imposed by their opponent. This case has been documented here for China, but it does not seem to be an isolated finding. In particular, Ossa (2016) documents a similar finding for the a world wide trade war for Japan. However, the inclusion of IO linkages as advocated by Caliendo and Parro (2015) and by Erbahar and Zi (2017) when considering trade policy, might have greatly contributed to the fact that optimal non-cooperative tariffs seem to be relatively low. As a result, better informed governments could in a trade war perhaps turn the proverbial other cheek and still win the trade war. An other result found over the course of this study, which seems particularly relevant to the modelling community, is that including multiple primary factors of production does seem, to some degree, alter optimal non-cooperative trade policy. This could be the result of the fact that countries are different in their resource endowments and, as a consequence, changes in tariffs are likely to have heterogeneous effects on the income received from different primary factors of production. This might affect the effective country size (which affects the ToT) to a different degree than if the primary factors of production were to be considered homogeneous. In addition, the primary factors of production are used in differing intensities across sectors and as a result a country might want to capitalise on this heterogeneity when setting its tariffs (changing imports will affect the marginal product of capital and labour as well given the fact that labour and capital are both used in production in combination with other composite goods). In either case, these economic mechanisms warrant additional research.

7.1 Limitations and Future Research

An important limitation is the fact that assessing the descriptive accuracy of the model utilised in this paper effectively resembles a joint test of whether the modelling assumptions, pertaining to the BRS-CFRT model as well as the objective function of the government, are sufficiently realistic to describe the interaction we have witnessed. Whilst it seems to perform well for the US during the first period, at least descriptively, the strategic interaction in the later rounds seems to be inconsistent with the modelling assumption that the government optimises the welfare of its citizens. Given the likely prominence of political economic motives during the second period, it seems like a fruitful area of research to include political economic motives when rationalising non-cooperative tariff setting. These could, for instance, be related to commitment (leaders cannot appear ”weak” as it would compromise re-election) or to inequality (which can readily incorporated into the BRS-CFRT model given the Melitz-style market structure and HO structure which are present in the model). In addition, it remains unclear to what extent the rationalisation provided in section 6 of the implied tariffs is correct. In order to rationalise the implied Nash tariffs, one would like to either derive general conclusions from numeric simulations (for varying parameters) or change the parameters for a specific case, such as the US-China trade war. The problem is that one cannot conduct such an exercise given the fact that the trade, output and income data are consistent with the *observed* IO linkages and expenditure shares such that changing these parameters would inevitably have changed the parameters which were calibrated using the data (in other words the δ parameters). A future endeavour could be to linearize the model in levels around the calibrated parameter set and convert the linear system into hat algebra (hopefully eradicating f). One could then back out the change in trade flows, production and factor income to get a revised starting point for the US-China trade war and one could then repeat the Nash tariff exercise above for a revised parameter to gauge the importance of, for instance, the vertical linkages.

Additionally, it would be interesting to examine whether the model can rationalise the tariffs set by the US during the first period at a less aggregated level using the BRS-CFRT model. At lower levels of aggregation, the fact that some primary resources (mineral deposits) are available in some countries whilst not in others should start to become more pronounced, hence increasing the need for HO or Specific Factors type trade models (in addition to vertical linkages). Another interesting avenue for future research would pertain to introducing dynamics to the model, although this would come with the increased difficulty of defining what Nash tariffs actually are in such a dynamic setting. Moreover, the BRS-CFRT model presented in sections 3 and 4 is static while we have used it to describe the unfolding of dynamic events (note that this would not be problematic for myopic governments). In particular, this may resolve an important problem regarding the implied sequencing of events in the model. If one were to restrict the US to leave its tariffs unchanged during the first period, China would nevertheless want to change its tariffs. This raises the question why China did not move first. A question which, at least with the BRS-CFRT model, we are unable to explain. Naturally, the introduction of dynamics could remedy this, or expose a problem with the modelling assumptions. In fact, if the introduction of dynamics would not remedy this observation, this may lead us to belief that considerations other than welfare motives are more important for the tariffs that are set by policy makers or that additional attention should be paid to modelling the negotiation stage of the game using Nash Bargaining.

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A Data Sources and Calibration Table

A.1 Data Sources

Table 1: Sources of the data used for the calibration of the BRS-CFRT model.

Data on:	Sourced from:	URL:
Bilateral trade flows and final demand between the US, China and the Rest of the World in 2014 on ISIC Rev. 4. (2 digits) level.	WIOD, version 2016	http://wiod.org/database/wiots16
Labour income and capital income (in national currency) on ISIC Rev. 4. (2 digits) level	SEA WIOD, version 2016	http://wiod.org/database/seas16
Exhange rates on 31/12/2014 for converting the Socio-Economic Accounts of the WIOD	Exchange Rates WIOD, version 2016	http://wiod.org/database/wiots16
Tariffs applied to tradeflows between US, China and the Rest of the World (trade flow weighted, group user defined and consistent with WIOD)	UNCTAD-TRAINS using WITS	https://wits.worldbank.org/WITS/WITS/Restricted/Login.aspx
Tariffs applied to trade flows between the US and China during the trade dispute (per round) on GTAP product level	Li (2018) [CARD]	https://www.card.iastate.edu/china/trade-war-data/
Trade flows in 2014 on the GTAP product level to weight changes in aplied tariffs	UN-COMTRADE	https://wits.worldbank.org/WITS/WITS/Restricted/Login.aspx
Country Invariant parameters (σ, ς, θ, ξ)	Caliëndro et al. (2017)	

A.2 Calibration Table

Table 2: Iteration invariant calibrated parameters on WIOD and UNCTAD-TRAINS data.

Parameter	Sectors			
	Agriculture	Mining	Manufacturing	Services
Country Invariant				
σ	6.7	10	6	2.8
ς	5.36	7.76	4.8	2.24
θ	8.6	13	5.1	2.7
ξ	0.68	0.70	0.76	0.60
Country Specific	US			
α	0.0045	0.0099	0.1617	0.8238
β (Agriculture)	0.2383	0.0002	0.0569	0.0013
β (Mining)	0.0073	0.0629	0.1072	0.0061
β (Manufacturing)	0.2080	0.0943	0.3691	0.1175
β (Services)	0.1987	0.1759	0.2428	0.4702
β (Labour)	0.1307	0.1499	0.1057	0.2389
β (Capital)	0.2170	0.5167	0.1183	0.1660
γ	0.3760	0.2249	0.4720	0.5901
	CHN			
α	0.0411	0.0008	0.2721	0.6860
β (Agriculture)	0.1583	0.0007	0.0755	0.0156
β (Mining)	0.0010	0.1159	0.0857	0.0296
β (Manufacturing)	0.2478	0.2539	0.6263	0.4072
β (Services)	0.0735	0.2250	0.1793	0.4265
β (Labour)	0.4939	0.1798	0.0148	0.0676
β (Capital)	0.0255	0.2246	0.0184	0.0535
γ	0.9509	0.4446	0.4457	0.5582
	ROW			
α	0.0300	0.0024	0.1985	0.7692
β (Agriculture)	0.2848	0.0005	0.0424	0.0058
β (Mining)	0.0025	0.4662	0.0574	0.0193
β (Manufacturing)	0.2217	0.1286	0.5944	0.1469
β (Services)	0.2042	0.2518	0.2329	0.5298
β (Labour)	0.1922	0.0312	0.0414	0.1803
β (Capital)	0.0946	0.1217	0.0315	0.1179
γ	0.6702	0.2043	0.5683	0.6046

B Derivations of Key Equations in Standard Levels Form

B.1 Indirect Utility Function

To be shown:

$$V_i = \mathcal{I}_i \cdot \prod_{s=1}^{S} \left(\frac{P_{i,s}}{\alpha_{i,s}}\right)^{-\alpha_{i,s}} \quad \text{and} \quad [Y_{i,s}]_{con} = \alpha_{i,s}\mathcal{I}_i$$

To show that the Cobb Douglass function yields said indirect utility function, we can readily derive the FOCs of the optimisation problem in equation (1) and find that (for some $s',\ s^* \in \mathcal{S}$):

$$Q_{i,s'} = \left(\frac{\alpha_{i,s'}}{\alpha_{i,s^*}}\right)\left(\frac{P_{i,s^*}}{P_{i,s'}}\right) Q_{i,s^*} \implies Q_{i,s^*} = \frac{\alpha_{i,s^*}\mathcal{I}_i}{P_{i,s^*}} \implies [Y_{i,s^*}]_{con} = \alpha_{i,s^*}\mathcal{I}_i$$

where the first implication follows from the fact that this holds for any arbitrary $s' \in \mathcal{S}$ and can then be readily substituted into the budget constraint to obtain the expression for Q_{i,s^*}. Given that this condition is also true for any arbitrary $s^* \in \mathcal{S}$, we substitute this expression into U_i and derive that:

$$V_i = \mathcal{I}_i \cdot \prod_{s=1}^{S} \left(\frac{P_{i,s}}{\alpha_{i,s}}\right)^{-\alpha_{i,s}} \quad \text{as required}$$

B.2 Marginal Costs of Intermediate Good Producers

To be shown:

$$c_{i,s}(\varphi) = \frac{c_{i,s}}{\varphi}; \quad c_{i,s} = \prod_{s'=1}^{S} \left(\frac{P_{i,s'}}{\beta_{i,s',s}}\right)^{\beta_{i,s',s}} \left(\frac{w_i}{\beta_{i,S+1,s}}\right)^{\beta_{i,S+1,s}} \left(\frac{r_i}{\beta_{i,S+2,s}}\right)^{\beta_{i,S+2,s}}$$

To show this we solve the constrained optimisation problem as jointly formulated by equations (3) and (4). This derivation yields similar conditions to those in B.1 due the similarity between functional form specification of utility and the production function.[34] Using the optimality conditions derived from the FOCs of the Lagrangian corresponding to the constrained optimisation problem described by equations (3) and (4), which have been stated in the footnote below, we can readily derive that:

$$q_{i,s}(\varphi) = \varphi \left(\frac{P_{i,s^*}}{\beta_{i,s^*,s}} Q_{i,s^*,s}\right) \prod_{s'=1}^{S} \left(\frac{P_{i,s'}}{\beta_{i,s',s}}\right)^{-\beta_{i,s's}} \left(\frac{w_i}{\beta_{i,S+1,s}}\right)^{-\beta_{i,S+1,s}} \left(\frac{r_i}{\beta_{i,S+2,s}}\right)^{-\beta_{i,S+2,s}}$$

which can be rewritten, after equating $q_{i,s}(\varphi) = 1$, to the obtain an expression for $P_{i,s^*}Q_{i,s^*,s}$. Rewriting and equating to unity yields the following expression:

$$[P_{i,s^*}Q_{i,s^*,s}] = \frac{\beta_{i,s^*,s}}{\varphi} \prod_{s'=1}^{S} \left(\frac{P_{i,s'}}{\beta_{i,s',s}}\right)^{\beta_{i,s's}} \left(\frac{w_i}{\beta_{i,S+1,s}}\right)^{\beta_{i,S+1,s}} \left(\frac{r_i}{\beta_{i,S+2,s}}\right)^{\beta_{i,S+2,s}} = \left(\frac{\beta_{i,s^*,s}}{\varphi}\right) c_{i,s}$$

[34] As a result, differentiating the Lagrangian with respect to $Q_{i,s^*,s}$ and also (not sequentially!) with respect to $Q_{i,s',s}$ for a specific s' yields the following relation between $Q_{i,s',s}$ and $Q_{i,s^*,s}$:

$$Q_{i,s',s}(\varphi) = \left(\frac{\beta_{i,s',s}}{\beta_{i,s^*,s}}\right)\left(\frac{P_{i,s^*}}{P_{i,s'}}\right) Q_{i,s^*,s}(\varphi)$$

Repeating the aforementioned derivation for labour and capital yields the following condition for the optimal amount of labour and capital used in the production of sector s in relation to $Q_{i,s^*,s}$, respectively:

$$L_{i,s} = \left(\frac{\beta_{i,S+1}}{\beta_{i,s^*,s}}\right)\left(\frac{P_{i,s^*}}{w_i}\right) Q_{i,s^*,s} \quad \text{and} \quad K_{i,s} = \left(\frac{\beta_{i,S+2}}{\beta_{i,s^*,s}}\right)\left(\frac{P_{i,s^*}}{r_i}\right) Q_{i,s^*,s}$$

We can, completely analogous to the derivation above, also obtain a similar expression for labour and capital where the LHS is replaced by $w_i L_{i,s}$ ($r_i K_{i,s}$) and where the $\beta_{i,s^*,s}$ on the RHS is replaced by $\beta_{i,S+1,s}$ ($\beta_{i,S+2,s}$). To obtain said expression we can simply express all other inputs in terms of labour or capital. Now using the aforementioned definition we can substitute back into the marginal cost function $c_{i,s}(\varphi)$ and obtain the following expression for marginal costs:

$$c_{i,s}(\varphi) = \sum_{s'=1}^{S} \left(\frac{\beta_{i,s',s}}{\varphi}\right) c_{i,s} + \left(\frac{\beta_{i,S+1,s}}{\varphi}\right) c_{i,s} + \left(\frac{\beta_{i,S+2,s}}{\varphi}\right) c_{i,s} = \frac{c_{i,s}}{\varphi} \quad \text{as required}$$

B.3 Price Indexes and Demand for a Particular Variety

To be shown:

$$q_{ii,s}(\varphi) = \left(\frac{p_{ii,s}(\varphi)}{P_{ii,s}}\right)^{-\sigma_s} \left(\frac{P_{ii,s}}{P_{i,s}}\right)^{-\varsigma_s} Q_{i,s}; \quad q_{ji,s}(\varphi) = \left(\frac{p_{ji,s}(\varphi)}{P^F_{i,s}}\right)^{-\sigma_s} \left(\frac{P^F_{i,s}}{P_{i,s}}\right)^{-\varsigma_s} Q_{i,s}$$

In order to derive the equations governing demand, it is convenient to explicitly define the minimisation problems which a final good producer faces in the first stage of aggregation:

$$\min_{\{q_{ii,s}\}} \quad P_{ii,s}(\vec{q}_{ii,s}(\varphi)) \quad \text{subject to} \quad Q_{ii,s} = \left(M_{i,s} \int_{\varphi^*_{ii,s}}^{\infty} q_{ii,s}(\varphi)^{\frac{\sigma_s - 1}{\sigma_s}} g_s(\varphi) d\varphi\right)^{\frac{\sigma_s}{\sigma_s - 1}} = 1$$

$$\min_{\{q_{ji,s}\}} \quad P^F_{i,s}(\vec{q}_{ji,s}(\varphi)) \quad \text{subject to} \quad Q^F_{i,s} = \left(\sum\nolimits_{j \neq i}^{N} M_{j,s} \int_{\varphi^*_{ji,s}}^{\infty} q_{ji,s}(\varphi)^{\frac{\sigma_s - 1}{\sigma_s}} g_s(\varphi) d\varphi\right)^{\frac{\sigma_s}{\sigma_s - 1}} = 1$$

where the price indexes are defined as follows:

$$P_{ii,s} = M_{i,s} \int_{\varphi^*_{ii,s}}^{\infty} p_{ii,s}(\varphi) q_{ii,s}(\varphi) g_s \varphi d\varphi \quad \text{and} \quad P^F_{i,s} = \sum_{j \neq i} M_{j,s} \int_{\varphi^*_{ji,s}}^{\infty} p_{ji,s}(\varphi) q_{ji,s}(\varphi) g_s(\varphi) d\varphi$$

Here $P_{ii,s}$ is the price per unit of the domestic composite good and $P^F_{i,s}$ is the price per unit of the foreign composite good. Moreover, $q_{ji,s}$ represents the quantity received in i from a producer in j (*id est* excluding the units sent to j but lost due to iceberg trade costs). To rationalise the objective functions in (9), one uses the fact that the FOCs are identical for intermediate good producers in j's sector s with the same productivity parameter φ when exporting to i and that as a result, these intermediate good producers set identical quantities ($q_{ji,s}(\varphi)$) and prices ($p_{ji,s}(\varphi)$). Now let $M_{j,s}$ denote the mass of entrants in the intermediate good production stage in j's sector s which all draw their φs at random from a distribution with as PDF $g_s(\varphi)$. Summing the total expenditure on intermediates from j's sector s with the same φ then yields $M_{j,s} p_{ji,s}(\varphi) q_{ji,s}(\varphi) g_s(\varphi)$. Analoguous to Melitz (2003), not all firms export and as a result, only those with a sufficiently high φ will actually produce for j's market. Let $\varphi^*_{ji,s}$ denote the minimum productivity required to obtain a non-negative profit from exporting from j to i in sector s. Then the total expenditure on intermediate good producers operating in j and selling in i (within sector s) can then simply be obtained by integrating φ over $\varphi^*_{ji,s}$ till infinity, given the fact that $g_s(\varphi)$ is non-zero for $\varphi \geq 1$. To complete the description of the constrained minimisation problems which the final good producer faces, we now also introduce the second stage constrained minimisation problem of the final good producer (which is rather self-explanatory):

$$\min_{\{Q_{ii,s}, Q^F_{i,s}\}} \quad P_{i,s} = P_{ii,s} Q_{ii,s} + P^F_{i,s} Q^F_{i,s} \quad \text{subject to} \quad Q_{i,s} = \left((Q_{ii,s})^{\frac{\varsigma_s - 1}{\varsigma_s}} + \left(Q^F_{i,s}\right)^{\frac{\varsigma_s - 1}{\varsigma_s}} \right)^{\frac{\varsigma_s}{\varsigma_s - 1}} = 1$$

The strategy to determine $q_{ii,s}$ and $q_{ji,s}$ in terms of total demand for sector s output is as follows:

1) determine the demand in terms of $Q_{ii,s}$ and $Q_{ji,s}$ using the first stage CMP[35];

2) determine the demand for $Q_{ii,s}$ and $Q^F_{i,s}$ in terms of $Q_{i,s}$ from the second stage CMP

To illustrate this, we will solve the second minimisation problem and exploit the similarity with the other minimisation problems to derive the desired demand equations. The FOC for some specific productivity $\varphi_k \geq \varphi^*_{ki,s}$ and $\varphi_j \geq \varphi^*_{ji,s}$ imported from some country $k \in \mathcal{N}$ is given by:

$$q_{ji,s}(\varphi_j) = \left(\frac{p_{ki,s}(\varphi_k)}{p_{ji,s}(\varphi_j)}\right)^{\sigma_s} q_{ki,s}(\varphi_k) \quad \forall\, j \in \mathcal{N}$$

for a certain exporting country $k \in \mathcal{N}$. Subsequently substituting this equilibrium condition back into the constraint $Q^F_{i,s} = 1$ yields (after some tedious rewriting):

$$Q^F_{i,s} = \left(\sum\nolimits_{j\neq i}^{N} p_{ki,s}(\varphi_k)^{\sigma_s-1} q_{ki,s}(\varphi_k)^{\frac{\sigma_s-1}{\sigma_s}} M_{j,s} \int_{\varphi^*_{ji,s}}^{\infty} p_{ji,s}(\varphi)^{1-\sigma_s} g_{j,s}(\varphi) d\varphi\right)^{\frac{\sigma_s}{\sigma_s-1}} = 1$$

$$\implies q_{ki,s}(\varphi_k) = p_{ki,s}(\varphi_k)^{-\sigma_s} \left(\sum\nolimits_{j\neq i}^{N} M_{j,s} \int_{\varphi^*_{ji,s}}^{\infty} p_{ji,s}(\varphi)^{1-\sigma_s} g_{j,s}(\varphi) d\varphi\right)^{\frac{\sigma_s}{1-\sigma_s}}$$

Inserting this definition for the optimum quantity of $q_{ki,s}(\varphi_k)$ per unit of the foreign composite good $Q^F_{i,s}$ into the definition of $P^F_{i,s}$ then yields the definition for the foreign price index:[36]

$$P^F_{i,s} = \sum_{j\neq i}^{N} M_{j,s} \int_{\varphi^*_{ji,s}}^{\infty} p_{ji,s}(\varphi)^{1-\sigma_s} \left(\sum\nolimits_{j\neq i}^{N} M_{j,s} \int_{\varphi^*_{ji,s}}^{\infty} p_{ji,s}(\varphi)^{1-\sigma_s} g_{j,s}(\varphi) d\varphi\right)^{\frac{\sigma_s}{1-\sigma_s}} g_s(\varphi) d\varphi$$

$$\implies P^F_{i,s} = \left(\sum\nolimits_{j\neq i}^{N} M_{j,s} \int_{\varphi^*_{ji,s}}^{\infty} p_{ji,s}(\varphi)^{1-\sigma_s} g_s(\varphi) d\varphi\right)^{\frac{1}{1-\sigma_s}}$$

It is important to note that the price index of the foreign composite good in i's sector s thus appears in the expression of the *per unit* (of $Q^F_{i,s}$) demand for variety φ_k on the RHS. As a result we can write the total demand $q_{ki,s}(\varphi_k)$ stemming from i's sector s final good producers for an intermediate variety, represented by φ_k, produced in country k as $q_{ki,s}(\varphi_k) = \left(p_{ki,s}(\varphi)/P^F_{i,s}\right)^{-\sigma_s} Q^F_{i,s}$. Using the second stage CMP of the final good producer in i we find that:

$$Q_{ii,s} = \left(\frac{P^F_{i,s}}{P_{ii,s}}\right)^{\varsigma_s} Q^F_{i,s} \implies Q^F_{i,s} = \left(\frac{P^F_{i,s}}{P_{i,s}}\right)^{-\varsigma} Q_{i,s} \quad \text{with} \quad P_{i,s} = \left(P_{ii,s}^{1-\varsigma_s} + P^{F\,1-\varsigma_s}_{i,s}\right)^{\frac{1}{1-\varsigma_s}}$$

which yields the demand for a particular variety produced with productivity parameter φ when combined with the definition for $q_{ji,s}(\varphi_j)$ in terms of $Q^F_{i,s}$ we derived before, as required. A similar exercise would yield the demand for a domestically produced variety. For completeness, the price index for the domestic variety $P_{ii,s}$ has been provided below:

$$P_{ii,s} = \left(M_{i,s} \int_{\varphi^*_{ii,s}}^{\infty} p_{ii,s}(\varphi)^{1-\sigma_s} g_s \varphi d\varphi\right)^{\frac{1}{(1-\sigma_s)}}$$

[35]CMP is an abbreviation I have adopted here to refer to a *Constrained Minimisation Problem*

[36]Note that $P^F_{i,s}$ is defined as the total expenditure for $Q^F_{i,s}$ and as such $P^F_{i,s} = \sum_{j\neq i}^{N} M_{j,s} \int_{\phi^*_{ji,s}}^{\infty} p_{ji,s}(\phi) q_{ji,s}(\phi) g_s(\phi) d\phi$

B.4 Zero Cut-off Profit (ZCP) Conditions

To be shown:

$$\varphi^*_{ii,s} = \left(\frac{\sigma_s}{\sigma_s - 1}\right)\left(\frac{\sigma_s(1+t_{ii,s})c_{i,s,f}f_{ii,s}}{Y_{i,s}}\right)^{\frac{1}{\sigma_s-1}}(P_{ii,s})^{-1}\left(\frac{P_{ii,s}}{P_{i,s}}\right)^{\frac{1-\varsigma_s}{1-\sigma_s}}\Big((1+t_{ii,s})\tau_{ii,s}c_{i,s}\Big)$$

$$\varphi^*_{ij,s} = \left(\frac{\sigma_s}{\sigma_s - 1}\right)\left(\frac{\sigma_s(1+t_{ij,s})c_{i,s,f}f_{ij,s}}{Y_{j,s}}\right)^{\frac{1}{\sigma_s-1}}(P^F_{j,s})^{-1}\left(\frac{P^F_{j,s}}{P_{j,s}}\right)^{\frac{1-\varsigma_s}{1-\sigma_s}}\Big((1+t_{ij,s})\tau_{ij,s}c_{i,s}\Big)$$

In order to show that these are in fact the Zero Cut-off Profit (ZCP) conditions, we simply have to equate the profits to a destination $j \in \mathcal{N}$ to zero and rewrite to obtain an expression for ϕ in terms of the other variables. As such we have that:

$$\pi_{ij,s}(\varphi^*_{ij,s}) = \left(\frac{1}{\sigma_s - 1}\right)\left(\frac{(1+t_{ij,s})\tau_{ij,s}c_{i,s}}{\varphi^*_{ij,s}}\right)\frac{q_{ij,s}(\varphi^*_{ij,s})}{(1+t_{ij,s})} - c_{i,s,f}f_{ij,s} = 0$$

Using the expression for the optimal price and quantity (see equation (8)) combination for an intermediate good supplier operating in i's sector s and exporting its product to j, we can rewrite the expression above to the following expression for $\varphi^*_{ij,s}$:

$$\implies \left(\frac{1}{\sigma_s - 1}\right)\left(\frac{(1+t_{ij,s})\tau_{ij,s}c_{i,s}}{\varphi^*_{ij,s}}\right)\frac{1}{(1+t_{ij,s})}(p_{ij,s})^{-\sigma_s}\left(P^F_{j,s}\right)^{\sigma_s-1}\left(\frac{P^F_{j,s}}{P_{j,s}}\right)^{1-\varsigma_s}Y_{j,s} = c_{i,s,f}f_{ij,s}$$

$$\implies \left(\frac{\sigma_s}{\sigma_s - 1}\right)^{1-\sigma_s}\left(\frac{(1+t_{ij,s})\tau_{ij,s}c_{i,s}}{\varphi^*_{ij,s}}\right)^{1-\sigma_s}\left(P^F_{j,s}\right)^{\sigma_s-1}\left(\frac{P^F_{j,s}}{P_{j,s}}\right)^{1-\varsigma_s}Y_{j,s} = \sigma_s(1+t_{ij,s})c_{i,s,f}f_{ij,s}$$

$$\implies \varphi^*_{ij,s}\left(\frac{\sigma_s}{\sigma_s - 1}\right)^{-1}((1+t_{ij,s})\tau_{ij,s}c_{i,s})^{-1}\left(P^F_{j,s}\right)\left(\frac{P^F_{j,s}}{P_{j,s}}\right)^{\frac{1-\varsigma_s}{\sigma_s-1}} = \left(\frac{\sigma_s(1+t_{ij,s})c_{i,s,f}f_{ij,s}}{Y_{j,s}}\right)^{\frac{1}{\sigma_s-1}}$$

$$\implies \varphi^*_{ij,s} = \left(\frac{\sigma_s}{\sigma_s - 1}\right)\left(\frac{\sigma_s(1+t_{ij,s})c_{i,s,f}f_{ij,s}}{Y_{j,s}}\right)^{\frac{1}{\sigma_s-1}}\left(P^F_{j,s}\right)^{-1}\left(\frac{P^F_{j,s}}{P_{j,s}}\right)^{\frac{1-\varsigma_s}{1-\sigma_s}}((1+t_{ij,s})\tau_{ij,s}c_{i,s})$$

As required. In a completely analogous way we can obtain the expression for the domestic ZCP condition (i.e. for $\varphi^*_{ij,s}$) and is left (as an exercise) to the readership.

B.5 Free Entry (FE) Conditions

To be shown:

$$\mathbb{E}\left[\pi_{ij,s}(\varphi)\right] = \int_{\varphi^*_{ij,s}}^{\infty}\pi_{ij,s}(\varphi)g_s(\varphi)d\varphi = \left(\frac{\sigma_s - 1}{\theta_s - (\sigma_s - 1)}\right)(c_{i,s,f}f_{ij,s})\left(\varphi^*_{ij,s}\right)^{-\theta_s}$$

To see that this holds true, we first note that the CDF of a Pareto distribution with minimum value $\varphi_{\text{min}} = 1$ and shape parameter θ_s is given by $1 - \varphi^{-\theta_s}$. To compute the expected value of the profits we now use the fact that combining equations (8) and (9) yields:

$$\pi_{ij,s}(\varphi) = \left(\frac{1}{\sigma_s - 1}\right)^{1-\sigma_s}\left(\frac{(1+t_{ij,s})\tau_{ij,s}c_{i,s}}{\varphi}\right)^{1-\sigma_s}\frac{\left(P^F_{j,s}\right)^{\sigma_s-1}}{(1+t_{ij,s})}\left(\frac{P^F_{j,s}}{P_{j,s}}\right)^{1-\varsigma_s}Y_{j,s} - c_{i,s,f}f_{ij,s}$$

Separating the part of $\pi_{ij,s}(\varphi)$ which contains φ and integrating then yields:

$$\mathbb{E}[\pi_{ij,s}(\varphi)] = \int_{\varphi^*_{ij,s}}^{\infty} \pi_{ij,s}(\varphi) g_s(\varphi)\mathrm{d}\varphi = A\int_{\varphi^*_{ij,s}}^{\infty} \left(\varphi^{\sigma_s-1}\right) g_s(\varphi)\mathrm{d}\varphi - (c_{i,s,f}f_{ij,s})\int_{\varphi^*_{ij,s}}^{\infty} g_s(\varphi)\mathrm{d}\varphi$$

$$= \theta_s A\int_{\varphi^*_{ij,s}}^{\infty} \left(\varphi^{\sigma_s-\theta_s-2}\right)\mathrm{d}\varphi - \theta_s(c_{i,s,f}f_{ij,s}\int_{\varphi^*_{ij,s}}^{\infty} \left(\varphi^{-\theta_s-1}\right)\mathrm{d}\varphi$$

$$= \theta_s A\left[\frac{1}{\sigma_s-\theta_s-1}\left(\varphi^{\sigma_s-\theta_s-1}\right)\right]_{\varphi^*_{ij,s}}^{\infty} + \theta_s(c_{i,s,f}f_{ij,s})\left[\frac{1}{\theta_s}\varphi^{-\theta_s}\right]_{\varphi^*_{ij,s}}^{\infty}$$

$$= \left(\frac{\theta_s}{\theta_s-(\sigma_s-1)}\right)\underbrace{A\left(\varphi^*_{ij,s}\right)^{\sigma_s-1}}_{=\pi_{ij,s}(\varphi^*_{ij,s})+c_{i,s,f}f_{ij,s}}\left(\varphi^*_{ij,s}\right)^{-\theta_s} - (c_{i,s,f}f_{ij,s})\left(\varphi^*_{ij,s}\right)^{-\theta_s}$$

$$= \left(\frac{\theta_s}{\theta_s-(\sigma_s-1)}-1\right)(c_{i,s,f}f_{ij,s})\left(\varphi^*_{ij,s}\right)^{-\theta_s} = \left(\frac{\sigma_s-1}{\theta_s-(\sigma_s-1)}\right)(c_{i,s,f}f_{ij,s})\left(\varphi^*_{ij,s}\right)^{-\theta_s}$$

As required. Here we have defined A (to keep the algebra and calculus relatively simple) as:

$$A \equiv \left(\frac{1}{\sigma_s-1}\right)^{1-\sigma_s}\left((1+t_{ij,s})\tau_{ij,s}c_{i,s}\right)^{1-\sigma_s}\frac{\left(P^F_{j,s}\right)^{\sigma_s-1}}{(1+t_{ij,s})}\left(\frac{P^F_{j,s}}{P_{j,s}}\right)^{1-\varsigma_s}Y_{j,s}$$

It is crucial to note that for the definite integrals to be finite, we require that $\theta_s > \sigma_s - 1$ for all $s \in \mathcal{S}$. The free entry condition then states that the expected profits across all potential markets (i.e. across all $i \in \mathcal{N}$) are equal to the entry costs $c_{i,s,f}f_{i,s,e}$ which a company incurs by setting up its business.

B.6 Sectoral Exports and Imports

To be shown:

$$X_{ij,s} \equiv \frac{\lambda_{ij,s}}{1+t_{ij,s}}Y_{j,s} \quad ; \quad X_{ji,s} \equiv \frac{\lambda_{ji,s}}{1+t_{ji,s}}Y_{i,s}$$

where $X_{ij,s}$ represents the value of exports of sector s from country i to country j (net of tariff and thus received by producers in i). Likewise, $X_{ji,s}$ represents the imports in sector s from country j of country i. The value received by a single firm, with production technology φ, of delivering $q_{ij,s}(\varphi)$ units in j from country i is then given by:

$$X_{ij,s}(\varphi) = \frac{p_{ij,s}(\varphi)q_{ij,s}(\varphi)}{1+t_{ij,s}}$$

In order to obtain total exports, one multiplies both the RHS and LHS by the mass of firms with a specific φ (*id est* $M_{i,s}g_s(\varphi)$) such that we have the total value of exports from i to j in sector s for all firms with a specific φ. Subsequently, aggregating over all φ's by taking the integral over φ at both the LHS and the RHS and renaming the left hand side as $X_{ij,s}$ yields the first part of definition 3.

$$X_{ij,s} = M_{i,s}\int_{\varphi^*_{ij,s}}^{\infty}\frac{p_{ij,s}(\varphi)q_{ij,s}(\varphi)}{1+t_{ij,s}}g_s(\varphi)d\varphi$$

$$= \frac{1}{1+t_{ij,s}}\left(M_{i,s}\int_{\varphi^*_{ij,s}}^{\infty}\frac{p_{ij,s}(\phi)q_{ij,s}(\varphi)}{Y_{j,s}}g_s(\varphi)d\varphi\right)Y_{j,s} = \frac{\lambda_{ij,s}}{1+t_{ij,s}}Y_{j,s}$$

As required. Simply interchanging j and i then yields the imports in sector s from country j.

B.7 Simplifying the Price Indexes

To be shown:

$$P_{i,s} = \left((\Lambda_{ii,s})^{\xi_s} + \left(\sum_{j \neq i} \Lambda_{ji,s} \right)^{\xi_s} \right)^{-\frac{1}{\xi_s \theta_s}} \quad \text{where} \quad \xi_s \equiv \frac{(\sigma_s - 1)(1 - \varsigma_s)}{(\sigma_s - 1)(1 - \varsigma_s) - \theta_s(\sigma_s - \varsigma_s)}$$

with $\Lambda_{ii,s}$ and $\Lambda_{ji,s}$ defined as follows:

$$\Lambda_{ii,s} \equiv \left(\frac{\sigma_s}{\sigma_s - 1} (1 + t_{ii,s}) \tau_{ii,s} c_{i,s} \right)^{-\theta_s} B_{ii,s} \quad ; \quad \Lambda_{ji,s} \equiv \left(\frac{\sigma_s}{\sigma_s - 1} (1 + t_{ji,s}) \tau_{ji,s} c_{j,s} \right)^{-\theta_s} B_{ji,s}$$

with $B_{ji,s}$ defined as:

$$B_{ji,s} \equiv \left(\left(\frac{\theta_s}{\theta_s - (\sigma_s - 1)} \right) M_{j,s} \right) \left(\frac{\sigma_s (1 + t_{ji,s}) c_{j,s,f} f_{ji,s}}{Y_{i,s}} \right)^{\frac{\sigma_s - 1 - \theta_s}{\sigma_s - 1}}$$

In order to show that we can rewrite the price indexes as they have been stated above, we first note that the price indexes for the domestically produced composite good ($P_{ii,s}$) and the price for the foreign composite good ($P^F_{i,s}$) are given by the equations in appendix B.3:

$$P^F_{i,s} = \left(\sum\nolimits_{j \neq i}^{N} M_{j,s} \int_{\varphi^*_{ji,s}}^{\infty} p_{ji,s}(\varphi)^{1-\sigma_s} g_s(\varphi) d\varphi \right)^{\frac{1}{1-\sigma_s}} \quad ; \quad P_{ii,s} = \left(M_{i,s} \int_{\varphi^*_{ii,s}}^{\infty} p_{ii,s}(\varphi)^{1-\sigma_s} g_s \varphi d\varphi \right)^{\frac{1}{(1-\sigma_s)}}$$

To condense this expression by implementing the ZCP conditions, we substitute in the expression for optimal price charged by an intermediate good producer of a variety ω produced with productivity φ. In addition we reduce the equation further by recognising the average productivity and substituting it with its explicit expression in terms of the ZCP productivity (see equation (16)). We do this only for $P^F_{i,s}$ (it is important to note that the domestic price index can be simplified in a completely analogous manner) and find that the foreign price index can be simplified as follows:

$$P^F_{i,s} = \left(\sum\nolimits_{j \neq i}^{N} {\varphi^*_{ji,s}}^{-\theta_s} M_{j,s} \int_{\varphi^*_{ji,s}}^{\infty} \left(\frac{\sigma_s}{\sigma_s - 1} (1 + t_{ji,s}) \tau_{ji,s} c_{j,s} \right)^{1-\sigma_s} \varphi^{\sigma_s - 1} \frac{g_s(\varphi)}{{\varphi^*_{ji,s}}^{-\theta_s}} d\varphi \right)^{\frac{1}{1-\sigma_s}}$$

$$\implies P^F_{i,s} = \left(\sum\nolimits_{j \neq i}^{N} {\varphi^*_{ji,s}}^{-\theta_s} M_{j,s} \left(\frac{\sigma_s}{\sigma_s - 1} (1 + t_{ji,s}) \tau_{ji,s} c_{j,s} \right)^{1-\sigma_s} \left(\frac{\theta_s}{\theta_s - (\sigma_s - 1)} \right) \left(\varphi^*_{ji,s} \right)^{\sigma_s - 1} \right)^{\frac{1}{1-\sigma_s}}$$

$$\implies P^F_{i,s} = \left(\sum\nolimits_{j \neq i}^{N} M_{j,s} \left(\frac{\sigma_s}{\sigma_s - 1} (1 + t_{ji,s}) \tau_{ji,s} c_{j,s} \right)^{1-\sigma_s} \left(\frac{\theta_s}{\theta_s - (\sigma_s - 1)} \right) \left(\varphi^*_{ji,s} \right)^{\sigma_s - 1 - \theta_s} \right)^{\frac{1}{1-\sigma_s}}$$

Now we can substitute in the appropriate ZCP equations (see equation (11), interchanging i and j

on the subscripts) and simplify to obtain the following expression for $P^F_{i,s}$:[37]

$$P^F_{i,s} = \Bigg(\sum_{j\neq i}^N \underbrace{\left(\left(\frac{\theta_s}{\theta_s-(\sigma_s-1)}\right) M_{j,s}\right)\left(\frac{\sigma_s(1+t_{ji,s})c_{j,s,f}f_{ji,s}}{Y_{i,s}}\right)^{\frac{\sigma_s-1-\theta_s}{\sigma_s-1}}}_{\equiv B_{ji,s}}$$

$$\left(\frac{\sigma_s}{\sigma_s-1}(1+t_{ji,s})\tau_{ji,s}c_{j,s}\right)^{-\theta_s} \left(P^F_{i,s}\right)^{\frac{(\sigma_s-\varsigma_s)(\sigma_s-1-\theta_s)}{1-\sigma_s}} \left(P_{i,s}\right)^{-\frac{(1-\varsigma_s)(\sigma_s-1-\theta_s)}{1-\sigma_s}} \Bigg)^{\frac{1}{1-\sigma_s}}$$

rearranging terms and bringing all terms containing $P^F_{i,s}$ to the LHS and bringing all terms containing $P_{i,s}$ out of brackets on the RHS yields[38]:

$$P^F_{i,s} = \left(\sum_{j\neq i}^N \Lambda_{ji,s}\right)^{\frac{1}{1-\sigma_s}} \left(P^F_{i,s}\right)^{\frac{(\sigma_s-\varsigma_s)(\sigma_s-1-\theta_s)}{(1-\sigma_s)(1-\sigma_s)}} \left(P_{i,s}\right)^{-\frac{(1-\varsigma_s)(\sigma_s-1-\theta_s)}{(1-\sigma_s)(1-\sigma_s)}}$$

$$\implies \left(P^F_{i,s}\right)^{\frac{(1-\sigma_s)(1-\sigma_s)}{(1-\sigma_s)}-\frac{(\sigma_s-\varsigma_s)(\sigma_s-1-\theta_s)}{(1-\sigma_s)}} = \left(\sum_{j\neq i}^N \Lambda_{ji,s}\right) \left(P_{i,s}\right)^{-\frac{(1-\varsigma_s)(\sigma_s-1-\theta_s)}{(1-\sigma_s)}}$$

$$\implies \left(P^F_{i,s}\right)^{(1-\varsigma_s)/\xi_s} = \left(\sum_{j\neq i}^N \Lambda_{ji,s}\right) \left(P_{i,s}\right)^{-\frac{(1-\varsigma_s)(\sigma_s-1-\theta_s)}{(1-\sigma_s)}}$$

$$\implies P^F_{i,s} = \left(\sum_{j\neq i}^N \Lambda_{ji,s}\right)^{\xi_s/(1-\varsigma_s)} \left(P_{i,s}\right)^{-\xi_s\frac{(\sigma_s-1-\theta_s)}{(1-\sigma_s)}}$$

In a completely analogous manner we can obtain the expression for $P_{ii,s}$ in terms of $P_{i,s}$, which has instead of a sum of Λs just $\Lambda_{ii,s}$ between brackets. Substituting these expression into the overall price index in country i for final goods in sector s yields:

$$P_{i,s} = \left((\Lambda_{ii,s})^{\xi_s} + \left(\sum_{j\neq i}^N \Lambda_{ji,s}\right)^{\xi_s}\right)^{\frac{1}{1-\varsigma_s}} \left(P_{i,s}\right)^{-\xi_s\frac{(\sigma_s-1-\theta_s)}{(1-\sigma_s)}}$$

$$\implies \left(P_{i,s}\right)^{1-\frac{\xi_s(\sigma_s-1-\theta_s)(1-\varsigma_s)}{(\sigma_s-1)(1-\varsigma_s)}} = \left((\Lambda_{ii,s})^{\xi_s} + \left(\sum_{j\neq i}^N \Lambda_{ji,s}\right)^{\xi_s}\right)^{\frac{1}{1-\varsigma_s}}$$

$$\implies \left(P_{i,s}\right)^{-\theta_s\xi_s/(1-\varsigma_s)} = \left((\Lambda_{ii,s})^{\xi_s} + \left(\sum_{j\neq i}^N \Lambda_{ji,s}\right)^{\xi_s}\right)^{\frac{1}{1-\varsigma_s}}$$

$$\implies P_{i,s} = \left((\Lambda_{ii,s})^{\xi_s} + \left(\sum_{j\neq i}^N \Lambda_{ji,s}\right)^{\xi_s}\right)^{-\frac{1}{\theta_s\xi_s}} \quad \text{as required}$$

[37] Note that simple rewriting of the ZCP condition for exporters yields:

$$\varphi^*_{ji,s} = \left(\frac{\sigma_s}{\sigma_s-1}(1+t_{ji,s})\tau_{ji,s}c_{j,s}\right)\left(\frac{\sigma_s(1+t_{ji,s})c_{j,s,f}f_{ji,s}}{Y_{i,s}}\right)^{\frac{1}{\sigma_s-1}} \left(P^F_{i,s}\right)^{\frac{\sigma_s-\varsigma_s}{1-\sigma_s}} \left(P_{i,s}\right)^{-\frac{1-\varsigma_s}{1-\sigma_s}}$$

[38] To see why the exponent simplifies in this way to ξ_s, we simply rewrite and find that :

$$\frac{(1-\sigma_s)(1-\sigma_s)}{(1-\sigma_s)} - \frac{(\sigma_s-\varsigma_s)(\sigma_s-1-\theta_s)}{(1-\sigma_s)} = \frac{1-\sigma_s+\sigma_s\theta_s+\sigma_s\varsigma_s-\varsigma_s-\varsigma_s\theta_s}{(1-\sigma_s)}$$

$$= \frac{\theta_s(\sigma_s-\varsigma_s)-(1-\sigma_s)(1-\varsigma_s)}{(1-\sigma_s)} = \frac{(\sigma_s-1)(1-\varsigma_s)-\theta_s(\sigma_s-\varsigma_s)}{(\sigma_s-1)(1-\varsigma_s)}(1-\varsigma_s) = \xi_s^{-1}(1-\varsigma_s)$$

B.8 Simplifying the Trade Shares

To be shown:

$$\lambda_{ij,s} = \Lambda_{ij,s} \left(\sum\nolimits_{i \neq j}^{N} \Lambda_{ij,s} \right)^{\xi_s - 1} (P_{j,s})^{\theta_s \xi_s}$$

In order to show that this is true we use the ZCP condition in equation (11)[39] in conjunction with the definition of average productivity in equation (16) and find that:

$$\lambda_{ij,s} = \frac{\theta_s}{\theta_s - (\sigma_s - 1)} \varphi_{ij,s}^{*\,\sigma_s - 1 - \theta_s} M_{i,s} \left(\frac{\sigma_s}{\sigma_s - 1} \tau_{ij,s} (1 + t_{ij,s}) c_{i,s} \right)^{1 - \sigma_s} \left(P_{j,s}^F\right)^{\sigma_s - \varsigma_s} (P_{j,s})^{\varsigma_s - 1}$$

$$\implies \lambda_{ij,s} = \Lambda_{ij,s} \left(P_{j,s}^F\right)^{\frac{(\sigma_s - \varsigma_s)(\sigma_s - 1 - \theta_s) + (\sigma_s - \varsigma_s)(1 - \sigma_s)}{1 - \sigma_s}} (P_{j,s})^{\frac{(\varsigma_s - 1)(1 - \sigma_s) - (1 - \varsigma_s)(\sigma_s - 1 - \theta_s)}{1 - \sigma_s}}$$

$$\implies \lambda_{ij,s} = \Lambda_{ij,s} \left(P_{j,s}^F\right)^{-\frac{\theta_s(\sigma_s - \varsigma_s)}{1 - \sigma_s}} (P_{j,s})^{\frac{\theta_s(1 - \varsigma_s)}{1 - \sigma_s}}$$

To simplify these trade shares further, we can now substitute in the price indexes and rewrite them to yield the following equations:

$$\implies \lambda_{ij,s} = \Lambda_{ij,s} \left(\sum\nolimits_{i \neq j}^{N} \Lambda_{ij,s} \right)^{\frac{\xi_s \theta_s (\sigma_s - \varsigma_s)}{(\sigma_s - 1)(1 - \varsigma_s)}} (P_{j,s})^{\frac{\xi_s \theta_s (\sigma_s - \varsigma_s)(\sigma_s - 1 - \theta_s) + \theta_s (1 - \varsigma_s)(1 - \sigma_s)}{(1 - \sigma_s)(1 - \sigma_s)}}$$

$$\implies \lambda_{ij,s} = \Lambda_{ij,s} \left(\sum\nolimits_{i \neq j}^{N} \Lambda_{ij,s} \right)^{\xi_s - 1} (P_{j,s})^{\xi_s \theta_s} \quad \text{as required}$$

B.9 Demand for Sectors from Intermediate Good Production

To be shown:

$$[P_{i,s} Q_{i,s}]_{\text{IO}} = \sum_{s'=1}^{S} \beta_{i,s,s'} \left(\frac{\sigma_{s'} - 1}{\sigma_{s'}} \right) \sum_{j=1}^{N} X_{ij,s'}$$

where $[P_{i,s} Q_{i,s}]_{\text{IO}}$ is the direct demand for goods produced by sector s from all other sectors $s' \in \mathcal{S}$. In order to see why this is the case, we use the equation which governs the amount spend by sector s^* on inputs from sector s per unit of output for a single intermediate good producer in country i's sector $s*$. Using the same equation as used in appendix B.2, multiplied by the total quantity sold by an intermediate good producer in country i's sector s^* to obtain the total expenditure on sector s by a sector s^* intermediate good producer, we find that:

$$[P_{i,s} Q_{i,s,s^*}] = \beta_{i,s,s^*} c_{i,s^*}(\varphi) q_{i,s^*}(\varphi)$$

Lets now decompose the general problem into the total expenditure on sector s by sector s^* for every destination j a producer in i ships its intermediate products to. In other words, what is the demand for sector s output which stems from sector s^* selling to final good producers in sector s^* at destination j. Additionally, note that selling $q_{ij,s}(\varphi)$ units abroad requires the production of $q_{i,s}(\varphi) = \tau_{ij,s} q_{ij,s}(\varphi)$

[39] Note that simple rewriting of the ZCP condition for exporters yields:

$$\varphi_{ij,s}^* = \left(\frac{\sigma_s}{\sigma_s - 1} (1 + t_{ij,s}) \tau_{ij,s} c_{i,s} \right) \left(\frac{\sigma_s (1 + t_{ij,s}) c_{i,s,f} f_{ij,s}}{Y_{j,s}} \right)^{\frac{1}{\sigma_s - 1}} \left(P_{j,s}^F\right)^{\frac{\sigma_s - \varsigma_s}{1 - \sigma_s}} (P_{j,s})^{-\frac{1 - \varsigma_s}{1 - \sigma_s}}$$

units at the production location in i. As such, we have that total expenditure on s by s^* with export destination j, denoted by $[P_{i,s}Q_{i,s,s^*}]_j$, is given by:

$$[P_{i,s}Q_{i,s,s^*}]_j = \int_{\varphi^*_{ij,s^*}}^{\infty} \beta_{i,s,s^*} c_{i,s^*}(\varphi)\tau_{ij,s^*} q_{ij,s^*}(\varphi) M_{i,s^*} g_{s^*}(\varphi) d\varphi$$

$$\implies [P_{i,s}Q_{i,s,s^*}]_j = \beta_{i,s,s^*}\left(\frac{\sigma_{s^*}-1}{\sigma_{s^*}}\right)\int_{\varphi^*_{ij,s^*}}^{\infty} \frac{p_{ij,s^*}(\varphi)q_{ij,s^*}(\varphi)}{(1+t_{ij,s^*})} M_{i,s^*} g_{s^*}(\varphi) d\varphi$$

$$\implies [P_{i,s}Q_{i,s,s^*}]_j = \beta_{i,s,s^*}\left(\frac{\sigma_{s^*}-1}{\sigma_{s^*}}\right) X_{ij,s^*}$$

where the first part is obtained by multiplying both sides of $P_{i,s}Q_{i,s,s^*}$ by the mass of firms with a specific φ (*id est* $M_{i,s^*}g_{s^*}(\varphi)$) such that we have the demand for the final good of sector s from sector s^* from exporting to j for all firms with a specific φ. Subsequently aggregating over all φ by taking the integral over φ at both the LHS and the RHS and renaming the left hand side as $[P_{i,s}Q_{i,s,s^*}]_j$ yields the first equality. Substituting in the definition for p_{ij,s^*} we recognise that the integral is the same as how we have defined X_{ij,s^*} and as a result we have the second and third equalities. In order to now obtain the total expenditure on sector s from IO linkages, one aggregates the last condition over all countries j and sectors s^*, for consistency replaced by s'. This yields the following condition (analogous to CFRT (2017)) for all sectors $s \in \mathcal{S}$:

$$[P_{i,s}Q_{i,s}]_{\text{IO}} = \sum_{s'=1}^{S} \beta_{i,s,s'}\left(\frac{\sigma_{s'}-1}{\sigma_{s'}}\right)\sum_{j=1}^{N} X_{ij,s'} \quad \text{as required}$$

B.10 Factor Market Clearing Conditions

To be shown:

$$w_i L_i = \sum_{s'=1}^{S} \gamma_{i,s'}\left[c_{i,s',f}\left(\frac{\theta_{s'}}{\sigma_{s'}-1}\right) M_{i,s'} f_{i,s',e}\right] + \sum_{s'=1}^{S} \beta_{i,S+1,s'}\left(\frac{\sigma_{s'}-1}{\sigma_{s'}}\right)\sum_{j=1}^{N} X_{ij,s'}$$

$$r_i K_i = \sum_{s'=1}^{S}(1-\gamma_{i,s'})\left[c_{i,s',f}\left(\frac{\theta_{s'}}{\sigma_{s'}-1}\right) M_{i,s'} f_{i,s',e}\right] + \sum_{s'=1}^{S} \beta_{i,S+2,s'}\left(\frac{\sigma_{s'}-1}{\sigma_{s'}}\right)\sum_{j=1}^{N} X_{ij,s'}$$

which represent the factor market clearing conditions for all countries $i \in \mathcal{N}$. To see why this is the clearing condition for labour in i, we have to realise that labour in i is used in the production of goods *as well as* covering the fixed (entry) costs. The fact that the primary factors of production are used in a similar fashion to the outputs of other sectors, yields a familiar expression for demand for labour required for producing the output in other sectors (see appendix B.9). As a result, we have that:

$$[w_i L_i]_{\text{IO}} = \sum_{s'=1}^{S} \beta_{i,S+1,s'}\left(\frac{\sigma_{s'}-1}{\sigma_{s'}}\right)\sum_{j=1}^{N} X_{ij,s'} \quad ; \quad [r_i K_i]_{\text{IO}} = \sum_{s'=1}^{S} \beta_{i,S+2,s'}\left(\frac{\sigma_{s'}-1}{\sigma_{s'}}\right)\sum_{j=1}^{N} X_{ij,s'}$$

To now obtain the total demand for labour, we need to also consider the labour used to cover the fixed costs. Using the definition given for $c_{i,s,f}$ in footnote (12) and the production function for fixed costs, it is straightforward to show that an individual firm exporting from j to i (in sector s) spends:

$$[w_i L_i(\varphi)]_{f,i\rightarrow j,s} = \gamma_{i,s}\left(c_{i,s,f}\right) f_{ij,s}$$

on labour to cover those fixed exporting costs. Once again, aggregating the fixed operating costs over the firms that have a similar φ (i.e. multiplying by $M_{i,s}g_s(\varphi)$) and then aggregating across all φ's that actively produce for the market of j in country i yields that:

$$[w_i L_i]_{f,i\to j,s} = \gamma_{i,s}\left(c_{i,s,f}\right) f_{ij,s} M_{i,s} \int_{\varphi^*_{ij,s}}^{\infty} g_s(\varphi)d\varphi \quad \text{where} \quad \int_{\varphi^*_{ij,s}}^{\infty} g_s(\varphi)d\varphi = \left(\varphi^*_{ij,s}\right)^{-\theta_s}$$

Aggregating the latter condition over the exporting countries $j \in \mathcal{N}$ and sectors $s \in \mathcal{S}$, yields the total labour requirement for the fixed operating costs for both domestic *and* foreign exporting operations. The labour requirement for the entry costs in equilibrium are simply equal to $[w_i L_i]_{fe,s} = \gamma_{i,s}(c_{i,s,f})f_{i,s,e}M_{i,s}$ in sector s in i (i.e. all entrants pay the entry fee). Aggregating over s then yields the total labour requirement for the fixed entry costs. As a result, we thus have that:

$$[w_i L_i]_{fo} = \sum_{s=1}^{S} \gamma_{i,s} M_{i,s} \sum_{j=1}^{N} (\phi^*_{ij,s})^{-\theta_s} c_{i,s,f} f_{ij,s} \quad ; \quad [w_i L_i]_{fe} = \sum_{s=1}^{S} M_{i,s}\gamma_{i,s}(c_{i,s,f}) f_{i,s,e}$$

We now also recognise that the sum across all countries $j \in \mathcal{N}$ is similar to the free entry condition, which can be substituted into this expression to simplify it further. The only thing remaining is to derive an expression for the total amount of labour demanded in i which can readily be obtained from the conditions derived before. Recognising that for factor markets to clear, $w_i L_i = [w_i L_i]_{fo} + [w_i L_i]_{fe} + [w_i L_i]_{IO}$, we obtain the following equilibrium condition for labour:

$$w_i L_i = \sum_{s'=1}^{S} \gamma_{i,s'} M_{i,s'} \left[\sum_{j=1}^{N} (\phi^*_{ij,s'})^{-\theta_{s'}} c_{i,s',f} f_{ij,s'} + c_{i,s',f} f_{i,s',e} \right] + \sum_{s'=1}^{S} \beta_{i,S+1,s'} \left(\frac{\sigma_{s'} - 1}{\sigma_{s'}} \right) \sum_{j=1}^{N} X_{ij,s'}$$

$$\implies w_i L_i = \sum_{s'=1}^{S} \gamma_{i,s'} M_{i,s'} \left(\frac{\theta_{s'}}{\sigma_{s'} - 1} \right) c_{i,s',f} f_{i,s',e} + \sum_{s'=1}^{S} \beta_{i,S+1,s'} \left(\frac{\sigma_{s'} - 1}{\sigma_{s'}} \right) \sum_{j=1}^{N} X_{ij,s'}$$

Importantly, an analogous derivation yields its counterpart for capital, which is equal to:

$$r_i K_i = \sum_{s'=1}^{S} (1 - \gamma_{i,s'}) M_{i,s'} \left(\frac{\theta_{s'}}{\sigma_{s'} - 1} \right) c_{i,s',f} f_{i,s',e} + \sum_{s'=1}^{S} \beta_{i,S+2,s'} \left(\frac{\sigma_{s'} - 1}{\sigma_{s'}} \right) \sum_{j=1}^{N} X_{ij,s'}$$

As required. As remarked, its derivation is somewhat tedious given the fact that one has to account for the demand for capital and labour from the production of goods *and* covering the fixed costs.

B.11 The Mass of Entering Firms

To be shown:

$$M_{i,s} = \sum_{j=1}^{N} X_{ij,s} \left[\left(\frac{\sigma_s \theta_s}{\sigma_s - 1} \right) (c_{i,s,f} f_{i,s,e}) \right]^{-1}$$

In order to obtain an explicit condition for entry and exit, we follow the exposition by CFRT (2017). Firstly, we obtain an explicit expression for $\tilde{\phi}_{ij,s}$ raised to the power of $(\sigma_s - 1)$, which is given by:

$$\tilde{\phi}^{\sigma_s - 1}_{ij,s} \equiv \left[\int_{\phi^*_{ij,s}}^{\infty} \phi^{\sigma_s - 1} g_s(\phi) d\phi \right] = \left(\frac{\theta_s}{\theta_s - (\sigma_s - 1)} \right) \left(\phi^*_{ij,s}\right)^{(\sigma_s - 1) - \theta_s}$$

Subsenquently, one inserts the aforementioned expression into equation (14) (for $\lambda_{ij,s}$ that is), such that the share of the value of intermediates from country i's sector s expressed in terms of the total value of final goods j's sector s is directly related to $\phi^*_{ij,s}$. This yields:

$$\lambda_{ij,s} = M_{i,s} \left(\frac{\theta_s}{\theta_s - (\sigma_s - 1)} \right) \left(\phi^*_{ij,s}\right)^{-\theta_s} \left[\frac{\sigma_s (1 + t_{ij,s}) c_{i,s,f} f_{ij,s}}{Y_{j,s}} \right]$$

In order to express the mass of firms in terms of exports and domestic sales in sector s, one multiplies by $Y_{j,s}/(1+t_{ij,s})$ and aggregates across all possible geographical markets j to obtain:

$$\sum_{j=1}^{N} X_{ij,s} = M_{i,s}\left(\frac{\sigma_s\theta_s}{\theta_s-(\sigma_s-1)}\right)\sum_{j=1}^{N}\left(\phi^*_{ij,s}\right)^{-\theta_s}\left[c_{i,s,f}f_{ij,s}\right]$$

By additionally using the free entry condition, we can further simplify this expression to:

$$\sum_{j=1}^{N} X_{ij,s} = M_{i,s}\left(\frac{\sigma_s\theta_s}{\sigma_s-1}\right)\left[c_{i,s,f}f_{i,s,e}\right] \implies M_{i,s} = \sum_{j=1}^{N} X_{ij,s}\left[\left(\frac{\sigma_s\theta_s}{\sigma_s-1}\right)\left(c_{i,s,f}f_{i,s,e}\right)\right]^{-1} \quad \text{as required.}$$

C Derivations of Key Equations in Calibrated Share Form

C.1 Indirect Utility in Hat Notation

To be shown:

$$\widehat{V}_i = \frac{\widehat{\mathcal{I}}_i}{\prod_{s=1}^{S}\left(\widehat{P}_{i,s}\right)^{\alpha_{i,s}}}$$

Let z' denote the value of the variable z under the revised trade policy vector and let z without the prime denote the value of the same variable under the current policy vector. As $\widehat{z} \equiv z'/z$, we have that the change in indirect utility, denoted by $\widehat{V}_i$, is given by the following expression:

$$\widehat{V}_i = \frac{\mathcal{I}'_i}{\prod_{s=1}^{S}\left(P'_{i,s}\right)^{\alpha_{i,s}}}\frac{\prod_{s=1}^{S}\left(P_{i,s}\right)^{\alpha_{i,s}}}{\mathcal{I}_i} = \frac{\mathcal{I}'_i/\mathcal{I}_i}{\prod_{s=1}^{S}\left(P'_{i,s}/P_{i,s}\right)^{\alpha_{i,s}}} = \frac{\widehat{\mathcal{I}}_i}{\prod_{s=1}^{S}\left(\widehat{P}_{i,s}\right)^{\alpha_{i,s}}} \quad \text{as required}$$

C.2 Trade Shares in Hat Notation

To be shown:

$$\widehat{\lambda}_{ij,s} = \widehat{\Lambda}_{ij,s}\left(\sum\nolimits_{i\neq j}^{N}\frac{\lambda_{ij,s}}{(1-\lambda_{jj,s})}\widehat{\Lambda}_{ij,s}\right)^{\xi_s-1}\left(\widehat{P}_{j,s}\right)^{\theta_s\xi_s}$$

To see why this is the case, we use the definition of trade shares in equation (21) and divide $\lambda'_{ij,s}$ by $\lambda_{ij,s}$, which yields the following expression:

$$\implies \widehat{\lambda}_{ij,s} = \widehat{\Lambda}_{ij,s}\left(\sum\nolimits_{i\neq j}^{N}\frac{\Lambda_{ij,s}}{\sum_{i\neq j}^{N}\Lambda_{ij,s}}\widehat{\Lambda}_{ij,s}\right)^{\xi_s-1}\left(\widehat{P}_{j,s}\right)^{\theta_s\xi_s}$$

where we recognise that $\Lambda_{ij,s}$ is equal to:

$$\Lambda_{ij,s} = \lambda_{ij,s}\left(P^F_{j,s}\right)^{\varsigma_s-\sigma_s}\left(P_{j,s}\right)^{1-\varsigma_s} \quad \text{for } i \neq j$$

And as a result we can thus simplify the condition for $\widehat{\lambda}_{ij,s}$ to:

$$\implies \widehat{\lambda}_{ij,s} = \widehat{\Lambda}_{ij,s}\left(\sum\nolimits_{i\neq j}^{N}\frac{\lambda_{ij,s}\left(P^F_{j,s}\right)^{\varsigma_s-\sigma_s}\left(P_{j,s}\right)^{1-\varsigma_s}}{\sum_{i\neq j}^{N}\lambda_{ij,s}\left(P^F_{j,s}\right)^{\varsigma_s-\sigma_s}\left(P_{j,s}\right)^{1-\varsigma_s}}\widehat{\Lambda}_{ij,s}\right)^{\xi_s-1}\left(\widehat{P}_{j,s}\right)^{\theta_s\xi_s}$$

$$\implies \widehat{\lambda}_{ij,s} = \widehat{\Lambda}_{ij,s}\left(\sum\nolimits_{i\neq j}^{N}\frac{\lambda_{ij,s}}{(1-\lambda_{jj,s})}\widehat{\Lambda}_{ij,s}\right)^{\xi_s-1}\left(\widehat{P}_{j,s}\right)^{\theta_s\xi_s} \quad \text{as required}$$

where we use the fact that all trade shares $\lambda_{ij,s}$ summed across all foreign countries i except j, plus the trade share from j to j is unity. As such we have that $\sum_{i\neq j}^{N}\lambda_{ij,s} = 1-\lambda_{jj,s}$.

C.3 The Price Indexes in Hat Notation

To be shown:

$$\widehat{P}_{i,s} = \left(\lambda_{ii,s} \widehat{\Lambda}_{ii,s}^{\xi_s} + (1-\lambda_{ii,s}) \left(\sum_{j \neq i} \frac{\lambda_{ji,s}}{1-\lambda_{ii,s}} \widehat{\Lambda}_{ji,s} \right)^{\xi_s} \right)^{-\frac{1}{\xi_s \theta_s}}$$

To see why this is true, we now use the definition of $P_{i,s}$ in terms of $\Lambda_{ji,s}$ and $\Lambda_{ii,s}$ as given in equation (18) and obtain the following expression for $\widehat{P}_{i,s}$:

$$\widehat{P}_{i,s} = \left(\left(\Lambda'_{ii,s}\right)^{\xi_s} + \left(\sum_{j \neq i} \Lambda'_{ji,s} \right)^{\xi_s} \right)^{-\frac{1}{\xi_s \theta_s}} P_{i,s}{}^{-\frac{1}{\xi_s \theta_s} \xi_s \theta_s} = \left(\frac{\left(\Lambda'_{ii,s}\right)^{\xi_s}}{P_{i,s}^{-\xi_s \theta_s}} + \frac{\left(\sum_{j \neq i} \Lambda'_{ji,s}\right)^{\xi_s}}{P_{i,s}^{-\xi_s \theta_s}} \right)^{-\frac{1}{\xi_s \theta_s}}$$

To proceed, we first note that the definition in appendix B.7 implies that we can write the trade shares $\lambda_{ii,s}$ and $1-\lambda_{ii,s}$ as:

$$\lambda_{ii,s} = \left(\Lambda_{ii,s}\right)^{\xi_s} \left(P_{i,s}\right)^{\xi_s \theta_s} \quad ; \quad (1-\lambda_{ii,s}) = \sum\nolimits_{i \neq j} \lambda_{ji,s} = \left(\sum\nolimits_{j \neq i} \Lambda_{ji,s} \right)^{\xi_s} \left(P_{j,s}\right)^{\xi_s \theta_s}$$

rewriting those expressions to isolate $P_{i,s}^{\xi_s \theta_s}$ and substituting those expressions in the expression for $\widehat{P}_{i,s}$ readily yields:

$$\widehat{P}_{i,s} = \left(\lambda_{ii,s} \frac{\left(\Lambda'_{ii,s}\right)^{\xi_s}}{\left(\Lambda_{ii,s}\right)^{\xi_s}} + (1-\lambda_{ii,s}) \frac{\left(\sum_{j \neq i} \Lambda'_{ji,s}\right)^{\xi_s}}{\left(\sum_{j \neq i} \Lambda_{ji,s}\right)^{\xi_s}} \right)^{-\frac{1}{\xi_s \theta_s}}$$

$$\implies \widehat{P}_{i,s} = \left(\lambda_{ii,s} \widehat{\Lambda}_{ii,s}^{\xi_s} + (1-\lambda_{ii,s}) \left(\sum_{j \neq i} \frac{\lambda_{ji,s}}{1-\lambda_{ii,s}} \widehat{\Lambda}_{ji,s} \right)^{\xi_s} \right)^{-\frac{1}{\xi_s \theta_s}} \qquad \text{as required}$$