Erasmus University Rotterdam
Erasmus School of Economics

Master Thesis, Financial Economics
Title: Defence Is The Best Offensive; Gold, Real Diversification and Portfolio Optimization

Name Student: Mattia Cremonini,
Student Number: 542312

Supervisor: Philippe J. P. M. Versijp
Second Assessor:

Date Final Version: 29-07-2021

Abstract

The research analyses gold's portfolio properties with the ultimate aim of determining its diversification abilities and contribution to a traditional equity-bond portfolio. During the study we look at gold's correlation structure with equity and how this changes in periods of positive and negative stresses. Moreover we look at gold contribution to the risk-adjusted performances of a long-only 60/40 equity bond portfolio. All our analysis are repeated for multiple timeframes and gold's results are compared to the ones of other diversifiers, namely; broad commodities indexes, TIPS and when possible Cryptocurrencies. Our findings suggest that gold offers a unique and superior portfolio proposition. During the past 20years the addition of a 20% gold holding to a traditional portfolio would have statistically significantly increased the sharp ratio of the latest by almost 40%.

Index

Section 1

A Market Blinded By Success

Introduction

For the past 50years USA investors have enjoyed a unique broad spread equity/bond bull-market that has rewarded aggressive allocations towards risk–on assets, while punishing austere and defensive investment styles. This unprecedented boom was initiated in the early 1980s with the peak of the Fed found rates at 19%, the highest in over 200years. Since then interest rates have been in a secular declined that drove them all the way down to their zero boundary and below, creating the perfect tailwind for equity. During this period the USA stock market, as tracked by the Wilshire 5000 index, has increased in value more than 44folds (4400%), which is roughly the equivalent of an 8% return compounded yearly for 50years.

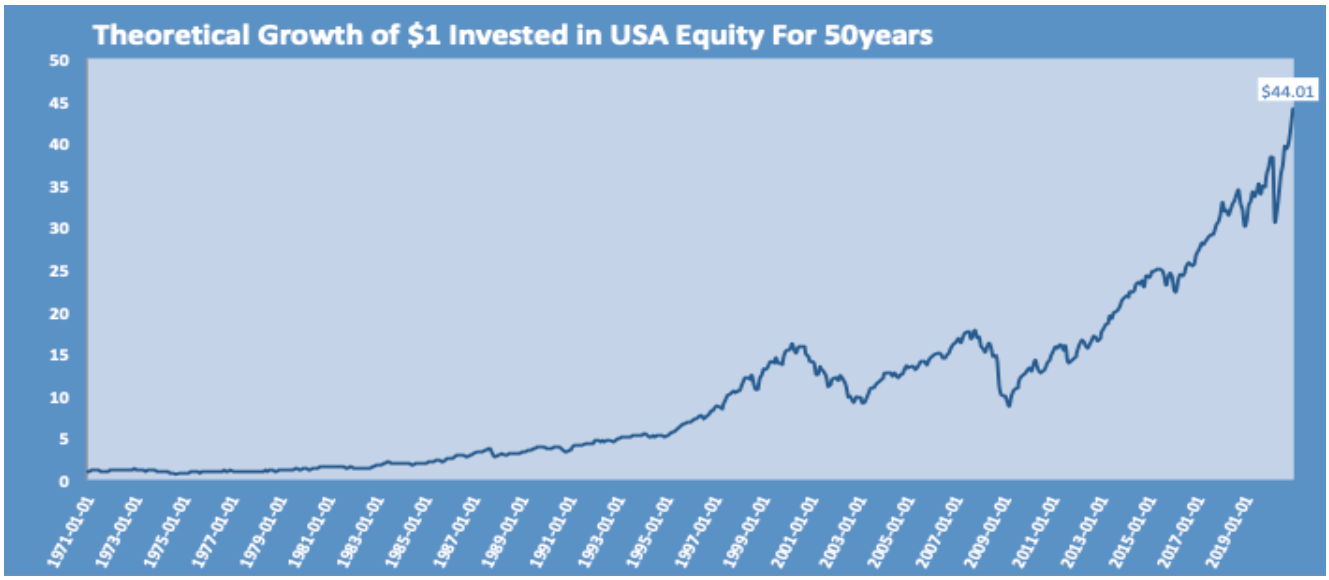

Chart 1: Theoretical growth of $1 invested in the Wilshire 5000 index from 01-01-1997 to 31-12-2020.

This environment has caused many investors to over-allocate to risk assets on the conviction that the current macroeconomic picture, characterized by abundant liquidity and near-zero interest rates, is one that can support a perpetual bull market (Arnott & West; 2021). Long-equity strategies have performed remarkably well in the past half a century (DeLong & Magin; 2009) reinforcing the nowadays popular belief that this asset class is one that nesver disappoints. In our opinion this is a dangerous view to hold and if Japan can teach us anything a passive investment in the Nikkei225 would still be down more than 30% from its peak in 1989. Even the current US bull market has been punctured by some sever losses that would likely be disastrous for most investors, especially the one involved with levered products. In

the 50years considered there have been 12months in which the market returned less than negative 10% and historically we can expect a 30% (or worst) correction roughly once every 10years. Ingenuously many investors believe that they can find protection from such catastrophic drawdowns either by trying their luck in timing the market or by embracing diversification as the key to safely navigate the ever-changing tides of financial markets.

While there exist an overwhelming amount of literature that proves timing the market to be neither a consistent nor feasible strategy (Chang & Lewellen; 1984 - Cuthbertson, Nitzsche & O'Sullivan; 2010), diversification surly is a core ingredient for financial success. However, achieving a well-diversified portfolio might not be as simple as one would think. An extensive amount of research has showed that correlation among assets can change dramatically and in truly unexpected ways during economic stresses (Engle & Colacito; 2006 - Baruník, Kočenda & Vácha; 2016). In particular the correlation between equity and bonds tends to turn positive when interest rates are close or below zero (like in the US in the 1930s or 1970s or now), which invalidates the main pillar on which traditional portfolio's diversification is based on (Connolly, Stivers & Sun; 2005 – Fan & Mitchell; 2017). Because of this volatile nature of assets' correlations many portfolios are left with no protection in periods of crisis, when protection is most needed (Szado; 2009). A recent study from Artemis Capital Management shows that during market crashes a classic well-diversified 60/40 equity-bonds portfolio is as exposed as a leveraged long-growth strategy (Cole; 2020). These findings suggest that the quest for a well-diversified portfolio really is a research for an anti-correlated asset, where with anti-correlated we refer to a consistently negative correlation that translates in material protection when equity and bonds disappoint.

We hypothesize that if such an asset was to be found, holding it regardless of its price action would greatly enhance the overall portfolio performance. The reason why this might be the case can be easily understood when considering a simple example taken from Christopher R. Cole (2016). Imagine you could chose two assets out of a basket of three. The first two assets (asset A & asset B) have positive returns, but they are strongly positive correlate between each other's and to the broad market. The third asset (asset C) loses money overall, but it's anti-correlated to the first two, meaning that it performs best when the others are struggling. Counter-intuitively a portfolio made of the two best performing assets (asset A + asset B) would heavily underperforms a portfolio of anti-correlated assets (asset A + asset C) once we normalize returns for the different levels of risk.

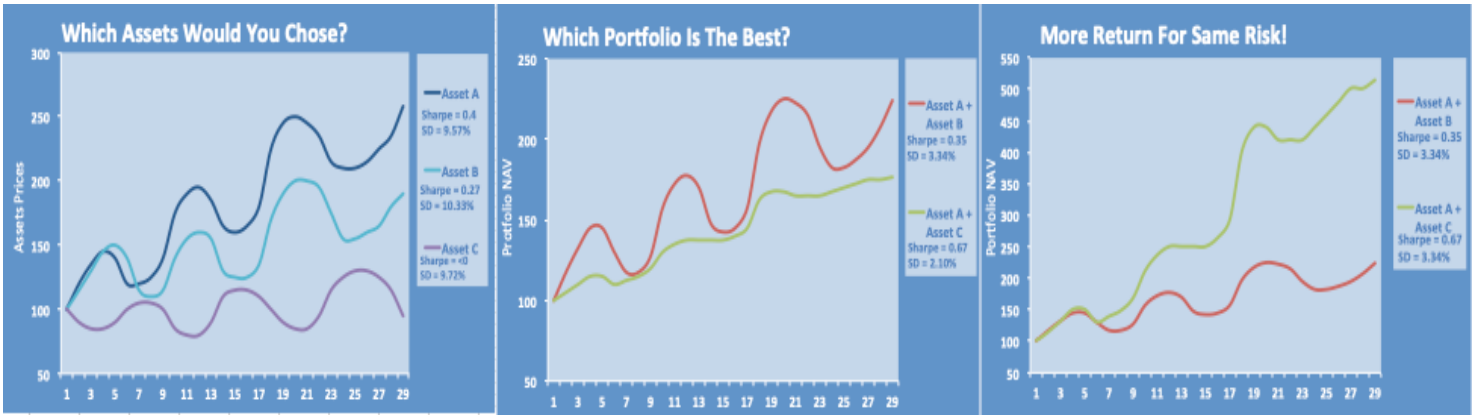

Chart 2: Example of theoretical portfolio constriction using 2 assets out of 3. Asset A; SD = 9.57%, Sharpe = 0.4 - Asset B; SD = 10.13%, Sharpe = 0.27; Asset C; SD = 9.72%, Sharpe < 0. The portfolios in the second panel are constructed by investing $50 in each of the chosen assets, Asset A + Asset B; SD = 3.34%, Sharpe = 0.35 - Asset A + Asset C; SD = 2.10%, Sharpe = 0.67. In the third panel we simply leverage the portfolio A+C up until the point in which it becomes as risky as the portfolio A+B.

The graphs above point to one clear conclusion: "anti-correlation is worth more than excess returns" (Cole; 2020). Therefore, to truly unlock the spoils of diversification we need to find and asset with similar characteristics to the ones of asset C. After extensively reviewing previous literature on the topic we are incline to think that gold might own the needed traits to be that asset and in the present research we aim to test such conviction.

In the following section we will review pervious literature over gold's portfolio properties and highlighting those qualities that makes us believe that the yellow metal has what it takes to be the ultimate portfolio diversifier. In section 3 we present the methodology applied and the dataset used. Section 4 is dedicated to empirically investigate gold's portfolio properties and compare them to the ones of other alternative asset classes, namely; TIPS, broad commodity indexes and (when possible) Cryptocurrencies. In section 6 we will analyse the contribution that each one of the just mentioned assets classes could give to a 60/40 equity-bond portfolio and we will compare their performances on a risk-adjusted base. In section 7 we will offer some concluding remarks over our findings and the study's limitation before closing the paper with the formal conclusion.

Section 2

A Cliff In The Storm

Literature Review

Gold has one of the richest and longest histories among all existing assets. Its versatility and unique properties granted it a place in the world economy as a consumer item, store of value and even as a currency since ancient times. While many other monetary and financial assets have come and gone the yellow metal has resiliently protected and generated wealth for over 4000years (Bernstein; 2012). In more recent history, starting from the nineteenth century, it has been used as the benchmark for one of the largest fixed exchange rate regime that humanity has ever witnessed, the Gold Standard. By the twentieth century the Gold Standard came to an end, but gold remained a vital component of the world monetary system by underpinning the formal exchange rate mechanism, the Bretton Woods system. In August 15th of 1971 the, at the time, President Richard Nixon terminated the Bretton Woods agreements and forever severed the direct link between fiat currencies and the yellow metal. That started a new chapter in gold's history, a chapter in which we have now been for the past 50 years and in which gold's price is left to freely float. It is a little known fact that during the past half a century, in the middle of a gigantic equity bull market, gold has actually outperformed most stock indexes on a price appreciation base. Moreover it has also held its value against all major fiat currencies which have depreciated anywhere between 80% and 90% in gold terms (Stoeferle & Valek; 2021).

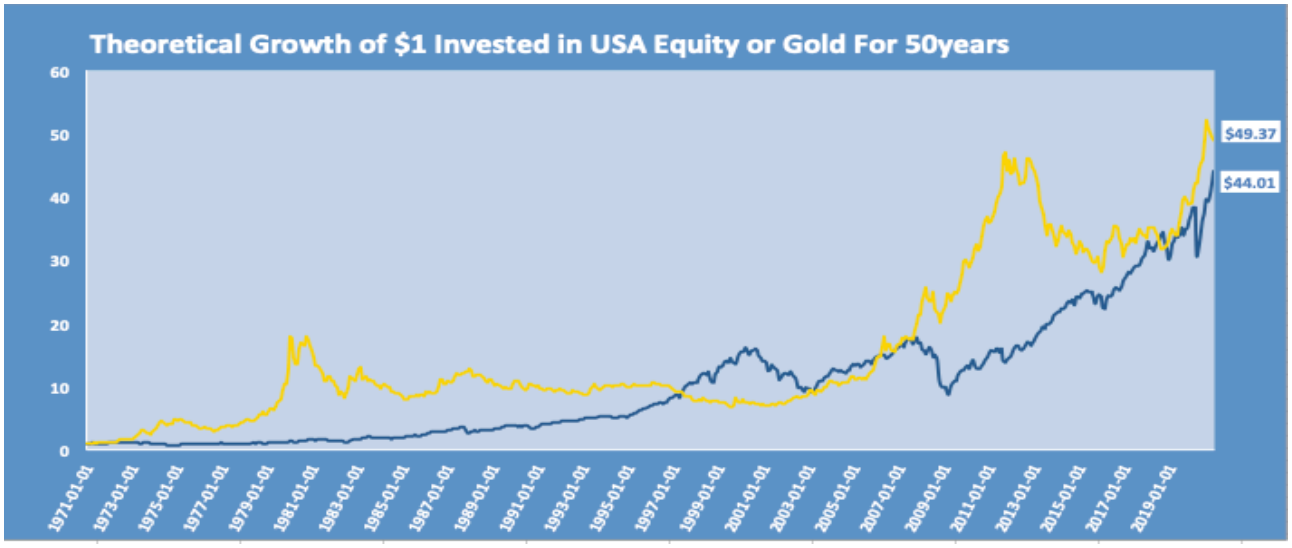

Chart 3: Theoretical growth of $1 invested in the Wilshire 5000 index and $1 invested in Gold from 01-01-1997 to 31-12-2020.

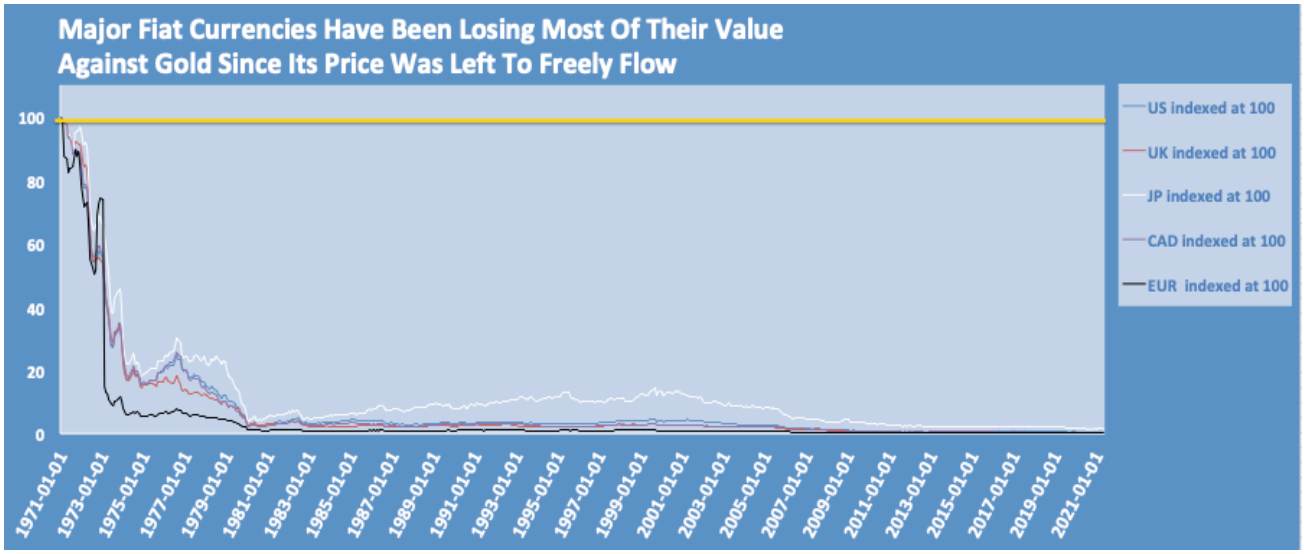

Chart 4: Value in gold of major FIAT currencies (USD, EUR, CAD, GBP and JPY) from 01-01-1971 to 01-01-2021. The value of each currency has been indexed at 100 at the start of the period. For the EUR for dates prior 01-01-1999 (the introduction of the euro) the ECU is used and for dates prior 13-03-1970 (the introduction of the ECU) the German Mark is used.

It is important to understand that gold's historic popularity as a store of value and its monetary properties are not simply due to chances, but are rather the result of unique physical characteristics that set it apart from any other asset. First, gold does not degrade nor perish and it can be modelled into any form one wishes making it possible and easy to store it for long periods of time (World Gold Council; 2019). Furthermore, it is one of the only commodities that can be said to be truly fungible in nature. Gold mined today is identical in properties and quality to the gold mined thousands of year ago, which makes it an incredibly reliable medium of exchange (Taskinsoy; 2019 – Berg; 2020). Moreover the combination of gold's durability with its fungibility also ensures great liquidity and stability to its market. This is the case since, as gold got mined through the centuries but never destroyed, its aboveground stock grew exponentially and now it far exceeds the yearly supply flows (Lawrence; 2003). This guarantees that a sudden surge in gold demand can be quickly and easily met through sales of existing holdings, like jewellery or investment products. At the same time a reduction in supply would have a lower impact on gold than it would have on perishable commodities highly dependent on their yearly production. This makes gold less subject to price spikes relatively to other physical assets and a fundamentally safer store of wealth (Harmston; 1998).

The yellow metal is surlygifted with specific physical characteristics that directly translate in unique financial properties and that set it apart from most (all) other assets. However, it is still to be determined whether these properties can materialize in a consistent anti-correlation with the equity markets. A first hint for answering this question can be found in gold's specific demand function. In the past 10 years roughly 41% of gold's demand came

from industrial applications (34% jewellery, 7% technology). Out of the remaining 59%, investment demands accounted for 42% while the last 17% comes from central banks (World Gold Council; 2020). This particular demand constitutions gives gold a dual nature of high end consumer item and safe haven investment that allows it to perform extraordinary well both in periods of economic prosperity (when jewellery's sales are high) and economic distress (when the demand for safety increases) (Oxford Economics; 2011 – World Gold Council; 2020).

Through the years gold has be the object of meticulous research that has ramified in three main areas of interest, namely its store of value, inflation hedge and safe haven properties. In the present research we mostly focus on gold's safe heaven status, as we believe this characteristic is the ones that most contributes to gold's diversification qualities. We adopt Baur and Lucey's (2010) definition of safe heave, which is a security that is uncorrelated with stocks and bonds during a market crash. From a theoretical stand point a safe heaven asset differs from an anti-correlated one as the latest is consistently zero to negative correlated with the broad market while the former becomes uncorrelated during periods of crashes in particular. We see safe heavens as a sub-category of anti-correlated assets, this is the case as, within an equity-bonds portfolio, they both serve a similar defensive propose. However, safe heavens offer a more attractive proposition as they provide for a similar level of protection without the need of scarifying as much returns during periods of market up-cycle.

To date the academic community is still dividend on this topic with both sides of the argument bring forward compelling evidences. From one side some authors have wondered, "how safe, safe heavens are?" (Kopyl & Lee; 2016). This line of work usually shows that during intense equity crashes everything sells off, even gold, as traders and investors run to liquidity. A recent research on the topic was conducted by Akhtaruzzaman, Boubaker, Lucey and Sensoy (2020), who analysed gold's defensive properties during the 2020 pullback and reached similar conclusions to the one above. Advocates of these researches would argue that cash is the only safe heaven during the storm and that gold hardly benefits from such adverse market events. (Erb & Harvey; 2013 – Erb, Harvey & Viskanta; 2020). On the other side of the argument the World Gold Council (2008, 2020, 2021) and many researchers working for this institution have argued that selling off during sharp market downturns is actually a hidden proof of the yellow metal's insurance abilities. The main idea here is that gold is one of the most liquid markets in the world, second only to oil in the commodity space, and therefore in moments of rough market downturns investors have the opportunity to sell their liquid gold positions (cashing in on their insurances) in order to meet margin calls and cover losses on

less liquid holdings. Thanks to this mechanism gold helps investors to better manage their risky portfolios offering benefits that cannot be quantified by usual metrics like risk-adjusted returns. Moreover there exist an abundant amount of literature showing that, while the yellow metal might very well sell off together with the broad market during the worst days of the crisis, historically it has the tendency to bounce of the lows faster and stronger than broad equity does, therefore creating the feeling of insurance and safety when looked in a broader timeframe (Gürgün & Ünalmış; 2014 - Bredin, Conlon & Potì; 2015 - World Gold Council; 2020).

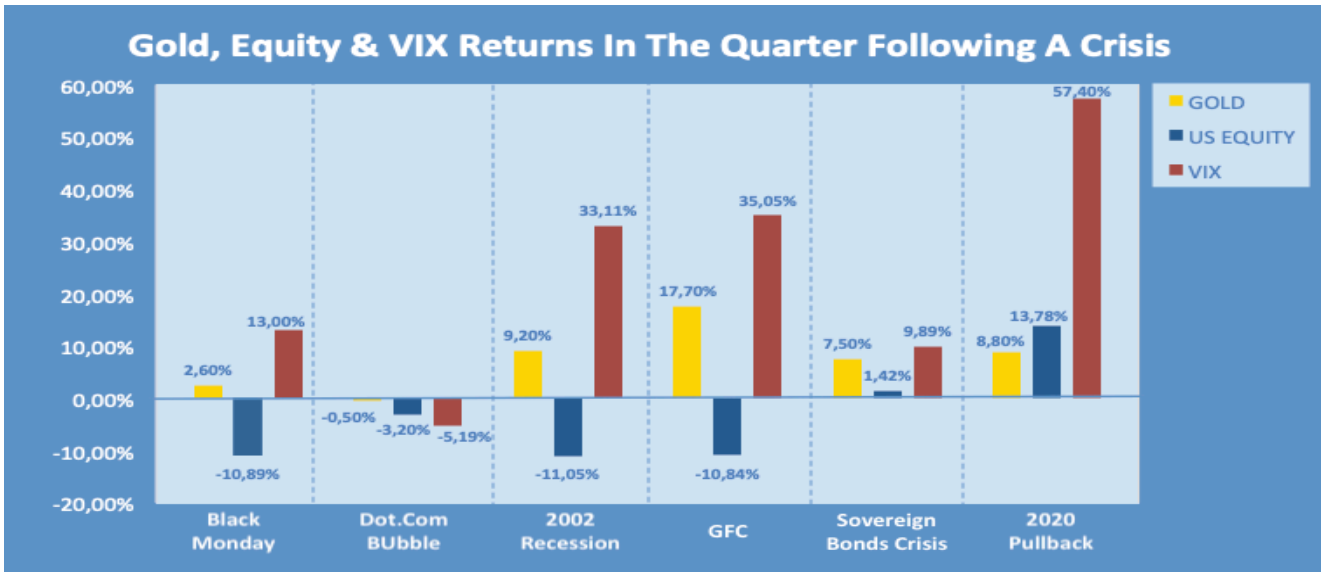

Chart 5: Realized returns of Gold, the Wilshire 5000 index and the VIX index in the quarter following a crisis. Back Monday; 09/1987 – 12/1987, Dot.com Bubble; 03/2000 – 06/2000, 2002 Recession; 03/2002 – 06/2002, Global Financial Crisis (GFC); 10/2007 – 01/2008, Sovereign Bonds Crisis; 02/2011 – 05/2011, 2020 Pullback; 03/2020 – 06/2020

Judging from the chart above in the quarter following a crisis gold has a tendency to outperform stocks and it seems to be more correlated to the VIX index rather than anything else. In fact some researches have showed how gold tends to track indicators of systemic risk like the VIX index or the Ted's spread, especially in periods of crisis. Because of this some have concluded that the yellow metal might very well be the ultimate diversifier able to offer protection even against those risks that are commonly perceived as unavoidable (Hathaway; 2020 - Oxford Economics; 2011).

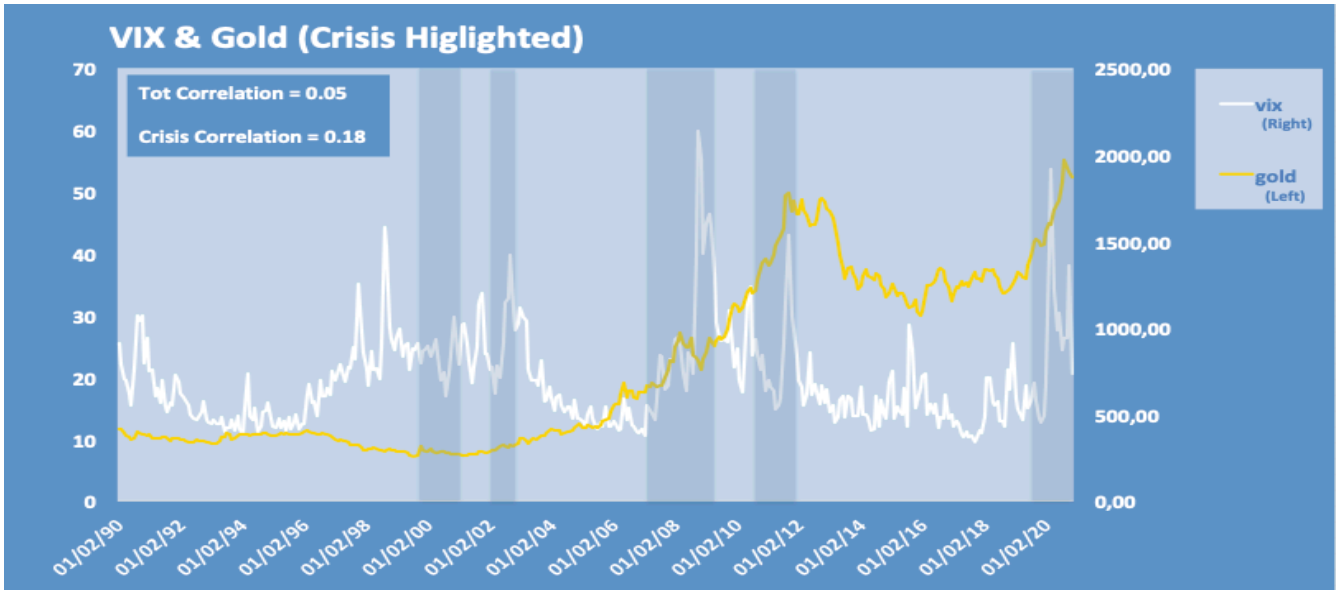

Chart 6: Gold and VIX index price chart from 01-02-1990 to 31-12-2020. Crisis correlation was calculated only for the returns within the highlighted areas, which coincide with full periods of crisis as reported by the Feds: the Dot.com Bubble; 03/2000 – 03/2001, 2002 Recession; 03/2002 – 07/2002, Global Financial Crisis (GFC); 10/2007 – 02/2009, Sovereign Bonds Crisis; 02/2011 – 10/2011, 2020 Pullback; 02/2020 – 12/2020

Finally there has been research that has looked into gold correlation structure with the broad market and its contribution to an equity-bond portfolio. The most striking result among this line of work is that gold not only is overall little correlated with the equity market but it actually grows in negative correlation with it as the market sells off (Baur & Lucey; 2010 - Gürgün & Ünalmış; 2014 - World Gold Council; 2009, 2019, 2020). Additionally such studies often revel that adding gold to a portfolio helps achieving a higher efficient frontier and Sharpe ratio (Jaffe; 1989 – Emmrich & McGroarty; 2013). While we are compelled by these researches in thinking that gold might be our "asset C" we also need to consider that most of the just-presented studies are static, in the meaning that they only look at specific time periods, and unfortunately metrics like correlations or Sharpe ratios are extremely time sensitive (Anand & Madhogaria; 2012). Moreover, little effort is usually put in statistically testing the significance and robustness of the results. Additionally a lot of these studies only look at gold compared to an equity/bond portfolio without even considering the existence of other diversifier. In the current research we wish to expand on the existing literature by using a multi dimensional approach and running our analysis for various time frames. Furthermore we will consider not only gold but also other alternative asset classes namely TIPS, broad commodity indexes and (when possible) Bitcoin. In this way we will obtain some benchmarks to use in the evaluation of gold's contribution to a traditional 60/40 equity-bonds portfolio.

Starting from the just reviewed literature we hypothesize that gold will prove itself as a safe heaven and as a valuable asset to hold both in periods of crashes and not. In particular we expect gold correlation with the broad market to be zero to negative during periods of

crisis and positive otherwise. Finally we hypothesize that a traditional 60/40 equity-bond portfolio would highly benefit from the addition of gold to its holdings and we expect the yellow metal to help the portfolio to achieve a statistically higher risk-adjusted returns.

Section 3

A Useful Road Map

Objectives & Methodology

In the current study we intend to contribute to the quest for a truly well diversified portfolio by presenting the case for gold and analysing its role within a traditional 60/40 equity-bonds portfolio. Our analyses will ramify following two main branches: The study of gold's correlation structure and its contribution to portfolio's performances.

Regarding our first theme, we will follow the work of previous literature (Lawrence; 2003 – Baur & Lucey; 2010 - Bredin Conlon & Potì; 2017) and analyse both gold's average correlation with the broad equity and how this relationship changes in periods of market rally or sell-off as proxied by standard deviation points deviation from the mean. This materializes in obtaining the historical returns distribution for the equity market, divide it in segments depending on their deviation from the mean and then analyse gold's correlation structure with each one of said segments. Practically, we divide the equity returns distribution in: returns 2.5σ points above/below the mean, returns 2σ points above/below the mean and overall returns. Then we look at gold's price action during the days in which those returns materialized and calculate the correlation. As previously said this type of analysis is extremely time sensitive reason why we ran our study in multiple time frames. We obtain data from the year 2000 to the year 2020. We run the analysis for the full period and for the two sub-periods going from 2000 to 2010 and from 2010 to 2020. This study will be done not only for gold but also for all the alternative assets classes considered in the research (for crypto the only possible time frame is the one from 2010 to 2020).

Additionally we will add a second dimension to the analysis by looking at some weekly trailing correlations between equity and gold. With this we wish to obtain a clearer picture of gold correlation structure and a graphical representation of its changes during time. From this first branch of our analysis we aim to understand whether gold can be considered a consistently anti-correlated asset with the broad equity market and whether its correlation structure really differs from the one of other so called alternative assets. The results of this analysis will answer our first hypothesis by revealing whether gold holds the right correlation structure to be considered a safe heaven asset. We expect gold to prove itself as a reliable safe heaven and a unique diversifier.

Regarding the second branch of our analysis we will empirically test gold's contribution to a traditional 60/40 equity-bond portfolio by comparing the performances of

various portfolios some of which holding gold. In total we will treat 7 different portfolios; the traditional portfolio, the traditional portfolio + gold (gold portfolio), the traditional portfolio + commodities (BCOM portfolio), the traditional portfolio + Cryptocurrencies (BTC portfolio), the traditional portfolio + TIPS (TIPS portfolio), the well-diversified portfolio and the well-diversified portfolio + gold (well-diversified gold portfolio). Where with well-diversified portfolio we refer to a portfolio that holds the traditional one plus all other diversifiers but gold. The specifics over the exact assets used for the study will be given in the following sub-section. The portfolios will be constructed Markowitz style (Markowitz; 1959, 1991), which means that we will assign weights to the various assets that compose the portfolios by maximizing returns given specific levels of variance on the base of historical returns and correlations.

In practical terms we need to find the optimal weights (w_i) to be given to each asset that compose our portfolio. Given a basket of assets; 1, 2, … , n with average returns; $\mu_1, \mu_2, \dots, \mu_n$ and average variance; $\sigma_1, \sigma_2, \dots, \sigma_n$ the portfolio return will equate the weighted average return of the assets included given their specific weight:

$$r_p = \sum_{i=1}^{n} r_i w_i$$

Where r_p is the portfolio's return. The portfolio variance can be obtained by multiplying the squared weight of each asset by the asset's variance and then adding two times the weighted average of the assets multiplied by the covariance between them:

$$\sigma_p^2 = \sum_{i=1}^{n} \sigma_i^2 w_i^2 + 2 \sum_{i=1}^{n} \sum_{j=i+1}^{n} w_i w_j \sigma_{ij}$$

Where σ_p^2 is the portfolio's variance and σ_{ij} is the covariance between the assets calculated as the product of the assets' standard deviations and their correlation (ρ): $\sigma_{ij} = \sigma_i \sigma_j \rho_{ij}$. Given all this by maximizing the portfolio return, r_p, for specific levels of variance, σ_p^2, using the portfolio weights will lead to the optimal portfolio for the chosen level of risk (variance).

However, following this procedure without imposing any further restriction would cause a couple of problems. First we would obtain an infinite number of portfolios all identical from a risk-reward prospective. Therefore to narrow it down to one specific portfolio we will chose the one among the obtained ones that maximizes the Sharpe ratio.

Where the Sharpe ratio is defined as the average return earned in excess of the risk-free rate per unite of volatility (Sharpe; 1994). That is to say that we will assign weights to the assets in the portfolios by maximizing the Sharpe ratio directly, which will lead to a single solution. Second, unless engineered otherwise the optimization process will likely fully allocate only in a couple of assets creating unrealistic portfolios. This is the case since the software used for the optimization constructs the portfolios in a purely backward looking manner and it's extremely dependent on the precise timeframe used. Therefore once it identifies the best performing asset it is usually optimal for it to over allocate in it. Moreover this problem is exacerbated by the fact that to construct our portfolios we use indexes rather than single assets; therefore these products already contain a certain degree of diversification. To address these shortcomings we optimize our portfolios following a number of restrictions like minimum and maximum weights for each asset. For the precise framework used for the optimization process see appendix A.

To obtain a first simple metric to evaluate the portfolios we will look at their achieved efficient frontiers, which is the theoretical curve that contains all the optimal portfolios that can be obtained from a given bucket of assets (Merton; 1972 - Markowitz; 1991). To obtain this we can simply construct various portfolios by assigning weights to the assets and maximizing returns given specific levels of variance following the procedure detailed above. Then, by simply keeping track of these portfolios and plotting them on a return-variance graph we will have obtained the efficient frontiers. The benefit of using efficient frontiers is that they give the possibility to make some quick graphical comparison among the various portfolios. This is the case because, by construction, any portfolio that lies below a given frontier is sub-optimal with respect to the frontier's portfolios as these achieve a higher risk reward trade off, with the opposite being true for any portfolio lying above the given frontier. Therefore by simply plotting the efficient frontiers from our 7 portfolios one against each other it will be easy to visually detect which asset blend leads to the best risk rewards trade-off. However in order to make any definitive claim on the findings, formal statistical testing of the various frontiers is required.

This is the case as the analysis of the frontiers alone can easily lead to "fake positives" due to the math underlining their construction. In fact since the precise risk-reward achieved by the portfolio is highly dependent on the covariance among the assets, simply adding one asset to a pre-existing portfolio will almost certainty shift the efficient frontier to the left, hence giving the impression of achieving an higher frontier. The only case in which this would not happen is if the asset was to be completely left out from the portfolio during the optimization process, which would result in achieving the same frontier of the considered

pre-existing portfolio. Because of this reason it is tremendously important to statistically test for differences between the various frontiers considered.

Before start commenting on the technical procedure we wish to underline that the introduction of the other alternative asset classes as comparable is extremely helpful to tackle this problem. This is the case since, while adding gold to the traditional portfolio could lead to a higher frontier simply because of the addition of an extra asset, when comparing various portfolios with the same amount of assets those concerns are minimized. However, while informative, this sort of comparison still cannot be considered a formal hypothesis test reason why we introduce the formal statistical tests next.

In order to test for statistical differences between portfolios performance we will mostly relay on the procedure described in Jobson & Korkie (1981) and then further improved by Memmel (2003). The first step is to decide how to capture portfolio performances. To this aim there are a few possibilities, namely: Sharpe Ratio, Information Ratio, the Treynor Measure, etc. However, given the ultimate scope of our research, previous literature points to the Sharpe ratio as the best metric to be used in this sort of statistical tests as differences in Sharpe ratios can be easily and robustly tested even in relatively small samples given the fact that Sharpe Ratios are asymptotically normal distributed, even if the returns are not (Jobson, Korkie; 1981 - Bailey, Lopez; 2012).

Following the choice of the Sharpe ratio Jobson & Korkie (1981) show that differences between the Sharpe ratios of two portfolios can be tested using a simple z-statistic under the null hypothesis that the two sharps are the same ($Sh_1 - Sh_2 = 0$). While to test multiple portfolios at once a Chi-squared statistic can be used under the null hypothesis that the Sharpe ratios are all equal ($Sh_1 = Sh_2 = ... = Sh_n$). The only modification that has to be made to the normal form of the z and chi-squared statistics is that when calculating them the asymptotic variance of the Sharpe ratios' distributions must be used instead of using the simple variance implied by the data. This modification needs to be made since, as pointed out above, Sharpe Ratios are asymptotically normal distributed. To compute the asymptotic variance we will use the calculation made by Memmel (2003), which correct for a typographical error that was made in the original study by Jobson & Korkie (1981).

Formally writing down the hypotheses testing, we test for the null-hypothesis that the Sharpe ratios, Sh, of the traditional portfolio, p_b, and the traditional portfolio plus gold, p_g, are the same, that is:

$$\mathbf{H_0}: Sh_g - Sh_b = 0$$

Where the Sharpe ratios are defined as the average return earned, Rp, in excess of the risk-free rate, Rf, per unite of volatility, σ, (Sharpe; 1994) or:

$$\text{Sh} = \frac{\text{Rp} - \text{Rf}}{\sigma}$$

Following Jobson & Korkie (1981) all we need to test for $\mathbf{H_0}$ is a simple z-statistic to test for the difference in the means between the two Sharpe Ratios' distributions[1]. For two normally distributed and independent sample the z-statistic can be simply obtained as:

$$\text{z} = \frac{\mu_1 - \mu_2 - \Delta}{\sqrt{\frac{\sigma_1}{n_1} + \frac{\sigma_2}{n_2}}}$$

Where μ_1 and μ_2 are the means of the two samples, Δ is the hypothesized difference between the population means (in our case 0 as we are testing for equal means), σ_1 and σ_2 are the standard deviations of the two populations, and n_1 and n_2 are the sizes of the two samples. However this formulation cannot be directly applied to the case in question, as Sharpe ratios are not normally distributed, but asymptotically normal distributed. However z-statistics can still be used by applying the modification showed in Memmel (2003), namely:

$$\text{Z} = \frac{\mu 1 - \mu 2 - \Delta}{\sqrt{\theta}}$$

Where θ is the asymptotic variance, which is defined as:

$$\theta = \frac{1}{T}\left[2\sigma_1^2\sigma_2^2 - 2\sigma_1\sigma_2\sigma_{1,2} + \frac{1}{2}\mu_1^2\sigma_2^2 + \frac{1}{2}\mu_2^2\sigma_1^2 - \frac{\mu_1\mu_2}{\sigma_1\sigma_2}\sigma_{1,2}^2\right]$$

Where T is the population size, μ_1 and σ_1 are the mean and variance of the first portfolio, μ_2 and σ_2 are the mean and variance of the second portfolio and $\sigma_{1,2}$ is the covariance among them.

The only detail left unsaid in the above description is how to obtain Sharpe ratio's distribution and obtain a sizable sample for running the statistical tests. This will be done through the mean of a Monte Carlo simulation. Starting from the historical returns we will

[1] To be more precise in their original study Jobson & Korkie (1981) suggest that it should be better to look at the transformed differences between the sharps instead of the nominal difference, however Memmel (2003) shows that this is superfluous and it will therefore not be done in the present study for sake of simplicity.

simulate multiple samples of returns, for different time horizon from, with which we will then calculate the Sharpe ratios and obtain the needed distribution.

Once again we are optimistic over gold performance and we believe the analysis will find evidences for it being a beneficial addition to any portfolio. Our results will not only be relevant for a single investor prospective but they will also be pertinent for the financial industry as a whole especially when it comes to the creation of financial retail products or financial consultancy. This is the case as, in most developed countries, when a retail investor wishes to open a brokerage account with a certified financial institution in order to obtain professional advice he is firstly requested by law to fill in a standardized survey (MIFID II1 in EU) with the purpose of capturing his level of risk tolerance. Based on the investor's risk profile revealed by the survey the financial advisor assigned to him will only be able to recommend financial products within the said risk profile. As a consequence it is not unusual for financial institution that offer such retail services to have a catalogue of pre-made products for the various levels of risk tolerance as detailed by MIFID II. Therefore understanding whether gold is a beneficial addition to a traditional portfolio could have important implication for financial advisors and help them design better-structured products.

Data

We will here detail the exact financial assets that we included in the study and the precise data type used. In general terms the assets classes included will be; broad equity indexes, bonds, gold, broad commodities indexes, TIPS and Cryptocurrencies. Everything that is said next is applicable to all these assets a part from Cryptocurrencies, which will be treated separately given the shorter horizon for which data are available. We will collect our data starting from the year 2000 up to the end of 2020, that will give us 20years of data that should be enough to run a robust and relevant study. We will use all data in weekly frequencies looking at the closing prices only. Therefore given the 20years horizon and the weakly frequency we expect to obtain roughly 1090 data points for each asset class. The only data type we need for each asset is the return data from which we can then extrapolate the volatility, correlation matrixes and calculate Sharpe ratios. For equity data we will use the Total Return index, RI instead of simply looking at the returns implied by nominal prices. This procedure is more accurate to follow as it also accounts for dividend payments that would otherwise being ignored. In practical terms the RI shows the theoretical growth in value of a share holding over the considered period, assuming that dividends are re-invested to purchase additional units of said equity at the closing price applicable on the ex-dividend date. The RI is constructed using the annualized dividend yield. This method adds increments

of 1/260th part of the dividend yield to the price each weekday. There are assumed to be 260 weekdays in a year, ignoring market's holidays.

Regarding the precise assets included in the study, to construct the traditional portfolio we use the following indexes for the equity part: MSCI USA; MSCI ex-USA, MSCI CORE EM. And for the bond share of the portfolio: Barclays US Aggregate, Barclays Global aggregate ex and the JPMorgan EM Global bond index. These assets were selected by following the example of previous literature that has already used portfolio structured in similar ways (Jaffe; 1989 – World Gold Council; 2021). Moreover Willis Tower (2019) and the Global Alternative Survey (2017) both point to this assets blend as a typical basic pension found portfolio. In addition to the components of the traditional portfolio we also have to choose the precise alternative assets to be included. We propose to use, gold spot price as quoted in the London metal exchange, the Bloomberg Commodity Index as our broad commodity index, US 10years TIPS and Bitcoin as a proxy for Cryptocurrencies. Additionally we will also use the Wilshire 5000 Index and the S&P 500index as a benchmarks and proxies for the broad equity market. A part for Cryptocurrencies all the afford-mentioned data have been obtained through Bloomberg.

With regards to Bitcoin it is impossible to aim for a 20years time frame as this asset has been in circulation only starting from 2007. It is also hard to find reliable data that date back to its origins. The best option seems to use the data provided by IntoTheBlock, which is one of the world largest Cryptocurrencies data provider. From here we managed to extrapolate reliable weekly price data for Bitcoin starting from 2010–01-01, up until the end of 2020. That is roughly a 10years time horizon that in weekly frequency provides a total of 545 data points. Has explained in the methodology section all analyses will be ran both on a long time horizon 20years and two shorter ones 10years (2000-2010; 2010-2020), Bitcoin will only be included in the latest of this two time frames. All the data used are reported in table 1 where in depth descriptive statistics are offered for the three time periods considered. We remind the reader that the data used are in weekly frequencies and so are the descriptive statistics offered next.

	2000-2020							
	Variables	**Mean**	**Min**	**Median**	**Max**	**SD**	**Skewness**	**Kurtosis**
EQUITY	MSCI USA	0.16%	-7.39%	0.27%	7.10%	0.025	-0.61	9.46
	MSCI EX-USA	0.11%	-7.93%	0.27%	7.10%	0.025	-0.96	11.04
	MSCI CORE EM	0.17%	-8.95%	0.38%	7.88%	0.029	-0.41	8.79
BONDS	Barclays Aggregate US	0.09%	-2.70%	0.06%	2.74%	0.011	-0.13	3.98
	Barclays Global aggregate ex USA	0.11%	-3.01%	0.18%	2.79%	0.011	-0.52	7.22
	JPMorgan Global EM	0.16%	-3.54%	0.21%	3.04%	0.012	-2.09	32.27
OTHER	TIPS	0.10%	-2.37%	0.16%	2.77%	0.009	-0.46	8.4
	BCOM	0.00%	-7.61%	0.07%	5.52%	0.022	-0.71	3.03
	GOLD	0.19%	-7.17%	0.27%	5.72%	0.024	-0.14	6.40
	2000-2010							
	Variables	**Mean**	**Min**	**Median**	**Max**	**SD**	**Skewness**	**Kurtosis**
EQUITY	MSCI USA	0.06%	-7.39%	0.18%	7.33%	0.028	-0.52	8.34
	MSCI EX-USA	0.11%	-8.53%	0.32%	6.99%	0.028	-0.99	10.68
	MSCI CORE EM	0.24%	-9.33%	0.47%	8.61%	0.033	-0.44	8.79
BONDS	Barclays Aggregate US	0.13%	-2.73%	0.05%	2.84%	0.012	0.00	3.32
	Barclays Global aggregate ex USA	0.14%	-3.12%	0.17%	2.91%	0.012	-0.34	6.82
	JPMorgan Global EM	0.16%	-3.63%	0.27%	3.23%	0.014	-1.70	29.22
OTHER	TIPS	0.12%	-2.55%	0.19%	3.04%	0.009	-0.44	8.51
	BCOM	0.11%	-7.61%	0.02%	5.52%	0.024	-0.76	5.58
	GOLD	0.30%	-7.74%	0.44%	5.84%	0.026	-0.20	7.00
	2010-2020							
	Variables	**Mean**	**Min**	**Median**	**Max**	**SD**	**Skewness**	**Kurtosis**
EQUITY	MSCI USA	0.29%	-6.87%	0.37%	5.61%	0.023	-0.70	11.00
	MSCI EX-USA	0.15%	-6.39%	0.25%	7.10%	0.022	-0.83	10.20
	MSCI CORE EM	0.13%	-7.14%	0.33%	6.82%	0.025	-0.38	5.84
BONDS	Barclays Aggregate US	0.05%	-2.92%	0.09%	2.37%	0.010	-0.38	4.70
	Barclays Global aggregate ex USA	0.10%	-3.01%	0.16%	3.01%	0.011	-0.45	6.98
	JPMorgan Global EM	0.12%	-2.16%	0.17%	2.20%	0.010	-2.99	29.62
OTHER	TIPS	0.08%	-2.30%	0.11%	2.54%	0.009	-0.50	7.93
	BCOM	0.09%	-7.08%	0.03%	5.57%	0.022	-0.86	6.55
	GOLD	0.10%	-5.85%	0.17%	5.52%	0.021	-0.12	4.38
	Bitcoin	3.28%	-35.8%	1.22%	69.7%	0.141	1.95	11.96

Section 4

The First Branch

Results & Discussion - Correlation's Structures On The Downside

We will here present the results and a preliminary discussion for the first part of our correlation analysis. Next you can find a summary of our results over the correlation between the S&P 500 with gold, commodities, TIPS and Bitcoin for the three timeframes considered. Each chart is divided in three sections, the total correlation between the assets, the correlation during days in which the S&P500 is down 2σ points or more and the correlation during days in which the S&P500 is down 2.5σ points or more. Given the timeframes considered (2000-2020, 2000-2010, 2010-2020) a downward 2σ points from the mean translates in a S&P 500 return respectively of; -2.43%, -2.71% and -2.09%. While a downward 2.5σ points from the mean translates in a return respectively of; -3.05%, -3.39% and -2.63%. We remind the reader that we seek to find evidences of anti-correlation between gold (or any other alternative asset) and the equity market, here proxied by the S&P 500. For an asset to prove itself as anti-correlated it needs to not only being overall zero/negative correlated with the market, but, more importantly, it also has to maintain such correlation when most needed; in periods of equity sells off.

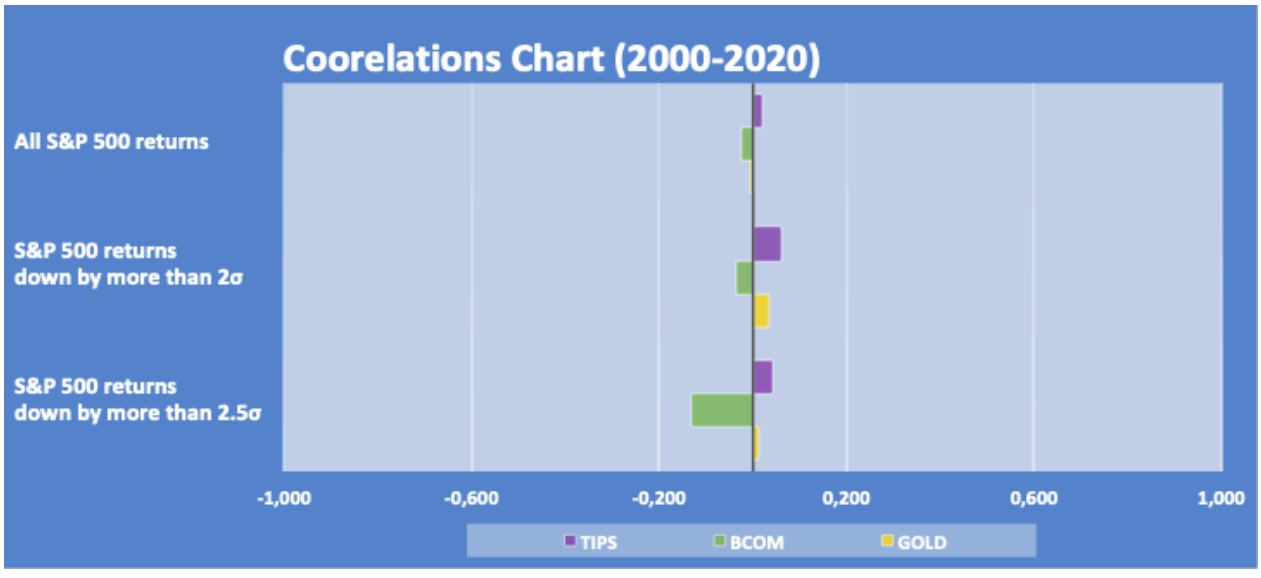

Chart 7: Daily Correlation chart between S&P 500 with gold, BCOM and TIPS for the full period 01-01-2000 to 31-12-2020, for those days in which S&P 500 returns are down 2σ points from the mean and for those days in which S&P 500 returns are down 2.5σ points from the mean.

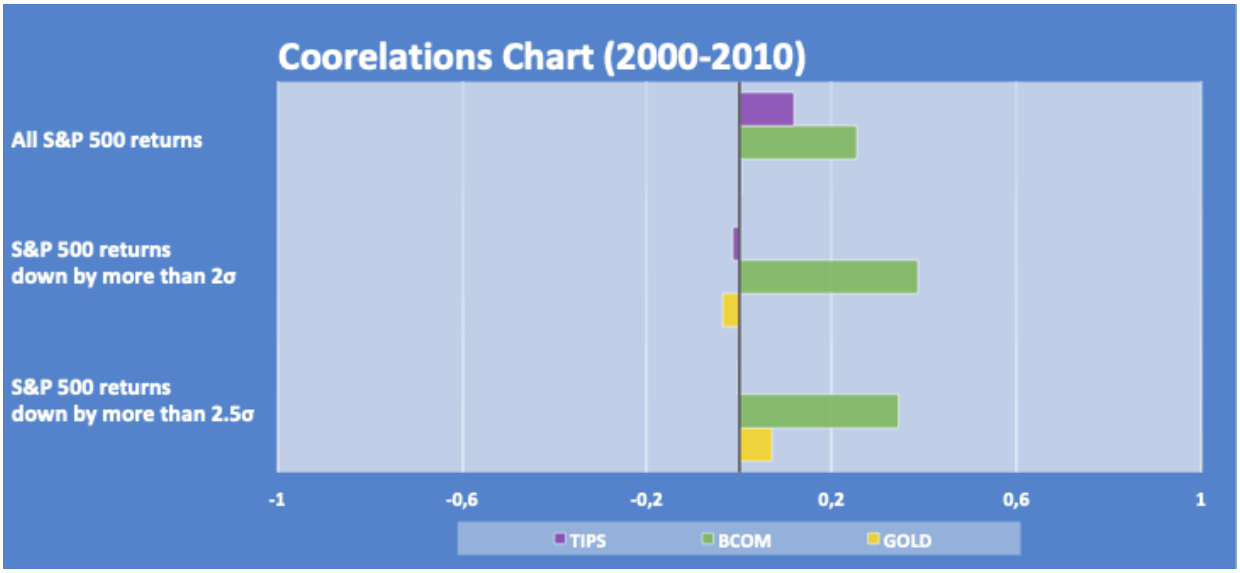

Chart 8: Daily Correlation chart between S&P 500 with gold, BCOM and TIPS for the full period 01-01-2000 to 01-01-2010, for those days in which S&P 500 returns are down 2σ points from the mean and for those days in which S&P 500 returns are down 2.5σ points from the mean.

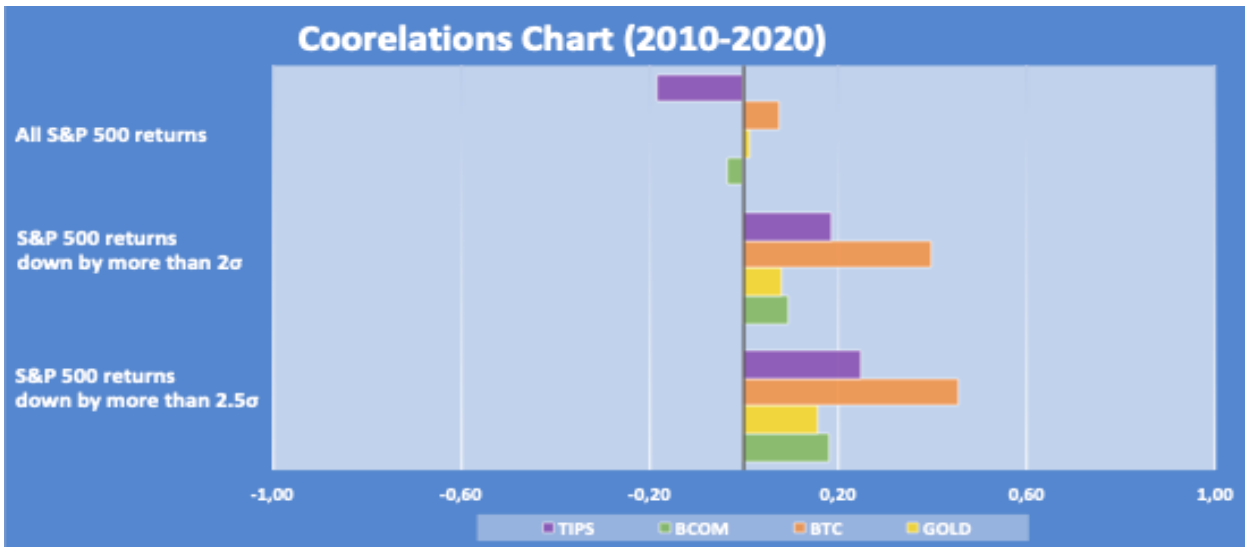

Chart 9: Daily Correlation chart between S&P 500 with gold, Bitcoin (BTC), BCOM and TIPS for the full period 01-01-2010 to 31-12-2020, for those days in which S&P 500 returns are down 2σ points from the mean and for those days in which S&P 500 returns are down 2.5σ points from the mean.

Looking at the full period (first chart), it seems that all the alternative assets considered can make a compelling case for themselves being solid diversifiers. Both gold and TIPS grow in positive correlation during market sells off, however these increases are minimal and economically irrelevant. For gold in particular the correlation fluctuates from -0.005 in the full returns case to 0.033 when the S&P 500 in down more than 2σ points and 0.011 when the equity index is down more than 2.5σ points. The TIPS follow a similar pattern. Judging from this first timeframe, the best diversifier seems to be the BCOM. This is the case as the commodity index does not simply maintain a zero correlation with the broad market, but it actually grows in negative correlation as the market sells off moving from a correlation of -

0.024 when compared to the full S&P 500 returns to a correlation of -0.13 when looking only at those returns down more than 2.5σ points from the mean.

However, the allure of the BCOM fades away in two sub-periods. We have already discussed extensively in the previous sections how correlations are a volatile metrics and extremely time sensitive. This becomes strikingly clear when looking at the charts above especially for the case of BCOM. While the commodity index performs well in the full period, in the 2000-2010 sub-sample it would be hard to even consider it a diversifier at all. The correlation with the full S&P 500 returns is 0.25 and it grows as high as 0.38 when looking at the S&P 500 returns down 2σ points from the mean.

The TIPS correlation structure found for the full period holds for the 2000-2010 sub-period, but it changes dramatically in the 2010-2020 timeframe. During this timespan the total correlation between the TIPS and S&P 500 is softly negative at -0.18, but it swinging positive when looking at the periods of sells off. In particular we find a correlations of 0.19 and 0.25 associate with the S&P 500 returns down respectively 2σ and 2.5σ points. This is possibly the worst scenario for a diversifier as it is on average negatively correlated with the broad market, hence it does not performs as well as equity does, and its correlation turns positive in periods of crisis, hence it does not even offer downside protection when needed.

When looking at the three timeframes together gold appears to be the most resilient and reliable diversifier among the one considered. The highest correlation achieved by gold is only 0.16 in the 2010-2020 for the case of equity returns down 2.5σ points. Worth saying that even in this unfavourable sub-sample gold still is the asset that achieves the lowest correlation. Moreover the correlations for the down 2.5σ points case in the 2010-2020 sub-period are highly driven by the 2020 pullback, which accounts for roughly 50%[2] of the total observations. This market crash was peculiar in nature and saw all assets quickly moving together substantially to the downside. Once we recalculate gold correlation with the S&P500 for the case of returns down 2.5σ points, excluding 2020 from the sample, we obtain a correlation of -0.36, which once again underlines the resilience of gold, but also the volatile nature of assets correlations.

Finally we spend some words on Bitcoin, for obvious reasons mentioned in the previous sections this asset could only be included in the second sub-period (2010-2020). Overall Bitcoin performs poorly in terms of correlation structure. Over the full period it is arguably little correlated with the broad market, 0.07, however this figure jumps to 0.41 and 0.47 when looking at market downturns. Those are by far the highest correlations achieved

[2] 20 out of the 43 observations were realized in the year 2020

by any of the assets considered in any of the time periods used. Moreover even when looking at the full returns Bitcoin still achieves the highest correlation, 0.07, against gold's 0.008, the second highest in the set. This evidences strongly challenge the nowadays-popular view of Bitcoin as a valuable diversifier revealing an unreliable nature and a clear tendency to struggle together with risk-on assets during periods of crisis. It is also true that it is challenging to take any definitive stand on this asset given its short life. However while this might shelter Bitcoin from any definitive judgement, it surly doesn't help in making a case for adding Cryptocurrencies to once portfolio's core holding.

Results & Discussion - Correlation's Structures On The Upside

In this second part of the correlation's structures analysis we will look at specular figures to the on just presented, but now focusing on periods of market rallies rather than sells off. To determine the market returns we will use the same cut points as of before but now on the positive side of the returns distribution. This materializes in looking at the correlation between the assets for the full period, the correlation during days in which the S&P500 is up 2σ points or more and the correlation during days in which the S&P500 is up 2.5σ points or more. Specularly to the previous subsection we will look at the same three time periods; 2000-2020, 2000-2010, 2010-2020. Given those an upward 2σ points from the mean translates in an S&P 500 return respectively of; 2.48%, 2.73% and 2.20%. While an upward 2.5σ points from the mean translates in a return respectively of; 3.10%, 3.41% and 2.73%. While the aim of the previous subsection was to test the defensive properties of the various diversifiers and their reliability, the current sub-section focuses on their offensive abilities. This is a topic broadly ignored by previous literature as usually diversifiers are mostly compared based on the protection they provide rather than their realized returns. However, we argue that an excellent diversifier should not only offer safety during periods of crisis, but it also shouldn't be a heavy burden in periods of prosperity. Therefore, while in the previous section we were looking for a negative/zero correlation with the market, we now value positive correlations the most as we are working with the positive side of the return distribution.

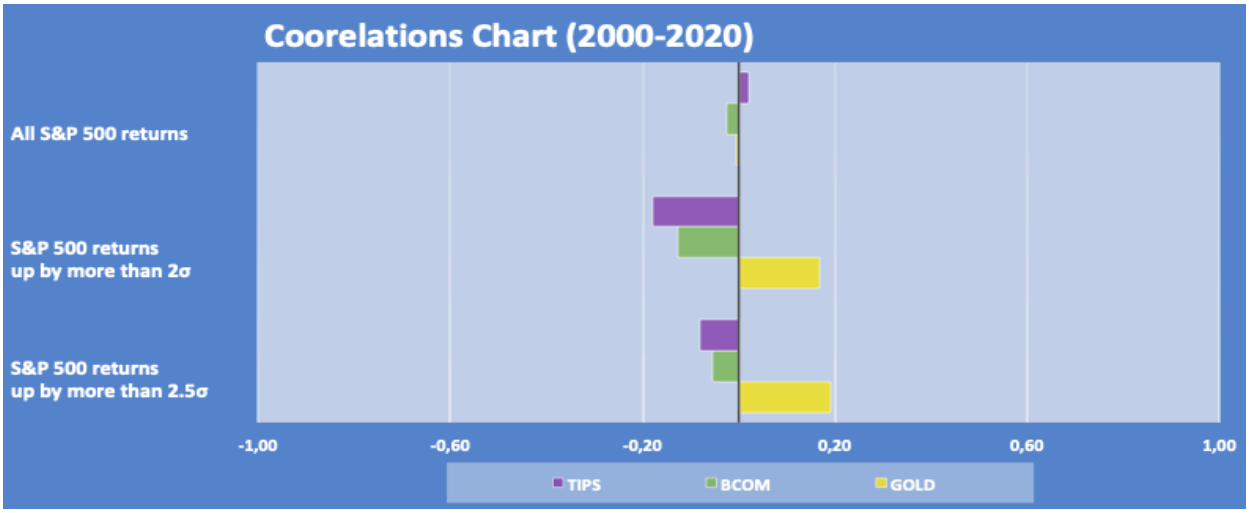

Chart 10: Daily Correlation chart between S&P 500 with gold, Bitcoin (BTC), BCOM and TIPS for the full period 01-01-2000 to 31-12-2020, for those days in which S&P 500 returns are up 2σ points from the mean and for those days in which S&P 500 returns are up 2.5σ points from the mean.

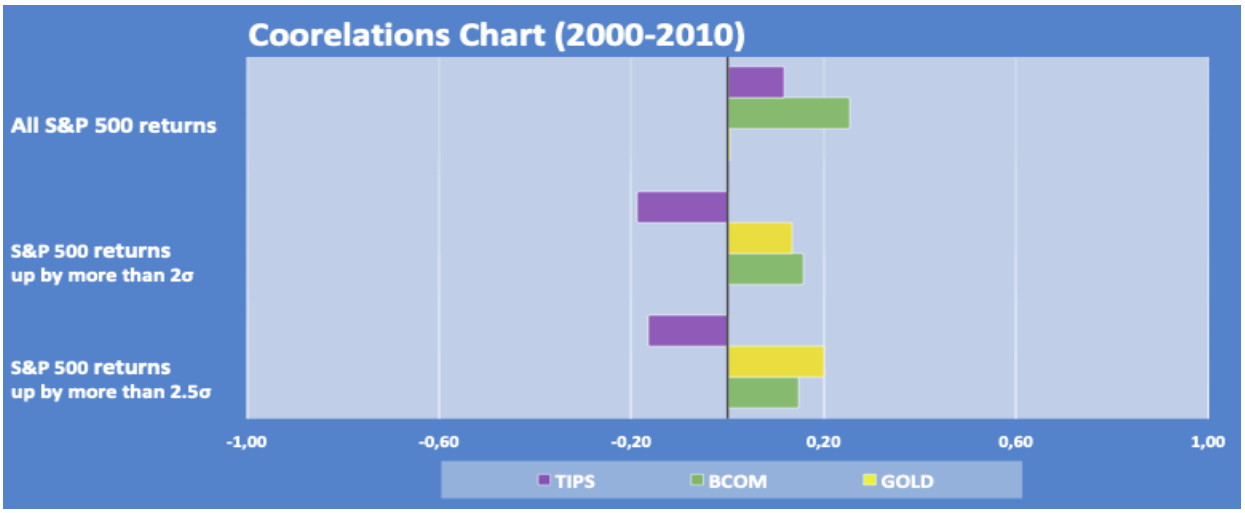

Chart 11: Daily Correlation chart between S&P 500 with gold, Bitcoin (BTC), BCOM and TIPS for the full period 01-01-2000 to 01-01-2020, for those days in which S&P 500 returns are up 2σ points from the mean and for those days in which S&P 500 returns are up 2.5σ points from the mean.

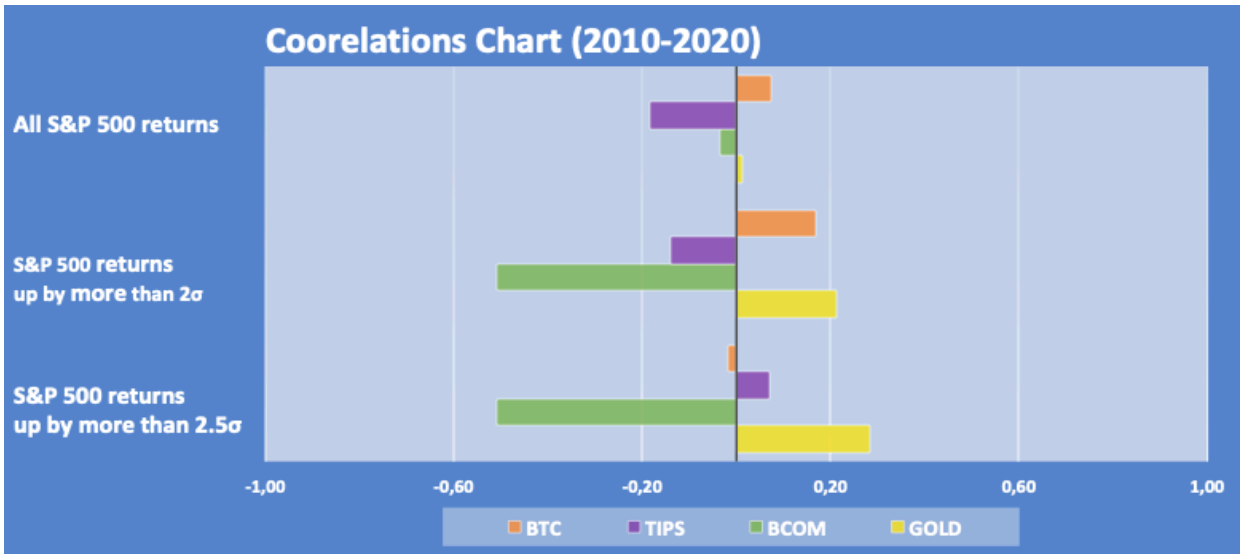

Chart 12: Daily Correlation chart between S&P 500 with gold, Bitcoin (BTC), BCOM and TIPS for the full period 01-01-2000 to 31-12-2020, for those days in which S&P 500 returns are up 2σ points from the mean and for those days in which S&P 500 returns are up 2.5σ points from the mean

Taken together these charts newly highlight the volatile nature of correlation and their time sensitivity. Overall the obtained findings points to gold being superior under two metrics. First, in any of the timeframe considered, gold is the most positively correlated assets with the broad equity market. Even more striking the correlation tends to grow positive in an ordinate fashion as equity rallies, usually starting from zero when looking at the full returns and rising as high as 0.28 in the case of equity returns up 2.5σ points for the 2010-2020 time span. Second, gold correlation structure seems somewhat more robust than any other. This is to say that its correlation stays fairly stable across all timeframes.

On the other hand BCOM's correlations remain the most volatile and pass from being broadly positive in the 2000-2010 sub-period to widely negative for the following 10years. Given the findings it could be argued that in specific time periods the BCOM behaves well and offers an interesting diversification proposition. However, when focusing on the longer run and on the need to find an asset that can become a portfolio's core holding together with equity and bonds, BCOM's correlation structure seems too volatile and unreliable to be trusted with such a fundamentally important role.

Looking at the TIPS they overall present a less volatile correlation than commodities, however, their correlation is mostly negative. This is problematic as it hints to TIPS being an anchor during market rallies and, therefore damaging the overall portfolio returns. TIPS might offer safety during some of the worst days in the market, but only by scarify returns during the best days. As discussed in the introductory section this does not necessary have to be neither a problem nor a reason to dislike this asset; anti-correlation is worth more than excess returns. However, even though they might matter less than anti-correlation, returns do matter. Therefore, given two diversifiers with similar correlation structures we will naturally be more attracted to the one that offers the highest returns or, better said, that sacrifices the least returns when not needed.

Finally Bitcoin is overall little positively correlated with the equity market, 0.07, its correlation slightly increases when looking at returns up 2σ points, 0.17, but it drops in negative territory, -0.03, when looking at returns up 2.5σ points. Once again it is hard to make any definitive stand on Cryptocurrencies given their limited life-spam, however Bitcoin correlations seems to be volatile, often in an unwanted direction and ultimately too unreliable just like the commodities' one.

Results & Discussion – Rolling Correlations

In this final section of the correlation analysis we look at 12months rolling correlations between gold and equity as proxied by the S&P 500. The aim is to obtain a clearer and more dynamic picture of gold's correlation structure in order to find support for the findings just presented in the previous section. In the chart we highlight periods of severe market downturn in order to distinguish those moments in which a negative correlation is most valuable. We will run this final analysis only for gold, as this will allow us to use a longer time horizon. The data used are monthly returns for gold and the S&P 500 from 1971 to 2020.

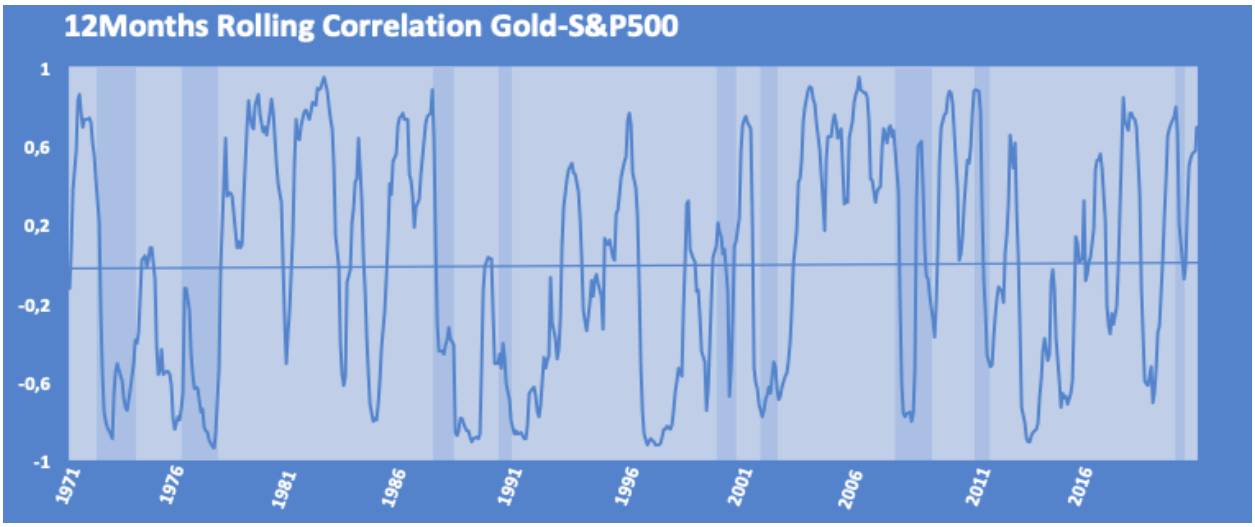

Chart 13: 12months-rolling correlation between S&P 500 and gold from 01-01-1971 to 31-12-2020. Highlighted are periods of sever equity market downturn; 01/1973-09/1974, 09/1976-03/1978, 08/1987-12/1987, 07/1990-10/1990, 03/2000-03/2001, 03/2002-07/2002, 10/2007-03/2009, 02/2011-10/2011, 02/2020-03/2020.

The correlation figures are clearly highly volatile and on the full sample it is hard to make any definitive statement. Overall gold's correlation spends 50.25% of the total period in the positive territory and the remaining 49.75% in the negative one. The fact that gold correlation is almost heavenly split between the positive and negative side is to be expected given its overall zero correlation with the market. However, when focusing on periods of equity downturns, gold's correlation have a tendency to turn negative. During the highlighted periods gold's correlation spends 68.75% of the time in the negative territory and only 31.25% in the positive. Moreover worth noticing that during downturns, even when the correlation remains positive it usually considerably falls in value, especially in the start of the period as first reaction to the crisis. We run some statistical analysis in order to quantify the magnitude and the significance of these results. In particular we create a dummy variable equal to 1 in periods of crisis and zero otherwise, we then regress gold/equity correlation against said dummy. The results indicate that the correlation in crisis periods is on average

0.25 points lower than in non-crisis periods. This result is statistically significant at the 99% confidence level with a P-value of 0.000.

Finally, we investigate what were the realized nominal returns for the S&P500 and gold in those highlighted periods. This is an interesting exercise as it adds an additional layer to the analysis by looking at returns rather than correlations. Moreover it can also function as a simple test for the conclusions reached over gold's safe heaven and insurance ability from the results of the previous sections.

Periods of crisis/recession	S&P 500's Nominal Return	Gold's Nominal Return
01/1973 - 09/1974	-46,0%	137,0%
09/1976 - 03/1978	-15,4%	51,8%
08/1987 - 12/1987	-25,5%	6,8%
07/1990 - 10/1990	-15,8%	7,8%
03/2000 - 03/2001	-22,6%	-7,6%
03/2002 - 07/2002	-20,5%	0,3%
10/2007 - 03/2009	-47,8%	26,7%
02/2011 - 10/2011	-14,8%	15,1%
02/2020 - 03/2020	-19,8%	-0,7%
AVERAGE	**-25,4%**	**26,4%**

The table above agrees with the previous findings over gold resilience as a defensive asset against market risks. During the worst market downturns gold decisively outperforms equity and often realizes wide gains. The Dot.com bubble burst (03/2000-03/2001) and the more recent 2020 pullback (02/2020-03/2020) are the only two periods among the 9 considered in which gold's delivers a negative return; -7.6% and -0.7% respectively. However, even for these two cases gold heavily outperforms equity, which realized a loss of 22.6% following the dot.com burst and a loss of 19.8% during the 2020 pullback. Moreover it is interesting to notice how the average S&P 500's return during the crisis periods is equal to -25.4% which almost perfectly cancels out with the 26.4% average return realized by gold. Following a similar procedure to the one detailed above we statistically test this findings as well. We run two regression analyses one having gold return as dependent variable and the other using the S&P 500 returns. Both regressions use the dummy crisis variable previously introduced. The results confirm the finding just exposed and reveal that on average during periods of crisis gold returns 1.67% more than otherwise. On the other hand the S&P500 returns during crisis are on average 3.66% lower than otherwise. Both these findings are statistically significant at the 99% confidence level with a P-value respectively of 0.008 of 0.000. Worth noticing that, during periods of crisis, gold does not only outperform the S&P500, but it also outperforms itself, 1.67% on average. This is an interesting finding that points to gold being exactly the

type of anti-correlated asset that we seek, with its biggest performances reserved for those times when traditional asset classes are struggling.

Finally, it is important to add that gold's returns are affected by a considerable dispersion. For example, while during 09/1976 to 03/1978 gold delivers a 51.8% return against an equity return of -15.4%, in 07/1990 to 10/1990 it only realizes a 7.8% gain against the same equity loss. The dispersion in gold's returns show that the yellow metal does not always offer the same level of protection and at times it might feels like it is not working as intended. However, when looking at the broader picture the evidences found so far point to gold being a valuable and reliable defensive asset, with a useful correlation structure that can offer great diversification benefits and the historical realized returns do confirm this narrative.

Section 5

The Second Branch

Results & Discussion – Portfolios Optimization

In this second branch of our analysis we look at the results of our portfolio optimizations. We start by reporting the results for the efficient frontiers and the graphical comparisons that can be made from there. We follow with the statistical testing in the subsequent section. The aim of this second branch of our analysis is to empirically test the contribution that the considered diversifiers bring to a traditional portfolio in practice. While the previous section has provided theoretical evidences over the portfolio properties of the assets considered, the current one intends to investigate whether this theory actually translates in meaningful performance increases.

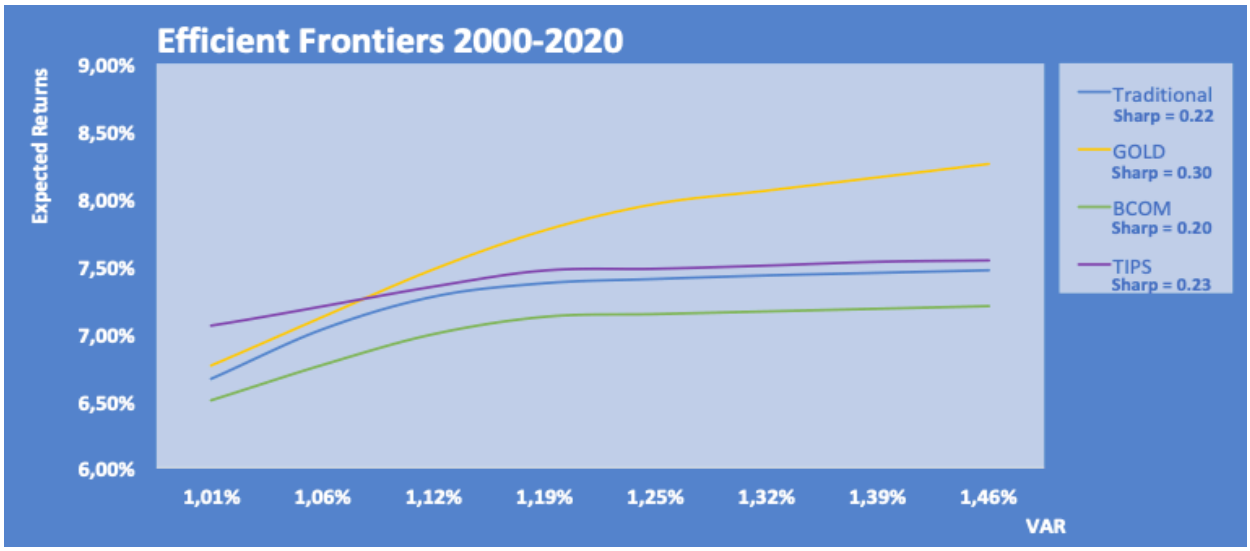

Chart 14: efficient frontiers for the traditional portfolio, traditional portfolio + gold, traditional portfolio + BCOM and traditional portfolio + TIPS. Weekly data with yearly rebalancing using data from 01-01-2000 to 31-12-2020

Looking at the full period the gold and TIPS portfolios' efficient frontiers consistently lay above the traditional portfolio's one. This hints to gold and TIPS to be a valuable addition to ones portfolio as they both help achieving a better risk-reward trade off. In particular the TIPS efficient frontier briefly lies above the gold's one for low variance values. The switching point between the two frontiers is at a portfolio's standard deviation value between 10.3% and 10.6%, after that value the difference between the two frontiers quickly widen in gold's favour. The maximum Sharpe ratios achieved by gold and TIPS are 0.30 and 0.23 respectively to be compared with the 0.22 realized by the traditional portfolio. Finally the BCOM efficient frontier lies substantially below the traditional portfolio's one suggesting that broad

commodities might not be a valuable addition for the longer run. Overall this negative result of the BCOM confirms what was concluded in the previous section where we underlined the fragility of BCOM correlation structure and therefore its dubious portfolio's ability. The maximum Sharpe achieved by the BCOM in the full period is only 0.20.

Chart 15: efficient frontiers for the traditional portfolio, traditional portfolio + gold, traditional portfolio + BCOM and traditional portfolio + TIPS. Weekly data with yearly rebalancing using data from 01-01-2000 to 01-01-2010

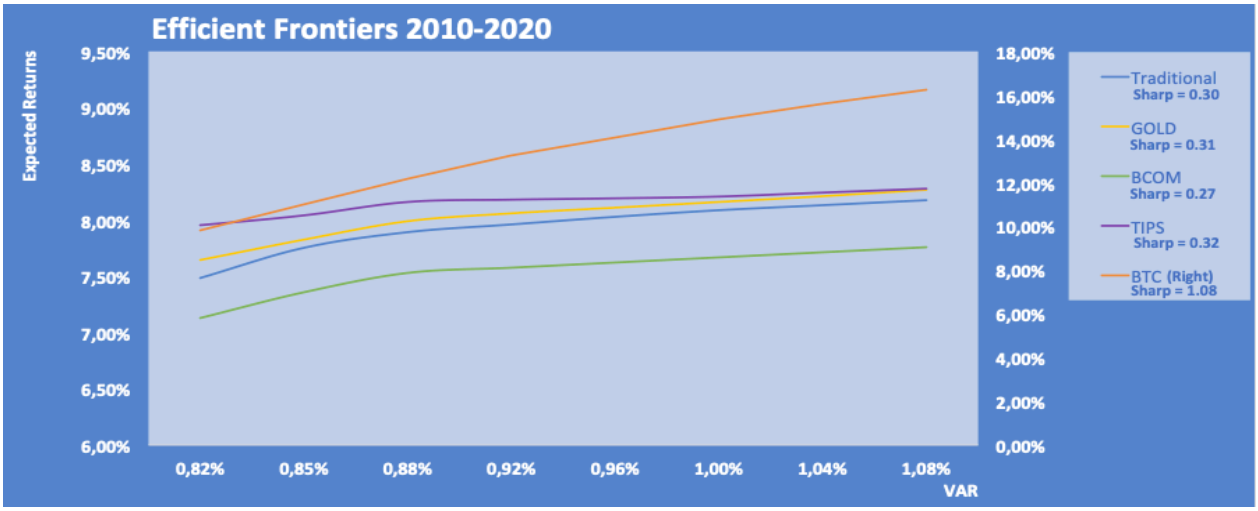

Chart 16: efficient frontiers for the traditional portfolio, traditional portfolio + gold, traditional portfolio + BCOM, traditional portfolio + Bitcoin and traditional portfolio + TIPS. Weekly data with yearly rebalancing using data from 01-01-2010 to 31-12-2020

Looking at the two sub periods the gold portfolio well behaves and always achieves a higher efficient frontier than the traditional one. In the sub-period 2000-2010 gold acts similarly to the full period starting at a disadvantaged for the lower volatility cases, but then growing sharply and widening the gap with all other portfolios. During this sub-period gold produces a Sharpe of 0.39. The second best Sharpe in the sample is 0.27 produced by the TIPS. In the final sub-period gold does not performs as well and comes in third after Bitcoin and the TIPS. It here produces a Sharpe of 0.31, just above the 0.30 realized by the traditional portfolio. The

worst performance of gold in the final sub-period is somewhat to be expected as this decade includes a 9years gold's bear market during which the price lost around 50% from its highs. During the first half of 2020 the price did recover, however gold still finished the year down around 15% from its peak.

Looking at the TIPS and BCOM during the two sub periods their performances do not vary much from the full period, BCOM's efficient frontier remains below the traditional portfolio's one while the TIPS always locates just above it. The Bitcoin portfolio visibly outperforms any other one for the sub-period in which it is included realizing a Sharpe of 1.2. This result is in strong contrast with the one found in the previous section where Bitcoin correlation structure hinted to it not being a valuable addition to ones portfolio. The diverges between the findings of these two sections can be attribute to Bitcoin short life-span, this is the case as the Sharpe ratio and efficient frontiers are purely backward looking measures and they have forecast abilities only dependently from the assumption that historical performance can be used to predict future ones. Clearly when looking at the 43000000% return realized by Bitcoin (from $0.06 to $30000) during the timespan considered it would be hard imaging it not improving the returns of any portfolio. However what investors should wonder about is whether such performances can ever be repeated and if Bitcoin can keep realizing high enough excess returns to out weight its unfavourable correlation structure.

Finally we present the results for the well-diversified portfolio and the well-diversified portfolio plus gold. This analysis is done only for the full period and therefore will not include Bitcoin.

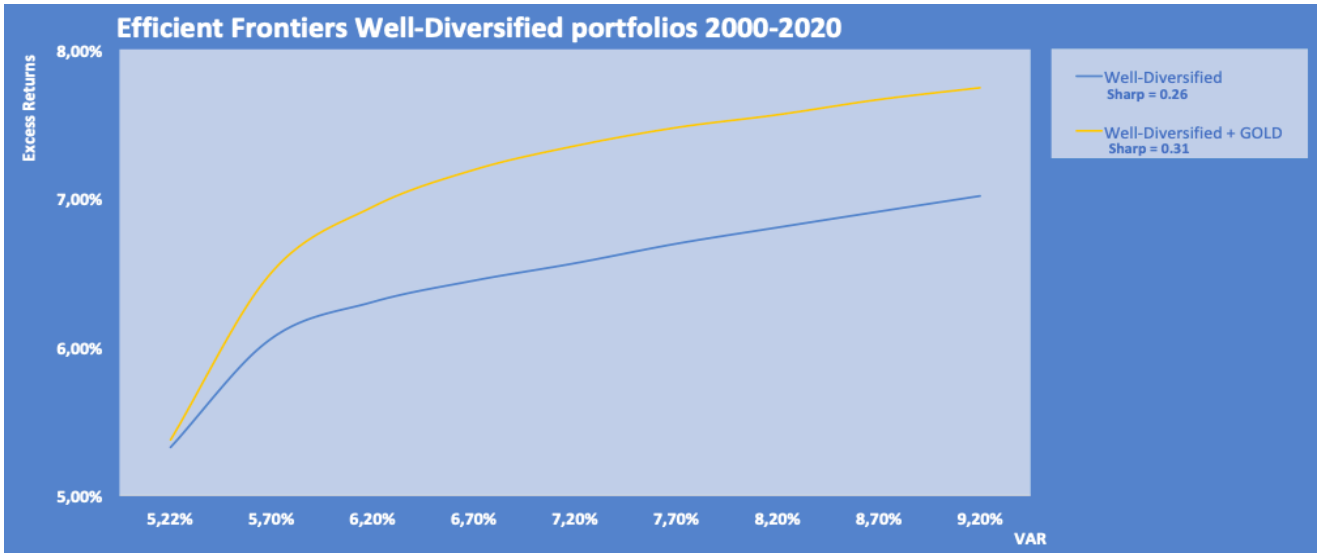

Chart 16: efficient frontiers for the well-diversified portfolio and the well-diversified portfolio + gold. Weekly data with yearly rebalancing using data from 01-01-2000 to 31-12-2020

The chart above makes a potent case for gold being a valuable portfolio's asset. Even when looking at the well-diversified portfolio the addition of gold seems to dramatically increase the overall risk-adjusted returns, hinting to gold offering a unique mix of diversification properties that cannot be replaced by any other assets considered. The Sharpe ratio of the well-diversified portfolio is 0.26 and it jumps to 0.31 when adding gold. It is also worth noticing how the Sharpe ratio achieved by the well-diversified portfolio is only slightly above the one of the traditional portfolio 0.26 against 0.22, but when adding gold the performances increase dramatically.

The value of gold to a portfolio's holdings is also suggested by the way in which the optimization process tends to fully allocate to gold within the restrictions given. In most cases, if the optimization was to be done free of constrains, the optimal portfolio would always contain anywhere between 30% and 50% of gold for the high volatility cases. The same cannot be said for all other diversifiers, which tend to remain just a smaller portion of the optimal portfolio; 5% for the BCOM and 15% for the TIPS. However we remind the reader that no definitive conclusion can be taken from the efficient frontiers alone, because of the reasons explained in section 3. The findings suggested by the current section can only be confirmed by the statistical testing that will be presented next.

Results & Discussion – Statistical Tests

The current section presents the statistical testing of the previously presented results; we use a Montecarlo simulation as a resampling technique in order to obtain the needed data points for the analysis. For each portfolio's mix, in each timeframe, we take the asset allocation that generates the highest Sharpe ratio and we use this information to simulate 1000 trials for every portfolio. It is important to notice that we use the Montecarlo as a simple resample technique rather than a forecast tool. This means that we are only back-testing the portfolios as we feed into the model data obtained through the backward looking portfolios optimization detailed above. Therefore this should not be considered an out of sample test and it remains uncertain whether an investors would have had any incentive to hold, before facts, the tested portfolio. We chose to not run an out of sample test as the simulation technique would have produced biased results, unaligned with the core topic of the research. This is the case as the Montecarlo process adopted generates results based only on expected returns and variance, completely ignoring the correlation between assets. Therefore we prefer to directly use the data obtained form the portfolios rather than the specific assets so that the various correlation structures have already been accounted for during the optimization process.

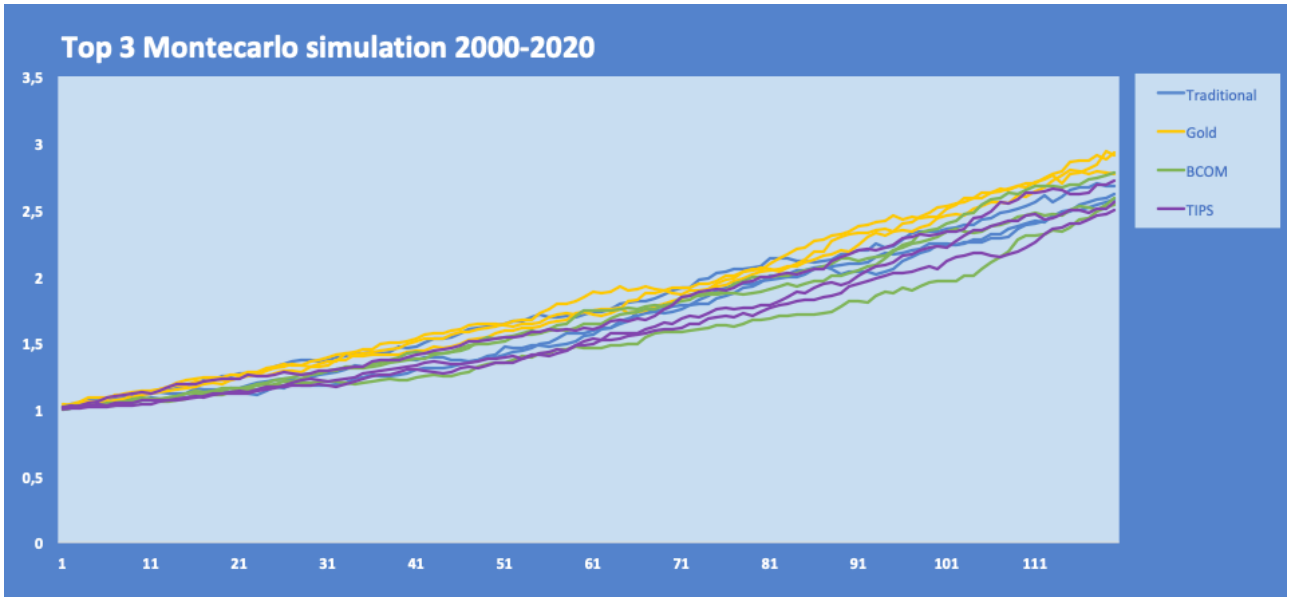

Chart 17: Top 3 Montecarlo performances for each portfolio type for the timeframe 2000-2020. Performances are measured in terms of nominal returns only. Monthly data are used and the process simulates 10years worth of performances, 120 month in total.

The chart above pictures a graphical representation of the top 3 simulations for each portfolio type for the 2000-2020 timeframe. The simulations are run with monthly data even though the statistical test will be done with yearly ones. In particular, the chart above tracks the theoretical growth of $1 invested in each simulated portfolio. As expected from the previous finding the gold portfolios broadly outperform all others. To test our result we generate 4 dummy variable, one for each portfolio type; traditional, gold, BCOM and TIPS. We then regress the simulated return against said variables, omitting the dummy for the traditional portfolio as a reference category. Therefore the results obtained should be interpreted in comparison to the traditional portfolio only. For the test we use yearly returns data. We find that adding gold to a traditional portfolio, on average, increases the yearly returns by 1%, this result is statistically significant within the 99% confidence interval with a P-value of 0.000. On the other hand, adding the BCOM to a traditional portfolio, on average, decreases the returns by 0.23%, this result is statistically significant within the 90% confidence interval with a P-value of 0.086. Finally the addition of the TIPS has both a statistically and economically insignificant impact. This portfolio produces on average returns that are 0.009% higher that the traditional one, with a P-value of 0.995. Finally we test for statistical differences across the Sharpe ratios. We do so by following the procedure lined out in section 3. We find that for the full period 2000-2020 the gold portfolio is the only one that produces Sharpe ratios statistically different from the traditional portfolio within the 90% confidence level and a P-value of 0.075. The P-values of the BCOM and TIPS are 0.125 and

0.720 respectively. We also run a simple regression analysis on the obtained Sharpe ratio following the same model used for the returns and find that on average the addition of gold to a traditional portfolio increases the yearly Sharpe ratio by 0.09.

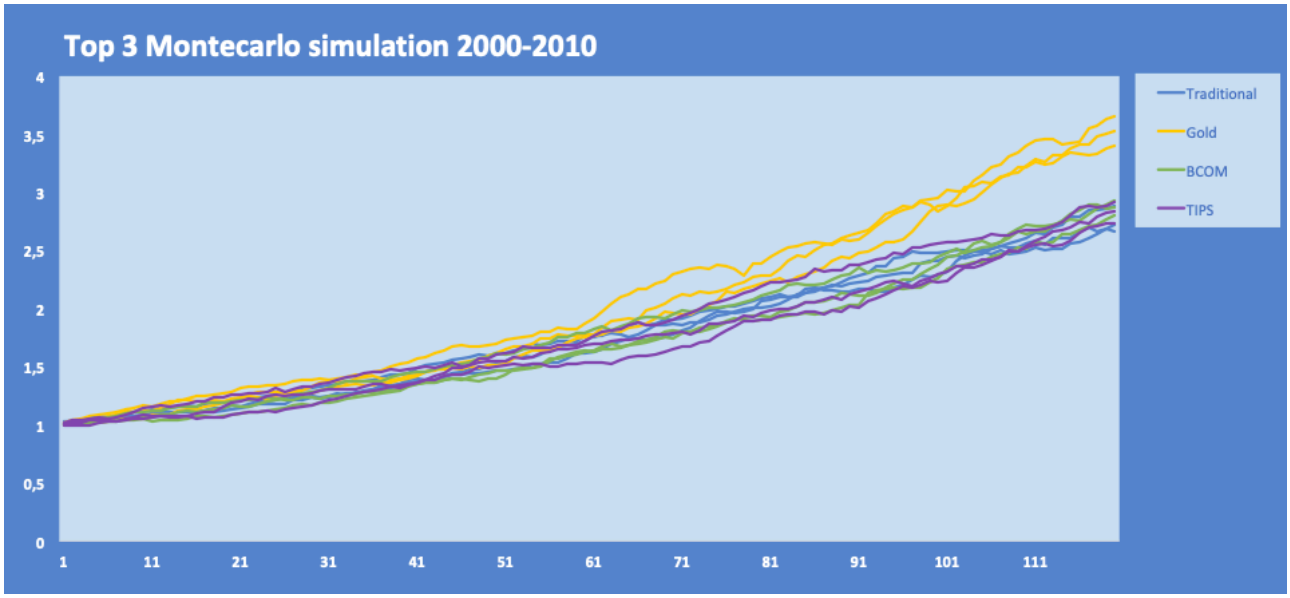

Chart 18: Top 3 Montecarlo performances for each portfolio type for the timeframe 2000-2010. Performances are measured in terms of nominal returns only. Monthly data are used and the process simulates 10years worth of performances, 120 month in total.

Looking at the top 3 Montecarlo simulations for the 2000-2010 sub period gold outperformance is visibly wider than for the full period. The worst performer between the 3 gold portfolios realizes an average yearly return of 13.06% against the 10.8% achieved by the best performing portfolio among of the others. The Montecarlo as well as the statistical tests were run following the same procedure detailed above. We find that adding gold to a traditional portfolio, on average, increases the yearly returns by 1.96%, this result is statistically significant within the 99% confidence interval with a P-value of 0.000. In this sub-period also the addition of the BCOM to a traditional portfolio, on average, increases yearly returns by 0.39%, this result is statistically significant within the 95% confidence interval with a P-value of 0.016. Finally the addition of the TIPS has once again a statistically and economically insignificant effect. This portfolio produces on average returns that are 0.008% higher that the traditional one, with a P-value of 0.960. When testing for the Sharpe ratios we find that for the 2000-2010 time frame both the gold portfolio and the BCOM produce Sharpe ratios statistically different than the traditional portfolio with a P-value of 0.000 and 0.081 respectively. The TIPS were associated with a P-value of 0.531. Both gold and BCOM have a positive effect on the Sharpe ratio, the addition of gold increases on average the yearly Sharpe

ratio of the traditional portfolio by 0.12 while the BCOM increases on average the yearly Sharpe ratio of the traditional portfolio by 0.06.

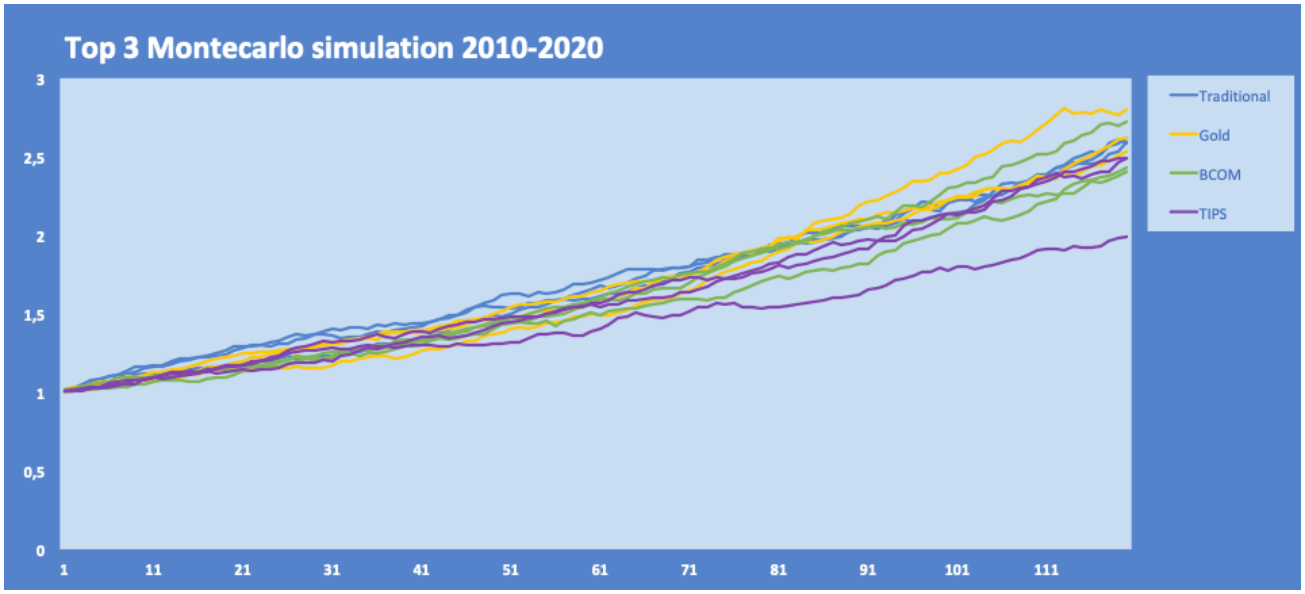

Chart 19: Top 3 Montecarlo performances for each portfolio type for the timeframe 2010-2020. Performances are measured in terms of nominal returns only. Monthly data are used and the process simulates 10years worth of performances, 120 month in total.

In the final period, 2010-2020, we obtain a more inordinate picture with the performances of the various portfolios overlapping and blending together. The best performing simulation is once again a gold one, but only for a slight margin. Moreover while in other time periods all gold simulations were clearly outperforming we here have a mixed outcome at best. The statistical tests show that this sub period is particularly adverse for the tested diversifiers as they all have negative effect on the average yearly returns of a traditional portfolio. However, when compared to the others, the gold portfolio still proves superior as its negative effect is neither economically nor statistically significant. Adding gold to a traditional portfolio, on average, decreases the yearly returns by 0.02%, this result is statistically insignificant within the 90% confidence interval with a P-value of 0.863. One the other hand both the BCOM and TIPS have statistically significant effects with a P-value of 0.000 and 0.001 respectively. Adding the BCOM to a traditional portfolio, on average, decreases the yearly returns by 0.68%, while the addition of the TIPS to a traditional portfolio, on average, decreases the yearly returns by 0.41%. When testing for the Sharpe ratios for this sub-period none of the considered portfolios leads to a statistically significant results with P-values of 0.198, 0.610 and 0.755 associate to gold, BCOM and TIPS respectively.

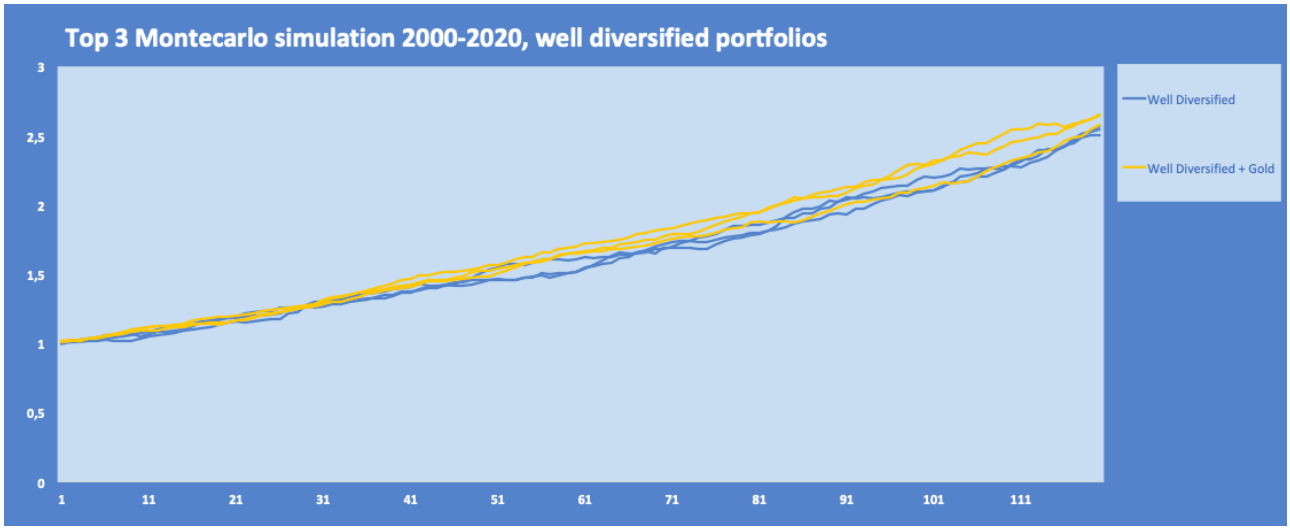

Chart 20: Top 3 Montecarlo performances for the well-diversified portfolio and the well-diversified portfolio + gold for the timeframe 2000-2020. Performances are measured in terms of nominal returns only. Monthly data are used and the process simulates 10years worth of performances, 120 month in total.

Finally we look at the well-diversified portfolio and the well-diversified portfolio plus gold. Our findings support what was previously said as adding gold to a well diversified portfolio, on average, increases the yearly returns of the latest by 1.1%. This result is statistically significant within the 99% confidence level with a P-value of 0.000. Moreover the addition of gold is found to have a positive effect on the yearly Sharpe ratio too, which, on average, increases by 0.11 after adding the yellow metal to the assets mix. This result is statistically significant within the 99% confidence level with a p-value of 0.002.

All in all the Montecarlo simulations and the statistical test ran through them broadly confirm the previously presented results. Gold proves to be the most reliable and effective diversifier as, a part from the last sub period, it's impact on a traditional portfolio is always statistically significant and in the wanted direction. On the other hand the TIPS, which from the study of the efficient frontiers seemed to be gold's first competitors are revealed to be an economically and statistically insignificant addition to a traditional portfolio. BCOM is confirmed to be an unreliable asset as its statistical significance is dubious and its impact on a portfolio is highly dependent from the timeframe chosen.

Section 6

Reflection

Full Discussion

To summarize the findings of the previous sections, the yellow metal makes a solid case for itself both in terms of correlation structure and empirical performances. Form the correlation study we find evidences of gold being little to zero correlated with the broad stock market in days of severe sells off. When looking at broader periods of equity downturns the case for gold fortifies as the yellow metal usually thrives and delivers wide gains during crisis events. Moreover gold does not only offer a reliable insurance against equity risks, it also performs well in periods of prosperity, as indicated by its correlation with the broad market turning strongly positive during equities' right tail events. In this way gold offers safety from left tail events while not compromising the right tail gains. The yellow metal proves to be an all around asset able to provide both safety and sizable returns. In fact gold nominal returns are comparable in size with equities' ones and, as mentioned in section 2, in the past 50years, gold has actually outperformed broad equity indexes. Furthermore gold correlation structure seems to be more robust and less time-sensitive than any other analysed, which is an additional proof of gold's reliability, a characteristic that surly cannot be ignored when trying to engineer a portfolio that can pass the prove of time.

All in all this favourable correlation structure makes of gold a great diversifier and the study of the efficient frontiers broadly confirms this result. In any time-span considered gold helps achieving a higher efficient frontier and a statistically significantly higher shape ratio. It is true that for specific sub-periods the gold portfolio becomes sub-optimal, however these are only a couple of isolated and weak cases. When looking at the broader picture gold results are consistent and prove it to be a valuable addition to any portfolio.

An interesting finding is that the gold portfolio is usually suboptimal for lower variance levels. This suggests that investors have to be willing to take a certain degree of risk in order to enjoy gold's diversification properties. This is the case as, counterintuitively, the yellow metal's volatility is actually elevated and more similar to the one of equity rather than bonds. Gold's safe heaven status does not come from gold being a riskless asset in itself, but rather comes from its insurance and diversification abilities just presented. Therefore one has to be ready to endure some volatility in order to be able to hold enough gold to "insure" his portfolio. We find that at a portfolio with a standard deviation value of 11% is needed for the

optimization process to be able to allocate 10% or more of the portfolio into gold. At this threshold the gold portfolio usually starts outperforming from a risk-reward prospective.

Looking at the other considered diversifiers the BCOM is the clear underperformer and our findings would recommend against the addition of broad commodities as core passive holding in ones portfolio. The TIPS offer an interesting defensive proposition, even though its negative correlation with equity during right tail events makes it so that its total contribution to a portfolio is only slight and at times dubious. However, thanks to their low volatility the TIPS usually produce the optimal global minimum variance portfolio and could be an attractive holding for highly risk-adverse individuals. Finally it is hard to say anything conclusive regarding Bitcoin given the limited life span with which to run the analysis. It is no surprise, given its historical performances, that adding Bitcoin to one's portfolio highly increases the expected returns. However it is worth mentioning that Bitcoin variance is as elevated as its returns. In the period from 2010 to 2020 Bitcoin realizes a variance above 100% to be compared with the second highest variance, 3.25%, offered by the Emerging Markets Equity index. Bitcoin risks have historically been well remunerated, however given its volatility, a regular investor would struggle to hold more than a couple of percentages of its total portfolio in this asset. Additionally, from the study of the correlation structures, Bitcoin seems to be delivering its returns with a poor timing as it sells off together with the equity markets during periods of crisis but it also struggles during favourable right tail events. Given the historical magnitude of Bitcoin's returns it little matters when the gains are delivered as far as they are indeed delivered. As mentioned in the previous section, it seems an unlikely speculation to bet on a repeat of the past 10 years in Bitcoin. History hardly repeats itself and 10 years are a too short period to base long-term commitments on. On the other hand gold has been proving itself for thousands of years helping investors to find safety from all sort of macroeconomic and market risks. Our formal analysis only looks as back as 20years however, in this timeframe, we find compelling evidences for gold to be a valuable long-term addition to most portfolio's mixes.

Study Limitations

The study offers robust evidences over gold's safe heaven properties and its consistent contribution to an equity-bond portfolio. However the type of analytical work just performed does have some limitations that are worth acknowledging. First of all any sort of quantitative work that like ours uses historical information in order to make future forecast or recommendations has an inescapable Achilles' talon, namely that we are assuming that historical data are representative of future price movements. This is a common assumption

made in countless studies, which should not be seen as a fundamental threat to our results. Nonetheless it is important to consider its implications. We can confirm with all certainty that in the 20years time span considered gold has indeed acted as a reliable diversifier and safe heaven asset, however whether this will keep being the case for the next 20years and more is at best a likely speculation based on historical observations.

Following this first limitation it is also important to consider that despite 20years being a lengthy period it is still quite limited when looking at the broader picture. As we said many times during the paper the metrics used are extremely time-sensitive and they could look significantly different if we were to add more data to the study. Moreover when running analysis on gold it would be ideal to start the sample in 1971, the year in which gold price was left to freely float. Unfortunately in the present research this was not possible, as some of the other financial instruments used do not date that back in time. When was possible we always tried to look at a 50years prospective, however the main analysis is performed on 20years and the results should be considered valid only within that time horizon.

Finally it is important to notice that the research for a diversifier is particular important for the long only portfolio case used in the paper, but it loses relevance when allowing for short-selling. This is the case as with a well-studied long/short strategy a capable investor could artificially create any risk profile or correlation structure he wished for. Moreover he could even look for protection in more exotic financial products like options or swaps, therefore eliminating the need for an anti-correlated asset. However it is worth saying that short positions are not absent of risks and costs that long only strategies do not bear. For example sometimes high recurrent fees are required, a margin is needed and it is needed to be maintained as prices fluctuates. All in all it is beyond the scope of the present research to compare long only with mixed strategy and the contribution (if any) that gold can have for the latest. We leave this topic together with the other aforementioned limitations as suggested material to future researches.

Conclusions

In the course of the present research we analyse gold's portfolio properties in order to understand whether the yellow metal is a valuable addition to a traditional equity-bond portfolio. We look at value in terms of portfolio performances proxied by risk-adjusted returns and argue that gold's correlation structure with equity allows for mitigating risks while enhancing returns, a result confirm during the course of our analysis. When compared with the other diversifiers included in the study gold set itself apart both in terms of performances, but most importantly, in terms of reliability. This is ultimately confirmed by

the portfolio optimization process, which shows that any portfolio, even one that already holds alternative assets, would benefit from the addition of the yellow metal to its core holdings. These results should be of interest to any investor concerned with market risks and looking for ways to consistently mitigate them. Moreover financial advisors and those involved with the creation of passive investment products could also learn from our findings and start normalizing the use of gold as a financial asset. A recent report from the CPM group reveals that the interest of the financial community in the yellow metal has strongly declined during the years and that gold now represents only 0.7% of global financial assets. After reviewing our findings we believe that such low interest in such an outstanding asset is truly shameful and that most investors, especially retail ones passively investing for retirement proposes, would greatly benefit from being introduced to gold and its properties.

Section 7

Supplementary Material

Bibliography

Anand, R., & Madhogaria, S. (2012). Is gold a 'safe-haven'?-An econometric analysis. Procedia Economics and Finance, 1, 24-33.

Arnott, R., & West, J. (2021). Is Diversification Dead?

Akhtaruzzaman, M., Boubaker, S., Lucey, B. M., & Sensoy, A. (2020). Is gold a hedge or safe haven asset during COVID–19 crisis?. Available at SSRN 3621358.

Bailey, D. H., & Lopez de Prado, M. (2012). The Sharpe ratio efficient frontier. Journal of Risk, 15(2), 13.

Baruník, J., Kočenda, E., & Vácha, L. (2016). Gold, oil, and stocks: Dynamic correlations. International Review of Economics & Finance, 42, 186-201.

Bauer Jr, R. J., & Dahlquist, J. R. (2001). Market timing and roulette wheels. Financial Analysts Journal, 57(1), 28-40.

Baur, D. G., & Lucey, B. M. (2010). Is gold a hedge or a safe haven? An analysis of stocks, bonds and gold. Financial Review, 45(2), 217-229.

Bernstein, P. L. (2012). The power of gold: the history of an obsession. John Wiley & Sons.

Berg, A. (2020). The Identity, Fungibility and Anonymity of Money. Economic Papers: A journal of applied economics and policy, 39(2), 104-117.

Bredin, D., Conlon, T., & Potì, V. (2015). Does gold glitter in the long-run? Gold as a hedge and safe haven across time and investment horizon. International Review of Financial Analysis, 41, 320-328.

Chang, E. C., & Lewellen, W. G. (1984). Market timing and mutual fund investment performance. Journal of Business, 57-72.

Cole, C., R. (2016). Dennis Rodman And The Art Of Portfolio Optimization. Artemis Capital Management, April, 1 – 6.

Cole, C., R. (2020). The Allegory Of The Hawk and Serpent How To Grow and Protect Wealth for 100 Years. Artemis Capital Management, January, 1 – 19.

Connolly, R., Stivers, C., & Sun, L. (2005). Stock market uncertainty and the stock-bond return relation. Journal of Financial and Quantitative Analysis, 161-194.

Cuthbertson, K., Nitzsche, D., & O'Sullivan, N. (2010). The market timing ability of UK mutual funds. Journal of Business Finance & Accounting, 37(1‐2), 270-289.

DeLong, J. B., & Magin, K. (2009). The US equity return premium: past, present, and future. Journal of Economic Perspectives, 23(1), 193-208.

Emmrich, O., & McGroarty, F. J. (2013). Should gold be included in institutional investment portfolios?. Applied Financial Economics, 23(19), 1553-1565.

Engle, R., & Colacito, R. (2006). Testing and valuing dynamic correlations for asset allocation. Journal of Business & Economic Statistics, 24(2), 238-253.

Erb, C. B., & Harvey, C. R. (2013). The golden dilemma. Financial Analysts Journal, 69(4), 10-42.

Erb, C. B., Harvey, C. R., & Viskanta, T. E. (2020). Gold, the Golden Constant, COVID-19,'Massive Passives' and Déjà Vu. COVID-19,'Massive Passives' and Déjà Vu (August 5, 2020).

Fan, J., & Mitchell, M. (2017). Equity-bond correlation: A historical perspective. Graeme Capital Management Research Note, September, 1-3.

Fisher, K. L., & Statman, M. (2006). Market timing in regressions and reality. Journal of Financial Research, 29(3), 293-304.

Gürgün, G., & Ünalmış, İ. (2014). Is gold a safe haven against equity market investment in emerging and developing countries?. Finance Research Letters, 11(4), 341-348.

Harmston, S., (1998). Gold as a store of value. Research Report, London: World Gold Councile

Hathaway, J., (2020). Systemic Risk Exposed. Research Report, Vancuver: Sprott

Jaffe, J. F. (1989). Gold and gold stocks as investments for institutional portfolios. Financial Analysts Journal, 45(2), 53-59.

Jiang, W. (2003). A nonparametric test of market timing. Journal of Empirical Finance, 10(4), 399-425.

Jobson, J. D., & Korkie, B. M. (1981). Performance hypothesis testing with the Sharpe and Treynor measures. Journal of Finance, 889-908.

Kopyl, K. A., & Lee, J. B. T. (2016). How safe are the safe haven assets?. Financial Markets and Portfolio Management, 30(4), 453-482.

Lawrence, C. (2003). Why is gold different from other assets? An empirical investigation. London, UK: The World Gold Council.

Markowitz, H. (1959). Portfolio selection. New York: J. Wiley

Markowitz, H. (1991). Portfolio selection, efficent diversification of investments. Cambridge, MA: Blackwell.

Markowitz, H. (1991). Portfolio selection, efficent diversification of investments. Cambridge, MA: Blackwell.

Memmel, C. (2003). Performance Hypothesis Testing with the Sharpe Ratio. Germany, EU: University of Cologne, financial letters 2003, 1, 21-23

Merton, R. C. (1972). An analytic derivation of the efficient portfolio frontier. Journal of financial and quantitative analysis, 1851-1872

Oxford Economics (2011). The impact of inflation and deflation on the case for gold. Research Report, London: World Gold Councile

World Gold Council (2009). Gold as a tactical inflation hedge and long-term strategic asset. Research Report, London: World Gold Councile.

World Gold Council (2019). Gold: the most effective commodity investment. Research Report, London: World Gold Councile.

World Gold Council (2020). Gold and Cryptocurrencies, How Gold's Role in a Portfolio Differs from cryptos. Research Report, London: World Gold Councile.

World Gold Council (2020). Gold, an Efficient Hedge. Research Report, London: World Gold Councile.

World Gold Council (2021). The relevance of gold as a strategic asset US edition. Research Report, London: World Gold Councile

Sharpe, W. F. (1994). The sharpe ratio. Journal of portfolio management, 21(1), 49-58

Stoeferle, R., & Valek, M., J. (2021). In Gold We Trust, Monetary Climate Change. Incrementum, May, 1-20.

Szado, E. (2009). VIX futures and options: A case study of portfolio diversification during the 2008 financial crisis. The Journal of Alternative Investments, 12(2), 68-85.

Taskinsoy, J. (2019). Pure Gold for Economic Freedom: A Supranational Medium of Exchange to End American Monetary Hegemony as the World's Main Reserve Currency. Available at SSRN 3377904.

Appendix A

As explained in section 3 of the main text we need to use a number of restrictions when running the portfolio optimization process in order to obtain realistic and usable results. First we ensure that the portfolios are long only by forcing the weights given to each asset class to be positive. In addition we impose that the sum of the weights is equal to 100%, which ensure that the portfolio will be fully invested and absent of leverage. Every asset, with the exception of Bitcoin, included into any portfolio had to be given a weight of at least 5% and maximum 20%. In the case of Bitcoin the lower boundary of the range was removed, as a 5% holding in Bitcoin would have already caused the portfolio to be incomparable to the others from a variance prospective. Moreover, to make sure that the portfolios will approximate the 60/40 equity-bonds structure, while remaining flexible enough to add other assets into the mix we force the equity side to fluctuate from 50% to 70% and the bond side from 30% to 50%. This leaves the possibility to add an additional asset class for a maximum weighting of 20%. In the case of the well-diversified portfolio we remove the 60/40 restriction as it makes little sense to have given the numerous extra asset classes we try to add in this step of the analysis. Every portfolio was optimized by maximizing returns given specific levels of variance as explained in the main text.