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Master thesis

Avoiding the dependency on exact inputs and incorporating realized measures in a portfolio construction setting with the TODIM algorithm

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Abstract

Mean-variance efficient portfolios constructed using the Markowitz framework generally suffer from estimation error. This often leads to severe underperformance of a portfolio. In this paper, the effect of avoiding exact return and covariance estimation is studied by incorporating a multi-criteria decision-making algorithm, TODIM. For this research, intraday data is utilised such that significant relations between realized measures and future returns can be implemented in the model, as this algorithm allows to incorporate additional characteristics. The results display that the TODIM algorithm is able to improve the Sharpe ratio of portfolio allocations in comparison to mean-variance efficient portfolios, even more so when the amount of risk an investor is willing to take increases. Therefore, I argue that avoiding the sensitivity to exact inputs and incorporating realized measures have a positive effect on the performance of a portfolio.

Keywords: portfolio construction, TODIM, plug-in estimation error, realized measures

1 Introduction

Since the dawn of financial markets, maximizing the return on one's capital has been a shared goal amongst investors. The stock market has ever since served as a platform in which they aspire to reach this goal. The decisions made by investors are mostly based on predictions using available data. In this modern era, despite the easy accessibility of intraday data for a range of investors, consistently accurate return and covariance predictions are hard to model. This paper will examine the possibility of avoiding the dependence on exact predictions of these moments for the allocation of portfolio weights and the incorporation of statistical properties of intraday data to enhance portfolio performance. This will be researched by implementing a variant of a multi-criteria decision-making algorithm called TODIM, which was originally introduced in Gomes and Lima (1992), based on prospect theory as introduced in Kahneman and Tversky (1979). The TODIM algorithm is able to give relative scores to alternatives based on chosen characteristics, where each characteristic can be given a certain weight, upon which the allocation is finally determined by taking the weighted scores of all characteristics into account. Instead of focusing solely on exact returns and volatility, this algorithm provides the ability to incorporate the division of these characteristics in quantiles and to implement any additional characteristic that could serve as beneficial in predicting returns. It has been implemented in a similar setting once before, in Alali and Tolga (2019), where daily returns are used, a constant weighting, and only returns, volatility, and correlation are considered. This paper extends former research by incorporating additional characteristics, intraday data, dynamic weighting, and adding a risk parameter to incorporate the desired risk of an investor.

In the past decades, portfolio allocation literature has been centered around the Markowitz framework, first introduced in Markowitz (1952), in which the mean-variance frontier is introduced by incorporating the benefits of diversification through the covariance of stocks. However, the allocation of portfolio weights is based on the expected return and covariance. As the conditional moments of the returns are unknown, they have to be estimated, which is usually done by using historical information. This can lead to estimation errors for both, which leads to a deviating return and volatility of a portfolio. Furthermore, the portfolio weights are unconstrained, which can lead to extreme positions. These extreme positions, in combination with errors in plug-in estimates, can lead to severe underperformance of a portfolio in terms of both return and volatility in comparison to the expected performance of the optimal portfolio.

The magnitude of the impact of estimation error on portfolio performance is examined for the first time in Jobson and Korkie (1980), which shows in a simulation study that portfolios constructed using the estimated moments perform far from optimal. They show that the aforementioned drawbacks of estimation error and extreme positions are not merely complementary, but that estimation error is the main cause of positions being more extreme. Stocks for which the return is estimated too high are assigned relatively high positive weights and vice versa. While Jobson and Korkie were the first to display the actual impact of plug-in estimation error extensively, dealing with estimation error had long been a topic of discussion.

In James and Stein (1961), the foundation for a shrinkage estimator of expected return is constructed, in which a prediction is shrunk to a benchmark return. A shrinkage estimator of the covariance matrix is proposed in Ledoit and Wolf (2003), with which a shrinkage target can be chosen based on one's own belief. In Black and Litterman (1992), a strategy is developed in which one can incorporate his own belief, to not completely rely on the estimate of expected return derived from historical data. Even though numerous techniques have been developed to decrease the impact of estimation error, several papers have shown that there still exists a large discrepancy between the results empirically attained and theoretically implied.

An extensive comparison of portfolio allocation strategies is displayed in DeMiguel et al. (2009), of which several incorporate shrinkage, and combinations of them. They find that none of the analyzed approaches can consistently outperform the benchmark equally weighted portfolio and that most of the strategies are unable to beat the benchmark during any period. However, this research has been conducted using daily return data.

In recent years, high-frequency data implementation has been gaining more significance. In Liu (2009) it is stated that under certain circumstances, using intraday data to estimate moments greatly improves the performance of a portfolio. Several papers, including Pooter et al. (2008) and Hautsch et al. (2015) focus on accurate estimation of the covariance matrix. Additionally, a significant correlation has been found between future returns and several characteristics of intraday data. A relation between future returns and both realized skewness and realized kurtosis is discussed in Amaya et al. (2015) and the relation between future returns and realized signed jump variation is discussed in Bollerslev et al. (2020).

Still, the research field of accurate return prediction remains scarce. The focus in portfolio construction literature lies mainly on the global minimum variance portfolio, which relies solely on the covariance matrix estimate. The disadvantage of this focus is that the possibilities for less risk-averse investors are limited. Theoretically well-performing strategies to incorporate a desired level of risk and return greatly underperform in practice because of their sensitivity to plug-in estimate errors. To remain able to incorporate a level of risk an investor is willing to take, a method might be required which is less sensitive to these errors. Methods that do not rely on exact estimates are less sensitive to plug-in errors, as slight deviations in accuracy do not directly impact the outcome of the allocation. One way to avoid this reliance is by incorporating quantiles instead of exact values. This research aims to find out whether a portfolio allocation method that is not based on exact inputs can take into account desired risk without being highly susceptible to the shortcomings of accurate estimation. Additionally, this research sheds light on whether incorporating the aforementioned relations between characteristics of high-frequency returns and future returns could enhance the performance of a portfolio allocation strategy.

Both the relationship between characteristics of stocks and future returns and incorporating quantiles instead of exact estimates are far from new in the field of portfolio construction. In Basu (1977), the relation between the Price/Earning (P/E) ratio and future returns is researched and portfolios are constructed based upon their respective quantile by sorting the stocks. Eugene and French (1992) research both size and book-to-market ratio to construct a portfolio based on the quantile of a certain stock with respect to a characteristic. In Lee and Swaminathan (2000) a double sort is implemented using both momentum and trading volume, and in Daniel et al. (1997) a triple sort is implemented using size, book-to-market ratio, and momentum. In these papers, portfolios are constructed based upon the quantile of a specific characteristic of a stock.

This approach seems fitting to solve the research questions, yet the framework brings along certain drawbacks. Firstly, implementing varying importance given to specific characteristics is hard to quantify when dividing the stocks into groups based upon quantiles of various characteristics. Secondly, when the number of characteristics taken into account increases, the sorting becomes increasingly complex as the number of portfolios taken into account increases exponentially based on the number of quantiles used and characteristics. When five characteristics are implemented with only 3 quantiles, there are already 243 different portfolios to be constructed. Finally, in most papers, a high minus low approach is normally considered, in which the returns of the best performing quantile portfolio are subtracted from the worst-performing quantile portfolio to determine what the return of a strategy potentially can be. This approach leads to only taking a minimal subset of stocks into account for the implementation of a strategy. With the remarkable performance of the equally weighted portfolio because of its diversification benefits in mind, a strategy that incorporates the entire set of stocks seems more desirable.

The solution to the main research problem of this paper seems to lie between the optimization framework laid down in Markowitz (1952), and the characteristic sorting framework described in Basu (1977). In the former, the goal is very clear, to minimize the volatility given a target return. In the latter, the focus lies on being able to capture a relationship between future return and a specific characteristic. While a straightforward optimization objective is less clearly represented in the characteristic sorting framework, the objective of a combination of these two research fields can be described as maximizing the return given a certain amount of desired risk determined by how much wealth an investor is willing to bet on a relation found between a certain characteristic and future returns. By increasing the number of characteristics taken into account, the risk of this bet decreases because of the benefits of diversification.

To find a method that provides the possibility to implement this objective, I divert to the field of multi-criteria decision-making (MCDM). The various characteristics which can be used to determine how much wealth to allocate to a stock can be compared to any MCDM problem, in which every characteristic resembles a criterion, and the decision that has to be made is how much wealth should be allocated to a specific alternative. The relation between MCDM and weight allocation in an investment problem has been identified already in Alali and Tolga (2019), where the TODIM algorithm is applied in a portfolio allocation setting. TODIM is a Portuguese acronym for interactive and multi-criteria decision-making, which was first proposed in Gomes and Lima (1992). With this algorithm, characteristics of various stocks can be incorporated, by calculating the respective value for every stock and then comparing the values for every pair of stocks. Based upon the relative score of each stock, a ranking is decided. This ranking is then used to allocate the wealth of an investor accordingly. The algorithm has been praised for being able to take into account prospect theory, an acclaimed way of incorporating human behavior towards different types of risk, as introduced in Kahneman and Tversky (1979). It has been implemented in several fields of research, ranging from environmental research in Soni et al. (2016) to an evaluation of rental properties in Gomes (2009), but so far it has only been implemented twice in an weight allocation setting, in Li et al. (2019) and Alali and Tolga (2019).

In Li et al. (2019), the focus lies on optimal decision-making with dual hesitant fuzzy linguistic information, while in Alali and Tolga (2019), the focus lies on optimal decision-making when allocating weight in a portfolio construction setting. As the research in Alali and Tolga (2019) is most relevant to the issue at hand, the framework from that paper will be followed and extended. That paper argues that the algorithm is well suited for investment purposes for the following reasons: Firstly, there is not a unique criterion that has to be optimized, leaving flexibility for the person who implements the algorithm, both in determining which criteria to take into account and which weights are allocated to each criterion. This provides a possibility to avoid the shortcoming as mentioned for sorted portfolios, in which a separation of criteria importance is hard to implement. Secondly, it avoids the sensitivity of optimal portfolio weights to exact inputs as the characteristics can be divided into quantiles.

Instead of implementing the algorithm because of its ability to incorporate personal preferences, it is proposed to adapt the algorithm to sacrifice this benefit to optimize performance. As the allocation is based on statistical measures, incorporating personal preference in relative weighting could lead to inferior results. However, by amplifying the difference between all allocated weights equally, a general measure of risk can still be implemented, giving the investor the choice of how much risk one is willing to take. I propose to add a risk parameter to the algorithm to enable this amplification of weights straightforwardly. While the original algorithm had no optimization component and a free choice in the weighting for each respective characteristic, I argue that an optimization procedure regarding the weights chosen for each characteristic could be beneficial for maximizing the performance of a portfolio in terms of Sharpe ratio, volatility and return. Lastly, in determining the characteristics for the implementation of the algorithm, the realized intraday measures will be taken into account to determine whether incorporating the relation found between these measures and future returns can serve to improve the performance of a portfolio.

This paper finds that the TODIM algorithm can achieve a very high return while maintaining relatively low volatility. This leads to a Sharpe ratio that exceeds all benchmarks, even when transaction costs are taken into account, yet not significantly. Additionally, it displays a more favorable scaling in terms of return, volatility, and turnover in comparison to the mean-variance portfolios with a target return. Furthermore, realized kurtosis, realized correlation, realized skewness, and realized return are taken into account by the TODIM algorithm, while it does not seem to be able to benefit from realized volatility nor from realized signed jump variation. Finally, a dynamic weighting approach outperforms a non-dynamic weighting approach, advocating the use of dynamic weighting. The results look promising, and a plethora of possibilities remain for extensions to the algorithm, which can be examined in future research.

The rest of the paper is organized as follows. In Section 2, the framework of the TODIM algorithm is discussed, including the deviations from the previous implementations. Additionally, the theory behind the benchmark strategies is elaborated on. In Section 3, the data is described on which the research is conducted and the process of cleaning the data is explained. Next, in Section 4, the results are explained, conducting a more in-depth analysis step by step on different aspects of the algorithm. Finally, in Section 5, I conclude and put the findings into perspective.

2 Methodology

2.1 Introduction to the TODIM algorithm

The TODIM algorithm is a MCDM algorithm that was originally constructed to provide the ability to implement prospect theory. Prospect theory is based on the assumption that individuals have a different perspective on down- and upside risk, even though they might be of the same magnitude. The algorithm takes predetermined characteristics into account for each alternative, calculates relative scores per characteristic per alternative, and afterward ranks them accordingly based on the weighting given to each characteristic. Using this final ranking, a weighting can be decided for every alternative. In this paper, an adaptation of the original algorithm is proposed, where a distinction is no longer made between upside and downside risk for an investor, but a general risk parameter is introduced which is argued to be more fitting in a weight allocation context. Additionally, a dynamic weighting approach is proposed to capture market dynamics. In the following sections, the algorithm is decomposed step by step, showing where certain adaptations are applied.

2.1.1 Introduction to the example case

To provide additional clarity on the structure of the algorithm, an example will be provided for each step using a limited set of alternatives and characteristics. In this example, six alternatives A to F are considered and two characteristics will be incorporated. The alternatives represent stocks and the characteristics taken into account for this example are return and volatility.

Table 1: Example stocks and their corresponding characteristics.

	A	B	C	D	E	F
Return	-1%	3%	7%	5%	2%	4%
Volatility	4%	5%	2%	6%	1%	3%

Note. This table displays the estimated annual return and volatility in the first and second row respectively, while the various stocks are displayed per column.

2.1.2 Setting up the criteria matrix

The first step of the algorithm is to set up a $N \times C$ criteria matrix CM. The N rows of this matrix represent all the stocks that are taken into account for the weight allocation. In the actual analysis, the C columns represent the following characteristics taken into account for the weight allocation: realized return, realized correlation, realized variance, realized signed jump variation, realized skewness, and realized kurtosis. The definition of these realized measures is given in Section 2.2. For the example case the criteria matrix CM is displayed in Equation 1.

$$CM = \begin{bmatrix} -0.01 & 0.04 \\ 0.03 & 0.05 \\ 0.07 & 0.02 \\ 0.05 & 0.06 \\ 0.02 & 0.01 \\ 0.04 & 0.03 \end{bmatrix} \tag{1}$$

2.1.3 Grouping and normalizing the criteria matrix

After the criteria matrix has been constructed, every value will be transformed to the respective quantile it belongs to per column. For example, if a stock has the highest realized return and the stocks are divided into ten categories, that stock will be given a value of 10 in the respective column. Following the same reasoning, the stocks with the lowest realized return will be given a value of 1 in the respective column. This way, we assign for each stock a relative score per characteristic. Subsequently, the criteria matrix will be normalized by using the formula in Equation 2 for characteristics that are deemed favorable to maximize, while Equation 3 is used for characteristics which are deemed favorable to minimize.

$$NM_{i,j} = \frac{CM_{i,j} - \min_i CM_{i,j}}{\max_i CM_{i,j} - \min_i CM_{i,j}} \tag{2}$$

$$NM_{i,j} = \frac{\max_i CM_{i,j} - CM_{i,j}}{\max_i CM_{i,j} - \min_i CM_{i,j}} \tag{3}$$

In Equation 2, NM represents the normalized matrix. In this paper, only realized return will be transformed using Equation 2, while the other characteristics are all transformed using Equation 3. After applying this transformation to every value in the criteria matrix, each entry of the matrix lies between 0 and 1.

$$\begin{bmatrix} -0.01 & 0.04 \\ 0.03 & 0.05 \\ 0.07 & 0.02 \\ 0.05 & 0.06 \\ 0.02 & 0.01 \\ 0.04 & 0.03 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 1 \\ 3 & 3 \\ 1 & 1 \\ 2 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \\ 1 & 1 \\ 1 & 0 \\ 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = NM \tag{4}$$

For the example case, the transformations are displayed in Equation 4 and the stocks are divided into three quantiles. Return is a characteristic that is favorable to maximize, while volatility is deemed to be favorable to minimize.

2.1.4 Comparison of alternatives

Having transformed the criteria matrix, the different stocks can be compared to each other. Lourenzutti and Krohling (2013) is followed for the comparison approach, which proposes a new variant to overcome certain pitfalls of the original approach. Firstly, a $N \times N$ criteria score matrix CS_c is created C times, once for every characteristic c. In these matrices, the rows and columns both represent the set of stocks, and the values in the matrix represent the score of the row stock relative to the column stock for its respective characteristic. These relative scores are computed by following the logic stated in Equation 5.

$$CS_{c,i,j} = \begin{cases} \frac{w_c\sqrt{(NM_{i,c}-NM_{j,c})}}{\sum_{c=1}^{C} w_c} & \text{if } NM_{i,c} > NM_{j,c} \\ 0 & \text{if } NM_{i,c} = NM_{j,c} \\ \frac{-1}{\theta}\frac{w_c\sqrt{(NM_{j,c}-NM_{i,c})}}{\sum_{c=1}^{C} w_c} & \text{if } NM_{i,c} < NM_{j,c} \end{cases} \tag{5}$$

In Equation 5, w_c represents the weight of each criterion and θ represents the attenuation factor. In the original implementation of the TODIM algorithm, prospect theory is applied during this step by imposing the attenuation factor θ on low-scoring entries which can be any value in the range $(0, \infty)$. This factor serves the purpose to either amplify or reduce the effect of a lower score, thus implementing a varying response to down- and upward risk. In a weight allocation environment, I argue to set this attenuation factor to one for two reasons. Firstly, as portfolio weights are allowed to be negative, amplifying negative scores can lead to extreme short positions, which is exactly what this implementation aims to avoid. Secondly, as the application of the algorithm is based on utilizing statistical properties, allocating scores on the same scale as the characteristics is desirable to maintain the benefits of these properties in a balanced manner. While the weights w_c are fixed in the original implementation, a dynamic weighting could be better suited to capture changing market dynamics. Each period, the weights will be recalculated based on a maximization of the Sharpe ratio in a previous period, where the weights are bound between 0 and 3. This range is chosen to avoid overfitting by either extreme weight allocation to a single characteristic or by assigning negative weights to certain characteristics. For the example case, the assumption is made that the optimal weight for the return has been determined to be 1, while the optimal weight for volatility has been determined to be 2. Then the corresponding criteria score matrices are constructed as displayed in Equations 6 and 7.

$$CS_{return} = \begin{bmatrix} 0 & -\frac{1}{3}\sqrt{\frac{1}{2}} & -\frac{1}{3} & -\frac{1}{3} & 0 & -\frac{1}{3}\sqrt{\frac{1}{2}} \\ \frac{1}{3}\sqrt{\frac{1}{2}} & 0 & -\frac{1}{3}\sqrt{\frac{1}{2}} & -\frac{1}{3}\sqrt{\frac{1}{2}} & \frac{1}{3}\sqrt{\frac{1}{2}} & 0 \\ \frac{1}{3} & \frac{1}{3}\sqrt{\frac{1}{2}} & 0 & 0 & \frac{1}{3} & \frac{1}{3}\sqrt{\frac{1}{2}} \\ \frac{1}{3} & \frac{1}{3}\sqrt{\frac{1}{2}} & 0 & 0 & \frac{1}{3} & \frac{1}{3}\sqrt{\frac{1}{2}} \\ 0 & -\frac{1}{3}\sqrt{\frac{1}{2}} & -\frac{1}{3} & -\frac{1}{3} & 0 & -\frac{1}{3}\sqrt{\frac{1}{2}} \\ \frac{1}{3}\sqrt{\frac{1}{2}} & 0 & -\frac{1}{3}\sqrt{\frac{1}{2}} & -\frac{1}{3}\sqrt{\frac{1}{2}} & \frac{1}{3}\sqrt{\frac{1}{2}} & 0 \end{bmatrix} \tag{6}$$

$$CS_{volatility} = \begin{bmatrix} 0 & \frac{2}{3}\sqrt{\frac{1}{2}} & -\frac{2}{3}\sqrt{\frac{1}{2}} & \frac{2}{3}\sqrt{\frac{1}{2}} & -\frac{1}{3}\sqrt{\frac{1}{2}} & 0 \\ -\frac{2}{3}\sqrt{\frac{1}{2}} & 0 & -\frac{2}{3} & 0 & -\frac{2}{3} & -\frac{2}{3}\sqrt{\frac{1}{2}} \\ \frac{2}{3}\sqrt{\frac{1}{2}} & \frac{2}{3} & 0 & \frac{2}{3} & 0 & \frac{2}{3}\sqrt{\frac{1}{2}} \\ -\frac{2}{3}\sqrt{\frac{1}{2}} & 0 & -\frac{2}{3} & 0 & -\frac{2}{3} & -\frac{2}{3}\sqrt{\frac{1}{2}} \\ \frac{2}{3}\sqrt{\frac{1}{2}} & \frac{2}{3} & 0 & \frac{2}{3} & 0 & \frac{2}{3}\sqrt{\frac{1}{2}} \\ 0 & \frac{2}{3}\sqrt{\frac{1}{2}} & -\frac{2}{3}\sqrt{\frac{1}{2}} & \frac{2}{3}\sqrt{\frac{1}{2}} & -\frac{2}{3}\sqrt{\frac{1}{2}} & 0 \end{bmatrix} \tag{7}$$

Afterward, the criteria matrices are summed together to create the final score matrix FS as shown in Equation 8, a $N \times N$ matrix which represents the score of each stock in comparison to the other stocks.

$$FS_{i,j} = \sum_{c=1}^{C} CS_{c,i,j} \tag{8}$$

In Equation 9, the final score matrix is constructed for the case example. Each value in the matrix represents the score of the row stock relative to the column stock.

$$FS = \begin{bmatrix} 0 & \frac{1}{3}\sqrt{\frac{1}{2}} & -\frac{1}{3}-\frac{2}{3}\sqrt{\frac{1}{2}} & -\frac{1}{3}+\frac{2}{3}\sqrt{\frac{1}{2}} & -\frac{2}{3}\sqrt{\frac{1}{2}} & -\frac{1}{3}\sqrt{\frac{1}{2}} \\ -\frac{1}{3}\sqrt{\frac{1}{2}} & 0 & -\frac{2}{3}-\frac{1}{3}\sqrt{\frac{1}{2}} & -\frac{1}{3}\sqrt{\frac{1}{2}} & -\frac{2}{3}+\frac{1}{3}\sqrt{\frac{1}{2}} & -\frac{2}{3}\sqrt{\frac{1}{2}} \\ \frac{1}{3}+\frac{2}{3}\sqrt{\frac{1}{2}} & \frac{2}{3}+\frac{1}{3}\sqrt{\frac{1}{2}} & 0 & \frac{2}{3} & \frac{1}{3} & \sqrt{\frac{1}{2}} \\ \frac{1}{3}-\frac{2}{3}\sqrt{\frac{1}{2}} & \frac{1}{3}\sqrt{\frac{1}{2}} & -\frac{2}{3} & 0 & -\frac{1}{3} & -\frac{1}{3}\sqrt{\frac{1}{2}} \\ \frac{2}{3}\sqrt{\frac{1}{2}} & \frac{2}{3}-\frac{1}{3}\sqrt{\frac{1}{2}} & -\frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3}\sqrt{\frac{1}{2}} \\ \frac{1}{3}\sqrt{\frac{1}{2}} & \frac{2}{3}\sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} & \frac{1}{3}\sqrt{\frac{1}{2}} & -\frac{1}{3}\sqrt{\frac{1}{2}} & 0 \end{bmatrix} \tag{9}$$

2.1.5 Ranking and weight allocation

After having computed the final score matrix, a normalized rank vector $\boldsymbol{R}$ is constructed by implementing the formula in Equation 10. In this vector, each stock is given a value between 0 and 1, which serves to represent the score of a stock.

$$\boldsymbol{R}_i = \frac{\sum_{j=1}^{N} FS_{i,j} - \min_i \sum_{j=1}^{N} FS_{i,j}}{\max_i \sum_{j=1}^{N} FS_{i,j} - \min_i \sum_{j=1}^{N} FS_{i,j}} \tag{10}$$

Then, the weights for the allocation are given by dividing the normalized rank of a stock by the sum of all normalized ranks. The weighting is determined for each stock i in weight vector $\boldsymbol{\pi}$, as shown in Equation 11.

$$\boldsymbol{\pi}_i = \frac{\boldsymbol{R}_i}{\sum_{j=1}^{N} \boldsymbol{R}_j} \tag{11}$$

The results of applying both transformations to the example final score matrix are shown in Equation 12.

$$\boldsymbol{R} = \begin{bmatrix} \frac{1}{5} \\ 0 \\ 1 \\ \frac{1}{5} \\ \frac{3}{5} \\ \frac{2}{5} \end{bmatrix} \qquad \boldsymbol{\pi} = \begin{bmatrix} \frac{1}{12} \\ 0 \\ \frac{5}{12} \\ \frac{1}{12} \\ \frac{3}{12} \\ \frac{2}{12} \end{bmatrix} \tag{12}$$

In Equation 12, the weight allocation is displayed resulting from the input given in Table 1. The difference in weight allocated to asset A and asset B is an example of how the TODIM approach differs from both a portfolio sort approach and the Markowitz approach. While asset B might look more favorable than asset A at first sight, the allocated weight implies the opposite. The reason for this difference is twofold. Firstly, while the return of asset A is substantially lower than asset B, the grouping of the characteristics in quantiles reduces the gap between both assets with respect to this characteristic. The return of asset A is not treated any differently than the return of asset E, which showcases the decreased dependency on exact inputs, as they are aggregated to quantiles in which they are assigned an equal value. Secondly, the optimization of the characteristic weights determined that volatility should be taken into account more than return. While asset A is grouped in a lower return quantile than asset B, asset A is still favored over asset B, as it is grouped in a lower volatility quantile as well. As asset B attains the lowest score in the ranking, it is assigned a weight of 0, while the rest of the assets are given a weight dependent on how good their score is compared to asset B. Thus, asset C is given the highest weight, which is to be expected as it belongs to the highest return quantile and the lowest volatility quantile, resulting in the highest score of all assets.

2.1.6 Risk parameter

The weight vector computation in Equation 11 implies a restriction in which short selling is not allowed. For an investor willing to take on more risk, short selling can be a valuable tool to generate leveraged wealth. In this step, I propose to add a risk parameter, which resembles how high an investor desires the sum of his absolute weight values to be. This risk parameter will be implemented as follows. Firstly, an investor determines the amount of risk one is willing to take as ρ, which roughly resembles the sum of absolute weights, dependent on the value of ρ and how the current weights of the weight vector are divided and is non-negative. Afterward, a scale parameter Ω is determined by implementing the risk parameter in the following formula:

$$\Omega = \frac{\rho}{\sum_{i=1}^{N} |\boldsymbol{\pi}_i - \mu_{\boldsymbol{\pi}}|} - 1 \tag{13}$$

In Equation 13, $\mu_{\boldsymbol{\pi}}$ represents the mean of the weight vector. After Ω has been determined, the weight vector is transformed using the following logic. If all stocks were ranked equally, the allocation would be the equally weighted allocation. Any discrepancy in rank is represented by the weight deviation from the equally weighted allocation. If an investor would like to amplify this deviation, one can incorporate Equation 14 to determine the weights needed.

$$\boldsymbol{\pi}_R = \boldsymbol{\pi} + (\Omega * (\boldsymbol{\pi} - \mu_{\boldsymbol{\pi}} * \boldsymbol{\iota})) \tag{14}$$

In Equation 14, $\boldsymbol{\pi}_R$ is the vector with risk-adjusted weights to construct the desired portfolio. For the example case, a risk parameter of 2 is determined to display what the effect is of incorporating this added feature in the allocation process. Then, by implementing Equation 13, Ω can be calculated and is equal to 3. Afterward, Ω is substituted in Equation 14 to compute $\boldsymbol{\pi}_R$, as shown in Equation 15.

$$\boldsymbol{\pi}_R = \begin{bmatrix} -\frac{2}{12} \\ -\frac{6}{12} \\ \frac{14}{12} \\ -\frac{2}{12} \\ \frac{6}{12} \\ \frac{2}{12} \end{bmatrix} \tag{15}$$

Contrary to a sorted portfolio approach, each stock is considered individually and given a respective weight. In the Markowitz framework, this is also done, but with a more clear optimization objective. As the current approach differs from both, while it inherits characteristics from both, it can be seen as a link between the portfolio sort field of research and the weight allocation field of research.

2.2 Realized measures

The characteristics taken into account when constructing the criteria matrix consist of various realized measures. The assumption is made that overnight return can be observed as noise, as taking them into account for constructing the realized measures greatly affects the values of the realized measures. For the incorporation of intraday data, avoiding a severe distortion of the realized measures is argued to be beneficial for utilizing the relationships found between intraday data and future return. To remain consistent, the overnight return will be excluded for all incorporated characteristics. A 5-minute interval is determined to calculate the return, as advocated in Liu et al. (2015), to optimize the tradeoff between the benefits of utilizing a higher frequency and the negative impact of microstructure noise. The 5-minute log-return of a stock i is defined as follows:

$$r_{i,T+\frac{k}{t}} = p_{t+\frac{k}{t}} - p_{t+\frac{k-1}{t}} \tag{16}$$

In Equation 16, T refers to the time in terms of days, t is the number of 5-minute intervals during day T, which can vary as exchanges have different opening times during several holidays, i refers to the specific 5-minute interval, and p is the log-price of a stock. Following the aforementioned set of assumptions, the realized log-return (RR) of stock i during one trading day is computed as displayed in Equation 17.

$$\mathrm{RR}_{i,T} = \sum_{k=1}^{t-1} r_{i,t+\frac{k}{t}} \tag{17}$$

In Equation 17, trading day T is defined as the time interval from T until $T-1+\frac{t-1}{t}$ and the log-return at time T is the overnight log-return. Having defined the RR, the higher moments can be calculated as well.

$$\mathrm{RV}_{i,T} = \sum_{k=1}^{t-1} r^2_{i,t+\frac{k}{t}} \tag{18}$$

In Equation 18, the realized variance (RV) is defined as the sum of the squared returns. Using the RV, the realized skewness (RSK) and kurtosis (RKT) can be computed as defined in Equations 19 and 20.

$$\mathrm{RSK}_{i,T} = \frac{\sqrt{n}\sum_{k=1}^{t-1} r^3_{i,T+\frac{k}{t}}}{\mathrm{RV}^{3/2}_{i,T}} \tag{19}$$

$$\mathrm{RKT}_{i,T} = \frac{n\sum_{k=1}^{t-1} r^4_{i,T+\frac{k}{t}}}{\mathrm{RV}^{2}_{i,T}} \tag{20}$$

Besides the four realized moments implemented in the algorithm, two less straightforward realized measures will be incorporated, being the realized correlation (RC) and the realized signed jump variation (RSJV). Realized correlation is incorporated to increase the diversification of the stocks to which weights are allocated. However, the realized correlation differs per stock pair, which makes it less straightforward to compute. The realized pair correlation RPC between stock i and j is computed by using the formula as displayed in Equation 21.

$$RPC_(i,j) = \frac{\sum_{k=1}^{t-1}\left(r_{i,T+\frac{k}{t}} - \bar{r}_{i,[T,T+\frac{t-1}{t}]}\right)\left(r_{j,T+\frac{k}{t}} - \bar{r}_{j,[T,T+\frac{t-1}{t}]}\right)}{\sqrt{\sum_{k=1}^{t-1}\left(r_{i,T+\frac{k}{t}} - \bar{r}_{i,[T,T+\frac{t-1}{t}]}\right)^2 \sum\left(r_{j,T+\frac{k}{t}} - \bar{r}_{j,[T,T+\frac{t-1}{t}]}\right)^2}} \tag{21}$$

In Equation 21, $\bar{r}_{i,[T,T+\frac{t-1}{t}]}$ corresponds to the mean of the five-minute log-returns of stock i between T and $T+\frac{t-1}{t}$. The total realized correlation (RC) of a stock can be calculated by summing the RPC of a single stock with every other stock and dividing it by $N-1$.

$$RC_i = \frac{\left(\sum_{j=1,j\neq i}^{N} RPC(i,j)\right)}{N-1} \tag{22}$$

In Equation 22, it is displayed how the RC is calculated, which will be implemented as a characteristic for the TODIM algorithm. The remaining characteristic is the RSJV, for which the intuition is explained in Appendix A, following the framework laid down in Bollerslev et al. (2020) and how it relates to future return.

In short, the expectation exists that information can be retrieved from the direction of volatility during a day. This can be done by looking at upside variance and downward variance, as proposed by Barndorff-Nielsen et al. (2008), by splitting the realized variance in a negative and a positive semi-variance, which are displayed in Equations 23 and 24.

$$\mathrm{RV}^{+}_{i,T} = \sum_{k=1}^{t-1} r^2_{T+\frac{k}{t}} \mathbf{1}_{\left\{r_{T+\frac{k}{t}}>0\right\}} \tag{23}$$

$$\mathrm{RV}^{-}_{i,T} = \sum_{k=1}^{t-1} r^2_{t+\frac{k}{t}} \mathbf{1}_{\left\{r_{T+\frac{k}{t}}<0\right\}} \tag{24}$$

By subtracting the negative semi-variance from the positive semi-variance, as proposed in Bollerslev et al. (2020), the signed jump variation can be computed, which is displayed in Equation 25.

$$\mathrm{SJ}_{i,T} = \mathrm{RV}^{+}_{i,T} - \mathrm{RV}^{-}_{i,T} \tag{25}$$

To make this measure scale-invariant, it can be divided by the realized variance to get a normalized measure ranging from -1 to 1. This leads to the realized signed jump variation, as displayed in Equation 26

$$\mathrm{RSJ}_{i,T} = \frac{\mathrm{SJ}_{i,T}}{\mathrm{RV}_{i,T}} \tag{26}$$

Depending on the period over which the realized measures are calculated, they might need to be adjusted to the respective timescale. To adapt a measure to a K day time frame, the following transformation formula can be used:

$$\mathrm{RM}^{K}_{i,T} = \frac{1}{K}\left(\sum_{k=0}^{K-1} \mathrm{RM}_{T-k}\right) \tag{27}$$

In Equation 27, RM represents the realized measure.

2.3 Evaluation

To evaluate the performance of the dynamic TODIM algorithm a comparison is made with several well-known weight allocation strategies. Following the benchmarks in this field of research, the strategies are the following: equally-weighted (EW), value-weighted market portfolio (VW), global minimum variance (GMV), mean-variance with a target annualized return x (MV $x\%$), global minimum variance using the shrinkage covariance estimator from Ledoit and Wolf (GMV-LW), and mean-variance using the James-Stein mean estimator with a target annualized return x (MV-JS $x\%$). This performance will be measured in terms of return, volatility, Sharpe ratio, and turnover.

2.3.1 Equally weighted portfolio

The EW portfolio also referred to as the $\frac{1}{N}$ portfolio, can be seen as the embodiment of naive diversification. For every dollar invested, exactly $\frac{1}{N}$ dollar is invested per stock, which can be rebalanced at any timescale. The effectiveness of the portfolio has been shown in DeMiguel et al. (2009), where it is argued that the strategy benefits from not being subjective to any estimation error and from fully diversifying one's wealth. Additionally, the turnover using this strategy is often very low.

2.3.2 Value-weighted market portfolio

The value-weighted market portfolio is constructed by dividing your wealth over all stocks in the market by taking the respective market capitalization into account and allocating your wealth proportionally. This portfolio is used as a benchmark to represent the return of the market, which is often used by stock managers as a goal to beat. The turnover of this portfolio is very low because rebalancing never has to occur as the growth of the individual amounts invested in each company is proportional to the change in market capitalization of each company.

2.3.3 Mean-variance efficient portfolios

The mean-variance efficient portfolios are derived from the seminal work presented in Markowitz (1952), where a framework is proposed to approach the weight allocation problem by solving the optimization problem displayed in Equation 28.

$$\min \frac{1}{2}\boldsymbol{\pi}'\boldsymbol{\Sigma}\boldsymbol{\pi} \quad \text{subject to} \quad \boldsymbol{\imath}'\boldsymbol{\pi} = 1 \quad \text{and} \quad \boldsymbol{\mu}'\boldsymbol{\pi} = \mu_{\text{targ}} \tag{28}$$

In Equation 28, $\boldsymbol{\pi}$ represents the vector of weights, $\boldsymbol{\mu}$ the vector of returns, $\boldsymbol{\Sigma}$ the covariance matrix and μ_{targ} the desired amount of return. The GMV portfolio is a simplified version of this optimization problem, in which the second constraint is not taken into account, leading to the problem displayed in Equation 29.

$$\min \frac{1}{2}\boldsymbol{\pi}'\boldsymbol{\Sigma}\boldsymbol{\pi} \quad \text{subject to} \quad \boldsymbol{\imath}'\boldsymbol{\pi} = 1 \tag{29}$$

This optimization problem has a closed-form solution which can be found by subtracting the constraint from the objective function, multiplying it with a Lagrangian term, and then solving the first-order conditions. The solution for the weight vector is shown in Equation 30.

$$\pi_{\text{gmv}} = \frac{1}{\boldsymbol{\imath}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\imath}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\imath} \tag{30}$$

If an investor wants to incorporate a desired return, the computation becomes a bit more complicated as an additional constraint is added to the maximization problem. Still, a closed-form solution exists, consisting of a linear combination of the weights of the global minimum variance portfolio and an additional mean-variance portfolio. To make the notation more clear, the following variables are defined beforehand:

$$A = \boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}, \quad B = \boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\imath}, \quad C = \boldsymbol{\imath}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\imath} \tag{31}$$

By applying the Lagrangian and solving the first-order conditions again, the weights can be found for the additional mean-variance portfolio $\boldsymbol{\pi}_{mu}$. Using the symbols defined in Equation 31, $\boldsymbol{\pi}_{mu}$ and $\boldsymbol{\pi}_{gmv}$ can be defined as in Equation 32.

$$\boldsymbol{\pi}_{\text{mu}} = \frac{1}{B}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} \quad \text{and} \quad \boldsymbol{\pi}_{\text{gmv}} = \frac{1}{\text{C}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\imath} \tag{32}$$

After having defined these initial mean-variance portfolios, a linear combination can be constructed to calculate the portfolio weights $\boldsymbol{\pi}$ depending on the desired return μ_{targ}, as shown in Equation 33

$$\lambda = \frac{BC\mu_{\text{targ}} - B^2}{AC - B^2} \text{ and } \boldsymbol{\pi} = \lambda\boldsymbol{\pi}_{\text{mu}} + (1-\lambda)\boldsymbol{\pi}_{\text{gmv}} \tag{33}$$

2.3.4 Shrinkage technique-based portfolios

The MV portfolios are constructed to optimize the Sharpe ratio based on incorporating the true moments. However, in reality, the true moments are not known, which is why they need to be estimated. Estimating moments introduces the risk of estimation error, which has been shown to lead to the underperformance of a constructed portfolio in comparison to its expected performance. Various shrinkage estimators have been constructed to decrease the estimation error for the return vector, as well as the estimation error for the covariance matrix. By implementing a shrinkage estimator, bias is introduced to decrease the variance of an estimate. In this research, the GMV portfolio will also be constructed by using the covariance matrix shrinkage estimator as proposed by Ledoit and Wolf (2003), while a MV portfolio with a desired return will be constructed by using the return vector shrinkage estimator as proposed by James and Stein (1961).

The estimator as proposed in Ledoit and Wolf (2003), uses the sample covariance Σ_{Sample}, a shrinkage target Σ_{Target} and shrinkage weight δ to construct the shrunken covariance matrix $\hat{\Sigma}_{Shrink}$, as shown in Equation 34.

$$\hat{\Sigma}_{Shrink} = \delta\Sigma_{Target} + (1-\delta)\,\Sigma_{Sample} \tag{34}$$

In this paper, the chosen shrinkage target for the covariance estimation is the constant correlation covariance matrix, as suggested in Ledoit and Wolf (2003), as it is more straightforward to compute than the single-factor model, while the difference in performance is shown to be insignificant. The shrinkage intensity δ is computed via the proposed method which can be found in Ledoit and Wolf (2003).

The estimator proposed by James and Stein (1961) follows a similar pattern, where the sample return vector $\boldsymbol{\mu}_{Sample}$ is shrunken towards a target return vector $\boldsymbol{\mu}_{Target}$ using shrinkage weight δ, to calculate the shrunken return vector $\hat{\boldsymbol{\mu}}_{Shrink}$ as shown in Equation 35.

$$\hat{\boldsymbol{\mu}}_{Shrink} = \delta\boldsymbol{\mu}_{Target} + (1-\delta)\boldsymbol{\mu}_{Sample} \tag{35}$$

In this paper, the shrinkage target $\boldsymbol{\mu}_{Target}$, which will be used to shrink the return vector, will be the grand mean vector in which every entry is equal to the mean of the sample mean vector. The shrinkage intensity δ is determined by using the formula in Equation 36.

$$\delta = \min\left[1, \frac{(N-2)/T}{\left(\boldsymbol{\mu}_{Sample} - \boldsymbol{\mu}_{Target}\right)' \Sigma^{-1}_{Sample} \left(\boldsymbol{\mu}_{Sample} - \boldsymbol{\mu}_{Target}\right)}\right] \tag{36}$$

After having computed the shrunken moments, these can be plugged into Equation 31 to estimate the portfolio weights using the techniques introduced in Section 2.3.3.

2.3.5 Evaluation metrics

The strategies will be compared using the excess return, volatility, Sharpe ratio, and turnover. The gross return is defined as μ_s for a strategy s and computed as displayed in Equation 37.

$$\mu_s = \frac{1}{T}\sum_{k=1}^{T} \boldsymbol{\mu_k}'\boldsymbol{\pi_{s,k}} \tag{37}$$

In Equation 37, T is the total number of days over which the return is evaluated, $\boldsymbol{\mu_k}$ is the mean vector of returns on day k and $\boldsymbol{\pi_{s,k}}$ is the weight allocation of strategy s on day k. Then the excess return $\mu{*}_s$ is defined as the gross return μ_s minus a risk-free rate Rf. The volatility is defined as σ_s and computed as displayed in Equation 38.

$$\sigma_s = \sqrt{\frac{\sum_{k=1}^{T} |\mu_{k,s} - \mu_s|^2}{T}} \tag{38}$$

In Equation 38, $\mu_{k,s}$ is the return for strategy s on day k. Having computed the excess return and the volatility, the Sharpe ratio of a strategy s can be computed straightforwardly as defined in Equation 39.

$$\mathrm{SR}_s = \frac{\mu{*}_s}{\sigma_s} \tag{39}$$

To test whether the Sharpe ratio and σ for any strategy s are significantly different from another strategy, a circular bootstrap block testing strategy is followed as advised in Ledoit and Wolf (2008) and Ledoit and Wolf (2011) for these two measures respectively. This approach is used as traditional approaches to test the Sharpe ratio for significance as introduced in Jobson and Korkie (1981) and later corrected in Memmel (2003) are shown to often yield invalid results.

The first three measures are focused on the return that is gained from a strategy. Additionally, transaction costs are involved when rebalancing the weights of a portfolio. A strategy can have a high Sharpe ratio, but the additional return gained by this strategy can be offset by high transaction costs, which is why an additional metric is incorporated in the evaluation. This metric is expressed as the turnover, which is defined in Equation 40.

$$\text{Turnover}_k = \frac{1}{R}\sum_{t=1}^{R}\sum_{j=1}^{N}\left(\left|\hat{w}_{k,j,t+1} - \hat{w}_{k,j,t^+}\right|\right) \tag{40}$$

In Equation 40, R is the number of times a portfolio is rebalanced, $w_{k,j,t}$ is the portfolio weight in stock j at time t under strategy k, w_{k,j,t^+} the portfolio weight before rebalancing at $t+1$, and $w_{k,j,t+1}$ is the desired portfolio weight at time $t+1$, after rebalancing.

To bring turnover into perspective, the bid-ask spread is used to calculate transaction costs, as proposed in Amihud and Mendelson (1986). By calculating the average bid-ask spread in the percentage of the value of a stock of all stockss used for the analysis during the respective period, an approximation can be made on how large the impact of transaction costs would be on the return of the portfolio by combining it with the turnover. By subtracting the transaction costs from the return, transaction cost adjusted return can be computed. However, as the strategies do not take into account transaction costs by definition, this is implemented in a separate analysis as the strategies which have a low turnover, which are unable to gain much by taking transaction costs into account, would have an unfair advantage over strategies that would be able to increase their transaction cost adjusted return significantly by incorporating this into the allocation.

3 Data

3.1 Introduction to the data

To perform analysis on portfolio construction, a sample of stocks should be chosen which ideally reflects the stock market in general. In this paper, a selection of 136 stocks is selected, consisting of all constituents of the NASDAQ 100 from 2014 until 2018 for which data is available on the Trade and Quote (TAQ) database, which are displayed in Appendix B. The NASDAQ 100 is an index that tracks the performance of the 100 most actively traded stocks on the NASDAQ. Therefore, selecting the sample of stocks based on the constituents of the NASDAQ 100 should realistically represent the stock market.

The data that are utilized for the analysis consist of all intraday trades of these 136 stocks over the period from the 2nd of January in 2014 until the 31st of December in 2018. In total, the number of trading days contained in the data is 1258. The stock series can be retrieved from the TAQ database, which can be accessed through the Wharton Research Data Services (WRDS). However, the raw data downloaded from this database still needs to be transformed to be able to conduct accurate analysis. The 'highfrequency' package in R is used for most of the transformations of the data.

To start, observations that are irrelevant to the goal of this research are removed. The decisions made whether observations are irrelevant can be divided into several categories. Firstly, only observations within the time slot of a trading day are taken into account. Any observations before 09 : 30 : 00 and after 16 : 00 : 00 are removed for regular trading days, while the timeframe is adjusted for days on which an exchange is only partly open. Secondly, the trades are filtered to only contain trades executed on the

NASDAQ. Additionally, some companies offer various classes of a stock. The difference between classes mainly lies in the distribution of voting rights. For this research, only the class which is included in the NASDAQ 100 is used and trades regarding the remaining classes are removed. Finally, trades with special conditions are removed. These special conditions apply mostly to warranted trades. When a trade is warranted, the price has been determined long before the trade is made. Leaving these trades in the data can result in distorted results as the warranted price often deviates from the actual price at the moment of the trade. Furthermore, warranted trades are not relevant for this research.

Following Liu et al. (2015) to maximize the benefits of the chosen interval, the data is aggregated to five-minute intervals. To do so, previous tick aggregation is implemented, which takes the value of the last trade in every five-minute interval, creates an observation at the exact fifth minute using this value, and removes the remaining trades. Through this process, the amount of data decreases substantially. A total of $98,995$ observations remain per time series. Not every stock has been traded in every five-minute interval over the entire set of series. In this case, the respective observation of this time interval is given an NA value. A distinction is made between NA values occurring between known values of a series and NA values occurring at the beginning or the end of the series. When dealing with the former, the NA values are imputated by carrying the last known value forward, to keep necessary analysis possible. In the latter case, the stock is not available on the exchange and should thus not be considered for the construction of a portfolio. In this case, the NA values are not imputated. Lastly, the prices are converted to log-returns, using Equation 16.

After having converted the stock prices to log-returns, additional transformations are applied. Several events can impact the return of a stock. A stock split does not directly change the return of an investor, but it does change the price of the stock. A shock on the other hand, whether it affects the entire market or just one company, impacts the return on a stock, while also changing the price of the stock. As TAQ data only shows the price of a stock, it is hard to differentiate between different events and what effect they have on the return. To account for this issue, the CRSP database is consulted to track specific events regarding stocks and adapt the log-returns accordingly. The events for which the log-return is changed are dividend issues, stock splits, and mergers. Finally, some of the stocks have been delisted from the exchange before the end of 2018. To avoid survivorship bias, these stocks have still been included, and the delisting log-return is added at the end of the log-return series for the respective stocks.

The risk-free rate is proxied by using the return of 1-month treasury bills, which is retrieved from the Fama and French data library. The CRSP database is consulted to retrieve the closing bid and ask price and the last traded price for each stock for each trading day in the timeframe. The difference between the bid and the ask price is then divided by the last traded price to compute the spread in the percentage of the stock price. This is computed for every stock and every day, after which the average is taken, which equals 0.0303%.

3.2 Descriptive analysis

To provide some clarity on the data, the minimum, maximum and mean values are computed of several characteristics of the 5-minute log-returns. These are displayed in Table 2.

Table 2: Descriptive statistics of 5-minute log-returns.

	Minimum	Mean	Maximum	SD	Skewness	Kurtosis
Min	-15.43%	-0.0032%	1.21%	0.00038	-8.47	13.99
Mean	-4.56%	-0.0001%	5.01%	0.00165	0.67	151.19
Max	-1.08%	0.0014%	16.78%	0.00452	26.94	3464.23

Note. This table displays descriptive statistics of the 5-minute log-returns. The minimum (Min), mean and maximum (Max) of each respective statistic is shown. Furthermore, the SD column displays the standard deviation.

In Table 2, the descriptive statistics of the 5-minute log-returns are shown. Firstly, some extreme values occur which could be caused by specific events in the market, leading to either up or downward shocks. Secondly, the standard deviation displays high values in comparison to the mean return, which can be partially explained by the extreme values. Furthermore, the kurtosis and skewness display very high values. This confirms that the 5-minute log-returns are distributed significantly different than a normal distribution. It is expected that these values become more stabilized when considering a lower frequency of returns.

Table 3: Descriptive statistics of daily returns.

	Minimum	Mean	Maximum	SD	Skewness	Kurtosis
Min	-55.18%	-0.17%	2.88%	0.00500	-10.61	4.12
Mean	-14.42%	0.03%	13.18%	0.01728	0.14	38.00
Max	-2.10 %	0.17%	54.19%	0.03847	29.57	988.80

Note. This table displays descriptive statistics of the daily log-returns. The minimum (Min), mean and maximum (Max) of each respective statistic is shown. Furthermore, the SD column displays the standard deviation.

In Table 3, the descriptive statistics of the daily log-returns are displayed. As expected, the minimum, mean, maximum and standard deviation display higher values than the 5-minute log-returns. Yet, the standard deviation decreases in magnitude relative to the mean return. The kurtosis and skewness have decreased in value, converging more towards a normal distribution.

In Table 4, the descriptive statistics of the monthly log-returns are displayed. In comparison to aforementioned frequencies, the minimum, mean and maximum values display higher values, which is to be expected. The standard deviation again decreased in magnitude relative to the mean return and the kurtosis and skew-

Table 4: Descriptive statistics of monthly returns.

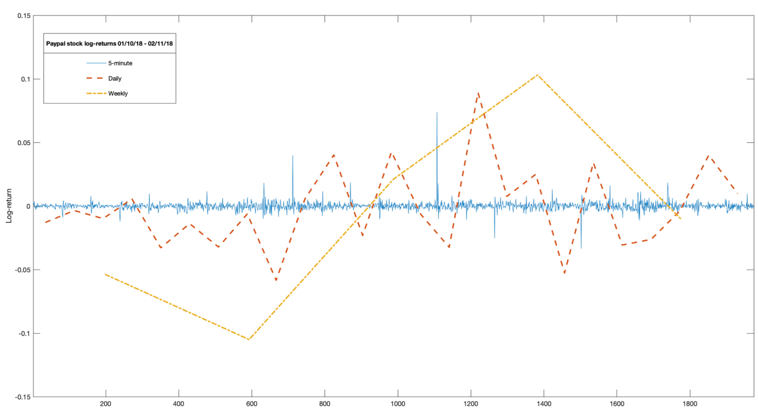
	Minimum	Mean	Maximum	SD	Skewness	Kurtosis
Min	-57.02%	-3.65%	6.24%	0.0195	-2.85	1.83
Mean	-19.37%	0.61%	19.52%	0.0731	0.01	5.20
Max	-1.03%	3.62%	54.09%	0.1749	6.97	52.06

Note. This table displays descriptive statistics of the monthly log-returns. The minimum (Min), mean and maximum (Max) of each respective statistic is shown. Furthermore, the SD column displays the standard deviation.

ness have converged even more towards a normal distribution. To gain additional insights into the statistical properties of the series, an Augmented Dickey-Fuller test to check for stationarity is conducted. The null hypothesis of unit root presence is rejected for all series at a 1% significance level for 5-minute log-returns. As every series is stationary, no additional transformations are required to avoid spurious results.

While descriptive statistics and tests serve as a way to increase the understanding of the data, a visual representation can improve the understanding of what the data consist of. The log-returns of two different stocks during the course of a month are displayed in 5-minute, daily and weekly intervals in Figure 1 and Figure 2.

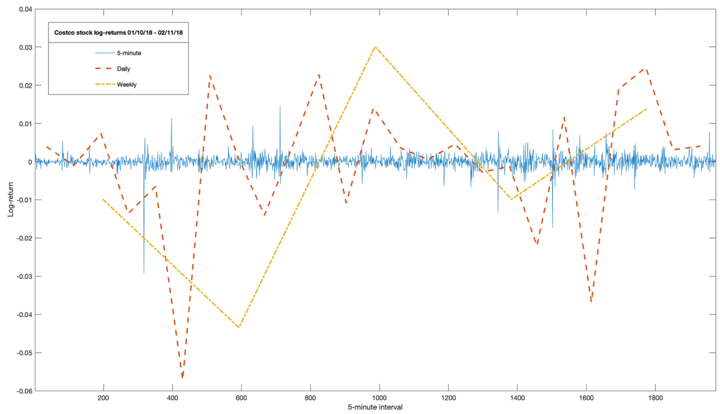
Figure 1: 5-minute, daily and weekly log-returns in October 2018 for Paypal



Note. In the above figures, 5-minute log-returns are displayed with the solid lines, the daily log-returns with the dashed lines and the weekly log-returns with the dashed-dotted lines. The daily and weekly returns are displayed at the midpoint of their respective time-period to clarify the visual comparison.

In Figure 1 and Figure 2, three different frequencies of log-returns are displayed for the month of October in 2018 for two stocks. When comparing both stocks the relative difference between weekly, daily, and 5-minute log-returns is visible. While the weekly log-returns mostly follow a comparable pattern for both stocks, the daily log-returns already display larger discrepancies, while the 5-minute log-returns seem even more distinct.

Figure 2: 5-minute, daily and weekly log-returns in October 2018 for Costco



Note. In the above figures, 5-minute log-returns are displayed with the solid lines, the daily log-returns with the dashed lines and the weekly log-returns with the dashed-dotted lines. The daily and weekly returns are displayed at the midpoint of their respective time-period to clarify the visual comparison.

Additionally, the difference in distributions is also visible in the graphs. While the weekly log-returns follow a fairly stable pattern, the data display relatively more extreme values as the frequency increases.

4 Results

In this section, the results of implementing the various strategies discussed in Section 2 are elaborated on. Firstly, the configurations and decisions for the implementation are elaborated on. Secondly, the weights of the characteristics will be evaluated. Thirdly, the overall results will be discussed and compared with the benchmark strategies. Fourthly, the returns are analyzed year by year to provide a deeper understanding of the performance of the strategies. Fifthly, a more in-depth analysis is conducted to determine the cause of over or underperformance of specific strategies during specific periods. Sixthly, the impact of transaction cost is measured by adding it to the overall results. Seventhly, a scalability analysis is conducted to evaluate the functionality of implementing risk appetite for relevant strategies. Finally, the implementation of the dynamic weighting aspect is evaluated by comparing it to a non-dynamic and an extended estimation window counterpart.

4.1 Implementation

The strategies that are compared are the EW, the value-weighted NASDAQ 100 (VW-N100) and the value-weighted S&P500 (VW-SP500) as the market portfolios, GMV, GMV-LW, MV 5%, MV 10%, MV-JS 5%, MV-JS 10%, and the TODIM algorithm with the risk parameter set at 1, 3 and 5.

The implementations of the TODIM strategies that are compared in this section utilize a rebalancing period of one week, which is implemented for the benchmark strategies as well for comparison purposes. This timeframe is chosen as the relations between realized measures and future return found in Bollerslev et al. (2020) and Amaya et al. (2015) are based on a weekly basis as well. A weekly basis in this paper corresponds to five consecutive trading days. I will refer to each set of five consecutive trading days as a trading week. However, this should not be confused with actual weeks. The exchange may be closed during a day of a normal week, which shifts the fifth trading day over the weekend. In testing the strategy, the trading weeks are not adjusted, so a weekend may occur in the middle of a trading week. As a consequence, the realized moments are also estimated using one trading week of observations. The same estimation window is used for the benchmark strategies to remain consistent. For robustness, the results of implementing a rebalancing period of one day is displayed in Appendix C.

Additionally, the weights of the characteristics in the TODIM method are recalculated after every year, by optimizing the Sharpe ratio in the previous two years. This optimization is done using a grid search over all possible combinations of weights, which results in 4096 possible combinations. In this case, the first two years cover the entire training set, after which a rolling window approach is incorporated by moving the two-year window up, instead of increasing the length of the optimization period. The reasoning behind this decision is that it is expected that the market is constantly developing, shifting the importance between the various characteristics in predicting future return, on which the allocation will be based. Furthermore, a shorter timeframe is not considered for the estimation to avoid overfitting the weights.

4.2 Characteristic weighting

For different risk parameters, a different weighting can serve to be optimal for maximizing the Sharpe ratio. By examining the weights allocated to each characteristic, it can be determined to what extent the TODIM strategy benefits from a certain characteristic. In Table 5, the corresponding weights for each characteristic are given per year during the testing period.

In Table 5 it is observed that RR, RC, and RKT are given large weights, while RV and RSJV are not given any weight at all. Comparing these weights to the conclusions stated in Amaya et al. (2015) and Bollerslev et al. (2020) leads to contrasting results. While those papers find a strong relationship between RSK and RSJV and future return, and only a weak relationship between RKT and future return, the weights given to the respective characteristics imply the opposite. Still, it is important to note that RV, RSJV, RSK, and RKT are closely related. While RV and RSJV have not been assigned any weight, some of their explanatory power can be found in both RSK and RKT. Therefore, it is not possible to conclude that any of the characteristics plays a significantly more important role in weighting than any other. Yet, the leading characteristics used by the TODIM algorithm when maximizing the Sharpe ratio are RR, RC, and RKT, and to a lesser extent RSK.

Table 5: Weights given to each characteristic in the various implementations of the TODIM algorithm.

	RR	RC	RV	RSJV	RSK	RKT
$\rho = 1$						
2016	1	0	0	0	0	1
2017	0	3	0	0	0	2
2018	3	3	0	0	2	2
$\rho = 3$						
2016	1	0	0	0	0	1
2017	0	2	0	0	0	1
2018	3	3	0	0	2	2
$\rho = 5$						
2016	2	0	0	0	0	3
2017	0	2	0	0	0	1
2018	3	3	0	0	2	3

Note. This table displays the given weights for each respective characteristic per time period. These weights were determined by calculating which set of weights would have attained the highest Sharpe ratio in the prior two years.

4.3 Overall results

In Table 6, the evaluation metrics are displayed for the different strategies and different configurations per strategy over the entire test period. In terms of return, it can be observed that the TODIM strategies can outperform most other strategies, with the only exception being that the return of the MV-JS strategies exceeds that of the TODIM-1 strategy. However, in terms of volatility, the TODIM algorithm displays the worst performance of all strategies. Notably, the return seems to scale at a higher rate than the volatility, to a certain extent. The MV portfolios with differing return only seem to scale minimally in both return and volatility. Higher volatility is expected with a higher return, so it can be more interesting to put things into perspective by observing the Sharpe Ratio. In terms of the Sharpe ratio, the TODIM strategies with risk parameters 3 and 5 outperform all benchmark strategies. The only strategies that achieve a comparable Sharpe ratio are the MV-JS strategies, with Sharpe ratios measured at 0.905 and 0.922 compared to the 0.927 and 1.021 for the TODIM-3 and TODIM-5 strategies respectively. Surprisingly, the EW strategy performs the worst in terms of Sharpe ratio, while it is generally considered to be a hard-to-beat benchmark. Furthermore, it is displayed that the shrinkage techniques increase the performance of the non-shrinkage MV and GMV

Table 6: Evaluation metrics for each strategy from 2016 until 2018

	EW	VW-N100	VW-SP500	GMV	GMV-LW	MV 5%	MV-JS 5%	MV 10%	MV-JS 10%	TODIM-1	TODIM-3	TODIM-5
Return	5.88%	4.95%	3.43%	5.49%	5.99%	6.38%	12.70%	6.39%	12.99%	10.16%	18.97%	26.40%
Volatility	15.41%	7.35%	5.65%	8.50%	8.01%	8.40%	14.48%	8.39%	14.53%	16.43%	20.69%	25.69%
TODIM-1	***(0.002)	***(0.000)	***(0.000)	***(0.001)	***(0.007)	***(0.003)	(0.244)	***(0.004)	(0.282)	(-)	***(0.001)	***(0.000)
TODIM-3	***(0.000)	***(0.000)	***(0.000)	***(0.000)	***(0.001)	***(0.000)	**(0.016)	***(0.000)	**(0.016)	***(0.001)	(-)	***(0.000)
TODIM-5	***(0.000)	***(0.000)	***(0.000)	***(0.000)	***(0.000)	***(0.000)	***(0.001)	***(0.000)	***(0.002)	***(0.000)	***(0.000)	(-)
Sharpe	0.382	0.674	0.607	0.645	0.747	0.760	0.877	0.762	0.894	0.619	0.917	1.028
TODIM-1	(0.200)	(0.839)	(0.969)	(0.969)	(0.859)	(0.862)	(0.790)	(0.842)	(0.783)	(-)	(0.283)	(0.341)
TODIM-3	(0.266)	(0.607)	(0.566)	(0.738)	(0.835)	(0.830)	(0.970)	(0.848)	(0.982)	(0.283)	(-)	(0.541)
TODIM-5	(0.301)	(0.561)	(0.499)	(0.656)	(0.719)	(0.761)	(0.878)	(0.755)	(0.895)	(0.341)	(0.541)	(-)
Turnover	0.036	0.007	0.007	3.190	1.934	3.233	6.907	3.232	6.918	1.383	4.059	6.684

*Note. This table displays the evaluation metrics for all benchmark strategies and the TODIM strategies with various configurations from the 1st trading day of 2016 until the last trading day of 2018. The return, volatility and Sharpe ratio are annualized. Underneath both volatility and Sharpe ratio the p-values are displayed in brackets as a result from testing for a significant difference compared to TODIM-1, TODIM-3 and TODIM-5 respectively. *, **, *** represent significance at a 10%, 5% and 1% level respectively.*

strategies. Additionally, nearly all strategies outperform the VW portfolios in terms of the Sharpe ratio. In terms of turnover, after the EW and VW portfolios, the TODIM-1 portfolio displays the best performance. However, the turnover also scales with the risk parameter, so the good performance in terms of turnover does not hold for the remaining TODIM portfolios. Still, the turnover of the TODIM strategies scales at a notably lower rate than the turnover of the MV-JS strategies.

Underneath the volatility and the Sharpe ratio in Table 6 the p-values are displayed from testing whether a significant difference exists regarding both measures. While the volatility of the TODIM strategies is significantly different from the volatility of almost every strategy at a 1% significance level, with the only exception being the MV-JS compared to the TODIM-1 and TODIM-3 strategies, the Sharpe ratio is not significantly different for any pair of strategies. Though the TODIM strategies achieve a high Sharpe ratio, their high volatility likely leads to the Sharpe ratio not being significantly different from any other strategy.

4.4 Results per year

The test set ranges from the beginning of 2016 until the end of 2018, in which the economy faced years with various market conditions. Besides analyzing a general overview of the performance, additional analysis of the various strategies in different environments can lead to more informed conclusions. To conduct a more in-depth analysis, the performance of the different strategies during the various years will be evaluated separately as well, starting with the results from 2016 in Table 7.

In Table 7, the evaluation metrics are displayed for the different strategies and configurations over 2016. In terms of return, the MV-JS strategies perform comparably to the TODIM-5 strategy. Though the volatility also increases when return increases, the MV-JS strategies seem able to achieve an increased return while the volatility remains relatively low, in contrast with the TODIM strategies for which the volatility does seem to scale. Consequently, in terms of the Sharpe ratio, the MV-JS strategies are the best performing strategies during this period. Furthermore, the GMV-LW strategy can achieve a higher Sharpe ratio than most strategies, with only the MV-JS strategies attaining a higher Sharpe ratio. In terms of turnover, the

Table 7: Evaluation metrics for each strategy in 2016

	EW	VW-N100	VW-SP500	GMV	GMV-LW	MV 5%	MV-JS 5%	MV 10%	MV-JS 10%	TODIM-1	TODIM-3	TODIM-5
Return	6.42%	3.66%	4.75%	4.99%	9.24%	6.64%	23.05%	6.54%	22.90%	8.98%	14.15%	19.37%
Volatility	16.60%	6.90%	5.68%	7.81%	8.18%	7.78%	12.21%	7.76%	12.11%	17.12%	20.22%	23.56%
TODIM-1	(0.193)	***(0.000)	***(0.001)	***(0.000)	***(0.000)	***(0.000)	(0.167)	***(0.000)	(0.178)	(-)	**(0.016)	***(0.001)
TODIM-3	***(0.006)	***(0.000)	***(0.000)	***(0.001)	***(0.000)	***(0.000)	**(0.025)	***(0.000)	**(0.028)	**(0.016)	(-)	(0.107)
TODIM-5	***(0.001)	***(0.000)	***(0.000)	***(0.000)	***(0.000)	***(0.000)	***(0.003)	***(0.000)	***(0.006)	***(0.001)	(0.107)	(-)
Sharpe	0.387	0.531	0.837	0.640	1.129	0.854	1.888	0.843	1.890	0.525	0.700	0.822
TODIM-1	(0.785)	(0.932)	(0.988)	(0.961)	(0.868)	(0.838)	(0..790)	(0.855)	(0.771)	(-)	(0.846)	(0.751)
TODIM-3	(0.642)	(0.918)	(0.869)	(0.911)	(0.990)	(1.000)	(0.916)	(0.998)	(0.885)	(0.846)	(-)	(0.954)
TODIM-5	(0.548)	(0.796)	(0.738)	(0.822)	(0.890)	(0.900)	(1.000)	(0.899)	(0.987)	(0.751)	(0.954)	(-)
Turnover	0.049	0.020	0.020	3.496	2.068	3.543	6.909	3.541	6.944	1.397	4.130	6.685

*Note. This table displays the evaluation metrics for all benchmark strategies and the TODIM strategies with various configurations from the 1st trading day of 2016 until the last trading day of 2016. The return, volatility and Sharpe ratio are annualized. Underneath both volatility and Sharpe ratio the p-values are displayed in brackets as a result from testing for a significant difference compared to TODIM-1, TODIM-3 and TODIM-5 respectively. *, **, *** represent significance at a 10%, 5% and 1% level respectively.*

results do not significantly differ from the full period results.

Underneath the volatility and the Sharpe ratio in Table 7 the p-values are displayed from testing whether a significant difference exists regarding both measures. The volatility of the TODIM strategies is significantly different from the volatility of most strategies at a 1% level, except for the EW and TODIM-1, TODIM-3 and TODIM-5, and the MV-JS strategies with the TODIM-1 and TODIM-3 strategies. The Sharpe ratio is again not significantly different for any strategy compared to the Sharpe ratio of the TODIM strategies. In Table 8, the results are displayed over 2017, a year in which the economy endured a steady increase.

Table 8: Evaluation metrics for each strategy in 2017

	EW	VW-N100	VW-SP500	GMV	GMV-LW	MV 5%	MV-JS 5%	MV 10%	MV-JS 10%	TODIM-1	TODIM-3	TODIM-5
Return	21.41%	12.17%	8.31%	19.94%	18.34%	20.74%	16.47%	20.90%	17.58%	27.03%	38.94%	50.41%
Volatility	9.15%	4.33%	2.90%	5.68%	5.43%	5.69%	9.73%	5.69%	9.74%	10.41%	15.55%	21.93%
TODIM-1	***(0.003)	**(0.030)	***(0.007)	**(0.042)	***(0.007)	**(0.025)	*(0.072)	**(0.022)	*(0.080)	(-)	(0.441)	**(0.028)
TODIM-3	***(0.000)	***(0.005)	***(0.001)	***(0.001)	***(0.001)	***(0.000)	(0.469)	***(0.001)	(0.523)	(0.441)	(-)	(0.454)
TODIM-5	***(0.000)	***(0.000)	***(0.001)	***(0.000)	***(0.000)	***(0.000)	**(0.021)	***(0.000)	**(0.029)	**(0.028)	(0.454)	(-)
Sharpe	2.338	2.808	2.871	3.510	3.378	3.645	1.692	3.674	1.804	2.597	2.504	2.299
TODIM-1	(0.121)	(0.150)	(0.126)	(0.109)	(0.111)	(0.123)	(0.253)	(0.120)	(0.239)	(-)	(0.111)	(0.113)
TODIM-3	(0.115)	(0.150)	(0.128)	(0.115)	(0.136)	(0.125)	(0.242)	(0.123)	(0.223)	(0.111)	(-)	(0.168)
TODIM-5	(0.108)	(0.183)	(0.156)	(0.141)	(0.151)	(0.166)	(0.253)	(0.189)	(0.248)	(0.113)	(0.168)	(-)
Turnover	0.051	0.020	0.020	2.535	1.506	2.569	5.215	2.564	5.225	1.356	3.901	6.495

*Note. This table displays the evaluation metrics for all benchmark strategies and the TODIM strategies with various configurations from the 1st trading day of 2017 until the last trading day of 2017. The return, volatility and Sharpe ratio are annualized. Underneath both volatility and Sharpe ratio the p-values are displayed in brackets as a result from testing for a significant difference compared to TODIM-1, TODIM-3 and TODIM-5 respectively. *, **, *** represent significance at a 10%, 5% and 1% level respectively.*

In Table 8, the evaluation metrics are displayed for the different strategies and configurations over 2017. In terms of return, every TODIM strategy outperforms the other strategies. However, the GMV and the GMV-LW strategies both display an exceptionally high return in comparison to the previously examined periods. In terms of volatility, most strategies display a strong performance during this period. This leads to the high Sharpe ratios during this period, where most strategies achieve comparable performance, but where the TODIM strategies are unable to outperform most other strategies in terms of Sharpe ratio with the only exception being the MV-JS strategies. In terms of turnover, the results remain comparable to the previous year.

Underneath the volatility and the Sharpe ratio in Table 8 the p-values are displayed from testing whether a significant difference exists regarding both measures. The same pattern persists as in the previous tables, although there are slightly more strategies for which the volatility is now only significantly different at a 5% significance level instead of at a 1% significance level compared to the TODIM strategies. Still, the Sharpe ratio remains insignificantly different at every significance level for every strategy. In Table 9, the results are displayed over 2018, a year in which the economic growth declined.

Table 9: Evaluation metrics for each strategy in 2018

	EW	VW-N100	VW-SP500	GMV	GMV-LW	MV 5%	MV-JS 5%	MV 10%	MV-JS 10%	TODIM-1	TODIM-3	TODIM-5
Return	-10.19%	-0.97%	-2.77%	-8.47%	-9.63%	-8.25%	-1.46%	-8.25%	-1.53%	-5.51%	3.83%	9.44%
Volatility	18.79%	9.78%	7.44%	11.07%	9.76%	10.84%	19.64%	10.84%	19.82%	20.21%	25.20%	30.75%
TODIM-1	**(0.030)	***(0.001)	***(0.001)	***(0.000)	***(0.000)	***(0.000)	***(0.006)	***(0.000)	**(0.011)	(-)	**(0.000)	***(0.002)
TODIM-3	***(0.008)	***(0.001)	***(0.000)	***(0.001)	***(0.000)	***(0.000)	***(0.001)	***(0.000)	***(0.001)	**(0.013)	(-)	**(0.013)
TODIM-5	***(0.003)	***(0.000)	***(0.000)	***(0.000)	***(0.000)	***(0.000)	***(0.001)	***(0.000)	***(0.000)	***(0.002)	**(0.013)	(-)
Sharpe	-0.542	-0.099	-0.373	-0.765	-0.987	-0.761	-0.074	-0.761	-0.077	-0.273	0.152	0.307
TODIM-1	(0.431)	(0.303)	(0.301)	(0.306)	(0.261)	(0.273)	(0.362)	(0.262)	(0.376)	(-)	(0.587)	(0.721)
TODIM-3	(0.804)	(0.551)	(0.628)	(0.585)	(0.489)	(0.527)	(0.516)	(0.512)	(0.524)	(0.587)	(-)	(0.513)
TODIM-5	(0.968)	(0.669)	(0.727)	(0.738)	(0.621)	(0.637)	(0.602)	(0.629)	(0.608)	(0.721)	(0.513)	(-)
Turnover	0.047	0.020	0.020	3.588	2.257	3.638	8.695	3.640	8.677	1.395	4.147	6.872

*Note. This table displays the evaluation metrics for all benchmark strategies and the TODIM strategies with various configurations from the 1st trading day of 2018 until the last trading day of 2018. The return, volatility and Sharpe ratio are annualized. Underneath both volatility and Sharpe ratio the p-values are displayed in brackets as a result from testing for a significant difference compared to TODIM-1, TODIM-3 and TODIM-5 respectively. *, **, *** represent significance at a 10%, 5% and 1% level respectively.*

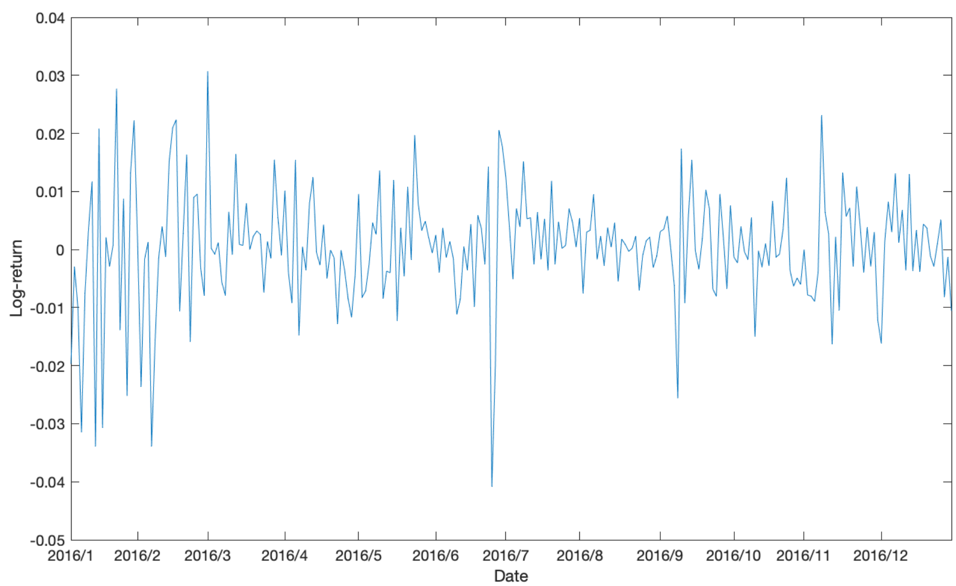
In Table 9, the evaluation metrics are displayed for the different strategies and configurations over 2018. In terms of return, it is shown that most strategies fail to achieve a positive result during this period. The only strategies that can do so are the TODIM-3 and TODIM-5 strategies. In terms of volatility, a sharp increase is displayed for every strategy in comparison to previous years. Furthermore, negative Sharpe ratios are not straightforward to interpret, which makes it hard to reliably compare them in this context. Still, it is surprising to see that the strategies with the highest volatility are the only strategies able to achieve a positive Sharpe ratio in this year of economic decline. In terms of turnover, an increase can be denoted for most strategies, except for the TODIM strategies. The TODIM-1 strategy achieves the lowest turnover besides the EW and VW portfolios, the TODIM-3 strategy achieves a comparable turnover to the GMV and MV portfolio, and the TODIM-5 strategy achieves a better turnover than the MV-JS strategies.

Underneath the volatility and the Sharpe ratio in Table 9 the p-values are displayed from testing whether a significant difference exists regarding both measures. During this year, the volatility of almost every strategy compared to the TODIM strategies is significantly different at a 1% significance level, with the single exception being the MV-JS 10% strategy compared to the TODIM-1 strategy, which has a p-value that is barely higher than 1%. Additionally, the volatility of the TODIM-3 strategy is only significantly different at a 5% significance level from the other two TODIM strategies. Again, the Sharpe ratio of not a single strategy is significantly different from the Sharpe ratio of the TODIM strategies.

4.5 In-depth return analysis

The analysis of individual years displays that the performance of the TODIM strategies compared to the benchmark strategies differs during different market conditions. To get a better understanding of how the TODIM strategy achieves different results, a more in-depth view will be given by comparing the return of selected strategies and determining what the cause could be of either under or overperformance of a strategy. The strategies chosen for this comparison are the MV-JS 10%, GMV-LW, and the TODIM-3 strategy. The MV-JS 10% is chosen as it represents a portfolio with increased risk, similar to the TODIM algorithm, and because it displays the highest Sharpe ratio of all benchmark strategies. the GMV-LW is chosen because of its ability to achieve minimal volatility compared to most strategies, and the TODIM-3 strategy is chosen because it resembles the basis of the algorithm, with moderate amplification by the risk parameter.

Figure 3: The value-weighted Nasdaq-100 returns in 2016.



Note. In this figure, the daily log-returns are displayed for the value weighted Nasdaq-100 portfolio from the first trading day of 2016 until the last trading day of 2016.

In Figure 3, the daily log-return in 2016 is plotted for the value-weighted NASDAQ 100. This specific portfolio is chosen as it represents the market portfolio of the stocks considered in the test set. The first months seem highly volatile, while the rest of the months seem more stable, except for a few sharp declines, which are all followed by a recovery. The expectation that the first months are more volatile than the rest of the year is confirmed by computing the volatility of these months, realizing a volatility of 1.87% and 1.45% for January and February respectively, and a volatility of 0.86% and 0.81% for March and April respectively. Even in June, in which the largest downward shock of the year is displayed, the realized volatility is equal to

1.30%, not exceeding the first two months. Looking at specific months can bring different insights as to how strategies perform in varying environments.

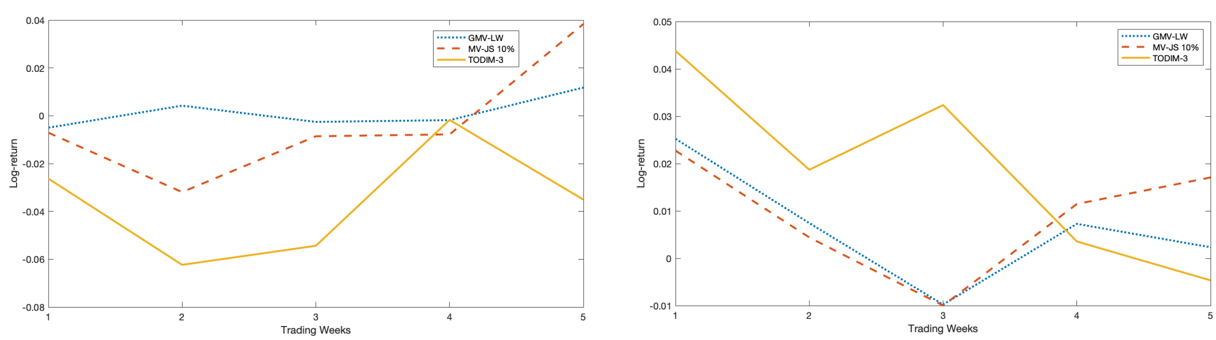
Table 10: Monthly return of compared strategies in 2016

	January	February	March	April	May	June	July	August	September	October	November	December
GMV-LW	0.10%	1.40%	4.71%	-3.14%	2.40%	3.13%	1.17%	-0.37%	0.16%	-1.35%	1.02%	0.10%
MV-JS 10%	2.02%	1.59%	4.93%	4.63%	0.22%	3.08%	2.27%	1.43%	0.07%	4.08%	1.95%	0.75%
TODIM-3	-15.16%	7.55%	5.42%	-0.11%	3.70%	-4.42%	8.13%	1.90%	0.51%	2.45%	4.87%	-0.61%

Note. This table displays monthly log-returns for the the global minimum variance portfolio estimated with the Ledoit and Wolf shrinkage estimator, the mean-variance portfolio with the James-Stein shrinkage estimator and a target annual return of 10%, and the TODIM algorithm with a risk parameter set at 3 for every month in 2016.

In Table 10, the log-return of the compared strategies is displayed per month in 2016. The GMV-LW and the MV JS 10% strategies follow a comparable pattern for some months, while the TODIM-3 strategy seems to deviate in most months. For months where the return deviates the most, a more in-depth analysis could provide useful insights into how the strategy differs from the benchmark strategies. In this year, January and July are examples of such months, in which the TODIM-3 strategy underperforms and overperforms respectively in comparison to the other strategies. Additionally, January is a month in which the VW-N100 displayed a high amount of volatility, while July, during which the VW-N100 realized a volatility of 0.52%, was a month in which there was a relatively low amount of volatility. Analyzing these months can lead to additional insights into the performance of the different strategies in environments with varying volatility. To conduct this analysis, the weekly log-returns of the strategies are plotted over the timespan of the respective month to determine whether the difference in return is caused by a consistent difference in the returns of a portfolio or by specific outliers during which the return differs significantly.

Figure 4: Weekly log-returns of the compared strategies during January and July in 2016.

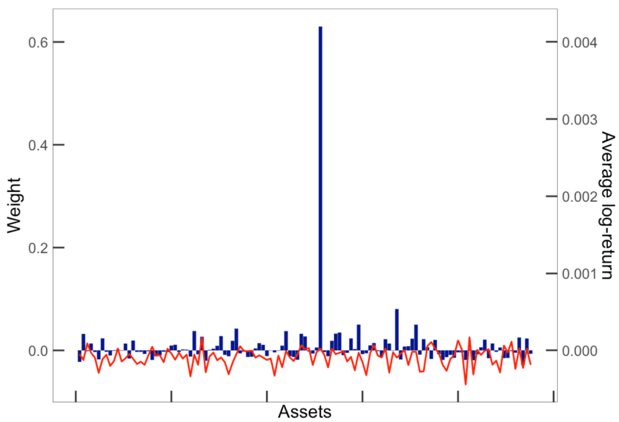


(a) January 2016 log-returns. (b) July 2016 log-returns.

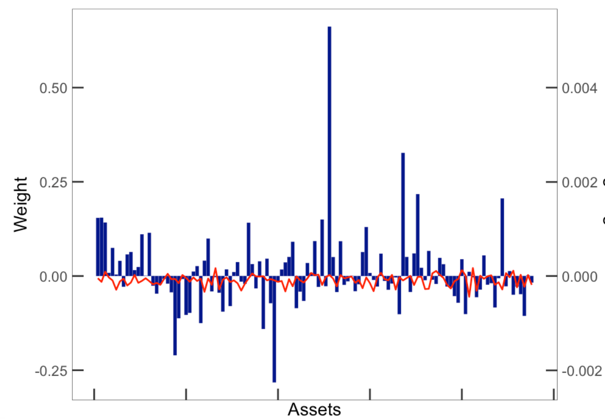
Note. In this figure, the weekly log-returns for January and July 2016 are plotted for the global minimum variance portfolio incorporating the Ledoit-Wolf shrinkage estimator for the covariance matrix, the mean-variance portfolio with a target return of 10% incorporating the James-Stein estimator for the mean vector, and the TODIM algorithm with a risk parameter of 3.

In Figure 4, the weekly log-returns for the compared strategies are plotted for both January and July. For January, it is shown that the TODIM-3 strategy follows a highly deviating pattern compared to the other strategies, incurring severe losses. Additionally, the GMV-LW and the MV-JS 10% follow a very similar pattern. In July, the TODIM-3 strategy achieves a comparable performance to the other strategies in some weeks, but also experiences weeks with a strongly deviating return. In both months, substantial outliers can be observed, which shall be analyzed more in-depth to determine the cause of the deviating performance. The weeks chosen for this in-depth analysis are trading week 2 in January and trading week 3 July, weeks in which the TODIM-3 strategy under- and overperformed in terms of return respectively. While the TODIM-3 strategy deviated more from the other strategies in week 3 of January, it can deliver additional insights to determine why the MV-JS 10% strategy deviated exceptionally much from the GMV-LW strategy this week.

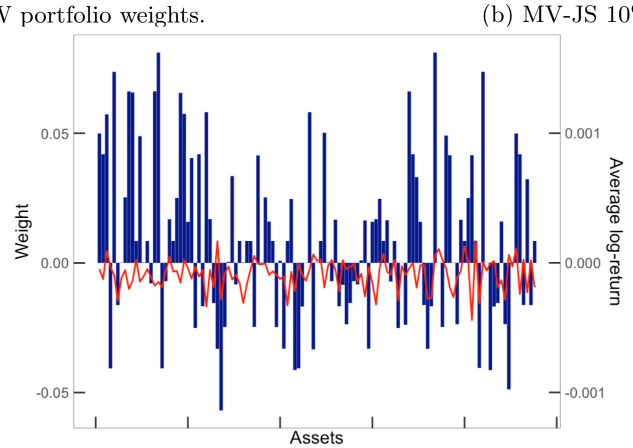
Figure 5: Portfolio weights per strategy and log-returns per stock in the second trading week of January 2016.



(a) GMV-LW portfolio weights.



(b) MV-JS 10% portfolio weights.



(c) TODIM-3 portfolio weights.

Note. In the above figures, the portfolio weights are shown for the compared strategies in the second trading week of January 2016 as a bar plot with the scale on the left axis, while the log-return in the corresponding week is displayed as a line plot with the scale on the right axis. Denote that the scale differs per graph.

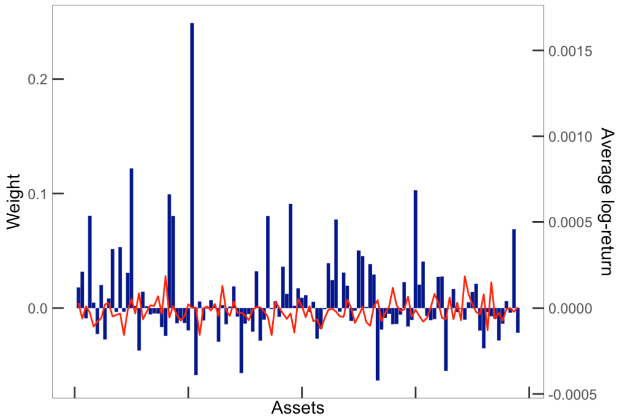
In Figure 5, the portfolio weights during the second trading week of January 2016, which are constructed using the different strategies, and the log-return of the stocks in the following week are shown. Firstly, it is displayed that the MV-JS 10% and the GMV-LW portfolios are very similar. The difference between the two strategies seems to be that the MV-JS 10% attributes more weight to some stocks, and has a more negative short position in certain stocks. The similarity can be explained by the fact that both strategies essentially strive for the same goal, minimizing the variance. The difference is that the MV-JS 10% portfolio additionally aims to achieve a target return, which can explain the increased short and long positions in certain stocks. Still, both strategies assign more than 60% of their initial budget into a single stock. The reason for this extreme allocation of weights is simple; the respective stock realized a volatility of 0.0000177% in the previous week. To put this number into perspective, the stock which experienced the second-lowest volatility in the same week realized a volatility of 0.000654%. The GMV-LW and the MV-JS 10% strategies do achieve exactly what they are designed for: constructing a portfolio with minimal volatility, with the MV-JS 10% portfolio striving for a target return. In the following week, the stock in which most of the wealth is allocated with these strategies remains the stock with the lowest volatility and additionally achieves a positive return. A positive return in this week is quite exceptional, in a week where only 8.4% of the stocks reported a positive return. By focusing on minimizing the volatility, the GMV-LW strategy seems able to avoid allocating large amounts of wealth into stocks that incur severe losses. For the MV 10% strategy, this argument does not hold, as it does allocate some large weights into losing stocks, which is also represented in the large discrepancy in return between the GMV-LW and the MV-JS during this trading week.

Instead of allocating most of the wealth in a few favorable stocks, the TODIM-3 strategy allocates its weights in a more diversified way. It does not solely focus on minimizing volatility but also takes different characteristics into account. By definition, it is highly improbable that the TODIM-3 strategy assigns the majority of wealth to a single stock because of the division into buckets instead of absolute values. This leads to a highly diversified portfolio, which tries to gain from taking short and long positions based on statistical properties. However, in a week where most stocks incur losses, this can lead to a position in which the strategy is unable to avoid severe losses. The algorithm seems unable to distinguish between good and bad performing stocks in terms of return, leading to long positions in stocks with a large negative log-return, and short positions in stocks with only a small negative log-return. When this is combined with the leveraged position in most stocks, the loss of the portfolio gets amplified, resulting in a -6% log-return for this week.

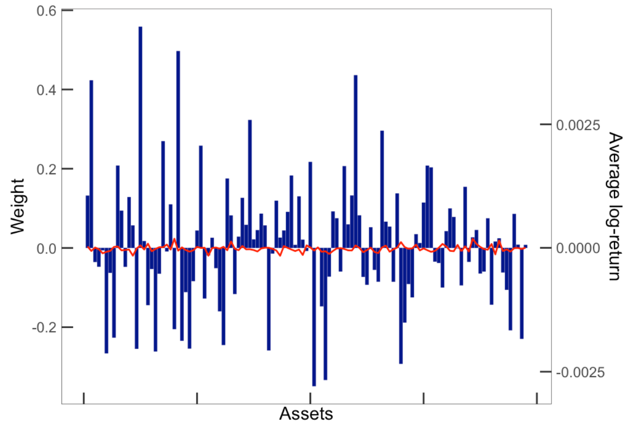
In this week, the underperformance of the TODIM-3 strategy can be explained by the majority of stocks incurring a loss and the algorithm being unable to distinguish between favorable and less favorable stocks. In combination with the leveraged positions taken by the strategy, this can lead to severe losses. However, the leveraging of position can also turn out in favor of the strategy. The third week of July is analyzed in-depth, to determine why the TODIM-3 strategy can outperform the other strategies during this period.

In Figure 6, the portfolio weights during the third trading week of July 2016, which are constructed using the different strategies, and the log-return of the stocks in the following week are shown. In contrast with the previous example, the GMV-LW and the MV-JS 10% strategy have a less similar allocation. While the

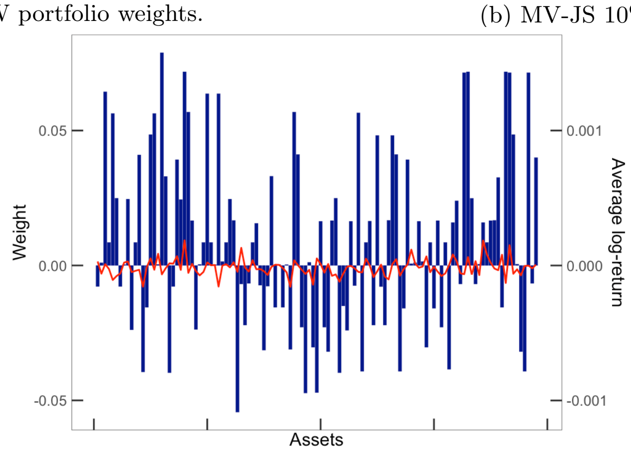
Figure 6: Portfolio weights per strategy and log-returns per stock in the third trading week of July 2016.



(a) GMV-LW portfolio weights.



(b) MV-JS 10% portfolio weights.



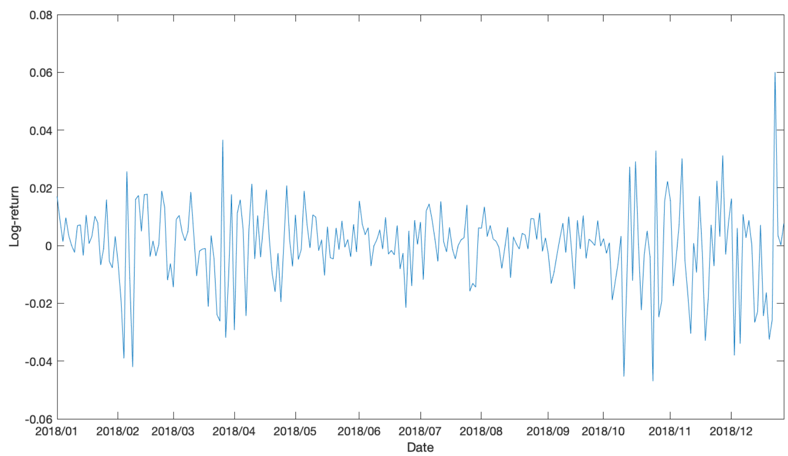
(c) TODIM-3 portfolio weights.

Note. In the above figures, the portfolio weights are shown for the compared strategies in the third trading week of July 2016 as a bar plot with the scale on the left axis, while the log-return in the corresponding week is displayed as a line plot with the scale on the right axis. Denote that the scale differs per graph.

GMV-LW strategy still has a clear stock which it favors, the MV-JS 10% strategy has multiple stocks to which it allocates a large weight, while it subsequently assigns a large negative weight to certain stocks. Interestingly, both the GMV-LW and the MV-JS 10% strategy assign a negative weight to the stock with the second-highest return in the following week, which is part of the reason why both strategies achieve a negative log-return during this week. On the contrary, the TODIM-3 strategy determines that that specific stock is a favorable stock, allocating a large positive weight to it. Overall, the TODIM-3 strategy manages to allocate large positive weights to stocks with a high positive return and a negative weight to stocks with a negative return for most of the allocated weight. Furthermore, 66.67% of the stocks achieve a positive result during this week. In this specific week, the TODIM algorithm seems able to utilize the statistical properties to assign positive and negative weights to profiting and losing stocks respectively in a decent manner. The benefit of being able to incorporate the statistical properties in combination with the leveraged positions leads to a log-return of 3.2% this week.

Only observing 2016 for an in-depth analysis can create a biased view on the performance of the compared strategies, as they might show different characteristics during varying market environments. To gain a broader perspective of the comparative performance of the TODIM strategy, an in-depth analysis of 2018, a year in which most strategies were unable to achieve a positive yearly log-return, while the TODIM-3 strategy did.

Figure 7: The value-weighted Nasdaq 100 returns in 2018.



Note. In this figure, the daily log-returns are displayed for the value weighted Nasdaq-100 portfolio from the first trading day of 2018 until the last trading day of 2018.

In Figure 7, the daily log-returns are plotted for the value-weighted NASDAQ 100 during 2018. It is displayed that 2018 experienced some extremely volatile months towards the end of the year. Still, the volatility during some other months seems relatively low. For example, October and December look more volatile than May and August. Computing the actual values confirms this, with a volatility of 2.10% and 2.37% for October and December respectively, and volatility of 0.69% and 1.08% for May and August respectively. To gain insights into how the different strategies perform during these months with varying volatility, the monthly return for each strategy is computed.

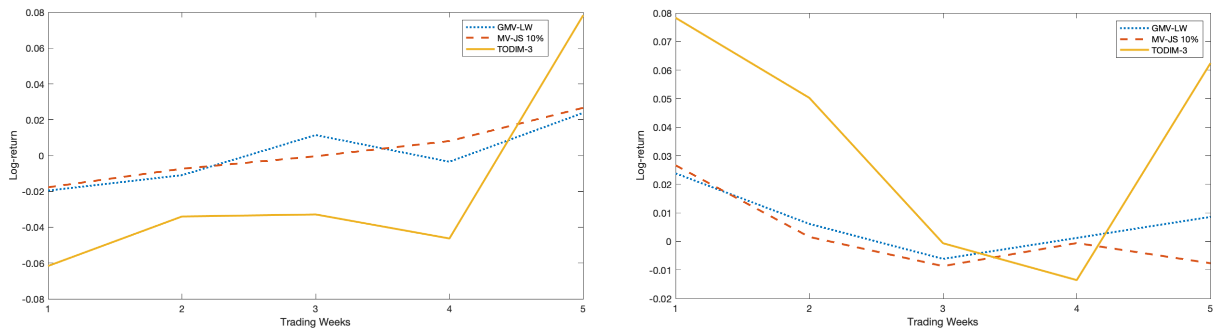
Table 11: Monthly return of compared strategies in 2018

	January	February	March	April	May	June	July	August	September	October	November	December
GMV-LW	4.84%	-4.22%	-3.72%	-2.94%	-2.75%	-0.65%	5.16%	-0.96%	-0.66%	0.16%	0.85%	-3.89%
MV 10%	4.73%	-21.40%	-4.00%	-3.42%	-1.40%	3.33%	8.86%	-0.21%	-0.65%	0.07%	-2.12%	15.46%
TODIM-3	14.19%	-3.35%	2.93%	0.10%	-3.85%	-5.19%	-1.29%	3.42%	6.05%	-14.63%	11.92%	-5.70%

Note. This table displays monthly log-returns for the the global minimum variance portfolio estimated with the Ledoit and Wolf shrinkage estimator, the mean-variance portfolio with the James-Stein shrinkage estimator and a target annual return of 10%, and the TODIM algorithm with a risk parameter set at 3 for every month in 2018.

In Table 11, the log-returns of the compared strategies are displayed per month in 2018. Contrary to 2016, the GMV-LW and MV-JS 10% strategies do not seem to follow a similar pattern, as all three strategies seem to follow a different pattern. Two months in which the performance of the TODIM-3 strategy stands out are October and November, in which the strategy under- and overperforms respectively compared to the other strategies. For these two months, an in-depth analysis of the weekly log-returns is conducted to clarify the difference in performance.

Figure 8: Weekly log-returns of the compared strategies during October and November in 2018.



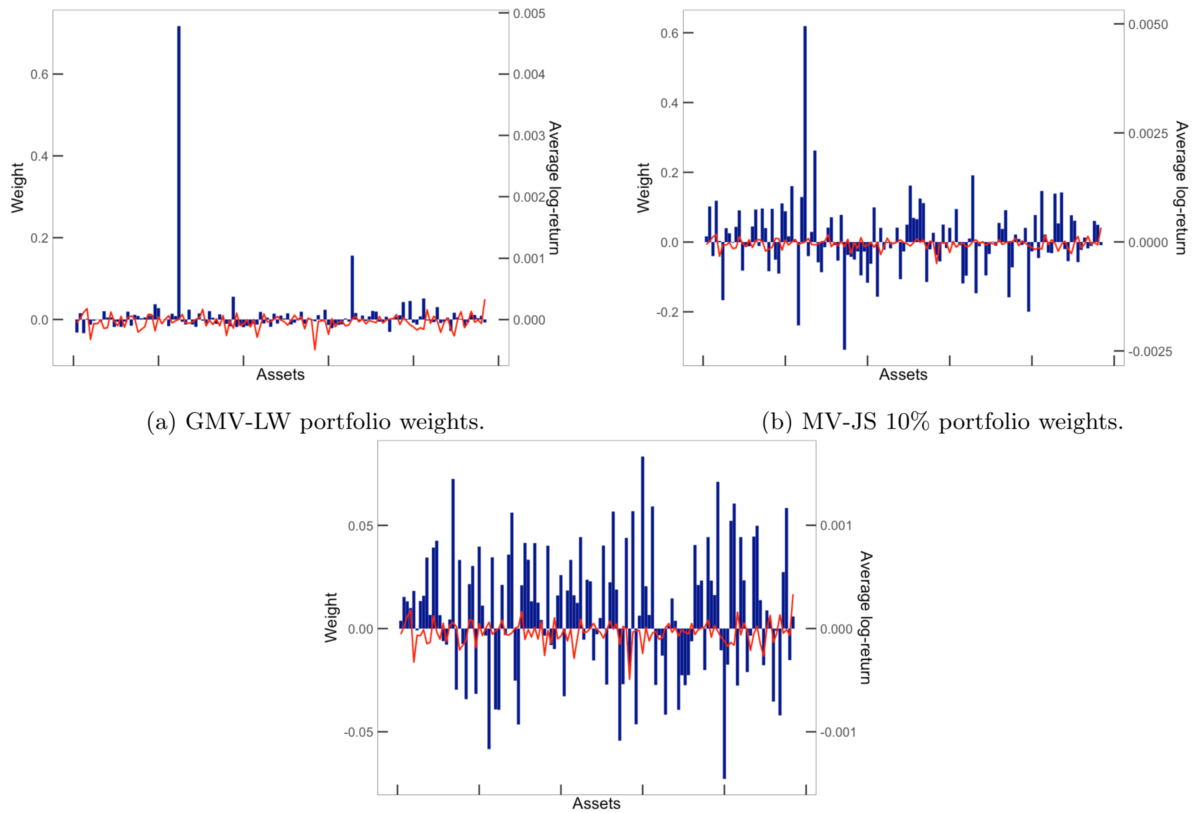
(a) October 2018 log-returns. (b) November 2018 log-returns.

Note. In this figure, the weekly log-returns for October and November 2018 are plotted for the global minimum variance portfolio incorporating the Ledoit-Wolf shrinkage estimator for the covariance matrix, the mean-variance portfolio with a target return of 10% *incorporating the James-Stein estimator for the mean vector, and the TODIM algorithm with a risk parameter of 3.*

In Figure 8, the weekly log-returns are shown for the compared strategies during October and November in 2018. In October, the TODIM-3 strategy follows mostly a similar pattern in comparison to the other strategies but steadily achieving a more amplified loss. The trading week that stands out the most during this month is the fourth, in which the TODIM-3 strategy achieves a log-return of -4.63%, while the MV-JS 10% strategy seems able to retain a positive return. In November, the TODIM-3 strategy achieves a higher return in almost every single week, except for the fourth week, in which it still achieves a comparable return. In these two months, the fourth trading week of October and the fifth trading week of November display the most deviating results for the TODIM-3 strategy. Analyzing these specific weeks in-depth can lead to additional insights into the over and underperformance of the TODIM-3 strategy in comparison to the other strategies in terms of return.

In Figure 9, the portfolio weights during the fourth trading week of October 2018, which are constructed using the different strategies, and the log-return of the stocks in the following week are shown. Again, it can be denoted that the GMV-LW portfolio allocates a large amount of weight to a single stock, while the MV-JS 10% strategy allocates its weights in a slightly more diversified manner. During this week, 64.5% of the stocks incurred a loss. In this example, it is displayed again that the Markowitz-derived strategies perform well in weeks during which most stocks incur losses. By allocating the majority of wealth to a single low volatility stock, it outperforms the TODIM-3 strategy greatly, which is unable to allocate most of its wealth to a single

Figure 9: Portfolio weights per strategy and log-returns per stock in the fourth trading week of October 2018.



(a) GMV-LW portfolio weights.

(b) MV-JS 10% portfolio weights.

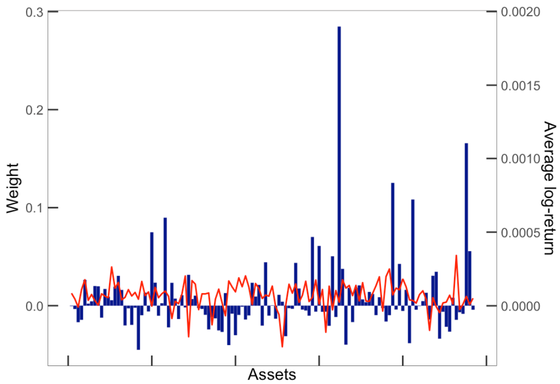
(c) TODIM-3 portfolio weights.

Note. In the above figures, the portfolio weights are shown for the compared strategies in the fourth trading week of October 2018 as a bar plot with the scale on the left axis, while the log-return in the corresponding week is displayed as a line plot with the scale on the right axis. Denote that the scale differs per graph.

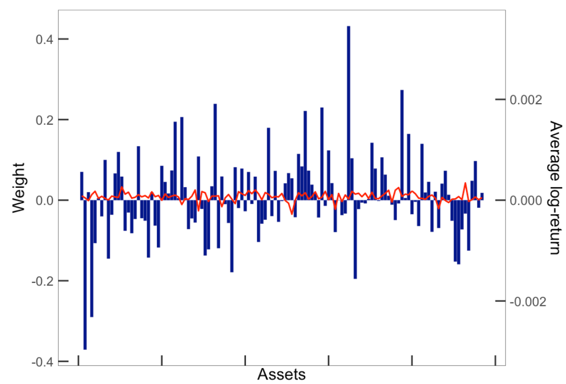
stock because of its diversifying character. Additionally, the algorithm seems unable to distinguish favorable and less favorable stocks decently, leading to large positions in stocks with negative log-returns. Therefore, by holding leveraged positions in a large subset of stocks incurring a loss, the TODIM-3 strategy attains a negative log-return of 4.63% during this week.

In Figure 10, the portfolio weights during the fifth trading week of November 2018, which are constructed using the different strategies, and the log-return of the stocks in the following week are shown. Firstly, it is displayed that the GMV-LW and the MV 10% strategies both take on an extreme position in a single stock. Secondly, 85.1% of the stocks report a positive log-return during this week. Contrary to the previous example, the TODIM-3 strategy can benefit greatly from leveraged positions in stocks with a positive return. Additionally, this example displays a downside of the Markowitz-derived strategies. The strategies are, by definition, focused on minimizing volatility. In this week this is displayed clearly, where both allocate more than 60% of wealth in a single stock. Even though this stock displays minimal volatility in both the previous

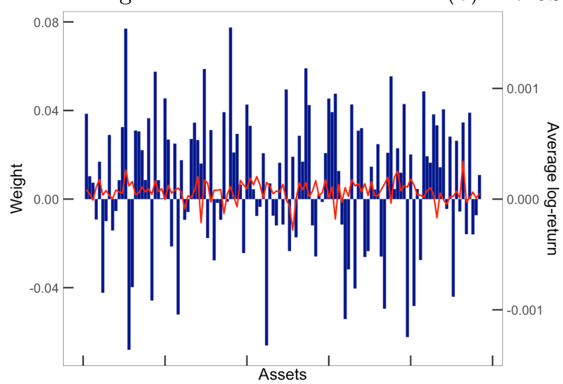
Figure 10: Portfolio weights per strategy and log-returns per stock in the fifth trading week of November 2018.



(a) GMV-LW portfolio weights.



(b) MV-JS 10% portfolio weights.



(c) TODIM-3 portfolio weights.

Note. In the above figures, the portfolio weights are shown for the compared strategies in the fifth trading week of November 2018 as a bar plot with the scale on the left axis, while the log-return in the corresponding week is displayed as a line plot with the scale on the right axis. Denote that the scale differs per graph.

and the following week, a lot of potential returns are missed by focusing solely on minimizing volatility. This shows the main difference between the Markowitz-derived strategies and the TODIM strategies. The former is designed to minimize volatility, while the latter is designed at capturing positive returns. Even with a target return, the MV strategies aim to exactly reach the target return, while minimizing the volatility in the process. While some examples showed weeks during which this worked effectively, avoiding severe losses by focusing on the least volatile stocks, other examples showed large lost opportunities, in weeks where most stocks profited substantially, but where the Markowitz derived strategies missed out on high returns by solely focusing on minimizing the volatility. Although focusing on minimizing volatility can prove to be more beneficial than focusing on maximizing return in certain weeks, the TODIM algorithm displays that in the long run, it does not necessarily lead to better results.

Overall, the TODIM algorithm seems to be able to capture and benefit from statistical properties in a proper manner, although this is not guaranteed for every week. This leads to severe losses in certain weeks, which is the main cause of the volatile behavior of the TODIM algorithm. Still, the high volatility is largely offset by the high returns, as it can achieve a Sharpe ratio that exceeds the Sharpe ratio of all benchmark strategies. Additionally, concerning the incentive for introducing the TODIM algorithm, namely being able to effectively incorporate desired risk, the strategy scales well in comparison to existing methods. While the MV portfolios with a target return display decent performance in terms of return, volatility, and Sharpe ratio, the scalability of desired risk seems absent, as the difference in target return only results in a minimal change in results.

4.6 Transaction cost adjusted returns

In the analysis conducted in the previous sections, turnover is considered as an additional metric, which gives an idea of how large the transaction costs would be. By incorporating the bid-ask spread of the stocks to approximate the actual transaction costs, the transaction cost adjusted returns can be computed. The average bid-ask spread of the stocks during the test period is 0.0303%, which can be translated to a log-return of -0.00881% of the total value of traded stocks per transaction. Thus, this log-return is multiplied by the turnover and by the number of times the portfolios are rebalanced to achieve the transaction costs over a specified period. As the entire set spans three years, the transaction costs are divided by three to annualize them.

Table 12: Evaluation metrics for each strategy from 2016 until 2018 with transaction cost adjusted returns

	EW	VW-N100	VW-SP500	GMV	GMV-LW	MV 5%	MV-JS 5%	MV 10%	MV-JS 10%	TODIM-1	TODIM-3	TODIM-5
Return	5.88%	4.95%	3.43%	5.49%	5.99%	6.38%	12.70%	6.39%	12.99%	10.16%	18.97%	26.40%
Volatility	15.41%	7.35%	5.65%	8.50%	8.01%	8.40%	14.48%	8.39%	14.53%	16.43%	20.69%	25.69%
Sharpe	0.382	0.674	0.607	0.645	0.747	0.760	0.877	0.762	0.894	0.619	0.917	1.028
Turnover	0.029	0.007	0.007	3.190	1.934	3.233	6.907	3.232	6.918	1.383	4.059	6.684
Transaction costs	0.02%	0.00%	0.00%	2.10%	1.27%	2.13%	4.55%	2.13%	4.56%	0.91%	2.67%	4.40%
Adjusted returns	5.86%	4.95%	3.43%	3.39%	4.72%	4.25%	8.15%	4.27%	8.44%	9.25%	16.29%	22.00%
Adjusted Sharpe ratio	0.380	0.673	0.606	0.398	0.588	0.506	0.563	0.509	0.581	0.563	0.787	0.856
TODIM-1	(0.204)	(0.839)	(0.968)	(0.758)	(0.862)	(0.884)	(0.884)	(0.951)	(0.961)	(-)	(0.204)	(0.276)
TODIM-3	(0.254)	(0.592)	(0.552)	(0.533)	(0.612)	(0.634)	(0.628)	(0.731)	(0.725)	(0.204)	(-)	(0.207)
TODIM-5	(0.267)	(0.553)	(0.511)	(0.439)	(0.523)	(0.551)	(0.553)	(0.605)	(0.646)	(0.276)	(0.207)	(-)

*Note. This table displays the evaluation metrics for all benchmark strategies and the TODIM strategies with various configurations from the 1st trading day of 2016 until the last trading day of 2018. The return, volatility and Sharpe ratio are annualized. The transaction costs are derived from the turnover and the average bid-ask spread. The adjusted return and adjusted Sharpe ratio are computed by subtracting the transaction costs from the return, and afterward computing the Sharpe ratio using the adjusted return. Underneath the Sharpe ratio the p-values are displayed in brackets as a result from testing for a significant difference compared to TODIM-1, TODIM-3 and TODIM-5 respectively. *, **, *** represent significance at a 10%, 5% and 1% level respectively.*

In Table 12, the transaction cost adjusted metrics are displayed for all strategies from 2016 until 2018. In Appendix D, the results are displayed per year. Firstly, the relative performance of the value-weighted portfolios in terms of both return and Sharpe ratio increases tremendously compared to the non-transaction adjusted returns. By quantifying the turnover, it is displayed that the transaction costs can impact the return substantially. Therefore, the Sharpe ratios of strategies with a high turnover decrease sharply as well. However, this negative impact is less present for the TODIM strategies. Because of the height of the returns, the transaction costs have a reduced impact on the Sharpe ratio. For the MV-JS portfolios, the decrease is the most apparent. While these strategies originally achieved the highest Sharpe ratio of all benchmarks, they are now outperformed by the VW and the GMV-LW strategies. The VW portfolios now belong to the best performers of all benchmark strategies as they only have minimal transaction costs. Yet, the Sharpe ratio remains substantially lower than the Sharpe ratio of the TODIM strategies with an increased risk parameter.

Underneath the adjusted Sharpe ratio the p-values in Table 12 are displayed from testing whether a significant difference exists regarding this measure. Even though the ratio is adjusted, there is not a single strategy with a significantly different Sharpe ratio than another strategy.

While the incorporation of transaction costs decreases the return of several strategies substantially, the TODIM strategies still remain able to outperform the benchmark models, even though their turnover is relatively high. The main reason that the strong performance persists is that the returns of the TODIM strategies are relatively high, thus proportionally they are less affected by the subtraction of these transaction fees. One could additionally argue that the turnover of the TODIM strategies is relatively low when it is put into perspective with the achieved return.

4.7 Scalability analysis

To come to a more informed conclusion about the difference in difference in the scalability of the strategies for which a desired risk can be incorporated, the MV-JS strategy can be analyzed with a higher desired return, to examine how the mean-variance efficient approach compares to the TODIM algorithm when different levels of return are desired.

Table 13: Performance of various MV-JS and TODIM strategies from 2016 until 2018

	MV-JS - 5%	MV-JS 10%	MV-JS 20%	MV-JS 50%	MV-JS 100%	MV-JS 200%	TODIM-1	TODIM-3	TODIM-5
Return	12.70%	12.99%	13.59%	15.36%	18.33%	24.25%	10.16%	18.97%	26.40%
Volatility	14.48%	14.53%	14.84%	17.17%	24.02%	41.64%	16.43%	20.69%	25.69%
TODIM-1	(0.732)	(0.783)	(0.971)	(0.303)	***(0.009)	***(0.001)	(-)	***(0.000)	***(0.001)
TODIM-3	*(0.099)	*(0.096)	(0.129)	(0.308)	(0.140)	***(0.003)	***(0.000)	(-)	***(0.002)
TODIM-5	**(0.021)	**(0.022)	**(0.014)	**(0.033)	(0.483)	**(0.011)	***(0.001)	***(0.002)	(-)
Sharpe	0.877	0.894	0.915	0.895	0.763	0.582	0.619	0.917	1.028
TODIM-1	(0.899)	(0.935)	(0.974)	(0.960)	(0.924)	(0.951)	(-)	(0.356)	(0.388)
TODIM-3	(0.691)	(0.678)	(0.727)	(0.807)	(0.825)	(0.781)	(0.356)	(-)	(0.616)
TODIM-5	(0.611)	(0.630)	(0.681)	(0.725)	(0.741)	(0.746)	(0.388)	(0.616)	(-)
Turnover	6.907	6.918	7.051	8.252	12.262	22.813	1.383	4.059	6.683

*Note. This table displays evaluation metrics for the mean-variance portfolio with several target annual returns and the TODIM algorithm with various configurations. The return, volatility and Sharpe ratio are annualized. Underneath both volatility and Sharpe ratio the p-values are displayed in brackets as a result from testing for a significant difference compared to TODIM-1, TODIM-3 and TODIM-5 respectively. *, **, *** represent significance at a 10%, 5% and 1% level respectively.*

In Table 13, the evaluation metrics are displayed for the MV-JS strategies with varying desired returns and the TODIM algorithm with varying risk parameters. Firstly, the ratio between desired returns and actual returns for the MV-JS strategies does not scale linearly, as the target return increases at an increasingly higher rate than the actual return. Secondly, as the return increases, the volatility of the MV-JS strategy increases at a higher rate than of the TODIM algorithm, leading to the Sharpe ratio of the TODIM-3 strategy exceeding the Sharpe ratio of all MV-JS strategies. If a comparable return as the TODIM-3 or TODIM-5 strategy was achieved using the MV-JS strategy, the volatility would be roughly 1.2 and 1.6 times as high respectively. Additionally, the lowest turnover of all MV-JS strategies exceeds the highest turnover of all TODIM strategies, strengthening the comparative position of the TODIM approach. By taking the quantification of the turnover in the previous section into account, it can be concluded that the transaction costs scale at a higher rate than the return of the MV-JS strategy, deeming it incapable of effectively achieving the same level of return as the TODIM strategies with an increased risk parameter.

Underneath the volatility and the Sharpe ratio in Table 13 the p-values are displayed from testing whether a significant difference exists regarding both measures. Again, the Sharpe ratio is not significantly different for any of the TODIM strategies compared to any of the MV-JS strategies. Additionally, for most strategies, the volatility is not significantly different from the volatility of the TODIM strategies at a 1% significance level, except for the MV-JS 100% compared to TODIM-1 and MV-JS 200% compared to TODIM-1 and TODIM-3. Furthermore, the volatility three TODIM strategies is also significantly different from each other at a 1% significance level.

Being able to input a desired return is more straightforward and easier quantifiable than having to input a risk parameter that roughly resembles the sum of absolute weights. However, this argument is weakened significantly by the results in Table 13, where it is displayed that the actual return deviates greatly from the desired return. Additionally, the volatility and turnover scale at a much higher rate than the return, which does not hold for the TODIM strategies.

4.8 Dynamic weighting analysis

One of the adaptations in comparison to the original TODIM algorithm is to implement a dynamic weighting algorithm instead of determining the weights once. While Table 5 displayed that the optimal set of weights differed greatly per year, an actual analysis of the difference in performance between the strategy incorporating a dynamic weighting approach with a two year estimation period, an approach with an increasing estimation window taking all observations so far into account, and a strategy determining the weights a single time can serve additional insights into the effect of dynamic weighting. As two years are used to determine the weights for the respective characteristic, 2016 can not be used for this comparison as all three strategies utilize the same weights for this period. However, in the following two years, differing weights are implemented which creates the possibility to compare the performance of each alternative.

In Table 14, the evaluation metrics are displayed for the three different weighting approaches. In Appendix E, an overview of the results per year is displayed. A clear distinction can be made between the differing approaches to the characteristic weighting. In terms of return, the dynamic weighting strategies outperform the non-dynamic weighting strategies for all three values of the risk parameter. In terms of volatility, the opposing conclusion can be made, as the volatility is lower for the non-dynamic weighting strategies. Still, in

Table 14: Evaluation metrics for the dynamic and non-dynamic weighting TODIM strategies from 2017 until 2018

	TODIM-1	TODIM-3	TODIM-5	TODIM-1	TODIM-3	TODIM-5	TODIM-1	TODIM-3	TODIM-5
	Non-dynamic			Dynamic			Increasing window		
Return	8.24%	12.38%	13.39%	11.32%	21.95%	30.49%	11.81%	23.09%	34.36%
Volatility	14.70%	17.29%	21.18%	16.09%	20.95%	26.71%	16.10%	20.79%	27.00%
ND-TODIM-1	(-)	**(0.012)	**(0.010)	**(0.011)	***(0.001)	***(0.001)	***(0.001)	***(0.001)	***(0.000)
ND-TODIM-3	**(0.012)	(-)	***(0.004)	(0.152)	***(0.005)	***(0.001)	(0.148)	**(0.010)	***(0.002)
ND-TODIM-5	**(0.010)	***(0.004)	(-)	**(0.014)	(0.913)	**(0.010)	**(0.023)	(0.764)	**(0.014)
D-TODIM-1	**(0.011)	(0.152)	**(0.014)	(-)	***(0.000)	***(0.000)	(0.993)	***(0.003)	***(0.000)
D-TODIM-3	***(0.001)	***(0.005)	(0.913)	***(0.000)	(-)	***(0.001)	***(0.001)	(0.615)	***(0.001)
D-TODIM-5	***(0.001)	***(0.001)	**(0.010)	***(0.000)	***(0.001)	(-)	***(0.001)	**(0.012)	(0.647)
IW-TODIM-1	***(0.001)	(0.148)	**(0.023)	(0.993)	***(0.001)	***(0.001)	(-)	***(0.002)	***(0.001)
IW-TODIM-3	***(0.001)	**(0.010)	(0.764)	***(0.003)	(0.615)	**(0.012)	***(0.002)	(-)	***(0.001)
IW-TODIM-5	***(0.000)	***(0.002)	**(0.014)	***(0.000)	***(0.001)	(0.647)	***(0.001)	***(0.001)	(-)
Sharpe	0.561	0.716	0.632	0.704	1.048	1.141	0.734	1.110	1.273
ND-TODIM-1	(-)	(0.719)	(0.919)	(0.490)	(0.331)	(0.368)	(0.300)	(0.211)	(0.241)
ND-TODIM-3	(0.719)	(-)	(0.709)	(0.966)	(0.537)	(0.464)	(0.916)	(0.345)	(0.262)
ND-TODIM-5	(0.919)	(0.709)	(-)	(0.866)	(0.512)	(0.450)	(0.837)	(0.369)	(0.257)
D-TODIM-1	(0.490)	(0.966)	(0.866)	(-)	(0.340)	(0.379)	(0.766)	(0.295)	(0.305)
D-TODIM-3	(0.331)	(0.537)	(0.512)	(0.340)	(-)	(0.635)	(0.400)	(0.820)	(0.437)
D-TODIM-5	(0.368)	(0.464)	(0.450)	(0.379)	(0.635)	(-)	(0.450)	(0.875)	(0.682)
IW-TODIM-1	(0.300)	(0.916)	(0.837)	(0.766)	(0.400)	(0.450)	(-)	(0.310)	(0.322)
IW-TODIM-3	(0.211)	(0.345)	(0.369)	(0.295)	(0.820)	(0.875)	(0.310)	(-)	(0.386)
IW-TODIM-5	(0.241)	(0.262)	(0.257)	(0.305)	(0.437)	(0.682)	(0.322)	(0.386)	(-)
Turnover	1.388	4.101	6.623	1.376	4.024	6.684	1.393	4.113	6.845

*Note. This table displays the evaluation metrics for the TODIM strategies with a non-dynamic weighting, a dynamic weighting using a moving estimation period of two years, and a dynamic weighting taking all observations into account, during the 1st trading day of 2017 until the last trading day of 2018. The return, volatility, and Sharpe ratio are annualized. Underneath both volatility and Sharpe ratio the p-values are displayed in brackets as a result from testing for a significant difference compared to the strategies displayed in the first column, where ND refers to the non-dynamic approach, D to the standard dynamic approach, and IW to the approach with an increasing window. *, **, *** represent significance at a 10%, 5% and 1% level respectively.*

terms of the Sharpe ratio, the dynamic weighting strategies outperform the non-dynamic weighting strategies. When comparing the dynamic strategies, a slight distinction can be made between the two-year estimation period and the increasing estimation window. Still, the performance of the strategies seems to benefit from an increased estimation period. Finally, the turnover is very comparable per value of the risk parameter.

Underneath the volatility and the Sharpe ratio in Table 14 the p-values are displayed from testing whether a significant difference exists regarding both measures. Again, for none of the strategies, a significant difference in the Sharpe ratio is displayed. For the volatility, most strategies display a significant difference at a 5% significance level, except for the two dynamic weighting strategies when the risk parameter is equal, and for the dynamic weighting strategies compared to the non-dynamic weighting strategy when the risk parameter is one level lower. While the difference in Sharpe ratio is insignificant, it does seem that utilizing a dynamic weighting approach benefits the performance of the TODIM strategies and increasing the estimation window to estimate the weights additionally has a beneficial impact on the performance of the strategy, compared to a fixed two-year estimation window.

5 Conclusion

The Markowitz framework has been in place for decades for optimizing portfolio performance. Due to the significant impact of estimation error, this framework is less well-suited for investors willing to incorporate more risk, as accurately estimating the return of assets remains notoriously hard. Additionally, intraday data has become available to a vast range of investors, with which several realized measures can be calculated which display significant relations with future return, as shown in Amaya et al. (2015) and Bollerslev et al. (2020). This paper implements an adaptation of TODIM, a multi-criteria decision-making algorithm, to research an alternative that is less sensitive to plug-in estimates and able to incorporate the relation of realized measures with future return. By not incorporating exact estimates, but by dividing assets into quantiles based upon realized return, realized correlation, realized volatility, realized signed jump variation, realized skewness, and realized kurtosis, a different approach is utilized to determine the allocation of weights in a portfolio. The implementation of quantiles and utilizing characteristics of assets for investment purposes is far from new, as there is an entire field of research focused on portfolio sorting. However, this research aims to incorporate properties from the portfolio sorting field of research to determine whether they can help overcome the shortcomings currently encountered in the weight allocation field of research. Additionally, two changes to the original TODIM algorithm are proposed, being a risk parameter with which an investor can incorporate desired risk and a dynamic weighting approach. The performance of the strategy based on the algorithm is evaluated using return, volatility, Sharpe ratio, and turnover and is compared to several well-known benchmarks, such as the equally-weighted portfolio, Markowitz framework derived portfolios incorporating shrinkage techniques, and value-weighted index portfolios.

The results show that in terms of return, the TODIM strategies with an increased risk parameter outperform all benchmarks. However, measuring only return gives a biased result, as the volatility is not taken into account. Hence, by combining them both and computing the Sharpe ratio, a better comparison can be made. Even with the increased volatility taken into account, the TODIM strategies with an increased risk parameter outperform all benchmark strategies in terms of Sharpe ratio during the testing period. However, the differences in Sharpe ratio compared to other strategies are insignificant. Thus, the conclusion can not be made that the TODIM strategies outperform the other strategies, but the results look promising. When transaction costs are quantified and subtracted from the return, the Sharpe ratio of the majority of the benchmark models decreases by a substantial amount. Yet, the TODIM strategies retain a similar Sharpe ratio as without the transaction costs, as the costs has a lower relative impact because of the height of the return. Additionally, if the same return as the TODIM strategies is to be achieved by using Markowitz-derived strategies, the Sharpe ratio deteriorates in comparison to that of the TODIM strategies. The turnover of the TODIM strategies scales linearly with the risk parameter, while the turnover of the Markowitz derived portfolios scales at a much higher rate. Finally, a dynamic weighting approach seems to improve the performance compared to a non-dynamic counterpart and an increasing estimation window benefits the return more than a fixed-length moving window. While no significant difference is displayed, the excellent performance of the

TODIM strategy with a dynamic weighting approach deems it a very solid alternative for an investor willing to take on an increased amount of risk, as the high return is accompanied by relatively low volatility levels, which is backed by the high Sharpe ratios. The TODIM algorithm aims to allocate positive weights to assets with a high return by leveraging wealth through taking on short positions in different assets. In weeks where the algorithm can distinguish high and low return assets decently, the strategies can achieve extremely high profits, exceeding a 10% log-return in a single week. On the contrary, during weeks where the algorithm is unable to do make this distinction, the leveraged position can lead to severe losses as well. Yet, the TODIM strategies with an increased risk parameter deliver a positive return during every single year of the testing period, displaying consistent yearly performance despite its increased volatility.

Overall, the algorithm seems to be able to benefit from the statistical relations found between realized measures and future return to some extent. However, realized volatility and realized signed jump variation are never implemented by the algorithm. While this does not imply that there exists no relation between those realized measures and future return, it does imply that the TODIM algorithm is unable to utilize this relation to improve its performance in terms of the Sharpe ratio. Additionally, it serves as an alternative for investors who desire more risk as it scales more favorably than the Markowitz framework-derived strategies. While the risk parameter is harder to interpret than a straightforward target return, its implementation in practice shows that the actual return deviates highly from the target return, decreasing the benefit of its straightforwardness. The performance displayed by the TODIM algorithm shows a promising potential, which can still be optimized and extended. The excellent performance of the TODIM strategies advocate the avoidance of the dependency on exact inputs and the incorporation of realized measures for allocating weights in a portfolio construction setting.

This paper only examines the NASDAQ-100 for a period of five years, of which only three are included in the test set. While this still leaves 754 daily observations of results, it does limit the testing of the strategy to the NASDAQ-100 environment in these specific years. The NASDAQ-100 is known to be quite tech-heavy, which might lead to a misrepresentation of the stock market in general by using this index as testing data.

the range of characteristics used for the TODIM strategies is very limited in this research and only the most recent week of observations is used to calculate these characteristics. The estimation window could be increased per characteristic, through which additional characteristics could be added by treating long-term, mid-term and short-term computations of these characteristics as different variables and different characteristics can be incorporated as well. Additionally, the weighting of these assets has been calculated using a restricted number of possibilities for the value of the weights. By imposing optimization algorithms with a broader range of weights, the results of the TODIM strategy might be improved. However, doing so might also lead to overfitting of the weights, which could in turn negatively impact the performance of the strategy.

Furthermore, the results presented in this paper could lay the foundation for a combination of the field of portfolio construction and the field of portfolio sorts, as it sheds light on a combination of characteristics of both fields. Instead of focusing on increasing the accuracy to estimate the expected moments, examining the possibilities to be less reliant on exact estimates and incorporate additional characteristics could serve to be beneficial for the development of new allocation strategies.

Future research can explore different MCDM algorithms to determine whether they can serve as a valuable addition to the weight allocation strategies by being able to implement various characteristics of intraday data. One can also explore whether the volatility accompanying the resulting return of implementing the TODIM algorithm can be decreased by taking into account additional characteristics. In the current setting, the attenuation factor has been fixed to -1 and a straightforward risk parameter has been introduced. In future research, the impact of this attenuation factor in a weight allocation environment can be measured, and various adaptations of the implementation of the risk parameter could lead to interesting results.

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Appendices

A Realized signed jump variation

For the logarithmic price of an asset i at time T, the price process in Equation 41 is assumed.

$$p_{i,T} = \int_0^T \mu_{i,\tau} d\tau + \int_0^T \sigma_{i,\tau} dW_\tau + J_{i,T} \tag{41}$$

This price process can be split up into three parts. μ represents the drift, σ represents the diffuse volatility, which is multiplied by a standard Brownian motion W, and J represents a jump process. A standard Brownian motion W satisfies the property that W_t is normally distributed with mean 0 and variance t. Using this property, the logic behind extracting the jump process can be explained.

By following the arguments in Andersen et al. (2003), it can be stated that the realized variance converges to the quadratic variation, which can further be decomposed into the sum of two components, consisting of a volatility part and a jump part, as shown in Equation 42.

$$\text{RV}_{i,T} \quad \rightarrow \quad \int_{T-1}^T \sigma_{i,s}^2 ds + \sum_{T-1 \leq \tau \leq t} J_{i,\tau}^2 \tag{42}$$

Observing that Equation 42 consists of two parts leads to the following idea. As there is both a jump process and a volatility process, it seems that the realized variance is being driven by two processes. The direction of the volatility process is random by definition as it is multiplied by the Brownian motion, but the direction of the jump process is not random, so the expectation exists that information can be retrieved from this specific part. To separate positive jumps from negative jumps, a division of the realized variance is required. Barndorff-Nielsen et al. (2008) propose a way to split the realized variance in a negative and a positive semi-variance, which are displayed in Equation 43 and Equation 44.

$$\text{RV}_{i,T}^{+} = \sum_{k=1}^{t-1} r_{T+\frac{k}{t}}^2 \mathbf{1}_{\left\{r_{T+\frac{k}{t}}>0\right\}} \tag{43}$$

$$\text{RV}_{i,T}^{-} = \sum_{k=1}^{t-1} r_{t+\frac{k}{t}}^2 \mathbf{1}_{\left\{r_{T+\frac{k}{t}}<0\right\}} \tag{44}$$

Without the assumption that a jump process is present, the distribution of the Brownian motion by definition leads to the conclusion that both of the realized semi-variances converge to Equation 45.

$$\frac{1}{2} \int_T^{T+1} \sigma_{i,s}^2 ds \tag{45}$$

However, with the inclusion of a jump process the realized semi-variances converge to two different values in which there is a separation in negative jumps and positive jumps. As proven in Barndorff-Nielsen et al. (2008), these measures converge as follows:

$$\begin{aligned} \mathrm{RV}_{i,T}^{+} &\rightarrow \tfrac{1}{2}\int_{T}^{T+1} \sigma_{i,s}^{2} ds + \textstyle\sum_{T \leq \tau \leq T+1} J_{i,\tau}^{2} \mathbf{1}_{(J_{i,\tau}>0)} \\ \mathrm{RV}_{i,T}^{-} &\rightarrow \tfrac{1}{2}\int_{T}^{T+1} \sigma_{i,s}^{2} ds + \textstyle\sum_{T \leq \tau \leq T+1} J_{i,\tau}^{2} \mathbf{1}_{(J_{i,\tau}<0)} \end{aligned} \tag{46}$$

The first parts of both equations are equal to each other. By subtracting the negative from the positive semi-variance, it is possible to extract the difference in jumps. In Bollerslev et al. (2020) a new measure is proposed, called the signed jump variation, which is displayed in Equation 47.

$$\mathrm{SJ}_{i,T} = \mathrm{RV}_{i,T}^{+} - \mathrm{RV}_{i,T}^{-} \tag{47}$$

By following the previously stated convergence of the semi-variances in equation 46, it can easily be shown that equation 47 converges to the following equation:

$$\sum_{T \leq \tau \leq T+1} J_{i,\tau}^{2} \mathbf{1}_{(J_{i,\tau}>0)} - J_{i,\tau}^{2} \mathbf{1}_{(J_{i,\tau}<0)} \tag{48}$$

With equation 48, it can be concluded that the signed jump variation converges to the difference in positive and negative jump processes. As such, a way has been found to quantify the jump process. To make the measure scale-invariant, it can be divided by the realized variance to get a normalized measure ranging from -1 to 1. This leads to the realized signed jump variation, as displayed in equation 49

$$\mathrm{RSJ}_{i,T} = \frac{\mathrm{SJ}_{i,T}}{\mathrm{RV}_{i,T}} \tag{49}$$

B Company tickers and PERMNO

Table 15: Exchange company tickers and PERMNO

Ticker	PERMNO	Ticker	PERMNO	Ticker	PERMNO
AABA	16752	EQIX	89617	NTAP	82598
AAL	21020	ESRX	77668	NTES	88362
AAPL	14593	EXPD	87717	NVDA	86580
ADBE	75510	EXPE	90808	NXPI	12084
ADI	60871	FAST	11618	ORLY	79103
ADP	44644	FB	13407	PAYX	61621
ADSK	85631	FISV	10696	PCAR	60506
AKAM	87299	FOX	90442	PEP	13856
ALGN	88860	FWONK	14811	PYPL	15488
ALTR	75577	GILD	77274	QCOM	77178
ALXN	83111	GLIBA	17334	QRTEA	91277
AMAT	14702	GMCR	79588	REGN	76614
AMGN	14008	GOOGL	90319	ROST	91556
AMZN	84788	GRMN	88837	SBAC	86996
ASML	81472	HAS	52978	SBUX	77702
ATVI	79678	HOLX	76095	SHPG	85888
AVGO	93002	HSIC	82581	SIAL	70536
BBBY	77659	IDXX	76709	SIRI	80924
BIDU	90857	ILMN	88446	SNDK	82618
BIIB	76841	INCY	79906	SNPS	77357
BKNG	86783	INTC	59328	SPLS	75489
BMRN	87056	INTU	78975	SRCL	83906
BRCM	85963	ISRG	88352	STX	89641
CA	25778	JBHT	42877	SWKS	45911
CDNS	11403	JD	14655	SYMC	75607
CELG	11552	KHC	15408	TMUS	91937
CERN	10909	KLAC	46886	TRIP	13168
CHKP	83639	KRFT	13598	TSCO	80286
CHRW	85459	LBTYK	90866	TSLA	93436
CHTR	12308	LILAK	15402	TTWO	84761
CMCSA	89525	LLTC	10299	TXN	15579
COST	87055	LRCX	48486	ULTA	92322
CSCO	76076	LSXMK	16000	VEON	93337
CSX	62148	MAR	85913	VIAB	91063
CTAS	23660	MAT	39538	VOD	75418
CTRP	89927	MCHP	78987	VRSK	93089
CTRX	91367	MDLZ	89006	VRTX	76744
CTSH	86158	MELI	92221	WBA	19502
CTXS	82686	MNST	88031	WDAY	13628
DISCA	90805	MSFT	10107	WDC	66384
DISH	81696	MU	53613	WFM	77281
DLTR	81481	MXIM	11896	WYNN	89533
DTV	89954	MYL	69550	XEL	23931
EA	75828	NCLH	13760	XLNX	76201
EBAY	86356	NFLX	89393	XRAY	11600
ENDP	88436				

C Results daily rebalancing

Table 16: Performance of each model from 2016 until 2018

	EW	VW-N100	VW-SP500	GMV	GMV-LW	MV 5%	MV-JS 5%	MV 10%	MV-JS 10%	TODIM-1	TODIM-3	TODIM-5
Return	0.064	0.054	0.038	-0.007	0.065	-0.006	-0.096	-0.008	-0.101	0.089	0.153	0.188
Volatility	0.155	0.073	0.057	0.082	0.077	0.082	0.168	0.082	0.168	0.169	0.203	0.294
Sharpe	0.414	0.729	0.679	-0.083	0.849	-0.079	-0.569	-0.092	-0.600	0.529	0.753	0.641
Turnover	0.029	0.000	0.000	1.724	0.873	1.746	4.682	1.754	4.699	0.547	1.993	2.662

Note. This table displays the evaluation metrics for all benchmark models and the TODIM models with various configurations and a rebalancing period of one day from the 1st trading day of 2016 until the last trading day of 2018. The return, volatility and Sharpe ratio are annualized.

Table 17: Performance of each model in 2016

	EW	VW-N100	VW-SP500	GMV	GMV-LW	MV 5%	MV-JS 5%	MV 10%	MV-JS 10%	TODIM-1	TODIM-3	TODIM-5
Return	0.068	0.037	0.048	-0.008	0.085	0.000	0.063	-0.001	0.057	0.110	0.178	0.284
Volatility	0.165	0.069	0.057	0.084	0.083	0.084	0.150	0.084	0.150	0.179	0.191	0.316
Sharpe	0.410	0.544	0.853	-0.096	1.033	-0.004	0.423	-0.017	0.382	0.615	0.928	0.898
Turnover	0.029	0.000	0.000	1.926	0.950	1.943	5.142	1.964	5.157	0.593	1.666	2.811

Note. This table displays the evaluation metrics for all benchmark models and the TODIM models with various configurations and a rebalancing period of one day from the 1st trading day of 2016 until the last trading day of 2016. The return, volatility and Sharpe ratio are annualized.

Table 18: Performance of each model in 2017

	EW	VW-N100	VW-SP500	GMV	GMV-LW	MV 5%	MV-JS 5%	MV 10%	MV-JS 10%	TODIM-1	TODIM-3	TODIM-5
Return	0.216	0.125	0.087	0.130	0.164	0.131	0.171	0.131	0.175	0.188	0.190	0.069
Volatility	0.091	0.043	0.029	0.055	0.050	0.056	0.098	0.056	0.097	0.116	0.166	0.258
Sharpe	2.365	2.888	2.990	2.355	3.258	2.351	1.740	2.358	1.808	1.622	1.144	0.269
Turnover	0.031	0.000	0.000	1.406	0.698	1.421	3.356	1.420	3.353	0.245	1.897	1.189

Note. This table displays the evaluation metrics for all benchmark models and the TODIM models with various configurations and a rebalancing period of one day from the 1st trading day of 2017 until the last trading day of 2017. The return, volatility and Sharpe ratio are annualized.

Table 19: Performance of each model in 2018

	EW	VW-N100	VW-SP500	GMV	GMV-LW	MV 5%	MV-JS 5%	MV 10%	MV-JS 10%	TODIM-1	TODIM-3	TODIM-5
Return	-0.091	-0.002	-0.020	-0.142	-0.054	-0.150	-0.522	-0.152	-0.535	-0.030	0.091	0.211
Volatility	0.191	0.098	0.074	0.099	0.091	0.098	0.228	0.098	0.228	0.201	0.244	0.306
Sharpe	-0.478	-0.020	-0.268	-1.434	-0.588	-1.528	-2.287	-1.550	-2.347	-0.150	0.371	0.691
Turnover	0.027	0.000	0.000	1.841	0.972	1.875	5.547	1.877	5.585	0.803	2.415	3.986

Note. This table displays the evaluation metrics for all benchmark models and the TODIM models with various configurations and a rebalancing period of one day from the 1st trading day of 2018 until the last trading day of 2018. The return, volatility and Sharpe ratio are annualized.

D Transaction cost adjusted returns per year

Table 20: Evaluation metrics for each model from 2016 with transaction cost adjusted returns

	EW	VW-N100	VW-SP500	GMV	GMV-LW	MV 5%	MV-JS 5%	MV 10%	MV-JS 10%	TODIM-1	TODIM-3	TODIM-5
Return	6.42%	3.66%	4.75%	4.99%	9.24%	6.64%	23.05%	6.54%	22.90%	8.98%	14.15%	19.37%
Volatility	16.60%	6.90%	5.68%	7.81%	8.18%	7.78%	12.21%	7.76%	12.11%	17.12%	20.22%	23.56%
Sharpe	0.387	0.531	0.837	0.640	1.129	0.854	1.888	0.843	1.890	0.525	0.700	0.822
Turnover	0.029	0.020	0.020	3.496	2.068	3.543	6.909	3.541	6.944	1.397	4.130	6.685
Transaction costs	0.02%	0.01%	0.01%	2.30%	1.36%	2.33%	4.55%	2.33%	4.57%	0.92%	2.72%	4.40%
Adjusted returns	6.40%	3.65%	4.74%	2.69%	7.88%	4.31%	18.50%	4.20%	18.32%	8.06%	11.43%	14.97%
Adjusted Sharpe ratio	0.386	0.529	0.835	0.345	0.963	0.554	1.515	0.542	1.513	0.471	0.565	0.635

Note. This table displays the evaluation metrics for all benchmark models and the TODIM models with various configurations from the 1st trading day of 2016 until the last trading day of 2016 with transaction cost adjusted returns. The return, volatility and Sharpe ratio are annualized and the transaction costs are derived from the average bid-ask spread.

Table 21: Evaluation metrics for each model from 2017 with transaction cost adjusted returns

	EW	VW-N100	VW-SP500	GMV	GMV-LW	MV 5%	MV-JS 5%	MV 10%	MV-JS 10%	TODIM-1	TODIM-3	TODIM-5
Return	21.41%	12.17%	8.31%	19.94%	18.34%	20.74%	16.47%	20.90%	17.58%	27.03%	38.94%	50.41%
Volatility	9.15%	4.33%	2.90%	5.68%	5.43%	5.69%	9.73%	5.69%	9.74%	10.41%	15.55%	21.93%
Sharpe	2.338	2.808	2.871	3.510	3.378	3.645	1.692	3.674	1.804	2.597	2.504	2.299
Turnover	0.031	0.020	0.020	2.535	1.506	2.569	5.215	2.564	5.225	1.356	3.901	6.495
Transaction costs	0.02%	0.01%	0.01%	1.67%	0.99%	1.69%	3.43%	1.69%	3.44%	0.89%	2.57%	4.28%
Adjusted returns	21.38%	12.15%	8.30%	18.27%	17.35%	19.05%	13.03%	19.21%	14.14%	26.14%	36.37%	46.13%
Adjusted Sharpe ratio	2.336	2.805	2.866	3.216	3.196	3.348	1.339	3.377	1.451	2.511	2.339	2.104

Note. This table displays the evaluation metrics for all benchmark models and the TODIM models with various configurations from the 1st trading day of 2017 until the last trading day of 2017 with transaction cost adjusted returns. The return, volatility and Sharpe ratio are annualized and the transaction costs are derived from the average bid-ask spread.

Table 22: Evaluation metrics for each model from 2018 with transaction cost adjusted returns

	EW	VW-N100	VW-SP500	GMV	GMV-LW	MV 5%	MV-JS 5%	MV 10%	MV-JS 10%	TODIM-1	TODIM-3	TODIM-5
Return	-10.19%	-0.97%	-2.77%	-8.47%	-9.63%	-8.25%	-1.46%	-8.25%	-1.53%	-5.51%	3.83%	9.44%
Volatility	18.79%	9.78%	7.44%	11.07%	9.76%	10.84%	19.64%	10.84%	19.82%	20.21%	25.20%	30.75%
Sharpe	-0.542	-0.099	-0.373	-0.765	-0.987	-0.761	-0.074	-0.761	-0.077	-0.273	0.152	0.307
Turnover	0.027	0.020	0.020	3.588	2.257	3.638	8.695	3.640	8.677	1.395	4.147	6.872
Transaction costs	0.02%	0.01%	0.01%	2.36%	1.49%	2.40%	5.73%	2.40%	5.72%	0.92%	2.73%	4.53%
Adjusted returns	-10.20%	-0.98%	-2.79%	-10.83%	-11.11%	-10.65%	-7.19%	-10.64%	-7.25%	-6.43%	1.10%	4.91%
Adjusted Sharpe ratio	-0.543	-0.101	-0.375	-0.978	-1.139	-0.982	-0.366	-0.982	-0.366	-0.318	0.044	0.160

Note. This table displays the evaluation metrics for all benchmark models and the TODIM models with various configurations from the 1st trading day of 2018 until the last trading day of 2018 with transaction cost adjusted returns. The return, volatility and Sharpe ratio are annualized and the transaction costs are derived from the average bid-ask spread.

E Results weighting analysis per year

Table 23: Evaluation metrics for the varying weighting TODIM models in 2017

	TODIM-1	TODIM-3	TODIM-5	TODIM-1	TODIM-3	TODIM-5	TODIM-1	TODIM-3	TODIM-5
	Non-dynamic			Dynamic			Increasing window		
Return	26.44%	35.81%	39.97%	27.38%	39.29%	50.76%	27.79%	39.87%	51.94%
Volatility	8.94%	12.10%	16.55%	10.41%	15.55%	21.93%	9.96%	14.19%	19.95%
Sharpe	2.957	2.959	2.415	2.630	2.526	2.315	2.790	2.810	2.603
Turnover	1.429	4.207	6.804	1.356	3.901	6.495	1.420	4.170	6.936

Note. This table displays the evaluation metrics for the TODIM models with and without dynamic weighting and with an extended estimation window from the 1st trading day of 2017 until the last trading day of 2018. The return, volatility, and Sharpe ratio are annualized.

Table 24: Evaluation metrics for the varying weighting TODIM models in 2018

	TODIM-1	TODIM-3	TODIM-5	TODIM-1	TODIM-3	TODIM-5	TODIM-1	TODIM-3	TODIM-5
	Non-dynamic			Dynamic			Increasing window		
Return	-9.96%	-11.06%	-13.19%	-4.73%	4.61%	10.22%	-4.17%	6.31%	16.78%
Volatility	18.72%	21.16%	24.88%	20.21%	25.20%	30.75%	20.44%	25.74%	32.56%
Sharpe	-0.532	-0.523	-0.530	-0.234	0.183	0.332	-0.204	0.245	0.515
Turnover	1.348	3.994	6.442	1.395	4.147	6.872	1.366	4.056	6.754

Note. This table displays the evaluation metrics for the TODIM models with and without dynamic weighting and with an extended estimation window from the 1st trading day of 2017 until the last trading day of 2018. The return, volatility, and Sharpe ratio are annualized.