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MSC QUANTITATIVE FINANCE
THESIS

Pension Scheme Transitions and Value-based ALM

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Abstract

In this paper, we examine the new value-based asset liability management method for pension funds. In The Netherlands, there will be a transition to a new pension scheme. The government prescribed to use value-based asset liability management to examine the effects of the transition, despite it being a relatively new method in a pension fund setting. Value-based asset liability management in combination with generational accounting allows us to gain understanding of value transfers between older and younger participants of the fund when transitioning to a new pension scheme for different economic scenarios. We set up a value-based asset liability management study and also re-calibrate the KNW model for economic scenario generation. The parameters of the KNW model have manually been changed by the Dutch Central Bank to obtain economic scenarios with lower long-term interest rates without re-calibrating the model. Instead we insert a similar restriction into calibration procedure, such that manual changes are not necessary anymore.

We show that the interest rate restriction on the KNW model leads to significantly different parameter estimates than the current calibration used by pension funds for asset liability studies. Additionally, by introducing the value-based asset liability management model, we are able to show that pension schemes that are less risky, do not necessarily lead to a fair redistribution of pension benefits. On average the new Dutch pension scheme leads to higher benefits than the current one, but there are greater potential losses.

Keywords: Value-based ALM, intergenerational accounting, KNW, pensions

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Contents

1 Introduction

Over the past years, there has been a lot of change in Dutch society with great impact on the current pension system. First of all, the life expectancy is increasing, causing the pension benefit payment horizon to increase. With the current system, this leads to low funding for pension funds. As a consequence, pension benefits could potentially be cut in order to compensate for the lack of assets for funding. Secondly, it is unusual to be employed at the same company for a long period of time. In The Netherlands, the pension system is based on a solidarity principle between the young and old generations: the participants of a pension fund form one collective and together they accrue joint capital which later is used to pay their pension. This indirectly means that participants that die early pay for participants that live longer. Flexible employment puts pressure on this principle, as participants do not stay within one sector with one pension fund during their working life. Lastly, during the financial crisis it became apparent that the current pension scheme is more sensitive to financial shocks and negative interest rates than initially thought. As an example, because of the low interest rates, pension funds need to have more assets to pay off future liabilities, since this means the expected return is low or even negative. It turned out that pension funds did not have the financial buffers to absorb these shocks. These challenges are examples of reasons why the current pension system is deficient. Therefore, the Future Pensions Act ('Wet Toekomst Pensioenen') enters into force in 2023. This will change the current pension system drastically. One of the major changes is going from a defined benefit (DB) pension scheme (secured benefit payment at retirement) to a defined contribution (DC) pension scheme (fixed premium to be invested by the pension fund).

To make an adequate trade-off between policies and schemes such as DB and DC, pension funds make use of asset liability management (ALM). It helps Boards of Trustees decide on the optimal funding, risk-sharing and investment strategies as well as indexation policies. Using ALM, pension funds analyse how different economic scenarios influence different features of the pension funds when looking at a specific strategy. The attractiveness of a strategy is often decided by the distribution of contributions, indexation and funding ratio (FR). Statistics such as the expected value and the value in best or worst case scenario are used to give summaries of these distributions (classical ALM). Despite that, an often seen critique is that classical ALM only teaches a well known truth that the more risk is taken, the higher expected return.

For this reason, Kortleve & Ponds (2006) introduced a value-based ALM method, which instead of summary statistics shows redistribution of assets over generations for a policy adjustment (generational accounting). Value-based ALM is frequently used by insurers, but rarely by pension funds. Nevertheless, with the introduction of the Future Pensions Act, this methodology must be used by pension funds too (de Groot & van Hoogdalem (2021) and Wijckmans (2020)). Value-based ALM is able to visualize hidden value transfers between generations when moving to a new pension scheme. This gives insight in the fairness and value of a pension contract per generation. A change in a pension plan always has a smaller or bigger effect on different generations as redistribution is inevitable (Lekniute, 2011). Therefore, it is important that the policymakers and Boards of Trustees are aware of the consequences following a change in pension scheme for each generation, such that those effects can be mitigated.

In order to conduct any ALM study, one needs to generate economic scenarios for the key determinants of pension risk, namely inflation, the stock return, and nominal interest rates. The Commission Parameters (Langejan, Gelauff, Nijman, Sleijpen, & Steenbeek (2014) and Dijsselbloem et al. (2019)) prescribe the KNW model by Koijen, Nijman, & Werker (2010) to simulate these scenarios for ALM studies. Surprisingly, in former research about value-based ALM (such as Lekniute (2011)), the underlying scenarios are not obtained from the prescribed KNW model, making the results of those researches less useful. To set up a realistic ALM study one needs to use the calibration as recommended by the Commission Parameters. Despite the good fit of the model on the data, The Dutch Central Bank (De Nederlandse Bank, DNB) recently made manual changes to the parameters of the model (DNB, 2021a). They set the long-term average interest rate equal to -0.01% instead of 2.41%, and thus going against the advice of the Commission Parameters in 2019. DNB states that they received signals from pension funds that the interest rates obtained from simulation from the 2019 calibration were not low enough. The changes made to the parameters were not based on the data or restrictions within the model, but were made manually after the calibration.

Especially the latter is problematic, as every parameter in the KNW model depends on the calibration of the other parameters. Therefore, in this study we re-estimate the KNW model and impose the -0.01% long-term interest rate as a restriction in the model to allow the other parameters to change to fit the restriction. We use the relatively new Kalman filter method introduced by Pelsser (2019) to calibrate the model. He showed that his results are statistically indistinguishable from the results obtained from the old method of simulated annealing by Draper (2014), but for completeness and to verify the robustness of this method, we check whether our model is able to obtain the same results as by the Commission Parameters in 2019. Then we re-estimate the model for more recent data (until 2020) with and without the interest rate restriction to inspect whether the restriction can be justified.

All in all, this paper mainly serves as a guideline on how to set up a value-based ALM model using scenarios obtained by the KNW model. We want to show what the added value is of this method in the setting of Future Pensions Act, while also carefully deciding on the best calibration of the KNW model for the economic scenarios. This leads to the research question of this thesis: To what extent does value-based ALM lead to different conclusions in terms of pension policies compared to classical ALM, based on scenarios obtained from the KNW model and more specifically, how does DB compare to DC?

For the calibration of the KNW to obtain economic scenarios on inflation, stock return and bond return, we use data of the Harmonized Index of Consumer Prices (HICP) for the euro area (European Central Bank), the yield curve of the euro area (Deutsche Bundesbank and DNB) and MSCI index (Morgan Stanley Capital International). Moreover, we need information about the Dutch population to set up the ALM model for which we use data from the CPB and Koninklijk Actuarieel Genootschap (2021).

The KNW results indicate that adding the new restriction to the KNW model leads to significantly different parameters than without the restriction. The in-sample fit of the restricted model shows that the low long-term interest rate restriction is not supported by the data. Especially the effect that this restriction has on the other parameters is a huge disadvantage as the parameters are far from the recommendation of the Commission Parameters in 2019. Instead the KNW model fitted on more recent data seems to be a good solution. This model leads to (slightly) lower long-term interest rates and has a good in-sample fit.

The value-based ALM results indicate that when going from DB to DC, especially the older participants (ages 50-60) need to be compensated. If they are not compensated, the older generation loses pension benefit they would have received in a DB scheme. This seems unfair, as they are close to retirement and do not have time to take precautionary measures anymore (such as setting aside savings to add to their pension benefit). In case of compensation, on average DC is more profitable than DB to the participants. On the other hand, the classical ALM results indicate that this kind of pension scheme is more risky than DB. There are greater potential losses. Together the two different methods give a good indication of the fairness and value for the participants (value-based ALM) and the risk for the pension fund (classical ALM). Usually when the classical ALM results give a positive outlook on the risk, the value-based ALM indicates a loss of value for most generations. From our research, we can conclude that value-based ALM helps to give a complete overview of the consequences of a pension scheme transition. It helps pension funds to find a good balance between risk and fairness.

The remainder of this paper is structured as follows. The relevant background information is described in Section 2. We describe the KNW model in Section 4. The set-up of the synthetic pension fund and the description of the (value-based) ALM analysis is discussed in Section 5. The results are shown in Section 6. Finally, the conclusion and discussion can bw found in Section 7.

2 Background Information

2.1 Dutch Pension System

The Dutch pension system is made up by three pillars (*Opbouw pensioenstelsel*). The first being public pension (Algemene Ouderdomswet, AOW). This is pension provided by the government for everyone over the age of 66 and 4 months as of 2021. The level of this pension is related to the minimum wage. Workers that earn middle to high wage need additional pension provision to maintain their standard of living. This is the role of the second pillar. About 90% of the Dutch population partakes in this form of pension. Next, there is the third pillar, which consists of voluntarily pension provisions, such as life insurance. In this paper we focus on the second pillar.

2.2 Pension Schemes

During their career, employees (and their employers) set aside a percentage of their wage (premium/contribution) that goes to their pension benefit. As if right now, the participants of the pension fund accrue joint capital together that will be used for everyone's individual pension benefit. However, some participants die early, which means that they will never profit from the premium they paid. On the contrary, there are participants that live longer than expected, which means that their benefit payment horizon increases and that they will receive more money than they saved by themselves. This is called the solidarity principle, as these two incidents balance each other out. Because of this and the fact that pension funds have many participants, solvency risks can be reduced and the participants are able to secure life-long pension benefit after retirement.

So fundamentally, pension is an agreement between generations and between the pension fund and its participants to share risk. In any scheme, the participant pays a premium per year and gets a benefit for the remainder of their life. Nonetheless, how the percentage of premium paid is defined, how the risk is shared between generations and how the benefit is paid out, can be different depending on the pension scheme considered.

There are two major categories of pension schemes to consider. In The Netherlands, the majority of pension funds offer a defined benefit (DB) plan. In this system pension, one has secured pension rights which are accrued during the working period and will be acquired at the start of the pension data (Bodie, Marcus, & Merton, 1988). Additional pension rights are built up every year, around 1.875% of the retirement payment in The Netherlands. Which implies that after 40 years, their pension will be 75% of the average salaries. The premium they have to pay to the pension fund, is set in such a way that with the expected return on total invested premium is expected to be able to pay out the benefits. In The Netherlands, most employees accrue benefits based on an average pay scheme. In this scheme, every wage increases influence the pension accrual. Another option would be a final pay scheme, but this scheme is rare nowadays. In that case, the final salary increase especially affects the right. Because of the drawbacks that come with DB contracts, mentioned at the beginning of Section 1, the Future Pensions Act enforces pension funds to introduce defined contribution (DC) contracts, the second category of pension schemes.

For this plan, the participant does not have a secure benefit at retirement and accrual factor, instead their premium is fixed and the return on the invested premium decides their pension benefit. Now, the level of the pension depends on how many years pension premium is paid and what the return on the pension funds investments are. This means that the pension benefit coincides with market movements. In bad economic times, the benefit decreases, while is good economic times the benefit increases. The accrual of 1.875% in pension benefit still services as an ambition.

When moving from the current pension system to the Future Pensions Act, pension funds roughly make a transition from DB to DC. The discussion of this transition and the effect on the benefit of the current workforce is a significant one.

3 Data

In order to estimate the KNW model, one needs to have data on price levels, yields and stock index. The Commission Parameters (Dijsselbloem et al., 2019) gives an overview on what data to use to estimate the KNW model in a Dutch setting. In this thesis, we make a few adjustments, because of the availability of the data. In our case the data is monthly, implying that $\Delta t = 1/12$, as the KNW parameters have yearly implications.

1. **Inflation**: We use the Harmonized Index of Consumer Prices (HICP) for the euro area. The data from 2000 on can be obtained from the European Central Bank (2021).

2. **Yields**: Draper (2014) recommends to use the three-month, one-year, two-year, three-year, five-year, and ten-year yield. Thus maturities $\tau = 0.25, 1, 2, 3, 5, 10$. The three-month yield can be obtained from the Deutsche Bundesbank (2021a). For the other maturities, from 2004 on, we use zero-coupon rates obtained from DNB (2021d). Before 2004, we use zero-coupon rates from the Deutsche Bundesbank (2021b).

3. **Stock return**: The MSCI index is used (Morgan Stanley Capital International, 2021). Returns are in euros and hedged for US dollar exposure.

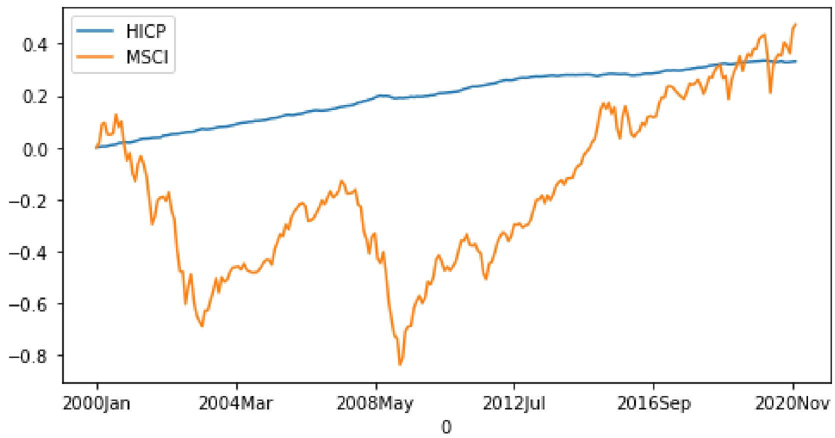


Figure 1: The blue line shows the logarithmic transformation of the HICP index over time. The orange line shows the logarithmic transformation of the MSCI index over time. The base value of the indices is set to 0 on January 2000.

In Figure 1 the (natural logarithm of the) HICP and MSCI indices are displayed. For estimation purposes, the HICP and MISCI indices have to start at a base value one at $t = 0$, which is why the logarithmic transformation of the series start at value zero in January 2000. As can be seen from Figure 1, the HICP is steadily increasing. There are a few drops (implying negative inflation) in the crisis years (2008-2010). This becomes more clear from Figure 2. On the other hand, the MSCI is more volatile and has steep declines more frequently, as can also be seen in Figure 2. We estimate using the log transformation of the MSCI and HICP index. For the ALM model, we have to create a simulation model that can directly obtain the inflation and stock return from these indices.

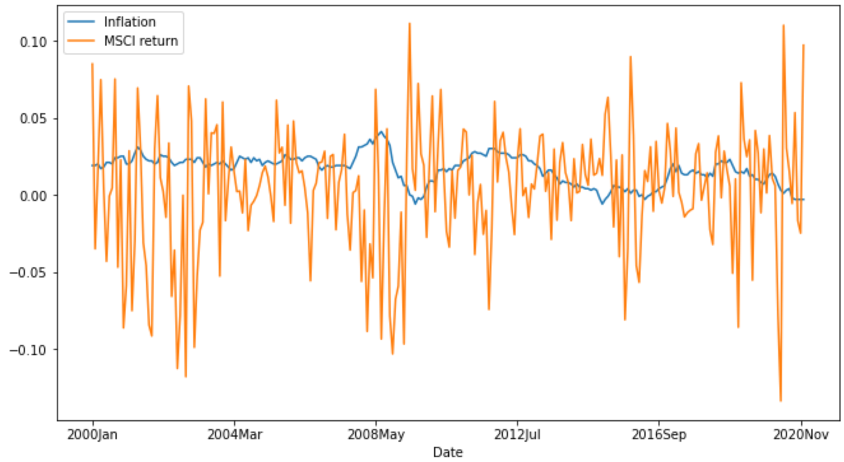


Figure 2: The blue line shows the inflation over time. The orange line shows the the monthly return of the MSCI index over time.

Figure 3 shows the nominal term structure of interest rates for maturities 0.25, 1, 2, 3, 5, and 10 years. From the figure it can be seen that from the crisis on, especially for the short maturities the interest rates are low or negative. Setting the long-term interest rates equal to -0.01% (as the DNB did) seems reasonable when looking at the data from 2018 on, because the interest rates are around zero from then on.

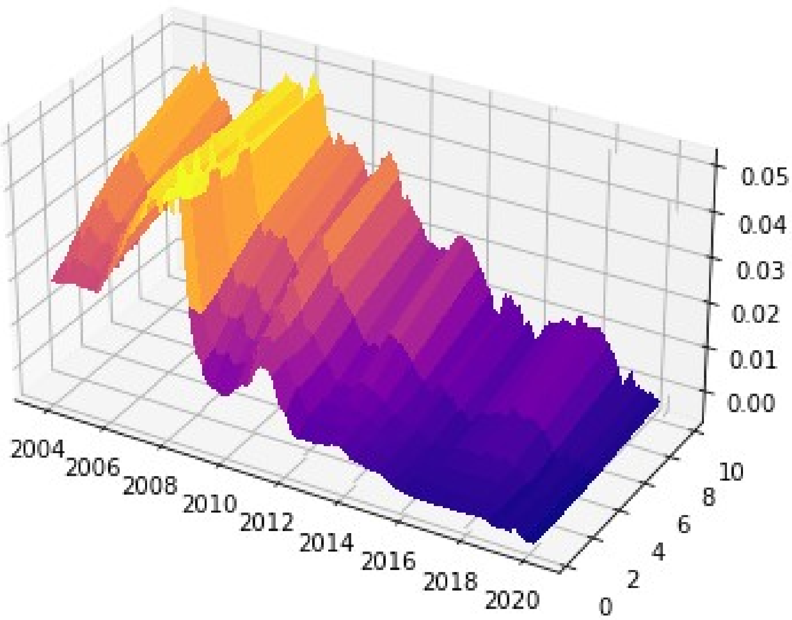


Figure 3: Nominal yield curve. The x-axis indicates the time, the y-axis shows the level of yield and the z-axis shows the time to maturity of the different yields.

To make an adequate ALM model one needs data on the demographics of the Dutch population. In order to correctly model the mortality in the workforce and the retired population of the synthetic pension fund, the mortality rates of the Koninklijk Actuarieel Genootschap

(2021) are used. In Figure 4, the observed and predicted life expectancy is shown, as estimated by Volksgezondheidenzorg (2021). By using the mortality rates of Koninklijk Actuarieel Genootschap (2021), this prognosis on the expected life expectancy is included. As an example, someone aged x in 10 years, has a higher life expectancy than someone aged x right now. Moreover, the data is gender specific. This is important as the female and male population differ in survival rates and thus for females the expected benefit payment horizon is longer.

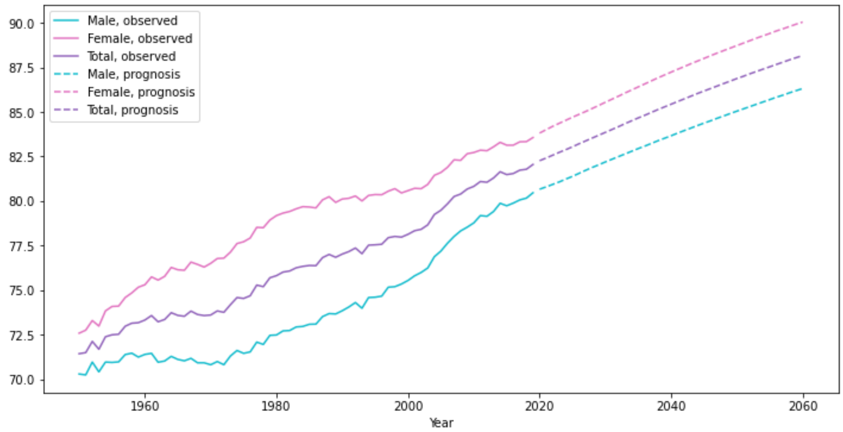


Figure 4: Observed life expectancy and the prognosis (dotted) from 2020 until 2060. Pink (blue) indicates the female (male) life expectancy and prognosis. In purple the weighted (there is not an equal number of females and males for each age) average of the total population is shown.

Furthermore, we start with an initial population in line with the current Dutch population (as of 2021). Data on the age distribution of the population is obtained from the CPB (*Bevolkingspiramide*). Figure 5 shows the percentage of females and males in each age cohort. To simulate future participants of the pension fund a prognosis of the future population of The Netherlands is used.

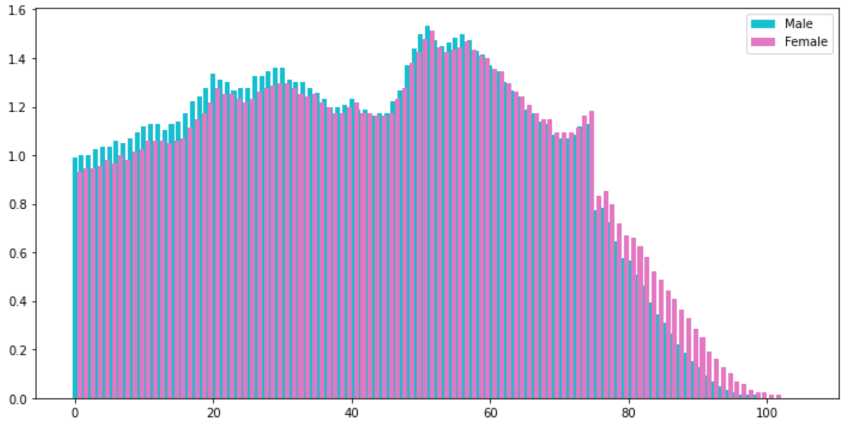


Figure 5: Age distribution in percentages. In pink (blue) the percentage of females (males) of a specific age is shown.

4 KNW model

In a pension setting, the major drivers of risk are demographic and financial developments. The pension results of the participants depend on return of their pension savings and to what extent the real value of their pension results is eroded by inflation. Moreover, decreasing mortality rates could lead to deficits, as there might be insufficient assets to pay out benefits over increasingly long horizons. Consequently, the uncertainty in mortality, stock return, inflation and interest rates cause the benefit payments and the purchasing power of these benefits to be uncertain as well. Therefore, in order to model pension accrual and payout and thus conduct an ALM study in a pension setting, one needs a model that can model these uncertainties. In this study however, we choose not to focus on modelling the demographics and instead use the mortality tables published by the Koninklijk Actuarieel Genootschap (2021). The focus of this paper is on the stochastic financial developments.

Because of its interpretability and ability to model the financial market adequately, despite being simplistic, the Commission Parameters (Langejan et al., 2014) recommends pension funds to generate these scenario sets based on the KNW capital market model. The KNW model was originally constructed by Koijen et al. (2010). It is a relatively simple model of the three major drivers of pension risk: interest rates, stock returns and inflation. Originally, Koijen et al. (2010) calibrated the KNW model on U.S. data. However, Draper (2014) re-estimated the model using data for The Netherlands. The model is revised on quarterly basis and the corresponding economic scenarios can be found on the site of DNB.[1]. They use simulated annealing (Goffe et al., 1994) to estimate the parameters. Nevertheless, recently Pelsser (2019) introduced a more straightforward method to estimate the KNW model using a Kalman filter. In terms of parameters, the results of Pelsser (2019) do not differ significantly from what is found by Dijsselbloem et al. (2019). Thus, this method is used in this paper. Besides, there have been many more extensions of the KNW model, such as Muns (2015) (how to impose restrictions in a continuous-time affine term structure model) and Bouwman & Lord (2016) (perfect calibration to the yield curve). Nevertheless, the objective of this paper is to create an usable and realistic model for ALM studies, hence we restrict ourselves to models advised by the Commission Parameters (Langejan et al. (2014) and Dijsselbloem et al. (2019)).

However, recently DNB went against the advice of the Commission Parameters and reset a few parameters of the model after calibration. The reason behind this was to obtain scenarios with lower long-term interest rates. Pension funds stated that the interest rates obtained from the current calibration of the KNW model (Dijsselbloem et al., 2019) are too high compared to what would be expected from more recent data (DNB, 2021a). Nevertheless, the manual change of a few parameters without re-calibration seems rather dubious, since all parameters are connected to each other within the estimation procedure. When changing one parameter, the rest needs to change as well. For this reason, we re-calibrate the KNW model but do impose a restriction on the long-term interest rate. In order to conduct a plausible study of the KNW model, we first re-estimate the model using the same data as used by the Commission Parameters (Dijsselbloem et al., 2019), to see if our model is able to replicate these findings and thus

[1]https://www.dnb.nl/voor-de-sector/open-boek-toezicht-sectoren/pensioenfondsen/haalbaarheidstoets/uitvoering-en-normen/scenarioset-haalbaarheidstoets-pensioenfondsen/

is correct and robust. Next, we look at a model with and without imposing a restriction on the long-term interest rate and examine how this affects the KNW parameters and thus scenarios.

In this section, an extensive description of the underlying assumptions, restrictions and estimation procedure of the KNW model is given.

4.1 Methodology

4.1.1 Assumptions

The KNW model takes the stock returns, interest rates, and inflation to be dependent on observed factors and two latent factors (Koijen et al., 2010). The uncertainty and the yearly dynamics of the instantaneous nominal interest rate R_t and the instantaneous expected inflation π_t are modelled using two unobserved state variables, collected in the vector $\boldsymbol{X}_t \in \mathbb{R}^2$.

$$R_t = R_0 + \boldsymbol{R}_1' \boldsymbol{X}_t. \tag{1}$$

With $R_0 \in \mathbb{R}$ and $\boldsymbol{R}_1 \in \mathbb{R}^2$. The stock return is assumed to be affine in the state variables.

$$\pi_t = \delta_{0\pi} + \boldsymbol{\delta}_{1\pi}' \boldsymbol{X}_t. \tag{2}$$

With $\boldsymbol{\delta}_{0\pi} \in \mathbb{R}$ and $\boldsymbol{\delta}_{1\pi} \in \mathbb{R}^2$. The inflation is assumed to be affine in the state variables. The coefficients $\boldsymbol{\delta}_{1r}$ and $\boldsymbol{\delta}_{1\pi}$ give insight in the correlation between the interest rate and inflation. The state variables follow a mean-reverting process around zero.

$$\mathrm{d}\boldsymbol{X}_t = -\boldsymbol{K}\boldsymbol{X}_t \mathrm{d}t + \boldsymbol{\Sigma}_{\boldsymbol{X}}' \mathrm{d}\boldsymbol{W}_t^{\mathbb{P}}. \tag{3}$$

$\boldsymbol{W}_t^{\mathbb{P}} \in \mathbb{R}^4$ is a vector of independent Brownian motions which model four sources of uncertainty of the financial market, namely uncertainty about the real interest rate, uncertainty about the instantaneous expected inflation, uncertainty about unexpected inflation and uncertainty about the stock return. $\boldsymbol{\Sigma}_{\boldsymbol{X}}'$ is $[\boldsymbol{I}_{2\times 2}\boldsymbol{0}_{2\times 2}]$. The matrix of coefficients $\boldsymbol{K} \in \mathbb{R}^{2\times 2}$ is used to control the dynamics of $\boldsymbol{X}_t$. The price index Π_t depends on the expected inflation π_t.

$$\frac{\mathrm{d}\Pi_t}{\Pi_t} = \pi_t \mathrm{d}t + \boldsymbol{\sigma}_{\Pi}' \mathrm{d}\boldsymbol{W}_t^{\mathbb{P}}. \tag{4}$$

Where $\boldsymbol{\sigma}_{\Pi} \in \mathbb{R}^4$ and $\Pi_0 = 1$. We do not observe π_t but do observe the price index Π_t. The stock index develops as:

$$\frac{\mathrm{d}S_t}{S_t} = (R_t + \eta_S)\mathrm{d}t + \boldsymbol{\sigma}_S' \mathrm{d}\boldsymbol{W}_t^{\mathbb{P}}. \tag{5}$$

With $\boldsymbol{\sigma}_s \in \mathbb{R}^4$ and $S_0 = 1$. Here, η_S is the stock-market risk premium. η_S is assumed to be fixed over time. Again, we do not observe R_t and η_S, but observe the stock index S_t. Finally, we have a nominal discount factor ϕ_t^N. ϕ_t^N can be used to determine the value of all assets.

$$\frac{\mathrm{d}\phi_t^N}{\phi_t^N} = -R_t \mathrm{d}t - \boldsymbol{\Lambda}_t' \mathrm{d}\boldsymbol{W}_t^{\mathbb{P}}. \tag{6}$$

Here we assume that the price of risk is time varying in $\boldsymbol{X}$.

$$\boldsymbol{\Lambda}_t = \boldsymbol{\Lambda}_0 + \boldsymbol{\Lambda}_1 \boldsymbol{X}_t. \tag{7}$$

Here, $\boldsymbol{\Lambda}_0$ and $\boldsymbol{\Lambda}_t. \in \mathbb{R}^4$ and $\boldsymbol{\Lambda}_1 \in \mathbb{R}^{4\times 2}$. This stochastic discount factor gives the trade-off between consumption today and tomorrow (marginal utility ratio as in Draper (2014)). We make the assumption of no risk premium for unexpected inflation (third row of $\boldsymbol{\Lambda}_0$ and $\boldsymbol{\Lambda}_1$ equal to zero), as one cannot identify unexpected inflation risk based on nominal data.

$$\boldsymbol{\Lambda}_0 = \begin{pmatrix} \Lambda_{0_{(1,1)}} \\ \Lambda_{0_{(2,1)}} \\ 0 \\ \Lambda_{0_{(4,1)}} \end{pmatrix}. \tag{8}$$

$$\boldsymbol{\Lambda}_1 = \begin{pmatrix} \Lambda_{1_{(1,1)}} & \Lambda_{1_{(1,2)}} \\ \Lambda_{1_{(2,1)}} & \Lambda_{1_{(2,2)}} \\ 0 & 0 \\ \Lambda_{1_{(4,1)}} & \Lambda_{1_{(4,2)}} \end{pmatrix}. \tag{9}$$

Without further restrictions $\Lambda_{0_{(4,1)}}$, $\Lambda_{1_{(4,1)}}$ and $\Lambda_{1_{(4,2)}}$ cannot be identified.

4.1.2 Nominal Term Structure

The above equations are relatively straightforward as they are assumptions by Koijen et al. (2010). However, it becomes more complicated as the KNW model considers the nominal term structure. For this it should be taken into account that the expected value of a price of a nominal zero coupon bond P should not change over time. This is implied by to the following fundamental pricing equation:

$$\mathbb{E}\,\mathrm{d}(\phi^N P^N) = 0. \tag{10}$$

Draper (2014) shows that Equation 10 can be approximated by:

$$\mathbb{E}(\mathrm{d}\phi^N P^N + \phi^N \mathrm{d}P^N + \mathrm{d}\phi^N \mathrm{d}P^N) = 0. \tag{11}$$

Now, using Itô Doeblin theorem and the assumption that bond prices dependent on the state of the economy and a time trend, $P^N = P^N(X,t)$, we get the following equation:

$$\begin{aligned} \mathrm{d}P^N &= \boldsymbol{P}_X^{N\prime}\mathrm{d}\boldsymbol{X} + P_t^N \mathrm{d}t + \frac{1}{2}\mathrm{d}\boldsymbol{X}'\boldsymbol{P}^N_{\boldsymbol{X}\boldsymbol{X}'}\mathrm{d}\boldsymbol{X} + \mathrm{d}\boldsymbol{X}'\boldsymbol{P}^N_{\boldsymbol{X}t}\mathrm{d}t + \frac{1}{2}\mathrm{d}tP^N_{tt}\mathrm{d}t, \\ &= \boldsymbol{P}_X^{N\prime}(-\boldsymbol{K}\boldsymbol{X}_t\mathrm{d}t + \boldsymbol{\Sigma}'_{\boldsymbol{X}}\mathrm{d}\boldsymbol{W}_t^{\mathbb{P}}) + P_t^N\mathrm{d}t + \frac{1}{2}(\mathrm{d}\boldsymbol{W}_t^{\mathbb{P}})\boldsymbol{\Sigma}_{\boldsymbol{X}}\boldsymbol{P}^N_{\boldsymbol{X}\boldsymbol{X}'}\boldsymbol{\Sigma}'_{\boldsymbol{X}}\mathrm{d}\boldsymbol{W}_t^{\mathbb{P}}. \end{aligned} \tag{12}$$

The lower equation is obtained from filling in Equation 3 into the first equation. $\mathrm{d}t^2$, $\mathrm{d}t\mathrm{d}\boldsymbol{W}^{\mathbb{P}}$ disappear, and $(\mathrm{d}\boldsymbol{W}^{\mathbb{P}})^2$ tends to $\mathrm{d}t$, we obtain

$$0 = \boldsymbol{P}_{\boldsymbol{X}}^{N\prime}(-\boldsymbol{K}\boldsymbol{X}_t) + P_t^N + \frac{1}{2}\mathrm{tr}(\boldsymbol{\Sigma}_{\boldsymbol{X}}\boldsymbol{P}^N_{\boldsymbol{X}\boldsymbol{X}'}\boldsymbol{\Sigma}'_{\boldsymbol{X}}) - P^N R_t - P_{\boldsymbol{X}}^{N\prime}\boldsymbol{\Sigma}'_{\boldsymbol{X}}\boldsymbol{\Lambda}_t. \tag{13}$$

This partial differential equation has a solution of the form:

$$P^N(\boldsymbol{X}_t, t, t+\tau) = e^{(A(\tau)+\boldsymbol{B}'(\tau)\boldsymbol{X}_t)}. \tag{14}$$

For zero coupon bonds there is a single pay-off at time T. This means $\tau = T - t$. Now substitute the derivatives.

$$
\begin{aligned}
\frac{1}{P^N}\boldsymbol{P}^N_{\boldsymbol{X}} &= \boldsymbol{B}, \\
\frac{1}{P^N}P^N_t &= -\frac{1}{P^N}P^N_t = -A - \boldsymbol{B}'\boldsymbol{X}_t, \\
\frac{1}{P^N}\boldsymbol{P}^N_{\boldsymbol{X}\boldsymbol{X}'} &= \boldsymbol{B}\boldsymbol{B}',
\end{aligned} \tag{15}
$$

into Equation 13,

$$
\begin{aligned}
0 &= \boldsymbol{B}'(-\boldsymbol{K}\boldsymbol{X}_t) + (-A - \boldsymbol{B}'\boldsymbol{X}_t) + \frac{1}{2}\mathrm{tr}(\boldsymbol{\Sigma_X}\boldsymbol{B}\boldsymbol{B}'\boldsymbol{\Sigma}'_{\boldsymbol{X}}) - R_0 - \boldsymbol{R}'_1\boldsymbol{X}_t - \boldsymbol{B}'\boldsymbol{\Sigma}'_{\boldsymbol{X}}(\boldsymbol{\Lambda}_0 + \boldsymbol{\Lambda}_1\boldsymbol{X}_t), \\
&= \underbrace{-A - R_0 - \boldsymbol{B}'\boldsymbol{\Sigma}'_{\boldsymbol{X}}\boldsymbol{\Lambda}_0 + \frac{1}{2}\mathrm{tr}(\boldsymbol{\Sigma_X}\boldsymbol{B}^N\boldsymbol{B}'\boldsymbol{\Sigma}'_{\boldsymbol{X}})}_{=0} + \underbrace{(-\boldsymbol{R}'_1 - \boldsymbol{B}'(\boldsymbol{K} + \boldsymbol{\Sigma}'_{\boldsymbol{X}}\boldsymbol{\Lambda}_1) - \boldsymbol{B})}_{=0}\boldsymbol{X}_t
\end{aligned} \tag{16}
$$

We have that $\mathrm{tr}(\boldsymbol{\Sigma_X}\boldsymbol{B}\boldsymbol{B}'\boldsymbol{\Sigma}'_{\boldsymbol{X}}) = \mathrm{tr}(\boldsymbol{B}'\boldsymbol{\Sigma}'_{\boldsymbol{X}}\boldsymbol{\Sigma_X}\boldsymbol{B}) = \boldsymbol{B}'\boldsymbol{\Sigma}'_{\boldsymbol{X}}\boldsymbol{\Sigma_X}\boldsymbol{B}$ because $\mathrm{tr}(\boldsymbol{A}\boldsymbol{B}) = \mathrm{tr}(\boldsymbol{B}\boldsymbol{A})$. In combination with the fact that the constant term and 'coefficient' term in front of $\boldsymbol{X}_t$ should both be equal to zero in Equation 16, this finally leads to:

$$
A(\tau) = -R_0 - (\boldsymbol{\Lambda}'_0\boldsymbol{\Sigma_X})\boldsymbol{B}(\tau) + \frac{1}{2}\boldsymbol{B}'(\tau)\boldsymbol{\Sigma}'_{\boldsymbol{X}}\boldsymbol{\Sigma_X}\boldsymbol{B}(\tau), \tag{17}
$$

$$
\boldsymbol{B}(\tau) = -\boldsymbol{R}_1 - (\boldsymbol{K}' + \boldsymbol{\Lambda}'_1\boldsymbol{\Sigma_X})\boldsymbol{B}(\tau). \tag{18}
$$

These differential equations can be solved to the following:

$$
A(\tau) = \int_0^\tau A(s)\,ds, \tag{19}
$$

$$
\boldsymbol{B}(\tau) = (\boldsymbol{K}' + \boldsymbol{\Lambda}'_1\boldsymbol{\Sigma_X})^{-1}(e^{-(\boldsymbol{K}' + \boldsymbol{\Lambda}'_1\boldsymbol{\Sigma_X})\tau} - \boldsymbol{I}_{2\times 2})\boldsymbol{R}_1, \tag{20}
$$

using the fact that a bond with payout 1 and maturity $\tau = 0$ has price $P^N(\boldsymbol{X}_t, t, t) = 1$ implying $A(0) = 0$, $\boldsymbol{B}(0) = \boldsymbol{0}_{2\times 1}$.

4.1.3 Restrictions

First of all, Koijen et al. (2010) creates a few restrictions to meet the fundamental valuation equation of the equity index:

$$
\mathbb{E}\,\mathrm{d}(\phi^N S) = 0. \tag{21}
$$

The expected discounted stock price should not change over time. Using the Itô Doeblin theorem, we get:

$$
\begin{aligned}
\frac{\mathrm{d}\phi^N S}{\phi^N S} &= \frac{\mathrm{d}\phi^N}{\phi^N} + \frac{\mathrm{d}S}{S} + \frac{\mathrm{d}\phi^N}{\phi^N}\frac{\mathrm{d}S}{S}, \\
&= (\eta_S - \boldsymbol{\Lambda}'_t\sigma'_S)\mathrm{d}t - (\boldsymbol{\Lambda}'_t - \boldsymbol{\sigma}'_S)\mathrm{d}\boldsymbol{W}^{\mathbb{P}}_t.
\end{aligned} \tag{22}
$$

Going from the first to the second part of the equation because $\mathrm{d}t^2$, $\mathrm{d}t\mathrm{d}\boldsymbol{W}^{\mathbb{P}}$ disappear, and $(\mathrm{d}\boldsymbol{W}^{\mathbb{P}})^2$ tends to $\mathrm{d}t$. Now by taking the expectations, this leads to:

$$\eta_S = \boldsymbol{\Lambda}_t'\boldsymbol{\sigma}_S. \tag{23}$$

Leading to $\boldsymbol{\sigma}_S'\boldsymbol{\Lambda_0} = \eta_S$ and $\boldsymbol{\sigma}_S'\boldsymbol{\Lambda}_1 = 0$. These two restrictions are not used during the estimation process, but to identify $\boldsymbol{\Lambda}_{0_{(4,1)}}$, $\boldsymbol{\Lambda}_{1_{(4,1)}}$ and $\boldsymbol{\Lambda}_{1_{(4,2)}}$.

In the methodological refinement, Muns (2015) adds a few restrictions to ensure that $\boldsymbol{W}^{\mathbb{P}}$ and $\boldsymbol{X}$ are identified. First of all, $\boldsymbol{K}$ should be a lower triangular matrix. This makes sure that the components of $\boldsymbol{X}$ and thus the first 2 components of $\boldsymbol{W}^{\mathbb{P}}$ do not switch. Additionally $\boldsymbol{\sigma}_{\Pi(3)} = 0$ this makes sure that the last two components of $\boldsymbol{W}^{\mathbb{P}}$ do not rotate. $\boldsymbol{W}^{\mathbb{P}}$ models the four sources of uncertainty of the financial market, if we allow for rotation, we are not sure which of the four Brownian motions indicates which source of uncertainty.

Moreover, the Commission Parameters (Dijsselbloem et al., 2019) decided on new restrictions on the KNW model. They want the Ultimate Forward Rate (UFR) and unconditional expected returns of inflation and stocks are equal to specific values. Therefore, Pelsser (2019) imposed the following restrictions on the model. The UFR is defined as the rate of a zero coupon bond with a maturity of $\tau \to \infty$. It is assumed that zero coupon rates gradually grow to the level of the UFR. Pension funds often use this rate to discount long term liabilities. Mathematically the UFR is defined as,

$$\ln(1+\mathrm{UFR}) = \lim_{\tau\to+\infty}\frac{A(\tau)}{\tau} = \delta_{0r} - \boldsymbol{\Lambda}_0'\boldsymbol{B}_\infty - \frac{1}{2}\boldsymbol{B}_\infty'\boldsymbol{B}_\infty, \tag{24}$$

leading to the following restriction:

$$\delta_{0r} = \ln(1+\mathrm{UFR}) + \tilde{\boldsymbol{\Lambda}}_0'\boldsymbol{B}_\infty + \frac{1}{2}\boldsymbol{B}_\infty'\boldsymbol{B}_\infty. \tag{25}$$

Here $\boldsymbol{B}_\infty = (\boldsymbol{K} + \tilde{\boldsymbol{\Lambda}}_\mathbf{1})^{-1}\boldsymbol{\delta}_{1r}$, with $\tilde{\boldsymbol{\Lambda}}_1$ being equal to the first two rows of $\boldsymbol{\Lambda}_1$. In the equation $\tilde{\boldsymbol{\Lambda}}_0$ is equal to the first two elements of $\boldsymbol{\Lambda}_0$. The unexpected geometric return of S_t over a period of 1 year is given by

$$\ln(1+r_S^g) = \lim_{t\to+\infty}\mathbb{E}(\ln\frac{S_{t+1}}{S_t}) = \delta_{0r} + \eta_S - \frac{1}{2}\boldsymbol{\sigma}_S'\boldsymbol{\sigma}_S. \tag{26}$$

After rewriting this leads to the following equation:

$$\eta_S = \ln(1+r_S^g) - \delta_{0r} + \frac{1}{2}\boldsymbol{\sigma}_S'\boldsymbol{\sigma}_S. \tag{27}$$

The unconditional geometric expected return of Π_t over a period of 1 year is given by

$$\ln(1+r_\Pi^g) = \lim_{t\to+\infty}\mathbb{E}(\ln\frac{\Pi_{t+1}}{\Pi_t}) = \delta_{0\pi} - \frac{1}{2}\boldsymbol{\sigma}_\Pi'\boldsymbol{\sigma}_\Pi, \tag{28}$$

which leads to

$$\delta_{0\pi} = \ln(1+r_\Pi^g) + \frac{1}{2}\boldsymbol{\sigma}_\Pi'\boldsymbol{\sigma}_\Pi. \tag{29}$$

The UFR, unexpected geometric return and unconditional geometric expected inflation are decided by the Commission Parameters on monthly basis, and in the latest scenario set they are set equal to 1.8 %, 5.6% and 1.9% respectively. This finally implies that we have to estimate $\boldsymbol{\delta_{1r}}$ (two parameters), $\boldsymbol{\delta_{1\pi}}$ (two parameters), $\boldsymbol{K}$ (lower triangular, so three parameters), $\boldsymbol{\Lambda_0}$ (with the restrictions, two parameters), $\boldsymbol{\Lambda_1}$ (with the restrictions, four parameters), $\boldsymbol{\sigma}_\Pi$ (with restriction, three parameters), $\boldsymbol{\sigma}_S$ (four parameters). Which are 20 parameters in total.

However, at the start of 2021, DNB decided to fix the long-term average interest rate at -0.01% after the calibration. In order to take into account this constraint during the estimation process, one can set δ_{0r} equal to -0.01% during the estimation process. This way Equation 25 can be omitted.

4.2 Estimation

4.2.1 Kalman Filter

Draper (2014) uses simulated annealing to estimate the KNW model and thus the scenario published by DNB are estimated this way. However, a more straightforward method is to use the Kalman filter procedure of Pelsser (2019). For more eleboration on how to derive the Kalman filter, we refer to Harvey (1990). To express the KNW model in state-space form, we add together Equations 3, 4 and 5 in matrix form, to get the following:

$$\mathrm{d}\begin{bmatrix}\boldsymbol{X}_t\\ \ln\Pi_t\\ \ln S_t\end{bmatrix} = \left(\begin{bmatrix}\boldsymbol{0}_{2\times1}\\ \delta_{0\pi}-\frac{1}{2}\boldsymbol{\sigma}_\Pi'\boldsymbol{\sigma}_\Pi\\ R_0+\eta_S-\frac{1}{2}\boldsymbol{\sigma}_S'\boldsymbol{\sigma}_S\end{bmatrix} + \begin{bmatrix}-\boldsymbol{K} & \boldsymbol{0}_{2\times2}\\ \boldsymbol{\delta}_{1\pi}' & \boldsymbol{0}_{1\times2}\\ \boldsymbol{R}_1' & \boldsymbol{0}_{1\times2}\end{bmatrix}\begin{bmatrix}\boldsymbol{X}_t\\ \ln\Pi_t\\ \ln S_t\end{bmatrix}\right)\mathrm{d}t + \begin{bmatrix}\boldsymbol{\Sigma}_{\boldsymbol{X}}'\\ \boldsymbol{\sigma}_\Pi'\\ \boldsymbol{\sigma}_S'\end{bmatrix}\mathrm{d}\boldsymbol{W}_t^{\mathbb{P}}. \tag{30}$$

We define the augmented state-vector $\tilde{\boldsymbol{X}}_t = [\boldsymbol{X}_t, \ln\Pi_t, \ln S_t]$, and we find that the dynamics of $\tilde{\boldsymbol{X}}_t$ are of the form $d\tilde{\boldsymbol{X}}_t = (\boldsymbol{a}+\boldsymbol{A}\tilde{\boldsymbol{X}}_t)\mathrm{d}t + \boldsymbol{C}\mathrm{d}\boldsymbol{W}_t$. From these dynamics, for a time-step Δt, Pelsser (2019) derives an expression for the multivariate Gaussian transition density.

$$f(\tilde{\boldsymbol{X}}_t|\tilde{\boldsymbol{X}}_{t-\Delta t}) \sim \mathcal{N}(e^{\boldsymbol{A}\Delta t}\tilde{\boldsymbol{X}}_{t-\Delta t} + \int_0^{\Delta t} e^{\boldsymbol{A}u}\boldsymbol{a}du, \int_0^{\Delta t} e^{\boldsymbol{A}u}\boldsymbol{C}\boldsymbol{C}'e^{\boldsymbol{A}'u}du). \tag{31}$$

From the transition density, we obtain the following representation of the development of the state vector $\tilde{\boldsymbol{X}}$.

$$\tilde{\boldsymbol{X}}_t = \boldsymbol{\omega} + \boldsymbol{\Omega}\tilde{\boldsymbol{X}}_{t-\Delta t} + \varepsilon_t, \qquad \mathrm{var}[\epsilon_t] = \boldsymbol{Q}, \tag{32}$$

with $\boldsymbol{\omega} = \int_0^{\Delta t} e^{\boldsymbol{A}u}\boldsymbol{a}du$, $\boldsymbol{\Omega} = e^{\boldsymbol{A}\Delta t}$ and $\boldsymbol{Q} = \int_0^{\Delta t} e^{\boldsymbol{A}u}\boldsymbol{C}\boldsymbol{C}'e^{\boldsymbol{A}'u}du$. Information about the state vector $\tilde{\boldsymbol{X}}_t$ can be obtained by using the yields $y_t(\tau_i)$ for different maturities τ_i and the price index Π_t and stock index S_t. In the KNW model these are an affine function of $\tilde{\boldsymbol{X}}_t$. This is shown in the measurement equation.

$$\tilde{\boldsymbol{y}}_t = d\begin{bmatrix}\boldsymbol{y}_t\\ \ln\Pi_t\\ \ln S_t\end{bmatrix} = \boldsymbol{a} + \boldsymbol{B}\tilde{\boldsymbol{X}}_{\boldsymbol{t}} + \boldsymbol{\eta}_{\boldsymbol{t}}, \qquad \mathrm{var}[\boldsymbol{\eta}_{\boldsymbol{t}}] = \boldsymbol{H}. \tag{33}$$

Here, the vector $\tilde{\boldsymbol{y}}_t$ consists of m zero-rates with maturities $\tau_1, \ldots, \tau_m$ together with $\ln\Pi_t$ and

$\ln S_t$. The vector $\boldsymbol{a} \in \mathbb{R}^{(m+2)\times 1}$ and $\boldsymbol{B} \in \mathbb{R}^{(m+2)\times 4}$ are as follows:

$$\boldsymbol{a} = \begin{bmatrix} A(\tau_1)/\tau_1 \\ \vdots \\ A(\tau_m)/\tau_m \\ 0 \\ 0 \end{bmatrix}. \qquad \boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}(\tau_1)'/\tau_1 & 0 & 0 \\ \vdots & \vdots & \vdots \\ \boldsymbol{B}(\tau_m)'/\tau_m & 0 & 0 \\ \boldsymbol{0}_{1\times 2} & 1 & 0 \\ \boldsymbol{0}_{1\times 2} & 0 & 1 \end{bmatrix}. \tag{34}$$

We know $A(\tau)$ and $\boldsymbol{B}(\tau)$ from Equation 19 and 20. However, Pelsser (2019) estimates these variables from the mean and variance of the process $\begin{bmatrix} \boldsymbol{X}_t & i_t \end{bmatrix}'$, where $i_t = \int_0^t R_s ds$. More details on this can be found in Pelsser (2019). The measurement errors are captured by $\boldsymbol{\eta}_t$. We make the assumption that $\boldsymbol{\eta}_t$ is assumed to follow an i.i.d. multivariate Gaussian distribution, $\mathcal{N}(0, \boldsymbol{H})$. $\boldsymbol{H} \in \mathbb{R}^{(m+2)\times(m+2)}$ is given by,

$$\boldsymbol{H} = \begin{bmatrix} \text{diag}(h_m^2) & \boldsymbol{0}_{m\times 2} \\ \boldsymbol{0}_{2\times m} & \boldsymbol{0}_{2\times 2} \end{bmatrix}. \tag{35}$$

This structures implies that $\ln\Pi_t$ and $\ln S_t$ are observed without measurement error. Moreover, $\boldsymbol{\eta}_t$ is independent from $\tilde{\boldsymbol{X}}_t$ and ε_t. As we do not know $\tilde{\boldsymbol{X}}_{t-\Delta t}$, the best information about $\tilde{\boldsymbol{X}}_{t-\Delta t}$ is the estimated state $\hat{\boldsymbol{X}}_{t-\Delta t}$. The covariance matrix of the estimation error is $\hat{\boldsymbol{X}}_{t-\Delta t} - \tilde{\boldsymbol{X}}_{t-\Delta t}$ is denoted by $\boldsymbol{P}_{t-\Delta t}$. Together Equation 32 and 33 lead to the following joint distribution of $\tilde{\boldsymbol{y}}_t$ and $\tilde{\boldsymbol{X}}_t$.

$$f\left[\begin{bmatrix} \tilde{\boldsymbol{X}}_t \\ \tilde{\boldsymbol{y}}_t \end{bmatrix} | \hat{\boldsymbol{X}}_{t-\Delta t}\right] \sim \mathcal{N}\left[\begin{bmatrix} \boldsymbol{\omega} + \boldsymbol{\Omega}\hat{\boldsymbol{X}}_{t-\Delta t} \\ \boldsymbol{a} + \boldsymbol{B}(\boldsymbol{\omega} + \boldsymbol{\Omega}\hat{\boldsymbol{X}}_{t-\Delta t}) \end{bmatrix}, \begin{bmatrix} \boldsymbol{P}_{t|t-\Delta t} & \boldsymbol{P}_{t|t-\Delta t}\boldsymbol{B}' \\ \boldsymbol{B}\boldsymbol{P}_{t|t-\Delta t} & \boldsymbol{V}_t \end{bmatrix}\right], \tag{36}$$

with,

$$\boldsymbol{P}_{t|t-\Delta t} = \boldsymbol{\Omega}\boldsymbol{P}_{t-\Delta t}\boldsymbol{\Omega}' + \boldsymbol{Q}, \tag{37}$$

$$\boldsymbol{V}_t = \boldsymbol{B}\boldsymbol{P}_{t|t-\Delta t}\boldsymbol{B}' + \boldsymbol{H}. \tag{38}$$

As $\tilde{\boldsymbol{y}}_t$ is observed at time t, we can compute the conditional distribution of $\tilde{\boldsymbol{X}}_t$ given $\tilde{\boldsymbol{y}}_t$ and $\hat{\boldsymbol{X}}_{t-\Delta t}$.

$$f(\tilde{\boldsymbol{X}}_t | \tilde{\boldsymbol{y}}_t, \hat{\boldsymbol{X}}_{t-\Delta t}) \sim \mathcal{N}(\boldsymbol{\omega} + \boldsymbol{\Omega}\hat{\boldsymbol{X}}_{t-\Delta t} + \boldsymbol{K}_t\boldsymbol{u}_t, \boldsymbol{P}_t), \tag{39}$$

where,

$$\boldsymbol{u}_t = \tilde{\boldsymbol{y}}_t - (\boldsymbol{a} + \boldsymbol{B}(\boldsymbol{\omega} + \boldsymbol{\Omega}\hat{\boldsymbol{X}}_{t-\Delta t}), \tag{40}$$

$$\boldsymbol{K}_t = \boldsymbol{P}_{t|t-\Delta t}\boldsymbol{B}'\boldsymbol{V}_t^{-1}, \tag{41}$$

$$\boldsymbol{P}_t = \boldsymbol{P}_{t|t-\Delta t} - \boldsymbol{P}_{t|t-\Delta t}\boldsymbol{B}'\boldsymbol{V}^{-1}\boldsymbol{B}\boldsymbol{P}_{t|t-\Delta t} = (\boldsymbol{I} - \boldsymbol{K}_t\boldsymbol{B})\boldsymbol{P}_{t|t-\Delta t}. \tag{42}$$

Finally, to complete the Kalman filter specification, the best estimate of $\tilde{\boldsymbol{X}}_t$ is the conditional expectation.

$$\hat{\boldsymbol{X}}_t = \mathbb{E}[\tilde{\boldsymbol{X}}_t | \tilde{\boldsymbol{y}}_t, \hat{\boldsymbol{X}}_{t-\Delta t}] = \boldsymbol{\omega} + \boldsymbol{\Omega}\hat{\boldsymbol{X}}_{t-\Delta t} + \boldsymbol{K}_t\boldsymbol{u}_t. \tag{43}$$

4.2.2 Maximum Likelihood Estimation

Until now, we have assumed that $\boldsymbol{\omega}, \boldsymbol{\Omega}, \boldsymbol{Q}, \boldsymbol{a}, \boldsymbol{B}, \boldsymbol{H}$ (augmented in $\boldsymbol{\delta_{1r}}$, $\boldsymbol{\delta_{1\pi}}$, $\boldsymbol{K}$, $\boldsymbol{\Lambda_0}$, $\boldsymbol{\Lambda_1}$, $\boldsymbol{\sigma}_\Pi$, $\boldsymbol{\sigma}_S$) are known. However, in fact we need to estimate these parameters. This can be done by maximising the likelihood of the observed data. The distribution of $(\tilde{\boldsymbol{y}}_t|\hat{\boldsymbol{X}}_{t-\Delta t})$ is given by,

$$f(\tilde{\boldsymbol{y}}_t|\hat{\boldsymbol{X}}_{t-\Delta t}) \sim \mathcal{N}(\boldsymbol{a} + \boldsymbol{B}(\boldsymbol{\omega} + \boldsymbol{\Omega}\hat{\boldsymbol{X}}_{t-\Delta t}), \boldsymbol{V}_t). \tag{44}$$

As it is a multivariate Gaussian distribution, the log-likelihood at t is given by,

$$\ell_t = -\frac{1}{2}\ln|\boldsymbol{V}_t| - \frac{1}{2}\boldsymbol{u}_t'\boldsymbol{V}_t^{-1}\boldsymbol{u}_t. \tag{45}$$

We can now run the Kalman filter. To initialize the Kalman filter, we use the stationary initialization as given in Pelsser (2019). As we can observe $\ln \Pi_t$ and $\ln S_t$, which are both non-stationary, we set $\hat{\boldsymbol{X}}_0 = [\mathbb{E}(X_\infty), \ln\Pi_0, \ln S_0]$. $t = 0$ is observations for December 1999. The initial estimation error has variance,

$$\boldsymbol{P_0} = \begin{bmatrix} \mathrm{var}(X_\infty) & \boldsymbol{0}_{(2\times2)} \\ \boldsymbol{0}_{(2\times2)} & \boldsymbol{0}_{(2\times2)} \end{bmatrix}.$$

The 0-matrices imply that we know $\ln \Pi_0$ and $\ln S_0$ exactly. The Kalman filter is run over the remaining observations $t = 1, 2, ...$ from January 2000 until December 2020. $\mathbb{E}(\boldsymbol{X}_\infty)$ and $\mathrm{var}(\boldsymbol{X}_\infty)$ are the unconditional expectation and variance of $\boldsymbol{X}_t$ respectively, which we do not know. Fortunately, this initialization is equal to replicating $\ln \Pi_t$ and $\ln S_t$ in the state-vector with their first differences. Then, we can use the unconditional mean and variance of the differences state-vector. The total log-likelihood is calculated as $\ell = \sum_{t=1}^T \ell_t$.

4.3 Simulation

For simulation, we need an exact discretization of the model. This is possible by writing the whole model as a multivariate Ornstein Uhlenbeck process, as in Equation 46.

$$d\boldsymbol{Y}_t = (\boldsymbol{\Theta}_0 + \boldsymbol{\Theta}_1\boldsymbol{Y}_t)\mathrm{d}t + \boldsymbol{\Sigma}_Y \mathrm{d}\boldsymbol{W}_t^{\mathbb{P}}. \tag{46}$$

With $\boldsymbol{Y}' = [\boldsymbol{X} \quad \ln\Pi \quad \ln S \quad \ln P^{N,0} \quad \ln P^{N,\tau}]$. $P^{N,\tau}$ is coupon rate for a nominal bond with maturity τ. After using the Itô Doeblin theorem on $\mathrm{d}\ln\Pi$, $\mathrm{d}\ln S$ and $d\ln P^{N,\tau}$, we get Equation 47. Elaboration on this can be found in Draper (2014).

$$d\begin{bmatrix} \boldsymbol{X} \\ \ln\Pi \\ \ln S \\ \ln P^{N,0} \\ \ln P^{N,\tau} \end{bmatrix} = \left(\begin{bmatrix} 0 \\ \delta_{0\pi} - \frac{1}{2}\boldsymbol{\sigma}_\Pi'\boldsymbol{\sigma}_\Pi \\ R_0 + \eta_S - \frac{1}{2}\boldsymbol{\sigma}_S'\boldsymbol{\sigma}_S \\ R_0 \\ R_0 + \boldsymbol{B}^N(\tau)'\boldsymbol{\Sigma}_X'\boldsymbol{\Lambda}_0 - \frac{1}{2}\boldsymbol{B}^{N\prime}\boldsymbol{\Sigma}_X'\boldsymbol{\Sigma}_X B^N \end{bmatrix} + \begin{bmatrix} -\boldsymbol{K} & 0 \\ \boldsymbol{\delta}_{1\pi}' & 0 \\ \boldsymbol{R}_1' & 0 \\ \boldsymbol{R}_1' & 0 \\ \boldsymbol{R}_1' + \boldsymbol{B}^N(\tau)'\boldsymbol{\Sigma}_{\boldsymbol{X}}'\boldsymbol{\Lambda}_1 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{X} \\ \ln\Pi \\ \ln S \\ \ln P^{N,0} \\ \ln P^{N.\tau} \end{bmatrix} \right) dt + \begin{bmatrix} \boldsymbol{\Sigma}_X' \\ \boldsymbol{\sigma}_\Pi' \\ \boldsymbol{\sigma}_S' \\ 0 \\ \boldsymbol{B}^N(\tau)'\boldsymbol{\Sigma}_X' \end{bmatrix} d\boldsymbol{W}_t^{\mathbb{P}}. \tag{47}$$

Together with nominal stochastic discount factor ϕ_t^N.

$$\frac{\phi_t^N}{\phi_t^N} = -R_t dt - \boldsymbol{\Lambda}_t' d\boldsymbol{W}_t^{\mathbb{P}}. \tag{48}$$

Now, using the eigenvalue decomposition. $\boldsymbol{\Theta}_1 = \boldsymbol{U}\boldsymbol{D}\boldsymbol{U}^{-1}$, the exact discretization is the following,

$$\boldsymbol{Y}_{t+h} = \boldsymbol{\mu} + \boldsymbol{\Gamma}\boldsymbol{Y}_t + \varepsilon_{t+1}, \qquad \varepsilon_{t+1} \sim \mathcal{N}(0, \boldsymbol{\Sigma}), \tag{49}$$

where $\boldsymbol{\Gamma}$, $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are defined as:

$$\boldsymbol{\Gamma} = \boldsymbol{\Theta} = \boldsymbol{U}e^{\boldsymbol{D}}\boldsymbol{U}^{-1}, \tag{50}$$

$$\boldsymbol{\mu} = \boldsymbol{U}\boldsymbol{F}\boldsymbol{U}^{-1}\boldsymbol{\Theta}_0, \tag{51}$$

$$\boldsymbol{\Sigma} = \boldsymbol{U}\boldsymbol{V}\boldsymbol{U}', \tag{52}$$

with $\boldsymbol{F}$, $\boldsymbol{V}$ are equal to,

$$\begin{aligned} \boldsymbol{F}_{ii} &= h\alpha(\boldsymbol{D}_{ii}), \\ \boldsymbol{V}_{ij} &= [\boldsymbol{U}^{-1}\boldsymbol{\Sigma}_{\boldsymbol{Y}}\boldsymbol{\Sigma}_{\boldsymbol{Y}}'(\boldsymbol{U}^{-1})']_{ij}\alpha([\boldsymbol{D}_{ii} + \boldsymbol{D}_{jj}]), \end{aligned} \tag{53}$$

$$\alpha(x) = \frac{e^x - 1}{x}, \qquad \alpha(0) = 1. \tag{54}$$

These relations are taken from Koijen et al. (2010) and Bergstrom (1984).

However, in case of value-based ALM, we need to have risk neutral scenarios. Therefore, we use the risk neutral formulation as described by Draper (2014). In this case, the expected discounted value of the price of an asset does change over time, making it possible to price derivatives. For more details about the derivation of this system of equations, we refer to the paper of Draper (2014).

$$d\begin{bmatrix} \boldsymbol{X} \\ \ln\Pi \\ \ln S \\ \ln P^{N,0} \\ \ln P^{N,\tau} \end{bmatrix} = \left(\begin{bmatrix} -\boldsymbol{\Sigma}'_{\boldsymbol{X}}\boldsymbol{\Lambda}_0 \\ \delta_{0\pi} - \frac{1}{2}\boldsymbol{\sigma}'_{\Pi}\boldsymbol{\sigma}_{\Pi} \\ R_0 + \eta_S - \frac{1}{2}\boldsymbol{\sigma}'_S\boldsymbol{\sigma}_S \\ R_0 \\ R_0 - \frac{1}{2}\boldsymbol{B}^{N\prime}\boldsymbol{\Sigma}'_X\boldsymbol{\Sigma}_X B^N \end{bmatrix} + \begin{bmatrix} -(\boldsymbol{K} + \boldsymbol{\Sigma}'_{\boldsymbol{X}}\boldsymbol{\Lambda}_1) & 0 \\ \delta'_{1\pi} & 0 \\ \boldsymbol{R}'_1 & 0 \\ \boldsymbol{R}'_1 & 0 \\ \boldsymbol{R}'_1 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{X} \\ \ln\Pi \\ \ln S \\ \ln P^{N,0} \\ \ln P^{N,\tau} \end{bmatrix} \right) dt + \begin{bmatrix} \boldsymbol{\Sigma}'_{\boldsymbol{X}} \\ \boldsymbol{\sigma}'_{\Pi} \\ \boldsymbol{\sigma}'_S \\ 0 \\ \boldsymbol{B}^N(\tau)'\boldsymbol{\Sigma}'_X \end{bmatrix} dW^{\mathbb{Q}}_t. \tag{55}$$

$$\frac{d\tilde{\phi}^N_t}{\tilde{\phi}^N_t} = -R_t dt. \tag{56}$$

Equation 55 and the stochastic discount factor in Equation 56, make it possible to simulate risk neutral scenarios necessary for value-based ALM. This makes it easy to valuate net pension benefits (a derivative product), because the discount factor for all assets is equal to R_t.

4.4 Fit of the Model

After we determine the KNW parameters, we examine the fit of the model to decide whether it is suitable to use for the ALM model. The fit is examined across three different areas: in-sample fit, simulation and compared to what was found by the Commission Parameters, DNB and Draper (2014).

First, we estimate the model using data from January 1999 until December 2018, such that we compare our estimates to the ones by the Commission Parameters in 2019. We compute the asymptotic standard errors such that we can obtain a z-score. After the log-likelihood ℓ, as in Equation 45, has converged to an optimum, we compute standard errors in the following way. We create the information matrix $\boldsymbol{\mathcal{I}}_\ell$ which consists of second-order partial derivatives of ℓ with respect to the model parameters. The inverse $\boldsymbol{\mathcal{I}}_\ell^{-1}$ is the covariance matrix of the maximum-likelihood parameter estimates (Pelsser, 2019). The standard errors are then calculated as the square-roots of the diagonal elements of ${\boldsymbol{\mathcal{I}}_\ell^{-1}}_{ii}$ where $i = 1, \ldots, 20$ as we have to estimate 20 parameters. The general fit of the model can be examined by means of the log-likelihood ℓ.

After confirming the robustness and correctness of the model, we re-estimate the model for data until 2020 with and without the new long-term interest rate restriction. To decide which model would be best to use for scenario generation in the ALM study, we again discuss the overall fit based on log-likelihood ℓ, the parameter implications and the corresponding standard errors. We can compare whether the difference in terms of log-likelihood between the restricted and non-restricted model is significant by means of a likelihood ratio test (Heij, de Boer, Franses, Kloek, & van Dijk, 2004). The likelihood ratio (LR) is equal to,

$$LR = 2(\ell_1 - \ell_2). \tag{57}$$

Here, ℓ_1 is the log-likelihood of the less restricted model and ℓ_2 the log-likelihood of the more restricted model (restrictions always make a model fit less well, thus ℓ_2 will always be lower than ℓ_1). LR is chi-squared distributed with as degrees of freedom the difference between the number of parameters of the more and less restricted model.

Moreover, we assess the in-sample fit of the newly calibrated KNW models. The estimation of the KNW model using the Kalman filter is based on the assumption that we observe the stock and customer price index without error. Therefore, we can best assess the goodness of fit by looking at the yield curve. For this reason, we asses whether the features of our model match features described in existing literature. Fama (1984), Koijen et al. (2010) and Dai & Singleton (2002) describe some stylized facts of the term structure. A summary of these facts is as follows:

1. The average yield curve over time is increasing and concave.
2. The long end of the yield curve is less volatile than the short end of the curve.
3. The yields have high auto-correlations, implying persistent yield dynamics.
4. There is more persistence for yields with long maturities than short maturities.
5. Across different yields, there are high cross-correlations.
6. The linear projection of R^{n-1}_{t+1} on $\frac{1}{n-1}(R^n_t - r_t)$ often have significant negative slope coefficients β_1.
7. β_1 becomes more negative with maturity.

The first two can be easily assessed by looking at a plot of the average yields and their corresponding volatility both for the estimated model and the data. Additionally, 3, 4 and 5 can be examined by looking at the (auto-)correlation matrix. Finally, 6 and 7 are from the paper of Dai & Singleton (2002) and are more complicated. Large yield spreads between a longer-term and short-term bonds forecasts declining yield on the long term bond over the maturity of the short-term bond. This empirical fact was first discussed in Campbell & Shiller (1991) and tested by Dai & Singleton (2002). In order to test whether our model replicates these empirical findings, we use the following Campbell-Shiller regression:

$$y_{t+m}(\tau_{n-m}) - y_t(\tau_n) = \beta_0 + \beta_1 \frac{m(y_t(\tau_n) - y_t(\tau_m))}{n-m} + \varepsilon_{t+m}, \tag{58}$$

where $n > m$ and $y_t(\tau_i)$ is the yield of a bond with maturity (τ_i) at time t. In Campbell & Shiller (1991) and Dai & Singleton (2002) it is shown that the coefficients are significantly negative, especially as n gets larger.

Finally, we also examine the models performance in terms of simulation paths. This is to give us an idea of how the stock and customer price path behave. From the data, we know that the stock index should be more volatile than the customer price index. We can assess this by simply looking at a plot of the simulated indices over time.

The careful examination of the models through the above mentioned methods should lead to a conclusion which scenarios to use for the ALM model.

5 Asset Liability Management

ALM is used by pension funds to evaluate current and alternative pension plans (Michielsen, 2015). These pension plans often consists of a pension policy, indexation policy, funding policy and investment policy. Figure 6 gives an overview of the different steps that need to be taken in an ALM study.

First, pension funds normally have participant files for which they can run their ALM model. However, in our case a synthetic pension fund has to be designed, because we do not have participant files. This is why demographic information is needed: Without making too many assumptions, a logical structure of the workforce is defined, specifically in terms of age and wage distribution. Additionally, the fund needs a fixed pension fund policy to be examined in the ALM model. These policies often are about the indexation, accrual and investment strategies. Each pension policy is examined for different economic scenarios. This is the last input for the ALM model. These scenarios are generated by an economic model, which is in our case the KNW model as described in Section 4. The scenarios should include major drivers of pension risk, namely asset return, interest rates and inflation.

After these three inputs are decided, the simulation is ran for an specified number of years. Finally, depending on the objective of the ALM study, the model gives different outputs. An especially important subject is the solvency of the pension fund. This is an indication of whether a pension fund is able to meet its long-term obligations. This solvency is measured as the FR, the ratio between the assets and liabilities. Underfunding occurs when the FR less than one. In this paper, for classical ALM the distribution of the FR is examined over the different scenarios.

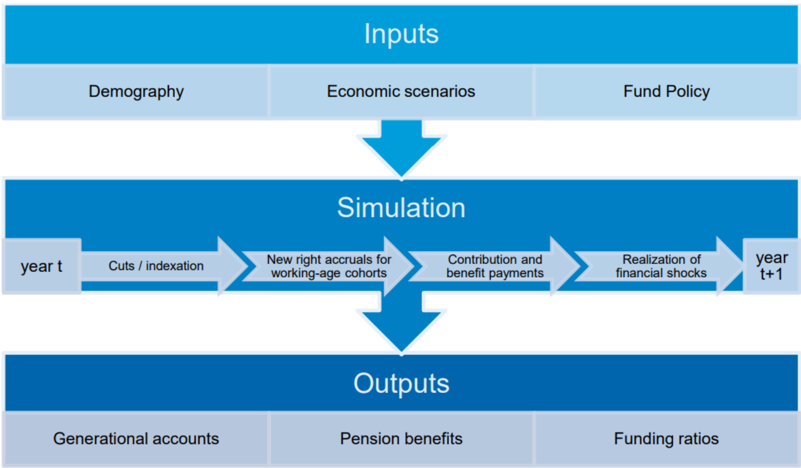


Figure 6: Overview of an ALM study, taken from Michielsen (2015).

Although classical ALM is a convenient tool to understand different possible future outcomes, Kortleve & Ponds (2006) have the critique that within this analysis high FRs are often explained by high risks. This leads to the first problem: Due to risk premiums, high expected returns do not necessarily mean that equities are more attractive. The second problem is that classical ALM misses an important insight. Classical ALM mostly focuses on risks for the pension fund and for the participants on an aggregate level, but not for the participants on an individual level.

To address the first issue, we employ value-based ALM. This means that risk neutral valuation is used to determine the value of a pension plan and that the scenarios obtained from the economic model are discounted back to present with a risk adjusted discount rate. Kortleve & Ponds (2006) names a few possibilities, namely deflators, risk neutral valuation or pricing kernels. Because of the compatibility with the KNW model, risk neutral valuation is used here, as in Section 4.3. To account for the second issue generational accounting is used (Ponds et al., 2003). This is a tool to explore intergenerational distributional effects of a policy. It gives an indication of generational value transfers of going from one policy to another. Using this approach one can show the benefits and contributions in the pension fund on age cohort level. This makes it possible to determine and compare the effects of a policy on the different generations in the fund.

5.1 Pension Fund Characteristics

Using our ALM model, we compare certain pension contracts. In order to do so, a synthetic pension fund is built. For this, certain assumptions are necessary. These assumptions account for every pension contract to be examined. In this section these assumptions will be described.

Table 1: Assumptions used for both classical ALM and value-based ALM.

Demographics	
Survival rates	Dutch population
Population distribution	Dutch population
Starters age	25
Working years	40
Maximum age	100
Wage	
Wage plan	Average pay plan
Development in pension basis	Inflation
Accrual rate, ε	1.875%
Assets	
Initial funding position	100%
Lower boundary FR, $\underline{FR}$	110%
Upper boundary FR, $\overline{FR}$	125%
Risk profile	
Investment policy	50% bonds, 50% stocks

Table 1 gives an overview of the most important assumptions. In the next sections, we explain what each entry means for our ALM model and how it is used.

5.1.1 Demographics

First of all, to make our synthetic pension fund as realistic as possible, the pension fund's workforce and retirement demographics are consistent with the Dutch population. For this reason data from the CBS is used, see Section 3. The initial age distribution within the synthetic pension is equal to the age distribution of the Dutch population in 2021. After the initialization, we use survival rates to update the population each year. As important factors, such as life expectancy, are gender specific, the data is gender specific. This leads to Equation 59.

$$
\begin{aligned}
M_t^x &= p_{t,t-1}^{m,x} M_t^x \\
F_t^x &= p_{t,t-1}^{f,x} F_t^x
\end{aligned}
\tag{59}
$$

The number of male and female participants in age cohort x at time t are denoted as M_t^x and F_t^x respectively. $p_{t,t-1}^{m,x}$, $p_{t,t-1}^{f,x}$ indicates probabilities male m, respectively female f, age $x-1$ in period $t-1$ will survive another year to age x in year t. Note that every year, new employees enter the workforce according to the prognosis of the future population of The Netherlands (*Bevolkingspiramide*). Thus, we model an *open* fund. As shown in Table 1, the minimum age of the workforce is 25 and the maximum age of a participant is 100. This implies that new participants enter the pension fund at age 25 and leave at age 100. This is a standard assumption usually made by the CPB, but an ALM should allow the minimum and maximum age to be changed. The retirement age is set to 65, but can be changed as well.

Some pension policies depend on the life expectancy, so here we show how to get from survival rates to life expectancy. We use the cohort life expectancy as defined in de Boer et al. (2020) (Koninklijk Actuarieel Genootschap), as they estimate the corresponding survival rates. The following formula expresses the remaining life expectancy of a person of year t under the assumption that this person was born on 1 January of year $t-x$,

$$
e_x^t = \frac{1}{2} + \sum_{k=0}^{\infty} \prod_{s=0}^{k} (p_{x+s,t+s}). \tag{60}
$$

Here, we assume that a person who dies is on average alive for half of that year.

5.1.2 Wage

Secondly, for each member of our pension fund, the wages are updated every year. How a member accrues pension depends on this wage and the corresponding wage plan. In Netherlands, one can choose between an average pay plan and a final pay plan. The latter, however, is very rare and means that the wage in the final working year is leading in the pension accrual. For this reason, we focus on average pay plan in this study. Average pay plan implies that the pension payment depends on the average salary during the working years. As for the wage itself, we assume the wage level of generation x at time t, W_t^x, evolves according to Equation 61 (Lekniute, 2011).

$$
\begin{aligned}
W_t^x &= W_{t-1}^{x-1} w_{t-1}^{x-1} \\
w_{t-1}^{x-1} &= (1+\pi_{t-1}) \tilde{w}_{t-1}^{x-1}
\end{aligned}
\tag{61}
$$

$\tilde{w}_{t-1}^{x-1}$ is the promotion rate from period $t-1$ to t for someone aged $x-1$ in period $t-1$. This promotion rate should be decreasing with age, as it is more difficult to make promotion as the one ages. We consider π_{t-1}, which is equal to the inflation at $t-1$. π_{t-1} is stochastic and determined by the economic model (see Section 4.1.1). Here we do not distinguish between female and male. At $t = 0$ the starters wage is equal to 1 and it increasing with inflation and the promotion rate over time as someone gets older. Normally, we would consider a social security offset ('franchise' in Dutch). In The Dutch pension system, one always receives public pension (the first pillar). Because of this regulation, in the second pension pillar, the pension accrual is adjusted. One only accrues pension benefits on their salary minus the social security offset. The social security offset also increases in line with inflation. However, as the salaries are fictional, it does not make sense to subtract a constant and thus we leave the social security offset out. This is possible, because the social security offset is the same level for all the participants.

Finally, we have the accrual rate ε, which is set equal to 1.875%. This implies that if someone is working 40 years, they can expect a pension benefit of 75% of their average salary. In case of a DB scheme, this benefit is secure. In case of a DC scheme, this is an ambition. More or less pension benefit is paid out depending on the returns of investment. A description of these schemes can be found in Section 5.2.

5.1.3 Assets

Michielsen (2015) names a few possibilities on how to initialize the assets of the pension fund. One of these options is to initialize the initial assets with zero. However, this is not realistic as in this thesis we want to show the effects of a change in policy on already existing pension funds. This is why it is decided to start with an initial funding ratio, FR_0, of 100%. The FR_t at time t can be calculated using the following formula.

$$\text{FR}_t = \frac{A_t}{L_t} \tag{62}$$

A_t and L_t are the assets and liabilities at time t respectively. This means that the pension fund has as much value in investment and premium payments as liabilities, such as benefit payments. Most of the pension funds in The Netherlands have an FR of around 100% (DNB, 2021c). From this initial funding position, we define the initial assets that the fund hold. This is done by multiplying the FR by the initial liabilities. These initial liabilities are calculated as the present value of the total accrued benefit claims. Here, we assume the initial wage level as described in Section 5.1.2 and that there was total indexation until now. The latter assumption is unavoidable but a bit unrealistic as DNB only allows for total indexation above a FR of 125%. This then means that the initial accrued benefits for each cohort are equal to the initial average pay of the workforce times the accrual rate ε times the number of years of service. Under the lower boundary is $\underline{FR} = 110\%$, indexation is not allowed. This boundary is set by DNB and makes sure that pension funds only index their liabilities when they have sufficient funds to do so. Between $\underline{FR}$ and $\overline{FR}$ partial indexation can take place. An overview of the indexation rules can be found on the site of the DNB.

After the initial assets are determined, we can now calculate the assets over time for each scenario s. For this the Equation 63.

$$A_{t+1,s} = A_{t,s} r_t + \sum_{x=25}^{64} (C_{t,s}^x (M_t^x + F_t^x) - \sum_{x=65}^{99} B_{t,s}^x (M_t^x + F_t^x)) r_t^{\frac{1}{2}} \tag{63}$$

r_t is the yearly return on investment. $C_{t,s}^x$ are the contributions paid in year t by the current work force in age cohort x in scenario s. $B_{t,s}^x$ are the benefits paid in year t to the pensioners in age cohort x in scenario s. $C_{t,s}^x$ and $B_{t,s}^x$ depend on the pension contract, described in Section 5.2. The assets are adjusted each year for these cash flows and then invested in stocks and bonds. The assets $A_{t,s}$ in year t are invested each year with return r_t. Because the model runs on yearly basis, and contribution and benefits are paid on monthly basis, return can only be made on average for half a year leading to the factor $r_t^{\frac{1}{2}}$.

5.2 Contracts

Here, we describe the pension contracts that are evaluated in this study.

5.2.1 Defined Benefit

In a defined benefit scheme, the benefit payment is secured (Zelinsky, 2004). However, this does mean that the premium paid can vary over time depending on new insights about the interest rates and mortality rates such that the pension fund can achieve the level of benefit payment financially. Therefore, every year the actuaries of a pension fund solve the following formula.

$$c_t \sum_{x=25}^{64} W_t^x (M_t^x + F_t^x) = \varepsilon \sum_{x=25}^{64} D_{c,t}^x. \tag{64}$$

With ε being the accrual factor as in Table 1. This then leads to:

$$c_t = \frac{\varepsilon \sum_{x=25}^{64} D_{c,t}^x}{\sum_{x=25}^{64} W_t^x (M_t^x + F_t^x)}, \tag{65}$$

where,

$$D_{c,t}^x = M_t^x \sum_{i=65-x}^{99-x} W_t^x p_{i,t}^{x,m} (R_t^{(i)})^{-i} + F_t^x \sum_{i=65-x}^{99-x} W_t^x p_{i,t}^{x,f} (R_t^{(i)})^{-i}. \tag{66}$$

D_p^x is used to calculate the future benefits to be paid-out discounted back to today. It calculates the total accrual of all the pension funds members and discounts it back to today. Obviously, the discount factor is gender specific, because it takes into account the mortality rates (why there are two separate sums in Equation 66), but the premium paid has to be same for both sexes. Moreover, in reality labor unions prevent large changes in premium level per year. Therefore, we set $|c_t - c_{t-1}| \leq 0.01$. This leads to the formula of contribution paid per year,

$$C_t^{x,DB} = c_t W_t^x \tag{67}$$

Additionally, we consider two kind of DB policies, one with unconditional indexation and another with conditional indexation. In case of the first, the pension benefits are always increased by the inflation, $ind = 1 + \pi_t$. This kind of policy is very rare. In reality, DNB set certain rules when it comes down to indexation. In this case, the benefits do not always grow with inflation, but it depends on the funding position of the pension fund.

$$ind_t = \begin{cases} 1 & \text{if } FR_t < \underline{FR}, \\ 1 + (\frac{FR_t - \underline{FR}}{\overline{FR} - \underline{FR}})\pi_t & \text{if } \underline{FR} \leq FR_t \leq \overline{FR}, \\ 1 + \pi_t & \text{if } \overline{FR} < FR_t. \end{cases} \tag{68}$$

As stated in Table 1, we assume the rate of accrual to be equal to $\varepsilon = 1.875\%$. For a DB policy this means that the pension benefit is accrued as,

$$B_t^{x,DB} = B_{t-1}^{x,DB} ind_t + W_t^x \varepsilon. \tag{69}$$

When a participant is age 25, $B_t^{25} = 0$. B_t is not be paid out until the participant turns 65.

5.2.2 Defined Contribution

As the name indicates for a DC scheme, the contributions paid every year are fixed. Nevertheless, the benefit payments are no longer secured. For the defined contribution plan, the premium is a fixed level c. This level is chosen in such that the present value at $t = 0$ of the premiums paid is equal to the value of the accrued benefits in the same year. Hence Equation 8 is only calculated once at $t = 0$. (as in Lekniute (2011)). This means that the premium paid every year is equal to,

$$C_t^{x,DC} = cW_t^x. \tag{70}$$

There is return on these premium payments, which determines the final yearly benefit payment after retirement. For each participant their total accrued benefit capital (BC) from which their yearly pension benefit is paid, is kept track of,

$$BC_t^{x,DC} = BC_{t-1}^{x,DC} r_t + C_{t-1}^{x,DC} r_t^{\frac{1}{2}} \tag{71}$$

Here r_t is the return on investment of the pension fund, a combination of the return on bonds and stocks. As the contributions are paid over the year, on average return can only be made for half a year, leading to the factor $r_t^{\frac{1}{2}}$.

$$BC_t^{x,DC} = B_t^{x,DC} \sum_{j=\max(0,65-x)}^{e_x^t} \frac{1}{(1 + r_{t,j})^j} \tag{72}$$

r is the discount interest rate at time t with maturity j. By solving this equation, the pension benefits B_t^x at time t for cohort x is found. Note that this allows us to keep investing even when someone is already retired: From the total capital, we calculate a yearly benefit. At retirement age, each year this benefit is paid, but what is left of the total capital can still be invested.

Finally, we look at two kinds of DC. One compensates the participants for going from DB to DC. This has to do with the fact that especially the older generation potentially does not profit from a DC policy, as they make only a few years of return over their invested contribution, while the younger generation does profit. Therefore, we compensate each generation as if they had a DC pension plan all their working time. We also consider DC without this compensation.

5.3 Output

5.3.1 Classical ALM

Classical ALM is mainly focused on the solvency of a pension fund over time. The FR is the most important indicator for this. The formula of the FR is shown in Equation 62, and can be calculated in both nominal and real terms. The distribution of the FR over the scenarios is of interest: What happens in the worst case and what is the corresponding risk? The 2.5%, 50% and 97.5% quantiles of the distribution of FR at certain points in time during the over the scenarios is used for this. Additionally, we look at the replacement ratio (RPR). This is the pension benefit divided by the average salary. As the accrual is set to 1.875% one should expect to have around 75% of their average salary as pension benefit after 40 years.

5.3.2 Generational Accounting and Value-based ALM

During the time loop, we store the received benefits and paid contributions for the different cohort in a matrix. In this way, at the end of the time horizon, we have a overview of the cash-flows per cohort over their lifetime. As stated before, we model an open fund. The youngest participant at $t = 0$ is 25 years old and as we model their lifetime, we need a time-loop of 75 years. In this way, the generational account of the youngest participating cohort can be completed. For scenario s the generational matrix is as follows:

$$\begin{bmatrix} C^{25}_{0,s} & C^{26}_{1,s} & C^{27}_{2,s} & \dots & C^{64}_{40,s} & B^{65}_{41,s} & \dots & B^{98}_{73,s} & B^{99}_{74,s} \\ C^{26}_{0,s} & C^{27}_{1,s} & C^{28}_{2,s} & \dots & C^{65}_{40,s} & B^{66}_{41,s} & \dots & B^{99}_{73,s} & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & & 0 & 0 \\ C^{64}_{0,s} & B^{65}_{1,s} & B^{66}_{2,s} & \dots & 0 & 0 & \dots & 0 & 0 \\ B^{65}_{0,s} & B^{66}_{1,s} & B^{67}_{2,s} & \dots & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & & 0 & 0 \\ B^{98}_{0,s} & B^{99}_{1,s} & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ B^{99}_{0,s} & 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \end{bmatrix}$$

As an illustration, the first row shows the generation that is 25 at $t = 0$, until they are 64 they pay contributions $C^x_{t,s}$ with $x = 25, \dots, 64$ and $t = 0, \dots, 40$. When they turn 65, they get benefits $B^x_{t,s}$ with $x = 65, \dots, 100$ and $t = 41, \dots, 74$. After creating the generational matrix, it is easy to calculate the generational account of each generation at time t (Ponds et al., 2003). The generational account of a person in age cohort x, is the expected value of all future cash flows under $\mathbb{Q}$-measure. For a person aged 65 or older:

$$V^x_t = E^{\mathbb{Q}}_t(GA^x_t) = E^{\mathbb{Q}}_t(\sum_{i=x}^{99}(p^x_{(t+(i-x),t})B^i_{t+(i-x)} \prod_{j=t}^{t+(i-x)}(R^f_j)^{-1})), \qquad x \geq 65 \tag{73}$$

For a person aged younger than 65:

$$V_t^x = E_t^{\mathbb{Q}}(GA_t^x) = E_t^{\mathbb{Q}}(\sum_{i=x}^{64}(p^x_{(t+(i-x),t})C^i_{t+(i-x)}\prod_{j=t}^{t+(i-x)}(R_j^f)^{-1}) \tag{74}$$

$$+\sum_{i=65}^{99}(p^x_{(t+65-x)+(i-65),t}B^i_{(t+65-x)+(i-65)}\prod_{j=t}^{(t+65-x)-(i-65)}(R_j^f)^{-1}), \qquad x \leq 65 \tag{75}$$

Note that the expectation under $\mathbb{Q}$ is the same as the average of the generational accounts if the risk neutral scenarios are used.

As we are interested in the effects of a change in pension contract, we are interested in,

$$\Delta V^x = V_t^x - V_t'^x \tag{76}$$

Here $V_t'^x$ is the generational account under a different pension contract.

6 Results

In this section, we first go through the results of the calibration of KNW model to decide on what scenarios to use for the ALM model. The second part of this section discusses the results of this ALM model.

6.1 KNW model

The KNW model results will be discussed in terms of calibration, in-sample fit and simulation to reach a decision on what is the most suitable calibration of the KNW model for economic scenario generation.

6.1.1 Calibration

First, we check whether our model is able to replicate the former calibrations. It is already shown that these old calibrations give the most optimal results (Langejan et al. (2014) and Dijsselbloem et al. (2019)). Therefore, we can check the correctness of our model by means of comparison. Table 2 shows the parameter estimates of different calibrations of the KNW model and the corresponding standard errors. The first column present the original results of the calibration by Draper (2014) on quarterly data from 1972 until 2013. We show these results for completeness and because there has not been a thorough comparison since the transition to the newly calibrated model in 2019. Moreover, the data we use in this thesis, is closer to what was available to Draper (2014) than to what was used by the Commission Parameters. The second column presents the result of the calibration by the Commission Parameters in 2019, based on quarterly data from 1999 until 2018. As we use (roughly) the same data, our estimates should be very close to what is seen in this column. The last three columns show our parameter estimation results, standard error and z-score respectively. The model is based on data from January 1999 until December 2018. In this section, we discuss the most relevant aspects of these results.

Table 2: (i) The estimates found by Draper (2014) using simulated annealing. For estimation quarterly data from 1972 until 2013 is used. (ii) The new advice of the Commission Parameters in 2019 (Dijsselbloem et al., 2019). For estimation data from 1999 until 2018 is used. (iii) The estimates obtained by our estimation procedure based on Pelsser (2019). For estimation data from 1999 until 2018 is used. The final row shows the overall fit of the model on historical data from 1999 until 2018 by means of the log-likelihood ℓ. * indicates significance at 5% level.

	(i) 1972-2013	**(ii) 1999-2018**	**(iii) 1999-2018**		
Parameter	Estimates	Estimates	Estimates	Std. Error	z-Score
Expected inflation $\pi_t = \delta_{0\pi} + \boldsymbol{\delta}_{1\pi}' \boldsymbol{X}_t$					
$\delta_{0\pi}$	2.00%	1.88%	1.89%		
$\boldsymbol{\delta}_{1\pi(1)}$	-0.63%	-0.21%	-0.23%	(0.0049)	-0.0482
$\boldsymbol{\delta}_{1\pi(2)}$	0.14%	0.00%	0.01%	(0.0048)	0.0208
Nominal interest rate $R_t = R_0 + \boldsymbol{R}_1' \boldsymbol{X}_t$					
R_0	2.40%	2.12%	2.17%		
$\boldsymbol{R}_{1(1)}$	-1.48%	-0.77%	-0.59%	(0.0017)	1.0588
$\boldsymbol{R}_{1(2)}$	0.53%	-0.08%	-0.07 %	(0.0052)	0.0192
Process real interest rate and expected inflation $\mathrm{d}\boldsymbol{X}_t = -\boldsymbol{K}\boldsymbol{X}_t \mathrm{d}t + \boldsymbol{\Sigma}_{\boldsymbol{X}} \mathrm{d}\boldsymbol{W}_t^{\mathbb{P}}$					
$\boldsymbol{\kappa}_{11}$	7.63%	6.56%	7.43%	(0.1761)	0.0494
$\boldsymbol{\kappa}_{22}$	35.25%	30.32%	30.78%	(0.2415)	0.0190
$\boldsymbol{\kappa}_{12}$	-19.00%	23.66%	19.87 %	(0.1210)	-0.3132
Realized inflation process $\frac{d\Pi_t}{\Pi_t} = \pi_t \mathrm{d}t + \boldsymbol{\sigma}_\Pi' \mathrm{d}\boldsymbol{W}_t^{\mathbb{P}}$					
$\boldsymbol{\sigma}_{\pi(1)}$	0.02%	-0.10%	-0.11%	(0.0010)	0.1000
$\boldsymbol{\sigma}_{\pi(2)}$	-0.01%	0.06%	0.08%	(0.0007)	0.2857
$\boldsymbol{\sigma}_{\pi(3)}$	0.61%	0.55%	0.55 %	(0.0005))	0.2000
Realized return process $\frac{\mathrm{d}S_t}{S_t} = (R_t + \eta_S)\mathrm{d}t + \boldsymbol{\sigma}_S' \mathrm{d}\boldsymbol{W}_t^{\mathbb{P}}$					
η_s	4.52%	4.33%	4.21%		
$\boldsymbol{\sigma}_{S(1)}$	-0.53%	-5.28%	-5.41%	(0.0090)	-0.1444
$\boldsymbol{\sigma}_{S(2)}$	-0.76%	-1.14%	2.29%	(0.0311)	1.1029
$\boldsymbol{\sigma}_{S(3)}$	-2.11%	0.05%	-0.26%	(0.0093)	-0.3333
$\boldsymbol{\sigma}_{S(4)}$	16.59%	13.07%	13.07%	(0.0037)	0.0000
Prices of risk $\boldsymbol{\Lambda}_t = \boldsymbol{\Lambda}_0 + \boldsymbol{\Lambda}_1 \boldsymbol{X}_t$					
$\boldsymbol{\Lambda}_{0(1)}$	0.176	0.673	0.189	(0.1972)	-2.4544*
$\boldsymbol{\Lambda}_{0(2)}$	0.017	0.118	0.010	(0.2981)	-0.3623
$\boldsymbol{\Lambda}_{1(1,1)}$	0.149	0.091	0.090	(0.0932)	-0.0107
$\boldsymbol{\Lambda}_{1(1,2)}$	-0.381	0.208	0.272	(0.0631)	-0.5705
$\boldsymbol{\Lambda}_{1(2,1)}$	0.089	-0.209	-0.245	(0.2873)	-0.5047
$\boldsymbol{\Lambda}_{1(2,2)}$	-0.083	-0.228	-0.194	(0.0988)	0.0506
ℓ	8976.50	10835.79	10897.62		

We start at the top of the table. The $\delta_{0\pi} = 1.89\%$ represents the long-term average expected inflation. Given Figure 2 this is not a surprising result. The average inflation in this figure is around 1.7%. This makes sense in terms of monetary policy, as the European Central Bank (ECB) enforces the inflation rate to be close but below 2.00%. The results of our Kalman filter estimation, do not deviate significantly from what was found by the Commission Parameters in 2019. $\delta_{0\pi}$ is essentially the rate at which the pension benefits lose their value. The average returns of the pension fund should be higher than this value, otherwise the pension fund cannot index the benefits of their participants. Indexation is when the pension fund increases the accrued benefit with the inflation. The higher value of $\boldsymbol{\delta}_{1\pi(1)}$ than $\boldsymbol{\delta}_{1\pi(2)}$ (in absolute terms) implies that real interest rate risk (the interpretation of the first entry of $\boldsymbol{X}_t$) has a stronger impact on the expected inflation than the expected inflation risk (the interpretation of the second entry of $\boldsymbol{X}_t$). The negative impact of $\boldsymbol{\delta}_{1\pi(1)}$ can be explained by the Fisher equation, which states that inflation $\approx$ nominal interest rate $-$ real interest rate.

The expectation of the nominal long-term money market rate (R_0) is around 2.17%. Again, not that different from what was found by the Commission Parameters in 2019. R_0 is set by a restriction which depends on $\boldsymbol{K}$, $\boldsymbol{\Lambda}_1$ and $\boldsymbol{\delta}_{1r}$ (Equation 25). Hence, because these parameters deviate a bit from the calibration of the Commission Parameters, it is not surprising that R_0 is slightly different. Compared to what was found in 2014 based on longer term data, we do see that the long-term interest rate R_0 is indeed lower for the more recent data set, which was the reason for the switch in calibration data (Dijsselbloem et al., 2019). Nevertheless, this decrease is not substantially. For pension funds, the level of this parameter is very important, as long-term interest is used to discount their liabilities. A high value of R_0 means that they need to hold less assets now to pay future (long-term) liabilities.

$\boldsymbol{\sigma}_{\pi 1}$ is almost the same as the results found by the Commission Parameters. We know for a fact that the data used to fit the customer price index is exactly equal to the data used for the Commission Parameters' calibration and therefore we should expect very similar results.

η_S is equal to the risk premium on equities. Our estimation shows it is equal to 4.21%. The result is close to what we have seen in the Commission Parameter estimation and even to the result of Draper (2014). This is a very useful result. As Koijen et al. (2010) assumed that the risk premium was stable over time, it is good to see that even based on data with different horizons, this is indeed approximately true. From $\boldsymbol{\sigma}_{S(4)} = 13.07\%$, we can see that most of the volatility in the realized return process is caused by uncertainty of the stock return. The result is equal to the result found by the Commission Parameters on two decimal places. A more outstanding result is the fact that effect of uncertainty in the instantaneous expected inflation from our estimation has a positive effect of 2.29% on the stock index level compared to -1.14%. Because of the large standard error, this result is not significant, but still one of the larger deviations in our results. Slight differences in initialization or data could explain this.

Next, as all the parameters in $\boldsymbol{K}$ of our estimation are positive, this means that $\boldsymbol{X}_t$ is a stationary time-series. This is a satisfactory result, because it is more realistic to model economic variables over a long time horizon using stationary underlying variables. In the first estimation by Draper (2014), $\boldsymbol{K}$ shows that $\boldsymbol{X}_t$ is actually not stationary (the negative value of κ_{12}). Whether $\boldsymbol{X}_t$ is stationary or not does not matter for the scenarios, but again it makes more sense to have a stationary model when looking at a long time-series of economic variables.

Finally, we have $\boldsymbol{\Lambda}_0$ and $\boldsymbol{\Lambda}_1$. $\boldsymbol{\Lambda}_{0(1)}$ is the first parameter that significantly differs from the estimates by the Commission Parameters in 2019. The unconditional price of risk with respect to the real interest rates is thus lower for our model. A noticeable fact to mention is that during the estimation process, both these parameters were highly dependent on the initialization. The model often finds local optima instead of the global optimum. This is a common problem for models with many parameters. When $\mathbf{\Lambda}_0$ and $\mathbf{\Lambda}_1$ change in signs implies that the interpretation of the entries of $\boldsymbol{X}_t$ rotates. Muns (2015) introduced some extra restrictions to prevent rotation, but the question remains whether these were enough. The model now seems to be weakly identified. This is verified by Pelsser (2019). He uses two different initialization, diffuse prior and stationary (as in this thesis), and both lead to very different $\mathbf{\Lambda}_0$ and $\mathbf{\Lambda}_1$. Luckily, because of rotation, both these initialization lead to a model with the same implications.

All in all, our results are not statistically different from what was found in 2019 by the Commission Parameters (except for $\mathbf{\Lambda}_{0(1)}$). Therefore, we can trust the accuracy and robustness of our model. We even see a slightly higher log-likelihood ℓ for our calibration. However, it should be taken into account that our data deviates a bit from what was recommended by the Commission Parameters.

We can now use the model for further analysis on more recent data with the newly introduced restriction of DNB. As can be seen in the second column of from Table 3, the restriction is equal to setting R_0 to -0.01%. For pension funds this implies that the long-term interest rates will be close to zero and negative, with as consequence that they have to discount their liabilities with low to negative rates. This leads to pension funds needing more assets now to pay off these future liabilities. As we have seen negative interest rates over the past few years, this seems plausible. However, the question remains whether this result can be obtained from the data (with or without a restriction).

Table 3 again shows the result of the calibration of the Commission Parameters in 2019 in the left column, such that we can clearly compare what DNB changed in 2021 in the second column. The only differences are R_0, η_S and $\mathbf{\Lambda}_0$. The focus should not be on Λ_0, as this vector of parameters is changed almost every new scenario set uploaded by DNB. The reason behind this is that DNB changes the initialization $\boldsymbol{X}_0$ such that the first simulated term structure is very close to the current observed term structure. For this $\mathbf{\Lambda}_0$ needs to change as well. On the other hand, η_S has to change because R_0 changes, because of Equation 27. The risk premium is now higher. This is an obvious result, because lower interest rates mean that the difference between stock returns and risk-free bond returns are now larger and thus the risk premium increases.

We now consider our calibration on more recent data to see whether we can justify $R_0 = -0.01\%$. Without the restriction, based on data until 2020, we do see a lower $R_0 = 2.04\%$. This is expected from the fact that when we increase the data span to 2020, the time span of low interest rates increases and puts more weight on the estimates. However, this R_0 is nowhere near -0.01%. The other parameters are very close to the results based on the 2019 data.

We now include the restriction into our estimation procedure and get the most right parameter estimates. Naturally R_0 is now equal to -0.01%. Besides, we see the risk premium increase again. However, we immediately observe that all the other parameters are now very distorted. The switch of sign for some parameters might have been caused by a rotation in $\boldsymbol{X}_t$. The differences are very substantial such that we cannot tell for sure.

Table 3: The left column represents the estimates found by the CPB using simulated annealing. The right columns represent the estimates for the updated model using the Kalman filter procedure of Pelsser (2019).

	(i) 1999-2018	**(ii) 1999-2018**	**(iii) 1999-2020**		**(iv) 1999-2020**	
Parameter	Estimates	Estimates	Estimates	Std. Error	Estimates	Std. Error
Expected inflation $\pi_t = \delta_{0\pi} + \boldsymbol{\delta}_{1\pi}' \boldsymbol{X}_t$.						
$\delta_{0\pi}$	1.88%	1.88%	1.89%		1.89%	
$\boldsymbol{\delta}_{1\pi(1)}$	-0.21%	-0.21%	-0.23%	(0.0031)	0.34%	(0.0019)
$\boldsymbol{\delta}_{1\pi(2)}$	0.00%	0.00%	0.01%	(0.0042)	0.15%	(0.0028)
Nominal interest rate $R_t = R_0 + \boldsymbol{R}_1' \boldsymbol{X}_t$						
R_0	2.12%	**-0.01**%	2.04%		**-0.01**%	
$\boldsymbol{R}_{1(1)}$	-0.77%	-0.77%	-0.63 %	(0.0015)	0.15%	(0.0007)
$\boldsymbol{R}_{1(2)}$	-0.08%%	-0.08%	-0.07 %	(0.0022)	-1.10%	(0.0014)
Process real interest rate and expected inflation $\mathrm{d}\boldsymbol{X}_t = -\boldsymbol{K}\boldsymbol{X}_t \mathrm{d}t + \boldsymbol{\Sigma}_{\boldsymbol{X}} \mathrm{d}\boldsymbol{W}_t^{\mathbb{P}}$						
$\boldsymbol{\kappa}_{11}$	6.56%	6.56%	7.34 %	(0.1291)	10.36%	(0.0918)
$\boldsymbol{\kappa}_{22}$	30.32%	30.32%	35.17%	(0.2501)	45.09%	(0.1876)
$\boldsymbol{\kappa}_{12}$	23.66%	23.66%	15.91 %	(0.1501)	10.81%	(0.1209)
Realized inflation process $\frac{\mathrm{d}\Pi_t}{\Pi_t} = \pi_t \mathrm{d}t + \boldsymbol{\sigma}_\Pi' \mathrm{d}\boldsymbol{W}_t^{\mathbb{P}}$						
$\boldsymbol{\sigma}_{\pi(1)}$	-0.10%	-0.10%	-0.08%	(0.0010)	0.08%	(0.0009)
$\boldsymbol{\sigma}_{\pi(2)}$	0.06%	0.06%	0.14%	(0.0007)	-0.16%	(0.0007)
$\boldsymbol{\sigma}_{\pi(3)}$	0.55%	0.55%	0.55 %	(0.0005))	0.57%	(0.0007)
Realized return process $\frac{\mathrm{d}S_t}{S_t} = (R_t + \eta_S)\mathrm{d}t + \boldsymbol{\sigma}_S' \mathrm{d}\boldsymbol{W}_t^{\mathbb{P}}$						
η_s	4.33%	**6.46**%	4.65%		**6.78**%	
$\boldsymbol{\sigma}_{S(1)}$	-5.28%	-5.28%	-3.67%	(0.0076)	3.70%	(0.0064)
$\boldsymbol{\sigma}_{S(2)}$	-1.14%	-1.14%	2.29%	(0.0252)	-5.86%	(0.0187)
$\boldsymbol{\sigma}_{S(3)}$	0.05%	0.05%	-0.03%	(0.0068)	-2.44%	(0.0032)
$\boldsymbol{\sigma}_{S(4)}$	13.07%	13.07%	13.95%	(0.0013)	14.64%	(0.0008)
Prices of risk $\boldsymbol{\Lambda}_t = \boldsymbol{\Lambda}_0 + \boldsymbol{\Lambda}_1 \boldsymbol{X}_t$						
$\boldsymbol{\Lambda}_{0(1)}$	0.428	0.673	0.159	(0.1832)	-0.168	(0.1671)
$\boldsymbol{\Lambda}_{0(2)}$	-0.052	0.118	0.129	(0.1711)	0.448	(0.1601)
$\boldsymbol{\Lambda}_{1(1,1)}$	0.091	0.091	0.089	(0.0623)	0.062	(0.0455)
$\boldsymbol{\Lambda}_{1(1,2)}$	0.208	0.208	0.186	(0.0589)	0.146	(0.0451)
$\boldsymbol{\Lambda}_{1(2,1)}$	-0.209	-0.209	-0.243	(0.2612)	-0.312	(0.1890)
$\boldsymbol{\Lambda}_{1(2,2)}$	-0.228	-0.228	-0.274	(0.0762)	-0.048	(0.0456)
ℓ	11869.63	10981.76	11878.07		11860.35	

When we look at the log-likelihood ℓ, we observe that our newly re-estimated model without restriction fit the best on the data from 1999 until 2020 with a log-likelihood $\ell = 11878.07$. This is not surprising as we used this data to calibrate the model, while the Commission Parameters version of the model was estimated on data until 2018. The model calibrated by the Commission Parameters is a very close second though, $\ell = 11869.63$. We see that setting R_0 to -0.01% after calibration leads to a surprisingly low likelihood of $\ell = 10981.76$. When setting $R_0 = -0.01\%$ as a restriction in the model, we see a higher log-likelihood at 11860.35, but the other parameters are unrecognizable. Adding restrictions to a model will always lead to a lower likelihood, therefore we should not reject the restricted model based on likelihood alone. So we perform a likelihood ratio test to see whether the difference between the restricted model and non-restricted model in terms of likelihood is significant. This test indicates that the non-restricted model fits significantly better (p-value of 0.0000).

DNB mentiones that setting the long term average interest rate in this way is only a temporary solution. Table 3 proves that it might not be wise to set the parameters after the calibration as it does not give a good fit to the underlying data. Setting the parameter during estimation does seem like a solution, since the model does not lose that much likelihood, but there are major changes in parameters to account for this restriction. Although the fit is good, this might be a problem to pension providers, because the model does not correspond with any of the recommendations of the Commission Parameters. Actually, the question is really about whether we believe that in the long-run interest rates will stay low or will increase again. The latter was the original objective of the European Central Bank. The fact that within the simulation the long-term interest rates are often not negative or low enough (as you would expect from the most recent data), can actually be a reasonable result, since these low interest rates are actually only supposed to be temporary and in the long run could increase again. Nevertheless, the signals from the Dutch pension funds did indicate that they think these low interest rates are long lasting.

6.1.2 In-sample Fit

Looking more closely at the in-sample fit in combination with some stylized facts might give us more insight in which model would be the most suitable. Therefore, we look more closely to the in-sample fit of our models. When estimating the KNW model using a Kalman filter it is assumed that we estimate the stock index and price index without error. Hence, the fit of the model can best be examined by looking at some of the stylized facts of interest rates.

First of all, the average yield curve is increasing and concave. From Table 4, we do see this is indeed the case for the data (panel C). The bond with lower maturities have a lower average yield. This also accounts for both the unrestricted and restricted model (panel A and B respectively). However, it can clearly be seen that even when imposing the negative long-term interest rate restriction, the yields are not negative. This might have been caused by the fact the model is only weakly identified and the restriction of $R_0 = -0.01\%$ is absorbed by the other parameters to still give a good fit to the data.

Secondly, the short end of the yield curve should be more volatile than the long end. For the data, this seems somewhat the case as the volatility of the 10-year bond is the lowest. Nevertheless, all the bond volatilities are relatively close together. This is the same for the calibrated model without restriction. With restriction, it actually seems that the long end of the curve is more volatile. This might have to do that the restriction in place mostly influences the estimated yields, leading to abnormal behaviour that is not consistent with the data.

Furthermore, for the data as well as the models, we observe persistence in yield dynamics. For both the models and data, we see higher auto-correlations on the yields with higher maturities, but the non-restricted model again suits better to the data.

In addition, Table 5 shows that there are high cross-correlations as expected from the stylized facts. We show that the stock return is negatively but not strongly correlated with the bonds. Especially not the long-term bonds.

Next we look at the last two stylized facts as in Section 4.4. Here we run the Campbell-Shiller regression to whether our models are able to replicate the empirical observation of Dai & Singleton (2002). For this we simulate 10.000 yield paths for the necessary maturities. Figure 7 shows that the models captures the negative coefficients that decrease with maturity. This means that spreads between long en short term bonds forecast a decline in yield on the long term bond over the life of the short-term bond. For the model without restriction we show a 95% confidence interval. This shows that the regression coefficients are significantly different from 1. One would expect the coefficients to be 1 from the Expectations Hypothesis. These bounds show that the coefficients are not significantly positive either. We also plotted the Campbell-Shiller coefficients of the data, we observe that both the unrestricted as unrestricted model is reasonably close to what is expected from the data.

Table 4: Mean, standard deviation and auto correlation of stock return and bond yield for bonds with maturity 3-month, 1-year, 2-year, 3-year,5-year and 10-year. Panel A shows these statistics as implied by the non-restricted model. Panel B shows the results for the restricted model. Panel C gives these restrictions as implied by the data.

A. KNW (iii)	Mean	St. Dev.	$\hat{\rho}_{0.25}$	$\hat{\rho}_{0.5}$	$\hat{\rho}_1$
Stock	0.0027	0.0411	0.1290	-0.0418	0.0129
3m bond	0.0154	0.0178	0.9909	0.9125	0.8112
1y bond	0.0167	0.0178	0.9912	0.9225	0.8362
2y bond	0.0182	0.0178	0.9914	0.9319	0.8598
3y bond	0.0197	0.0178	0.9915	0.9382	0.8760
5y bond	0.0225	0.0180	0.9916	0.9028	0.8956
10y bond	0.0283	0.0179	0.9920	0.9547	0.9167
B. KNW (iv)	Mean	St. Dev.	$\hat{\rho}_{0.25}$	$\hat{\rho}_{0.5}$	$\hat{\rho}_1$
Stock	0.0027	0.0411	0.1290	-0.0418	0.0129
3m bond	0.0063	0.0174	0.9942	0.9343	0.8443
1y bond	0.0074	0.0175	0.9950	0.9461	0.8741
2y bond	0.0088	0.0178	0.9955	0.9533	0.8936
3y bond	0.01037	0.0180	0.9956	0.9565	0.9030
5y bond	0.0130	0.0182	0.9957	0.9595	0.9121
10y bond	0.0192	0.0178	0.9960	0.9646	0.9243
C. Data	Mean	St. Dev.	$\hat{\rho}_{0.25}$	$\hat{\rho}_{0.5}$	$\hat{\rho}_1$
Stock	0.0027	0.0412	0.1290	-0.0418	0.0129
3m bond	0.0159	0.0178	0.9943	0.9327	0.8301
1y bond	0.0171	0.0178	0.9926	0.9297	0.8416
2y bond	0.0182	0.0178	0.9914	0.9319	0.8598
3y bond	0.0196	0.0180	0.9912	0.9358	0.8747
5y bond	0.0226	0.0180	0.9916	0.9459	0.8956
10y bond	0.0282	0.0176	0.9929	0.9265	0.9208

Table 5: Auto correlation between stock and bond return. Panel A shows the correlations between the estimated stock returns and 3-month, 1-year, 2-year, 3-year,5-year and 10-year nominal bonds as implied by the updated KNW model of this thesis without restriction. Panel B reports the correlations as implied by the updated KNW model with restriction. Panel C reports the correlations based on the data.

A. KNW (iii)	Stock	3m bond	1y bond	2y bond	3y bond	5y bond	10y bond
Stock	1	-0.19	-0.19	-0.19	-0.18	-0.17	-0.16
3m bond		1	1	0.99	0.97	0.94	0.89
1y bond			1	1	0.99	0.97	0.92
2y bond				1	1	0.98	0.95
3y bond					1	0.99	0.97
5y bond						1	0.99
10y bond							1
B. KNW restr	Stock	3m bond	1y bond	2y bond	3y bond	5y bond	10y bond
Stock	1	-0.17	-0.16	-0.15	-0.14	-0.14	-0.13
3m bond		1	0.99	0.97	0.96	0.93	0.89
1y bond			1	0.99	0.98	0.96	0.94
2y bond				1	1	0.99	0.97
3y bond					1	1	0.99
5y bond						1	0.99
10y bond							1
B. Data (iv)	Stock	3m bond	1y bond	2y bond	3y bond	5y bond	10y bond
Stock	1	-0.23	-0.20	-0.19	-0.18	-0.17	-0.16
3m bond		1	0.99	0.97	0.97	0.95	0.91
1y bond			1	1	0.99	0.97	0.93
2y bond				1	1	0.98	0.95
3y bond					1	0.99	0.97
5y bond						1	0.99
10y bond							1

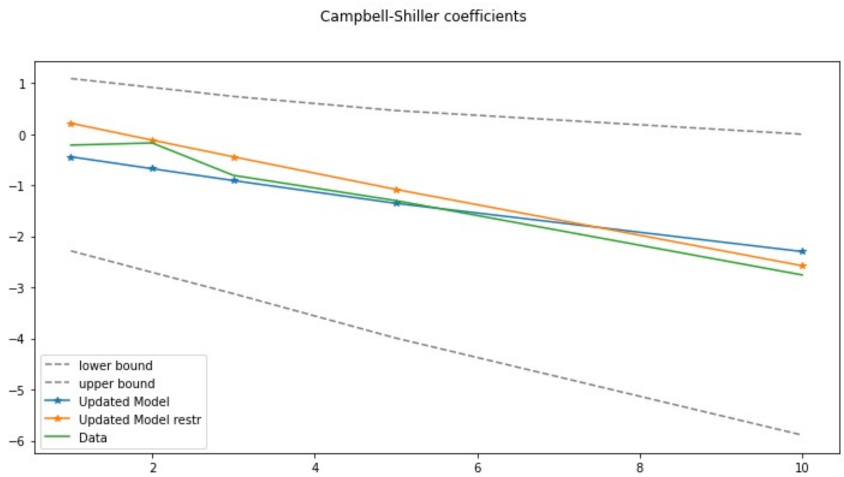
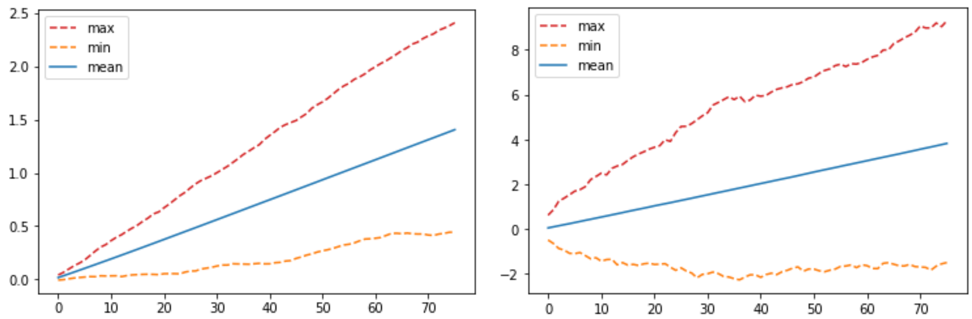


Figure 7: Campbell-Shiller coefficients of the updated model without restriction (blue), updated model with restriction (orange) and data (green).

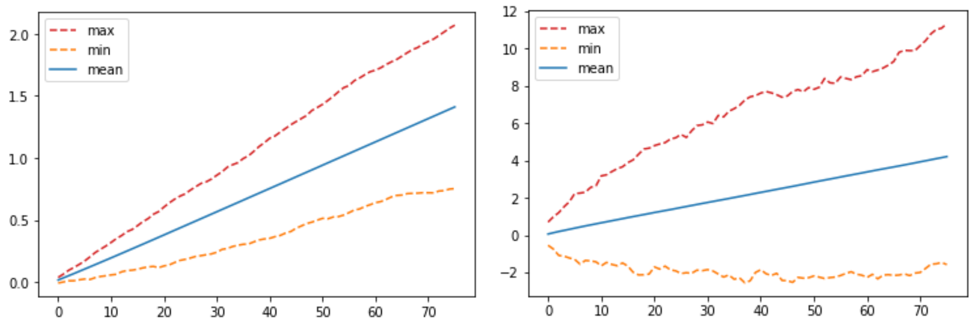
6.1.3 Simulation

Because we need to decide which model to use for the ALM study, the simulation is important as well. We do not want the differences in parameter to significantly affect our ALM simulation results. Therefore, we take a closer look at the resulting simulation paths from the models. We simulate the 10.000 economic scenarios.



(a) Price index simulation path. **(b)** Stock index simulation path.

Figure 8: Simulation paths using the parameters of the model updated in this thesis without restriction. The simulation time is 75 years. The red dotted line is the sequence of all maximum values of the price index over time. The blue line is the average over time. Note that in reality one path is not a straight line but fluctuates over time.

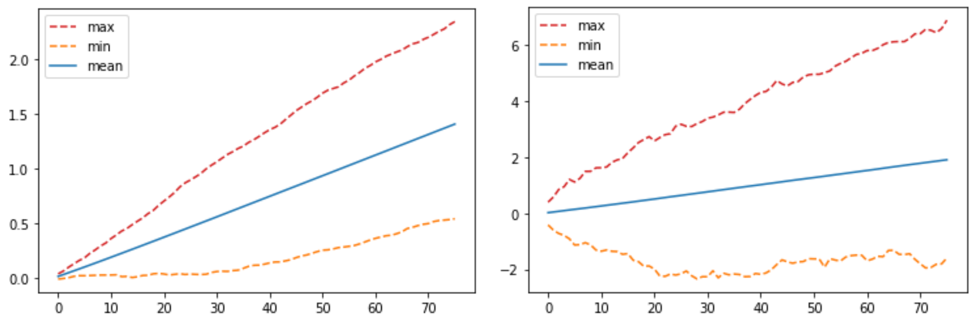


(a) Price Index simulation path. **(b)** Stock index simulation path.

Figure 9: Simulation paths using the parameters estimated of the model with restriction. The simulation time is 75 years. The red dotted line is the sequence of all maximum values of the price index over time. The blue line is the average over time. Note that in reality one path is not a straight line but fluctuates over time.

In Figure 8 we display the average simulation paths of the model without restrictions. When comparing the price index with the stock index, we observe a few differences. First of all, the price index is less volatile than the stock index, as expected from the data. The red and orange dotted line that indicate the maximum and minimum of the stock index over time for the 10.000 scenarios, deviate less from the average, than is the case for the stock index. Although on average the price index is increasing, per scenario it can either increase or decrease per year. If we now turn to Figure 9, the restricted model, we observe an almost identical figure for the price index. The stock index, however, has a higher maximum. This might have to do with a combination of the high risk premium and the more extreme value of $\boldsymbol{\sigma}_{S(4)}$. Both models seem to be very close to each other in terms of scenarios, the model with restriction does seem to lead to more extreme scenario paths though.

The restricted model has a few drawbacks. First of all, the parameters are not even close to the recommendation of the Commission Parameters. Additionally, the significantly poorer fit to the data. On the other hand, restrictions always lead to a somewhat poorer fit. It makes more sense to make a decision on what we predict to happen to the term structure in the future. This is why we looked more closely to the stylized facts of interest rates. Surprisingly, yields obtained from the restricted model were only a little bit lower. This makes us doubt whether the estimation procedure only found a local optimum or that the other parameters were able to absorb the restriction. Apparently a single parameter restriction may not have a huge effect on the final simulated interest rates, because the other parameters can still change to fit the data. This is why we have decided to use the non-restricted model for the ALM study. As discussed before, the low interest rates that we currently observe, should at some point increase again. Hence, an interest rate greater than -0.01% is not necessarily a bad result. As the risk neutral scenarios are especially important for the value-based ALM part, we take a look at the risk neutral scenarios in Figure 10. The price index remained relatively the same, but what is especially interesting is the fact that the stock index is now on average only slightly increasing. This makes sense, since we removed the risk premium of the index.



(a) Price index simulation path.

(b) Stock index simulation path.

Figure 10: Risk neutral simulation paths using DNB estimated parameters. The simulation time is 75 years. The red dotted line is the sequence of all maximum values of the price index over time. The blue line is the average over time. Note that in reality one path is not a straight line but fluctuates over time.

6.2 Asset Liability Management

Now that we have decided to use a newly calibrated KNW model without restriction but estimated on more recent data for economic scenario generation, we can turn to our ALM analysis.

6.2.1 Classical ALM analysis

We first turn to the results of our classical ALM analysis. We only show the results for 25 and 75 years in the future, because this gives a good overview of what short-term and long-term effects. The results in terms on the FR and RPR are displayed in terms of 5%, 50% and 95% quantiles. These give an indication of the worst case, median and best case scenario. Additionally, we give an indication of the reliability of these results by means of the variance. We also show the probability of having insufficient assets implying an FR lower than 100%, and of having less pension result than promised implying an RPR lower than 75%.

In Table 6, the results of unconditional DB are displayed in terms of FR and RPR. By definition, unconditional DB has great indexation results. This is made visible by the RPR being close to 0.75. Given this policy, one can expect a pension benefit as has been agreed on (75% of the average pay). However, whether this kind of generous indexation is really sustainable is still the question. Table 6, indicates low funding ratios in the worst case scenario. This is caused by the fact that even in bad economic scenarios full indexation still has to be granted. The probability of the FR to be lower than 1 is relatively high. Another interesting result is the fact that a longer horizon leads to more extreme results. Obviously, the further into one simulation, the more extreme the simulation path can get. Besides, the FR on average always increases because of small surpluses, because the expected return (based on the yield curve) is on average lower than the real return on assets, as we can see from the results of the KNW model in Section 6.1 and the fact that participants sometimes die without benefiting from their pension accrual. The FR equal to 169.72% is actually not very high considering this. This proves that on the long term unconditional indexation might not be sustainable. This is not only because of the fact that unconditional indexation is expensive, but also because the increasing benefit horizon caused by the increase life expectancy over the years. This is indeed one of the problems considered by the Dutch government which has led to a revision of the current pension scheme (see Section 1). For the horizon of 75, we even see that the indexation is no longer full for the quantile of 5%. This is because in extremely bad scenarios benefits have to be cut. In reality, situations such as that are unlikely, because the pension fund would take more measures when the FR approaches zero. Nevertheless, this shows how unsustainable unconditional indexation can be.

Table 6: Classical ALM results for unconditional DB.

Horizon	$FR^{0.05}$	$FR^{0.5}$	$FR^{0.95}$	Variance FR	$p(FR < 1)$
25	0.5642	1.4118	3.5811	0.1302	0.2949
75	0.4593	1.6972	6.1730	0.9045	0.3132
Horizon	$RPR^{0.05}$	$RPR^{0.5}$	$RPR^{0.95}$	Variance RPR	$p(RPR < 0.75)$
25	0.7547	0.7664	0.7777	0.0000	0.0040
75	0.7391	0.7544	0.7687	0.0000	0.3209

When we move to conditional indexation DB, we immediately observe an improvement in terms of FR for the worst case scenarios. The upper quantiles remain in line with what we saw previously, but the results of the worst case scenarios (for both the 25 and 75 horizon) improve a lot. It increases from 56.42% to 67.35% for a time horizon of 25, and from 45.93% to 61.17% for a time horizon of 75 years. The probability of a low FR has decreased. On the other hand, the RPR does decrease for the worst case scenarios especially. Now the RPR and the FR both absorb economic shocks. This is in line with expectation, because conditional indexation implies that the pension fund can save assets at the expense of the pension benefit in bad economic times, such that the FR does not have to decrease substantially.

Table 7: Classical ALM results for conditional DB.

Horizon	$FR^{0.05}$	$FR^{0.5}$	$FR^{0.95}$	Variance FR	$p(FR < 1)$
25	0.6735	1.4598	3.8035	0.2578	0.1951
75	0.6117	1.7660	6.1730	0.8494	0.2499
Horizon	$RPR^{0.05}$	$RPR^{0.5}$	$RPR^{0.95}$	Variance RPR	$p(RPR < 0.75)$
25	0.6084	0.7611	0.7754	0.0003	0.3281
75	0.5094	0.7364	0.7669	0.0008	0.4780

We now turn to the DC contracts. The results of both DC without compensation and with compensation are displayed in Table 8. What stands out about this table is the fact that there is no FR component anymore. This has to do with the fact that the FR does not give additional insights in these type of contracts. The pension benefit is no longer secured, and the pension fund no longer is obliged to pay out 75% of the average pay. In case of bad economic scenarios the participant's benefit is simply cut. Because of this, the FR is always equal to 1. This is why we only look at RPR.

Table 8: DC

DC without compensation					
Horizon	$RPR^{0.05}$	$RPR^{0.5}$	$RPR^{0.95}$	Variance RPR	$p(RPR < 0.75)$
25	0.3265	0.5930	0.8234	0.0015	0.7609
75	0.4195	0.7929	0.9561	0.0055	0.5010
DC with compensation					
Horizon	$RPR^{0.05}$	$RPR^{0.5}$	$RPR^{0.95}$	Variance RPR	$p(RPR < 0.75)$
25	0.5197	0.8062	0.9575	0.0054	0.2724
75	0.4195	0.9288	1.1534	0.0055	0.5012

For both contracts the RPR is a lot more spread. Among other things, this is shown by the high variance compared to DB. In the best case scenario, the RPR can even be a little bit above 100%, implying that one gets more benefit than the average pay. In reality, the pension fund will do a fiscal check, such that when too much pension benefit is accrued, the pension benefit flows party flows back to the pension fund. This, however, is difficult to model, because there is no clearly defined manner of how and when to do this as of right now.

In contrast, the RPR in the worst scenario can even drop to only 54.62% of the average pay. If we compare the two DC variants, we see that for DC without compensation the RPR rate is lower on average. This makes sense, as the participants are not compensated for the change from DB to DC. This could have to do with the fact that the older generation cannot generate high enough investment return in the short run to compensate themselves for the loss on DB accrual. This would explain why in a larger horizon the RPR are almost the same as DB with compensation.

It is difficult to formulate a conclusion based on this analysis without knowing what goals the pension fund persuades. If the pension fund is willing to take more risk, with the probability of being able to give the participants a higher benefit, DC is certainly an option. From these figures conditional DB is still a good option, since the RPR are more stable and the funding ratio is promising. Unconditional DB allows for higher accrual, but is unsustainable in the long run. In fact, we partly see the disadvantage Kortleve & Ponds (2006) mentions as a reason to introduce value-based ALM, namely that higher FRs go together with higher volatility in FR over the scenarios.

6.2.2 Value-based ALM analysis

In this section, we discuss the ALM results. This result is summarized by showing the generational gain or loss when going from the old contract to a new contract. We take conditional DB as a base contract, because it is the closest to the current Dutch pension scheme.

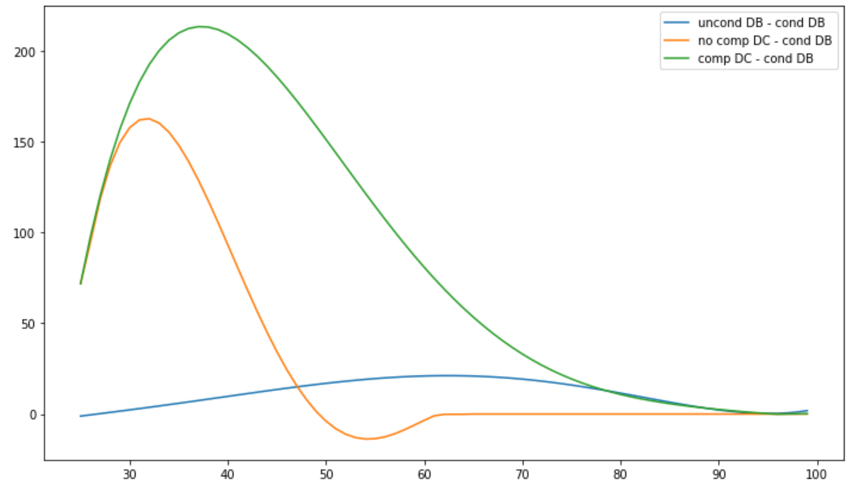


Figure 11: The age of the cohort is on the x-axis, the gain or loss per cohort is displayed on the y-axis.

From Figure 11 we observe that going from conditional DB to unconditional DB, there is gain in value for every participant in the pension fund. This is not surprising, because conditional DB means that the pension benefit does not always increase with the inflation. This is always the case for unconditional DB. This means that unconditional DB will always lead to higher pension benefits. What we do see is that the difference is not great. That probably has to do with the fact that the FR will mostly grow over the years, because of small surpluses every year, and indexation depends on this FR. This implies that in the long-run, indexation is always possible. Hence, our assumption of indexation until $t = 0$ has small to no effects on our results.

Next, we observe the value transfer of going form conditional DB to a DC pension without being compensated. We observe that this is especially profitable for participants around 30 years old. What is surprising is that the minimum lies at around 30 years old, and then for younger generation participants going to DC is slightly less profitable. This probably has to do with the fact that younger generations have a higher the chance that any missed indexation in the past is caught up. This leads to higher benefits for a DB policy for this generation. After age 65, there is no difference in value between conditional DB and DC without compensation. This is because at $t = 0$ the generation of age 65 already has accrued pension benefit and therefore they will not be affected by a new accrual scheme.

If we do compensate the generations for the switch from DB to DC, we get the green line in Figure 11. We observe that is it now profitable for all generations to have a DC pension policy. This is somewhat expected, because the expected return on stocks is higher than the accrual of a DB policy. This means that especially young generations have a lot to win from these kind of policies, because the contributions they put in each year have a long period to make high return. This is why it is less profitable for the older generations. Again, we do notice that the minimum lies around 35 years old, which we give the same reasoning as before. Understandable is the fact that DC without compensation and DC with compensation give the same value for the 25 year old cohort: they do not need to be compensated because they just started working. Another remarkable fact is the fact that the difference between DC with compensation and conditional DB for the 65 year old generation is positive. This is because we compensate this generation for the transition as they potentially miss the benefits from the ability to continue investing after retirement.

All in all, from these results, DC schemes seem to be more profitable for the pension fund's participants than DB. Especially compensated DC shows promising results in terms of the value-based ALM. What is important to note though, is the fact that the compensation might be too high in order to be paid by the pension fund alone. In reality, total compensation might be unattainable due to insufficient funding.

We have now looked into both classical ALM and value-based ALM. They both have their own insights about how different pension schemes influence the pension fund and it's participants. The classical ALM mostly gives insights about potential risk for the pension fund, while value-based ALM mostly gives insights about what value goes to which generation due to a pension scheme transition. Because of the fact that the FR is useless for DC schemes (as there are no liabilities anymore), it is understandable that a new method of pension scheme assessment had to be introduced. The added value of value-based ALM over classical ALM is the fact that is visualizes the pension funds participants: even though from classical ALM DB seems to be less risky, there are huge losses in benefits for it's participants. Moreover, we can now see which generation is especially affected by the transition. The transition to DC in The Netherlands means the disappearance of the solidarity principle, implying that all the generations now save for themselves and risk is not spread over these generations anymore. Being able to visualize these generations apart from each other is therefore very convenient.

The conclusion from the comparison between DC and DB would be that DC has high potential benefits but there are higher risks of big losses tied to it. The average value of a DB scheme is a lot lower than DC schemes for all participants if all participants are compensated. Nevertheless, it is questionable if compensation is affordable for the pension funds. DB does seem a good and stable pension scheme with lower risk. However, the change to DC is inevitable, because of the negative effects of the increasing life expectancy and low interest rates as mentioned in Section 1 and seen in these results.

7 Conclusion & Discussion

In this thesis, we answered the following question: To what extent does value-based ALM lead to different conclusions in terms of pension policies compared to classical ALM, based on scenarios obtained from the KNW model and more specifically, how does DB compare to DC? We set up a value-based ALM model to show the added value of this model compared to classical ALM. Additionally, we carefully examined the KNW model to choose a suitable calibration to generate economic scenarios. More specifically, we discussed a new restriction introduced by DNB to see if it made sense to incorporate it in our ALM study.

First, we re-estimated the KNW model. Here we have shown that the estimation method by Pelsser (2019) is a correct and robust estimation method for the KNW model by comparing it to what has been found in the past. More importantly, we show that adding the newly introduced restriction of low long-term interest rates into the model leads to significantly different parameters. Since the model is weakly identified, the new restriction is absorbed by the other parameters to fit better to the data. Therefore, the restriction did not have a huge effect on the simulation. Nevertheless, the Commission Parameters recommends very different parameters. This in combination with the fact that the newly restricted model did not fit the current term structure well, showed that it was not suitable for an ALM study. A newly re-estimated model on more recent data, but without an interest rate restriction, turned out to be a good solution. It led to a model with lower interest rates and higher risk premium (as DNB requires) but still fitted the data very well. As the parameters are still reasonably similar to the calibration of the Commission Parameters in 2019, this model was suitable for the ALM study.

The first limitation of our KNW model is the fact that the data used differs a bit from what was originally used by the Commission Parameters. Besides, we have used monthly data, while the Commission Parameters uses quarterly data. This could have led to small differences in the estimation results and worse in-sample fit of the model estimated by the Commission Parameters and DNB. It would be interesting to conduct a similar study but with the recommended data.

Furthermore, DNB wants lower long-term interest rates. In this thesis, we estimated two new models that potentially fix this problem: a restriction and more recent data. Although the latter model only decreased the interest rate a little. This effect is probably due to the low interest rates after the financial crisis that now make up a large part of our data set. Hence, another solution would be to use an even smaller time-span for estimation. From the start of 2020 the long-term interest rates are actually negative, estimating from an even shorter period would give this period more weight in the estimation process, potentially leading to lower interest rates. The Commission Parameters would need to use monthly data in this case though, as there would otherwise be insufficient data to get reliable results.

In the second part of this thesis, we conducted an ALM study based around the Future Pensions Act. The Future Pensions Act implies a transition from DB to DC. Now that the plan has been designed, it is the question how it will be put into practise. Obviously, this new DC scheme should improve the (long-term) solvency of the fund without leading to unfair distribution over the generations. Using value-based ALM in combination with classical ALM, we have shown that there is not one plan that has all the advantages and none of the disadvantages. Nonetheless, in this study we have presented the extra dimension that value-based ALM adds to ALM studies, namely the visualisation the pension fund's participants in terms of intergenerational transfers. Currently the intergenerational effects are only qualitatively evaluated (if at all) (Lekniute, 2011).

Value-based models will help with the decision by the following implications that we have shown in this thesis. First of all, when going from DB to DC, especially the participants need to be compensated. We have seen that especially the participants from 50 to 65 miss out on benefits when there is no compensation. Even so, from the classical analysis we saw that the DC can be more risky, because there is no security in pension benefit. The probability of having a low pension benefit is higher. On the other hand, in best case scenario the participants pension benefit will be higher. Therefore, we can conclude that value-based ALM does add an extra dimension to the classical analysis. It leads to more insight in generational fairness as it considers its participants on individual level. We have seen that high funding ratios do not necessarily mean a fair generational balance: DB gives high funding ratios, but the participants, especially the younger generations, lose value. This could start discussions about "fairness" in a pension setting. All in all, the real added value of value-based ALM is when combining it with classical ALM. This way, a complete picture can be painted of what the risks are for the pension fund and the different generations.

Conversely, as of right now, the value-based ALM tool has a few limitations. Currently, the tool assumes that there are no intragenerational differences, e.g. everyone in one age cohort is exactly the same (except for gender). As stated in Section 1, these intragenerational differences have caused the revision of the current pension system: most people are not employed in the same industry over their life-time, this for example leads to differences in accrual.

Moreover, in this study, the initialization is close to an average Dutch pension fund. For instance, it could be interesting to look at how these results differ for 'green' ('grey') pension funds, which mostly have young (old) participants, or rich versus old pension funds.

Besides, we saw how the view on the low interest rates differs. The interest rates have a huge influence on the results on an ALM study. Pension funds use interest rates to discount their liabilities, lower interest would lead to worse solvency. It would be interesting to look into how interest rate models affect an ALM model.

Finally, after the introduction of the Future Pensions Act, there is probably going to be a new pension system after some time. Here we looked at what would happen for the next 75 years if the new pension scheme was stable over those years. In reality, the new scheme is probably not going to last for that long. There are some studies that look at 'closed' pension funds (Lekniute, 2011), that for example cease to exist after 25 years. This gives more insight in what it would look like if the new scheme only held for the next 25 years.

However, the value-based ALM model is a flexible model that is able to incorporate different time horizons and initializations. Therefore, future research on these small drawbacks should be no problem. This study has shown how value-based ALM can lead to different conclusions about a pension scheme, which shows the importance of combining it with classical ALM in future analysis. It makes it possible to not only discuss financial risk of the pension fund but also intergenerational fairness.

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