Erasmus University Rotterdam

Erasmus School of Economics

Master Thesis Econometrics and Management Science

Combining Copula Density Forecasts for Hedging Cryptocurrencies using Bitcoin Futures

Name Student	*Student ID Number*	*Supervisor*	*Second Assessor*
Dylan J. van Gemeren	477690	Anastasija Tetereva	Chen Zhou

Date: August 16, 2021

Abstract

Since their introduction, cryptocurrencies have become popular investment products and have recently attracted the interest of large firms. However, cryptocurrencies are known to be very volatile and large losses can be incurred. With the recent launch of Bitcoin futures in December 2017 it is possible to hedge against adverse price movements. This paper evaluates the daily hedge effectiveness in terms of Variance, Semivariance, Value-at-Risk and Expected Shortfall of the Bitcoin futures for the Bitcoin as well as the Ethereum, Ripple, Cardano and Litecoin using several well-known bivariate copulas. Moreover, this paper proposes to combine the density forecasts of different copulas to construct more robust hedges. Whereas the individual copula methods are generally unable to outperform the OLS hedge and DCC-GARCH hedge, the combined density forecast hedging methods are able to compete with and outperform these benchmark hedges for alternative cryptocurrencies.

Contents

1 Introduction

The Bitcoin, developed by Satoshi Nakamoto back in 2009, was the first cryptocurrency to be created. According to the Oxford Dictionaries Online, a cryptocurrency is defined as “a digital currency in which encryption techniques are used to regulate the generation of units of currency and verify the transfer of funds, operating independently of a central bank”. This contrasts regular currencies such as the US dollar and the euro which can be printed by central banks at any time. One of the main advantages of cryptocurrencies over regular currencies are the low transactions costs as no intermediary institutions are involved.

Despite the original intentions of cryptocurrencies, the major driver of their increasingly growing popularity has been their performance as alternative investments. According to Šurda (2014), the Bitcoin was worth \$0.01 when it was first used, and has peaked at a value of about \$60,000 at the start of 2021. To put this in perspective, this equals an average annual return of about 250%. And the future of the Bitcoin and other cryptocurrencies is looking bright, with large firms like Tesla planning to accept the Bitcoin as a valid method of payment in the future if the Bitcoin gets more energy efficient.

However, one of the drawbacks of cryptocurrencies is their unstable price nature due to the lack of regulation and relatively small volumes, which causes even small buys and sells to influence the price. Many papers have confirmed that the prices of cryptocurrencies are very volatile. Therefore holding cryptocurrencies involves a lot of risk and might lead to large losses. To reduce this risk, cryptocurrency owners can hedge their positions by taking positions in other assets such as Bitcoin futures. This might be particularly interesting for those firms who wish to accept cryptocurrencies as method of payment, but do not want to be exposed to the exchange risk. For that reason it is important to correctly calculate the optimal hedge ratio, which depends on the amount of comovement between the assets.

This paper focuses on estimating those hedge ratios that minimize the Expected Shortfall of holding the Bitcoin, Ethereum, Ripple, Cardano and Litecoin using Bitcoin futures. To accurately capture the dependence structure of the cryptocurrencies and the Bitcoin future, this paper investigates the hedging potential of different copula methods. Moreover, this study examines whether a combination of density forecasts from different copulas can improve the hedging performance. In addition, this paper explores whether changes in the dependence structure are present in the data.

At the time of writing, the only cryptocurrency with its own futures derivatives and sufficient amount of data is the Bitcoin. Bitcoin futures were introduced on 18 December 2017 by the CME as a way to hedge against price movements of the Bitcoin. However, as the interconnectedness between different cryptocurrencies is relatively large, Bitcoin futures might also be useful for hedging alternative cryptocurrencies. Therefore, the data considered in this paper consists of daily price data on five of the largest cryptocurrencies: Bitcoin, Ethereum, Ripple, Cardano, Litecoin, as well as price data on Bitcoin futures. The data set ranges from 18 December 2017 to 30 April 2021 and includes 879 observations.

A great amount of papers has investigated similar hedging problems for other applications over the years. For example, Lien et al. (2002) use futures to hedge several currencies, commodities and stock indices. However, many of these papers simply assume a multivariate Normal or Student-t distribution for modelling the dependence across the assets, even though they lack the ability to cope with some important data characteristics. The Normal distribution is unable to capture tail dependence and both the Normal distribution and the Student-t distribution assume symmetry in the data. For that reason, copulas have gained a lot of popularity in the recent years. For instance, Awudu et al. (2016), Chen et al. (2016) and Sukcharoen & Leatham (2017) adopt copulas for constructing optimal hedge strategies for ethanol processors, grain processors and oil refineries respectively. Nonetheless, no research has been conducted to investigate the application of copulas to construct hedge strategies for cryptocurrencies. Moreover, the literature on hedging cryptocurrencies using Bitcoin futures is rather limited and just consists of Corbet et al. (2018), who attempt to hedge the Bitcoin using the simple OLS hedge, and Sebastião & Godinho (2020), who use the DCC-GARCH model for hedging several cryptocurrencies. Therefore, this leaves a gap in the literature for more advanced copula models.

In addition, the majority of the literature on hedging problems considers minimum-Variance as objective. Yet, it is argued that the Variance is not an appropriate risk measure as it not only takes the downside risk into account, but the upside risk as well. Therefore, several papers have proposed the use of alternative risk measures such as Value-at-Risk (VaR), Expected Shortfall (ES) and Lower Partial Moments (LPM). For instance, Sukcharoen & Leatham (2017) construct and compare minimum-LPM, minimum-VaR and minimum-ES hedges. Moreover, the Expected Shortfall in particular has received more attention over the past few years as it exhibits some nice properties. For that reason, this paper focuses on estimating those hedge ratios that minimize the Expected Shortfall.

Finally, since the work of Bates & Granger (1969), it is well-known in financial literature that combinations of forecasts generally perform better than individual forecasts. For example, Diebold et al. (2021) construct mixtures of density forecasts for the inflation based on the out-of-sample likelihood, and show that these combined density forecasts outperform the individual density forecasts. Nonetheless, to the best of the author's knowledge this idea has not yet been applied to copulas. Therefore, this paper proposes to combine copula density forecasts in order to calculate more robust hedge strategies. Moreover, this paper considers several different methods for determining the combination weights suggested by Diebold et al. (2021), which are an equal weighting scheme, a simplex weighting scheme and a best 3 average weighting scheme, where the latter two methods assign weights based on past out-of-sample fit.

This paper finds that Bitcoin futures are effective hedging tools for the Bitcoin and are able to reduce the risk by over 40%. This is in line with the findings of Sebastião & Godinho (2020). However, in contrast to Sebastião & Godinho (2020), this paper also finds that Bitcoin futures are effective for reducing the tail risk of alternative cryptocurrencies such as Ethereum, Ripple, Cardano and Litecoin by 10% to 40%.

Moreover, the hedging results show that although individual copula hedges are generally unable to outperform the benchmark OLS and DCC-GARCH hedges for the Bitcoin and alternative cryptocurrencies, combining the density forecasts from the different copulas results in hedge strategies that do outperform these benchmarks for alternative cryptocurrencies. This illustrates the robustness of combined density forecasts, which in return benefits the effectiveness of the hedge. Moreover, the results show combining density forecasts with weights based on past out-of-sample performance can be beneficial for hedging purposes. In addition, these methods are able to incorporate different copula densities in different periods and therefore are well able to detect changes in the dependence structure. However, these hedges do generally come with slightly higher transaction costs as they require more rebalancing.

The remainder of the thesis is structured as follows. Section 2 gives an overview on the literature on copulas, optimal hedge strategies and cryptocurrencies. In Section 3 and Section 4 the data and the methods used in this paper are described. Section 5 shows the results of the different models. Finally, Section 6 provides a discussion and conclusion.

2 Literature

Modelling the dependence structure for a set of variables has been a topic of interest in many papers through history. An intuitive way of modelling the dependence structure is by assuming some multivariate distribution such as the multivariate Normal distribution or multivariate Student-t distribution. These distributions assume dependence between the different variables through the regular Pearson correlation (1895) and an additional second parameter for the Student-t distribution. However, Embrechts et al. (1999) argue that using correlation as measure of dependence might give an inaccurate view on the actual dependence structure whenever the true distribution is non-normal. For example, the multivariate Normal distribution does not exhibit tail dependence which is often present in the data, and both the multivariate Normal and multivariate Student-t distribution assume symmetry in the data. Therefore, they advocate the use of alternative dependence measures, such as the Kendall's τ or Spearman's ρ, which measure rank correlation in combination with copulas to model the dependence between different variables.

The idea of copulas was introduced by Sklar (1959) as a way to model dependence in the data for higher dimensions. A copula is defined as a multivariate distribution function with uniform marginals that links the different variables. According to Sklar's theorem, each joint distribution function can be decomposed into some marginal distribution functions and a copula. In contrast to the multivariate Normal and multivariate Student-t distributions, some copulas can incorporate features such as tail dependence and asymmetry. This flexibility makes copulas an interesting and useful tool for modelling different series.

Over time, different copula methods have been proposed. Some of the most popular copulas according to Dorey & Joubert (2005) are the Gaussian copula, the Student-t copula, and the Archimedean Gumbel and Clayton copulas. The Gaussian copula and the Student-t copula are derived from their distributional counterparts and therefore suffer from the same drawbacks. The Archimedean copulas on the other hand, can allow for both tail dependence and asymmetry. Additionally, in contrast to the elliptical copulas they enjoy the benefit of having an explicit formula. Estimation of these copulas has been widely researched by Scaillet & Fermanian (2002), Chen et al. (2006), Genest et al. (1995) and others, and goodness-of-fit tests have been proposed by Chen & Fan (2005) and Genest et al. (2006). Joe (1997) and Nelsen (2007) give a good overview on the different copulas in the literature.

One particular area of research that has some special interest for copulas is the field of financial risk management as discussed by Embrechts et al. (2002) and Junker & May (2005). A popular topic in financial risk management is constructing hedge strategies. The reason is that in order to construct an efficient hedge strategy it is important to know how the assets co-behave and react to different news. Many papers have evaluated hedging strategies by applying different specifications of the GARCH model introduced by Bollerslev (1986) and Engle (1982). These models take into account volatility clustering which is a well-known feature for financial returns. For example, Lien et al. (2002) and Chang et al. (2011) use the constant conditional correlation GARCH from Bollerslev (1990) and dynamic conditional correlation GARCH model from Engle (2002) and Tse & Tsui (2002) to model the dependence structure and to construct hedge strategies with futures. However, these models generally assume a multivariate Normal or Student-t distribution for the errors. As discussed above, these distributions might not be appropriate for modelling dependencies in financial return data.

To overcome this problem, Patton (2006) proposes dynamic copula GARCH models, which relax this distributional assumption and allow for greater flexibility. Similar to the dynamic conditional correlation GARCH model, dynamic copula GARCH models can incorporate changing dependence structures of the assets. That is, the parameters are allowed to evolve over time according to some evolution equation. Hsu et al. (2008), Van den Goorbergh et al. (2005), Lai et al. (2009) and others use these methods to construct hedge strategies. They find that dynamic hedging strategies using copula GARCH are generally more effective than those based on regular GARCH models and can greatly reduce risk.

Next to choosing the right method for finding the dependence relation between several assets, it is important to use the right criteria for calculating the hedging ratios. As hedges are constructed in order to reduce the amount of risk for the holder, the objective is usually to minimize with respect to a certain risk measure. In the literature on constructing hedge strategies, the Variance is widely used as objective. This includes some of the above-mentioned papers, as well as for example Haigh & Holt (2002) and Ji & Fan (2011), who construct hedge strategies for oil refineries. However, it is often argued that the Variance is not a good risk measure as it not only takes the downside risk into account, but the upside risk as well.

For that reason, risk measures that solely focus on the downside risk might be more suited for the task. For example, Chen et al. (2016) and Awudu et al. (2016) construct hedge strategies by minimizing the Value-at-Risk (VaR). Power & Vedenov (2010) minimize Lower Partial Moments (LPM) introduced by Fishburn (1977) and compare the results to those of a minimum-Variance (MV) hedge. They find that the MV approach leads to overhedging compared to the minimum LPM hedge. Sukcharoen & Leatham (2017) construct optimal hedge strategies for several risk minimizing objectives independently, including LPM, VaR and Expected Shortfall (ES). They find consistent hedging results across the different hedging objectives.

A relatively new field of research in finance is the use of cryptocurrencies. Cryptocurrencies bear the advantage over regular currencies of little regulatory rules and low transaction costs. The popularity of cryptocurrencies however is mainly driven by their exceptional performance as investments. Moreover, Bouri et al. (2017) and Wang et al. (2019) argue that cryptocurrencies are a safe haven during financial turmoils. Due to the success of the Bitcoin and other cryptocurrencies such as Ethereum, the cryptocurrency market has been growing rapidly over the past few years and more and more large companies are getting involved. For instance, Tesla bought $1.5 billion worth of bitcoin at the start of 2021 and is planning to accept Bitcoin as matter of payment if it achieves to become more energy efficient, see Kovach (2021). However, the prices of cryptocurrencies are generally very volatile, and holding them might lead to large losses, as found by Ardia et al. (2019) and Conrad et al. (2018). Additionally, other features of data on cryptocurrencies are revealed by Osterrieder & Lorenz (2017), Phillip et al. (2018), Alvarez-Ramirez et al. (2018) who find that returns of cryptocurrencies are non-normally distributed, have asymmetric correlations and volatility clustering, and exhibit heavy-tail behaviour.

Therefore, hedging cryptocurrencies might become an interesting and important topic in finance. The hedging capabilities of the Bitcoin and other cryptocurrencies on commodities such as oil have been shown in Dyhrberg (2016) and Okorie & Lin (2020) who apply regular GARCH specifications. Nonetheless, little research has yet been done regarding the hedging of Bitcoin and other cryptocurrencies using Bitcoin futures, which were recently introduced in December 2017 by the CME. Corbet et al. (2018) use Bitcoin futures to hedge price movements of the Bitcoin using OLS, but find that this actually increases the portfolio's volatility. Furthermore, Sebastião & Godinho (2020) calculate the optimal hedge ratios of Bitcoin futures for both Bitcoin and other cryptocurrencies using regular GARCH specifications. They show that these futures can be effective tools for hedging price risk for the Bitcoin as well as for other cryptocurrencies. However, they also find that the tail risk of the alternative cryptocurrencies increases when using Bitcoin futures. All in all, this poses a gap in the literature as at the moment of writing there exist no papers that consider copula methods for this particular problem. For that reason, it is interesting to investigate whether copula methods can improve the hedging performance of Bitcoin futures for different cryptocurrencies.

Finally, ever since the work of Bates & Granger (1969) it is well known in financial literature that combining different forecasts might lead to better and more robust forecasts. That is, whereas individual forecasts can be very sensitive to certain behaviour and may make some larger errors, a combination of different forecasts is likely to average out these errors. Often even the simple average forecast proves to be hard to outperform. However, whereas most forecasting papers are interested in point forecasts, the goal of this paper is to reduce risk which depends on the lower tail of the distribution and therefore density forecasts should be considered

Diebold et al. (2021) propose several regularized combinations of density forecasts based on density score functions such as the log-score function by Good (1992) and Winkler & Murphy (1968). They compare the combined density forecasts with the individual density forecasts and find that combined density forecasts outperform even the best individual density forecast. Therefore, it might be interesting to explore whether combining the density forecasts from different copula models can also improve the hedging performance.

All in all, the contribution of this paper to the literature is twofold. First of all, this paper is the first to consider copulas for hedging cryptocurrencies with Bitcoin futures. Secondly, this paper combines density forecasts resulting from different copulas and investigates whether this can improve hedging performance.

3 Data

The data considered in this paper consists of daily price data on both cryptocurrencies and CME Bitcoin futures (BTF) and is obtained from YahooFinance. All prices are in US dollar. The cryptocurrencies are traded 24/7, and the spot prices are taken as the closing prices at 20:00 EDT. Bitcoin futures were introduced by the CME on 18 December 2017 and are traded from Sunday to Friday 18:00 to 17:00 EDT with an one-hour break each day between 17:00 EDT to 18:00 EDT. This poses a time difference between the spot and futures closing prices of 3 hours. Similar to Sebastião & Godinho (2020) and Zhang & Choudhry (2015), the futures price series is constructed from monthly future contracts which are rolled over at maturity at the start of each month. Five of the largest cryptocurrencies in terms of market capitalization as of 30 April 2021 that already existed on 18 December 2017 are chosen. From large to small, this includes the Bitcoin (BTC), Ethereum (ETH), Ripple (XRP), Cardano (ADA), and Litecoin (LTC).

The price movements over time are shown in Figure 1, where the log-prices are displayed. Obviously, the Bitcoin has by far the highest value as its price has skyrocketed since its introduction back in 2009. Furthermore, in this figure already some comovement can be spotted. For example, in general the prices of the cryptocurrencies have been declining in 2018. Hale et al. (2018) and others argue that this crash might have been the result of the introduction of the Bitcoin futures, as it allowed investors to bet against the Bitcoin more easily. However, this reasoning is still under debate as for example Hattori & Ishida (2021) reject this hypothesis. After 2018, most cryptocurrencies have been increasing in value again. Moreover, the outbreak of the corona virus at the start of 2020 seems to have boosted the popularity of the different cryptocurrencies even more. This is likely due to people being no longer able to spend their money elsewhere, and hence are more willing to get involved with investing in cryptocurrencies. Nonetheless, the Ripple and the Litecoin still have not yet reached their peak value from the start of 2018.

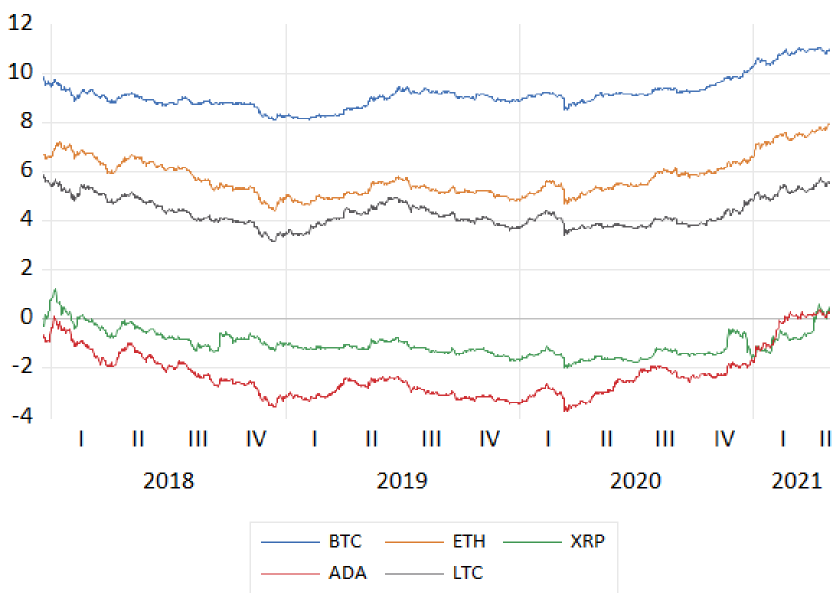


Figure 1: Log-prices of the Bitcoin, Ethereum, Ripple, Cardano and Litecoin for the period December 2017 - April 2021

As the Bitcoin futures are not traded during weekends, these observations are omitted from the data. Missing observations during trading days are filled using the next day opening prices if available, and are linearly interpolated otherwise, similar to Sebastião & Godinho (2020). The log-returns are calculated for each asset $i \in \{BTC, ETH, XRP, ADA, LTC, BTF\}$ at each observation $t = 1, ..., T$ as $r_{i,t} = \log(P_{i,t}) - \log(P_{i,t-1})$, where $P_{i,t}$ denotes the price.

The final data set contains $T = 879$ observations on the log-returns. The log-returns of the cryptocurrencies and the Bitcoin futures over time are displayed in Figure 2. As expected, the figure shows that the returns of Bitcoin and Bitcoin futures are very much related over time with returns often moving in the same direction. In addition there seems to be some volatility clustering present in the data, which was also found by Phillip et al. (2018). In particular, at the start of 2018 just after the introduction of the Bitcoin

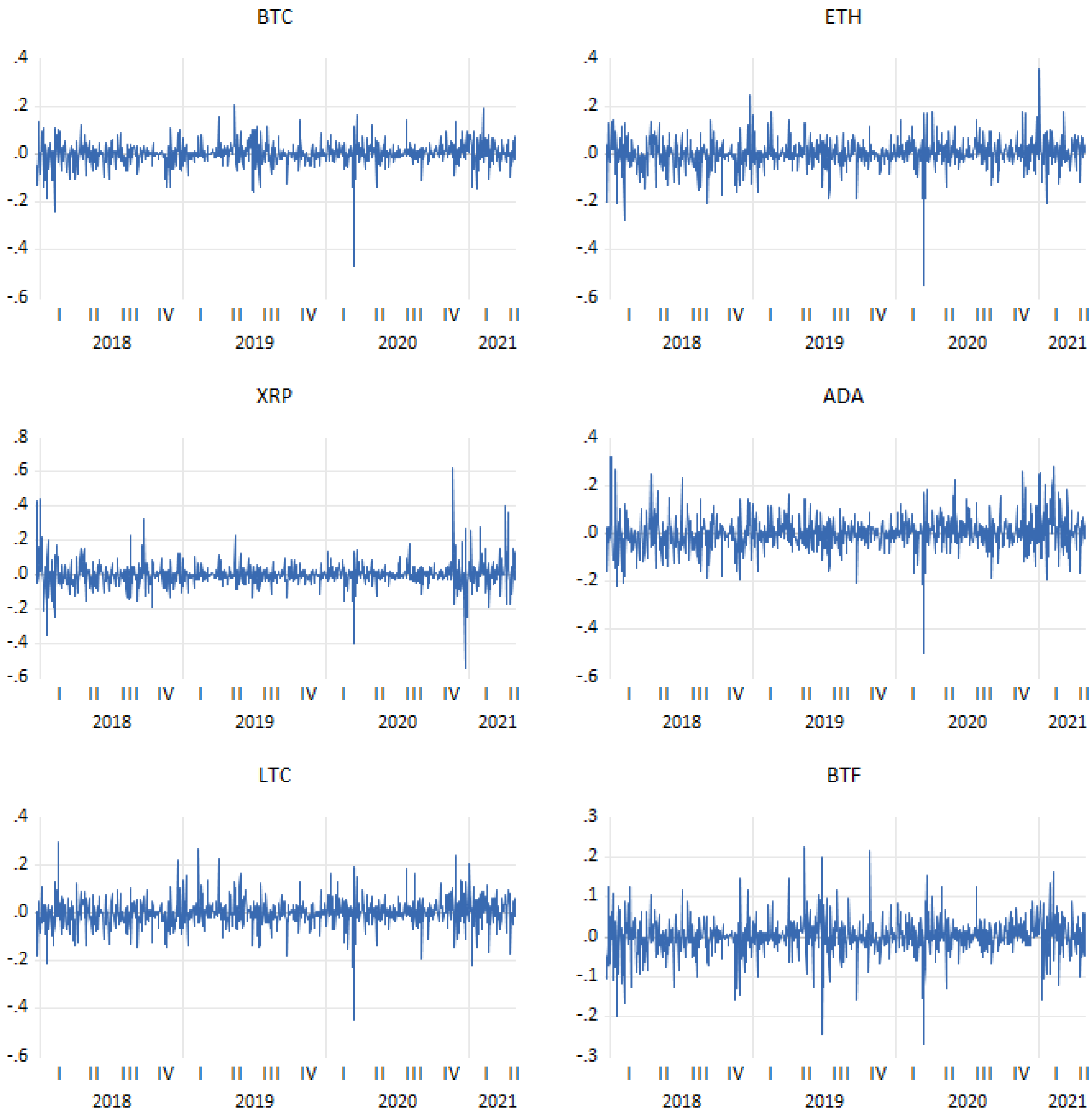


Figure 2: Log-returns of the Bitcoin, Ethereum, Ripple, Cardano, Litecoin and Bitcoin futures for the period December 2017 - April 2021

futures and at the end of 2020 with the corona pandemic, the log-returns seem to be more volatile than during most other periods. Furthermore there are a few large spikes visible which indicate huge profits and losses of sometimes over 20%. One observation at the start of 2020 especially stands out with a log-return of about -0.45. The timing of this observation suggests that it occurred during the start of the corona pandemic when there was an extreme amount of uncertainty about the future state of the economy.

As for the other cryptocurrencies, the returns generally seem to be in the same direction as well. Moreover, the periods with higher volatility coincide with those of the Bitcoin, with relatively high volatility at the start of 2018 and at the end of 2020. The Ripple in particular has some huge profits and losses at the end of 2020, with a positive log-return of about 0.6 followed by a negative log-return of almost -0.6 a few days later. Furthermore, similar to the Bitcoin, all alternative cryptocurrencies show a large negative spike at the start of the corona pandemic, which indicate the large losses at this date. It can also be seen from the graph that the other cryptocurrencies are generally somewhat more volatile than the Bitcoin with larger positive and negative returns.

Some summary statistics on the log-returns are shown in Table 1. The average log-returns are relatively low, since daily returns are considered. However, looking at the maximum and minimum log-returns, there are quite a few extreme returns with log-returns as high as 0.627 for the Ripple and as low as -0.551 for the Ethereum and the Ripple. The volatility of the returns is further indicated by the relatively high standard deviations which are between 0.04 to 0.08. As suggested by Figure 2, the volatility of the alternative cryptocurrencies is indeed somewhat larger than that of the Bitcoin. This makes sense, as the Bitcoin was the first cryptocurrency to be introduced and therefore has had the most time to stabilize.

Table 1: Summary statistics of the log-returns of the Bitcoin, Ethereum, Ripple, Cardano, Litecoin and Bitcoin futures for the period December 2017 - April 2021

Asset	BTC	ETH	XRP	ADA	LTC	BTF
Mean	0.001	0.001	0.001	0.001	0.000	0.001
Maximum	0.203	0.354	0.627	0.322	0.291	0.222
Minimum	-0.465	-0.551	-0.551	-0.504	-0.449	-0.268
Std. Dev.	0.047	0.061	0.076	0.073	0.060	0.047
Skewness	-1.194	-0.789	0.846	0.134	-0.268	-0.330
Kurtosis	15.645	12.923	16.984	7.629	8.763	7.782
JB Stat.	6065	3698	7267	787	1227	853

Furthermore, similar to Osterrieder & Lorenz (2017), non-normality of the log-returns is clearly shown by the Jarque-Bera test statistic, as $JB = \frac{T}{6}[\hat{S}^2 + \frac{1}{4}(\hat{K} - 3)^2] \sim \chi^2_2$ has a critical value of about 6 at 5% significance level. This is mainly due to all log-returns exhibiting excess kurtosis indicating fat tails. In particular the log-returns of the Bitcoin and Ripple have fat tails with kurtosis of 15.645 and 16.984 respectively, which is in line with the findings of Alvarez-Ramirez et al. (2018). Skewness poses less of a problem in the data, as only the Bitcoin, Ethereum and Ripple are moderately skewed. Histograms of the log-returns are shown in Appendix 8.1.

The correlations and rank correlations between the daily log-returns are displayed in Table 2. Obviously, the (rank) correlation between the Bitcoin and its derivative, the Bitcoin futures, is the highest. The table confirms that the price movements of the different cryptocurrencies are very much related as they are relatively high and positive. For example, the Bitcoin has correlations with the other cryptocurrencies between 0.531 and 0.794 and similarly the Kendall's τ is between 0.487 and 0.611. This is line with the conclusions from Figure 1 and is presumably because the Bitcoin is a dominant factor in the cryptocurrency market and therefore impacts the other cryptocurrencies significantly. As for the Bitcoin futures, the observations are similar, but with slightly lower (rank) correlations. This means that the futures are somewhat less related to price changes of different cryptocurrencies. This makes sense as with futures there is the opportunity for the prices to correct before maturity. However, even then the comovement with the other cryptocurrencies is relatively high with correlations between 0.424 and 0.660 and Kendall's τ between 0.386 and 0.470. This suggests that the Bitcoin futures are likely to also be useful for hedging against adverse price movements of alternative cryptocurrencies.

Table 2: Correlations (upper-right) and Kendall's τ (lower-left) of the log-returns of the Bitcoin, Ethereum, Ripple, Cardano, Litecoin and Bitcoin futures over the period December 2017 - April 2021

Asset	BTC	ETH	XRP	ADA	LTC	BTF
BTC	1.000	0.793	0.531	0.689	0.794	0.840
ETH	0.611	1.000	0.603	0.761	0.814	0.636
XRP	0.487	0.578	1.000	0.648	0.585	0.424
ADA	0.518	0.595	0.564	1.000	0.710	0.554
LTC	0.591	0.630	0.536	0.558	1.000	0.660
BTF	0.647	0.470	0.386	0.395	0.464	1.000

4 Methodology

This section discusses the methods for modelling the dependence of the assets and constructing the hedge strategies. For the purpose of incorporating possible time dependencies a rolling window of $W = 200$ observations is used, which is standard in literature. This section kicks off by specifying the GARCH model which is used to model serial dependence in the data in Section 4.1. Then different copula methods are discussed in Section 4.2. The optimization of the hedge ratios is explained in Section 4.3, and the combination methods are in Section 4.4. Finally, performance measures and benchmark models are discussed in Section 4.5.

4.1 GARCH Model

Before modelling the dependence among the different assets, the log-returns of the individual series need to be cleaned of time dependencies. A popular way of fitting time series (log-)return data in the literature is by using the Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) model of Engle (1982) and Bollerslev (1986). This model is able to capture volatility clustering, which is a well-known feature in financial data. Volatility clustering refers to the observation that extreme returns in one period are followed relatively often by extreme returns in subsequent periods and vice versa. This behaviour is present for the log-returns of the different cryptocurrencies as discussed in Section 3 and was also found by Phillip et al. (2018). The regular GARCH model assumes that this effect is symmetric meaning that high positive returns have just as much of an impact on the volatility as high negative returns. The GJR-GARCH(1,1) by Glosten et al. (1993) on the other hand is able to incorporate this so-called leverage effect. However, a quick estimate of this model on the entire data set shows that this effect is not present in this data set.

Therefore, in this paper a simple GARCH(1,1) model is adopted to fit the log-returns of the assets. This model is given by

$$\begin{aligned} r_{i,t} &= \mu_i + \sigma_{i,t} z_{i,t} \\ z_{i,t} &\sim t_{\nu_i}(z_{i,t}) \\ \sigma_{i,t}^2 &= \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2. \end{aligned} \tag{1}$$

Here $r_{i,t}$ is the log-return of asset $i \in \{BTC, ETH, XRP, ADA, LTC, BTF\}$ at time $t = 1, .., W$, and μ_i and $\sigma_{i,t}$ are the corresponding mean and conditional volatility respectively. Furthermore, $z_{i,t}$ are the errors and $\varepsilon_{i,t} = \sigma_{i,t} z_{i,t}$ are the residuals. ω_i, α_i and β_i are the parameters to be estimated in the volatility equation, where the latter two determine the persistence of the volatility.

Since the log-returns seem to exhibit heavy-tails, as was found by Alvarez-Ramirez et al. (2018) and the excess kurtosis in Section 3, the GARCH(1,1) model is implemented with $z_{i,t}$ following the standard Student-t distribution denoted by $t_{\nu_i}(x)$ with shape parameter ν_i, which has the ability to cope with heavy-tails in the data. This distribution includes the Normal distribution as a special case when $\nu_i \to \infty$.

The model is estimated with Maximum Likelihood using the 'rugarch' package by Ghalanos (2014) in the statistical software R . From here the fitted errors can be easily obtained as $\hat{z}_{i,t} = \frac{r_{i,t} - \hat{\mu}_i}{\hat{\sigma}_{i,t}}$. These fitted errors are used to calculate the dependence among different assets, which will be discussed in the next section.

4.2 Copulas

Essential for constructing efficient hedge strategies is correctly modelling the dependence structure of the different assets. The dependence structure is a characteristic of the joint density of the assets, and a flexible way to model the joint density is by adopting copulas. According to Sklar (1959), any joint distribution function can be decomposed into some marginal distribution functions and a copula. That is,

$$F(z_1, ..., z_n) = C[F_1(z_1), ..., F_n(z_n)] \tag{2}$$

where $F(z_1, ..., z_n)$ denotes the CDF of the joint distribution function, $C(u_1, ..., u_n)$ denotes the copula and $F_i(z_i)$ denotes the CDF of the marginal distribution function of asset i. The copula $C(u_1, ..., u_n)$ is effectively a joint distribution function with uniform marginals as $u_i = F_i(z_i) \sim U(0, 1)$.

As the density of the multivariate distribution can be written as

$$\begin{aligned} f(z_1, ..., z_n) &= \frac{\partial^n}{\partial z_1 \cdots \partial z_n} F(z_1, ..., z_n) \\ &= \frac{\partial^n}{\partial z_1 \cdots \partial z_n} C[F_1(z_1), ..., F_n(z_n)] \\ &= c[F_1(z_1), ..., F_n(z_n)] f_1(z_1) \bullet \cdots \bullet f_n(z_n) \end{aligned} \tag{3}$$

where

$$c(u_1, ..., u_n) = \frac{\partial^n}{\partial u_1 \cdots \partial u_n} C(u_1, ..., u_n) \tag{4}$$

the log-likelihood can be written as

$$\ell = \sum_{t=1}^{T} \log\{c[F_1(z_{1,t}), ..., F_n(z_{n,t})]\} + \sum_{t=1}^{T} \log[f_1(z_{1,t})] + ... + \sum_{t=1}^{T} \log[f_n(z_{n,t})] \tag{5}$$

Therefore, it is common in practice to use two-stage estimation as described by Joe (1997). This means that first the marginal distributions are estimated, for example with Student-t distributed marginals as in Section 4.1, after which the pseudo-sample $u_{i,t}$ can be easily constructed and the copula can be estimated using maximum likelihood on the pseudo-sample.[1]

One characteristic that might be particularly important for hedging is lower tail dependence, which measures how much assets co-behave in case of large negative returns. The exact definition of tail dependence for copulas and an overview of the tail dependence of the different copulas considered in this paper are discussed Appendix 8.2. As the main goal in this paper is to hedge an individual cryptocurrency with Bitcoin futures and hence

[1] When Maximum Likelihood is unable to converge, the copula is fitted using the method of moments with Kendall's τ.

$n = 2$, the remainder of this section will elaborate on some well-known bivariate copulas to model the dependence. All copulas are implemented in the statistical software R using the 'copula' package by Hofert et al. (2020).

4.2.1 Elliptical Copulas

The first and most well-known class of copulas is the class of elliptical copulas. This class contains the Gaussian copula and the Student-t copula and is popular due to its simple structure. As the names suggest, these copulas are derived from the Normal (Gaussian) and Student-t distribution respectively. The Gaussian copula and the Student-t copula are defined as

$$C_{Gauss}(u_1, u_2; \rho) = \Phi[\Phi^{-1}(u_1), \Phi^{-1}(u_2); \rho] \tag{6}$$

$$C_{Student-t}(u_1, u_2; \rho, \nu) = t_\nu[t_\nu^{-1}(u_1), t_\nu^{-1}(u_n); \rho] \tag{7}$$

where $\Phi(x)$ denotes the CDF of a (multivariate) standard Normal distribution (with correlation ρ), and $t_\nu(x)$ denotes the CDF of the (multivariate) standard Student-t distribution with ν degrees of freedom (and scale parameter ρ). However, similar to its distributional counterparts, the Gaussian copula is unable to cope with tail dependence and in addition both elliptical copulas assume symmetry in the data.

4.2.2 Archimedean Copulas

Another popular class of copulas is the class of Archimedean copulas. This class is especially popular for modelling bivariate dependencies as they can allow for asymmetry as well as tail dependence, while only being specified by one dependence parameter. The general structure of the bivariate Archimedean copula is

$$C_\psi(u_1, u_2; \theta) = \psi[\psi^{-1}(u_1; \theta) + \psi^{-1}(u_2; \theta); \theta]. \tag{8}$$

Here $\psi(x; \theta)$ is the Archimedean generator function of the copula, which has to be a decreasing, continuous, and convex function with domain $\psi : [0, \infty) \to [0, 1]$. Moreover, it needs to satisfy the conditions $\psi(0; \theta) = 1$ and $\lim_{x\to\infty}\psi(x; \theta) = 0$. θ is called the dependence parameter of the generator.

Two popular choices for the generator function that are considered in this paper are the Gumbel generator with $\psi(x; \theta) = \exp(-x^{\frac{1}{\theta}})$ and the Clayton generator with $\psi(x; \theta) = (1 + \theta x)^{-\frac{1}{\theta}}$. The Gumbel copula on the one hand only possesses upper tail dependence, whereas the Clayton copula on the other hand only possesses lower tail dependence.

Other relatively well-known Archimedean copulas include the Frank copula with generator function $\psi(x; \theta) = -\frac{1}{\theta}\log\{1 + \exp(-x)[\exp(-\theta) - 1]\}$, the Joe copula with generator $\psi(x; \theta) = 1 - [1 - \exp(-x)]^{\frac{1}{\theta}}$, and the Ali-Mikhail-Haq (AMH) copula with generator

$\psi(x;\theta) = \frac{1-\theta}{\exp(x)-\theta}$. The Frank and the AMH copula exhibit no upper and lower tail dependence, whereas the Joe copula only exhibits upper tail dependence.

4.2.3 Rotated Copulas

Alternatively, there also exist rotated forms of Archimedean copulas known as survival copulas. Popular examples here again include the rotated Clayton copula and rotated Gumbel copula. As opposed to their counterparts, the rotated Clayton copula only exhibits upper tail dependence, whereas the rotated Gumbel copula only exhibits lower tail dependence. The general structure of rotated copulas is given by

$$C_{rotated-\psi}(u_1, u_2; \theta) = u_1 + u_2 - 1 + C_\psi(1-u_1, 1-u_2; \theta). \tag{9}$$

This means that the rotated Archimedean copulas are not Archimedean copulas themselves, as they do not necessarily satisfy the regularity conditions of Archimedean copulas.

4.3 Optimal Hedge Ratio Estimation

4.3.1 Monte Carlo Simulation

Together, the estimations of the GARCH(1,1) model and the copula give the estimated joint distribution of the errors. However, as this distribution is hard if not impossible to calculate analytically, the idea is to use Monte Carlo simulation. This means that a joint sample of returns is simulated from the copula. In this paper, $M = 10,000$ replications are used each simulation. The construction of the simulated sample of returns is as follows:

1. Simulate jointly the pseudo observations $u_{c,i}$ and u_{c,BTF_i} from copula c for asset i.

2. Convert the simulated pseudo observations to simulated errors by using $z_{c,i} = t_{\hat{\nu}_i}^{-1}(u_i)$ which is the inverse CDF of the fitted skewed Student-t distribution.

3. Calculate the simulated residuals $\varepsilon_{c,i} = \hat{\sigma}_{i,W+1} z_{c,i}$ where $\hat{\sigma}_{i,W+1}$ is the forecasted volatility which results from the volatility equation of the fitted GARCH(1,1) model. That is, $\hat{\sigma}^2_{i,W+1} = \hat{\omega}_i + \hat{\alpha}_i \hat{\varepsilon}^2_{i,W} + \hat{\beta}_i \hat{\sigma}^2_{i,W}$, where W denotes the last observation of the window.

4. Obtain the simulated log-returns from the mean equation of the fitted GARCH(1,1) model as $r_{c,i} = \hat{\mu}_i + \hat{\sigma}_{i,W+1} z_{c,i}$.

5. Convert the simulated log-returns to returns, $R_{c,i} = \exp(r_{c,i}) - 1$.

After simulating the asset returns $R_{c,i}$ from the different copulas, the simulated portfolio returns can be calculated as

$$R_{c,p_i} = R_{c,i} + h_{c,i} R_{c,BTF_i} \tag{10}$$

where $h_{c,i}$ denotes the hedge ratio which is to be estimated. The R_{c,BTF_i} has an additional subscript i since the simulated Bitcoin futures returns differ from asset to asset. For example, the simulated Bitcoin futures returns of the Gaussian copula when fitted together with the Bitcoin are different from the simulated Bitcoin futures returns of the Gaussian copula when fitted together with Ethereum. The $h_{c,i}$ can be interpreted as the amount of dollars to invest in the Bitcoin futures for holding one dollar of asset i. That is, the portfolio goes long one dollar in asset i, for example the Bitcoin or Ethereum, and short $-h_{c,i}$ dollar in Bitcoin futures. For optimization purposes, $h_{c,i}$ is restricted between 0 and -1.5.

4.3.2 Risk Minimizing Objective

As the eventual goal of hedging is to reduce the risk, many papers consider the Variance as the risk minimizing objective. However, Sukcharoen & Leatham (2017) and others argue that Variance may not be a suitable risk measure since it also takes upside risk into account. Therefore in this paper the Expected Shortfall (ES) is considered as the objective. The ES is defined as the expected loss above the Value-at-Risk (VaR), where the VaR is a quantile indicating the maximum amount to be lost at a given probability level α. The ES is chosen over the VaR itself as it exhibits some nice properties and therefore has become more popular in the financial literature in recent years. The VaR and ES of the returns are defined as

$$VaR_\alpha = \sup\{R : F(R) \leq \alpha\} \tag{11}$$

$$ES_\alpha = \frac{1}{\alpha}\int_0^\alpha VaR_u du \tag{12}$$

where R denotes some return or profit with corresponding distribution $F(R)$. This paper uses a probability level of $\alpha = 0.05$.

4.3.3 Optimization

Finally, the optimal hedge ratios should be chosen such that the amount of downside risk is minimal. However, as the distribution of future returns is unknown, the simulated copula returns are used to evaluate the amount of risk. Therefore, using the ES the optimal hedge ratio is calculated by

$$h^*_{c,i} = \underset{h_{c,i}}{\operatorname{argmin}} \quad \widehat{ES_\alpha} \tag{13}$$

where $\widehat{ES}_\alpha$ is estimated from the simulated portfolio returns as

$$\widehat{ES}_\alpha = \frac{1}{M\alpha} \sum_{j=1}^{\lceil M\alpha \rceil} R_{c,p_i,(j)} \tag{14}$$

where $R_{c,p_i,(1)} \leq R_{c,p_i,(2)} \leq ... \leq R_{c,p_i,(M)}$ denotes the order statistics of the simulated portfolio returns from Equation (10) which are dependent on the hedge ratio $h_{c,i}$, and $M = 10,000$ is the number of simulations.

4.4 Copula Density Forecast Combination

Ever since the work of Bates & Granger (1969) it is well-known in financial literature that combinations of forecasts are generally more accurate than individual forecasts as they are more robust. Even a simple average forecast is often able to outperform individual forecasts. However, the goal in this paper is to reduce risk, which does not just depend on a point forecast but rather on the entire distribution. For example, the VaR is a quantile of a given density. Therefore, this paper aims on constructing a weighted average density forecast. Note that the simulated samples of returns from the different copulas as in Section 4.3 are essentially density forecasts. How much an individual copula contributes to the joint density forecast depends on its given weight. For that purpose, again simulation is used to determine from which copula to simulate an observation for the combined density forecast. That is, denote the weights corresponding to each copula model by ω_c, then

$$R^m_{comb} = \begin{cases} R_1 & \text{if } w^m \leq \omega_1 \\ R_k & \text{if } \sum_{c=1}^{k-1} \omega_c < w^m \leq \sum_{c=1}^{k} \omega_c, \text{ for } k = 2, ..., J \end{cases} \tag{15}$$

where R_k denotes a vector of jointly simulated returns from copula c, and $w^m \sim U(0, 1)$ is a simulated uniform variable that determines from which copula to simulate. When the number of simulated samples is sufficiently large, this simulated sample should converge to the joint mixture density. From here, the optimal hedge ratios can be calculated in similar fashion as before.

The question then remains how to choose the weights ω_c. As stated above, simple average point forecasts often already tend to outperform individual forecasts. Therefore, the first and most obvious choice for the weights is the equally weighted or average density forecast with $\omega_c = \frac{1}{J}$ for all c.

However, this choice may be too simplistic as some density forecasts might be significantly better than others. How good density forecasts are can be assessed using score functions. A popular choice for score functions in this context is the log-score function by Good (1992) and Winkler & Murphy (1968). This score function is given by $L(x) = -\log[f(x)]$, where $f(x)$ is the predicted density and x is the realization. Smaller

$L(x)$ are preferred over larger $L(x)$. This means that the joint density as given by Equation (3) is evaluated, thereby taking the fitted Student-t distribution and the fitted copula together. These log-scores can then be used for calculating the optimal weights which turns out to be the regularized estimator with log-objective of Geweke & Amisano (2011)

$$\begin{gathered} \omega^* = \underset{\omega}{\operatorname{argmin}} \sum_{t=1}^{D} -\log[\sum_{c=1}^{J} \omega_c f_{c,t}(z_{1,t}, z_{2,t})] \\ w_c \geq 0, \quad \sum_{c=1}^{J} \omega_c = 1 \end{gathered} \tag{16}$$

where D is the amount of past predictions that is used for determining the optimal weights. In this paper $D = 100$ is used. This means that the construction of the combined joint density forecast is based on the out-of-sample fit of the different copulas over approximately the last four months. Furthermore, Diebold et al. (2021) show that the restrictions regarding ω_c also act as some sort of LASSO regularization, which was introduced by Tibshirani (1996), and shrinks the weights towards zero. Due to its constraints, this combination will be referred to as the simplex weighted combination.

Lastly, Diebold et al. (2021) show that best subset averaging can also perform quite well. Subset averaging is a special case of partially egalitarian penalization by Diebold & Shin (2019) with ridge. The partially egalitarian penalization with ridge problem is given by

$$\begin{gathered} \omega^* = \underset{\omega}{\operatorname{argmin}} \sum_{t=1}^{D} -\log[\sum_{c=1}^{k} \omega_c f_{c,t}(z_{1,t}, z_{2,t})] + \lambda \sum_{c=1}^{k} [\omega_c - \frac{1}{\delta(\omega)}]^2 \\ w_c \geq 0, \quad \sum_{c=1}^{k} \omega_c = 1. \end{gathered} \tag{17}$$

$\delta(\omega)$ is the number of nonzero elements of ω, and the ridge penalty ensures that the remaining weights are shrunk towards their average weight. Best subset averaging happens if $\lambda \to \infty$ as the penalty of deviating from the average dominates the objective. In this paper best 3 averaging is considered, which is the case when $\delta(\omega) = 3$. The choice for $\delta(\omega) = 3$ is somewhat arbitrary as no good estimation methods exist for determining the optimal $\delta(\omega)$. However, as the total number of copula density forecasts equals twelve, $\delta(\omega) = 3$ ensures that only the best density forecasts are considered, without losing the benefit of averaging.

4.5 Performance Evaluation

4.5.1 Performance Measures

The main goal of this paper is to develop hedge strategies that reduce the risk in cryptocurrency portfolios. For that reason, the focus is on good hedging performance rather than high profitability. To evaluate the hedging performance again risk measures are considered. That is, the hedges are evaluated by calculating several risk measures for the realized out-of-sample returns of the hedged portfolios. The set of risk measures considered in this paper consists of the Variance, Semivariance, and the VaR and ES at probability levels $\alpha \in \{0.1, 0.05, 0.025, 0.01\}$. It is common in the literature to compare the results of the hedged portfolio with those of the unhedged portfolio. This comparison is summarized by the Hedging Effectiveness (HE) which is given by

$$HE = 1 - \frac{\text{Risk}(R_{hedged})}{\text{Risk}(R_{unhedged})} \tag{18}$$

where Risk denotes one of the above-mentioned risk measures. Furthermore, R_{hedged} are the realized returns of the hedged portfolio, and $R_{unhedged}$ are the realized returns of the unhedged portfolio, or in other words the realized returns of the cryptocurrency. Obviously the hedge strategy should reduce the amount of risk as much as possible and therefore higher hedge effectiveness is preferred. The significance between the hedging performances of different models is assessed using the Wilcoxon Ranked Sum test on the different hedge effectivenesses.

However, although reducing the risk is the main goal of the hedge, there are some important limitations. Most notably, since dynamic hedge strategies are considered, there are costs involved with the buying and selling of Bitcoin futures given by the bid-ask spread and other fees. For that reason, the turnover which measures the amount of changes in positions, or in other words the volatility of the hedge ratio, is also considered. The turnover is defined as

$$TO = \frac{1}{T - W - 1} \sum_{t=W+1}^{T} |h_t - h^*_{t-1}| \tag{19}$$

where $T = 879$ is the total number of observations, and $W = 200$ is the moving window. h_t denotes the estimated hedge ratio at time t and h^*_{t-1} is the estimated hedge ratio at time $t-1$ corrected by the realized returns at time t. That is, $h^*_{t-1} = h_{t-1}\frac{1+R_{BTF,t}}{1+R_{i,t}}$. Models that produce lower turnover are preferred over models with higher turnover.

In this paper the average trading fee is assumed to be 0.1% which is relatively low. More information on trading fees of Bitcoin futures is available on the respective websites of BitMEX and Okex. In addition, the bid-ask spread obtained from YahooFinance is approximately 0.05%. This leads to a total transactions cost of about 0.15%. This means that when an investor wishes to buy or sell \$100 of Bitcoin futures, he has to pay an additional \$0.15. These low transaction costs are one of the advantages of Bitcoin futures mentioned by among others Corbet et al. (2018).

4.5.2 Benchmark Models

Finally, the models are compared to three more simplistic hedge strategies. The first of these hedge strategies is the naive hedge which takes the opposite position in the hedging asset. This means a constant hedge ratio of $h_{naive,i} = -1$.

The second strategy is the OLS hedge. This method simply estimates the hedge ratio by performing OLS of the hedgeable asset on the hedging asset,

$$R_{i,t} = a_i + h_{OLS,i} R_{BTF,t} + \varepsilon_{i,t} \tag{20}$$

Subsequently, $h_{OLS,i}$ turns out to be the hedge ratio that minimizes the in-sample unconditional Variance.

The third benchmark model is the Dynamic Conditional Correlation GARCH (DCC-GARCH) model proposed by Engle (2002), which is also used by Sebastião & Godinho (2020) for hedging several different cryptocurrencies using Bitcoin futures. The DCC-GARCH model is a multivariate extension of the univariate GARCH model where the conditional correlation between assets is allowed to vary over time similar to the volatility. It is different from the copula methods considered in this paper since the way it constructs the dependence structure between assets is still restricted.

The model is given by

$$\begin{aligned}
r_t &= \mu_t + \varepsilon_t \\
\varepsilon_t &\sim \Phi(0, D_t \Sigma_t D_t) \\
D_t^2 &= \text{diag}\{\omega_i\} + \text{diag}\{\alpha_i\} \circ \varepsilon_{t-1}\varepsilon_{t-1}^{'} + \text{diag}\{\beta_i\} \circ D_{t-1}^2 \\
\Sigma_t &= \text{diag}\{Q_t\}^{-1} Q_t \ \text{diag}\{Q_t\}^{-1} \\
Q_t &= S \circ (\iota\iota^{'} - A - B) + A \circ \varepsilon_{t-1}\varepsilon_{t-1}^{'} + B \circ Q_{t-1}
\end{aligned} \tag{21}$$

where r_t denotes the log-returns at time t with corresponding means μ_t and residuals ε_t respectively. Following Sebastião & Godinho (2020) these errors follow a multivariate Normal distribution with conditional covariance matrix $D_t \Sigma_t D_t$. Here, D_t denotes the diagonal matrix of conditional standard deviations which follow from univariate GARCH processes, and Σ_t denotes the conditional correlation matrix. The conditional correlation matrix is constructed from the covariance matrix Q_t which also follows a GARCH process. In particular, S is the unconditional correlation matrix, ι denotes a vector of ones, and A and B are the parameter matrices that determine the persistence of Q_t. The model is implemented in the statistical software R using the 'rmgarch' package from Ghalanos (2019). Both OLS and DCC-GARCH are estimated using the same moving window of $W = 200$.

5 Results

5.1 GARCH(1,1) Volatility Forecasts

This section analyzes how the GARCH(1,1) model behaves over time, as the errors resulting from the GARCH(1,1) model are used to get rid of serial correlation. The average estimates of the GARCH(1,1) for all different cryptocurrencies and the Bitcoin futures are shown in Appendix 8.3.

The volatility equation is particularly interesting as it determines the amount of uncertainty about the next period returns. This might affect the optimal hedging strategy quite significantly, as more uncertainty about the future returns generally means higher risk, and reducing this risk is exactly the goal of this paper. In addition, it gives an indication of the how economic circumstances affect the different cryptocurrencies. For that reason the forecasted next period volatilities of the different assets over time are displayed in Figure 3.

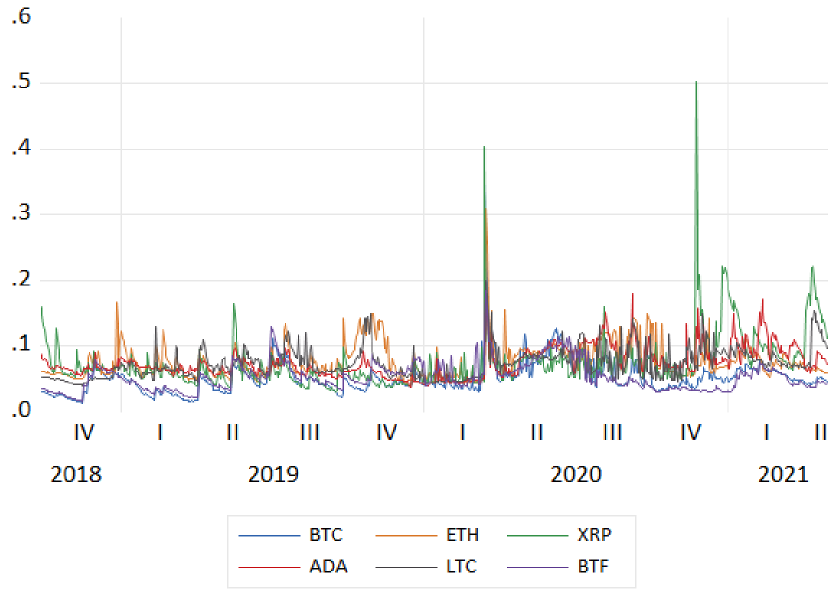


Figure 3: GARCH(1,1) next day volatility forecasts of the Bitcoin, Ethereum, Ripple, Cardano, Litecoin and Bitcoin futures with a moving window of $W = 200$ observations for the period September 2018 - April 2021

As expected, the Bitcoin and the Bitcoin futures show very similar behaviour in terms of forecasted volatility. The volatility starts off relatively low, but then rapidly increases at the end of the first quarter of 2020. Around that time the corona virus arrived in Western countries and there was a lot of uncertainty about the future state of the economy. However, the Bitcoin reacts much stronger then the Bitcoin futures, peaking at a volatility of about 0.35 compared to a volatility of about 0.18. Such observations will likely affect the optimal hedge ratios significantly, as in terms of risk it is probably beneficial to take a relatively larger position in the Bitcoin futures to control for the higher uncertainty in the Bitcoin. The volatility then drops quickly to about the same level as before. The forecasted volatility then gradually increases during the last months of 2020 and the first months of 2021. A possible explanation might be the hype around the Bitcoin at that time due to investments and announcements of Tesla and their CEO Elon Musk. In general these findings support the behaviour of the log-returns found in Section 3.

As for the volatility of the alternative cryptocurrencies, the general patterns are mostly the same as those of the Bitcoin. This is in line with the expectations since it was already found that the different cryptocurrencies are interconnected and show very similar behaviour. However, the forecasted volatilities of these cryptocurrencies are significantly larger than those of the Bitcoin and Bitcoin futures. Considering the peak at the start of the corona pandemic however, Cardano and Litecoin only reach a peak volatility of approximately 0.19 and 0.20 respectively. The Ethereum and the Ripple on the other hand are more volatile with peaks of 0.31 and 0.40. In addition, the Ripple shows extreme volatility towards the end of the data set. What is more, the GARCH(1,1) model produces a forecasted volatility of about 0.5 for the Ripple at some point after a large positive return. All in all, these findings are again in line with the findings in Section 3.

5.2 Copula Fit

As discussed in Section 4.2, this paper adopts different copulas in order to see which copulas capture the joint dependence structure of the cryptocurrencies and the Bitcoin futures best and provide a good hedge. In particular, this paper includes two elliptical copulas, five Archimedean copulas and five rotated Archimedean copulas. The general fit of these copulas indicated by their average log-likelihoods is displayed in Table 3. For illustrative purposes, simulated pseudo-samples of these copulas are plotted in Appendix 8.2.

For the Bitcoin, the Student-t copula gives the best average fit with an average log-likelihood of 145. In contrast to the other copulas that all include only one parameter, the Student-t copula includes two parameters that determine the dependence structure. This gives the Student-t copula an advantage in terms of flexibility which is probably one of the reasons for its good fit. The Gumbel and rotated Gumbel copulas also model the joint dependence structure relatively well with log-likelihoods of about 128 and 135 respectively. The next best fit is given by the Gaussian copula with an average log-likelihood of 125. Looking at the tail characteristics of these four copulas, the Student-t copula possesses both upper and lower tail dependence, the Gaussian copula possesses neither lower nor upper tail dependence, and the Gumbel and rotated Gumbel copula posses only upper and lower tail dependence respectively. This suggests that tail dependence is not an important feature for the general fit of the copula which makes sense as these characteristics are only relevant for a few observations. Lastly, the AMH copula and rotated AMH copula give the worst fits of all copulas with log-likelihoods of 81 and 78 respectively, showing that the general dependence structure of these copulas is not accurate.

Table 3: Average log-likelihoods of the different copulas for the Bitcoin, Ethereum, Ripple, Cardano, Litecoin with Bitcoin futures for the period December 2017 - April 2021 with a moving window of $W = 200$ observations

Crypto	BTC	ETH	XRP	ADA	LTC
Gauss	125.497	65.259	46.091	46.906	61.660
Student-t	145.364	71.326	51.283	50.965	66.315
Clayton	113.291	63.848	50.172	47.626	58.248
Gumbel	127.753	58.280	38.550	40.801	55.329
Frank	120.352	62.645	45.327	42.787	54.901
Joe	101.084	40.032	23.941	27.258	39.187
AMH	81.278	58.921	48.785	46.897	54.890
R-Clayton	101.689	42.881	26.728	24.795	41.949
R-Gumbel	135.218	71.773	54.253	52.344	66.087
R-Frank[2]	120.352	62.645	45.327	42.787	54.901
R-Joe	113.054	62.746	49.639	46.580	57.132
R-AMH	78.176	44.418	31.759	32.213	42.647

As for the alternative cryptocurrencies, the copula fits are significantly lower. This makes sense as these cryptocurrencies are not as strongly related to the Bitcoin futures as the Bitcoin itself, and hence there is less structure in their joint dependence. The average log-likelihoods are the highest for the Ethereum and the Litecoin, followed by the Ripple and the Cardano. In general, the same copulas that provide a good fit for the Bitcoin, also provide a good fit for the alternative cryptocurrencies. For example, the rotated Gumbel copula and the Student-t copula fit the data best and second best respectively for all cryptocurrencies. An exception here is the AMH copula, which provides a bad fit for the Bitcoin, but a relatively good fit for the alternative cryptocurrencies.

Important determinants for the structure of the copulas are their dependence parameters. These dependence parameters are related to Kendall's τ, or the rank correlation, by

$$\tau = 1 + 4\int_0^1 \frac{\psi^{-1}(x;\theta)}{(\psi^{-1})'(x;\theta)}dx$$

for Archimedean copulas, and similar relations exist for other copulas. Hence, the Kendall's τ can be used to summarize the dependence structure of these copulas. The Kendall's τ is preferred over the regular Pearson correlation as it is more robust in correctly describing the dependence structure. That is, rank correlation does not depend on the

[2] Due to its symmetric structure, the Frank copula and rotated Frank copula are actually the same. However, because of time restrictions the rotated Frank copula is still left in the analysis. Moreover, it can actually be used to check the sensitivity of the hedging performance by the simulated sample by comparing the hedge results of the Frank copula and rotated Frank copula.

marginal distributions as Pearson correlation does. In addition, it is better able to deal with heavy-tails in the data.

For that reason, Figure 4 shows the estimated Kendall's τ for the various cryptocurrencies linked with the Bitcoin futures over time. As expected, the rank correlation is higher and more stable for the Bitcoin than for the alternative cryptocurrencies, ranging between 0.6 and 0.7, and might even be slightly increasing over time. This probably benefits the hedging performance as more stable estimates indicate that the dependence structure next period is more likely to be similar to the dependence structure this period, and thus the joint density forecasts provided by the copulas are probably more accurate.

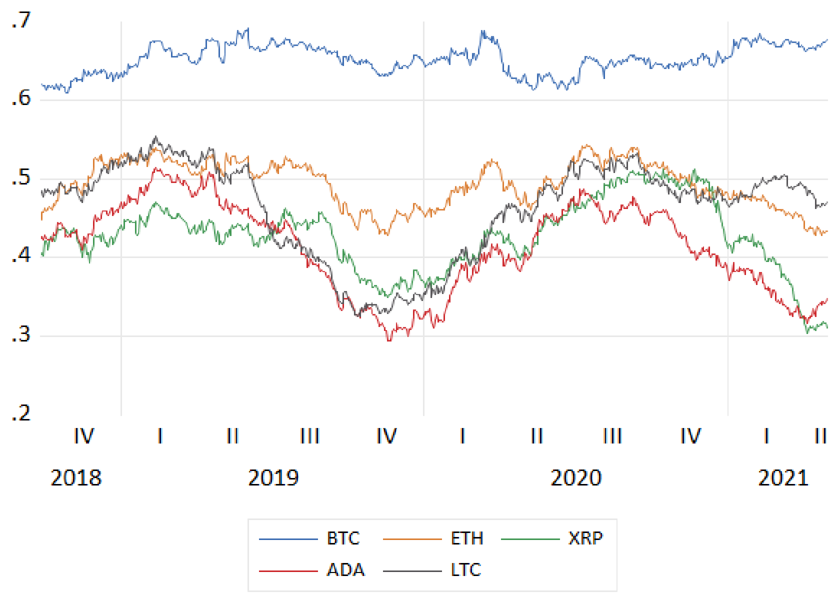


Figure 4: Estimated Kendall's τ of the Bitcoin, Ethereum, Ripple, Cardano and Litecoin with the Bitcoin futures for the period September 2018 - April 2021 with a rolling window of $W = 200$ observations

The Kendall's τ of the alternative cryptocurrencies on the other hand show some more time-varying behaviour. The Ethereum, which is most related to the Bitcoin and the Bitcoin futures as indicated by its correlation and Kendall's τ estimates in Section 3, have the highest and most stable estimates of the alternative cryptocurrencies. Its Kendall's τ ranges between 0.45 and 0.55. Furthermore, the Kendall's τ of the Ripple, Cardano and Litecoin show a clear pattern where first the dependence decreases at the end of 2019, then increases during the first months of 2020, and then decreases again at the end of that year. Looking back at the GARCH(1,1) volatility forecasts in Section 5.1, the last decline at the end of 2020 might be explained by the fact that the volatility of these cryptocurrencies during this period increased significantly. This hints that this more volatile behaviour is probably not picked up as well by the Bitcoin futures and might negatively impact the hedging capabilities of the Bitcoin futures for these cryptocurrencies during this period.

5.3 Hedging Results

In this section, the various hedge strategies with minimum ES objective are evaluated. Since the amount of different hedge strategies is quite large, only the performances of the unhedged portfolio, the best benchmark hedge strategy, the three best individual copula hedge strategies and the three density forecast combination hedge strategies are discussed for each cryptocurrency. The best performing models are chosen based on their average ranking across the different risk measures. The performance of the remaining methods as well as the results of the Wilcoxon Signed Rank test are shown in Appendix 8.4.

5.3.1 Hedging Results Bitcoin

First, the performances of the direct hedging strategies of the Bitcoin futures for the Bitcoin are discussed. Table 4 shows some statistics as well as measures of riskiness with corresponding hedge effectiveness for the different portfolios. In general the table confirms that hedging Bitcoin using Bitcoin futures is effective for reducing risk, which is in line with the findings of Sebastião & Godinho (2020). The hedging effectiveness for all methods and all risk measures is positive and this also holds for the remaining methods which are shown in Appendix 8.4. Moreover, the hedging effectiveness generally ranges between 40% and 50% for these models, indicating that the risk reduction is quite significant. The relative decrease of the Variance is even higher with hedging effectiveness just below 70%. This is particularly interesting since this paper did not target minimum-Variance hedging, but instead minimizes the ES at a 5% level. However, it is well-known in literature that aiming to minimize a certain objective, in this case the ES, might actually benefit other objectives more, in this case the Variance. Additionally, it might be less difficult to reduce the Variance than the ES, as the ES relies on extreme observations which are generally harder to predict.

Another obvious result of hedging is that the mean return of the hedged portfolios are lower than those of the unhedged portfolio. That is, the average daily return of the unhedged portfolio equals about 0.006, whereas the average returns of the hedged portfolios are significantly lower around 0.001. In addition the minimum return also increases. This makes sense as the hedged portfolios take a position in the Bitcoin futures in the opposite direction, meaning that when the price of the Bitcoin increases, the price of the Bitcoin futures likely also increases, hence the value of the hedged portfolio increases less than the value of the Bitcoin itself. Interestingly however, there are also hedge strategies where the maximum return increases, such as the hedge strategies resulting from the Gaussian copula and the equally weighted combination method.

The OLS benchmark model provides the best hedging performance of all benchmark models based on its average rank of 2.0, thereby outperforming the Naive hedge and the DCC-GARCH hedge. This is in line with the findings of Sebastião & Godinho (2020) who find that the DCC-GARCH model is not able to improve upon the OLS benchmark in most cases. Moreover, the OLS model provides the best risk reduction of all models for more extreme observations, reducing the ES by approximately 40% on all levels.

Table 4: Risk measures for evaluating the out-of-sample hedging performance for the Bitcoin portfolio using different hedge strategies. The hedging effectiveness is displayed between brackets. The results are based on the period September 2018 - April 2021. For convenience only the best models are shown in this table. In addition the average model ranks for the 10 risk measures is calculated. The best models are indicated in bold.

Method	Unhedged	OLS	Gauss	R-Gumbel	R-Joe	C-Equal	C-Simplex	C-Best 3
Mean	0.006	0.001	0.002	0.002	0.003	0.001	0.001	0.001
Max	0.225	0.214	0.289	0.142	0.144	0.241	0.224	0.132
Min	-0.372	-0.196	-0.185	-0.195	-0.224	-0.178	-0.181	-0.158
Variance	0.002	0.001	0.001	0.001	0.001	0.001	0.001	**0.001**
	(-)	(68.246)	(60.023)	(63.124)	(58.298)	(63.921)	(65.521)	**(69.047)**
Semivariance	0.043	**0.025**	0.026	0.027	0.028	0.026	0.026	0.025
	(-)	**(42.220)**	(39.219)	(36.839)	(35.900)	(39.167)	(39.861)	(40.802)
VaR 0.1	-0.039	-0.022	-0.023	-0.024	-0.024	-0.022	**-0.020**	-0.022
	(-)	(43.065)	(42.179)	(39.142)	(39.631)	(44.485)	**(48.781)**	(44.485)
VaR 0.05	-0.057	-0.034	-0.036	-0.036	-0.037	-0.034	**-0.034**	-0.034
	(-)	(39.768)	(35.962)	(35.796)	(34.340)	(39.124)	**(40.069)**	(39.850)
VaR 0.025	-0.078	-0.055	**-0.053**	-0.054	-0.057	-0.058	-0.055	-0.056
	(-)	(29.922)	**(31.592)**	(31.013)	(27.007)	(25.683)	(28.832)	(28.526)
VaR 0.01	-0.130	**-0.068**	-0.077	-0.077	-0.074	-0.080	-0.076	-0.080
	(-)	**(47.852)**	(40.512)	(41.012)	(43.057)	(38.834)	(41.490)	(38.621)
ES 0.1	-0.073	**-0.044**	-0.046	-0.047	-0.048	-0.047	-0.045	-0.045
	(-)	**(39.573)**	(36.289)	(35.053)	(34.633)	(35.849)	(37.534)	(37.712)
ES 0.05	-0.101	**-0.061**	-0.065	-0.066	-0.066	-0.067	-0.065	-0.064
	(-)	**(39.049)**	(35.547)	(34.731)	(34.437)	(33.480)	(35.498)	(36.455)
ES 0.025	-0.136	**-0.078**	-0.084	-0.086	-0.085	-0.086	-0.084	-0.083
	(-)	**(42.954)**	(38.397)	(37.028)	(37.322)	(36.372)	(38.059)	(38.714)
ES 0.01	-0.185	**-0.109**	-0.122	-0.123	-0.121	-0.121	-0.121	-0.115
	(-)	**(40.954)**	(34.107)	(33.553)	(34.747)	(34.581)	(34.587)	(37.719)
Rank	-	2.0	5.0	6.9	7.1	7.1	3.5	3.5

Furthermore, the hedging results suggest that the Gaussian copula, rotated Gumbel copula and the rotated Joe copula are the copulas most suited for hedging the Bitcoin. However, looking back at the average copula fits in Section 5.2, the Student-t copula actually fitted this particular data best. This goes to show that a good fit does not necessarily lead to high hedge effectiveness as certain characteristics of the distribution that relate to downside risk, such as tail dependence, are most important for hedging. Hence a copula that fits the data well but is not able to accurately capture those particular data characteristics might still give bad hedging results.

The rotated Gumbel and rotated Joe copula both incorporate lower tail dependence. This hints that this characteristic might indeed be an important characteristic for hedging. However, the Gaussian copula performs best of all copulas based on its average rank of 5.0 and does not incorporate any tail dependence. Comparing the individual copula hedges to the benchmark OLS hedge, the OLS hedge outperforms these methods for all risk measures except the VaR at 2.5% level. This shows that although individual copulas have some potential for hedging Bitcoin in terms of hedge effectiveness, they are generally unable to do better than the simple OLS hedge. Moreover, this suggests that the OLS hedge might actually provide quite a competitive and hard-to-beat benchmark.

Turning to the density forecast combination hedge strategies, recall that these are constructed by combining the various density forecasts resulting from the individual copulas according to different weighing schemes. This also includes the aforementioned Gaussian copula, rotated Gumbel copula and rotated Joe copula. The idea is that these combined density forecasts are more robust and take some good characteristics from each individual density forecast, while averaging out the bad characteristics.

First, the equally weighted combination hedge strategy has the worst overall hedging performance of the combination methods with an average ranking of 7.1. This is similar to the best performing individual copula methods displayed in the table. Therefore this immediately shows the great benefit of combining density forecasts for hedging purposes. However, in general the OLS hedge still seems to provide a better hedge than the equally weighted combination method

It might be that the equally weighted combination method assigns too much weight to bad density forecasts which in return can hurt the combined density forecast and with that its hedging ability. For that reason it could be beneficial to consider some alternative weighting schemes which are based on out-of-sample fit as discussed in Section 4.4. The simplex weighting combination strategy has an overall performance that is indeed closer to that of the OLS benchmark based on its average rank of 3.5. Moreover, it completely dominates the equally weighted combination method hinting that this weighting scheme leads to a more accurate density forecast. This is in line with the findings of Diebold et al. (2021), and again confirms the benefits of combining different density forecasts. However, this method still gives relatively low hedge effectiveness compared to the OLS hedge. Especially its ability to hedge against large losses is lackluster with for example a hedge effectiveness of 34.6% for the ES at 1% level versus 41% for the OLS hedge.

Another weighting scheme is the best 3 average combination. This method achieves the same average rank as the simplex weighting scheme at 3.5. Overall, its performance is fairly similar to that of the simplex weights combination strategy, and similar conclusions hold. It shows that these methods can compete with the OLS hedge benchmark, which has been shown to be hard to outperform, but might not improve upon the OLS hedge, especially not for reducing extreme losses.

As argued in Section 4.5, next to looking at the hedging abilities of different hedge strategies, it is also important to consider how feasible the hedge strategies are. That is, a certain hedge strategy might give an excellent performance, but when the costs of this strategy are large, the hedge might still not be worth pursuing. Therefore some summary statistics on the hedge ratio as well as the turnover are displayed in Table 5. Recall that the turnover is the average magnitude of rebalancing, which typically involves costs.

First of all, the mean hedge ratios of the different hedge strategies differ quite a lot. The mean hedge ratios of the OLS hedge and the combination methods are about -0.8, whereas the mean hedge ratios of the individual copula hedges are between -0.5 and -0.7. This suggests that the individual copula models likely underestimate the amount of risk in the Bitcoin or underestimate the amount of tail dependence between the Bitcoin and Bitcoin futures, which leads to lower hedge ratios and thus worse hedge performance. Furthermore, the ranges of the different hedge ratios differ quite a lot between the methods. The OLS hedge ratio is very stable between -0.729 and -0.954, whereas the other methods have a far greater variation in hedge ratios. This is likely the result of the GARCH(1,1) specification, since the volatility forecasts shown in Section 5.1 can differ significantly over time. Moreover, these strategies even attain hedge ratios of approximately zero which indicates that it is expected that holding additional units in Bitcoin futures would increase the risk. This may have happened in times where the volatility of the Bitcoin is relatively low compared to the volatility of the Bitcoin futures and therefore there is little potential for hedging.

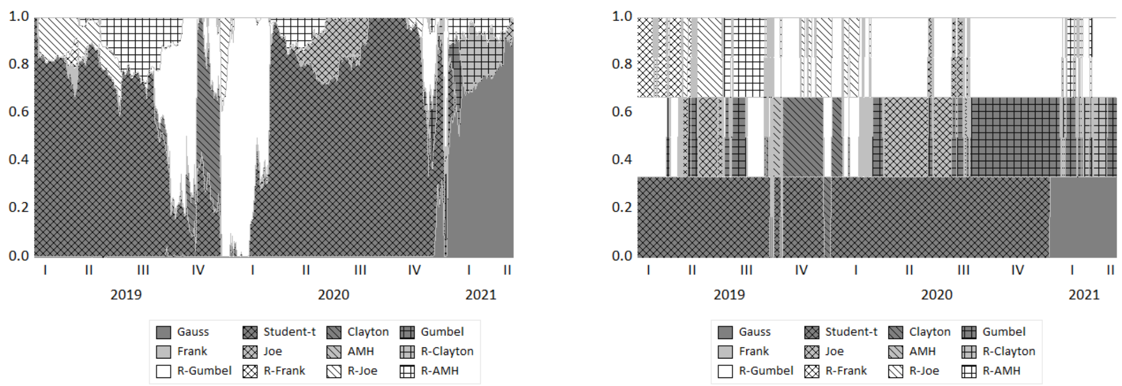
Table 5: Summary statistics of the hedge ratios of different hedge strategies for the Bitcoin. The results are based on the period September 2018 - April 2021. For convenience only the best models are displayed in this table.

Method	OLS	Gauss	R-Gumbel	R-Joe	C-Equal	C-Simplex	C-Best 3
Mean	-0.827	-0.685	-0.626	-0.500	-0.794	-0.809	-0.826
Max	-0.729	0.000	0.000	0.000	0.000	0.000	0.000
Min	-0.954	-1.500	-1.425	-1.153	-1.439	-1.500	-1.500
Turnover	0.015	0.095	0.090	0.078	0.086	0.074	0.070

Focusing on the turnover, the OLS method clearly has the lowest turnover of 0.015. This means that on average the absolute difference between the adjusted previous hedge ratio and the new hedge ratio is 0.015. Again, the combination methods do seem somewhat more robust than the individual methods with all three combination methods having lower or almost as low turnover as the best individual copula methods. The second best method in terms of turnover is the best 3 average combination method with a turnover of 0.070 which is over four times as high as that of OLS. Assuming transaction costs of 0.15% as argued in Section 5.5, this boils down to average yearly transaction costs of $\$1,000,000*0.015*0.0015*250 = \$5,625$ and $\$1,000,000*0.070*0.0015*250 = \$26,250$ respectively, for holding $\$1,000,000$ in Bitcoin futures.

Taking the results together, the combination hedge strategies do perform relatively well compared to the individual copula hedge strategies and in addition are less costly. However, they can not outperform the OLS benchmark hedge strategy and tend to have higher turnover. Therefore they should not be considered by investors for hedging Bitcoin with Bitcoin futures.

To analyze these combination methods in more detail, the weights assigned to the different copula density forecasts over time for the Bitcoin are plotted in Figure 5. Note that only the simplex weights and the best 3 average weights are considered, as by definition the equally weighted combination does not change. As for the remaining two methods, in general their weights are rather consistent for extended periods of time. This makes sense as the density in one period is likely to be very similar to that of the previous period. In addition, the weights are obtained from the optimal combined density fit over the last 100 observations, hence the weights in the previous period are based on 99 out of 100 of the same observations.



(a) Simplex **(b)** Best 3 average

Figure 5: Weights assigned to the copula density forecasts for the Bitcoin by the simplex method and the best 3 average method for the period September 2018 - April 2021

For the simplex weights, the first three quarters of 2019 are dominated by the Student-t copula density forecasts as a significant amount of the weight, often even above 70%, is assigned to this copula. This indicates that this joint density fits the data well during this period, which is in line with the log-likelihoods displayed in Section 5.2. Then at the end of 2019 and the start of 2020 there is a change point, where the rotated Gumbel and Clayton copula start to dominate. This partially coincides with the start of the corona pandemic which makes sense as this came with a lot of uncertainty about the future state of the economy, and both copulas are able to incorporate lower tail dependence.

After the huge negative returns in March 2020, the cryptocurrency market calms down again and the Student-t copula starts to dominate again. This goes on till the end of 2020 when again uncertainty hits the market, as indicated by the forecasted volatilities in Section 5.1, and a new changing point appears. This might be partially the result of the hype from announcements and large investments of Tesla. In addition, this was around the time that the Bitcoin finally reached its peak value of about \$20,000 which had happened only once, namely just before the launch of the Bitcoin futures back in December 2017. The Gaussian copula starts dominating from this point. All in all, the weights hint that the Student-t copula fits the data particularly well when the market is relatively stable. On the other hand, during times of higher uncertainty other copulas such as the rotated Gumbel and Clayton copula as well as the Gaussian copula become more important.

As for the best 3 average combination method, the Student-t copula is again incorporated for the majority of the time and is followed by the Gaussian copula during the last period. Moreover, the rotated Gumbel copula is also incorporated in the combined density forecast relatively often. In general these weights seem to be somewhat more consistent and robust than those of the simplex weighting scheme, which can be beneficial in some cases.

5.3.2 Hedging Results Ethereum

The second hedge considered in this paper is the cross-hedge of Bitcoin futures on Ethereum. In general, the hedging results are somewhat worse than those of the Bitcoin, which makes sense as the Bitcoin futures are specifically designed for hedging the Bitcoin. Nonetheless, the results of the hedges are quite significant. The hedging effectivenesses for most methods range between 20% to 30%. A notable exception here is the VaR on 10% level, where the hedging effectiveness is considerably lower at about 5% to 15%, indicating that the different hedge strategies are unable to reduce this particular type of risk very well. All together, these hedging results are different from the results found by Sebastião & Godinho (2020), as they conclude that hedging with Bitcoin futures increases the tail risk for alternative cryptocurrencies such as Ethereum, whereas this paper finds that Bitcoin futures can reduce the ES at 1% level by up to 34%.

Furthermore, this paper finds that the DCC-GARCH model provides the best benchmark for the Ethereum. This is different from the hedge results of the Bitcoin where OLS was found to give the best benchmark hedge. A possible explanation is that the DCC-GARCH model is more flexible and is better able to incorporate changes in the dependence structure as it models the conditional correlation. For the Bitcoin the Kendall's τ is relatively stable as can be seen in Section 5.2, indicating that the dependence structure does probably not change a lot, which benefits the OLS hedge. However, for the Ethereum the Kendall's τ is less stable over time hinting at changes in the dependence structure, and hence modelling the conditional correlation might be beneficial.

The average rank of the DCC-GARCH hedge is 3.4 which is lower than those of the best individual copula hedge strategies. In particular its ability to hedge against more extreme events is superior. For example, it attains an hedging effectiveness of about 31% for the ES on 1% level, whereas the best three individual copula methods reach an hedging effectiveness of just 19%, 24% and 20% respectively. The best individual copula hedges are obtained from the Gaussian copula, the Joe copula and the rotated Joe copula. These copulas are almost the same as the ones for the Bitcoin with the exception

Table 6: Risk measures for evaluating the out-of-sample hedging performance for the Ethereum portfolio using different hedge strategies. The hedging effectiveness is displayed between brackets. The results are based on the period September 2018 - April 2021. For convenience only the best models are shown in this table. In addition the average model ranks for the 10 risk measures is calculated. The best models are indicated in bold.

Method	Unhedged	DCC	Gauss	Joe	R-Joe	C-Equal	C-Simplex	C-Best 3
Mean	0.007	0.002	0.002	0.004	0.003	0.004	0.003	0.003
Max	0.424	0.406	0.394	0.414	0.400	0.409	0.409	0.400
Min	-0.423	-0.231	-0.301	-0.325	-0.313	-0.244	-0.300	-0.246
Variance	0.003	**0.002**	0.002	0.002	0.002	0.002	0.002	0.002
	(-)	**(35.385)**	(32.980)	(32.384)	(32.717)	(35.184)	(32.104)	(33.972)
Semivariance	0.053	**0.039**	0.042	0.041	0.042	0.039	0.041	0.040
	(-)	**(27.407)**	(21.398)	(22.657)	(21.653)	(27.016)	(22.546)	(25.103)
VaR 0.1	-0.049	**-0.041**	-0.046	-0.047	-0.044	-0.044	-0.043	-0.043
	(-)	**(14.829)**	(5.578)	(3.965)	(9.895)	(9.474)	(10.618)	(12.320)
VaR 0.05	-0.077	-0.066	-0.066	**-0.062**	-0.065	-0.062	-0.065	-0.063
	(-)	(13.419)	(13.416)	**(18.904)**	(15.539)	(18.760)	(15.069)	(17.172)
VaR 0.025	-0.106	-0.079	-0.078	-0.078	-0.078	-0.077	-0.077	-0.077
	(-)	(25.517)	(25.916)	(26.102)	(26.309)	(27.271)	(27.003)	(27.300)
VaR 0.01	-0.167	-0.094	-0.116	**-0.091**	-0.127	-0.098	-0.099	-0.100
	(-)	(43.782)	(30.700)	**(45.795)**	(24.416)	(41.700)	(40.675)	(40.411)
ES 0.1	-0.093	-0.071	-0.077	-0.073	-0.075	**-0.070**	-0.074	-0.072
	(-)	(23.052)	(17.551)	(21.235)	(19.265)	**(24.489)**	(20.261)	(22.373)
ES 0.05	-0.123	-0.092	-0.099	-0.093	-0.096	**-0.089**	-0.095	-0.092
	(-)	(25.396)	(19.672)	(24.256)	(21.561)	**(27.910)**	(22.482)	(24.857)
ES 0.025	-0.160	-0.111	-0.124	-0.115	-0.122	**-0.108**	-0.119	-0.114
	(-)	(30.623)	(22.872)	(28.278)	(23.923)	**(32.769)**	(25.857)	(28.669)
ES 0.01	-0.224	-0.154	-0.182	-0.170	-0.179	**-0.147**	-0.176	-0.163
	(-)	(31.200)	(18.794)	(23.988)	(20.038)	**(34.490)**	(21.402)	(27.109)
Rank	-	3.4	9.7	5.3	7.6	2.2	5.7	3.3

of the Joe copula which replaces the rotated Gumbel copula. In contrast to the rotated Gumbel copula and the rotated Joe copula, the Joe copula only incorporates upper tail dependence. This suggests that lower tail dependence is a less important feature of the joint density for hedging the Ethereum. In addition, these results again emphasize that good fit is not equal to good hedging performance as the rotated Gumbel copula and Student-t copula, which fitted this data best on average, do not give the highest hedging effectivenesses.

As for the combination methods, the simplex weighted hedge gives only mediocre hedging results and rank at 5.7 on average. The best 3 average weighted hedge does better ranking at 3.3 on average, which is even lower than the DCC-GARCH hedge. However, this is only due to the DCC-GARCH performing relatively bad for the VaR at 5% and 2.5% level respectively. What is more, similar to the individual copula hedge strategies, both the simplex weighted hedge and the best 3 average weighted hedge are dominated by the DCC-GARCH benchmark hedge for all other risk measures. This suggests that these weighted density forecasts do not correctly capture the tail structure. Nonetheless, these hedges perform similar if not better than the best individual copula hedge which does show the increased robustness of these methods compared to the individual copula methods.

Interestingly, the more naive equally weighted combination method provides a better hedge and is even able to outcompete the DCC-GARCH hedge. That is, this hedge strategy attains the lowest average rank of 2.2 and achieves the highest hedge effectiveness for reducing the more extreme negative returns. A partial explanation as to why the equally weighted density forecast combination gives a better hedge than more advanced weighted density forecasts might be that those weights are based on the fit rather than hedging performance. In addition, the Ethereum and Bitcoin futures might be simply harder to model correctly, and for that reason the more naive equally weighted combination can give better hedging results than more advanced weighted combinations.

Turning to statistics on the hedge ratios in Table 7, the hedge ratios of the combination methods are considerably lower than those of the DCC-GARCH model. For example, the mean hedge ratio of the equally weighted combination is 0.563 and that of the DCC-GARCH is 0.808. This might provide a benefit for the equally weighted combination as this method requires a smaller short position in Bitcoin futures to hedge a long position in Ethereum. What is more, looking at the average returns in Table 6, the portfolio of the equally weighted combination hedge achieves an average daily return of 0.004 which is higher than the average portfolio return of the DCC-GARCH hedge which is just 0.002. Hence the equally weighted combination method seems to hedge the risk associated with the Ethereum more efficiently.

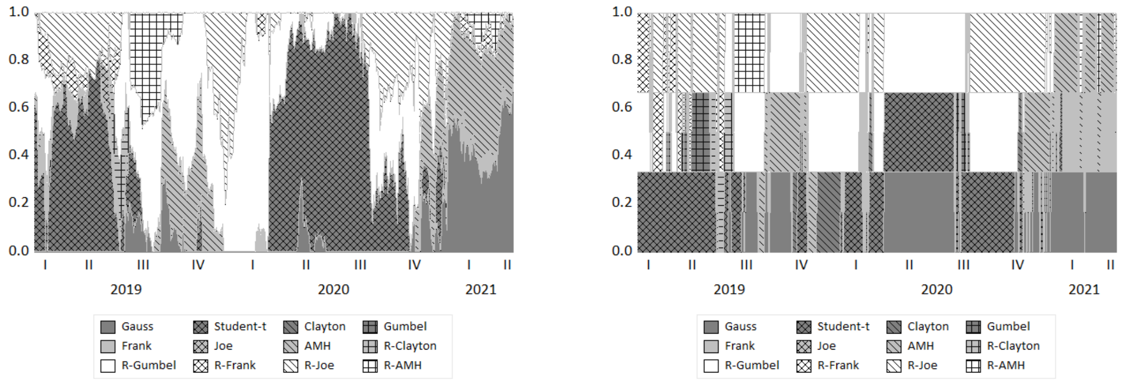
Table 7: Summary statistics of the hedge ratios of different hedge strategies for the Ethereum. The results are based on the period September 2018 - April 2021. For convenience only the best models are displayed in this table.

Method	DCC	Gauss	Joe	R-Joe	C-Equal	C-Simplex	C-Best 3
Mean	-0.808	-0.749	-0.560	-0.619	-0.563	-0.620	-0.614
Max	-0.204	0.000	-0.012	0.000	0.000	0.000	0.000
Min	-1.500	-1.500	-1.500	-1.500	-0.993	-1.098	-1.127
Turnover	0.072	0.116	0.112	0.106	0.092	0.089	0.099

However, this strategy also comes with higher transaction costs as indicated by the turnover. The equally weighted combination method comes with an average turnover of 0.092 and the DCC-GARCH with an average turnover of 0.072. This means that for each $1,000,000 in Bitcoin futures, the DCC-GARCH hedge costs approximately $27,000 per year and the equally weighted density forecast combination hedge costs $34,500 per year, which slightly favors the DCC-GARCH hedge.

Taken together, the equally weighted combination hedge provides a better hedge than the DCC-GARCH hedge in terms of reducing the most extreme losses, but comes with slightly higher transaction costs. Therefore the equally weighted combination hedge might have the edge over the DCC-GARCH hedge as it also achieves higher average return and a smaller short position in Bitcoin futures is required.

To get an idea about the relatively lackluster hedge performance of the alternative combination methods, the combining weights are displayed in Figure 7. Where on the one hand the density combination weights for the Bitcoin seemed quite stable over time, those for the Ethereum look more fluctuating. Similar to the Bitcoin however, the Student-t copula dominates the first two quarters of 2019 and the start of 2020. However, its dominance is less prominent. During these times, the Student-t copula is mainly mixed with the rotated Joe copula, which adds lower tail dependence to the density. At the end of 2019 and then end of 2020 on the other hand, many other copulas are being incorporated such as the rotated Gumbel and the Gaussian copula. This indicates that there are some clear change points in the dependence structure of the Ethereum and Bitcoin futures which coincides with periods where the Kendall's τ is lower. This means that at times where there is less dependence in the data, these copulas fit the data better and become more important for the combined density forecast, whereas in more stable times the Student-t copula often provides a good fit. Interestingly, the Joe copula, which gives the best hedge of the individual copulas methods, is not really incorporated in the combined density forecast at any point in time.



(a) Simplex

(b) Best 3 average

Figure 7: Weights assigned to the copula density forecasts for the Ethereum by the simplex method and the best 3 average method for the period September 2018 - April 2021

All in all, the relatively unstable nature of the mixed density forecast together with the omission of densities that provide good hedges such as the Joe copula, results in combined density forecasts that are not able to accurately capture those characteristics that are important for hedging.

5.3.3 Hedging Results Ripple

The second cross-hedge in this paper regards the Ripple and the hedging results are shown in Table 8. The effectiveness of this hedge is lower than that of Ethereum and even negative in some cases. In particular, the ES on 1% level is -4% for the rotated Joe copula and -3% for the best 3 average combination, implying that the taking a short position in Bitcoin futures only amplifies the most extreme losses. In addition, the minimum returns of all hedged portfolio are all lower than the -0.423 of the unhedged portfolio. The remaining hedge effectivenesses generally range between 0% and 15%. A possible explanation as to why these hedges perform relatively bad is that there is less dependence between the Ripple and the Bitcoin futures, as indicated by Kendall's τ in Section 5.2.

Table 8: Risk measures for evaluating the out-of-sample hedging performance for the Ripple portfolio using different hedge strategies. The hedging effectiveness is displayed between brackets. The results are based on the period September 2018 - April 2021. For convenience only the best models are shown in this table. In addition the average model ranks for the 10 risk measures is calculated. The best models are indicated in bold.

Method	Unhedged	OLS	Joe	R-Clayton	R-Joe	C-Equal	C-Simplex	C-Best 3
Mean	0.006	0.001	0.003	0.003	0.001	0.003	0.002	0.001
Max	0.872	0.883	0.882	0.881	0.884	0.879	0.884	0.885
Min	-0.423	-0.425	-0.424	-0.425	-0.427	-0.425	-0.425	-0.425
Variance	0.006	0.005	0.005	0.005	**0.005**	0.005	0.005	0.005
	(-)	(9.608)	(7.316)	(7.309)	**(9.651)**	(9.633)	(9.419)	(9.614)
Semivariance	0.058	**0.051**	0.053	0.053	0.054	0.051	0.052	0.054
	(-)	**(12.402)**	(7.833)	(8.681)	(7.531)	(12.160)	(10.842)	(7.225)
VaR 0.1	-0.054	-0.050	-0.054	-0.054	-0.048	-0.049	-0.050	-0.049
	(-)	(8.512)	(1.441)	(1.441)	(11.748)	(9.541)	(7.615)	(10.686)
VaR 0.05	-0.089	-0.075	-0.079	-0.080	-0.081	-0.076	**-0.074**	-0.076
	(-)	(15.337)	(11.248)	(9.894)	(9.230)	(14.845)	**(17.416)**	(14.112)
VaR 0.025	-0.118	**-0.111**	-0.112	-0.112	-0.116	-0.111	-0.116	-0.119
	(-)	**(5.409)**	(4.928)	(4.914)	(1.129)	(5.179)	(0.929)	-(1.502)
VaR 0.01	-0.161	-0.147	**-0.147**	-0.152	-0.155	-0.150	-0.149	-0.149
	(-)	(8.877)	**(9.101)**	(5.746)	(3.935)	(7.172)	(7.869)	(7.864)
ES 0.1	-0.106	-0.097	-0.100	-0.100	-0.102	**-0.095**	-0.098	-0.101
	(-)	(8.219)	(5.331)	(4.946)	(3.550)	**(9.700)**	(7.020)	(3.877)
ES 0.05	-0.144	-0.133	-0.136	-0.136	-0.142	**-0.130**	-0.137	-0.144
	(-)	(7.624)	(5.485)	(5.203)	(1.007)	**(9.863)**	(4.669)	(0.131)
ES 0.025	-0.181	-0.169	-0.174	-0.174	-0.182	**-0.169**	-0.174	-0.184
	(-)	(6.409)	(3.664)	(3.675)	-(0.280)	**(6.635)**	(3.765)	-(1.458)
ES 0.01	-0.262	-0.244	-0.257	-0.255	-0.273	-0.245	-0.253	-0.270
	(-)	(6.633)	(1.689)	(2.505)	-(4.211)	(6.335)	(3.438)	-(3.396)
Rank	-	2.7	6.6	7.9	8.5	3.1	5.5	8.6

The OLS hedge is the best benchmark model based on its average rank, thereby outperforming the DCC-GARCH hedge although by a small margin. It has an average rank of 2.7 and therefore outperforms the best individual copula hedges which are the Joe copula, rotated Clayton copula and rotated Joe copula. These copulas are similar to those for Ethereum and only the rotated Joe copula possesses lower tail dependence. This suggests that lower tail dependence is hardly present in the data, which provides another explanation as to why the different hedges do not perform well.

The hedge strategies for the Ripple do underline again the potential of the combined density forecast hedge strategies, as both the equally weighted method and the simplex weighted method lead to an improvement in hedge effectiveness based on average rank compared to the best individual copula hedges. In particular the naive and more robust equally weighted method displays a relatively strong hedging performance with an average ranking of 3.1 and relatively high hedge effectiveness for reducing large losses. What is more, this method is well able to compete with the OLS benchmark hedge in terms of the different risk measure. The hedging results of the best 3 average method on the other hand are rather disappointing with low and sometimes negative hedge effectiveness and therefore ranks at only 8.6 on average.

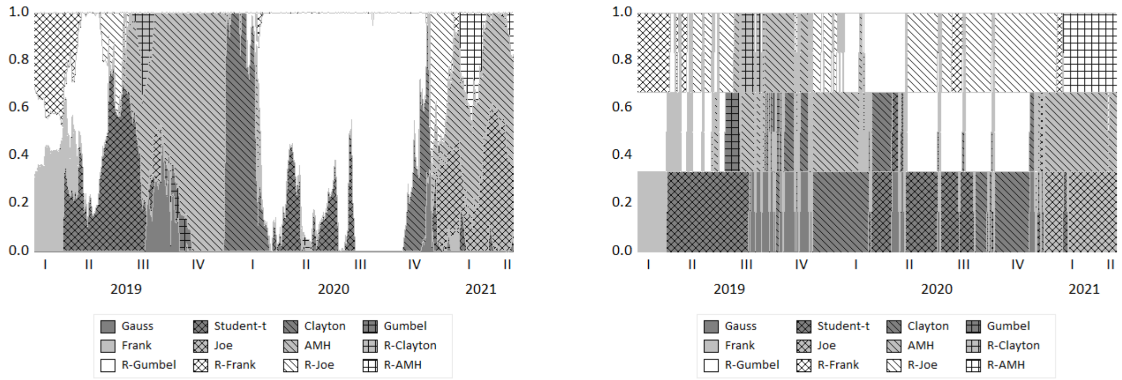
The average hedge ratios shown in Table 9 are around -0.3 to -0.6, which is considerably lower than those of the Bitcoin and Ethereum hedge strategies, which is presumably due to the low dependence. Comparing the turnovers of the OLS hedge and the equally weighted combination hedge, the latter does have a much higher turnover of 0.091 versus 0.025. This means that this hedge comes with slightly higher transaction costs. On the other hand, this hedge also comes with a lower mean hedge ratio and therefore the holder of the Ripple portfolio generally needs a smaller short position in Bitcoin futures when adopting this strategy, which in return also leads to higher average daily returns.

Table 9: Summary statistics of the hedge ratios of different hedge strategies for the Ripple. The results are based on the period September 2018 - April 2021. For convenience only the best models are displayed in this table.

Method	OLS	Joe	R-Clayton	R-Joe	C-Equal	C-Simplex	C-Best 3
Mean	-0.618	-0.337	-0.390	-0.566	-0.485	-0.578	-0.582
Max	-0.389	0.000	0.000	0.000	0.000	0.000	0.000
Min	-0.862	-1.500	-1.500	-1.500	-0.808	-1.500	-1.500
Turnover	0.025	0.100	0.106	0.110	0.091	0.116	0.120

All in all, it remains the question whether hedging the Ripple using Bitcoin futures is worth pursuing at all as the hedge effectiveness is rather limited. However, if one decides to perform this hedge, the equally weighted combination hedge strategy might be worth taking into consideration as it is well able to compete with the OLS benchmark hedge both in terms of reducing extreme losses and less extreme losses, and in addition requires a smaller short position on average resulting in higher average daily returns. However, this hedge also comes with significantly higher transaction costs.

Finally, the weights of the combination strategies are shown in Figure 9, where again some clear changing points for the density can be observed. In 2019 several different copulas make up the combined density forecasts including the Student-t copula, rotated Gumbel copula, AMH copula, and Clayton copula. At the start of the corona pandemic on the other hand, the rotated Gumbel copula clearly starts to dominate as the huge losses at the start of this period probably require lower tail dependence which the rotated Gumbel copula is able to incorporate. Finally, at the end of 2020 when the Ripple becomes very volatile as shown in Section 5.1, other copulas start do dominate the joint density again. The lack of consistency during the first and the last period where the construction of the joint density differs frequently might be one of the reasons for the worse hedging results of these more advanced weighting schemes.



(a) Simplex **(b)** Best 3 average

Figure 9: Weights assigned to the copula density forecasts for the Ripple by the simplex method and the best 3 average method for the period September 2018 - April 2021

5.3.4 Hedging Results Cardano

The third cross-hedge considered in this paper involves the Cardano and Bitcoin futures. The general risk reduction is somewhat stronger than for Ripple, even though Kendall's τ is usually found to be lower. The hedge effectiveness mostly ranges between 10% and 30%.

The DCC-GARCH hedge is again the best benchmark model. Nonetheless, it has an average rank of 7.1, which is higher than the hedges resulting from the Clayton copula and rotated Joe copula which rank at 6.7 and 6.0 on average respectively. However, it still seems to outperform those individual copula methods for hedging against extreme losses as for example it achieves a hedge effectiveness of 26% compared to 17% to 19% for the ES on 1% level. As for the individual copulas methods, both the Clayton and rotated Joe copula posses lower tail dependence, which therefore suggests that lower tail dependence is present in the data and is important for hedging.

Overall, the density forecast combination hedge strategies perform best. The equally weighted density forecast hedge strategy ranks 5.1 on average which is lower than the best individual copula hedge, and the simplex weighted and best 3 average weighted methods perform even better with average ranks of 2.2 and 1.7 respectively. Moreover, their respective hedge effectivenesses actually are significantly higher than those of the DCC-GARCH benchmark hedge and the individual copula hedges with improvements between 5% and 15% for several risk measures. In contrast to the hedge results for the Ethereum and Ripple, this also shows that weighting schemes based on the overall fit can benefit the hedging performance compared to a more naive equal weighting scheme.

Table 10: Risk measures for evaluating the out-of-sample hedging performance for the Cardano portfolio using different hedge strategies. The hedging effectiveness is displayed between brackets. The results are based on the period September 2018 - April 2021. For convenience only the best models are shown in this table. In addition the average model ranks for the 10 risk measures is calculated. The best models are indicated in bold.

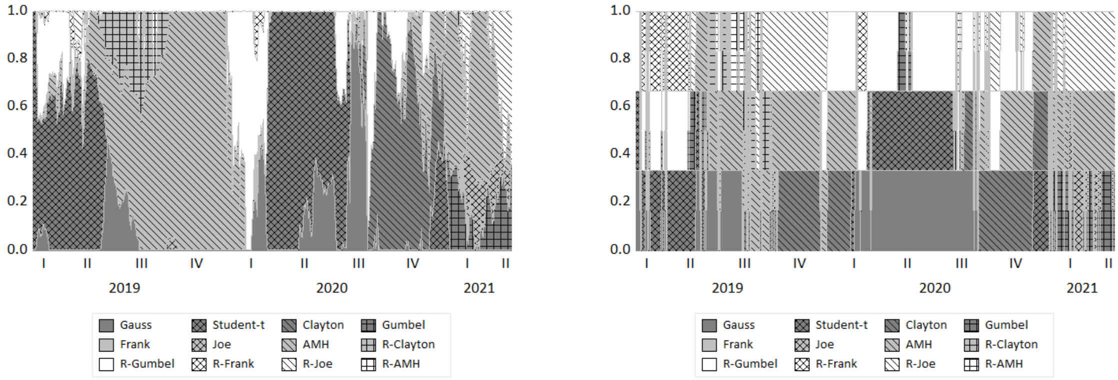
Method	Unhedged	DCC	Clayton	AMH	R-Joe	C-Equal	C-Simplex	C-Best 3
Mean	0.008	0.003	0.003	0.003	0.003	0.005	0.004	0.004
Max	0.322	0.352	0.364	0.361	0.362	0.356	0.361	0.362
Min	-0.396	-0.219	-0.303	-0.308	-0.307	-0.247	-0.235	-0.251
Variance	0.005	0.004	0.004	0.004	0.004	0.004	**0.004**	0.004
	(-)	(21.031)	(21.658)	(20.800)	(21.563)	(21.310)	**(23.819)**	(23.830)
Semivariance	0.061	0.049	0.051	0.051	0.051	0.050	**0.048**	0.049
	(-)	(18.799)	(15.591)	(15.351)	(15.712)	(17.213)	**(20.195)**	(19.831)
VaR 0.1	-0.063	-0.058	-0.061	-0.064	-0.062	-0.061	-0.058	**-0.057**
	(-)	(7.529)	(3.802)	-(1.173)	(1.249)	(2.999)	(7.608)	**(9.231)**
VaR 0.05	-0.089	-0.082	-0.080	-0.079	-0.079	-0.079	-0.079	-0.078
	(-)	(8.359)	(10.501)	(11.171)	(11.717)	(11.131)	(11.681)	(12.645)
VaR 0.025	-0.117	-0.104	-0.098	-0.098	-0.101	-0.100	**-0.092**	-0.093
	(-)	(11.165)	(16.368)	(16.459)	(13.662)	(14.665)	**(20.933)**	(20.848)
VaR 0.01	-0.155	-0.130	-0.123	-0.127	**-0.117**	-0.128	-0.125	-0.121
	(-)	(15.832)	(20.693)	(17.965)	**(23.998)**	(17.079)	(19.310)	(21.481)
ES 0.1	-0.105	-0.090	-0.093	-0.092	-0.092	-0.089	**-0.086**	-0.086
	(-)	(13.847)	(11.603)	(12.046)	(12.577)	(15.320)	**(18.132)**	(17.806)
ES 0.05	-0.135	-0.112	-0.115	-0.114	-0.113	-0.110	-0.105	**-0.104**
	(-)	(16.990)	(15.198)	(15.467)	(16.353)	(18.762)	(22.536)	**(22.830)**
ES 0.025	-0.170	-0.134	-0.139	-0.140	-0.137	-0.132	-0.124	**-0.123**
	(-)	(21.130)	(17.995)	(17.755)	(19.089)	(22.441)	(27.185)	**(27.372)**
ES 0.01	-0.224	-0.165	-0.186	-0.183	-0.181	-0.160	**-0.153**	-0.155
	(-)	(26.199)	(16.947)	(18.441)	(19.259)	(28.548)	**(31.853)**	(30.914)
Rank	-	7.1	6.7	8.4	6.0	5.1	2.2	1.7

Table 11 shows that the turnover of DCC-GARCH is lower than those of the combination hedge strategies, whereas its mean hedge ratio is higher. However, given the significant improvements in terms of hedge effectiveness, the combined density forecast hedge strategies, and in particular the simplex weighted and best 3 average combined density forecast hedge strategies, are recommended for those investors who wish to hedge their position in Cardano with Bitcoin futures.

Table 11: Summary statistics of the hedge ratios of different hedge strategies for the Cardano. The results are based on the period September 2018 - April 2021. For convenience only the best models are displayed in this table.

Method	DCC	Clayton	AMH	R-Joe	C-Equal	C-Simplex	C-Best 3
Mean	-0.767	-0.653	-0.588	-0.638	-0.568	-0.636	-0.638
Max	-0.078	0.000	0.000	0.000	0.000	0.000	0.000
Min	-1.500	-1.500	-1.500	-1.500	-1.056	-1.500	-1.500
Turnover	0.082	0.106	0.125	0.110	0.107	0.105	0.093

The weights of the simplex combination and the best 3 average combination are displayed in Figure 11. The usual regime switch can be seen at the start of 2020. The weights are in general relatively stable before the third quarter of 2020 after which the weights start to vary. This again coincides with the period with higher volatility and lower dependence.



(a) Simplex

(b) Best 3 average

Figure 11: Weights assigned to the copula density forecasts for the Cardano by the simplex method and the best 3 average method for the period September 2018 - April 2021

5.3.5 Hedging Results Litecoin

The last cross-hedge in this paper regards the Litecoin. This hedge actually proves to be relatively effective as shown in Table 12, with hedge effectiveness ranging between 20% and 40% for the best performing methods, with the exception of the lower levels VaR. This was to be expected as its dependence with the Bitcoin futures as indicated by Kendall's τ is relatively high compared to other alternative cryptocurrencies.

The DCC-GARCH hedge is again the best benchmark and attains an average rank of 5.1. This is better than the best individual copula hedge strategies. The best individual copulas include the Clayton, rotated Gumbel and rotated Joe copula with respective average ranks of 6.1, 7.7 and 7.1. All three copulas exhibit lower tail dependence. The DCC-GARCH hedge provides the best hedge for reducing the largest losses with hedge effectiveness of 31% and 39% for the ES at 2.5% and 1% level. However, the DCC-GARCH hedge achieves a negative portfolio return on average which means that this hedged portfolio costs the holder.

Interestingly, the simplex weighted combination method has a relatively lackluster performance given its average rank of 7.6. This is mainly due to it being unable to reduce the more extreme losses as it achieves a hedge effectiveness of only 17% and 25% for the VaR and ES on 1% level respectively. The other combination hedge strategies perform relatively well based on their average ranks. That is, the equally weighted method and the best 3 average weighted method achieve lower average ranks than the DCC-GARCH benchmark. Besides, they do not yield negative daily returns as the DCC-GARCH hedge does, but at the cost of being less capable for reducing the most extreme losses. This probably is the result of the relatively high mean hedge ratio of the DCC-GARCH hedge

Table 12: Risk measures for evaluating the out-of-sample hedging performance for the Litecoin portfolio using different hedge strategies. The hedging effectiveness is displayed between brackets. The results are based on the period September 2018 - April 2021. For convenience only the best models are shown in this table. In addition the average model ranks for the 10 risk measures is calculated. The best models are indicated in bold.

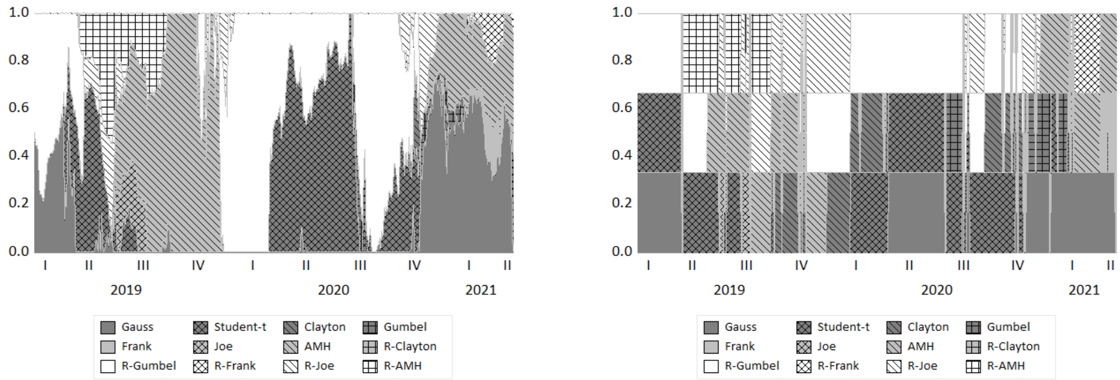
Method	Unhedged	DCC	Clayton	R-Gumbel	R-Joe	C-Equal	C-Simplex	C-Best 3
Mean	0.005	-0.001	0.000	-0.001	0.000	0.002	0.000	0.001
Max	0.267	0.352	0.220	0.233	0.240	0.269	0.248	0.270
Min	-0.362	-0.154	-0.234	-0.231	-0.263	-0.193	-0.229	-0.210
Variance	0.003	**0.002**	0.002	0.002	0.002	0.002	0.002	0.002
	(-)	**(37.247)**	(35.534)	(36.522)	(35.033)	(35.414)	(36.838)	(36.735)
Semivariance	0.056	0.042	0.044	0.043	0.044	**0.042**	0.044	0.043
	(-)	(24.438)	(21.433)	(22.676)	(20.881)	**(25.028)**	(21.584)	(23.618)
VaR 0.1	-0.055	-0.053	-0.051	-0.050	-0.051	**-0.048**	-0.050	-0.051
	(-)	(2.452)	(7.219)	(7.765)	(7.476)	**(11.272)**	(7.917)	(7.484)
VaR 0.05	-0.082	-0.076	-0.075	-0.076	-0.077	-0.075	-0.076	-0.077
	(-)	(7.306)	(8.410)	(7.734)	(6.329)	(8.897)	(7.409)	(6.813)
VaR 0.025	-0.122	-0.095	-0.087	-0.101	-0.088	-0.091	-0.088	**-0.086**
	(-)	(22.438)	(28.858)	(17.014)	(28.037)	(25.240)	(27.796)	**(29.020)**
VaR 0.01	-0.157	**-0.113**	-0.127	-0.126	-0.120	-0.119	-0.129	-0.114
	(-)	**(28.054)**	(19.014)	(19.557)	(23.504)	(24.077)	(17.482)	(27.229)
ES 0.1	-0.101	-0.082	-0.083	-0.084	-0.084	**-0.079**	-0.083	-0.080
	(-)	(18.533)	(17.878)	(16.497)	(17.132)	**(21.758)**	(17.834)	(20.321)
ES 0.05	-0.135	-0.101	-0.103	-0.106	-0.104	-0.100	-0.105	**-0.100**
	(-)	(25.242)	(23.169)	(21.356)	(22.598)	(25.614)	(21.988)	**(25.689)**
ES 0.025	-0.167	**-0.116**	-0.125	-0.126	-0.125	-0.119	-0.128	-0.119
	(-)	**(30.756)**	(25.318)	(24.880)	(25.116)	(28.968)	(23.248)	(29.058)
ES 0.01	-0.221	**-0.135**	-0.155	-0.153	-0.162	-0.145	-0.166	-0.153
	(-)	**(38.911)**	(29.729)	(30.816)	(26.654)	(34.257)	(24.760)	(30.767)
Rank	-	5.1	6.1	7.7	7.1	3.2	7.6	3.8

of -0.833. In addition, the DCC-GARCH hedge has lower turnover. For example, in terms of average yearly transaction costs, the DCC-GARCH hedge strategy costs approximately $27,375 per $1,000,000 in Bitcoin futures, whereas the equally weighed density forecast hedge costs approximately $36,375. All in all, the DCC-GARCH hedge and the combined density forecast hedges both have their benefits and drawbacks.

Table 13: Summary statistics of the hedge ratios of different hedge strategies for the Litecoin. The results are based on the period September 2018 - April 2021. For convenience only the best models are displayed in this table.

Method	DCC	Clayton	R-Gumbel	R-Joe	C-Equal	C-Simplex	C-Best 3
Mean	-0.833	-0.628	-0.710	-0.619	-0.556	-0.672	-0.620
Max	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Min	-1.500	-1.500	-1.500	-1.500	-0.983	-1.500	-1.500
Turnover	0.073	0.108	0.121	0.098	0.097	0.120	0.111

Figure 13 shows that quite a few different density forecasts are incorporated at the start of the sample in the simplex weighted density forecast, after which the AMH copula starts to dominate. At the start of 2020, the rotated Gumbel receives most of the weight and is then mixed with the Student-t copula, thereby again suggesting lower tail dependence during this period. Finally in 2021, the Gaussian copula together with the AMH copula makes up most of the mixture density. Compared to the simplex weights the best 3 average weights seems to incorporate the rotated Gumbel and Student-t copula more frequently. Therefore, this method exhibits more lower tail dependence during these periods, which has shown to be an important feature for hedging the Ripple and might explain why this method performs better.



(a) Simplex **(b)** Best 3 average

Figure 13: Weights assigned to the copula density forecasts for the Litecoin by the simplex method and the best 3 average method for the period September 2018 - April 2021

6 Conclusion

In this paper the daily hedging capabilities of the Bitcoin futures are investigated. Moreover, as different cryptocurrencies are shown to be very related, both its hedging potential as a direct hedge on the Bitcoin as well as its hedging potential as a cross-hedge on alternative cryptocurrencies is evaluated. These cryptocurrencies include the Ethereum, Ripple, Cardano and Litecoin. Daily price data is considered from the introduction of the Bitcoin futures in December 2017 to April 2021.

Moreover, this paper is the first to investigate the hedge effectiveness of the Bitcoin futures on different cryptocurrencies using copulas. Previous papers on this topic, such as Sebastião & Godinho (2020), adopted more simplistic hedge strategies like the one-to-one hedge, the OLS hedge and the DCC-GARCH hedge. However, copulas might give a more accurate representation of the underlying dependence structure of the particular cryptocurrency and the Bitcoin future as their flexibility can allow for certain data characteristics such as tail dependence, which might positively impact the hedging performance. Therefore, different bivariate copulas, such as the Gaussian copula, Student-t copula and several Archimedean copulas and their rotated forms, have been used to model this dependence. Additionally, this paper innovates on the copula hedges by combining the simulated copula density forecasts to create more robust forecasted densities and with that more robust hedges. For that purpose several different weighting schemes have been considered based on past out-of-sample fit of the combined density forecasts. These weighting schemes include the equal weights, optimal simplex weights and best 3 average weights as described by Diebold et al. (2021).

Similar to Sebastião & Godinho (2020), the hedging results confirm that Bitcoin futures are effective tools for reducing the risk of Bitcoin portfolios. Different from Sebastião & Godinho (2020) however, this paper finds evidence that the Bitcoin futures are also able to reduce the tail risk for portfolios of alternative cryptocurrencies.

Interestingly, this paper finds that the OLS hedge provides the best benchmark for the Bitcoin, whereas the DCC-GARCH hedge often provides the best benchmark for alternative cryptocurrencies. This is probably due to the dependence structure of the Bitcoin and Bitcoin futures being relatively stable while that of the alternative cryptocurrencies is changing over time. The individual copulas generally have a hard time beating these two benchmark models. However, combining the density forecasts of the different copulas does lead to promising results. Although for the Bitcoin these hedge strategies are not able to outperform the OLS hedge and in addition have higher transaction costs, they do outperform the individual copula hedges. Moreover, for the alternative cryptocurrencies these combined density forecasts hedge strategies do often prove to be more effective than the OLS and DCC-GARCH benchmarks, and additionally require lower hedge ratios on average which means hat the holder of these cryptocurrencies generally needs to take a smaller short position to reduce its portfolio risk. However, these strategies do require slightly higher rebalancing costs. Moreover, the results show that weights based on out-of-sample fit can provide a good way to construct the joint density forecast and allows for incorporating important data characteristics for hedging such as tail dependence. In addition, these combined density forecasts are able to incorporate some regime switching as during different economic times different copula densities are integrated. For example, the Student-t copula gives a good overall fit for the joint density during stable economic periods and therefore is often incorporated in the combined density forecasts during these periods. However, when there is more uncertainty other copulas, such as the rotated Gumbel copula, which exhibit lower tail dependence are often incorporated.

However, the results found in this paper depend on optimization with respect to a certain risk measure of a sample of returns that is simulated from a copula, where for computational purposes the number of simulated observations is set to 10,000. Therefore it might be the case that by chance a sample of returns is simulated that does not give an accurate representation of the tail behaviour which can influence the estimation of the hedge ratio. When this happens at a time of large negative returns, this might impact the hedge effectiveness significantly.

In addition, as the Bitcoin futures are a relatively new hedging instrument, the amount of data is relatively small. In particular, the out-of-sample period in this paper consists of approximately 30 months, and therefore the hedging results found here might not be representative for different periods of time. This issue might be further amplified by the fact that this period consists for a large part of a time where the corona virus greatly impacts the economy, which might bias the results. Furthermore, there is a time gap of three hours between the timing of the spot prices and the timing of the futures prices. This might result in some inconsistencies especially since these cryptocurrencies are known to be very volatile and their prices can go up and down significantly in the matter of only hours, which as a consequence can negatively impact the hedging results.

Concludingly, this paper contributes to the literature by investigating whether Bitcoin futures are viable hedge instruments for different cryptocurrencies and constructing minimum tail risk hedge ratios using (combinations of) copulas. Similar to Sebastião & Godinho (2020), this paper finds that Bitcoin futures are able to reduce risk for portfolios of cryptocurrencies. Moreover, the combined copula density forecasts proposed in this paper are generally more effective for hedging alternative cryptocurrencies than the OLS hedge and the DCC-GARCH hedge adopted by Sebastião & Godinho (2020). Therefore these techniques might be interesting for firms and investors who wish to hedge against exchange risk in their cryptocurrency portfolio.

For future research, it would be interesting to see whether copula density forecast combination methods can also be effective for hedging other assets such as oil or regular currencies. Further research could also focus on finding whether alternative weighting schemes such as some of the alternatives described by Diebold et al. (2021) can improve hedging performance even more.

7 Bibliography

Ali, M. M., Mikhail, N., & Haq, M. S. (1978). A class of bivariate distributions including the bivariate logistic. *Journal of multivariate analysis*, *8*(3), 405–412.

Alvarez-Ramirez, J., Rodriguez, E., & Ibarra-Valdez, C. (2018). Long-range correlations and asymmetry in the bitcoin market. *Physica A: Statistical Mechanics and its Applications*, *492*, 948–955.

Ardia, D., Bluteau, K., & Rüede, M. (2019). Regime changes in bitcoin garch volatility dynamics. *Finance Research Letters*, *29*, 266–271.

Awudu, I., Wilson, W., & Dahl, B. (2016). Hedging strategy for ethanol processing with copula distributions. *Energy Economics*, *57*, 59–65.

Bates, J. M., & Granger, C. W. (1969). The combination of forecasts. *Journal of the Operational Research Society*, *20*(4), 451–468.

BitMEX. (2021). Retrieved from `https://www.bitmex.com/app/fees#Other-Fees`

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, *31*(3), 307–327.

Bollerslev, T. (1990). Modelling the coherence in short-run nominal exchange rates: a multivariate generalized arch model. *The review of economics and statistics*, 498–505.

Bouri, E., Molnár, P., Azzi, G., Roubaud, D., & Hagfors, L. I. (2017). On the hedge and safe haven properties of bitcoin: Is it really more than a diversifier? *Finance Research Letters*, *20*, 192–198.

Chang, C.-L., McAleer, M., & Tansuchat, R. (2011). Crude oil hedging strategies using dynamic multivariate garch. *Energy Economics*, *33*(5), 912–923.

Chen, & Fan, Y. (2005). Pseudo-likelihood ratio tests for semiparametric multivariate copula model selection. *Canadian Journal of Statistics*, *33*(3), 389–414.

Chen, Fan, Y., & Tsyrennikov, V. (2006). Efficient estimation of semiparametric multivariate copula models. *Journal of the American Statistical Association*, *101*(475), 1228–1240.

Chen, Wilson, W., Larsen, R., & Dahl, B. (2016). Risk management for grain processors and “copulas”. *Canadian Journal of Agricultural Economics/Revue canadienne d’agroeconomie*, *64*(2), 365–382.

Clayton, D. G. (1978). A model for association in bivariate life tables and its application in epidemiological studies of familial tendency in chronic disease incidence. *Biometrika*, *65*(1), 141–151.

Conrad, C., Custovic, A., & Ghysels, E. (2018). Long-and short-term cryptocurrency volatility components: A garch-midas analysis. *Journal of Risk and Financial Management*, *11*(2), 23.

Corbet, S., Lucey, B., Peat, M., & Vigne, S. (2018). Bitcoin futures—what use are they? *Economics Letters*, *172*, 23–27.

Diebold, F. X., & Shin, M. (2019). Machine learning for regularized survey forecast combination: Partially-egalitarian lasso and its derivatives. *International Journal of Forecasting*, *35*(4), 1679–1691.

Diebold, F. X., Shin, M., & Zhang, B. (2021). On the aggregation of probability assessments: Regularized mixtures of predictive densities for eurozone inflation and real interest rates.

Dorey, M., & Joubert, P. (2005). Modelling copulas: an overview. *The Staple Inn Actuarial Society*, 1–27.

Dyhrberg, A. H. (2016). Hedging capabilities of bitcoin. is it the virtual gold? *Finance Research Letters*, *16*, 139–144.

Embrechts, P., McNeil, A., & Straumann, D. (2002). Correlation and dependence in risk management: properties and pitfalls. *Risk management: value at risk and beyond*, *1*, 176–223.

Embrechts, P., Mcneil, E., & Straumann, D. (1999). Correlation: pitfalls and alternatives. In *Risk magazine*.

Engle, R. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. *Econometrica: Journal of the econometric society*, 987–1007.

Engle, R. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics*, *20*(3), 339–350.

Fishburn, P. C. (1977). Mean-risk analysis with risk associated with below-target returns. *The American Economic Review*, *67*(2), 116–126.

Genest, C., Ghoudi, K., & Rivest, L.-P. (1995). A semiparametric estimation procedure of dependence parameters in multivariate families of distributions. *Biometrika*, *82*(3), 543–552.

Genest, C., Quessy, J.-F., & Rémillard, B. (2006). Goodness-of-fit procedures for copula models based on the probability integral transformation. *Scandinavian Journal of Statistics*, *33*(2), 337–366.

Geweke, J., & Amisano, G. (2011). Optimal prediction pools. *Journal of Econometrics*, *164*(1), 130–141.

Ghalanos, A. (2014). rugarch: Univariate garch models. [Computer software manual]. (R package version 1.4-0.)

Ghalanos, A. (2019). rmgarch: Multivariate garch models. [Computer software manual]. (R package version 1.3-6.)

Glosten, L. R., Jagannathan, R., & Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *The journal of finance*, *48*(5), 1779–1801.

Good, I. J. (1992). Rational decisions. In *Breakthroughs in statistics* (pp. 365–377). Springer.

Haigh, M. S., & Holt, M. T. (2002). Crack spread hedging: Accounting for time-varying volatility spillovers in the energy futures markets. *Journal of Applied Econometrics*, *17*(3), 269–289.

Hale, G., Krishnamurthy, A., Kudlyak, M., Shultz, P., et al. (2018). How futures trading changed bitcoin prices. *FRBSF Economic Letter*, *12*, 1–5.

Hattori, T., & Ishida, R. (2021). Did the introduction of bitcoin futures crash the bitcoin market at the end of 2017? *The North American Journal of Economics and Finance*, *56*, 101322.

Hofert, M., Kojadinovic, I., Maechler, M., & Yan, J. (2020). copula: Multivariate dependence with copulas [Computer software manual]. Retrieved from `https://CRAN.R-project.org/package=copula` (R package version 1.0-1)

Hsu, C.-C., Tseng, C.-P., & Wang, Y.-H. (2008). Dynamic hedging with futures: A copula-based garch model. *Journal of Futures Markets: Futures, Options, and Other Derivative Products*, *28*(11), 1095–1116.

Jarque, C. M., & Bera, A. K. (1980). Efficient tests for normality, homoscedasticity and serial independence of regression residuals. *Economics letters*, *6*(3), 255–259.

Ji, Q., & Fan, Y. (2011). A dynamic hedging approach for refineries in multiproduct oil markets. *Energy*, *36*(2), 881–887.

Joe, H. (1997). *Multivariate models and multivariate dependence concepts.* CRC Press.

Junker, M., & May, A. (2005). Measurement of aggregate risk with copulas. *The Econometrics Journal*, *8*(3), 428–454.

Kendall, M. G. (1948). Rank correlation methods.

Kovach, S. (2021). Tesla buys 1.5 billion dollars in bitcoin, plans to accept it as payment. Retrieved from `https://www.cnbc.com/2021/02/08/tesla-buys-1point5-billion-in-bitcoin.html`

Lai, Y., Chen, C. W., & Gerlach, R. (2009). Optimal dynamic hedging via copula-threshold-garch models. *Mathematics and Computers in Simulation*, *79*(8), 2609–2624.

Lien, D., Tse, Y. K., & Tsui, A. K. (2002). Evaluating the hedging performance of the constant-correlation garch model. *Applied Financial Economics*, *12*(11), 791–798.

Nelsen, R. B. (2007). *An introduction to copulas.* Springer Science & Business Media.

Okex. (2021). Retrieved from `https://www.okex.com/en/fees.html`

Okorie, D. I., & Lin, B. (2020). Crude oil price and cryptocurrencies: Evidence of volatility connectedness and hedging strategy. *Energy Economics*, *87*, 104703.

Osterrieder, J., & Lorenz, J. (2017). A statistical risk assessment of bitcoin and its extreme tail behavior. *Annals of Financial Economics*, *12*(01), 1750003.

Patton, A. J. (2006). Modelling asymmetric exchange rate dependence. *International economic review*, *47*(2), 527–556.

Pearson, K. (1895). *Notes on regression and inheritance in the case of two parents proceedings of the royal society of london, 58, 240-242.* ed.

Phillip, A., Chan, J. S., & Peiris, S. (2018). A new look at cryptocurrencies. *Economics Letters*, *163*, 6–9.

Power, G. J., & Vedenov, D. (2010). Dealing with downside risk in a multi-commodity setting: A case for a “texas hedge”? *Journal of Futures Markets: Futures, Options, and Other Derivative Products*, *30*(3), 290–304.

R Core Team. (2020). R: A language and environment for statistical computing [Computer software manual]. Vienna, Austria. Retrieved from `https://www.R-project.org/`

Scaillet, O., & Fermanian, J.-D. (2002). Nonparametric estimation of copulas for time series. *FAME Research paper*(57).

Sebastião, H., & Godinho, P. (2020). Bitcoin futures: An effective tool for hedging cryptocurrencies. *Finance Research Letters*, *33*, 101230.

Sklar, M. (1959). Fonctions de repartition an dimensions et leurs marges. *Publ. inst. statist. univ. Paris*, *8*, 229–231.

Spearman, C. (1904). Proof and measurement of association between two things. *American Journal of Psychology*.

Sukcharoen, K., & Leatham, D. J. (2017). Hedging downside risk of oil refineries: A vine copula approach. *Energy Economics*, *66*, 493–507.

Šurda, P. (2014). The origin, classification and utility of bitcoin. *Classification and Utility of Bitcoin (May 14, 2014)*.

Tibshirani, R. (1996). Regression shrinkage and selection via the lasso. *Journal of the Royal Statistical Society: Series B (Methodological)*, *58*(1), 267–288.

Tse, Y. K., & Tsui, A. K. C. (2002). A multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations. *Journal of Business & Economic Statistics*, *20*(3), 351–362.

Van den Goorbergh, R. W., Genest, C., & Werker, B. J. (2005). Bivariate option pricing using dynamic copula models. *Insurance: Mathematics and Economics*, *37*(1), 101–114.

Wang, G., Tang, Y., Xie, C., & Chen, S. (2019). Is bitcoin a safe haven or a hedging asset? evidence from china. *Journal of Management Science and Engineering*, *4*(3), 173–188.

Wilcoxon, F. (1992). Individual comparisons by ranking methods. In *Breakthroughs in statistics* (pp. 196–202). Springer.

Winkler, R. L., & Murphy, A. H. (1968). “good” probability assessors. *Journal of Applied Meteorology and Climatology*, *7*(5), 751–758.

YahooFinance. (2021). Retrieved from `https://finance.yahoo.com/`

Zhang, Y., & Choudhry, T. (2015). Forecasting the daily dynamic hedge ratios by garch models: evidence from the agricultural futures markets. *The European Journal of Finance*, *21*(4), 376–399.

8 Appendix

8.1 Data Characteristics

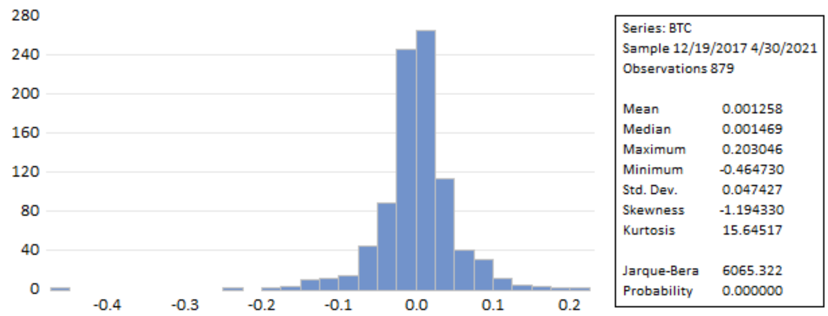


Figure 15: Histogram and summary statistics of the Bitcoin log-returns

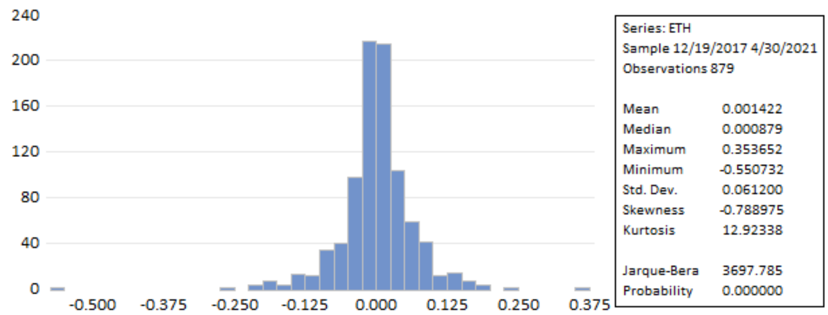


Figure 16: Histogram and summary statistics of the Ethereum log-returns

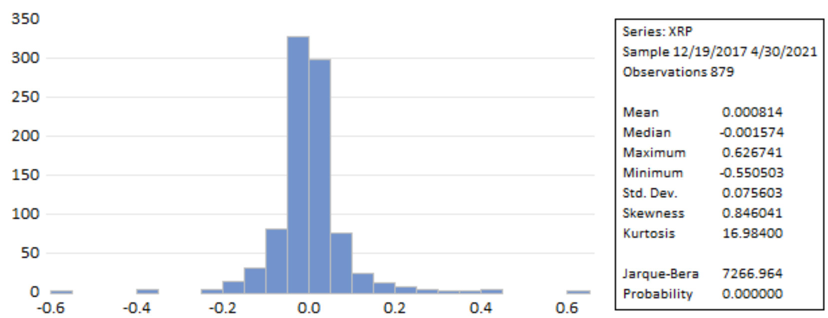


Figure 17: Histogram and summary statistics of the Ripple log-returns

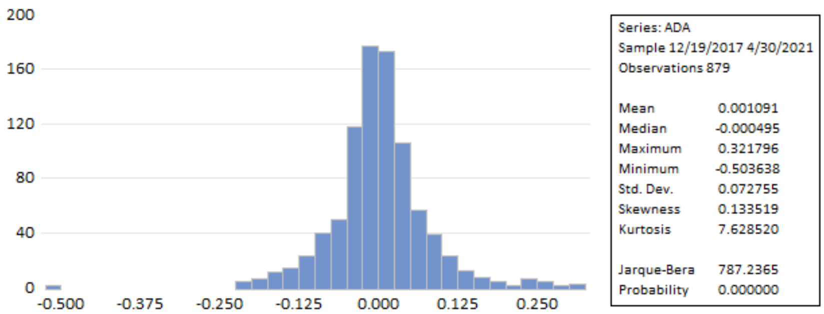


Figure 18: Histogram and summary statistics of the Cardano log-returns

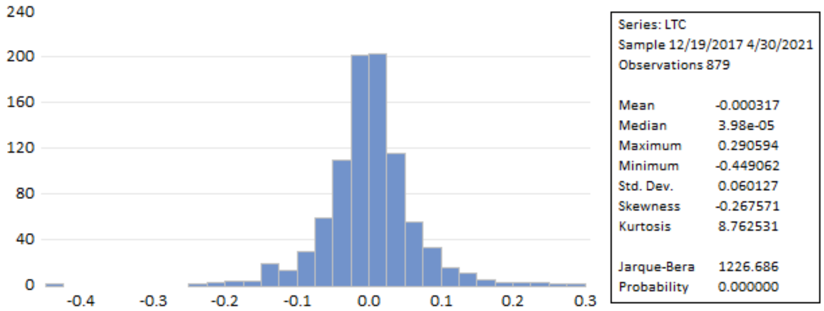


Figure 19: Histogram and summary statistics of the Litecoin log-returns

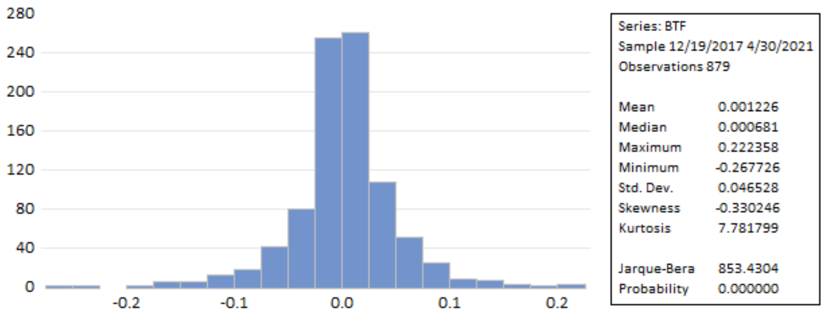


Figure 20: Histogram and summary statistics of the Bitcoin futures log-returns

8.2 Copula Tail Dependence and Dependence Structure

One of the characteristics some copulas are able to capture in the data is tail dependence. Opposed to regular dependence measures such as correlation, tail dependence solely focuses on the amount of dependence when extreme events occur. In the case of asset returns this means that upper tail dependence measures the amount of comovement between the assets whenever large positive returns occur and similarly lower tail dependence measures the amount of comovement between asset returns whenever large negative returns occur. The tail dependence can be calculated for different copulas as

$$\lambda_L = \lim_{q\to 0} \frac{C(q,q)}{q} \tag{22}$$

$$\lambda_U = 2 - \lim_{q\to 0} \frac{1 - C(1-q, 1-q)}{q} \tag{23}$$

where λ_L denotes the lower tail dependence and λ_U denotes the upper tail dependence. The tail dependence of several well-known copula is shown in the table below. In addition, the figures below show simulated samples of pseudo observations for the different copulas with a Kendall's τ of 0.6.

Table 14: Tail dependence imposed by the different copulas considered in this paper. λ_L denotes the lower tail dependence and λ_U denotes the upper tail dependence.

Copula	λ_L	λ_U
Gaussian	0	0
Student-t	$2\ t_{\nu+1}(\sqrt{\nu+1}\frac{\sqrt{1-\rho}}{\sqrt{1+\rho}})$	$2\ t_{\nu+1}(\sqrt{\nu+1}\frac{\sqrt{1-\rho}}{\sqrt{1+\rho}})$
Clayton	$2^{-\frac{1}{\theta}}$	0
Gumbel	0	2 - $2^{\frac{1}{\theta}}$
Frank	0	0
Joe	0	2 - $2^{\frac{1}{\theta}}$
AMH	0	0
R-Clayton	0	$2^{-\frac{1}{\theta}}$
R-Gumbel	2 - $2^{\frac{1}{\theta}}$	0
R-Frank	0	0
R-Joe	2 - $2^{\frac{1}{\theta}}$	0
R-AMH	0	0

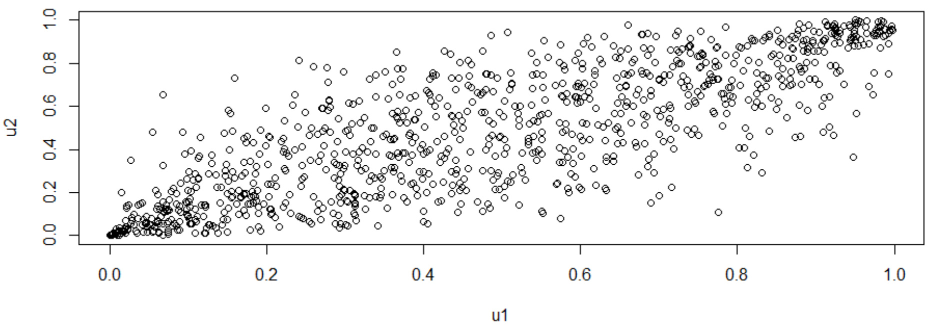


Figure 21: Simulated pseudo-sample from the Gaussian copula with $\tau = 0.6$

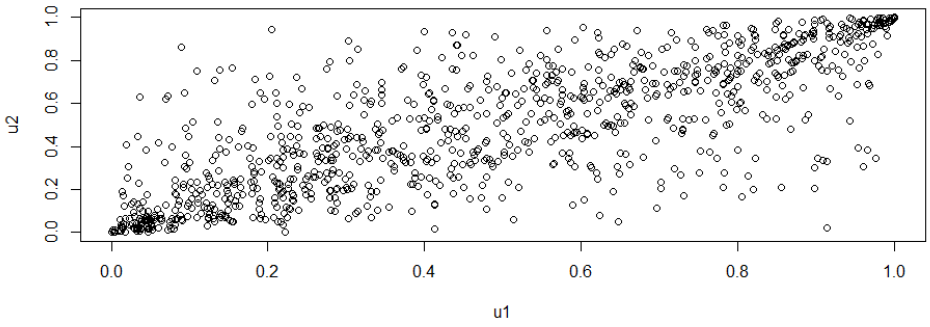


Figure 22: Simulated pseudo-sample from the Student-t copula with $\tau = 0.6$

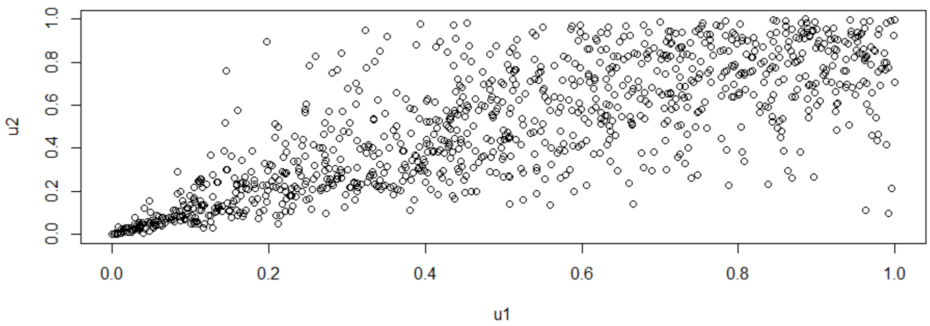


Figure 23: Simulated pseudo-sample from the Clayton copula with $\tau = 0.6$

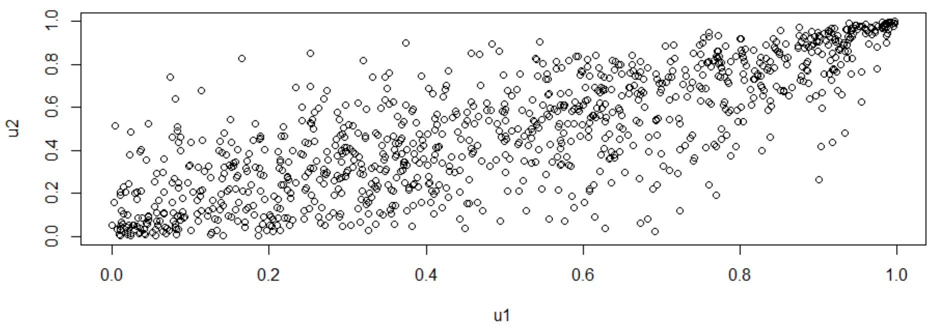


Figure 24: Simulated pseudo-sample from the Gumbel copula with $\tau = 0.6$

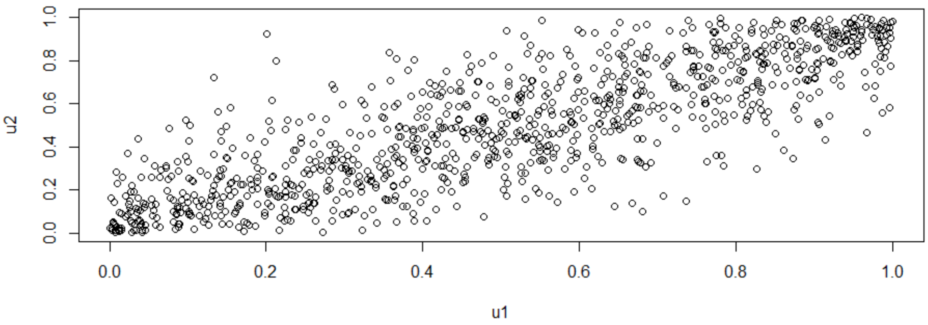


Figure 25: Simulated pseudo-sample from the Frank copula with $\tau = 0.6$

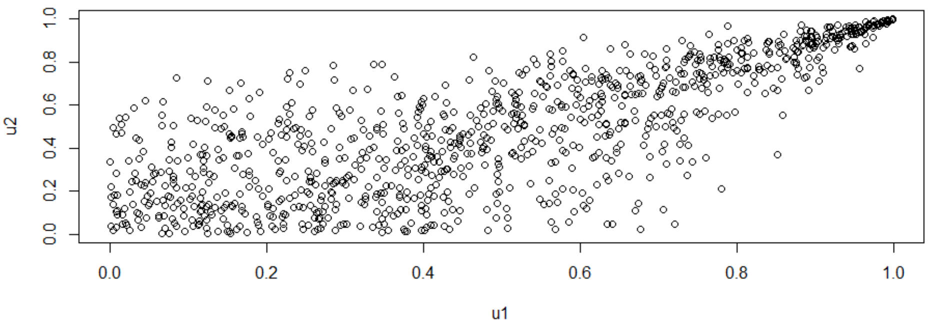


Figure 26: Simulated pseudo-sample from the Joe copula with $\tau = 0.6$

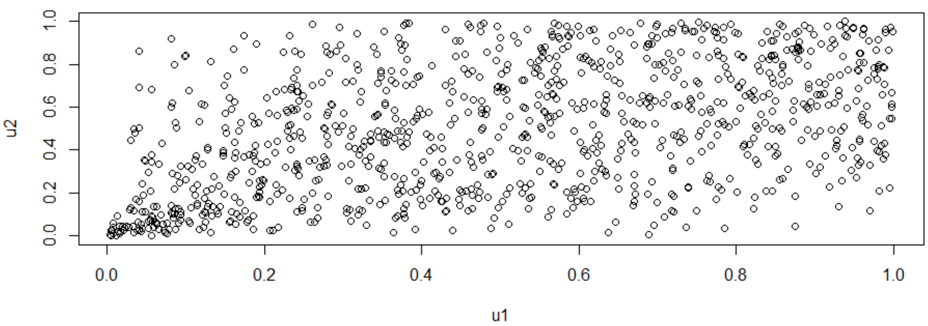


Figure 27: Simulated pseudo-sample from the AMH copula with $\tau = 0.6$

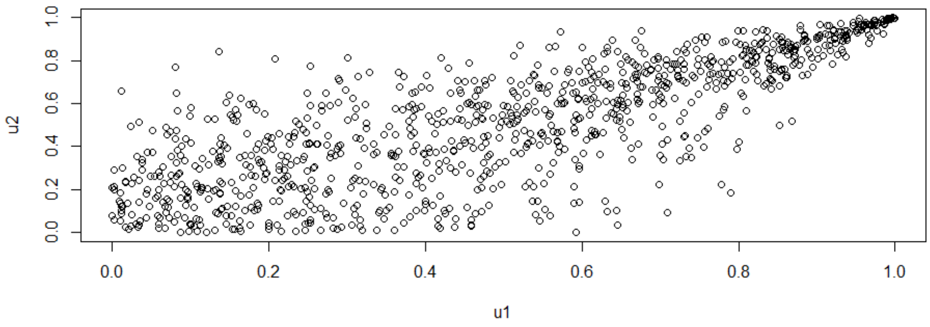


Figure 28: Simulated pseudo-sample from the rotated Clayton copula with $\tau = 0.6$

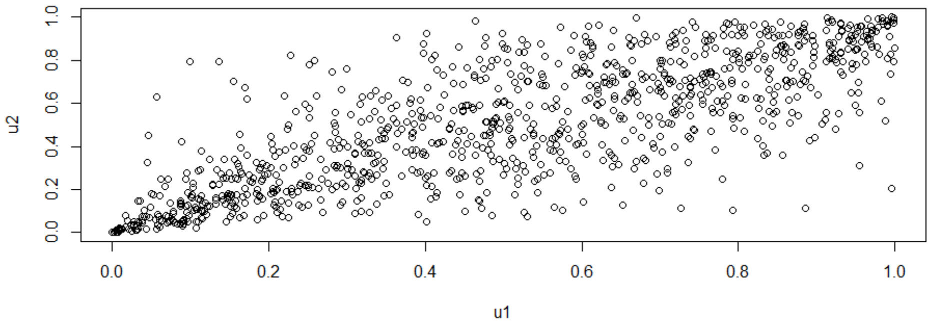


Figure 29: Simulated pseudo-sample from the rotated Gumbel copula with $\tau = 0.6$

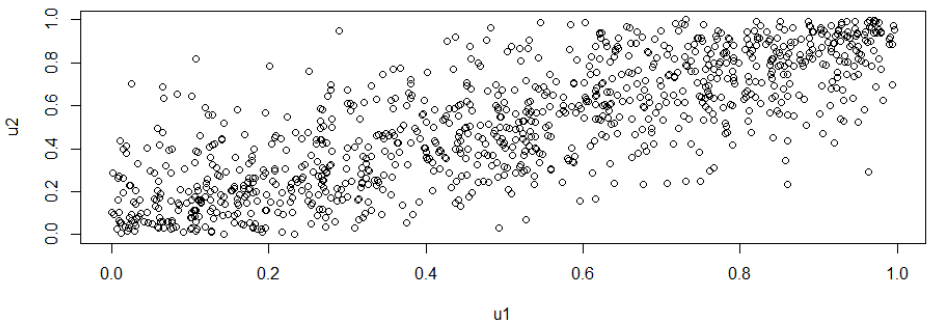


Figure 30: Simulated pseudo-sample from the rotated Frank copula with $\tau = 0.6$

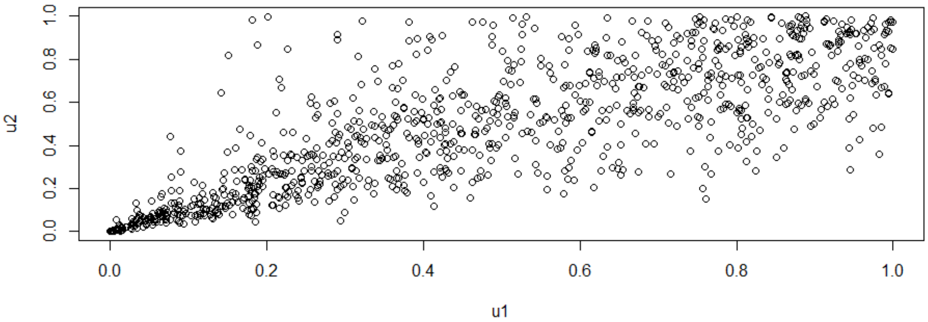


Figure 31: Simulated pseudo-sample from the rotated Joe copula with $\tau = 0.6$

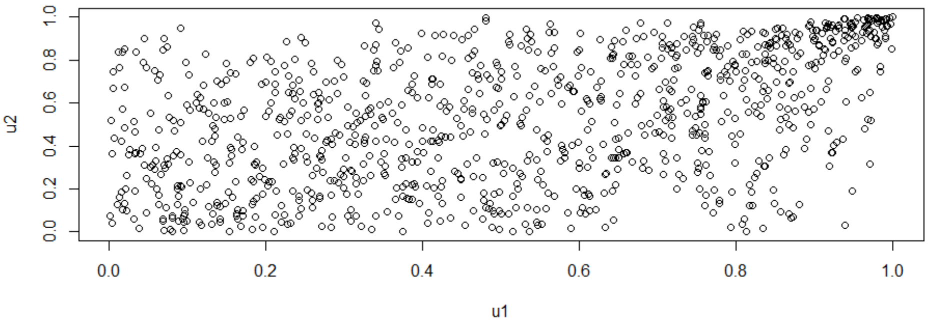


Figure 32: Simulated pseudo-sample from the rotated AMH copula with $\tau = 0.6$

8.3 GARCH(1,1) estimates

Table 15: Average parameter estimates for the GARCH(1,1) model with Student-t errors calculated with a rolling window of $W = 200$ daily observations for the period December 2017 - April 2021

Asset	BTC	ETH	XRP	ADA	LTC	BTF
μ	0.001	0.000	-0.002	-0.001	-0.001	0.001
ω	0.000	0.001	0.001	0.001	0.001	0.000
α	0.156	0.173	0.346	0.082	0.115	0.083
β	0.825	0.774	0.563	0.824	0.831	0.903
ν	2.917	2.824	2.694	3.893	3.475	3.056

8.4 Hedging Results

Table 16: Upper: Risk measures for evaluating out-of-sample hedging performance for the Bitcoin portfolio using different hedging strategies. The hedging effectiveness is displayed between brackets. The results are based on the period September 2018 - April 2021. In addition the average model rank for the 10 risk measures is calculated. The best model is indicated in bold.
Lower: Hedge ratio statistics for the Bitcoin portfolio for the different hedge strategies for the period September 2018 - April 2021.

Method	Unhedged	Naive	OLS	DCC	Gauss	Student-t	Clayton	Gumbel	Frank	Joe	AMH	R-Clayton	R-Gumbel	R-Frank	R-Joe	R-AMH	C-Equal	C-Simplex	C-Best 3
Mean	0.006	0.000	0.001	0.001	0.002	0.001	0.002	0.000	0.003	0.000	0.003	0.001	0.002	0.003	0.003	0.003	0.001	0.001	0.001
Max	0.225	0.232	0.214	0.261	0.289	0.289	0.243	0.289	0.196	0.289	0.174	0.289	0.142	0.289	0.144	0.256	0.241	0.224	0.132
Min	-0.372	-0.172	-0.196	-0.289	-0.185	-0.221	-0.223	-0.214	-0.259	-0.222	-0.282	-0.199	-0.195	-0.224	-0.224	-0.310	-0.178	-0.181	-0.158
Variance	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	**0.001**
	(-)	(65.992)	(68.246)	(59.238)	(60.023)	(56.794)	(53.367)	(57.368)	(52.533)	(56.062)	(46.010)	(56.872)	(63.124)	(48.132)	(58.298)	(41.115)	(63.921)	(65.521)	**(69.047)**
Semivariance	0.043	0.026	**0.025**	0.029	0.026	0.029	0.029	0.030	0.029	0.030	0.032	0.029	0.027	0.029	0.028	0.032	0.026	0.026	0.025
	(-)	(38.898)	**(42.220)**	(32.462)	(39.219)	(32.830)	(32.824)	(30.544)	(33.158)	(30.929)	(26.520)	(33.390)	(36.839)	(33.422)	(35.900)	(24.868)	(39.167)	(39.861)	(40.802)
VaR 0.1	-0.039	-0.023	-0.022	-0.022	-0.023	-0.023	-0.024	-0.024	-0.025	-0.024	-0.027	-0.024	-0.024	-0.026	-0.024	-0.026	-0.022	**-0.020**	-0.022
	(-)	(42.551)	(43.065)	(43.343)	(42.179)	(41.722)	(38.891)	(38.292)	(35.989)	(39.457)	(31.799)	(38.192)	(39.142)	(33.902)	(39.631)	(33.258)	(44.485)	**(48.781)**	(44.485)
VaR 0.05	-0.057	-0.038	-0.034	-0.034	-0.036	-0.040	-0.041	-0.044	-0.037	-0.043	-0.048	-0.043	-0.036	-0.043	-0.037	-0.047	-0.034	**-0.034**	-0.034
	(-)	(33.171)	(39.768)	(39.592)	(35.962)	(29.880)	(27.864)	(22.133)	(34.834)	(23.728)	(15.919)	(24.358)	(35.796)	(24.311)	(34.340)	(16.544)	(39.124)	**(40.069)**	(39.850)
VaR 0.025	-0.078	-0.055	-0.055	-0.054	**-0.053**	-0.057	-0.057	-0.060	-0.057	-0.058	-0.057	-0.059	-0.054	-0.057	-0.057	-0.063	-0.058	-0.055	-0.056
	(-)	(28.860)	(29.922)	(31.074)	**(31.592)**	(26.811)	(26.644)	(23.240)	(26.685)	(26.134)	(27.383)	(24.259)	(31.013)	(26.459)	(27.007)	(18.853)	(25.683)	(28.832)	(28.526)
VaR 0.01	-0.130	-0.077	**-0.068**	-0.074	-0.077	-0.079	-0.079	-0.088	-0.077	-0.086	-0.092	-0.078	-0.077	-0.084	-0.074	-0.083	-0.080	-0.076	-0.080
	(-)	(40.666)	**(47.852)**	(42.950)	(40.512)	(39.346)	(38.979)	(32.605)	(40.594)	(33.819)	(29.655)	(40.110)	(41.012)	(35.503)	(43.057)	(35.827)	(38.834)	(41.490)	(38.621)
ES 0.1	-0.073	-0.047	**-0.044**	-0.047	-0.046	-0.050	-0.050	-0.053	-0.049	-0.052	-0.055	-0.051	-0.047	-0.050	-0.048	-0.055	-0.047	-0.045	-0.045
	(-)	(35.568)	**(39.573)**	(35.204)	(36.289)	(31.896)	(30.910)	(26.640)	(32.250)	(27.974)	(25.039)	(29.442)	(35.053)	(30.690)	(34.633)	(24.265)	(35.849)	(37.534)	(37.712)
ES 0.05	-0.101	-0.066	**-0.061**	-0.068	-0.065	-0.071	-0.070	-0.075	-0.068	-0.074	-0.074	-0.071	-0.066	-0.070	-0.066	-0.076	-0.067	-0.065	-0.064
	(-)	(34.837)	**(39.049)**	(32.395)	(35.547)	(29.843)	(30.752)	(25.690)	(32.159)	(26.644)	(26.248)	(29.370)	(34.731)	(30.512)	(34.437)	(24.471)	(33.480)	(35.498)	(36.455)
ES 0.025	-0.136	-0.085	**-0.078**	-0.092	-0.084	-0.092	-0.091	-0.096	-0.089	-0.096	-0.096	-0.091	-0.086	-0.090	-0.085	-0.098	-0.086	-0.084	-0.083
	(-)	(37.202)	**(42.954)**	(32.521)	(38.397)	(32.185)	(33.003)	(29.113)	(34.757)	(29.019)	(29.041)	(33.112)	(37.028)	(33.793)	(37.322)	(27.717)	(36.372)	(38.059)	(38.714)
ES 0.01	-0.185	-0.119	**-0.109**	-0.144	-0.122	-0.137	-0.129	-0.136	-0.126	-0.143	-0.144	-0.129	-0.123	-0.123	-0.121	-0.144	-0.121	-0.121	-0.115
	(-)	(35.615)	**(40.954)**	(22.289)	(34.107)	(25.720)	(29.915)	(26.561)	(31.812)	(22.777)	(21.898)	(30.055)	(33.553)	(33.694)	(34.747)	(21.923)	(34.581)	(34.587)	(37.719)
Rank	-	5.6	2.0	8.0	5.0	11.5	12.1	15.0	10.6	14.5	16.3	12.3	6.9	12.6	7.1	17.3	7.1	3.5	3.5

Method	Unhedged	Naive	OLS	DCC	Gauss	Student-t	Clayton	Gumbel	Frank	Joe	AMH	R-Clayton	R-Gumbel	R-Frank	R-Joe	R-AMH	C-Equal	C-Simplex	C-Best 3
Mean	0.000	-1.000	-0.827	-0.773	-0.685	-0.738	-0.507	-0.825	-0.472	-0.780	-0.371	-0.776	-0.626	-0.468	-0.500	-0.378	-0.794	-0.809	-0.826
Max	0.000	-1.000	-0.729	-0.260	0.000	0.000	0.000	-0.027	0.000	-0.010	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Min	0.000	-1.000	-0.954	-1.483	-1.500	-1.500	-1.193	-1.500	-1.284	-1.500	-0.749	-1.500	-1.425	-1.500	-1.153	-1.206	-1.439	-1.500	-1.500
Turnover	0.000	0.016	0.015	0.077	0.095	0.107	0.072	0.094	0.102	0.120	0.071	0.122	0.090	0.096	0.078	0.093	0.086	0.074	0.070

Table 17: Upper: Risk measures for evaluating out-of-sample hedging performance for the Ethereum portfolio using different hedging strategies. The hedging effectiveness is displayed between brackets. The results are based on the period September 2018 - April 2021. In addition the average model rank for the 10 risk measures is calculated. The best model is indicated in bold.

Lower: Hedge ratio statistics for the Ethereum portfolio for the different hedge strategies for the period September 2018 - April 2021.

Method	Unhedged	Naive	OLS	DCC	Gauss	Student-t	Clayton	Gumbel	Frank	Joe	AMH	R-Clayton	R-Gumbel	R-Frank	R-Joe	R-AMH	C-Equal	C-Simplex	C-Best 3
Mean	0.007	0.001	0.001	0.002	0.002	0.001	0.003	0.002	0.003	0.004	0.003	0.003	0.002	0.003	0.003	0.004	0.004	0.003	0.003
Max	0.424	0.397	0.398	0.406	0.394	0.393	0.399	0.398	0.399	0.414	0.403	0.409	0.397	0.399	0.400	0.410	0.409	0.409	0.400
Min	-0.423	-0.235	-0.271	-0.231	-0.301	-0.301	-0.319	-0.397	-0.369	-0.325	-0.423	-0.326	-0.308	-0.308	-0.313	-0.392	-0.244	-0.300	-0.246
Variance	0.003	0.002	0.002	**0.002**	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.002	0.002
	(-)	(32.916)	(33.868)	**(35.385)**	(32.980)	(32.063)	(31.957)	(28.021)	(28.022)	(32.384)	(26.603)	(31.771)	(32.932)	(29.759)	(32.717)	(20.831)	(35.184)	(32.104)	(33.972)
Semivariance	0.053	0.041	0.041	**0.039**	0.042	0.042	0.042	0.044	0.044	0.041	0.045	0.042	0.042	0.043	0.042	0.046	0.039	0.041	0.040
	(-)	(22.327)	(23.756)	**(27.407)**	(21.398)	(20.458)	(20.563)	(16.783)	(16.918)	(22.657)	(15.270)	(20.569)	(21.309)	(19.027)	(21.653)	(13.049)	(27.016)	(22.546)	(25.103)
VaR 0.1	-0.049	-0.045	-0.046	**-0.041**	-0.046	-0.048	-0.047	-0.046	-0.046	-0.047	-0.045	-0.046	-0.047	-0.047	-0.044	-0.047	-0.044	-0.043	-0.043
	(-)	(7.860)	(5.147)	**(14.829)**	(5.578)	(1.592)	(2.414)	(5.616)	(4.704)	(3.965)	(6.646)	(5.722)	(2.943)	(3.420)	(9.895)	(2.872)	(9.474)	(10.618)	(12.320)
VaR 0.05	-0.077	-0.068	-0.065	-0.066	-0.066	-0.066	-0.065	-0.068	-0.066	**-0.062**	-0.066	-0.067	-0.067	-0.064	-0.065	-0.069	-0.062	-0.065	-0.063
	(-)	(10.759)	(14.905)	(13.419)	(13.416)	(14.114)	(15.425)	(11.015)	(14.362)	**(18.904)**	(13.869)	(12.422)	(12.097)	(16.884)	(15.539)	(9.603)	(18.760)	(15.069)	(17.172)
VaR 0.025	-0.106	-0.083	-0.082	-0.079	-0.078	-0.082	**-0.077**	-0.078	-0.084	-0.078	-0.080	-0.081	-0.079	-0.083	-0.078	-0.087	-0.077	-0.077	-0.077
	(-)	(21.313)	(22.207)	(25.517)	(25.916)	(22.742)	**(27.340)**	(26.753)	(20.907)	(26.102)	(24.055)	(23.664)	(24.985)	(21.764)	(26.309)	(18.306)	(27.271)	(27.003)	(27.300)
VaR 0.01	-0.167	-0.100	-0.116	-0.094	-0.116	-0.119	-0.126	-0.118	-0.114	**-0.091**	-0.133	-0.096	-0.122	-0.131	-0.127	-0.126	-0.098	-0.099	-0.100
	(-)	(40.462)	(30.961)	(43.782)	(30.700)	(29.211)	(24.740)	(29.255)	(31.954)	**(45.795)**	(20.489)	(42.708)	(26.945)	(21.804)	(24.416)	(24.792)	(41.700)	(40.675)	(40.411)
ES 0.1	-0.093	-0.077	-0.075	-0.071	-0.077	-0.078	-0.076	-0.078	-0.077	-0.073	-0.078	-0.076	-0.077	-0.077	-0.075	-0.081	**-0.070**	-0.074	-0.072
	(-)	(17.249)	(18.813)	(23.052)	(17.551)	(16.093)	(18.349)	(15.909)	(16.657)	(21.235)	(15.543)	(18.235)	(17.113)	(17.362)	(19.265)	(12.279)	**(24.489)**	(20.261)	(22.373)
ES 0.05	-0.123	-0.099	-0.097	-0.092	-0.099	-0.102	-0.097	-0.103	-0.101	-0.093	-0.102	-0.098	-0.099	-0.099	-0.096	-0.107	**-0.089**	-0.095	-0.092
	(-)	(19.458)	(20.995)	(25.396)	(19.672)	(16.810)	(20.996)	(16.425)	(17.985)	(24.256)	(16.640)	(20.047)	(19.280)	(19.650)	(21.561)	(12.567)	**(27.910)**	(22.482)	(24.857)
ES 0.025	-0.160	-0.122	-0.121	-0.111	-0.124	-0.129	-0.123	-0.133	-0.127	-0.115	-0.132	-0.122	-0.125	-0.126	-0.122	-0.136	**-0.108**	-0.119	-0.114
	(-)	(23.936)	(24.858)	(30.623)	(22.872)	(19.466)	(23.098)	(17.248)	(20.729)	(28.278)	(18.006)	(24.068)	(22.387)	(21.594)	(23.923)	(15.535)	**(32.769)**	(25.857)	(28.669)
ES 0.01	-0.224	-0.172	-0.163	-0.154	-0.182	-0.194	-0.181	-0.202	-0.192	-0.170	-0.201	-0.182	-0.184	-0.182	-0.179	-0.200	**-0.147**	-0.176	-0.163
	(-)	(23.420)	(27.215)	(31.200)	(18.794)	(13.451)	(19.310)	(9.833)	(14.388)	(23.988)	(10.385)	(18.695)	(17.748)	(18.917)	(20.038)	(10.674)	**(34.490)**	(21.402)	(27.109)
Rank	-	9.7	7.4	3.4	9.7	13.7	9.7	14.1	13.2	5.3	14.7	9.8	12.0	12.3	7.6	17.2	2.2	5.7	3.3

Method	Unhedged	Naive	OLS	DCC	Gauss	Student-t	Clayton	Gumbel	Frank	Joe	AMH	R-Clayton	R-Gumbel	R-Frank	R-Joe	R-AMH	C-Equal	C-Simplex	C-Best 3
Mean	0.000	-1.000	-0.837	-0.808	-0.749	-0.829	-0.626	-0.823	-0.512	-0.560	-0.525	-0.601	-0.727	-0.514	-0.619	-0.346	-0.563	-0.620	-0.614
Max	0.000	-1.000	-0.561	-0.204	0.000	0.000	0.000	-0.009	0.000	-0.012	0.000	-0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Min	0.000	-1.000	-1.068	-1.500	-1.500	-1.500	-1.500	-1.500	-1.500	-1.500	-1.444	-1.500	-1.500	-1.500	-1.500	-1.500	-0.993	-1.098	-1.127
Turnover	0.000	0.030	0.028	0.072	0.116	0.121	0.108	0.120	0.118	0.112	0.109	0.118	0.112	0.116	0.106	0.125	0.092	0.089	0.099

Table 18: Upper: Risk measures for evaluating out-of-sample hedging performance for the Ripple portfolio using different hedging strategies. The hedging effectiveness is displayed between brackets. The results are based on the period September 2018 - April 2021. In addition the average model rank for the 10 risk measures is calculated. The best model is indicated in bold.
Lower: Hedge ratio statistics for the Ripple portfolio for the different hedge strategies for the period September 2018 - April 2021.

Method	Unhedged	Naive	OLS	DCC	Gauss	Student-t	Clayton	Gumbel	Frank	Joe	AMH	R-Clayton	R-Gumbel	R-Frank	R-Joe	R-AMH	C-Equal	C-Simplex	C-Best 3
Mean	0.006	0.000	0.001	0.002	0.001	0.001	0.001	0.001	0.002	0.003	0.002	0.003	0.001	0.002	0.001	0.003	0.003	0.002	0.001
Max	0.872	0.886	0.883	0.889	0.887	0.887	0.883	0.888	0.881	0.882	0.882	0.881	0.885	0.882	0.884	0.878	0.879	0.884	0.885
Min	-0.423	-0.426	-0.425	-0.426	-0.427	-0.427	-0.427	-0.427	-0.427	-0.424	-0.427	-0.425	-0.427	-0.427	-0.427	-0.425	-0.425	-0.425	-0.425
Variance	0.006	0.006	0.005	**0.005**	0.005	0.005	0.005	0.006	0.005	0.005	0.005	0.005	0.005	0.005	**0.005**	0.005	0.005	0.005	0.005
	(-)	(4.317)	(9.608)	**(9.469)**	(8.115)	(7.052)	(8.858)	(5.972)	(7.879)	(7.316)	(8.664)	(7.309)	(7.834)	(6.608)	**(9.651)**	(6.677)	(9.633)	(9.419)	(9.614)
Semivariance	0.058	0.056	**0.051**	**0.051**	0.054	0.054	0.054	0.055	0.055	0.053	0.054	0.053	0.054	0.056	0.054	0.055	0.051	0.052	0.054
	(-)	(3.509)	**(12.402)**	**(11.207)**	(6.329)	(6.012)	(7.109)	(4.691)	(4.348)	(7.833)	(6.649)	(8.681)	(6.363)	(3.992)	(7.531)	(4.732)	(12.160)	(10.842)	(7.225)
VaR 0.1	-0.054	-0.051	-0.050	**-0.047**	-0.050	-0.049	-0.049	-0.050	-0.051	-0.054	-0.050	-0.054	-0.048	-0.053	-0.048	-0.055	-0.049	-0.050	-0.049
	(-)	(6.809)	(8.512)	**(14.098)**	(8.244)	(10.059)	(10.093)	(7.547)	(6.494)	(1.441)	(7.577)	(1.441)	(11.498)	(3.000)	(11.748)	-(1.651)	(9.541)	(7.615)	(10.686)
VaR 0.05	-0.089	-0.086	-0.075	-0.076	-0.086	-0.082	-0.080	-0.089	-0.083	**-0.079**	-0.081	-0.080	-0.077	-0.090	-0.081	-0.083	-0.076	**-0.074**	-0.076
	(-)	(3.162)	(15.337)	(14.912)	(3.209)	(7.937)	(9.937)	-(0.449)	(7.068)	**(11.248)**	(8.563)	(9.894)	(13.400)	-(1.053)	(9.230)	(6.459)	(14.845)	**(17.416)**	(14.112)
VaR 0.025	-0.118	-0.116	**-0.111**	-0.113	-0.118	-0.117	**-0.116**	-0.119	-0.115	-0.112	-0.116	-0.112	-0.118	-0.116	-0.116	-0.114	-0.111	-0.116	-0.119
	(-)	(1.203)	**(5.409)**	(4.088)	-(0.483)	(0.157)	**(1.100)**	-(1.084)	(2.373)	(4.928)	(1.632)	(4.914)	-(0.055)	(1.174)	(1.129)	(3.412)	(5.179)	(0.929)	-(1.502)
VaR 0.01	-0.161	-0.165	-0.147	-0.152	-0.159	-0.169	-0.155	-0.163	-0.149	**-0.147**	-0.149	-0.152	-0.165	-0.149	-0.155	-0.150	-0.150	-0.149	-0.149
	(-)	-(2.643)	(8.877)	(5.857)	(1.293)	-(4.674)	(3.897)	-(1.125)	(7.864)	**(9.101)**	(7.448)	(5.746)	-(2.609)	(7.701)	(3.935)	(7.005)	(7.172)	(7.869)	(7.864)
ES 0.1	-0.106	-0.105	-0.097	-0.096	-0.103	-0.104	-0.103	-0.105	-0.103	-0.100	-0.103	-0.100	-0.104	-0.105	-0.102	-0.102	**-0.095**	-0.098	-0.101
	(-)	(1.013)	(8.219)	(8.996)	(2.639)	(1.555)	(2.298)	(0.903)	(2.639)	(5.331)	(2.661)	(4.946)	(1.909)	(1.019)	(3.550)	(3.484)	**(9.700)**	(7.020)	(3.877)
ES 0.05	-0.144	-0.144	-0.133	-0.134	-0.144	-0.148	-0.144	-0.147	-0.142	-0.136	-0.142	-0.136	-0.146	-0.144	-0.142	-0.137	**-0.130**	-0.137	-0.144
	(-)	(0.021)	(7.624)	(6.720)	-(0.188)	-(2.912)	(0.169)	-(1.959)	(1.547)	(5.485)	(1.160)	(5.203)	-(1.849)	(0.147)	(1.007)	(4.433)	**(9.863)**	(4.669)	(0.131)
ES 0.025	-0.181	-0.189	-0.169	-0.172	-0.181	-0.188	-0.183	-0.186	-0.182	-0.174	-0.180	-0.174	-0.186	-0.184	-0.182	-0.176	**-0.169**	-0.174	-0.184
	(-)	-(4.272)	(6.409)	(4.819)	-(0.089)	-(3.612)	-(0.954)	-(2.511)	-(0.578)	(3.664)	(0.548)	(3.675)	-(2.907)	-(1.442)	-(0.280)	(2.885)	**(6.635)**	(3.765)	-(1.458)
ES 0.01	-0.262	-0.276	-0.244	**-0.242**	-0.268	-0.269	-0.273	-0.272	-0.274	-0.257	-0.272	-0.255	-0.270	-0.279	-0.273	-0.264	**-0.245**	-0.253	-0.270
	(-)	-(5.558)	(6.633)	**(7.394)**	-(2.646)	-(2.943)	-(4.261)	-(3.913)	-(4.867)	(1.689)	-(4.122)	(2.505)	-(3.420)	-(6.818)	-(4.211)	-(0.890)	**(6.335)**	(3.438)	-(3.396)
Rank	-	15.7	2.7	3.6	11.9	13.6	10.5	15.5	11.0	6.6	9.5	7.9	11.9	14.1	8.5	10.6	3.1	5.5	8.6

Method	Unhedged	Naive	OLS	DCC	Gauss	Student-t	Clayton	Gumbel	Frank	Joe	AMH	R-Clayton	R-Gumbel	R-Frank	R-Joe	R-AMH	C-Equal	C-Simplex	C-Best 3
Mean	0.000	-1.000	-0.618	-0.591	-0.594	-0.667	-0.570	-0.614	-0.434	-0.337	-0.509	-0.390	-0.628	-0.435	-0.566	-0.263	-0.485	-0.578	-0.582
Max	0.000	-1.000	-0.389	-0.096	0.000	0.000	0.000	-0.017	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Min	0.000	-1.000	-0.862	-1.500	-1.500	-1.500	-1.500	-1.500	-1.500	-1.500	-1.500	-1.500	-1.500	-1.500	-1.500	-1.500	-0.808	-1.500	-1.500
Turnover	0.000	0.038	0.025	0.084	0.124	0.123	0.114	0.128	0.118	0.100	0.107	0.106	0.122	0.118	0.110	0.107	0.091	0.116	0.120

Table 19: Upper: Risk measures for evaluating out-of-sample hedging performance for the Cardano portfolio using different hedging strategies. The hedging effectiveness is displayed between brackets. The results are based on the period September 2018 - April 2021. In addition the average model rank for the 10 risk measures is calculated. The best model is indicated in bold.
Lower: Hedge ratio statistics for the Cardano portfolio for the different hedge strategies for the period September 2018 - April 2021.

Method	Unhedged	Naive	OLS	DCC	Gauss	Student-t	Clayton	Gumbel	Frank	Joe	AMH	R-Clayton	R-Gumbel	R-Frank	R-Joe	R-AMH	C-Equal	C-Simplex	C-Best 3
Mean	0.008	0.003	0.002	0.003	0.002	0.002	0.003	0.002	0.004	0.005	0.003	0.005	0.003	0.004	0.003	0.005	0.005	0.004	0.004
Max	0.322	0.382	0.377	0.352	0.368	0.372	0.364	0.358	0.358	0.337	0.361	0.341	0.373	0.356	0.362	0.347	0.356	0.361	0.362
Min	-0.396	-0.215	-0.259	-0.219	-0.296	-0.288	-0.303	-0.311	-0.348	-0.330	-0.308	-0.363	-0.300	-0.333	-0.307	-0.344	-0.247	-0.235	-0.251
Variance	0.005	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	**0.004**	0.004
	(-)	(20.112)	(20.046)	(21.031)	(19.192)	(20.754)	(21.658)	(19.847)	(17.224)	(14.921)	(20.800)	(16.184)	(20.856)	(17.496)	(21.563)	(14.560)	(21.310)	**(23.819)**	(23.830)
Semivariance	0.061	0.051	0.051	0.049	0.053	0.052	0.051	0.052	0.054	0.053	0.051	0.053	0.052	0.053	0.051	0.054	0.050	**0.048**	0.049
	(-)	(16.549)	(15.497)	(18.799)	(12.537)	(13.954)	(15.591)	(14.017)	(11.495)	(12.035)	(15.351)	(11.915)	(14.121)	(12.702)	(15.712)	(11.080)	(17.213)	**(20.195)**	(19.831)
VaR 0.1	-0.063	-0.061	-0.062	-0.058	-0.063	-0.062	-0.061	-0.062	-0.064	-0.062	-0.064	-0.064	-0.061	-0.062	-0.062	-0.065	-0.061	-0.058	**-0.057**
	(-)	(2.402)	(1.776)	(7.529)	-(0.878)	(1.931)	(3.802)	(0.772)	-(2.360)	(1.798)	-(1.173)	-(1.045)	(2.535)	(1.002)	(1.249)	-(3.258)	(2.999)	(7.608)	**(9.231)**
VaR 0.05	-0.089	-0.081	-0.085	-0.082	-0.081	-0.080	-0.080	-0.082	-0.079	-0.078	-0.079	**-0.078**	-0.082	-0.080	-0.079	-0.080	-0.079	-0.079	-0.078
	(-)	(8.710)	(4.738)	(8.359)	(8.685)	(10.451)	(10.501)	(7.381)	(11.524)	(11.800)	(11.171)	**(12.820)**	(7.788)	(10.313)	(11.717)	(9.803)	(11.131)	(11.681)	(12.645)
VaR 0.025	-0.117	-0.098	-0.102	-0.104	-0.103	-0.098	-0.098	-0.103	-0.104	-0.111	-0.098	-0.105	-0.097	-0.103	-0.101	-0.109	-0.100	**-0.092**	-0.093
	(-)	(15.880)	(12.790)	(11.165)	(12.350)	(16.052)	(16.368)	(12.229)	(11.226)	(4.677)	(16.459)	(10.164)	(17.118)	(11.802)	(13.662)	(6.880)	(14.665)	**(20.933)**	(20.848)
VaR 0.01	-0.155	-0.129	-0.120	-0.130	-0.132	-0.130	-0.123	-0.127	-0.131	-0.148	-0.127	-0.138	-0.127	-0.134	**-0.117**	-0.148	-0.128	-0.125	-0.121
	(-)	(16.238)	(22.268)	(15.832)	(14.364)	(16.053)	(20.693)	(17.938)	(15.192)	(4.457)	(17.965)	(10.571)	(18.104)	(13.432)	**(23.998)**	(4.386)	(17.079)	(19.310)	(21.481)
ES 0.1	-0.105	-0.092	-0.094	-0.090	-0.096	-0.094	-0.093	-0.094	-0.095	-0.095	-0.092	-0.094	-0.094	-0.095	-0.092	-0.096	-0.089	**-0.086**	-0.086
	(-)	(12.210)	(10.296)	(13.847)	(8.353)	(10.768)	(11.603)	(10.684)	(9.091)	(9.771)	(12.046)	(10.479)	(10.906)	(9.726)	(12.577)	(8.767)	(15.320)	**(18.132)**	(17.806)
ES 0.05	-0.135	-0.112	-0.115	-0.112	-0.120	-0.117	-0.115	-0.116	-0.120	-0.120	-0.114	-0.119	-0.116	-0.120	-0.113	-0.121	-0.110	-0.105	**-0.104**
	(-)	(17.148)	(15.075)	(16.990)	(11.161)	(13.544)	(15.198)	(14.115)	(11.032)	(11.114)	(15.467)	(11.869)	(14.506)	(11.566)	(16.353)	(10.480)	(18.762)	(22.536)	**(22.830)**
ES 0.025	-0.170	-0.134	-0.135	-0.134	-0.148	-0.143	-0.139	-0.138	-0.149	-0.150	-0.140	-0.151	-0.143	-0.147	-0.137	-0.152	-0.132	-0.124	**-0.123**
	(-)	(21.032)	(20.348)	(21.130)	(12.705)	(15.978)	(17.995)	(18.890)	(12.023)	(11.609)	(17.755)	(11.069)	(15.993)	(13.314)	(19.089)	(10.662)	(22.441)	(27.185)	**(27.372)**
ES 0.01	-0.224	-0.181	-0.178	-0.165	-0.194	-0.189	-0.186	-0.179	-0.205	-0.187	-0.183	-0.192	-0.190	-0.196	-0.181	-0.189	-0.160	**-0.153**	-0.155
	(-)	(19.018)	(20.372)	(26.199)	(13.423)	(15.379)	(16.947)	(20.059)	(8.289)	(16.238)	(18.441)	(14.312)	(15.290)	(12.490)	(19.259)	(15.623)	(28.548)	**(31.853)**	(30.914)
Rank	-	7.5	9.2	7.1	14.3	10.3	6.7	10.9	14.8	13.6	8.4	13.7	9.2	13.7	6.0	16.6	5.1	2.2	1.7

Method	Unhedged	Naive	OLS	DCC	Gauss	Student-t	Clayton	Gumbel	Frank	Joe	AMH	R-Clayton	R-Gumbel	R-Frank	R-Joe	R-AMH	C-Equal	C-Simplex	C-Best 3
Mean	0.000	-1.000	-0.835	-0.767	-0.716	-0.744	-0.653	-0.705	-0.492	-0.433	-0.588	-0.489	-0.709	-0.483	-0.638	-0.340	-0.568	-0.636	-0.638
Max	0.000	-1.000	-0.501	-0.078	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Min	0.000	-1.000	-1.235	-1.500	-1.500	-1.500	-1.500	-1.500	-1.500	-1.500	-1.500	-1.500	-1.500	-1.430	-1.500	-1.500	-1.056	-1.500	-1.500
Turnover	0.000	0.040	0.036	0.082	0.122	0.121	0.106	0.135	0.139	0.114	0.125	0.118	0.127	0.134	0.110	0.127	0.107	0.105	0.093

Table 20: Upper: Risk measures for evaluating out-of-sample hedging performance for the Litecoin portfolio using different hedging strategies. The hedging effectiveness is displayed between brackets. The results are based on the period September 2018 - April 2021. In addition the average model rank for the 10 risk measures is calculated. The best model is indicated in bold.
Lower: Hedge ratio statistics for the Litecoin portfolio for the different hedge strategies for the period September 2018 - April 2021.

Method	Unhedged	Naive	OLS	DCC	Gauss	Student-t	Clayton	Gumbel	Frank	Joe	AMH	R-Clayton	R-Gumbel	R-Frank	R-Joe	R-AMH	C-Equal	C-Simplex	C-Best 3
Mean	0.005	-0.001	-0.001	-0.001	-0.001	-0.001	0.000	-0.002	0.001	0.000	0.001	0.000	-0.001	0.001	0.000	0.001	0.002	0.000	0.001
Max	0.267	0.324	0.288	0.352	0.243	0.236	0.220	0.245	0.216	0.231	0.228	0.229	0.233	0.218	0.240	0.223	0.269	0.248	0.270
Min	-0.362	-0.258	-0.242	-0.154	-0.264	-0.221	-0.234	-0.233	-0.275	-0.288	-0.267	-0.272	-0.231	-0.268	-0.263	-0.361	-0.193	-0.229	-0.210
Variance	0.003	0.002	0.002	**0.002**	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.002	**0.002**	0.002
	(-)	(34.108)	(33.302)	**(37.247)**	(33.987)	(36.211)	(35.534)	(34.941)	(31.133)	(30.570)	(33.141)	(32.931)	(36.522)	(31.410)	(35.033)	(25.550)	(35.414)	**(36.838)**	(36.735)
Semivariance	0.056	0.044	0.045	0.042	0.045	0.045	0.044	0.045	0.045	0.046	0.045	0.045	0.043	0.046	0.044	0.048	**0.042**	**0.044**	0.043
	(-)	(20.814)	(19.259)	(24.438)	(19.459)	(19.902)	(21.433)	(19.270)	(19.018)	(18.241)	(19.383)	(19.785)	(22.676)	(17.911)	(20.881)	(13.628)	**(25.028)**	**(21.584)**	(23.618)
VaR 0.1	-0.055	-0.051	-0.049	-0.053	-0.052	-0.052	-0.051	-0.051	-0.051	-0.050	-0.050	-0.050	-0.050	-0.053	-0.051	-0.051	**-0.048**	-0.050	**-0.051**
	(-)	(7.096)	(10.628)	(2.452)	(5.130)	(4.540)	(7.219)	(6.164)	(7.213)	(9.202)	(8.016)	(8.942)	(7.765)	(3.584)	(7.476)	(6.598)	**(11.272)**	(7.917)	**(7.484)**
VaR 0.05	-0.082	**-0.073**	-0.075	-0.076	-0.082	-0.076	-0.075	-0.079	-0.078	-0.080	-0.077	**-0.075**	-0.076	-0.078	-0.077	-0.078	-0.075	-0.076	-0.077
	(-)	**(11.525)**	(8.671)	(7.306)	(0.300)	(7.526)	(8.410)	(4.423)	(4.713)	(3.054)	(6.345)	**(8.253)**	(7.734)	(5.375)	(6.329)	(4.978)	(8.897)	(7.409)	(6.813)
VaR 0.025	-0.122	-0.089	-0.091	-0.095	-0.100	-0.097	-0.087	-0.099	-0.092	-0.100	-0.093	-0.093	-0.101	-0.092	-0.088	-0.102	-0.091	**-0.088**	**-0.086**
	(-)	(27.327)	(25.303)	(22.438)	(17.677)	(20.179)	(28.858)	(18.366)	(24.253)	(18.086)	(23.971)	(23.332)	(17.014)	(24.828)	(28.037)	(16.283)	(25.240)	**(27.796)**	**(29.020)**
VaR 0.01	-0.157	-0.113	-0.126	**-0.113**	-0.124	-0.123	-0.127	-0.126	-0.113	-0.128	-0.122	-0.127	-0.126	-0.117	**-0.120**	-0.126	-0.119	-0.129	-0.114
	(-)	(27.823)	(19.340)	**(28.054)**	(20.673)	(21.344)	(19.014)	(19.768)	(27.741)	(18.128)	(22.318)	(19.029)	(19.557)	(25.441)	**(23.504)**	(19.782)	(24.077)	(17.482)	(27.229)
ES 0.1	-0.101	-0.084	-0.086	-0.082	-0.087	-0.086	-0.083	-0.088	-0.085	-0.088	-0.085	-0.085	-0.084	-0.086	-0.084	-0.088	**-0.079**	**-0.083**	-0.080
	(-)	(17.282)	(14.891)	(18.533)	(13.601)	(14.736)	(17.878)	(13.147)	(16.179)	(13.314)	(16.045)	(15.537)	(16.497)	(14.885)	(17.132)	(12.473)	**(21.758)**	**(17.834)**	(20.321)
ES 0.05	-0.135	-0.105	-0.109	-0.101	-0.109	-0.108	-0.103	-0.110	-0.106	-0.112	-0.106	-0.107	-0.106	-0.105	-0.104	-0.114	-0.100	-0.105	**-0.100**
	(-)	(22.248)	(19.048)	(25.242)	(18.900)	(19.885)	(23.169)	(18.132)	(21.203)	(16.778)	(21.077)	(20.598)	(21.356)	(21.722)	(22.598)	(15.548)	(25.614)	(21.988)	**(25.689)**
ES 0.025	-0.167	-0.127	-0.134	**-0.116**	-0.129	-0.130	-0.125	-0.131	-0.127	-0.134	-0.129	-0.129	-0.126	-0.126	-0.125	-0.139	-0.119	-0.128	**-0.119**
	(-)	(23.984)	(19.797)	**(30.756)**	(22.937)	(22.442)	(25.318)	(21.383)	(24.200)	(19.705)	(23.032)	(23.077)	(24.880)	(24.744)	(25.116)	(17.146)	(28.968)	(23.248)	**(29.058)**
ES 0.01	-0.221	-0.170	-0.174	**-0.135**	-0.162	-0.163	-0.155	-0.159	-0.166	-0.170	-0.168	-0.167	-0.153	-0.160	-0.162	-0.182	-0.145	**-0.166**	-0.153
	(-)	(22.845)	(21.091)	**(38.911)**	(26.529)	(26.041)	(29.729)	(28.141)	(24.652)	(23.028)	(23.963)	(24.468)	(30.816)	(27.729)	(26.654)	(17.768)	(34.257)	**(24.760)**	(30.767)
Rank	-	7.5	11.0	5.1	13.3	11.0	6.1	13.2	10.8	15.0	10.6	10.6	7.7	11.0	7.1	16.4	3.2	7.6	3.8

Method	Unhedged	Naive	OLS	DCC	Gauss	Student-t	Clayton	Gumbel	Frank	Joe	AMH	R-Clayton	R-Gumbel	R-Frank	R-Joe	R-AMH	C-Equal	C-Simplex	C-Best 3
Mean	0.000	-1.000	-0.853	-0.833	-0.741	-0.792	-0.628	-0.793	-0.490	-0.561	-0.554	-0.612	-0.710	-0.500	-0.619	-0.411	-0.556	-0.672	-0.620
Max	0.000	-1.000	-0.504	0.000	0.000	0.000	0.000	-0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Min	0.000	-1.000	-1.149	-1.500	-1.500	-1.500	-1.500	-1.500	-1.468	-1.500	-1.444	-1.500	-1.500	-1.500	-1.500	-1.500	-0.983	-1.500	-1.500
Turnover	0.000	0.033	0.031	0.073	0.127	0.117	0.108	0.128	0.137	0.114	0.106	0.112	0.121	0.124	0.098	0.130	0.097	0.120	0.111

Table 21: Model comparison by the Wilcoxon Signed Rank test on the hedging effectiveness for the minimum Expected Shortfall objective for the Bitcoin. *, ** and *** denote significance on 10%, 5% and 1% level respectively.

Method	Naive	OLS	DCC	Gauss	Student-t	Clayton	Gumbel	Frank	Joe	AMH	R-Clayton	R-Gumbel	R-Frank	R-Joe	R-AMH	C-Equal	C-Simplex	C-Best 3
Unhedged	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***
Naive		0***	39	21	55***	55***	55***	53***	55***	55***	55***	37	55***	42	55***	35	10*	7**
OLS			50**	53***	55***	55***	55***	55***	55***	55***	55***	54***	55***	55***	55***	53***	46*	47**
DCC				16	46*	43	51**	40	54***	55***	43	18	43	27	55***	15	5**	10*
Gauss					55***	55***	55***	54***	55***	55***	55***	43	55***	47**	55***	28	10*	12
Student-t						36	52***	17	55***	54***	33	2***	36	3***	55***	3***	0***	1***
Clayton							49**	12	49**	54***	35	0***	38	0***	55***	3***	0***	1***
Gumbel								5**	17	47**	3***	0***	16	0***	51**	0***	0***	0***
Frank									48**	54***	40	0***	47**	2***	55***	5**	0***	2***
Joe										50**	8**	1***	17	0***	52***	1***	0***	0***
AHM											2***	0***	1***	1***	38	1***	0***	0***
R-Clayton												0***	29	0***	55***	1***	0***	1***
R-Gumbel													54***	35	55***	21	5**	7**
R-Frank														0***	54***	1***	0***	0***
R-Joe															55***	18	5**	6**
R-AHM																0***	0***	0***
C-Equal																	0***	1**
C-Simplex																		22

Table 22: Model comparison by the Wilcoxon Signed Rank test on the hedging effectiveness for the minimum Expected Shortfall objective for the Ethereum. *, ** and *** denote significance on 10%, 5% and 1% level respectively.

Method	Naive	OLS	DCC	Gauss	Student-t	Clayton	Gumbel	Frank	Joe	AMH	R-Clayton	R-Gumbel	R-Frank	R-Joe	R-AMH	C-Equal	C-Simplex	C-Best 3
Unhedged	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***
Naive		17	0***	34	46*	32	48**	49**	6**	47**	27	42	40	23	55***	0***	8**	1***
OLS			1***	44	54***	43	49**	52***	11	50**	39	48**	50**	34	55***	0***	18	2***
DCC				53***	54***	52***	54***	54***	42	54***	55***	55***	54***	52***	55***	18	52***	42
Gauss					54***	28	52***	48**	6**	52***	29	55***	45*	12	55***	0***	1***	0***
Student-t						10*	37	25	0***	44	6**	12	20	8**	54***	0***	0***	0***
Clayton							45*	42	2***	51**	30	39	47**	10*	52***	1***	5**	1***
Gumbel								14	3***	33	4**	12	19	7**	54***	0***	0***	0***
Frank									1***	41	3***	19	14	10*	55***	0***	0***	0***
Joe										53***	53***	54***	55***	43	55***	7**	42	15
AHM											6**	8**	10*	0***	46*	0***	0***	0***
R-Clayton												40	45*	11	55***	1***	5**	2***
R-Gumbel													35	9*	55***	0***	1***	0***
R-Frank														2***	53***	0***	1***	0***
R-Joe															54***	1***	3***	0***
R-AHM																0***	0***	0***
C-Equal																	52***	47**
C-Simplex																		1***

Table 23: Model comparison by the Wilcoxon Signed Rank test on the hedging effectiveness for the minimum Expected Shortfall objective for the Ripple. *, ** and *** denote significance on 10%, 5% and 1% level respectively.

Method	Naive	OLS	DCC	Gauss	Student-t	Clayton	Gumbel	Frank	Joe	AMH	R-Clayton	R-Gumbel	R-Frank	R-Joe	R-AMH	C-Equal	C-Simplex	C-Best 3
Unhedged	20	0***	0***	12	18	8**	26	7**	0***	5**	0***	18	17	7**	3***	0***	0***	9*
Naive		0***	0***	7**	15	1***	28	1***	5**	0***	6**	8**	24	1***	9*	0***	1***	3***
OLS			38	55***	54***	53***	55***	55***	54***	55***	55***	52***	55***	52***	55***	27	49**	50**
DCC				55***	55***	55***	55***	52***	49**	53***	53***	55***	54***	54***	53***	16	43	51**
Gauss					37	13	55***	23	9*	12	11	33	39	7**	15	0***	1***	12
Student-t						5**	39	19	9*	11	9*	5**	31	2***	16	1***	3***	5**
Clayton							54***	31	13	22	13	37	45*	6**	23	1***	4**	12
Gumbel								6**	6**	2***	5**	8**	23	1***	10*	0***	0***	1***
Frank									10*	9*	12	31	55***	16	22	1***	3***	11
Joe										43	30	42	52***	41	55***	2***	13	36
AHM											14	36	54***	20	26	1***	3***	21
R-Clayton												40	50**	40	51**	0***	10*	33
R-Gumbel													34	11	22	3***	3***	8**
R-Frank														9*	10*	1***	2***	7**
R-Joe															23	3***	11	29
R-AHM																0***	4**	25
C-Equal																	48**	49**
C-Simplex																		49**

Table 24: Model comparison by the Wilcoxon Signed Rank test on the hedging effectiveness for the minimum Expected Shortfall objective for the Cardano. *, ** and *** denote significance on 10%, 5% and 1% level respectively.

Method	Naive	OLS	DCC	Gauss	Student-t	Clayton	Gumbel	Frank	Joe	AMH	R-Clayton	R-Gumbel	R-Frank	R-Joe	R-AMH	C-Equal	C-Simplex	C-Best 3
Unhedged	0***	0***	0***	1***	0***	0***	0***	1***	0***	1***	1***	0***	0***	0***	1***	0***	0***	0***
Naive		40	17	55***	44	29	47**	53***	51**	33	51**	42	53***	27	54***	5**	0***	0***
OLS			13	48**	34	19	43	48**	47**	28	46*	32	48**	11	49**	8**	1***	1***
DCC				52***	44	34	49**	51**	54***	38	52***	43	52***	32	54***	15	0***	0***
Gauss					0***	0***	4**	39	33	1***	32	1***	23	0***	45*	0***	0***	0***
Student-t						0***	28	53***	49**	10*	50**	12	55***	7**	54***	5**	0***	0***
Clayton							46*	54***	53***	31	53***	52***	55***	18	55***	16	2***	0***
Gumbel								47**	47**	18	48**	17	47**	3***	53***	1***	0***	0***
Frank									27	1***	21	6**	14	0***	46*	1***	0***	0***
Joe										5**	18	8**	18	3***	46*	1***	1***	0***
AHM											51**	39	53***	9*	55***	8**	0***	0***
R-Clayton												8**	18	1***	52***	1***	1***	1***
R-Gumbel													51**	9*	52***	7**	0***	0***
R-Frank														0***	48**	0***	0***	0***
R-Joe															55***	12	5**	3***
R-AHM																0***	0***	0***
C-Equal																	0***	0***
C-Simplex																		20

Table 25: Model comparison by the Wilcoxon Signed Rank test on the hedging effectiveness for the minimum Expected Shortfall objective for the Litecoin. *, ** and *** denote significance on 10%, 5% and 1% level respectively.

Method	Naive	OLS	DCC	Gauss	Student-t	Clayton	Gumbel	Frank	Joe	AMH	R-Clayton	R-Gumbel	R-Frank	R-Joe	R-AMH	C-Equal	C-Simplex	C-Best 3
Unhedged	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***	0***
Naive		47**	21	48**	45*	18	47**	43	52***	50**	44	31	44	21	55***	10*	24	10*
OLS			10*	32	24	9*	33	22	49**	19	21	16	26	10*	54***	1***	8**	3***
DCC				54***	51**	39	52***	47**	49**	46*	45*	48**	51**	40	53***	26	41	30
Gauss					9*	2***	20	11	37	11	14	3***	13	0***	45*	0***	9*	0***
Student-t						5**	40	20	46*	20	23	13	23	7**	53***	1***	14	2***
Clayton							53***	45*	53***	46*	50**	32	45*	43	53***	6**	43	5**
Gumbel								16	46*	12	14	3***	16	3***	49**	0***	11	0***
Frank									50**	28	27	19	31	10*	54***	2***	14	2***
Joe										3***	1***	4**	9*	1***	47**	0***	3***	1***
AHM											29	17	29	3***	55***	0***	11	2***
R-Clayton												15	27	6**	54***	0***	13	3***
R-Gumbel													34	29	54***	1***	32	8**
R-Frank														9*	52***	2***	17	0***
R-Joe															55***	4**	31	0***
R-AHM																0***	2***	0***
C-Equal																	51**	33
C-Simplex																		6**