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Exploring the European economic heterogeneity in 21st century

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Abstract

This research analyses the European heterogeneity in 21st century for the overall economic situation and three separate spheres. We use quarterly observations for 13 financial and economic variables from 2000 to 2020. We consider 11 countries that are divided into groups by means of Principal Component Analysis, resulting into two groups: core, including Austria, Denmark, France, Germany, the Netherlands and Sweden, and periphery with Portugal, Ireland, Italy, Greece and Spain. We construct a hierarchical dynamic factor model with three levels of latent factors to explain the relations between the nations. In particular, we find different behavior of the factors for two groups based on overall economic situation. We observe that the effect of shocks on periphery countries is much stronger and takes more time to be absorbed, leading to a Europe of two speeds situation. Real economy sector has similar characteristics to the overall situation, while fiscal sector is very volatile but has the most integration and for financial markets one most of the variation comes from individual countries with limited differences between groups. In this way, policies of central European institutions targeting every country should differ based on the economic sphere they are applied to.

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1 Introduction

Around the start of the 21st century European countries were aiming to integrate their economies and create one powerful union to become an important player on the world economic map. Moves such as expansion of the European Union and introduction of the Euro as the sole currency seemed as the finals steps on the way to this goal. However, the Great Financial Crisis of 2008 and European Debt Crisis in 2010 brought significant damage to the unified European Union. There were clear differences between the ways how the countries were affected with some suffering much more than the others (Landesmann (2015)). Recovery plans and policies introduced by the central EU governments were meant to bring back the equal distribution of economic progress and help the most affected ones to catch up in the very short time. In reality, the opposite happened as the measures including economic help only worsened the divide and poorer countries struggled to recover fast enough. This process lead to a huge debate regarding the growing heterogeneity of Europe and the end of the idea of integrated economies.

The division can be seen in both political and economic outlooks of the countries. There is a growing concern for European Union Institutions that the heterogeneity leads to more conflicts inside the bloc regarding its monetary and fiscal policies as well as other financial regulations. These differences in economic behavior of different countries can be observed from recent works of the European institutions such as European Central Bank (2020), European Investment Bank (2020, 2021). One possible solution to this new problem is to use different approaches when designing the policies for each group of countries based on their characteristics. In this way, the damage that caused existing economic split can be limited and the current economic tensions between countries potentially eased. However, in order for this to be efficient we need to have a clear picture regarding the European heterogeneity and understand the economic spheres where the differences are the most striking.

In this paper, we investigate whether there is heterogeneity inside Europe and in what areas this split is the most evident. The aim is to use data from each country to find out the answer to this question from a quantitative point of view. The focus is on both financial data such as stock and bond markets data of the countries researched and macroeconomic indicators describing the state real economy, monetary policies and output characteristics.

We consider information from eleven countries in this research - Spain, Portugal, Italy, Ireland, Greece, France, Austria, Sweden, Denmark, Germany and the Netherlands. Division of these countries into groups is done by looking at the common behavior of their data by means of Principal Component Analysis (PCA). In this way, we do not restrict the groups to only theoretical ideas from the literature or our personal views, but allow the data to drive the decision.

Next, several hierarchical dynamic factor models are constructed in order to research the presence of common factors between the indicators inside the countries and between the countries in each group. In our case, the models allow for four sources of movement for every series. Potential factors are: European component - single factor for all countries in the research; group component - unique for each group of countries; country component - one factor for each country and idiosyncratic component - present only for one separate series. Looking for the percent of variation explained by each component for a series provides us with the information regarding what forces are the most important in moving this series and what is its level of integration with other series from different levels. All models are estimated by means of a Kalman filter that allows us to observe potentially different level of integration throughout the 21st century and compare the output as time changes.

First, we are interested in the common factors present for countries inside each group that move their economies differently from the countries in the other group. In this way, we aim to find whether there is an empirical evidence for a split between the two sets of countries in the last 20 years. Answering this question opens the possibility to design different policies and regulations by the European institutions for the countries in each group. Therefore, we construct a model using all data series that are considered to describe the most general behavior of each country's economy.

Next, we are interested in more details regarding the split between the countries. We research different financial and economic areas individually to find out what spheres have the strongest presence of the common factor in each group and what is the difference between the effect of the split for each area. The first sphere considered is output and real economy indicators as this is the target of investment decisions and loans or grants provided by the European agencies. Discovery of the common behavior between the countries in a certain group would help to create more specific and efficient goals and conditions to obtain money for these countries. We follow by looking at monetary indicators and fiscal policies in order to create the base some common monetary policies and debt regulations for countries with some common factors in this area. Finally, we consider the financial markets data to research whether there are some common trends for both individual and corporate investors interested in countries from the same group. In this way, we are trying to find the insights for investment strategies and diversification of risks depending on a group each country belongs to. In this part of the research, we construct three separate model with each one using the limited number of series that is related to the indicators from the corresponding economic sphere only. The outcomes of the three models are compared to answer the question about the area with the most evident economic split between European countries.

After completing the data driven group selection process we find a split between core and periphery countries. The first groups, which consists of Portugal, Ireland, Italy, Greece and Spain, can be called PIIGS based on the names of its nations. The remaining countries - Austria, Denmark, France, Germany, the Netherlands and Sweden complete the core group. Analysing the behavior of the overall economic conditions in two groups we can see clear differences in the behavior of the group level factors throughout the whole observation period. We observe a situation of a Europe of two speeds where the PIIGS countries take much longer to digest shocks leading to their larger and more persistent effect. Additionally, there is only limited level of European integration as the highest level factor does not explain most of the variation in the variables.

Looking at the three economic sectors separately we find that the evidence for European heterogeneity is different in each the area. The real economy sector acts the most similar to the overall situation with different reaction to shocks by the factors in the two groups during the whole observation period, leading to existence of Europe of two speeds. In this way, there is the most evidence for heterogeneity between the two groups in this sector as comparing to the other two. The fiscal sector is the most volatile one meaning that the factors do not behave in a stable way and there is no pattern to make reasonable conclusions about the split. However, there is the most integration in this area as the European factor is able to explain most of variation in the data comparing to the remaining sectors. Lastly, the financial markets sector exhibits only limited heterogeneity with slightly different magnitude of the two group factors but similar behavior regarding the incoming shocks and their persistence. Moreover, the country level factors seem to be the most important in explaining the variation in this area leading to both limited integration and small effect of the groups.

We conclude that the presence of heterogeneity between European economies may help the central government policymakers to create targeted interventions specifically for each country and group. Meanwhile, lack of integration of the financial markets allows investors to efficiently diversify their portfolios by looking at several countries based on their individual characteristics. Additionally, the underlying factors and the related posterior distributions can be used in forecasting in order to predict the behavior of different economic variables in the future. Finally, in this paper we implement a hierarchical factor model by means of a state space representation and Kalman filter estimation procedure. In this way, it is possible to observe the whole time series of factors and estimate their values at every point in time as opposed to most other approaches that provide static results looking at the whole period in a single manner.

This paper proceeds with the discussion of the literature and previous research in the area of European heterogeneity and applications of hierarchical dynamic factor models in Section 2. Next, we provide a summary of the data and some starting analysis of the relation between the variables in Section 3. Section 4.1 describes the data driven method to determine the grouping of the countries, while Section 4.2 describes in details the main factor model of this research. In Section 4.3 we provide the metrics that are used to evaluate the performance of the model and its implications for the economic divide. Section 5.1 describes the results of a group division process, while we discuss the findings of the research using these groups for all variables combined and three separate economic sectors in Sections 5.2 to 5.5 respectively. Section 6 concludes the paper by summarizing the main outcomes and discussing the limitations and potential areas for future research.

2 Literature

The topic of European economic heterogeneity is widely discussed in the literature both from academic and political points of view. It is covered in plenty of papers and documents issued by European leaders and corresponding institutions. The origin of the split from the time of the Great Financial Crisis is explained by Landesmann (2015). There he investigates different behavior of the European countries before and after the crisis and what areas is the heterogeneity most present. Our paper continues this idea by looking at the time period around the crisis but using more recent data up until 2020 and building a more statistically oriented factor model. Farina and Tamborini (2015) look at the heterogeneity from a more social and political point of view, discovering the potential drivers for the divide as coming from the residents of different countries. We follow their idea of not restricting the model to two predefined groups as North and South but start from the broader selection of countries to let data determine the actual split.

The leaders of European institutions are aware of the existing heterogeneity and have different ideas on handling the issue. Juncker (2015) discusses the necessary steps in order to complete a European integration and looks at the benefits of the more united and cohesive European Union. Meanwhile, Policy Department for Budgetary Affairs of European Parliament (2020) considers the existing policies and money flows that are targeted specifically to mitigate the impact of the split and benefit all European countries equally. Our paper develops a data driven grouping of the countries that should help both approaches to be more specific in their policies. The information about what economic sectors are the most affected by the split and what countries are more evident to be a part of a certain group is important to create the most efficient and specifically targeted policies by European institutions.

Kuhn and Stoeckel (2014) and Bolstad (2015) collect the people's opinions regarding the process of integration. The former focuses on the European governance during the Euro crisis and the economic outcomes for different countries, while the latter looks at the historical public opinion on the general trend towards more integration that is often promoted by the European institutions. The public views on the European divide are not only important as an indicator in what countries and what spheres is the divide more evident but also as a potential reactions to any policies proposed by the EU. Therefore, any EU policies should not only be based on the data and macroeconomic picture that the model, such as ours, provides but also on the public opinion regarding the heterogeneity.

Finally, Spolaore (2013) and de Grauwe (2013) consider the broader picture of the integration versus heterogeneity and discuss both sides as well as the reasons behind them. de Grauwe (2013) focuses on the monetary side discussing the effect of the Euro as the single currency and European Central Bank as its main regulator. We follow his idea by choosing the banking and monetary sector to be one of the three separate areas to discover the presence of the split. Spolaore (2013) discusses even broader picture by looking at both political and economic sides of the European integration and how the idea was originated. We build up on his research by moving from the starting point of the integration ideas as it develops in the opposite direction of more heterogeneity in the recent years. In this way, we complete the history of European economic ideas and reality including the most recent information.

The central model of our research - dynamic hierarchical factor model, is discussed in literature with regards of the macroeconomic usage. The closest to our idea is Kose et al. (2003), where the mode finds the underlying factors that create the business cycles on different levels. We create a similar four level structure with regions and countries as divisions between groups but use a slightly different estimation approach. We follow Moench et al. (2009) by considering the factor model in a state space representation, therefore we are able to apply Kalman filter and smoother to find the factors using the PCA as a starting point. However, we restrict the number of factors at each level to only one, so that our model can be seen as the scalar version of a more general case. Another model that assumes hierarchy in factors is presented by Diebold et al. (2008), where they explain the variation of the yield curves in a country by means of a global and local variables. However, they estimate their model in two steps obtaining the factors of each level separately by different procedures such as OLS. Finally, Halka and Szafranski (2015) create a five level factor model to explain inflation in Central and Eastern European countries. Their approach adds another level of factors to the model leading to more complexity and more detailed decomposition of the variation. Additionally, they use a Principal Components (PCs) based method to estimate the whole model, while we make use of a state space representation to update the factors with the starting values taken from PCA.

3 Data

The data used for this research consists of several economic and financial characteristics for eleven European countries, these being Germany, Denmark, Sweden, the Netherlands, Spain, Portugal, Italy, Greece, Ireland, France and Austria. The time period used is from 2000Q1 to 2020Q2 as this covers the most recent years when the debate regarding the integration of the financial and economic system and the following divide between some countries inside Europe takes place. In this way every series used in this research has 82 quarterly observations. This period includes a time of economic growth until 2007, Great Financial Crisis followed by the European Debt crisis, recovery starting from 2013 and the beginning of the ongoing Covid-19 pandemic and the corresponding economic uncertainty in 2020. In this way, we can observe the interaction between the economies and financial markets of European countries during different surrounding conditions. Allowing the dynamics in side the models, provides us with ability to see how these conditions affect the behavior of the economies. As most of the data consists of different economic indicators it is published on the quarterly basis only, the remaining series that published with the higher frequency are transformed into quarterly observations to match the dimensions. The transformation is done by taking the average of the monthly observations in each quarter. Finally, as the data is a part of the national statistics obtained by central governments we do not expect any missing values to occur in the the dataset.

3.1 Data description

Table 3.1 displays the economic indicators used in this research. The data is taken either from the database of Federal Reserve Bank of Saint Louis (FRED) or from the statistics libraries of the national central banks depending on the availability. All series except for the interest rates and unemployment rate are transformed to be stationary and represent the growth rates of the corresponding variables. This is done in order to have all values scaled to a similar level such that there is no additional effect of a level difference between the variables. In this way, all countries have the same effect on the resulting factors independently of the size of their economies or financial markets.

Table 3.1: Glossary of the dataset

ID	Type	Description
		Real economy
GDP	Percent change	Real Gross Domestic Product
INF	Percent change	Inflation based on the Consumer Price Index
UMPL	Percent	Harmonized unemployment rate
PRD	Percent change	Total industrial production
NIT	Percent change	Net international trade
		Fiscal sector
NID	Percent change	Net international debt
IRS	Percent	Overnight interest rate
DEB	Percent change	Total central government debt
CRE	Percent change	Total credit to central government
		Financial markets
IR3	Percent	3-Month interest rate
IR10	Percent	10-Year interest rate
SMC	Percent change	Total capitalization of the stock market
SMI	Percent change	Stock market index

For the first part of the research, we use all series together to find the general interaction between the economies of the considered countries. Next, we split the data into three groups as displayed in Table 3.1 and consider each one separately. In this way, we aim to find whether the interaction is different depending on a certain part of the economy. The considered groups are real economic indicators, fiscal sector and financial markets characteristics. The first group includes the variables such as GDP, unemployment rate, production index and consumer prices inflation among others. It covers the general state of a country's economy and is the most interesting for individuals as it directly affects them. The second group covers the fiscal and monetary sectors consisting of internal and external debt and central government credit information. This area is the most useful for policy makers of each individual country and the central European government as it covers the tools that they use to influence the state of the underlying economies. Finally, the last group includes the information about the financial markets including change of the stock index, the market capitalization and several interest rates. This group is the most important for the investors that are willing to diversify their positions across different markets in terms of both securities and their locations.

Table 3.2 displays the standard deviations of all series for every country. We focus on analysing the volatility of the data and not the average level as most of the series represent percentage change of the underlying values. In this way, the actual level of the variables can not be seen and the most important for us are movements of the data as opposed to its actual values. From the statistic, we can see the overall level of variation of every variable and look for potential common patterns for a certain selection of countries. We observe that the magnitude of the values depends mostly on the variable it represents and not the country that the data is originated from. Therefore, we can make some comparisons only based on volatility of certain sectors and the overall state of each country economy. One such observation is the fact that for all series representing behavior of financial markets, Greece has the highest volatility. This can be explained by both having the least developed market system from the countries in the group and being hit the hardest by the economic crisis during the corresponding time period. The most extreme example of this behavior comes in the SMI series where the standard deviation for Greece, taking the value of 1036.24 is more than 100 times above the other values. While the Greek values for SMC and DEB are not as much contrasting to the other countries they are still well above the overall level with corresponding standard deviations of 13.05 and 15.32. This may be resulted from the suspension of aid from the IMF in 2014 to Greece or the suspension of Greek sovereign bonds by the ECB in 2015, with both events adding uncertainty into local financial markets. Another interesting observation is very high overall level of volatility for all countries for series NIT and NID. This can be resulted from the fact that these two variables look at the net values of the underlying characteristics leading to both negative and positive values without a certain trend. Due to the netting effect there is no stable level of the values overtime as it might be expected for economic series leading to huge variation in the data. As a result of such high variation these two series might be the most problematic for the factor model to capture possible relations between the countries.

Table 3.2: Standard deviations of the data series

Series	**Aut**	**Den**	**Fra**	**Ger**	**Gre**	**Irl**	**Ita**	**Ned**	**Por**	**Spa**	**Swe**
GDP	1.91	1.27	2.65	1.43	2.19	4.38	1.70	1.19	1.80	2.21	1.25
INF	0.45	0.47	0.43	0.43	1.42	0.86	0.38	0.56	0.85	1.06	0.57
UMPL	0.63	1.34	0.81	2.42	6.74	4.13	1.97	1.30	3.32	5.72	0.91
PRD	2.70	2.50	3.32	3.10	2.37	6.35	2.82	2.24	3.16	2.52	2.67
NIT	212.61	82.36	90.64	16.46	12.00	16.23	111.15	17.31	15.45	28.10	198.11
NID	1329.20	750.14	637.08	4467.13	585.38	474.53	4443.78	1700.51	3331.13	1176.81	577.27
IRS	1.49	1.60	1.72	1.72	2.20	1.72	1.72	1.72	1.72	1.72	1.43
DEB	2.23	4.69	2.23	5.00	15.32	7.82	2.85	5.39	2.42	2.38	3.30
CRE	5.15	7.72	5.31	2.07	4.80	7.75	2.35	3.68	2.49	5.82	6.08
IR3	1.78	1.93	1.78	1.78	2.17	1.78	1.62	1.78	1.78	1.68	1.72
IR10	1.79	1.89	1.70	1.81	4.81	2.34	1.36	1.77	2.49	1.64	1.77
SMC	9.57	7.99	7.65	8.76	13.05	9.18	8.51	8.29	8.37	8.28	8.81
SMI	10.55	8.19	7.75	9.01	1036.24	9.39	8.70	8.31	8.87	8.50	7.90

3.2 Correlation analysis

We perform correlation analysis to observe the interaction between the movement of the same series for different countries. While such an analysis gives only a limited information about the behavior of the economies it can be a good start before applying the factor model to get some general insights from the data. Additionally, it can provide us with some initial ideas regarding the countries that can be grouped together and compared to the results of the PCA later. Correlation analysis is performed for the whole time period using all 82 observations and therefore does not account for any possible dynamics in the model. Three economic series are used to compute correlations with one from each of the subgroups. The series concerned are Real GDP, Net international debt and 10-Year interest rate. For the first two series we use the transformed percent change values, while the last one consists of actual percent values. The correlation matrices for the remaining series are displayed in Appendix B.

Table 3.3: Correlation of Real GDP across countries

Country	**Aut**	**Den**	**Fra**	**Ger**	**Gre**	**Irl**	**Ita**	**Ned**	**Por**	**Spa**	**Swe**
Austria	-										
Denmark	.75	-									
France	.94	.76	-								
Germany	.84	.69	.86	-							
Greece	.66	.47	.65	.58	-						
Ireland	.20	.27	.25	.17	.14	-					
Italy	.93	.78	.97	.89	.67	.27	-				
Netherlands	.90	.77	.89	.88	.70	.23	.90	-			
Portugal	.89	.76	.92	.82	.68	.24	.92	.90	-		
Spain	.92	.76	.96	.85	.74	.26	.96	.91	.95	-	
Sweden	.79	.72	.77	.76	.51	.30	.78	.77	.76	.78	-

Table 3.3 displays the correlation values for Real GDP percentage change for all countries in this research. We can observe a high overall level of integration as most of the values are well above 50%. The only exception is Ireland with the average correlation value of 21% which is well below the average of other countries. This can be explained by the fact that Irish economy is much more tied to the one of the United Kingdom and not the continental Europe. Another observation is the lack of a clear division of countries into groups as most countries have the values close to their averages. This means that the correlation of a certain economy to others mostly depends on the general integration of this specific country into European economic system and not the fact of being a member of some smaller group of countries. We can conclude that it seems possible to have some

common underlying factor moving the change in real economy sector for all European countries.

Table 3.4: Correlation of Net international debt across countries

Country	Aut	Den	Fra	Ger	Gre	Irl	Ita	Ned	Por	Spa	Swe
Austria	-										
Denmark	.03	-									
France	.00	.00	-								
Germany	-.02	.05	.04	-							
Greece	-.07	.13	-.01	.03	-						
Ireland	.11	.04	.03	.03	.03	-					
Italy	.10	-.04	-.01	-.01	.03	-.02	-				
Netherlands	.00	-.05	-.30	-.02	-.01	.09	-.01	-			
Portugal	.09	-.03	.08	-.04	.04	-.03	-.01	-.08	-		
Spain	.01	-.04	-.01	.00	-.09	-.06	.00	-.08	-.01	-	
Sweden	.08	.04	-.13	-.22	.10	.14	.18	.06	.06	-.07	-

Next, we look at the fiscal sector by means of correlation between countries Net international debt as displayed in Table 3.4. We observe that most correlations are very low with the highest absolute value of only 30%. This concludes that the fiscal decisions of every government are mostly independent and do not seem to depend on the decisions done by other counties or some central European strategy. Additionally, we can see several negative correlation values meaning that the international debt of different countries might often move in opposite directions. However, this fact does have a certain trend that would allow us to divide some countries into separate groups based on their fiscal decisions. Therefore, while we still perform the factor analysis for this sector, from the correlation values we do not see a clear underlying factor model for this economic area.

Table 3.5: Correlation of 10-Year interest rate across countries

Country	Aut	Den	Fra	Ger	Gre	Irl	Ita	Ned	Por	Spa	Swe
Austria	-										
Denmark	.99	-									
France	.99	.99	-								
Germany	.99	.99	.99	-							
Greece	-.09	-.20	-.07	-.19	-						
Ireland	.72	.63	.72	.65	.49	-					
Italy	.85	.77	.86	.78	.34	.87	-				
Netherlands	.99	.99	.99	.99	-.14	.68	.82	-			
Portugal	.41	.30	.43	.32	.82	.84	.76	.36	-		
Spain	.83	.76	.84	.78	.42	.90	.96	.81	.80	-	
Sweden	.98	.99	.98	.99	-.21	.61	.76	.99	.29	.76	-

Finally, we perform the correlation analysis for the financial markets sector looking at the 10-Year interest rates of different countries in Table A.8. We can see a very high overall level of correlation with a lot of countries reaching 99%. This means that the monetary policies of such countries and the stability of their financial markets seems to behave very similarly. However, the values of correlation for Greece is much lower than for other countries, with the average of 11% and even several negative values. This might be cause by the huge spike in Greece interest rates and its problems with repaying the debt during the European crisis of 2010-2012. Additionally, we can observe some sort of division into two groups with higher correlations between the members of each groups as comparing to the countries from another combination. The first group covers countries that are classified as more safe according to official ratings, including Austria, Denmark, France, Germany, the Netherlands and Sweden. While the second one consists of more risky financial markets of Greece, Ireland, Italy, Portugal and Spain. This division into two groups based on the correlation analysis of the financial markets sector seems to follow the idea of the recent debates about the European divide into northern and southern economic areas. Therefore, while the correlation values offer only limited information regarding the actual behavior of the underlying data, we can see the base for the factor model where the latent factor is different based on each group.

To conclude the correlation analysis of the economic and financial information for 11 countries used in this research, we can see the possibility of factor structure for this data in general. However, different economic spheres seem to have different relations between countries, therefore we expect different performance of this actor model depending on the area it is applied to. Additionally, while we do not see a clear divide into two groups for all spheres, there are some sectors where the behavior of northern and southern economies seem to differ.

4 Methodology

In this section, the methods of the research are defined. Firstly, the data based method to divide the European countries into groups based on their economic relations is discussed. Next, the main method of the analysis is assessed, this being dynamic hierarchical factor model, creating several levels of latent factors describing the data. Finally, the metrics needed to analyse the performance of the models and the resulting factors are described.

4.1 Principal Component Analysis

In order to divide the countries considered in this research into groups based on their economic behavior we use Principal Component Analysis. PCA is a dimension reduction technique that that computes orthogonal vectors that explain most of the variation in the original dataset. In our framework these vectors are used to display the differences in behavior of the data between the countries. While the complete goal of the PCA is to explain all the variation in the data we are only interested in the first few vectors that provide most of the information. After performing the analysis, we compare its outcome to the ideas from the literature to create the most clear groups of countries for the factor model to be applied to.

Mathematically, we can describe the idea of PCA as finding some factors, these being linear combinations of the data matrix, that have maximum variance and are uncorrelated. The data matrix consists of N observations and P variables, therefore, the realizations of the PCs can be displayed as

$$z_{ij} = \gamma_{1j}x_{i1} + \gamma_{2j}x_{i2} + ... + \gamma_{pj}x_{ip}, \tag{1}$$

where z_{ij} are the scores of the j-th PC and γ_{ij} are its loadings. In order to find the values of PCs, the maximization procedure is performed. The first PC $\boldsymbol{z}_1$ is the linear combination $z_{i1} = \boldsymbol{\gamma}_1'\boldsymbol{x}_i$, that maximizes

$$Var(\boldsymbol{z}_1) = \boldsymbol{\gamma}_1'\hat{\boldsymbol{\Sigma}}_x\gamma_1, \tag{2}$$

such that $\boldsymbol{\gamma}_1'\gamma_1 = 1$. The k-th PC is the linear combination $\boldsymbol{z}_{ik} = \boldsymbol{\gamma}_k'\boldsymbol{x}_i$, that maximizes

$$Var(\boldsymbol{z}_k) = \boldsymbol{\gamma}_k'\hat{\boldsymbol{\Sigma}}_x\gamma_k, \tag{3}$$

such that $\boldsymbol{\gamma}_k'\gamma_k = 1$ and $\boldsymbol{\gamma}_k'\gamma_j = 0$ for $j = 1, ..., k-1$. After completing the procedure, we look at the loadings γ_{ij} of the first few PCs in order to find the intuition for different behavior of some countries that can then be split into several groups.

4.2 Hierarchical Dynamic Factor Model

4.2.1 Model Description

The main method used in this research is a hierarchical dynamic factor model. The dynamic characteristic of the model allows to describe the contemporaneous movement of the variables and their covariance at every point in time through some common factors (Stock and Watson (2016)). The hierarchical part allows to create the factors that are specific to a certain group of variables united in a block or sub-block. Due to the dynamic nature of the model every covariance stationary time series x_t at time t is explained in terms of the set of dynamic factors. Following Kose et al. (2003) we can represent this process in the time domain as

$$\boldsymbol{x}_t = \boldsymbol{A}(L)\boldsymbol{f}_t + \boldsymbol{u}_t, \tag{4}$$

where $\boldsymbol{A}(L)$ is the matrix of polynomials in lag operators, $\boldsymbol{f}_t$ - stochastic process of the factors and $\boldsymbol{u}_t$ cross-sectionally uncorrelated error terms.

In our case, a four-level model is created, where each level explains a part of the variation in each variable. The first level is the main European factor (f) that affects all the economic variables for all countries. Next, there are two blocks in the second level that represent countries in each group separately generating factors (d_g, with $g = 1, 2$), that are unique for countries only in one group. The third level consists of sub-blocks representing each country, where the common factors (h_c, with $c = 1, ..., 11$) combine the information from all economic variables from that country in one national common variable. Finally, the last level is the idiosyncratic part (ε_i, with $i = 1, ..., 13$) for each variable in each country that explains the unique variation that can not be aggregated to any other level and is not combined with any other variable. In this way, the model for each observable variable i can be seen as

$$x_{i,t} = \beta_i^e f_t + \beta_i^g d_{g,t} + \beta_i^c h_{c,t} + \varepsilon_{i,t}. \tag{5}$$

The series specific terms $\epsilon_{i,t}$ are assumed to be normally distributed and follow s_i-order autoregression:

$$\varepsilon_{i,t} = \phi_{i,1}\varepsilon_{i,t-1} + \phi_{i,2}\varepsilon_{i,t-2} + ... + \phi_{i,s_i}\varepsilon_{i,t-s_i} + u_{i,t}. \tag{6}$$

All other level factors are also assumed to move following an autoregression process of order q_k, where $k = 1, ..., 14$ and normally distributed error terms:

$$\varepsilon_{f_k,t} = \phi_{f_k,1}\varepsilon_{f_k,t-1} + \phi_{f_k,2}\varepsilon_{f_k,t-1} + ... + \phi_{f_k,q_k}\varepsilon_{f_k,t-q_k} + u_{f_k,t}. \tag{7}$$

4.2.2 State Space Representation

Additionally, the model can be seen in a State Space representation at each level as displayed in Moench et al. (2009). In this case, the measurement equation relates the factors between the factors from different levels and the transition equation displays the dynamic of the factors based on their lags. Combining all factors from the same level at time t into vectors we obtain $\boldsymbol{f}_t$, $\boldsymbol{d}_t$ and $\boldsymbol{h}_t$ with corresponding sets of parameters $\boldsymbol{\theta}^e$, $\boldsymbol{\theta}^g$ and $\boldsymbol{\theta}^c$. While the European level factor is only one and therefore it can be seen as a scalar, we refer to it as a vector (1×1) for consistence of notation. Additionally, we consider a vector of observations from all data series at time t as $\boldsymbol{x}_t$. In this case, we can clearly see the hierarchical structure of the model, where each level represents a certain type of factors. First, the European level factor is constructed from all available information. Next, it is followed by the group level factors that are conditioned on the European one and use only the remaining information. This procedure is continued until the idiosyncratic variation of each specific series remains given all factors of the higher levels. From this model it is possible to identify a measurement and state space equations at each level separately. We use only one lag for all autoregressive processes in the model, however, it can be generalized in a similar way to include more information from the past at any stage.

We start with the main European factor dynamics. As it is the highest level and does not depend on any other factor, there is no transition equation and the factor follows a simple autoregression process

$$\boldsymbol{\Phi}^e(L)\boldsymbol{f}_t = \boldsymbol{u}_t^e, \tag{8}$$

where $\boldsymbol{\Phi}^e(L)$ is a single lag polynomial of $\phi^e(L) = 1 - \phi_1^e L$ and errors are normally distributed.

For the group level factors we use their dependence on the higher level factors to formulate the (pseudo) observation equation as

$$\boldsymbol{\Phi}^g(L)\boldsymbol{d}_t = \boldsymbol{\Phi}^g(L)\boldsymbol{\beta}^e(L)\boldsymbol{f}_t + \boldsymbol{u}_t^g, \tag{9}$$

where $\Phi^g(L)$ is a diagonal matrix of lag polynomials with elements $\phi^g(L) = 1 - \phi_1^g L$, $\boldsymbol{\beta}^e$ is the vector of factor loadings and errors are normally distributed. The group level transition equation becomes

$$\boldsymbol{d}_t = \boldsymbol{a}_t^e + \boldsymbol{\Phi}_1^g \boldsymbol{d}_{t-1}, \tag{10}$$

where $\boldsymbol{a}_t^e = \boldsymbol{\Phi}^g(L)\boldsymbol{\beta}^e \boldsymbol{f}_t$. Here we introduce a time-varying intercept $\boldsymbol{a}_t^e$, which captures the relation between the two groups in our model.

The country level state space representation follows similarly to the one on the group level resulting into corresponding observation and transition equations

$$\boldsymbol{\Phi}^c(L)\boldsymbol{h}_t = \boldsymbol{\Phi}^c(L)\boldsymbol{\beta}^g(L)\boldsymbol{d}_t + \boldsymbol{u}_t^c, \tag{11}$$

$$\boldsymbol{h}_t = \boldsymbol{a}_t^g + \boldsymbol{\Phi}_1^c \boldsymbol{h}_{t-1}, \tag{12}$$

where $\boldsymbol{\Phi}^c(L)$ is a diagonal matrix of lag polynomials with elements $\phi^c(L) = 1 - \phi_1^c L$ and errors are normally distributed. Finally, the individual series dynamics can be explained using measurement equation only as it represents purely individual movements of each variable conditional on all previous factors. Here, we again use the relation dependence between the levels to end up with

$$\boldsymbol{\Phi}^x(L)\boldsymbol{x}_t = \boldsymbol{\Phi}^x(L)\boldsymbol{\beta}^c(L)\boldsymbol{h}_t + \boldsymbol{\varepsilon}_t^x. \tag{13}$$

Combining the representations of each level we can see the dependence between them and what parameters need to be known at each point to estimate every factor.

To ensure identification of the model we need to add some restrictions to the parameter vector $\boldsymbol{\beta}$ for every factor. For the European level factor this means setting the first element of the $\boldsymbol{\beta}^e$ to be 1. For the country level we split the coefficient vector into parts representing only one factor at a time, leading to 11 smaller vectors representing one country each. Next, we set the first element of all these smaller vectors to be 1. After recombining the vectors back to the original form, we obtain 11 elements of $\boldsymbol{\beta}^c$ are restricted to be 1 for identification purposes. However, for the group level factors we use the identification restrictions on the variance of the factor disturbances in order to not affect the magnitude of the resulting values. We do it by imposing the unit variance of the shocks to both group factors. In this way, we are able to analyse and compare the resulting time series of factors and the interaction between them. All these restrictions are set individually per block or sub-block as they can be considered independently during the estimation procedure.

4.2.3 Model estimation

To estimate a hierarchical dynamic factor model we use Markov Chain Monte Carlo (MCMC) approach following Moench et al. (2009). In this approach the starting values of the factors are treated as given and are estimated using PCA. Next for every step of the Markov Chain the following algorithm is considered:

1. Sample from the conditional distribution of parameters based on the estimated factors.
2. Given the estimated parameters and higher level factors, sample each country factor from its corresponding distribution.
3. Given the estimated parameters and first and third level factors, sample each group factor from its corresponding distribution.

4. Given the estimated parameters and lower level factors, sample from the distribution of the main European factor.

Repeating this procedure for 27 000 times, the resulting elements converge to their actual posterior distributions. First 2000 runs are discarded as a burn-in leading to the remaining 25 000 draws being used to produce estimates of the distributions. We take every 50th iteration to store all the values and take the average of the resulting 500 datapoints as the estimated parameters of the model. Next, we discuss in details how every set of factors is sampled given the available information in steps 2-4. In these steps we rely on the State Space representation as discussed before in order to be able to apply Kalman filter and smoother. Using these techniques allows us to formulate clear updating equations at every level of the model.

For the European level factor we start with pre-whitening the observation equation of the group level to obtain i.i.d. errors. This gives

$$\tilde{\boldsymbol{d}}_t = \tilde{\boldsymbol{\beta}}^e(L)\boldsymbol{f}_t + \boldsymbol{u}_t^g, \tag{14}$$

where $\tilde{\boldsymbol{d}}_t = \boldsymbol{\Phi}^g(L)\boldsymbol{d}_t$ and $\tilde{\boldsymbol{\beta}}^e = \boldsymbol{\Phi}^g(L)\boldsymbol{\beta}^e$. Next, we apply the Kalman filter to obtain estimates of $\boldsymbol{f}^e_{T|T}$ and covariance estimate of $\boldsymbol{P}^e_{T|T}$ in period T resulting in

$$\boldsymbol{f}_{t+1|t} = \boldsymbol{\Phi}^e \boldsymbol{f}_{t|t} \tag{15}$$

$$\boldsymbol{P}^e_{t+1|t} = \boldsymbol{\Phi}^e \boldsymbol{P}^e_{t|t} \boldsymbol{\Phi}^{e\prime} + \boldsymbol{\Sigma}^e \tag{16}$$

$$\boldsymbol{f}_{t|t} = \boldsymbol{f}_{t|t-1} + \boldsymbol{P}^e_{t|t-1}\tilde{\boldsymbol{\beta}}^{e\prime}(\tilde{\boldsymbol{\beta}}^e \boldsymbol{P}^e_{t|t-1}\tilde{\boldsymbol{\beta}}^{e\prime} + \boldsymbol{\Sigma}^g)^{-1}(\tilde{\boldsymbol{d}}_t - \tilde{\boldsymbol{\beta}}^e \boldsymbol{f}_{t|t-1}) \tag{17}$$

$$\boldsymbol{P}^e_{t|t} = \boldsymbol{P}^e_{t|t-1} - \boldsymbol{P}^e_{t|t-1}\tilde{\boldsymbol{\beta}}^{e\prime}(\tilde{\boldsymbol{\beta}}^e \boldsymbol{P}^e_{t|t-1}\tilde{\boldsymbol{\beta}}^{e\prime} + \boldsymbol{\Sigma}^g)^{-1}\tilde{\boldsymbol{\beta}}^e \boldsymbol{P}^e_{t|t-1}, \tag{18}$$

where $\boldsymbol{\Sigma}$ is the variance of the corresponding error terms. We follow by applying the Kalman smoother to generate draws of the factors using the whole time series available. The procedure goes backwards from the last observation to draw $\boldsymbol{f}_{t|T}$ from $N(\hat{\boldsymbol{f}}_{t|t}, \hat{\boldsymbol{P}}^e_{t|t})$ with

$$\hat{\boldsymbol{f}}_{t|t} = \boldsymbol{f}_{t|t} + \boldsymbol{P}^e_{t|t}\boldsymbol{\phi}^{e\prime}(\boldsymbol{\phi}^e \boldsymbol{P}^e_{t|t}\boldsymbol{\phi}^{e\prime} + \boldsymbol{\Sigma}^e)^{-1}(f_{t+1} - \boldsymbol{\phi}^e \boldsymbol{f}_{t|t}) \tag{19}$$

$$\hat{\boldsymbol{P}}^e_{t|t} = \boldsymbol{P}^e_{t|t} - \boldsymbol{P}^e_{t|t}\boldsymbol{\phi}^{e\prime}(\boldsymbol{\phi}^e \boldsymbol{P}^e_{t|t}\boldsymbol{\phi}^{e\prime} + \boldsymbol{\Sigma}^e)^{-1}\boldsymbol{\phi}^e \boldsymbol{P}^e_{t|t}, \tag{20}$$

where f and $\boldsymbol{\phi}^e$ are the first rows of $\boldsymbol{f}$ and $\boldsymbol{\Phi}^e$ respectively.

To sample the factors on a group level we follow the similar procedure of Kalman filter and smoother. After running the filter for each block separately we obtain

$$\boldsymbol{d}_{t+1|t} = \boldsymbol{a}^e_t + \boldsymbol{\Phi}^g \boldsymbol{d}_{t|t} \tag{21}$$

$$\boldsymbol{P}^g_{t+1|t} = \boldsymbol{\Phi}^g \boldsymbol{P}^g_{t|t} \boldsymbol{\Phi}^{g\prime} + \boldsymbol{\Sigma}^g \tag{22}$$

$$\boldsymbol{d}_{t|t} = \boldsymbol{d}_{t|t-1} + \boldsymbol{P}^g_{t|t-1} \tilde{\boldsymbol{\beta}}^{g\prime} (\tilde{\boldsymbol{\beta}}^g \boldsymbol{P}^g_{t|t-1} \tilde{\boldsymbol{\beta}}^{g\prime} + \boldsymbol{\Sigma}^c)^{-1} (\tilde{\boldsymbol{h}}_t - \tilde{\boldsymbol{\beta}}^g \boldsymbol{d}_{t|t-1}) \tag{23}$$

$$\boldsymbol{P}^g_{t|t} = \boldsymbol{P}^g_{t|t-1} - \boldsymbol{P}^g_{t|t-1} \tilde{\boldsymbol{\beta}}^{g\prime} (\tilde{\boldsymbol{\beta}}^g \boldsymbol{P}^g_{t|t-1} \tilde{\boldsymbol{\beta}}^{g\prime} + \boldsymbol{\Sigma}^c)^{-1} \tilde{\boldsymbol{\beta}}^g \boldsymbol{P}^g_{t|t-1}. \tag{24}$$

Next, applying the smoother, we sample the updated values of $\boldsymbol{d}_{t|T}$ from $N(\hat{\boldsymbol{d}}_{t|t}, \hat{\boldsymbol{P}}^g_{t|t})$ with

$$\hat{\boldsymbol{d}}_{t|t} = \boldsymbol{d}_{t|t} + \boldsymbol{P}^g_{t|t} \boldsymbol{\phi}^{g\prime} (\boldsymbol{\phi}^g \boldsymbol{P}^g_{t|t} \boldsymbol{\phi}^{g\prime} + \boldsymbol{\Sigma}^g)^{-1} (d_{t+1} - \boldsymbol{a}^e_{t+1} - \boldsymbol{\phi}^g \boldsymbol{d}_{t|t}) \tag{25}$$

$$\hat{\boldsymbol{P}}^g_{t|t} = \boldsymbol{P}^g_{t|t} - \boldsymbol{P}^g_{t|t} \boldsymbol{\phi}^{g\prime} (\boldsymbol{\phi}^g \boldsymbol{P}^g_{t|t} \boldsymbol{\phi}^{g\prime} + \boldsymbol{\Sigma}^g)^{-1} \boldsymbol{\phi}^g \boldsymbol{P}^g_{t|t}, \tag{26}$$

where d and $\boldsymbol{\phi}^g$ are the first rows of $\boldsymbol{d}$ and $\boldsymbol{\Phi}^g$ respectively.

The final step of the estimation is to get the country level factors. The procedure here is the same as for the group level factors. We start by running the Kalman filter for each sub-block to obtain

$$\boldsymbol{h}_{t+1|t} = \boldsymbol{a}^g_t + \boldsymbol{\Phi}^c \boldsymbol{h}_{t|t} \tag{27}$$

$$\boldsymbol{P}^c_{t+1|t} = \boldsymbol{\Phi}^c \boldsymbol{P}^c_{t|t} \boldsymbol{\Phi}^{c\prime} + \boldsymbol{\Sigma}^c \tag{28}$$

$$\boldsymbol{h}_{t|t} = \boldsymbol{h}_{t|t-1} + \boldsymbol{P}^c_{t|t-1} \tilde{\boldsymbol{\beta}}^{c\prime} (\tilde{\boldsymbol{\beta}}^c \boldsymbol{P}^c_{t|t-1} \tilde{\boldsymbol{\beta}}^{c\prime} + \boldsymbol{\Sigma}^x)^{-1} (\tilde{\boldsymbol{x}}_t - \tilde{\boldsymbol{\beta}}^c \boldsymbol{h}_{t|t-1}) \tag{29}$$

$$\boldsymbol{P}^c_{t|t} = \boldsymbol{P}^c_{t|t-1} - \boldsymbol{P}^c_{t|t-1} \tilde{\boldsymbol{\beta}}^{c\prime} (\tilde{\boldsymbol{\beta}}^c \boldsymbol{P}^c_{t|t-1} \tilde{\boldsymbol{\beta}}^{c\prime} + \boldsymbol{\Sigma}^x)^{-1} \tilde{\boldsymbol{\beta}}^c \boldsymbol{P}^c_{t|t-1}. \tag{30}$$

Next, we follow with the smoothing procedure to draw $\boldsymbol{h}_{t|T}$ from $N(\hat{\boldsymbol{h}}_{t|t}, \hat{\boldsymbol{P}}^c_{t|t})$ with

$$\hat{\boldsymbol{h}}_{t|t} = \boldsymbol{h}_{t|t} + \boldsymbol{P}^c_{t|t} \boldsymbol{\phi}^{c\prime} (\boldsymbol{\phi}^c \boldsymbol{P}^c_{t|t} \boldsymbol{\phi}^{c\prime} + \boldsymbol{\Sigma}^c)^{-1} (h_{t+1} - \boldsymbol{a}^g_{t+1} - \boldsymbol{\phi}^c \boldsymbol{h}_{t|t}) \tag{31}$$

$$\hat{\boldsymbol{P}}^c_{t|t} = \boldsymbol{P}^c_{t|t} - \boldsymbol{P}^c_{t|t} \boldsymbol{\phi}^{c\prime} (\boldsymbol{\phi}^c \boldsymbol{P}^c_{t|t} \boldsymbol{\phi}^{c\prime} + \boldsymbol{\Sigma}^c)^{-1} \boldsymbol{\phi}^c \boldsymbol{P}^c_{t|t}, \tag{32}$$

where h and $\boldsymbol{\phi}^c$ are the first rows of $\boldsymbol{h}$ and $\boldsymbol{\Phi}^c$ respectively. The prior distribution of all parameters in the model is assumed to be standard normal and the prior of the variance parameters is set to be chi-squared with 4 degrees of freedom following Moench et al. (2009).

4.3 Metrics

To evaluate the outcome of the model two ways are considered. The first one is to look at the dynamic of the resulting factors and try to interpret the values. Here, we use the autoregressive behavior of the factors and look at the corresponding parameters to evaluate their behavior over time. As the factors are unobserved it may be very difficult to do so and the result is nothing more than an educated guess. However, it is possible to look at persistence of the factors by means of the autocorrelations based on the calculated parameters. In this way, we are able to see the behavior of the factors inside a single time period of a moving window.

More statistically informative approach is to look at the variance decomposition of all variables. In this way, we can see how much of the variation each common factor and an idiosyncratic component can explain. The variance decomposition can be calculated as the sum of variances of individual factors including an idiosyncratic one multiplied by the squared corresponding loadings. In our four level case it becomes

$$var(x_{i,t}) = (\beta_i^e)^2 var(f_t^e) + (\beta_i^g) var(f_t^g)^2 + (\beta_i^c)^2 var(f_t^c) + var(\varepsilon_{i,t}). \tag{33}$$

Then the fraction of the volatility explained by each factor is the its individual variance times the squared coefficient divided by the total variance of the observation. Using the state space representation, the variance decomposition can be performed at every iteration of the estimation algorithm to observe the convergence of the model. The larger proportion explained by a certain factor meaning that there is large comovement between the variables in the corresponding level. For example, if the group factors explain most of the variance comparing to the main European one, we can confirm the clear split between Northern and Southern countries. Completing the same procedure for models based on one sector data only can give more ideas whether the factors are the same for all economic and financial areas or not.

5 Results

This section discusses the outcomes produced by the models described previously. Firstly, the grouping process is investigated based on the data driven methods and some literature background. Next, the performance of the dynamic hierarchical factor model is assessed for the resulting groups by means of the factors behavior and variance decomposition of the underlying series. The analysis of the factor model is performed starting from all variables combined and following with the three economic sectors individually.

5.1 Principal Components

We use Principal Components Analysis (PCA) for every series as a data driven method to determine the initial differences between economic behavior of the countries and create groups to be used in the main part of the research. The PCA results build on the correlation analysis displayed earlier and are combined with the literature based evidence. We compute the PCs based on the whole time period of 82 observation across all countries for every series individually. Due to a short observation sample, we do not base the analysis on all 13 series at the same time as such a simple method as PCA is not able to capture a large amount of difficult relations between plenty of very different economic variables with such an amount of observations. Here we display the results for one series from each group, this being Real GDP, Net international debt and 10-Year interest rate.

The results are displayed in form of loadings of the first two PCs as they cover most of the variation for majority of the series. The values for the remaining 10 variables can be found in Appendix B.

Table 5.1 displays the values of the loadings for the first two PCs for all countries based on Real GDP series. We observe that the second PC can be interpreted as the relation between changes in GDP of Greece and all other countries as the loading for Ireland takes a large value of 0.806 with all other values smaller in magnitude and negative. Therefore, we focus on the values regarding the first PC for the purpose of determining the split between the countries based on their GDP changes. Firstly, we can see that Ireland has the highest coefficient values from all nations of 0.589 which is well above the average. Next, we observe several countries that have very similar values and are below the average, this being Austria, Denmark, Germany, the Netherlands and Sweden. Such a split follows closely the most popular division into groups based on literature, separating the periphery countries known as PIIGS (Portugal, Ireland, Italy, Greece and Spain) from the rest. The only outlier from this group based on our PCA is France which is often considered as a core European country. However, looking at PCA for other series in the Real economy group such as Inflation and Unemployment we can see that France is indeed closer to the core group than to PIIGS. With the remaining two series in the group, being Total production and Net international trade, providing little intuition based on their coefficients, we can conclude that the data driven group selection based on the Real economy sector supports the theoretical ideas from the literature separating the PIIGS countries from the rest of the European core.

Table 5.1: Principal components loadings of Real GDP series

	Aut	**Den**	**Fra**	**Ger**	**Gre**	**Irl**	**Ita**	**Ned**	**Por**	**Spa**	**Swe**
PC1	0.225	0.162	0.284	0.207	0.277	0.589	0.284	0.191	0.292	0.373	0.184
PC2	-0.179	-0.090	-0.204	-0.172	-0.239	0.806	-0.195	-0.140	-0.211	-0.266	-0.095

Next, we look at the fiscal PCA outcomes of the data from the fiscal sector. Table 5.2 shows the values of PC loadings of Net international debt series for all countries. We can clearly see that both PCs do not have a clear interpretation with almost all values very close to 0. Only Greece and Italy have coefficients that are different from 0 in both cases. Such values of the coefficients show that there is little interaction between the countries based on their Net international debt. It is confirmed by very low correlation values for all nations as displayed in the Data section. Additionally, it may be cause by very high volatility of this series as displayed by extremely large standard deviation values for all nations in Table 3.2. Looking at the remaining series of the fiscal sector, we again do not observe a clear pattern or established relation between some groups of countries. Therefore, we can conclude that these variables do not bring any useful information into the group selection process due to their large volatility and low correlation.

Table 5.2: Principal components loadings of Net international debt series

	Aut	**Den**	**Fra**	**Ger**	**Gre**	**Irl**	**Ita**	**Ned**	**Por**	**Spa**	**Swe**
PC1	-0.024	0.011	0.005	0.857	0.002	0.004	-0.511	-0.005	-0.048	0.000	-0.036
PC2	0.022	-0.001	0.002	0.510	0.005	0.001	0.859	-0.009	-0.044	-0.001	0.005

Finally, we focus on the PCA results for the Financial markets data. From Table 5.3 we can see the loadings of PCs of the 10-Year interest rate series for all countries. We can interpret the first principal component as the level of the yields with a clear distinction into two groups based on the magnitude of the coefficients. This divide follows our findings from the Real economy sector and the literature by proposing the separation between PIIGS and the rest of core European countries. In this case, the PIIGS countries have a higher level of coefficients ranging from 0.184 of Italy up to 0.717 of Greece. While the highest coefficient of the second group is only 0.139 for Austria with the rest separated around 0.11. Additionally, the second PC can be seen as the interaction between the interest rate of Greece and the remaining countries with the Greek coefficient being by far the largest one of 0.515. Here, the coefficients of the core countries are more negative than the ones for PIIGS (excluding Greece) with the corresponding averages of -0.328 and -0.127. While the remaining three series in the Financial markets sector do not lead to a significant distinction between any groups of countries, the correlation analysis displayed in the Data section confirms the proposed division.

Table 5.3: Principal components loadings of 10-Year interest rate series

	Aut	**Den**	**Fra**	**Ger**	**Gre**	**Irl**	**Ita**	**Ned**	**Por**	**Spa**	**Swe**
PC1	0.139	0.111	0.137	0.113	0.717	0.352	0.184	0.125	0.428	0.234	0.102
PC2	-0.318	-0.352	-0.298	-0.340	0.515	-0.189	-0.157	-0.326	0.008	-0.170	-0.332

Concluding the PCA results for all sectors combined with the correlation analysis and some literature backed information, we distinguish two groups of countries for the main stage of our research. The first group includes the periphery countries such as Greece, Ireland, Italy, Portugal and Spain. The remaining core countries are selected to form the second group of Austria, Denmark, France, Germany, the Netherlands and Sweden. While the results of the PCA are not treated as the final findings of our research, they are used for the initial values of the factors input to the model. Combining this fact with the group selection procedure, PCA is an important aspect of this research and has a high impact on the final conclusions.

5.2 General economic situation

To explore the overall economic relations between European countries we construct a dynamic hierarchical factor model using all 13 series describing different sectors combined. We split the countries in two groups based on data-driven PCA and evidence from literature as described in the previous section. We call the resulting groups PIIGS and core countries. The first group includes Portugal, Ireland, Italy, Greece and Spain and is also referred as periphery countries due to their location on the map of Europe. The second group consists of Austria, Denmark, France, Germany, the Netherlands and Sweden with most of the countries located in the center of Europe. After completing the model we obtain underlying latent factors for every point in time on European, group and country levels.

We start the analysis by looking at the values of the factors and their dynamics over time. Figure 5.1 displays the behavior of the two top level factors over the whole observation period, being a European one and a factor for each group of countries. First, we observe that the European factor takes the smallest values in magnitude and is the least volatile. This can be expected as it affects a larger amount of countries, therefore is not as affected by an individual shocks as the group level ones. However, there are several periods of increased volatility that are mostly connected to the recessions in the European area. In this way, we see movements in 2003, 2011 and 2020 when the economy was either coming out or entering a recession period. We can conclude that periods of economic instability affect the European factor by changing its values meaning that all economies tend to move in the same direction. Additionally, we can see high starting values for the first observation point for all factors. This may be explained by a post recession period after 1999 or a simple modelling issue where the estimation procedure is not the most accurate for the starting values.

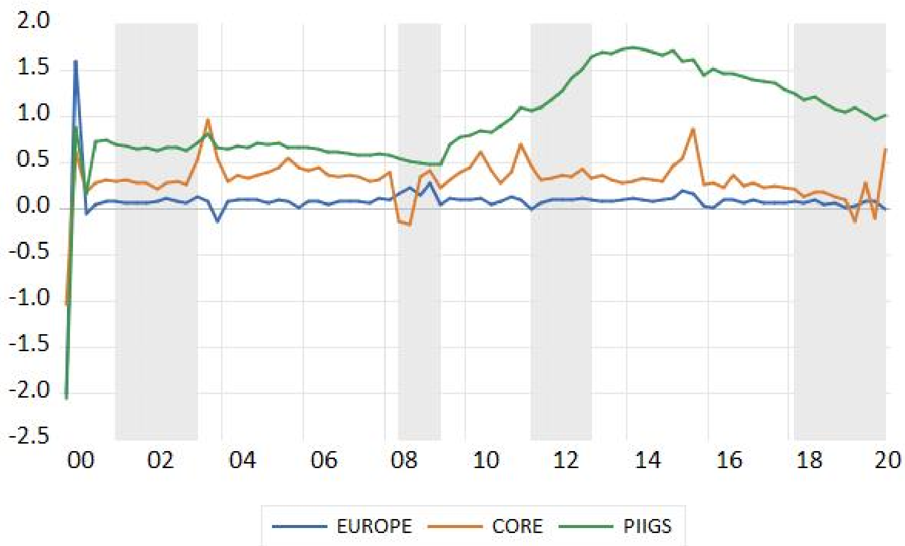

Figure 5.1: Values of European and two group level factors for all series combined (shaded areas are OECD-defined recession periods in Europe)

Next, we look at the two group level factors as displayed in Figure 5.1. We observe that overall level for both groups is similar and is slightly above the one for the European factor. However, the behavior of the factors over time varies with core group factor being more affected by spikes around the recession periods. This may be explained by the fact that such periods affect countries in the core group similarly, while the PIIGS countries tend to move more individually. Additionally, the periphery group factor exhibits a steady change of its level from 2009, slowly reverting back to the previous average continuing into 2020. We can see this as a period of increased heterogeneity in Europe after the Great Financial crisis of 2008. In this period, the behavior of economies of PIIGS countries seems to be very different from the core ones reaching its peek around 2013 and starting a gradual decline from late 2014. Additionally, we see some but not complete reversion of this trend towards the current time with two factors having similar values by 2020. In this way, we observe a different reaction speed of two groups to the major shock of the great financial crisis of 2008 with the PIIGS countries taking much longer to rebuild their trust and economic growth. We can call this trend as Europe of two speeds where there are countries that rebound fast after any shocks and there are other that need much longer time to get back to normal state.. We can conclude that European heterogeneity has increased after 2008 but the integration trend seems to be stronger after 2014 leading to more similar behavior of both groups closer to 2020.

After looking at the behavior of the factors overtime, we examine their persistence by means of the autoregressive coefficients. Persistence determines the level by which the factors are determined by their previous values contrary to the individual shocks of every time period. The European factor has a very low autoregresive coefficient of 0.06 meaning that only limited information comes from its previous value. This can be explained by the fact that it is responsible for behavior of many countries that may differ at every period leading to low persistence. However, the group level factors are much more persistence with values of 0.92 and 0.29 for PIIGS and core groups respectively. Here, most of the changes in the factors values come from their previous values with shocks having only limited effect. Additionally, the periphery countries have an autoregressive coefficient close to 1 meaning that the time series is close to being not stationary. This can be explained by the change of the overall level of the factor starting from around 2009. Next to the difference in the mean of the posterior distribution of the coefficients, we observe that the standard deviations for two groups also behave differently. The PIIGS factors coefficients tend to be close together with standard deviation of only 0.12 while the core group ones are more volatile taking the value of 0.60. We can conclude that by covering a smaller number of countries and more specific information both group factors are less affected by independent shocks and are more predictable.

Finally, we look at the country level factors persistence with the parameters of the posterior distribution of the coefficients displayed in Table 5.4. We can see that all country level factors are very persistent with the lowest value of ϕ being 0.731 for the Netherlands. Again there are several factors with coefficients above 1, most of them belonging to the PIIGS group with the highest value of 1.505 for Greece. In general, we see a large difference between the average means for two groups, where the one for PIIGS is 1.23 and for core - 0.89. Additionally, we observe that the distribution of core group coefficients is more spread out with the average standard deviation of 0.42 comparing to 0.15 for periphery countries. Overall, the difference between the country and group level coefficients for core and PIIGS countries suggest that the behavior of two groups is indeed different and there is evidence for the European heterogeneity.

Table 5.4: Posterior parameters of autoregressive coefficients of country level factors for all series

	Gre	**Irl**	**Ita**	**Por**	**Spa**	**PIIGS**	**Aut**	**Den**	**Fra**	**Ger**	**Ned**	**Swe**	**Core**
mean	1.505	1.173	0.998	1.097	1.372	1.229	0.811	0.956	0.997	0.836	0.731	1.012	0.890
median	1.506	1.182	1.031	1.122	1.371	1.242	0.971	0.999	0.997	1.031	0.948	0.992	0.990
sdev	0.066	0.154	0.220	0.256	0.035	0.146	0.815	0.187	0.018	0.586	0.519	0.364	0.415

We continue by looking at the importance of each factor for every series by means of variance decomposition. In this way, we can see what percent of variation for every variable is explained by each level factor with detailed results displayed in Appendix C. The summary of the values including average percentage explained by each level factor and number of series that a factor is the most important one are displayed in Table 5.5. First, we observe very small importance of the European level factor for all series as it is not resulted as the most important one for any variable. This leads to a conclusion that there is limited integration between the countries for the general economic outlook and most of the importance comes from lower level factors. However, looking at the behavior of group level factors and their similarities during the most volatile periods we can expect both level factors to share some information and, therefore, take away some of the variation explained by the European one. Additionally, this may be resulted from the large number of series that have to be explained by the highest level factor. In this way, it has to take values that incorporate some information from very broad data leading to it being less specific and not able to explain well the changes in every particular variable.

Table 5.5: Summary of the variance decomposition for all series

Level	**Average variation explained**		**Number of series a factor is dominant**	
	PIIGS	*Core*	*PIIGS*	*Core*
I	0.121	0.024	0	0
II	0.417	0.310	33	26
III	0.233	0.505	18	39
IV	0.229	0.161	14	13

Similarly, the idiosyncratic factor also has limited importance on the data in general with being the dominant one for only 27 out of 143 cases. This means that there are important relations between most of the series at least on the country or group levels. Additionally, this supports the idea of constructing a factor model for the data as there is a lot of commonality present. The only variable which is mostly explained by idiosyncratic factor is production, with the lowest level factor being the most important for 6 of 11 countries. This can be explained by the fact that industrial production in each country does not depend a lot on the international state of economy or even other variables in the same country. The main shocks to total production usually come from raw materials or some legislative changes that were not included in this research and may be very country specific.

Finally, we do not observe a clear pattern between group and country level factors where one is being the most important in a certain scenario for all cases. It is expected here due to a large number of series that come from different sectors leading to a more difficult task for a factor model to combine all the information. Therefore, the most important factor between the group or county level ones should be decided specifically for each variable in each country depending on some additional causes and can not be collected into one general trend. Despite this lack of consistency, the fact that the group level factor is the most important in explaining the variation for 59 cases out of 143 even when combining all the series in one model supports the idea of some level of European heterogeneity present during the observation period. Additionally, higher importance of group factor as comparing to European one shows some support for a limited amount of integration present between European countries.

5.3 Real economy

After exploring the European heterogeneity between overall economic conditions of the selected countries, we dive deeper into three separate sectors to analyze how the divide is developed on a smaller scale. We start by looking at the real economy sector which includes variables as real GDP, inflation, unemployment, production and international trade. All these characteristics are important for a country as a whole and affect its every individual. Therefore, exploring this sector individually is beneficial for national and European policymakers that aim to build a stable economic situation with equal development of all countries. The groups of countries are the same as in the previous part of the research resulting in the same amount of latent factors on three levels.

First, we look at the dynamics of the underlying latent factors over time as presented in Figure 5.2. We can see that the overall situation of the factors in the real economy sector is similar to the factors applied to all data with the PIIGS group factor taking higher values in magnitude comparing to the core countries and the European one. Additionally, we again observe a change of level of the PIIGS factor starting from 2009 and reaching its peak values around 2014. Again, the reversion to the average is not completed by 2020 and is even starting to level off completely in 2020. Therefore, we can conclude that the divide between the groups seems to present throughout the whole observation period being even stronger from 2009 until nowadays. Lastly, the values of the core group factor is even below the European factor ones contrary to the results for all series combined. This can be resulted from the fact that most variables in the real economy sector for the countries in the core group are below the European averages, while the other sectors do not exhibit the same situation.

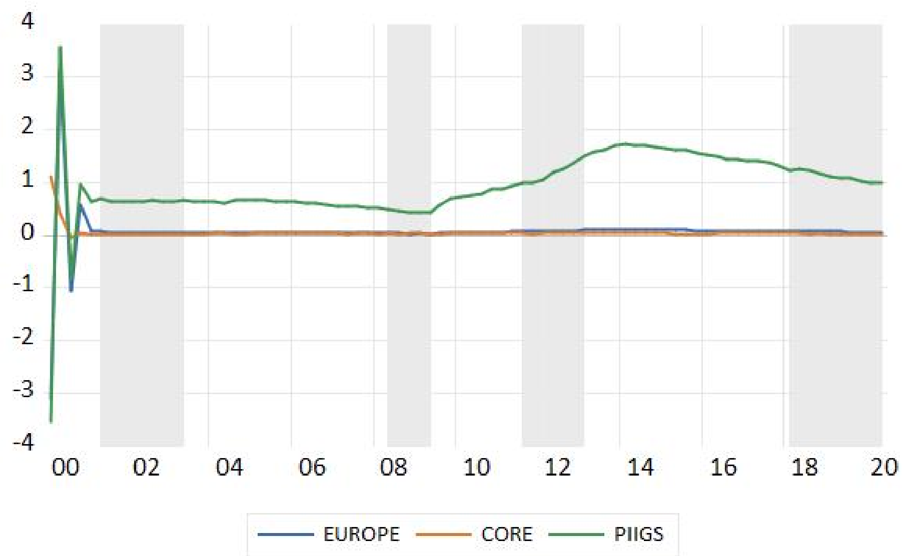

Figure 5.2: Values of European and two group level factors for series in the real economy sector (shaded areas are OECD-defined recession periods in Europe)

Another observation from Figure 5.2 is overall lower level of volatility of the factors. We can see that all three factors tend to be on one level for the whole duration of the observation period without many spikes or intersections of the corresponding lines. The only exception to this is the very starting period in 2000 where the values for all factors start well above the level of the ones for all data. However, this could be due to the start of the estimation window and not represent the actual situation in the underlying economies. The lower level of volatility can be explained by low volatility of the series in the real economy sector as measured by standard deviation in the Data section. Additionally, variables such as GDP or inflation are expected to change very slowly over time comparing to more dynamic financial markets or debt positions. In this way, we can expect the factors of real economy sector to be not only less volatile comparing to the overall ones but also to the remaining two groups.

We continue the analysis of the real economy sector by looking at the persistence of the factors using their autoregressive coefficients. The coefficient value of 0.08 for the European level factor leads to it being not persistent and around the level of the same factor when applied to all series. The standard deviation of the posterior distribution of the European level coefficients is again close to the value for all data case, being equal to 0.36. This can be resulted by the fact that the factors shows similar trends and behavior comparing to the all series situation and real economy sector is a fundamental one to the overall situation in a country. Looking at the group level factors, we can see even larger gap between their behavior as compared to the all data case with the values of 0.88 for PIIGS and 0.05 for core countries. In this way, the periphery group factor seems to be again very persistent and close the stationarity bound of 1. This can be explained by the same shift of the factor level from 2009 with only some reversion back to the starting mean towards 2020. Interestingly, we can see very low level of persistence of the core group factor meaning that shocks are the most important driver of its values. We can explain this by the fact that the average magnitude of the factor is very low leading to its values being very dependant on the external movements. Concluding the differences in the persistence of the group factors are in line with the dynamic observations and lead to a well present heterogeneity between the countries.

Lastly, we look at the persistence of country level factors individually as displayed by the parameters of their posterior distributions in Figure 5.6. We again can see a difference in the general trend between two groups with the PIIGS countries having higher average of 1.25 which well above 1. Looking at the standard deviation of the posterior distributions we do not observe any large differences between the two groups with very low values in both cases. The lower level of volatility comparing to the all data case can be caused by the fact that fewer series are considered and, therefore, it is easier for the model to estimate the factors more precisely. One interesting exception to the general trend is Italy with its value of 1.03 closer to the average of the core group, this being 1.01 and even below one of its member - Germany with value of 1.06. This is similar to the behavior of the Italian factor when applied to all series which may lead to some discussion about the correctness of our split into groups. However, as there is only limited information provided by the autoregressive coefficients about the actual behavior of the countries and correctness of the split we believe that we can not make such conclusions with certainty.

Table 5.6: Posterior parameters of autoregressive coefficients of country level factors for real economy series

	Gre	**Irl**	**Ita**	**Por**	**Spa**	**PIIGS**	**Aut**	**Den**	**Fra**	**Ger**	**Ned**	**Swe**	**Core**
mean	1.521	1.190	1.034	1.121	1.374	1.248	0.974	1.004	0.997	1.059	1.008	0.989	1.005
median	1.520	1.189	1.035	1.121	1.372	1.247	0.987	1.007	0.997	1.059	1.008	0.992	1.008
sdev	0.052	0.030	0.009	0.015	0.034	0.028	0.169	0.052	0.017	0.012	0.007	0.020	0.046

We complete the analysis of the real economy sector looking at the variance decomposition results. The proportions of each series explained by every level factor for each country and the group averages are displayed in Table 5.7. We can clearly see a different behavior between the countries in two groups for all five series with different level factors being the most important ones. The series regarding countries in the core group are dominated by the country level factor with it explaining the most variation for 25 out of 30 cases. The 4 of remaining cases come from the production series when the idiosyncratic part explains the most variation. The last outlier is the inflation series for Germany when there is a close situation between the country and individual sector levels being the most important. The average percentage of variation explained by the country factor in he core group varies from 40 up to 98 percent, while only reaching 20 in the PIIGS group. The dominance of the country level factor for core group can be explained by a low magnitude if the corresponding group factor and smaller values of the underlying data in general. This leads to the situation when European factor that affects all countries follows more precisely the movements in more volatile and higher absolute level PIIGS countries.

Table 5.7: Proportion of variance explained by each level factor for real economy sector (level I - Europe, level II - group, level III - country, level IV - individual series)

Series	Level	Gre	Irl	Ita	Spa	Por	PIIGS	Aut	Den	Fra	Ger	Ned	Swe	Core
GDP	*I*	.64	.52	.33	.67	.55	.54	.01	.00	.00	.01	.00	.00	.00
	II	.24	.19	.13	.25	.22	.21	.03	.00	.00	.04	.00	.01	.01
	III	.05	.09	.53	.04	.19	.18	.86	.98	.98	.85	.98	.95	.93
	IV	.07	.21	.00	.04	.05	.07	.10	.02	.01	.10	.02	.03	.05
INF	*I*	.47	.51	.27	.54	.45	.45	.01	.00	.00	.01	.00	.00	.00
	II	.18	.18	.10	.23	.17	.17	.02	.00	.00	.02	.00	.01	.01
	III	.04	.08	.43	.04	.15	.15	.59	.89	.87	.48	.90	.74	.75
	IV	.32	.23	.19	.19	.23	.23	.39	.11	.13	.50	.09	.25	.25
UMPL	*I*	.68	.65	.34	.68	.57	.58	.01	.00	.00	.01	.00	.00	.00
	II	.26	.24	.13	.28	.23	.23	.03	.00	.00	.04	.00	.01	.01
	III	.05	.11	.53	.05	.20	.19	.96	.99	.99	.95	.99	.99	.98
	IV	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
PRD	*I*	.52	.73	.29	.62	.57	.55	.01	.00	.00	.00	.00	.00	.00
	II	.15	.13	.11	.17	.14	.14	.01	.01	.00	.01	.00	.00	.01
	III	.03	.06	.45	.03	.12	.14	.57	.30	.83	.13	.32	.24	.40
	IV	.30	.09	.15	.18	.17	.18	.42	.69	.17	.86	.68	.75	.60
NIT	*I*	.03	.62	.18	.52	.73	.42	.07	.00	.00	.02	.00	.00	.02
	II	.45	.24	.13	.39	.11	.26	.07	.01	.01	.11	.01	.02	.04
	III	.09	.10	.62	.07	.10	.20	.81	.97	.98	.85	.99	.96	.93
	IV	.43	.04	.07	.02	.06	.12	.05	.01	.01	.03	.00	.02	.02

In contrast to the core group, the countries in PIIGS group mostly have the European level factor explaining largest share of the variation. In 19 out of 25 cases the top level factor is dominant across all variables. Here the average values of the variation explained highest level factor are between 42 and 58 percent, much above the values for core group that are all very close to 0. In this way, we can see how the European level factor actually reacts heavily to the more volatile series of the periphery countries and is not as important for core group ones. Interestingly, 5 out of 6 remaining cases come from Italy which becomes an outlier of the group again. Here, the country level factor is the most important for all 5 series, similarly to the situation of the core group countries. However, we can see that the shares explained by the European and country level factors for Italy are relatively close while the difference between these two values are very large for other countries. Therefore, Italy seems to act as not completely belonging to one of the two groups of countries and being an individual entity that has some features from both. Overall, we can conclude that there is a striking difference in behavior of the countries between two groups for all series in the real economy sector leading to further support of the European heterogeneity in this area.

To complete the analysis of the real economy sector, we can conclude that we have found some evidence of the divide between two groups of countries present in the area. This split seems to be more present in this sector if comparing to the overall economic situation with its effect magnified starting from 2009 and continuing into 2020. The only variable in the real economy area that is not hugely affected by the integration or groups of countries is production as its changes are often come from idiosyncratic shocks independent of the country or higher level factors. Finally, while most countries concerned in this research can be clearly split into two groups, Italy seems to be in between taking some features from both.

5.4 Fiscal sector

Next, we discuss the results of the analysis for the fiscal sector. This sector consists of the series such as net international debt, overnight interest rate, government debt and total credit to government. In this way, it mostly affects bank and other financial institutions and is controlled by the decisions done by central banks corresponding central banks. Therefore, the implications of this research are important for the most efficient distinction between the tasks of European and national central banks in terms of regulations and other fiscal decisions. We continue with the same groups as discussed in all previous parts and create a model with the same structure of factor levels.

We start by discussing the dynamics of the underlying factors that are displayed in Figure 5.3. We can clearly see a very different situation comparing to both real economy sector and overall economic situation with no clear pattern between the factors. In this case, we do not observe a consistent pattern for any of the three factors or any meaningful interaction between each other. Moreover, there is a very high degree of volatility of the values present during most of the observation periods. While some of the volatility spikes as in 2003 or 2008 can be explained by entering or leaving a recession period, the one happening from 2014 to 2016 does not seem to be related to a certain economic event. Additionally, there were no changes in the factors behavior during time period for the all data case or real economy series. High level of volatility of the factors can be explained by the fact that the data in this sector is the most volatile of the all variables used in this research as was discussed in the Data part with net international debt being particularly extreme. Due to this inconsistency of factors behavior we can not make any conclusions regarding the development or existence of the heterogeneity between the countries for variables in the fiscal sector.

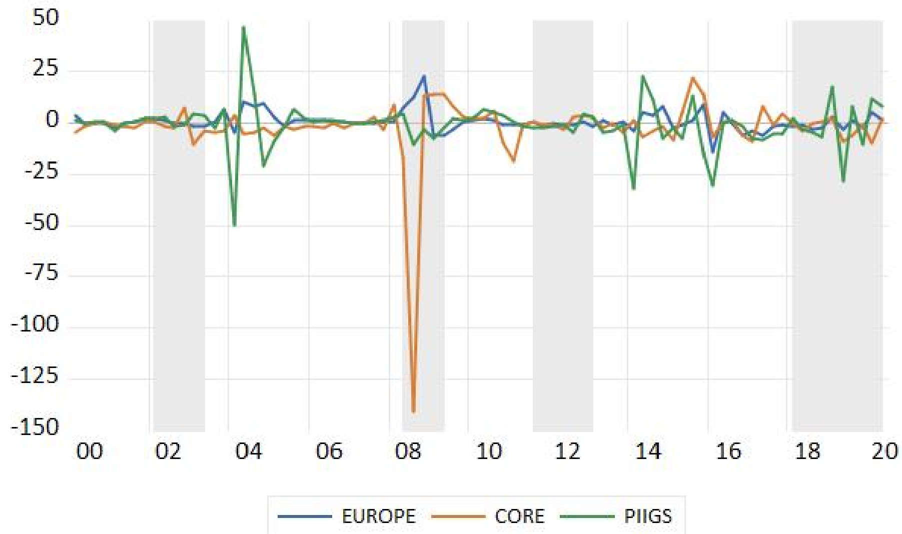

Figure 5.3: Values of European and two group level factors for series in the fiscal sector (shaded areas are OECD-defined recession periods in Europe)

We continue the discussion of the factors behavior by looking at their persistence throughout the observation period. As expected from the previous discussion we observe a very low value of autoregressive coefficient for the European level factor with the value of 0.13. Such high importance of the shock is in line with no particular trend line and very high volatility of the factor realizations. Interestingly, the value here is above the one for the real economy sector despite the latter one having more stable behavior throughout the observation period. The group level factors are also not persistent with values of 0.00 for PIIGS and -0.05 for core countries. Such values very close to 0 mean that there is almost no persistence in the series and most of the changes are coming from idiosyncratic shocks at every time point. Next to very low values, all three level factors have very high posterior standard deviation with the values close to 1. This is the result of high level of uncertainty in the data leading to extra difficulty for the model to create a precise factor representation. This corresponds with the previous findings where none of the higher level factors produced any pattern and were very volatile.

To complete the analysis of the factors behavior we look at the country level factors autoregressive coefficients with their posterior parameters displayed in Table 5.8. Here we can see that all countries have very low values close to 0 independently of their group with averages of 0.02 and -0.01 for PIIGS and core groups respectively. Therefore, we conclude that none of the three levels have any persistence on them and are mostly driven by the shocks coming outside of the common factors. Looking at the standard deviation of the posterior distributions we again observe all values very close to 1 leading to high model uncertainty and no clear factor structure. Additionally, we do not see any distinction between the factors behavior depending on the group of countries, therefore, we can not make any conclusions regarding the level of integration or heterogeneity in the fiscal sector. Lastly, the behavior of the factors in this sector is completely different from both real economy and the all data cases leading to a conclusion that the presence of economic divide in Europe depends on the sector that is examined.

Table 5.8: Posterior parameters of autoregressive coefficients of country level factors for fiscal sector series

	Gre	**Irl**	**Ita**	**Por**	**Spa**	**PIIGS**	**Aut**	**Den**	**Fra**	**Ger**	**Ned**	**Swe**	**Core**
mean	0.063	0.018	-0.014	0.047	-0.004	0.022	0.016	0.008	-0.042	-0.003	-0.012	-0.033	-0.011
median	0.081	0.038	-0.029	-0.016	-0.063	0.002	0.062	0.053	-0.011	-0.027	-0.032	-0.012	0.006
sdev	0.967	1.036	1.009	1.034	0.975	1.004	0.975	1.023	0.987	0.957	0.993	1.059	0.999

We complete the analysis of the fiscal sector by looking at the variance decomposition of every series as shown in Table 5.9. First, we can see that there is no domination by one single level in terms of its importance for all series. The European factor occurs as the most important one in around half of the cases, this being 23 out of 44 times. In this way, it seems to be slightly more informative than the rest in general leading to some level of integration across the variables in the fiscal sector. Additionally, the idiosyncratic part is the most important in only 8 cases meaning that even in this volatile case there is some room for underlying common factors in the data. From these two facts we can conclude that despite almost no persistence of the factors and their volatile behavior there is quite some level of integration in the fiscal sector and common factors are important for most variables and countries.

Table 5.9: Proportion of variance explained by each level factor for fiscal sector (level I - Europe, level II - group, level III - country, level IV - individual series)

Series	Level	Gre	Irl	Ita	Por	Spa	PIIGS	Aut	Den	Fra	Ger	Ned	Swe	Core
NID	*I*	.25	.24	.44	.05	.41	.28	.10	.10	.23	.25	.51	.62	.30
	II	.02	.19	.09	.03	.10	.09	.09	.09	.19	.19	.12	.15	.14
	III	.11	.08	.30	.71	.26	.29	.56	.56	.35	.14	.23	.09	.32
	IV	.61	.49	.17	.20	.24	.34	.24	.24	.23	.42	.14	.14	.24
IRS	*I*	.37	.39	.02	.29	.85	.38	.19	.60	.80	.55	.84	.57	.59
	II	.01	.14	.01	.06	.01	.05	.02	.15	.08	.04	.02	.22	.09
	III	.58	.22	.96	.60	.08	.49	.01	.15	.05	.22	.02	.08	.09
	IV	.04	.25	.02	.05	.06	.08	.79	.10	.07	.18	.12	.13	.23
DEB	*I*	.33	.52	.02	.14	.83	.37	.56	.49	.74	.58	.73	.50	.60
	II	.02	.18	.00	.03	.01	.05	.04	.12	.09	.03	.01	.20	.08
	III	.52	.30	.89	.26	.08	.41	.02	.12	.05	.24	.02	.07	.09
	IV	.13	.00	.09	.57	.09	.18	.38	.26	.12	.15	.24	.23	.23
CRE	*I*	.53	.02	.02	.31	.79	.33	.59	.22	.79	.22	.32	.05	.37
	II	.01	.04	.00	.08	.02	.03	.04	.07	.10	.02	.04	.01	.05
	III	.05	.66	.83	.59	.07	.44	.04	.08	.00	.09	.08	.12	.07
	IV	.42	.28	.15	.02	.12	.20	.33	.62	.11	.67	.57	.82	.52

Looking at series coming from the two groups separately, we do not observe a particular trend in their behavior. The only slight difference occurs in the importance of the country level factors, where the averages for the PIIGS group take values around 40 percent which is above the highest one for the core countries of 32 and much above most of the averages of that group. However, even the country level factor for the periphery countries does not act the same for every nation and usually can not explain more than half of the variation. Additionally, the group level factor fails to be the most important one for any variable and generally explains the lowest amount of variation. Concluding from these facts, we do not observe any evidence for the European heterogeneity in the fiscal sector. This can be caused by the fact that the European Central Bank is the most important entity in the sector for all countries and tends to control most of the regulation and banking behavior. Finally, issues as government debt or international loans of any single country are usually connected to the rest of European Union as most of credit moves inside it. Therefore, the most significant decisions (such as European Covid recovery fund) are discussed and agreed by all the countries together, such that they do not lead to an increased level of heterogeneity.

Overall, we can conclude that the behavior of the variables in the fiscal sector are very different from the general economic situation or real economy series. The factors in this sector are very volatile with almost no persistence and no clear division between groups. Additionally, there is no single pattern throughout the whole observation period with few several spikes of the underlying values. However, we can see some level of integration in the data as the European level factor is more dominant in terms of variance decomposition comparing to any other level. Combining lack of specific behavior present in each of the groups with the low importance of group level factor we conclude that there is very low level of heterogeneity present in the fiscal sector.

5.5 Financial markets

The final individual area that we explore in this research is financial markets sector. For this part we consider variables as 3-month interest rate, 10-year interest rate, stock market capitalization and stock market index. This area is the most important for different kind of investors as it covers stock and bod markets that are the most used tools to invest into. The issue of heterogeneity in the sector plays an important role in risk management and portfolio diversification when investing in markets of different countries. As before, we continue using the same two groups of countries, this being PIIGS and core countries, resulting in the same three levels of underlying factors.

We start by examining the behavior of European and group level factors dynamics over time as displayed in Figure 5.4. We observe that factors behave more similarly to the real economy sector than to the fiscal one, having a certain trend throughout the observation period and low level of volatility. We again can see a big jump in values at the very beginning of 2000 due to the start of the estimation window. Additionally, the PIIGS group factor again takes the highest values comparing to the remaining two for the whole time period. This factor is also the most volatile one with several spikes well above its average level, which can be explained by the fact that the underlying data coming from the PIIGS countries is more volatile than the core one.

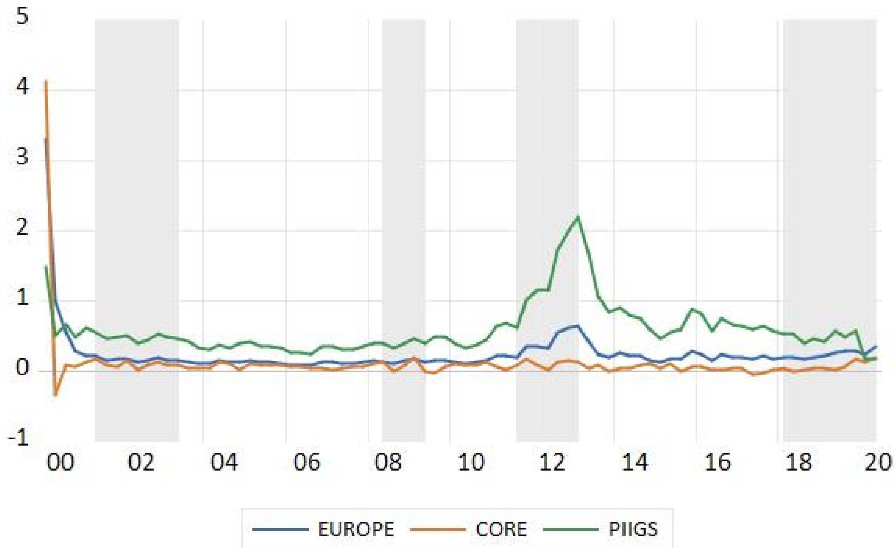

Figure 5.4: Values of European and two group level factors for series in the financial markets sector (shaded areas are OECD-defined recession periods in Europe)

However, there are several interesting differences here as comparing to the real economy or all data situations. First, we can see that while the factors do not exhibit a lot of large changes in their values, there are much more movement around the average level. In this way, we do not see a smooth line as it was the case for real economy factors. This happens as financial markets change their values at every time point and, therefore, are more volatile than real economy variables that are usually updated only on a quarterly basis. As we can see this additional volatility persists even if we build all variables on the same scale by using percentage change and with the same frequency. Additionally, dynamics of the PIIGS factor is slightly different from the previous situations as this time there is only a short time spike around 2012 and not a long term change of level. This also leads to a smaller drop in 2020 which starts with the Covid related restrictions as the factor is on its normal times level shortly before it. This can be explained by the fact that financial markets are usually not prone to large structural changes and have short term memory. On the other hand, the reaction of the real economy sector is more prolonged and may persist for long period of time. Therefore, the idea of Europe of two speeds is mostly present in the real economy variables and can not be applied to a more fast moving financial markets ones. Despite these differences, as in the real economy sector we can see some evidence for European heterogeneity in the data as the behavior of the two factors is very different.

Next, we look at the persistence of the factors by means of the autoregressive coefficients. We observe the European level factor being somewhat persistent with its coefficient taking a value of 0.23. This is the highest value comparing to all previous European level ones. We can conclude that shocks have the least effect on the highest level factor in this case as comparing to the real economy one and fiscal sectors with the lowest standard deviation of 0.24. As in the most previous cases we can see a large difference between the coefficients of the two group level factors leading to values of 0.56 and 0.12 for PIIGS and core groups respectively. However, this time the value for periphery group is well below 1 leading to a stationary time series of factors. This is indeed supported by the lack of any structural change of level throughout the time period and only several spikes of large magnitude as comparing to the two other factors fro this sector.

We continue the analysis of the factor persistence by looking at the coefficients of country level ones with their posterior parameters displayed in Table 5.10. We observe that all values are very close to each other and very slightly above 1. We do not see any difference between the average means for two groups with values of 1.06 and 1.05 for PIIGS and core countries respectively. This means that all 11 factors do not seem to be stationary and do change their level contrary to the higher level ones. Additionally, this is the first case when all country level coefficients are above 1 and show very similar behavior independently of their group. The standard deviation of the posteriors for both groups are also similar and all close to 0 meaning that there is high certainty of the model and it is able to make precise estimation of the factors. Therefore, we can conclude that the lower level factors do not seem to be affected by a group that a certain country belongs and move differently comparing to higher level ones for financial markets sector.

Table 5.10: Posterior parameters of autoregressive coefficients of country level factors for financial markets series

	Gre	**Irl**	**Ita**	**Por**	**Spa**	**PIIGS**	**Aut**	**Den**	**Fra**	**Ger**	**Ned**	**Swe**	**Core**
mean	1.149	1.060	1.012	1.067	1.024	1.063	1.047	1.082	1.024	1.045	1.032	1.038	1.045
median	1.246	1.061	1.013	1.067	1.026	1.082	1.047	1.079	1.028	1.049	1.037	1.039	1.046
sdev	0.420	0.022	0.011	0.019	0.014	0.097	0.024	0.027	0.025	0.032	0.028	0.022	0.026

We complete the analysis of the financial markets area by looking at the proportion of variation that every factor explains for all variables. The outcomes of the variance decomposition procedure are displayed in Table 5.11. We observe that country level factor is by far the most important one for all series explaining most of the variation in 43 out of 44 cases. The only outlier of this rule is stock market capitalization variable for Greece where the European level factor is only slightly above the country one. This may be explained as the similarities between all variables in the financial sector inside one country are much larger than even the similarities between the markets of neighboring countries. Therefore, we can expect that most of the changes in the values of the variables in this sphere are country specific and might not affect the other nations markets. In this way, we can conclude that the most of diversification and risk mitigation for investors can be achieved by spreading their wealth across several countries independently of the groups they belong to.

Table 5.11: Proportion of variance explained by each level factor for financial markets sector (level I - Europe, level II - group, level III - country, level IV - individual series)

Series	Level	Gre	Irl	Ita	Por	Spa	PIIGS	Aut	Den	Fra	Ger	Ned	Swe	Core
IR3	*I*	.21	.09	.02	.11	.05	.10	.00	.01	.00	.00	.01	.00	.00
	II	.23	.10	.02	.11	.05	.10	.03	.14	.06	.06	.19	.06	.09
	III	.45	.76	.96	.71	.88	.75	.95	.80	.90	.90	.69	.88	.85
	IV	.10	.05	.01	.06	.02	.05	.02	.06	.04	.04	.12	.05	.06
IR10	*I*	.24	.10	.02	.12	.05	.11	.00	.01	.00	.00	.01	.00	.00
	II	.26	.10	.02	.12	.05	.11	.03	.15	.06	.06	.23	.07	.10
	III	.51	.80	.97	.76	.90	.79	.96	.81	.94	.93	.76	.93	.89
	IV	.00	.00	.00	.00	.00	.00	.00	.04	.00	.00	.01	.00	.01
SMC	*I*	.37	.08	.03	.02	.04	.11	.02	.01	.00	.01	.00	.01	.01
	II	.08	.16	.11	.05	.03	.09	.14	.15	.05	.10	.03	.06	.09
	III	.32	.55	.72	.18	.75	.50	.83	.81	.73	.75	.55	.88	.76
	IV	.22	.21	.15	.75	.18	.30	.00	.04	.22	.14	.42	.06	.15
SMI	*I*	.24	.06	.01	.08	.06	.09	.01	.01	.00	.00	.00	.00	.00
	II	.22	.06	.01	.02	.01	.06	.06	.15	.07	.07	.06	.06	.08
	III	.32	.73	.98	.40	.83	.65	.74	.83	.48	.83	.58	.76	.70
	IV	.22	.15	.01	.51	.10	.20	.19	.02	.45	.10	.36	.18	.22

Another observation from Table 5.11 is a very different behavior of all series fro Greece comparing to other nations. In Greek case we can see that all four levels explain some of the variation in most cases and he differences between the most of them are small. For stock market capitalization this even leads to a different level being the most important comparing to all other situations in the sector. This may be caused by he fact that the local financial markets system in Greece is the least developed from all the countries considered in this research, therefore, it is affected more by different external issues coming from European level or other nations. Additionally, Greek stock market seems to be by far the most volatile one as was discussed in the Data section. Such a high level of volatility can lead to some difficulties for the factor model to capture all relations correctly and produce smooth results as we could see from the fiscal sector. Combining these two issues, we realize that we can not draw the same conclusions for Greek financial markets as we do for all the other countries in this sector.

Finally, after completing the analysis of the financial markets sector we can see that there is some evidence for the European heterogeneity in the data looking at the behavior of the factors. However, we do not observe the same structural changes in the underlying factors as we did for real economy sector or the overall case. Additionally, the country level factors that seem to be the most important ones in explaining the changes of the variables do not behave differently based on the group they belong to. In this way, we conclude that while there is some split between the core and periphery countries financial markets, most of the diversification for investors can be achieved by selecting several countries to invest in independently of the divide into groups. Finally, due to its underdevelopment and higher level of volatility, the Greek financial market does not completely follow the same pattern as the other countries ones. This leads to both potentially more diversification and higher level of risk when investing into this country.

6 Conclusion

The discussion about rising heterogeneity inside European Union after the Great Financial Crisis is one of the main topics for European policy makers and politicians in the last 10 years. The presence of such a split can affect both social and economic decisions of the central government as well as the situations inside every single country concerned. Hence, in this paper we examine the presence of European heterogeneity in 21st century and investigate what economic spheres are affected the most. We construct a dynamic hierarchical factor model that makes use of three levels of latent factors to explain the relations between the countries economies. The model starts from the highest level European factor that affects every nation, moving down into group and country levels that influence only a certain selection of variables.

We use 11 European countries in this research consisting of Austria, Denmark, France, Germany, Greece, Ireland, Italy, the Netherlands, Portugal, Spain and Sweden. As we are interested in the economic divide in 21st century, the dataset includes 82 quarters of observations from 2000Q1 until 2020Q2 covering the most of the period when the debate about European heterogeneity is present. The dataset includes 13 economic and financial variables that are divided into three sectors, this being real economy, fiscal sector and financial markets. In this way, the factor model is first applied to all variables combined followed by a separate analysis for each economic sector. To determine the groups of countries we make use of both a quantitative method of PCA and some insights from the literature. After completing this analysis, we find support from the data for the two groups of countries most used in the debate on this topic. The first group includes the periphery countries and is called PIIGS, based on the fact that it includes Portugal, Ireland, Italy, Greece and Spain. The second group are the remaining nations of Austria, Denmark, France, Germany, the Netherlands and Sweden that can be called core European countries. We make use of this split into groups for the rest of the research that is based on the behavior of the underlying factors and their importance in explaining the variation of the economic variables.

Exploring the general economic situation of the countries we find out some evidence for presence of the European heterogeneity. We can see it from both behavior of the factors that is different for the two groups and high importance of the group level factor in explaining the variation of the series. Additionally, not only the magnitude of the two group factors differs but also their response to the incoming shocks with much slower decay of the past information for the PIIGS one. In this way, we observe a Europe of two speeds where the reaction time for any big news is much larger for the periphery countries and it needs much more time for them to restore trust and return to growing ways. On the other hand, the core countries are much less volatile and are not affected by the shocks for a long time leading to a faster recovery from any crisis. Next to this heterogeneity, we observe a low level of overall integration as the European level factor fails to explain a lot of the variation and most of the importance comes from the lower level factors.

Looking at the three economic spheres separately, we observe plenty of differences between them. The real economy sector seems to be the closest to the general outlook with the same differences in factors behavior and a present situation of a Europe of two speeds where shocks affect the countries in two groups for different amount of time. Despite the lower volatility for the factors in this area, we can see the highest level of heterogeneity when comparing to the remaining two sectors that is present throughout the whole observation period. The fiscal sector includes the most volatile series and, therefore, is the most difficult for a factors model to capture the underlying relations. In this way, we are not able to determine as much from the behavior of the factors in this area or the effect of the external shocks on different countries. However, we can conclude that there is some evidence for integration in this sector as the European level factor is more dominant in terms of explaining the variation of the data than any other level. Lastly, for the financial markets sector we see only limited support for the European heterogeneity as the magnitude of the two group level factors differ but their behavior and impact of shocks is similar. Additionally, the country level factors seem to explain most of the variation in the data leading to low level of both overall European integration and importance of a certain group.

The outcomes of this research can be used as the first step of a larger analysis and economic application of the exploration of European heterogeneity. In this paper we focus on looking at the past data to determine whether the divide is present and how it evolved in the last years. However, it is possible to make use of these findings to first forecast the underlying factors and later make use of the posterior distributions of the parameters to link them to some economic variables such as GDP similar to Adrian et al. (2019). Additionally, our findings regarding the presence and level of heterogeneity between European countries can be used by policy makers to create more targeted and efficient interventions for every country or a certain group. Finally, the lack of integration and importance of every individual country for the financial markets sector can be included by investors when building the most efficient and profitable strategies and portfolios. In this way, the diversification of portfolio exposure can be obtained by simply investing in several countries that are even in the same group and have similar real economy characteristics.

There are several parts of this research that can be extended or some assumptions that can be explored more in the future. First, we use only a limited amount of countries and variables, therefore, not focusing on central and eastern European nations as well as more in depth analysis of several economic areas. Next, we assume that the factors follow a simple autoregressive process with only one lag enough to explain their variation. However, it may be useful to relax this assumption and explore how adding more lags would change the outcomes and behavior of the factors. Similarly, we restrict every level to have only one factor per group or country, while it may be the case that a few more ones can be added to describe better the underlying relations. Lastly, the current framework can be extended to include real time monitoring that would allow to update the factors and the parameters once the new data on some economic variables becomes available. In this way, it would be possible to check for the latest developments on the heterogeneity between European countries and incorporate all the latest information into potential forecasting.

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A Correlation tables

Table A.1: Correlation of Inflation across countries

Country	**Aut**	**Den**	**Fra**	**Ger**	**Gre**	**Irl**	**Ita**	**Ned**	**Por**	**Spa**	**Swe**
Austria	-										
Denmark	.55	-									
France	.74	.75	-								
Germany	.42	.44	.64	-							
Greece	.67	.53	.66	.17	-						
Ireland	.46	.54	.67	.52	.44	-					
Italy	.38	.66	.69	.60	.25	.67	-				
Netherlands	.49	.65	.68	.62	.42	.54	.60	-			
Portugal	.73	.59	.76	.44	.78	.61	.43	.56	-		
Spain	.76	.45	.71	.32	.83	.53	.33	.47	.84	-	
Sweden	.60	.51	.70	.43	.68	.62	.39	.50	.68	.71	-

Table A.2: Correlation of Unemployment across countries

Country	**Aut**	**Den**	**Fra**	**Ger**	**Gre**	**Irl**	**Ita**	**Ned**	**Por**	**Spa**	**Swe**
Austria	-										
Denmark	.37	-									
France	.50	.65	-								
Germany	-.14	-.40	-.35	-							
Greece	.47	.65	.75	-.77	-						
Ireland	.22	.89	.55	-.36	.55	-					
Italy	.30	.47	.79	-.68	.87	.30	-				
Netherlands	.73	.60	.74	-.09	.65	.52	.48	-			
Portugal	.47	.78	.62	-.30	.67	.88	.37	.79	-		
Spain	.37	.88	.74	-.65	.86	.87	.70	.64	.84	-	
Sweden	.57	.78	.41	-.11	.37	.74	.10	.62	.72	.63	-

Table A.3: Correlation of Total industrial production across countries

Country	Aut	Den	Fra	Ger	Gre	Irl	Ita	Ned	Por	Spa	Swe
Austria	-										
Denmark	.36	-									
France	.77	.38	-								
Germany	.77	.39	.88	-							
Greece	.32	.08	.36	.41	-						
Ireland	.26	.12	.24	.18	.27	-					
Italy	.77	.42	.91	.88	.34	.19	-				
Netherlands	.42	.26	.56	.51	.39	.21	.45	-			
Portugal	.60	.29	.76	.72	.32	.13	.70	.34	-		
Spain	.72	.40	.87	.81	.43	.21	.86	.48	.70	-	
Sweden	.70	.51	.72	.80	.39	.10	.76	.39	.62	.77	-

Table A.4: Correlation of Net international trade across countries

Country	Aut	Den	Fra	Ger	Gre	Irl	Ita	Ned	Por	Spa	Swe
Austria	-										
Denmark	.03	-									
France	.13	.09	-								
Germany	-.07	-.05	-.17	-							
Greece	.06	-.03	-.08	.29	-						
Ireland	.08	.01	.06	.15	.00	-					
Italy	-.04	-.03	.16	-.41	.02	-.11	-				
Netherlands	-.02	-.06	.02	.35	.42	-.18	.02	-			
Portugal	.13	-.04	.05	.15	.23	-.01	-.03	-.05	-		
Spain	.06	.11	.04	.14	.25	.10	.03	.03	.33	-	
Sweden	-.38	-.03	-.03	-.02	.00	-.07	-.05	-.21	-.14	-.03	-

Table A.5: Correlation of Overnight interest rate across countries

Country	Aut	Den	Fra	Ger	Gre	Irl	Ita	Ned	Por	Spa	Swe
Austria	-										
Denmark	.98	-									
France	.99	.99	-								
Germany	.99	.99	.99	-							
Greece	.92	.91	.91	.91	-						
Ireland	.99	.99	.99	.99	.91	-					
Italy	.99	.99	.99	.99	.91	.99	-				
Netherlands	.99	.99	.99	.99	.91	.99	.99	-			
Portugal	.99	.99	.99	.99	.91	.99	.99	.99	-		
Spain	.99	.99	.99	.99	.91	.99	.99	.99	.99	-	
Sweden	.81	.81	.83	.83	.69	.83	.83	.83	.83	.83	-

Table A.6: Correlation of Central government debt across countries

Country	Aut	Den	Fra	Ger	Gre	Irl	Ita	Ned	Por	Spa	Swe
Austria	-										
Denmark	.36	-									
France	.29	.56	-								
Germany	-.09	.19	.13	-							
Greece	-.17	-.29	-.18	-.05	-						
Ireland	.03	.27	.31	.59	-.04	-					
Italy	.11	.05	.02	-.03	-.03	-.21	-				
Netherlands	.01	.42	.27	.82	-.09	.59	.10	-			
Portugal	.14	.39	.37	.19	-.12	.24	-.02	.19	-		
Spain	.27	.45	.45	.04	-.09	.19	.20	.17	.41	-	
Sweden	.00	.06	.16	.02	-.25	-.06	.21	.06	.11	.32	-

Table A.7: Correlation of Total credit to government across countries

Country	Aut	Den	Fra	Ger	Gre	Irl	Ita	Ned	Por	Spa	Swe
Austria	-										
Denmark	.67	-									
France	.79	.69	-								
Germany	.10	.30	.07	-							
Greece	.09	.14	.16	-.08	-						
Ireland	.19	.36	.13	.31	.08	-					
Italy	.28	.28	.22	.22	.12	.13	-				
Netherlands	.10	.51	-.01	.49	-.01	.37	.28	-			
Portugal	.07	.27	.20	.35	.26	.29	.36	.23	-		
Spain	.70	.65	.90	.00	.22	.15	.18	.03	.18	-	
Sweden	.48	.31	.66	-.02	.10	-.06	-.14	-.27	.04	.66	-

Table A.8: Correlation of 3-Month interest rate across countries

Country	Aut	Den	Fra	Ger	Gre	Irl	Ita	Ned	Por	Spa	Swe
Austria	-										
Denmark	.98	-									
France	.99	.98	-								
Germany	.99	.98	.99	-							
Greece	.93	.92	.93	.93	-						
Ireland	.99	.98	.99	.99	.93	-					
Italy	.94	.91	.94	.94	.90	.94	-				
Netherlands	.99	.98	.99	.99	.93	.99	.94	-			
Portugal	.99	.98	.99	.99	.93	.99	.94	.99	-		
Spain	.91	.88	.91	.91	.87	.91	.98	.91	.91	-	
Sweden	.95	.92	.95	.95	.89	.95	.93	.95	.95	.92	-

Table A.9: Correlation of Stock market capitalization across countries

Country	Aut	Den	Fra	Ger	Gre	Irl	Ita	Ned	Por	Spa	Swe
Austria	-										
Denmark	.75	-									
France	.80	.84	-								
Germany	.74	.84	.95	-							
Greece	.74	.58	.68	.64	-						
Ireland	.79	.80	.79	.74	.57	-					
Italy	.78	.81	.94	.88	.70	.79	-				
Netherlands	.71	.79	.85	.82	.59	.73	.75	-			
Portugal	.72	.70	.83	.80	.71	.64	.84	.63	-		
Spain	.77	.68	.86	.80	.77	.68	.88	.70	.81	-	
Sweden	.69	.81	.91	.92	.57	.72	.85	.72	.83	.78	-

Table A.10: Correlation of Stock market index across countries

Country	Aut	Den	Fra	Ger	Gre	Irl	Ita	Ned	Por	Spa	Swe
Austria	-										
Denmark	.56	-									
France	.80	.57	-								
Germany	.74	.58	.93	-							
Greece	-.19	.03	-.17	-.16	-						
Ireland	.74	.53	.78	.74	-.13	-					
Italy	.80	.54	.93	.84	-.12	.79	-				
Netherlands	.75	.61	.93	.90	-.13	.77	.85	-			
Portugal	.66	.35	.75	.73	-.19	.64	.78	.70	-		
Spain	.77	.43	.85	.79	-.15	.66	.88	.78	.79	-	
Sweden	.60	.76	.66	.65	.04	.52	.61	.63	.50	.57	-

B Principal Components

Table B.1: Principal components loadings of Inflation series

	Aut	Den	Fra	Ger	Gre	Irl	Ita	Ned	Por	Spa	Swe
PC1	0.171	0.146	0.168	0.085	0.633	0.274	0.086	0.162	0.372	0.470	0.214
PC2	0.022	0.177	0.165	0.305	-0.510	0.619	0.286	0.302	0.084	-0.119	0.098

Table B.2: Principal components loadings of Unemployment series

	Aut	Den	Fra	Ger	Gre	Irl	Ita	Ned	Por	Spa	Swe
PC1	0.028	0.112	0.064	-0.171	0.641	0.331	0.152	0.090	0.281	0.566	0.052
PC2	-0.012	0.129	-0.016	0.264	-0.550	0.607	-0.252	0.037	0.344	0.205	0.122

Table B.3: Principal components loadings of Total industrial production series

	Aut	Den	Fra	Ger	Gre	Irl	Ita	Ned	Por	Spa	Swe
PC1	0.226	0.144	0.279	0.336	0.160	0.606	0.305	0.164	0.291	0.270	0.251
PC2	-0.145	-0.104	-0.196	-0.274	-0.029	0.789	-0.242	-0.076	-0.257	-0.203	-0.244

Table B.4: Principal components loadings of Net international trade series

	Aut	Den	Fra	Ger	Gre	Irl	Ita	Ned	Por	Spa	Swe
PC1	0.774	0.014	0.046	-0.003	0.002	0.006	0.001	0.007	0.011	0.007	-0.631
PC2	0.626	0.001	0.058	-0.006	0.003	0.000	-0.099	-0.022	-0.002	0.004	0.772

Table B.5: Principal components loadings of Overnight interest rate series

	Aut	Den	Fra	Ger	Gre	Irl	Ita	Ned	Por	Spa	Swe
PC1	0.266	0.285	0.308	0.308	0.370	0.308	0.308	0.308	0.308	0.308	0.214
PC2	0.000	-0.032	-0.052	-0.052	0.720	-0.052	-0.052	-0.052	-0.052	-0.052	-0.680

Table B.6: Principal components loadings of Central government debt series

	Aut	Den	Fra	Ger	Gre	Irl	Ita	Ned	Por	Spa	Swe
PC1	-0.027	-0.102	-0.031	-0.030	0.989	-0.047	-0.006	-0.049	-0.024	-0.018	-0.055
PC2	0.005	0.187	0.079	0.417	0.094	0.744	-0.034	0.461	0.071	0.054	-0.017

Table B.7: Principal components loadings of Total credit to government series

	Aut	Den	Fra	Ger	Gre	Irl	Ita	Ned	Por	Spa	Swe
PC1	0.358	0.582	0.402	0.037	0.085	0.267	0.051	0.084	0.058	0.431	0.306
PC2	-0.103	0.163	-0.204	0.092	-0.006	0.789	0.047	0.256	0.076	-0.213	-0.413

Table B.8: Principal components loadings of 3-Month interest rate series

	Aut	**Den**	**Fra**	**Ger**	**Gre**	**Irl**	**Ita**	**Ned**	**Por**	**Spa**	**Swe**
PC1	0.302	0.324	0.302	0.302	0.352	0.302	0.266	0.302	0.302	0.269	0.282
PC2	-0.113	-0.317	-0.154	-0.154	0.901	-0.154	0.062	-0.154	-0.154	0.092	-0.119

Table B.9: Principal components loadings of Stock market capitalization series

	Aut	**Den**	**Fra**	**Ger**	**Gre**	**Irl**	**Ita**	**Ned**	**Por**	**Spa**	**Swe**
PC1	0.323	0.262	0.278	0.306	0.405	0.292	0.304	0.263	0.278	0.284	0.295
PC2	0.104	-0.236	-0.161	-0.225	0.817	-0.222	-0.098	-0.191	0.036	0.107	-0.285

Table B.10: Principal components loadings of Stock market index series

	Aut	**Den**	**Fra**	**Ger**	**Gre**	**Irl**	**Ita**	**Ned**	**Por**	**Spa**	**Swe**
PC1	-0.002	0.000	-0.001	-0.001	0.999	-0.001	-0.001	-0.001	-0.002	-0.001	0.000
PC2	0.385	0.233	0.310	0.348	0.003	0.333	0.343	0.322	0.296	0.313	0.249

C Variance decomposition

Table C.1: Proportion of variance explained by each level factor for all series (level I - Europe, level II - group, level III - country, level IV - individual series)

Series	**Level**	**Aut**	**Den**	**Fra**	**Ger**	**Gre**	**Irl**	**Ita**	**Ned**	**Por**	**Spa**	**Swe**
GDP	*I*	.02	.01	.00	.01	.03	.08	.02	.02	.00	.04	.02
	II	.19	.02	.16	.33	.09	.35	.05	.21	.01	.10	.41
	III	.21	.76	.26	.03	.02	.28	.22	.29	.00	.02	.27
	IV	.59	.21	.57	.63	.87	.30	.71	.48	.99	.83	.31
INF	*I*	.02	.01	.00	.02	.09	.08	.06	.02	.10	.13	.02
	II	.40	.02	.08	.44	.31	.26	.15	.27	.42	.44	.34
	III	.32	.87	.83	.08	.07	.30	.43	.55	.04	.07	.19
	IV	.26	.11	.08	.45	.54	.36	.36	.16	.44	.35	.45
UMPL	*I*	.03	.01	.01	.03	.20	.12	.09	.03	.19	.21	.02
	II	.54	.02	.09	.82	.65	.42	.24	.31	.75	.67	.66
	III	.43	.97	.91	.15	.14	.46	.68	.66	.07	.12	.31
	IV	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00

Table C.2: Proportion of variance explained by each level factor for all series (level I - Europe, level II - group, level III - country, level IV - individual series), continued

Series	Level	Aut	Den	Fra	Ger	Gre	Irl	Ita	Ned	Por	Spa	Swe
PRD	*I*	.02	.00	.01	.01	.08	.10	.06	.00	.10	.12	.00
	II	.36	.00	.06	.31	.22	.38	.18	.04	.45	.36	.01
	III	.35	.45	.83	.02	.07	.45	.44	.44	.02	.07	.06
	IV	.26	.55	.11	.66	.64	.08	.32	.51	.43	.45	.93
NIT	*I*	.02	.16	.01	.11	.18	.13	.05	.01	.09	.10	.06
	II	.44	.05	.05	.27	.67	.60	.08	.13	.72	.61	.74
	III	.53	.77	.94	.17	.09	.25	.83	.86	.03	.06	.02
	IV	.01	.02	.00	.45	.06	.02	.04	.00	.16	.22	.18
NID	*I*	.03	.00	.02	.08	.15	.06	.02	.02	.19	.30	.02
	II	.85	.13	.15	.65	.53	.35	.06	.84	.65	.48	.10
	III	.00	.86	.80	.14	.09	.55	.12	.09	.15	.10	.86
	IV	.12	.01	.03	.13	.23	.04	.80	.05	.01	.11	.01
IRS	*I*	.01	.01	.01	.04	.19	.11	.08	.03	.17	.20	.03
	II	.25	.03	.10	.78	.63	.40	.22	.36	.69	.62	.68
	III	.19	.96	.89	.14	.13	.44	.65	.59	.07	.11	.26
	IV	.54	.01	.01	.05	.05	.04	.04	.02	.07	.05	.04
DEB	*I*	.03	.01	.01	.04	.16	.11	.07	.03	.17	.19	.03
	II	.48	.02	.09	.69	.52	.38	.20	.30	.66	.62	.62
	III	.38	.96	.90	.13	.11	.44	.56	.64	.06	.11	.29
	IV	.11	.01	.01	.15	.20	.07	.17	.04	.11	.08	.07
CRE	*I*	.04	.02	.01	.03	.12	.08	.06	.01	.17	.21	.05
	II	.42	.11	.05	.56	.39	.24	.16	.19	.73	.59	.24
	III	.41	.74	.93	.11	.10	.65	.51	.72	.07	.12	.45
	IV	.13	.13	.01	.31	.40	.04	.26	.08	.03	.08	.26
IR3	*I*	.03	.01	.01	.03	.19	.12	.09	.03	.19	.21	.03
	II	.51	.02	.09	.80	.64	.42	.24	.32	.72	.66	.63
	III	.44	.97	.91	.14	.14	.44	.66	.65	.06	.11	.32
	IV	.02	.00	.00	.00	.00	.02	.01	.01	.04	.01	.03
IR10	*I*	.03	.01	.01	.03	.20	.12	.09	.03	.19	.21	.03
	II	.54	.02	.09	.82	.66	.41	.23	.31	.75	.68	.71
	III	.42	.97	.91	.15	,14	.47	.68	.65	.06	.11	.26
	IV	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00

Table C.3: Proportion of variance explained by each level factor for all series (level I - Europe, level II - group, level III - country, level IV - individual series), continued

Series	Level	Aut	Den	Fra	Ger	Gre	Irl	Ita	Ned	Por	Spa	Swe
SMC	*I*	.02	.00	.02	.03	.10	.07	.08	.00	.08	.18	.03
	II	.00	.08	.08	.71	.28	.26	.23	.03	.46	.02	.70
	III	.97	.90	.58	.15	.06	.12	.37	.85	.02	.12	.08
	IV	.00	.01	.32	.11	.56	.55	.33	.12	.44	.68	.19
SMI	*I*	.07	.01	.01	.10	.08	.06	.07	.02	.18	.11	.03
	II	.06	.01	.05	.34	.52	.40	.26	.37	.56	.41	.39
	III	.54	.93	.86	.10	.07	.34	.47	.34	.26	.08	.40
	IV	.33	.05	.08	.46	.32	.19	.20	.27	.00	.40	.18

D Main MATLAB code

D.1 main.m

```
%Read the data
input = readtable('Financial markets.xlsx', 'ReadVariableNames',0);
data = table2array(input);

%Extract data size
t = size(data,1);
N = size(data,2);
p = N/11;
finalfE = zeros(t,500);
finalfG = zeros(t,1000);
finalfC = zeros(t,5500);
finalphiE = zeros(500,1);
finalphiG = zeros(500,2);
finalphiC = zeros(500,11);
finalvarE = zeros(500,N);
finalvarG = zeros(500,N);
finalvarC = zeros(500,N);
finalvarX = zeros(500,N);
partE = zeros(1,N);
partG = zeros(1,N);
partC = zeros(1,N);
partX = zeros(1,N);

%Compute initial values using PCA
[coeff, score, latent] = pca(data);
```

```
Fc = score(:,1:11);
[coeff, score, latent] = pca(Fc);
Fg = score(:,1:2);
[coeff, score, latent] = pca(Fg);
Fe = score(:,1);

%Initialize beta parameters
lambdaE = zeros(2,1);
lambdaG = zeros(11,1);
lambdaC = zeros(N,1);

lambdaE(1,1) = 1;
[mu, sigma] = PosteriorN(0, 1, Fg(:,2));
lambdaE(2,1) = normrnd(mu,sigma);


for i = 1:11
    [mu, sigma] = PosteriorN(0, 1, Fc(:,i));
    lambdaG(i,1) = normrnd(mu,sigma);
end
%Alternative regularization:
%lambdaG(1) = 1;
%lambdaG(6) = 1;

for i = 1:N
    [mu, sigma] = PosteriorN(0, 1, data(:,i));
    lambdaC(i,1) = normrnd(mu,sigma);
end
for i = 1:11
    lambdaC(p*i-(p-1)) = 1;
end

%Initialize autoregressive parameters
phiG = zeros(2,1);
phiC = zeros(11,1);
phiX = zeros(N,1);

[mu, sigma] = PosteriorN(0, 1, Fe);
phiE = normrnd(mu,sigma);
for i = 1:2
    [mu, sigma] = PosteriorN(0, 1, Fg(:,i));
    phiG(i,1) = normrnd(mu,sigma);
end
for i = 1:11
    [mu, sigma] = PosteriorN(0, 1, Fc(:,i));
```

```
    phiC(i,1) = normrnd(mu,sigma);
end
for i = 1:N
    [mu, sigma] = PosteriorN(0, 1, data(:,i));
    phiX(i,1) = normrnd(mu,sigma);
end

%Initialize variance parameters
sigmaG = eye(2);
sigmaC = eye(11);
sigmaX = eye(44);

[a, b] = posteriorC(4,0.01,Fe);
sigmaE = 1/gamrnd(a,b);
%In case aternative regularization is used:
%for i = 1:2
%     [a, b] = posteriorC(4,0.01,Fg(:,i));
%     sigmaG(i,i) = 1/gamrnd(a,b);
%end
for i = 1:11
    [a, b] = posteriorC(4,0.01,Fc(:,i));
    sigmaC(i,i) = 1/gamrnd(a,b);
end
for i = 1:N
    [a, b] = posteriorC(4,0.01,data(:,i));
    sigmaX(i,i) = 1/gamrnd(a,b);
end

%Run the iterative process and store all values of posterior distributions
for d = 1:27000
    %Generate country level factors
for j =1:11
    [Fc(:,j),Pc(:,j)]=KalmanSmootherC(j, lambdaC(p*j-(p-1):p*j,1), lambdaG(j,1), phiC(j,1), phiX(p*j-(p-1):p*j,1), sigmaC(j,j), sigmaX(p*j-(p-1):p*j,p*j-(p-1):p*j), Fg, Fc(:,j),data(:,p*j-(p-1):p*j));
end

%Generate group level factors
[Fg1,Pg1]=KalmanSmootherG(lambdaG(1:5,1), lambdaE(1,1), phiG(1,1), phiC(1:5,1), sigmaG(1,1), sigmaC(1:5,1:5), Fe, Fg(:,1),Fc(:,1:5));
[Fg2,Pg2]=KalmanSmootherG(lambdaG(6:11,1), lambdaE(2,1), phiG(2,1), phiC(6:11,1), sigmaG(2,2), sigmaC(6:11,6:11), Fe, Fg(:,2),Fc(:,6:11));
Fg = [Fg1, Fg2];
Pg = [Pg1, Pg2];

```

```
%Generate European level factor
[Fe,Pe]=KalmanSmootherE(lambdaE, phiE, phiG, sigmaE, sigmaG, Fg, Fe);
Pe = transpose(Pe);
Fe = transpose(Fe);

%Update beta parameters
lambdaE(1,1) = 1;
[mu, sigma] = PosteriorN(0, 1, Fg(:,2));
lambdaE(2,1) = normrnd(mu,sigma);

for i = 1:11
    [mu, sigma] = PosteriorN(0, 1, Fc(:,i));
    lambdaG(i,1) = normrnd(mu,sigma);
end
%Alternative regularization:
%lambdaG(1) = 1;
%lambdaG(6) = 1;

for i = 1:N
    [mu, sigma] = PosteriorN(0, 1, data(:,i));
    lambdaC(i,1) = normrnd(mu,sigma);
end
for i = 1:11
    lambdaC(p*i-(p-1)) = 1;
end

%Update autoregressive parameters
[mu, sigma] = PosteriorN(0, 1, Fe);
phiE = normrnd(mu,sigma);
for i = 1:2
    [mu, sigma] = PosteriorN(0, 1, Fg(:,i));
    phiG(i,1) = normrnd(mu,sigma);
end
for i = 1:11
    [mu, sigma] = PosteriorN(0, 1, Fc(:,i));
    phiC(i,1) = normrnd(mu,sigma);
end
for i = 1:N
    [mu, sigma] = PosteriorN(0, 1, data(:,i));
    phiX(i,1) = normrnd(mu,sigma);
end

%Update variance parameters
[a, b] = posteriorC(4,0.01,Fe);
sigmaE = 1/gamrnd(a,b);
```

```
%In case alternative regularization is used:
%for i = 1:2
%     [a, b] = posteriorC(4,0.01,Fg(:,i));
%     sigmaG(i,i) = 1/gamrnd(a,b);
%end
for i = 1:11
    [a, b] = posteriorC(4,0.01,Fc(:,i));
    sigmaC(i,i) = 1/gamrnd(a,b);
end
for i = 1:N
    [a, b] = posteriorC(4,0.01,data(:,i));
    sigmaX(i,i) = 1/gamrnd(a,b);
end

%Calculate variance decomposition parameters
for i = 1:N
    if i/5 > p
        s = 2;
    else
        s = 1;
    end
    q = ceil(i/p);
gammaE = lambdaE(s,1)^2 * lambdaG(q,1)^2 * lambdaC(i,1)^2;
gammaG = lambdaG(q,1)^2 * lambdaC(i,1)^2;
gammaC = lambdaC(i,1)^2;
varE = sigmaE/(1-phiE^2);
varG = sigmaG(s,s)/(1-phiG(s,1)^2);
varC = sigmaC(q,q)/(1-phiC(q,1)^2);
varX = sigmaX(i,i)/(1-phiX(i,1)^2);
partE(1,i) = gammaE*varE;
partG(1,i) = gammaG*varG;
partC(1,i) = gammaC*varC;
partX(1,i) = varX;
end

%Store every iteration values after the burnout
if d>2000
    if rem(d,50) ==0
        v = (d-2000)/50;
finalfE(:,v) = Fe;
finalfG(:,2*v-1:2*v) = Fg;
finalfC(:,11*v-10:11*v) = Fc;
finalphiE(v,1) = phiE;
finalphiG(v,:) = transpose(phiG);
finalphiC(v,:) = transpose(phiC);
```

```
finalvarE(v,:) = partE;
finalvarG(v,:) = partG;
finalvarC(v,:) = partC;
finalvarX(v,:) = partX;
        end
end
end
```