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Optimal Copula in a downside-risk hedging framework for oil refineries

Master thesis Quantitative Finance

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Abstract

In this research, we use the Hierarchical Archimedean Copula (HAC) to obtain optimal hedge ratios that minimize the downside-risk of oil refineries. Spot and futures prices of three oil products are used. These three oil products, crude oil, gasoline and heating oil are used by oil refineries to make a profit. We compare the profit series of oil refineries that only buy and sell the oil products and oil refineries that also buy and sell futures of the oil products. Different copulas are used to compare the performance of the HAC in finding optimal hedge ratios, denoted as the percentage we need to invest in futures relative to the underlying oil product. We evaluate and compare these copula-models using different downside-risk measures. The performance measure that we use is the hedging effectiveness, denoted as the reduction in downside-risk when we use the optimal hedge ratios following from the copula-model. We conclude that, in general, the HAC with Gumbel generator function is the best copula model for hedging downside-risk.

Keywords: Copula, GARCH, Downside-Risk, C-vine, D-vine, HAC, Optimal Hedge Ratios

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Contents

Acronyms

AIC Akaike Information Criterion. 13, 38

AMH Ali-Mikhail-Haq. 15, 17, 18, 19, 21, 24

DCC Dynamic Conditional Correlation. 3

ECM Error Correction Model. 3

ES Expected-Shortfall. 1, 3, 7, 21, 23, 25, 39, 40, 48, 49, 51, 53, 54

GARCH Generalized Autoregressive Conditional Heteroskedasticity. 1, 2, 3, 8, 9, 29, 36, 37

HAC Hierarchical Archimedean Copula. , i, 1, 2, 3, 4, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 38, 39, 40, 46, 47, 48, 49, 50, 51, 52, 53, 54

HE Hedging Effectiveness. i, 1, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54

JB Jarque-Bera. 5, 6

LPM Lower Partial Moment. 1, 3, 7, 21, 23, 24, 25, 29, 39, 40, 47, 49, 52

MVGARCH Multivariate Generalized Autoregressive Conditional Heteroskedasticity. 3

OLS Ordinary Least Squares. 3

SCAD Smoothly Clipped Absolute Deviation. 16, 31

SV Semi-Variance. 1, 2, 7, 21, 23, 24, 25, 27, 29, 39, 40, 46, 49, 52

VaR Value-At-Risk. 1, 3, 7, 8, 9, 21, 23, 27, 28, 29, 39, 40, 47, 48, 50, 52, 53

1 Introduction

In the financial world investing always comes with a certain degree of risk. Some investors try different hedging strategies against certain investment risk. All hedging strategies make use of risk measures. Sukcharoen and Leatham (2017) state that variance is not a good risk measure for investment risk, although it is mostly used to measure the investment risk. Especially when the returns on investments are not normally distributed (which is mostly the case), variance is not a valid risk measure. This is because investors are only interested in the downside of risk, as upwards movements are beneficial to the investor. This research is all about improving the effectiveness of the hedging strategies. We look at hedging strategies that make use of futures to cover a certain position. The problem for this hedging strategy is to find the optimal amount of futures an investor needs to buy to cover the position. The proportion of futures needed to cover a certain position is commonly denoted as a hedge ratio (Haigh & Holt, 2002). In this research, we aim to find optimal hedge ratios that minimize the downside-risk. For this purpose, we use the downside-risk measures Semi-Variance (SV), Lower Partial Moment (LPM), Value-At-Risk (VaR) and Expected-Shortfall (ES) rather than the commonly used risk measure variance. The data set that we use contains spot and futures prices of three oil products, namely crude oil, gasoline and heating oil. With this data series, we make a profit equation for refineries that buy and sell these oil products. To calculate the downside-risk measures, we thus need the joint distribution of the spot and futures price changes of the three oil products. To model this joint distribution we make use of copulas, which capture the dependence structure of the spot and futures price changes, so that we decompose the joint distribution into multiple marginal distributions. We, however, do not model the dependence structure directly, but first apply a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) filter to the data series to model the conditional volatility. The copulas are then used for the filtered series. By doing this, we create a so called copula-GARCH model.

We investigate what the best multivariate copula is in the copula-GARCH model to find optimal hedge ratios for the downside-risk measures. The copulas that we consider are the standard Gaussian and Student t copula. In addition, the more advanced and nested C-vine and D-vine copulas and the (penalized) Hierarchical Archimedean Copula (HAC) with different copula generator functions are used. To see which copula is the best copula for our data set, we measure the performance using the Hedging Effectiveness (HE) that we define in section 3. This measure is also used by Sukcharoen (2017) to classify the performance of copulas in a hedging framework. Besides this, we check if the results are consistent for the various risk measures that we use.

Using spot and futures price changes of crude oil, gasoline and heating oil in the copula-GARCH hedging framework makes this research especially relevant to oil refineries, as exactly these three oil products are used by oil refineries. The oil refineries may use the hedging framework to hedge against unexpected and unfavorable price changes. Other companies that may want to hedge against unfavorable price movements of crude oil, gasoline or heating oil are airlines, cruise lines and trucking companies (Turner & Lim, 2015). Airlines can also hedge the price of jet fuel using crude oil prices, as these prices are correlated with each other. Next to oil products, the hedging framework can also be used for sectors where different commodities are produced and processed. Take for example the major American beverage industry players, such as Coca-Cola. As described by Dewally and Marriott (2008), this industry demands all the six base metals (aluminum, copper, lead, nickel, tin, and zinc) in their production process. They may use our approach to find optimal hedging ratios for the base metal futures price changes. Furthermore, Tejeda and Goodwin (2014) give us the soybean complex industry (where soybeans are converted to soybean oil and soybean meal) as yet another example where our proposed multiproduct approach is relevant to.

For all our methods, we use spot and futures prices obtained from the Datastream database (Refinitiv, 2021). Both spot and futures prices are used for the commodities crude oil (WTI), unleaded gasoline, and number 2 heating oil. We use first differences of the price series for the analysis to obtain a sort of return series. The period that we consider runs from 2005 until 2021. The further analysis of the data can be found in section 2.

Our motivation to use the (penalized) HAC model comes from the fact that it is an innovative model and not yet used in solving the empirical problem of finding optimal hedge ratios. The reason for using the commodity products crude oil, gasoline and heating oil is that the data is easily accessible and obviously perfectly suited for investigating multiproduct hedge ratios. The oil products are also considered volatile, so the data is suited for hedging.

We conclude that, generally, the best copula is the HAC with Gumbel generator function, as this copula outperforms the standard Gaussian, Student t copula and the vine copulas. We further show the potential of using multiple generator functions in a HAC. Unfortunately, it is also shown that penalizing the HAC structure does not influence the performance. However, a larger rolling window size does improve the performance. To conclude, all the different downside-risk measures obtain similar hedge ratios for each copula model.

To describe our contribution, we should first place our research in the current literature by comparing it with other papers surrounding the estimation of optimal (mulitproduct) hedge ratios. A first contribution could be the use of the downside-risk measures Semi-Variance (SV), Lower Partial Moment (LPM), Value-At-Risk (VaR) and Expected-Shortfall (ES), as most papers do not consider these downside-risk measures. For example, Ji and Fan (2011) and Haigh and Holt (2002) all search for an optimal hedge ratio that minimizes the variance. Ji and Fan (2011) however do use a comparable procedure to obtain the hedge ratios for oil refineries. They give us the profit equation at time t of oil refineries considering the changes of spot and futures prices of the oil products and the hedge ratios. Only the hedge ratios are unknown in this profit equation. Then, they take the variance of the profit equation and calculate the optimal hedge ratios by taking partial derivatives of the profit variance. Furthermore, they model the variance and covariance for the profit variance at time t, using a Dynamic Conditional Correlation (DCC)-Error Correction Model (ECM)-Multivariate Generalized Autoregressive Conditional Heteroskedasticity (MVGARCH) model. Sukcharoen and Leatham (2017) use a similar procedure by first giving the profit equation of an oil refinery, but in contrast to Ji and Fan (2011) do use the downside-risk measures and more importantly, make use of copulas in the procedure. We closely follow the approach of Sukcharoen and Leatham (2017), but we do the following things different. First, their data is sampled weekly, while we use daily data. Another difference is that we use different copulas in addition to the other copulas that they use. The last and most important difference is that Sukcharoen and Leatham (2017) directly use the original data series to estimate the copula distributions. We, however, first use the GARCH filter to obtain a filtered series and then use a copula to capture the dependence structure. In this aspect, we follow the copula-GARCH model proposed by Yuan et al. (2020). However, they do not use the model to obtain hedge ratios.

In the literature there is also an elementary different procedure which is a more simple way to obtain optimal hedge ratios. This procedure is described by Power and Vedenov (2008), Louhichi and Rais (2019), Hsu et al. (2008) and Tejeda and Goodwin (2014). They all assume that the optimal hedge ratio is given by the the covariance between spot and futures prices divided by the variance of the futures prices. They propose several methods to model the variance and covariance of the spot and futures prices. The simplest method proposed to estimate the covariance is by regressing the spot prices on futures prices using Ordinary Least Squares (OLS). This is thus another approach compared to the ones described by Sukcharoen and Leatham (2017), Ji and Fan (2011) and Haigh and Holt (2002). The difference is in the fact that this simpler approach assumes that the optimal hedge ratio is already defined, while the more empirical approach of Sukcharoen and Leatham (2017) searches the value of the optimal hedge ratio that minimize the risk measure.

Coming back to our contribution to the literature, we consider the use of the innovative HAC and penalized HAC framework proposed by Okhrin et al. (2015) as maybe our most important contribution. The (penalized) HAC is not often used in an empirical setup, so the use of the HAC is certainly a contribution to the current literature on multiproduct hedging. In the literature, vine-copulas (which are also nested and very flexible) are used to optimize hedge ratios, but the HAC is not. Furthermore, there are some papers that describe the HAC, but do not compare the performance with other copulas. We compare the performance of different HAC by using different copula generator functions in each HAC structure. Furthermore, we consider one HAC that uses multiple copula generator functions in its structure to investigate if this improves the performance. Therefore, our contribution can be considered as a new application of the framework of Okhrin et al. (2015).

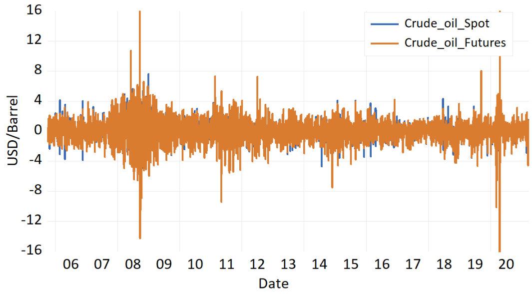
The remainder of this paper is structured as follows. First, we describe the data in section 2 and highlight some of the characteristics of the price changes series of crude oil, gasoline and heating oil. Then, we describe the procedure to obtain optimal hedge ratios in section 3, where we also explain the used methods in detail. In section 4 the results are presented and discussed. Finally, we conclude our research by providing a discussion of the obtained results and by giving suggestions for future research.

2 Data description

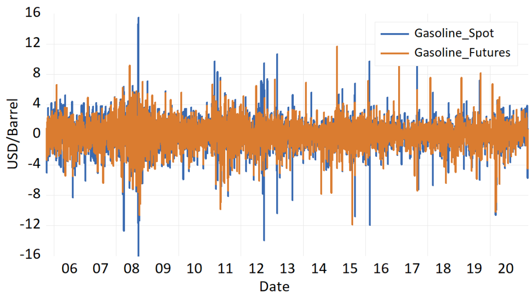
Our data consists of spot and futures prices for:

1. Crude oil, WTI - Cushing
2. Unleaded Gasoline, New York Harbor, Regular
3. Number 2 heating oil, New York Harbor

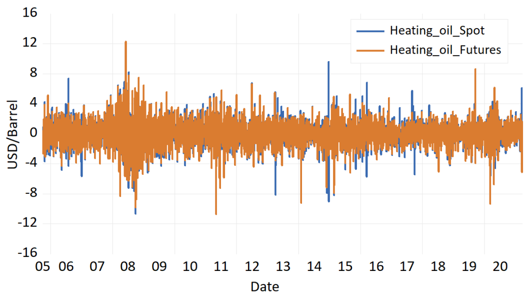
We use daily prices from 04/10/2005 until 23/03/2021, using only weekday information. The data sample thus consists of 4036 data points. All prices are converted to USD/Barrel, using that a barrel of oil contains 42 gallons. The data comes from the Datastream database and is presented to us as an Excel file (Refinitiv, 2021). For our analysis we are not interested in the prices itself, but only in the price changes. Therefore, we consider the first differences of the oil product prices. The figure below shows the first differences of the oil product spot and futures prices.



(a) Crude oil



(b) Gasoline



(c) Heating oil

Figure 1: plots of spot and futures price changes for the different oil products

Figure 1 shows three plots, where in each plot the differences in spot and futures prices are displayed for the respective oil product. We know that spot and futures prices of oil products can be volatile and we observe different periods of high volatility interchanged by periods of low volatility. Furthermore, the graphs show that all oil products are closely correlated. This is supported by Table 7 in Appendix A, as this table shows the correlations between all the data series. To give some more characteristics of the data series, we show summary statistics of the series in the table below.

Table 1: Summary statistics of first differences of spot and futures prices of all oil products

Statistics	Crude oil		Gasoline		Heating oil	
	Spot	Futures	Spot	Futures	Spot	Futures
Mean	-1.537 $\times 10^{-3}$	-1.522 $\times 10^{-3}$	-2.045 $\times 10^{-3}$	5.270 $\times 10^{-4}$	-2.718 $\times 10^{-3}$	-3.131 $\times 10^{-3}$
SD	1.936	1.929	2.025	1.796	1.707	1.679
Skewness	-2.543	-2.360	-0.339	-0.334	-0.104	-0.157
Kurtosis	259.9	271.9	9.535	7.188	6.563	6.691
JB	1110* $\times 10^{4}$	1216* $\times 10^{4}$	7256*	3024*	2142*	2306*
$Q^2(12)$	881.26*	913.1*	1071.9*	380.25*	673.65*	985.7*

** Indicates rejection of the null-hypothesis at 1% level.*

Table 1 shows summary statistics of the first differences of all oil product spot and futures prices. An important characteristic that we need to check is the distribution of the price changes series.

We can clearly see that all series are skewed and leptokurtic. We notice that the kurtosis is much higher for the crude oil prices, as this series contains more outliers. Removing the two largest outliers from the crude oil spot and futures price series results in a skewness of -0.0407 and -0.125, a kurtosis of 10.8 and 11.6 and a JB statistic of 10259 and 12456, respectively. Jarque-Bera (JB) denotes the test for normality with the null-hypothesis that the data is normally distributed. The test statistic is denoted as

$$JB = \frac{n}{6}(S^2 + \frac{1}{4}(K-3)^2), \tag{1}$$

where the skewness is indicated with an S and the kurtosis with a K. The null hypothesis states that the skewness is zero and the excess kurtosis is zero. From the test outcomes we conclude that all the price changes series are not normally distributed. Sukcharoen and Leatham (2017) state that if returns are non-normal, variance is not a good measure to capture the downside-risk. Therefore, the statistics in Table 1 can be used to support the use of other risk measures than variance.

Furthermore, as proposed by Louhichi and Rais (2019), we use the Ljung–Box statistics of order 12 applied to squared differences. The test statistic of the Ljung-box test is denoted as

$$Q = n(n+2)\sum_{k=1}^{h} \frac{\widehat{\rho}_k^2}{(n-k)}, \tag{2}$$

where n denotes the sample size, $\widehat{\rho}_k^2$ denotes the auto estimated autocorrelation at lag k between the squared differences. The number of lags that we test is denoted by h (in our case 12). The null-hypothesis states that the correlations in the squared data series are zero. The critical region for rejection of the null-hypothesis for a given probability level α is $Q > \chi^2_{(1-\alpha),h}$, as Q asymptotically follows a $\chi^2_{(h)}$ under the null-hypothesis. The null hypothesis is rejected, indicating that there is serial correlation over time. We use the outcome of the Ljung-Box statistics to give certain specifications to the model that we present in section 3.

3 Methodology

To find optimal hedge ratios we need to solve the hedging problem. Following the notation of Sukcharoen and Leatham (2017) and Ji and Fan (2011), we describe our hedging problem as follows. Our hedging problem focuses on the problem of a regular oil refinery, where two barrels of gasoline and one barrel of heating oil are created from three barrels of crude oil (NewYorkMercantileExchange, 2021). The refinery hedges downside-risk using the crude oil, gasoline and heating oil futures. Therefore, we need to estimate $y_t(b)$, denoted as the profit/loss at time t

$$y_t(b) = -\Delta S_t^C + \frac{2}{3}\Delta S_t^G + \frac{1}{3}\Delta S_t^H + b_C \Delta F_t^C - \frac{2}{3} b_G \Delta F_t^G - \frac{1}{3} b_H \Delta F_t^H, \tag{3}$$

where S_t^C, S_t^G and S_t^H are the spot prices of crude oil, gasoline and heating oil, respectively. F_t^C, F_t^G and F_t^H are futures prices for the same products at time t. $\Delta S_t^C = S_t^C - S_{t-1}^C$, so this indicates that we use price changes instead of the normal series. Furthermore, $b = \{b_C, b_G, b_H\}$ are hedge ratios at time t determined at time $t-1$. In (3), we only consider the costs of buying the oil products and ignore other costs that influence the profits and losses of the refinery.

We want to know the profit/loss at time t, using historical data. Therefore, we will use a rolling window to estimate the profit/loss. We use different rolling window sizes of $\{262, 786, 1310\}$ days or $\{1, 3, 5\}$ years. For the rolling window size of 262 observations, the first hedge ratio that we estimate is thus the hedge ratio at time $t = 263$.

The objective of our research is to find optimal b^*, that minimize downside-risk measures. We formulate this by

$$b^* = \underset{b}{\operatorname{argmin}}\ Risk(y_t(b)), \tag{4}$$

where $Risk(y_t(b))$ denotes the value of the downside-risk measure given $y_t(b)$. The risk measures that we consider are displayed in Table 2.

Table 2: The four risk measures and their definition

Risk Measure	Definition
SV	$\int_{-\infty}^{0}(x)^2 dF_x$
LPM	$\int_{-\infty}^{0}(-x)^n dF_x$
VaR	$F_x^{-1}(1-p)$
ES	$E[x \mid x \le VaR_p]$

Note:
F_x denotes the distribution function of variable x. n denotes the value of risk appetite and is set equal to 3, meaning it represents a risk-averse hedger. The number p denotes the probability level and is set equal to 0.90, 0.95 or 0.99.

3.1 Copula-GARCH approach

From the table and graphs in section 2 we conclude that our data series show the characteristics of heteroskedastic, clustered and fat-tailed volatility. This terms are further explained by Engle and Kroner (1995). Furthermore, Yuan et al. (2020) state that these characteristics are common for daily commodity returns. To explain and fit these characteristics we use the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model that is widely used in the financial world, as stated by Louhichi and Rais (2019). GARCH was first introduced by Bollerslev (1990) and following this model, we denote the conditional variance as a linear function of lagged squared error terms and lagged conditional variance terms

$$\begin{aligned} y_t &= \mu + \xi_t \\ \xi_t &= \sigma_t z_t, \text{ where } z_t \sim \text{Skewed } t(\eta, \lambda) \\ \sigma_t^2 &= w + \alpha \xi_{t-1}^2 + \beta \sigma_{t-1}, \end{aligned} \tag{5}$$

where ξ_t denotes the random error at time t, μ denotes the conditional mean and σ_t the conditional variance at time t. Furthermore, α and β denote the GARCH parameters with the restriction that $\alpha + \beta < 1$ to ensure stationarity. The parameters η and λ denote the kurtosis parameter and skewness respectively. We do not make the assumption that $z_t \sim N(0,1)$, as the skewed student t distribution would be a better fit to our data. It makes it a bit more complicated, as the skewed student t distribution has more parameters that need to be estimated. The degrees of freedom are determined by the sample kurtosis, as they are related and the skewness parameter is determined by the sample skewness. Let y_t denote the first differences at time t for the series of (3), such that $y_t \in \{\Delta S_t^C, \Delta S_t^G, \Delta S_t^H, \Delta F_t^C, \Delta F_t^G, \Delta F_t^H\}$. We model the heteroskedasticity and other features of the marginal distribution for each series in y. As we can see from the formula of the VaR in Table 2 for example, the distribution of $y_t(b)$ is needed and therefore we need to model the entire joint distribution of all spot and futures prices mentioned before.

As commodities appear to co-move symmetrically, which is also shown by the graphs of section 2, we use a copula to capture these co-movements and interdependence structures. Let C be a copula and let $F(x_1, \ldots, x_n)$ be the entire distribution. We can transform the joint distribution to $C(F(x_1), \ldots, F(x_n))$, where the copula C describes the dependence structure. A copula thus can be considered as a sort of function that decomposes the joint distribution into various marginal distributions (Sukcharoen, 2017). After obtaining all necessary parameters in the copula, we use the same procedure as Sukcharoen and Leatham (2017) to compute the four downside-risk measures. We start by simulating six vectors of 10.000 standard uniform variables from the copula, such that we get $\{u_{1,s}, u_{2,s}, \ldots, u_{6,s}\}_{s=1}^{10.000}$. We choose 10.000 simulations as this a sufficient large number for which we can incorporate all dependency structures implied by the copulas. By using the copula density functions for the simulations, these six vectors have the same dependency structure as modeled by the copula. As the values of the uniform variables lie between zero and one, we use the inverse marginal distribution of the price change series to get simulated series of the spot and futures price changes. By using the profit/losses equation as in (3), the simulated series are used to calculate $y_t(b)$. This simulated series from the simulated joint distribution of $y_t(b)$ is then used to compute the risk measures.

Instead of using the marginal distributions of the original series to get the simulated series, as done by Sukcharoen (2017), we follow the approach that that is not implemented, but nevertheless mentioned by Sukcharoen (2017). We summarize all the steps of the approach as follows: First, a GARCH filter is used to filter out the the white noise process implied by z_t from (5) for each price change series. Second, the marginal distributions with corresponding parameters for the white noise series are determined (in our case the assumed skewed t distribution). Third, the white noise series are transformed to uniform variables using the marginal distribution. Then, we estimate the copula distribution (parameters etc.) using the uniform variables based on white noise series. After this is done, we simulate a number of standard uniform variables from the copula distribution. Thereafter, the inverse marginal distribution is used to obtain simulated $\widehat{z}_t$. Then, we use the estimated $\widehat{\sigma}_{t+1}$ from GARCH and $\widehat{z}_t$ to obtain $\widehat{y}_t$ estimates. Finally, we use the simulated $\widehat{y}_t$ to obtain risk measures (99% percentile of $\widehat{y}_t$ for VaR, for example). Combining a copula and the forecast function of the GARCH model gives us the copula-GARCH approach.

3.1.1 Standard copulas

The standard copulas that we use in this research are the multivariate Gaussian copula and Student t copula. An arhimedean copula of dimension n is defined as
$C(x_1, \ldots, x_n) = \phi(\phi^{-1}(x_1), \ldots, \phi^{-1}(x_n))$, where ϕ is a so called copula generator function. The copula generator function is defined by Louhichi and Rais (2019) as

$$\phi \in \mathscr{L} = \left\{ \phi : [0; \infty) \longrightarrow [0, 1] \mid \phi(0) = 1,\ \phi(\infty) = 0;\ (-1)^k \phi^{(k)} \geq 0;\ k \in \mathbb{N} \right\}. \tag{6}$$

The copulas are archimedian copulas with copula generator function $\phi = \Phi$ and $\phi = t$. The Gaussian copula is defined as follows

$$C(x_1, \ldots, x_n) = \Phi(\Phi^{-1}(x_1), \ldots, \Phi^{-1}(x_n)), \tag{7}$$

where $x_1, \ldots, x_n$ denote the marginal distributions and Φ the Gaussian distribution function and Φ^{-1} its inverse. For the standard Student t copula, we take the Student t distribution function t and its inverse t^{-1} instead of Φ and Φ^{-1}. We generate 10.000 uniform variables from these copulas using the following procedure, as described by Torrent-Gironella and Fortiana (2011)

Algorithm 1: Simulation algorithm for the standard Gaussian and Student t copula

Input: Matrix of n data series $X = \{x_1, \dots, x_n\}$;
Find correlation matrix P of $X = \{x_1, \dots, x_n\}$;
Find Cholesky matrix A of P ;
if *Gaussian copula* **then**
  $Y = AZ$, with $Z \sim N(0, \Sigma)$ of length 10.000 ;
  $U = \Phi(Y)$;
end
if *Student t copula* **then**
  $Y = \sqrt{\frac{v}{s}} AZ$, with $s \sim \chi(v)$, $Z \sim N(0, \Sigma)$ of length 10.000 ;
  $U = t(Y)$
end
return U

3.1.2 Vine copulas

Standard Gaussian and Student t copulas restrict the tail dependence between all input variables, or in our case the six spot and futures price changes series, to be the same. These restrictions on the dependence structure are obviously not optimal. It is therefore beneficial to look at more advanced ways to model dependency structures. One way is the use of vine copulas. The first to introduce the vine copulas were Bedford and Cooke (2002). They state that the key advantage of this approach is that these copulas "allow heterogeneous dependence structures between input variables during normal and extreme market conditions". This means that we can use different dependency structures for the 6 input variables. The vine copula describes the dependence of these variables by mixing a group of different bivariate copulas. There exist several vine structures, but we make use of the C-vine and D-vine structures, as described by Sukcharoen and Leatham (2017).

To give the general explanation of a vine copula structure, we represent the joint distribution function

$$\begin{aligned} f(x_1, x_2, x_3, x_4, x_5, x_6) = & f_1(x_1) f_{2|1}(x_2|x_1) f_{3|1,2}(x_3|x_1, x_2) f_{4|1,2,3}(x_4|x_1, x_2, x_3) \\ & f_{5|1,2,3,4}(x_5|x_1, x_2, x_3, x_4) f_{6|1,2,3,4,5}(x_6|x_1, x_2, x_3, x_4, x_5), \end{aligned} \tag{8}$$

where $f_{i|j,k} = f_{i|j,k}(x_i|x_j, x_k)$.

From (8) we see that the joint distribution is split into six conditional marginal distributions. Each marginal distribution is conditioned on the previous variable(s), where the first marginal distribution is obviously unconditional. Using the conditional probability formula we can write every conditional distribution as the joint distribution divided by its marginals. If we then use copulas we can decompose each conditional marginal distribution in (8) into six unconditional marginal distributions and accessory bivariate copula distributions to model the dependency between each of the six variables. There are several combinations of bivariate copulas that we can use to model the dependence. For example, for the C-vine structure, we first model the dependence of the first variable with all the other variables by taking the bivariate copulas of the first variable with all the other variables. This is also called a tree, were the first variable is the ”root node” that is connected to the other variables. Then, conditioned on the first variable, we take the bivariate copulas of the second variable with the the remaining variables. This way, we create a new tree where the second variable is the ”root node”. This goes on until we only have the bivariate copula of the fifth and sixth variables left, conditioned on all other variables. This bivariate copula is the fifth and last tree where we use the bivariate copula of the first and sixt variable, conditional on all other variables. This is also illustrated in Figure 2.

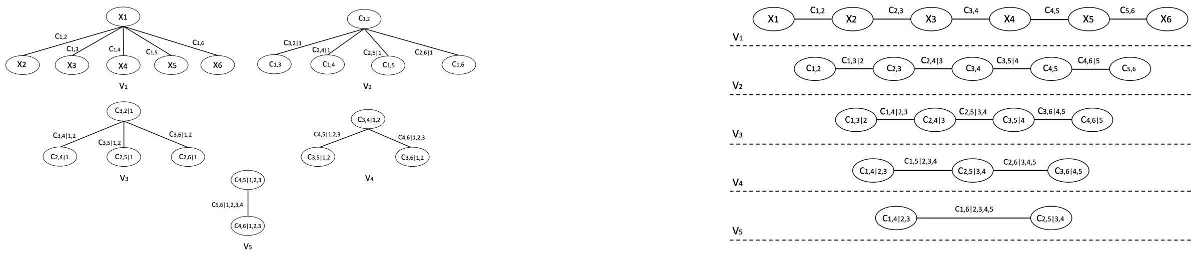
For the D-vine, the dependency structure is modelled a bit different by using other bivariate copulas and therefore creating different trees. For instance, we first take the bivariate copulas of the first and second variable, the second and third variable, third and fourth, fourth and fifth and finally the fifth and sixth variables. This is also our first tree, where in contrast to the C-vine, we do not have a root node with one variable that connects all the others, but rather a ”path structure” where more variables are connected directly with each other. The second tree consists of the bivariate copulas of the first and third variable, third and fifth variable, second and fourth variable and fourth and sixth variable, conditional on the variable that lies in between. In the third tree, we take the bivariate copula of the first and fourth variable, the second and fifth variable and the third and sixth variable. Again we take the bivariate copulas conditional on the variables that lie in between. The fourth tree consists of two bivariate copulas, namely between the first and fifth variable and the second and sixth variable again conditional on the variables in between. To illustrate, we can rewrite (8) as a C-vine structured copula

$$\begin{aligned} f(x_1,x_2,x_3,x_4,x_5,x_6) =& f_1 f_2 f_3 f_4 f_5 f_6 C_{1,2} C_{1,3} C_{1,4} C_{1,5} C_{1,6} C_{2,3|1} C_{2,4|1} C_{2,5|1} C_{2,6|1} \\ & C_{3,4|1,2} C_{3,5|1,2} C_{3,6|1,2} C_{4,5|1,2,3} C_{4,6|1,2,3} C_{5,6|1,2,3,4}, \end{aligned} \tag{9}$$

where f_i denotes the marginal distribution of x_i and $C_{i,j|k}$ denotes the conditional bivariate copula of x_i and x_j conditional on information of x_k. If we decompose the joint distribution in (8) slightly different, we can give the D-vine structured copula as

$$\begin{aligned} f(x_1,x_2,x_3,x_4,x_5,x_6) =& f_1 f_2 f_3 f_4 f_5 f_6 C_{1,2} C_{2,3} C_{3,4} C_{4,5} C_{5,6} C_{1,3|2} C_{2,4|3} C_{3,5|4} C_{4,6|5} \\ & C_{1,4|2,3} C_{2,5|3,4} C_{3,6|4,5} C_{1,5|2,3,4} C_{2,6|3,4,5} C_{1,6|2,3,4,5}, \end{aligned} \tag{10}$$

where we use the same notation as in (9), but clearly observe a different structure. To give some more clarification on the equations above and to get a more vivid picture of the structures of the D-vine and C-vine, we present the following figures. These figures are inspired by the figures presented earlier by Sukcharoen and Leatham (2017).



(a) C-vine **(b)** D-vine

Figure 2: plots of six dimensional vine structures

From Figure 2 we observe again that the C-vine and D-vine copulas have different structures. If we look for example at V_1 in both figures. The C-vine models the dependence between the first variable and all other variables, while the D-vine models the dependence of all successive variables. This way, only one variable is directly connected other variables for the C-vine, while for the D-vine we have multiple variables that are directly connected with other variables. An advantage of the vine copulas overall is that we do not have to use one standard copula, but we can use multiple different bivariate copulas for each pair of variables in the vine trees. Therefore, the vine copulas can model different types of the dependence structures instead of just one.

For both vine copulas it matters which order you choose for $x_1, \ldots, x_6$. We follow the proposed method of Sukcharoen and Leatham (2017) to obtain an optimal order. We look at the variable which is most connected or better said has the highest degree of association with the other variables and select this variable as x_1. Then we select the variable with the second highest degree of association to be x_2 and so on. We measure the degree of association for each variable i

$$\Theta_\tau^i = \sum_{j=1, i \neq j}^{6} |\tau_{i,j}|, \tag{11}$$

where $\tau_{i,j}$ denotes the Kendall's tau coefficient between a pair of variables i, j (Sukcharoen & Leatham, 2017). For the D-vine copula, we use a slightly different measure for the degree of association, namely

$$\Psi_\tau^{(k)} = \sum_{i=1}^{5} |\tau_{i,i+1}|, \tag{12}$$

where $\tau_{i,i+1}$ is the Kendell's tau coefficients for each variable pair $i, i+1$. We have k possible orders for the six variables and determine for each order Ψ_τ. Then, we choose the order k such that Ψ_τ is maximized. We already observe that the C-vine finds it important that one variable has a high degree of association with all the other variables. The D-vine only finds it important that the degree of association is high between two successive pairs of variables.

The last step to complete the procedure of fitting both C-vine and D-vine copulas is to choose the suited bivariate copula for each copula pair in (9) and (10). We consider various bivariate copulas and use the Akaike Information Criterion (AIC) to choose which bivariate copula is best suited for each pair of variables. After obtaining the optimal order of the variables and after fitting the bivariate copulas we can simulate data from the C-vines and D-vines. To perform the simulations, we use the algorithms proposed by Aas et al. (2009). algorithm 2 shows us the procedure to simulate 10.000 uniform values for our 6 input variables from the C-vine. We start the algorithm with 6 vectors containing uniformly distributed values between 0 and 1 and change the random values with the use of copula functions to get 6 simulated series that follow the dependency structure of the C-vine.

Algorithm 2: Simulation algorithm for a C-vine

Input: Simulate vectors $u_1, \ldots, u_6$ containing n independent uniform values between [0,1];

$y_1 = w_{1,1} = u_1$;
for $i \in \{2, \ldots, 6\}$ **do**
    $w_{i,1} = u_i$;
    for $k \in \{i-1, i-2, \ldots, 1\}$ **do**
        $w_{i,1} = h^{-1}(w_{i,1}, w_{k,k}, \Theta_{k,i-k})$;
    end
    $y_i = w_{i,1}$;
    if $i = 6$ **then**
        Stop;
    end
    for $j \leftarrow 1, \ldots, i-1$ **do**
        $w_{i,j+1} = h(w_{i,j}, w_{j,j}, \Theta_{j,i-j})$;
    end
end
return $y_1, \ldots, y_6$

$y_1, \ldots, y_6$ denote the final simulated values from the C-vine. For our research we use 10.000 simulated values, so in our case $n = 10.000$. These simulated values for $y_1, \ldots, y_6$ are used to obtain a simulated profit/loss series as in (3) for which we can calculate the downside-risk measures. In the algorithm, we repeatedly make use of the h-function and the inverse of this function. This function denotes the density function of a specified bivariate copula. The first two arguments of the h-function denote the input data for the copula function and the last argument $\Theta_{j,i}$ denotes the set of parameters for the copula function $C_{j,j+i|1,\ldots,j-1}$ from (9). The algorithm to sample from the D-vine is a bit longer as more conditional copula distributions need to be calculated. Also, the h-function is defined a bit different, as $\Theta_{j,i}$ denotes the parameters for the copula function $C_{i,i+j|i+1,\ldots,i+j-1}$, which is different to the C-vine. The algorithm for the D-vine is shown in Appendix C

3.1.3 Hierarchical Archimedean Copula (HAC)

Another type of copula that we consider is the Hierarchical Archimedean Copula (HAC), as proposed by Okhrin et al. (2013). Similar to vine copulas, these copulas are also nested and have an advanced dependence structure. To explain the HAC and especially its structure better we look at the following figure.

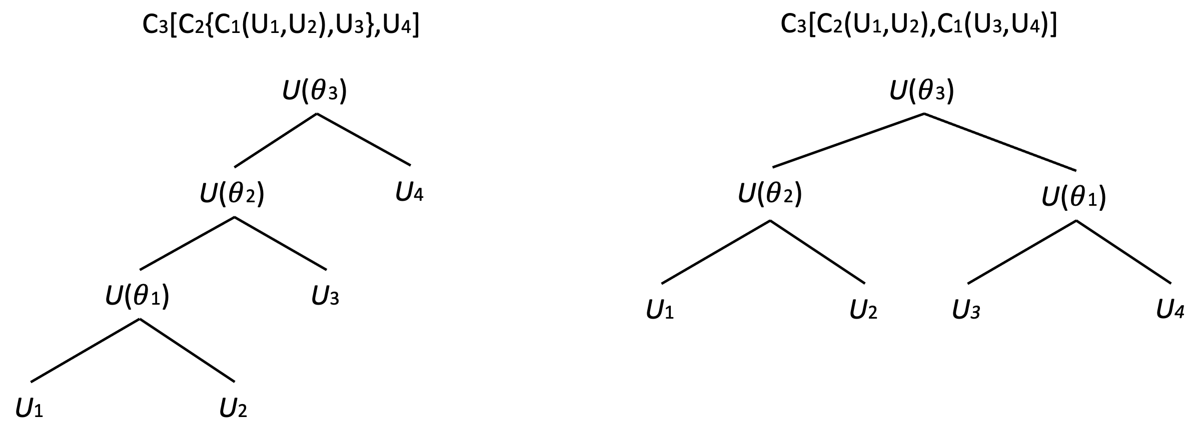


Figure 3: Four-dimensional fully nested (left) and partially nested (right) HAC

There are numerous different structures proposed in the literature, such as partially or fully nested structures or binary nested structures. Figure 3 clearly shows the differences between fully nested and partially nested HAC. Both displayed HAC have binary structures, as each (sub-)copula has at most two inputs. This way, the Archimedean (sub-)copulas in the structure are all bivariate copulas. The HAC with binary nested structure for our research purposes can look as follows

$$
\begin{aligned}
C_{l1} &= \phi_{l1}(\phi_{l1}^{-1}(x1) + \phi_{l1}^{-1}(x_2)) \\
C_{l2} &= \phi_{l2}(\phi_{l2}^{-1}(x_3) + \phi_{l2}^{-1}(x_4)) \\
C_{l3} &= \phi_{l3}(\phi_{l3}^{-1}(x_5) + \phi_{l3}^{-1}(x_6)) \\
C_{l4} &= \phi_{l4}(\phi_{l4}^{-1}(C_{l1}) + \phi_{l4}^{-1}(C_{l2})) \\
C_{l5} &= \phi_{l5}(\phi_{l5}^{-1}(C_{l4}) + \phi_{l5}^{-1}(C_{l3})),
\end{aligned}
\tag{13}
$$

where we observe the use of five different Archimedean copulas to model the dependency of our six input variables. We can see that it is a nested copula with different generator functions and different sub-copulas. To give an example of a non-binary structure, we can simply look at the fully nested copula in (13) and merge C_{l4} and C_{l3} such that the structure becomes $C_{l5} = \phi_{l5}(\phi_{l5}^{-1}(C_{l1}) + \phi_{l5}^{-1}(C_{l2}) + \phi_{l5}^{-1}(C_{l3}))$ and therefore non-binary. The non-binary HAC do have advantages compared to binary HAC. The non-binary HAC structure has less (sub-)copulas and coming with that it has less parameters that need to be estimated. It is therefore beneficial to also look at non-binary structures in finding the optimal structure of the HAC. However, we follow the approach of Okhrin et al. (2013) and only look at binary structured HAC to find the optimal structure. We follow their procedure, as it is computationally efficient. For now, we also take just one Archimedean copula that we use during the whole procedure of finding an optimal structure. The copulas that we consider are the Archimedean copulas with the Clayton, Gumbel, Frank, Joe or Ali-Mikhail-Haq (AMH) generator function. We present the generator functions for the copulas in the table below.

Table 3: The copula generator functions that we use

Copula	Parameter	$\phi(t)$
Clayton	$\theta \in (0, \infty)$	$(1+t)^{-\frac{1}{\theta}}$
Gumbel	$\theta \in (1, \infty)$	$e^{-t^{\frac{1}{\theta}}}$
Frank	$\theta \in (0, \infty)$	$-\frac{1}{\theta}\log(1-(1-e^{-\theta})e^{-t})$
Joe	$\theta \in (1, \infty)$	$1-(1-e^{-t})^{\frac{1}{\theta}}$
AMH	$\theta \in (0, 1)$	$\frac{1-\theta}{e^t-\theta}$

Note:
For each copula we report the parameter range and the generator function.

We observe that all reported copulas have quite different generator functions and accept different parameter values. The possible parameter values for the AMH copula are between 0 and 1, which seems quite restrictive. When we have decided which copula to use in the HAC, we need to choose an optimal structure. At each step of this procedure we join two variables in the structure with the highest value of the dependence parameter θ of the chosen copula. We estimate θ using maximum likelihood. This is a very simple and fast approach to find an optimal hierarchical structure, as this approach does not ”seek and try” every possible structure. Be that as it may, if the ”true” HAC is not a binary copula we have a misspecified model. Okhrin et al. (2013) come with a solution for this possible problem as they state that while the difference in the structure may exist between a binary and non-binary structure, the difference in distribution functions is small under certain conditions. For example, consider the binary copula $C_2[C_1\{u_1, u_2\}, u_3]$ with parameters θ_1 for copula C_1 and θ_2 for copula C_2. Furthermore, consider the non-binary copula $C_3\{u_1, u_2, u_3\}$ with parameter θ_3 and the same variables $\{u_1, u_2, u_3\}$. Assuming that the copulas C_1, C_2, C_3 all have the same generator function and following the property of associativity of the HAC, if θ_1 and θ_2 are not significantly different than the copula distribution functions $C_2[C_1\{u_1, u_2\}, u_3]$ and $C_3\{u_1, u_2, u_3\}$ are not significantly different and therefore the binary and non-binary structures are not significantly different (Nelsen, 2007). Using this theory we can evaluate the optimal binary structure and transform it to a standard

HAC. To combine three variables in the hierarchical structure we must evaluate the difference of the two copula dependency parameters at subsequent hierarchy levels. Okhrin et al. (2013) propose several methods to test if two subsequent parameters are significantly different. We however, follow Okhrin et al. (2015) and use the proposed penalized framework to test for significant differences. Okhrin et al. (2015) propose to combine two subsequent variables if $\theta_1 - \theta_2 \leq \epsilon$. Now we use an empirical method to specify the value for ϵ (Okhrin et al., 2015).

$$\epsilon = \widehat{\mathbb{I}}(\widehat{\theta}_2)^{-1} p'_{\lambda_n}(\widehat{\theta}_1 - \widehat{\theta}_2) \tag{14}$$

Where we denote $\widehat{\mathbb{I}}$ as

$$\widehat{\mathbb{I}}(\widehat{\theta}_2) = -n^{-1} \sum_{i=1}^{n} l''_i(\theta_2), \tag{15}$$

where $l_i(\theta_2)$ are the log-likelihood contributions. The function $p_{\lambda n}$ denotes the Smoothly Clipped Absolute Deviation (SCAD) penalty function and $p'_{\lambda n}$ its derivative. We denote this derivative penalty function as

$$p'_{\lambda_n}(x) = \lambda \left[\mathbb{1}\{x \leq \lambda_n\} + \frac{\max\{a\lambda_n - x, 0\}}{\lambda_n(a-1)} \mathbb{1}\{x > \lambda_n\} \right], \tag{16}$$

where λ_n is a tuning parameter and $a = 3.7$ as suggested by Fan and Li (2001). $\widehat{\mathbb{I}}(\widehat{\theta})$ is computationally hard to evaluate, as it is complicated to obtain second derivatives of log-likelihood contributions. The Clayton copula is an exception, as we can write the second order derivative as

$$\mathrm{l}''_i(\theta) = -\textstyle\sum_{k=0}^{d-1}(\frac{k}{\theta k+1})^2 + \frac{2}{\theta^2}\left[\frac{t'_\theta(u)}{1+t_\theta(u)} - \frac{1}{\theta}\log(1+t_\theta(u))\right] + (d+\frac{1}{\theta})\left[(\frac{t'_\theta(u)}{1+t_\theta(u)})^2 - \frac{\sum_{j=1}^{d}(\log(u_j))^2 u_j^{-\theta}}{1+t_\theta(u)}\right], \tag{17}$$

where d denotes the number of variables in the copula and u_j denotes the data series of variable j. $t_\theta(u) = \sum_{j=1}^{d} u_j^{(-\theta)} - d + 1$ and $t_\theta(u)' = \sum_{j=1}^{d}(-\log(u_j))u_j^{-\theta}$. Now we only need to choose a suitable value for the tuning parameter λ_n. We choose λ_n such that

$$\lambda_n = \underset{\lambda_n}{\operatorname{argmax}}\, 2\sum_{i=1}^{n} l_i\{\widehat{\theta}_2 + \epsilon(\lambda_n)\} - q\log(n), \tag{18}$$

where $\widehat{\theta}_2$ denotes the parameter at the highest hierarchical level. ϵ denotes the function in Equation 14 with inputs λ_n, $\widehat{\theta}_1$ and $\widehat{\theta}_2$. q denotes the number of estimated parameters up the current hierarchical level and n the numbers of observations. Finally, we have l_i denoting the log-likelihood contributions, which for the Clayton copula is denoted as

$$l_i(\theta) = d\log(\theta) + \log(\Gamma(\frac{1}{\theta} + d)) - \log(\Gamma(\frac{1}{\theta})) - (\theta+1)\sum_{j=1}^{d} u_j - (\frac{1}{\theta} + d)\log(\sum_{j=1}^{d} u_j^{-\theta} - d + 1), \tag{19}$$

where we use the same notation as in Equation 17. For computational reasons we only use consider this penalization on the hierarchical structure for the Clayton HAC.

3.1.4 Mixing copulas in HAC structure

For now we only considered HAC where one copula generator is used in the entire structure. This way, we can only model different dependency structures between our input variables by choosing a different dependence parameter θ. However, if we use different copulas in the hierarchical structure, we can model completely different dependencies between the input variables. Each copula there is has its unique properties and to take the most suitable property of a copula for each dependency in the nested structure of the HAC can be a huge advantage. A part of this research about optimal copulas should therefore address and investigate the use of different copula generators in the HAC structure. How beneficial it might be to combine several copulas in the HAC structure, it is not possible to combine every copula. To explain this, we need to consider two different copulas with generators ϕ_0 and ϕ_1. With these generators we can write a simple HAC as $\phi_0[\phi_0^{-1}(\phi_1[\phi_1^{-1}(u_3) + \phi_1^{-1}(u_2)]) + \phi_0^{-1}(u_1)]$. To obtain a valid copula and to be able to sample from this copula, the derivative of $\phi_0^{-1}\phi_1[\ldots]$ needs to be completely d-monotone for all parameter values in the parameter range. If ϕ is d-monotone, it has to hold that, for all $d \in \mathbb{N}$, ϕ is $d-2$ times differentiable. All these derivatives have to satisfy $(-1)^k\phi^{(k)}$, for $k = 1, \ldots, d-2$ as stated before in Equation 6. Furthermore, $(-1)^{(d-2)}\phi^{(d-2)}$ has to be non-increasing and convex. Unfortunately, not all combinations of copula generators lead to a valid copula. For example, if we take a Clayton generator for ϕ_0 and a Gumbel generator ϕ_1 in the example above, we do not obtain a monotone function $\phi_0^{-1}(\phi_1[\ldots])$. Hofert (2010) discusses sampling methods for nested Archimedean copulas. There are several combinations named of two different copula generators that result in a valid nested copula. The combinations are

$$\{(A, C); (A, 19); (A, 20); (C, 12); (C, 14); (C, 19); (C, 20)\}, \tag{20}$$

where the first element of the combination denotes the copula at the higher hierarchical level of the nested copula. ”A” denotes the AMH copula and ”C” denotes the Clayton copula. The rest of the numbers correspond to the numbered copulas presented by Nelsen (2007). To implement these combinations in our HAC framework we need to determine the structure first. We do this by combining two data series with the highest degree of association, which is measured by Kendall's tau. Then, we can perform a goodness-of-fit test to see which copula is the best choice from Equation 20 for these two data series. However, this goodness-of-fit test is computationally inefficient. To be able to evaluate HAC with different copula generators, we consider the predetermined binary structure $A(A(C(C(12))))$. This means that the copula of the highest hierarchical level is the bivariate AMH copula and the copula at the lowest hierarchical level is the number 12 copula. Now that we know what copula to use at each level of the structure we need to estimate the copula dependence parameters. We can use Kendall's tau to estimate the dependence parameter, as for each copula it is possible to write Kendall's tau as a function of the dependency parameter.

3.1.5 Simulate from HAC

Like with the C-vine and D-vine, we want to simulate data from the HAC to find optimal hedge ratios in our hedging framework. To simulate six data series for each of our input variables from the HAC that we construct we use the algorithm below.

Algorithm 3: Simulation algorithm HAC

Input: HAC C with copula C_0 at the highest hierarchical level with generator ϕ_0;
1 **Simulate** $V_0 \sim F_0 = \mathbb{LS}^{-1}(\phi_0)$ and set $i = 1$;
2 **for** *Inputs* u_1 *and/or* u_2 *of copula* C_0 *that are data series* **do**
3   **Simulate** $w \sim U[0, 1]$;
4   **Set** $x_i = \phi_0(\frac{-\log(w)}{V_0})$;
5   **Set** $i = i + 1$
6 **end**
7 **for** *Inputs* u_1 *and/or* u_2 *of copula* C_0 *that are copulas itself* **do**
8   **Set** C_1, with generator ϕ_1 to be copula u_1 and/or u_2 ;
9   **Simulate** $V_{01} \sim F_{01} = \mathbb{LS}^{-1}(\phi_{01}(\ldots|V_0))$, where $\phi_{01} = \phi_0^{-1}(\phi_1[\ldots])$;
10   **Set** $C_0 = C_1$, $\phi_0 = \phi_1$, $V_0 = V_{01}$;
11   Continue to next hierarchical level and go to step 2 ;
12 **end**
13 **return** $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$

We have to explain some more details about algorithm 3. For example, for each ϕ_0 we need the corresponding distribution function F_0. This distribution can be considered as the inverse Laplace-Stieltjes transform, denoted as $\mathbb{LS}^{-1}$, of the generator function ϕ_0. In the literature, several algorithms are proposed to numerically inverse the Laplace-Stieltjes transform of ϕ_0. Unfortunately, these algorithms again prove to be computationally too inefficient. Therefore, we make use of assumptions about the distributions of F_0. This way we can sample from the assumed distribution functions for the Clayton, AMH and the number 12 copula for our mixed structure.

Table 4: Copula generators that are used for the mixed HAC

Copula	Parameter	$\phi(t)$	$V_0 \sim F_0$	$V_{01} \sim F_{01}$	τ
12	$\theta \in (1, \infty)$	$\frac{1}{(1+t^{1/\theta})}$	$S(\frac{1}{\theta}, 1, (cos(\frac{\pi}{2\theta}))^{\theta}, \mathbb{I}_{\{\theta=1\}}; 1)e^{\theta}$	-	$1 - \frac{2}{3\theta}$
Clayton	$\theta \in (0, \infty)$	$(1+t)^{-\frac{1}{\theta}}$	$\Gamma(\frac{1}{\theta}, 1)$	$\widetilde{S}(\alpha, 1, (cos(\frac{\pi\alpha}{2}))^{\frac{1}{\alpha}}, V_0\mathbb{I}_{\{\alpha=1\}}, \mathbb{I}_{\{\alpha\neq1\}}; 1)$	$\frac{\theta}{2+\theta}$
AMH	$\theta \in (0, 1)$	$\frac{1-\theta}{e^t-\theta}$	$Geo(1-\theta)$	$V_0 + NB(V_0, \frac{1-\theta_1}{1-\theta_0})$	$1 - \frac{2(\theta+(1-\theta)^2 \log(1-\theta))}{3\theta^2}$

Note:
$S(\dots)$ *denotes the stable distribution,* $\widetilde{S}(\dots)$ *denotes the exponentially tilted stable distribution.*
$Geo(\dots)$ *denotes the geometric distribution,* $NB(\dots)$ *the negative binomial distribution and*
Γ *of course denotes the gamma distribution. Furthermore, note* $\alpha = \frac{\theta_0}{\theta_1}$

To simulate from our mixed HAC structure, described by $A(A(C(C(12))))$, we first sample V_0 from the AMH copula of the highest hierarchical level. Then we sample V_{01}, again from the AMH copula. Then, we sample V_{01} from the Clayton copula, but we cannot take V_{01} from Table 4, as the copula at the higher level is not a Clayton copula, but the AMH copula. We therefore use the formula

$$V_{01} = \widetilde{S}V^{\theta_1}, \tag{21}$$

where $V \sim \Gamma(V_0, \frac{1}{1-\theta_0})$ and $\widetilde{S} \sim \widetilde{S}(\frac{1}{\theta_1}, 1, (cos(\frac{\pi}{2\theta_1}))^{\theta_1}, \mathbb{I}_{\{\theta_1=1\}}, V^{\theta_1}\mathbb{I}_{\{\theta_1\neq1\}}; 1)$. θ_0 is the parameter of the higher hierarchical level and θ_1 is the parameter at the lower level. In the next step there again follows a Clayton copula function, but since the copula in the level above is also a clayton copula, we use $\widetilde{S}(\alpha, 1, (cos(\frac{\pi\alpha}{2}))^{\frac{1}{\alpha}}, V_0\mathbb{I}_{\{\alpha=1\}}, \mathbb{I}_{\{\alpha\neq1\}}; 1)$ from Table 4 to sample V_{01}. At the last hierarchical level, we have the number 12 copula. We can sample V_{01} through

$$V_{01} = S\widetilde{S}^{\theta_1}, \tag{22}$$

where $S \sim S(\frac{1}{\theta_1}, 1, (cos(\frac{\pi}{2\theta_1}))^{\theta_1}, \mathbb{I}_{\{\theta_1=1\}}; 1)$ and $\widetilde{S} \sim \widetilde{S}(\theta_0, 1, (cos(\frac{\pi\theta_0}{2}))^{\frac{1}{\theta_0}}, \mathbb{I}_{\{\theta_0\neq1\}}, V_0\mathbb{I}_{\{\theta_0=1\}}; 1)$. θ_0 is again the parameter of the higher hierarchical level and θ_1 is the parameter at the lower level. Now that we have all V_0 and V_{01}, we can use algorithm 3 to simulate our six data series from the mixed HAC.

3.2 Performance measure

Using the different copulas to estimate the joint distribution of $y_t(b)$ from (3), we calculate each risk measure for different hedge ratios b. We find the optimal hedge ratios for which $Risk(y_t(b))$ is minimized, in accordance with the minimization problem in (4). We find the optimal hedge ratios b^* by solving the minimization problem using the Nelder-Mead direct search method, as proposed by Sukcharoen and Leatham (2017). To assess all the hedge ratios that we find from each copula model, we use the Hedging Effectiveness (HE). The HE is proposed by Sukcharoen and Leatham (2017). We use the following formula

$$HE_t = \left(1 - \frac{Risk(y_t(b^*))}{Risk(y_t(0))}\right) \times 100, \tag{23}$$

where $y_t(b^*)$ is the hedged profit/loss of (3), where we fill in the optimal hedge values. $y_t(0)$ is the unhedged profit/loss, where we fill in 0 for all b in (3). The HE is presented in percentages and we want a highest possible value for this measure, as we want the hedged risk measure $Risk(y_t(b^*))$ as low as possible compared to the unhedged risk measure $Risk(y_t(0))$. As we work with dynamic hedge ratios and a rolling window approach, we take the average of all estimated HE. We compare the performance of the standard multivariate Gaussian and Student t copula with the C-vine and D-vine copula model and the different (penalized) HAC.

4 Results

4.1 Optimal hedge ratios

The goal of this research is to find optimal hedge ratios using different copulas. Table 5 shows the optimal hedge ratios for each copula and for each downside-risk measure. In Table 5 a rolling window size of 262 observations is used, which means that we forecast the hedge ratios 3773 times. The numbers in the table are averages of all estimated hedge ratios in the forecast window. The numbers between brackets denote the standard deviation of the forecasted optimal hedge ratios.

Table 5: The optimal average hedge ratios for different downside-risk measures

Rolling window size 262	SV	LPM	VaR			ES		
			0.99	**0.95**	**0.90**	**0.99**	**0.95**	**0.90**
Model								
Crude oil hedge ratios								
SGC	0.777	0.777	0.812	0.799	0.795	0.799	0.789	0.786
	(0.185)	(0.199)	(0.211)	(0.189)	(0.185)	(0.229)	(0.199)	(0.190)
STC	0.566	0.547	0.534	0.575	0.586	0.501	0.541	0.556
	(0.191)	(0.200)	(0.213)	(0.191)	(0.184)	(0.239)	(0.205)	(0.194)
C-vine	0.870	0.828	1.083	0.983	1.000	0.980	0.869	0.906
	(0.079)	(0.096)	(0.122)	(0.079)	(0.082)	(0.134)	(0.091)	(0.099)
D-vine	0.691	0.660	0.666	0.725	0.742	0.628	0.685	0.695
	(0.270)	(0.286)	(0.262)	(0.253)	(0.272)	(0.275)	(0.263)	(0.271)
HAC								
Clayton	0.612	0.644	0.377	0.435	0.477	0.356	0.426	0.490
	(0.395)	(0.420)	(0.357)	(0.368)	(0.377)	(0.337)	(0.360)	(0.385)
Gumbel	0.505	0.485	0.600	0.582	0.570	0.572	0.541	0.523
	(0.495)	(0.491)	(0.501)	(0.504)	(0.503)	(0.498)	(0.500)	(0.497)
Frank	0.531	0.490	0.533	0.584	0.609	0.478	0.517	0.543
	(0.259)	(0.263)	(0.262)	(0.270)	(0.280)	(0.260)	(0.257)	(0.257)
Joe	0.293	0.242	0.544	0.496	0.473	0.456	0.400	0.356
	(0.514)	(0.521)	(0.535)	(0.518)	(0.521)	(0.494)	(0.504)	(0.514)
AMH	-0.073	-0.090	0.053	0.014	-0.010	0.036	-0.013	-0.040
	(0.102)	(0.116)	(0.133)	(0.109)	(0.107)	(0.169)	(0.129)	(0.118)
Mixed	1.857	2.044	-0.558	-0.429	-0.466	0.181	0.389	0.486
	(1.758)	(1.759)	(3.031)	(2.637)	(2.401)	(0.336)	(0.239)	(0.222)
Gasoline hedge ratios								
SGC	0.691	0.711	0.627	0.634	0.640	0.641	0.673	0.685
	(0.246)	(0.271)	(0.268)	(0.247)	(0.250)	(0.287)	(0.261)	(0.259)
STC	0.471	0.479	0.470	0.508	0.522	0.449	0.493	0.506
	(0.224)	(0.287)	(0.265)	(0.236)	(0.235)	(0.306)	(0.247)	(0.239)
C-vine	0.928	0.859	0.755	0.934	1.004	0.683	0.851	1.026
	(0.247)	(0.264)	(0.239)	(0.229)	(0.244)	(0.250)	(0.242)	(0.260)
D-vine	0.723	0.706	0.548	0.678	0.737	0.531	0.657	0.706
	(0.333)	(0.342)	(0.305)	(0.334)	(0.360)	(0.330)	(0.327)	(0.337)
HAC								
Clayton	0.638	0.609	0.820	0.803	0.778	0.661	0.664	0.647
	(0.299)	(0.309)	(0.479)	(0.410)	(0.391)	(0.436)	(0.353)	(0.326)
Gumbel	0.789	0.781	0.583	0.665	0.713	0.601	0.702	0.758
	(0.403)	(0.426)	(0.374)	(0.394)	(0.410)	(0.376)	(0.384)	(0.401)
Frank	0.720	0.682	0.548	0.700	0.773	0.502	0.636	0.706
	(0.313)	(0.400)	(0.334)	(0.343)	(0.361)	(0.383)	(0.339)	(0.336)
Joe	0.657	0.633	0.355	0.459	0.540	0.304	0.438	0.538
	(0.408)	(0.469)	(0.256)	(0.303)	(0.355)	(0.377)	(0.292)	(0.336)
AMH	0.206	0.240	0.043	0.094	0.116	0.114	0.167	0.183
	(0.114)	(0.201)	(0.214)	(0.149)	(0.135)	(0.268)	(0.172)	(0.147)
Mixed	0.215	0.229	1.450	1.282	1.389	1.005	0.807	0.735
	(0.402)	(0.470)	(0.866)	(1.134)	(1.557)	(0.632)	(0.469)	(0.430)
Heating oil hedge ratios								
SGC	0.691	0.711	0.627	0.634	0.640	0.641	0.673	0.685
	(0.246)	(0.271)	(0.268)	(0.247)	(0.250)	(0.287)	(0.261)	(0.259)
STC	0.764	0.682	0.737	0.741	0.726	0.719	0.718	0.716
	(0.414)	(0.806)	(0.548)	(0.410)	(0.373)	(0.724)	(0.515)	(0.459)
C-vine	0.929	1.121	1.235	0.741	0.786	1.339	0.858	0.766
	(0.188)	(0.316)	(0.465)	(0.267)	(0.196)	(0.505)	(0.363)	(0.275)
D-vine	0.998	1.050	0.999	0.909	0.887	0.965	0.943	0.944
	(0.643)	(0.773)	(0.669)	(0.580)	(0.559)	(0.754)	(0.627)	(0.626)
HAC								
Clayton	0.631	0.635	0.673	0.654	0.645	0.808	0.719	0.664
	(0.343)	(0.514)	(0.519)	(0.403)	(0.384)	(0.742)	(0.484)	(0.382)
Gumbel	0.836	0.891	0.622	0.615	0.633	0.622	0.652	0.728
	(0.480)	(0.591)	(0.480)	(0.429)	(0.436)	(0.523)	(0.439)	(0.438)
Frank	0.761	0.750	0.690	0.689	0.684	0.591	0.673	0.714
	(0.471)	(0.778)	(0.535)	(0.373)	(0.340)	(0.767)	(0.480)	(0.434)
Joe	0.765	0.783	0.538	0.524	0.531	0.408	0.463	0.521
	(0.518)	(0.703)	(0.537)	(0.435)	(0.422)	(0.699)	(0.456)	(0.398)
AMH	0.334	0.338	0.038	0.131	0.184	0.095	0.224	0.277
	(0.301)	(0.678)	(0.468)	(0.309)	(0.294)	(0.648)	(0.427)	(0.349)
Mixed	-0.209	-0.109	3.006	2.922	3.110	1.934	1.631	1.431
	(1.609)	(1.671)	(4.290)	(3.671)	(4.506)	(1.240)	(1.143)	(1.149)

SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC.

We observe that all hedge ratios are fairly similar. Almost all averages of the hedge ratios are between 0.5 and 1.1. Only the hedge ratios of the AMH copula and the mixed HAC are different from the hedge ratios of the rest of the copulas. The standard deviation of the hedge ratios are also quite similar for the different oil products. These results are in line with Figure 1, as the price changes series co-move quite similar and have similar summary statistics. Observe also that in some cases the hedge ratios are larger than one, meaning that we need to buy more futures than we have in the position of the spot prices. The optimal hedge ratios for the other two rolling windows are presented in Appendix D. The hedge ratios for these rolling window sizes are a bit different, but the differences are not big enough to go into more detail. The hedge ratios of the rolling window size of 786 observations are really close to the rolling window size of 1310 observations.

4.2 Hedging effectiveness

Using all optimal hedge ratios from Table 5 and Table 8, Table 9 from Appendix D we calculate the HE and report them in the table below.

Table 6: The hedging effectiveness using the optimal hedge ratios from different hedging models and different downside-risk measures

Risk measures	**SV**	**LPM**	**VaR**			**ES**		
			0.99	**0.95**	**0.90**	**0.99**	**0.95**	**0.90**
Model								
Rolling window size 262								
SGC	38.97	51.69	24.43	23.32	23.14	24.87	23.43	23.15
STC	23.15	28.69	12.85	14.48	15.44	12.12	12.73	13.34
C-vine	63.66	75.93	34.27	41.48	43.85	32.12	35.35	38.25
D-vine	50.25	59.48	28.50	32.72	34.34	26.16	28.71	30.17
HAC								
Clayton	42.87	50.39	29.27	31.45	32.10	24.16	24.79	25.64
Gumbel	63.10	76.35	**37.95**	36.69	36.99	**36.37**	**35.93**	**37.58**
Frank	47.34	53.59	24.12	30.83	34.20	20.75	25.47	28.69
Joe	47.80	58.25	24.78	26.44	27.43	20.49	20.47	21.68
AMH	9.03	14.84	3.04	2.12	2.43	4.28	3.55	4.04
Mixed	**68.63**	**77.87**	37.69	**47.07**	**53.93**	29.56	33.04	34.93
Rolling window size 786								
SGC	38.94	52.33	24.19	22.99	22.63	24.54	23.07	22.68
STC	22.25	27.77	12.09	13.67	14.46	10.97	11.74	12.41
C-vine	43.42	57.00	26.25	27.04	28.01	27.58	26.08	26.54
D-vine	50.05	59.16	28.60	32.77	34.47	25.05	27.90	29.81
HAC								
Clayton	40.83	47.59	31.73	33.12	32.81	25.49	24.98	24.76
Gumbel	68.41	**81.49**	**43.98**	**42.25**	**42.07**	**41.49**	**40.80**	**42.19**
Frank	50.95	56.48	26.62	34.52	38.00	22.04	27.94	31.26
Joe	52.71	62.66	27.45	30.21	31.66	21.71	22.56	23.98
AMH	6.87	11.39	3.27	2.47	2.62	4.53	3.41	3.46
Mixed	**70.24**	80.29	28.33	35.97	41.82	27.98	31.69	33.43
Rolling window size 1310								
SGC	39.40	52.87	24.31	23.34	23.08	24.61	23.45	23.09
STC	23.07	29.00	12.31	14.09	14.94	11.05	12.22	12.90
C-vine	46.83	57.26	29.41	30.32	30.77	28.04	28.68	29.15
D-vine	50.42	58.24	29.22	34.67	36.28	24.86	27.94	30.72
HAC								
Clayton	40.10	47.27	32.66	33.72	33.10	26.18	25.24	24.53
Gumbel	70.39	**83.28**	**44.49**	**43.56**	**43.73**	**41.89**	**41.96**	**43.63**
Frank	52.63	58.78	26.50	34.83	38.52	22.22	28.77	32.18
Joe	56.69	65.88	31.49	35.19	37.02	24.83	26.09	27.40
AMH	7.39	12.04	3.27	2.73	2.90	4.53	3.66	3.77
Mixed	**71.08**	82.03	26.64	33.78	39.56	27.39	32.08	33.85

SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The values for the hedging effectiveness are presented in percent.

Table 6 shows us the average of the HE for all downside-risk measures and all copula models.

The HE denotes the percentage of risk reduction if we hedge our profit/loss from Equation 3. For every copula model, there is an average reduction of risk. The greatest improvement is observable for the downside-risk measure LPM. The best copula model for each downside-risk measure, having the highest HE, is indicated in bold. To compare different copula models, we need to test whether the values for the HE are significantly different from each other. Therefore, a paired t-test is performed to test for significant differences. The statistics from all the tests are displayed in Appendix F. We observe that for each downside-risk measure, either the HAC with Gumbel generator or the HAC with mixed generators performs best. The differences in the HE between the Gumbel HAC and the mixed HAC are not significant for the downside-risk measure LPM considering the three rolling window sizes. The performance of the mixed HAC is remarkable after considering the contrasting optimal hedge ratios from for example Table 5. The hedge ratios tend to differ for each downside-risk measure, especially compared to the other copulas. The standard deviation of the hedge ratios is also much higher for the mixed HAC, but this does not seem to influence the performance. From Table 6 we can conclude that the only copula that is competitive with these best performing copulas is the C-vine copula, as the values for the HE are quite close for the rolling window size of 262 observations. Especially after considering that the difference between the Gumbel HAC and the C-vine is not significant for the downside-risk measures SV and LPM.

If we look at all the nested copulas, the AMH has the worst performance of all models, without a clear explanation and in contrast to our hypothesis. It is even stranger, as this copula is used in the mixed HAC structure. The AMH copula is however used in the top two levels of the hierarchical structure of the mixed HAC, which means that this copula mostly models the dependence between two (sub-)copulas. If the AMH copula is used at every hierarchical level of the HAC, also the dependency between the input variables is modelled. Maybe this does not suit the AMH copula. The same reasoning could be used to explain the increase in performance when using a mixed HAC instead of the Clayton HAC, as the Clayton copula is used in the HAC for the second and third hierarchical level. This way, the Clayton copula mostly models the two pairs of input variables, which probably suits the Clayton copula better than modelling the dependency at higher hierarchical levels.

To elaborate more on the Clayton HAC, remember that this is the only HAC where we use a penalization on the hierarhical structure. This penalty, as described in section 3, does not improve the performance significantly. This penalization gives us a systematic way to overcome our shortcoming of only considering binary structures, but cannot help us by improving the performance.

Looking at the standard copulas, the standard Student t copula is clearly a benchmark copula that is outperformed by the nested copulas. The standard Gaussian copula is much more competitive and even outperforms the Clayton HAC for the downside-risk measure LPM. Furthermore, the standard Gaussian copula sometimes significantly outperforms the Frank and Joe HAC for the downside-risk measure ES. The Gumbel HAC and the mixed HAC are superior to the benchmark copulas considering all downside-risk measures.

Addressing the vine copulas, we first observe that they outperform the standard copulas. The vine copulas also show that they are tough competitors for the different HAC. It is remarkable that the performance between the vines is quite different, as the C-vine significantly outperforms the D-vine for all downside-risk measures in the rolling window size of 262 observations, but not for the other rolling window sizes. In the literature, the D-vine mostly outperforms the C-vine in a similar setting to ours. This is because the D-vines can directly model the dependence between each spot and futures price, while the C-vine can only model the dependence of one spot price with all other spot and futures prices directly (see Figure 2).

We observe that the results are a bit different when using different rolling window sizes. For example, we find the largest HE to be 83.28% for the Gumbel HAC and the downside-risk measure LPM for the rolling window size of 1310 observations. This value is significantly higher than the largest value for the HE of the rolling window size of 262 observations.

On average, the HE for the Gumbel HAC are higher for all downside-risk measures when the rolling window size of 786 or 1310 observations is used instead of the rolling window size of 262 observations. In contrast to the Gumbel HAC, the mixed HAC only performs better considering the downside-risk measure SV and LPM, as the performance for other downside-risk measures significantly drop. An explanation for this can be the high standard deviation of the optimal hedge ratios of the mixed HAC. Another copula with diminishing performance for larger rolling window sizes is the C-vine. In the case of the two largest rolling window sizes, the C-vine cannot be considered as a competitor of the Gumbel HAC. In addition, the performances of the C-vine and D-vine are much closer to each other. The D-vine outperforms the C-vine for seven out of eight downside-risk measures for the rolling window size of 786 observations, and five out of eight times for the rolling window size of 1310 observations. These results are more in line with results from the literature and more in line with our hypothesis. For the standard Gaussian copula and the standard Student t copula, the performance seems unrelated to the choice for rolling window size. The HAC easily outperform these standard copulas, but only the Gumbel HAC and mixed HAC outperform the vine copulas significantly.

The results for the rolling window size of 1310 observations are very similar to the results of the rolling window size of 786 observations. This rolling window size comes closest to the one used in Sukcharoen and Leatham (2017). They use a rolling window size of 261 weekly observations, which comes closest to the 1310 daily observations that we use. The results for the SGC, STC and vine copulas that they present are a bit different to our results. First, the HE of the four copulas lie closer to each other. Moreover, the STC in their results shows a slightly better performance than the SGC, while the vines have superior performance. That vines are superior to the SGC and STC is also shown by Table 6, but the STC is not performing as good as, or better than the SGC.

Looking at all the HE, we can conclude that the HE improves when using a larger rolling window size. A reason for this is the fact that a larger rolling window size means more observations that we can use. This way, the copulas have more information about the data series and therefore also more information about the dependency structure that they can use to simulate the profit series of the oil refineries. To give a small restraint, this effect is not really (clearly) seen for the Clayton HAC, C-vine and mixed HAC.

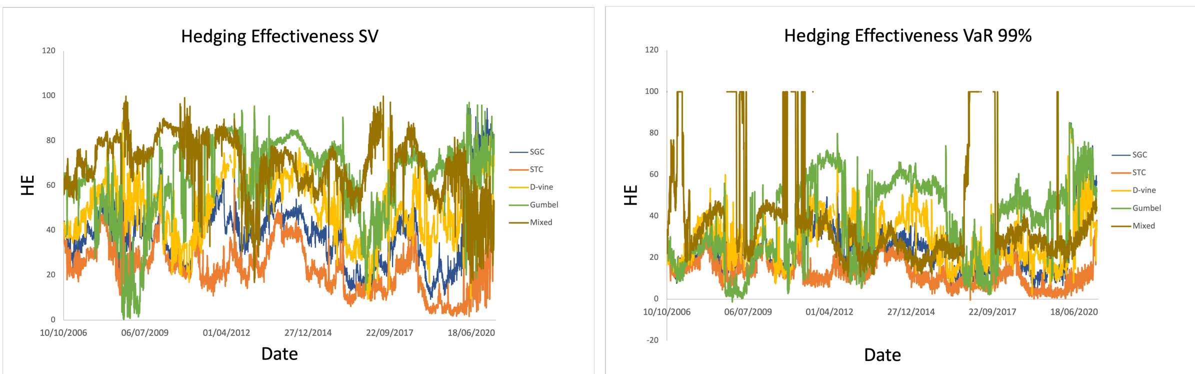
4.2.1 Development of HE over time

To obtain a comprehensive view of the performance of each copula, we present and discuss the development of the HE with the use of the figures below in addition to the reported averages of the HE. The development of the HE of all models and for all rolling window sizes is displayed in Appendix E. The first thing to observe is that for all copulas (except the C-vine), the HE is less volatile for larger rolling window sizes. There seems to be a direct relation between the standard deviation of the estimated optimal hedge ratios from Table 5, Table 8, Table 9 and the volatility of the HE, which is also logical. Some copulas have stable performance for the whole forecasting period, while others, deviate a lot. When we make a choice for the best copula model, we need to keep this in mind.

Looking at Figure 1, we observe three periods of high volatility in the price changes. The first and most notable period is of course the well-known crisis between 2007-2009. For the rolling window size of 262 observations, we observe from the figures in subsection E.1 a drop in HE. Especially the HAC copulas suffer from this drop in HE, as the SGC and STC show a minor drop and the vine copulas only show some increased volatility in the HE. This is maybe a strange results, as we would expect that hedging would be more effective during high volatility periods. The reason for this could be that the downside-risk of an unhedged profit series and a hedged profit series increases sharply during high volatility periods. It is in this case much harder to improve with 50% than it is when the downdside-risk measures have lower values for example.

The other rolling window forecasts do not cover the crisis period, as the observations of the crisis period are used in the rolling window to make predictions, so they cannot be in the forecast window. Next to the big crisis between 2007-2009, there is the period between 2011-2013 that records a period of high volatility in the price changes of the oil products (not as high as the 2007-2009 period). For the rolling window of 262 observations, we cannot observe a significant influence on the HE in the figures from subsection E.1. Only the Gumbel and Joe copula record a notable dip in HE during this period, while STC has an increased HE for this period. For the rolling window of 786 observations, we observe from subsection E.2 a large but short dip for the Gumbel and Joe HAC for this period. These results are not found for the rolling window size of 1310 observations, as the high volatility in the price changes for the period 2011-2013 seems to have no effect on the HE. The last period with high volatility is the period at the beginning of 2020. This small period records huge spikes in the price change series. In this period the oil prices dropped below 0 USD/barrel, which is a unique situation. This price recovers quite fast, explaining the short period of huge spikes in the price changes series. We observe the same effect for all copulas and for all rolling window sizes, namely an increase in the HE. This is expected, as the sudden price changes do not effect the hedged profit series as much as the unhedged profit series.

To compare the copula models with each other, we plot the HE of different copulas in the figures below. To keep a good overview we only plot the two benchmark copulas SGC and STC, the best performing vine copula, the best performing HAC with a single generator function and the HAC with multiple mixed generators. We only show the development of the HE for the downside-risk measures SV and VaR with 99% probability level to keep the results clear and concise. Moreover, these downside-risk measures are a good representation of the other downside-risk measures.



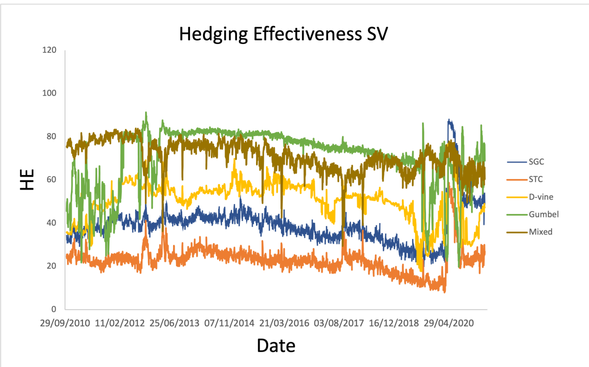
(a) HE minimizing SV **(b)** HE minimizing VaR

Figure 4: plots of the development of the HE using a rolling window of 262 observations

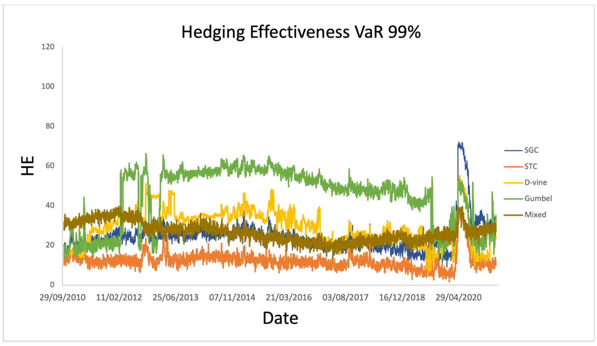
From Figure 4a we observe that the development of the series of HE for the copula models have a similar pattern. All series of HE can be stacked on top of each other. The HE for the STC at the lowest part of the figure and the mixed HAC on top. As the mixed HAC has the highest volatility, there are some periods where the other copula models have a better performance. Despite this, both HAC outperform the standard copulas and the D-vine. Looking at the final period of the figure, we observe that the SGC performs really well, which is quite remarkable.

Considering the HE of the downside-risk measure VaR with a probability level of 99%, the first thing we observe from Figure 4b are the periods where the mixed HAC has an extremely high HE. There is no clear explanation or theory for this behaviour. Despite this periods of high HE, the mixed HAC is outperformed by the Gumbel HAC (as well as all other copulas), but its performance is still much better than the performance of the standard copulas and the D-vine. Overall, the Gumbel HAC seems the best performing copula, but its performance is not impeccable. We observe some periods where other copulas outperform the Gumbel HAC. Especially during the crisis period between 2007-2009, the Gumbel HE lacks performance. During the high volatility period in the beginning of 2020, this lack of performance is not noticeable.

To evaluate the performance of all the copulas for the larger rolling window of 1310 observations, we use the figure below.



(a) HE minimizing SV



(b) HE minimizing VaR

Figure 5: plots of the development of the HE using a rolling window of 1310 observations

We immediately observe less volatile HE series for all copula models for this rolling window of 1310 observations. The figures are much clearer and they become much more interpretable. It is clear to see which copula performs best. From Figure 5a we observe that there is a period from 2013 until the end of 2018, where the Gumbel HAC is superior to the other copulas. However, difference in performance between the mixed HAC and the Gumbel HAC is not significant. Only at the beginning of the forecast window and at the end, there are other copulas that outperform the Gumbel HAC, with the mixed HAC is the best performing copula. Looking at Figure 5b, we notice that the HE is lower for all copula models when minimizing the VaR instead of the SV. The order of best copula models is also quite different for the VaR. The Gumbel HAC is now the best performing copula while the STC is still the worst performing copula. However, the mixed HAC is outperformed by the D-vine and almost by the SGC. Again, in the beginning and ending period, the Gumbel HAC is outperformed. A remarkable thing to notice is the last period of the series, where the highest HE is obtained by the SGC. This period denotes the short and extremely high volatility period at the beginning of 2020. The SGC shows to make use of this period of extreme volatility to boost its performance.

5 Conclusion

The main goal of this research is to find optimal hedge ratios for three oil products that are processed in oil refineries. The copula-GARCH model is used to find the optimal hedge ratios. We do this, by first estimating a GARCH model for the spot- and futures price change series to capture the heteroskedastic volatility that is present in our price changes series. We then use the copulas to model the dependence between the white noise processes from the GARCH and simulate data series accordingly. After this, we find optimal hedge ratios that minimize a certain downside-risk and check how effective these optimal hedge ratios are. The main research question is to find the best copula that finds the most effective hedge ratios. It is important to choose the best copula to model the dependency structure and this research evaluates the Standard Gaussian copula (SGC) and Standard Student t copula (STC), the C-vine and D-vine and the HAC. In the literature, there is the least attention for the HAC. Especially the use of the HAC in a practical manner such as our described hedging framework is underexposed in the current literature. We therefore construct different HAC that make use of one generator function and a HAC that makes use of multiple generator functions. Furthermore, the penalty framework of Okhrin et al. (2015) is used to improve the performance of the HAC.

To see how the different copulas in our hedging framework perform, we show the HE of all models in section 4. We conclude that the best copula for our data set is the Gumbel HAC. This result is not, however, consistent for all the different downside-risk measures that we use. For the downside-risk measures SV and LPM, the mixed HAC shows it can outperform the Gumbel HAC. The Gumbel HAC is still competitive for these risk measures, while the mixed HAC is not competitive for the other risk measures. Therefore, we conclude that the best copula model overall is the Gumbel HAC. This copula only uses only the Gumbel generator function for the whole HAC structure. We also use multiple generators in the HAC structure (mixed HAC), as in the D-vine structure multiple copulas from different families are used, which seems a huge benefit. Due to computational reasons, we only use one predetermined mix of different generators for the mixed HAC, while it is preferable to choose a copula generator function that fits for that particularly copula in the structure. We can realise this with a goodness-of-fit test for every copula in the structure, but this is too computationally expensive. Although we are limited by the predetermined structure with mixed copulas, we show the potential of this copula, as the mixed HAC outperforms the Gumbel HAC and vine copulas several times. The use of different generator functions in one HAC structure is to our knowledge never evaluated and compared with other copulas using a certain performance measure. We therefore cautiously conclude that this copula has got a high potential.

To place our result in the existing literature, let us first compare this result with Sukcharoen and Leatham (2017), as they perform a similar research. Their main conclusion is that the D-vine performs best in terms of HE, considering the SGC, STC and C-vine among others. They do not consider HAC, so this research tries to add a competitor for their research question. Although we show that the D-vine generally outperforms the SGC, STC and C-vine, the D-vine is not the best performing copula. We should however mention that, they use a different data set where the data is also sampled weekly instead of daily. We should also mention that not all HAC outperform the D-vine, as the AMH HAC is the worst copula with terrible performance, for example. Thus, we cannot make the statement that HAC in general are better copulas in hedging downside-risk. Based on our results we can, however, advise the use of the Gumbel HAC in hedging downside-risk. Especially oil refineries, for which our research and our data set should be very relevant could try the Gumbel HAC to estimate optimal hedge ratios. We would also propose the use of the rolling window size of 1310 observations (approximately 5 years of data), as this results in the highest HE and the most stable predictions. We cannot point out one certain downside-risk measure that results in the best estimated hedge ratios. We do see that, although the HE differs per downside-risk measure because of different scaling of the value of the downside-risk measures, the estimated hedge ratios are really similar. We therefore propose the use of all downside-risk measures and taking the average of all estimated hedge ratios.

If we try to explain the superior performance of the (Gumbel) HAC compared to the D-vine we should look at the structures of both copulas. For the HAC, we construct a binary structure where we put a pair of variables together in the structure if they have the highest value for the estimated dependence parameter. The structure for our six input variables that we find most is $C_5[C_4(C_1\{u_1, u_2\}, C_2\{u_3, u_4\}), C_3\{u_5, u_6\}]$. We observe that each pair of variables (mostly the spot and futures variables of the same oil product) are put together and are given a dependency parameter. Then, the dependency of two pairs of variables are described by a copula and then the last pair is added to complete the sturcture of the HAC. This way, we mostly model the dependency of one spot price series and one futures price series directly and then model the dependency of the three pairs. A huge advantage of the HAC is that we can use different dependency parameters for each copula in the structure. The superior performance of the HAC with Gumbel generator function, compared to the other generator functions can only be explained by the simple fact that the dependencies of our data series suits the Gumbel generator function best. Considering the D-vine, the structure is already predetermined and we use the copula variables u_1 and u_2 in one copula and the copula variables u_2 and u_3 in one copula etc. This structures thus considers the dependence between variables u_1, u_2, but also for u_2 and u_3. For our set up of three variable pairs (three oil products) of spot and futures prices, it is maybe more beneficial to only look at the dependence between the pairs u_1, u_2 and of u_3, u_4 and then model the dependency between these pairs instead of modelling the dependence between each input variable separately. If we do however model the dependency of one input variable with more than one other input variable, the D-vine is preferable to the C-vine. We see in Figure 2 and (10) that each spot price input variable is directly linked to its futures price input variable, while in for example the C-vine structure, they are not. This could be the reason why the D-vine has the best performance for this data compared to the C-vine.

In our framework, we only use six input variables. For other commodity hedging problems, more input variables could be needed. In that case, the D-vine has a clear advantage compared to the HAC, as the structure of the D-vine is predetermined, which is not the case for the HAC. Determining the structure of the HAC can be a difficult task, whereas for the D-vine, only the order of the input variables need to be determined. We show a fast method to obtain that optimal order in section 3.

To deal with another part of our research, we should address the penalization of the HAC. In section 3 we show a systematic method to aggregate the binary structure of a HAC, so that more than two variables can be modeled by one (sub-)copula in the HAC. This systematic method aggregates two subsequent copula if the difference between the dependency parameter of the two copulas at the subsequent hierarchical levels is smaller than a threshold. This threshold is calculated with the use of the SCAD derivative penalty function. We implement this approach but find no significant difference in performance.

This brings us to the limitations of our research. Due to the computationally hard derivation of the second derivative of the copula log-likelihood functions, we only consider the penalized HAC for the Clayton copula. This did not result in an improved performance, but it might for other copulas. Moreover, we could not perform a goodness-of-fit test to determine which mix of copula generator functions is optimal in the HAC structure. We showed the potential of a mixed HAC, but can most certainly improve the performance by using the goodness-of-fit test.

To add a final point, the results show that the Gumbel HAC is outperformed by other copulas in periods with high volatility in the price changes series. For example, the D-vine performance is stable in this high volatility periods and therefore outperforms the Gumbel HAC. As these periods of high volatility are short and our used data set is large, the average performance of the Gumbel HAC is still superior. A solution for the decrease in performance during high volatility periods can be the use of a threshold system, that uses the D-vine when we enter a economic turbulent time, and for the other periods the Gumbel HAC.

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Appendices

A Appendix A

A.1 Graph of level

We know that the oil prices can be very volatile, as we show in the graph below.

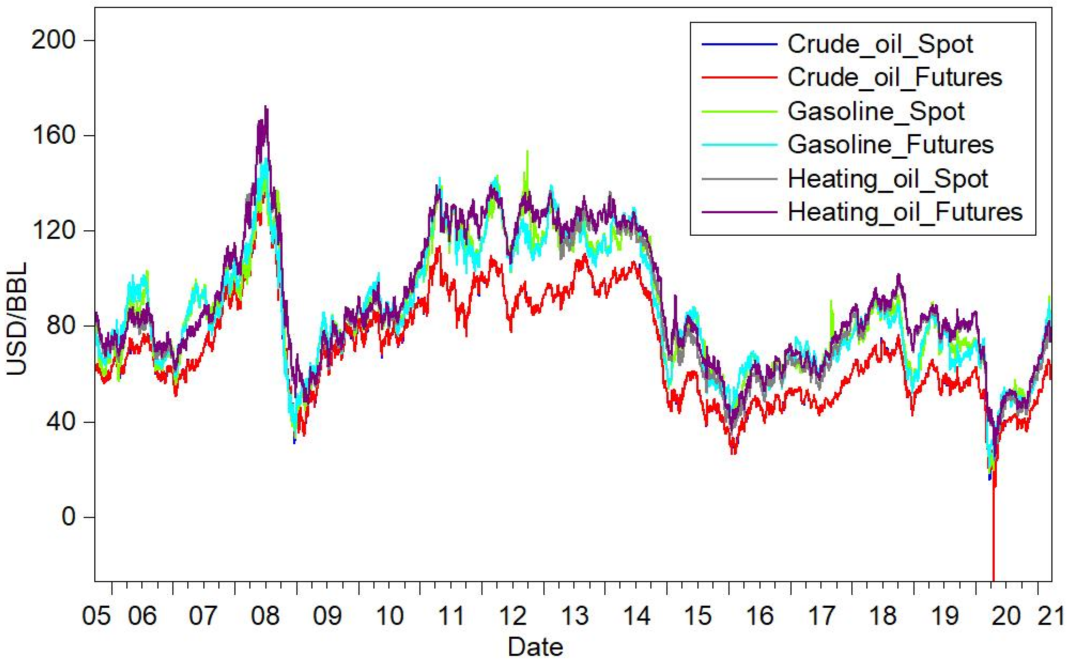


Figure 6: The development of spot and futures prices for our products crude oil, unleaded gasoline and number 2 heating oil.

Figure 6 shows the price development from 2005 until 2021. We observe from the graph that all oil products are closely correlated.

A.2 Correlation matrix

The table below shows the correlations between all data series that we use in this research.

Table 7: Correlation of first differences in spot and futures prices of oil product

		Crude oil		Gasoline		Heating_oil	
		Spot	Futures	Spot	Futures	Spot	Futures
Crude oil	Spot	1					
	Futures	0.964	1				
Gasoline	Spot	0.479	0.481	1			
	Futures	0.592	0.602	0.758	1		
Heating oil	Spot	0.587	0.600	0.655	0.743	1	
	Futures	0.616	0.634	0.643	0.787	0.894	1

B Appendix B

B.1 Algorithm to simulate from a D-vine

The algorithm below shows us how to simulate series from a D-vine copula. We observe that the algorithm is larger than the simulation algorithm of the C-vine.

Algorithm 4: Simulation algorithm for a D-vine

Input: Simulate vectors $u_1, \ldots, u_6$ containing n independent uniform values between [0,1];

$y_1 = w_{1,1} = u_1$;
$y_2 = w_{2,1} = h^{-1}(u_2, w_{1,1}, \Theta_{1,1})$;
for $i \in \{3, \ldots, 6\}$ **do**
  $w_{i,1} = u_i$;
  for $k \in \{i-1, i-2, \ldots, 2\}$ **do**
    $w_{i,1} = h^{-1}(w_{i,1}, w_{i-1,2k-2}, \Theta_{k,i-k})$;
  end
  $w_{i,1} = h^{-1}(w_{i,1}, w_{i-1,1}, \Theta_{1,i-1})$;
  $y_i = w_{i,1}$;
  if $i = 6$ **then**
    Stop
  end
  $w_{i,2} = h(w_{i-1,1}, w_{i,1}, \Theta_{1,i-1})$;
  $w_{i,3} = h(w_{i,1}, w_{i-1,1}, \Theta_{1,i-1})$;
  if $i > 3$ **then**
    for $j \leftarrow 2, \ldots, i-2$ **do**
      $w_{i,2j} = h(w_{i-1,2j-2}, w_{i,2j-1}, \Theta_{j,i-j})$;
      $w_{i,2j+1} = h(w_{i,2j-1}, w_{i-1,2j-2}, \Theta_{j,i-j})$;
    end
  end
  $w_{i,2i-2} = h(w_{i-1,2i-4}, w_{i,2i-3}, \Theta_{i-1,1})$;
end
return $y_1, \ldots, y_6$

B.2 Nelder-Mead search method

After we use the copula-GARCH approach to model the first difference series of the six spot and futures prices, we can find $y_t(b)$ in (3) for which we only need the hedge ratios $b = \{b_C, b_G, b_H\}$. As said before, we use the Nelder-Mead direct search algorithm to find the optimal values for $\{b_C, b_G, b_H\}$ for which a certain risk measure $Risk(y_t(b))$ is minimized. Therefore, we create a function $f()$ that uses b as input and has an output of $Risk(y_t(b))$, so $f(b) = Risk(y_t(b))$. The Nelder-Mead algorithm can be summarized as follows:

Algorithm 5: Nelder-Mead direct search method algorithm

Input: Test sets $\mathbf{b}_1, \ldots, \mathbf{b}_{n+1}$ containing possible hedge ratio solutions ;
$f(\widehat{\mathbf{b}})$ is standard deviation of function values $f(\mathbf{b}_1), \ldots, f(\mathbf{b}_n)$;

1 **while** $f(\widehat{\boldsymbol{b}}) > \epsilon$ **do**
2   Order test sets such that $f(\mathbf{b}_1) \leq \cdots < f(\mathbf{b}_n)$;
3   Calculate $f(\widehat{\mathbf{b}})$;
4   Calculate $\mathbf{b}_0$ as the average of all sets except $\mathbf{b}_{n+1}$;
5   Compute reflected set $\mathbf{b}_r = \mathbf{b}_0 + \alpha(\mathbf{b}_0 - \mathbf{b}_{n+1})$;
6   **if** $f(\boldsymbol{b}_1) \leq f(\boldsymbol{b}_r) \leq f(\boldsymbol{b}_{n+1})$ **then**
7   Replace $\mathbf{b}_{n+1}$ with $\mathbf{b}_r$ and go back to line 2
8   **end**
9   **if** $f(\boldsymbol{b}_r) < f(\boldsymbol{b}_1)$ **then**
10   Compute expand set $\mathbf{b}_e = \mathbf{b}_0 + \gamma(\mathbf{b}_r - \mathbf{b}_0)$
11   **if** $f(\boldsymbol{b}_e) < f(\boldsymbol{b}_r)$ **then**
12   Replace $\mathbf{b}_{n+1}$ with $\mathbf{b}_e$ and go back to line 2
13   **else**
14   Replace $\mathbf{b}_{n+1}$ with $\mathbf{b}_r$ and go back to line 2
15   **end**
16   **end**
17   **end**
18   **if** $f(\boldsymbol{b}_r) \geq f(\boldsymbol{b}_n)$ **then**
19   Compute contracted set $\mathbf{b}_c = \mathbf{b}_0 + \rho(\mathbf{b}_{n+1} - \mathbf{b}_0)$ **if** $f(\boldsymbol{b}_c) < f(\boldsymbol{b}_{n+1})$ **then**
20   Replace $\mathbf{b}_{n+1}$ with $\mathbf{b}_c$ and go back to line 2
21   **end**
22   **end**
23   Replace all values except $\mathbf{b}_1$ with $\mathbf{b}_i = \mathbf{b}_1 + \sigma(\mathbf{b}_i - \mathbf{b}_1)$ and go back to line 2
24 **end**
25 Set $\mathbf{b}^* = \mathbf{b}_1$;
26 **return** $\mathbf{b}^*$

The Nelder-Mead direct search method is a local method that does not contain derivatives and does not handle explicit constraints. Furthermore, the method is suited for a maximum of ten dimensions and in our case we only need three. The parameters that we use are the standard parameters $\alpha = 1, \gamma = 2, \rho = \frac{1}{2}$ and $\sigma = \frac{1}{2}$ (Nelder & Mead, 1965).

C Appendix C

C.1 R coding

The methods of this research are all implemented in the software from R Core Team (2013). The data that we collect are from Refinitiv (2021) and given to us in an ”Excel” file. The first modifications to the data, converting to USD/Barrel and taking first differences, are done before we load the data into R. To read all the data into R, we make use of the package ”readxl” from Wickham et al. (2019). Then, for all methods, we first fit a GARCH model to the six data series using the package ”rmgarch” from Ghalanos (2019). To fit the standard Gaussian and Student t copulas, we do not download a specific package, but use our own code. However, for the C-vine and D-vine copulas, after obtaining the correct order of the six input variables, we use the package ”CDVineCopulaConditional” from Bevacqua (2017). This package tracks down which bivariate copulas in the vine structure are optimal using the AIC. Then, these bivariate copulas are used in the algorithms from Aas et al. (2009) that we write ourselves to simulate data from the vine structures. For the HAC, we use the ”copula” package to obtain the binary copula parameter, needed to obtain the optimal structure. Then, we use the structure as an input for the ”HAC” package from Okhrin (2020) to simulate observations from the HAC. For the mixed HAC, we write our own code to simulate observations. Finally, all simulated series are used to calculate the risk measures with and without optimal hedge ratios. We find the optimal hedge ratios by using the Nelder-Mead algorithm from ”optimization” package (Husmann & Lange, 2017).

D Appendix D

Table 8: The optimal average hedge ratios for different downside-risk measures

Rolling window size 786	SV	LPM	VaR			ES		
			0.99	**0.95**	**0.90**	**0.99**	**0.95**	**0.90**
Model								
Crude oil hedge ratios								
SGC	0.793	0.787	0.829	0.820	0.815	0.813	0.808	0.804
	(0.104)	(0.116)	(0.134)	(0.110)	(0.110)	(0.154)	(0.117)	(0.110)
STC	0.533	0.503	0.556	0.592	0.599	0.493	0.533	0.545
	(0.132)	(0.151)	(0.156)	(0.117)	(0.111)	(0.184)	(0.141)	(0.132)
C-vine	-0.002	-0.014	0.103	0.071	0.056	0.130	0.070	0.045
	(0.671)	(0.757)	(0.544)	(0.515)	(0.506)	(0.574)	(0.550)	(0.568)
D-vine	0.712	0.685	0.747	0.787	0.804	0.723	0.725	0.729
	(0.209)	(0.228)	(0.218)	(0.208)	(0.209)	(0.224)	(0.210)	(0.196)
HAC								
Clayton	0.686	0.714	0.430	0.482	0.525	0.398	0.459	0.526
	(0.174)	(0.203)	(0.158)	(0.181)	(0.188)	(0.180)	(0.200)	(0.220)
Gumbel	0.661	0.638	0.760	0.739	0.726	0.734	0.699	0.680
	(0.276)	(0.278)	(0.281)	(0.292)	(0.294)	(0.288)	(0.295)	(0.289)
Frank	0.605	0.549	0.606	0.665	0.693	0.546	0.593	0.619
	(0.172)	(0.185)	(0.207)	(0.209)	(0.215)	(0.208)	(0.192)	(0.191)
Joe	0.406	0.350	0.674	0.623	0.599	0.574	0.517	0.473
	(0.426)	(0.443)	(0.432)	(0.416)	(0.422)	(0.396)	(0.406)	(0.427)
AMH	-0.059	-0.064	0.002	-0.022	-0.034	0.010	-0.031	-0.045
	(0.079)	(0.090)	(0.122)	(0.095)	(0.089)	(0.142)	(0.108)	(0.098)
Mixed	1.413	1.597	0.370	0.381	0.340	0.378	0.492	0.621
	(0.124)	(0.307)	(0.200)	(0.354)	(0.317)	(0.153)	(0.111)	(0.129)
Gasoline hedge ratios								
SGC	0.697	0.715	0.632	0.635	0.634	0.642	0.678	0.687
	(0.194)	(0.225)	(0.210)	(0.184)	(0.181)	(0.248)	(0.210)	(0.201)
STC	0.487	0.484	0.400	0.452	0.465	0.364	0.438	0.469
	(0.174)	(0.214)	(0.279)	(0.184)	(0.169)	(0.368)	(0.236)	(0.203)
C-vine	0.162	0.157	0.091	0.135	0.133	0.090	0.155	0.166
	(0.579)	(0.617)	(0.553)	(0.542)	(0.554)	(0.569)	(0.563)	(0.590)
D-vine	0.707	0.638	0.559	0.703	0.732	0.512	0.653	0.686
	(0.204)	(0.211)	(0.197)	(0.212)	(0.225)	(0.233)	(0.217)	(0.218)
HAC								
Clayton	0.721	0.668	0.934	0.909	0.882	0.770	0.757	0.730
	(0.180)	(0.198)	(0.347)	(0.264)	(0.241)	(0.301)	(0.215)	(0.204)
Gumbel	0.975	0.975	0.752	0.840	0.891	0.768	0.875	0.938
	(0.306)	(0.336)	(0.272)	(0.290)	(0.303)	(0.285)	(0.291)	(0.306)
Frank	0.780	0.711	0.631	0.801	0.878	0.557	0.710	0.781
	(0.219)	(0.247)	(0.281)	(0.252)	(0.255)	(0.405)	(0.273)	(0.239)
Joe	0.813	0.815	0.453	0.582	0.682	0.389	0.539	0.654
	(0.383)	(0.446)	(0.241)	(0.277)	(0.314)	(0.407)	(0.323)	(0.331)
AMH	0.107	0.130	0.012	0.033	0.051	0.072	0.080	0.090
	(0.212)	(0.238)	(0.362)	(0.300)	(0.262)	(0.380)	(0.293)	(0.266)
Mixed	0.202	0.213	1.517	1.413	1.454	0.701	0.609	0.595
	(0.150)	(0.242)	(0.433)	(0.540)	(0.595)	(0.164)	(0.064)	(0.072)
Heating oil hedge ratios								
SGC	0.984	1.024	1.125	1.004	0.970	1.187	1.056	1.011
	(0.225)	(0.303)	(0.365)	(0.271)	(0.252)	(0.467)	(0.323)	(0.271)
STC	0.731	0.707	0.732	0.764	0.773	0.674	0.727	0.740
	(0.304)	(0.464)	(0.576)	(0.322)	(0.255)	(0.854)	(0.500)	(0.376)
C-vine	0.364	0.410	0.851	0.601	0.553	0.999	0.603	0.493
	(1.363)	(1.542)	(1.249)	(1.190)	(1.146)	(1.343)	(1.240)	(1.231)
D-vine	1.088	1.261	1.127	0.948	0.957	1.183	1.020	1.056
	(0.531)	(0.641)	(0.571)	(0.494)	(0.439)	(0.594)	(0.562)	(0.515)
HAC								
Clayton	0.538	0.574	0.527	0.509	0.506	0.732	0.627	0.562
	(0.298)	(0.329)	(0.420)	(0.335)	(0.316)	(0.638)	(0.454)	(0.349)
Gumbel	0.793	0.856	0.582	0.576	0.595	0.554	0.594	0.682
	(0.394)	(0.451)	(0.476)	(0.409)	(0.396)	(0.501)	(0.415)	(0.388)
Frank	0.769	0.813	0.717	0.670	0.653	0.651	0.687	0.708
	(0.358)	(0.543)	(0.520)	(0.360)	(0.322)	(0.718)	(0.458)	(0.397)
Joe	0.717	0.755	0.552	0.503	0.499	0.409	0.375	0.434
	(0.456)	(0.547)	(0.569)	(0.447)	(0.415)	(0.689)	(0.552)	(0.420)
AMH	0.211	0.222	0.069	0.099	0.123	0.111	0.156	0.183
	(0.382)	(0.518)	(0.607)	(0.461)	(0.412)	(0.735)	(0.517)	(0.432)
Mixed	0.151	0.215	2.326	2.115	2.219	1.288	0.954	1.087
	(0.637)	(0.673)	(2.997)	(1.800)	(1.129)	(0.327)	(0.113)	(0.214)

SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The values for the hedging effectiveness are presented in percent.

Table 9: The optimal average hedge ratios for different downside-risk measures

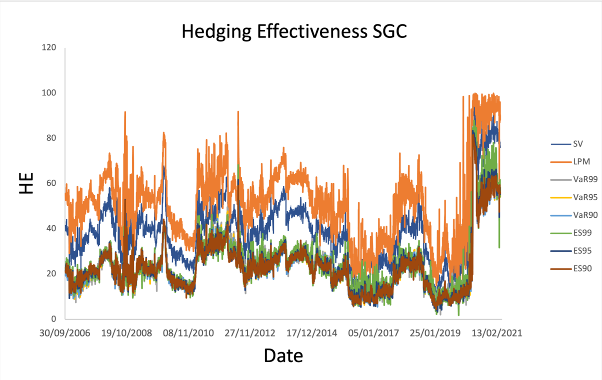
Rolling window size 1310	SV	LPM	VaR			ES		
			0.99	**0.95**	**0.90**	**0.99**	**0.95**	**0.90**
Model								
Crude oil hedge ratios								
SGC	0.791	0.785	0.830	0.825	0.817	0.815	0.811	0.804
	(0.085)	(0.094)	(0.113)	(0.095)	(0.093)	(0.132)	(0.104)	(0.097)
STC	0.537	0.507	0.559	0.598	0.605	0.501	0.542	0.552
	(0.090)	(0.103)	(0.128)	(0.101)	(0.098)	(0.153)	(0.112)	(0.103)
C-vine	-0.133	-0.150	-0.073	-0.096	-0.099	-0.073	-0.098	-0.104
	(0.530)	(0.587)	(0.531)	(0.498)	(0.468)	(0.564)	(0.522)	(0.498)
D-vine	0.739	0.711	0.757	0.785	0.816	0.726	0.759	0.755
	(0.194)	(0.217)	(0.178)	(0.174)	(0.184)	(0.197)	(0.190)	(0.201)
HAC								
Clayton	0.664	0.688	0.412	0.457	0.501	0.373	0.426	0.488
	(0.146)	(0.166)	(0.142)	(0.155)	(0.159)	(0.154)	(0.160)	(0.180)
Gumbel	0.697	0.677	0.805	0.784	0.771	0.778	0.746	0.724
	(0.220)	(0.224)	(0.225)	(0.235)	(0.236)	(0.246)	(0.248)	(0.236)
Frank	0.613	0.560	0.606	0.667	0.696	0.564	0.604	0.626
	(0.187)	(0.187)	(0.220)	(0.221)	(0.224)	(0.216)	(0.211)	(0.209)
Joe	0.593	0.540	0.860	0.810	0.791	0.751	0.700	0.664
	(0.175)	(0.171)	(0.218)	(0.195)	(0.193)	(0.201)	(0.182)	(0.186)
AMH	-0.079	-0.084	-0.016	-0.034	-0.048	-0.008	-0.045	-0.059
	(0.096)	(0.102)	(0.129)	(0.106)	(0.104)	(0.154)	(0.122)	(0.110)
Mixed	1.410	1.575	0.375	0.401	0.351	0.349	0.556	0.622
	(0.092)	(0.174)	(0.164)	(0.129)	(0.147)	(0.111)	(0.063)	(0.115)
Gasoline hedge ratios								
SGC	0.685	0.704	0.637	0.635	0.631	0.649	0.677	0.682
	(0.199)	(0.225)	(0.207)	(0.186)	(0.183)	(0.241)	(0.211)	(0.204)
STC	0.485	0.480	0.406	0.452	0.468	0.369	0.446	0.470
	(0.169)	(0.196)	(0.253)	(0.181)	(0.163)	(0.355)	(0.227)	(0.195)
C-vine	0.044	0.040	0.032	0.013	0.004	0.038	0.026	0.038
	(0.708)	(0.712)	(0.694)	(0.646)	(0.674)	(0.677)	(0.644)	(0.701)
D-vine	0.738	0.684	0.553	0.695	0.753	0.483	0.645	0.710
	(0.213)	(0.237)	(0.229)	(0.210)	(0.242)	(0.287)	(0.226)	(0.222)
HAC								
Clayton	0.709	0.653	0.950	0.899	0.867	0.783	0.747	0.713
	(0.191)	(0.174)	(0.395)	(0.321)	(0.293)	(0.345)	(0.287)	(0.263)
Gumbel	0.974	0.976	0.746	0.837	0.893	0.766	0.872	0.939
	(0.309)	(0.340)	(0.345)	(0.339)	(0.333)	(0.338)	(0.319)	(0.314)
Frank	0.762	0.693	0.617	0.780	0.852	0.570	0.698	0.763
	(0.233)	(0.252)	(0.341)	(0.297)	(0.276)	(0.361)	(0.306)	(0.275)
Joe	0.937	0.954	0.516	0.669	0.790	0.467	0.628	0.752
	(0.361)	(0.408)	(0.337)	(0.338)	(0.351)	(0.348)	(0.327)	(0.329)
AMH	0.103	0.122	0.026	0.045	0.048	0.087	0.085	0.090
	(0.225)	(0.246)	(0.338)	(0.301)	(0.296)	(0.343)	(0.296)	(0.275)
Mixed	0.191	0.188	1.497	1.345	1.359	0.761	0.734	0.685
	(0.072)	(0.119)	(0.415)	(0.335)	(0.357)	(0.115)	(0.138)	(0.120)
Heating oil hedge ratios								
SGC	0.992	1.032	1.121	1.012	0.975	1.189	1.058	1.010
	(0.192)	(0.246)	(0.312)	(0.223)	(0.213)	(0.414)	(0.262)	(0.233)
STC	0.731	0.711	0.753	0.776	0.779	0.735	0.738	0.748
	(0.322)	(0.519)	(0.573)	(0.312)	(0.265)	(0.723)	(0.472)	(0.383)
C-vine	0.270	0.275	0.551	0.456	0.372	0.561	0.458	0.347
	(1.501)	(1.527)	(1.582)	(1.513)	(1.473)	(1.651)	(1.498)	(1.478)
D-vine	0.997	1.110	1.126	1.008	0.950	1.126	1.104	1.001
	(0.537)	(0.752)	(0.598)	(0.358)	(0.326)	(0.805)	(0.517)	(0.457)
HAC								
Clayton	0.487	0.522	0.411	0.419	0.425	0.616	0.513	0.462
	(0.308)	(0.384)	(0.459)	(0.376)	(0.335)	(0.586)	(0.438)	(0.368)
Gumbel	0.792	0.857	0.538	0.553	0.583	0.502	0.580	0.667
	(0.506)	(0.574)	(0.739)	(0.631)	(0.563)	(0.859)	(0.659)	(0.584)
Frank	0.754	0.789	0.649	0.631	0.638	0.632	0.670	0.693
	(0.521)	(0.761)	(0.808)	(0.544)	(0.455)	(0.965)	(0.697)	(0.580)
Joe	0.849	0.898	0.728	0.644	0.620	0.575	0.524	0.543
	(0.505)	(0.574)	(0.693)	(0.626)	(0.541)	(0.913)	(0.633)	(0.521)
AMH	0.205	0.219	0.082	0.104	0.127	0.137	0.167	0.185
	(0.438)	(0.514)	(0.633)	(0.532)	(0.503)	(0.736)	(0.566)	(0.507)
Mixed	0.194	0.223	2.004	1.928	2.079	1.456	1.390	1.171
	(0.130)	(0.209)	(0.506)	(0.382)	(0.427)	(0.250)	(0.268)	(0.167)

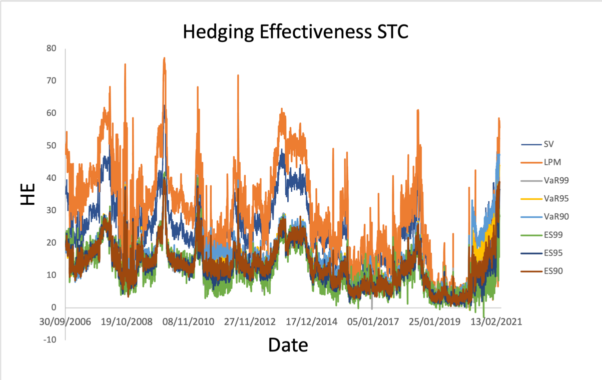
SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The values for the hedging effectiveness are presented in percent.

E Appendix E

This section displays the development of the HE for every rolling window size. Each figure shows the development of the HE for one copula and for all downside-risk measures.

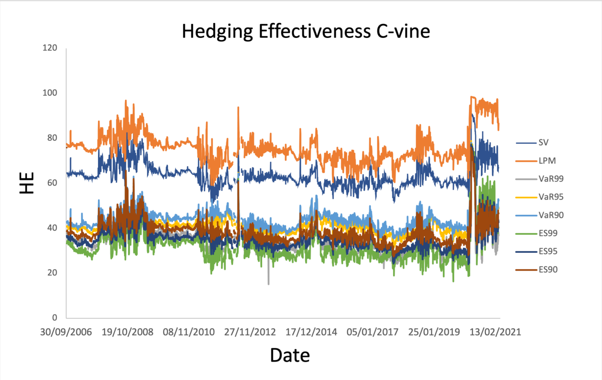
E.1 Rolling window size of 262 observations

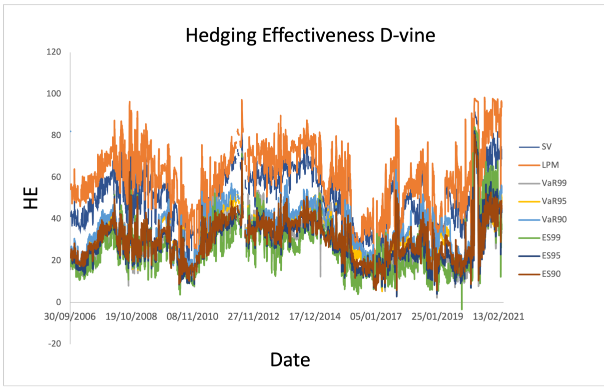




(a) **(b)**

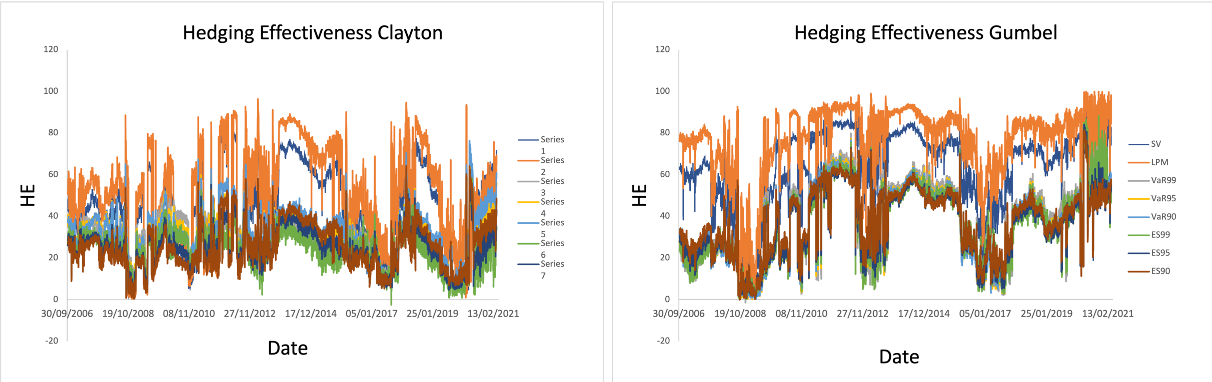
Figure 7: plots of the development of the HE using a rolling window of 262 observations





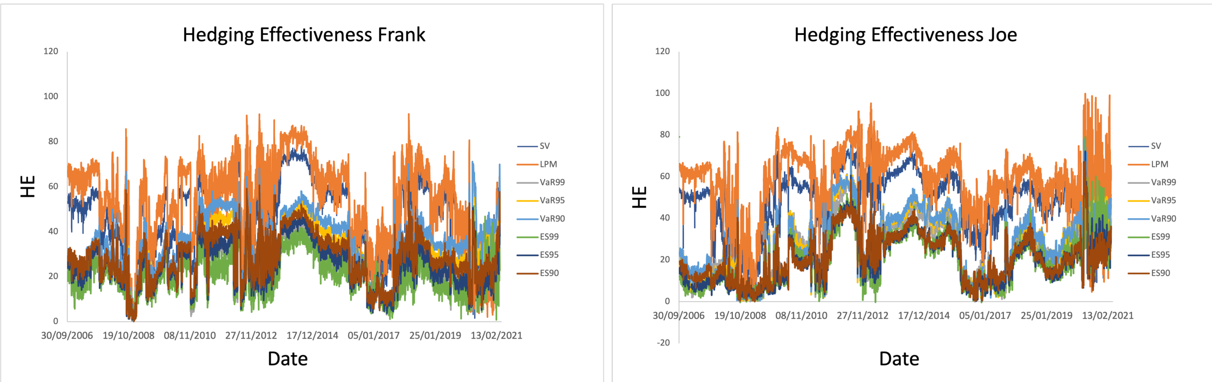
(a) **(b)**

Figure 8: plots of the development of the HE using a rolling window of 262 observations



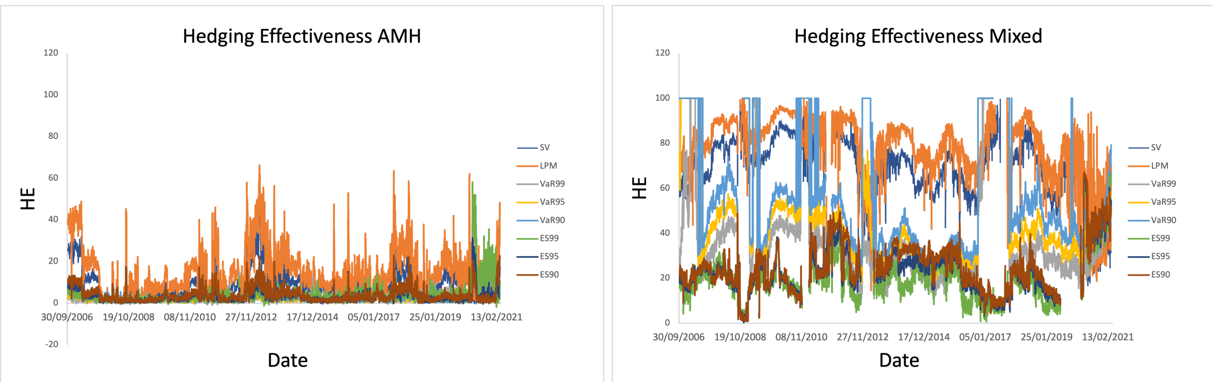
(a) (b)

Figure 9: plots of the development of the HE using a rolling window of 262 observations



(a) (b)

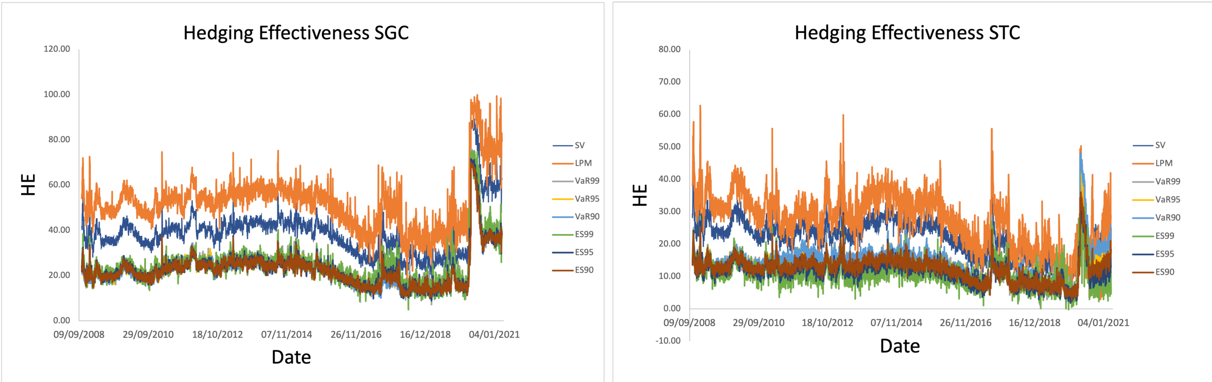
Figure 10: plots of the development of the HE using a rolling window of 262 observations



(a) (b)

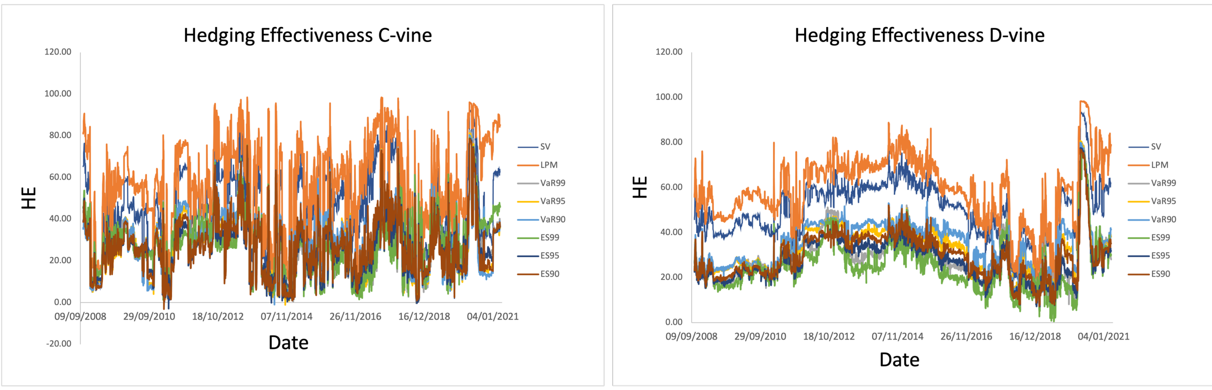
Figure 11: plots of the development of the HE using a rolling window of 262 observations

E.2 Rolling window size of 786



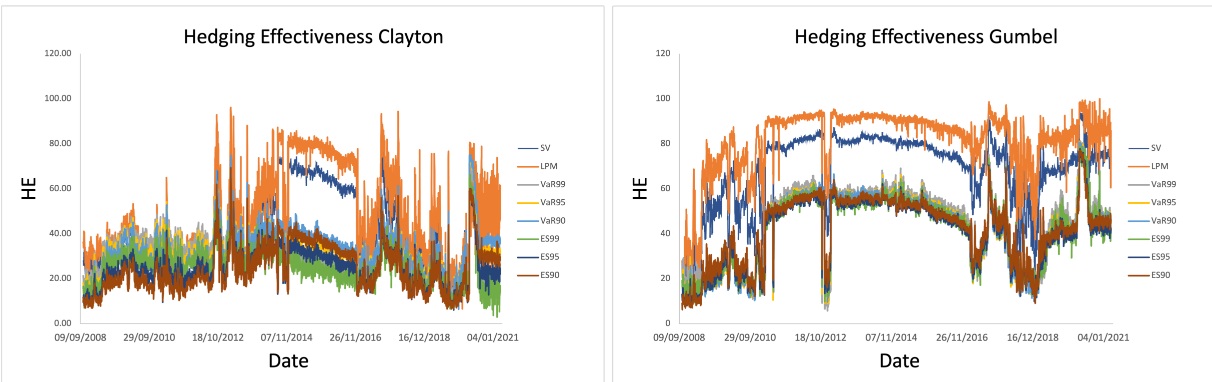
(a) **(b)**

Figure 12: plots of the development of the HE using a rolling window of 786 observations



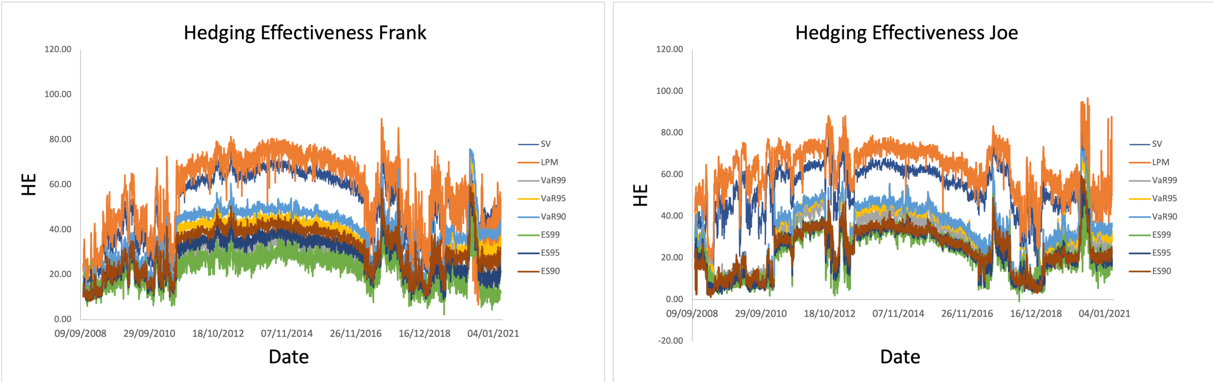
(a) **(b)**

Figure 13: plots of the development of the HE using a rolling window of 786 observations



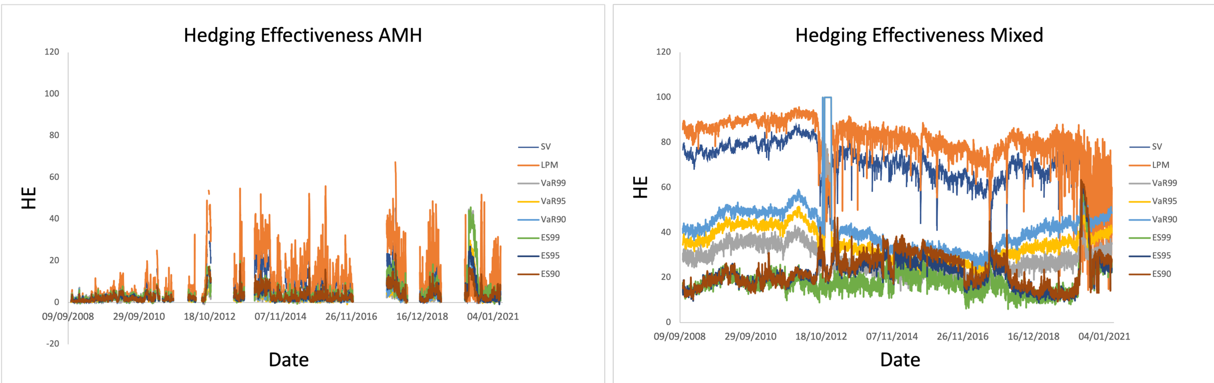
(a) **(b)**

Figure 14: plots of the development of the HE using a rolling window of 786 observations



(a) **(b)**

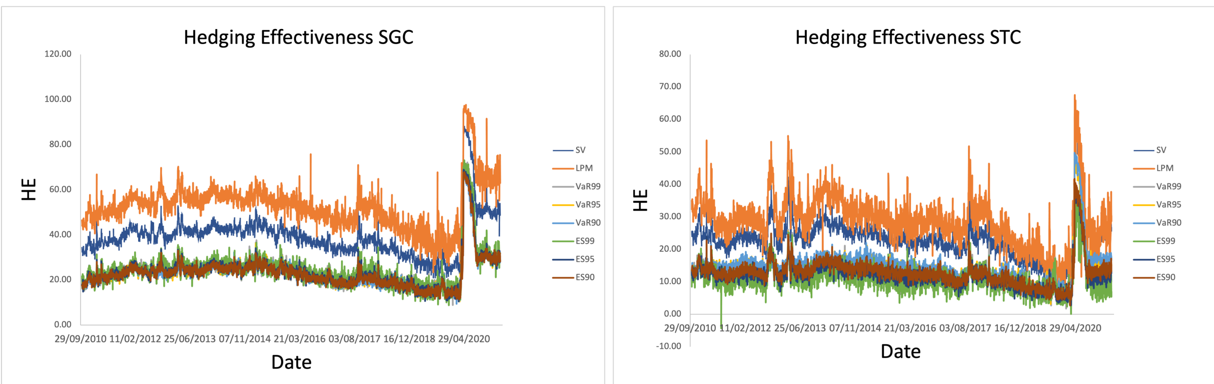
Figure 15: plots of the development of the HE using a rolling window of 786 observations



(a) **(b)**

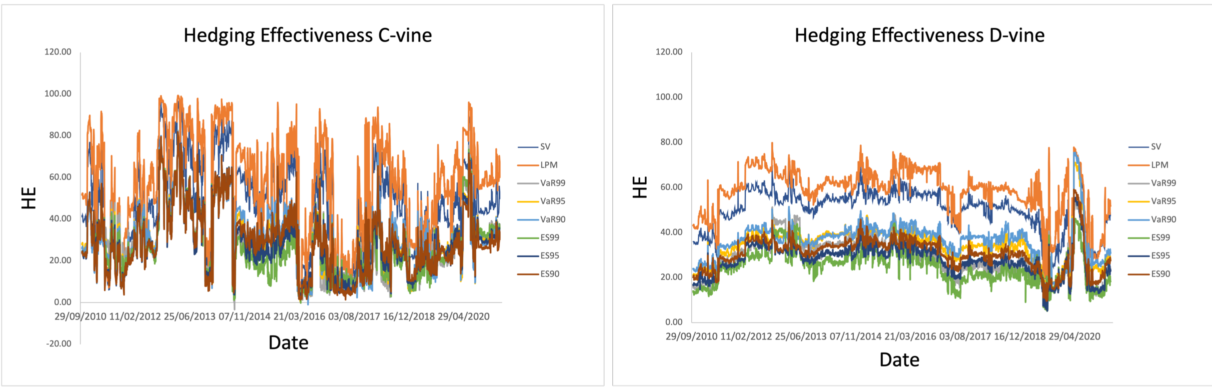
Figure 16: plots of the development of the HE using a rolling window of 786 observations

E.3 Rolling window size of 1310 observations



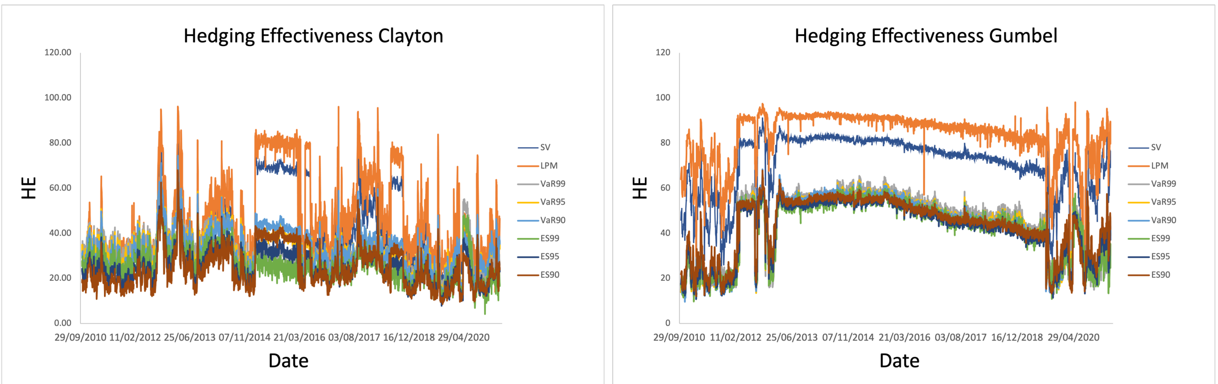
(a) **(b)**

Figure 17: plots of the development of the HE using a rolling window of 1310 observations



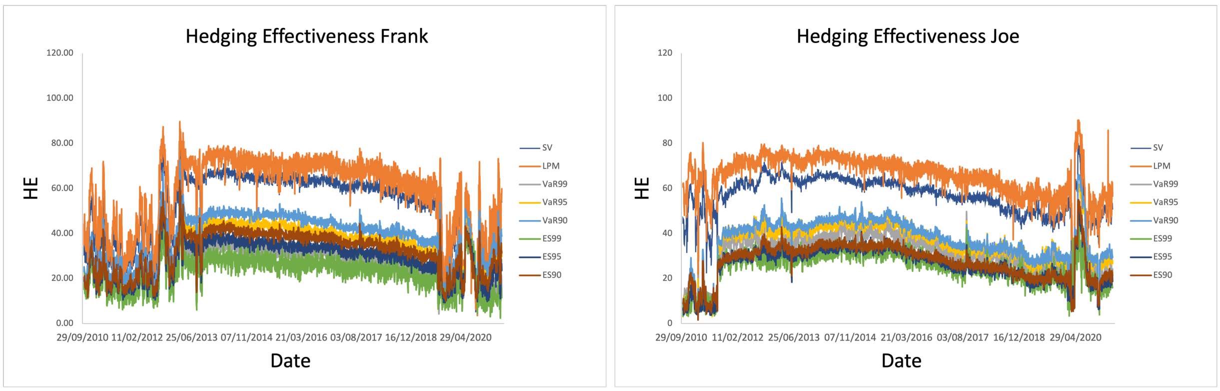
(a) **(b)**

Figure 18: plots of the development of the HE using a rolling window of 1310 observations



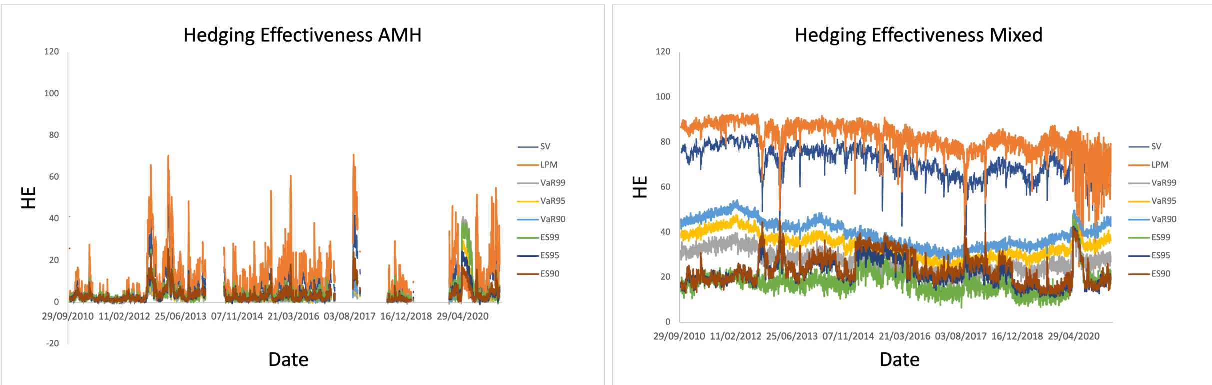
(a) **(b)**

Figure 19: plots of the development of the HE using a rolling window of 1310 observations



(a) **(b)**

Figure 20: plots of the development of the HE using a rolling window of 1310 observations



(a) **(b)**

Figure 21: plots of the development of the HE using a rolling window of 1310 observations

F Appendix F

In this section, we present the paired t-test statistics for tables Table 6. The paired t-test compares the means of the HE between two different copula models. For each rolling-window period and for each downside-risk measure we present the test statistics in the tables below.

F.1 Rolling window size of 262 observations

Table 10: Paired t-test statistics of equal HE of two copula models

Rolling window size 262
Downside-risk measure: **SV**

	SST	C-vine	D-vine	HAC					
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	70.44	-110.94	-56.69	-11.83	-70.47	-23.02	-25.02	102.93	-80.40
STC		-218.61	-123.22	-66.22	-119.74	-91.20	-92.32	66.11	-183.29
C-vine			62.88	63.18	2.55*	50.94	54.41	308.23	-19.25
D-vine				25.07	-38.49	9.03	9.21	163.04	-53.10
Clayton					-64.33	-16.21	-15.98	107.13	-58.96
Gumbel						62.46	80.85	182.30	-12.02
Frank							-2.15*	151.43	-59.46
Joe								170.23	-62.35
AMH									-199.63

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

Table 11: Paired t-test statistics of equal HE of two copula models

Rolling window size 262									
Downside-risk measure: **LPM**									
	SST	C-vine	D-vine	HAC					
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	77.85	-105.00	-36.73	3.15	-67.88	-4.47	-17.98	105.24	-62.50
STC		-177.07	-102.75	-61.56	-126.91	-85.46	-98.00	46.25	-180.37
C-vine			69.97	63.87	-0.47**	58.05	57.39	238.42	-5.09
D-vine				22.98	-43.82	14.40	4.32	134.84	-44.62
Clayton					-69.69	-10.14	-21.93	94.89	-59.70
Gumbel						74.02	88.16	206.03	-1.71**
Frank							-17.23	131.11	-67.25
Joe								169.97	-55.59
AMH									-179.97

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

Table 12: Paired t-test statistics of equal HE of two copula models

Rolling window size 262									
Downside-risk measure: **VaR 0.99**									
	SST	C-vine	D-vine	HAC					
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	56.08	-57.06	-28.46	-22.23	-49.28	1.40**	-1.49**	119.97	-27.53
STC		-155.35	-80.22	-115.75	-74.69	-73.46	-47.70	76.29	-61.98
C-vine			34.04	31.04	-10.09	55.87	39.90	386.09	-8.80
D-vine				-3.19	-32.28	22.46	18.11	145.59	-19.23
Clayton					-27.46	32.87	17.68	161.44	-18.08
Gumbel						53.51	73.74	114.88	2.23*
Frank							-3.25	133.66	-28.16
Joe								94.07	-23.07
AMH									-87.07

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

Table 13: Paired t-test statistics of equal HE of two copula models

Rolling window size 262									
Downside-risk measure: **VaR 0.95**									
	SST	C-vine	D-vine	HAC					
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	58.32	-106.00	-64.91	-43.16	-50.03	-35.94	-12.95	113.05	-48.58
STC		-220.64	-117.23	-117.06	-73.79	-89.40	-46.24	114.56	-76.81
C-vine			53.49	57.11	17.01	52.84	59.94	534.60	-13.89
D-vine				7.52	-14.24	10.20	30.97	179.83	-28.39
Clayton					-19.71	3.83	20.68	179.67	-31.58
Gumbel						27.54	66.28	114.62	-15.04
Frank							21.51	153.77	-30.28
Joe								94.85	-34.38
AMH									-110.77

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

Table 14: Paired t-test statistics of equal HE of two copula models

Rolling window size 262										
Downside-risk measure: **VaR 0.90**										
	SST	C-vine	D-vine	HAC						
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	61.19	-121.30	-75.06	-49.27	-52.88	-52.61	-17.69	109.54	-65.54
STC		-237.80	-126.92	-108.10	-76.15	-95.02	-47.65	116.89	-92.28
C-vine			56.52	61.73	24.15	43.75	64.40	518.89	-25.47
D-vine				13.07	-9.41	-0.67**	34.40	183.29	-40.01
Clayton					-20.36	-12.52	20.74	166.01	-44.02
Gumbel						14.66	65.90	117.48	-26.60
Frank							34.88	153.83	-36.75
Joe								97.13	-45.61
AMH									-132.32

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

Table 15: Paired t-test statistics of equal HE of two copula models

Rolling window size 262										
Downside-risk measure: **ES 0.99**										
	SST	C-vine	D-vine	HAC						
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	57.86	-44.99	-9.59	3.21	-45.61	19.32	21.18	119.34	45.58
STC		-130.02	-66.34	-92.63	-77.23	-63.77	-37.31	56.07	-38.74
C-vine			38.72	44.77	-13.00	63.74	55.41	286.66	83.73
D-vine				9.38	-37.29	26.83	29.42	127.61	40.73
Clayton					-42.87	26.89	16.85	135.80	31.95
Gumbel						64.07	95.11	118.63	65.66
Frank							2.03**	114.69	10.54
Joe								84.19	4.65
AMH									-103.91

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

Table 16: Paired t-test statistics of equal HE of two copula models

Rolling window size 262										
Downside-risk measure: **ES 0.95**										
	SST	C-vine	D-vine	HAC						
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	59.34	-73.58	-41.02	-7.01	-50.88	-9.29	14.71	104.57	9.68
STC		-177.63	-101.01	-87.42	-84.49	-85.93	-37.42	83.86	-57.82
C-vine			45.95	61.61	-0.86	50.97	72.80	309.69	77.18
D-vine				23.44	-28.98	17.48	48.89	152.04	42.76
Clayton					-50.82	-5.64	25.57	153.59	19.10
Gumbel						50.67	107.17	124.02	61.09
Frank							30.48	140.56	17.38
Joe								88.74	-14.46
AMH									-120.39

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

Table 17: Paired t-test statistics of equal HE of two copula models

Rolling window size 262										
Downside-risk measure: **ES 0.90**										
	SST	C-vine	D-vine			HAC				
				Clayton	Gumbel	Frank	Joe	AMH	Mixed	
SGC	61.46	-92.08	-52.21	-12.98	-59.14	-24.93	7.76	98.47	-7.27	
STC		-216.40	-117.85	-73.27	-94.12	-93.28	-44.03	84.38	-63.41	
C-vine			54.65	64.98	3.59	46.22	82.74	308.25	73.21	
D-vine				27.41	-31.06	7.37	54.43	158.67	38.53	
Clayton					-60.57	-21.18	25.88	125.08	11.86	
Gumbel						48.27	112.11	132.84	63.25	
Frank							46.87	147.39	23.58	
Joe								98.69	-19.72	
AMH									-112.78	

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

F.2 Rolling window size of 786 observations

Table 18: Paired t-test statistics of equal HE of two copula models

Rolling window size 786									
Downside-risk measure: **SV**									
	SST	C-vine	D-vine			HAC			
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	116.24	-12.33	-71.71	-6.65	-101.80	-41.36	-56.42	124.10	-125.04
STC		-59.49	-145.17	-67.07	-154.11	-110.24	-145.04	98.89	-270.44
C-vine			-17.38	6.53	-65.01	-20.40	-26.68	88.38	-63.00
D-vine				33.87	-70.30	-3.85	-12.06	148.31	-71.00
Clayton					-94.20	-37.72	-44.25	113.05	-72.60
Gumbel						105.33	100.16	175.97	-5.15
Frank							-10.68	149.11	-57.69
Joe								183.78	-63.15
AMH									-233.92

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

Table 19: Paired t-test statistics of equal HE of two copula models

Rolling window size 786									
Downside-risk measure: **LPM**									
	SST	C-vine	D-vine			HAC			
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	117.13	-12.71	-38.36	13.46	-101.88	-11.83	-38.82	117.81	-86.04
STC		-71.74	-122.06	-61.47	-177.68	-100.67	-161.15	64.22	-249.89
C-vine			-5.14	20.06	-61.00	1.13**	-14.07	93.04	-48.76
D-vine				33.16	-75.71	8.24	-12.93	122.94	-58.82
Clayton					-101.84	-28.24	-47.34	105.86	-74.76
Gumbel						114.01	104.13	212.21	3.48
Frank							-28.97	133.84	-69.86
Joe								170.77	-63.48
AMH									-185.81

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

Table 20: Paired t-test statistics of equal HE of two copula models

Rolling window size 786									
Downside-risk measure: **VaR 0.99**									
	SST	C-vine	D-vine	HAC					
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	90.80	-9.10	-35.09	-40.88	-80.71	-15.05	-16.23	148.88	-21.73
STC		-62.52	-101.95	-139.15	-114.11	-110.47	-75.69	91.28	-109.92
C-vine			-9.47	-24.52	-53.20	-1.54**	-3.97	90.36	-8.39
D-vine				-17.25	-64.29	13.56	5.65	145.93	1.28**
Clayton					-41.07	31.81	16.84	173.28	18.03
Gumbel						84.55	106.95	116.75	43.68
Frank							-5.01	131.12	-7.84
Joe								91.02	-3.07
AMH									-158.57

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

Table 21: Paired t-test statistics of equal HE of two copula models

Rolling window size 786									
Downside-risk measure: **VaR 0.95**									
	SST	C-vine	D-vine	HAC					
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	99.82	-16.26	-81.08	-61.51	-77.13	-67.14	-34.80	134.41	-61.76
STC		-56.74	-136.34	-139.89	-105.09	-123.97	-74.24	174.09	-118.97
C-vine			-22.21	-27.40	-46.79	-31.41	-9.94	89.46	-31.19
D-vine				-1.98**	-46.13	-13.98	15.04	149.07	-13.11
Clayton					-35.09	-9.17	11.59	203.45	-11.63
Gumbel						46.37	88.24	109.26	16.36
Frank							25.37	143.34	-5.25
Joe								90.41	-16.84
AMH									-234.96

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

Table 22: Paired t-test statistics of equal HE of two copula models

Rolling window size 786									
Downside-risk measure: **VaR 0.90**									
	SST	C-vine	D-vine	HAC					
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	107.88	-20.46	-87.27	-61.61	-77.98	-83.21	-42.81	124.71	-94.92
STC		-55.07	-135.59	-123.10	-103.77	-125.48	-76.37	148.37	-147.50
C-vine			-23.63	-20.19	-43.64	-39.81	-11.56	87.62	-46.69
D-vine				9.18	-38.81	-26.65	17.37	137.89	-28.55
Clayton					-38.85	-32.05	4.95	174.50	-33.49
Gumbel						28.20	82.72	107.83	0.68**
Frank							40.51	136.77	-13.15
Joe								90.05	-29.56
AMH									-259.26

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

Table 23: Paired t-test statistics of equal HE of two copula models

Rolling window size 786									
Downside-risk measure: **ES 0.99**									
	SST	C-vine	D-vine	HAC					
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	96.46	-13.84	-4.07	-5.44	-73.80	15.60	15.29	132.95	67.13
STC		-69.66	-81.77	-112.33	-116.91	-96.15	-62.37	58.90	-71.10
C-vine			9.82	8.79	-43.25	21.52	20.17	88.11	42.33
D-vine				-2.38*	-70.45	18.08	17.21	116.15	47.87
Clayton					-62.94	27.09	18.72	146.82	54.24
Gumbel						98.40	122.83	116.64	95.99
Frank							2.38*	112.26	30.23
Joe								75.24	21.01
AMH									-123.05

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

Table 24: Paired t-test statistics of equal HE of two copula models

Rolling window size 786									
Downside-risk measure: **ES 0.95**									
	SST	C-vine	D-vine	HAC					
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	99.77	-13.88	-44.41	-12.52	-78.08	-28.62	3.05	119.67	13.93
STC		-66.98	-113.21	-105.40	-117.36	-114.83	-63.76	110.56	-107.61
C-vine			-7.63	5.18	-50.04	-8.21	13.20	92.50	21.02
D-vine				19.71	-67.26	-0.31	39.88	126.32	55.86
Clayton					-77.91	-26.44	14.59	166.58	39.65
Gumbel						81.62	138.81	117.80	93.73
Frank							44.08	138.77	53.73
Joe								82.57	5.92
AMH									-149.99

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

Table 25: Paired t-test statistics of equal HE of two copula models

Rolling window size 786									
Downside-risk measure: **ES 0.90**									
	SST	C-vine	D-vine	HAC					
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	101.55	-16.33	-61.69	-12.50	-86.71	-47.87	-7.87	112.12	-6.40
STC		-62.44	-122.62	-78.21	-123.78	-116.83	-70.79	115.09	-98.71
C-vine			-12.90	7.33	-53.93	-19.93	9.99	86.32	13.79
D-vine				31.35	-67.31	-10.71	49.74	123.02	51.73
Clayton					-90.95	-47.80	5.22	118.80	16.87
Gumbel						81.72	150.96	124.18	97.70
Frank							71.99	137.10	61.53
Joe								90.36	4.03
AMH									-131.96

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

F.3 Rolling window size of 1310 observations

Table 26: Paired t-test statistics of equal HE of two copula models

Rolling window size 1310									
Downside-risk measure: **SV**									
	SST	C-vine	D-vine	HAC					
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	158.87	-18.84	-59.13	-2.08*	-97.16	-39.33	-83.66	135.09	-152.17
STC		-62.83	-184.72	-57.97	-175.72	-105.80	-217.32	97.74	-284.34
C-vine			-9.07	15.78	-57.62	-13.57	-26.58	101.97	-55.90
D-vine				37.19	-87.64	-8.85	-44.74	191.64	-101.53
Clayton					-108.93	-48.32	-58.54	101.95	-83.48
Gumbel						123.60	70.37	210.72	-2.18*
Frank							-17.65	148.41	-53.79
Joe								229.40	-73.59
AMH									-223.45

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

Table 27: Paired t-test statistics of equal HE of two copula models

Rolling window size 1310									
Downside-risk measure: **LPM**									
	SST	C-vine	D-vine	HAC					
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	162.64	-10.47	-22.12	14.57	-104.26	-15.68	-59.18	134.34	-115.46
STC		-69.19	-151.09	-54.49	-234.15	-98.77	-235.53	70.84	-267.79
C-vine			-2.15*	19.39	-58.54	-3.08	-20.42	103.13	-52.62
D-vine				33.14	-114.30	-1.87**	-46.19	144.51	-108.29
Clayton					-112.77	-37.40	-53.79	91.62	-86.74
Gumbel						129.82	96.73	244.55	4.79
Frank							-26.25	130.07	-67.46
Joe								177.29	-86.65
AMH									-188.80

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

Table 28: Paired t-test statistics of equal HE of two copula models

Rolling window size 1310									
Downside-risk measure: **VaR 0.99**									
	SST	C-vine	D-vine	HAC					
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	116.64	-18.12	-30.19	-44.85	-65.86	-10.96	-36.55	207.85	-15.22
STC		-61.93	-115.61	-153.56	-116.19	-96.05	-111.75	130.38	-140.08
C-vine			0.67**	-13.96	-37.13	9.28	-6.32	90.22	9.95
D-vine				-20.61	-63.20	16.41	-17.12	147.19	14.92
Clayton					-41.15	37.43	5.64	165.29	40.50
Gumbel						96.96	63.94	110.89	54.75
Frank							-30.29	113.30	-0.70**
Joe								119.09	23.80
AMH									-162.52

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

Table 29: Paired t-test statistics of equal HE of two copula models

Rolling window size 1310
Downside-risk measure: **VaR 0.95**

	SST	C-vine	D-vine			HAC			
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	131.42	-24.30	-94.70	-52.29	-68.08	-52.80	-61.63	180.44	-73.24
STC		-58.39	-187.04	-124.93	-106.49	-112.41	-113.88	206.70	-180.01
C-vine			-14.93	-14.34	-35.47	-14.49	-15.23	96.02	-12.38
D-vine				5.38	-39.16	-1.01**	-3.73	234.28	5.17
Clayton					-38.14	-6.72	-6.73	182.35	-0.31**
Gumbel						56.05	40.69	116.92	29.77
Frank							-1.91**	144.39	4.32
Joe								133.62	6.20
AMH									-240.32

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

Table 30: Paired t-test statistics of equal HE of two copula models

Rolling window size 1310
Downside-risk measure: **VaR 0.90**

	SST	C-vine	D-vine			HAC			
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	137.24	-26.24	-107.15	-47.92	-71.09	-67.04	-73.30	161.21	-113.66
STC		-55.68	-185.85	-102.53	-105.41	-115.27	-118.91	165.80	-203.61
C-vine			-17.98	-9.32	-36.79	-24.45	-20.51	97.02	-31.16
D-vine				16.48	-31.96	-11.88	-4.76	226.17	-17.05
Clayton					-44.95	-31.53	-18.61	168.09	-32.11
Gumbel						37.19	33.46	122.52	12.76
Frank							8.05	149.51	-3.87
Joe								141.97	-10.83
AMH									-274.41

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

Table 31: Paired t-test statistics of equal HE of two copula models

Rolling window size 1310
Downside-risk measure: **ES 0.99**

	SST	C-vine	D-vine			HAC			
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	116.34	-12.74	-1.52**	-9.20	-60.69	13.54	-1.27**	170.10	64.83
STC		-62.52	-102.79	-127.28	-121.87	-89.60	-94.57	78.95	-76.12
C-vine			10.99	7.86	-36.08	19.40	9.99	84.33	37.28
D-vine				-8.56	-74.09	17.50	0.22**	125.85	59.12
Clayton					-63.09	28.60	7.84	145.96	65.08
Gumbel						108.96	91.42	113.67	94.89
Frank							-18.73	108.06	33.31
Joe								103.46	50.80
AMH									-126.50

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

Table 32: Paired t-test statistics of equal HE of two copula models

Rolling window size 1310 Downside-risk measure: **ES 0.95**									
	SST	C-vine	D-vine	HAC					
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	128.04	-19.23	-35.00	-9.84	-68.92	-27.34	-16.47	147.91	11.13
STC		-62.27	-148.22	-97.34	-123.61	-110.58	-97.78	129.84	-120.69
C-vine			2.67*	14.24	-39.83	-0.30	8.82	92.31	25.24
D-vine			18.30	-69.79	-5.80	18.09	176.24	61.13	
Clayton					-81.60	-28.35	-5.19	156.93	34.90
Gumbel						93.55	89.81	130.09	98.25
Frank							18.90	143.26	56.48
Joe								121.23	33.16
AMH									-184.06

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*

Table 33: Paired t-test statistics of equal HE of two copula models

Rolling window size 1310 Downside-risk measure: **ES 0.90**									
	SST	C-vine	D-vine	HAC					
				Clayton	Gumbel	Frank	Joe	AMH	Mixed
SGC	130.41	-21.80	-61.76	-6.95	-78.24	-42.92	-28.34	132.35	-5.29
STC		-60.58	-172.22	-69.13	-131.66	-112.72	-108.38	116.49	-102.51
C-vine			-5.60	17.60	-46.32	-10.56	6.39	93.39	20.03
D-vine				36.87	-66.51	-9.28	34.61	188.57	65.37
Clayton					-99.46	-54.17	-16.86	117.86	7.41
Gumbel						92.90	94.82	144.48	106.32
Frank							32.78	146.96	65.15
Joe								134.35	29.85
AMH									-160.09

Note:
*SGC denotes the standard Gaussian copula and STC denotes the standard student t copula. AMH denotes the Ali-Michail-Haq HAC. The numbers in the graph denote the statistic of the paired t-test. If the number is positive, the model on the left obtained a higher HE. If the number is negative, the model above obtained a higher HE. * and ** denote that the hypothesis of equal HE is not rejected at 1% and 5% significance level, respectively.*