An analysis on the dynamic relationship between Bitcoin and European financial assets

MSc Thesis Quantitative Finance

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Abstract

This paper uses the VAR-GARCH-BEKK model to investigate the return and volatility spillover effects between Bitcoin, AEX, DAX, FTSE100, EUR/USD and GBP/USD. Furthermore, the copula-GARCH model is used to analyze safe haven and hedging properties of Bitcoin during the COVID-19 pandemic. It is found that EUR/USD only shows one-directional volatility spillover effects. All other assets show one-directional return spillover effects and bidirectional volatility spillover effects. The return spillover effects are all positive and relatively small, while the volatility spillover effects differ. Furthermore, it appears that the spillover effects of Bitcoin on the assets is significantly weaker than the spillover effects of the assets on Bitcoin, indicating that Bitcoin is still a small asset. In case of the safe haven and hedging properties, it is found that Bitcoin does not act as a hedge against the assets during the COVID-19 pandemic. Furthermore, it is found that Bitcoin acts as a safe haven against AEX and EUR/USD, while it does not act as a safe haven against DAX, FTSE100 and GBP/USD.

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1 Introduction

Bitcoin is a decentralised digital currency that was outlined in a paper by Nakamoto, Bitcoin (2008) and went online in 2009. In the years after its establishment, it has become increasingly popular and its price has been growing rapidly. Fidelity (2020) conducted a survey of many investors, advisors and institutions to investigate how they think of digital assets in general and as a apart of an investment portfolio. This survey finds that 36% of the respondents invests in digital assets and 26% has exposure to Bitcoin. Almost 60% of the respondents has a positive or neutral perception towards digital assets and another 60% feel digital assets have a place in portfolios. Furthermore, almost all respondents that are interested in investing in digital assets expect that digital assets will be 0.5% of their investment portfolio in the next five years. These results show that bitcoin has become increasingly popular among institutional investors. A reason for its popularity can be its extremely high level of return and volatility. Since there is an increasing interest in Bitcoin as an investment and more institutional investors are accepting it as a legitimate investment asset, there is a need to examine the relation between Bitcoin and other asset classes. This research investigates the relation between Bitcoin and European financial assets in two different ways. First, the return and volatility spillover effects in the period between june 2013 and june 2021 are analyzed. Then, the safe haven and hedging capabilities of Bitcoin against European financial assets during the COVID-19 pandemic are explored.

Thus far, many existing studies investigate Bitcoin its safe haven and hedging capabilities. Most studies, such as (Kliber et al., 2019), (Stenså s et al., 2019), (Bouri et al., 2017), (Dyhrberg, 2016), use a Dynamic Conditional Correlation (DCC) based method to analyze the dependency between Bitcoin and other stock market prices and find that it acts as a hedge against other stock prices. As the DCC model assumes bivariate normality on the join distribtuion and it only captures linear relationship between marginals of different time series, it might not be the most appropriate model to model dependencies between Bitcoin and European financial assets. Taking this into account, this paper adds to existing literature by investigating the safe haven and hedging properties of Bitcoin through a copula analysis that appropriately describes the average and tail dependence structure between financial assets (Reboredo, 2013), (Junker et al., 2006) and (Liu et al., 2016). Main advantages of copula models are that they are able to capture complex and non linear dependency structure between different assets, that the marginal behaviour and the dependence structure are separated by the framework of copulas and the fact that copulas are invariant to increasing and continuous transformations, such as taking logarithm returns, which is commonly used in the field of finance. Only a few studies investigate volatility spillover effects between bitcoin and other assets by using Multivariate Generalized Autoregressive Conditional Heteroscedasticity (MGARCH) models. Bouri, Das, Gupta, and Roubaud (2018) study the spillover effects between Bitcoin and assets in different market conditions. However, the study only considers the MSCI World, MSCI Emerging market and MSCI China equity index when examining the spillover effects between Bitcoin and stocks indices. Wang et al. (2019) use the multivariate VAR-GARCH-BEKK model to investigate return and volatility spillover effects. However, this study focuses only on the Chinese market.

Existing studies mainly focus on the Chinese market and major world stock indices. As results may vary between different regions, this study extends existing literature by focusing on European financial assets. To the best of the author's knowledge, this is the first paper to focus on this region. The dynamic relation between Bitcoin and European financial assets may be of interest for investors who invest in European assets. Therefore, the aim of this research is to investigate mean and volatility spillover effects between bitcoin and European assets and explore its hedge and safe haven capabilities during the COVID-19 pandemic. Hence, some research questions are: Are there any significant return and volatility spillover effects between Bitcoin and the European financial assets? What is the direction and magnitude of these significant volatility spillover effects? Are there tail dependencies? If the tail dependence exist, are they symmetric or asymmetric? How is the dependence structure between Bitcoin and the assets on average? Which copula model captures the dependency with the best fit? Does Bitcoin act as a hedge or a safe haven during the COVID-19 pandemic?

This paper first measures return and volatility spillover effects by using the VAR-GARCH-BEKK model. We use AIC values to determine the parameters of the VAR and GARCH-BEKK model. Additionally, to check for adequate model specification, we perform a multivariate Ljung-Box test. This test checks whether serial correlation exists in the standardized residuals. We use the Cholesky decomposition method to standardize the residuals of the GARCH-BEKK model. By recursively fitting the VAR and the GARCH-BEKK models with different combination of parameters, the optimal model can be chosen. We determine the direction and magnitude of the spillover effects based on statistical significance. The safe haven and the hedging properties during the COVID-19 pandemic are examined by using the five most frequently used copulas in this kind of study: Gaussian, Student-t, Clayton, Gumbel and Frank. For the marginals, we follow Garcia-Jorcano, Muela (2020) and use the univariate APARCH model to fit the returns. The APARCH model not only captures all relevant stylized facts of financial returns such as volatility clustering, leptokurtotic and the leverage effect, but has also great flexibility. The distribution of the innovations, student-t or normal, that is used to estimate the APARCH model is selected based on AIC, log-likelihood and a visual inspection of the QQ-plot. Since the copulas are different, because each of them capture a different pattern in dependence, and consequently have non comparable dependence parameter, the Kendals τ coefficient, which is a rank correlation measure, is used to measure the dependence structure on average. Unlike traditional methods that are used to measure dependency between asset returns, a copula function also provides information about dependencies in the tails of the distribution. The dependence in the tail of the distribution provides information during times of extreme market movements and is therefore useful to assess the safe haven property of Bitcoin against European assets. Also here, each copula captures a different pattern in the tail dependence: tail independence, symmetric tail dependence and asymmetric tail dependence. To find out which of the five different copulas suit the data best, we perform a model selection process based on AIC, mean absolute error (MAE) and the Cramér-von Mises test with parametric bootstrapping. This research uses daily data of Bitcoin, the Amsterdam Stock Exchange Index (AEX), the Deutscher Aktien index (DAX), the Financial Times Stock Exchange index (FTSE100), the Euro (EUR/USD) and the Pond (GBP/USD) between June 2013 and june 2021. These equity indices and exchange rates are among the most important prosperity indicators of Europe.

This study provides a detailed literature review in section 2 and will further deepen the analysis in our data in section 3. This is followed by the methodology in section 4, where the models are described. The results are shown in section 5 and the paper is concluded with a brief summary of our findings in section 6.

2 Literature review

Wang et al. (2019) use the the multivariate VAR-GARCH-BEKK model to investigate return and volatility spillover effects between Bitcoin and six major Chinese financial assets from 2013 to 2017. This study uses the VAR model to assess the mean spillover effects and the GARCH-BEKK model to measure the volatility spillover effects. Empirical results show that only the monetary market has a mean spillover effect on Bitcoin. Gold, monetary and bond markets affect the volatility of Bitcoin. Bitcoin itself has spillover effects on the volatility of gold. Furthermore, Bitcoin is a hedge against stocks and bonds and a safe haven against monetary market during times of stress.

Vardar, Aydogan (2019) examined the volatility and return spillover effects between Bitcoin and other asset classes from turkey by also using the multivariate VAR-GARCH-BEKK model, where the VAR model is used to estimate the mean equation and the multivariate GARCH-BEKK model is used to model the conditional variance. Firstly, the study finds return spillover effects in only one direction. The return of the bond market affects the return of Bitcoin, while the return of Bitcoin does not affect the return of the bond market. Secondly, the study finds evidence for the existence of bidirectional cross-market volatility spillover effects. The volatility of Bitcoin and all other financial assets except the U.S. dollar affect each other. Their findings support the position of Bitcoin as a new investment asset class, since it is connected with other financial assets. According to Vardar, Aydogan (2019), Bitcoin offers opportunities for diversification swings. This is because of its relatively high return and low correlation with other financial assets. They also concluded that the continued rapid growth of Bitcoin and its unregulated nature could create new vulnerabilities in the international finance system. Therefore, the study recommends regulators and policy makers to closely monitor the Bitcoin market and be aware of the return and volatility spillover effects among Bitcoin and other asset classes for selected and specific countries.

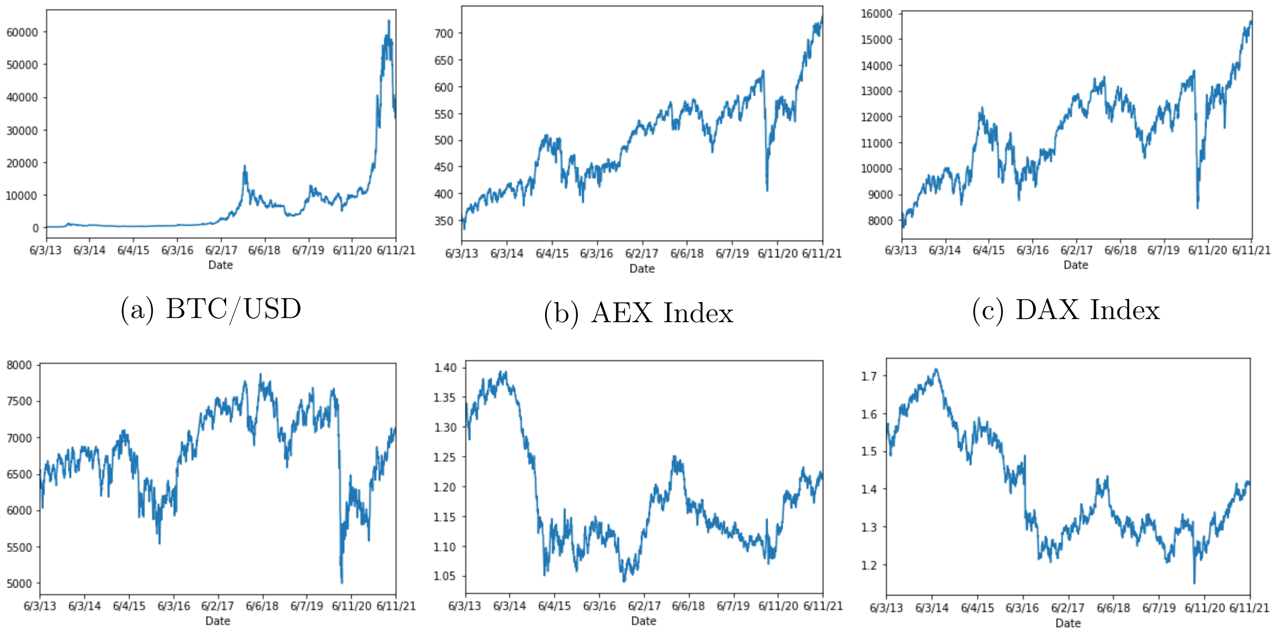
Many studies investigate the safe haven and hedging properties of Bitcoin. However, most of them, such as (Stensås et al., 2019), (Bouri et al., 2017), (Kliber et al., 2019) and (Dyhrberg, 2016) used a Dynamic Conditional Correlation (DCC) method to analyze these properties. Existing literature based on a copula approach to assess the safe haven and hedging properties of Bitcoin is limited. However, more research is available related to other assets such as gold. Reboredo (2013) uses a copula approach to investigate average movements across marginals as well as upper and lower tail dependence (joint extreme movements) in order to determine the hedging and safe haven capabilities of gold against oil price movements. In order to capture different patterns of dependence and tail dependence different copula functions are used. More specifically, the Gaussian, Student-t, Clayton, Gumbel and Clayton-Gumbel copula functions are used. Additionally, as in Patton (2006), the time-varying dependence is measured by allowing the correlation parameter of the Student-t and Gaussian copula to vary according to the ARMA(1,q)-type process. In order to fit the marginal densities, the threshold generalised autoregressive conditional heteroskedasticity (TGARCH) is used. Based on Akaike information criterion (AIC) and the pseudo-likelihood ratio test of Chen, Fan (2006), the Gaussian copula with time varying parameters (TVP Gaussian) is selected as the best performing model. Furthermore, they find that gold does not act as a hedge against oil price movements, as the copula functions reveal a positive and significant dependence between the two assets on average. On the other hand, they find evidence that gold and oil prices are independent in the tail of the distribution, indicating that gold acts as a safe haven in periods of extreme downward price movements of the oil market.

Reboredo (2013) also investigates the safe haven and hedging properties of gold against the US dollar (USD) using a copula approach to measure the average and extreme market dependence between gold and USD. In their research, they considers different copula specifications in order to capture different patterns of dependence and tail dependence such as tail independence, tail dependence, asymmetric tail dependence or time-varying dependence. They consider the Gaussian, Student-t, Clayton, Gumbel, Symmetrized Joe-Clayton (SJC), TVP Gaussian and TVP Student-t copulas. For the marginal distribution, the ARMA (p,q) model with TGARCH is considered with the aim to account for the most important stylized features of gold such as fat tails and the leverage effect. Based on AIC it appears that the Student-t copula is the one that most adequately represents the dependence structure between gold and the USD. The Student-t copula was the best performing copula for all exchange rates except CAD and JPY where the SJC and TVP Gaussian performed best. Furthermore, this study finds evidence that gold is a hedge against USD on average and a safe haven during periods of USD market stress.

Garcia-Jorcano, Muela (2020) use the Gaussian, Student-t, TVP Gaussian, TVP Student-t, Clayton, Gumbel and Frank copulas to investigate the role of Bitcoin as a hedge or diversifier against the following international market stock indexes: SP500 (US), STOXX50 (EU), NIKKEI (Japan), CSI300 (Shanghai), and HSI (Hong Kong). For the marginals, this research considers the univariate APARCH model because of its great flexibility, having as special cases, among other, the GARCH and GJR-GARCH models. In order to select the copula that fits the data best, Garcia-Jorcano, Muela (2020) use a grapic method that compares the empirical and parametric distribution of the copulas, multiple information criteria and goodness of fit tests. All methods appear to rule out Clayton, Gumbel and Frank, which are also known as the Archimedean copulas, as an appropriate model to describe the dependencies between Bitcoin and the stock indices analyzed. In the case of the US, European and Hong Kong markets both Gaussian and Stundent-t copulas appear to describe the dependencies adequately with Bitcoin. In case of the Japanese market and the Chinese market, the Gaussian copula appears to be the one that describes the dependency structure best. Furthermore, Garcia-Jorcano, Muela (2020) find that Bitcoin acts as a hedge against all stock indexes analyzed on average and as an diversifier during periods of market stress. The TVP Gaussian gives the same result as the constant copula models. However, according to the Student-t copula the hedging property of Bitcoin fails on a high percentage of days. Our research extends the work of Garcia-Jorcano, Muela (2020) by also taking into account the safe haven properties of Bitcoin. e.tex

3 Data

This research uses daily data of Bitcoin, European equity indices and exchange rates. The equity indices are represented by the Amsterdam Stock Exchange (AEX), the Deutscher Aktien (DAX) and the Financial Times Stock Exchange (FTSE100). The exchange rates are the EUR/USD and the GBP/USD. Bitcoin data, consisting of daily closing prices, is extracted from Coinmetrics whereas the daily closing prices of equity indices and exchange rates are extracted from Yahoo Finance. Figure 7 illustrates daily prices of all assets from June 2013 to june 2021.



(a) BTC/USD

(b) AEX Index

(c) DAX Index

(d) FTSE100 Index

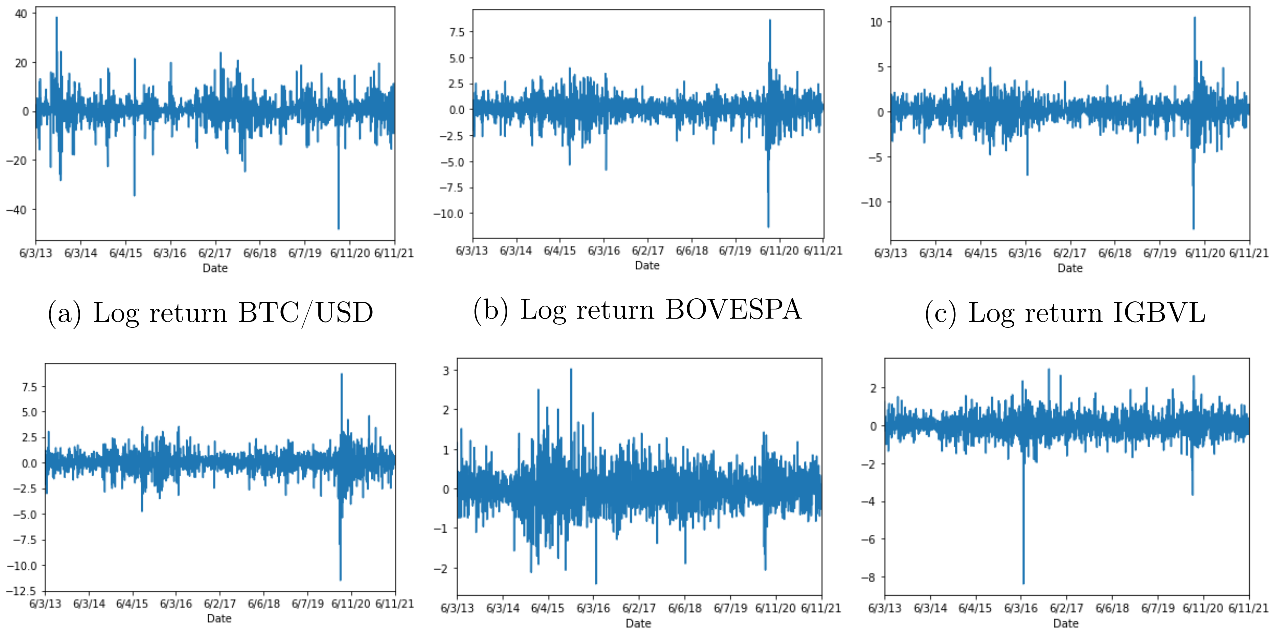
(e) EUR/USD

(f) GBP/USD

Figure 1: **Daily prices of Bitcoin (BTC), the Amsterdam Stock Exchange Index (AEX), the Deutscher Aktien Index (DAX), the Financial Times Stock Exchange Index (FTSE100), the Euro (EUR/USD) and the Pond (GBP/USD)**

Since bitcoin is traded every day and the other assets are not, there is more bitcoin data available than the other assets. To make sure that data of all assets are available at each given date, all dates that contain missing values of at least one of the assets are omitted. This results in a sample of 2001 observations. This research focuses on log-returns which is calculated as follows: $r_i = ln(price_{i,t}/price_{i,t-1}) * 100$, where i denotes the asset i at time t and t-1. Figure 2 shows the log-returns. It is clear that all assets exhibit volatility clustering. Table 1 shows summary statistics of the data. It can be observed that Bitcoin has a higher mean and standard deviation than the other assets. The Jarque Bera test rejects the null-hypothesis in all cases, which indicates that the returns are not normally distributed for all assets.[1] The high kurtosis

[1]See appendix A.1 for the specification of the Jarque Bera test



(a) Log return BTC/USD (b) Log return BOVESPA (c) Log return IGBVL

(d) Log return IPC (e) Log return BRL/USD (f) Log return PEN/USD

Figure 2: **Log returns of Bitcoin (BTC), the Amsterdam Stock Exchange Index (AEX), the Deutscher Aktien index (DAX), the Financial Times Stock Exchange Index (FTSE100), the Euro (EUR/USD) and the Pond (GBP/USD)**

values indicate heavy tails and the negative skewness means that negative returns are more likely to occur than positive returns. The univariate Ljung-Box and squared Ljung-Box tests have been performed to check for auto-correlation of the returns and the squared returns.[2] It can be observed that all assets except the GBP/USD reject the null hypothesis of the Ljung-Box test at the 1% significance level. This indicates that there is evidence for serial correlation for all assets except GPB/USD. After performing the squared Ljung-Box test, we observe that all assets reject the null hypothesis at the 1% significance level. This suggests that all assets exhibit volatility clustering and thus can be modelled by GARCH-type models. The ADF test is performed to check the presence of stationary time-series.[3] It can be observed that there is evidence for stationarity for all assets at the 1% significance level. This allows the Vector Autoregressive (VAR) model to be used for modelling.

[2]See appendix A.2 for the specification of the Ljung-Box Q-test

[3]See appendix A.4 for the specification of the ADF test

Table 1: Summary statistics

Variable	BTC	AEX	DAX	FTSE100	EUR/USD	GBP/USD
N	2001	2001	2001	2001	2001	2001
Mean	0.291	0.035	0.032	0.005	-0.003	-0.004
Std	5.130	1.103	1.251	1.032	0.494	0.588
Min	-48.090	-11.376	-13.055	-11.512	-2.417	-8.401
Max	38.049	8.59074	10.414	8.667	3.025	2.985
Skewness	-0.502	-0.851	-0.722	-0.887	0.049	-1.482
Kurtosis	12.651	13.548	14.055	16.747	5.507	25.939
Jarque Bera	7849.7*	9517.5*	10364*	16018*	524.92*	44604*
Ljung-Box-Q(10)	36.463*	22.662*	23.331*	40.357*	21.749**	13.461
Ljung-Box-S-Q(10)	135.73*	876.27*	647.48*	935.4*	260.96*	91.672*
ADF	-10.605*	-13.517*	-12.962*	-13.555*	-13.449*	-14.011*

The Jarque Bera statistic tests for normality. LB-Q(10) and the LB-S-Q(10) are the Ljung-Box Q-test statistics of returns and squared returns up to tenth lag and are used to test serial auto correlation. ADF is a statistic of the Augmented Dickey Fuller and is performed to test for unit roots. * and ** indicate that the null hypothesis is rejected at the 1% and 5% significance levels, respectively.

4 Methodology

This section focuses on the methodology of the research. The first half of this section presents VAR-GARCH-BEKK model that is used to measure the return and volatility spillover effects. The second half of the methodology presents the copula-GARCH model with APARCH for the marginals and several copulas for the dependence structure. This model is used to analyze the safe haven and hedging properties of Bitcoin.

In order to decide whether Bitcoin acts as a hedge or a safe haven we follow the definitions given by Baur, McDermott (2010) and Kaul, Sapp (2006). According to them, an asset is a hedge if it is uncorrelated or negatively correlated with another asset or portfolio on average and a safe haven if it is uncorrelated or negatively correlated with another asset or portfolio in times of extreme downwards market movements. Hence, we can formulate the following two conditions to determine whether Bitcoin can serve as a hedge or a safe haven against European financial assets: **condition 1**. $\tau \leq 0$ (Bitcoin is a hedge). **condition 2**. $\lambda_L \leq 0$ (Bitcoin is a safe haven). The parameter τ is the Kendall's tau coefficient and is used to assess the dependence structure between Bitcoin and the European financial assets on average. λ_L is the lower tail dependence coefficient and is used to assess the dependence structure between Bitcoin and the European assets in times of extreme downwards market movements. The parameter of the copulas are estimated by using the Maximum Likelihood (ML) method. The standard errors and the estimated coefficients are used to calculate the t-statistics and p-values. Statistical significance of the parameters are tested based on the p-values that are obtained. Based on the copula parameter, the Kendall's tau and the tail dependence coefficients are calculated.

4.1 The VAR-GARCH-BEKK model

We use the multivariate VAR-GARCH-BEKK model to analyze the return and volatility spillover effects between Bitcoin and European financial assets. The Vector Autoregressive (VAR) model is used to analyze the return spillover effects, while the GARCH-BEKK model is used to examine the volatility spillover effects.

4.1.1 The Autoregressive (VAR) model

The Vector Autoregressive (VAR) model is specified as follows:

$$\begin{bmatrix} r_{1,t} \\ r_{2,t} \\ r_{3,t} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} + \sum_{i=1}^{k} \begin{bmatrix} \beta_{11,i} & \beta_{12,i} & \beta_{13,i} \\ \beta_{21,i} & \beta_{22,i} & \beta_{23,i} \\ \beta_{31,i} & \beta_{32,i} & \beta_{33,i} \end{bmatrix} \begin{bmatrix} r_{1,t-i} \\ r_{2,t-i} \\ r_{3,t-i} \end{bmatrix} + \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \end{bmatrix},$$

where r_t denotes the return vector, r_{t-i} the lagged return vector, α a vector of constants, β a matrix containing parameters that are associated with the lagged return vector and ϵ_t a vector that contains the error terms. The return spillover effects between Bitcoin and the European assets are measured by the coefficients of the β matrix and their statistical significance. For example, if β_{13} is significant and equal to zero, the lagged return of the third time series has no return spillover effect on the return of the first time series.

Another model that could be considered to analyze the return spillover effects is the Vector Autoregressive Moving Average (VARMA) model. As Tsay (2005) mentions that the VAR model is adequate in financial applications and the VARMA model may experience issues such as the identification problem, this research uses the VAR model to determine the return spillover effects between Bitcoin and European financial assets.

4.1.2 Baba, Engle, Kroner and Kraft (BEKK) MGARCH

The variance equation is the MGARCH-BEKK model of Engle, Kroner (1995). The error terms of the mean equation are assumed to have a conditional distribution with mean zero and variance H_t. These residuals are used to model the conditional variance in the BEKK model, which is explicitly given by

$$\mathrm{H}_t = \begin{bmatrix} h_{11,t} & h_{12,t} & h_{13,t} \\ h_{21,t} & h_{22,t} & h_{23,t} \\ h_{31,t} & h_{32,t} & h_{33,t} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ 0 & c_{22} & c_{12} \\ 0 & 0 & c_{33} \end{bmatrix}' \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ 0 & c_{22} & c_{12} \\ 0 & 0 & c_{33} \end{bmatrix} +$$

$$\sum_{i=1}^{p} \begin{bmatrix} a_{11,i} & a_{12,i} & a_{13,i} \\ a_{21,i} & a_{22,i} & a_{23,i} \\ a_{31,i} & a_{32,i} & a_{33,i} \end{bmatrix}' \begin{bmatrix} \epsilon^2_{11,t-i} & \epsilon_{1,t-i}\epsilon_{2,t-i} & \epsilon_{1,t-i}\epsilon_{3,t-i} \\ \epsilon_{2,t-i}\epsilon_{1,t-i} & \epsilon^2_{22,t-i} & \epsilon_{2,t-i}\epsilon_{3,t-i} \\ \epsilon_{3,t-i}\epsilon_{1,t-i} & \epsilon_{3,t-i}\epsilon_{2,t-i} & \epsilon^2_{33,t-i} \end{bmatrix} \begin{bmatrix} a_{11,i} & a_{12,i} & a_{13,i} \\ a_{21,i} & a_{22,i} & a_{23,i} \\ a_{31,i} & a_{32,i} & a_{33,i} \end{bmatrix} +$$

$$\sum_{j=1}^{q} \begin{bmatrix} g_{11,j} & g_{12,j} & g_{13,j} \\ g_{21,j} & g_{22,j} & g_{23,j} \\ g_{31,j} & g_{32,j} & g_{33,j} \end{bmatrix}' \begin{bmatrix} h_{11,t-j} & h_{12,t-j} & h_{13,t-j} \\ h_{21,t-j} & h_{22,t-j} & h_{23,t-j} \\ h_{31,t-j} & h_{32,t-j} & h_{33,t-j} \end{bmatrix} \begin{bmatrix} g_{11,j} & g_{12,j} & g_{13,j} \\ g_{21,j} & g_{22,j} & g_{23,j} \\ g_{31,j} & g_{32,j} & g_{33,j} \end{bmatrix},$$

where H_t denotes the time-varying conditional variance-covariance matrix. C is the upper triangular matrix of constants. The elements of matrix A, also called the ARCH coefficient matrix, measure the lagged and cross shocks from asset i to j. These shocks are spillover effects. The cross-asset spillover effects are represented by the off-diagonal coefficients, while the effect of the lagged shocks are represented by the diagonal coefficients. The elements of matrix g, also called the GARCH coefficients matrix, measure the volatility spillover effects. To be more specific, the coefficients in matrix G measures the conditional volatility spillover effects from asset i to asset j. Here again, the cross-asset volatility spillover effects are represented by the off-diagonal coefficients, while the effect of the lagged volatility are represented by the diagonal coefficients. Furthermore, Maximum Likelihood Estimation (MLE) is used to estimate the parameters of the model.

The main drawback of the BEKK model is that the number of free parameters go up very fast with the number of time series. For a number of time series N, the number of parameters is equal to (N(N+ 1))2/2 +N(N+ 1)/2. Despite this drawback, this research still chooses to use the BEKK model. One of the advantages of using the BEKK model for volatility spillover effects is the fact that it enforces H_t to be positive definite (Doan, 2013). Another important advantage of the BEKK model is that it allows all complicated interactions among different time series and thus, in contrast to other multivariate GARCH models, all volatility spillover effects can be measured. The full VECH model is not chosen because it has more parameters than the BEKK model and only enforces positive definite H_t under certain restrictions. The DCC model performs well in capturing volatility, correlation and time-varying correlation, but its main drawback in this application is that it does not capture volatility spillover effects nor is DCC closed under linear transformation (Basher, Sadorsky, 2016).

Bauwens et al. (2006) state that the BEKK model is a flexible model, but requires too many parameters when more than four time series are estimated. To prevent difficulties that come with estimating too many parameters, this paper considers two different multivariate VAR-GARCH-BEKK models. The first model analyzes volatility spillover effects between Bitcoin and European stock indices and the second model analyzes volatility spillover effects between Bitcoin and European exchange rates.

4.2 Introduction to copula

A copula is a multivariate cumulative distribution function (CDF) that is introduced by Sklar (1959). It joins univariate distributed functions to a multivariate distribution and extracts the dependence structure from the joint distribution, independent of marginal distributions. The multivariate CDF is denoted by C in the following equation:

$$F(x_1, ...x_d) = C(F_1(x_1), ..., F_d(x_d)), \tag{1}$$

where $F_i(x_i)$, i = 1,..., d are uniform marginals on [0,1]. Alternatively, a copula can be written as the multivariate distribution, C, of a vector of random variables with uniformly distributed marginals U(0,1).

$$C(u_1, ..., c_d) = F(F_1^{-1}(u_1), ..., F_d^{-1}(u_d)) \tag{2}$$

where the $u_i = F(X_i)$'s are the quantile functions of the marginals. Equation 2 represents the Elliptical copula family. Elliptical copulas are the ones that are derived from elliptical contoured multivariate distributions (Frahm et al., 2003). As elliptical distributions are characterized by radial symmetry, the upper and lower tail dependence are equal. The normal and student-t distributions are the most commonly used elliptical distributions in financial application. Another important class of copulas is the Archimedean copula. This type of copula can be written in the following form:

$$C(u_1, u_2) = \psi^{-1}(\psi(u_1) + \psi(u_2)), \tag{3}$$

where $\psi(u_t)$ is the decreasing, continuous and convex generator that maps [0,1] into [0,∞]. An attractive feature of this function is the single parameter θ that allows modelling in high dimensions without increasing the number of parameters. Archimdedean copulas that are frequently used in financial application are the Clayton, Gumbel and Frank copula.

4.3 Kendall's tau

The Kendall's τ is a rank correlation matrix which fulfills the scale invariant property and is a commonly used measure to investigate dependence. The scale invariant property means that it remains unchanged given a strictly increasing transformation of the random variables. Given a random vector (X_1, X_2), consider an independent copy $(\tilde{X}_1, \tilde{X}_2)$. Denote Y = $(\mathrm{X}_1 - \tilde{X}_2)(X_2 - \tilde{X}_2)$. The Kendall's τ is then specified as follows:

$$\tau(X_1, X_2) = \mathbb{E}(sign(Y)) = Pr(Y > 0) - Pr(Y < 0) \tag{4}$$

The general idea is that it measures concordance. (X_1, X_2) and $(\tilde{X_1}, \tilde{X_2})$ are concordant if $X_1 < \tilde{X_1}$ and $X_2 < \tilde{X_2}$ or if $X_1 > \tilde{X_1}$ and $X_2 > \tilde{X_2}$. Alternatively, this can be formulated as $Y = (X_1 - \tilde{X_2})(X_2 - \tilde{X_2}) > 0$. (X_1, X_2) and $(\tilde{X_1}, \tilde{X_2})$ are disconcordant if $Y = (X_1 - \tilde{X_2})(X_2 - \tilde{X_2}) < 0$. As we can observe, the definition of Kendalls τ in equation 5 is equal to the difference between the probability of concordance and the probability of disconcordance. The kendall's τ is linked to the copula theory through the following definition[4]:

$$\tau(C_1, C_2) = 4 \int \int C(u_1, u_2) dC(u_1, u_2) - 1, \tag{5}$$

where C_1 and C_2 are the copulas and u_1 and u_2 are the univariate marginals.

4.4 Tail dependence

Tail dependence measures co-movements in the tails of the distribution between a pair of random variables. It is a characteristic of copula functions that is not captured by traditional linear correlation functions. Many researchers, such as Hu (2006) and Lin (2011), have shown that financial assets exhibit tail dependence. The upper tail dependence coefficient between a pair of random variables X_1 and X_2 is specified as follows[5]:

$$\lambda_u = \lim_{q \to 1} Pr(X_2 > VaR_q(X_2) | X_1 > VaR_q(X_1)) \tag{6}$$

The lower tail dependence is given by

$$\lambda_l = \lim_{q \to 0} Pr(X_2 > F_2^{-1}(q) | X_1 > F_1^{-1}(q)) \tag{7}$$

4.5 The copula-APARCH model

Let r_1 and r_2 denote the two asset returns series. The specification of the copula-APARCH is then given by the following characteristics:

[4]See appendix C.1 for the Kendall's tau of each copula

[5]See appendix C.1 for tail dependence features of each copula

The dynamics of r_1 is described by the following APARCH process:

$$\begin{cases} r_{1,t} = \mu_1 + \epsilon_{1,t} \\ \epsilon_{1,t} = \sigma_{1,t} z_{1,t}, & z \sim \mathcal{N}(0,1) \\ \sigma_{1,t}^{\delta} = \omega_1 + \sum_{i=1}^{q} \alpha_i(|\epsilon_{1,t-1}| + \gamma_i \epsilon_{1,t-i})^{\delta} + \sum_{j=1}^{p} \beta_j \sigma_{1,t-j}^{\delta} \end{cases}$$

The dynamics of r_2 is described by the following APARCH process:

$$\begin{cases} r_{2,t} = \mu_2 + \epsilon_{2,t} \\ \epsilon_{2,t} = \sigma_{2,t} z_{2,t}, & z \sim \mathcal{N}(0,1) \\ \sigma_{2,t}^{\delta} = \omega_2 + \sum_{i=1}^{q} \alpha_i(|\epsilon_{2,t-1}| + \gamma_i \epsilon_{2,t-i})^{\delta} + \sum_{j=1}^{p} \beta_j \sigma_{2,t-j}^{\delta}, \end{cases}$$

where μ_i, ω_i,γ_i, α_i, β_j and δ are parameters that need to be estimated. The paramater $\gamma \in$(-1,1) represents the leverage effect. A positive (negative) value of γ means that past positive (negative) shocks have more effect on the current conditional volatitlity than past negative (positive) shocks. The parameter δ is the Box-Cox transformation of the conditional variance and the asymmetric absolute residuals. It is used to linearise non-linear models.

The dependence structure between innovations z_1 and z_2 is modeled by using bivariate copula functions, characterized by the copula parameter θ. This paper employs the following copula functions to model the dependency structure: Gaussian ($\theta := \rho$), Student-t (θ:=(v,ρ)), Clayton (θ:= $\alpha, \alpha > 0$), Gumbel ($\theta := \alpha, \alpha > 1$), Frank ($\theta := \alpha, \alpha \neq 0$).[6]

4.6 Estimation

The Maximum Likelihood (ML) method chooses C and the marginals $F_1, ..., F_n$, such that the probability of observing the data is maximized. Given a data set $(x_{1,t}, ..., x_{n,t})$, the ML function calculates for which θ the following likelihood function is maximized:

$$l(\theta) = \prod_{t=1}^{T} \left(c(F_1(x_{1t}), ..., F_n(x_{nt}; \theta) \prod_{i=1}^{n} f_i(x_{i,t}); \theta) \right), \tag{8}$$

where θ is the parameter vector of the copula function that is used. θ also maximizes the following log-likelihood function:

$$log\ l(\theta) = \sum_{t=1}^{T} log\ c(F_1(x_{1t}), ..., F_n(x_{nt}); \theta) + \sum_{t=1}^{T} \sum_{i=1}^{n} log\ f_i(x_{i,t}); \theta). \tag{9}$$

[6]See appendix B for specification of the Gaussian, Student-t, Clayton, Gumbel and Frank copula

The ML estimator is then given by:

$$\hat{\theta}_{ML} := arg \max_{\theta} l(\theta). \tag{10}$$

4.7 Goodness of fit

A commonly used method to analyze the best fitting copula is comparing the log-likelihood and Akaike information criterion (AIC) values. The AIC criteria is specified as follows:

$$AIC = 2k - 2ln(\hat{L}), \tag{11}$$

where k denotes the number of parameters and $\hat{L}$ is the maximum value of the likelihood function for the model. Beyond this, Genest et al. (2009) review different "blanket tests" which are goodness of fit tests of copula models. These "blankets tests" test the hypothesis $H_0 : C \in C_0$, where C_0 represents a specific parametric family of copulas. As the underlying copula of a random vector is invariant by continuous, strictly increasing transformations of its components, the only reasonable option for testing H_0 is using a function of a collection pseudo observations that can be interpreted as random variables from the underlying C: $U_1 = (U_{11}, ..., U_{1d}), ..., U_n = (U_{n1}, ..., U_{nd})$..

Genest et al. (2009) investigate five different goodness of fit tests and find that the Cramér-von Mises test with parametric bootstrapping is the best performing test. To test H_0, the "distance" between the empirical copula C_n and the parametric copula C_{θ_n} is measured. The empirical copula summarizes the information contained in $U_1, ..., U_n$ and is given by

$$C_n(u) = \frac{1}{n} \sum_{i=1}^{n} 1(U_{i1} \leq u_1, ..., U_{id} \leq u_d), \tag{12}$$

where $u = (u_1, ..., u_d) \in [0, 1]^d$. The study measures the distance considering the rank-based versions of the familiar Cramer–von Mises and Kolmogorov–Smirnov statistics. The Cramér-von Mises test appears to be more powerful than the Kolmogorov–Smirnov test and is specified as:

$$S_n = \int_{[0,1]^d} C_n(u)^2 dC_n(u), \tag{13}$$

where $C_n(u) = (\sqrt{n}(C_n(u) - C_{\theta_n}(u))$. The hypothesis H_0 is rejected for large values of S_n. Furthermore, S_n is a consistent statistic. This means that if C $\notin C_0$, H_0 is rejected with probability 1 as n $\rightarrow \infty$.

In addition, the goodness of fit between the empirical copula and the parametric copula is measured by using the mean absolute error (MAE) evaluated on all data points of the empirical copula:

$$MAE = \frac{1}{n}\sum_{i=1}^{n} |C_n(u_1, ..., u_d) - C_{\theta_n}(u_1, ..., u_d)|, \tag{14}$$

where C_n is the empirical copula, C_{θ_n} the parametric copula, d the dimension of the data and n the number of data points.

5 Results

This section presents the results of the paper. Firstly, The VAR-GARCH-BEKK model is used to measure the return and volatility spillover effects. The return spillover effects are measured by using the VAR model, while the volatility spillover effects are measured by using the GARCH-BEKK model. Secondly, the results of the copula-APARCH model are presented. Lastly, a model selection strategy is performed to find the copula that fits the data best. The hedging and safe haven properties of Bitcoin are measured based on the chosen copula.

5.1 Spillover effects between Bitcoin and European Stock Indices

The VAR model is used to examine return spillover effects between Bitcoin and European stock indices. Before constructing the model, the optimal lag order is determined by using Akaike Information Criterion (AIC). Based on AIC, the second order lag is chosen to be optimal. Subsequently, the multivariate Ljung-Box (LB) test is used to check for adequate model specification. However, the multivariate LB test rejects the null hypothesis which means that the residuals of the model are serially correlated. Therefore, by recursively fitting the model for other lag values, we find that VAR(4) is the optimal model, where no serial correlation exists in the residuals up to lag 6.

The VAR(4) model is fitted in order to obtain the estimates of the VAR model that is used for the return spillover effects between Bitcoin, AEX, DAX and FTSE100. Table 2 shows all significant results. It appears that the return of bitcoin is only affected by its own lagged returns. The return of AEX is affected by the fourth lag of Bitcoin return. The return of DAX is affected by the second and fourth lag of Bitcoin return. Lastly, the return of FTSE100 is affected by the fourth lag of Bitcoin return. As we can observe, Bitcoin and the stock indices show return spillover effects in only one direction. Bitcoin affects all stock indices, while the stock indices don't affect Bitcoin. All significant effects are positive, which means that if the return of Bitcoin increases, the return of the stock indices also increase. Furthermore, the effects are more or less of the same magnitude, which indicates that no clear regional differences exist. All coefficients are relatively small, indicating a weak relation between the returns of Bitcoin and the stock indices.

The GARCH-BEKK(p,q) model is used to analyze the volatility spillover effects between Bitcoin and European stock indices. The parameters of the GARCH-BEKK(p,q) model, where p and q are the lag orders of the ARCH and GARCH coefficients, are selected based on AIC. The GARCH-BEKK model assumes that no serial correlation exists in the standardized residuals and squared standardized residuals. To check whether this assumption is violated, the multivariate Ljung-Box (LB) test is performed on the standardized and squared standardized residuals. The standardized residuals are calculated by using the Cholesky decomposition. The Cholesky decomposition maps the covariance matrix of the GARCH-BEKK model into the product of a lower triangular matrix and its transpose. Then the standardized residuals are obtained by the product of the inverse of the lower triangular matrix and the residuals.[7] It appears that the GARCH-BEKK(2,1) has the lowest AIC value. However, the multivariate LB test reveals that serial correlations exists in the standardized residuals. Although the GARCH-BEKK(1,2) has higher AIC value than the other models, it is the only model that shows no serial correlation in the standardized residuals and the squared standardized residuals up to lag 6. Therefore, the GARCH-BEKK(1,2) is chosen as an adequate model to analyze volatility spillover effects between Bitcoin and European stock indices.

Table 2 also shows the estimation results of the GARCH-BEKK(1,2) model. As the coefficients of the constant matrix in the GARCH-BEKK model do not measure spillover effects, it is not shown in the table. From the results, it can be observed that the volatility of Bitcoin is negatively affected by its own lagged shocks (ARCH coefficients) and volatility (GARCH coefficient) at the 1% significance level. All past shocks and past volatility of DAX impact the volatility of Bitcoin. In case of FTSE100, only its past shocks impact Bitcoin its volatility. Additionally, the past volatility of AEX and its shock at lag 2 affects Bitcoin its volatility at a 1% significance level. It appears that the past volatility of FTSE100 and AEX negatively affects Bitcoin its volatility, while the past volatility of DAX positively affects Bitcoin. Furthermore, we can observe that AEX has a significantly bigger impact on the volatility of Bitcoin than the other assets.

On the other hand, we can observe that Bitcoin its shocks at lag 1 impacts the volatility of DAX and FTSE at a 5% significance level. However, it affects DAX negatively, while it affects FTSE100 positively. Additionally, the results show that Bitcoin its past volatility positively affects the volatility of AEX, DAX and FTSE100 at 1% significance level. Unlike the return spillover effects, the volatility spillover effects between Bitcoin and all stock indices are bidirectional. Overall, it appears that the volatility spillover effects between Bitcoin and the stock indices are stronger than the return spillover effects between Bitcoin and the stock indices. We can observe more and stronger volatility spillover effects compared to the return spillover effects. When we look at the magnitude of the volatility spillover effects, it also appears that all effects of Bitcoin on the stock indices are much weaker than the effects of stock indices on

[7] See appendix A.5 for the specification of the Cholesky decomposition

Bitcoin. This is in line with the findings of Wang et al. (2019) and indicates that Bitcoin is still a rather small asset compared to the European stock indices.

Table 2: Estimation results of the VAR(4)-GARCH-BEKK(1,2) model

Parameters	Coefficient	Parameters	Coefficient
$\beta_{11,t-1}$	-0.06786***	$\beta_{21,t-4}$	0.013902***
	(0.02253)		(0.004853)
$\beta_{11,t-2}$	0.05533***	$\beta_{31,t-2}$	0.0121025***
	(0.02255)		(0.0054856)
$\beta_{11,t-3}$	0.06034***	$\beta_{31,t-4}$	0.0170193***
	(0.02258)		(0.0054936)
$\beta_{11,t-4}$	0.06841***	$\beta_{41,t-4}$	0.0126937***
	(0.02258)		(0.0045453)
$\alpha_{11,t-1}$	-0.197***	$\alpha_{31,t-1}$	-0.008**
	(-0.082)		(0.007)
$\alpha_{12,t-1}$	0.615	$\alpha_{32,t-1}$	0.401**
	(1.561)		(0.144)
$\alpha_{13,t-1}$	-3.808***	$\alpha_{33,t-1}$	0.117
	(1.291)		(0.102)
$\alpha_{14,t-1}$	-2.300**	$\alpha_{34,t-1}$	0.060
	(1.309)		(0.127)
$\alpha_{21,t-1}$	-0.003	$\alpha_{41,t-1}$	0.015**
	(0.139)		(0.006)
$\alpha_{22,t-1}$	0.216	$\alpha_{42,t-1}$	0.193
	(0.109)		(0.131)
$\alpha_{23,t-1}$	0.326**	$\alpha_{43,t-1}$	0.216
	(0.132)		(0.119)
$\alpha_{24,t-1}$	0.304**	$\alpha_{44,t-1}$	0.364***
	(0.132)		(0.121)
$\alpha_{11,t-2}$	-0.348***	$\alpha_{31,t-2}$	0.016
	(0.125)		(0.011)
$\alpha_{12,t-2}$	-4.285***	$\alpha_{32,t-2}$	0.606***
	(1.613)		(0.157)

$\alpha_{13,t-2}$	-1.932**	$\alpha_{33,t-2}$	-0.146**
	(1.265)		(0.133)
$\alpha_{14,t-2}$	2.929**	$\alpha_{34,t-2}$	-0.455***
	(1.332)		(0.123)
$\alpha_{21,t-2}$	0.016	$\alpha_{41,t-2}$	0.013
	(0.012)		(0.016)
$\alpha_{22,t-2}$	0.760***	$\alpha_{42,t-2}$	0.682***
	(0.152)		(0.144)
$\alpha_{23,t-2}$	-0.068	$\alpha_{43,t-2}$	-0.042
	(0.123)		(0.100)
$\alpha_{24,t-2}$	-0.571***	$\alpha_{44,t-2}$	-0.663***
	(0.138)		(0.125)
$g_{11,t-1}$	-0.561***	$g_{31,t-1}$	-0.054***
	(0.017)		(0.004)
$g_{12,t-1}$	-7.000***	$g_{32,t-1}$	-0.796***
	(0.907)		(0.106)
$g_{13,t-1}$	-1.905***	$g_{33,t-1}$	0.096
	(0.475)		(0.105)
$g_{14,t-1}$	-0.274	$g_{34,t-1}$	-0.101**
	(1.422)		(0.100)
$g_{21,t-1}$	-0.003***	$g_{41,t-1}$	-0.003***
	(0.0009)		(0.001)
$g_{22,t-1}$	0.211**	$g_{42,t-1}$	0.928***
	(0.085)		(0.102)
$g_{23,t-1}$	-0.612***	$g_{43,t-1}$	-0.691***
	(0.062)		(0.053)
$g_{24,t-1}$	-0.052	$g_{44,t-1}$	-0.660***
	(0.105)		(0.049)

Table 2: Return and spillover effects between Bitcoin, AEX, DAX and FTSE100. $\beta_{ij,t-k}$ denote the parameter that belongs to the returns, where i $\in (1, .., 4)$ is the asset, j $\in (1, .., 4)$ the lagged asset and k the lag order. $a_{i,j}$ and $g_{i,j}$ correspond to the coefficients in the ARCH and GARCH matrix. The numbers 1,2,3,4 represent Bitcoin, AEX, DAX and FTSE100, respectively. * and ** and *** indicate the 10%, 5% and 1% significance levels.

5.2 Spillover effects between Bitcoin and European Exchange Rates

Based on AIC, VAR(2) is chosen as the optimal model. However, the multivairate Ljung-Box (LB) test rejects the null hypothesis, which means that the residuals are serially correlated. In order to find an adequate model that measures return spillover effects, we recursively fit all models up to lag 10. Again, VAR(4) appears to be optimal. It has the lowest AIC value and no serial correlation exists in the residuals up to lag 6.

Again, the VAR(4) model is fitted in order to obtain the estimates of the VAR model that is used to measure return spillover effects between Bitcoin, EUR/USD and GBP/USD. It appears that that there are no spillover effects between Bitcoin and EUR/USD, while one-directional effects exists between Bitcoin and GBP/USD. The fact that the returns of Bitcoin and EUR/USD do not impact each other might indicate a hedging property of Bitcoin against EUR/USD. We can observe that the past return of Bitcoin at lag 2 and 3 significantly impact the return of GBP/USD. Furthermore, it appears that return spillover effects between Bitcoin and exchange rates is even smaller than the ones between Bitcoin and the stock indices. This indicates that there is a even weaker relation between the returns of Bitcoin and the exchange rates.

For the exchange rates, we again recursively fit the BEKK(p,q) model. Based on AIC, the BEKK(1,1) model slightly outperforms BEKK(1,2) and BEKK(2,1) models. However, the multivariate Ljung-Box (LB) reveals that BEKK(1,2) is the only model that shows no serial correlation in the standardized residuals and squared standardized residuals. Therefore, the BEKK(1,2) model is chosen as an adequate model to assess spillover effects between Bitcoin and European exchange rates.

From the results shown in table 5, it appears that there is one-directional volatility spillover effects between Bitcoin and EUR/USD, while there is a bidirectional volatility spillover effect between Bitcoin and GBP/USD. It appears that Bitcoin its past shocks affects the volatility of the GBP/USD negatively, while it affects the volatility of EUR/USD positively. Furthermore, we can observe that Bitcoin has less volatility spillover effects with the exchange rates than with the stock indices, indicating that there is a weaker relation between the volatility of Bitcoin and the exchange rates. Lastly, The effect of Bitcoin on the exchange rates is smaller than the effect of the exchange rates on Bitcoin. This can be explained by Bitcoin its small market cap and trading volume compared to the exchange rates and proves again that Bitcoin is still a small asset that has not much impact on major European financial assets.

Table 3: Estimation results of the VAR(1)-GARCH-BEKK(1,2) model

Parameters	Coefficient	Parameters	Coefficient
$\beta_{11,t-1}$	-0.07068***	$\beta_{11,t-4}$	0.06992***
	(0.02241)		(0.02248)
$\beta_{11,t-2}$	0.05958***	$\beta_{31,t-2}$	0.004255**
	(0.02243)		(0.002580)
$\beta_{11,t-3}$	0.06240***	$\beta_{31,t-3}$	0.006908***
	(0.02244)		(0.002581)
$\alpha_{11,t-1}$	0.309***	$\alpha_{31,t-1}$	-0.003**
	(0.038)		(0.003)
$\alpha_{12,t-1}$	0.252	$\alpha_{32,t-1}$	-0.148***
	(0.430)		(0.046)
$\alpha_{13,t-1}$	-0.652**	$\alpha_{33,t-1}$	-0.398***
	(0.334)		(0.045)
$\alpha_{21,t-1}$	0.000	$\alpha_{11,t-2}$	0.527***
	(0.001)		(0.046)
$\alpha_{22,t-1}$	-0.0443**	$\alpha_{12,t-2}$	0.480
	(0.026)		(0.672)
$\alpha_{23,t-1}$	-0.082***	$\alpha_{13,t-2}$	0.470
	(0.019)		(0.324)
$\alpha_{21,t-2}$	0.004**	$\alpha_{31,t-2}$	-0.005**
	(0.002)		(0.003)
$\alpha_{22,t-2}$	0.213***	$\alpha_{32,t-2}$	-0.077**
	(0.020)		(0.057)
$\alpha_{23,t-2}$	-0.024**	$\alpha_{33,t-2}$	0.278***
	(0.015)		(0.046)
$g_{11,t-2}$	-0.546***	$g_{21,t-2}$	0.005***
	(0.072)		(0.002)
$g_{12,t-2}$	0.551	$g_{22,t-2}$	-0.993***
	(0.283)		(0.012)
$g_{13,t-2}$	0.371	$g_{23,t-2}$	0.049**
	(0.403)		(0.018)
$g_{31,t-1}$	-0.009***		

	(0.004)
$g_{32,t-1}$	-0.300***
	(0.035)
$g_{33,t-1}$	-0.492***
	(0.081)

Table 3: Return and volatility spillover effects between Bitcoin, EUR/USD and GBP/USD. $\beta_{ij,t-k}$ denote the parameter that belongs to the returns, where i $\in (1,..,3)$ is the asset, j $\in (1,..,3)$ the lagged asset and k the lag order. $a_{i,j}$ and $g_{i,j}$ correspond to the coefficients in the ARCH and GARCH matrix. The numbers 1,2 and 3 represent Bitcoin, EUR/USD and GBP/USD, respectively. * and ** and *** indicate the 10%, 5% and 1% significance levels.

5.3 Marginal modeling

The APARCH(p,q) model is used to model the marginals of each asset return series separately. For the lags, we use $p = 1$ and $q = 1$. According to literature, this specification usually fits time series data best. The ARCH test is used to check for adequate model specifications. The ARCH test rejects the null hypothesis in all cases, indicating that the volatility of the innovations are adequately captured. In table 6 estimation results of the APARCH(1,1) model with ν degrees of freedom are reported. For each asset return except EUR/USD, we can observe that the Log-Likelihood of the student-t distribution is higher than the Log-Likelihood of the normal distribution. This indicates that the student-t distribution better fits the data. The Akaike Information Criterion (AIC) provides the same result. Additionally, in order to examine the goodness-of-fit of the distribution, a visual analysis of the QQ-plot is made.[8] Also here, for each asset except EUR/USD, we can observe that the student-t distribution better fits the data, especially the left and right tails. As the student-t distribution is better at capturing the fat tails of the return distribution, this result was expected.

[8]See appendix C.3 for the QQ-plot of each asset return series.

. Table 6: Estimates of the APARCH(1,1) model

Asset	Distribution	μ	ω	α	β	γ	δ	ν	LL	AIC
Bitcoin	Normal	0.45*	6.00	0.17**	0.70***	0.26**	2.40***		-1085.04	6.06
	Student-t	0.45***	0.17	0.11***	0.92***	0.19*	1.43***	2.86***	**-1024.61**	**5.73**
AEX	Normal	0.13***	0.03***	0.10***	0.91***	-0.99***	0.61***		-583.61	3.27
	Student-t	0.13***	0.03	0.11**	0.91***	-0.96***	0.66**	5.80***	**-576.17**	**3.24**
DAX	Normal	0.09***	0.02***	0.11***	0.91***	-0.97***	0.40***		-616.08	3.45
	Student-t	0.08***	0.01*	0.11***	0.93***	-1***	0.62 ***	3.88***	**-603.48**	**3.39**
FTSE100	Normal	-0.009	0.027***	0.082***	0.93***	-0.99***	0.53***		-593.68	3.33
	Student-t	0.014***	0.013**	0.079***	0.94 ***	-0.99***	0.61***	4.68***	**-579.13**	**3.25**
EUR/USD	Normal	0.027	0.014	0.093**	0.82***	-0.21	2.07*		**-199.62**	**1.14**
	Student-t	0.027	0.014	0.094**	0.82***	-0.21	2.00*	99	-199.70	1.15
GBP/USD	Normal	0.031	0.028*	0.13***	0.78***	-0.08	1.98***		-299.18	1.70
	Student-t	0.035	0.026**	0.11	0.81***	-0.13	2.01***	11.12**	**-297.6**	**1.69**

*LL represents the Log-likelihood value. * and ** and *** indicate the 10%, 5% and 1% significance levels.*

Before explaining what the parameters indicate, it should be mentioned that not all parameters are significant, especially in case of the exchange rates. This is in line with the QQ-plot analysis in which we observed that that the student-t and normal distribution fit the data of the stock indices better than the exchange rates.

Bitcoin its μ and ω parameters are higher than the stock indices and exchange rates. This is because Bitcoin its returns and volatility is relatively high. We also find that β is slightly higher for Bitcoin and the stock indices compared to the exchange rates. This means that volatility of Bitcoin and the stock indices is slightly more persistent compared to the exchange rates. Furthermore, the γ parameter, which measures the leverage effect, is positive for Bitcoin, while it is negative for the European financial assets. In case of Bitcoin, this means that past positive shocks have more effect on the current conditional volatility than pas negative shocks. This is called the inverse leverage effect and is a characteristics of gold and commodities in general. Baur (2012) states that the inverse leverage effect might be an indication of the safe haven property. In case of the European financial assets, it means that past negative shocks have more effect on the current conditional volatility than past positive shocks. δ, which is the power parameter, is around 1 for Bitcoin and the stock exchanges and around 2 for the exchange rates. Apparently, in case of Bitcoin and the stock indices, modeling the standard deviation is better than modeling the variance.

5.4 Dependence modeling

Table 6 reports estimations of copula models that are used in this paper. The Kendall's τ and the tail dependence parameters are derived from the copula functions and included in the table.

Table 4: Elliptical and Archimedean copula specifications and estimations

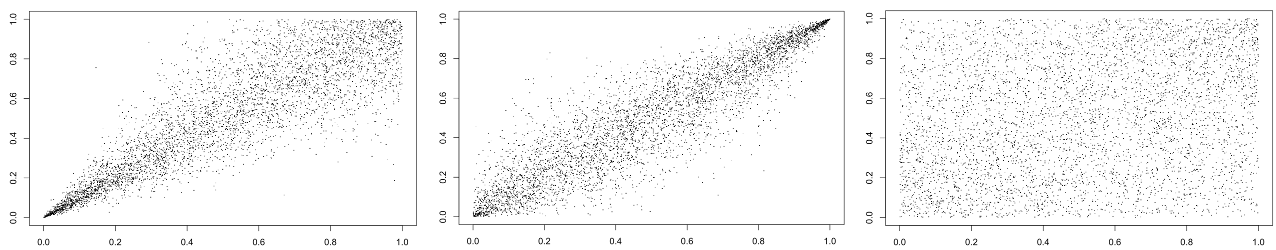
Asset Pair	Copula	Copula parameter	df	Kendall's τ	Lower tail	Upper tail
BTC-AEX	Gaussian	0.221***	-	0.141	0	0
	Student-t	0.218***	16.72	0.140	0.0034	0.0034
	Clayton	0.289***	-	0.126	0.091	0
	Gumbel	1.131***	-	0.116	0	0.154
	Frank	1.218***	-	0.024	0	0
BTC-DAX	Gaussian	0.204***	-	0.131	0	0
	Student-t	0.199***	9.39	0.128	0.024	0.024
	Clayton	0.290***	-	0.127	0.092	0
	Gumbel	1.121***	-	0.107	0	0.144
	Frank	1.123***	-	0.123	0	0
BTC-FTSE100	Gaussian	0.178***	-	0.114	0	0
	Student-t	0.163***	9.41	0.104	0.020	0.020
	Clayton	0.244***	-	0.109	0.058	0
	Gumbel	1.097***	-	0.088	0	0.118
	Frank	0.872**	-	0.096	0	0
BTC-EUR/USD	Gaussian	0.106** (0.053)	-	0.068	0	0
	Student-t	0.113**	10.83	0.072	0.0099	0.0099
	Clayton	0.108*	-	0.351	0.0016	0
	Gumbel	1.066***	-	0.062	0	0.084
	Frank	0.669**	-	0.074	0	0
BTC-GBP/USD	Gaussian	0.146***	-	0.093	0	0
	Student-t	0.140**	9.22	0.089	0.019	0.019
	Clayton	0.234***	-	0.105	0.052	0
	Gumbel	1.069***	-	0.065	0	0.088
	Frank	0.791***	-	0.087	0	0

Note: * and ** and *** indicate the 10%, 5% and 1% significance levels. The first column denotes the structure dependence (copula) between Bitcoin and one of the assets.

For the copula of the Bitcoin and AEX pair, we can observe that the Kendall's tau of the Gaussian, Student-t, Clayton, Gumbel and Frank copulas are 0.141, 0.140, 0.126, 0.116 and 0.024, respectively. Following the condition for the hedging property, it appears that all copulas indicate that Bitcoin can not be used as a hedge against AEX index during the COVID-19 pandemic. In order to assess the safe haven property of Bitcoin against the AEX index, we analyze the co-movement in times of extreme downward movements. The lower tail dependence parameter provides information about the probability of a joint crash. As we can observe, the Gaussian and Frank copula do not exhibit tail dependence, the Student-t copula captures symmetric tail dependence that is close to zero, the Clayton copula captures lower tail dependence which is equal to 0.091 and the Gumbel copula captures a positive upper tail dependence which is equal to 0.154. Hence, the lower tail dependence parameter is zero or close to zero for all copulas except the Clayton copula. Following the conditions for the safe haven property, it appears that the Gaussian, Student-t, Gumbel and Frank copula indicate that Bitcoin acts as a safe haven, while the Clayton copula indicates that Bitcoin does not act as a safe haven during the COVID-19 pandemic. The copula of all other asset pairs can be analyzed in a similar way.

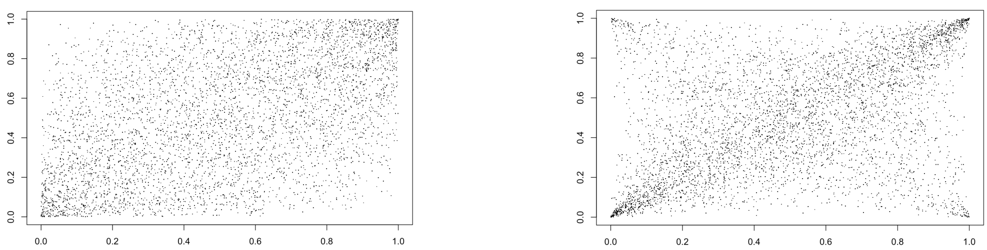
5.5 Model selection

As the copulas imply a disparity of results, this section explains step by step how the best fitting copula is chosen. The best fitting copula is eventually used to determine the hedging and safe haven property of Bitcoin against the assets. First, we make pre-estimation observations by presenting the empirical copulas and analyzing whether they exhibit well known properties of the copulas that are proposed in this paper. Figure 3 gives informative examples of the well known properties of the copulas that are proposed in this paper.



(a) Clayton with $\alpha = 0.5$ (b) Gumbel copula with $\alpha = 0.5$ (c) Frank copula with $\alpha = 5$

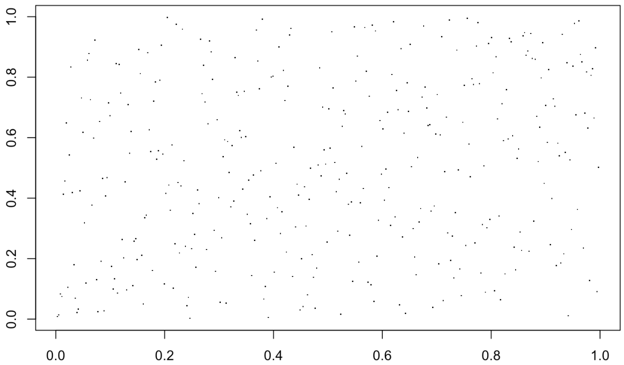
Figure 3: **An example of the bivariate Archimedean copulas with N=5000**



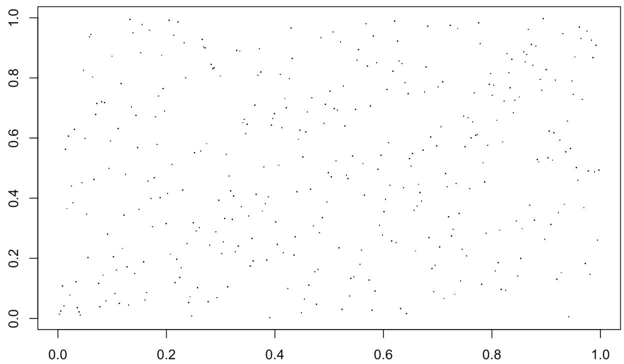
(a) Gaussian copula with $\rho = 4$ (b) Student-t copula with $\rho = 1.5$

Figure 4: **An example of the bivariate Elliptical copulas with N=5000**

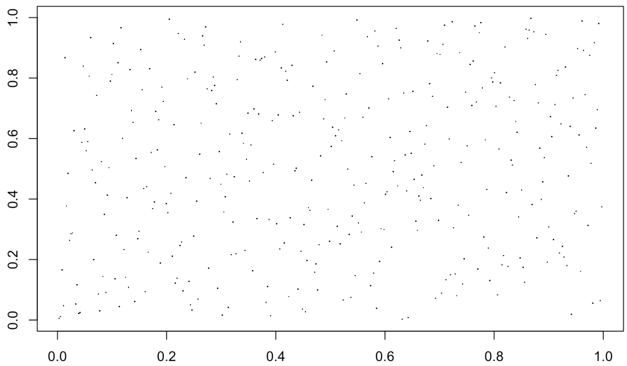
Figure 4 and 5 present the bivariate empirical copulas, which are also called the true or non-parametric copulas. The bivariate empirical copulas are the pairs of standardized residuals from the APARCH(1,1) model that are mapped within the uniform distributions, as an input to fit the copulas. The empirical copulas contain 360 observations, which is rather small compared to the simulated copulas shown in figure 3 and 4.



(a) BTC-AEX

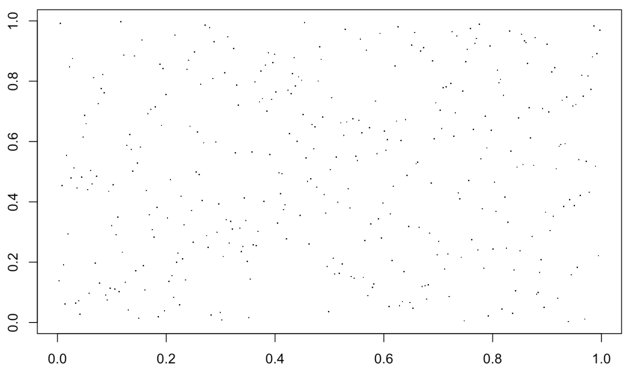


(b) BTC-DAX

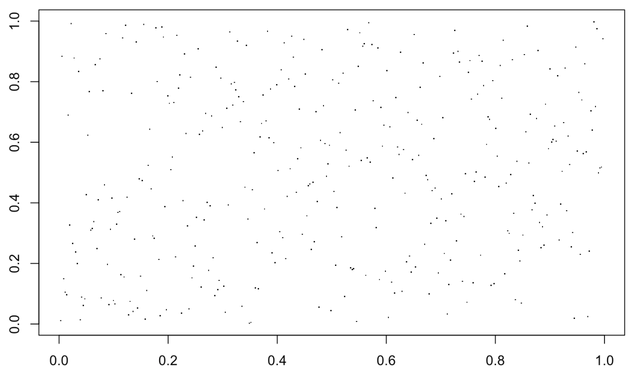


(c) BTC-FTSE100

Figure 5: **Empirical copula of Bitcoin-AEX, Bitcoin-DAX and Bitcoin-FTSE100 with N=360**



(a) BTC-EUR



(b) BTC-GBP

Figure 6: **Empirical copula of Bitcoin-EUR/USD, Bitcoin-DAX and Bitcoin-GBP with N=360**

As the sample size during the Covid 19 pandemic is rather small, the plots are almost indistinguishable and it is really hard to recognize the well known properties shown in figure 3. This proves that, in this case, observing the plots does not help and statistical inference is needed. Therefore, we will not further discuss and compare the plots of the parametric (fitted) copulas with the ones of the empirical copulas. We further analyze the copulas by using statistical inference based on the Akaike Information Criterion (AIC), the Cramér-von Mises test with parametric bootstrapping and the Mean Absolute Error (MAE). Statistical results of the AIC, Cramér-von Mises test and MAE are reported in table 5.

Table 5: AIC, Cramér-von Mises and MAE results of the copula models

Asset Pair	Copula	Copula parameter	AIC	S_n	p-value	MAE
BTC-AEX	Gaussian	0.221***	-15.07	0.014	0.78	0.00494
	Student-t	0.218***	-13.90	0.013	**0.88**	**0.00492**
	Clayton	0.289***	**-17.09**	0.026	0.24	0.00652
	Gumbel	1.131***	-9.57	0.018	0.57	0.00550
	Frank	1.218***	-12.07	0.018	0.51	0.00556
BTC-DAX	Gaussian	0.204***	-12.42	0.015	0.73	0.00517
	Student-t	0.199***	-12.82	0.013	**0.84**	**0.00498**
	Clayton	0.290***	**-17.28**	0.017	0.57	0.00540
	Gumbel	1.121***	-8.30	0.025	0.26	0.00677
	Frank	1.123***	-9.88	0.018	0.48	0.00571
BTC-FTSE100	Gaussian	0.178***	-8.91	0.011	0.96	0.00457
	Student-t	0.163***	-9.55	0.009	**0.99**	**0.00455**
	Clayton	0.244***	**-12.71**	0.013	0.87	0.00482
	Gumbel	1.097***	-5.88	0.013	0.88	0.00495
	Frank	0.872***	-5.29	0.012	0.92	0.00492
BTC-EUR/USD	Gaussian	0.106**	-1.84	0.017	0.58	0.00550
	Student-t	0.113**	**-2.46**	0.015	**0.76**	**0.00530**
	Clayton	0.108*	-0.81	0.024	0.33	0.00665
	Gumbel	1.066***	-2.32	0.020	0.50	0.00571
	Frank	0.669**	-2.32	0.017	0.59	0.00549
BTC-GBP/USD	Gaussian	0.146***	-5.35	0.045	0.012	0.00904
	Student-t	0.140**	-6.52	0.044	0.009	0.00913
	Clayton	0.234***	**-11.30**	0.028	**0.2**	**0.00702**
	Gumbel	1.069***	-2.83	0.059	0.002	0.01034
	Frank	0.791***	-4.01	0.047	0.004	0.00930

Note: * and ** and *** indicate the 10%, 5% and 1% significance levels. S_n is the test statistic of Cramér-von Mises test. The test uses 1000 repetitions of the parametric bootstrap. The lowest AIC values, highest p-values and lowest MAE values are in bold.

For the copula of the BTC and AEX pair, we can observe that the Gaussian, Student-t and Clayton copulas are among the three best performing copulas based on AIC. They have the lowest AIC values which are close to each other. To test the goodness-of-fit and thus check whether the dependence structure of the empirical copula is well represented by the specific parametric family of copulas, we use the Cramér-von Mises test with 1000 repetitions of the parametric bootstrap and the Mean Absolute Error (MAE) metric. The Cramér-von Mises test and the MAE both indicate that the Student-t copula fits the data best. Therefore, we choose the Student-t copula as the most appropriate copula to model the dependency between Bitcoin and the AEX index. This indicates that the data of the Bitcoin and AEX pair is characterized by a symmetric dependence structure. Furthermore, The Kendall's tau (τ) and the lower tail dependence parameter (λ_L) of the Student-t copula are 0.140 and 0.0034, respectively. The τ parameter is positive , while λ_L is close to zero. Following the two conditions for the hedging and safe haven property, we can conclude that Bitcoin does not act as a hedge against AEX, while it does act as a safe haven against AEX during the Covid-19 pandemic.

For the copula of the BTC and DAX pair, we observe the same results. Based on AIC, the Gaussian, Student-t and Clayton copulas are among the three best performing copulas. The Cramér-von Mises and MAE criteria both select the Student-t copula as the one that fits the data best. Therefore, the Student-t copula is chosen as the most appropriate copula to model the dependency between Bitcoin and DAX. This indicates that the data of the pair Bitcoin and DAX is characterized by a symmetric dependence structure. Furthermore, the Kendall's tau (τ) and the lower tail dependence parameter (λ_L) of the Student-t copula are 0.128 and 0.092, respectively. The τ and λ_L parameters are both positive. Following the two conditions for the hedging and safe haven property, we can conclude that Bitcoin does not act as a hedge or a safe haven against DAX during the Covid-19 pandemic.

For the copula of the Bitcoin and FTSE100 pair, we also observe the same results. The Gaussian, Student-t and Clayton are the ones with the lowest AIC values. Their values are close to each other and therefore these copulas are further investigated by using the goodness of fit test and MAE. The Cramer-von Mises test and the MAE both select the Student-t copula as the copula that fits the data best. Therefore, again the Student-t copula is selected as the most appropriate copula. This indicates that the data of the pair Bitcoin and FTSE100 is characterized by a symmetric dependence structure that captures both upper and lower tail dependence. Furthermore, the Kendall's tau (τ) and the lower tail dependence parameter (λ_L) of the Student-t copula are 0.104 and 0.020, respectively. The τ and λ_L parameters are both positive. Following the two conditions for the hedging and safe haven property, we can conclude that Bitcoin does not act as a hedge or a safe haven against FTSE100 during the Covid-19 pandemic.

For the copula of the Bitcoin and EUR/USD pair, the AIC, Cramér-von Mises test and MAE provide the same results. They all select the Student-t copula as the copula that fits the data best. Therefore, the Student-t copula is chosen as the most appropriate one to measure the dependency between Bitcoin and EUR/USD. This indicates that the data of the pair Bitcoin and EUR/USD is characterized by a symmetric dependence structure that exhibits both upper and lower tail dependence. Furthermore, the Kendall's tau (τ) and the lower tail dependence parameter (λ_L) of the Student-t copula are 0.072 and 0.0099, respectively. The τ parameter is positive , while λ_L is close to zero. Following the two conditions for the hedging and safe haven property, we can conclude that Bitcoin does not act as a hedge against EUR/USD, while it does act as a safe haven against EUR/USD during the Covid-19 pandemic.

For the copula of the Bitcoin and GBP/USD pair, the AIC, Cramer-von Mises test and MAE also provide the same results. They all select the Clayton copula as the best performing copula. Therefore the Clayton copula is chosen as the most appropriate copula to measure the dependency between Bitcoin and GBP/USD. This indicates that the data of the pair Bitcoin and GBP/USD is characterized by a asymmetric dependence structure in which only lower tail dependecy exists. The Kendall's tau (τ) and the lower tail dependence parameter (λ_L) of the Student-t copula are 0.105 and 0.052, respectively. The τ and λ_L parameters are both positive. Following the two conditions for the hedging and safe haven property, we can conclude that Bitcoin does not act as a hedge or a safe haven against GBP/USD during the Covid-19 pandemic. Furthermore it appears that in case of the Gaussian, Student-t, Gumbel and Frank copulas, the null hypothesis of the Cramér-von mises test is rejected. This means that the dependence structure of Bitcoin and GBP/USD is not well represented by these copulas. The MAE measure provides the same results.

6 Discussion and Conclusion

This paper uses the VAR-GARCH-BEKK model to analyze return and volatility spillover effects between Bitcoin and major European financial assets. Furthermore, the copula-APARCH model is used to investigate the safe haven and hedging properties of Bitcoin against the assets.

It is found that the return spillover effects between Bitcoin, AEX, DAX, FTSE100 and GBP/USD are one-directional. The returns of the assets do not impact the return of Bitcoin, while past returns of Bitcoin do have a positive impact on the assets. However, these return spillover effects are weak and therefore we can conclude that there is a weak relation between the returns of Bitcoin and the assets. Furthermore, it is found that there are no return spillover effects between Bitcoin and EUR/USD. The volatility spillover effects between Bitcoin and all assets except EUR/USD are found to be bidirectional. It appears that FTSE100, AEX, DAX and GBP/USD have different effects on the volatility of Bitcoin and vice versa. In case of the EUR/USD rate, it appears that there are only one-directional volatility spillover effects. Bitcoin affects EUR/USD, while EUR/USD does not affect Bitcoin. It is found that there are more volatility spillover effects than return spillover effects. These volatility spillover effects are also stronger than the return spillover effects. Furthermore, the effects of Bitcoin on the assets appear to be small compared to the effect of the assets on Bitcoin. This indicates that Bitcoin is still a small asset in comparison to the European assets.

For the safe haven and hedging properties, we found that in all cases except the BTC-GBP pair the Student-t copula fits the data best. This indicates that the data of these asset pairs is characterized by a symmetric dependence structure in which both upper and lower tail dependence can be captured. For the Bitcoin and GBP/USD pair, the Clayton copula fits the data best, indicating that the dependence structure between Bitcoin and GBP/USD is best described by an asymmetric copula where the upper tail dependence is equal to zero. Based on the Student-t and the Clayton copula it appears that Bitcoin does not act as a hedge against the assets. Furthermore, it is found that Bitcoin does act as a safe haven against EUR/USD and AEX, while it does not act as a safe haven against DAX, FTSE100 and GBP/USD.

As these findings have implications for portfolio selection decisions, investors can benefit from it. However, they should be aware that Bitcoin is still a highly volatile and thus risky asset. A limitation of this paper is that the APARCH(1,1) model does not fit the EUR/USD and GBP/USD rates well. Therefore, future research could try to improve the goodness of fit of these two marginals. Furthermore, it might be interesting to investigate how to allocate Bitcoin in a portfolio with the European financial assets.

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8 Appendix

8.1 A

A.1 Jarque-Bera

The Jarque-Bera (JB) test statistic is given by

$$JB = \frac{n}{6} * (S^2 + \frac{(K-3)^2}{4}), \tag{15}$$

where S is the skewness, K the kurtosis and n the sample size. JB has an asymptotically chi-squared dsitribution with two degrees of freedom. H_0 is rejected if JB $\geq X_{1-\alpha,2}$

A.2 Univariate Ljung-Box-Q

The Ljung-Box-Q test statistic is given by

$$Q(h) = N(N+2)\sum_{i=1}^{h} \frac{\hat{p}_i^2}{N-i}, \tag{16}$$

where N is the number of observations, h the lag order and p_i represents the auto-correlation of the observation at lag i. H_0 is rejected if $Q > X^2_{1-\alpha,h}$

A.3 Multivariate Ljung-Box-Q

The multivariate test statistic is given by

$$Q(m) = N^2 \sum_{i=1}^{h} \frac{1}{N-i} tr({\Gamma'}_m {\Gamma_0}^{-1} {\Gamma}_m {\Gamma_0}^{-1}), \tag{17}$$

where $H_0 = \Gamma_m = 0$ for $m > 0$

A.4 Augmented Dickey–Fuller

The Augmented Dickey–Fuller (ADF) test statistic is given by:

$$\Delta y_t = \alpha + +\gamma y_{t-1} + \delta \Delta y_{t-1} + ... + \delta_{p-1} \Delta y_{t-p+1} + \epsilon_t, \tag{18}$$

where α is a constant, δ is the coefficient of the time trend and p is the lag order. If $H_0 : \gamma = 0$ is rejected, it means that no unit root exists.

The ADF **A.5 Cholesky decomposition**

The Cholesky decomposition is given by

$$H_t = L * L^H, \tag{19}$$

where H_t is the covariance matrix, L is the lower triangular matrix and L^H is its transpose. The standardized residuals are derived from the following equation:

$$Lz_t = \epsilon_t, \tag{20}$$

where z_t represents the standardized residuals and ϵ the residuals. The standardized residuals is then given by

$$z_t = L^{-1} * \epsilon_t \tag{21}$$

8.2 B

B.1 Gaussian copula

The bivariate Gaussian copula can be written in the following form:

$$C(u_1, u_2; p) = \phi_2(\phi^{-1}(u_1), \phi^{-1}(u_2)), \tag{22}$$

where u_1 and u_2 are marginal probabilities, ϕ_2 is a multivariate CDF of a standard normal distribution with correlation coefficient p and ϕ^{-1} is the inverse of the standard normal distribution. After deriving equation 8, we obtain the following equation:

$$c(u_1, u_2; p) = \frac{1}{\sqrt{1-p^2}} exp(-\frac{1}{2(1-p^2)}(x_1^2 - 2px_1x_2 + x_2^2)), \tag{23}$$

where p is the linear correlation coefficient between the two time series. For p=0 it is a independence copula, For p=1 it is a comonotonicity copula and for p=-1 it is the countermonotonicity copula. Furthermore, the Gaussian copula does not exhibit tail dependence.

B.2 Student-t copula

The bivariate Student-t copula is specified as follows:

$$C(u_1, u_2; p, v) = T_v(t_{v_1}^{-1}(u_1), t_{v_2}^{-1}(u_2)), \tag{24}$$

Where T_v is the multivariate CDF of the student-t distribution with correlation coefficient p and v degrees of freedom, $t_{v_i}^{-1}$ is the inverse univariate CDF of the Student-t distribution. After deriving equation 10, we obtain the following expression:

$$C(u_1, u_2; p, v) = \frac{K}{\sqrt{1-p^2}}[1 + \frac{1}{v(1-p^2)}(\xi_1^2 - 2p\xi_1\xi_2 + \xi_2^2)]^{\overline{\frac{v+2}{2}}}[(1+v^{-1}\xi_1^2)(1+v^{-1}\xi_2^2)]^{\overline{\frac{v+2}{2}}}, \tag{25}$$

where p is the correlation coefficient, v the degrees of freedom of the student-t distribution, $K = \Gamma(\frac{v}{2})\Gamma(\frac{v+1}{2})^{-2}\Gamma(\frac{v}{2}+1)$ and $\xi_i = t_{v_i}^{-1}(u_i)$. The Student-t copula is characterized by the fact that it is symmetric and has a nonzero tail dependence.

B.3 Clayton copula

The Clayton copula belongs to the Archimedean copula family and is specified as follows:

$$C(u_1, u_2; \alpha) = (\sum_{i=1}^{n} u_i^{-\alpha} - n + 1)^{-1/\alpha}, \tag{26}$$

where α is the parameter of the copula that describes the dependence structure among the variables. for $\alpha = 0$ it means that it is a independence copula and for $\alpha = \inf$ it is a comonotonicity copula. After deriving equation 13, we get the following expression:

$$c(u_1, u_2, \alpha) = (1+\alpha)(u_1^{-\alpha} + u_2^{-\alpha} - 1)^{-1/\alpha-2}(u_1u_2)^{-\alpha-1} \tag{27}$$

The tail dependence of the Clayton copula is asymmetric. It only exhibits lower tail dependence.

B.4 Gumbel copula

The distribution of the Gumbel copula is specified as follows:

$$C(u_1, u_2; \alpha) = exp(-\sum_{i=1}^{n}(-ln(u_i))^{\alpha})^{1/\alpha}, \tag{28}$$

where α is the dependence parameter. $\alpha = 1$ implies independence and $\alpha = \inf$ implies perfect positive dependence. After deriving equation 15, we obtain the following expression:

$$c(u_1, u_2; \alpha) = (A + \alpha - 1)A^{1-\alpha}exp(-A)(u_1u_2)^-1(-lnu_1)^{\alpha-1}(-lnu_2)^{\alpha-1}, \tag{29}$$

where $A = [(-lnu_1)^{\alpha}(-lnu_2)^{\alpha}]^{1/\alpha}$. Also the Gumbel copula has asymmetric tail dependence. It only exhibits upper tail dependence.

B.5 Frank copula

The Frank copula is given by the following equation:

$$C(u_1, u_2; \alpha) = -\frac{1}{\alpha}ln(1 + \frac{\sum_{i=1}^{n} exp(-\alpha u_i) - 1}{exp(-\alpha) - 1}), \tag{30}$$

where α is the copula parameter that satisfies $\alpha \in (-\inf, +\inf)$. This copula allows positive as well as negative dependence values. Furthermore, $\alpha = 0$ implies independence copula. After deriving equation 17, we obtain the following expression:

$$c(u_1, u_2; \alpha) = \frac{\alpha[1 - exp(-\alpha)exp(-\alpha u_1u_2)]}{([1 - exp(-\alpha)] - (1 - exp(-\alpha u_1))(1 - exp(-\alpha u_2))^2} \tag{31}$$

This copula has no tail dependence and thus the lower and upper tail dependencies are equal to zero.

8.3 C

C.1 Kendall's tau and tail dependency

Table 6: Kendall's τ

Copula	Kendall's τ	range of tau
Gaussian	$\tau = (2/\pi) \times arcsin(p_t)$	[-1,1]
Student-t	$\tau = (2/\pi) \times arcsin(p_t)$	[-1,1]
Clayton	$\tau = \alpha_t/(\alpha_t + 2)$	[0,1]
Gumbel	$\tau = 1 - \delta_t^{-1}$	[0,1]
Frank	$\tau = 1 - 4[1 + D_1(\theta_t)]/\theta$	[-1,1]

Note: $D_1(\theta_t) = \frac{1}{\theta_t} \int_0^{\theta_t} \frac{1}{exp(t) - 1} dt$ (Debye function)

Table 7: Tail dependence

Copula	Lower tail dependence (λ_L)	Upper tail dependence (λ_U)
Gaussian	0.00	0.00
Student-t	$\lambda = 2t_{v+1}(\frac{\sqrt{v+1}\sqrt{1-p_t}}{\sqrt{1+p_t}})$	$\lambda = 2t_{v+1}(\frac{\sqrt{v+1}\sqrt{1-p}}{\sqrt{1+p}})$
Clayton	$2^{-1/\alpha_t}$	0.00
Gumbel	0.00	$2 - 2^{1/\delta_t}$
Frank	0.00	0.00

Note: λ_L of the Clayton copula only exists if $\alpha_t > 0$. Also, λ_U of the Gumbel copula only exists if $\delta_t > 1$

C.2 VAR-GARCH-BEKK test statistics

Table 8: Multivariate LB test statistic: BEKK(1,2) model between BTC and the exchange rates

m	Q(m)	df	p-value
1.0	4.5	3.0	1.00
2.0	17.5	12.0	0.13
3.0	25.4	21.0	0.23
4.0	36.2	30.0	0.20
5.0	47.3	39.0	0.17
6.0	59.8	48.0	0.12

Table 9: Multivariate LB test statistic: BEKK(1,2) model between BTC and the stock indices

m	Q(m)	df	p-value
1.0	17.9	10.0	1.00
2.0	28.0	26.0	0.36
3.0	40.6	42.0	0.53
4.0	68.1	58.0	0.17
5.0	76.2	74.0	0.41
6.0	90.8	90.0	0.46

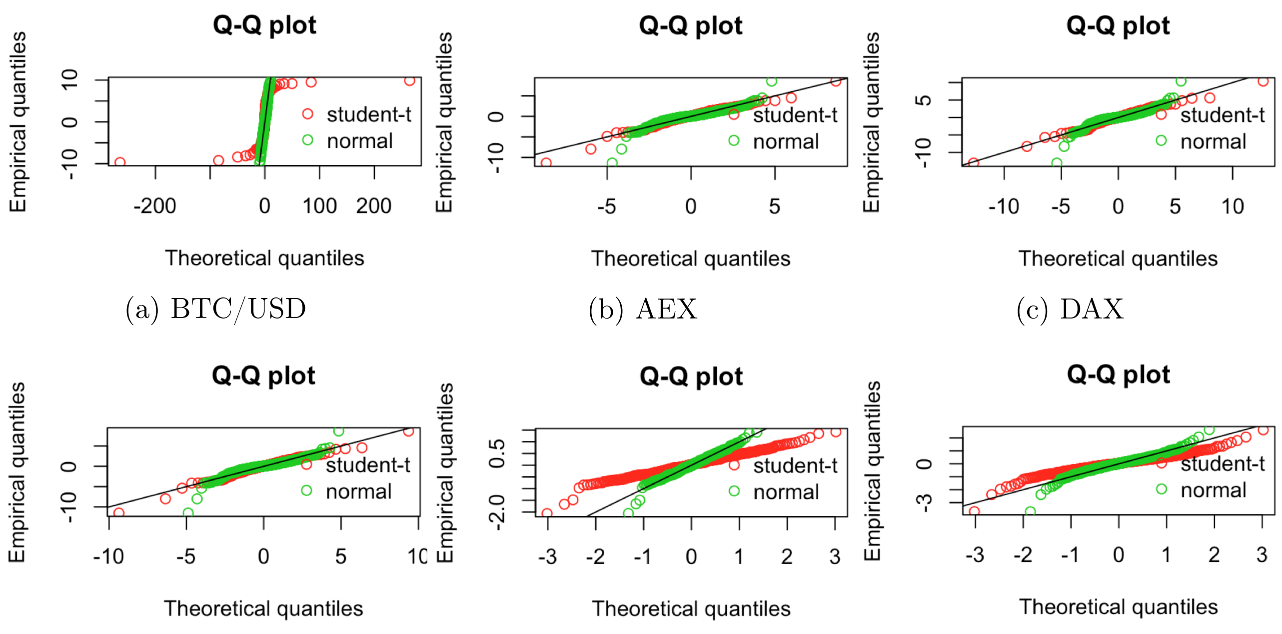
Table 10: Multivariate LB Statistic: VAR(4) model between BTC and the stock indices

m	Q(m)	df	p-value
1.0	0.06	-48.0	1.00
2.0	0.17	-32.0	1.00
3.0	0.53	-16.0	1.00
4.0	0.86	0	1.00
5.0	23.36	16.0	0.10
6.0	43.21	32.0	0.09

Table 11: Multivariate LB test statistics: VAR(4) model between BTC and the exchange rates

m	Q(m)	df	p-value
1.0	0.014	-27.0	1.00
2.0	0.039	-18.0	1.00
3.0	0.1	-9.0	1.00
4.0	0.25	0	1.00
5.0	11.53	9	0.24
6.0	28.46	18	0.06

C.3 QQ-plot marginals



(a) BTC/USD (b) AEX (c) DAX

(d) FTSE100 (e) EUR/USD (f) GBP/USD

Figure 7: **QQ-plot of Bitcoin (BTC), the Amsterdam Stock Exchange Index (AEX), the Deutscher Aktien Index (DAX), the Financial Times Stock Exchange Index (FTSE100), the Euro (EUR/USD) and the Pond (GBP/USD)**