

ERASMUS UNIVERSITY ROTTERDAM

Erasmus school of Economics

Bachelor thesis [Economie en Bedrijfseconomie]

Sustainability certifications in a market with sustainable products, a game theoretic view.

Name student: Michiel Bekkers.

Student ID number: 483584

Supervisor: Karamychev, V.

Second assessor: Delfgaauw, J.

Date final version: 22-08-2021

The views stated in this thesis are those of the author and not necessarily those of the supervisor, second assessor, Erasmus School of Economics or Erasmus University Rotterdam.

Table of Contents

1. Introduction.....	3
2. Literature review	6
2.1. Consumers reaction to sustainability	6
2.2. Transparency	6
2.3. Greenwashing and communication of CSR marketing	7
3. Mathematical formulation and analysis.....	9
3.1. Notation.....	9
3.2. Problem formulation:	10
3.3. Dual-product competitive pricing	15
3.4. Sustainability effort as a choice variable.....	18
4. Insights and limitations	21
4.1. Insights	21
4.2. Limitations	22
5. Conclusion	23
Sources:	24
Appendix.....	28

1. Introduction

Messages about new heat records being set all over the world and reports about workers' rights scandals are part of the daily life of a news reader in 2021. And they signal the huge challenges that we will have to solve as a society going forward. As our economies globalize, solving these problems becomes very difficult, as the number of stakeholders in each issue is enormous. Luckily, because of the internet and modern media, people can be made more aware than ever of these issues. And as of now, 64% of surveyed people in 50 countries by the United Nations Development Programme in 2020 said they think climate change is an emergency. Additionally, reports like the one by The Guardian (2021) about the 6500 deaths already incurred in the construction of the world cup of football, or reports about the Uyghur situation in China, (Business insider, 2021) whip up needed discussion about our responsibility in these issues. The question of how to solve this remains. Should the government interfere? And how? Or should companies and consumers themselves take more responsibility for their actions?

Theory about government intervention to these externality problems have been around for a long time. Most notably the Pigouvian tax, which we find often in current day policy in the form as for example the carbon tax. As well as Coase Theorem, on which the emissions trading system we see in most western countries is based on. These government interventions have proven to be successful, (Lu et al, 2012) but there could be other ways to improve the situation. This paper aims to explore the consumer side of the problem, as the previously mentioned awareness of problems could help to steer our economies into the right direction.

Consumers could be one of the keys to solving the problem. As in a worldwide survey by IBM in 2020, they found that 57% of consumers are willing to help better the environment by changing their purchasing habits. Of these 57%, more than 70% say they are fine with paying more for a product if the production process of that product is done in a sustainable way. And not only consumers are interested. Morningstar (2020) found in a report that investment in companies that practice sustainability has doubled since 2019 to \$51 billion in 2020. Where the companies were measured in terms of environment, social influence, and governance (ESG). These discussed developments open a possibility for business strategy where practicing sustainability in production can become a comparative advantage.

But do consumers have sufficient information in the market to actually change their habit? Unfortunately, this seems to not be the case, as The Sustainability Consortium found in 2016 that in a survey of 1.700 companies only one-fifth had a comprehensive view of their supply chains'

sustainability performance. With more than half reporting that they have no view at all on sustainability issues within their supply chains. Many companies themselves are not aware of how sustainable their supply chains are, and because of this transparency issue, it is hard for consumers to make a choice that fits their concern of certain issues. This could mean that consumers are not able to maximize their utility properly, as they often cannot consider their desire for sustainability when buying a product. Which would imply that the economy currently produces lower social utility than it would if consumers would have the information they need.

As earlier stated, sustainability is a way to differentiate for companies. But as discussed in the previous paragraph, transparency could have the same effect. This is shown in a study by Kraft et al. in 2019. The study shows that when companies improve their visibility of actions in an experiment, this always leads to the consumer having more trust in that company. And that this could lead to more sales. Although there is a problem with this kind of transparency in the form of greenwashing. It could be that transparency of actions by the companies themselves is less reliable than when a third party would provide it. A solution to this could be independent certification companies that provide information on other companies' supply chains. An example of this is B-lab. B-lab is a non-profit organization that gives companies a score based on their performance regarding the environment, governance, worker rights and the effect the company has in their community. This thesis will explore these kinds of certification companies and their role in the economy.

To do this I will evaluate two types of models. Firstly, a Stackelberg type game where sustainable companies choose to get certified based on the price certification companies ask for this service. Secondly, a differentiated Bertrand version of the first game where regular and sustainable producers choose their price simultaneously. And lastly a version of the second model where producers choose the level of sustainability in their production. From this I can analyze which variables are important when looking at the value these certifications have for sustainable producers. I will also address what effect this has on the demand for sustainable products, as well as the optimal effort sustainable producers put towards their sustainability level. The contributions of this thesis can be explained as follows:

- Considering the possibility of sustainability certification in a game theory model of a market.
- Evaluating the important factors in the value of these certifications for companies.
- Evaluating the effect that the possibility of certification has on the demand for sustainable products.
- Evaluating the effect of the possibility of certification on the optimal level of sustainability in the production process of sustainable producers.

To do this the following questions will be answered:

What factors influence the value of sustainability certifications for companies?

Does the possibility of being certified increase the demand for sustainable products?

Does the possibility of certification increase the optimal level of sustainability in the production process of sustainable producers?

The remaining of this thesis will be structured as follows. Section 2 will contain a literature review of relevant literature in this field. Then, Section 3 will describe the mathematical formulation of the different models and present the results and propositions that follow. Section 4 will describe the insights that come out of the analysis of the models, as well as limitations and suggestions for future research will be discussed. And finally, section 5 will conclude the thesis. In the appendix you will find the code used to make numerical examples in python.

2. Literature review

When looking at the interest of this thesis, two fields of study are very relevant. These are the reaction of consumers on sustainability, and the role of transparency in these topics. I will now discuss the relevant literature in this field.

2.1. Consumers reaction to sustainability

The field of study about whether consumers are willing to pay more for sustainable products dates back to just about 1990. Where many find that ecological concerns are great and that consumers prefer products that are sustainably made. (Kerr, 1990; and Donaton and Fitzgerald, 1992) Others find that many companies are already adapting to this movement of consumers. (Williams, 1992; Kassaye and Verma, 1992) Although evidence of the economic relationship between sustainability and financial performance was weak, the field of study proved to be very relevant going forward. Going from mostly ecological to a broader definition of sustainability called Corporate Social Responsibility (CSR), which also takes into account concerns like worker rights and the development of the community. Orlitzky et al. in 2003 established the first link between CSR and financial performance in a meta-analysis of 52 studies, which solidified CSR concerns as a possible comparative advantage.

This opened the door for analysis on how sustainability or CSR concerns influence the decisions of players in a market using game theory models. With Conrad (2005) using a spatial duopoly model to evaluate the choices competing firms make under different levels of environmental concerns by consumers. And Liu et al. (2012) look at the effect of consumers' green considerations under a competitive supply chain, and their results show that with increasing environmental concerns by consumers, retailers and producers with superior ecological operations have an advantage. But these models have a fundamental problem that Mohr et al. already noted in 2001, which is that they either assume that consumers have a perfect view on which firms produce sustainably and which do not, or artificially make sure they do in experimental settings. This is where transparency comes into the analysis.

2.2. Transparency

The comment by Mohr et al. (2001) on the transparency and awareness of consumers of CSR behavior inspired more studies into this problem. For instance, Augur et al. (2003) find that from surveyed subjects from Hong Kong and Australia, only 5 and 10 percent respectively could recall the ethical attributes of their athletic shoes. And Bhattacharya and Sen (2004) report from survey

research that most consumers seem to not be aware of CSR behavior of companies, and that most response comes from a small number of consumers. Augur et al. (2003) show apart from consumers not being aware of ethical attributes of the products they consume, that consumers are largely not aware of the social issues companies try to help by employing CSR behavior in general. Therefore, the problem could lie in the corporate communication stage, Dawkins (2004) argues. From that study Dawkins (2004) does show that consumers are willing to and interested in learning about CSR initiatives, and that therefore CSR and the communication thereof can be used to differentiate brand image.

In a broader view, transparency on issues on country level also show effects on economies. Li and Li (2012) argue from a survey study on Chinese corporations that when environmental transparency by the Chinese government increases, this leads to increases in the production of ecological products in China. Environmental transparency in the form of a transparency-based platform for environmental data can also encourage brand-sensitive supply chains, consumers and even governments to behave more ecologically. (Haddad, 2015)

This idea of transparency being relevant in the demand curve used in sustainable supply chain models is first used by Khosroshahi et al. in 2019. Where they perform game theory analysis on a supply chain model where producers and manufacturers of different levels of sustainability choose their level of transparency based on a convex cost curve of transparency. This model is a step forward in the modeling of CSR marketing in markets but has a fundamental problem. This problem lies in the assumption that actors in the model can choose their own level of transparency and therefore communicate their sustainability level properly. This assumption does not seem to hold as will be explained in the next paragraph.

2.3. Greenwashing and communication of CSR marketing

The issue with companies communicating their own sustainability level lies in greenwashing and the signal CSR marketing gives off to consumers. Greenwashing is when companies use marketing to deceptively persuade consumers that their products are sustainable, even when this is not the case. The existence of companies that do this can make consumers skeptical of the claims of other, possibly truly sustainably producing companies. (Dahl, 2010) This is also seen in a Danish study by Schultz and Morsing (2003), where they found that some consumers react hostile towards CSR communication in marketing. Because of this, they conclude that CSR communication in marketing cannot be done in the same way as traditional marketing. Morsing (2006, p. 176) later found in a study on the rebranding of a Danish telecom company from a shareholder driven company to a more

CSR focused company that their efforts were 'met with skepticism, disbelief and accusations of window-dressing.' For CSR communication to be successful, consumers need to trust the source of the information. (Maignan and Ferrell, 2001) And Obermiller and Spangenberg found in 1998 that consumers are more skeptical of advertisement information in general.

These challenges make it hard for companies to use CSR to differentiate themselves. But a possible solution for these problems could be to have the communication come from a third party, in the form of the previously mentioned certification for instance. At the time of writing this, there exist more than 455 so called 'ecolabels' according to the tracking website ecolabelindex.com. (2021). Them mostly being successful in the food industry (de Andrade Silva et al., 2017; Rousseau, 2015). But individual labels also show success across industries in the form of increased short-term sales. As Paelman et al. (2020) found from their difference in difference study into companies before and after their certification by the certification company B-Lab. Although concrete and long-term evidence is scarce, the studies indicate that consumers react to the information these certifications provide positively.

Taking the positive results of certifications and their possible solution to the problem that the Khosroshahi et al. (2019) model faces, this thesis adds the choice of certification to a basic model of a market between consumers and both sustainable as well as regular producers. By doing this, this thesis will try to explain the factors that are important in the success of certifications and their effect on the strategies of actors in a market.

3. Mathematical formulation and analysis

3.1. Notation

Parameters (all parameters fall between (0, 1)):

a = Total market for the product

c = Marginal costs

k = Sensitivity of consumers for sustainability

e = Sustainability rating of the product

t = Transparency penalty of information in the market, 0 being no penalty and total transparency.

ψ = Costs of sustainable producing

β = Substitution parameter

Decision variables:

V = Cost of getting a certification

p_s = Price sustainable product

p_r = Price regular product.

Dependent variables:

D_s = Demand for the sustainable product

D_r = Demand for the regular product

π_{s0} = Profit the sustainable producer makes if they do not get certified

π_{s1} = Profit the sustainable producer makes if they do get certified

π_r = Profit the regular producer makes

π_{cert} = Profit the certification company makes

3.2. Problem formulation

I evaluate a market which is roughly based on a model by Hong et al (2018), with two producers of a homogenous product. One which sells a non-sustainable version of the product (called the regular producer), and one that sells a sustainable variant of the product (called the sustainable producer). The products are identical in their functionality, but the sustainable version is produced in a way that considers sustainability (CSR) concerns. The effort put into these concerns is denoted as e . The first two models evaluate the short term and therefore it is assumed that e is static and cannot be changed.

In this model it is assumed consumers are aware of environmental and human rights problems in the industry evaluated and derive extra utility from a product if they know that these problems are considered when producing. Therefore, demand for these products will rise with the terms ke . Where k is the sensitivity consumers have towards sustainable products. This utility is assumed to be a linear function based on papers by Chen (2001) and Yalabik and Fairchild (2011). This utility does run into a problem in the form of a transparency penalty, denoted as t . If the transparency in a market is very bad, claims made by sustainable companies are not received well and the terms ke decrease in the term t .

As earlier done in papers by Ferrer and Swaminathan (2006) and Yenipazarli (2016), it is assumed that the market size α is one, and the decision variables p_s and p_r are limited to $(0, 1)$. This way the market shares of the products indicate the choice consumers make between the two products. Because 54% of surveyed people say they do not buy sustainable products because of the increased price (AT Kearney, 2019), the parameter β indicates the level of substitution that is present between the demand of the two products. This term is in favor of the regular product as in this model it is assumed the sustainable product is going to be more expensive in all cases.

The choice consumers take are given by the following demand functions:

$$D_s(p_s, p_r) = a - p_s + ke(1 - t) - \beta(p_s - p_r) \quad | \text{ For sustainable producers} \quad (1)$$

$$D_r(p_r, p_s) = a - p_r + \beta(p_s - p_r) \quad | \text{ For regular producers} \quad (2)$$

Producing sustainably does come with extra costs. As Yalabik and Fairchild (2011) also show, the investment in producing sustainably follows a convex shape implying increasing costs the more sustainably produced a product is. This cost is denoted as ψ in the model, where ψe^2 denotes the total costs the sustainable producer incurs from producing sustainably.

This model introduces a new player, certification companies. These are external companies that evaluate the sustainability of a producer and communicate this to consumers. This way the information on producers is from an external party and can be trusted more. This is used in the model as a way for sustainable producers to remove the negative effect they endure from transparency issues. The producer can pay a certification company a price V , and the certification company will in return will give consumers an accurate view on the sustainability of a producer. This way the transparency penalty parameter t will become 0 and have no effect on the terms ke .

Sustainable producers maximize the following profit function:

$$\pi_{s0} = (p_s - c_s)D(p_s, p_r) - \psi e^2 \quad | \quad \text{In case they do not get certified} \quad (3)$$

$$\pi_{s1} = (p_s - c_s)D(p_s, p_r) - \psi e^2 - V \quad | \quad \text{In case they do get certified} \quad (4)$$

For regular companies this follows a basic function:

$$\pi_r = (p_r - c_r)D(p_s, p_r) \quad (5)$$

Certification companies maximize their profit in V , the price they ask for certification. For the sake of this analysis, it is assumed they incur no costs. Therefore, the profit function is given by:

$$\pi_{cert} = V \quad (6)$$

Timeline of events:

1. The price of the regular product is determined by the perfect competition on the market.
2. Certification companies have perfect information on the other actors and choose the price to get certified V first.
3. The sustainable producer chooses whether to get certified or not and their price. They have perfect information on the price of the regular product.
4. Consumers act according to the previous events.

Because certification companies move first, they can set their price V based on backwards induction. For them their maximum price is at the point where the sustainable producer is indifferent between getting a certification and not getting one. And so, they will set their price just under this point.

At this point the following equation must hold:

$$\pi_{s0} = \pi_{s1} \quad (7)$$

To set up this equation we must first know the equilibrium price producers are going to set in either case. To find this price I look at the game from the sustainable producer's perspective where the price of the regular product is exogenous. This is the case when the regular market is a mature market in which price is set by perfect competition, but the sustainable producer can act as a monopoly. This assumption is made because in most markets sustainable products are still a niche. The problem is to maximize π_{s0} and π_{s1} in p . This gives the following values.

$$p_{s0}^* = \frac{(1+ke-ket+\beta p_r+c_s+\beta c_s)}{(2+2\beta)} \quad (8)$$

$$p_{s1}^* = \frac{(1+ke+\beta p_r+c_s+\beta c_s)}{(2+2\beta)} \quad (9)$$

From these optimal decisions of the producers, we can derive the following propositions about the effects of changes in the parameters on the decisions of producers:

Proposition 1

$\frac{dp_{s0}^*}{dk} = \frac{e-et}{2\beta+2} > 0$ for $t \in (0, 1)$ and $\frac{dp_{s1}^*}{dk} = \frac{e}{2\beta+2} > 0$. This means that for this model, when consumers react more heavily on sustainability, both uncertified as well as certified sustainable producers will increase their price. Although certified producers can increase their price more, depending on the transparency of information in the market.

Proposition 2

$\frac{dp_{s0}^*}{de} = \frac{k-kt}{2\beta+2} > 0$ and $\frac{dp_{s1}^*}{de} = \frac{k}{2\beta+2} > 0$. This means that when producers increase their efforts into producing sustainably, both uncertified as well as certified sustainable producers should increase their price. Again, certified producers can increase their price more, depending on the transparency of information in the market.

Proposition 3

$\frac{dp_{s0}^*}{dt} = -\frac{ke}{2\beta+2} < 0$. This shows that when the transparency on the market worsens, sustainable producers that are not certified must decrease their price.

From these parameter effects it is clear that certification has effects on the pricing strategy for sustainable producers that are mostly dependent on the transparency level of information in the market. To evaluate what the effect is of getting certified on the profit of sustainable producers, I will now derive the optimal pricing that certification companies will ask for their service. In equilibrium, this should be exactly the benefit that sustainable producers gain from the certification.

Entering both optimal price functions as well as the demand function into $\pi_0 = \pi_1$ gives the following result for V :

$$V^* = -\frac{ekt(2\beta c_s - 2\beta_r + 2c_s + ekt - 2ek - 2)}{4(\beta + 1)} \quad (10)$$

And so, the price certification companies will set for providing certification is equal to the result of (10).

To analyze this equilibrium price of certification I will look at six different scenarios of numerical examples. These will include three different levels of sensitivity of consumers to sustainable products k for two different levels of sustainable producing e , and will be plotted using python. The price of the regular product p_r will be set to 0,4 for these examples, as even in the least sustainable producing product, this will mean that the sustainable product is about 20% more expensive. This is in line with a report by A.T Kearney (2020) which states that the average sustainably produced product is about 80% more expensive than their regular peers, and that this markup ranges from about 20% to 250%. The variable costs c_s will be set to 0,2 for all examples as this is not a variable that this paper focusses on. And the substitution effect β will be set to the maximum value 1 to make sure this effect is present in the analysis. By doing these numerical examples I can determine in what scenarios certification is most valuable for producers.

Table 1: The parameter values in the numerical example A, B and C.

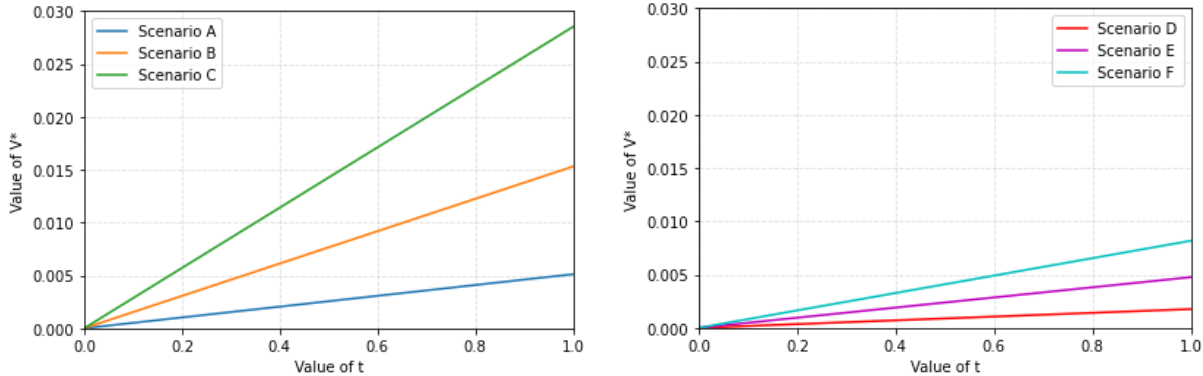
Scenarios	A	B	C
Values	$k = 0,2$	$k = 0,5$	$k = 0,8$
	$e = 0,8$	$e = 0,8$	$e = 0,8$

Table 2: The parameter values in the numerical example D, E and C

Scenarios	D	E	F
Values	$k = 0,2$	$k = 0,5$	$k = 0,8$
	$e = 0,3$	$e = 0,3$	$e = 0,3$

I will now plot the value of V^* for with regards to t for the different scenarios described. After, I will also plot the derivative of V^* with regards to t for all different scenarios.

Figure 1 and 2: The value of V^* for all described scenarios for different levels of t



The following propositions about the effect of the parameters are derived from these plots.

Proposition 4

From figures 1 and 2 it is seen that V^* is strictly rising for all scenarios. From this it is clear that for these scenarios, the value of certifications for sustainable companies rises when the transparency on the market worsens.

Proposition 5

From comparing the scenarios A, B and C and the scenarios D, E and F in figures 1 and 2, it is seen that for a higher level of k , V^* will always be higher. This suggests that the value of certifications rises when the sensitivity of consumers towards sustainability increases.

Also seen in figure 1 and 2 is that in every scenario, certification provides a positive value for sustainable producers, which implies that sustainable producers will always certify if the price of certification is lower than V^* . Of course, this price V^* should be higher than the costs that the certification companies incur for investigating the producer and providing the certification. But the situation where this is not the case is not evaluated in this model. In the evaluated case, where sustainable producers will always choose to certify, we derive the following proposition about the effect on demand.

Proposition 6

$$D_{\text{certified}} > D_{\text{non-certified}}$$

It is seen that in the case where sustainable producers get certified, their demand in the evaluated scenarios is always higher than when they do not certify. So, in the evaluated case where they always get certified, the presence of certification companies leads to more demand for sustainable products.

3.3. Dual-product competitive pricing

The previous section evaluated a game where the sustainable producer could behave monopolistic, and the price of the regular product was determined by a perfectly competitive market and was therefore stable and exogenous in the model. But this is not always the case. If the regular producer can choose price freely, how do the strategies change? And what is the effect of the existence of certifications in that case? To evaluate this, I will set up a differentiated Bertrand game where the regular and sustainable producer compete directly and choose their price simultaneously. The timeline of this game looks as follows:

1. Certification companies have perfect information on the other actors and choose the price to get certified V first.
2. The sustainable producer chooses whether to get certified or not.
3. The sustainable and regular producers choose their respective prices simultaneously.
4. Consumers act according to the previous events.

The demand functions in this game are the same as in the previous game and are given by (1) and (2). The profit functions are also the same and are given by (3) and (4). For this game I assume that both firms do not have fixed costs and variable costs will also be set to 0, this is done to make the math less cluttered. Again, the total market size will be set to 1 and both prices fall under the

window (0,1). To evaluate the price strategies in equilibrium, I will now determine the reaction functions of both producers by solving (3), (4) and (5) with regards to p_s or p_r .

$$p_{s0}^r = \frac{(1+ke(1-t)+\beta p_r)}{(2+2\beta)} \quad | \text{ For uncertified sustainable producers} \quad (13)$$

$$p_{s1}^r = \frac{(1+ke+\beta p_r)}{(2+2\beta)} \quad | \text{ For certified sustainable producers} \quad (14)$$

$$p_r^r = \frac{(1+\beta p_s)}{2(\beta+1)} \quad | \text{ For regular producers} \quad (15)$$

The optimal strategies for the sustainable producers are given by $p_{s0}^r(p_r^r)$ and $p_{s1}^r(p_r^r)$ and solving for p_s :

$$p_{s0}^{**} = \frac{\beta(3-2ek(t-1))-2ek(t-1)+2}{3\beta^2+8\beta+4} \quad | \text{ For the uncertified producer} \quad (16)$$

$$p_{s1}^{**} = \frac{2e\beta k+3\beta+2ek+2}{3\beta^2+8\beta+4} \quad | \text{ For the certified producer} \quad (17)$$

For the regular producer the optimal strategies are given by $p_r^r(p_{s0}^r)$ and $p_r^r(p_{s1}^r)$ and solving for p_r :

$$p_{r0}^{**} = \frac{\beta(e(k-kt)+3)+2}{3\beta^2+8\beta+4} \quad | \text{ If the sustainable producer does not certify} \quad (18)$$

$$p_{r1}^{**} = \frac{e\beta k+3\beta+2}{3\beta^2+8\beta+4} \quad | \text{ If the sustainable producer certifies} \quad (19)$$

From these optimal price strategies, we can again derive a couple propositions.

Proposition 7

$\frac{dp_{s0}^{**}}{dk} = -\frac{2e(\beta+1)(t-1)}{3\beta^2+8\beta+4} > 0$ for $t \in (0, 1)$ and $\frac{dp_{s1}^{**}}{dk} = \frac{\beta e}{3\beta^2+8\beta+4} > 0$. This means that again for this model, when consumers react more heavily on sustainability, both uncertified as well as certified sustainable producers will increase their price. Although certified producers can increase their price more, depending on the transparency of information in the market.

Proposition 8

$\frac{dp_{s0}^{**}}{de} = -\frac{2(\beta+1)k(t-1)}{3\beta^2+8\beta+4} > 0$ and $\frac{dp_{s1}^{**}}{de} = \frac{\beta k}{3\beta^2+8\beta+4} > 0$. This means that when producers increase their efforts into producing sustainably, both uncertified as well as certified sustainable producers should increase their price. Again, certified producers can increase their price more, depending on the transparency of information in the market. This result is consistent with the first model.

Proposition 9

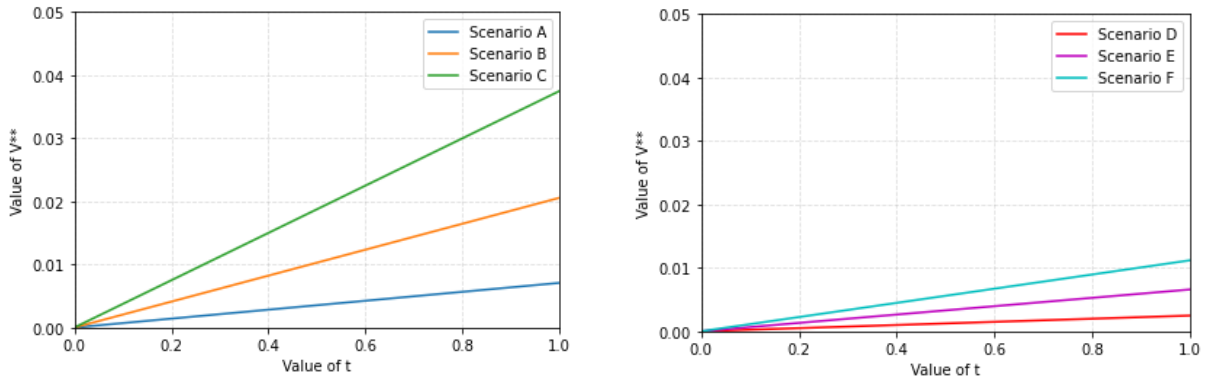
$\frac{dp_{s0}^{**}}{dt} = -\frac{2ek(\beta+1)}{3\beta^2+8\beta+4} < 0$. Similarly to the first model, when the transparency on the market worsens, sustainable producers that are not certified have to decrease their price.

To determine the value of certifications, I will again solve equation (7) but this time I will use p_{s0}^{**} , p_{s1}^{**} , p_{r0}^{**} and p_{r1}^{**} to plug into the profit equations. This gives the following result for V^{**} , structured differently for layout purposes:

$$V^{**} = -\beta(p_{s1}^2 - p_{s1}p_{r1} - p_{s0}^2 + p_{s0}p_{r0}) - ke(p_{s0}(1-t) - p_{s1}) - p_{s1}^2 + p_{s1} + p_{s0}^2 - p_{s0} \quad (20)$$

Like in the previously evaluated game, I will provide a numerical example of 6 possible scenarios of values of the different parameters present in the model. The values of the different parameters are explained in the previous section and are given by table 1 and 2.

Figure 3 and 4: The value of V^{**} for all described scenarios and different levels of t



From these plots flow the following propositions:

Proposition 10

From figures 3 and 4 it is seen that V^{**} is strictly rising in t for all scenarios. From this it is clear that for these scenarios, the value of certifications for sustainable companies rises when the transparency on the market becomes less good. These results are consistent with the previous model.

Proposition 11

From comparing the scenarios A, B and C and the scenarios D, E and F, it is seen that for a higher level of k , V^{**} will always be higher. This suggests that the value of certifications rises when the sensitivity of consumers towards sustainability increases. These results are also consistent with the previous model.

Same as in the previous model, it is seen in figure 3 and 4 is that in every scenario, certification provides a positive value for sustainable producers. So again, in the evaluated situation where certification companies will always offer certification for a price sustainable producers will take, it gives the following proposition about demand for sustainable products.

Proposition 12

$D_{s_{certified}} > D_{s_{non-certified}}$ Similarly to the first model, in this model the existence of certification companies in the market shows a positive effect on the demand for sustainable products.

3.4. Sustainability effort as a choice variable

In the previous two models it is assumed that the effort producers lay into producing sustainably is set and cannot be changed within the game. This describes the short-term choice of producers getting certified or not. But on the long term, producers will be able to change their sustainability effort. The question I will investigate in this section is whether the possibility of certification move producers towards producing more sustainably. For this I will assume that sustainable producers always get certified if this is a possibility.

To analyze this question, I will set up an extended version of the second model, with the extension being that in this version the sustainability effort e is chosen by the producer. The timeline of events will look as follows:

1. Certification companies have perfect information on the other actors and choose the price to get certified V first.
2. The sustainable producer chooses whether to get certified or not.
3. The sustainable producer chooses their sustainability level.
4. The sustainable and regular producers choose their respective prices simultaneously.
5. Consumers act according to the previous events.

To determine how this change affects the results, we must find the optimal sustainability level. This is done by maximizing (3) and (4) with regards to e . But as (3) and (4) are affected by p_s and p_r , their respective optimal values should be entered into the respective profit functions. These optimal price values are given by (16), (17), (18) and (19).

Plugging (16) and (18) into (3) and (17) and (19) into (4) and maximizing both in e will give the optimal sustainability efforts given the optimal price strategies:

$$e_0^* = -\frac{2(\beta+1)^2(3\beta+2)(t-1)k}{(3\beta^2+8\beta+4)^2\psi-4(\beta+1)^3(t-1)^2k^2} \quad | \text{ not certified} \quad (21)$$

$$e_1^* = \frac{2(\beta+1)^2(3\beta+2)k}{(3\beta^2+8\beta+4)^2\psi-4(\beta+1)^3k^2} \quad | \text{ certified} \quad (22)$$

As these equations can give the irrational result of a negative effort level e for sustainable producers, it is important that ψ must be above the following thresholds:

$$\psi > \frac{4(\beta+1)^3(t-1)^2k^2}{(3\beta^2+8\beta+4)^2} \quad | \text{ For } e_0^* \quad (23)$$

$$\psi > \frac{4(\beta+1)^3k^2}{(3\beta^2+8\beta+4)^2} \quad | \text{ For } e_1^* \quad (24)$$

The interest of this version of the model is to see whether the possibility of certification increases the optimal effort into sustainable producing e . In This case $e_1^* > e_0^*$. To evaluate this, I will plot the value of the equation $e_1^* - e_0^*$ in numerical examples that might occur. The numerical examples are given in the following tables 3 and 4. The values ψ indicate two hypothetical markets in which it is relatively expensive or cheap to become more sustainable in producing. β will again be set to 1 for all scenarios in this section.

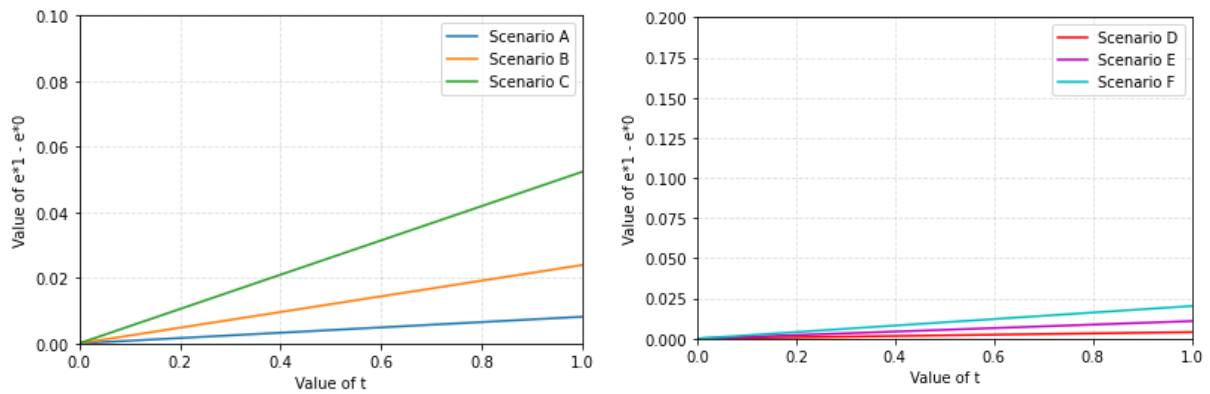
Table 3: The parameter values in the numerical examples A, B and C in section 3.4

Scenarios	A	B	C
Values	$k = 0,2$	$k = 0,5$	$k = 0,8$
	$\psi = 0,5$	$\psi = 0,5$	$\psi = 0,5$

Table 4: The parameter values in the numerical examples D, E and F in section 3.4

Scenarios	D	E	F
Values	$k = 0,2$	$k = 0,5$	$k = 0,8$
	$\psi = 1$	$\psi = 1$	$\psi = 1$

Figure 5 and 6: The value of $e_1^* - e_0^*$ for all described scenarios and different levels of t



From these plots flow a last and important proposition.

Proposition 13

For every scenario evaluated in figures 5 and 6, $e_1^* - e_0^* > 0$. Which implies that for the evaluated scenarios in this model, under the taken assumptions, the possibility of getting certificated increases the optimal effort sustainable producers are going to put into producing sustainably.

It is also seen that this effect is larger when consumers are more sensitive towards sustainable products, when the premium on producing sustainably is less steep and when the sustainability communication penalty t is stronger.

4. Insights and limitations

4.1. Insights

The most important insight for producers of sustainable products is that it is important for them to be aware of the characteristics of their market regarding sustainability, when assessing the choice to become certified. From proposition 1 and 7 it is shown that in both models, when the reaction consumers show towards sustainable products increases, those producers can increase their price. Propositions 5 and 11 suggest that in this case the value a sustainable producer gets from a certification increases, which could make it a more attractive option. A real-world example of this could be when environmental concerns in the target audience increase, or when scandals like the earlier discussed Uyghur situation in China get brought to light and consumers react to the news. Another important characteristic is the transparency level in the market. From proposition 3 and 9 it is found that when the transparency in the market decreases, this should lead sustainable producers to reduce their price, indicating that transparency is important for sustainable producers to realize their advantage. In propositions 4 and 10 it is shown that when the transparency in a market worsens, certifications rise in value. Which could make getting certificated a more attractive option. This could happen when other firms in a market start to employ misleading marketing about their own sustainability, discrediting the claims of truly sustainable producing manufacturers. (Dahl, 2010) On the market level the following insights are given. From propositions 6 and 12 it is apparent that for all described situations where sustainable companies get certified, demand for those companies increases. This result suggests that if the information inefficiency is solved, sustainable producers can differentiate themselves better and capture more market share in markets where consumers get utility out of the sustainability of their product. As well as this, proposition 13 suggests that when certification is possible, the optimal effort that is exerted into producing sustainably by sustainable producers will rise. These insights together answer the research questions of this thesis. Propositions 4, 5, 10 and 11 together suggest that the reaction of consumers towards sustainable products, as well as the transparency of the market are the most important variables that influence the value of certifications. And propositions 6, 12 and 13 explain that the possibility of getting certificated and solving the information problem that consumers face could increase the demand for sustainable products, as well as increase the optimal sustainability level sustainable producers will make their products at. And therefore, this might mean that these certifications could help steer our economy towards a more sustainable way of producing.

4.2. Limitations

There are a couple limitations in this study, however. Firstly, this thesis makes a strong assumption that certifications provide perfect information to consumers. Which is unlikely, as there is a large incentive for corruption present, and scandals have happened. (Yale Environment, 2018) Secondly, this model assumes there are no costs to certification and sustainable producers will always get certified if this is a possibility, which is also not the case in reality. Third, this model leaves out important parts of a market. Including supply chain actors like a retailer and a supplier, or even a financial actor like an investment fund, could make for interesting additions to the models, as this would come closer to reality. And lastly, because of the success of certifications as of now, it could be worthwhile to investigate how governments could play a role in this environment. As this thesis provides a suggestion that certifications could help steer our economies towards a more sustainable way of producing. An example of this could be to investigate whether subsidizing the certification costs could be worthwhile. Or it could be that providing regulation to make sure certifications are not corrupt is an idea worth further research.

5. Conclusion

This thesis evaluated two types of models based on producers of a homogenous product, where one of the two products is produced in a sustainable way. The structure of the first model was so that the market of the regular product is mature, and the sustainable product behaves monopolistically, as is the case in many markets where sustainable products differ from the status quo. Whereas the second model drops this assumption and has the sustainable and regular producer compete directly and choose their price simultaneously. The extended second model evaluated the same type of game as the second model, only in this version the producers can set their sustainability level. This is a more long-term view on the situation described. The models' uniqueness comes from the added option to become certified by a third party, which provides perfect information to consumers about the producer's sustainability level. Which is different from when companies do not certify, as there are transparency issues which make the claims of producers about themselves unreliable. The focus of the models was to evaluate which parameters are important in determining the value these certifications have for producers, as well as determining the effect of the possibility of certification on the sustainability level of a market in general. This was done by deriving the optimal price strategy of sustainable producers and having the certification companies price their certification based on backwards induction of the strategy of the producers. The analysis of the models gave a couple insights that held under both types of models. For sustainable producers most notably that it is important to be aware of the transparency as well as the sensitivity of consumers towards sustainability. And in general, the models suggest that the existence of certifications could increase the demand for sustainable products, as well as increase the optimal sustainability level on which sustainable producers will make their products. Which could mean that these certifications can play a role in redirecting our economy towards more sustainable products and help fight global problems like climate change and workers' rights violations. Because of these insights it could be an interesting area for future study. Most importantly in more in dept and realistic scenarios, as well as the influence governments could have regarding the subject.

Sources:

UNDP and University of Oxford. (2021, January). The Peoples' Climate Vote.

<https://www.undp.org/publications/peoples-climate-vote>

Pattison, P., & McIntyre, N. (2021, February 23). Revealed: 6,500 migrant workers have died in Qatar since World Cup awarded. The Guardian. <https://www.theguardian.com/global-development/2021/feb/23/revealed-migrant-worker-deaths-qatar-fifa-world-cup-2022>

Canales, K. (2021, May 10). 7 Apple suppliers in China have links to forced labor programs, including the use of Uyghur Muslims from Xinjiang, according to a new report. Business Insider.

<https://www.businessinsider.com/apple-china-suppliers-uyghur-muslims-forced-labor-report-2021-5?international=true&r=US&IR=T>

Yujie Lu, Xinyuan Zhu, Qingbin Cui,

Effectiveness and equity implications of carbon policies in the United States construction industry, Building and Environment,

Volume 49,

2012,

Pages 259-269,

ISSN 0360-1323,

<https://doi.org/10.1016/j.buildenv.2011.10.002>.

(<https://www.sciencedirect.com/science/article/pii/S0360132311003519>)

Institute For Business Value. (2020). Meet the 2020 consumers driving change.

<https://www.ibm.com/downloads/cas/EXK4XKX8>

Greening global supply chains: From blind spots to hot spots to action, The Sustainability Consortium, 2016, sustainabilityconsortium.org.

Kraft, Tim and Valdés, León and Zheng, Yanchong, Consumer Trust in Social Responsibility Communications: The Role of Supply Chain Visibility (October 27, 2020).

Kerr, Kevin, "Thinking Green is No Longer a Hippie Dream," Ad Weeks Marketing Week, (1990), Vol. 31, 18-19.

Donaton, Scott and Kate Fitzgerald, "Polls Show Ecological Concerns is Strong," Advertising Age, (1992), Vol. 63, 49.

Kassaye, Wossen and D. Verma, "Balancing Traditional Packaging Functions with the New Green Packaging Concerns," Advanced Management Journal, (1992), Vol. 57, 15-23.

Williams, Monte, "Environmentally Safe can Enhance Sales," Advertising Age, (1992), Vol. 63, 8-9.

Conrad, K. (2005). Price competition and product differentiation when consumers care for the environment. *Environmental and Resource Economics*, 31(1), 1-19.

Z.L. Liu, T.D. Anderson, J.M. Cruz

Consumer environmental awareness and competition in two-stage supply chains

Eur. J. Oper. Res., 218 (3) (2012), pp. 602-613

Mohr, L. A., D. J. Webb and K. E. Harris: 2001, 'Do Consumers Expect Companies to be Socially Responsible? The Impact of Corporate Social Responsibility on Buying Behavior', *Journal of Consumer Affairs* 35(1), 45–72.

Auger, P., P. F. Burke, T. M. Devinney and J. J. Louviere: 2003, 'What will Consumers Pay for Social Product Features?', *Journal of Business Ethics* 42(3), 281–304.

Bhattacharya, C. B. and S. Sen: 2004, 'Doing Better at Doing Good: Why, Why, and How Consumers Respond to Corporate Social Initiatives', *California Management Review* 47(1), 9–24.

Dawkins, J.: 2004, 'The Public's Views of Corporate Responsibility 2003', White Paper Series, MORI. Available from <http://mori.com>, accessed 31 May 2006.

W. Li, D. Li
Environmental information transparency and implications for green growth in China
Publ. Adm. Dev., 32 (3) (2012), pp. 324-334

M.A. Haddad
Increasing environmental performance in a context of low governmental enforcement: evidence from China
J. Environ. Dev., 24 (1) (2015), pp. 3-25

Dahl, Richard (1 June 2010). "Greenwashing: Do You Know What You're Buying?". *Environmental Health Perspectives*. 118 (6): a246–a252. doi:10.1289/ehp.118-a246. PMC 2898878. PMID 20515714.

Schultz, M. and M. Morsing: 2003, 'The Catch 22 of Integrating CSR and Marketing: Findings from a Reputation Study of Danish Companies', *Conference Proceedings, MSI, Boston University*, 17–19 September 2003.

Maignan, I. and O. C. Ferrell: 2001, 'Corporate Citizenship as a Marketing Instrument – Concepts, Evidence, and Research Directions', *European Journal of Marketing* 35(3–4), 457–484.

Obermiller, C. and E. R. Spangenberg: 1998, 'Development of a Scale to Measure Consumer Scepticism Toward Advertising', *Journal of Consumer Psychology* 7(2), 159–186.

De Andrade Silva, A.R., Bioto, A.S., Efraim, P. and de Castilho Queiroz, G. (2017), "Impact of sustainability labeling in the perception of sensory quality and purchase intention of chocolate consumers", *Journal of Cleaner Production*, Vol. 141 No. 10, pp. 11-21.

Rousseau, S. (2015), "The role of organic and fair trade labels when choosing chocolate", *Food Quality and Preference*, Vol. 44, pp. 92-100.

Paelman V, Van Cauwenberge P, Vander Bauwhede H. Effect of B Corp Certification on Short-Term Growth: European Evidence. *Sustainability*. 2020; 12(20):8459.

Hossein Khosroshahi, Morteza Rasti-Barzoki, Seyed Reza Hejazi,
A game theoretic approach for pricing decisions considering CSR and a new consumer satisfaction
index using transparency-dependent demand in sustainable supply chains,
Journal of Cleaner Production,
Volume 208,
2019,
Pages 1065-1080,
ISSN 0959-6526,

Chen, 2001

C. Chen

Design for the environment: a quality-based model for green product development
Manage. Sci., 47 (2) (2001), pp. 250-263

Yalabik and Fairchild, 2011

B. Yalabik, R.J. Fairchild

Customer, regulatory, and competitive pressure as drivers of environmental innovation
Int. J. Prod. Econ., 131 (2) (2011), pp. 519-527

Zhaofu Hong, Hao Wang, Yugang Yu,

Green product pricing with non-green product reference,

Transportation Research Part E: Logistics and Transportation Review,

Volume 115,

2018,

Pages 1-15,

ISSN 1366-5545,

<https://doi.org/10.1016/j.tre.2018.03.013>.

(<https://www.sciencedirect.com/science/article/pii/S1366554517311067>)

Ferrer and Swaminathan, 2006

G. Ferrer, J.M. Swaminathan

Managing new and remanufactured products

Manage. Sci., 52 (1) (2006), pp. 15-26

Yenipazarli, 2016

A. Yenipazarli

Managing new and remanufactured products to mitigate environmental damage under emissions
regulation

Eur. J. Oper. Res., 249 (1) (2016), pp. 117-130

Read @Kearney: Harvesting the green opportunity for CPGs and retailers. (2019). AT Kearney.

<https://www.nl.kenarney.com/consumer-retail/article/?/a/harvesting-the-green-opportunity-for-cpgs-and-retailers>

Read @Kearney: Why today's pricing is sabotaging sustainability. (2020). AT Kearney.

<https://www.nl.kenarney.com/consumer-retail/article/?/a/why-todays-pricing-is-sabotaging-sustainability>

Conniff, R. (2018, 20 februari).

Greenwashed Timber: How Sustainable Forest Certification Has Failed. Yale E360.

<https://e360.yale.edu/features/greenwashed-timber-how-sustainable-forest-certification-has-failed>

Appendix

Python do-file:

```
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd

t = np.linspace(0, 1, 10, endpoint=True)

# scenario A
c = 0.2
e=0.8
k=0.2

# scenario B
w = 0.5

#scenario C
u = 0.8

#scenario DEF
q = 0.3

b = 1
o = 0.4
h = .5
j = 1

v1= -(e*k*t*(2*b*c-2*b*o+2*c+e*k*t-2*e*k-2))/(4*(b+1))
v2= -(e*w*t*(2*b*c-2*b*o+2*c+e*w*t-2*e*w-2))/(4*(b+1))
```

$$v3 = -(e^{*u}t^{*t}(2^{*b}c-2^{*b}o+2^{*c}+e^{*u}t-2^{*e}u-2))/(4^{*}(b+1))$$

$$v4 = -(q^{*k}t^{*t}(2^{*b}c-2^{*b}o+2^{*c}+q^{*k}t-2^{*q}k-2))/(4^{*}(b+1))$$

$$v5 = -(q^{*w}t^{*t}(2^{*b}c-2^{*b}o+2^{*c}+q^{*w}t-2^{*q}w-2))/(4^{*}(b+1))$$

$$v6 = -(q^{*u}t^{*t}(2^{*b}c-2^{*b}o+2^{*c}+q^{*u}t-2^{*q}u-2))/(4^{*}(b+1))$$

$$g1 = -(e^{*k}(c^{*}(b+1)-b^{*}o+e^{*k}(t-1)-1))/(2^{*}(b+1))$$

$$g2 = -(e^{*w}(c^{*}(b+1)-b^{*}o+e^{*w}(t-1)-1))/(2^{*}(b+1))$$

$$g3 = -(e^{*u}(c^{*}(b+1)-b^{*}o+e^{*u}(t-1)-1))/(2^{*}(b+1))$$

$$g4 = -(q^{*k}(c^{*}(b+1)-b^{*}o+q^{*k}(t-1)-1))/(2^{*}(b+1))$$

$$g5 = -(q^{*w}(c^{*}(b+1)-b^{*}o+q^{*w}(t-1)-1))/(2^{*}(b+1))$$

$$g6 = -(q^{*u}(c^{*}(b+1)-b^{*}o+q^{*u}(t-1)-1))/(2^{*}(b+1))$$

$$ps0A = (b^{*}(3-2^{*e}k^{*}(t-1))-2^{*e}k^{*}(t-1)+2)/(3^{*}b^{**2}+8^{*}b+4)$$

$$pr0A = (b^{*}(e^{*}(k-k^{*}t)+3)+2)/(3^{*}b^{**2}+8^{*}b+4)$$

$$ps1A = (2^{*e}b^{*}k+3^{*}b+2^{*e}k+2)/(3^{*}b^{**2}+8^{*}b+4)$$

$$pr1A = (e^{*}b^{*}k+3^{*}b+2)/(3^{*}b^{**2}+8^{*}b+4)$$

$$ps0B = (b^{*}(3-2^{*e}w^{*}(t-1))-2^{*e}w^{*}(t-1)+2)/(3^{*}b^{**2}+8^{*}b+4)$$

$$pr0B = (b^{*}(e^{*}(w-w^{*}t)+3)+2)/(3^{*}b^{**2}+8^{*}b+4)$$

$$ps1B = (2^{*e}b^{*}w+3^{*}b+2^{*e}w+2)/(3^{*}b^{**2}+8^{*}b+4)$$

$$pr1B = (e^{*}b^{*}w+3^{*}b+2)/(3^{*}b^{**2}+8^{*}b+4)$$

$$ps0C = (b^{*}(3-2^{*e}u^{*}(t-1))-2^{*e}u^{*}(t-1)+2)/(3^{*}b^{**2}+8^{*}b+4)$$

$$pr0C = (b^{*}(e^{*}(u-u^{*}t)+3)+2)/(3^{*}b^{**2}+8^{*}b+4)$$

$$ps1C = (2^{*e}b^{*}u+3^{*}b+2^{*e}u+2)/(3^{*}b^{**2}+8^{*}b+4)$$

$$pr1C = (e^{*}b^{*}u+3^{*}b+2)/(3^{*}b^{**2}+8^{*}b+4)$$

$$ps0D = (b^{*}(3-2^{*q}k^{*}(t-1))-2^{*q}k^{*}(t-1)+2)/(3^{*}b^{**2}+8^{*}b+4)$$

$$pr0D = (b^{*}(q^{*}(k-k^{*}t)+3)+2)/(3^{*}b^{**2}+8^{*}b+4)$$

$$ps1D = (2^{*q}b^{*}k+3^{*}b+2^{*q}k+2)/(3^{*}b^{**2}+8^{*}b+4)$$

$$pr1D = (q^{*}b^{*}k+3^{*}b+2)/(3^{*}b^{**2}+8^{*}b+4)$$

$$ps0E = (b^{*}(3-2^{*q}w^{*}(t-1))-2^{*q}w^{*}(t-1)+2)/(3^{*}b^{**2}+8^{*}b+4)$$

$$pr0E = (b^{*}(q^{*}(w-w^{*}t)+3)+2)/(3^{*}b^{**2}+8^{*}b+4)$$

$$ps1E = (2^{*q}b^{*}w+3^{*}b+2^{*q}w+2)/(3^{*}b^{**2}+8^{*}b+4)$$

$$pr1E = (q^{*}b^{*}w+3^{*}b+2)/(3^{*}b^{**2}+8^{*}b+4)$$

$$ps0F = (b*(3-2*q*u*(t-1))-2*q*u*(t-1)+2)/(3*b**2+8*b+4)$$

$$pr0F = (b*(q*(u-u*t)+3)+2)/(3*b**2+8*b+4)$$

$$ps1F = (2*q*b*u+3*b+2*q*u+2)/(3*b**2+8*b+4)$$

$$pr1F = (q*b*u+3*b+2)/(3*b**2+8*b+4)$$

$$vbA = -b*(ps1A**2-ps1A*pr1A-ps0A**2+ps0A*pr0A)-k*e*(ps0A*(1-t)-ps1A)-ps1A**2+ps1A+ps0A**2-ps0A$$

$$vbB = -b*(ps1B**2-ps1B*pr1B-ps0B**2+ps0B*pr0B)-w*e*(ps0B*(1-t)-ps1B)-ps1B**2+ps1B+ps0B**2-ps0B$$

$$vbC = -b*(ps1C**2-ps1C*pr1C-ps0C**2+ps0C*pr0C)-u*e*(ps0C*(1-t)-ps1C)-ps1C**2+ps1C+ps0C**2-ps0C$$

$$vbD = -b*(ps1D**2-ps1D*pr1D-ps0D**2+ps0D*pr0D)-k*q*(ps0D*(1-t)-ps1D)-ps1D**2+ps1D+ps0D**2-ps0D$$

$$vbE = -b*(ps1E**2-ps1E*pr1E-ps0E**2+ps0E*pr0E)-w*q*(ps0E*(1-t)-ps1E)-ps1E**2+ps1E+ps0E**2-ps0E$$

$$vbF = -b*(ps1F**2-ps1F*pr1F-ps0F**2+ps0F*pr0F)-u*q*(ps0F*(1-t)-ps1F)-ps1F**2+ps1F+ps0F**2-ps0F$$

$$eA = ((2*((b+1)**2)*(3*b+2)*k)/(h*(3*b**2+8*b+4)**2-k**2*4*(b+1)**3))-((2*(b+1)**2*(3*b+2)*(t-1)*k)/((3*b**2+8*b+4)**2*h-4*(b+1)**3*(t-1)**2*k**2))$$

$$eB = ((2*((b+1)**2)*(3*b+2)*w)/(h*(3*b**2+8*b+4)**2-w**2*4*(b+1)**3))-((2*(b+1)**2*(3*b+2)*(t-1)*w)/((3*b**2+8*b+4)**2*h-4*(b+1)**3*(t-1)**2*w**2))$$

$$eC = ((2*((b+1)**2)*(3*b+2)*u)/(h*(3*b**2+8*b+4)**2-u**2*4*(b+1)**3))-((2*(b+1)**2*(3*b+2)*(t-1)*u)/((3*b**2+8*b+4)**2*h-4*(b+1)**3*(t-1)**2*u**2))$$

$$eD = ((2*((b+1)**2)*(3*b+2)*k)/(j*(3*b**2+8*b+4)**2-k**2*4*(b+1)**3))-((2*(b+1)**2*(3*b+2)*(t-1)*k)/((3*b**2+8*b+4)**2*j-4*(b+1)**3*(t-1)**2*k**2))$$

$$eE = ((2*((b+1)**2)*(3*b+2)*w)/(j*(3*b**2+8*b+4)**2-w**2*4*(b+1)**3))-((2*(b+1)**2*(3*b+2)*(t-1)*w)/((3*b**2+8*b+4)**2*j-4*(b+1)**3*(t-1)**2*w**2))$$

$$eF = ((2*((b+1)**2)*(3*b+2)*u)/(j*(3*b**2+8*b+4)**2-u**2*4*(b+1)**3))-((2*(b+1)**2*(3*b+2)*(t-1)*u)/((3*b**2+8*b+4)**2*j-4*(b+1)**3*(t-1)**2*u**2))$$

plt.xlabel('t -axis')

plt.ylabel('y - axis')

plt.plot(v1,label='Scenario A')

plt.plot(v2,label='Scenario B')

plt.plot(v3,label='Scenario C')

plt.xlim(0,1)

```
plt.ylim(0,.03)
plt.grid(alpha=.4,linestyle='--')
plt.legend()
plt.show()

plt.xlabel('t -axis')
plt.ylabel('y - axis')
plt.plot(v4,label='Scenario D',color = 'red')
plt.plot(v5,label='Scenario E',color = 'm')
plt.plot(v6,label='Scenario F',color = 'c')
plt.xlim(0,1)
plt.ylim(0,.03)
plt.grid(alpha=.4,linestyle='--')
plt.legend()
plt.show()
```

```
plt.xlabel('t -axis')
plt.ylabel('y - axis')
plt.plot(g1,label='Scenario A')
plt.plot(g2,label='Scenario B')
plt.plot(g3,label='Scenario C')
plt.xlim(0,1)
plt.ylim(0,0.3)
plt.grid(alpha=.4,linestyle='--')
plt.legend()
plt.show()
```

```
plt.xlabel('t -axis')
plt.ylabel('y - axis')
plt.plot(g4,label='Scenario D',color = 'red')
```

```
plt.plot(g5,label='Scenario E',color = 'm')
plt.plot(g6,label='Scenario F',color = 'c')
plt.xlim(0,1)
plt.ylim(0,.3)
plt.grid(alpha=.4,linestyle='--')
plt.legend()
plt.show()
```

```
plt.xlabel('Value of t')
plt.ylabel('Value of V*')
plt.plot(vbA,label='Scenario A')
plt.plot(vbB,label='Scenario B')
plt.plot(vbC,label='Scenario C')
plt.xlim(0,1)
plt.ylim(0,.05)
plt.grid(alpha=.4,linestyle='--')
plt.legend()
plt.show()
```

```
plt.xlabel('Value of t')
plt.ylabel('Value of V*')
plt.plot(vbD,label='Scenario D', color = 'red')
plt.plot(vbE,label='Scenario E', color = 'm')
plt.plot(vbF,label='Scenario F', color = 'c')
plt.xlim(0,1)
plt.ylim(0,.05)
plt.grid(alpha=.4,linestyle='--')
plt.legend()
plt.show()
```



```
plt.xlabel('Value of t')
plt.ylabel('Value of e*1 - e*0')
plt.plot(eA,label='Scenario A')
plt.plot(eB,label='Scenario B')
plt.plot(eC,label='Scenario C')
plt.xlim(0,1)
plt.ylim(0,.7)
plt.grid(alpha=.4,linestyle='--')
plt.legend()
plt.show()
```

```
plt.xlabel('Value of t')
plt.ylabel('Value of e*1 - e*0')
plt.plot(eD,label='Scenario D', color = 'red')
plt.plot(eE,label='Scenario E', color = 'm')
plt.plot(eF,label='Scenario F', color = 'c')
plt.xlim(0,1)
plt.ylim(0,.7)
plt.grid(alpha=.4,linestyle='--')
plt.legend()
plt.show()
```