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Scheduling Container Liners between Asia and Europe with different speeds

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1. Introduction

By far most of the goods traded internationally are transported by sea. Within this transport mode, containers have a large part of the total market. Because Asia is still growing (even under the current economic situation), more and more goods are transported between Asia and Europe. The picture below depicts the development of container shipping until 2008, from the website of Hofstra (Hofstra, 2009), which took it from drewry shipping consultants.

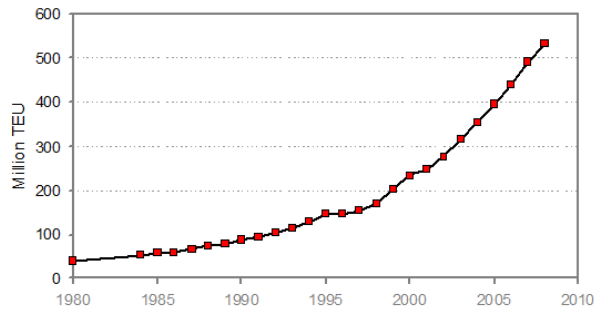


Figure 1: World container traffic 1980-2008, from drewry shipping consultants.

Further, fuel consumption is a hot issue nowadays. People are getting more aware of the dangers of too much air pollution, so many initiatives are set up to limit damage to the environment.

Today, many planning problems in the liner shipping business are solved by hand, using simple calculations in excel spreadsheets, based on a lot of simplification (Man, 2007). Because of this, much gain can come from solving the planning problems in a mathematical way. The main question of this thesis will be:

- What is an easy-to-use, straight-forward way to optimize cyclical liner shipping routes between Asia and Europe?

This thesis introduces a method too tackle the problem of optimizing schedules for shipping lines carrying containers between Asia and Europe. These ships can stop at some mayor stops in the Far East, Middle East and Europe. The idea is that from these world ports, feeder lines will transport goods to smaller ports, not covered by these Asia-Europe lines. This topic is widely covered by many previous authors.

One aspect of shipping routes is the time it takes; this is directly dependent on the speed. So the question raises which speed a ship should follow. I this thesis an answer will be given to the question:

- What is the influence of different speeds on the optimization shipping?

The thesis will start with a description of the problem, where we will give a short definition of the problem, assumptions, data for an example and some insight in previous literature around this subject. Next, we will describe the methods used in this

project, where after we present the results of this methods in application to the given example. At last, the results will be discussed and some conclusions are drawn. In the last chapter of this paper, some suggestions for further research and a list of used literature can be found.

In short, this paper will present a straight-forward algorithm to optimize those schedules and will present an extension to this algorithm to make it even better. Further, it will give some insight in the influence of speed on optimizing transport schedules in the liner shipping business.

2 Problem formulation

The idea of this thesis is to present a way to optimize shipping routes between Asia and Europe. Ships will sail according to static routes, which have their own ports to stop at. A route is described as a collection of ports where the ships following stop. Routes are return trips: from Asia to Europe and back. An important requirement to the routes is that they have to stop at a port on the same day of every week. This means that a route requires as much ships as a ship needs weeks to sail the route (rounded up and with safety days). If a route takes eight weeks for example, eight ships have to sail on that route to meet the requirement of weekly service in each service (see also *assumptions*).

In each port where a certain route stops, a ship will unload all containers which have the current port as destination and load as much containers as possible, to transport them to other ports on its route. The amount of containers transported between two ports can not be bigger then the transportation demands between those ports and can not be bigger then the capacity of the ships going between those two ports.

The goal is to maximize profit, or maximize the difference between total revenue and total costs. The costs I take into account are the capital and operating costs of ships, fuel costs and port costs.

2.1 The main problem

In words, profit has to be maximized, under the condition that the containers that are to be transported to make that profit, don't exceed the transport demand and don't exceed the total capacity of the routes that are in service. A special feature of the problem in this thesis is the variation of speed, which I will describe in the comments about the fuel costs.

The problem is a Mixed Integer Programming problem (MIP), because it needs an integer decision variable and a binary decision variable next to continuous variables. It can be formulated in a mathematical way as follows:

$$\begin{aligned} \max \sum_{i \in P} \sum_{j \in P, j \neq i} \sum_{s \in S} RT_{sij} - C_C A_s - C_F T_{sij} - C_P T_{sij} \\ \sum_{i < k < j} T_{sij} + \sum_{j > k} T_{skj} \leq cap_s A_s & \text{for all } k \text{ in route } s & (1) \\ \sum_s T_{sij} \leq D_{ij} & \text{for all } i, j & (2) \\ T_{sij} \text{ integer} & \text{for all } s, i, j & (3) \\ A_s \text{ binary} & \text{for all } s & (4) \end{aligned}$$

The decision variables here are:

T_{sij} the total amount of goods in TEU transported over a year from port i to port j by route s
 A_s 1 if route s is in service, 0 elsewhere

Parameters:

R revenue per delivered TEU of goods per year
 D_{ij} transport demand in TEU's from port i to port j
 C_C capital and operating costs per ship per year
 C_F fuel costs per nautical mile
 C_P port costs per stopover

Explanation restrictions:

- (1) ships may never carry more goods than their capacity allows them to, and the used ships must be in service (so that the capital and operating costs are paid).
- (2) The total amount of goods carried by all used ships coming from any ports i and going to any port j cannot be higher than the total transport demand from port i to port j .

2.2 Remarks about variables

To make things more clear, some remarks about the main problem. First, the number of routes s can be as large as needed. The number of ships that can be used to realize all routes is also unlimited. Further, I like to make some things clear about the Revenue and costs:

Revenue

There will be assumed that the revenue is the same for each TEU transported, no matter what distance it must travel (see also *assumptions*).

Capital and operating costs

The costs per ship per year. The total costs for a return trip per year can be multiplied by the duration of the trip in week, because of the assumption that each port must be visited once a week (Man).

Fuel costs

Man worked with an assumed speed of the vessel, with a speed from Asia to Europe a bit higher than on the way back. I will use with varying speeds. For the fuel costs per nautical mile, I will use a function introduced by Rommert Dekker during personal communication in 2009.

Port costs

The port costs are assumed to be the same for each port.

2.3 NP-hardness

It is defensible that the main problem is NP-hard by comparing it with the Travelling Salesman Problem (TSP), which can be formulated like this:

$$\text{Min } \sum_{i,j} c_{ij} x_{ij}$$

$$\sum_i x_{ij} \geq 1 \quad \text{for all } j \quad (1)$$

$$\sum_j x_{ij} \geq 1 \quad \text{for all } i \quad (2)$$

$$u_i - u_j + nx_{ij} \leq n - 1 \quad \text{for } i, j \geq 2 \text{ and } i \text{ not } j \quad (3)$$

$$x_{ij} \text{ binary} \quad \text{for all } i, j \quad (4)$$

One can see that the objective function contains

$$\sum_{i \in P} \sum_{j \in P, j \neq i} \sum_{s \in S} RT_{sij},$$

which is a complicated version of the objective function of the standard TSP above, when one takes the minus of the objective function.

Further, when one sets constraints 1 and two of the main problem formulation in minus, one gets complicated versions of constraints 1 and 2 of the formulation of TSP above.

Because the main problem can be seen as a complicated version of TSP and TSP is known as a NP-hard problem, the main problem of this thesis is shown to be a NP-hard problem.

2.4 Assumptions main problem

To limit the main problem to doable proportions, some assumptions are made. We will first state the assumptions Man made and give the differences between his assumptions and mine. In sub 2.2.3, I will give some assumption Man did not mention, but which are made in both mine and his thesis. In the last part of this sub-chapter, we will stress the impact of the most important assumptions.

2.4.1 Assumptions previous author

Man made the following assumptions (Man, 2007):

1. The yearly demands in TEU to be transported between each pair of ports are given.
2. There are no demands between ports that are in the same area. Assumed is that these are serviced by other lines, for transport over shorter distances.
3. The time that a ship spends in a seaport is constant. It takes the time into account for berthing and exiting, as well as the time for loading and unloading the containers. This same duration is also applied in case of transshipment in a hub.
4. All vessels are of the same type, that is, they have the same speed, capacity and costs.
5. For each route there is a call at least once a week, at fixed days. This means that if the duration of a trip is n days (including times spent in ports), then $\lceil n/7 \rceil$ ships are needed.
6. The slack of a route (= the time between the duration of a round trip and the next integer number of weeks) should be between a minimum and a maximum.
7. The total number of routes that can be used, and therefore the total number of ships, is unlimited.
8. The shipping costs are divided into capital and operating costs, fuel costs, port charges and transshipment costs. Capital and operating costs are the total costs for using the ship each day. For example, the cost of owning the ship, crew wages, maintenance costs and insurance. Fuel costs depend on the total distance covered on the route. Port charges represent the costs for pilotage, towage, berth occupancy, and container handling; excluding transshipment handlings; the port charges are the same for all ports, independent of the amount handled. Transshipment costs are costs for transshipping containers from one ship to another. These costs are per transshipment handling (either loading or offloading), and are independent of the amount handled.
9. The revenues of satisfying the demand per container are constant per origin and destination, and the same over all pairs of ports.
10. All routes have a certain direction in which a ship sails. Routes do not need to be back-and-forth, so a ship is allowed to visit a port only once.
11. The demands may be served by multiple routes.

As will be seen, most of these assumptions will stay intact, but some will be changed to expand investigation on this subject.

2.4.2 Difference between with previous assumptions

The first difference between the assumptions Man (Man, 2007) made and the assumptions behind this project is that I have changed assumption 6. I demand a slack of at least one day, but I don't set a maximum slack, for it doesn't seem to be a problem if a ship has to be at sea for a few more days, if that gives a more optimal solution. Further, assumption 4 is different, because I will vary speed and fuel consumption as a consequence of this speed changes. I will come back on this point later in this thesis.

2.4.3 Further general assumptions

Further assumptions not mentioned are:

12. there is no timeframe for the delivery of the goods
13. the demand of goods to be transported are available throughout the whole year
14. the size of the vessels is constant: 10.000 TEU per piece.
15. the demand in transport will not have to be satisfied fully. It is allowed to let containers stay where they are.

2.4.4 Impact Assumptions

Most of the assumptions are quite straight-forward and don't need much extra explanation. However, I want to focus on some of the more important assumptions and their impact on the optimization process:

No timeframe

Assumption 12 states that there is no timeframe for the delivery of the containers. In practice this assumption is, within certain boundaries, quite reasonable. When people want their goods delivered in a short notice, they usually choose direct shipping, where they hire a ship especially for their delivery. Further, once a schedule is made, people can see when their goods are expected to arrive and plan their transports taking that schedule into account. Further, feeder lines and hinterland transports should take that planning into account, too. This seems reasonable, because it does not seem practical to change the system of a long line, because a small feeder line can have planning problems otherwise. It seems more practical to change the feeder line planning (or hinterland planning), because this has a smaller impact on the whole world-system.

Availability throughout the year

Another quite reasonable assumption is that the goods are available throughout the year (assumption 13). Most of the products transported between Asia and Europe are non-food or food with a long expiration date (so that they can have the long journey to Europe). The amounts of these goods are not varying very much in production throughout the year, because factories will produce the whole year through (it is not profitable to close down for some time). A problem can theoretically emerge just before demand peaks (like times around holidays). But I don't expect this to be a huge problem: goods will be sold throughout the whole year and ships can encourage import companies to spread by making transport around peak times more expensive (I will not do further investigation after this option in this paper).

Size of vessels

The size of vessels is held constant in this research project, as stated in assumption 14. In practice, this can be a difficult point: shipping companies often have a fleet to start with which not only has ships of one certain size. When we change the sizes (and capacities) of ships, certain routes can become less profitable or more profitable (because less or more ships are needed). In this project I don't focus on this subject, but I have, as stated, made the assumption of having only ships of 10.000 TEU.

Given demands

The assumption of given demands, stated in assumption 1, is of course quite drastic. Shipping companies never know the demand for a coming year exactly and this demand always changes because of a changing economic climate, better or more competitors on certain routes, or other external factors. I will come back to this point when I discuss input and data in section 1.4.

Constant time in port

Assumption 3, that a ship always spends the same amount of time in every port where it stops, is not very drastic. Of course it is not realistic that a ship needs exactly one day in each port to load and unload freight, but a few hours more or less aren't expected to have a substantial influence on the total planning horizon.

Unlimited amount of ships

The assumption that there is an unlimited amount of ships is not substantial in the long-term, but is in the short-term. In the long-term, ships can be built or bought from other companies, but in the short term companies have to count on their fleet and hiring or buying extra ships can be very expensive (because of opportunity costs), so that the optimization problem can change.

2.5 Literature

Much literature study is done by Man (Man, 2007). From different literature he gets difference between shipping and trucking (shorter trips, no port fees, not always international trade) and aircrafts (mainly passengers, don't operate around the clock).

Hsu and Hsieh (Hsu and Hsieh, 2006) used a pareto optimum between shipping and warehousing costs. Comments on their method is that shippers don't have to care about operating costs and (the most important point) that they make the assumption that all ports must be served by the chosen set of routes. For shippers it is almost certainly better to use multiple lines and not to stop at each port, because for some ports it might not be profitable to serve them. Another problem with the method is that it does not take into account the wish of customers for a reliable timetable at the ports (to stop in each port once a week, for example).

Fagerholt has written two papers on optimizing shipping routes in Scandinavia. They include some cyclicity, but are taking less than a week to cover, so the models are less useable for intercontinental routes.

Man (Man, 2007) presents an algorithm where he sorts possible routes by their length. He chooses the shortest ones (under certain lower bounds in stops to be made in different regions) and assigns as much demand to it as possible. A weak point about this method is that it doesn't have to be optimal to go for the shortest routes. Later on in this thesis, we will see that this is indeed not the case.

In this thesis I will use an example constructed in this thesis by Man.

Alvarez (2008) presents an approach covering many subjects in the route-optimizing problem. He decomposes the problem into two tiers, where the first creates new routes and adds new vessels to routes and the second tier assigns container flows to the routes and determines transshipment points. With this technique, there is a danger of getting into a suboptimal solution. By evaluating all possible routes, I'll try to avoid this.

3 Data & example

In this section I will explain which data is needed and how this can be gathered, or what problems exist in gathering certain information. Later on, the example of Man (Man, 2008) is introduced, with which the used methods will be illustrated.

3.1 Data needed

To optimize routes for shipping lines, several data are needed.

First of all, one must know some specifications of ships that can follow the chosen routes. It is important to know something about capacities, possible speeds and associated fuel consumption. As I will show in section 4.3 on Speed, it is not very difficult to find some of these properties to work with.

Another important part of the data needed concerns the distances between several pairs of ports. These distances can be taken from www.searates.com.

To optimize, it is also needed to know (or estimate) the demand in transport between pairs of ports. This is very difficult. As stated earlier, these demands are hard to predict and can vary under different circumstance. This is why I will work with an artificial example, introduces by Man in 2007.

3.2 Used example

In this study, I'll focus on 10 world ports, spreaded along Asia, Middle East and Europe, just like Man. These ports are (with their short names for the matrices):

- Tokyo (TO)
- Shanghai (SH)
- Hong Kong (HK)
- Singapore (SI)
- Jebel Ali (JA)
- Port Saïd (PS)
- Goia Tauro (GT)
- Antwerp (AN)
- Rotterdam (RO)
- Hamburg (HA)

In the picture below, these ports can be seen in the world map (picture from Man):



figure 2: ports in the example on the map

As stated earlier, I will focus on an artificial example constructed by Man (Man, 2007). In this example, the yearly demand between the chosen ports looks as follows:

O/D	TO	SH	HK	SI	JA	PS	GT	AN	RO	HA	TS
TO	0	0	0	87	23	11	185	100	223	138	767
SH	0	0	0	92	18	16	211	151	631	364	1483
HK	0	0	0	80	20	14	194	116	312	185	921
SI	118	131	75	0	24	10	420	149	358	277	1562
JA	42	28	36	21	0	0	59	46	51	39	322
PS	34	45	24	18	0	0	64	38	40	24	287
GT	98	168	18	28	9	12	0	0	0	0	333
AN	102	132	113	72	0	10	0	0	0	0	429
RO	110	501	175	155	8	13	0	0	0	0	962
HA	98	280	164	123	3	12	0	0	0	0	680
TD	602	1285	605	676	105	98	1133	600	1615	1027	7746

Table 1: yearly demand between the ports

As can be seen, the example only covers demand between the regions Far East, Middle East and Europe. Demands between ports within a certain region are set to zero.

Using www.searates.com, Man (Man, 2007) constructed a matrix of distances between the ports as follows:

O/D	TO	SH	HK	SI	JA	PS	GT	AN	RO	HA
TO	0	1048	1596	2904	6353	7914	8873	11191	10966	11439
SH	1048	0	845	2237	5686	7247	8101	10524	10519	10772
HK	1596	845	0	1460	4909	6470	7374	9747	9742	9995
SI	2904	2237	1460	0	3449	5016	5957	8293	8068	8541
JA	6353	5686	4909	3449	0	2908	3843	6187	6182	6435
PS	7914	7247	6470	5016	2908	0	951	3279	3274	3527
GT	8873	8101	7374	5957	3843	951	0	2371	2378	2635
AN	11191	10524	9747	8293	6187	3279	2371	0	149	405
RO	10966	10519	9742	8068	6182	3274	2378	149	0	305
HA	11439	10772	9995	8541	6435	3527	2635	405	305	0

Table 2: distances between ports in nautical miles

For the other parameters we also choose for the same values as Man:

Symbol	Description	Value
R	the revenues per TEU transported	\$ 500
C_C	Capitol and operating costs per ship per year	\$ 18,000,000
C_F	Fuel costs per nautical mile	\$ 100
C_P	Port costs per stopover	\$ 200,000

3.3 Type of ships used

On internet (<http://www.marinelog.com/DOCS/NEWSMMVII/2007jul00260.html>) we found the COSCO ASIA, a ship with a capacity of 10.050 TEU (close enough to the supposed capacity of 10.000 TEU), which had a structural draft of 47,57 feet (14,5 metres). This ship can be found in the picture below and is used as the type of ships used in the example to show the working of the chosen algorithms.



Figure 3: COSCO ASIA

4. Methods

In this section the used methods will be described. First of all, I will explain why and how I don't focus on the travelling salesman problem in this study. Then I present the heuristic used, which forms a basic approach, where speed is still held constant. In the last part of this section one can read how varying speeds and fuel consumption are approached.

4.1 Travelling Salesman Problem

One of the aspects that must be looked at in the problem of optimizing shipping routes is the shortest path between a string of ports. This shortest path can be found by solving a travelling salesman problem, which can become quite complicated at the turning points (points in Europe and Asia where a ship ends the journey in one direction and starts the journey in the other direction). in Europe and Asia (Man, 2007).

Over time, many algorithms are constructed to find solutions close to the theoretical optimum.

The easiest (and one of the most famous) constructing heuristics is the nearest neighbour algorithm (NN). This algorithm chooses the closest point after each point, until all points are reached. On average, this problem comes to a path 1.25 times the length of the theoretical optimum. However, one can construct examples where NN gives the worst solution (G. Gutin, A. Yeo and A. Zverovich, 2002).

Further, much progress has been made with iterative or randomized improvement methods, where one starts with a feasible solution and then tries to improve it by changing the solution step by step.

Match, Twice and Stitch (Kahng, Reda 2004) created a heuristic for which it has been shown that it outperforms all other heuristic until now. They work with two sequential matchings, which yield a set of cycles over a given set of points. These cycles are then stitched to create a tour. This method outperforms all constructing algorithms, but is dominated by several improvement heuristics.

In recent scientific history, random path change algorithms are used quite often. They can come with reasonable results for problems with up to 100.000 points. The idea is to choose a random path (for the total collection), choose for nearby points and swap their ways to create a new random path, while decreasing the upper bound for the length of the path at the same time.

To apply TSP in the problem of this paper, one has to realise that distance between ports is not the only factor which has to be taken into account when optimizing liner shipping routes. Transport demand has an equal (if not bigger) influence on the way optimal routes should look like. The shortest path has a more complicated meaning in this problem than in a theoretical TSP.

In this paper, I don't focus on TSP, which is why I use an example where TSP is not needed, because the ports are in a quite logical order. One can see that there is only one logical way to go past the ten ports (see picture in section 1.4), namely TO – SH – HK – SI – JA – PS – GT – AN – RO – HA. It would be irrational to go for example from Hong Kong to Port Saïd, then back to Jebel Ali and then on to Europe again. The possibility in this case that there is a big enough demand from Port Saïd to Jebel Ali (which can make the detour rational) can be captured by hitting this route on the way back from Europe to Asia.

Further, the already mentioned turning points are not a problem in the method discussed in this paper because, as will become clear, I use single routes which are then connected.

4.2 Heuristic

Because the problem is, as mentioned earlier, NP-hard, I will try to get a solution as great as possible using a heuristic. I will first describe a basic form and the idea of this basic form, where after I will bring some extra features to the heuristic, to let it have better results.

4.2.1 Basic Heuristic

The main idea behind the basic heuristic can be explained quite shortly. Because I work with only ten world ports, it is possible to calculate the profits (or losses) of all possible routes (given a matrix with transport demands and distances between the ports) in a fairly acceptable time. The heuristic then selects the route giving the highest profit and changes the demand matrix by subtracting the demand covered by that route. After this, a new list of the possible route is made where their profits are calculated (under de new demand-matrix) and again the most profitable route is selected. This is done iteratively until the new list of profits of routes contains no profitable routes anymore.

To limit the possibilities to doable proportions and to avoid the problem of the turn-around point, described earlier in this thesis and by Man (Man, 2007), I have chosen to split the routes into Asia-Europe and Europe-Asia. In each iteration, every route from Asia to Europe will be attached to the most profitable route from Europe tot Asia, with the condition that the last port in the first one has to be the same as the first from the second one and vice versa (they must be connected). From the list of return trips which is then created the algorithm chooses the return trip with the highest profit.

The profit of each route is calculated by letting a ship on the route pick up all freight that is waiting in a port *and* fits in the available space in the ship at that moment. This is done after all goods in the ship, which where to be transported to the port in question, are unloaded, so that the available space is bigger. Hereby I calculate (as already stated by me and Man) with a yearly available space and a yearly demand in the ports.

This basic heuristic can be formulated as follows:

- Step 0: Read in the demand- and distances matrix. Also, initialise a matrix for the figures of chosen routes in the end.
- Step 1: set the counter z (1 for Asia – Europe and 2 for Europe – Asia) on 1.
- Step 2: list all possible single routes of collection z in matrix R . Hereby a single route is a route from Asia to Europe or from Europe to Asia, stopping at at least 2 ports. Further, initialise a 3d-matrix to keep track of the change in OD for each route.
- Step 3: set the route counter i on 1.
- Step 4: initialize ship capacity at the standard starting capacity of 520. Further, initialize number of ports visited, units transported and time travelled to zero.
- Step 5: set the port counter j on 1
- Step 6: add as much capacity to the route as there must be delivered in the current port *and* is on board. Add the unloaded freight to the total number of units transported. Further, subtract this freight from the OD-matrix for this particular route.
- Step 7: let route i load as much cargo waiting in the j 'th stop in the z 'th direction of the chosen route as possible, where freight going to a port closer to the current port will be given priority to freight heading to a port further on the line. All this loaded freight is subtracted from the capacity.
- Step 8: if $j > 1$, add the distance between the previous port visited and the current port (from the distance matrix) to the total distance travelled. In all cases, add 1 to the total number of ports visited and go to step 9.
- Step 9: if there are still ports ahead in the route, set $j = j + 1$ and go to step 6. Otherwise, go to step 10.
- Step 10: if there is a route $i + 1$ in R , set $i = i + 1$ and go to step 4. If all routes are done, sort the routes to profit, descending, and go to step 11.
- Step 11: is $z = 1$, set $z = 2$ and go to step 2. Otherwise, go to step 12.
- Step 12: connect the 100 routes from Asia to Europe to the routes back with the highest profit, with the property that the last port of the 1st category is the same as the first one of the second category.
- Step 13: sort the coupled routes to profit, descending. If the highest profit is positive, go to step 14. Otherwise, go to step 15

Step 14: add the coupled route with the highest profit to the matrix with chosen routes (from step 0) and change the standard OD-matrix to the one from this route (after the delivered amount of goods are subtracted from the OD-matrix). Go to step 1

Step 15: end. Create list with figures about chosen routes as output.

4.2.3 Randomized selecting

One of the main problems of the above heuristic, is that always selecting the best individual route, won't have to give the best result in selecting a set of routes. Very often, a combination of routes that are not in first place in the ranking list, can give better results. Because of this, I have worked with a program which doesn't always choose the best route in step 14. Instead, it chooses a random route from a collection of the best R routes. This procedure is done iteratively, where at each step the new set-up is chosen only if this generates a higher profit then its predecessor. Otherwise, it takes the values of it's predecessor. The iteration is done until the last N solutions didn't give a profit which was higher then the maximum of the previous ones (so that the last N values have taken the same values).

R and N are chosen by experimenting. There comes a point where a higher R is not giving better results anymore. After trying out different R's, I chose one. With this R, I chose a high N and evaluated the results.

This way, far better results are reached. I will stress this point by applying randomized selecting in the case Man created.

4.3 Speed

To investigate the optimal speed for the vessels, I used the algorithm above with different speeds. The profitability of routes changed under this variation, mainly because of two reasons.

First of all, by going faster, some routes needed fewer ships. By going a bit faster, it can happen that a route needs (rounded up) a week less, so that less vessels are needed to meet the requirement that each port is visited once. This can save huge amounts of money.

On the other hand, fuel costs per nautical mile go up as speed goes up, because fuel consumptions then become higher (Dekker, personal communication). For this, Dekker (Dekker, 2009) created the following formula:

$$F(v) = be^{av} / v$$

Where F is the fuel consumption per nautical mile, v the speed in knots and a and b are constants determined by factors like vessel draught, hull shape, engine characteristics and wind force (Dekker, personal communication). For determining a and b , the following data are used:

draft (ft)	A	B	R ²
34	0,1267	0,3729	0.9986
36	0,1272	0,3640	0.9984
38	0,1281	0,3542	0.9981
40	0,1299	0,3395	0.9978
42	0,1314	0,3278	0.9975
44	0,1339	0,3153	0.9970
46	0,1373	0,3025	0.9961
47,6	0,1393	0,2970	0.9958
49,2	0,1430	0,2831	0.9947

By least squares regression with Excel with the data concerning the COSCO ASIA from section 3.3, we estimated that this gives an a of 0,140727 and a b of 0,296837. With this two figures I calculated the fuel consumption in each case.

We have tried to change the speed within the algorithm, so that the different routes have different lengths. This is done by running the basic heuristic (see 4.2.1) with varying speeds.

Hereby we have chosen a lower bound of 5 knots, because the program gave profits of 0 (no vessels going out) below this. Further, below this value fuel consumption rises as the speed of vessels is lower, so that a lower speed only gives cost increase: higher fuel consumption *and* more ships needed because of a longer travel time.

The upper bound we took was 26 knots per hour, because the ship above has a maximum speed of 25,8 knots (to be found on the website mentioned above). As a step size I took 1, because this seemed (and has showed) to generate enough observations to get an insight in the effect of speed and fuel consumption on the optimisation of shipping routes.

5. Results

Here I will present the results of the methods used above. I will make some comparisons with the results of Man (Man, 2007). His example will be used to show the effect of randomized selection. Further, I will show the effect of speed changes and I will close this section with some remarks about the results.

5.1 Comparison with previous research

5.1.1 Basic algorithm

First, I present results of the heuristic with the same assumptions as Man, so that the speed from Asia to Europe is 28 nautical miles per hour and the speed on the way back is 25. Further, the costs of fuel are fixed: 100 dollar per nautical mile. The outcome then becomes as follows:

route	Profit x10 ⁵	Revenue x10 ⁶	To	Sh	Hk	Si	Ja	Ps	Gt	An	Ro	Ha	Ha	Ro	Ant	Gt	Ps	Ja	Si	Hk	Sh	To	miles	days	weeks
1	3.65	7.48	1	1	0	1	1	1	0	0	1	0	0	1	1	0	0	0	1	1	0	1	24317	50.2	8
2	2.86	5.98	0	1	1	1	0	0	0	0	0	1	1	0	0	1	1	0	1	0	1	0	21678	43.2	7
3	2.50	5.21	0	1	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	1	0	21038	37.2	6
4	1.88	5.47	1	0	0	1	0	0	1	0	0	0	0	0	1	1	1	1	1	1	1	1	20845	43.2	7
5	1.61	5.27	1	0	0	1	0	0	0	0	1	1	1	0	1	0	0	0	0	1	0	1	23322	45.9	7
6	0.57	2.67	0	1	0	1	0	0	1	1	0	0	0	0	1	0	0	0	0	0	1	0	11019	23.5	4
total	13.1	32.1	3	4	1	5	1	1	2	1	3	2	2	2	3	2	3	1	3	3	4	3	122219	243	39

Table 3: outcome with Man's assumptions

The table above can be read as follows:

- The first column numbers the routes
- the second column gives profit per route (and total profit in the last row)
- the third column gives revenue per route (and total revenue in the last row)
- the fourth to the 23th column mention the stops per route (where 1 means that a route stops in that port and 0 means that it does not). All ports are mentioned two times, one time on the route Asia-Europe and once on the way back. One can see that these two routes always connect, as already mentioned. The last row indicates the total number of stops.
- The column "miles" gives the travelled number of miles per route and in total
- The column "days" gives the number of days needed per route and in total to travel the distances.
- The column "weeks" gives the number of weeks needed per route and in total to travel the distances. According to assumption 5, a route must have a stop in a port once a week, so that the number of needed weeks to travel the route is also the number of ships needed.

Further, this kind of calculations takes between 30 and 40 seconds.

We see that a profit of 1.31 billion dollar is made. This is more than two and a half times the profit Man made (0,521 billion dollars). Further, Man needed more revenues to make this profit, so that his relative profit is even lower. In this solution, 6.420.000 TEU is transported.

5.1.2 Randomized selection

When we include the randomized selection from section 4.2.3, the resulting schedule looks like this (for explanation about the table, I refer to subsection 5.1.1):

route	Profit x 10 ⁸	Revenue x 10 ⁸	To	Sh	Hk	Si	Ja	Ps	Gt	An	Ro	Ha	Ha	Ro	Ant	Gt	Ps	Ja	Si	Hk	Sh	To	miles	days	weeks
1	3,54	6,79	1	0	1	1	1	1	0	1	0	1	1	0	1	0	1	0	1	1	0	1	24595	50,7	8
2	3,33	6,32	0	1	0	1	0	0	0	0	1	0	0	1	0	0	1	0	1	0	1	0	20610	38,5	6
3	2,33	5,11	0	1	0	1	0	1	0	0	1	0	0	1	0	1	0	0	0	0	1	0	21046	39,2	6
4	2,09	5,20	0	1	0	0	0	0	1	0	0	1	1	0	1	0	0	0	0	1	1	0	21613	41,1	7
5	1,88	5,19	1	0	0	1	0	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	20637	42,8	7
6	1,88	5,20	1	0	1	0	0	0	0	0	1	0	0	1	1	1	0	0	0	1	1	1	23127	44,5	7
total	15,0	33,8	3	3	2	4	1	3	2	1	3	2	2	3	3	3	3	1	3	4	5	3	131628	257	41

Table 4: outcome with Man's assumptions after randomized selection

It is easy to see that great progress is made by randomized selection. It took 108 iteration to reach the 50 times of no improvement. The profit now is almost three times the profit Man could make. In the figure below one sees this happening. Here, I created 200 random selections, where the graph gives the highest profit achieved until that point. From this view, I concluded that a bound of 50 iterations (mentioned in methods) would be sufficient.

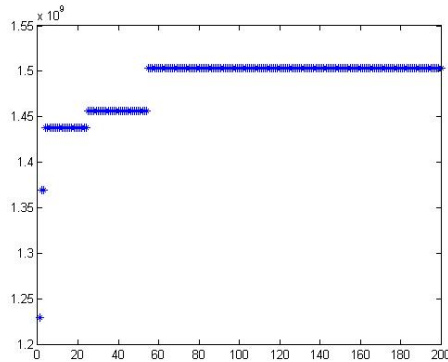


Figure 4: outcomes of iterations of randomized selection

In the appendix, I've added some further analysis on different numbers for the choice out of the ranking list of profits. You can see that if we choose from the highest five routes, the profit clearly becomes lower. When choosing from the highest 15, we get a better result, while choosing from the best 20 does not give better results than with 15. This is why I did the same analysis again with 15, but now with a bound of 200 iterations (the algorithm stopped when there was no improvement during 200 iterations in a row). See also A5 of the appendix. This left the following schedule with profits (for explanation about the table, I refer to subsection 5.1.1):

route	profit	Revenue	To	Sh	Hk	Si	Ja	Ps	Gt	An	Ro	Ha	Ha	Ro	Ant	Gt	Ps	Ja	Si	Hk	Sh	To	miles	days	weeks
1	3,65E+08	7,48E+08	1	1	0	1	1	1	0	0	1	0	0	1	1	0	0	0	1	1	0	1	24317	50,2	8
2	2,59E+08	5,83E+08	1	1	1	1	0	0	1	0	0	0	0	0	0	1	1	1	1	0	1	1	21431	43,1	7
3	2,71E+08	5,89E+08	0	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	1	0	21434	40,8	6
4	1,91E+08	5,27E+08	0	1	0	1	0	0	1	0	0	1	1	1	0	0	0	0	0	0	1	0	21502	40,9	6
5	1,72E+08	5,30E+08	1	0	0	1	0	0	0	0	1	1	1	1	0	0	0	0	0	0	1	1	23145	44,6	7
6	1,78E+08	5,21E+08	0	1	1	0	0	1	0	0	1	1	1	1	0	0	1	0	1	0	1	0	21481	41,9	7
7	1,90E+08	5,21E+08	0	1	1	1	0	0	1	1	1	0	0	1	0	0	0	0	1	0	1	0	21301	41,6	7
8	9,78E+07	4,96E+08	1	0	1	1	1	0	1	1	0	1	1	0	1	0	0	0	1	1	1	1	24919	52,2	8
total	1,72E+09	4,51E+09	4	6	4	6	2	2	4	2	5	5	5	5	2	1	2	1	6	3	7	4	179530	355,2	56

Table 5: best result from the randomized selection

One can see that for the first time (and in this whole thesis the only time) eight routes are chosen, with 56 ships needed. With this configuration, we get a profit of $1,72 * 10^9$ dollar, almost 3,5 times the profit of Man (Man, 2007). Further, we see that the routes become shorter (but still, not the shortest routes are chosen). We see that containers are better spreaded in this outcome, so that more routes become profitable. This way, 9.030.000 TEU is transported, a lot more then the 6.420.000 in the basic algorithm (see 3.1.1).

5.2 Speed

5.2.1 Effect of speed on fuel consumption

The situation above is quite unrealistic. When we look at the chart below, we see costs of 100 dollars per mile are, (with a ship of 10.000 TEU, see ‘methods’ for further specifications) only reasonable with a very low speed, and certainly not with a speed of 25 or 28 miles per hour. This is why I decided to take this change in fuel consumption into account.

In the graph below, one can see how fuel costs per nautical mile depend on sailing speed. This graph is constructed using the formula and assumptions explained in section 2.3.

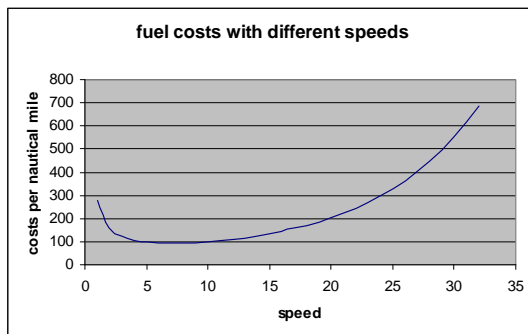


figure 5: fuel costs with different speeds

5.2.2 results including speeds and different fuel usage

With the different speeds, the following profits and revenues came from the algorithm:

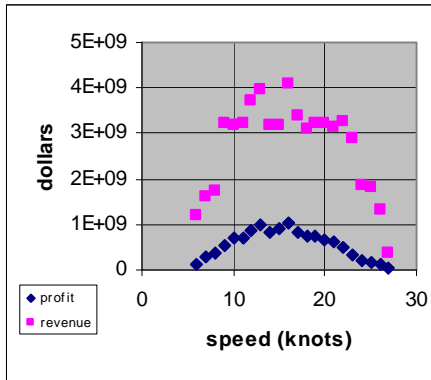


figure 6: profits with different speeds

For the speed with the highest profit, 16 knots, the collection of routes are as follows (for explanation about the table, I refer to subsection 3.3.1):

route	Profit x 10 ⁸	Revenue x 10 ⁸	To	Sh	Hk	Si	Ja	Ps	Gt	An	Ro	Ha	Ha	Ro	Ant	Gt	Ps	Ja	Si	Hk	Sh	To	miles	days	weeks
1	2,55	7,33	1	1	0	1	0	0	0	0	1	0	0	1	0	0	0	0	1	0	1	0	21726	64,6	10
2	2,13	6,22	0	0	1	1	1	0	1	0	0	0	0	0	0	1	1	1	1	0	1	1	20646	62,8	10
3	1,90	6,28	0	0	0	1	0	1	0	0	1	1	1	0	1	0	0	0	1	1	0	1	20351	63	10
4	1,23	5,24	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	1	1	0	21038	58,8	9
5	1,16	5,20	0	1	0	0	0	0	1	0	0	1	1	1	0	0	0	0	0	0	1	0	21409	61,8	9
6	1,12	4,99	0	0	1	1	0	0	0	1	1	0	0	1	1	0	0	0	0	0	1	0	20421	59,2	9
7	1,47	5,55	1	0	1	0	0	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	23486	73,2	11
total	10,2	40,8	2	3	3	4	1	2	3	1	4	3	3	4	2	1	1	1	3	2	5	2	149077	443	68

Table 6: chosen routes with a speed of 16 knots

One can see that with this speed, 7 routes are chosen, 68 ships are needed, the total revenue is $4,08 \cdot 10^9$ and the total profit is a little more than 1 billion dollar. Further, Singapore is the most visited port on the westbound route and Shanghai the most visited eastern port on the eastbound routes. In Europe, Rotterdam is called the most times on either ways.

If an investor wants to know at which speed his profit is the highest in percentages of investment, the graph looks like in figure 6.

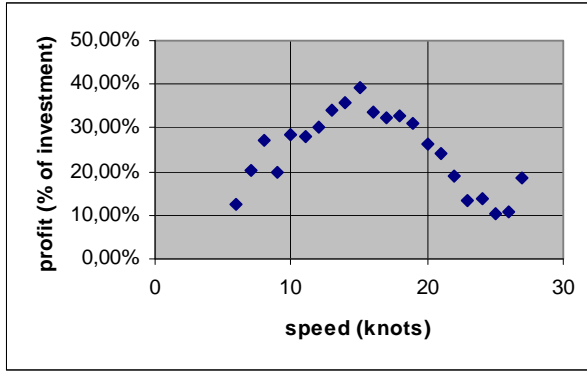


figure 7: profit in percentages of investment with different speeds

Here, 15 knots seems to be the best speed to choose. The collection of route now looks like this (for explanation about the table, I refer to subsection 5.1.1):

route	profit	revenue	To	Sh	Hk	Si	Ja	Ps	Gt	An	Ro	Ha	Ha	Ro	Ant	Gt	Ps	Ja	Si	Hk	Sh	To	miles	days	weeks
1	2,58E+08	6,65E+08	1	1	0	1	0	1	0	0	1	0	0	1	0	0	0	0	1	0	1	0	21948	70	11
2	2,15E+08	6,67E+08	0	0	1	1	0	0	0	0	1	1	1	0	1	0	0	0	1	1	0	1	21589	70	11
3	1,69E+08	5,27E+08	0	0	0	1	0	0	1	0	1	0	0	1	0	0	0	0	0	0	1	0	18856	58,4	9
4	1,06E+08	5,37E+08	0	0	0	1	1	0	0	1	0	1	1	1	0	0	1	0	0	1	1	0	21015	67,4	10
5	8,72E+07	5,13E+08	0	1	1	0	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	21635	71,1	11
6	5,54E+07	2,60E+08	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	10519	31,2	5
total	8,91E+08	3,17E+09	1	3	2	4	2	2	2	1	4	2	2	4	1	1	2	1	3	3	4	2	115562	368	57

table 7: collection of routes with a speed of 15 knots

Here one can see that with this speed, 6 routes are chosen, 57 ships are needed, the total revenue is $3,17 \cdot 10^9$ and the total profit is a little less than 900 million dollar. Further, Singapore is again the most visited port on the westbound route and Shanghai again the most visited eastern port on the eastbound routes. In Europe, Rotterdam is still called the most times on either ways.

In the comparison between the two solutions, one clearly sees the effect of going faster: because the higher speed in the outcome with $v = 16$, one extra route making a profit can be added. Because the speed is higher, more routes become profitable, because in some routes less vessels are needed (see also assumption 5). The last route in the solution with a speed of 16 knots would need more then 76 days, so that the number of weeks (and therefore the number of ships) would go to 12 weeks instead of 11 (including the slack, see assumption 6), so that an extra ship is needed. Given that this last route makes a profit of $1,47 \cdot 10^7$ and the capital and operating costs are assumed to be $1,8 \cdot 10^7$ per ship per year, it is not hard to see that the route is not profitable anymore with a speed of 15.

On the other hand, because of the higher fuel consumption, the relative profit of the routes in the slower variant is higher then the one with a higher speed.

5.2.3 Different speeds for different routes

Another idea was to change the speed for every route, so that not all routes have to have the same speeds. Here I chose an optimal speed for every single route, so the first route chosen is the one which has the highest profit, under an optimal speed. If we do this, the following set of routes comes out of the algorithm (for explanation about the table, I refer to subsection 5.1.1 and as one can see, a column for speed is added):

route	Profit x 10 ⁶	revenue x 10 ⁷	To	Sh	Hk	Si	Ja	Ps	Gt	An	Ro	Ha	Ha	Ro	Ant	Gt	Ps	Ja	Si	Hk	Sh	To	miles	days	weeks	Speed
1	2.65	6.87	1	1	0	1	0	1	0	0	1	0	0	1	0	1	1	0	1	1	0	1	21880	73.1	11	14
2	1.97	6.44	0	1	1	1	0	0	0	0	0	1	1	0	1	0	0	0	1	0	1	0	21783	65.7	10	16
3	1.49	5.23	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	1	0	1	0	21038	66.6	10	14
4	9.44	5.31	1	0	0	1	0	0	1	0	0	0	0	0	0	1	1	1	0	1	1	1	20908	63.4	10	16
5	5.89	5.30	1	0	0	1	0	0	0	0	1	1	1	0	1	0	1	0	0	1	0	1	23322	78.4	12	14
total	7.64	29.1	3	3	1	4	0	1	1	0	3	2	2	2	2	2	3	1	3	3	3	3	108931	347.3	53	

Table 8: collection of routes when allowing different speeds for different routes

We see that the total profit is much lower: only 764 million dollars. It seems that there is too much focus on the first route in this method, leaving less profitability for other routes. Comparing, it would be a better idea to leave the speed the same for all routes, so that there is a limitation to the first route, so that the goods are better balance between the routes (which gives better results).

5.3 Remarks and interpretation

Note that, with all solutions, the total demand is never met, just as it happened in Man's method. Some demand is not met, because the costs of opening another line meeting this demand will cost more then the revenues it provides.

Further, if you compare chosen speeds and fuel costs in 5.2.1, it seems that the best (most profitable) speeds are a bit above the speed with minimal fuel consumption (which lies between 6 and 9). This indicates that the profit that is made by going faster (because fewer ships are needed) has a stronger effect then the cost savings which are reached by reducing speed (because of less friction).

A last important note is that the profits made are based on the assumptions of this thesis. From personal communication with staff from the port authority of Rotterdam, I learned that in real liner-shipping configurations, margins are not as big as in this thesis, due to competitors, external risks, fluctuations in demand and clients demanding lower and lower prices.

6. Discussion

As already stated, the method introduced in this paper gives better outcomes than the method of Man, because it doesn't make the choice to work with the shortest routes. Clearly, the more profitable collection of routes coming out of my method is not the shortest routes. Further, because of the variation in speed, I have found even better solutions on lower speeds. Also, the problem of how to handle turning points is tackled by working with separated west- and eastbound routes.

Further, in contrast with Alvarez, this algorithm looks at the full collection of routes, so that the danger of getting into local optimums is tackled. However, there are also some critical points that can be made about the method here presented.

First of all, it doesn't have to be optimal to choose the route with the highest profit. It is very well possible that a combination of more than one routes (not including the best one) on a list of routes together create more profit than the collection that is created by choosing the first one and then search for routes that create a good profit in combination with that first one.

Further, the presented algorithm loads the ships in each port as full as possible, while there can be optimal solutions where this is not the case, for example because the waiting goods have to travel a long time, while in the next port goods are waiting for a shorter distance, but won't fit because of the longer-distance freight on board. For most part, this problem is covered because ships picking up relatively short-distance freight will come on a higher place on the profit-rating list, but this is not guaranteed for the full hundred percent.

Another point is that the method presented in this paper will take a lot of time when one chooses more than 10 ports. This could be solved by splitting the set of ports in smaller groups and link the groups to each other (just like I linked the outgoing and return trip to each other in this thesis?)

7. Conclusions

This paper introduced a quite straight-forward way to optimize routes for liner ships between Asia and Europe. This method is extended with variation in speed and working with randomly selected routes from a ranking list. The following clear conclusions came from this method:

First of all, optimal routing schedules don't necessarily need to be shortest routes. With routes that take more time, often bigger profits can be made.

Secondly, the speed of the vessels in a transport system has a major influence on the profit that can be made. It can be quite dangerous to assume the fuel costs per nautical mile as a fixed value, independent of the speed. We have seen that this can lead to very unrealistic models, with ships sailing with a too high speed to make any profit at all. Further, it became clear that with realistic fuel consumptions, the ships would sail at speeds that were clearly lower than assumed in the model with assumed fuel consumption. However, the chosen speeds were above the speeds where fuel consumption would be the lowest. An important conclusion from the outcomes with different speeds is that higher earnings by choosing a higher speed have a stronger effect than fuel cost reducing when slowing down.

Further, random selection from a ranking list of single routes after their profit is a good idea: it gives better results than always selecting the one with the best profit and then choosing the next one. We have seen by applying random selection on the case of Man, that this method gave a far better result.

The conclusions can be summarised as follows:

- An optimal routing system for liner ship doesn't have to consist of short routes
- Fuel consumption under influence of speed is a factor not to be neglected
- Optimal speeds lie above speeds where fuel consumption is optimal (lowest).
- Random selection in ranking lists of routes is a good idea

8. Further research

An important idea for further research is implementing hubs in the above algorithm, like Man did in his. With one or more hubs, one might find even better collections of routes.

The shortcoming of choosing the best of a list every time can be tackled by creating a kind of pyramid system. One can think of choosing the first 100 and compare the combinations of that collection. The best of these combinations can then be combined with another list, and so on. This could give better result than the random selection used in this paper. To do this, there only has to be a smart way of dealing with the demand matrix.

The problem of picking up too many long-distance goods could be tackled by first choosing short-distance goods in every route. For this, the picking up of good must not be in the geographical order of ports, but in order of distance between them. To do this, one must think about creative ways to deal with the available space in a ship in each port.

Further, it could be possible that more optimal solutions are reached by choosing other selection criteria than individual profit. Also, one can optimize ship size or vary fuel prices.

Also, some more strict circumstances can be created. One can think of demands in delivery time, variable demand in delivery (throughout the year) or different revenues for different types of goods or different distances. One can put limits on the availability of ships. Ships can become more expensive (or impossible to get) when needed on a short notice or in bigger amounts.

Especially optimizing ship size will become an important topic in the future. There has been a trend of increasing ship size to have a better service to the customers in comparison with the competition (Notteboom & Vernimmen, 2008). But these bigger vessels create too much supply in transport, much capacity is unemployed. Therefore, and because of the recent economic developments, transport prices through liner ships are dropping dramatically, causing Shipment CV's to find themselves in financial problems (De Volkskrant, 17 June 2009) and possibly causing shipping companies to go bankrupt in the future. It is quite interesting to see what will happen in the coming future: will ships keep on growing or will there be a trend of going to (relatively) smaller ships again? Time will tell...

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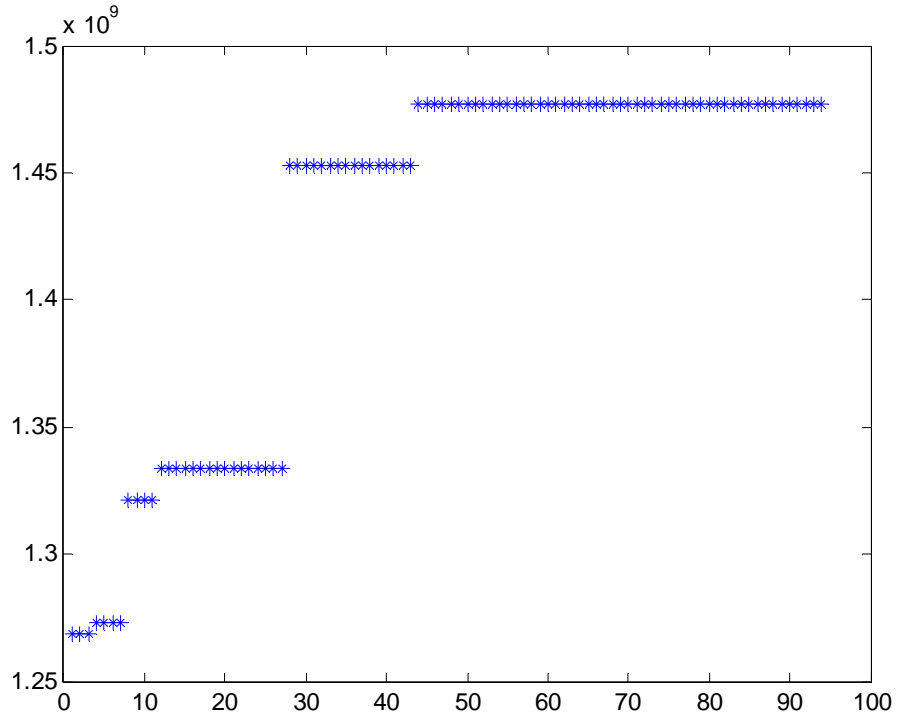
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Appendix

A1



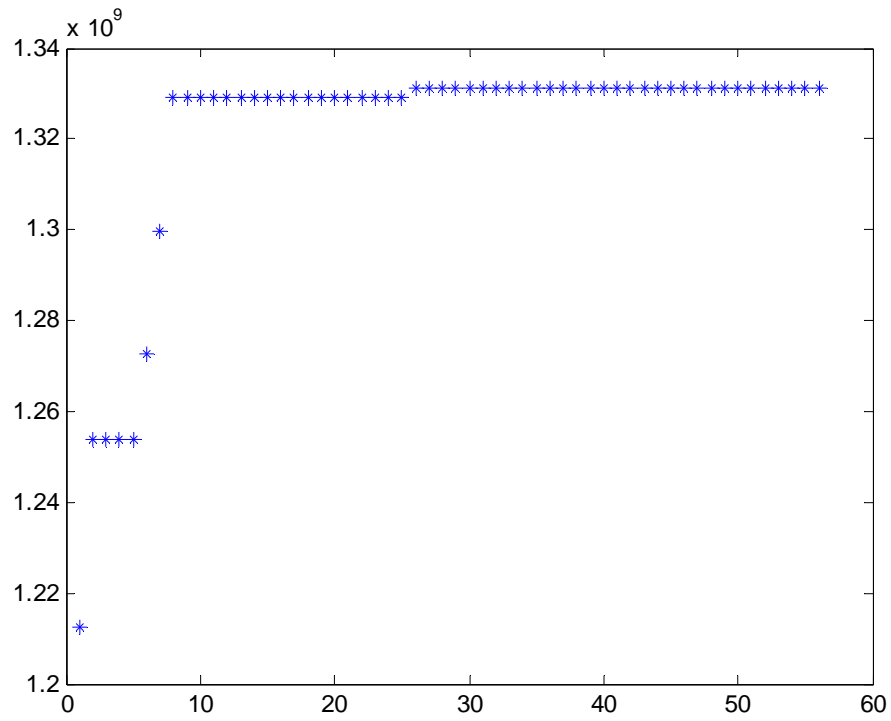
(10, 50)

i =

94

Elapsed time is 3112.562339 seconds.

A2

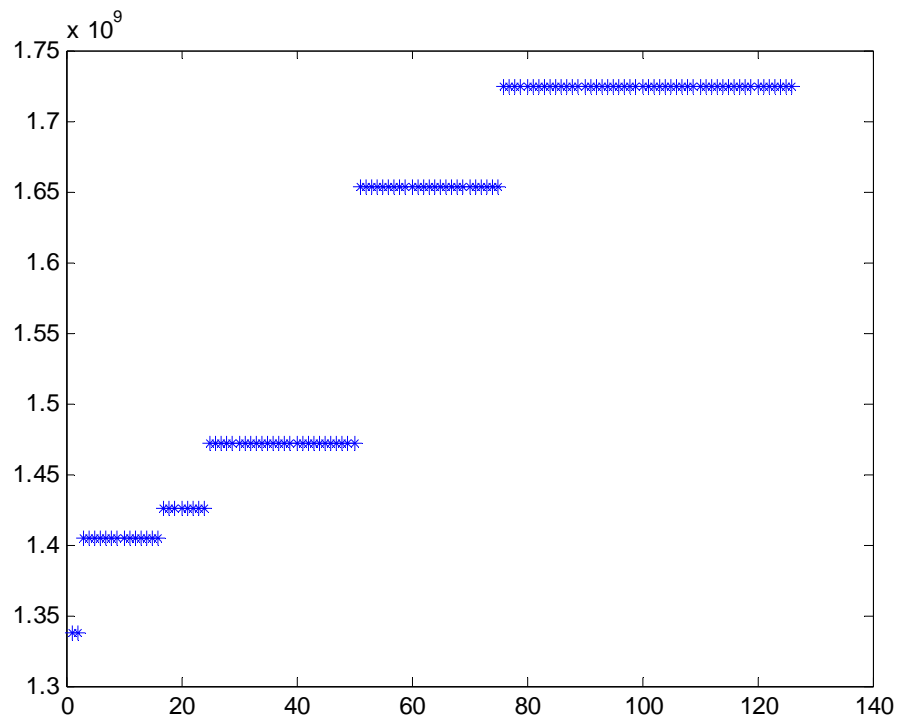


highest 5 routes of each ranking list, with 30 as bound (after 30 times with no increase in profit, the iteration stops).

56 iterations

1939.014378 seconds needed.

A3

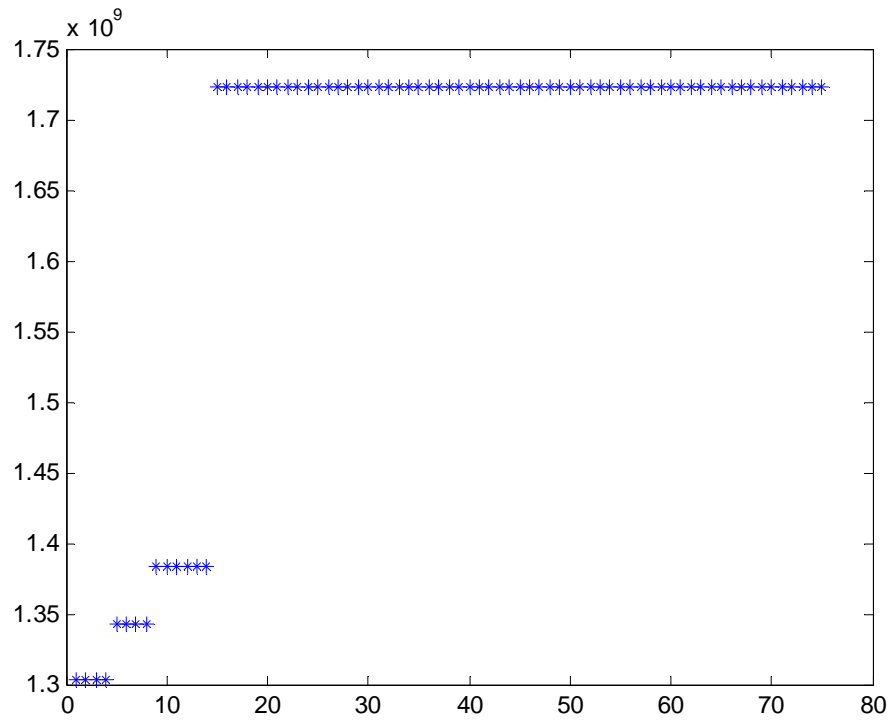


highest 15 routes of each ranking list, with 50 as bound (after 50 times with no increase in profit, the iteration stops).

126 iterations

4414.144407 seconds needed.

A4

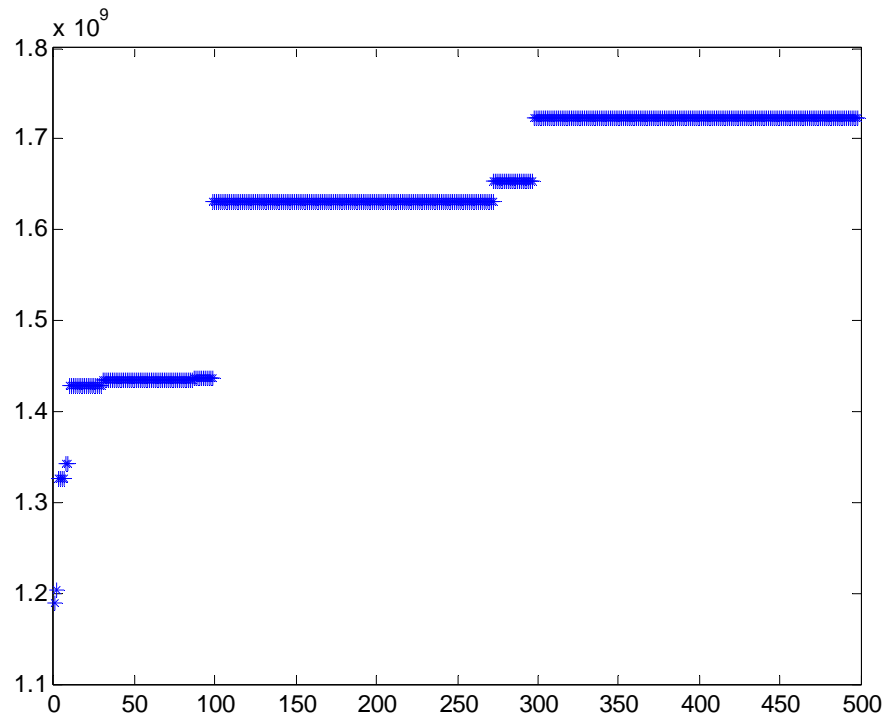


highest 20 routes of each ranking list, with 60 as bound (after 60 times with no increase in profit, the iteration stops).

75 iterations

2575.692801 seconds needed.

A5



highest 15 routes of each ranking list, with 200 as bound (after 200 times with no increase in profit, the iteration stops).

499 iterations

25506 seconds needed.