ERASMUS UNIVERSITY ROTTERDAM

Erasmus School of Economics

Bachelor Thesis International Bachelor Economics and Business Economics

Hedging Downside Risk using the VIX

Name student: Koen Kranenburg

Student ID number: 503769

Supervisor: Ricardo Barahona

Second Assessor:

Date final version: 13-7-2021

Abstract

The Cboe volatility index (VIX) has great potential in reducing downside risk, however, it is not directly investable. Therefore, this paper examines how a VIX exchange traded fund (ETF) performance compares to the actual VIX index in hedging downside risk while keeping risk-adjusted returns high. It is well-established that systematic risk is hedged relatively easily with small exposures and that there is almost no decline in returns. This paper expects that the VIX index outperforms the VIX ETF. In order to test this, several stock portfolios with different small exposures to the VIX are created. These portfolios are analyzed in a theoretical exercise on the VIX index from 1990-2020 and a practical exercise on the VIX ETF from 2015-2020. The portfolios are examined on their relative downside betas, Sharpe Ratios, and CAPM Alphas. Moreover, the results are also compared to a standard 60% stock 40% bond portfolio. To obtain these results annually and full sample CAPM regressions are performed. The results indicate that a small exposure to the VIX index can hedge downside risk effectively while improving risk-adjusted returns, and a small exposure to the VIX ETF cannot. The reason for the failure of the VIX ETF as a long-term trading strategy is mainly due to the high trading costs while being mean-reverting.

Contents

Introduction

The Covid-19 pandemic triggered a freefall in share prices (Bradley & Schumpner, 2021). It also caused a great deal of fear on the market and hence volatility levels rose to an all-time high (Davitt, 2021). This paper examines two important related concepts; downside risk and the Cboe volatility index (VIX). Risk, in general, is an extensively well-researched topic in financial economics, and academics constantly publish new insights in this area. Furthermore, scholars try to find new ways on how to diminish these risks by, for example, diversifying and hedging. Downside risk is the probability of losses caused by a downward movement on the market. Therefore, it is different from the variance since variance refers to both the upward variance and the downward variance. In contrast, downside risk refers only to the downward variance. This paper will focus on hedging downside risk. In particular by investigating which role the VIX can play in decreasing a portfolio's downside risk. The VIX is an index of implied volatility on the S&P 500, which predicts the expected volatility of the next 30 days. Additionally, the paper also examines risk-adjusted performance measures to see if the VIX is a viable trading product. In this way, the paper will analyze if a forward-looking component that measures volatility is able to improve a portfolio by reducing downside risk and keeping returns high. These considerations lead to the following research question: **does a small exposure to the VIX in a stock portfolio reduce downside risk while improving risk-adjusted returns?**

This research is of great relevance as downside risk is complicated to predict even though it is crucial for investors. If this VIX strategy proves itself, these agents could reduce unnecessary exposure to this risk and make a safer investment. This implication holds for both the individual investor as the institutional investor.

The analysis of this question starts by discussing the relevant scientific literature, followed by the description of the data and the methodology. Afterwards, a theoretical and a practical exercise are performed. Next, the underlying questions are discussed. Finally, a conclusion in which the research question is answered, and limitations and further research are presented.

Literature review

A great deal of prior research has been performed on downside risk and hedging risk. The idea of downside risk originally comes from "safety first and the holding of assets" (Roy, 1952). He presents a selection criterion for portfolios based on the chance that the portfolio return is below the expected return. This criterion later was named the SFRatio (Roy's Safety First criterion). The following important paper on downside risk is "Portfolio Selection: Efficient Diversification of Investments" (Markowitz, 1959). Markowitz uses the semivariance instead of the normal variance for portfolio selection since downside risk, as previously mentioned, only consists of the downward variance. A more recent paper and still highly relevant paper is "Downside risk" (Ang et al, 2006). In which it is proven that "that the cross-section of stock returns reflects a premium for bearing downside risk" as "stocks with high downside betas have, on average, high unconditional average return" (Ang et al, 2006). Hence, taking a larger downside risk is rewarded with a higher return. This is interesting for this paper as the VIX strategy will potentially still be rewarded for the higher downside risk stock while the portfolio is partially hedged against the market downside risk by the VIX. Furthermore, in the paper "Can unpredictable exposure to risk be priced?" (Barahona et al, 2021), Barahona et al. take a similar approach to downside risk as Ang et al. (2006) by measuring relative downside risk instead of standard downside risk since the latter is highly correlated with market risk. Barahona et al. (2021) show that beta unpredictability causes a decline in hedging demand and a lower risk premium for downside risk.

"Hedging risks" by (Herskovic et al, 2019) shows an extensive empirical study on hedging standard risks. The paper shows that macro factors and reduced form factors are hedged relatively easy and that there is almost no decline in returns (Herskovic et al, 2019). The low factor exposed stocks yield homogenous results compared with the high factor exposed stocks. Therefore, a long-short portfolio hedges the systematic risk. This paper adds to the paper of "Hedging risks" by hedging risk and keeping the returns as high as possible. It is different as derivatives in the form of a VIX ETF will be used in the portfolios, and it interesting to see how the results will differ from the results found by Herskovic et al. (2019). A specific paper on hedging downside risk in the currency market with futures and options is written by Lien and Tse (2001). In the paper, Lien and Tse measure downside risk by lower partial moments (LPM) and find that future contracts on the S&P 500 outperform options in reducing the variance. This paper matches Lien and Tse's research well as it also analyzes hedging downside risk with derivative instruments. This paper adds to Lien and Tse since this paper will not focus on the currency markets and uses a different downside risk measure. Moreover, VIX derivatives are used instead of standard S&P 500 index derivatives.

Next, there is substantive prior research on volatility and the VIX. Theoretically, the VIX is an excellent candidate to hedge downside risk. It is an excellent candidate because volatility in equity markets behaves asymmetrically (Bekaert & Wu, 2000). Dennis et al. (2006) write "The asymmetric volatility phenomenon (AVP) refers to the stylized fact that negative return shocks tend to imply higher future volatility than do positive return shocks of the same magnitude". This is of great importance since the primary goal of this paper is to hedge downside risk. The asymmetric relationship between volatility and market returns is stronger when the market falls, making a long position in the VIX a perfect theoretical investment strategy to hedge downside risk. Moreover, another promising aspect of the VIX is that "asymmetric volatility is primarily attributed to systematic market-wide factors rather than aggregated firm-level effects" (Dennis et al., 2006). This indicates that possibly with a small exposure to the VIX, systematic risk can be hedged.

Finally, prior research on the VIX being used in portfolios as Dash and Moran (2005) with 'VIX a companion of hedge funds' shows that VIX futures help hedge funds substantially in reducing risk and increasing returns. A different paper by (Deng & McCan, 2012) finds that the benefit of including a long position of a VIX ETP in a portfolio of stocks is highly doubtable. Moreover, Berkowitz & Delisle (2018) find that a 10% portfolio stake in the VIX and Moran & Dash (2007) a 5% stake outperforms a 100% stock portfolio in terms of risk-adjusted return (Sharpe ratio). However, they both find that tradeable VIX products like futures and options result in negative returns relative to the market. Furthermore, for the theoretically maximum stake in the VIX, this should not be higher than 10-20% "as a practical matter, volatility is mean reverting, has no intrinsic value, and has an expected long-term value of zero" (Moran & Dash, 2007). Higher rates such as 30% decreased Sharpe Ratios significantly.

Considering prior research, this paper adds perfectly to the existing literature because the previous VIX papers do not look specifically at downside risk. Furthermore, this paper uses a longer time frame, and VIX trading products are now more liquid. The VIX futures and options were not effective in practice during those timeframes. However, the theoretical implication of the VIX is promising, and this paper looks at a VIX ETF of which the outcomes were not researched so extensively that there is a clear answer whether it reduces risk while keeping returns high. This means that this research will extend this prior research and shows whether it works in this specific setting.

Lastly, this paper is different from all the papers discussed in the literature review fundamentally. This paper considers the general investor, which implies that it analyses a trading strategy that should not be only useful for sophisticated investors. Therefore, a buy-hold strategy is chosen to prevent that trading every day is necessary. Furthermore, a VIX ETF as a VIX tradeable product is chosen as this is tremendously less complicated than constructing a similar exposure to the VIX by buying futures with a margin account.

Research question

- Does a small exposure to the VIX in a stock portfolio reduce downside risk while improving risk-adjusted returns?

Sub-questions

- How do we measure downside risk?
- Does a small exposure to the theoretical VIX index reduce downside risk?
- How does the VIX ETF in a portfolio perform compared to the VIX index?
- How does the performance of VIX portfolios relate to a standard 60% stock and 40% bond portfolio?

Hypothesis

According to the previous literature, if a market portfolio includes a small exposure to the VIX index, the portfolio's risk is reduced, and the Sharpe Ratios outperforms the market Sharpe Ratio. Therefore, the expectation is that the theoretical approach which uses the VIX index will outperform the practical approach with the VIX ETF. This is because the VIX ETF does not track the VIX directly since it is constructed out of futures contracts. Hence, it replicates the VIX index not efficiently. A larger exposure to the VIX will hedge more downside risk, but this will come at the cost of a lower return since the VIX is mean reverting and increases variance as the VIX is more volatile than the market. Compared to a 60/40 portfolio, the VIX is expected to excel in reducing downside risk. However, Sharpe Ratios can favour the 60/40 portfolio as bonds produce a long-term positive return and are less volatile than stocks and VIX products.

Data

The following section is divided into four parts. Firstly, the data is presented. Secondly, the methodology of measuring downside risk and risk-adjusted returns are explained. Thirdly, the analysis is split into a theoretical exercise, and fourthly, in a practical exercise. A long window analysis is performed in the theoretical exercise from a theoretical view that is not entirely tradeable. Next, the practical exercise looks at the last six years for which a complete tradeable strategy is available with a VIX ETF. The reason for this split is that the VIX index is not directly investable, while a long-term analysis is required for a solid empirical argument. Therefore, a large timeframe is strictly preferred over a short time frame with fewer observations. Furthermore, VIX tradeable products such as options, futures and ETFs were not available during a large timeframe of the VIX index. In this way, the split yields an opportunity to analyze the hypothesis statistically, shows what the current real-world applications are, and provides the possibility to compare these two outcomes.

The data used in the theoretical exercise consists of 3 leading indices. The Standard and Poor's 500 (S&P 500), The Chicago Board Options Exchange's CBOE Volatility Index (VIX) and the Bloomberg Barclays U.S. aggregate bond index. The data on the S&P 500, the VIX and the bond index are collected through Bloomberg. The data used is daily data as this is the most precise way to analyze the theoretical exercise. The starting point of the data is January 1986 because since then the VIX index was launched. However, the first four years of data are empty, and therefore the starting point of data in this paper is the 2nd of January 1990. The last available data point for all indices was the 31st of December 2020, this is the end of this paper's time-series data. This horizon yields 7811 observations over 31 years, and for the practical exercise, the last six years are used, containing 1511 observations. For all indices, price-level data is used, and specifically, the closing prices are used. For the analysis, return data is needed, and therefore, this data needs to be transformed. The returns are calculated in a continuous compounded way by dividing the natural logarithm of the closing price at time t by the natural logarithm of the closing price at time t-1. The returns are also market value-weighted which means every stock has a weight in the portfolio of the market cap it has in the index. This is preferred over an equal value-weighted approach, which uses the same weight for every stock in the index. A market value-weighted approach is exactly what would be obtained if the index was directly investable. Furthermore, no false outliers were detected, and the only adjustments to the data were deleting no trading days present in the dataset.

For the practical exercise, three mimics of the three original indices were needed. The three mimics are all Exchange Traded Funds (ETFs). To replicate the S&P 500, the Vanguard S&P 500 ETF (VOO) is used. This is an ETF that tracks the S&P 500, and it is chosen since it has the lowest fund fees of the highly liquid S&P 500 ETF's. Next, to replicate the Barclays us aggregate bond index, the iShares Core U.S. Aggregate Bond ETF (AGG) is chosen. Lastly, for the VIX index, the ProShares Trust VIX Short-Term (VIXY) is selected. This is a short-term futures commodity pool. This specific VIX ETF is chosen as short-term futures mimic the VIX index better than mid-term futures. Furthermore, the VXX, the most liquid VIX ETF, is not selected as ETF matured in 2018. However, the VIXY, with assets in use of more than 300 million U.S. dollars it is liquid enough to use. The timespan for the practical exercise ranges from January 2015 until the end of December 2020. The closing price data of these three indices were also collected from Bloomberg, and the returns are calculated in the same way. For both the theoretical and the practical exercises, the risk-free rate is needed. In this paper and standard in finance daily U.S., one-month Treasury bills are used as a proxy for the risk-free rate. The data is retrieved from the CRSP database.

Methods

To start the analysis, several portfolios are created. The first portfolio is the market portfolio which is the S&P 500 index in the theoretical exercise and the VOO ETF in the practical exercise. This market portfolio is called S&P throughout the paper. The benchmark portfolio is made by combining the S&P 500 and Bloomberg Barclays U.S. aggregate bond index as well as their ETFs for the practical analysis. The S&P 500 returns have a weight of 0.6, and the bond returns of 0.4 and the name of this portfolio is 60/40 B. The VIX index is included in the theoretical exercise and the VIX ETF in the practical exercise for the other portfolios. According to previous literature, the VIX stake should not be higher than 20%. Furthermore, a small weight proved theoretically in the prior research to effectively hedge systematic risk with a slight decline in returns. Therefore, five different weighted VIX portfolios are constructed. Namely, a 1, 2, 5, 10 and 20 percent exposure are used. These portfolios are called 99/1 V, 98/2 V, 95/5 V, 90/10 V, and 80/20 V.

Downside Risk

The literature review made it clear that downside risk is a well-researched topic and that multiple approaches are possible. This paper aims to analyze whether the VIX index/options can improve portfolio performance when the market is doing poorly. Therefore, this paper follows the steps of Ang et al (2006) and Barahona et al (2021). In those papers, downside risk is measured by calculating the standard downside risk followed by the relative downside risk. This method is the most suitable for this purpose compared to other methods like the semivariance, max drawdown and lower partial moments.

The market beta follows the traditional CAPM framework by Sharpe (1964) and Treynor (1961). The following Equations 1 and 2 show the CAPM model and the mathematical definition of the standard market beta.

$$ERi = Rf + \beta i(ERm - Rf) \tag{1}$$

$$\beta = \frac{Cov(Ri, Rm)}{Var(Rm)} \tag{2}$$

In which ERi is the expected return, Rf the risk-free rate, β the standard market beta, ERm the expected market return, Cov the covariance and Var the variance. In this paper, asset prices are not predicted. In contrast, this paper analyses historical prices, and therefore all the values in these formulas are realized returns.

In practice, however, the market beta is not calculated following Equation 2. The risk-free rate is subtracted from the market return to obtain the excess market return as a time series. Then the same is done for the different portfolios to obtain their excess return. Finally, the portfolio excess return is regressed on the market excess return (Equation 3), which provides the exact β coefficient as in Equation 2.

$$Ri,t - Rf,t = \alpha i + \beta(Rm,t - Rf,t) + \epsilon i,t \tag{3}$$

In which the left-hand side is the excess return of the portfolio at time t. The right-hand side consists of the CAPM α, the CAPM β times the market excess return at time t and ε the error term at time t.

Then, the standard downside beta (noted as β^-) is calculated. For this, the data needs to be adjusted only to contain the excess returns below average. The average of that particular year is used as a threshold. See Equation 4 for a mathematical expression of the standard downside beta and equation 5 for the regression used to obtain the standard downside beta.

$$\beta^- = \frac{Cov(\,Ri, Rm \mid Rm < \mu m\,)}{Var(\,Rm \mid Rm < \mu m\,)} \tag{4}$$

$$Ri,t - Rf,t = \alpha^{-}i + \beta^{-}(Rm,t - Rf,t \mid Rm < \mu m) + \epsilon i,t \tag{5}$$

In Equation 4 and 5, the variables are equal to those before except the β^{-} and α^{-}, which represent the standard downside beta, alpha and the µm which is the yearly average market return.

The final step calculates the relative downside beta by subtracting the market beta from the standard downside beta. The relative downside beta shows how the CAPM beta reduces when the sample only contains poor market days. This provides a measure of how effective the downside risk is hedged. When the relative downside beta is smaller than the market beta, it implies that the portfolio behaves less volatile on poor market days and hence is less risky. This results in Equation 6, where β^{DR} is the relative downside beta.

$$\beta^{DR} = \beta^{-} - \beta \tag{6}$$

The betas are calculated over two different timespans. First, the betas are calculated over the full 31 years for the theoretical exercise and over the full 6 years for the practical exercise. These regressions are performed over the full-time horizon to allow an interpretation of how much risk is reduced based on the entire sample. The full sample CAPM regressions are performed with robust standard errors to control for time-varying residuals. Secondly, to analyze whether the downside risk is reduced on a shorter timeframe and to check whether the betas are stable across the years, the CAPM regressions are performed on a 12-month timespan from 1990 until 2020 for the theoretical exercise and from 2015 until 2020 for the practical exercise. As the number of observations drastically declines when sorting the returns on below average, a larger horizon than one month or half a year is needed to have a sufficient number of observations to make a well-grounded argument. Furthermore, Ang et al (2006) and Fama and French (2005) also suggest that a time horizon of one year is preferred when calculating betas. This also has to do with the fact that an even longer horizon is not ideal as "market risk exposures are time-varying" which implies that a large timespan can lead to a noisy beta estimate (Ang et al, 2006).

Performance Measures

After analyzing whether the VIX helps reduce downside risk, it is still unclear if the VIX portfolios provide this hedge at the cost of a significant decline in returns. To analyze the performance, this paper examines the Sharpe Ratio (Equation 7) (Sharpe, 1994).

$$S = \frac{\bar{d}}{\sigma d} \tag{7}$$

In which S stand for the Sharpe Ratio, the $\bar{d}$ is the differential return and σd is the standard deviation of the differential return. The differential return consists of an asset's return minus a benchmark return (Sharpe, 1994). The differential return is equal to the portfolio or market excess return in this paper since this is an asset return (portfolio return) minus a benchmark (risk-free rate).

The Sharpe Ratio is risk-adjusted as it expresses the excess return per unit of risk. This is an interesting measure for this paper as the hypotheses expect that downside risk is reduced, and hence the total risk compared to the market risk should be lower, which would yield a higher Sharpe Ratio. Sharpe Ratios are the most informative over the entire sample or investment period. Moreover, the Sharpe Ratio is time-dependent, and since the full sample Sharpe Ratios are the most informative, the Sharpe Ratios need to be calculated over the entire time horizon. The data is, however, daily and therefore, the Sharpe Ratio is adjusted from a daily format to the full sample time horizon. This is done properly by adding the excess returns since continuously compounded returns have an additive property. The standard deviation is correctly adjusted according to Sharpe (1994) by multiplying the daily standard deviation by the square root of the time horizon. In the full sample setting, this is the number of trading days over the 31 years for the theoretical exercise and over the last 6 years for the practical exercise.

Furthermore, the CAPM alpha (Equation 8) is also examined as a performance measure.

$$\alpha i = Ri,t - Rf,t - \beta(Rm,t - Rf,t) + \epsilon i,t \tag{8}$$

It reveals how much an assets excess return over or underperforms compared to the market excess return. If positive, the asset beats the market and vice versa. In practice, the CAPM alpha is obtained as constant in the regression of Equation 3. This paper uses daily excess returns, and the betas are estimated annually as well as over the full sample. Hence, the full sample CAPM alpha is the average daily over or underperformance over the entire time horizon. The full sample CAPM alphas are the most informative as the other performance measures are also based on the full sample. Lastly, the CAPM alpha is a risk-adjusted performance measure like the Sharpe Ratio since the CAPM framework already controls for risk.

Theoretical Exercise

The full sample from 1990-2020 with the real indices is used to measure the theoretical exercise's downside risk. The full sample contains 7811 observations. After removing the above yearly average returns to calculate the standard downside betas, the same sample contains 3817 observations. The market betas together with their significance levels and T-statistics are stored in Table A1. The same holds for the standard downside beta in Table A2. These two tables show that, in general, the 60/40 B portfolio and all VIX portfolios except the 80/20 V portfolio are positively correlated with the market.

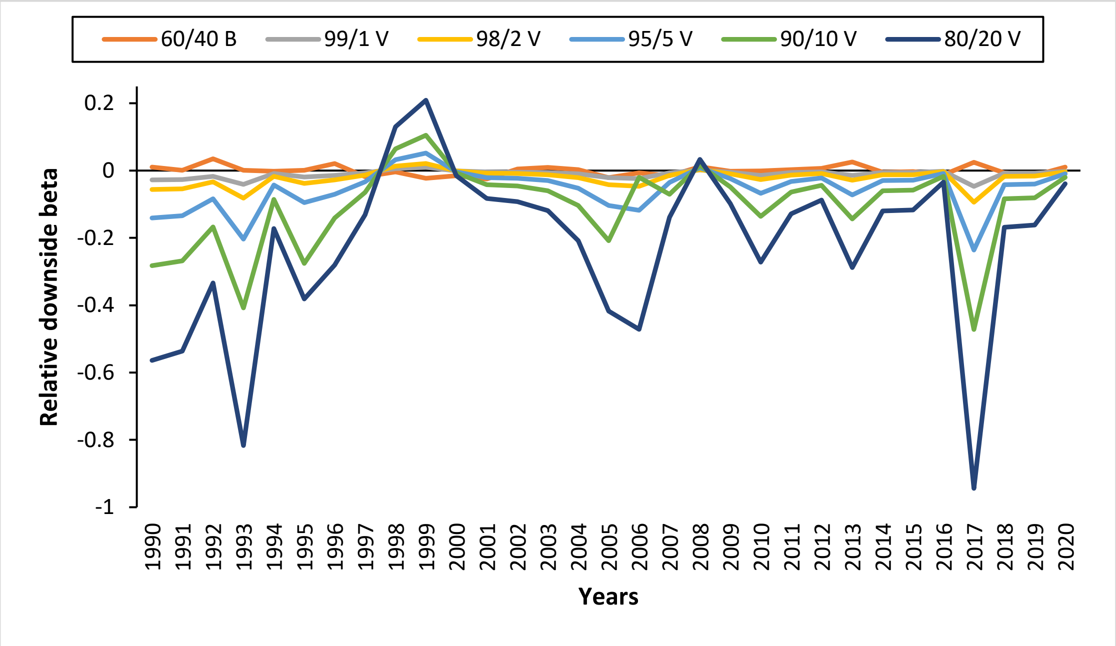

Figure 1. Annual estimated relative downside betas for all theoretical portfolios.
Note. All corresponding standard errors and significance of the market betas and standard downside betas are shown in Table A1 and A2.

Figure 1 shows the annual relative downside betas (β^{DR}). As visible in Figure 1, the 60/40 B portfolio has a consistent β^{DR} that is around 0 for the entire sample. When the β^{DR} spikes, the magnitude is small. This is consistent with the expectation of the benchmark portfolio. However, in contrast to the 60/40 B portfolio, the VIX proves to have a more volatile β^{DR}. It also proves on an annual basis to hedge more downside risk. All VIX portfolios yield a negative downside beta every year except for 1998, 1999, and 2008. Moreover, the negative β^{DR} in the VIX portfolios have a substantially larger magnitude than the 60/40 B portfolio. The magnitude of the β^{DR} increases in the size of the VIX exposure. Therefore, it appears that 95/5 V, 90/10 V, and 80/20 V outperform the 99/1 V and 98/2 V in hedging downside risk. Nevertheless, the 80/20 V portfolio is often riskier in the downside sample, as is visible in Table A1 and A2. The 80/20 V portfolio often has an insignificant beta estimation or has already a negative beta coefficient. Hence, a large β^{DR} shows that the 80/20 V becomes riskier. This is not the case for the 95/5 V and 90/10 V portfolios. Therefore, Figure 1 shows that the 95/5 V and 90/10 V are the most promising annual relative downside betas considering the annual market betas.

Table 1: Theoretical full sample betas and performance measures

Portfolios	**Annual excess return**	**Annual St dev**	**Sharpe ratio**	**CAPM α**	**CAPM β**	**CAPM β⁻**	$\boldsymbol{\beta^{DR}}$
S&P	0.050	0.033	1.515				
60/40 B	0.043	0.020	2.177	5.25E-05***	0.591***	0.586***	-0.004
				(4.85)	(394.73)	(194.72)	
99/1 V	0.049	0.031	1.573	7.37E-06	0.949***	0.949***	-0.000
				(1.38)	(949.05)	(491.03)	
98/2 V	0.048	0.030	1.634	1.47E-05	0.898***	0.898***	-0.000
				(1.38)	(449.1)	(232.31)	
95/5 V	0.046	0.025	1.827	3.69E-05	0.746***	0.745***	-0.001
				(1.38)	(149.14)	(77.08)	
90/10 V	0.043	0.021	2.052	7.37E-05	0.492***	0.490***	-0.002
				(1.38)	(49.15)	(25.34)	
80/20 V	0.036	0.027	1.355	1.47E-04	-0.017	-0.021	-0.004
				(1.38)	(-0.85)	(-0.54)	

Note. T-Statistics are presented in parentheses. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively. Sharpe Ratios are calculated through dividing the corresponding annual excess return by the annual standard deviation.

In Table 1 all important full sample data is shown. Based on the full sample betas, it is clear that the 60/40 B portfolio is effective in reducing risk. The bond portfolio reduces a substantial market risk since the CAPM β is much lower than one. Furthermore, the downside beta is even lower, hence it also hedges downside risk. The VIX portfolios are effective in hedging risk as well, however, over the full 31 years, the relative downside betas of the VIX portfolios have a smaller magnitude than on an annual basis. The relative downside betas being small while the CAPM beta is low implies that market risk is effectively reduced, although the betas for these assets are not very different in the full sample compared to the downside sample. Table 1 shows that all VIX portfolios decrease risk. Nonetheless, the 99/1 V and 98/2 V portfolios with their small exposure are the least effective in reducing risk. All betas of the VIX portfolios decline as exposure to the VIX increases. Therefore, the 95/5 V, 90/10 V, and 80/20 V are more effective in reducing downside risk. Moreover, the 90/10 V and 80/20 V beat the 60/40 portfolio on absolute values of the market beta and standard downside beta. Yet, when comparing the relative downside beta, even the 80/20 V portfolio does not reduce downside risk more effectively than the 60/40 B.

The findings are consistent with the previous literature since small exposures already hedge systematic risk. In addition, the fact that the decline in risk is increasing in the VIX exposure is consistent with the annual findings in Figure 1 and the previous literature. Moreover, it is also consistent that 95/5 V and 90/10 V are the most promising portfolios. Lastly, the findings confirm that a large VIX exposure of 20% or more leads to a more volatile portfolio as well.

Theoretical performance measures

To analyze the risk-adjusted performance, Table 1 provides the full sample Sharpe Ratios. Next to the Sharpe Ratios, Table 1 shows the excess returns and standard deviations from which the Sharpe Ratios are calculated. Evident from Table 1 is that both the 60/40B and the VIX portfolios (except 80/20 V) outperform the market. The VIX index is an effective hedge since market risk and downside risk are reduced while keeping returns high. In Table 1, it is shown that the 60/40 B portfolio, as well as the 95/5 V and 90/10 V, outperform the other portfolios. The full-sample Sharpe Ratio reveals that the 90/10 V portfolio is the highest-scoring VIX portfolio while the 60/40 B scores the highest overall. The annual excess returns show that the market yields the highest return. For the VIX portfolios, the annual excess return decreases as exposure to the VIX increases. Furthermore, the 60/40 B portfolio yields the same annual excess return as the 90/10 V portfolio. This pattern is similar for the annual standard deviation except for the 80/20 V portfolio. The market has the highest risk. The VIX portfolios succeed in decreasing the standard deviation significantly. The standard deviation decreases as VIX exposure increases as far as the 90/10 V portfolio. After the 90/10 V portfolio, the relationship turns around and the 80/20 V has a much higher standard deviation. Lastly, the 60/40 B portfolio has the lowest standard deviation.

The Sharpe Ratio findings are consistent with the hypothesis and prior literature. The theoretical VIX portfolios are capable of outperforming the market. Furthermore, the negative relationship between return and exposure to the VIX is consistent with the nature of the VIX as it is mean-reverting. Finally, consistent with the hypothesis the 60/40 B yields the highest Sharpe Ratio of all.

Table 1 also states the CAPM alphas, which show a similar result to the Sharpe ratios for the positively correlated portfolios. The full sample CAPM alphas are positive for all portfolios. The 95/5 V and 90/10 V show that compared to the 99/1 V and 99/2 V, the magnitude is larger. The 80/20 V portfolio has the highest CAPM alpha, which is inconsistent with the actual annual excess return, and can be explained by the insignificant CAPM beta estimation of the 80/20V portfolio. The Sharpe Ratio and CAPM alpha findings are consistent with the hypothesis and the prior literature. The 95/5 V and 90/10 V are the best performing VIX portfolios since the one- and two-percent portfolios do not hedge enough. Next, the 60/40 B portfolio performs similarly to the 90/10 V portfolio.

Considering the findings, if the VIX index were directly investable, it would be a great addition to the portfolio of a general investor. It reduces market risk and hedges some downside risk. Furthermore, the VIX index does not hurt the returns substantially on good trading days because of the small exposure. This results in Sharpe Ratios for the 5% and 10% exposure that exceed the market Sharpe Ratio substantially.

Practical Exercise

To measure the performance and downside risk for the practical exercise, the sample from 2015 until 2020 is used. Instead of the non-tradeable indices, the ETFs are used, which are entirely tradeable. The full sample contains 1510 trading days, and the downside sample 750 trading days. Table 2, 3 and 4 present the results. The format of the tables is different compared to the theoretical exercise as the theoretical portfolios for the 2015-2020 sample are also included in the tables. Therefore, the practical portfolios can be directly compared to the theoretical portfolios over the same sample.

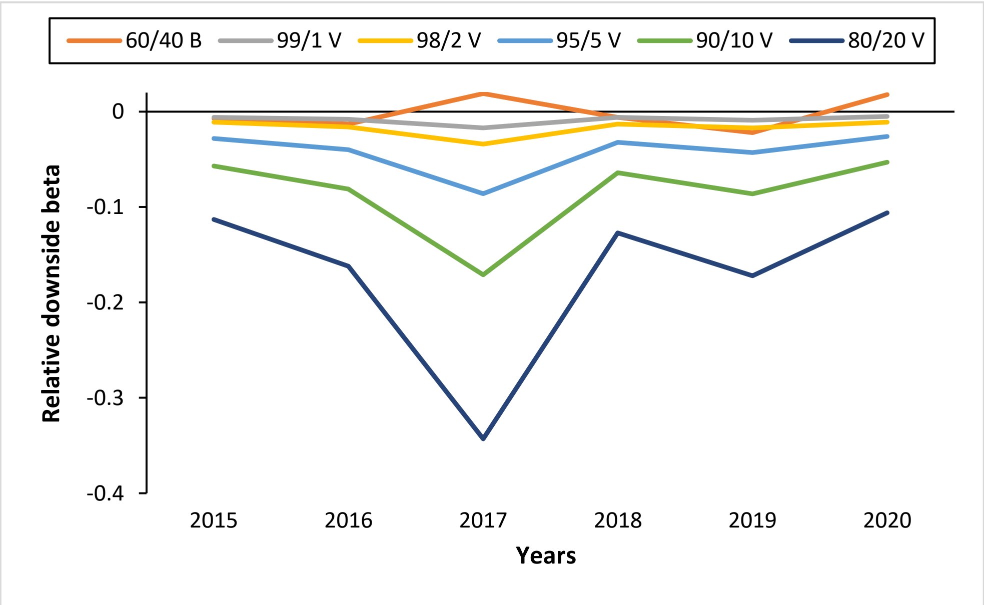

Figure 2. Annual estimated relative downside betas for all practical portfolios.
Note. Scale differs from Figure 1. All corresponding standard errors and significance of the market betas and standard downside betas are shown in Table B1 and B2.

Figure 2 presents the annual relative downside betas. These are calculated on the annual market and standard downside betas in Table B1 and B2. Figure 2 shows a generally similar pattern as Figure 1. This similar pattern consists of the β^{DR} decreasing further as exposure to the VIX (in this case ETF) increases and the 60/40 B portfolio moving around 0. However, there is a major difference. The practical annual relative downside betas have a much smaller magnitude than the theoretical annual relative downside betas. This difference mainly holds for the 80/20 V, as well as the 90/10 V and the 95/5 V portfolios. For example, as visible in Figure 1 and 2, there is a large downward spike in 2017. The theoretical 80/20 V portfolio in Figure 1 reaches a β^{DR} of -0.944. While its corresponding practical portfolio only reaches a β^{DR} of -0.343. The difference implies that the VIX ETF reduces downside risk less effectively than the VIX index. Therefore, the portfolios with a larger exposure to the VIX yield a significantly smaller annual relative downside beta.

Table 2: Practical and theoretical 2015-2020 sample Betas

	Beta		Downside beta		Relative downside beta	
Portfolios	**Practical**	**Theoretical**	**Practical**	**Theoretical**	**Practical**	**Theoretical**
60/40 B	0.599***	0.579***	0.609***	0.588***	0.010	0.009
	(53.52)	(304.83)	(25.54)	(69.65)		
99/1 V	0.960***	0.948***	0.957***	0.941***	-0.004	-0.007
	(571.13)	(697.97)	(364.42)	(148.86)		
98/2 V	0.921***	0.896***	0.914***	0.882***	-0.007	-0.014
	(273.80)	(329.91)	(173.98)	(69.77)		
95/5 V	0.802***	0.741***	0.784***	0.705***	-0.018	-0.035
	(95.40)	(109.07)	(59.71)	(22.32)		
90/10 V	0.604***	0.482***	0.568***	0.411***	-0.037	-0.071
	(35.93)	(35.46)	(21.62)	(6.50)		
80/20 V	0.208***	-0.037	0.135**	-0.178	-0.073	-0.142
	(6.20)	(-1.35)	(2.57)	(-1.41)		

Note. T-Statistics are presented in parentheses. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.

Table 2 presents the practical full sample (2015-2020) betas for the practical and theoretical portfolios. The first thing that Table 2 shows is that the 60/40 B portfolio yields a positive β^{DR}. The positive β^{DR} suggests that the portfolio became riskier during the poor market days. A note on this is that it also holds for the theoretical portfolio which proved over 31 years to actually reduce downside risk. Therefore, it is likely that it is due to the specific and shorter time horizon that the results are different. The short time horizon causes the year 2017 with a large negative spike for the VIX portfolios and an upward spike for the 60/40 B portfolio to have a large effect on the beta estimations.

The 99/1 V and 98/2 V portfolios only hedge downside risk with a small magnitude. Moreover, the pattern is the same as in Table 1: the relative downside betas increase in absolute terms when the VIX exposure is larger. A difference is that the magnitude of the relative downside betas over this 6 years estimation is larger. Which is likely due to the 2017 downward spike. The magnitude of the β^{DR} is lower for all practical VIX portfolios compared to the theoretical portfolios over the 2015-2020 sample. The theoretical portfolios have around twice as large relative downside betas. Therefore, it is clear that the VIX index hedges downside risk better than a VIX ETF, which is consistent with the findings in Figure 2. The 80/20 V portfolio has a positive correlation in the practical portfolio while it is negatively correlated to the market in the theoretical portfolio. These findings align with the hypothesis and prior research as a VIX ETF appears to be a significantly worse product to hedge downside risk than the VIX index.

Practical performance measures

Table 3: Practical and theoretical 2015-2020 sample performance measures

	Annual excess return		**Annual St dev**		**Sharpe Ratio**	
Portfolios	**Practical**	**Theoretical**	**Practical**	**Theoretical**	**Practical**	**Theoretical**
S&P	0.091	0.091	0.076	0.076	1.195	1.195
60/40 B	0.056	0.066	0.046	0.045	1.203	1.470
99/1 V	0.085	0.091	0.073	0.072	1.153	1.262
98/2 V	0.078	0.090	0.070	0.068	1.107	1.334
95/5 V	0.058	0.088	0.062	0.056	0.936	1.566
90/10 V	0.025	0.085	0.050	0.048	0.497	1.781
80/20 V	-0.042	0.080	0.039	0.077	-1.072	1.039

Note. Sharpe Ratios are calculated through dividing the corresponding annual excess return by the annual standard deviation.

To analyze the risk-adjusted performance, Table 3 shows the practical full sample (2015-2020) Sharpe Ratios for both the practical and theoretical portfolios. Table 3 also contains the excess returns and standard deviations. Table 3 shows that in terms of Sharpe Ratio the VIX portfolios underperform substantially compared to the market and a 60/40 B portfolio in the real world. Moreover, they perform even worse compared to their matching theoretical portfolios. In general, there is a clear pattern that the Sharpe Ratio decreases as exposure to the VIX increases. There is an entirely different pattern for the theoretical portfolios as VIX portfolio Sharpe ratios increase when the exposure is larger until the 90/10 V portfolio and afterward decrease. The 90/10 V and 80/20 V show that a larger exposure to the VIX ETF is not viable as a strategy to hedge downside risk while keeping returns high.

However, Table 3 shows that the VIX ETF is capable of reducing risk since the practical standard deviations are similar to theoretical standard deviations. Yet, the problem arises in the first columns where the annual excess returns are shown. The returns drop drastically, therefore the Sharpe Ratios are low. The practical results are inconsistent with the theoretical hypothesis and prior research that the 95/5 V and 90/10V would perform best. Therefore, they prove that the VIX Index is capable of keeping returns high while the VIX ETF is not. In practice, the smaller exposure portfolios hurt the returns less. Lastly, the practical 60/40 B portfolio performs worse than the theoretical while still beating the market and all other portfolios.

Table 4: 2015-2020 CAPM Alphas

	Samples	
Portfolios	**Practical**	**Theoretical**
60/40 B	4.33E-06	5.64E-05***
	(0.14)	(4.34)
99/1 V	-1.21E-05*	8.34E-06
	(-1.70)	(-1.16)
98/2 V	-2.43E-05*	1.67E-05
	(-1.70)	(-1.16)
95/5 V	-6.07E-05*	4.17E-05
	(-1.70)	(-1.16)
90/10 V	-1.21E-04*	4.82E-01
	(-1.70)	(-1.16)
80/20 V	-2.43E-04*	1.67E-04
	(-1.70)	(-1.16)

Note. T-Statistics are presented in parentheses. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.

Finally, Table 4 compares the practical CAPM alphas directly to the theoretical alphas. It becomes evident that the theoretical model shows that the VIX index is capable of beating the market while the VIX ETF is not, because for all VIX portfolios the theoretical VIX has positive alphas and the practical has negative alphas. Table 4 presents similar results as Table

3 as the VIX patterns mentioned in Table 3 are exactly the same. Lastly, only the 60/40 B portfolio beats the market in both the practical and theoretical portfolios.

Discussion

This discussion focuses on two main questions that arise from the results. Firstly, can using a trading strategy that holds a VIX ETF still be worth it to a general investor? Secondly, why does the VIX ETF fail in replicating the VIX index?

After analyzing the results, it is evident that the VIX has great potential in improving portfolios. However, the VIX index outperforms the current VIX ETF's. Therefore, the practical implementation of the VIX in a long-term trading strategy is limited. Table 3 and 4 show the performance measures from 2015 until 2020 for the VIX ETF and the VIX index. It becomes clear that a VIX ETF is very costly and therefore the performance measures of the theoretical exercise are much better. Nonetheless, what is visible in table 3 is that only the Sharpe Ratio of the 80/20 V portfolio is negative. The Sharpe ratios are reduced, but the other VIX portfolios are still profitable.

When would using these VIX portfolios be worth it to a general investor? Since risk preferences differ significantly among investors, a risk-averse investor may be willing to accept the reduction in returns for a risk reduction. Two opposing strategies are possible in this case. Firstly, a moderately risk-averse investor would prefer a small exposure to the VIX such as the 99/1 V or 98/2 V, to reduce a small amount of risk while still obtaining high returns. The 99/1 V yearly returns are reduced by 6.5%, while the standard deviation and the market beta are reduced by 4%. The 98/2 V portfolio reduces returns by 14%, while the market beta and standard deviation are reduced by around 8%. This strategy is indeed viable if the investor's utility function matches this risk-averse behaviour.

Secondly, an even more risk-averse investor wants a larger exposure to the VIX to reduce risk further. The 95/5 V and 90/10 V are suitable candidates for this purpose. The beta of the 95/5 V portfolio is 20% lower than the market, and the standard deviation is 18% lower. Moreover, the 90/10 V portfolio reduces it further to a beta that is 40% lower than the market and a standard deviation that is 34% lower. However, the annual returns are further decreased with a reduction of 36% and 72%, respectively. Therefore, the 95/5 V and certainly the 90/10 V are less likely to be used in practice and necessitate a highly risk-averse investor.

Both strategies are applicable in practice. The major problem is not that the portfolios are not profitable, instead the limitation of their usability is that there are better alternatives to risk-averse investors. This is visible in table 2, the 60/40 B portfolio outperforms the 95/5 V and

90/10V portfolios. The standard deviation and the market beta of the 60/40 B portfolio are even lower than the 90/10 V portfolio while providing similar returns to the 95/5 V portfolio. Hence, 60/40 B gives the most return per unit of risk and even beats the market in terms of Sharpe Ratio. This implies that using the 60/40 B is a more suitable long-term trading strategy for a general investor. There are only two situations in which a general investor would prefer a VIX portfolio over the 60/40 B. Firstly, if the investor's major concern is downside risk, because the practical bond portfolio does not reduce downside risk. Secondly, if the investor is more risk-seeking and therefore prefers the 99/1 or 98/2 with a higher return.

Lastly, the results section confirms that the VIX ETF does not replicate the VIX index well. The VIX index reduces downside risk as well as the standard deviation significantly more than the VIX ETF. However, the VIX ETF is still relatively good at reducing risk, while its major drawback is profitability. A VIX ETF is so costly to hold over a long investment horizon that the returns decline immensely. A VIX ETF behaves differently because the VIX is not directly investable, and therefore cannot directly be tracked. A VIX ETF tries to replicate the VIX index by creating a fund of futures contracts on the VIX index. These future contracts are very different from the actual VIX index, so the risk reduction is not as efficient. In particular, the VIXY ETF from Proshares 'measures the returns of a portfolio of monthly VIX futures contracts that rolls positions from first-month contracts into second-month contracts on a daily basis' (Proshares, 2021).
The costs are high because the fund needs trading and rebalancing each day to keep the same weight and time to maturity on the futures contracts. For this reason, Proshares itself says the VIX ETF's 'are intended for short-term use. When holding the funds beyond short-term periods, investors risk potentially losing a substantial portion of their investment. The longer the holding period, the greater the potential for loss. Investors should actively manage and monitor their investments, as frequently as daily' (Proshares, 2021). A VIX ETF performs better as a short-term investment product because, as mentioned earlier, it is mean-reverting and has high everyday trading costs, which causes the value of the investment to decline over time.

Conclusion

Considering all the different betas for both the theoretical and practical exercise, the VIX index can hedge downside risk more effectively than the VIX ETF. However, for both the theoretical and practical portfolios the relative downside betas are small. Although, they both succeed in lowering the CAPM beta significantly. Furthermore, it implies that market risk is effectively reduced while the betas are not very different for these assets in the full sample compared to the downside sample. When looking at the returns, a much larger difference appears between the VIX index and the VIX ETF. Stock portfolios in the theoretical exercise with a 5% or 10% exposure to the VIX index improve Sharpe Ratios and CAPM alphas to a great extent. In contrast, the practical portfolios with the VIX ETF actually reduce the Sharpe Ratios substantially and the CAPM alphas become negative. Therefore, a small exposure to the VIX index can hedge downside risk effectively while improving risk-adjusted returns and a small exposure to the VIX ETF cannot. The reason for the failure of the VIX ETF as a long-term trading strategy is mainly due to the high trading costs while being mean-reverting. Thus, only the 99/1 V and 98/2 V portfolios seem actually viable for a risk-averse investor. A note on the full sample performance (relative downside betas and Sharpe Ratios) of the VIX index is that in both the theoretical and practical exercise, except for the relative downside beta between 2015-2020, the 60/40 B portfolio beats every VIX portfolio. Only on an annual beta estimation, the VIX portfolios show that they can hedge more downside risk. Hence, a general risk-averse investor would prefer a 60/40 B over the practical VIX portfolios. The findings are consistent with prior research because the theoretical implication of the VIX is viable and a small exposure can hedge systematic risk. The findings are also consistent with the hypothesis, except that no VIX portfolio outperformed the 60/40 B in hedging downside risk over the theoretical full sample. The findings of this paper add to the prior research because they diminish the usefulness of VIX ETF in keeping returns high. Hence, it implies that VIX tradeable products are not yet practical enough for the general investor.

This paper's limitation is that it does not control for all transaction costs such as the ETF's holding costs. These transaction costs would even further reduce the profitability of the practical portfolios. A further limitation is the fact that only the CAPM model is used and that more comprehensive models such as the Fama-French three-factor or five-factor models could have been used in addition to the CAPM framework in the analysis. A suggestion for further research is using a mathematical model that calculates the optimal VIX exposure for a certain level of risk aversion. In this way, performance measures should increase, and a VIX ETF's practical usage becomes more likely to succeed. However, this will only be of actual use if VIX ETF's become significantly better in reducing costs. In addition, an interesting follow-up would be to focus on a specific investor such as an institutional investor who is more sophisticated and could use a dynamic trading strategy.

Bibliography

Ang, A., Chen, J., & Xing, Y. (2006). Downside Risk. *Review of Financial Studies*, *19*(4), 1191–1239. https://doi.org/10.1093/rfs/hhj035

Barahona, R., Driessen, J., & Frehen, R. (2021). Can unpredictable risk exposure be priced? *Journal of Financial Economics*, *139*(2), 522–544. https://doi.org/10.1016/j.jfineco.2020.08.006

Bekaert, G., & Wu, G. (2000). Asymmetric Volatility and Risk in Equity Markets. *Review of Financial Studies*, *13*(1), 1–42. https://doi.org/10.1093/rfs/13.1.1

Berkowitz, J. P., & DeLisle, R. J. (2018). Volatility as an Asset Class: Holding VIX in a Portfolio. *The Journal of Alternative Investments*, *21*(2), 52–64. https://doi.org/10.3905/jai.2018.21.2.052

Bradley, C., & Stumpner, P. (2021, March 22). *The impact of COVID-19 on capital markets, one year in*. McKinsey & Company. https://www.mckinsey.com/business-functions/strategy-and-corporate-finance/our-insights/the-impact-of-covid-19-on-capital-markets-one-year-in#

Coulter, J., & Martínez-Garcia, E. (2021, March 2). *COVID-19 Risks Expose Vulnerabilities, Downside Risks to U.S. Outlook*. Dallasfed.Org. https://www.dallasfed.org/research/economics/2021/0302

Dash, S., & Moran, M. T. (2005). VIX as a Companion for Hedge Fund Portfolios. *The Journal of Alternative Investments*, *8*(3), 75–80. https://doi.org/10.3905/jai.2005.608034

Deng, G., McCann, C. J., & Wang, O. (2012). Are VIX Futures ETPs Effective Hedges? *SSRN Electronic Journal*, 1–21. https://doi.org/10.2139/ssrn.2094624

Dennis, P., Mayhew, S., & Stivers, C. (2006). Stock Returns, Implied Volatility Innovations, and the Asymmetric Volatility Phenomenon. *Journal of Financial and Quantitative Analysis*, *41*(2), 381–406. https://doi.org/10.1017/s0022109000002118

Herskovic, B., Moreira, A., & Muir, T. (2019). Hedging Risk Factors. *SSRN Electronic Journal*, 1–82. https://doi.org/10.2139/ssrn.3148693

Kawa, L. (2020, March 16). *Stock Market Volatility Tops Financial Crisis With VIX at Record*. Bloomberg. https://www.bloomberg.com/news/articles/2020-03-16/stock-market-vola

Lien, D., & Tse, Y. K. (2001). Hedging downside risk: futures vs. options. *International Review of Economics & Finance*, *10*(2), 159–169. https://doi.org/10.1016/s1059-0560(00)00074-5

Moran, M. T., & Dash, S. (2007). VIX Futures and Options. *The Journal of Trading*, *2*(3), 96–105. https://doi.org/10.3905/jot.2007.688954

Proshares. (2021). *ProShares VIX Short-Term Futures ETF | VIXY | Index/Benchmark | ProShares ETFs*. https://www.proshares.com/funds/vixy_index.html

Roy, A. D. (1952). Safety First and the Holding of Assets. *Econometrica*, *20*(3), 431. https://doi.org/10.2307/1907413

Sharpe, W. F. (1994). The Sharpe Ratio. *The Journal of Portfolio Management*, *21*(1), 49–58. https://doi.org/10.3905/jpm.1994.409501

Stuart, A., & Markowitz, H. M. (1959). Portfolio Selection: Efficient Diversification of Investments. *OR*, *10*(4), 253. https://doi.org/10.2307/3006625

Appendix A: Theoretical exercise

Table A1: Annual Market Betas

	Portfolios						
Years	**S&P**	**60/40 B**	**99/1 V**	**98/2 V**	**95/5 V**	**90/10 V**	**80/20 V**
1990		0.648***	0.951***	0.901***	0.753***	0.507***	0.013
		(107.030)	(242.88)	(115.14)	(38.49)	(12.94)	(0.17)
1991		0.633***	0.951***	0.902***	0.755***	0.510***	0.020
		(109.290)	(261.94)	(124.22)	(41.59)	(14.05)	(0.28)
1992		0.629***	0.951***	0.902***	0.755***	0.510***	0.020
		(66.43)	(246.14)	(116.73)	(39.08)	(13.20)	(0.26)
1993		0.649***	0.940***	0.879***	0.698***	0.396***	-0.207*
		(72.45)	(175.50)	(82.12)	(26.08)	(7.41)	(-1.93)
1994		0.733***	0.917***	0.833***	0.584***	0.167***	-0.666***
		(72.10)	(207.68)	(94.41)	(26.44)	(3.79)	(-7.54)
1995		0.694***	0.947***	0.893***	0.733***	0.466***	-0.068
		(59.47)	(174.12)	(82.15)	(26.97)	(8.58)	(-0.62)
1996		0.702***	0.937***	0.874***	0.684***	0.369***	-0.263***
		(88.80)	(264.60)	(123.38)	(38.65)	(10.41)	(-3.71)
1997		0.619***	0.958***	0.917***	0.791***	0.583***	0.166***
		(138.64)	(470.97)	(225.23)	(77.79)	(28.64)	(4.07)
1998		0.584***	0.949***	0.899***	0.747***	0.493***	-0.013
		(134.08)	(526.25)	249.09	(82.79)	(27.36)	(-0.36)
1999		0.626***	0.950***	0.900***	0.750***	0.500***	-0.001
		(118.58)	(497.63)	(235.71)	(78.55)	(26.17)	(-0.02)
2000		0.596***	0.959***	0.918***	0.795***	0.591***	0.181***
		(148.16)	(619.91)	(296.72)	(102.81)	(38.17)	(5.86)
2001		0.582***	0.957***	0.914***	0.785***	0.570***	0.140***
		(113.39)	(651.00)	(310.87)	(106.79)	(38.76)	(4.75)
2002		0.569***	0.961***	0.923***	0.807***	0.614***	0.227***
		(179.25)	(753.25)	(361.48)	(126.42)	(48.07)	(8.89)
2003		0.569***	0.967***	0.933***	0.834***	0.667***	0.334***
		(94.50)	(550.08)	(265.57)	(94.87)	(37.97)	(9.51)
2004		0.596***	0.937***	0.874***	0.686***	0.371***	-0.258***
		(67.26)	(326.44)	(152.27)	(47.76)	(12.93)	(-4.49)
2005		0.602***	0.923***	0.845***	0.613***	0.227***	-0.546***
		(84.78)	(323.46)	(148.18)	(43.01)	(7.95)	(-9.58)

2006	0.616*** (88.49)	0.913*** (269.60)	0.826*** (121.92)	0.564*** (33.31)	0.128*** (3.78)	-0.744*** (-10.99)
2007	0.569*** (109.34)	0.920*** (335.10)	0.840*** (152.97)	0.600*** (43.69)	0.199*** (7.27)	-0.601*** (-10.95)
2008	0.585*** (166.35)	0.964*** (949.69)	0.929*** (457.29)	0.822*** (161.85)	0.644*** (63.37)	0.287*** (14.14)
2009	0.583*** (144.87)	0.965*** (714.01)	0.931*** (344.20)	0.827*** (122.31)	0.654*** (48.35)	0.307*** (11.36)
2010	0.567*** (118.12)	0.936*** (435.06)	0.871*** (202.57)	0.678*** (63.08)	0.357*** (16.58)	-0.287*** (-6.67)
2011	0.564*** (162.09)	0.941*** (526.11)	0.882*** (246.52)	0.704*** (78.77)	0.409*** (22.85)	-0.183*** (-5.11)
2012	0.557*** (131.36)	0.932*** (294.86)	0.863*** (136.60)	0.658*** (41.64)	0.316*** (9.99)	-0.369*** (-5.83)
2013	0.596*** (78.94)	0.909*** (265.56)	0.817*** (118.07)	0.543*** (31.38)	0.086** (2.48)	-0.828*** (-11.97)
2014	0.564*** (103.73)	0.901*** (262.70)	0.802*** (116.95)	0.506*** (29.50)	0.01 (0.35)	-0.976*** (-14.33)
2015	0.572*** (97.46)	0.912*** (339.96)	0.825*** (153.64)	0.562*** (41.85)	0.123*** (4.59)	-0.754*** (-14.05)
2016	0.571*** (97.14)	0.914*** (266.16)	0.828*** (120.58)	0.570*** (33.22)	0.141*** (4.11)	-0.718*** (-10.45)
2017	0.545*** (54.33)	0.869*** (126.64)	0.737*** (53.66)	0.343*** (9.98)	-0.314*** (-4.85)	-1.629*** (-11.86)
2018	0.586*** (144.70)	0.915*** (262.89)	0.829*** (119.18)	0.573*** (32.95)	0.147*** (4.21)	-0.707*** (-10.16)
2019	0.551*** (90.88)	0.911*** (274.13)	0.821*** (123.59)	0.553*** (33.27)	0.105*** (3.16)	-0.790*** (-11.89)
2020	0.591*** (174.22)	0.960*** (548.93)	0.921*** 263.18	0.802*** (91.73)	0.605*** (34.57)	0.210*** (6.00)

Note. T-Statistics are presented in parentheses. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.

Table A2: Annual Standard Downside Betas

	Portfolios						
Years	**S&P**	**60/40 B**	**99/1 V**	**98/2 V**	**95/5 V**	**90/10 V**	**80/20 V**
1990		0.658***	0.922***	0.845***	0.612***	0.225**	-0.551***
		(47.69)	(93.85)	(42.98)	(12.46)	(2.28)	(-2.80)
1991		0.634***	0.924***	0.848***	0.621***	0.242***	-0.516***
		(48.16)	(121.67)	(55.84)	(16.35)	(3.18)	(-3.40)
1992		0.664***	0.934***	0.869***	0.671***	0.343***	-0.314*
		(29.02)	(117.39)	(54.57)	(16.87)	(4.31)	(-1.97)
1993		0.650***	0.899***	0.798***	0.494***	-0.012	-1.024***
		(34.93)	(89.21)	(39.58)	(9.81)	(-0.12)	(-5.08)
1994		0.731***	0.908***	0.816***	0.540***	0.081	-0.838***
		(30.62)	(103.51)	(46.52)	(12.32)	(0.92)	(-4.78)
1995		0.695***	0.928***	0.855***	0.638***	0.276**	-0.449**
		(28.62)	(85.18)	(39.26)	(11.71)	(2.53)	(-2.06)
1996		0.723***	0.923***	0.846***	0.614***	0.228***	-0.544***
		(42.56)	(152.26)	(69.76)	(20.26)	(3.76)	(-4.49)
1997		0.601***	0.952***	0.903***	0.759***	0.517***	0.035
		(65.57)	(260.17)	(123.49)	(41.48)	(14.14)	(0.48)
1998		0.580***	0.956***	0.912***	0.779***	0.558***	0.117*
		(68.50)	(273.06)	(130.22)	(44.52)	(15.95)	(1.67)
1999		0.603***	0.960***	0.921***	0.802***	0.604***	0.209**
		(45.47)	(215.81)	(103.46)	(36.05)	(13.58)	(2.34)
2000		0.580***	0.958	0.917***	0.792***	0.583***	0.166**
		(67.25)	(278.76)	(133.32)	(46.05)	(16.96)	(2.42)
2001		0.556***	0.953***	0.906***	0.764***	0.528***	0.056
		(54.90)	(287.58)	(136.67)	(46.12)	(15.94)	(0.85)
2002		0.574***	0.957***	0.913***	0.784***	0.567***	0.135**
		(74.94)	(319.09)	(152.33)	(52.28)	(18.93)	(2.25)
2003		0.578***	0.961***	0.922***	0.804***	0.608***	0.215***
		(39.82)	(265.06)	(127.12)	(44.35)	(16.76)	(2.97)
2004		0.599***	0.927***	0.853***	0.634***	0.267***	-0.466***
		(29.30)	(144.52)	(66.54)	(19.76)	(4.16)	(-3.63)
2005		0.581***	0.902***	0.804***	0.509***	0.018	-0.963***
		(31.62)	(124.56)	(55.50)	(14.07)	(0.25)	(-6.65)
2006		0.609***	0.889***	0.778***	0.446***	0.108	-1.216***
		(37.53)	(122.27)	(53.52)	(12.27)	(-1.48)	(-8.36)

2007	0.554***	0.913***	0.826***	0.565***	0.130**	-0.740***
	(53.27)	(176.04)	(79.63)	(21.79)	(2.50)	(-7.14)
2008	0.596***	0.966***	0.932***	0.830***	0.660***	0.320***
	(89.86)	(444.91)	(214.62)	(76.45)	(30.39)	(7.37)
2009	0.581***	0.960***	0.921***	0.802***	0.605***	0.210***
	(74.00)	(350.75)	(168.16)	(58.61)	(22.09)	(3.83)
2010	0.566***	0.922***	0.844***	0.610***	0.221***	-0.559***
	(64.40)	(214.00)	(97.96)	(28.33)	(5.12)	(-6.48)
2011	0.567***	0.934***	0.869***	0.672	0.345***	-0.310***
	(90.39)	(248.42)	(115.50)	(35.75)	(9.17)	(-4.12)
2012	0.564***	0.927***	0.854***	0.636***	0.272***	-0.457***
	(60.98)	(132.85)	(61.20)	(18.22)	(3.89)	(-3.27)
2013	0.621***	0.894***	0.788***	0.471***	-0.580	-1.116***
	(49.63)	(125.96)	(55.53)	(13.27)	(-0.82)	(-7.86)
2014	0.559***	0.895***	0.790***	0.476***	-0.048	1.096***
	(51.29)	(126.98)	(56.06)	(13.50)	(-0.68)	(-7.78)
2015	0.569***	0.906***	0.813***	0.532***	0.065	-0.870***
	(47.57)	(162.69)	(72.95)	(19.11)	(1.16)	(-7.81)
2016	0.558***	0.912***	0.825***	0.562***	0.124*	-0.752***
	(51.24)	(139.90)	(63.23)	(17.23)	(1.90)	(-5.77)
2017	0.569***	0.821***	0.643***	0.107*	-0.787***	-2.573***
	(29.78)	(72.23)	(28.26)	(1.88)	(-6.92)	(-11.31)
2018	0.579***	0.906***	0.812***	0.531***	0.0625	-0.875***
	(73.97)	(123.93)	(55.56)	(14.53)	(0.85)	(-5.98)
2019	0.537***	0.902***	0.805***	0.512***	0.024	-0.952***
	(47.67)	(157.02)	(70.02)	(17.82)	(0.42)	(-8.28)
2020	0.601***	0.959***	0.917***	0.793***	0.585***	0.171**
	(93.29)	(275.06)	(131.58)	(45.50)	(16.80)	(2.45)

Note. T-Statistics are presented in parentheses. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.

Appendix B: Practical Exercise

Table B1: Annual market betas

	Portfolios						
Year	**S&P**	**60/40 B**	**99/1 V**	**98/2 V**	**95/5 V**	**90/10 V**	**80/20 V**
2015		0.575***	0.950***	0.899***	0.748***	0.495***	-0.009
		(94.62)	(630.42)	(298.45)	(99.28)	(32.89)	(-0.31)
2016		0.586***	0.948***	0.896***	0.740***	0.479***	-0.042
		(9.54)	(601.29)	(284.13)	(93.83)	(30.39)	(-1.32)
2017		0.550***	0.937***	0.874***	0.685***	0.370***	-0.259***
		(53.15)	(326.00)	(152.05)	(47.68)	(12.89)	(-4.51)
2018		0.591***	0.953***	0.905***	0.763***	0.526***	0.053*
		(148.39)	(641.17)	(304.65)	(102.73)	(35.43)	(1.77)
2019		0.567***	0.949***	0.898***	0.745***	0.491***	-0.018
		(88.59)	(650.06)	(307.60)	(102.11)	(33.62)	(-0.63)
2020		0.613***	0.969***	0.937***	0.843***	0.687***	0.374***
		(100.30)	(866.81)	(419.39)	(150.94)	(61.45)	(16.71)

Note. T-Statistics are presented in parentheses. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.

Table B2: Annual standard downside betas

	Portfolios						
Year	**S&P**	**60/40 B**	**99/1 V**	**98/2 V**	**95/5 V**	**90/10 V**	**80/20 V**
2015		0.568***	0.944***	0.888***	0.719***	0.439***	-0.123**
		(47.26)	(340.90)	(160.31)	(51.96)	(15.84)	(-2.22)
2016		0.573***	0.940***	0.880***	0.699***	0.398***	-0.203***
		(48.60)	(314.22)	(147.05)	(46.75)	(13.32)	(-3.40)
							-
2017		0.569***	0.920***	0.840***	0.600***	0.199***	-0.602***
		(30.16)	(184.39)	(84.17)	(24.03)	(3.99)	(-6.03)
2018		0.584***	0.946***	0.893***	0.731***	0.463***	-0.075
		(75.72)	(333.17)	(157.13)	(51.50)	(16.29)	(-1.31)
2019		0.545***	0.941***	0.881***	0.703***	0.405***	-0.190***
		(48.33)	(378.42)	(177.24)	(56.53)	(16.30)	(-3.82)
2020		0.631***	0.963***	0.927***	0.817***	0.634***	0.268***
		(50.88)	(439.86)	(211.57)	(74.60)	(28.94)	(6.11)

Note. T-Statistics are presented in parentheses. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.