Forecasting Macroeconomic Risks

BSc2 Econometrics / Economics: Bachelor Thesis

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Abstract

In this research it was found that the inclusion of a financial conditions index in forecasting models can be informative. Moreover, this research showed that downside risk is larger in financial tight periods for real GDP growth while the upside risk is much more stable. For the inflation and unemployment rate the upside risk is more volatile. This all holds for a linear quantile regression and a non-parametric quantile regression and for both the US and the Euro area. Finally, it was found that the linear quantile regression is the best model for one-quarter-ahead forecasts in terms of predictive power. For the four-quarters-ahead forecasts it is unclear which model is the best.

Contents

1 Introduction

In the current research, a study by Adams et al. (2021) will be replicated and extended. In their research, Adams et al. (2021) applied a method to quantify risks of time-varying macroeconomic variables. Moreover, this method was used to create density forecasts for these variables. These macroeconomic variables are real GDP growth, unemployment and the inflation. To construct these density forecasts, conditioning information is used to determine the forecast error distribution. The forecasts that are used are based on the Survey of Professional Forecasters (SPF) while the financial conditioning information set is the Federal Reserve Bank of Chicago's National Financial Conditions Index (NFCI). In the replication part, this research is going to verify these conclusions. The extension part of this research will have two main directions. The first part is using the same method on the macroeconomic variables in the Euro area and a conditioning European financial conditions index, and the second direction is using a non-parametric method to forecast the macroeconomic variable distributions. The first direction is going to review the robustness of the results by implementing the methods on a different data set and the second direction proposes a new less restricted method and this method is analysed based on the predictive power.

Researching ways to forecast macroeconomic variables is both scientifically and practically relevant. By conducting research, new methods for forecasting will be developed and these methods can be applied in e.g. public policy making. New methods for forecasting lead to better forecasts and a better understanding of what is going on. Public policy making can use these forecasts to decide which policy is the most effective, based on the financial conditions. The decision makers could use the found results to develop and extent the existing macroeconomic models. According to Blanchard (2018), the macroeconomic models that are being used in the contemporary literature are inaccurate. The macroeconomic models that are being used in the existing literature are dynamic stochastic general equilibrium (DSGE) models. These DSGE-models only contain the right foundations to build on in the future. Improved forecasting could be helpful in improving the performance of these macroeconomic models, which is why this thesis could be of great value.

Another reason why this thesis is relevant is because it applies new statistical techniques on data to find new relations between variables. These newly developed techniques can reveal new or highlight existing relations between macroeconomic variables. Moreover, they allow for finding more complex connections in the data set that is used. The first statistical method that is being used is a distribution regression. This technique is described by Foresi and Peracchi (1995), and is used to estimate the distribution of the variables. This is useful for macroeconomic forecasting because it estimates the risk profile of a certain variable. The increased use of distribution forecasting can alter the existing economic models that are being used. Timmermann (2000) states that using distribution forecasting in empirical studies can be used for correct model specification and also for discovering non-linear relations between variables. However, this model construction is based on economic theory and economic definitions which is largely influenced by subjectivity of researches. In the scope of this problem Heilemann and Stekler (2007) state that economic theory is likely to play a smaller role in the forecasting. This is due to the fact of the larger availability of data and the larger demand for accuracy. This thesis will reduce this subjectivity by using non-parametric regressions, as used by Adrian et al. (2019), to find more complex structures in the data set that are not bound to restrictions such as linearity. Another technique that could have been used to find these complex relations is machine learning. Hall et al. (2018) stated that machine learning will be more influential in the future. Machine learning and non-parametric regressions will - for the most part - remove the uncertainty among assumptions stated in earlier studies and enhance model building in the future. This will increase forecasts for the macroeconomic variables in the future.

Finally, this thesis will research structural differences between the US and the Euro area in terms of the macroeconomic variables. Agresti and Mojon (2001) describe the relation between the business cycle in the US and the Euro area. The business cycles are similar to a high degree in terms of magnitude and fluctuations. The GDP growth and inflation are co-moving with the business cycle, which indicates that these are similar in the US and Euro area which makes the variables in this research comparable. The difference in unemployment rate is analysed by Netšunajev and Glass (2017), which showed that the shocks on the US employment are influential on the local and foreign labor market. However, the shocks in the Euro area are only locally influential. The second main difference is that the US labor market is better in absorbing shocks, so it is less rigid. This can be explained by the better labour rights in the Euro area. The labour market in the US is therefore more volatile. In this thesis, these results will be verified by using different methods, which can possibly support future literature.

This thesis will contribute to the current literature because the original research is limited to conclusions in the United States. This thesis aims to expand this research to more regions which make the existing research more robust. This will also show if the variables in the US and the Euro area behave in similar ways under financial tight and loose conditions. Moreover, a new method is proposed which will be analysed on the forecasting performance. If this new method performs better than the used method in Adams et al. (2021), it can be used for more accurate forecasts in the future.

It was found that including financial conditions in the forecasting model can increase the forecast accuracy. Moreover, the macroeconomic variables have asymmetric risk patterns. Real GDP growth has a lot more downside risk, while inflation and unemployment have much more upside risk. When comparing the linear quantile regression against the non-parametric quantile regression it is clear that the linear quantile regression performs the best for the one-quarter-ahead forecasts in terms of prediction accuracy. However, for the four-quarters-ahead forecast is is not clear which model is the best in terms of forecasting.

This thesis is structured as follows: the data that is going to be used is outlined in Section 2. In Section 3 the methods and theoretical concepts are defined which are going to be used in the research. The results are presented in Section 4 and the economic impact is discussed in Section 5. Finally the thesis is concluded and directions for further research are proposed in Section 6.

2 Data

The data set that is used for the United States part of the paper consists of real survey forecasts for unemployment, GDP price index inflation and real GDP growth which are published in the Survey of Professional Forecasters (SPF). This quarterly data is available since 1968. Until 1990 the survey was conducted by the American Statistical Association and the National Bureau of Economic Research. Since then it has been managed by the Federal Reserve Bank of Philadelphia. In this survey, a variety of forecasters participated, institutions as well as individual professional forecasters. Each quarter these forecasters forecasted many variables for different forecast horizons. In this thesis only three variables with two different forecast horizons will be used, which are the real GDP growth, GDP price index inflation and the unemployment rate for one-quarter and four-quarters-ahead forecasts. In this research the median of each forecast by every institution is used, like in Adams et al. (2021), to get a balanced forecast out of the SPF. The conditioning variable used in the density forecasts is the Federal Reserve Bank of Chicago's National Financial Conditions Index (NFCI), as fully explained by Brave and Butters (2018). This variable is a robust and precise summary index which aims to give a value to the financial conditions in the United States. Its average value is 0 for normal financial conditions, positive values indicate tighter than average financial times while negative value indicate looser than average financial times. The period that will be used is 1971Q1 until 2018Q4, because all data from this period is available.

A similar data set is used for the Euro area in the current research. The Euro area's SPF for real GDP growth, unemployment and price index inflation are used. This survey is published every quarter since 1999Q1 by the European Central Bank (ECB), which is explained in detail by Garcia (2003). This survey contains information for countries in the Euro area, which are shown in Figure 17 in the Appendix. The Euro area's SPF contains the same basis as the SPF conducted in the USA in the sense that a lot of different professional forecasters are forecasting for different macroeconomic variables. The only difference is that for the ECB survey only contains the four-quarters-ahead forecast and only the mean of these forecasts is available. The Euro area SPF data can only be compared to the American four-quarters-ahead data. According to Mboup et al. (2018), the difference between the median SPF and mean SPF are statistically indistinguishable for the real GDP growth, unemployment rate and the price index inflation. This justifies the uses of the SPF mean in the Euro area's data set. The second difference is that the inflation used in the American SPF is the GDP price index inflation while the Euro area's SPF uses the Harmonized Index of Consumer Prices (HICP). The GDP price index and the HICP have a high degree of similarity, both in the short-term and the long-term, as stated by Crawford et al. (1998). Which indicates that the HICP is also a sufficient measure for the inflation.

There is no direct Euro area financial condition index controlled by the ECB, which has the same properties as the NFCI. This is the case because Europa consists of different countries with their own financial system. However A Financial Conditions Index, developed by Petronevich and Sahuc (2019) help of *Banque de France* is going to be used. By making use of a principal component analysis this index is compiled out of time-varying financial series monitored by *Banque de France*. This index has the same properties as the NFCI used by the Federal Reserve Bank of Chicago. So normal financial conditions have value 0. However this index is only available from 2008Q4, that is why the Euro area's data set ranges from 2008Q4 until 2021Q1 which is much more recent compared to the US data set.

Both the American SPF and European SPF data sets did not need any transformations to be used in the research. The statistics of both the SPFs and Financial condition indices are listed in Table 1. It becomes clear that the European data set is much smaller than the US data set, which may lead to unbalanced results. However, looking at the descriptive statistics, they are statistically similar. What stands out in the data are the large peaks in the realized real GDP growth in the Euro area. The values -38.61 and 60.65 are both recent numbers from the COVID-19 crises where the economy got a major setback and quickly recovered after. This is how these numbers can be explained. This is also mentioned by Gräbner et al. (2020), who also stated that the southern countries in the Euro area (Italy, Greece, Spain) were hit harder. This is not visible in the used data set. Removing these from the data set does not change the coefficients of the quantile regression significantly which is why these observations are not removed from the data set.

Table 1: Descriptive statistics of the macroeconomic variables and financial conditions indices

	Observations	Mean	St. dev	Skewness	Kurtosis	Min	Max
1Q forecast Real GDP growth US	192	2.68	1.76	-0.76	2.66	-4.04	7.44
4Q forecast Real GDP growth US	192	2.96	1.11	0.40	1.65	-0.45	6.17
Realized Real GDP growth US	192	2.88	3.24	-0.09	2.53	-8.40	16.40
1Q Unemployment rate US	192	6.33	1.54	0.69	-0.16	3.73	10.3
4Q Unemployment rate US	192	6.23	1.38	0.71	-0.17	4.03	9.8
Realized Unemployment rate US	192	6.27	1.57	0.72	-0.14	3.8	10.7
1Q GDP price index inflation US	192	3.48	2.14	1.28	0.85	1.08	10.19
4Q GDP price index inflation US	192	3.52	1.96	1.19	0.55	1.25	9.37
Realized GDP price index inflation US	192	3.46	2.54	1.44	1.81	-0.40	12.90
4Q Real GDP growth Euro area	50	1.352	0.48	-0.84	0.13	0.2	2.1
Realized Real GDP growth Euro area	50	0.857	10.78	2.48	22.88	-38.61	60.65
4Q Unemployment rate Euro area	50	9.646	1.59	-0.20	-1.09	6.7	12.4
Realized Unemployment rate Euro area	50	9.758	1.48	-0.14	-1.17	7.3	12
4Q Price index inflation Euro area	50	1.548	0.33	0.78	0.96	1	2.6
Realized Price index inflation Euro area	50	1.476	0.998	0.16	-0.65	-0.5	3.7
NFCI US	192	0.02	1.007	2.007	4.315	-1.06	4.86
Financial conditions index Euro area	50	-0.098	0.967	0.542	-0.13	-1.84	2.58

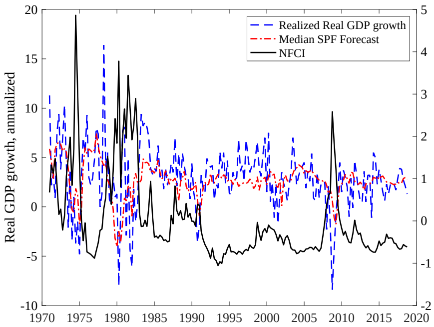
Note. 1Q and 4Q denote One-quarter-ahead forecast and Four-quarters-ahead forecast respectively

In Figure 1a to Figure 2c the macroeconomic variables are plotted together with the financial condition indices. From these figures it seems that there is no relation between the financial condition and the Real GDP growth forecasts which hold both for the Euro area and the US because the median (mean) SPF forecast is almost constant over time. It does not move with or against the financial conditions indices. Moreover, there is a positive relation between the unemployment rate and the indices in both areas. So if the financial conditions are tight, the unemployment rate is higher. And finally, the relation between the indices and the inflation is not clear. In the US and Euro area the relation seems positive in the beginning of the period. However, this relationship is not visible at the end of the data period. These relations are going to be quantified and used to construct forecasts in the current research.

Figure 1: Raw data US

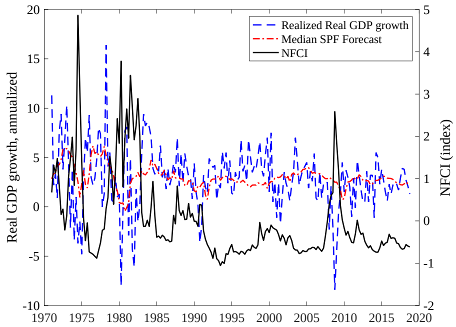
(a) Real GDP growth

One quarter ahead



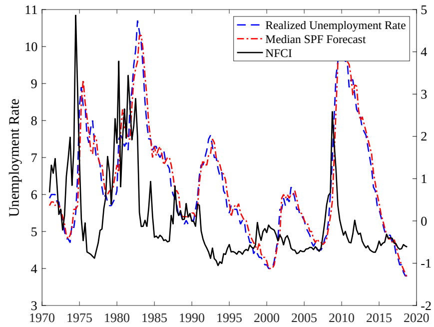
(b) Real GDP growth

Four quarters ahead



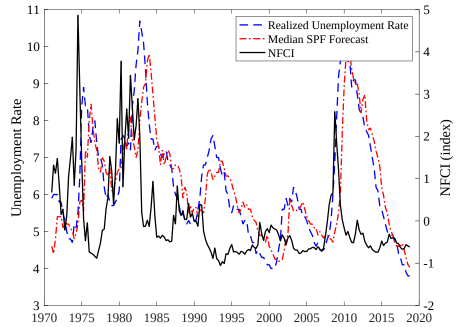
(c) Unemployment

One quarter ahead



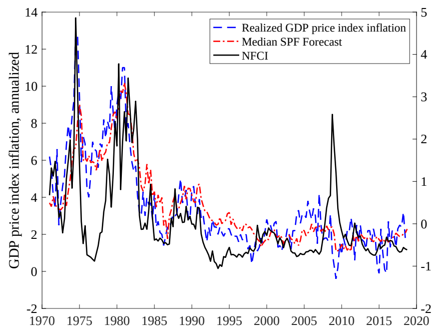
(d) Unemployment

Four quarters ahead



(e) GDP price index inflation

One quarter ahead



(f) GDP price index inflation

Four quarters ahead

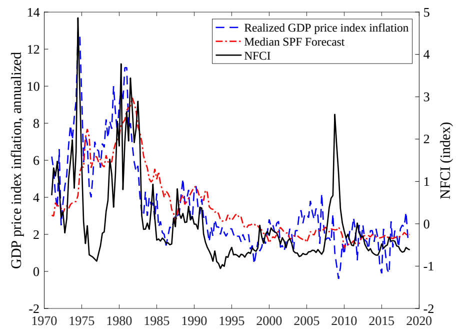
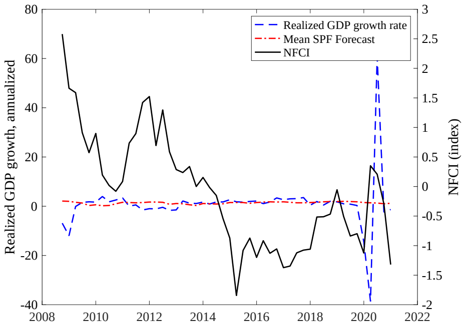


Figure 2: Raw data Euro area

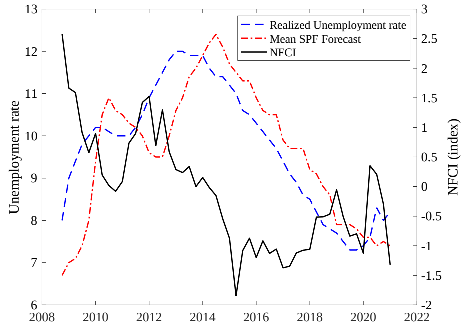
(a) Euro area real GDP growth

Four quarters ahead



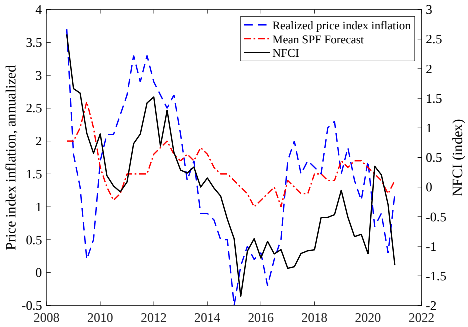
(b) Euro area unemployment

Four quarters ahead



(c) Euro area price index inflation

Four quarters ahead



3 Methodology

This research consists of multiple main theoretical concepts. The key concepts are: quantile regression, non-parametric quantile regression, predictive distributions, downside and upside risk measures. These concepts are explained in detail below. Furthermore, an out-of-sample evaluation is conducted to evaluate how important it is to include a conditioning variable in forecasting the variables. Firstly the notation will be explained that will be used throughout the entire research. Starting with the variables, y_{t+h} is the value of the variable in period $t+h$. The forecasted variables for period $t+h$ in period t by the median SPF are denoted as $\hat{y}^{SPF}_{t+h|t}$. From the forecast follows automatically the forecast error which is $e^{SPF}_{t+h|t} = y_{t+h} - \hat{y}^{SPF}_{t+h|t}$, which can be used to estimate the accuracy of the forecasts.

3.1 Quantile Regression

Quantile regression, developed by Koenker and Bassett Jr (1978), is a method similar to Ordinary Least Squares (OLS) to estimate quantiles. However instead of minimizing the sum of squared residuals, the quantile regression minimizes the sum of quantile-weighted absolute residuals. Formula 2 shows how the regression coefficients are chosen. The goal of the quantile regression is to construct linear estimates for chosen quantile values for the macroeconomic variables.

A τ-quantile, as explained by Koenker and Hallock (2001), is when a proportion τ is above this value and proportion $(1-\tau)$ below this value. This means that $\tau \in (0,1)$. For example the 50th quantile is the median of the data set. The τ-quantile of h-quarter-ahead forecast error distribution conditional on x_t, denoted as $F_{e^{SPF}_{t+h|x_t}}$, is shown in formula 1.

$$Q_{e^{SPF}_{t+h|t}}(\tau|x_t) = inf\{q \in \mathbb{R}|F_{e^{SPF}_{t+h|x_t}}(q|x_t) \geq \tau\} \tag{1}$$

$$\hat{\beta_\tau} = \underset{\beta_\tau \in \mathbb{R}^k}{argmin} \sum_{t=1}^{T-h} (\tau \cdot \mathbb{1}_{\{e^{SPF}_{t+h|t} > x'_t\beta_\tau\}} |e^{SPF}_{t+h|t} - x'_t\beta_\tau| + (1-\tau) \cdot \mathbb{1}_{\{e^{SPF}_{t+h|t} < x'_t\beta_\tau\}} |e^{SPF}_{t+h|t} - x'_t\beta_\tau|) \tag{2}$$

Where $\hat{\beta_\tau}$ is the estimated coefficient corresponding to the τ-th quantile. Important to notice is that every quantile has another coefficient conditional on the financial conditions indices, this is a characteristic of a quantile regression. $\mathbb{1}_{\{e^{SPF}_{t+h|t} > x'_t\beta_\tau\}}$ is an indicator function which is 1 if the forecast error is larger than the predicted quantile regression value and 0 otherwise. $\mathbb{1}_{\{e^{SPF}_{t+h|t} < x'_t\beta_\tau\}}$ is the indicator function which is 1 if the forecast error is smaller than the predicted quantile regression value and 0 otherwise. x'_t is the vector containing the financial conditions index corresponding to the observation and a constant. This total sum of quantile-weighted absolute residuals (forecast-errors) is minimized with respect to β_τ.

$$\hat{Q}_{e^{SPF}_{t+h|t}|x_t}(\tau|x_t) = x'_t\hat{\beta}_\tau \tag{3}$$

Formula 3 shows how the linear estimation is constructed for the τ-quantile of $e^{SPF}_{t+h|t}$ (forecast errors) conditional on x_t. Which is used for the construction of the quantiles of y_{t+h}. This construction is shown in equation 4.

$$\hat{Q}_{y_{t+h}|x_t}(\tau|x_t) = \hat{Q}_{e^{SPF}_{t+h|t}|x_t}(\tau|x_t) + \hat{y}^{SPF}_{t+h|t} = x'_t\hat{\beta}_\tau + \hat{y}^{SPF}_{t+h|t} \tag{4}$$

3.2 Non-parametric Quantile Regression

The quantile regression, explained in section 3.1, is linear in the conditioning variable. This is a limit on the regression coefficients. The linearity can be seen in equation 3, $\hat{Q}_{e^{SPF}_{t+h|t}|x_t}(\tau|x_t)$ is a linear combination of $x'_t\hat{\beta}_\tau$. In the non-parametric quantile regression this relationship is not limited to a linear relation. This method is based on the general dependence of the quantiles on the conditioning variables. The non-parametric method that is going to be used is based on the method developed by Li et al. (2013). The method is based around selecting the parameters in such way that a weighted estimation mean squared error of the conditional quantile function is minimized. Following this method, the conditional cumulative density function of one-quarter ahead SPF forecasts is,

$$\hat{F}_{y_{t+1}|x_t}(y|x) = \frac{\frac{1}{T-1}\sum_{t=1}^{T-1} \Phi(\frac{y-y_{t+1}}{\omega_0})K_\omega(x_t,x)}{\frac{1}{T-1}\sum_{t=1}^{T-1} K_\omega(x_t,x)} \tag{5}$$

where

$$K_\omega(x_t,x) = \prod_{i=1}^{n} \frac{1}{\omega_i}\phi(\frac{x_i - x_{i,t}}{\omega_i})$$

and $\phi(.), \Phi(.)$ are the PDF and CDF of a standard normal distribution respectively, which are shown in equations 13 and 14 in the Appendix. x_t is the conditioning variable, which is the financial conditions index in this case. The inverse of the non-parametric cumulative density function is the non-parametric quantile function. The conditional τ-th quantile is defined as,

$$q_\tau(x) = inf(y : F(y|x) \geq \tau) = F^{-1}(\tau|x)$$

$F^{-1}(\tau|x)$ (inverse CDF) is the quantile function, T is the number of observations, y is the forecast error defined in Section 3, x is the conditioning variable and $\omega_0, ..., \omega_n$ are the bandwidths. These bandwidths are determined by cross-validation in the research conducted by Li et al. (2013). Cross-validation, as described by Mosier (1951), is a method which is used to estimate true parameters of the model by making use of re-sampling of the data and estimating true prediction errors. Its main purpose is to validate the predictive power of a model. The cross-validation technique that is going to be used in this research is the least-squares cross-validation. The optimal bandwidths are obtained by minimizing the least-squares cross validation function, which uses leave-one-out cross validation, shown below.

$$\hat{\omega}_0 = \underset{\omega_0}{argmin}\ CV = \underset{\omega_0}{argmin} \frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{j \neq i}^{n} (I_{(y_i \leq y_j)} - \hat{F}_{-i}(y_j|x_i))^2$$

where $\hat{F}_{-i}(y_j|x_i)$ is equal to equation 5. $\hat{\omega}_0$ are optimally chosen, which are the bandwidths used in the research.

3.3 Predictive Distributions

Distribution prediction is the second theoretical concept that is used in the research conducted by Adams et al. (2021). After constructing the linear estimates of the quantiles, it is more informative to forecast the distributions of the different macroeconomic variables. A distribution is fitted on the estimated conditional quantiles in order to construct a full conditional probability distribution. The probability distribution that is going to be fitter is the four-parameter skew t-family developed by Azzalini and Capitanio (2003). The probability density function is given in formula 6,

$$f(y; \mu, \sigma, \alpha, \nu) = \frac{2}{\sigma} t(\frac{y-\mu}{\sigma}; \nu) \times T(\alpha(\frac{y-\mu}{\sigma})\sqrt{\frac{\nu+1}{\nu + (\frac{y-\mu}{\sigma})}}; \nu+1) \tag{6}$$

where μ is the location parameter, σ is the scale parameter, α is the shape parameter and ν are the degrees of freedom. Moreover, $t(-; n)$ and $T(-; n)$ are the probability density function (PDF) and cumulative distribution function (CDF) of the Student's t-distribution respectively, with n degrees for freedom. The PDF and CDF of a Student's t-distribution can be found in the Appendix, equation 11 and 12 respectively.

The parameters are estimated based on a selection of different quantiles, by minimizing the squared differences between the skew t-implied quantiles and the estimated quantile regression estimates. This process is shown in formula 7, where S is a selection of different quantiles and $F^{-1}(\tau; \mu, \sigma, \alpha, \nu)$ is the quantile function (inverse CDF) of the skew t-distribution. Since the skew t distribution has four parameters, the fitting is also going to be based around the value of four different quantiles. This will make the parameters exactly identified and there is no over- or under-identification problem. The four quantiles are going to be the 5th, 25th 75th and the 95th quantile, $S = (0.05, 0.25, 0.75, 0.95)$.

$$\{\hat{\mu}_{t+h|t}, \hat{\sigma}_{t+h|t}, \hat{\alpha}_{t+h|t}, \hat{\nu}_{t+h|t}\} = \underset{\mu,\sigma,\alpha,\nu}{argmin} \sum_{\tau=S} (\hat{Q}_{y_{t+h}|x_t}(\tau|x_t) - F^{-1}(\tau; \mu, \sigma, \alpha, \nu))^2 \tag{7}$$

This is how distributions are estimated for the macroeconomic variables in the research.

3.4 Downside and Upside Risk Measures

The two risk measures that are used in the research are the expected shortfall, firstly used by Rockafellar et al. (2000), and the expected longrise. The expected shortfall and expected longrise are shown in formulas 8 and 9 respectively, where $\hat{F}^{-1}_{y_{t+h}|x_t}(\tau|x_t)$ is the fitted quantile function of the predictive distribution. In the research by Adams et al. (2021), the chosen values for the shortfall and longrise are 5% and 95% respectively which are common values in financial research. The downside risk measures gives an indication on how much a variable can decrease and the upside risk measure gives an indication on how much a variable can increase, both in the extreme cases.

$$SF_{t+h|t} = \frac{1}{0.05} \int_0^{0.05} \hat{F}^{-1}_{y_{t+h}|x_t}(\tau|x_t) d\tau \tag{8}$$

$$LR_{t+h|t} = \frac{1}{0.05} \int_{0.95}^{1} \hat{F}^{-1}_{y_{t+h}|x_t}(\tau|x_t) d\tau \tag{9}$$

3.5 Out-of-sample Evaluation

An out-of-sample evaluation is conducted to show and quantify the importance of the conditioning variables in this research. The out-of-sample period in the US data starts in 1992Q2 and for the EU data this starts in 2016Q1, this results in 26 years of out-of-sample data for the US and 4 years for the Euro area. The choice for the out-of-sample set is arbitrary and is between half and three quarters of the available sample period for both the US and the Euro area. This out-of-sample situation tries to replicate the real life by mimicking the timing of the availability of the variables for the forecasters. So the quantiles are estimated based on the information available to the forecasters in that period. The quantiles which are conditional on the financial condition indices are compared to quantiles which are unconditionally constructed. The unconditional quantile regression is based on Reifschneider and Tulip (2019), which is equation 2 with only a constant as conditioning variable.

The presentation and quantification of the differences between the two different quantiles is done in two separate ways. The first method used predictive scores and the second method used probability integral transforms (PIT).

3.5.1 Predictive Scores

Predictive scores are used to compare the accuracy of the out-of-sample density forecasts. The predictive score for a give h-period-ahead predictive density is defined as follows, $PS_{\hat{f}_{t+h}|I_t}(y^0_{t+h}) = \hat{f}_{t+h}|I_t(y^0_{t+h})$. The predictive score evaluates the predictive density at the realized value of the variable. So the higher the predictive score is, the higher the accuracy is. For the comparison between two forecast methods, the difference in log predictive scores is used.

$$\frac{1}{T-h-t_{start}} \times \sum_{t=t_{start}}^{T-h} (\log PS_{\hat{f}_{t+h}|I_t}(y^0_{t+h}) - \log PS_{\hat{g}_{t+h}|I_t}(y^0_{t+h})) \tag{10}$$

where $\hat{f}_{t+h}|I_t(x)$ and $\hat{g}_{t+h}|I_t(x)$ are two different forecasts for observation x. If equation 10 is positive, than forecast $\hat{f}_{t+h}|I_t(x)$ is better and vice versa. The difference in means of the predictive scores will be tested with a t-test, this will verify if the results are significant.

3.5.2 Probability Integral Transforms

Probability Integral Transforms (PITs) are used to evaluate the calibration of the predictive distributions. This method as described by Angus (1994) can detect inefficiencies in the intensity and dependence structure of the models that are used. The PITs work by evaluating the estimated CDF ($\hat{F}_{y_{t+h}|I_t}(.)$) at the realized value of the variable.

$$PIT_{\hat{F}_{y_{t+h}|I_t}}(y^0_{t+h}) = \hat{F}_{y_{t+h}|I_t}(y^0_{t+h})$$

If the model is specified perfectly, the PITs will be uniformly distributed. In the figures, uniform distribution corresponds with the 45°-line. Confidence bands are constructed around the 45°-line to evaluate the uniformity of the distribution. The bounds are constructed using the method proposed by Rossi and Sekhposyan (2019). For the one-quarter-ahead forecasts the bands are based on the critical values derived under the null hypothesis of uniformity of the PIT, which is 1.34 in this case. For the four-quarters-ahead forecasts the confidence bands are computed by bootstrapping the 5% critical values. Bootstrapping, according to Efron and Tibshirani (1994), is a method that replicates the sampling process multiple times to find certain measures, the 5% critical value in this case.

4 Results

4.1 Linear Quantile Regression

4.1.1 Estimated Forecast Error Quantiles

Figures 3a until Figures 4c show the heterogeneity in the forecast errors of the different variables. These figures in general show that the uncertainty in the forecasts increase when the financial conditions tighten, this holds for both the US and the Euro area. The quantiles become wider if the NFCI or financial conditions index becomes higher. Looking at the figures for US real GDP growth, it is clear that there is more down side risk for the four-quarters-ahead forecast when financial conditions are tight. The 95th conditional quantile line is almost flat while the 5th quantile line decreases more sharply. Looking at the unemployment rate, it becomes clear that the forecast errors are leaning towards the positive side when the financial conditions index is higher in the US case. This is highlighted by the upward trend in both the 5th and 95th quantile lines in Figures 3c until Figure 3f.

For the Euro area, Figure 4a to Figure 4c, the same conclusions can be derived. There is more downside risk in the real GDP growth when financial conditions are tighter. Moreover, the unemployment rate forecast errors are more positive when the index is higher like in the US case. The price index inflation has more upside risk because the 95th quantile line is almost flat and the 5th quantile line has an upward trend, when the financial conditions are more narrow. This is the main difference compared to the US case.

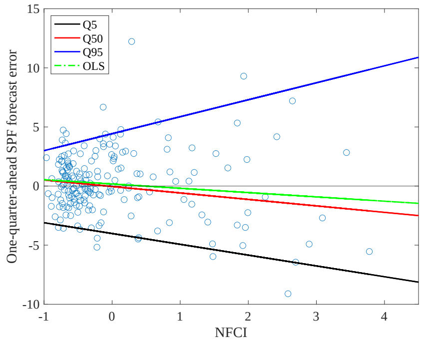
4.1.2 Quantile regression coefficients

The coefficients of the quantile regressions are plotted in Figure 5a to Figure 6c. All these figures show an upward sloping pattern in the in-sample fit. This indicates that tightening financial conditions pushes one or both risk tails of the forecast error distributions outward. This means that the uncertainty around the median or mean forecast becomes bigger with more narrow financial conditions. In these figures the 95 confidence bands are plotted, which indicate statistical significance. The bands that are plotted are the 68%, 90% and the 95% confidence bounds under the null hypothesis that the true data generating process (DGP) is a linear vector autoregressive model containing the target variable, four lags, the median/mean SPF forecast and the financial conditions indices. When values fall outside these confidence bands than the vector autoregressive model can be rejected. This is the case in almost all figures, however not in Figure 6a. So there there is not a indication to reject autoregressive model in this case.
Although these figures indicate that the uncertainty around the forecasts becomes larger with tight financial conditions, they do not give a causal relation. It is not clear which variable causes which. Do tight financial times generate higher risk or does higher risk generate tight financial times? This could be solved by finding a sufficient instrumental variable, but this is beyond the scope of this thesis.

Figure 3: SPF forecast errors and financial conditions US

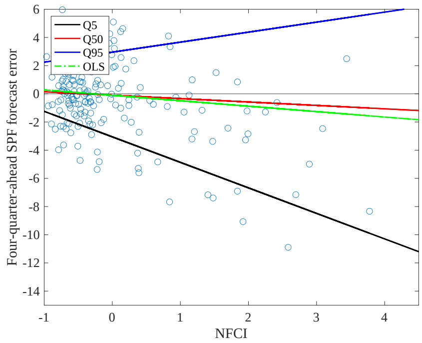
(a) Real GDP growth

One quarter ahead



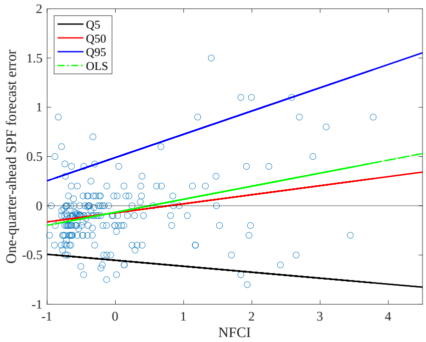
(b) Real GDP growth

Four quarters ahead



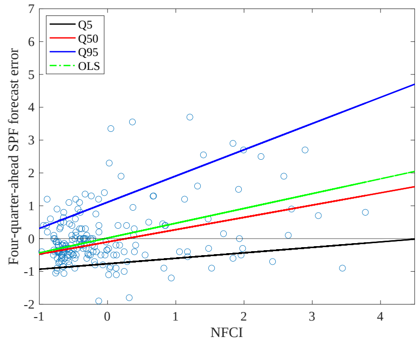
(c) Unemployment

One quarter ahead



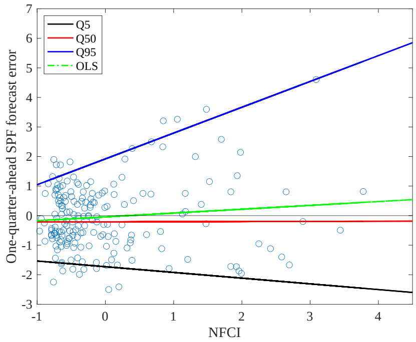
(d) Unemployment

Four quarters ahead



(e) GDP price index inflation

One quarter ahead



(f) GDP price index inflation

Four quarters ahead

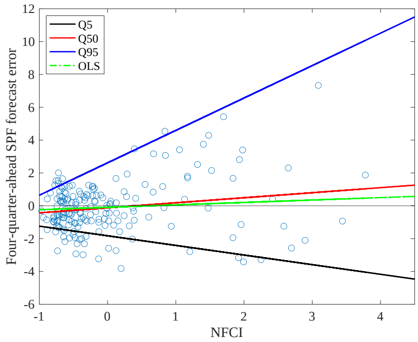
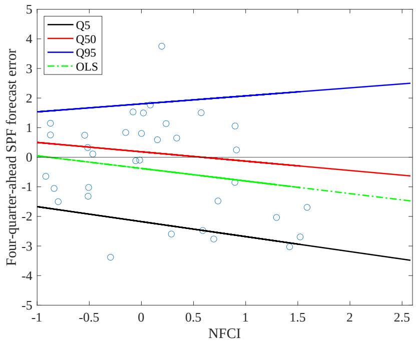


Figure 4: SPF forecast errors and financial conditions Euro area

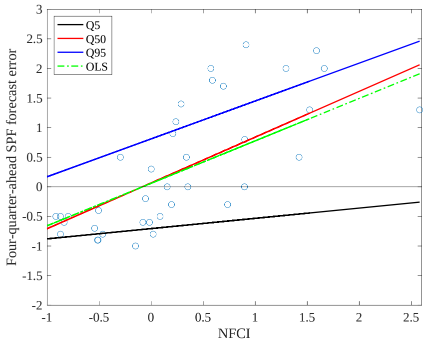
(a) Euro area real GDP growth

Four quarters ahead



(b) Euro area unemployment

Four quarters ahead



(c) Euro area price index inflation

Four quarters ahead

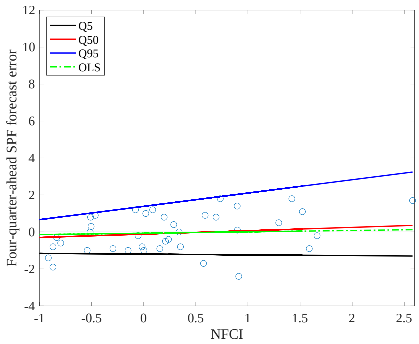
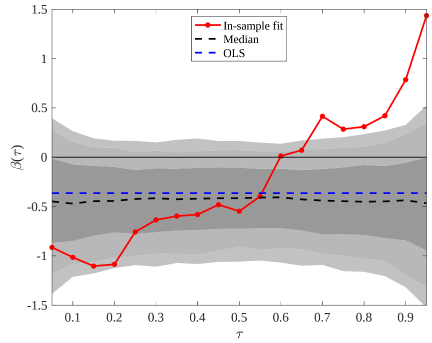


Figure 5: Quantile regression coefficients US

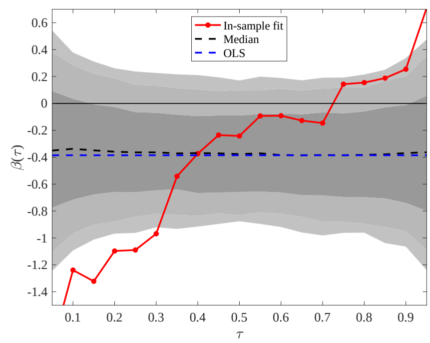
(a) Real GDP growth

One quarter ahead



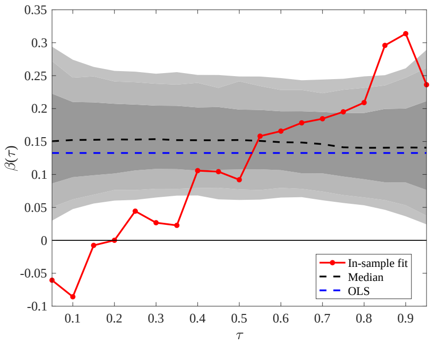
(b) Real GDP growth

Four quarters ahead



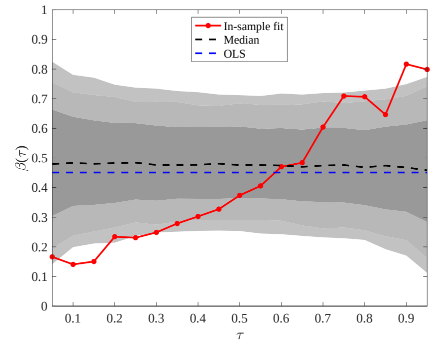
(c) Unemployment

One quarter ahead



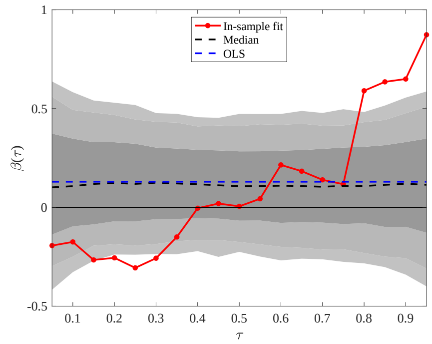
(d) Unemployment

Four quarters ahead



(e) GDP price index inflation

One quarter ahead



(f) GDP price index inflation

Four quarters ahead

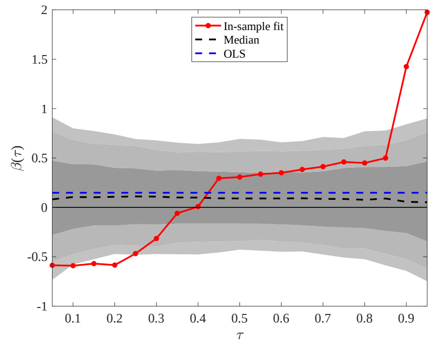
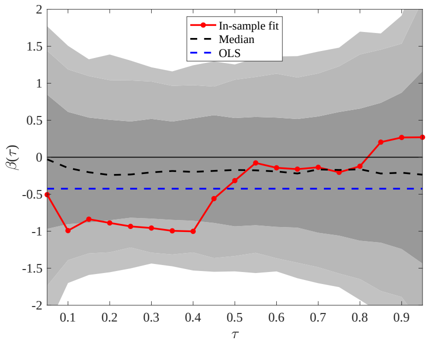


Figure 6: Quantile regression coefficients Euro area

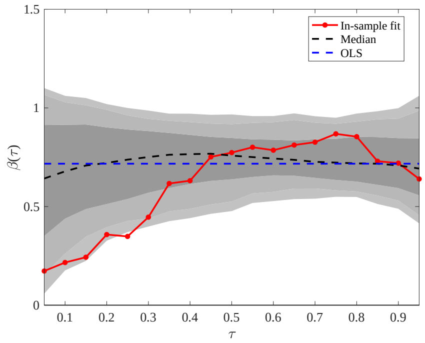
(a) Euro area real GDP growth

Four quarters ahead



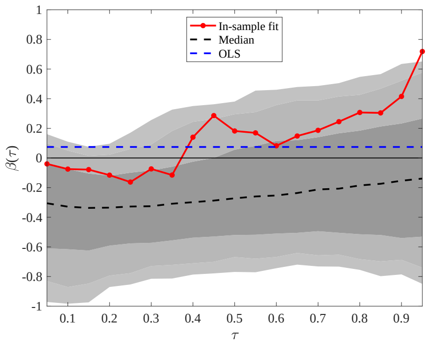
(b) Euro area unemployment

Four quarters ahead



(c) Euro area price index inflation

Four quarters ahead



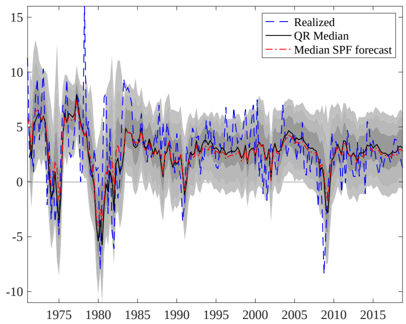
4.1.3 Estimated Predicted Distributions

Figures 7a-8c depict the fitted distributions around the SPF forecasts. For the Real GDP growth the lower tail of the quantile distributions varies much more than the upper tail. The lower tail increases a lot in times of financial tight conditions, while the upper tail stays stable over the time in times of financial stress and financial loose conditions. Moreover, this holds for both for the one-quarter-ahead and four-quarters-ahead forecasts in the US. The increase of the lower tail is not as much present in the Euro area. For the unemployment rate both in the US as the Euro area, the upside tails of the distribution are less stable and peak in the periods with financial tight conditions, this is more present in the four-quarters-ahead forecast. The forecasts are much more uncertain for a longer forecast horizon. The same conclusion can be seen in the figures for the GDP price index inflation in the US and the price index inflation in the Euro area. The lower quantiles are more stable than the higher quantiles. There is more upside risk in periods with financial stress, while the downside risk is much more stable in the total period. Also the principle that there is much more uncertainty for a longer forecast horizon holds here. Another interesting detail can be seen in Figure 8a, which shows that the real GDP growth decrease as a cause of the COVID-19 pandemic is a real shock and falls outside any of the predicted quantile distribution.

Figure 7: Estimated predicted distributions US

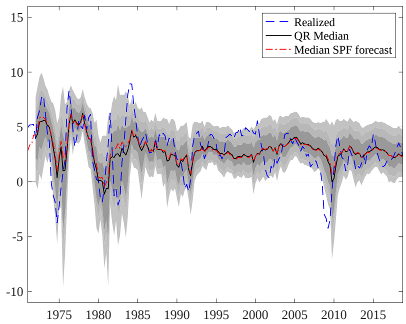
(a) Real GDP growth

One quarter ahead



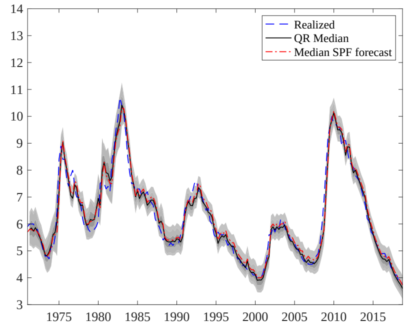
(b) Real GDP growth

Four quarters ahead



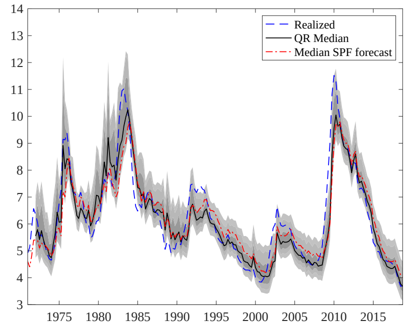
(c) Unemployment

One quarter ahead



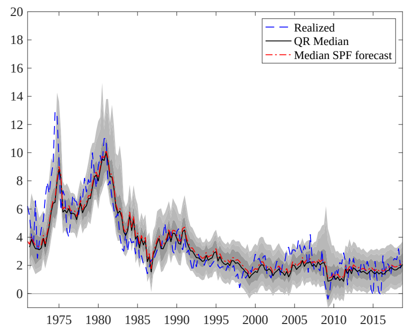
(d) Unemployment

Four quarters ahead



(e) GDP price index inflation

One quarter ahead



(f) GDP price index inflation

Four quarters ahead

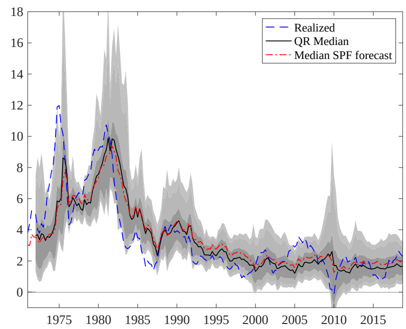
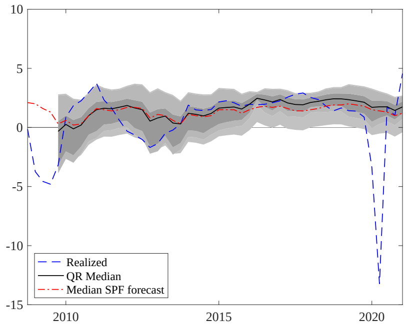


Figure 8: Estimated predicted distributions Euro area

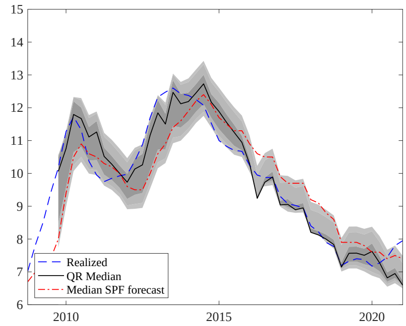
(a) Euro area real GDP growth

Four quarters ahead



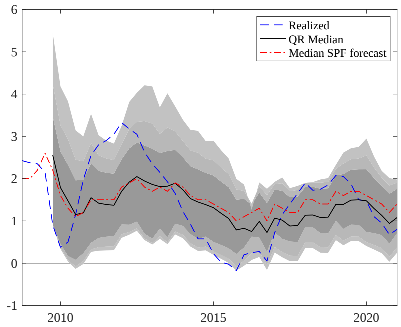
(b) Euro area unemployment

Four quarters ahead



(c) Euro area price index inflation

Four quarters ahead



4.1.4 Interquartile Range

With Figures 18a to Figure 19c in the appendix, the influence of the business cycle on the uncertainty in forecasts is shown. The difference between the 25th and 75th percentile is plotted against the SPF point forecast. If the difference is higher, the uncertainty is larger. For GDP growth, the measure of uncertainty moves counter cyclically since the value of uncertainty becomes larger as the forecasted values become smaller. Which indicates a negative correlation. This is true for both the US and the Euro area as well as for the one-quarter-ahead and the four-quarters-ahead forecasts. The unemployment rate SPF point forecasts has a positive correlation with the uncertainty measure. This means that also this uncertainty measure moves counter cyclically with the business cycle. Since unemployment has also has a negative correlation with the business cycle as well. This correlation is less visible in the Euro area, Figure 19b.

For inflation SPF point forecasts the correlation with the uncertainty measure changed in 1985 in the US. Before 1985 there was a clear positive correlation between the level of inflation forecasts and the uncertainty. The relationship is unclear after 1985 in the US. For the Euro area there seems to be a positive correlation between the uncertainty and the level of inflation. Looking at the data there seemed to be a shift in relation after 2017, but Figure 19c denies this. Inflation co-moves with the business cycle which means that the uncertainty measure for inflation is moves cyclically.

4.1.5 Predictive densities

After the skewed student's t-distribution are fitted, two periods are highlighted. These plotted predictive densities show that conditioning on financial indices is informative for the risks around the SPF point forecasts. These figures are shown in the appendix. Figures 24a, 24c and 24e show the densities in 2008Q3 which was a financial tight period. During this period the uncertainty increases compared to the unconditional predicted densities. The tails are fatter which is equal to higher probabilities for more extreme events. In more loose financial conditions, which are Figures 24b, 24d and 24f, the densities are more concentrated and have less uncertainty. This is also the case for the Euro area, where Figure 25a, 25c and 25e in the financial tight period (2009Q1).
The skewness of these predicted densities are also informative. This highlights the shift in risk in the different periods, for example Figure 24a has a much larger left tail compared to Figure 24b. This indicates that the downside risk is much higher in the financial tight period, 2008Q3. The skewness is much more visible in the Euro area where all figures show an high degree of asymmetry. If Figures 25a and 25b are compared than the financial tight period is peaked on the left tail. This same pattern is visible in the other variables, where the direction of the risk is more visible in the financial tight period, while the densities in the financial loose period are much more central around the mean.

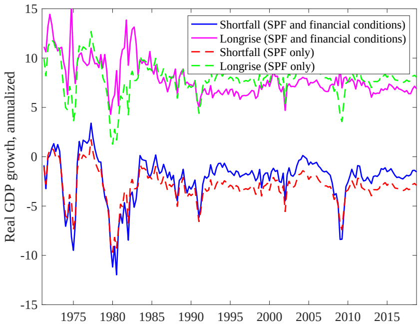
4.1.6 Expected shortfall and longrise

Figures 9a until Figures 10c plot the expected shortfall and longrise of the predicted distributions based on the financial indices and the expected shortfall and longrise of the unconditional predicted distributions. The first thing to notice is that the expected shortfall is much less stable than the expected longrise for the real GDP growth in the US, this is less visible in the Euro area. For both the unemployment and the inflation the longrise is more unstable, for both the US and the Euro area. These results are in line with the conclusions in Section 4.1.3.
Moreover, comparing the unconditional expected shortfall and longrise with the expected shortfall and longrise based on the financial condition indices it is found that these indices are not always informative in predicting tail risk. For example, when comparing the shortfall in both the unemployment and the inflation, the financial condition indices is not informative. These expected shortfalls are equal in every case, also for both the US and the Euro area. The expected longrises show that the financial condition indices are informative, since the unconditional expected longrises underestimate the risks during financial tight periods and the other way around in financial loose periods. For the real GDP the expected shortfalls and longrise both differ when comparing the unconditional against the conditional. The unconditional distribution for the one-quarter-ahead forecast underestimates the upward risk in the financial tight periods, while for the four-quarters-ahead forecast the shortfall is much more unstable. For the lower tail, the unconditional distribution underestimates the tail risk. However, the longrise in this case is much more equal. This can also be seen in Figures 7a and 7b.

Figure 9: Expected shortfall and longrise US

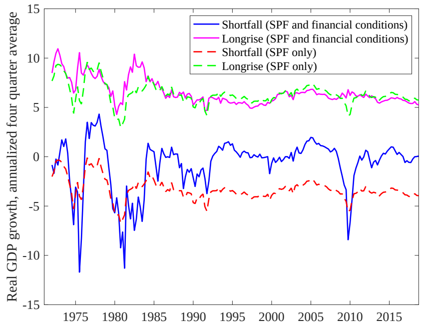
(a) Real GDP growth

One quarter ahead



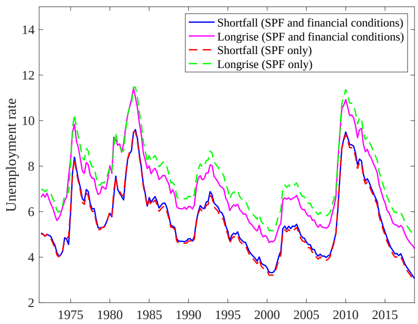
(b) Real GDP growth

Four quarters ahead



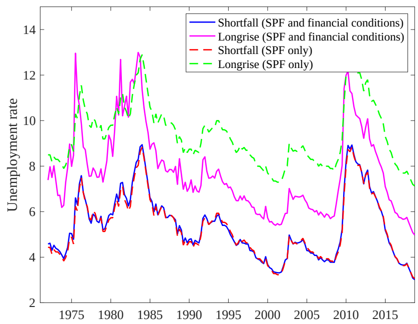
(c) Unemployment

One quarter ahead



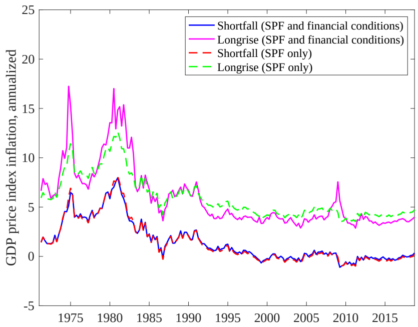
(d) Unemployment

Four quarters ahead



(e) GDP price index inflation

One quarter ahead



(f) GDP price index inflation

Four quarters ahead

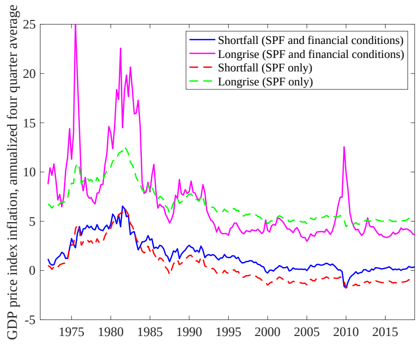
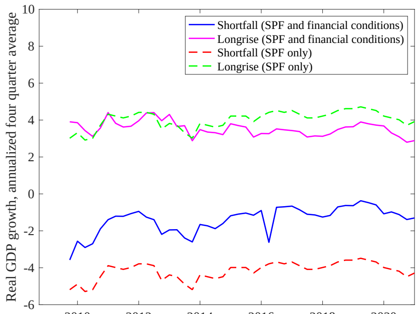


Figure 10: Expected shortfall and longrise Euro area

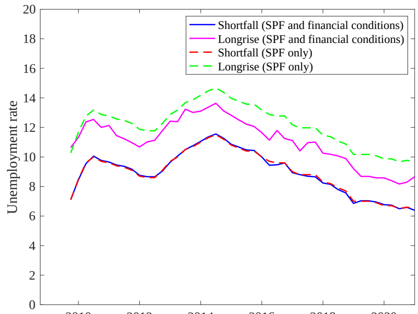
(a) Euro area real GDP growth

Four quarters ahead



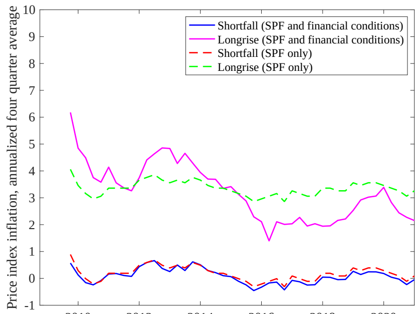
(b) Euro area unemployment

Four quarters ahead



(c) Euro area price index inflation

Four quarters ahead



4.1.7 Out-of-sample Evaluation

Figure 20a until Figure 21c have plotted the predictive scores for the different variables. However a direct conclusion cannot be seen from these figures. However, by using Equation 10 these figures can be quantified. The value is positive if the conditional linear quantile regression has higher predictive power compared tot the unconditional model, and vice versa. From Table 2 it is clear that including the financial conditions index is not always better. It certainly is better when using the one-step-ahead forecasts, since all results are significant at the 5 % level. However, using the four-quarters-ahead forecast it is unclear whether the financial conditions index increases the predictive power.

The Probability Integral Transforms (PIT) show that the empirical distributions do not significantly deviate from the 45°-line. This proofs that there is evidence for good forecast calibration. The models that are used for forecasting have sufficient intensity and dependence structure.

Table 2: Average difference in log scores:

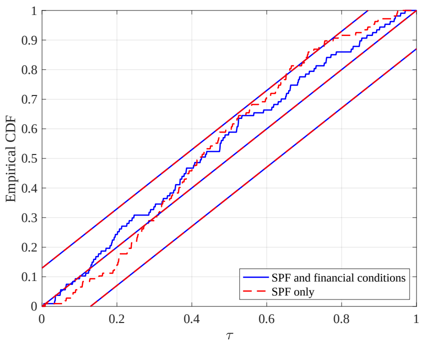
SPF and financial conditions (linear quantile regression) - SPF only

	US Real GDP	US GDP inflation	US Unemployment	EU Real GDP	EU Inflation	EU Unemployment
h=1	0.030***	0.004**	0.029**			
h=4	-0.121	-0.010	0.017**	-9.458	0.083	-2.64

*** denotes statistical significance at the 1% level, ** denotes significance at the 5% level

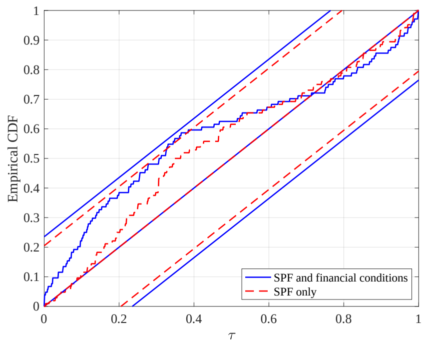
(a) Real GDP growth

One quarter ahead



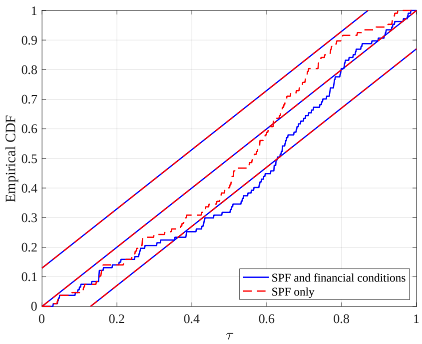
(b) Real GDP growth

Four quarters ahead



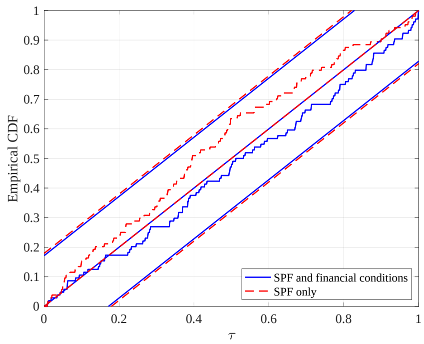
(c) Unemployment

One quarter ahead



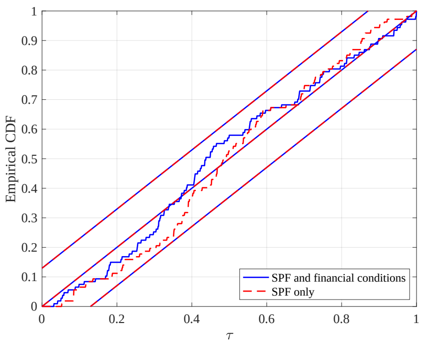
(d) Unemployment

Four quarters ahead



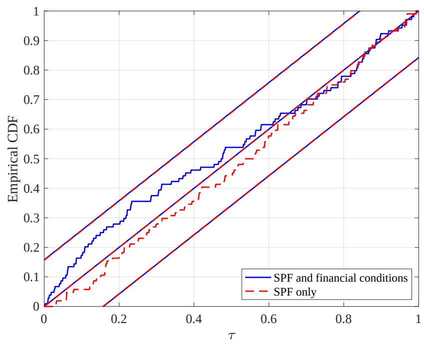
(e) GDP price index inflation

One quarter ahead



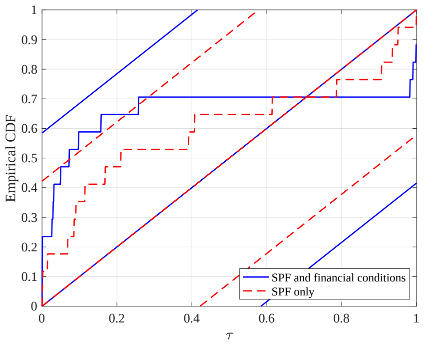
(f) GDP price index inflation

Four quarters ahead



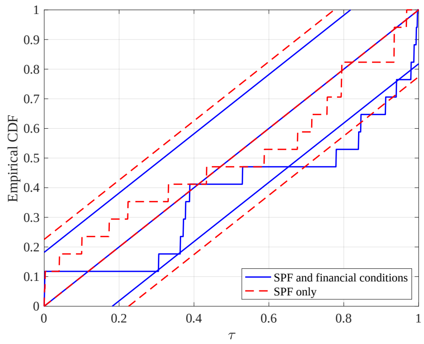
(a) Euro area real GDP growth

Four quarters ahead



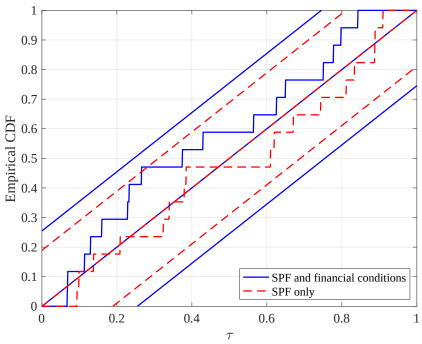
(b) Euro area unemployment

Four quarters ahead



(c) Euro area price index inflation

Four quarters ahead



4.2 Non-parametric Quantile Regression

4.2.1 Estimated Predicted Distributions

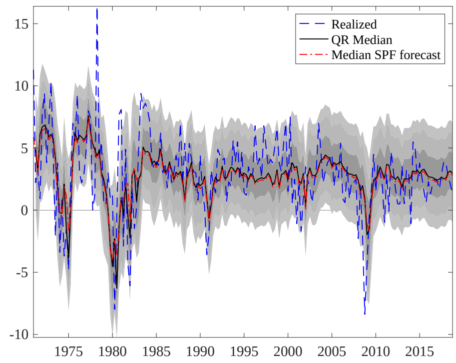
Figures 13a until 14c have plotted the non-parametric estimated predictive distributions. These figures have the same general conclusions as the linear quantile method. For the real GDP growth the lower tail risk varies much more than the upper tail risk. For the unemployment, the upside risk is much more unstable than the downside risk. Moreover, the four-quarters-ahead risk is much larger than the one-quarter-ahead forecast. Finally, the risk around the inflation both of the risk tails vary substantially. It is unclear which side varies more.

However, comparing the non-parametric method to the linear quantile regression, it is interesting to notice some differences. For the real GDP growth, the non-parametric method estimates much smaller risks around the SPF forecast. This is mainly for the four-quarters-ahead forecasts, both for US and the Euro area. The uncertainty is much smaller mainly on the downside risk side. For the unemployment the estimated uncertainty around the SPF forecast it is much smaller for the non parametric method, mainly on the upside risk side. Finally, for the estimated uncertainty around the inflation SPF forecast it is divided. For the one-quarter-ahead forecast it is larger for the non-parametric method, while for the four-quarters-ahead forecast this is smaller. Generally, the non-parametric method estimates much smaller uncertainty around the SPF forecast for the different variables.

Figure 13: Non-parametric estimated predicted distributions US

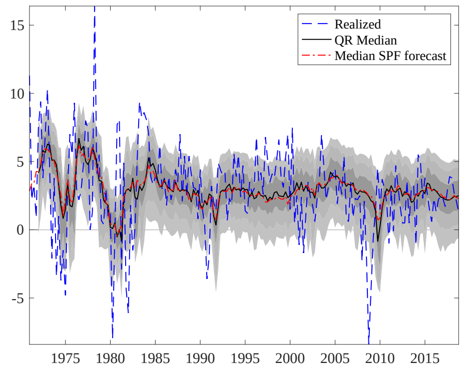
(a) Real GDP growth

One quarter ahead



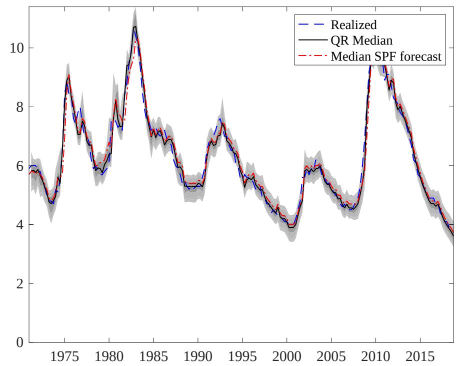
(b) Real GDP growth

Four quarters ahead



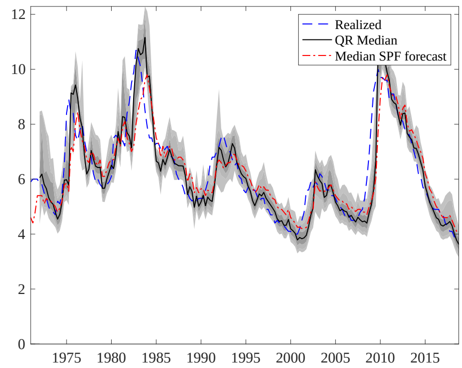
(c) Unemployment

One quarter ahead



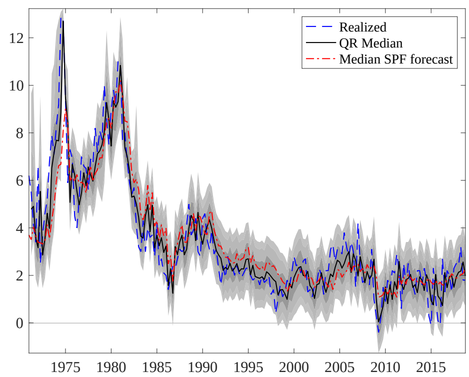
(d) Unemployment

Four quarters ahead



(e) GDP price index inflation

One quarter ahead



(f) GDP price index inflation

Four quarters ahead

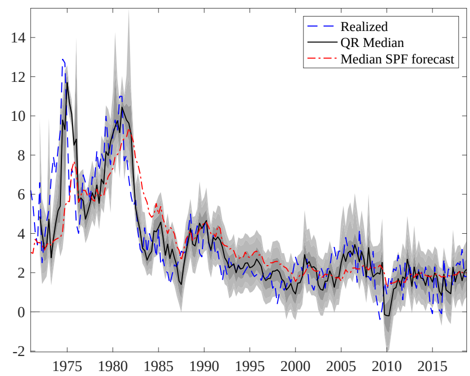
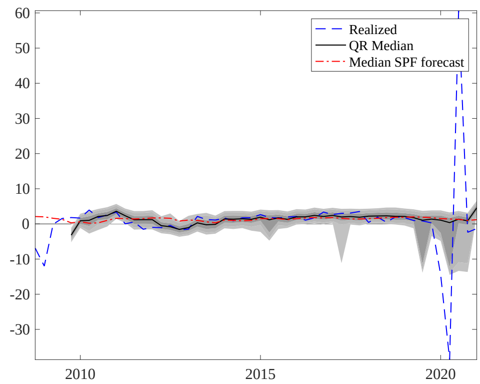


Figure 14: Non-parametric estimated predicted distributions Euro area

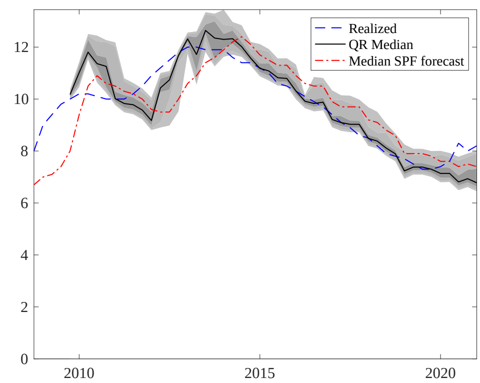
(a) Euro area real GDP growth

Four quarters ahead



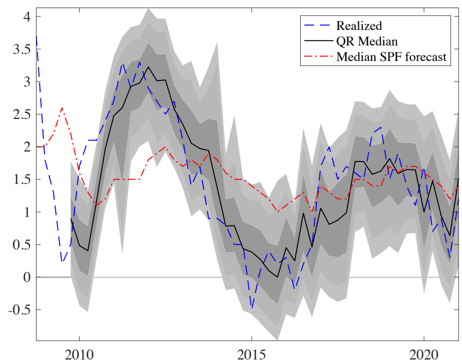
(b) Euro area unemployment

Four quarters ahead



(c) Euro area price index inflation

Four quarters ahead



4.2.2 Out-of-sample Evaluation

Figures 22a until 23c have plotted the predictive scores for the conditional and the unconditional out-of-sample estimation for the non-parametric method. The results are seen in Table 3 indicate that the non-parametric method is useful for the one-quarter-ahead forecasts. However the null hypothesis of equal means for the prediction errors cannot be rejected for all the macroeconomic variables. This gives no evidence that the non-parametric quantile regression yields better forecasts than the unconditional quantile regression. For the four-quarters-ahead forecasts the unconditional model outperforms the non-parametric quantile regression for almost every variable. Again, almost all results are not significant, only for the four-quarters-ahead forecast for the inflation in the Euro area the unconditional model is significantly better.

Furthermore, Table 4 shows the comparison between the non-parametric quantile regression against the linear quantile regression. For the one-step-ahead forecasts, the linear quantile regression model is significantly better than the non-parametric model for the real GDP growth and the unemployment rate but the non-parametric quantile regression is significantly better for forecasting the GDP price index inflation. The results for the four-quarters-ahead forecasts show that only for the US unemployment rate and the Euro area inflation the linear quantile regression is significantly better for forecasting. For the other variables the null hypothesis for equal prediction power can not be rejected. The results show that it depends on the variable and forecast horizon when the linear quantile model outperforms the non-parametric quantile model in terms of predictive power. The large negative values for the real GDP growth in the Euro area are dominated by one observation, which is completely predicted wrong. These results were insignificant, so no conclusion can be derived from this. From the probability integral transforms (PIT) in Figures 15a-16c it is clear that the null hypothesis of uniformity can not be rejected. The empirical distribution do not significantly deviate from the 45°-line. This indicates that the models are sufficiently calibrated.

Table 3: Average difference in log scores:
SPF and financial conditions (non-parametric quantile regression) - SPF only

	US Real GDP	US GDP inflation	US Unemployment	EU Real GDP	EU Inflation	EU Unemployment
h=1	0.012	0.011	0.011			
h=4	0.019	-0.030	-0.052	-2.082	-0.796**	-1.935

*** denotes statistical significance at the 1% level, ** denotes significance at the 5% level

Table 4: Average difference in log scores:
SPF and financial conditions (linear quantile regression) - SPF and financial conditions (non-parametric quantile regression)

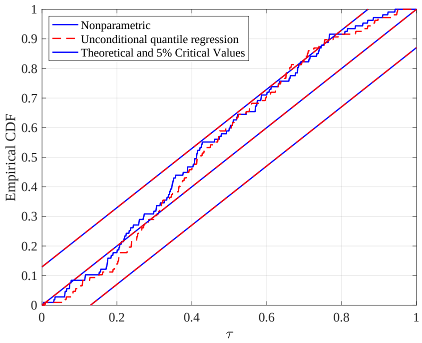
	US Real GDP	US GDP inflation	US Unemployment	EU Real GDP	EU Inflation	EU Unemployment
h=1	0.019**	-0.007*	0.018*			
h=4	-0.140	0.020	0.069*	-7.258	0.879***	1.670

*** denotes statistical significance at the 1% level, ** denotes significance at the 5% level, * denotes significance at the 10% level

Figure 15: Probability Integral Transforms US

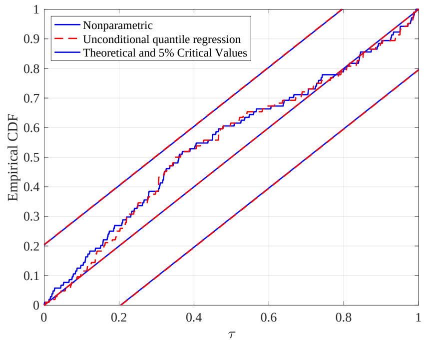
(a) Real GDP growth

One quarter ahead



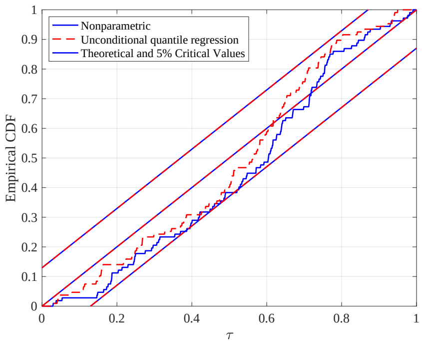
(b) Real GDP growth

Four quarters ahead



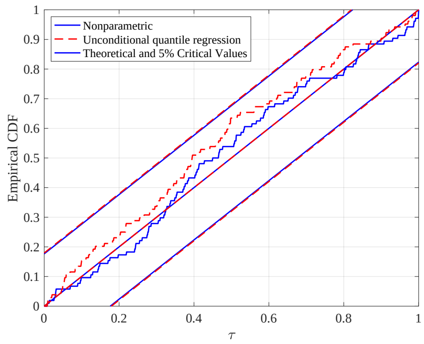
(c) Unemployment

One quarter ahead



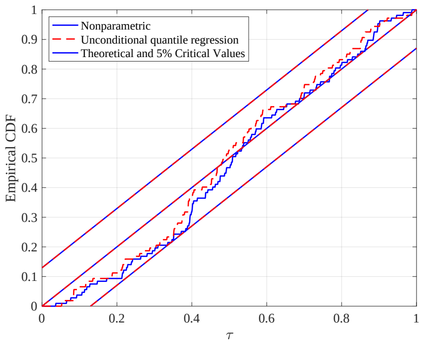
(d) Unemployment

Four quarters ahead



(e) GDP price index inflation

One quarter ahead



(f) GDP price index inflation

Four quarters ahead

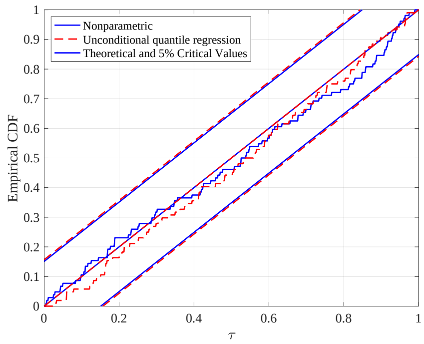
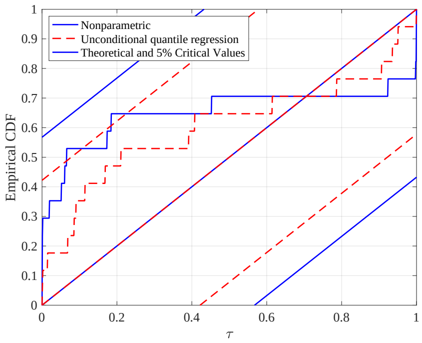


Figure 16: Probability Integral Transforms Euro area

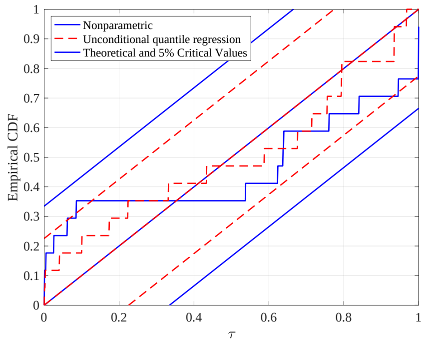
(a) Euro area real GDP growth

Four quarters ahead



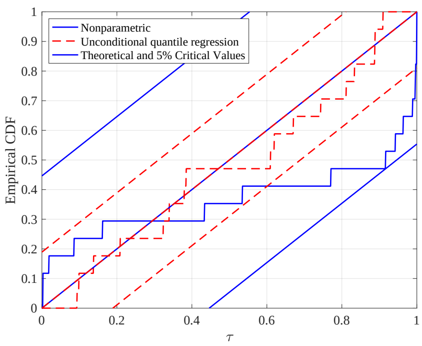
(b) Euro area unemployment

Four quarters ahead



(c) Euro area price index inflation

Four quarters ahead



5 Economic Impact

This section is going to describe how the found results can impact economic choices that are made. The main result is that economic forecasts sometimes can be improved by including the financial conditions as a conditional variable. According to Ramos et al. (2013), better forecasts do not always lead to better decision-making. The most important variable is the combination between the uncertainty and the forecast. Uncertainty in forecasts lead to more optimal decision-making because risk has been evaluated. Without the uncertainty decision makers are more often risk averse which lead to sub-optimal outcomes. Risk aversion leads to overcompensating the amount of risk there is, which is not optimal since the overcompensated amount could be used invested in something else. The density forecasts in this research can help decision-makers to come to optimal outcomes instead of using the simple point forecasts by the median SPF.

The discovered relations between the variables and the business cycle are supported by this research. These relations are supported by Diebold and Rudebusch (2021), who developed a modern and empirical view on the business cycle. The co-movements and counter-movements of these variables are important factors, but the exact relationship is still unclear. Diebold and Rudebusch (2021) propose a regime-switching model to model the relation between the variables and the business cycle. This view can also be supported by this research because of the different reactions to financial tight and loose periods. For example, the upside risk is much more volatile for unemployment compared to the downside risk. This would imply that the unemployment rate could be an large indicator in financial tight times but very small in a financial accommodate period in a regime switching model. Both these reasons is how this research will most likely impact the economy and the used macroeconomic models.

6 Conclusion

In this research, macroeconomic risk of three variables is quantified. This is analysed by means of a linear quantile regression as well as a non-parametric quantile regression. It is found that real GDP growth has a much more unstable downside risk, which is particularly high in financial tight periods. For the unemployment rate and the inflation, the upside risk is much more unstable in these periods. The total uncertainty is also larger in financial tight periods. This holds for the US as well as the Euro area for both of the models. Moreover, the found results support the existing literature in the sense that real GDP and inflation both co-move with the business cycle contrary to the unemployment rate, which moves counter-cyclically.

If the linear quantile regression conditional on the financial conditions indices against the unconditional quantile regression than it is found out that the unconditional regression fails to incorporates all the risk in the model. This results indicates that including the conditioning variable is informative for the model. Finally, an out-of-sample evaluation is used to try and copy the real situation for the forecasters. Using the out-of-sample evaluation it is found that for the one-quarter-ahead forecasted variables, the linear quantile regression model is the best. For the four-quarters-ahead forecasts, there is no significantly better model for all the variables. So per macroeconomic variable different models should be used to get the highest predictive power.

All these results can have an impact on the economy by supporting existing knowledge about business cycles and by improving regime-switching macroeconomic models.

One of the main limitations of the current research is that it is unclear whether or not the found results are reliable all over the world. The found result may not hold true in emerging countries which have different growth patterns and different business cycles. Moreover, these results may not hold in the future as data gets re-balanced or adjusted due to measurement errors or new variable determination methods.

The first main direction for further research would be to use this method in different economies around the world to see if the results are universally true. Moreover, new macroeconomic variables could be used in this method. This could give interesting new results and relations which could impact the economic models. The final direction of further research would be to examine more types of non-parametric quantile regressions which may possibly beat the linear quantile regression in terms of predictive power.

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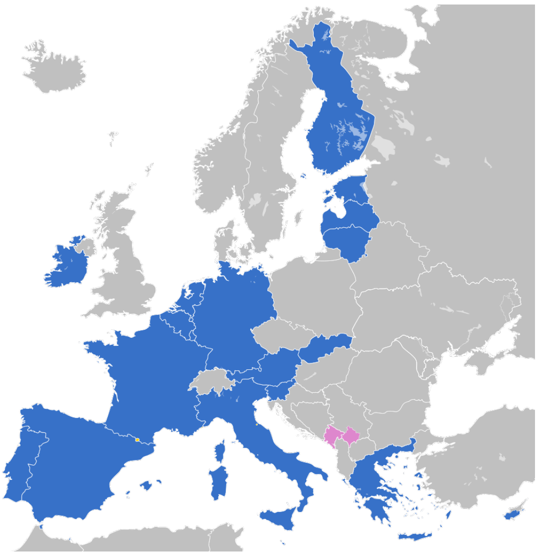
7 Appendix

Figure 17: Euro area



PDF and CDF of Student's t-distribution

$$t(x;n) = \frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac{n}{2})}(1+\frac{x^2}{n})^{-\frac{n+1}{2}} \tag{11}$$

$$T(x;n) = \frac{1}{2} + x\Gamma(\frac{n+1}{2}) \times \frac{{}_2F_1(\frac{1}{2}, \frac{n+1}{2}; \frac{3}{2}; -\frac{x^2}{n})}{\sqrt{n\pi}\Gamma(\frac{n}{2})} \tag{12}$$

where $\Gamma(x)$ is the Gamma function, defined as $\Gamma(x) = (x-1)!$, and ${}_2F_1(a,b;c;z)$ is the hypergeometric function, defined as ${}_2F_1(a,b;c;z) = \sum_{n=0}^{\infty} \frac{(a)_n(b)_n}{(c)_n}\frac{z^n}{n!}$

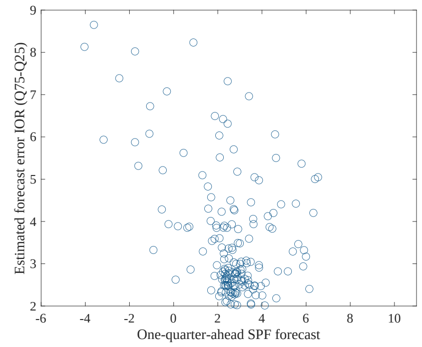
PDF and CDF of Standard Normal distribution

$$\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}} \tag{13}$$

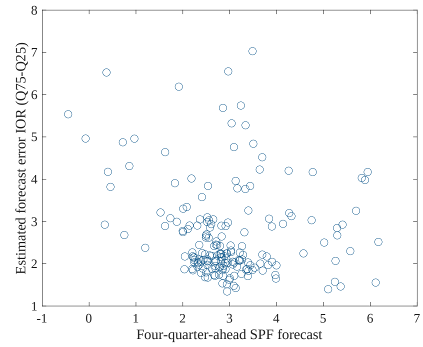
$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{t^2}{2}}\,dt \tag{14}$$

Figure 18: Interquartile Range US

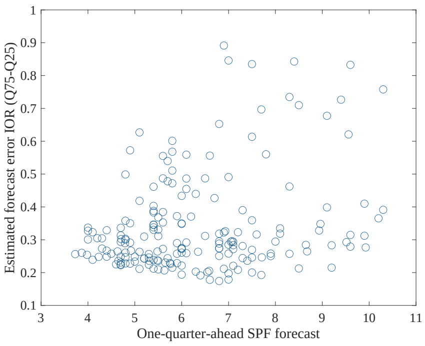
(a) Real GDP growth
One quarter ahead



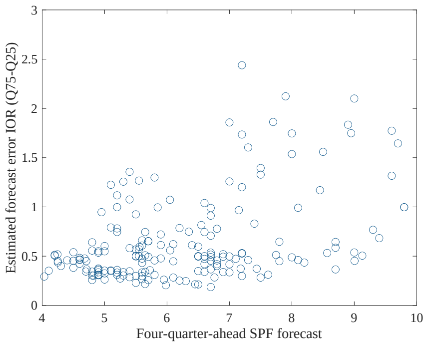
(b) Real GDP growth
Four quarters ahead



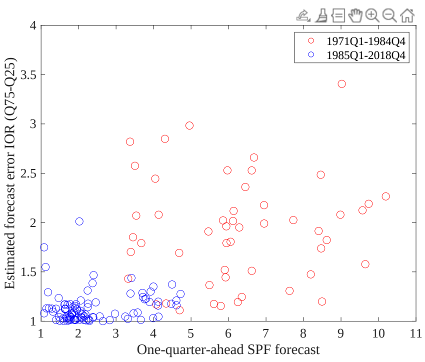
(c) Unemployment
One quarter ahead



(d) Unemployment
Four quarters ahead



(e) GDP price index inflation
One quarter ahead



(f) GDP price index inflation
Four quarters ahead

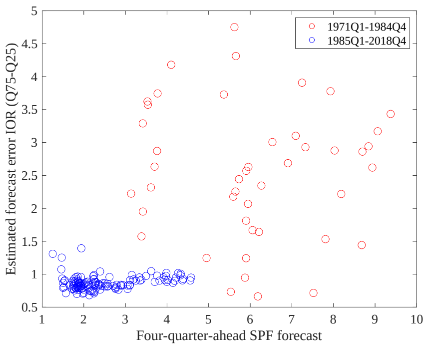
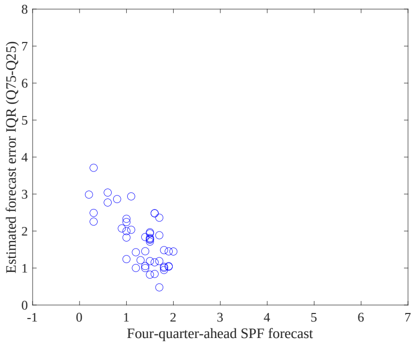
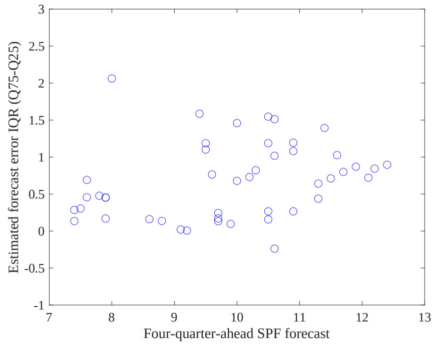


Figure 19: Interquartile Range Euro area

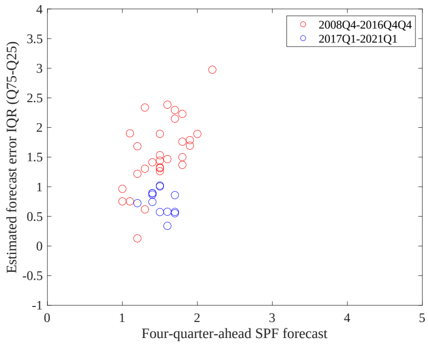
(a) Euro area real GDP growth
Four quarters ahead



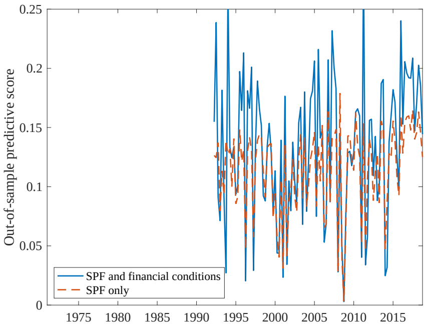
(b) Euro area unemployment
Four quarters ahead



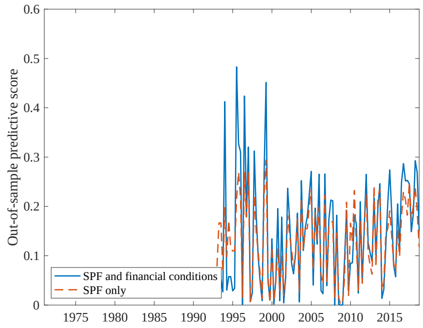
(c) Euro area price index inflation
Four quarters ahead



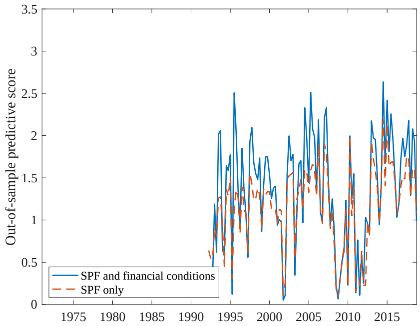
(a) Real GDP growth
One quarter ahead



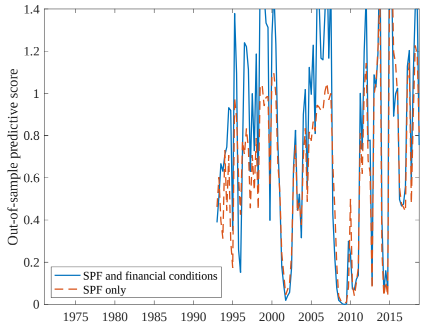
(b) Real GDP growth
Four quarters ahead



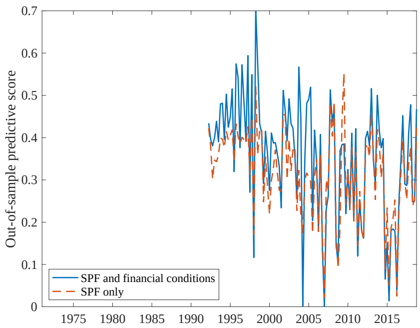
(c) Unemployment
One quarter ahead



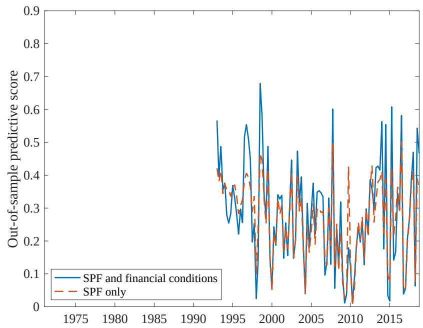
(d) Unemployment
Four quarters ahead



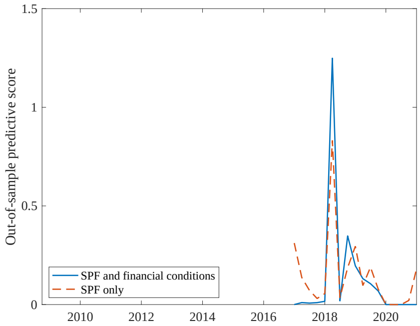
(e) GDP price index inflation
One quarter ahead



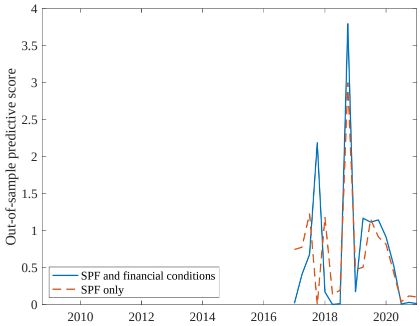
(f) GDP price index inflation
Four quarters ahead



(a) Euro area real GDP growth
Four quarters ahead



(b) Euro area unemployment
Four quarters ahead



(c) Euro area price index inflation
Four quarters ahead

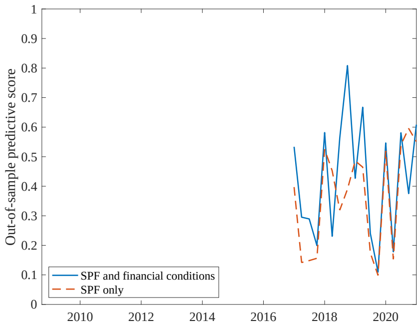
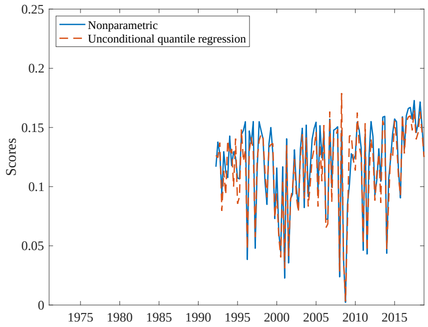
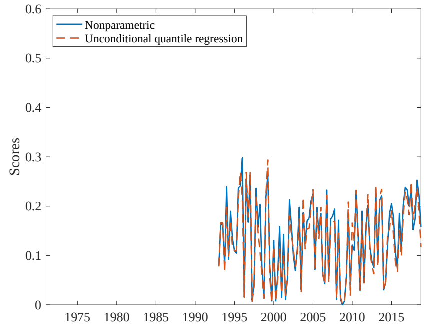


Figure 22: Out-of-sample predictive scores US (non-parametric)

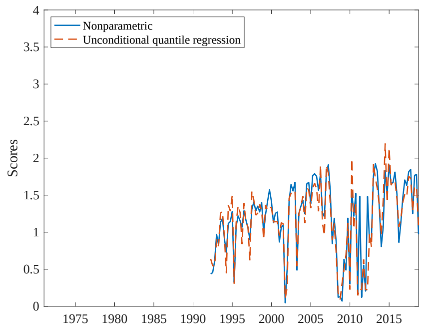
(a) Real GDP growth
One quarter ahead



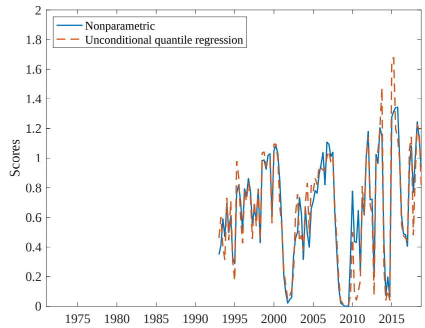
(b) Real GDP growth
Four quarters ahead



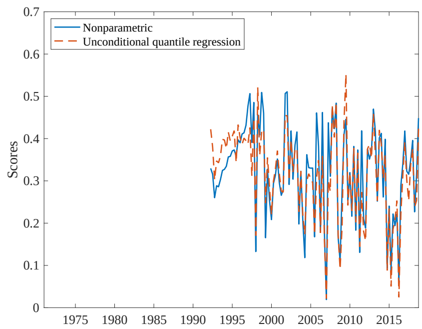
(c) Unemployment
One quarter ahead



(d) Unemployment
Four quarters ahead



(e) GDP price index inflation
One quarter ahead



(f) GDP price index inflation
Four quarters ahead

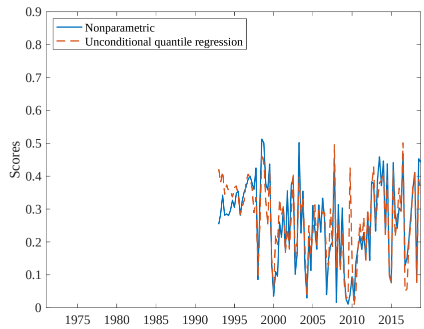
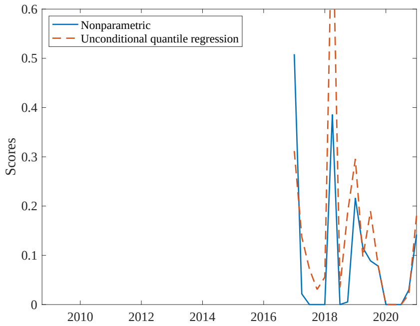
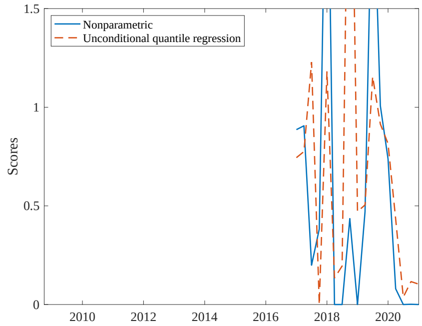


Figure 23: Out-of-sample predictive scores Euro area (non-parametric)

(a) Euro area real GDP growth
Four quarters ahead



(b) Euro area unemployment
Four quarters ahead



(c) Euro area price index inflation
Four quarters ahead

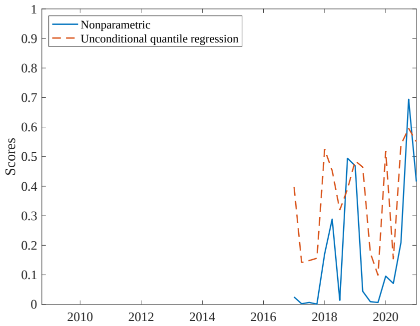
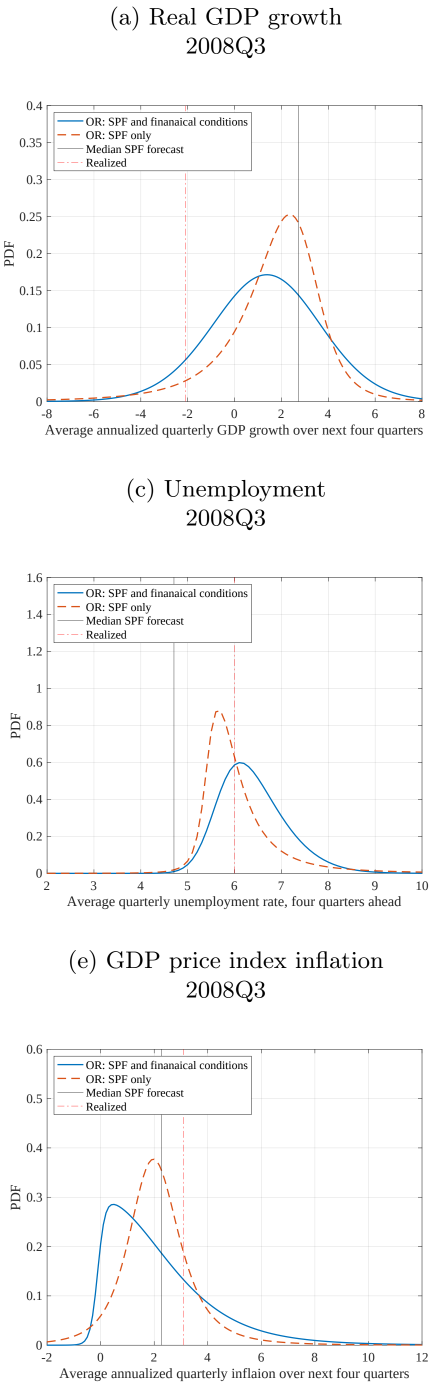


Figure 24: Predicted densities US



(a) Real GDP growth 2008Q3

(b) Real GDP growth 2017Q4

(c) Unemployment 2008Q3

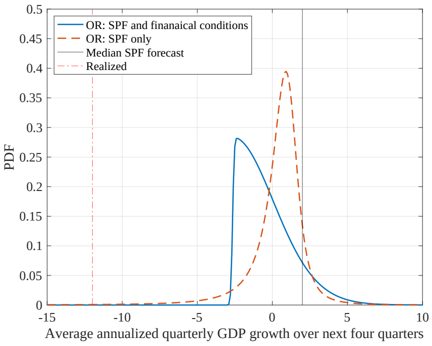
(d) Unemployment 2017Q4

(e) GDP price index inflation 2008Q3

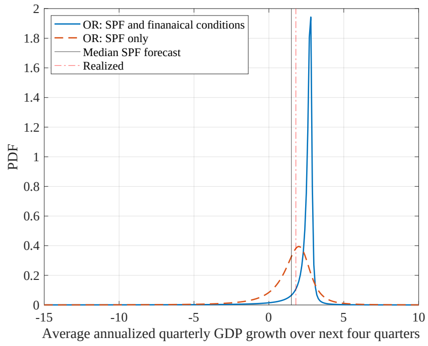
(f) GDP price index inflation 2017Q4

Figure 25: Predicted densities Euro area

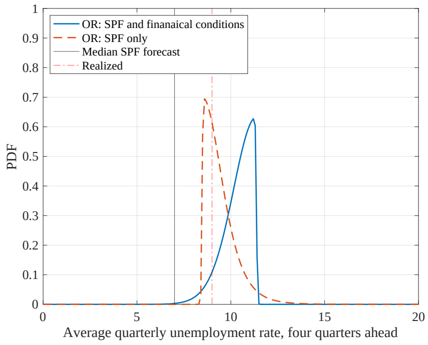
(a) Euro area real GDP growth
2009Q1



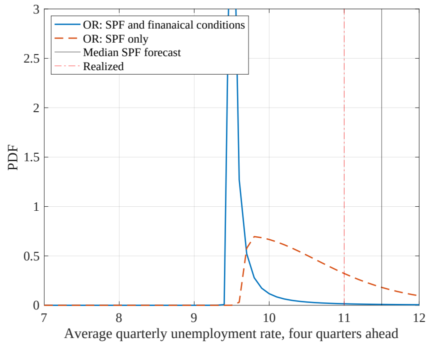
(b) Euro area real GDP growth
2015Q2



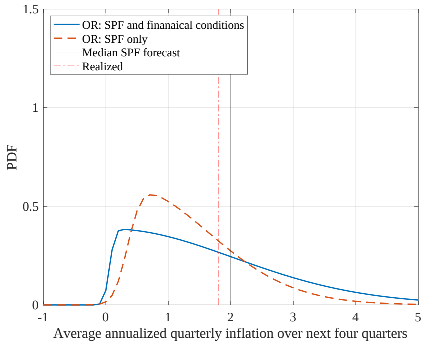
(c) Euro area unemployment
2009Q1



(d) Euro area unemployment
2015Q2



(e) Euro area price index inflation
2009Q1



(f) Euro area price index inflation
2015Q2