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## The emission allocation game: the ecological trade-off in the routing approach

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### Abstract

Business enterprises are increasingly expected to comply with emission regulations and emission targets. However, distribution routes are regularly based on minimizing the total travel time. Therefore, a discrepancy could occur between the used distribution route and the distribution route with the least amount of emission. This discrepancy is caused by a certain trade-off that has to be made in the routing approach regarding the travel time and the emitted emission. In this paper, we identify this ecological trade-off by constructing both an emission minimizing and travel time minimizing distribution route in similar instances. Additionally, we identify the consequences on the allocated amount of emission by means of the emission allocation game. We compare for both distribution routes the allocations for several allocation methods, and we evaluate the performance of all allocation methods by means of four evaluation criteria. We find no significant objection to divert from a travel time minimizing approach to an approach considering emission minimization.

*The views stated in this thesis are those of the author and not necessarily those of the supervisor, second assessor, Erasmus School of Economics or Erasmus University Rotterdam*

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# 1 Introduction

Business enterprises are under pressure to reduce the emissions across their supply chain and are increasingly expected to comply with environmental standards, see Kumar et al. (2014) and Lai & Wong (2012). One way in which these companies affect the environment is by emitting carbon dioxide ( $\text{CO}_2$ ). Canadell et al. (2007) state that the emission of  $\text{CO}_2$  is the largest human contributor to human-induced climate change. A common-used way of holding companies responsible for their contribution to the emission of  $\text{CO}_2$  is by using the carbon footprint of their products. By means of this measuring instrument, governments can verify the compliance of certain emission caps. Additionally, Chopra (2019) states that corporate social responsibility (CSR) has become more important for companies along with the traditional focus on economic performance. Therefore, it becomes more common for companies to set emission targets themselves.

Pandey et al. (2011) state that for the calculation of the carbon footprint, the total amount of  $\text{CO}_2$  emitted in the life cycle of the product has to be estimated and added. The life cycle includes all stages involved for the product. However, in this paper, we consider particularly the distribution stage used to distribute the products among the customers. Ligterink et al. (2012) have shown that the total emission is affected by several factors of the distribution route, such as vehicle type, speed, and payload. These factors make it difficult to identify the contribution of each customer to this total emission. Though, a fair and transparent manner of allocating the emission is important, due to the recent emission caps and set emission targets.

This paper follows up on Naber et al. (2015), where the allocation of  $\text{CO}_2$  emission to customers on a distribution route is investigated. In this paper, part of their research will be replicated and extended. Naber et al. (2015) have introduced the emission allocation game based on cooperative game theory. Several game-theoretic concepts are used to allocate emission, which they refer to as allocation methods. These allocation methods have been evaluated based on several criteria concerning the practicality. However, Naber et al. (2015) state that in general the routes are designed to minimize total distance or travel time. Consequently, there could be a discrepancy between the route that is used in practice and the route with the least amount of emission. It could become questionable to what extent this discrepancy could be justified in the current time. Solomon et al. (2009) have shown that climate change due to the emission of  $\text{CO}_2$  is irreversible for the next 1000 years. However, to limit the increase in global temperature to  $2^\circ\text{C}$ , as strongly recommended by the Intergovernmental Panel on Climate Change, every change seems necessary.

Therefore, it could become more relevant to consider a routing approach that minimizes the emission of CO<sub>2</sub>. Such distribution routes could reduce the overall emission to be distributed, making it easier for companies to comply with certain regulations. Though, an increase in monetary costs or travel distance could be observed for such an emission minimizing route. Hence, a trade-off has to be made while selecting the routing approach by the company. Porter & Kramer (2006) have shown that companies can develop a competitive advantage by incorporating CSR efforts into the strategy of the firm, such that the increased costs could be discounted.

By addressing the discrepancy between the two routing approaches, this ecological trade-off could be clarified for the companies. Additionally, the effects on the allocations could become clear, as well as how the performance of the allocation methods is affected by the routing approach. Therefore, we make a distinction between two types of routing approaches. The first approach determines the distribution route by minimizing the total emission of CO<sub>2</sub>. The second approach considers the distribution routes to be minimized based on travel distance, and it could be seen as a reproduction of the paper by Naber et al. (2015). For both types, five allocations methods are discussed and applied. Additionally, four evaluation criteria are used to assess the practical performance of the methods.

By diverting from distance minimizing routes to emission minimizing routes, we find on average a stronger percentual decrease in emission compared with the increase in distance. Therefore, the trade-off is in favor of the routing approach that minimizes the emission. Additionally, as less CO<sub>2</sub> is emitted, we find on average a smaller allocated amount of CO<sub>2</sub> to the customers for each allocation method. Finally, we find the practicality based on the four evaluation criteria of the allocation methods for both routing approaches to be relatively similar.

The remainder of this report is structured as follows: in Section 2 a more detailed analysis of the relevant literature can be found. Section 3 presents the problem definitions for both routing approaches. Additionally, we develop the emission allocation game and present the five allocation methods. In Section 4 the results are presented and discussed. Finally, in Section 5 a conclusion is given as well as suggestions for further research.

## 2 Literature Review

In this section, we first discuss the routing aspect of the problem, since the used distribution route determines the total emission to be allocated. In particular, we consider several models minimizing the total emission. Additionally, we present information in selected literature on the general cost allocation game, that can be translated and used to allocate the emission of CO<sub>2</sub> among the customers.

## 2.1 Routing problems concerning the minimization of emission

The used route is of influence on the total emission to be allocated. Ligterink et al. (2012) have shown that the total emission is affected by several factors, such as type of vehicle, speed, and payload. Therefore, the objective considered in this report is to design a distribution route, such that all customers are visited in an emission minimizing manner. As all customers need to be visited by a single vehicle, the Traveling Salesman Problem (TSP) seems a starting point for further research. The goal of the TSP is to formulate a route visiting all customers in a cost-minimizing manner (Flood 1956). These costs are generally given in terms of travel time or distance. The extension of the TSP, in which a fleet of vehicles is considered, is known as the Vehicle Routing Problem (VRP), see Toth & Vigo (2002). Both the TSP and the VRP are shown to be NP-hard, see Archetti et al. (2003) and Prins (2004).

However, both problems consider only an objective in terms of cost minimization. Therefore, the objective of the VRP has been adjusted to consider the minimization of emitted CO<sub>2</sub>. A particular formulation concerning this adjusted optimization problem is referred to as the Carbon Emission Based Vehicle Routing Problem (C-VRP). Many variants of the C-VRP are formulated, due to various characteristics influencing the emission. Behnke et al. (2021) and Kwon et al. (2013) propose different implementations of a C-VRP concerning the load of the vehicle and heterogeneous vehicles. Other implementations by Kuo (2010) and Behnke & Kirschstein (2017) also consider speed and acceleration of the vehicles. Kwon et al. (2013) state that this problem is considered as NP-hard. Therefore, Behnke et al. (2021) propose a column-generation approach for solving the problem, while Kwon et al. (2013) consider a tabu search algorithm.

The formulations for the objective for the C-VRP are often based on emission models incorporating the particular aspects considered in the respective problem formulation. In recent years, different emission models have been proposed due to accurate on-road emission measurements. Ligterink et al. (2012) have proposed a model based on measurements of individual vehicles under varying payloads. Therefore, their model takes into account the velocity and certain vehicle characteristics. Additional characteristics are taken into account in the emission model presented in Kirschstein & Meisel (2015). Factors such as acceleration and driving resistance are considered. Consequently, the effects of different traffic conditions are evaluated in this model. These models are validated by comparing them to empirical data as well as various other emission models. Both Ligterink et al. (2012) and Kirschstein & Meisel (2015) find that their emission models can predict the emission accurately given the particular situation that is considered.

Additionally, another problem concerning the reduction of emission on a route is known as the Pollution Routing Problem (PRP). Bektaş & Laporte (2011) have stated that the PRP is formulated to minimize the emission of different greenhouse gasses. In addition to the C-VRP, Bektaş & Laporte (2011) have translated the emission in terms of monetary costs for the PRP. Therefore, they are able to include other monetary factors often present in the routing approach, such as labor and fuel costs. Bektaş & Laporte (2011) find that the PRP is significantly more difficult to solve to optimality but has the potential of yielding savings in total costs. Zhang et al. (2015) propose the Low-Carbon Routing Problem (LCRP), and mainly relates to the PRP. However, they use a more effective method for the calculation of the fuel consumption and propose a tabu search algorithm due to the complexity of the problem.

## 2.2 The cost allocation game

After identifying the used distribution route, the total emission of CO<sub>2</sub> needs to be allocated among the customers. This can be seen as a specific implementation of a cost allocation game. The cost allocation game solves problems arising in situations, where individuals, all with their own purposes, decide to work together (Tijds & Driessen 1986). Cooperative game theory provides the tools for analyzing these problems (Young 1994). A cooperative game is constituted by a set of players and a characteristic function (Branzei et al. 2008). Using this characteristic function, for each possible subset of players the realized cost can be calculated. Such a subset of players is referred to as a coalition, and the goal of the game is to find a coalition that minimizes this total cost.

Cost allocation games are a specific implementation of cooperative game theory. Tijds & Driessen (1986) have stated that the goal of a cost allocation game is to fairly allocate the total cost of the cooperation among the players. Again, this total cost for each coalition is calculated by means of a characteristic function. For cost allocation games, the characteristic function is typically specified to be subadditive. This condition ensures that the cost of two joint coalitions never exceeds the sum of the costs of the two coalitions separately (Rosenthal 2017), and therefore encourages cooperation among the players.

Other concepts arising from the field of cooperative game theory find their use in cost allocation games. A particular concept is the definition of the core. In cost allocation games, a core allocation discourages a coalition to leave the grand coalition due to obtaining a smaller allocated cost when leaving (Tamir 1993). This guarantees the acceptability of the allocation, such that a core allocation is referred to as stable. To be in the core of an allocation game, the allocation has to satisfy two conditions, which are explained in a general setting in Potters

(1992). The first condition is referred to as the efficiency condition and ensures that the sum of all allocated costs adds up to the total costs. Secondly, an allocation has to satisfy the individual rationality condition. This means that each coalition is willing to pay at most their stand-alone cost. The stand-alone cost depicts the cost of a player if this coalition does not cooperate with anyone else.

The costs to be allocated in the cost allocation game do not have to be specified in terms of monetary costs. The costs can be translated into other units increasing its applicability. A recent application is the allocation of emissions to customers on a distribution route. Therefore, Naber et al. (2015) have introduced the emission allocation game. The game aims to propose different methods that allocate the emission of transport to individual customers for a given distribution route. A similar problem is considered in Xu et al. (2012), where a fair allocation of both the transportation costs and the CO<sub>2</sub> emission is desired for pooled supply chains. Other approaches additionally incorporate the routing aspect of the problem, as given in the pollution routing game in Kellner & Schneiderbauer (2019).

### 3 Methodology

In this section, we present the methodology used in this paper. In Section 3.1, the formulation for the C-VRP is translated to consider only a single vehicle. We refer to this problem as the Carbon Emission Based Traveling Salesman Problem (C-TSP). Next, in Section 3.2 the emission allocation game is developed similarly to Naber et al. (2015). This game is used to tackle the emission allocation problem considered in this paper. Thereafter, in Section 3.3 the allocation methods and their implementations are discussed.

#### 3.1 The Carbon Emission Based Traveling Salesman Problem

Initially, the distribution route used to visit all customers is determined. Therefore, we first introduce some notation. Let  $N = \{1, \dots, n\}$  be the set of customers in a complete undirected graph. Additionally, the central depot is denoted by 0. An Euclidian distance  $d_{ij}$  in kilometers is associated with every edge  $(i, j) \in A$ , where  $A$  is the edge set. Additionally, for each customer  $i \in N$  a demand  $q_i$  in loading units is given which has to be fully satisfied when serving that customer. This means that the vehicle has to carry at least  $q_i$  loading units when visiting customer  $i$ . However, as a limitation, each vehicle has a certain capacity  $Q$  which cannot be exceeded. To ensure that the vehicle can visit all customers on a single route, we use a similar assumption as proposed in Anily & Mosheiov (1994). We assume that the sum of all customer

demands does not exceed the capacity of the vehicle, such that the following condition holds:

$$\sum_{i \in N} q_i \leq Q. \quad (1)$$

In contrast to the regular TSP, for the C-TSP the demands of the customers and capacity of the vehicle are of interest because it enables us to monitor the load on the vehicle on each edge. This is an important factor for the calculation of the emitted CO<sub>2</sub>.

For the routing approach, we distinguish two different types. Type 1 considers a route such that the total emission is minimized. To find this particular route, we formulate the model for the C-TSP. The objective is based on the emission model used in Behnke et al. (2021) since this model measures the emission based on a linear model. More specifically, the total emission is divided into a certain fixed emission and a load-dependent emission. They define the parameters  $c^{\text{fix}}$  and  $c^{\text{load}}$ , that represent the fixed and load-dependent emission coefficients, respectively. In particular, the parameter  $c^{\text{fix}}$  depends on the various factors needed to overcome air and rolling resistance of the unloaded vehicle. The parameter  $c^{\text{load}}$  takes into account the additional resistance caused by the additional payload. An overview of the required notation and the equations for both parameters are given in Appendix A.

Additionally, the constraints are based on the model for the C-VRP given in Kwon et al. (2013), however, they are adjusted such that a single vehicle is considered. Let  $x_{ij}$  be a binary variable set to 1 if the arc  $(i, j)$  is used, otherwise 0. Additionally, we define the variable  $f_{ij}$  for every  $(i, j) \in A$  indicating the number of loading units on the truck on that edge. The C-TSP is then formulated as follows:

$$\text{minimize} \quad \sum_{(i,j) \in A} d_{ij}(c^{\text{fix}}x_{ij} + c^{\text{load}}f_{ij}), \quad (2a)$$

$$\text{subject to} \quad \sum_{j \in N \cup \{0\}, i \neq j} x_{ji} = \sum_{j \in N \cup \{0\}, i \neq j} x_{ij} = 1 \quad i \in N \cup \{0\}, \quad (2b)$$

$$\sum_{j \in N \cup \{0\}} f_{ji} - \sum_{j \in N \cup \{0\}} f_{ij} = q_i \quad i \in N, \quad (2c)$$

$$q_j x_{ij} \leq f_{ij} \leq (Q - q_i) x_{ij} \quad (i, j) \in A, \quad (2d)$$

$$f_{ij} \geq 0 \quad (i, j) \in A, \quad (2e)$$

$$x_{ij} \in \mathbb{B} \quad (i, j) \in A. \quad (2f)$$

The objective function (2a) establishes the minimization of the total emission for the distribution route. In addition, constraints (2b) ensure the connectivity of the solution, i.e., each location has an incoming and outgoing edge. Additionally, constraints (2c) ensures that the demand of



each customer is satisfied while simultaneously removing subtours. Constraints (2d) represent the vehicle's capacity. Finally, constraints (2e) and (2f) are domain constraints on the variables  $f_{ij}$  and  $x_{ij}$ , respectively.

Additionally, for the routing approach Type 2 is also defined. This type can be seen as a replication of the original paper by Naber et al. (2015). They consider the distribution route, such that the total distance is minimized. Therefore, for Type 2, the objective of the previous model is adjusted to:

$$\text{minimize} \quad \sum_{(i,j) \in A} d_{ij}x_{ij}, \quad (3)$$

while the constraints are kept similar. This objective ensures the minimization of the total travel distance on the distribution route. Finally, it must be noted that due to the correlation between the traveled distance and the emitted emission, similar routes could be obtained for both types.

### 3.2 The emission allocation game

Cooperative game theory is used to develop the emission allocation game used for solving the emission allocation problem. A similar description is used as introduced by Naber et al. (2015). A distribution route visiting customers  $S \subseteq N$  is represented by a permutation  $\sigma(S)$ . As discussed before, we distinguish two possible implementations for the used distribution route. The route  $\tilde{\sigma}(N)$  visits all customers in an emission minimizing way as given by Type 1, and is calculated by the Model (2a) - (2f). Additionally, we define the route  $\hat{\sigma}(N)$  that visits all the customers in a manner such that the total travel distance is minimized. This routing approach is considered in Type 2, and is calculated with a similar model. Only the objective is adjusted as given in Equation (3). As in Naber et al. (2015), we assume that only the routes  $\tilde{\sigma}(N)$  and  $\hat{\sigma}(N)$  are available. This means that each subroute  $\tilde{\sigma}(S)$  or  $\hat{\sigma}(S)$  cannot be determined for all  $S \subset N$ . Therefore, we determine  $\tilde{\sigma}(S)$  by simply visiting the customers in the order as prescribed by  $\tilde{\sigma}(N)$ , while omitting the customers not in  $S$ . A similar approach is used for determining the routes  $\hat{\sigma}(S)$ .

Consequently, let  $g(\sigma(S))$  be the total emission of CO<sub>2</sub> if customers  $S$  are served in the order  $\sigma(S)$ . To compute  $g(\sigma(S))$  the emission model is used as given in Behnke et al. (2021). As mentioned before, a full description of this emission model can be found in Appendix A. Given the route  $\sigma(S)$  the characteristic function  $e(S)$  for this cooperative game is given by:

$$e(S) = g(\sigma(S)). \quad (4)$$

Furthermore, for the characteristic function we assume that  $\sum_{i \in N} e(\{i\}) \geq e(N)$ . This way, it is possible that all emission is allocated to the customers such that each customer gets at most its stand-alone emission. Indeed, this does not occur for any of the instances considered in this paper. With the characteristic function fully specified, solution concepts from cooperative game theory can be applied to find the allocated amount of emission for each customer. An allocation of the emission is given as  $x = (x_i)_{i \in N}$ . For convenience, we use  $x(S) = \sum_{i \in S} x_i$ . Finally, the core of the game is defined as:

$$\text{core}(e) = \{x \in \mathbb{R}^n : x(N) = e(N); x(S) \leq e(S), \forall S \subset N\}. \quad (5)$$

It can be seen that the core is based on the efficiency condition and the individual rationality condition as discussed in Section 2.2. When these two conditions cannot be satisfied for a given instance, it could occur that the core is empty. Naber et al. (2015) present a simple example of an instance with an empty core. Moreover, observe that by definition of the core negative allocations are allowed.

### 3.3 The allocation methods

For the allocation of the emission, we use the same five allocation methods as used in Naber et al. (2015). The first method is a proportional allocation method based on common practice, referred to as the Star method. The next four methods are based on allocation concepts from cooperative game theory. These four methods are the Shapley value, the Nucleolus, the Lorenz Allocation, and the Equal Profit Method (EPM). For the latter two, we use the same adjusted versions as in Naber et al. (2015), such that we refer to them as Lorenz+ Allocation and Equal Profit Method+ (EPM+).

#### 3.3.1 Star method

The Star method allocates the amount of emission proportionally to the stand-alone emission of customers. Therefore, for every customer  $i$  the allocated amount of emission is equal to:

$$x_i = \frac{e(\{i\})}{\sum_{i \in N} e(\{i\})} e(N). \quad (6)$$

Allocations generated by the Star method are not guaranteed to be in the core even if the core is non-empty. Furthermore, note that the allocation generated by the Star method is independent of the order in which the customers are visited and the distances between them. It is dependent on the route only through the total emission that it yields.

### 3.3.2 Shapley value

The Shapley value allocates emission to each customer equal to the average marginal emission over all coalitions (Shapley 1953). The emission allocated to customer  $i$  is calculated as:

$$x_i = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(n - |S| - 1)!}{n!} m_i(S), \quad (7)$$

for which  $m_i(S)$  denotes the marginal emission of adding customer  $i$  to subset  $S$ , such that  $m_i(S) = e(S \cup \{i\}) - e(S)$ . Shapley (1971) has shown that allocations based on the Shapley value are not necessarily in the core. This only occurs for specific games, such as convex cooperative games. As discussed before, it is shown that for the emission allocation game instances could occur with an empty core, such that this game is non-convex.

### 3.3.3 Nucleolus

The Nucleolus is introduced by Schmeidler (1969). We define the excess of an allocation as  $e(S) - x(S)$ . With the Nucleolus, we calculate the allocation with the largest excess among all coalitions. As stated by Schmeidler (1969) the excess of a coalition reflects the “attitude” towards the allocation, and the Nucleolus can be interpreted as the most accepted allocation. To find this allocation, a similar approach is used as proposed in Engevall et al. (1998). They use a method in which different linear programming (LP) problems are solved successively. First, an initial allocation is found by maximizing the smallest excess. If the obtained allocation is not unique, fix the excess for those coalitions with strictly positive dual variables. From Duality Theory, when the optimal value of a dual variable is strictly positive, the inequality constraint associated to this variable must hold with equality.

The decision variable  $w_t$  represents the optimal value in iteration  $t$  for the excess for all coalitions for which the excess is not yet fixed. Furthermore, collection  $F_t$  represents the coalitions that are fixed after iteration  $t$  with  $F_1 = \emptyset$ . The problem solved in iteration  $t$  is given as:

$$\text{maximize } w_t, \quad (8a)$$

$$\text{subject to } x(\{i\}) \leq e(\{i\}) \quad i \in N, \quad (8b)$$

$$x(S) + w_t \leq e(S) \quad S \subset N, S \notin (\cup_{q < t} F_q), \quad (8c)$$

$$x(S) + w_q = e(S) \quad q < t, S \in F_q, \quad (8d)$$

$$x(N) = e(N). \quad (8e)$$

The objective function (8a) establishes the maximization of the smallest excess in iteration  $t$ . Constraints (8b) ensures the individual rationality condition of each customer. Constraints (8c) and (8d) represent the constraints for the non-fixed and fixed excesses of each coalition, respectively. Finally, constraint (8e) satisfies the efficiency condition.

Let  $\lambda_t$  be the vector of optimal dual variables associated with constraint (8c), such that  $F_t = \{S \subset N | \lambda_t(S) > 0\}$ . The algorithm terminates if a unique solution is found. This corresponds with the constraint matrix corresponding to the fixed coalitions (8d) and the efficiency constraint (8e) having rank  $|N|$ . The Nucleolus generates an allocation in the core if the core is non-empty.

### 3.3.4 Lorenz+ Allocation

The Lorenz Allocation was introduced as “Leximin” by Arin (2003). For the general Lorenz Allocation, we solve the following LP problem, in which the absolute difference  $f$  between all allocations is minimized:

$$\text{minimize } f, \tag{9a}$$

$$\text{subject to } x_i - x_j \leq f \quad i, j \in N, \tag{9b}$$

$$x(S) \leq e(S) \quad S \subset N, \tag{9c}$$

$$x(N) = e(N). \tag{9d}$$

The objective function (9a) establishes the minimization of the difference between the allocations. This difference is defined in constraint (9b). Constraints (9c) and (9d) ensure the validation of both core conditions. It could be seen that the Lorenz Allocation cannot produce an allocation when the core is empty. Therefore, if the core is empty, the Nucleolus is used to generate the allocations. This method is referred to as the Lorenz+ Allocation. The Lorenz+ Allocation is guaranteed to produce an allocation in the core if the core is non-empty, as this holds for both the general Lorenz Allocation and the Nucleolus. Furthermore, the Lorenz+ Allocation is in general not unique, as the solution to the LP problem (9a) - (9d) is not unique in general.

### 3.3.5 Equal Profit Method+

For the EPM, as introduced by Frisk et al. (2010), a similar LP problem is solved as for the Lorenz Allocation. However, in this case we minimize the largest difference between allocations relative to the stand-alone emission of a customer. The LP problem is given as:

$$\text{minimize } g, \tag{10a}$$

$$\text{subject to } \frac{x_i}{e(\{i\})} - \frac{x_j}{e(\{j\})} \leq g \quad i, j \in N, \tag{10b}$$

$$x(S) \leq e(S) \quad S \subset N, \tag{10c}$$

$$x(N) = e(N). \tag{10d}$$

The objective function (10a) establishes the minimization of the relative difference between the allocations, that is defined in constraint (10b). Again, constraints (10c) and (10d) ensure the validation of both core conditions. Similar to the Lorentz Allocation, the EPM does not give an allocation when the core is empty. Therefore, the Nucleolus is used to generate an answer when this occurs. The allocation obtained by this method is referred to as the EPM+. The EPM+ is guaranteed to generate an allocation in the core if the core is non-empty. Again this allocation is not unique in general.

## 4 Results

In this section, we present the results of the experiment. As there is no data set available, we will first discuss the experimental design, similar as used in Naber et al. (2015). This design includes the evaluation criteria for the various methods and the data generation process. This process generates several distinct instances with each having its own properties. Thereafter, on each instance both routing approaches as given for Type 1 and Type 2 are applied, followed by the five allocation methods. Hence, we obtain allocations for each allocation method and each routing approach in every instance. We first present certain characteristics of the used distribution routes and the allocations. Next, the evaluation criteria are applied to the allocations.

All computations were performed on an Intel Core i5-8257U @ 4.1 GHz with 4 cores and 8 threads. For the randomly generated instances, both the routing methods and the allocation methods are implemented in Eclipse 4.18.0 using the Java programming language. We use CPLEX 20.10 to solve the LP problems, and EViews 11 is used for the regression analysis.

### 4.1 Experimental design

In this experiment, the five allocation methods are applied to both the routing approaches in different instances. This way we are able to assess the differences between Type 1 and Type 2. The practical performance of each allocation method is evaluated by means of four evaluation

criteria. Firstly, an allocation method is preferred to be stable. To assess the stability property, we check per instance whether the allocation is in the core of the game as defined in Equation (5). Secondly, the criterion “consistency” evaluates whether an allocation changes similarly to the underlying data. A regression analysis is performed which is explained in more detail in Section 4.3. Thirdly, an allocation method is evaluated based on the robustness. If a customer receives a similar shipment periodically, it will resent being allocated significantly different amounts of emission. Therefore, to assess the robustness, we evaluate the allocation to a single customer, referred to as the target customer. The properties of the target customer do not change over the instances, while the properties of other customers do change. Finally, the computation time is considered for each allocation method. Clearly, companies that frequently have to use allocation methods prefer low computation times.

Therefore, we have generated random instances based on the procedure used in Naber et al. (2015). In this generation process, each instance has different properties, such that the results and performance can be evaluated in different circumstances. We consider three possibilities for the number of customers (5, 10 and 15), 10 configurations of customer locations discussed in more detail later, and 3 types of demand sizes (low, average and high). By applying a full combinatorial design, a total of 90 instances is obtained.

Firstly, the generation process for the customer and depot locations is discussed. We distinguish between three groups: the depot, the target customer, and the rest of the customers. These groups can be located close together or further apart, such that the total emission is influenced by the relative distance between the groups. The specific locations for each group are generated as follows. We have constructed five squares with sides of length 30 km. The first square D contains the depot directly in the center. The second square T contains the target customer whose position is generated by means of a uniform distribution over the square. Finally, the last three squares contain either the remaining 4, 9 or 14 customers, and are given by the names C4, C9 or C14, respectively. The positions of these other customers are also uniformly distributed over the square. We have increased the length of the sides compared to the method used in Naber et al. (2015), to increase the distances between the other customers. Each square is generated only once at the start of the procedure.

Next, a large square is considered with sides of length 100 km and we place the squares D, T, and either one of the squares C4, C9 or C14 in the square. (For brevity, we indicate the chosen square of either C4, C9 or C14 simply with C hereafter.) This way, each group is represented in the instance. Each square is either positioned in the bottom left corner, the center or the top right corner, see Figure 1. These positions are referred to as regions, and we allow different squares to be located in the same region. This leads to a total of 10 distinct configurations of

the squares. Finally, we assume a constant driving speed of 50 km/h between each region.

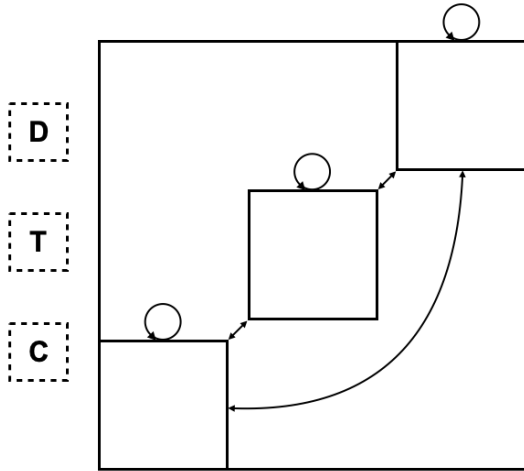


Figure 1: Visualization of the position of each square used for the randomly generated instances.

Table 1: All possible configurations of locations for each scenario.

Scenario type	Bottom left	Center	Top right
Scenario 1	D, T, C		
	D, T	C	
	D, T		C
Scenario 2	D, C	T	
	D	T, C	
	D	T	C
	C	D	T
Scenario 3	D, C		T
	D	C	T
	D		T, C

We subdivide the instances into three scenarios based on the distance between the depot and the target customer. This way, when evaluating robustness, the property defined as the distance to the depot remains fixed for the target customers per scenario. For Scenario 1, the depot and the target customer are in the same region. However, for Scenario 2 both locations are one region apart, and finally, in Scenario 3 the depot and the target customer are 2 regions apart. The configurations for each scenario are given in Table 1.

Finally, certain vehicle characteristics are assumed. The weight of the vehicle is set to 5 tons and the vehicle capacity is equal to 507 loading units. A single loading unit is assumed to weigh 0.01 tons. Additional characteristics are considered for the vehicles in comparison with Naber et al. (2015). These can be found in Appendix A. The demand for the target customers is fixed and set to medium demand, equivalent to 7.1 loading units. The demand for the other customers is changed dependent on the instance. As discussed before, three possible demands are possible: all other customers have either low (1.4), medium (7.1), or high demand (35.7). This way the summed demand of all customers never exceeds the vehicle's capacity, and we ensure that each customer can be visited in a single route.

## 4.2 Routing and allocation characteristics

We applied both routing approaches given as Type 1 and Type 2 to each instance. For 58 of the 90 instances, different distribution routes in the same instance are found. The differences are

evenly distributed over the three scenarios as this is also dependent on the locations of the other customers, and not only on the location of the target customer. For Type 1, an average travel distance of 208.65 km per distribution route is found. Additionally, per distribution route, an average amount of emission equal to 70,351 grams is realized in terms of emitted CO<sub>2</sub>. For Type 2, the average travel distance decreases to 208.50 km. Therefore, the total difference in travel distance among all instances is equal to approximately 13 km. The average emission for a distribution route of Type 2 increases to 70,738 grams. This results in a total of 34,836 grams of CO<sub>2</sub> emission saved among all instances when using emission minimizing routes.

To gain an insight into the ecological trade-off, we use the percentual deviation of both route characteristics. When diverting from a Type 1 routing approach to a Type 2 routing approach, we find an increase of 0.6% for the average emission. However, the decrease in terms of average distance traveled is only 0.1%, such that a relatively smaller benefit in terms of travel distance is obtained. Therefore, if we equally value both aspects of the distribution route, we find the trade-off generally to be in favor of the emission minimizing route.

Next, we discuss the differences between the allocations for Type 1 and Type 2. As expected, the allocated values are generally smaller for distribution routes of Type 1. We find on average a reduction of 38.7 grams in the allocated amount of CO<sub>2</sub>. The Star method performs best in fairly distributing the total reduction among the customers. Percentage wise each customer obtains a similar reduction. However, we find for 20.8% of all allocations generated by the four other methods an increase in allocated values, such that certain customers are negatively affected by the reduced emission. This suggests that properties such as visit order and the distances between customers are affecting the allocations of these four methods more substantially than total emission. Additional details of the allocations are presented in Appendix B.

### 4.3 Practical performance

Having obtained the allocations for the distribution routes of both Type 1 and Type 2 for all instances, we can assess the practicality of the allocation methods by means of the four different evaluation criteria previously discussed.

#### 4.3.1 Stability

Firstly, we discuss the stability of each method. As discussed before, an allocation method is referred to as stable if it is able to generate an allocation in the core as defined by Equation (5). The core turned out to be empty for 4 out of the 90 instances for Type 1 (zero for Scenario 1, two for Scenario 2, and two for Scenario 3). The instances with an empty core for Type 2 doubled,



such that 8 out of the 90 instances turned out to have an empty core (zero for Scenario 1, four for Scenario 2, and four for Scenario 3). In Table 2 for all allocation methods, the percentage of generated allocations that are in the core are given for both types per scenario. First is the percentage given including the instances with empty cores. Thereafter, these instances are excluded.

Table 2: Percentage of allocations in the core for both types for all allocation methods.

Scenario type	Allocation method	% alloc. in core incl.		% alloc. in core excl.	
		Type 1	Type 2	Type 1	Type 2
Scenario 1	Star method	0.0	0.0	0.0	0.0
	Shapley	100.0	92.6	100.0	92.6
	Nucleolus	100.0	100.0	100.0	100.0
	Lorentz+	100.0	100.0	100.0	100.0
	EPM+	100.0	100.0	100.0	100.0
Scenario 2	Star method	44.4	44.4	47.1	50.0
	Shapley	91.7	80.6	97.1	90.6
	Nucleolus	94.4	88.9	100.0	100.0
	Lorentz+	94.4	88.9	100.0	100.0
	EPM+	94.4	88.9	100.0	100.0
Scenario 3	Star method	33.3	33.3	36.0	39.1
	Shapley	85.2	85.2	92.0	100.0
	Nucleolus	92.6	85.2	100.0	100.0
	Lorentz+	92.6	85.2	100.0	100.0
	EPM+	92.6	85.2	100.0	100.0

As discussed before, the Nucleolus, Lorentz+, and EPM+ are guaranteed to generate a stable allocation if the core is non-empty. This can be observed in Table 2, as for these methods core allocations were found for all instances with a non-empty core. Even though the Shapley value is not constructed by implicitly using any of the core criteria, it was a core solution for 92.2% of the instances for Type 1. This means that for only 3 instances with a non-empty core the Shapley value was not able to generate an allocation in the core. For Type 2 this percentage decreases to 85.6% of all instances. On the contrary, for both types, the Star method was able to generate a core allocation in only 27.8% of these instances. Additionally, in Table 2 a substantial difference in terms of stability can be noted per scenario, suggesting an inconsistent performance for the Star method in different circumstances. Other allocation methods perform similarly for all scenarios.

If we consider all instances, the allocation methods for Type 2 generally produce fewer allocations in the core compared with the allocation methods for Type 1. However, this can be caused by the increased number of empty cores for Type 2. Namely, if we consider only those instances with a non-empty core, the differences in stability are minimal between the two routing

approaches. Therefore, the performances in terms of stability are for both routing approaches relatively similar. For both types the Star method performs worst in terms of stability, followed by the Shapley value. The Nucleolus, Lorentz+, and EPM+ perform similarly as these are guaranteed to find a core allocation for instances with a non-empty core. Similar conclusions in terms of stability are derived in Naber et al. (2015).

### 4.3.2 Consistency

Secondly, we assess the consistency of each allocation method. This criterion is evaluated similarly as used in Naber et al. (2015). The effect of the distance to the depot, the average distance to other customers, and the order size are evaluated on the allocated emission. Therefore, an ordinary least squares analysis was performed, where the allocated emission is the dependent variable. The explanatory variables are a constant, the distance to the depot, the average distance to other customers, and the order size. Additionally, Naber et al. (2015) include two cross-product terms as they expect an interaction effect between pairs of explanatory variables. Namely, they performed their regression analysis on a concrete case study concerning realistic data. Therefore, they include a cross-product term of the variables “distance to the depot” and “order size”. Similarly, “average distance to the other customers” and “order size” are expected to interact. However, as we use a combinatorial design to generate all instances no real interaction is to be expected among these variables. The regression results for Type 1 are shown in Table 3, with for each allocation method the estimated coefficient and the one-sided  $p$ -value reported.

Table 3: Regression results of the allocation methods for Type 1.

Explanatory variable	Star		Shapley		Nucleolus		Lorentz+		EPM+	
	Coeff.	$p$ val.	Coeff.	$p$ val.	Coeff.	$p$ val.	Coeff.	$p$ val.	Coeff.	$p$ val.
Constant	7115.15	0.00	7007.21	0.00	7033.59	0.00	7028.79	0.00	7014.09	0.00
Avg. dist to other cust.	157.72	0.00	290.88	0.00	313.24	0.00	301.45	0.00	274.88	0.00
Dist. to depot	75.01	0.00	93.06	0.00	81.83	0.00	76.54	0.00	87.33	0.00
Order size	30.02	0.00	9.54	0.43	12.19	0.28	10.78	0.30	10.02	0.38
Avg. dist to other cust. $\times$ order size	3.43	0.01	-1.26	0.43	-0.12	0.94	-0.32	0.82	-0.96	0.53
Dist. to depot $\times$ order size	-0.32	0.20	-0.61	0.05	-0.57	0.04	-0.50	0.06	-0.49	0.09
$R^2$	0.43		0.59		0.63		0.64		0.59	

We observe a positive relationship between the allocated emission and the three explanatory variables “average distance to the other customers”, “distance to the depot”, and “order size”. However, the latter variable is only significant at a 5% significance level for the Star method. Additionally, it can be observed that the cross-product terms are significant at a 5% significance level only for three cases, suggesting an insignificant interaction between the variables.

Similarly, the regression results for Type 2 can be observed in Table 4. Similar results can be observed for Type 2 in terms of significance of the coefficients. In Naber et al. (2015) less insignificant coefficients are found. More specifically, the cross-product terms are significant for all but one case. As to be expected, this suggests a smaller interaction between the various variables used for the cross-product terms.

Table 4: Regression results of the allocation methods for Type 2.

Explanatory variable	Star		Shapley		Nucleolus		Lorenz+		EPM+	
	Coeff.	<i>p</i> val.	Coeff.	<i>p</i> val.	Coeff.	<i>p</i> val.	Coeff.	<i>p</i> val.	Coeff.	<i>p</i> val.
Constant	7156.02	0.00	7048.76	0.00	7077.04	0.00	7071.18	0.00	7056.86	0.00
Avg. dist to other cust.	159.74	0.00	293.89	0.00	316.26	0.00	305.20	0.00	279.35	0.00
Dist. to depot	74.93	0.00	92.59	0.00	81.44	0.00	76.52	0.00	86.98	0.00
Order size	32.71	0.00	12.20	0.31	15.05	0.18	13.22	0.21	12.34	0.28
Avg. dist to other cust. $\times$ order size	3.52	0.01	-1.14	0.47	0.08	0.96	-0.16	0.91	-0.78	0.61
Dist. to depot $\times$ order size	-0.35	0.17	-0.64	0.04	-0.60	0.04	-0.50	0.06	-0.51	0.08
$R^2$	0.43		0.59		0.63		0.64		0.59	

Additionally, for both types, the  $R^2$  is measured. The  $R^2$  is the proportion of variation in the dependent variable that is explained by the explanatory variables (Heij et al. 2004). Therefore, the  $R^2$  can be seen as the overall consistency of the allocation method. For both types, the Lorenz+ has the highest  $R^2$ , while the Star method has the lowest value for  $R^2$ . Hence, the Lorenz+ method performs best in terms of consistency for both types. The Star method performs the worst.

This conflicts with the conclusion given in Naber et al. (2015). They find the Star method to perform best in terms of consistency. The downturn of consistency for the Star method can be explained by the insignificance of the cross-product term concerning the variables “distance to the depot” and “order size”, which are both important for the calculation of the stand-alone emission. This stand-alone emission is particularly crucial for the allocations of the Star method as can be seen in Equation (6). Therefore, as the coefficient of a major explanatory variable is insignificant, the  $R^2$  turns out substantially less. If we consider a real-life data set other conclusions could be derived.

### 4.3.3 Robustness

Thirdly, the robustness of the allocation methods is assessed. As discussed before, a target customer was identified for each instance. For this target customer, all properties concerning the distance to the depot and order size are kept constant. This way we assess whether a customer is allocated a similar amount of emission for different instances. We measure the robustness of a method in terms of the coefficient of variation (CoV). The CoV can be calculated as the

standard deviation of the allocated emission to the target customer relative to the average allocated emission.

The results are presented in Table 5 and Table 6 for Type 1 and Type 2, respectively. The results are presented per scenario as for each scenario the distance to the depot changes for the target customer. Observe that the CoV is relatively high for all cases. This is mainly due to the fact that we aggregate over instances for which the positions of the other customers differ substantially, which has an impact on the allocated emission. To see which allocation performs best in terms of robustness, we compare the CoV for the different methods.

Table 5: Coefficient of Variation (CoV) for all instances for Type 1.

Scenario type	Allocation method	Average Emission (g)	Std. dev. of emission	CoV	Average comp. time (s)
Scenario 1	Star method	2,067	1,111	0.537	0.0
	Shapley	4,061	1,178	0.290	1.0
	Nucleolus	4,964	1,442	0.290	74.1
	Lorentz+	5,422	1,944	0.359	0.5
	EPM+	3,923	730	0.186	1.5
Scenario 2	Star method	9,343	5,530	0.592	0.0
	Shapley	16,785	12,727	0.758	1.0
	Nucleolus	18,143	11,536	0.636	28.2
	Lorentz+	17,940	11,127	0.620	4.2 <sup>1</sup>
	EPM+	16,599	12,149	0.732	4.4 <sup>1</sup>
Scenario 3	Star method	21,249	14,600	0.687	0.0
	Shapley	30,927	19,658	0.636	1.0
	Nucleolus	30,558	19,150	0.627	20.8
	Lorentz+	28,522	19,056	0.668	0.9 <sup>1</sup>
	EPM+	28,373	19,192	0.676	1.2 <sup>1</sup>

<sup>1</sup> Includes computation time of the nucleolus for instances with an empty core

Table 6: Coefficient of Variation (CoV) for all instances for Type 2.

Scenario type	Allocation method	Average Emission (g)	Std. dev. of emission	CoV	Average comp. time (s)
Scenario 1	Star method	2,080	1,126	0.541	0.0
	Shapley	4,106	1,200	0.292	1.0
	Nucleolus	5,016	1,470	0.293	53.2
	Lorenz+	5,453	1,927	0.353	0.5
	EPM+	4,046	782	0.193	1.4
Scenario 2	Star method	9,413	5,585	0.593	0.0
	Shapley	16,878	12,840	0.761	1.0
	Nucleolus	18,168	11,536	0.635	31.5
	Lorentz+	17,991	11,202	0.623	4.6 <sup>1</sup>
	EPM+	16,648	12,224	0.734	4.8 <sup>1</sup>
Scenario 3	Star method	21,391	14,716	0.688	0.0
	Shapley	30,970	19,798	0.639	1.0
	Nucleolus	30,656	19,381	0.632	31.8
	Lorentz+	28,701	19,413	0.676	1.4 <sup>1</sup>
	EPM+	28,551	19,548	0.685	1.8 <sup>1</sup>

<sup>1</sup> Includes computation time of the nucleolus for instances with an empty core

First, note the large difference in average emission between the Star method and the other four allocation methods. This can be explained by the fact that the Star method only tends to evenly distribute the total emission. However, the other methods are dependent on much more factors, such as visit order and distance to other customers. As the target customer is often isolated from the other customers, this substantially increases the allocated amount of emission for the other four allocation methods.

Thereafter, we see that for both types that the Star method performs the worst in Scenario 1 and Scenario 3. For Scenario 2 the Shapley value performs the worst, closely followed by the EPM+. No allocation method clearly stands out. However, as the Nucleolus never performs worst or second worst for both types, it can be considered as the best performing allocation method in terms of robustness. This conclusion matches with the results shown in Naber et al. (2015).

#### 4.3.4 Computation time

Finally, we discuss the computation time of each allocation method. In Table 5 and Table 6, the computation times per scenario are presented for Type 1 and Type 2, respectively. The computation time heavily depends on the number of customers, as this increases the computational complexity. It can be noted that the computational complexity of the Star method is polynomial in the number of customers. For the other methods, it is exponential, as these methods make use of the subsets for the computation.

Additionally, it must be noted that the computation times of the allocation methods are presented excluding the computation time needed for the distribution routes. On average, the distribution routes are constructed in 0.2 and 0.1 seconds for Type 1 and Type 2, respectively. In this paper, only the general distribution route visiting all customers is calculated by means of this optimization problem. If we would calculate the optimal route for all subsets of customers, this could significantly increase the computation time of certain allocation methods. Only the computation time of the Star method would remain unchanged, as this is the only allocation method independent of the particular subsets.

The Nucleolus specifically stands out, as the computation time is substantially higher compared with other methods. The Nucleolus solves multiple LP problems, which causes this high computation time. Additionally, if the core is empty the nucleolus was used to find an allocation for the Lorenz+ and EPM+. This negatively affects the average computation time of both methods, as the Nucleolus generally has a higher computation time. Therefore, the Nucleolus performs worst in terms of computation time. The Star method generally performs best.

## 5 Conclusion

In this paper, we examined the discrepancy between two different routing approaches concerning the emission minimization or travel distance minimization of the distribution route. For both approaches, a different optimization problem was given to obtain the distribution routes. Additionally, we applied different allocation methods to both distribution routes to identify the difference in terms of the allocated values and compared the performance of the methods based on four evaluation criteria.

We find that a certain ecological trade-off has to be made when selecting the routing approach. However, when diverting from a travel distance minimizing route to an emission minimizing route, percentage wise the decrease in emission is greater than the increase in travel distance. This suggests the trade-off to be in favor of the emission minimizing route. Additionally, on average we find the allocations to the customers to be slightly smaller for the emission minimizing routes compared with the travel distance minimizing routes. However, for certain methods, this decrease in emission is not as evenly distributed among the customers.

In terms of performance, all allocation methods perform similarly for both routing approaches. Hence, we compare the various allocations methods between each other to identify the best method. We find that there is no allocation method that performs best for all evaluation criteria. Therefore, the choice of allocation method depends on which criterion is set to be most important. If computation time is the most important criterion, then the Star method seems the best choice. If stability and robustness are the most important criteria, the Nucleolus is recommended. However, overall the Lorentz+ and EPM+ could be considered as the best options as these methods never perform the worst in any of the criteria.

Taking all these findings into account, we find no objection to divert from a travel distance minimizing approach to an approach considering emission minimization. The ecological trade-off to be made benefits the emission more than the increase in travel distance. Additionally, as a result from the lower allocations, companies can decrease their carbon footprint. Therefore, companies can easier comply with recent emission regulations and emission targets. This helps to achieve a more corporate social responsible approach for the companies that could benefit the economic performance of the companies. Additionally, an environmental approach positively contributes to the necessary attainment regarding the limitation of the global temperature.

Finally, the question still remains how the discrepancy changes in a concrete situation. In this paper, several assumptions are made regarding the emission calculation and the different instances. We, therefore, see it as a valuable direction for further research to consider this research in a concrete case study.

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## Appendix A Emission function

To compute emission on a distribution route, we use the model used in Behnke et al. (2021). More specifically, their computation method of the several parameters is based on the extensive model given by Kirschstein & Meisel (2015). This emission model takes into account certain vehicle characteristics, the number of loading units, and edge-specific characteristics. The latter concerns the driving speed and acceleration frequency on an edge.

In Table A1 an overview is given of all necessary parameters, as well as a short description and their assigned values. The vehicle’s weight and payload are chosen similarly as used in Naber et al. (2015). However, additional vehicle parameters are based on the values given in Behnke & Kirschstein (2017), and chosen in such a way that they conform to the chosen vehicle size. Finally, the road parameters are assumed ourselves.

Table A1: Parameters for the emission estimation model and their corresponding values

Parameter	Unit	Value	Description
<i>Vehicle parameters</i>			
$m^{\text{tare}}$	[t]	5.00	tare weight of the vehicle
cap	[t]	5.07	payload capacity of the vehicle
$P$	[kW]	150.00	rated power of the engine in the vehicle
$S$	[m <sup>2</sup> ]	6.50	front surface are of the vehicle
$r^{\text{idle}}$	[l/h]	1.50	the vehicle’s minimum fuel consumption rate (in idle mode)
$r^{\text{full}}$	[l/h]	38.00	the vehicle’s maximum fuel consumption rate (in full throttle)
$c^{\text{air}}$		0.64	air resistance coefficient for the vehicle
$c^{\text{roll}}$		0.007	rolling resistance coefficient for the vehicle
<i>Road parameters</i>			
$v$	[km/h]	50.00	driving speed for the vehicle
$\eta^{\text{acc}}$	[#/km]	0.50	number of acceleration processes per kilometer
<i>Physical constants</i>			
$\rho$	[kg/m <sup>3</sup> ]	1.20	density of air
$g$	[m/s <sup>2</sup> ]	9.81	gravitational acceleration
$e$	[kg CO <sub>2</sub> e/l]	3.15	(well-to-wheel) emission coefficient for diesel fuel

The emission model is given as a linear formulation, in which the total emission is calculated in terms of a fixed amount and a load-dependent amount. Therefore, the coefficients  $c^{\text{fix}}$  and  $c^{\text{load}}$ , based on Equations (2) and (3) in Behnke et al. (2021), are constructed. For these formulations, we first construct the following parameter:

$$\alpha = \frac{r^{\text{full}} - r^{\text{idle}}}{P \cdot (0.88 - 0.72 \cdot \exp(-0.077 \cdot v^{1.41}))}. \quad (\text{A1})$$

Thereafter, both parameters  $c^{\text{fix}}$  and  $c^{\text{load}}$  can be calculated as follows:

$$c^{\text{fix}} = e \cdot \left( \frac{r^{\text{idle}}}{v} + \alpha \cdot \left( \frac{1}{2000} \cdot \frac{c^{\text{air}}}{3.6^3} \cdot \rho \cdot S \cdot v^2 + m^{\text{tare}} \cdot \left( \frac{c^{\text{roll}}}{3.6} \cdot g + \frac{0.504}{2 \cdot 3600 \cdot 3.6^2} \cdot \eta^{\text{acc}} \cdot v^2 \right) \right) \right), \quad (\text{A2})$$

$$c^{\text{load}} = \frac{1}{100} \cdot e \cdot \alpha \cdot \left( \frac{c^{\text{roll}}}{3.6} \cdot g + \frac{0.504}{2 \cdot 3600 \cdot 3.6^2} \cdot \eta^{\text{acc}} \cdot v^2 \right). \quad (\text{A3})$$

It can be noted that the parameter  $c^{\text{load}}$  is multiplied with  $\frac{1}{100}$ . It is assumed that a single loading unit weighs approximately 0.01 ton. Therefore, by multiplying with this factor, the parameter depicts the kilograms of emission per kilometer and per loading unit. Thus, by combining both parameters, the amount of emission in kilograms for traveling one kilometer with the number of loading units  $f$  can be calculated by means of:

$$\text{emis}(f) = c^{\text{fix}} + c^{\text{load}} \cdot f \quad (\text{A4})$$

## Appendix B Allocation details

In order to obtain are more detailed insight in the resulting allocations, we present several characteristics of the exact allocations and discuss other properties affecting the resulting allocations. In Table B1, an overview of certain properties of the allocations is presented distinguished between allocation method, scenario and type.

Table B1: Comparison of certain characteristics of the allocated values in grams.

		Star		Shapley		Nucleolus		Lorentz+		EPM+	
Scenario type	Feature	Type 1	Type 2	Type 1	Type 2	Type 1	Type 2	Type 1	Type 2	Type 1	Type 2
Scenario 1	Average	5,969	5,993	5,969	5,993	5,969	5,993	5,969	5,993	5,969	5,993
	Min. value	729	729	1,181	1,196	1,412	1,405	2,218	2,219	807	798
	Max. value	22,240	22,240	22,318	22,318	20,869	20,869	19,094	19,094	21,577	21,577
Scenario 2	Average	6,953	6,998	6,953	6,998	6,953	6,998	6,953	6,998	6,953	6,998
	Min. value	1,158	1,159	1,118	1,137	1,202	1,213	1,256	1,213	816	805
	Max. value	21,365	21,718	33,922	36,819	33,912	34,017	33,808	34,017	33,808	34,017
Scenario 3	Average	8,210	8,255	8,210	8,255	8,210	8,255	8,210	8,255	8,210	8,255
	Max. value	1,474	1,475	1,123	1,157	1,192	1,031	1,818	1,171	776	813
	Min. value	49,654	49,894	56,841	58,218	57,152	58,963	56,734	58,964	56,734	58,964

We find in every case on average a lower allocated amount of CO<sub>2</sub> for Type 1 compared with Type 2. Additionally, in six cases a lower maximum or minimum value is observed for Type 2. However, the differences are generally not substantial. Additionally, it can be noted that for Scenario 2 and Scenario 3 the Star method has a much lower maximum value compared with the other allocation methods, suggesting a more even distribution of the total CO<sub>2</sub> emission.

However, certain properties of instances could affect the resulting allocations. First we evaluate the effect of the distance to the depot. To do this, we only consider the allocations of the target customer as the distance to the depot is kept constant per scenario. We find that if the customer is located in the same area as the depot, the allocated amount of CO<sub>2</sub> is less than 7,500 grams. On average the target customer is allocated 4,113 grams. When the depot and the target customer are one region apart, the allocated amount ranges from 2,000 to 37,000 grams with an average allocated amount equal to 15,791 grams. This is an average increase of 283.9%. Finally, when they are two regions apart the allocated amount ranges from 5,000 to 59,000 grams and on average is equal to 27,990 grams. This results in average increase equal to 77.3%. Therefore, the increase tends to be marginally decreasing over the additional distance to the depot.

Another important property concerning the allocated amount is the number of customers. As we consider instances with similar locations and order sizes, however, differ the number of customers we observe a strong decrease in the allocated emission of CO<sub>2</sub>. If we consider the allocations of the target customer, an average decrease is observed of 15.7% when increasing

the number of customers from 5 to 10. This average decrease is equal to 5.2% when we go from 10 to 15 customers. This can be explained by the fact that the total emission to be distributed does not increase in a similar manner as the number of customers do. Therefore, relatively less emission has to be distributed among the customers.

Finally, we consider the possible effect of the order size to the allocations. Therefore, we only consider the allocations of the other customers as the order size of the target customer is kept constant. When the other customers have an order size of 1.4 demand units, we find an average allocated amount of CO<sub>2</sub> equal to 5,815 grams. If the order size is increased to 7.1 loading units, the average allocated amount slightly increases to 5,916 grams. Finally, we increase the loading units to 35.7, we find a average allocated amount equal to 6,469 grams. When changing from the smallest to the largest order size, we find an increase of only 11.2%. This suggest a relatively small effect of the order size on the allocated emission.