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Shelf attractiveness in an integrated assortment planning and shelf-space allocation model

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Abstract

Assortment planning and shelf-space allocation are two of the most important decisions to make for retailers. The attractiveness levels of different sorts of shelves play a critical role in these decisions. In this paper, a mixed-integer programming (MIP) model, called APSA, is used to simultaneously choose what product categories to select in the assortment and where to allocate them in a retail store. APSA is used in an optimization-based heuristic and the computational performance of the heuristic is presented and assessed. Furthermore, the effects of varying shelf attractiveness coefficients on the model are investigated in the form of a sensitivity analysis. The selected assortment, the allocation of the products and the tractability of the model are discussed. In addition, a theoretical framework of how to combine the heuristic with short-term retailing decisions such as endcap allocation and sales promotions in a multi-period model is presented. For the study on computational effects a simulated dataset is used and for the sensitivity analysis a realistic dataset is used to obtain results that come close to reality.

Contents

1 Introduction

Walmart, the biggest retailer in the world, is on top of the Fortune 500 list in 2021, the list with America's biggest companies ranked on yearly revenue. Clearly, supermarkets and other retailers are powerful entities who are immensely important for the world economy and have the ability to greatly influence customer behaviour. Retailers have some important decisions to make in order to maximize customer footprint (number of customers in the store), conversion from visiting to purchasing and profit. According to Cachon and Kök (2007), the customer footprint is closely related to the assortment offered by the retailer. The in-store conversion rate, which is the percentage of customers that buys a particular product when visiting the store, is strongly dependent on the store layout and different kinds of promotions (i.e., special displays, price discounts and promotional signage) (Lam et al., 2001). The store layout combined with the different promotion strategies increases purchase likelihood, which increases the in-store conversion rate (Bogomolova et al., 2017). Evident from the above, assortment planning, shelf-space allocation and promotions are mutual dependent and important aspects of decision making for retailers in order to reach commercial success.

A widely-used in-store promotion technique is making use of high attractiveness displays such as endcaps. Endcaps are the displays at the end of the aisles and can be seen as the most prominent shelf-spaces in the store. Endcap allocation is a short-term promotion technique as the products that are displayed on the endcaps change every time period t. Approximately 2% of the total assortment in a supermarket is allocated to endcaps or other displays, this small percentage of products is responsible for around 30% of the total supermarket sales (Sorensen, 2009). On top of that, Tan et al. (2018) states that the sales of products that are displayed on a rear endcap and front endcap increase with 416% and 346%, respectively. The reason for endcap effectiveness is the fact that these locations in the supermarket have higher customer traffic and therefore higher visual attention (Larson et al., 2005). It is clear that endcaps and other high attractiveness shelves are a powerful promotion tools which can be used by retailers to maximize in-store conversion and boost sales.

Another effective and important promotion technique employed by retailers is sales or price promotions. Similar to endcap allocation, sales promotions is a short-term promotion techniques as the products that are put on sale often change every time period t. Recent study shows that packaged goods manufacturers spend more money on sales promotions than on television and radio advertising combined (McColl et al., 2020). Sales promotions such as coupons and "two-for-one" have a strong impact on short-term consumption behaviour and increase the likelihood of impulse purchases (Laroche et al., 2003). Additionally, sales promotions have a positive effect on the number of shopping trips to the store (Van den Poel et al., 2003).

In Flamand et al. (2018), an optimization-based approach for integrated assortment planning and store-wide shelf-space allocation is proposed. The goal is to maximize the overall store profit which is not only dependent on the assortment selection, but also on the shelf space attractiveness, product category profitability, impulse purchase potential and expected demand volumes. Flamand et al. (2018) propose a mixed-integer programming model as a standalone approach which is embedded in an optimization-based heuristic. Much has been written on the importance of different kinds of shelves and shelf attractiveness, so I want to investigate the effects of shelf attractiveness on the outcome and the tractability of the heuristic. In this paper, I will first replicate the results in Flamand et al. (2018). Next, I will use the proposed heuristic to perform a shelf attractiveness sensitivity analysis with a realistic dataset and analyse these results. In addition, I will present a theoretical approach of how to combine short-term retailing decisions (endcaps and sales promotions) with long-term retailing decisions (assortment planning and shelf-space allocation) in one multi-period model.

The remainder of this paper will be structured as follows: Section 2 will discuss the relevant academic literature related to this research. Section 3 discusses the different datasets used in this paper. Consecutively, Section 4 discusses the relevant notation, the used heuristic, the methodology of the sensitivity analysis and the theoretical framework of the multi-period model. Next, the results of the replication and the sensitivity analysis are discussed in Section 5. Last, Section 6 summarizes the results and discusses the suggestions for future research.

2 Related Literature

Assortment planning, shelf-space allocation, display promotions and sales promotions have mostly been investigated separately, instead as one integrated problem. Most literature in the field solely focuses on one of the above mentioned aspects of retail decision making, without combining these aspects in one analysis. First, I will present the relevant literature on assortment planning and shelf-space allocation. Next, I will discuss some studies that investigate the effects of short-term promotion techniques such as displays, high attractiveness shelves and sales promotions. Last, I will present some relevant literature on integrated approaches.

2.1 Assortment Planning

Assortment planning is a decision that has to be made by retailers to determine the optimal mix of products that is offered in a store in order to maximize certain objectives, such as profit. Efficient assortment planning is one of the most crucial decisions a retailer has to make (Mantrala et al., 2009). A lot of research in the assortment planning field investigates the trade-offs related to demand substitution across the assortment (A. Hübner et al., 2016; A. Hübner and Schaal, 2017; A. H. Hübner and Kuhn, 2012). The model proposed in A. Hübner et al. (2016) considers out-of-assortment (OOA) and out-of-stock (OOS) demand substitution effects when there is only limited shelf-space available, and is solved via a heuristic procedure. In Chong et al. (2001) a local improvement heuristic is proposed that generates an alternative category assortment based on the lost sales implication of the alternatives. Apart from Chong et al. (2001), not a lot of research has been done related to which product categories to offer in a store, most assortment planning studies consider individual products. Flamand et al. (2018) is one of the first researches that considers trade-offs related to balancing slow-moving, high-profit-margin, impulsive product categories and fast-moving, low-profit-margin, product categories.

2.2 Shelf-space Allocation

Another decision that has to be made by retailers is allocating the products to shelf-spaces. Shelf-space allocation is one of the most important resources to attract more consumers to the store, and therefore a critical decision for retailers (Yang and Chen, 1999). Additionally, well shelf-space management can strengthen vendor relationships and increase customer satisfaction (Erol et al., 2015). A widely-used approach in experimental studies where shelf-space allocation is optimized is space elasticity - the ratio of percentage change in unit sales to the percentage change in shelf-space - (Amrouche and Zaccour, 2007; Curhan, 1973; Desmet and Renaudin, 1998; Eisend, 2014; Schaal and Hübner, 2018). Schaal and Hübner (2018) investigates the phenomenon of cross-space elasticity, but the authors show that cross-space effects have minor impact on solution structures and profits. Zufryden (1986) proposes an approach where the objective function accounts for space elasticity, cost of sales and other marketing variables.

2.3 Display and Sales Promotions

Allocating a product to a display shelf or other high attractiveness shelves (such as endcaps) and applying sales promotions are powerful techniques for retailers as well. Past studies on the effect of retail techniques on individual product performance show that both promotion techniques have a positive effect on the total sales. Caruso et al. (2018) shows that products placed on endcaps experience a boost in sales, especially when placed on front endcaps. Also, recently a lot of research has been done on the use of endcaps to improve targeted healthy purchases (Payne and Niculescu, 2018). Evidently, the use of high attractiveness shelves, such as endcaps, is a relevant promotional tool to promote the contemporary growing awareness of healthy diets. Studies show that the combination of promotional techniques (such as on-display and price promotions) have a greater impact on unit sales of the promoted product than when the promotional techniques are used individually (Bennett and Wilkinson, 1974; Hawkins, 1957; Wilkinson et al., 1982; Woodside and Waddle, 1975). Most studies on the effects of displays investigate the relation between brand substitution effects and brand shelf-space, as in Anderson (1979). There are only a few studies that investigate the effects of endcap allocation, but none of these studies proposes an integrated approach where assortment planning or sales promotions are included as well. In this work, the effects of having different levels of shelf attractiveness in the integrated framework of Flamand et al. (2018) will be investigated.

2.4 Integrated Approaches

A. Hübner and Schaal (2017) is the first to propose a model that integrates assortment planning decisions with shelf-space allocation decisions. Recently, a lot of research is done on the integration of these two important retailing decisions. Some studies that consider integrated shelf-space allocation and assortment planning are Anderson and Amato (1974), Bayındır and Gülsaç (2006), Borin et al. (1994), Hariga et al. (2007), and Urban (1998). The difference with the model discussed in this paper is the fact that these papers focus on constructing planograms and allocating a single product category to a single shelf. In this work, multiple product categories are considered which can be allocated to store-wide shelf-spaces. Other studies propose integrated models that combine other promotional variables together with assortment planning and/or shelf-space allocation. Murray et al. (2010) proposes a model that jointly optimizes the decisions for product prices, display facing area, display orientations and shelf-space locations, using a branch-and-bound based MINLP algorithm. Unlike in Flamand et al. (2018), the assortment is pre-determined and considered as given in Murray et al. (2010). Additionally, this research considers the width and the height of the display shelves, unlike a lot of other studies in the field where only the width of the display shelves is considered. The model of Murray et al. (2010) is unique because it also takes into account pricing decisions, which can be used to interpret sales promotions. Hariga et al. (2007) also proposes an integrated model that jointly optimizes the retailer's profit by determining optimal product assortment, inventory replenishment, display area and shelf-space allocation. However, this proposed model does not take into account sales promotions decisions and high attractiveness displays, such as endcaps, specifically. Most of the proposed integrated approaches use mathematical (mixed-integer) programming in a deterministic setting to reach an optimal solution (Flamand et al., 2018; Ghoniem and Maddah, 2015; Ghoniem et al., 2014; A. Hübner and Schaal, 2017).

3 Data

In this Section, I will first discuss how the data used for the replication of the tables in Flamand et al. (2018) is generated. Consecutively, I will discuss the realistic dataset used for the sensitivity analysis.

3.1 Computational Study

For testing the heuristic and replicating the tables in Flamand et al. (2018) I use simulated data. The data is generated in a similar way as in the computational study in Flamand et al. (2018). All shelves have the same total capacity C_i

= 18. Every shelf consists of 3 segments, where each segment has maximum capacity $c^{max} = 6$. The minimum space requirement, l_j, and the maximum space requirement, u_j, of product category j are randomly generated using a uniform distribution over the ranges $[1, \frac{C_i}{6}]$ and $[l_j, \frac{C_i}{3}]$, respectively. The largest possible profit of product category j, Φ_j, is randomly drawn from a uniform distribution over the range [1, 25]. The minimum allocated space for product category j, ϕ_j, is set to 0.1 for all product categories. The attractiveness of shelf segment k, f_k, is generated in such a way that there are 5 different levels, t, of shelf attractiveness. Every 20% of the shelves shares the same level of attractiveness. The attractiveness levels are as follows: $t = 5\%$, 25%, 45%, 65% and 85%. Furthermore, the middle shelf segments have lower attractiveness than the end-of-aisle shelf segments. The attractiveness of the middle shelf segments and attractiveness of the end-of-aisle shelf segments is generated using a uniform distribution over the ranges $[t, t + 0.05]$ and $[t + 0.06, t + 0.1]$, respectively.

3.2 Sensitivity Analysis

To perform a case study I need product data that reflects the real world. A convenient choice is to use the same 100 product categories and 31 product groups, as presented in Table 1 and their demand volumes, v_j, as used in Flamand et al. (2018). This dataset is readily available and it is difficult to find alternative datasets. Moreover, most of these product categories are present in a research on impulse purchase potential of Kollat and Willett (1967), which is convenient as the data on impulse purchase potential is not provided in Flamand et al. (2018). The probability of purchase of certain product categories in Kollat and Willett (1967) can be used to approximate the impulse purchase potential of the product categories in the dataset used for the case study. In this study, the impulse purchase potential is divided into 3 levels: low [0 , 0.1), medium [0.1, 0.4) and high [0.4, 0.5]. The 20 fast-movers in the dataset are assigned an impulse purchase potential equal to 1. Furthermore, the gross profit margin of product category j, ρ_j, is necessary to calculate the largest possible profit of product category j, Φ_j. Unfortunately, this data is not provided in Flamand et al. (2018) and grocery retailers are not willing to share this information for this research. I am forced to approximate these parameters myself. Similar to the impulse purchase potential, the gross profit margin is divided into 3 levels: low [0 , 0.2), medium [0.2, 0.3) and high [0.3, 0.55]. The gross profit margin of product category is based on the indicated gross profit margins per industry and product groups in this web article [1]

20 out of the 100 product categories are considered to be fast-movers. Fast-movers are high-sales product categories that often have a low profit margin (e.g. milk, bread, coffee). The 20 fast-movers in our realistic dataset are responsible for approximately 80% of the total sales. Logically, fast-movers have high demand volumes, v_i, and their impulse purchase potential is set equal to 1, ($\gamma_j = 1$), which results in a relatively high maximum possible profit of product

[1] https://www.naveocommerce.com/on-demand-grocery-what-to-consider-chapter-2/.

category j, Φ_j. The fast-movers in our dataset are: Bread (9), Canned Vegetables (17), Cigarettes (23), Juice (26), Soda (27), Cheese (32), Milk (34), Packed Cheese (35), Specialty Cheese (36), Unpacked Meat (44), Coffee (45), Vegetables (63), Water (64), Ice (70), Dinners (71), Pizza (72), Household Cleaner (82), Salad Dressings (90), Pasta sauce (92) and Canned Fruit (95).

On the other hand, we have high-impulse product categories. High-impulse product categories have a strong impulse purchase potential and are often unplanned purchases. Examples of high-impulse product categories in our realistic dataset are: Chewing gum (13), Lollipop (14), Marshmallows (15), Candy (16), Chocolate (22), Sliced Deli (38), Chips (47), Nuts (48), Popcorn (49), Ice Cream (69), Snacks (91), Rice Cakes (94).

Table 1: Product categories and groups from Flamand et al. (2018)

#	Group	Category
1	Alcohol	Liquor (1), Champagne (3), Vodka (5), Whiskey (6). Wine (7)
2	Light Alcohol	Beer (2), Energy Drinks (4)
3	Bread	Croissant (8), Bread (9), Sandwich (10), Bagel (11), Toast (12), Bread Crumbs (73)
4	Candy	Chewing Gum (13), Lollipop (14), Marshmallow (15), Candy (16), Chocolate (21), Chocolate Chips (22)
5	RM Food	Ready Made Food (18), Frozen Sea Food (62), Dinners (71), Pizza (72)
6	Breakfast	Hot Cereals (19), Cold Cereals (20), Peanut Butter & Jelly (74), Honey & Sirups (97).
7	Cigarettes	Cigarettes (23), Cigars (24)
8	Cold Beverages	Iced Tea (25), Juice (26), Soda (27), Water (64)
9	Dairy 1	Butter (31), Eggs (33), Milk (34), Cookie Dough (65)
10	Dairy 2	Sour Cream (66), Yoghurt (67)
11	Cheese	Cheese (32), Packed Cheese (35), Specialty Cheese (36)
12	Canned Food	Canned Meat (37), Canned Sea Food (61)
13	Desserts	Boxed Desserts (39), Cakes (40), Ready Made Desserts (41), Spreaded Desserts (42), Pies & Toppings (68)
14	Meat	Sliced Deli (38), Packed Meat (43), Unpacked Meat (44)
15	Hot beverages & Cookies	Cookies (28), Gourmet Cookies (29), Biscuits (30), Coffee (45), Tea (46), Herbal Tea (93)
16	Nuts & Chips	Chips (47), Nuts (48), Popcorn (49), Snacks (91), Rice Cakes (94)
17	Pasta	Pasta (50), Pasta Sauce (92)
18	Powders	Grain (51), Rice (52), Soup (53), Spice (54), Sugar-Salt (55), Flour (56)
19	Sauces & Syrups	Creams (57), Dips (58), Oil (59), Sweet Sauce (60)
20	Vegetables	Canned Vegetables (17), Vegetables (63)
21	Frozen	Ice Cream (69), Ice (70)
22	Bath Tissue	Bath Tissue (77)
23	Paper Towels	Paper Towels (79)
24	Bath Needs	Facial Tissue (76), Bath Needs (80)
25	Plastic needs	Cups & Plates (75), Wraps & Bags (78)
26	Cleaning Supplies	Fabric Softeners (81), Laundry Detergents (85)
27	Household	Household Cleaner (82), Bleach (83), Wipes (84), Dish Detergents (86)
28	Condiments	Vinegar (87), Ketchup (88), Pickles & Olives (89), Salad Dressings (90)
29	Canned Fruit	Canned Fruit (95)
30	Cake Supplies	Cake Decorations (86), Cake Mixes (98)
31	Baby needs	Baby Food (99), Diapers (100)

Moreover, product category pairs (j, j') can have four different affinity relations. The affinity relations will be used extensively in the rest of the paper and are defined as follows:

- Allocation disaffinity L: Two product categories that have allocation disaffinity should not be allocated to the same shelf. However, please note that the product categories can be chosen in the assortment simultaneously (e.g. cigarettes and baby food).

- Symmetric assortment affinity H_1: Two product categories that have symmetric assortment affinity must be selected together in the assortment or neither of them should be selected (e.g. coffee and coffee filter).
- Assymmetric assortment affinity H_2: If product category pair (j, j') have assymmetric assortment affinity, then if product category j is selected, product category j' must be selected in the assortment as well (e.g. pasta sauce and pasta).
- Allocation affinity H_3: Two product categories that have allocation affinity should be allocated to the same shelf when both selected in the assortment. Please note that both product categories can be selected in the assortment individually (e.g. milk and yoghurt).

4 Methodology

First, I will introduce some notation in Subsection 4.1 that will be used throughout the paper. Second, In Subsection 4.2 I will describe the Mixed-Integer Programmming problem called APSA. In Subsection 4.3 I will describe the heuristic proposed by Flamand et al. (2018). Consecutively, in Subsection 4.4 I will describe the methodology of the sensitivity analysis based on changes in shelf attractiveness. Last, in Subsection 4.5 I will present the theoretical framework of how to combine short-term promotion techniques such as endcap allocation and sales promotion with the proposed heuristic of Flamand et al. (2018) in a multi-period model.

4.1 Notation

In this Subsection, I will introduce the mathematical notation that is necessary to explain the methods used in this work. Most of the notation is similar to the notation in Flamand et al. (2018). The sets and variables used in the model are as follows:

- $N \equiv \{1, ..., n\}$: The set of product categories, indexed by j.
- $F \subset N$: Set of fast-movers.
- $I \equiv N \setminus F$: Set of slow-movers.
- L: Set of product category pairs (j, j') where $j \epsilon N$, which are related by allocation disaffinity.
- H_1: Set of product category pairs (j, j') where $j \epsilon N$, which are related by symmetric assortment affinity.

- H_2: Set of product category pairs (j, j') where $j \epsilon N$, which are related by asymmetric assortment affinity.
- H_3: Set of product category pairs (j, j') where $j \epsilon N$, which are related by allocation affinity.
- $B \equiv \{1, ..., m\}$: Set of shelves, indexed by i.
- K_i: Set of consecutive shelf segments along shelf i, indexed by k. If shelf $i \epsilon E$, then $|K_i| = 1$.
- $K \equiv \cup_{i \epsilon B} K_i$: Set of all the shelf segments in the store.

The following parameters are used in the model:

- ρ_j: The profit margin for product category j, $j \epsilon N$.
- v_j: The expected demand volume of product category j, $j \epsilon N$.
- $\gamma_j \epsilon (0, 1]$: Parameter that reflects the impulse purchase potential.
- f_k: The traffic density of segment k. This can be seen as a parameter that reflects the attractiveness of segment k.
- $\Phi_j = \gamma_j \times \rho_j \times v_j$: The largest possible profit of product category j.
- l_j/u_j: Lower/upper bound on the space requirement for product category j.
- ϕ_j: The minimum space to be allocated to product category j.
- α_i/β_i: Smallest/largest index index of a segment that belongs to shelf i.
- c_k: The capacity of segment k.
- c^{max}: The maximum capacity of segment k.
- $C_i \equiv \sum_{k \epsilon K_i} c_k$: The capacity of shelf i.

The decision variables of the model are defined as follows:

- $x_{ij} \epsilon \{0, 1\}$: $x_{ij} = 1$ if and only if product category j is allocated to shelf i, $\forall i \epsilon B$, $j \epsilon N$.
- $y_{kj} \epsilon \{0, 1\}$: $y_{kj} = 1$ if and only if product category j is allocated to shelf segment k, $\forall k \epsilon K$, $j \epsilon N$.
- s_{kj}: amount of shelf space allocated to product category j along shelf segment k, $\forall k \epsilon K$, $j \epsilon N$.
- $z_{jj'} \epsilon \{0, 1\}$: $z_{jj'} = 1$ if and only if product categories j and j' are selected in the assortment simultaneously, $\forall j, j' \epsilon N$
- $q_{kj} \epsilon \{0, 1\}$: $q_{kj} = 1$ if and only if product category j is allocated to both shelf segments k and $k + 1$, $\forall k \epsilon K \setminus \{\beta_i : i \epsilon B\}$, $j \epsilon N$.

4.2 APSA

The full APSA model can optimize a pre-chosen set of shelves and product categories at the same time. The objective function and the constraints are defined as follows:

$$\text{Maximize} \quad \sum_{k \epsilon K} \sum_{j \epsilon N} \Phi_j \frac{f_k s_{kj}}{c_k} \tag{1a}$$

$$\text{subject to} \sum_{i \epsilon B} x_{ij} \leq 1, \quad \forall j \epsilon N \tag{1b}$$

$$\sum_{j \epsilon N} s_{kj} \leq c_k, \quad \forall k \epsilon K \tag{1c}$$

$$l_j \sum_{i \epsilon B} x_{ij} \leq \sum_{k \epsilon K} s_{kj} \leq u_j \sum_{i \epsilon B} x_{ij}, \quad \forall j \epsilon N \tag{1d}$$

$$\phi_j y_{kj} \leq s_{kj} \leq min(c_k, u_j) y_{kj}, \quad \forall j \epsilon N,\ k \epsilon K \tag{1e}$$

$$y_{kj} \leq x_{ij} \quad \forall i \epsilon B,\ j \epsilon N,\ k \epsilon K_i \tag{1f}$$

$$x_{ij} \leq \sum_{k \epsilon K_i} y_{kj}, \quad \forall i \epsilon B,\ j \epsilon N,\ k \epsilon K_i \setminus \{\beta_i\} \tag{1g}$$

$$q_{kj} \geq y_{kj} + y_{k+1,j} - 1, \quad \forall i \epsilon B,\ j \epsilon N,\ k \epsilon K_i \setminus \{\beta_i\} \tag{1h}$$

$$\sum_{j \epsilon N} q_{kj} \leq 1, \quad \forall i \epsilon B,\ k \epsilon K_i \setminus \{\beta_i\} \tag{1i}$$

$$x_{ij} + x_{ij'} \leq 1, \quad \forall (j, j') \epsilon L,\ i \epsilon B \tag{1j}$$

$$x_{ij} - x_{ij'} = 0, \quad \forall (j, j') \epsilon H_1,\ i \epsilon B \tag{1k}$$

$$x_{ij} \leq x_{ij'}, \quad \forall (j, j') \epsilon H_2,\ i \epsilon B \tag{1l}$$

$$x_{ij} - x_{ij'} \leq 1 - z_{jj'}, \quad \forall (j, j') \epsilon H_3,\ i \epsilon B \tag{1m}$$

$$x_{ij} - x_{ij'} \geq -1 + z_{jj'}, \quad \forall (j,j') \epsilon H_3,\ i \epsilon B \tag{1n}$$

$$z_{jj'} \leq \sum_{i \epsilon B} x_{ij}, \quad \forall (j,j') \epsilon H_3 \tag{1o}$$

$$z_{jj'} \leq \sum_{i \epsilon B} x_{ij'}, \quad \forall (j,j') \epsilon H_3 \tag{1p}$$

$$z_{jj'} \geq \sum_{i \epsilon B} x_{ij} + \sum_{i \epsilon B} x_{ij'} - 1, \quad \forall (j,j') \epsilon H_3 \tag{1q}$$

$$y_{k_1 j} + y_{k_2 j} \leq 1, \quad \forall (k_1, k_2, j) \epsilon R \tag{1r}$$

$$x, y, z \text{ binary}, \quad s, q \geq 0 \tag{1s}$$

I will shortly explain the constraints of the full APSA model. For more detail and elaborate explanations please refer to Flamand et al. (2018). The objective function (1a) maximizes the overall store profit, using maximum possible profit of product category j, Φ_j, the attractiveness of shelf segment k, f_k, the capacity of shelf segment k, c_k, and the decision variable s_{kj}. Constraint (1b) makes sure that every product category is allocated to at most one shelf. Constraint (1c) guarantees that the total space assigned to product categories in segment k is at most the capacity of segment k. Constraint (1d) makes sure that the total space allocated to a product category lies between its minimum and maximum space requirements. Constraint (1e) ensures that the space allocated to a product category lies between the minimum space requirement for any product category and the maximum space requirement for that particular product category, which either is its maximum space requirement or the capacity of the segment. Constraint (1f) ensures that a product category can only be allocated to a shelf segment of shelf i, if the product category is placed on shelf i. Constraint (1g) ensures that if a product category is assigned to a shelf i, it is also placed in a shelf segment on that particular shelf i. Constraint (1h) and Constraint (1i) make sure that one product category can run over two adjacent shelf segments. Constraint (1j) ensures that two product categories which have allocation disaffinity cannot be allocated to the same shelf. Constraint (1k) guarantees that product categories which have symmetric assortment affinity are assigned to the same shelf if both selected. If not, both product categories should not be in the assortment. Constraint (1l) addresses the situation when product categories (j, j') have assymmetric assortment affinity. The constraint ensures that when product category j is selected, product category j' should be selected as well and allocated to the same shelf as product category j. Constraint (1m) and Constraint (1n) ensure that product categories which have allocation affinity are allocated to the same shelf when both selected in the assortment. Constraint

(1o), Constraint (1p) and Constraint (1q) are linearization constraints. Together with the assumptions that the segment capacity equals 6 for all segments and the maximum space requirement for all product categories is at most 6, Constraint (1r) ensures that a product category is allocated to at most 2 adjacent shelf segments. Constraint (1s) forces binary and non-negativity restrictions on the decision variables.

Constraint 1f in Flamand et al. (2018) is defined as follows:

$$s_{k2,j} \geq c_{k2}(y_{k1,j} + y_{k3,j} - 1), \quad \forall j \epsilon N,\ i \epsilon B,\ k_1, k_2, k_3 \epsilon K | k_1 < k_2 < k_3 \tag{2}$$

Constraint 2 ensures that any product category that is allocated to shelf segments k_1 and k_3 along the same shelf also entirely fills the the shelf segment k_2 in between. In both the computational study and the sensitivity analysis it is not necessary to include this constraint. The fact that the maximum space requirement of product category j, u_j, is maximally 6, the segment capacity, $c^{max} = 6$ and the inclusion of constraint 1r ensure that constraint 2 is not necessary. Constraint 1r ensures that a product category is allocated to at most two adjacent shelves. Therefore, the situation where a product category is allocated to shelf segments k_1 and k_3 never occurs. Also, when allocated to multiple shelf segments on the same shelf, product categories need to allocated to adjacent segments. These product categories can never be allocated over 3 adjacent segments as the maximum space requirement, u_j, is maximally 6 which is equal to the maximum segment capacity, $c^{max} = 6$. For the above reasons, constraint 2 is redundant and is removed from the model in the benefit of time.

4.3 Heuristic

The IBM program CPLEX can be used to solve optimization problems such as the integrated assortment planning and shelf-space allocation problem presented in Flamand et al. (2018). As CPLEX is a time consuming program, Flamand et al. (2018) propose a heuristic that can solve the problem much faster than CPLEX. Throughout this paper the heuristic will be used for large parts of the replication and the sensitivity analysis. Standalone CPLEX will only be used for the replication of the regular model APSA runs. The proposed heuristic consists of two parts: the initialization procedure (Algorithm 1) and the MIP-based reoptimization procedure (Algorithm 2). I will briefly explain both algorithms in this Subsection. For more detail and the pseudocodes of both algorithms please refer to Flamand et al. (2018).

4.3.1 Intitialization Procedure (Algorithm 1)

SSP, the single-shelf variant of model APSA, is used in the initialization procedure. The objective and the constraints of SSP can be found in Appendix A.1. There is no need to individually explain the constraints of SSP as they are very similar to the constraints of APSA.

The initialization procedure works as follows: first all the shelves are sorted in decreasing order based on their relative attractiveness, in this order the shelves will be optimized. The set of selected product categories $S = \emptyset$, is initialized before the initialization procedure starts. For the shelf with the highest relative attractiveness an initial solution is found by using model SSP, the single-shelf variant of model APSA. SSP restricts APSA to the chosen shelf and not yet selected product categories. The selected product categories on this first shelf are then added to set S, in order to prevent that these product categories are allocated to other shelves in later iterations. After the initialization by SSP, there is a check if any of the selected product categories j has an allocation affinity relation with any other product category j'. If yes, the product category j' which has an allocation affinity relation with the selected product category j is also added to the set S, to prevent that product category j' is allocated to other shelves in later iterations. Next, the same procedure starts for the shelf with the second-highest attractiveness. Again, SSP is used to choose an initial allocation out of the still available product categories. This procedure continues until all the shelves are initialized or until all the product categories are allocated to a shelf.

4.3.2 MIP-based re-optimization Procedure (Algorithm 2)

In order to calculate an optimality gap, we need an upper bound. This upper bound is obtained by solving the continuous relaxation of APSA. The continuous relaxation of APSA is the exact same MIP-problem as described in Subsection 4.2, but the binary constraints of the decision variables are relaxed. Then, the initial solution is obtained by running Algorithm 1. Now that all the shelves are initialized by Algorithm 1, the shelves are again sorted in decreasing order based on their objective value contribution. Then, every iteration τ (initially equal to 4) shelves are selected to be re-optimized. The sorted shelves are divided in τ levels, where the first level contains the shelves with the highest objective contributions and the last level the shelves with the lowest objective contributions. From every level a shelf is randomly chosen and those shelves are removed from the set of available shelves thereafter. The τ selected shelves are re-optimized by solving model APSA with the set of available product categories. Next, τ of the remaining shelves are randomly selected by choosing one shelf from every level again. These shelves are again re-optimized by solving model APSA. This procedure continues until the set of available shelves for re-optimization is smaller than τ. When this point in the algorithm is reached, the below defined stopping conditions are checked. If one of the stopping conditions is met, the algorithm terminates. If not, all shelves are made available for re-optimization again and the procedure starts over.

The stopping conditions are defined as follows:

- Stop when the relative gap between the continuous relaxation upper bound and the objective of the incumbent solution is less than or equal to $\epsilon\%$.
- Stop when the algorithm traverses all shelves (makes all shelves available for re-optimization again) a specified amount of times.

- Stop when the algorithm reaches a specified time limit.

4.4 Sensitivity Analysis Attractiveness

I perform a sensitivity analysis based on changes in the attractiveness of the shelf segments to see the computational effects of different varieties of shelf segment attractiveness on the heuristic. Every one of the fifteen considered shelves has an attractiveness coefficient vector $[f_{k1}, f_{k2}, f_{k3}]$, where f_{k1} is the attractiveness coefficient of the shelf segment near to the checkouts, f_{k2} the attractiveness coefficient of the middle segment and f_{k3} the attractiveness coefficient of the segment furthest away from the checkouts. I consider 4 different situations:

- Situation 1: Every shelf has an attractiveness coefficient vector of [0.5, 0.5, 0.5].

- Situation 2: Every shelf has an attractiveness coefficient vector of [0.55, 0.45, 0.50]. This vector represents the fact that segments on the outside of the shelf in general have a higher attractiveness than the middle segment(s) (Flamand et al., 2018), and the fact that segments near the checkouts in general have a higher attractiveness than the other segment (Sigurdsson et al., 2014).

- Situation 3: I use 3 different levels of attractiveness and include the same assumptions as in Situation 2. Level 1: [0.25, 0.15, 0.20] Level 2: [0.55, 0.45, 0.50] Level 3: [0.85, 0.75, 0.80]

- Situation 4: I use 5 different levels of attractiveness and include the same assumptions as in Situation 2 and Situation 3. Level 1: [0.15, 0.05, 0.10] Level 2: [0.35, 0.25, 0.30] Level 3: [0.55, 0.45, 0.50] Level 4: [0.75, 0.65, 0.70] Level 5: [0.95, 0.85, 0.90]

Please note that the average shelf segment attractiveness equals 0.50 in all the four situations.

I run the heuristic as described Subsection 4.3 and use the realistic data as described in Section 3 as an input. In Section 5 the effects of the different situations on the output and the tractability of the model will be presented and thoroughly discussed.

4.5 Theoretical Framework Multi-period Model

In this Subsection, I will present the theoretical framework of how to combine endcap allocation and sales promotions in a multi-period model.

4.5.1 Additional Notation

Some additions to Subsection 4.1 are needed to explain the multi-period model.

- $E \subset B$: The set of endcap displays.

- $Y \subset E \subset B$: The set of rear endcap displays.
- $Z \equiv E \setminus Y$: The set of front endcap displays.
- $R \equiv B \setminus E$: The set of regular shelf-spaces.
- $P \subset N$: The set of product categories that are reduced in price because of sales promotions.
- $L \equiv N \setminus P$: The set of product categories that do not have a sales promotions.

4.5.2 Endcap Allocation

In Figure 1 the endcap displays at the front and the rear of the shelves are visualized by light-grey squares and dark-grey squares, respectively. For every time period t, a set of product categories E will be displayed on the endcaps. Note that the product categories which are selected to be displayed on the endcaps are placed on the regular shelves as well. The front and rear endcaps are added to the model as individual shelves with only one segment. The endcaps have higher attractiveness than the regular shelf-spaces. Moreover, the rear endcaps will have higher attractivenesse than the front endcaps (Tan et al., 2018).

The new endcap segments that are added to the model will have a higher segment attractiveness f_k than the regular shelf-spaces. Let's say we have segment p on shelf e where $e \epsilon E$ and segment q on shelf r where $r \epsilon R$, then:

- $f_p > f_q$

In words, segment p which is on an endcap will have a higher attractiveness than segment q which is on a regular shelf.

Moreover, the rear endcaps will have higher segment attractiveness than the front endcaps. Let's say we have segment g on shelf y where $y \epsilon Y$ and segment h on shelf z where $z \epsilon Z$, then:

- $f_g > f_h$

The set E of product categories which are displayed on the endcaps changes every time period t as endcap allocation is a short-term promotion technique. Furthermore, the product categories that are displayed on endcaps in earlier periods cannot be included in set E again for a specified period of time.

4.5.3 Sales Promotions

Some parameters in the model can be altered/fixed to investigate the effects of sales promotions on total profit. Parameters such as the profit margin, ρ_j, the expected demand, v_j, and the impulse purchase potential, γ_j, for $j \epsilon P$ can be altered and fixed in such a way that a sales promotion in the form of a price reduction is mimiced. The profit margin, ρ_j, and expected demand, v_j, can be

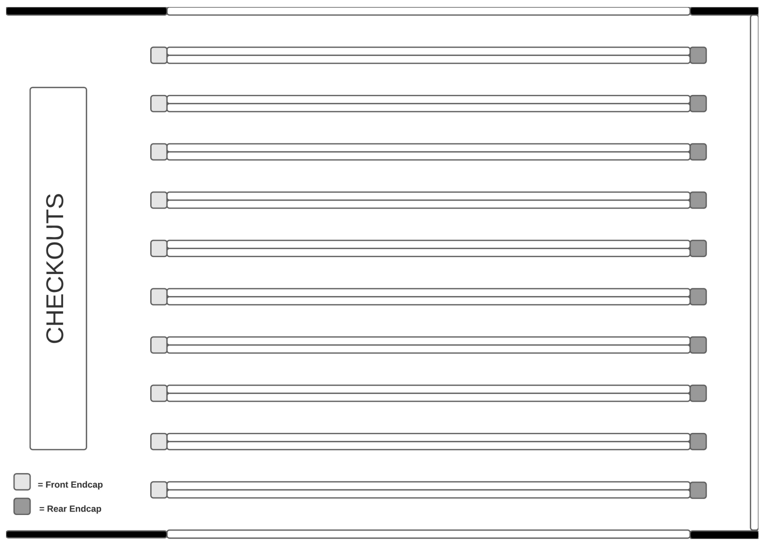

Figure 1: Planogram of a retail store with regular shelves and endcaps.

increased/decreased with different percentages to analyse the effects of offering particular product categories on sale. Set P contains the product categories which are put on sales, for these product categories the above mentioned variables will be altered to mimic a price promotion. As the product categories which are put on sales often change every time period t in a retail store, the set P will also change every time period t. Similar to the set of endcaps, the product categories that are put on sale in earlier periods cannot be included in set P again for a specified period of time. One can analyse both the scenarios that product categories are offered on sale when placed on endcaps and when not placed on endcaps. Comparing these two scenarios can retrieve information on whether these promotional techniques strengthen each other when combined.

4.5.4 Multi-period Model

Promotion techniques such as endcaps and sales promotions are short-term decisions that change every time period t. The product categories that are displayed on the endcaps and the product categories that are on sale change every time period t (often weekly). On the other hand, assortment planning and (regular) shelf-space allocation are long-term decisions that do not change and are considered as given after the initial optimization. A multi-period model is proposed to link short-term with long-term retail decisions.

For every time period t, a store with a layout as in Figure 1 will be optimized. More specifically, several "different" stores are created and optimized, where every store represents the store layout of a particular time period t. For the first period ($t = 1$), the assortment planning, shelf-space allocation, endcaps and sales promotions decisions will be jointly optimized by the optimization heuristic proposed by Flamand et al. (2018). The selected assortment and the shelf-space allocation of the regular shelves (excluding endcaps) are frozen and will not change in the remainder of the model as they are long-term retailing decisions. For every time period thereafter ($t > 1$), the short-term retailing decisions (endcaps and sales promotions) will be jointly optimized, given the selected assortment and the regular shelf-space allocation from time period $t = 1$. The short-term decisions are optimized under the constraint that the product categories displayed on endcaps and the product categories on sale in earlier periods cannot be displayed on endcaps and put on sale again in later periods. Again, note that the product categories which are displayed on the endcaps will always be placed on regular shelves as well, meaning that the regular shelf-space allocation never changes in this model after time period $t = 1$.

Unfortunately, due to time constraint it was not achievable to implement the multi-period with the heuristic. However, the proposed multi-period model is a good approach to include short-term promotion techniques such as endcap allocations and sales promotions in the heuristic proposed by Flamand et al. (2018) and is recommended for further research.

5 Results

In this Section, the results of the replication of the computational study in Flamand et al. (2018) are presented and shortly discussed. Consecutively, the results of the sensitivity analysis are presented and discussed.

Model APSA and the heuristic are coded in the JAVA eclipse environment and solved using the IBM program CPLEX 20.1 The computational runs and the runs for the sensitivity analysis were all made on a Dell Inspirion 7370 having a Intel Core i7 8th Gen processor and 8GB of RAM.

I use normal elapsed time in seconds (abbreviated as Time in the tables) instead of CPU(s) in all the replication and extension tables as it is complicated to measure CPU(s) in Java.

5.1 Replication Computational Study

In this Subsection, I will present the replication results and compare them with the results in Table 3, Table 4, Table 5 and Table 6 in Flamand et al. (2018). The input data used for the replication part is described in Subsection 3.1.

Please note that there can be some irregularities in the trends of elapsed time in the below presented tables. The irregularities are simply caused by the fact that the data is generated randomly and the heuristic handles one dataset better than the other.

Due to time constraints I decided to only run 3 instances instead of 10 and to only run the first 4 sets. Including Set 5 and 10 instances per set for all the tables would simply take too much time. The amount of considered shelves and the amount of considered product categories for Set 1, Set 2, Set 3 and Set 4 are (30, 240), (40, 320), (50, 400) and (60, 480), respectively.

The computational impact of using different neighborhood sizes ($\tau = 2, \tau = 3$ & $\tau = 4$) are presented in Table 2. The elapsed times in seconds and the optimality gaps for 3 instances per set are reported. The optimality gap is defined as the gap between the objective of the continuous relaxation and the objective of the final solution. For $\tau = 3$ and $\tau = 4$ the heuristic terminated within 305 seconds for all instances. This is faster than the heuristic in Flamand et al. (2018), where all the instances terminated within 592 seconds. This is due to the fact that constraint 2 is not included in the models used in this paper. As discussed in Subsection 4.2, constraint 2 is redundant and time consuming and therefore removed from the heuristic.

For $\tau = 2$, the heuristic terminates within 1000 seconds for almost all the cases, except 2 out of the 3 instances of Set 4. In Flamand et al. (2018), almost all instances are terminated because of a time limit of 1000 seconds. In both results the elapsed time increases when the neighborhood size decreases, this makes sense as fewer shelves are re-optimized per iteration.

Just like in Flamand et al. (2018), CPLEX (Model APSA) failed to solve the problem within the preset time limit of 3600 seconds. The optimality gaps reached after 3600 seconds are more or less similar to the optimality gaps in Flamand et al. (2018). Logically, the optimality gaps increase as the number of shelves and the number of product categories increase.

Table 2: Effect of neighborhood size on performance of the heuristic.

Set	Inst	Model APSA		Heuristic		Heuristic		Heuristic	
				$\tau = 2$	$\epsilon = \frac{1}{2}\%$	$\tau = 3$	$\epsilon = \frac{1}{2}\%$	$\tau = 4$	$\epsilon = \frac{1}{2}\%$
		Time	Gap(%)	Time	Gap(%)	Time	Gap(%)	Time	Gap(%)
Set1	1	3600	0.23	59	0.45	52	0.50	38	0.46
	2	3600	0.09	77	0.30	55	0.07	57	0.48
	3	3600	0.43	49	0.45	37	0.47	128	0.31
Set2	1	3600	0.56	46	0.47	123	0.50	112	0.44
	2	3600	0.73	72	0.48	77	0.47	59	0.47
	3	3600	0.49	115	0.44	119	0.38	332	0.45
Set3	1	3600	1.40	234	0.47	212	0.46	218	0.43
	2	3600	1.27	188	0.37	218	0.47	223	0.44
	3	3600	1.33	221	0.49	153	0.49	194	0.43
Set4	1	3600	1.99	754	0.38	299	0.47	221	0.46
	2	3600	2.37	1000	0.54	219	0.50	305	0.46
	3	3600	1.84	1000	0.63	194	0.46	268	0.49

In Table 3 the computational impact of different optimality gaps is presented ($\epsilon = 1.5\%$, $\epsilon = 1.0\%$, $\epsilon = 0.5\%$). The elapsed time of most instances is slightly lower than the elapsed time of the instances in Flamand et al. (2018). Again, this is due to the fact that Constraint 2 is not included in the models used in this paper. Naturally, elapsed time increases as the optimality gap gets tighter, this is visible in both results. The results for model APSA are exactly the same as reported in Table 2.

No affinity relations were used in Table 2 and Table 3. However, in Table 4 and Table 5 the different affinity relations are employed and the computational impact of adding these affinity relations are presented. For every affinity relation

Table 3: Effect of optimality gap on performance of the heuristic.

Set	Inst.	Model APSA		Heuristic		Heuristic		Heuristic	
				$\tau = 4$	$\epsilon = \frac{3}{2}\%$	$\tau = 4$	$\epsilon = 1\%$	$\tau = 4$	$\epsilon = \frac{1}{2}\%$
		Time	Gap(%)	Time	Gap(%)	Time	Gap(%)	Time	Gap(%)
Set1	1	3600	0.23	26	0.82	51	0.82	38	0.46
	2	3600	0.09	30	0.66	27	0.94	57	0.48
	3	3600	0.43	42	0.76	58	0.49	128	0.31
Set2	1	3600	0.56	39	1.04	57	0.94	112	0.44
	2	3600	0.73	58	0.73	50	0.85	59	0.47
	3	3600	0.49	57	1.06	78	0.88	332	0.45
Set3	1	3600	1.40	111	0.52	135	0.72	218	0.43
	2	3600	1.27	151	0.89	150	0.59	223	0.44
	3	3600	1.33	113	0.85	117	0.82	194	0.43
Set4	1	3600	1.99	167	0.83	148	0.94	221	0.46
	2	3600	2.37	143	0.70	230	0.64	305	0.46
	3	3600	1.84	169	0.80	149	0.89	268	0.49

type, 5 product pairs (j, j') are randomly selected. For the same reason as before, the elapsed times in the tables in this paper are slightly less than the elapsed times in Table 6 in Flamand et al. (2018). Especially for the situation where all affinity relations are included the elapsed times in this paper are significantly shorter. There was no need to set a time limit as all instances terminated within 773 seconds.

Table 4: Effect of affinity relations on performance of the heuristic.

Set	Inst.	No Affinity		L		H_1	
		Time	Gap(%)	Time	Gap(%)	Time	Gap(%)
Set 1	1	38	0.46	34	0.46	54	0.35
	2	57	0.48	32	0.44	54	0.40
	3	128	0.31	40	0.38	38	0.42
Set 2	1	112	0.44	111	0.49	113	0.47
	2	59	0.47	108	0.44	131	0.38
	3	332	0.45	99	0.30	101	0.44
Set 3	1	218	0.43	144	0.42	241	0.37
	2	223	0.44	145	0.41	210	0.30
	3	194	0.43	175	0.45	193	0.44
Set 4	1	221	0.46	266	0.50	419	0.46
	2	305	0.46	373	0.35	368	0.46
	3	268	0.49	325	0.39	177	0.38

5.1.1 Analysis of Assumptions

There are some assumptions in Flamand et al. (2018) that are rather questionable and important to note before discussing the results of the sensitivity analysis. First, the objective function is the sum of weighted largest possible profit, Φ_j, so the results cannot be interpreted as real profits. The model is

Table 5: Effect of affinity relations on performance of the heuristic.

Set	Inst.	H_2		H_3		(L, H_1, H_2, H_3)	
		Time	Gap(%)	Time	Gap(%)	Time	Gap(%)
Set 1	1	75	0.38	44	0.41	85	0.41
	2	34	0.32	46	0.23	68	0.35
	3	183	0.47	50	0.41	98	0.41
Set 2	1	176	0.36	145	0.44	254	0.31
	2	87	0.46	124	0.48	227	0.46
	3	104	0.45	190	0.47	270	0.46
Set 3	1	194	0.44	174	0.37	117	0.47
	2	191	0.40	188	0.47	528	0.29
	3	266	0.49	237	0.42	372	0.48
Set 4	1	439	0.20	248	0.42	773	0.19
	2	529	0.25	411	0.43	836	0.36
	3	422	0.31	283	0.35	331	0.47

developed specifically for the purpose to maximize this weighted sum and not necessarily to maximize the "real" overall store profit. In the sensitivity analysis I will not comment on the objective as overall store profit as it has no economic value.

Second, the largest possible profit of product category j, Φ_j, is determined as follows: $\Phi_j = \gamma_j \times \rho_j \times v_j$. In the paper it is mentioned that real data is used for γ_j and ρ_j, but unfortunately this data is not given in the paper. Moreover, it is not mentioned in the paper whether the profit margin of product category j, ρ_j, is gross or net profit margin. Also, the demand volumes, v_j, are given in units instead of in a currency amount. The high demand volumes, v_j, of fast-movers which are not fully compensated by smaller profit margins, ρ_j, cause excessively high largest possible profits, Φ_j, for fast-movers. Therefore, the model depicts fast-movers as more profitable product categories than is the case in reality. In addition, the high demand volumes (in units) cause the objective to rise when the attractiveness of the shelves is increased. The before mentioned reasons make it impossible to interpret the objective function as "real" store profit when realistic data is used. Also, Flamand et al. (2018) state that they use the real profit margin of product category j, ρ_j, in their case study and that they approximate the impulse purchase rate of product category j, γ_j. Unfortunately, they do not present this data in their paper as well, so it is difficult for me to analyse their assumptions for these parameters.

5.2 Senstivity Analysis Segment Attractiveness

In this Subsection I will discuss and interpret the results of the heuristic in the four situations as described in Subsection 4.4. The selected product categories and the allocated shelves of these product categories can be found in Table 6 for Situation 1 and 2, and Table 7 for Situation 3 and 4. The outcome of Situation

3 is visualized in a planogram and can be found in Figure 2. The objective, elapsed time, optimality gap and the amount of selected product categories are presented in Table 8.

The following affinity relations are used in all four situations:

- Allocation disaffinity L: Croissants (8) & Bleach (83), Popcorn (49) & Vegetables (63), Cigarettes (23) & Baby Food (99), Cigars (24) & Diapers (100).
- Assymmetric assortment affinity H_2: if Cake decorations (96) also Cake mixes (98), if Cake decorations (96) also Cakes (40), if Pasta sauce (92) also Pasta (50), if Salad dressings (90) also Vegetables (50).
- Allocation affinity H_3: Milk (34) & Yoghurt (67).

5.2.1 Situation 1

In Situation 1, every shelf has a shelf attractiveness coefficient vector equal to $[0.50, 0.50, 0.50]$. When I ran Situation 1 without a time constraint, the heuristic did not even terminate within 24 hours, it got stuck at one of the first iterations. The heuristic probably got stuck due to some symmetry imposed by equal attractiveness coefficients of the shelf segments f_k. I set a time limit of 100 seconds per iteration, because we see convergence for all iterations after 100 seconds. The selected assortment and the shelf-space allocation can be found in Table 6.

Almost all, 19 out of the 20 fast-movers are included in the assortment. This is due to the fact that these product categories all have a relatively high largest possible profit, Φ_j, because they all have high demand volumes, v_i. For a retailer it is important to include fast-movers in the assortment, as high-sales products are said to increase customer loyalty (Flamand et al., 2018). As all the shelf segments have the same attractiveness, there is no pattern in which product categories are assigned where. The product categories that are selected by the heuristic are just the 60 product categories with the highest largest possible profit, Φ_j. So when all the shelf segments have equal attractiveness, the heuristic just randomly assigns the 60 most profitable product categories to the shelves.

5.2.2 Situation 2

In Situation 2, every shelf has a shelf attractiveness coefficient vector equal to $[0.55, 0.45, 0.50]$. 19 out of the 20 fast-movers are selected in the assortment in Situation 2. In addition, we see that most of the fast-movers and especially the fast-movers with the highest demand volumes (e.g. bread, soda, coffee) are allocated to the segments near the checkouts, as these segments have the highest attractiveness in Situation 2. Allocating fast-movers to segments with a high attractiveness is beneficial in this model as attractiveness and largest possible profit, Φ_j, are linearly related in the objective function. As explained earlier, the largest possible profit, Φ_j, is often excessively larger for fast-movers than for the other product categories, because the demand volumes, v_i, of fast-movers are so much larger. This is not very intuitive as fast-movers often have small profit margins, but in our dataset the smaller profit margins do not compensate for the much larger demand volumes.

The high-impulse product categories are often placed on the medium attractive shelf segments as the most attractive spots are already taken by the fast-movers. It does make sense for a retailer to allocate fast-movers to the same shelf as high-impulse product categories. The fast-movers will attract the customer to a particular shelf, coming across the high-impulse product categories, which might boost the sales of these product categories.

Table 6: Sensitivity Analysis: Allocated product categories per shelf Situation 1 & 2

Shelves	Situation 1	Situation 2
	Products	Products
1	4, 66, 75, 91	34, 67, 72, 85
2	33, 59, 68, 81	4, 30, 58, 89
3	8, 24, 54, 93	36, 44, 74, 81
4	11, 23, 47, 82	8, 11, 54, 93
5	48, 77, 79, 83	65, 78, 83, 94
6	10, 28, 45, 88	41, 66, 71, 77
7	14, 17, 38, 85	16, 45, 48, 69
8	22, 36, 64, 65	9, 22, 68, 95
9	26, 30, 69, 72	13, 14, 35, 75
10	13, 21, 71, 94	17, 21, 28, 91
11	34, 67, 76, 89	26, 70, 79, 88
12	9, 78, 86, 95	27, 33, 47, 82
13	35, 41, 63, 90	10, 38, 63, 76
14	32, 58, 70, 92	24, 32, 50, 90, 92
15	16, 27, 44, 74	23, 59, 64, 86

5.2.3 Situation 3

In Situation 3 there are 3 different levels of shelf attractiveness as described in Subsection 4.4. In Figure 2 the selected product categories and their allocated shelf-spaces are visualized in a planogram. The green coloured shelves have the highest attractiveness coefficient vector, the orange shelves the medium attractiveness coefficient vector and the yellow shelves the lowest attractiveness coefficient vector. The product categories in a circle are fast-movers, product categories in a triangle are high-impulse categories and product categories in a rectangle are regular product categories. Please note that I designed this particular retail shop, but the shelves can be placed in any other preferred composition as well. Which product categories are allocated to which shelves is also presented in Table 7

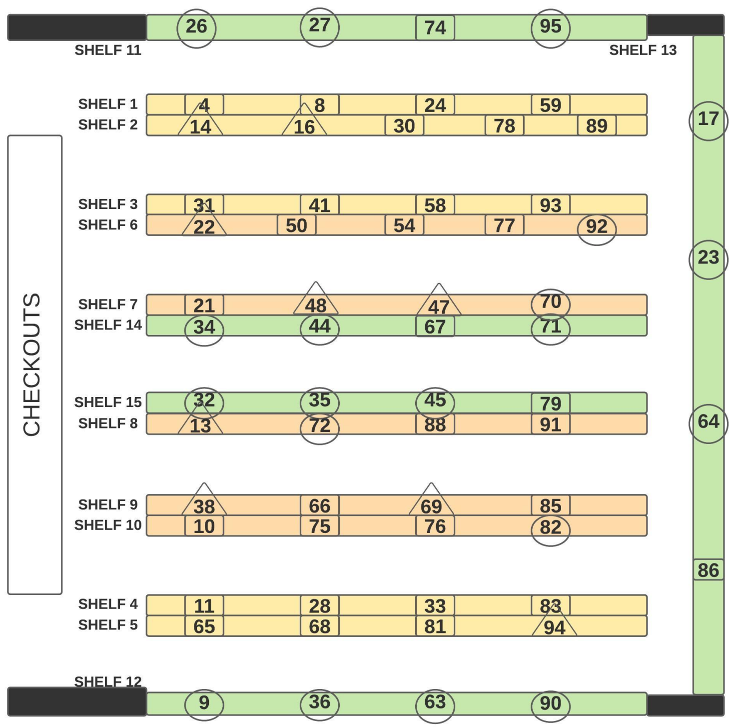

Figure 2: Planogram of Situation 3

Again, 19 out of the 20 fast-movers are selected in the assortment. 14 out of the 17 selected fast-movers are allocated to shelf 11 - shelf 15, the 5 most attractive shelves in this situation. Remarkable is the fact that the remaining spots on the high attractiveness shelves are not filled up by high-impulse product categories but rather by high-volume product categories that just did not make it to the fast-movers group (e.g. Yoghurt (67) & Peanut Butter Jelly (74)). It would be better for the retailer to divide the fast-movers over all the shelves and surround them by high-impulse product categories. Fast-movers can attract the customers to certain shelves where they hopefully buy high-impulse products after encountering them.

Another interesting observation is the fact that no fast-movers or high-impulse product categories are allocated to the low attractiveness shelves. This is due to the fact that fast-movers and high-impulse product categories all have a high largest possible profit, Φ_j. Fast-movers because of the high demand volumes, v_i, and high-impulse product categories because of the high impulse purchase potential, γ_j.

Noteworthy is the fact that Pasta sauce (92), a fast-mover, is not placed on a high attractiveness shelf, this is possibly explained by the fact that Pasta sauce (92) has an assymmetric assortment affinity relation with Pasta (50).

5.2.4 Situation 4

In Situation 4 there are 5 different levels of shelf attractiveness as described in Subsection 4.4. In Table 7 we see that again 19 out of the 20 fast-movers are selected in the assortment. 9 of these fast-movers are allocated to the 3 most attractive shelves. In contrast to Situation 3, a high-impulse product category (Chewing Gum (13)) is allocated to one of the most attractive shelves together with fast-movers. Remarkable is the fact that 11 of the fast-movers are allocated to the 3 medium-high attractiveness shelves, more than allocated to the 3 high attractiveness shelves. This could be explained by the fact that that Milk (34), a fast-mover with very high demand volume, has a allocation affinity relation with Yoghurt (67), which takes the place of other fast-movers.

Table 7: Sensitivity Analysis: Allocated product categories per shelf Situation 3 & 4

Shelf	Situation 3		Situation 4	
	Products	Attractiveness	Products	Attractiveness
1	4, 8, 24, 59	Low	4, 8, 24, 68	Low
2	14, 16, 30, 78, 89	Low	30, 54, 89, 93	Low
3	31, 41, 58, 93	Low	11, 59, 73, 78, 81	Low
4	11, 28, 33, 83	Low	14, 16, 86, 91	Low-Medium
5	65, 68, 81, 94	Low	46, 65, 74, 79	Low-Medium
6	22, 50, 54, 77, 92	Medium	4, 10, 83, 85	Low-Medium
7	21, 47, 48, 70	Medium	21, 47, 48, 88	Medium
8	13, 72, 88, 91	Medium	38, 41, 66, 69	Medium
9	38, 66, 69, 85	Medium	28, 70, 75, 76	Medium
10	10, 75, 76, 82	Medium	22, 71, 72, 95	Medium-High
11	26, 27, 74, 95	High	35, 50, 82, 92, 94	Medium-High
12	9, 36, 63, 90	High	36, 63, 64, 90	Medium-High
13	17, 23, 64, 86	High	17, 34, 44, 67	High
14	34, 44, 67, 71	High	26, 32, 45, 77	High
15	32, 35, 45, 79	High	9, 13, 23, 27	High

5.2.5 Model Tractability

In Table 8 some parameters on the performance of the model are presented. Some model parameters change significantly as the variety in the attractiveness of shelves increases. The objective increases quite sharply when the variety in attractiveness increases. In this model, it seems like more variety in the attractiveness of shelves is beneficial for the objective of the problem. As explained earlier in Subsection 5.1.1, this does not imply that more variety in shelf attractiveness is beneficial for total store profit in reality.

Secondly, the elapsed time is decreasing sharply as the variety in attractiveness of shelves increases. Situation 1 ran for more than 24 hours but did not terminate, the algorithm got stuck at the first iteration of 4-shelf APSA. I decided to set a time limit of 100 seconds per iteration, as this is approximately the point we see convergence of the objective. With the time limit the heuristic terminated in 530 seconds, so it seems like the first iteration of APSA was the bottleneck. The heuristic (especially the first iteration of 4-shelf APSA) has trouble solving situations where the attractiveness of the shelves are very similar, like in Situation 1 and Situation 2. The heuristic terminated after approximately 3.2 hours in Situation 2. Please note that no time limit for the iterations of Algorithm 2 was set in Situations 2, 3, & 4. Just like in Situation 1 the heuristic was stuck quite long at the first iteration in Situation 2, but managed to get out within reasonable time. Situation 3 terminated withing an hour and Situation 4 even terminated within 6 minutes. Having larger differences in shelf attractiveness, like in Situation 3 and Situation 4, seems to benefit the speed of the heuristic.

Lastly, the optimality gap shows a positive relation with the variety in shelf attractiveness. The optimality gaps of the 4 situations in the sensitivity analysis are significantly larger than the presented optimality gaps in the replication part. This is due to the algorithm traversion stopping condition. The runs in all the 4 situations were done with the criteria to stop the heuristic after all shelves are traversed 25 times. A remarkable observation is the fact that the optimality gap is better in Situation 1 where a time limit of 100 seconds for every iteration is set. It seems beneficial for the final outcome that a time limit is set and that the heuristic does not get stuck at the first iteration for too long.

Table 8: Heuristic performance in all 4 situations.

	Situation 1	Situation 2	Situation 3	Situation 4
Objective	7356.6	7769.77	10697.00	11259.12
Gap (%)	0.49	2.40	2.85	3.00
Time(s)	530	11686	3427	351
#Products	60	61	62	62

5.2.6 Vertical Interpretation of Model

The model can be interpreted vertically as well. In our model we have shelf attractiveness coefficient vector $[f_{k1}, f_{k2}, f_{k3}]$, where f_{k1} is the attractiveness coefficient of the shelf segment near to the checkouts, f_{k2} the attractiveness coefficient of the middle segment and f_{k3} the attractiveness coefficient of the segment furthest away from the checkouts. The shelf attractiveness coefficient vector $[f_{k1}, f_{k2}, f_{k3}]$ could also be interpreted vertically, where f_{k1} would be the highest segment on the shelf, f_{k2} would be the middle segment of the shelf and f_{k3} would be the lowest segment of the shelf. It is generally agreed on that shelves on eye level are best visible and generate the best sales (Adam et al., 2017). A retailer could set f_{k2} higher than f_{k1} and f_{k3} to model this assumption. The model is quite versatile as the model works exactly the same in this case, but is interpreted differently. This is convenient for retailers as they will be able to customize the model according to their own store layout, without having to adjust the mathematics behind the model.

Another option is to incorporate both vertical and horizontal segments for every shelf. One would need to replace the 3x1 shelf attractiveness coefficient vector by a 3x3 shelf attractiveness coefficient matrix. Additionally, some of the constraints of APSA and SSP need to be adjusted accordingly. Unfortunately, this was not possible in this paper due to time constraints, so I recommend it for further research.

6 Conclusion

Retail shops and supermarkets are daily part of life for billions of people in this world and immensely important for the world economy. Assortment planning and shelf-space allocation are critical decisions for all these retailers to maximize profit and reach other goals. Therefore it is of high importance that integrated models such as the heuristic proposed in Flamand et al. (2018) keep being developed and are tested thoroughly.

For the replication part, one main common tendency can be derived from the obtained elapsed times of the instances. The elapsed times in this paper are slightly lower than the elapsed times in Flamand et al. (2018). The reason behind the slight difference is the exclusion of redundant constraint 2. I tested the heuristic including constraint 2 and this indeed increases the elapsed time.

From the shelf segment attractiveness sensitivity analysis it is evident that more variety in shelf segment attractiveness significantly increases the objective of the model. Please note that this cannot be interpreted as an increase in overall store profit, as explained in Subsection 5.1.1. Moreover, I conclude that in this model fast-movers are often allocated to shelves with a high attractiveness coefficient. Sometimes a high-impulse product category is added to high attractiveness shelves, but not very often. In a real world situation, it would probably be better to spread the fast-movers more evenly throughout the store and surround them by high-impulse product categories to obtain maximum customer footprint.

Some conclusions about the computational impact of varying the shelf attractiveness can be drawn as well. The more variety in the shelf attractiveness the easier it is for the computer to solve the model. The heuristic has trouble when there is too much symmetry imposed in the attractiveness coefficient vectors. The size of optimality gap is affected by the variety in shelf attractiveness as well: The more variety, the larger the optimality gap.

For further research I recommend testing a even more realistic data set with more realistic affinity relations. For example, in Situation 3 Bleach (83) and Eggs (33) are allocated to the same shelf, this is not a good reflection of a real world supermarket. If not constrained by time, it would be good to analyse our realistic dataset with more, realistic affinity relations. In addition to the affinity relations, it would be good to implement the theoretical framework as described in Subsection 4.5 in the heuristic proposed by Flamand et al. (2018). A multi-period model is a good approach to combine short-term retailing decisions as sales promotions and endcap allocations with long-term retailing decisions as assortment planning and shelf-space allocation. I believe that this multi-period model comes close to reality.

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A Appendix

A.1 SSP Model

For the initialization procedure (Algorithm 1) the single-shelf variant of model APSA, called SSP is needed. The objective function and the constraints are defined as follows:

$$\text{Maximize} \quad \sum_{k \epsilon K_i} \sum_{j \epsilon N \setminus S} \Phi_j \frac{f_k s_{kj}}{c_k} \tag{3a}$$

$$subjectto \quad \sum_{j \epsilon N \setminus S} s_{kj} \leq c_k, \quad \forall k \epsilon K_i \tag{3b}$$

$$l_j w_j \leq \sum_{j \epsilon N \setminus S} s_{kj} \leq u_j w_j, \quad \forall j \epsilon N \setminus S \tag{3c}$$

$$\phi_j y_{kj} \leq s_{kj} \leq min(c_k, u_j) y_{kj}, \quad \forall j \epsilon N \setminus S, \; k \epsilon K_i \tag{3d}$$

$$y_{kj} \leq w_j, \quad \forall j \epsilon N \setminus S, \; k \epsilon K_i \tag{3e}$$

$$w_j \leq \sum_{k \epsilon K_i} y_{kj} \quad \forall j \epsilon N \setminus S, \; k \epsilon K_i \tag{3f}$$

$$q_{kj} \geq y_{kj} + y_{k+1,j} - 1, \quad \forall j \epsilon N \setminus S, \; k \epsilon K_i \setminus \{\beta_i\} \tag{3g}$$

$$\sum_{j \epsilon N \setminus S} q_{kj} \leq 1, \quad \forall k \epsilon K_i \setminus \{\beta_i\} \tag{3h}$$

$$w_j + wj' \leq 1, \quad \forall (j, j') \epsilon L \tag{3i}$$

$$w_j - wj' = 0, \quad \forall (j, j') \epsilon H_1 \tag{3j}$$

$$w_j \leq w_{j'}, \quad \forall (j, j') \epsilon H_2 \tag{3k}$$

$$y_{k_1 j} + y_{k_2 j} \leq 1, \quad \forall (k_1, k_2, j) \epsilon R | k_1, k_2 \epsilon K_i \tag{3l}$$

$$w, y, q \quad \text{binary}, \quad s \geq 0. \tag{3m}$$

A.2 Guide to Code

In this Section of the Appendix I will briefly explain the different classes and functions used in my Java code. Please note that there are additional comments present in the code files.

Class Main: In the main class you can run the heuristic. In the main function of the main class there are some functions called from other classes which can set the data for the computational study or the sensitivity analysis. If you want to run the heuristic the functions under the comment "Use this methods to generate the variables of the computational study" should be run to generate the data. Next, you should run the 2 functions of the algorithms *Algorithm1new.initializationProcedure* and *Algorithm2.reoptimizationProcedure* if you want to run the heuristic. The computer will automatically print the objective, optimality gap and the decision vairables. Furthermore, there are some methods that help printing the output in .txt files.

Class General: In this class we have a lot of class variables that are used as input for the heuristic. When the function to set the input data are called, these functions update the class variables in this class. The algorithms again call this class variables.

Class DecisionVar: This is an object class that stores the values of the decision variables. After every iteration the variables in this class are updated. In this way we can always access the decision variables and the objectives.

Class APSAnew: In this class the APSA model for 4 shelves is coded with IBM CPLEX. At the beginning of the class the right input variables needed to optimize the 4 shelves are extracted from the total input data. The objective, constraints and functions to update the DecisionVar object can be found in this class.

Class SSPcopy: In this class you find the code of the single-shelf variant of APSA, SSP. This is very similar to the APSA class but this model works for one shelf only.

Class APSAregular: In this class the full model APSA is coded. It is similar to APSA and SSP class but for all shelves.

Class APSAcontinuousNew: In this class the continuous relaxtion of the APSA model is coded. This is used in algorithm 2 to calculate the optimality gap.

Class Algorithm1new: Algorithm 1 of the proposed heuristic by Flamand et al. (2018) is coded in this class. Function *Algorithm1new.initializationProcedure* runs Algorithm 1.

Class Algorithm2: Algorithm 2 of the proposed heuristic by Flamand et al. (2018) is coded in this class. Function *Algorithm2.reoptimizationProcedure* runs Algorithm 2.

Class HelpMethods: In this class there are some functions that are needed to perform some of the steps in the algorithms. For example, there is a function that removes zeroes from vectors, a function that sorts vectors in decreasing order and a function that reverses the order of a vector. These methods are called in Algorithm 1 and Algorithm 2.

Class RealLifeData: This class sets the variables in the General class to the data needed to run the sensitivity analysis.