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BSc^2 Econometrics/Economics

The Impact of Bi-Objective Optimization for Green Vehicle Routing Problems on CO_2 Emission and Its Allocation to Customers

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Abstract

In this thesis, we present a bi-objective green vehicle routing problem (bi-GVRP) to evaluate the effects of considering CO_2 emission during the formulation of the distribution routes of a logistics service provider. We assign different weights to the objectives and compare relative changes of total distance and CO_2 emission for all randomly generated test instances. Furthermore, we investigate the impact on the allocation of CO_2 emission to customers by following the approach of Naber et al. (2015). We assess the emission allocations of five methods – being the Star method, the Shapley value, the Nucleolus, the Lorenz+ Allocation and the Equal Profit Method+ – and investigate whether their behaviour changes based on three criteria: stability, consistency and computation time. Our numerical experiments show that the criteria do not detect significant changes in the behaviour of the allocation methods. However, it is still worthwhile to consider bi-objective optimization for economic and environmental factors in the determination of distribution routes as a significant decrease in CO_2 emission can be achieved by a relatively smaller increase in total distance travelled.

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1 Introduction

Global warming is an unavoidable challenge that the world is continuously dealing with. The correlation between human emissions of greenhouse gases (GHGs) and global temperatures has always been there and is undisputed (Lacis et al., 2010). Over the last century, global surface temperatures have, on average, increased by 1.3°C due to the remarkable surge in the concentration of GHGs (Ritchie and Roser, 2020). One well-known gas that contributes to anthropogenic climate change is carbon dioxide, CO_2. The impact of its concentration is tremendous and, even if CO_2 emission would come to an end immediately, it is already irreversible for the next millennium, as the study by Solomon et al. (2009) depicts.

Due to the prominent position of environmental effects in the current world, companies value their carbon footprint and devote more attention to it. The awareness of the externalities of their services is of interest for several underlying reasons. One reason for this is the role and influence of the government. On the one hand, companies might be subsidized or be offered tax reductions when making use of environmentally friendly processes, where on the other hand, specific industries are restricted by 'relative' or 'intensity' emission caps (Quirion, 2005). The presence of such environmental measures in the logistics sector makes sense, as the International Energy Agency (2009) quantifies the contribution of transportation activities in the emission of GHGs to be around 23% of the world's total. Another reason why businesses evaluate their total emission is the exhibition of their corporate social responsibility. They set their own emission targets to provide themselves with a more competitive position in the market. In this way, it is more likely that consumers who value their positive contribution to the environment will prefer them because of their lower carbon footprint.

In this thesis, we study a setting where a logistics service provider (LSP) visits its customers on different distribution routes. It might be the case that these customers ask the LSP to inform them about their contribution of the total emitted CO_2 on the driven distribution route. They cannot estimate this part of their carbon footprint themselves as it is dependent on the LSP supplying them. The driven route is vital in allocating CO_2 emission as the total distance travelled and the weights of the vehicle's load determine the total emission of transport. Mainly, a LSP designs its routes from an economic (profit-maximizing) perspective and uses basic transportation models such as the vehicle routing problem (VRP) (Toth and Vigo, 2002). However, these models neglect factors like emission or environmental impact. Hence, this may cause a discrepancy between the constructed route and the hypothetical route which would be optimal from an environmental perspective.

Therefore, we consider a trade-off between economic and environmental aspects in the optimization of distribution routes. Subsequently, we evaluate the impact of this objective on total CO_2 emission and the behaviour of selected allocation methods. Our aim is not only to determine to what extent emission should be incorporated in optimizing routes, but also to see what influence this has on the allocated emission to the customers.

To establish a transparent and fair allocation of CO_2 emission to a single customer on a given distribution route, Naber et al. (2015) propose and compare five different methods. Their approach models the allocation problem with concepts from cooperative game theory, which results in the so-called emission allocation game. Then, they apply allocation methods to allocate CO_2 emission on a case study in which the routes are constructed following a cost-minimizing objective. To evaluate the allocation methods, they use several important criteria from a customer perspective: stability, consistency, robustness and computation time. The five allocation methods and the emission allocation game will form the basis of this research.

This thesis aims to evaluate the effects of considering CO_2 emission during the formulation of the distribution routes of a LSP. We implement environmental aspects by presenting a bi-objective green vehicle routing problem (bi-GVRP), based on the approach of Sawik et al. (2016) and El Bouzekri El Idrissi and Elhilali Alaoui (2014), which finds optimal routes in the context of total distance travelled and expected emission. To conclude whether including environmental externalities adds value or generates extra costs for the LSP, we compare relative changes in CO_2 emission and the total distance of multiple instances. In this comparison, different weights are assigned to both objectives. In addition, we follow the approach of Naber et al. (2015) and allocate CO_2 emission for all instances using both their proportional and game-theoretic models. We evaluate the influence of the bi-objective optimization on the proposed allocation methods by comparing the allocated amounts of CO_2 emission. Finally, we investigate a possible change in behaviour of the methods by assessing the criteria stability, consistency and computation time (Naber et al., 2015).

By means of numerical experiments, we show that it is worthwhile to consider bi-objective optimization for economic and environmental factors in the formulation of an optimal route schedule. Even though the criteria do not show significant changes in the behaviour of the allocation methods when the objective varies, bi-objective optimization provides incentives as CO_2 emission can significantly be decreased by a relatively smaller increase in total distance travelled.

The remainder of this thesis is as follows. Section 2 gives an overview of the relevant literature. In Section 3, a formal problem description and a mathematical formulation for the bi-GVRP are presented, together with the emission allocation game and the five allocation methods. Subsequently, Section 4 presents the randomly generated test instances whereafter Section 5 contains the computational results. Finally, the thesis finishes off with a conclusion and suggestions for further research in Section 6.

2 Literature review

2.1 Vehicle routing problems

In the field of operations research, the VRP plays a central role and "is one of the most important, and studied, combinatorial optimization problems" (Toth and Vigo, 2002, p. xvii). The problem was introduced by Dantzig and Ramser (1959), who generalized the classic travelling salesman problem (TSP) of Flood (1956). Their real-world application on handling the supply of gasoline initiated intensively investigated derivatives and many adaptations. The reason for the prominent position of the VRP in literature is its widespread applicability. The VRP belongs to the class of NP-hard problems and is therefore restricted to a size of 50 - 100 customers when it needs to be solved to optimality within a reasonable timespan (Kumar and Panneerselvam, 2012). The specific maximum size is dependent on the used VRP variant.

Until recently, companies only cared about their economic costs while maximizing the profitability of their business (McKinnon et al., 2015). However, since public and government interest in sustainability rose at the beginning of the 21st century, logistic processes are now under the pressure of considering environmental aspects, too. There are claims that environmental benefits can be achieved by simply reducing the total distance travelled as this consequently means less fuel consumed (Sbihi and Eglese, 2007). Nevertheless, VRP variants incorporating environmental externalities like pollution and generated noises are widely developed (Dekker et al., 2012). The significant presence of literature studies that integrate both logistics and environmental concerns into one model, as for example Palmer (2007) does, proves that more can be done besides the minimization of distance only. These models that consider environmental aspects are categorized as green vehicle routing problems (GVRP).

In most applications, the focus of the GVRP is similar and the differences are to be found in the formulation of the objective function and solving procedures. For example, Ubeda et al. (2011) and Elbouzekri et al. (2013) solve for a single objective function that only minimizes

CO_2 emission. The difference in these studies is the solving method: the first-mentioned solves its objective heuristically in two steps as emission is estimated first, where the latter applies a hybrid ant colony system. Another study that also implements a single objective is the one of Bektaş and Laporte (2011). Their paper can be seen as one of the most explicit in the field of GVRPs as its objective not only incorporates distance travelled and emissions, but also the corresponding costs of fuel used and the duration of travelling. Instead of solving one single objective, other studies partition the multiobjective into separate functions. Where Kim et al. (2009) use one function for the freight transport costs and one for the CO_2 emission, Molina et al. (2014) follow a similar approach with the addition of a third objective to also consider the emission of the air pollutant NO_x. Furthermore, there are studies, like those of Sawik et al. (2016) and El Bouzekri El Idrissi and Elhilali Alaoui (2014), that combine the intentions of green logistics and formulate a bi-objective function that is solved with respect to the standard constraints belonging to a VRP, resulting in a bi-GVRP. In this thesis, we will apply green logistics following the principles of this variant.

2.2 Cost allocation problems

Besides the optimization of distribution routes, this thesis inspects fair and transparent allocations of CO_2 emission amongst the customers on these routes. The roots of these methods are to be found in cost allocation problems. Reviewing the literature reveals a distinction between these problems; some apply basic proportional rules to allocate costs, where others incorporate theoretical concepts derived from game theory (Tijs and Driessen, 1986; Guajardo and Rönnqvist, 2016). Often, these two categories appear together in literature for the sake of comparison.

In proportional methods, players are assigned a share of the total costs incurred. The value of each individual's share can be quantified by using different factors. If every player gets assigned an equal cost share, it is referred to as the egalitarian method. It is applied in a broad scope of research; Dror (1990) uses this method in a TSP setting and Lehoux et al. (2011) implement this equal allocation in a supply chain of a pulp and paper supplier. The relative share of the players can also be based on criteria such as the stand-alone costs – resulting costs from a single delivery to a player – or units demanded. The first-mentioned criteria is used by Sun et al. (2015) and Özener (2014) in a transportation network with fixed routes. The latter criteria is, for example, exerted in the case study of Flisberg et al. (2015), who evaluate proportional cost allocations on forest fuel transportation in Sweden.

When cooperative concepts from game theory are involved in allocating costs, a well-known and frequently used method is the Shapley value (Shapley, 2016). See, for instance, the study of Dror (1990), which implements this method next to its proportional allocations. Zakharov and Shchegryaev (2015) provide a research where the method is used solely in a dynamic VRP. However, the Shapley allocations do not need to be in the core of a game by definition (see Section 3.2 and Equation (3.4) for details), which calls for other cost allocation methods that do follow this major concept of game theory. The Nucleolus, introduced by Schmeidler (1969), is such a method that guarantees an allocation in the core when it is non-empty. For example, Frisk et al. (2010) apply this method, next to other methods such as the Equal Profit Method (EPM), in their study on collaborative forest transportation. Furthermore, the study by Engevall et al. (2004) focuses on the Nucleolus and places it into a VRP setting. Due to its complex computation, various algorithms have been suggested; Puerto and Perea (2013) propose a single linear program (LP) problem to find the Nucleolus, where others, as Fromen (1997), successively solve multiple LP problems.

Allocation methods are not only used to allocate costs as Sakawa et al. (2001) show by also allocating profit within a cooperative game setting. Both proportional and game theoretic methods can be adjusted to an environmental setting to allocate emission with the same reasoning. For instance, Rosen (2008) compares different output-based proportional allocation methods for CO_2 emissions. Moreover, Dai et al. (2014) implement a proportional method to allocate tradable emissions permits and illustrate their approach in a case study over 30 regions in China. Naber et al. (2015) set the proportional Star method as a benchmark in the comparison with four other (game-theoretic) allocation methods, which are all examined in this thesis as well (see Section 3.3). Other models that rely on concepts from cooperative game theory focus mainly on cap-and-trade schemes in which trade caps are being allocated on an international level (Endres and Finus, 2002; Böhringer and Rosendahl, 2009). Research that does allocate emission is carried out by Zhu et al. (2014), who allocate emission within a maritime logistics chain by taking into account specific characteristics as volume and weight.

3 Methodology

This section introduces and elaborates on the bi-GVRP model, the emission allocation game and the allocation methods treated in this thesis. First, the general problem description and notation for the bi-GVRP are given, followed by its mathematical formulation (see Section 3.1). The bi-objective model aims to construct an optimal number of routes, from both an economic and environmental perspective, visiting all customers. Secondly, in Section 3.2, the emission allocation game is developed by using cooperative game theory. This game forms the base for solving the allocation problem. Finally, Section 3.3 discusses the five allocation methods. These are similar to the methods used in Naber et al. (2015): the Star method, the Shapley value, the Nucleolus, the Lorenz+ Allocation and the Equal Profit Method+.

3.1 Problem description and formulation bi-GVRP

The optimization of the bi-GVRP determines an optimal set of routes that visits all customers once while satisfying their demands. To attain an optimal solution, the total travelled distance and the amount of CO_2 emission are minimized. As this problem is an extension of the standard VRP, it displays identical characteristics: (a) homogeneous fleet, (b) known fleet size, (c) single depot, (d) deterministic demand and (e) oriented network (Ubeda et al., 2011).

3.1.1 General model

To formulate the model, we introduce $V = \{0, 1, 2, ..., v\}$ a set of v nodes representing the locations to be visited. The index 0 corresponds to the depot, where the other nodes 1 to n represent the customers. We denote the set $V' = V \setminus \{0\}$, which refers to all nodes except the depot. The bi-GVRP can now be defined on the complete directed graph $G = (V, A)$, where $A = \{(i, j) \,|\, i, j \in V, i \neq j\}$ is the set of arcs. Also, transport is performed by a fleet of k homogeneous vehicles represented by set $K = \{1, 2, ..., k\}$. All vehicles k have a maximum holding capacity of W.

The parameters used in the formulation are defined as follows. We define $\lambda \in [0, 1]$ as a weight for the objectives. Here, $\lambda = 1$ corresponds to the situation where only total distance is minimized and $\lambda = 0$ represents minimization of total CO_2 emission only. Next, $d_{i,j}$ denotes the Euclidean distance between every pair of nodes i and j. Each such arc has its emission ratio when utilized, $e_{i,j}$ (g CO_2). These amounts are the distance of the arc multiplied by the emission per driven kilometer when the vehicle is fully loaded. The emission function developed by Ligterink et al. (2012) determines the emission per kilometer (see Appendix A). We assume a fully-loaded vehicle as we cannot determine yet how many loading units the vehicle will be carrying on arc (i,j) as this is dependent on the distribution route. Furthermore, each customer $i \in V'$ has a nonzero demand of q_i units. Hence, the depot has a demand of 0.

3.1.2 Mathematical formulation

To formulate the bi-GVRP correctly, we define decision variables, based on El Bouzekri El Idrissi and Elhilali Alaoui (2014), next to the introduced sets and parameters. The binary decision variables $x_{i,j}^k$ are equal to 1 if vehicle k visits node j directly after node i, and 0 otherwise. In addition, binary decision variables y_i^k are equal to 1 if vehicle k serves node i, and 0 otherwise. We also add decision variables u_i^k for all customers $i \in V'$ to use the Miller-Tucker-Zemlin (MTZ) formulation that eliminates possible subtours (Miller et al., 1960).

We are dealing with a bi-objective model and, therefore, we consider two different objectives while solving to optimality. The first part, Equation (3.1), represents the economic perspective and calculates the total travelled distance of all routes driven by vehicles k. The environmental equation, Equation (3.2), sums the CO_2 emission of the retrieved driving schedule.

$$TD = \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} d_{i,j} x_{i,j}^k \tag{3.1}$$

$$TE = \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} e_{i,j} x_{i,j}^k \tag{3.2}$$

The objectives described by Equation (3.1) and (3.2) are not on the same scale; it is case dependent which level of importance is assigned to which perspective. As such, we include the weights λ in the transformation from single objectives to the bi-objective function (Sawik et al., 2016):

$$\min \quad \lambda \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} d_{i,j} x_{i,j}^k + (1 - \lambda) \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} e_{i,j} x_{i,j}^k \tag{1}$$

In the bi-GVRP, the objective function (Equation (1)) is minimized with respect to the constraints based on Ubeda et al. (2011):

$$\sum_{k \in K} y_0^k = |K|, \tag{2}$$

$$\sum_{k \in K} y_i^k = 1 \qquad \forall i \in V', \tag{3}$$

$$\sum_{i \in V} x_{i,j}^k = y_j^k \qquad \forall j \in V, \forall k \in K, i \neq j, \tag{4}$$

$$\sum_{i \in V} x_{j,i}^k = y_j^k \qquad \forall j \in V, \forall k \in K, i \neq j, \tag{5}$$

$$\sum_{i \in V} q_i y_i^k \leq W \qquad \forall k \in K, \tag{6}$$

$$u_i^k - u_j^k + W x_{i,j}^k \leq W - q_j \qquad \forall i,j \in V', \forall k \in K, i \neq j, \tag{7}$$

$$q_i \leq u_i^k \leq W \qquad \forall i \in V', \forall k \in K, \tag{8}$$

$$x_{i,j}^k \in \{0,1\} \qquad \forall i,j \in V, \forall k \in K, \tag{9}$$

$$y_i^k \in \{0,1\} \qquad \forall i \in V, \forall k \in K, \tag{10}$$

$$u_i^k \in \mathbb{R} \qquad \forall i \in V, \forall k \in K. \tag{11}$$

Constraint (2) guarantees that all vehicles are assigned a route that begins and ends in the depot. The following constraints (3) ensure that every customer is visited exactly once, or in other words, only a single vehicle drives past each customer. Subsequently, constraints (4) and (5) link all nodes, excluding the depot, to a predecessor and a successor to construct correctly defined routes. The maximum holding capacity of each vehicle is not exceeded because of constraints (6). We introduce the MTZ constraints (7) and (8) to avoid subtours. Lastly, we have constraints (9) - (11) that restrict the domain of all decision variables used.

3.2 The emission allocation game

In allocating emission over customers on a distribution route, we use the so-called emission allocation game derived from cooperative game theory. This field of game theory deals with situations in which players – in this case customers of the LSP – can benefit more from collaborating with other players than rather acting on their own (Hart, 1997). This subsection introduces the specific emission allocation game formulation, as earlier done by Naber et al. (2015). The notation and interpretation for our application are described below.

We deal with a bi-GVRP that formulates multiple distribution routes for the LSP to visit all its customers. Note, however, that we define the emission allocation game for a single route as we eventually allocate CO_2 emission on a single distribution route. Therefore, we represent the set of n customers that the LSP serves on a single distribution route by $N = \{1, 2, \ldots, n\}$. This set can also be referred to as the grand-coalition, and the route driven to visit all customers is denoted by $\sigma(N)$. This notation also specifies in which order the LSP serves these customers.

In our application, the routes $\sigma(N)$ are known as we solve the problem (1)-(11). We assume that only the grand-coalition route $\hat{\sigma}(N)$ is known and that it is not possible to determine a subroute $\hat{\sigma}(S)$ for all $S \subset N$ in this emission allocation game. Hereafter, we specify route $\hat{\sigma}(S)$ by applying the order of $\hat{\sigma}(N)$; we simply leave out all customers from the original route that are not in S. This method also prevents the game from being skewed significantly as routes $\hat{\sigma}(S)$ cannot outperform $\hat{\sigma}(N)$. This avoidance is essential as we aim to allocate emission based on the original route.

Next to determining the routes driven by the LSP, we also need to calculate the corresponding total emissions. Let $g(\sigma(N))$ be the emission function of route $\sigma(N)$, calculated by the formula used by Ligterink et al. (2012) (see Appendix A). In this calculation, the weight of the goods transported, the distance travelled, and the driving speed are considered. We denote the total emission of route $\sigma(N)$ by Equation (3.3) for ease of notation.

$$e(N) = g(\sigma(N)) \tag{3.3}$$

Now that we defined the emission function, we introduce another assumption that $\sum_{i \in N} e(\{i\}) \geq e(N)$ to ensure that no customer gets allocated more than its stand-alone emission. In other words, it is always possible to determine an emission allocation that is individually rational and efficient. This assumption is coherent as a LSP will rarely emit more CO_2 when it visits all its customers individually.

We have now wholly identified the characteristic function $e(N)$ and can realize emission allocations based on existing solution concepts from cooperative game theory. Despite that there are several ways to define a stable allocation among the players, we will use a core-like approach (Hart, 1997). An allocation of emission, defined as $x = (x_i)_{i \in N}$, is a stable allocation if it is located in the core of the game (N, e). To define the core of this game, we let $x(N) = \sum_{i \in N} x_i$ be the total allocated emission of all customers in N. The core is then as follows:

$$\text{core}(e) = \{x \in \mathbb{R}^n : x(N) = e(N); x(S) \leq e(S), \forall S \subset N\} \tag{3.4}$$

Equation (3.4) represents the set of all feasible allocation vectors for which no player or coalition has an incentive to leave the game as it cannot improve by acting alone (Kannai, 1992). However, this verification of stability does have some remarks in this context. First of all, it is possible by definition that a negative allocation occurs. If an additional customer makes the distribution route more efficient when looking at total emission, such negatives allocations are feasible. Nevertheless, as we are constructing the routes beforehand from an environmental perspective and emission likely grows with an extra customer, it is improbable that this will happen in this research. Another observation that could occur in our setting is the emptiness of the core. See Naber et al. (2015) for an illustrative example of such an empty core.

3.3 The allocation methods

This subsection does not only discuss the five allocation methods that are used to allocate CO_2 emission among customers on a distribution route, it also introduces the criteria on which we evaluate the performance of the methods. We follow Naber et al. (2015) and present one proportional allocation method, the Star method, and four other methods that originate from cooperative game theory: the Shapley value, the Nucleolus, the Lorenz+ Allocation and the Equal Profit Method+.

3.3.1 Star method

The Star method can be compared with standard allocation methods that allocate costs proportionally. The only difference is that the Star method allocates CO_2 emission proportional to the stand-alone emission of the customers on a distribution route (Naber et al., 2015). A graphical view of all these single trips provides a star-shaped figure initiating the name for this method. The allocated emission x_i for every customer $i \in N$ can be calculated by Equation (3.5).

$$x_i = \frac{e(\{i\})}{\sum_{i \in N} e(\{i\})} e(N) \tag{3.5}$$

It is a straightforward computation and depends on just one factor. Therefore, it easily provides insights for both the LSP and its customers. The stand-alone emission can also be replaced by other shipment parameters like volume or tonne-kilometers. Nevertheless, the method also has its downsides. Even when the core is non-empty, it is not necessary that the allocation solution is a core solution and with that stable. Next to that, the method only depends on total emission and ignores, for example, the distances between customers and the visited order.

3.3.2 Shapley value

The Shapley value incorporates game-theoretic concepts and considers marginal emissions while allocating CO_2 emission over the customers (Shapley, 2016). When a customer i is added to subset S, which is a permutation of the distribution route, the marginal emission is denoted by $m_i(S) = e(S \cup \{i\}) - e(S)$. Now, a customer's allocated emission x_i is equal to its average marginal emission over all subsets S and computed by Equation (3.6).

$$x_i = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(n - |S| - 1)!}{n!} m_i(S) \tag{3.6}$$

Even though a core solution is in general not guaranteed, even when the core is non-empty, Shapley (1971) noted that allocations following the Shapley value are definitely in the core of the game when it is convex. In our case, the emission allocation game is not convex as our computational results show Shapley value allocations that are not a core solution.

3.3.3 Nucleolus

Schmeidler (1969) introduced the Nucleolus and stated that it reveals the attitude of a coalition S towards its allocated emission and therefore provides the most accepted CO_2 emission allocation. For the allocation x, we define a vector $\theta(x)$ containing all excesses for each coalition S in decreasing order, where the excess is computed as $e(S) - x(S)$. The Nucleolus is defined as the individual rational and efficient allocation x for which $\theta(x)$ is the lexicographic minimum. We apply the approach by Naber et al. (2015) and iteratively solve different LP problems to find this 'optimal' allocation x. The first step is maximizing the smallest excess – with an empty core this excess is negative – to find a suitable allocation. If this x is unique, the Nucleolus allocation is found. However, if this is not the case, a successive LP problem is solved after fixing the acquired excess for all coalitions with strictly positive dual variables. As the Nucleolus is unique and lies in the core, if non-empty, this termination is repeated until the LP formulation finds a unique solution.

To translate this into a LP problem, we introduce decision variable δ indicating the excess of allocation x. As we iteratively generate δ, we define its optimal value in iteration l as δ_l. We also denote the set F_l consisting of all fixed coalitions after iteration l. Obviously, in the first iteration this set is empty, i.e. $F_1 = \emptyset$. Now we formulate the following LP problem:

$$\delta_l = \max \quad \delta \tag{12}$$

$$\text{s.t.} \quad x(\{i\}) \leq e(\{i\}) \qquad \forall i \in N, \tag{13}$$

$$x(S) + \delta \leq e(S) \qquad \forall S \subset N, S \notin (\cup_{m<l} F_m), \tag{14}$$

$$x(S) + \delta_m = e(S) \qquad \forall m < l, S \in F_m, \tag{15}$$

$$x(N) = e(N). \tag{16}$$

Constraints (13) are included to ensure the individual rationality of the allocations for all customers i. Constraints (14) - (16) guarantee a core solution when the core is non-empty. Note that constraints (15) are only included after the first iteration. To construct the set F_l in iteration l, we define the vector μ_l of optimal dual variables of constraints (14). We fix the excess for all coalitions with strictly positive dual variables. Hence, the collection of coalitions that need their excess fixed in iteration l is defined as $F_l = \{S \subset N \mid \mu_l(S) > 0\}$.

3.3.4 Lorenz+ Allocation

The Lorenz+ Allocation, introduced by Naber et al. (2015), is slightly different from the original variant by Arin Aguirre (2003). The latter is a leximin and allocates emission by minimizing the smallest difference f between the lowest and highest allocated CO_2 emission among all customers $i \in N$. Additionally, it always requires a core solution as the following LP problem (17)-(20) is solved:

$$\min \quad f \tag{17}$$

$$\text{s.t.} \quad x_i - x_j \leq f \qquad \forall i, j \in N, \tag{18}$$

$$x(S) \leq e(S) \qquad \forall S \subset N, \tag{19}$$

$$x(N) = e(N). \tag{20}$$

Constraints (18) define the differences in allocation between any location i and j. The solution in the core is ensured by constraints (19) and (20); an empty core does not allow a Lorenz Allocation. This is also the point where the upgraded Lorenz+ Allocation comes into scope. When no solution is generated, we apply the Nucleolus instead to maintain continuity. In general, the solution of the LP problem is non-unique if the core is non-empty, implying that the Lorenz+ Allocation is not unique, too.

3.3.5 Equal Profit Method+

The Equal Profit Method (EPM) applies an approach similar to the initial Lorenz Allocation and is introduced by Frisk et al. (2010). Both try to find an emission allocation in the core, but the EPM does this with a dissimilar minimization. For each customer, we determine the ratio of its CO_2 emission and its stand-alone emission. Subsequently, the largest difference g between all these ratios is to be minimized. To determine the EPM solution, the following LP problem (21)-(24) is solved to optimality:

$$\min \quad g \tag{21}$$

$$\text{s.t.} \quad \frac{x_i}{e(\{i\})} - \frac{x_j}{e(\{j\})} \le g \qquad \forall i, j \in N, \tag{22}$$

$$x(S) \le e(S) \qquad \forall S \subset N, \tag{23}$$

$$x(N) = e(N). \tag{24}$$

Constraints (22) define the differences of the allocation relative to the stand-alone emission between any location i and j. Again, constraints (23) and (24) are included to ensure that no EPM allocation exists when the core is empty. If the core is empty, we use the Nucleolus and refer to this allocation as the Equal Profit Method+ (EPM+) allocation. Just as with the Lorenz+ allocation, the optimal EPM+ allocation does not have to be unique if the core is non-empty.

3.3.6 Performance criteria

All described methods in Section 3.3.1-3.3.5 are assessed on criteria that are important from a customer's perspective as they are the ones that eventually get the CO_2 emission allocated. In total, we use three criteria adopted from Naber et al. (2015). The first one is stability. We define an allocation to be stable when it is in the core of the game. In other words, a stable allocation does not create an incentive for any (subset of) customer(s) to withdraw their order and leave the distribution route. Additionally, an emission allocation method needs to be consistent. If any factor of influence changes for a particular customer, their allocated emission should vary accordingly. It would not make sense that, for example, one's allocated emission would decrease when its order size is enlarged. An ordinary least squares (OLS) regression is used to analyze the consistencies (see Section 5.2.2 for details). Finally, computation times are compared as the frequent determination of allocations by the LSP requires relatively fast methods.

4 Data

We have generated different test instances with varying characteristics to perform our approach and evaluate the proposed methods. Each random instance consists of a depot, n customers with corresponding demand q_i, where $i \in \{1, 2, ..., n\}$ represents the customer, and a fleet of k homogeneous vehicles with maximum holding capacity W. In total, we consider six different instances where the total number of customers is set equal to $n \in \{18, 19, 20, 21, 22, 23\}$. For each test instance, all locations are randomly generated in a square with sides of length 100 km. In other words, each customer is drawn a random x-coordinate and y-coordinate from a range [0,100]. This method is also followed for the depot in each instance, such that this location is random, too. Subsequently, the Euclidean distance between each pair of locations is calculated. Next, each location is assigned a random demand from a uniform distribution [0,200], such that the average demand of all locations is approximately 100. The depots all have a demand of 0.

While we keep the locations and demands fixed after the random generation, we assign four different configurations of vehicles k to each test instance. We set the number of vehicles equal to $k \in \{3, 4, 5, 6\}$, where each vehicle has a (proportionally) corresponding maximum holding capacity of $W \in \{833, 625, 500, 417\}$. In this way, the total capacity of all vehicles is always equal to 2500 loading units. We set this capacity after calculating the maximum sum of demands over all instances (2394 loading units) to ensure the feasibility of the problem. As we now have the Euclidean distances and the capacity of the vehicles, we can also calculate the maximum emission ratio between every pair of locations by Equation (A.1).

Hence, we have a total of 24 instances combining the sets of locations and vehicles. We will refer to each instance by calling the number of customers and vehicles used. For example, the notation N19-k4 refers to the instance where $n = 19$ and $k = 4$.

5 Computational results

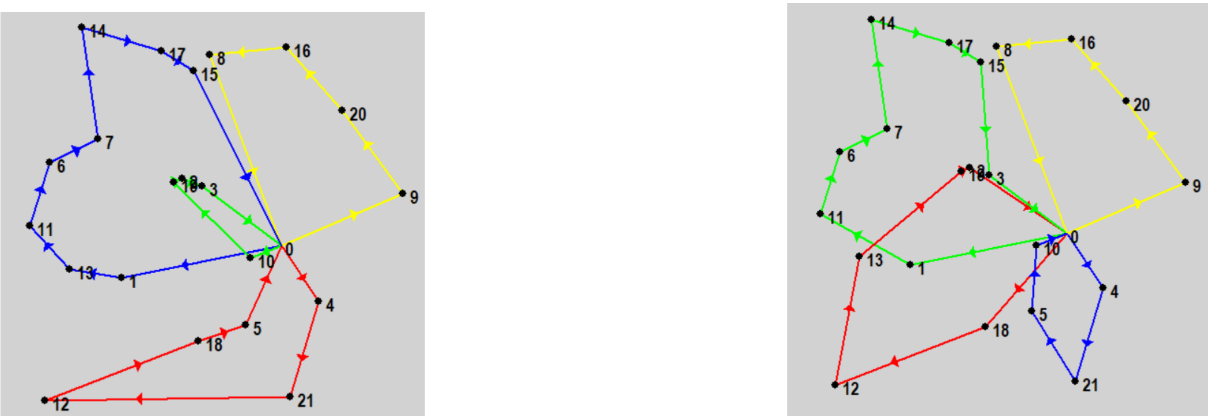
This section contains the computational results of applying the bi-GVRP model (Section 3.1) and the allocation methods (Section 3.3) to all randomly generated test instances. The results are obtained by using an Intel(R) Core(TM) i7 CPU @ 2.60GHz with 16 GB of RAM.

We constructed the route schedule for all possible instances by solving the bi-GVRP model. This model is implemented in AIMMS 4.77 by slightly modifying the Capacitated Vehicle Routing Problem (CVRP) library to fit our context. CPLEX 20.1 is used as a solver for our MIP model. The five allocation methods and the corresponding emission allocation game are implemented by using Python 3.8.8. Here CPLEX 20.1 is also used to solve the LP formulations of the Nucleolus, the Lorenz+ Allocation and the EPM+.

5.1 Impact of bi-objective optimizing

To evaluate the impact of taking environmental aspects into account in the determination of an optimal route schedule, we solved the bi-GVRP model for all different test instances where

λ was set to varying values. Each of the instances (24 in total) was successively solved five times for $\lambda \in \{1; 0.75; 0.5; 0.25; 0\}$. Logically, $\lambda = 1$ equals the situation where only distance is minimized, and $\lambda = 0$ represents the situation in which only the CO_2 emission is of interest in the optimization (see Equation (1)). We choose this particular set of values to show the route schedule changes gradually with varying importance of the economic and environmental objectives. In total, we have $5 \times 24 = 120$ instances that provide $6 \times 90 = 540$ routes. As we now know the optimal route schedule and loading units on each arc (i,j) precisely, we recalculated the total emission TE by Equation (3.2), where $e_{i,j}$ is determined by using the emission function in Appendix A.



(a) $\lambda = 1$, optimal distance

(b) $\lambda = 0$, optimal CO_2 emission

Figure 1: Optimal route schedule for different objective weights, instance N21-k4

Before looking at the effects on distance and total CO_2 emission of varying values of λ, we give a visual example of the change in routes when the objective differs. For this illustration, we use the N21-k4 instance. Figure 1a shows the determined routes for $\lambda = 1$, where on the contrary, Figure 1b shows the route schedule for $\lambda = 0$. Observe that the weights of the objectives establish a different route schedule, where only one route is identical for the same instance. The effects of the changing route schedule are reflected by the increase in total distance, from 603 km for $\lambda = 1$ to 621 km for $\lambda = 0$, and by the reduced amount of emission; the total of 27437 grams CO_2 for $\lambda = 1$ decreases to 27349 grams CO_2 for $\lambda = 0$.

Table 1 shows the total distance travelled in kilometers and corresponding emission in grams CO_2 for all 120 instances. It is not easy to compare absolute amounts for different values of λ as distance and emission are not linearly related. Therefore, Table 2 presents the relative changes of total distance and CO_2 emission compared to the situation where $\lambda = 1$. We set this value as a baseline because this represents the 'standard' used minimization of distance only.

Table 1: Total distance and CO_2 emission

		$\lambda = 1$		$\lambda = 0.75$		$\lambda = 0.5$		$\lambda = 0.25$		$\lambda = 0$	
Instance	k	TD (km)	TE (g CO_2)	TD (km)	TE (g CO_2)	TD (km)	TE (g CO_2)	TD (km)	TE (g CO_2)	TD (km)	TE (g CO_2)
N18	3	470	20277	470	20277	474	20133	480	20075	480	20075
	4	527	24835	536	24072	536	24072	538	23904	538	23904
	5	604	28502	609	28228	609	28228	611	28110	611	28110
	6	638	31028	643	30712	643	30712	645	30353	645	30353
N19	3	443	20360	459	20391	459	20391	459	20391	459	20391
	4	504	24142	513	23259	513	23259	513	23259	513	23259
	5	558	27172	562	27147	562	27147	562	27147	578	27367
	6	615	30465	615	30465	615	30465	615	30465	615	30465
N20	3	576	22527	576	22527	576	22509	576	22509	576	22509
	4	681	29462	696	29264	696	29264	691	28536	691	28536
	5	687	29583	687	29266	688	29120	695	29030	695	29030
	6	872	39836	799	36260	799	36260	799	36260	795	35591
N21	3	569	23748	569	23748	567	22908	567	22908	544	21648
	4	603	27437	609	28423	621	27349	621	27349	621	27349
	5	687	32318	687	32318	673	30872	677	30851	677	30851
	6	730	34428	733	34197	733	34197	733	34197	733	34197
N22	3	493	21322	499	21043	499	21043	499	21043	499	21043
	4	543	24828	543	24828	537	24264	537	24264	524	23923
	5	568	27249	584	27367	584	27367	576	27088	576	27088
	6	614	30727	627	30787	627	30787	635	30480	635	30480
N23	3	482	22401	485	21561	489	21253	489	21253	489	21253
	4	565	27395	567	26295	567	26295	567	26295	567	26295
	5	610	29014	623	29047	623	29047	623	29047	623	29047
	6	688	35754	715	38223	701	30913	701	30913	701	30913

Table 2: Changes of total distance and CO_2 emission relative to baseline $\lambda = 1$

		$\lambda = 0.75$		$\lambda = 0.5$		$\lambda = 0.25$		$\lambda = 0$	
Instance	k	TD (%)	TE (%)	TD (%)	TE (%)	TD (%)	TE (%)	TD (%)	TE (%)
N18	3	0.0	0.0	0.9	-0.7	2.1	-1.0	2.1	-1.0
	4	1.7	-3.1	1.7	-3.1	2.1	-3.7	2.1	-3.7
	5	0.8	-1.0	0.8	-1.0	1.2	-1.4	1.2	-1.4
	6	0.8	-1.0	0.8	-1.0	1.1	-2.2	1.1	-2.2
N19	3	3.6	0.2	3.6	0.2	3.6	0.2	3.6	0.2
	4	1.8	-3.7	1.8	-3.7	1.8	-3.7	1.8	-3.7
	5	0.7	-0.1	0.7	-0.1	0.7	-0.1	3.6	0.7
	6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N20	3	0.0	0.0	0.0	-0.1	0.0	-0.1	0.0	-0.1
	4	2.2	-0.7	2.2	-0.7	1.5	-3.1	1.5	-3.1
	5	0.0	-1.1	0.1	-1.6	1.2	-1.9	1.2	-1.9
	6	-8.4	-9.0	-8.4	-9.0	-8.4	-9.0	-8.8	-10.7
N21	3	0.0	0.0	-0.4	-3.5	-0.4	-3.5	-4.4	-8.8
	4	1.0	3.6	3.0	-0.3	3.0	-0.3	3.0	-0.3
	5	0.0	0.0	-2.0	-4.5	-1.5	-4.5	-1.5	-4.5
	6	0.4	-0.7	0.4	-0.7	0.4	-0.7	0.4	-0.7
N22	3	1.2	-1.3	1.2	-1.3	1.2	-1.3	1.2	-1.3
	4	0.0	0.0	-1.1	-2.3	-1.1	-2.3	-3.5	-3.6
	5	2.8	0.4	2.8	0.4	1.4	-0.6	1.4	-0.6
	6	2.1	0.2	2.1	0.2	3.4	-0.8	3.4	-0.8
N23	3	0.6	-3.7	1.5	-5.1	1.5	-5.1	1.5	-5.1
	4	0.4	-4.0	0.4	-4.0	0.4	-4.0	0.4	-4.0
	5	2.1	0.1	2.1	0.1	2.1	0.1	2.1	0.1
	6	3.9	6.9	1.9	-13.5	1.9	-13.5	1.9	-13.5
Average		0.7	-0.7	0.7	-2.3	0.8	-2.6	0.6	-2.9

Intuitively, when the value of λ decreases, i.e. environmental aspects are more dominant, it is expected that total distance increases and emission reduces in the optimal solutions. Looking at the relative changes of distance reveals that, indeed, distance grows when total emission becomes more leading in the objective. See, for example, instances N19-k4 where total distance increases with 1.8% for all λ or instance N22-k6 where we have changes of 2.1% and 3.4% for $\lambda = 0.75$ and $\lambda = 0.25$, respectively. Even though there are some instances for which distance does decrease when $\lambda = 0$, the changes of -8.8% and -1.5% for instances N20-k6 and N21-k5 are such examples, we observe that emission drops with an even higher percentage in these cases.

The percentages of emission changes show some more fluctuations. For $\lambda = 0.75$ we see, for example, that emission rises by 3.6% and 6.9% for N21-k4 and N23-k6, respectively. Nevertheless, for both cases it can be seen that total emission eventually decreases for lower values of λ. In the end, there are three instances for which EM increases when comparing $\lambda = 1$ to $\lambda = 0$. These changes are minor as they are quantified to be only 0.2%, 0.7% and 0.1%.

The percentage changes of total distance and CO_2 emission show that taking emission into account has its (positive) effects. On average, distance increases around 0.7% when $\lambda = 0.5$ and 0.6% when the distribution routes are optimized solely from an environmental perspective, where total emission decreases by 2.3% and 2.9%, respectively. As the costs for driving extra kilometers are unknown, it is case-specific for a LSP how beneficial this bi-objective optimization can be.

5.2 Allocation of CO_2 emission

After we have constructed the distribution routes, we applied all five allocation methods to each route to get insight into the allocated emission to each customer. We want to see if the different objectives in optimizing the driven routes influence the allocated emission to a customer or the behaviour of the allocation methods. To do so, we do not only compare the retrieved allocations per instance, we also evaluate the behaviour of the allocation methods for the different values of λ and compare this to the conclusions drawn by Naber et al. (2015).

The absolute and relative comparison of the allocated emission to the customers in each instance does not give much insight into what extent the bi-objective optimization has its influence. It is difficult to draw any conclusions on this matter as we deal with allocations over a single distribution route, i.e. allocated CO_2 emission is independent of other routes within the same instance. When λ changes and the routes change accordingly, it might be that customers are switched over the routes. Their allocated emission is now dependent on other customers' demand and location, which makes mutual comparisons invalid. Therefore, the following results and conclusions are based on comparisons between the different instances with the same value of λ. This way of evaluating the results also provides a clear view of the allocations based on an environmental perspective and the allocations based on an economic perspective.

5.2.1 Stability

The first criteria on which we evaluate the emission allocation methods is stability. The allocation is preferably stable as this provides no incentive for any (subset of) customer(s) to leave the coalition. We define an allocation to be stable when it finds itself in the core of the game (see Equation (3.4)). For any route in our random test instances, the core turned out to be non-empty. A reason for this may be the location of the depot. Even though its location was randomly generated, it appeared to be in the centre of all customers for all instances.

In Table 3, the total percentage of allocations in the core for each method and all values λ are given. The results are mainly of interest for the Star method and the Shapley value as the non-empty cores always provide a core allocation for the Nucleolus, the Lorenz+ Allocation and the EPM+ by definition. The Shapley value outperforms the Star method in all cases and provides core allocations for 87%-91% of the instances. Nevertheless, the Star method still has acceptable stability and results in a core solution in 74%-80% of the instances. The percentages are high given that both methods do not implicitly account for the core criteria when establishing the allocations.

Table 3: Core allocations per allocation method for all λ

Allocation method	$\lambda = 1$ (%)	$\lambda = 0.75$ (%)	$\lambda = 0.5$ (%)	$\lambda = 0.25$ (%)	$\lambda = 0$ (%)
Star method	78.70	74.07	75.93	77.78	79.63
Shapley value	87.04	88.88	90.74	90.74	89.81
Nucleolus	100	100	100	100	100
Lorenz+ Allocation	100	100	100	100	100
EPM+	100	100	100	100	100

The findings are in line with Naber et al. (2015), where the Shapley value also performs significantly better than the Star method. However, it is noteworthy, that their case study shows a greater difference in the stability of the two methods. Their results present poor stability of 34.9% for the Star method, where, on the contrary, the Shapley value constructs a core solution for almost all instances (98.7%).

It can also be concluded that the performance of the methods, in terms of stability, does not change much when the economic and environmental objectives are assigned different weights. The comparison of the situations in which either distance or emission is minimized ($\lambda = 1$ and $\lambda = 0$) shows that this leads to an increase in stability of 1% for the Star method and 3% for the Shapley value. Therefore, we conclude that the overall stability of the methods provides little incentive to determine the optimal route schedule based on the minimization of CO_2 emission.

5.2.2 Consistency

An allocation method should also be consistent; a change in any factor influencing a customer's allocated emission should lead to a corresponding proportional change in CO_2 emission allocated. We evaluate consistency by performing an OLS regression for all methods and each value of λ. We follow the approach of Naber et al. (2015) and set the customer's allocated emission as the dependent variable. Next to a constant, we add three explanatory variables that affect the allocation: the distance from the customer to the depot, its average distance to the other customers and its demand. In addition, we add two cross-product terms including the demand and the two other variables. In advance, we expect the first three variables to positively affect the allocated emission as, intuitively, the CO_2 emission of a customer should increase together with a rise of one of these variables. In Table 4 and Table 5, the regression results for $\lambda = 1$ and $\lambda = 0$ are presented, respectively. The results for the other three values of λ can be found in Table 8 - 10 in Appendix B. Each table contains the coefficients and corresponding one-sided p-values of each explanatory variable. Also, the R^2 of each model is given.

Table 4: OLS regression results for all allocation methods, $\lambda = 1$

	Star method		Shapley value		Nucleolus		Lorenz+ Allocation		EPM+	
Explanatory variable	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
Constant	662.75	0.00	342.19	0.04	538.90	0.00	1028.30	0.00	661.44	0.00
Dist. to depot	21.90	0.00	28.44	0.00	14.63	0.00	5.40	0.01	21.50	0.00
Avg. dist. to other cust.	-10.28	0.00	-4.51	0.31	5.23	0.27	0.19	0.85	-4.49	0.00
Demand	0.96	0.34	-0.47	0.76	4.48	0.01	1.07	0.27	0.94	0.36
Dist. to depot × Demand	0.00	0.23	0.00	0.93	-0.03	0.30	0.03	0.12	0.01	0.69
Avg. dist. to other cust. × Demand	-0.02	0.52	-0.01	0.78	-0.10	0.03	-0.05	0.09	-0.02	0.40
R^2	0.501	-	0.429	-	0.106	-	0.140	-	0.494	-

Table 5: OLS regression results for all allocation methods, $\lambda = 0$

	Star method		Shapley value		Nucleolus		Lorenz+ Allocation		EPM+	
Explanatory variable	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
Constant	686.53	0.00	857.06	0.00	688.47	0.00	1034.47	0.00	687.12	0.00
Dist. to depot	21.00	0.00	13.25	0.00	12.01	0.00	5.50	0.00	20.97	0.00
Avg. dist. to other cust.	-10.03	0.00	-4.28	0.49	2.71	0.66	-0.63	0.83	-10.00	0.00
Demand	0.46	0.66	0.94	0.63	4.34	0.03	0.65	0.49	0.44	0.67
Dist. to depot × Demand	-0.01	0.57	0.01	0.85	-0.03	0.34	0.01	0.47	-0.01	0.58
Avg. dist. to other cust. × Demand	0.01	0.76	-0.02	0.67	-0.09	0.11	-0.02	0.52	0.01	0.76
R^2	0.441	-	0.101	-	0.052	-	0.107	-	0.440	-

The outcomes of the OLS regressions show different results for all methods. The only thing they have in common is that the constant term and the variable "distance to depot" differ significantly from zero at a 5% significance level. Next to that, the coefficient of the variable "average distance to other customers" is significantly different from zero at the 5% significance level for the Star method and the EPM+. A thing that is noticeable and hard to explain is the negative effect of this variable on allocated emission. The random generation of the instances may be a reason explaining this unexpected effect. Furthermore, we notice insignificant effects for the variable "demand" for all methods except the Nucleolus. This may seem odd, but the random generation of demand and the fact that none of the methods literally take the customer's order size into account while allocating emission, can explain this insignificance. As demand is insignificant, both cross-terms are as well for all methods. Here, the Nucleolus is again an exception as the cross-term "average distance to other customers × demand" significantly affects the dependent variable for all values of λ (except for $\lambda = 0$).

From these results, we cannot conclude that the chosen variables can explain allocated emission well. This contradicts the findings of Naber et al. (2015), who conclude that the variables are able to do so. Nevertheless, if we compare the values of the R^2, which represent the proportion of the variance of the allocated emission that is explained by the independent variables, we come to a similar conclusion that the Star method performs best on consistency. This method has the highest value of R^2, ranging from 0.501 to 0.441 for all values of λ. The consistency of the EPM+ follows closely with a R^2 ranging from 0.494 to 0.440.

Based on our test instances, we conclude that optimizing for environmental aspects has negative effects on the consistency of the methods, as all values of R^2 drop when $\lambda = 1$ changes to $\lambda = 0$. Even though the decreases in R^2 are tiny, the minimization of the total distance to determine an optimal route schedule provides the best consistency.

5.2.3 Computation time

The third criteria that compares the allocation methods on performance is computation time. As the allocations need to be determined frequently, low computation times are preferred. We computed the average computation time of all instances for each allocation method. These times in seconds are shown in Table 6. There are no excessive computation times as we are dealing with relatively small test instances. From the definition, we can deduct that the Star method performs a polynomial number of elementary calculations. With an average time of 0.005 seconds this is reflected in the lowest computation time of all methods. The Shapley value follows with a slightly higher time of 0.02 seconds. This difference is coherent with the exponential number of elementary calculations in N that the Shapley value requires. The other three methods are approximately similar and share the highest computation times, which are still more than acceptable (0.33 and 0.34 seconds). Their higher times can be explained by the exponential number of constraints that need to be solved in the LP formulations. Even though the difference is slight, our average computation times are less than the ones from Naber et al. (2015). Their computation times are higher as the distribution routes in their case study visit more customers. We do have the same conclusion that the Star method is the fastest allocation method in terms of computation and that the computation times of the other methods are also acceptable.

Table 6: Average computation time in seconds per allocation method over all instances

Allocation method	Average time (sec)
Star method	0.005
Shapley value	0.02
Nucleolus	0.34
Lorenz+ Allocation	0.33
EPM+	0.33

We also want to evaluate if changing the weights in the bi-objective optimization affects total solving time. The average computation times of the allocation methods are insufficient to do so, as CO_2 emission is allocated based on a given distribution route. Therefore, we look at the time in seconds needed to solve the bi-GVRP that determines the optimal route schedule. We solved each instance for all possible values of k vehicles and computed the average solving time for each value of λ (see Table 7). None of the instances shows significantly different solving times when the different values of λ are compared. This leads to the conclusion that changing the weights for the objectives in the determination of distribution routes does not have negative influences on solving time. However, we should note the exponential growth of the solving time when the number of customers in the test instance is increased. Our test instances could all be solved within a reasonable time, but exact solving will probably not be usable when routes for larger instances (N > 30) should be determined.

Table 7: Average solving time in seconds for the bi-GVRP

	Average time (sec)					
λ	N18	N19	N20	N21	N22	N23
1	19	60	180	430	921	1973
0.75	21	62	209	418	913	1899
0.5	18	58	178	422	897	1978
0.25	20	64	193	403	946	1954
0	18	55	182	412	923	2012

6 Conclusion and discussion

In this thesis, we evaluate the effects of taking CO_2 emission into account while determining the distribution routes of a LSP. We solve a bi-GVRP in which the objective assigns different weights λ to the total distance travelled and expected emission. Subsequently, we build on the framework of Naber et al. (2015) and present the emission allocation game where we allocate CO_2 emission to customers on a single distribution route. Five different methods are used to perform the allocation: one proportional method, the Star method, and four methods, being the Shapley value, the Nucleolus, the Lorenz+ Allocation and the EPM+, that rely on solution concepts from cooperative game theory. We compare relative changes in total distance and emission over the different weights to see the impact of bi-objective optimization for economic and environmental aspects. Furthermore, we compare the allocated amounts of CO_2 emission and evaluate the behaviour of the five allocation methods based on the criteria stability, consistency and computation time.

Our computational results show that the bi-objective optimization has its positive effects as the relative decrease in CO_2 emission is higher than the relative increase in total distance. This conclusion can be drawn for all different weights λ that we evaluated. When it comes to the criteria all five methods are evaluated on, we conclude that the Star method performs worst on stability, followed by the Shapley value. The other three methods are stable by definition as they use the core criteria while determining the allocations. In addition, the Star method is, as expected, the fastest in allocating emission. Nevertheless, the other methods have acceptable computation times as well. All conclusions are similar to the ones from Naber et al. (2015), also with respect to consistency where the Star method and the EPM+ are the best performing methods. Furthermore, the behaviour of the methods does not significantly change when the weights λ differ. The only change can be found in the consistency of the methods. We observe that, even though the differences are slight, consistency diminishes if CO_2 emission gets assigned more weight in the objective of the bi-GVRP.

In conclusion, the implementation of environmental aspects in the optimization of routes does not make a significant difference when we look at the behaviour of allocation methods. However, the (positive) changes in total distance and CO_2 emission, together with the public attention to environmental effects in the current world, show that bi-objective optimization for economic and environmental factors should be considered. To what extent it should be incorporated is case-specific for a LSP as it depends on its targets and the costs for driving extra kilometers.

We would like to remark that in the objective function of the bi-GVRP we assign weights to total distance and CO_2 emission on a proportional base. However, the two terms are not of the same order and, therefore, we are not optimizing on scale. We were still able to draw conclusions on this matter, but future research should take the implementation and magnitudes of the weights into consideration. In addition, we optimize the environmental part of the objective by taking the maximum emission ratio on each arc between any pair of locations. This is because the CO_2 emission cannot be determined exactly before planning the routes as it depends on the carrying load of the vehicle. We recommend to further investigate a better approximation of total CO_2 emission when travelling between two locations before determining the optimal route schedule. Moreover, future research could evaluate the proposed models on a case study instead of using randomly generated test instances. This could improve the reliability of the results and lead to more generic conclusions, especially when larger instances are considered. Note that it would be worthwhile to investigate heuristics to solve the bi-GVRP when the number of customers increases as exact solving will not provide a solution within a reasonable time. Lastly, as discussed in Section 5.2, it might be the case that the allocated emission of a customer is dependent on the demand or location of a customer on another distribution route. Therefore, an interesting extension of this research could be the examination of allocation methods that allocate CO_2 emission over multiple routes in a vehicle routing setting instead of allocating emission on one distribution route only.

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A Emission function

We compute the emission on a distribution route by following the same approach as Naber et al. (2015). From the paper of Ligterink et al. (2012), we combine Equation (2) – (3) and the CO_2 parameters in Table 1 to formulate our emission function (see Equation (A.1)).

We assume the following: the total mass of each vehicle is 5 ton, a single loading unit has a weight of 0.01 ton and the maximum capacity is dependent on the instance (see Section 4). Hence, we estimate the specific power to be $KWt = 131.25/(5 + 0.01d)$, where d is the total number of units loaded on the vehicle. The total CO_2 emission in grams of a vehicle travelling one kilometer with d loading units is denoted by EM and calculated as follows:

$$\begin{aligned} EM = & \frac{465.390 + 48.143KWt}{V} + 32.389 \\ & + 0.8931KWt - (0.4771 + 0.02559KWt)V \\ & + (0.0008889 + 0.0004055KWt)V^2 \end{aligned} \tag{A.1}$$

In Equation (A.1), V denotes the velocity of the vehicle in kilometers per hour. As we do not have estimates of the actual velocity between two locations, we assume the velocity to be 35 km/h when two locations are within 15 kilometers of each other and 70 km/h if they are further apart. This limit corresponds to the distinction of Naber et al. (2015), where locations are defined as close when they are in the same square region with side lengths of 10 km.

B OLS regression results

Table 8: OLS regression results for all allocation methods, $\lambda = 0.75$

	Star method		Shapley value		Nucleolus		Lorenz+ Allocation		EPM+	
Explanatory variable	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
Constant	645.48	0.00	778.76	0.00	440.01	0.04	999.48	0.00	642.42	0.00
Dist. to depot	21.42	0.00	17.47	0.00	13.71	0.00	5.68	0.00	20.99	0.00
Avg. dist. to other cust.	-9.09	0.00	-5.17	0.36	8.50	0.14	0.80	0.78	-8.20	0.01
Demand	1.27	0.21	0.04	0.98	6.46	0.00	1.49	0.11	1.26	0.22
Dist. to depot × Demand	0.00	0.79	0.00	0.95	-0.04	0.24	0.01	0.46	0.00	0.93
Avg. dist. to other cust. × Demand	-0.02	0.44	-0.01	0.82	-0.14	0.01	-0.04	0.08	-0.03	0.33
R^2	0.479	-	0.159	-	0.075	-	0.115	-	0.472	-

Table 9: OLS regression results for all allocation methods, $\lambda = 0.5$

	Star method		Shapley value		Nucleolus		Lorenz+ Allocation		EPM+	
Explanatory variable	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
Constant	684.57	0.00	835.42	0.00	563.45	0.01	1029.28	0.00	681.29	0.00
Dist. to depot	21.16	0.00	16.94	0.00	12.40	0.00	6.04	0.00	20.84	0.00
Avg. dist. to other cust.	-10.12	0.00	-6.35	0.27	5.58	0.34	-0.97	0.74	-9.40	0.00
Demand	0.82	0.41	-0.26	0.89	5.05	0.01	1.02	0.28	0.82	0.41
Dist. to depot × Demand	0.00	0.77	0.00	0.96	-0.03	0.45	0.01	0.52	0.00	0.87
Avg. dist. to other cust. × Demand	-0.01	0.74	0.00	0.99	-0.11	0.03	-0.03	0.27	-0.01	0.60
R^2	0.472	-	0.149	-	0.070	-	0.115	-	0.467	-

Table 10: OLS regression results for all allocation methods, $\lambda = 0.25$

	Star method		Shapley value		Nucleolus		Lorenz+ Allocation		EPM+	
Explanatory variable	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value	Coeff.	p-value
Constant	697.38	0.00	857.51	0.00	540.35	0.01	1040.20	0.00	693.29	0.00
Dist. to depot	21.07	0.00	13.29	0.00	12.99	0.00	5.52	0.00	20.74	0.00
Avg. dist. to other cust.	-10.17	0.00	-3.69	0.53	6.96	0.24	-0.32	0.91	-9.44	0.00
Demand	0.62	0.55	-0.02	0.99	5.23	0.01	0.89	0.34	0.63	0.54
Dist. to depot × Demand	-0.01	0.60	0.02	0.56	-0.04	0.26	0.01	0.47	-0.01	0.70
Avg. dist. to other cust. × Demand	0.00	0.96	-0.02	0.77	-0.12	0.03	-0.03	0.26	0.00	0.88
R^2	0.452	-	0.121	-	0.063	-	0.107	-	0.448	-