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Passenger Satisfaction in Railway Timetabling

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Abstract

In practice rail transport services are susceptible to stochastic disturbances, whereas the timetable of these transport services is deterministic. Hence timetables should be constructed in a way that it can incorporate these disturbances. That is, timetables should contain extra times on top of the 'normal' amount of time it would take for a train to go from point A to point B to be able to absorb shocks. These are buffer times and supplements on run time and dwell time. This thesis will first discuss and replicate the stochastic optimization model of Kroon, Dekker, et al. (2005) for the optimal allocation of run time supplements of a single train on a single line. Then the model is used to compare different delay distributions, after which the model is extended to incorporate passenger satisfaction. Computational results indicate that other delay distributions give a better relative improvement. They also show that passenger satisfaction can be improved by allocating the supplements in a different way.

Keywords: *Timetabling, Railway, OR, Passenger Satisfaction, Delay distributions, Thesis*

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Contents

1 Introduction

Public transportation plays a big role in daily life, even more so since society is pressing for a more sustainable life style. The European Union is pushing for 90% reduction in greenhouse gas emissions by 2050 (European Union 2021). A way to cut down on these emissions is by having more people take public transportation. Punctuality and reliability of public transport services play a big role in the satisfaction of passengers. Which is why it is important that timetables are as robust as possible. Satisfaction in turn plays a role in how likely people are to use the service again. Although the Netherlands has one of the best performing railway services in the world, the Dutch still have a lot of complaints (nieuws 2019).

Different aspects play a role in the punctuality of trains, such as rolling stock, crew and infrastructure. However the main focus of this thesis will be the allocation of time supplements in such a way that if delays occur these can be compensated but if they do not occur, not too much extra time is scheduled on the route.

Railway timetables are deterministic plans, yet in real-time stochastic disturbances can occur. The goal then becomes to create a timetable in such a way that it is equipped to handle these disturbances. In this thesis an extension on the research proposed by Kroon, Dekker, et al. (2005) is discussed. This research entails the timetabling of trains in such a way that it is better equipped to absorb stochastic shocks. To do so they created a model to best allocate supplements for running times. In the first part of the research, a model is discussed to minimize the average delay by finding an optimal allocation of the running time supplements. This is the model discussed in Kroon, Dekker, et al. (2005). Based on this model the computational results obtained in the aforementioned research will be replicated. The reason behind this replication is to check the credibility of this paper, a good research paper needs to be replicable and give approximately the same results. There are two reasons why the credibility of this research is important in this paper. First, that this thesis serves as prove that the research is credible and thus applicable in practice. Second, this thesis discusses an extension on the model that is replicated. In the second part said model will be applied to different disturbance distributions and extended to incorporate passenger satisfaction.

There are two extensions that this paper will discuss. The first extension is to take a look at the current model based on different disturbance distributions than the ones used in the original research. The first extension will analyse how the different delay distributions affect the allocation of the time supplements. The second extension is extending the current model discussed by Kroon, Dekker, et al. (2005) with the maximisation of passenger satisfaction. The second extension should result in a minor modification to the linear model that used for the replication. This modification entails adding the objective of maximizing passenger satisfaction. The goal of this extension is to see if and how the time supplements are redistributed among the trips. This extension could then also be used to evaluate the way time supplements are distributed based on the different delay distributions.

The research that will be done in this thesis could be of interest to the national railway company in the Netherlands for passenger trains, particularly the part about passenger satisfaction. Moreover, they could benefit from knowing the results proposed by Kroon, Dekker, et al. (2005) are credible. Multiple companies that make use of freight trains could be interested in this aspect as well. Furthermore this topic could also be relevant for further research.

This thesis will focus on the research question: "*How does passenger satisfaction affect the distribution of run time supplements?*". With the sub-questions: "*Is the research done by Kroon, Dekker, et al. (2005) credible?*" and "*Do different distributions for delays effect the way running time supplements distributed?*"

The remainder of this thesis is organised as follows. First we provide a literature review in Section 2. After which Section 3 will give a description of the models. Next in Section 4 we will give the computational results of the replication part. Then Section 5 will provide the results of the extension. Lastly, Section 6 concludes the paper and discusses possible future research.

2 Relevant Literature

The topic of railway scheduling has been widely discussed over the years. Many authors have discussed the improvement and evaluation of railway service (see Bergmark (1970), Dotoli et al. (2013), Huisman and Boucherie (2001), Kroon, Dekker, et al. (2005), Kroon and Peeters (2003), Kroon, Peeters, et al. (2014), Lee et al. (2016), Liebchen (2008), Petersen and Taylor (1982), Vromans (2005), and Weeda et al. (2008)). However, most of them consider other aspects of the timetable, like rolling stock, where in this paper the focus lies on the allocation of running time supplements. If we look at more recent papers regarding railway timetabling, a popular topic is the sustainability of rail transport services (see Chevrier et al. (2013), Liu et al. (2020), and Tian et al. (2017). Another topic related to this paper is the distribution of delays on railway infrastructures (see Büker and Seybold (2012), Goverde et al. (2001), and Yang et al. (2019)). The last topic which is of interest for this thesis is passenger satisfaction (see Dotoli et al. (2013), Robenek, Maknoon, et al. (2016), and Robenek, Sharif Azadeh, et al. (2015))

An interesting paper to look at with regard to robust timetabling is Polinder et al. (2019). They consider the timetable of a base period, for example one hour, that follows a repeating pattern throughout the day. The disturbances they consider are periodic as well, these disturbances can occur hourly, daily, weekly or even less frequent like monthly, quarterly or even yearly. They look at three aspects of the timetable they analyse: recovery budget, robustness and efficiency. The recovery budget stands for the maximum time the timetable can be adjusted for.

Vansteenwegen and Van Oudheusden (2006) also discussed creating a more robust timetable under uncertain disturbances. However, they focused on calculating optimal buffer times as opposed to running time supplements. They calculate ideal buffer times for each transfer. After which these buffer times are used in a linear program in which they minimize a generalised waiting cost function.

Another approach incorporating disturbances timetable is discussed by Zhu and Goverde (2020). They proposed a rolling horizon two-stage stochastic method to handle uncertain disruptions. Contrary to creating a cyclic timetable that can incorporate delays, they focus on rescheduling when a delay occurs. The model offers better rescheduling solutions for uncertain disruptions which leads to fewer train cancellations and/or delays.

Büker and Seybold (2012) analyse the delay propagation of large scale railway networks. By doing this they try to assign distributions to the delays that occur. In the paper they established that the delays follow an extended exponential distribution. This distribution has a very similar form to the exponential distribution that is used by Kroon, Dekker, et al. (2005). Which indicates that using an exponential distribution for the delays will give a relatively accurate depiction of the real delays.

Yang et al. (2019) also look at the probability functions that most accurately describe railway disturbances. Aside from the exponential distribution that the aforementioned paper found most fitting, the lognormal and gamma distributions were found to be a good fit as well. All three distributions were evaluated with a Kolmogorov–Smirnov test. Which showed that the they had good practical applicability. This indicates that not only the exponential distribution, but also the lognormal and gamma distributions are worth looking into.

Another paper that discusses the distribution of delays on railway structures is that of Goverde et al. (2001). Aside from the exponential distribution, which seems to be a common find in the literature when discussing this topic, The normal distribution is also deemed to be a good fit. It was found that both distributions gave reliable forecasts of train delays.

Dotoli et al. (2013) address the railway timetabling problem for a regional railway by using a cyclic approach. Based on a mixed integer linear programming model for offline timetable optimization they enhance the model using a discrete event formulation. This paper deals with the generic and complex railway network as opposed to the scheduling problem for a corridor. Contrary to the problem discussed in Kroon, Dekker, et al. (2005) where the robustness of the timetable is prioritized, Dotoli et al. (2013) consider the minimization of the total travel time for passengers. They do this by using a single objective function,that is typically used to measure the service level.

Robenek, Maknoon, et al. (2016) introduce the Passenger Centric Train Timetabling Problem. They account for the passenger satisfaction in the design of the timetable. The paper considers both types of timetable(s): cyclic and non-cyclic. For this they give a Mixed Integer Linear Programming (MILP) problem with an objective of maximizing the train operating company's profit while maintaining a certain level of passenger satisfaction. Although they do not work with time supplements, this paper gives an interesting perspective on passenger satisfaction which we would like to incorporate in our extension. The satisfaction they focus on is on all aspects of a journey whereas this thesis will only focus on the satisfaction that is related to delays that occur. The run time supplements will be allocated to the trips such that the satisfaction of the passengers will be maximized. We will take the same approach as Robenek, Maknoon, et al. (2016) with regard to the penalties of a delay or if the train is earlier than scheduled.

Robenek, Sharif Azadeh, et al. (2015) looks at the evaluation of passenger satisfaction. Where they also look at how to value delays form a passengers perspective. This is done in a similar way as the aforementioned paper. However, as opposed to this, in our extension we won't measure the satisfaction per traveler but an overall dissatisfaction that occurs on the given route. As for the value of time the measures mentioned by Kouwenhoven et al. (2014) are used which came to be based on research done in the Netherlands, as opposed to the ones used in Robenek, Maknoon, et al. (2016) which looks at a amount proposed by Swiss research (Axhausen et al. 2008). They choose this manner of evaluation because they test the applicability of the approach on a case study of a Swiss railway company.

3 Method

The methods used in this thesis will be discussed in three parts. First the model for the replication is discussed, which can also be found in Section 3 of Kroon, Dekker, et al. (2005). Second the different kinds of probability distributions that can be used for the disturbances will be discussed. Then the third and final part will discuss the method for the extension that looks at passenger satisfaction.

3.1 Single train single line

First the model that is used for the replication part of this thesis will be discussed. In Table 1 a short overview of the inputs of the model is shown.

Table 1: Inputs of the model

Name	**Description**
R	the number of realisations r
N	the number of trips t
$\delta_{t,r}$	the external disturbance on trip t in realisation r
s_t	the running time supplement on trip t
w_t	the weight of the delay at the end of trip t
$D_{t,r}$	the resulting delay of realisation r of the train by the end of trip t

The complete model as given by Kroon, Dekker, et al. (2005) is formulated as follows:

$$min\ D = \sum_{t=0}^{N}\sum_{r=1}^{R} w_t D_{t,r}/R \tag{1}$$

$$s.t.\ D_{t-1,r} + \delta_{t,r} - s_t \leq D_{t,r}\ for\ t = 1, ..., N;\ r = 1, ..., R \tag{2}$$

$$\sum_{t=1}^{N} s_t \leq S \tag{3}$$

$$D_{t,r} \geq 0\ for\ t = 0, ..., N;\ r = 1, ..., R \tag{4}$$

$$s_t \geq 0\ for\ t = 0, ..., N; \tag{5}$$

Equation (1) is the objective function which minimizes the average weighted delay. Where w_t indicates the weight of the delay at the end of trip t. However, in the computational results an assumption was made of equal weights. Which means this part of the equation can be neglected in practice. Constraints (2) and (4) depict the linearisation of the following equation that calculates the balance of disturbance

$$D_{t,r} = max\{0, D_{t-1,r} + \delta_{t,r} - s_t\}\ for\ t = 1, ..., N;\ r = 1, ..., R$$

The linearisation is done by relating delay at the end of the current trip t to the delay at the end of the preceding trip *t-1*. Then the fact that there can only be a fixed amount of supplement to be distributed amongst is conveyed in Constraint (3). Lastly, Constraints (4) and (5) specify that the variables need to be non-negative.

When applying the model the assumption is made that the results of the model converge to the optimal allocation of the running time supplement if the number of realisations goes to infinity. This is proven by Kroon, Dekker, et al. (2005) in Section 4 of their paper, hence the proof will not be shown in this paper. Also in the original paper disturbances are distributed according to an exponential distribution with a mean of 1. Therefore, the same will be done for the replication.

3.2 Delay Distributions

In the paper by Kroon, Dekker, et al. (2005) a few delay distributions have been looked at to compare punctuality gain for different disturbance distributions. In this extension other delay distributions will be looked into. Specifically some distributions that the literature deemed a good fit when compared to reality (see Goverde et al. (2001) and Yang et al. (2019)). Hence, in this paper we will examine how the model proposed by Kroon, Dekker, et al. (2005) would react if these distributions are used. The distributions that are looked at are the Log-normal, Gamma and Half-normal distribution.

The parameters of each distribution will chosen in such a way that the theoretical mean of each distribution equals 1. This makes it easier to draw a comparison between the different distributions. First we will look at the Log-normal distribution. Which has the following pdf:

$$\frac{1}{x\sigma\sqrt{2\pi}}e^{-(ln\ x-\mu)^2/2\sigma^2} \tag{6}$$

Where we take $\mu = -0.5$ and $\sigma = 1$. Second is the Gamma distribution, with shape parameter $\alpha = 2$ and scale parameter $\beta = 0.5$.

$$\frac{x^{(\alpha-1)}(\beta/1)^{\alpha}x^{(-x/\beta)}}{\Gamma(\alpha)} \tag{7}$$

Last there is the Normal distribution. Since we can only have positive values we will use the Half-normal distribution. Where $\mu = 0$ and $\sigma = \sqrt{\frac{\pi}{2}}$.

$$\sqrt{\frac{2}{\pi}}\frac{1}{\sigma}e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} \tag{8}$$

These distributions will be compared with each other and with the "original" distribution, the exponential distribution, that is used in the replication part of this thesis. They will be compared based on how the supplements are distributed as well as the minimal obtained delay.

3.3 Passenger Satisfaction

To evaluate the allocation of the time supplements in a general case, we look at the passenger satisfaction. The idea behind this extension is to see if the passenger satisfaction plays a role in the distribution of time supplements. To model the dissatisfaction given by delays and excess time supplement we use the penalties as discussed by Robenek, Maknoon, et al. (2016) and Robenek, Sharif Azadeh, et al. (2015). However, we will not look at the penalty of having to transfer trains or the travel time. Which leaves us with the following penalties:

- $\beta_E = -0.5$ is the willingness to arrive to the destination earlier than the preferred arrival time. Thus satisfaction would decrease with half the amount of time a train is early.
- $\beta_L = -1$ is the willingness to arrive to the destination later than the preferred arrival time. Here satisfaction will decrease with the amount of minutes the train arrives late.
- $\beta_{WT} = -2.5$ is the willingness to wait for a delayed train. This means passenger satisfaction decreases with 2.5 time the minutes they have to wait.

The dissatisfaction of the groups of passengers are measured for deviations from the timetable, not deviations from personal preferences of arrival and departure. Both the waiting time and late arrivals can be measured with $D_{t,r}$. Which will have the same meaning as in the original model. The early arrival time will be calculated in a similar fashion to $D_{t,r}$.

$$E_{t,r} abs(min(D_{t-1,r} + \delta_{t,r} - s_t, 0)) \tag{9}$$

The total cumulative waiting time, for all passengers on a trip t for realisation r, will be expressed as $WT_{t,r}$. The total delay and total early arrival time, for all passengers on a trip t for realisation r, are defined as $TD_{r,t}$ and $E_{t,r}$ respectively. Where Where $WT_{t,r} = TD_{t,r} = D_{t,r}$ The inconvenience of delays per realization can then be calculated as follows:

$$I_{t,r} = \beta_{WT} * WT_{t,r} + \beta_E * E_{t,r} + \beta_D * TD_{t,r} \tag{10}$$

This means that the total satisfaction of a trip will decrease by I. This can be translated to a certain amount of money if we take into account the value of time (VOT). Which is €9.25 an hour for train travel according to Kouwenhoven et al. (2014). Since time is measured in minutes these will need to be divided by 60. As such the cost can be defined as $C_{t,r} = -VOT * I_{t,r}$. This results in the following objective for the extended problem.

$$minimize \sum_{t \in N} \sum_{r \in R} n_{t,r} C_{t,r} / R \tag{11}$$

The function shown above minimizes the average dissatisfaction. Or in other words, maximizes the average satisfaction of passengers. Where $n_{t,r}$ denotes the number of passengers that make trip t in realization r.

The goal of this extension is to see if passenger satisfaction can impact the way time supplements are distributed, by minimizing the possible dissatisfaction. Meaning that if the satisfaction is minimized the time supplements might be distributed differently across the trips. Which could result in a longer average delay.

As it is unknown how passengers are distributed among a single line we use three different ways to generate said passengers. The passengers will be generated according to a Weibull distribution. Which means that the amount of passengers will differ per realisation. However, the same number passengers will be used per case. Thus although the realisations differ in number of passengers the three cases that will be discussed do not. This is to give a general idea of how the model could work in practice. As mentioned before all three options the same probability distribution will be used. The difference lies in where the passengers are centered.

In the first case most of the passengers are grouped at the last few trips. This means that in the first few trips few passengers will take the train where in the last trips a lot of passengers will want to take the train. The number of passengers is increasing over the trips. Then the second case will then have the passengers mostly located in the first few stations. This depicts the opposite of the first case. Then the last case will generate the passengers along the trajectory at random. This means that the passengers will be distributed relatively evenly amongst the trips in all realisations. That is on average each station will have approximately the same number of passengers.

4 Computational Results of the Replication

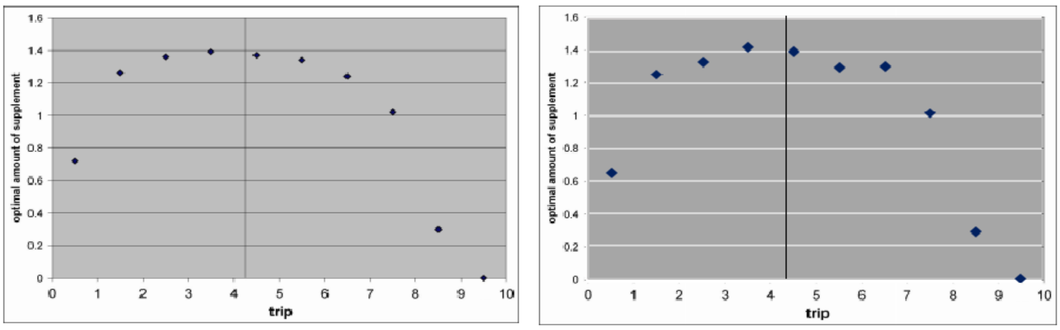
This section describes the computational results of the replication part of this paper. These results were obtained by applying the model described in Section 3 to several theoretical cases. Like in the research by Kroon, Dekker, et al. (2005), all results in this section are based on equally and exponentially distributed disturbances and on equally weighted delays.

The results discussed in this section were obtained by implementing the model in CPLEX Studio IDE 20 1.0 on an Intel CORE i7 10th generation with 1.30GHz processor speed, 475 MB internal memory and 8.0 GB ram. The disturbances were generated in MATLAB R2020b according to the needed distributions and exported to excel to be used in the model.

The number of realisations R for all instances in this section is 1000. This was deemed large enough for Kroon, Dekker, et al. (2005) to generate stable results and lead to an acceptable running of a few seconds. Hence, as to most accurately replicate the results the same sample size was used.

4.1 Optimal allocation of running time supplements

For this part of the replication the running times are subject to exponentially distributed disturbances with a mean of 1 minute. In this section the total amount of time supplement S is equal to the average total disturbances. The goal is to distribute the time supplements in such a way that the average delay is minimal. In the proportional allocation each trip gets a running time supplement of 1 minute. Figure 1 shows the optimal allocation of the running time supplements for 10 trips.



a) Original

b) Replication

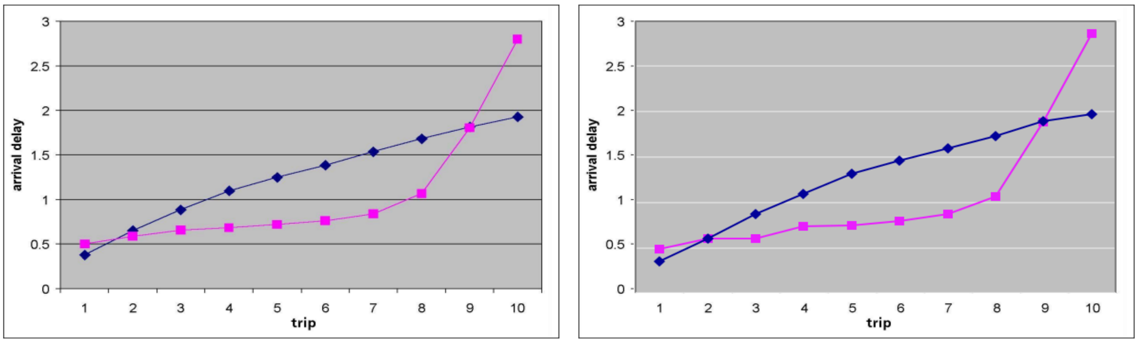
Figure 1: The optimal allocation of the running time supplements for 10 trips.

As can be seen from the figure above the allocation of the time supplements is relatively similar. There are a few slight deviations between trips 6 and 7, here one is slightly higher and the other is slightly lower. As opposed to the original in the replication these trips get an almost equal amount of supplement. This difference however seem negligible as the deviations are less than 10 seconds. The horizontal line in the figures represent the weighted average delay (WAD), which is defined by

$$WAD = \sum_{t=1}^{N} \frac{2t-1}{2N} * s_t$$

The WAD for the original is equal to approximately 0.425 while that of the replication is about 0.421. In both instances the WAD of the proportionally allocated running time supplements is exactly 0.5. This also indicates a slight deviation from the original experiment. However, due to how small of a difference this is, it again seems negligible.

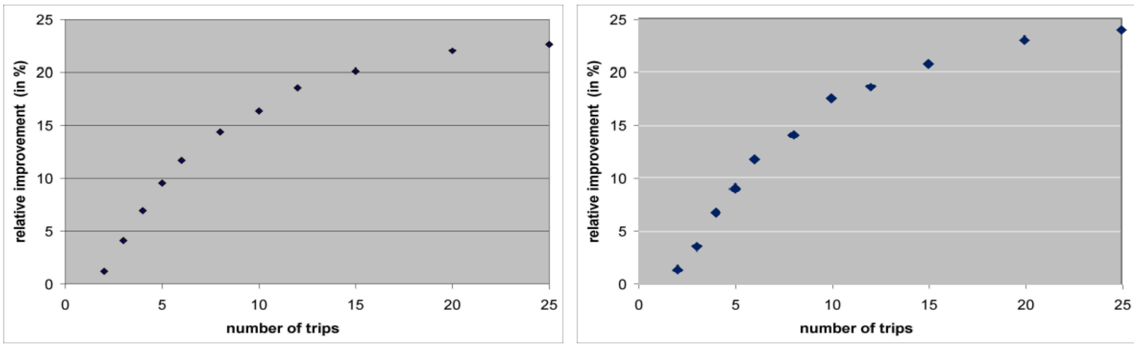
To further evaluate the replicability of the research we look at the other evaluations as well. In Figure 2, the average delays are shown. The convex line shows the average delay by the end of each trip for the optimal allocation of the running time supplements. The nearly diagonal line in this figure shows the average delay by the end of the trips for the proportional allocation of the running time supplements. As can be seen from both the original and the replication, the optimal allocation performs better on almost all trips. Except for the last trips, the average delay increases rapidly for the optimal allocation. This is a result of the fact that the supplements have been shifted from the last trips towards the first trips.



a) Original b) Replication

Figure 2: The average delay by the end of the trips for 10 trips

Figure 3 shows the relative improvement when the model is applied to 2 to 25 consecutive trips. The average delay of the optimal improvement and the average delay of proportional allocation of the running time supplements are compared with one-another. By doing this we find, like in the original research, that the relative decreases in the average delay are not at all equivalent for the different cases.

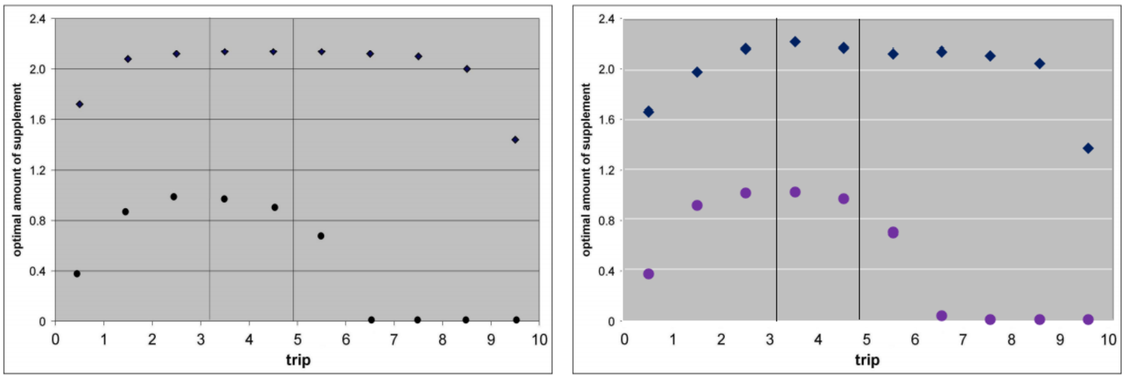


a) Original b) Replication

Figure 3: The decrease in average delay for the optimal supplement allocation in comparison with the average delay for the proportional supplement allocation

4.2 Different amounts of running time supplements

The results shown in the previous subsection show that part of the research is replicable and thus credible. To be able to determine if it is fully credible one last thing needs do be checked, which is the effect of a different total amount of running time supplement. In Figure 4 a case with 10 trips is considered with exponentially distributed disturbances with mean 1 like in the previous case. However this time the total amount of running time supplement that can be allocated is different. In this case either half a minute or two minutes of time supplement per trip is available. In other words S=5(dots) or S=20(diamonds).



a) Original b) Replication

Figure 4: The optimal allocation for half and double the total amount of supplement

As can be seen from Figure 4, the replication shows almost exactly the same distribution of the supplements in both cases. The same can be said for the WAD, which also seems identical. The similarity in these results also shows that the research is replicable and thus credible.

5 Computational Results of the Extension

In this section the computational results of the extension will be described. These results were obtained by implementing the models and methods discussed in Section 3 to various theoretical cases. For this section the number of realisations R will again be 1000 for all instances in this subsection. Also all results are based on equally distributed disturbances and equally weighted delays. The model was also again implemented in CPLEX Studio IDE 20 1.0 on an Intel CORE i7 10th generation with 1.30GHz processor speed, 475 MB internal memory and 8.0 GB ram. The disturbances were generated in MATLAB R2020b according to the needed distributions and exported to excel to be used in the model.

5.1 Different Delay distributions

In this section the total amount of time supplement S will be equal to the average total disturbances, which is 10. This applies to all distributions as each has a mean of 1 minute. Like in the replication the goal is to distribute time supplements in such a way that it minimizes the total average delay. The first thing to look at is how the supplements are distributed amongst the trips. The allocation of the optimal time supplement for each distribution is shown in Figure 5.

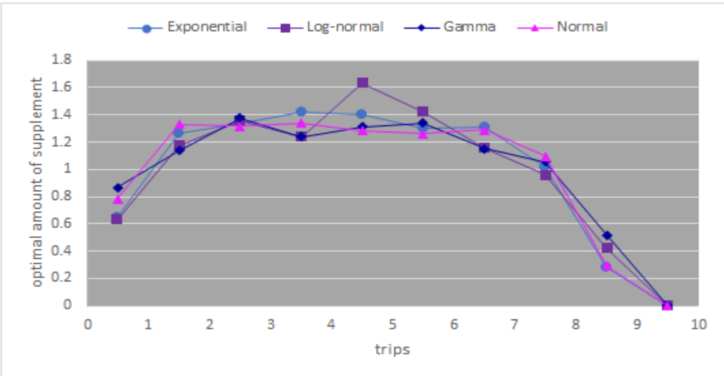


Figure 5: Comparison of the optimal allocation of the running time supplements for 10 trips for all distributions.

From this figure we can see that the distribution of the supplements is relatively similar. All distributions give no time supplement to the last trip and the trips taking place in the middle have relatively the most supplement allocated. The only outlier can be seen for trip 5, here the amount of time supplement is relatively much for the Log-normal distribution compared to the other distributions. For the Log-normal this trip gets 1.632 minutes time supplement, whereas the other distributions get between the 1.286 and 1.400 minutes. This outlier could be explained by that during the generation of the disturbances, trip 5 gets a relatively large disturbance in each realisation. This can be seen when we look at the average disturbances. For the Log-normal distribution trip 5 has the highest average disturbance namely 1.112, which is approximately 5% higher than the second highest disturbance. While for the other distributions this difference is less than 3.5%. Although this figure does give an idea of how the supplements are distributed, further comparisons need to be made to see which disturbance distributions results in the least delay and gives the best allocation of supplements when compared to proportional allocation.

In Figure 6 we compare the average delay of the optimal allocation with that of the proportional allocation. In each figure the convex line with squares represents the optimal allocation and the diagonal line with diamonds the proportional allocation. Each of the distributions shows that the optimal allocation preforms better than the proportional allocation. Except for the last trip of course. What can be seen from Figure 6 (c) and (d) is that the total amount of delay seems less. Hence, when using either the Gamma or Normal distribution one expects that the duration of delays is less. Another interesting thing is that the Log-normal distribution 5 (b) seems to have the highest disturbances on each trip. This indicates that the average total delay from this distribution will also be the highest out of all the distributions. Furthermore, the improvement compared to the proportional allocation also seems less than for the other distributions as the both lines are a lot closer to one another. For the other three distributions it is hard to see which does the best compared to the proportional allocation.

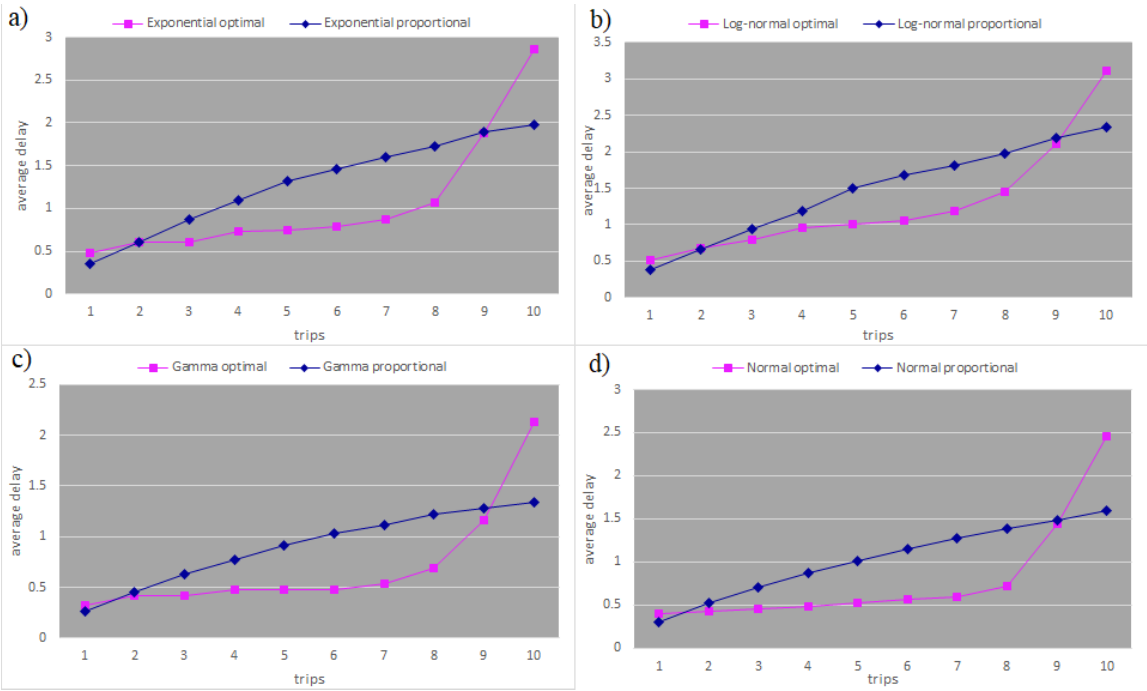


Figure 6: The average delay by the end of the trips for 10 trips for all distributions

Next in Figure 7 the decrease in average delays for applying the model to 2 to 25 consecutive trips is shown. Here a comparison is made between the average delays of optimal allocation and the proportional allocation. From this we can see that all distributions follow the same kind of increasing concave line. As we thought based on the figure discussed in the previous paragraph, the Log-normal distribution has the least improvement of the optimal allocation of time supplements compared to the proportional allocation. Then the distributions, for which the average delay for the optimal supplement allocation that seem to decrease the most compared to the proportional allocation are the Gamma distribution and the Normal distribution. If we compare these two, it seems that the Normal distribution has a slightly higher improvement.

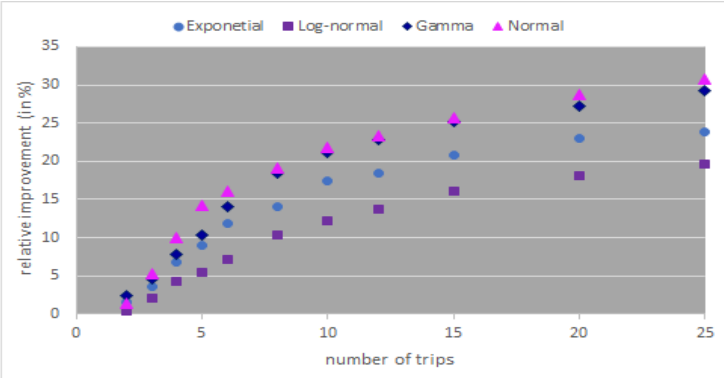


Figure 7: Comparison of the decrease in average delay for the optimal supplement allocation in comparison with the average delay for the proportional supplement allocation for all distributions

Lastly, we look at the optimal average delays shown in Table 2. It is hard to draw a definite conclusion on which distribution is better. If the only thing that matters is the minimization of the average delay then the Gamma distribution would be best overall. From what can be seen in previous figures as well, the delays that occur for each distribution vary in duration. In Table 2 we can see that, the Log-normal distribution gives longer disturbances on average. With the optimal values sometimes being 2 minutes longer than the second highest. While the Gamma distribution on average gives shorter disturbances. This could again be because of the average disturbances. As can be seen in the table below the average disturbance for the Gamma distribution is the lowest, whilst that of the Log-normal distribution is the highest. A noteworthy thing to look at is the Exponential distribution compared to the Normal distribution. This is noteworthy because the average disturbance is lower for the Exponential distribution but, the optimal average delay is lower for the Normal distribution. An explanation for this could be that the disturbances have a smaller variance in the Normal distribution.

Table 2: Optimal average delay with different amounts of time supplement

Distribution	**S=5**	**S=10**	**S=20**	**Average disturbance**
Exponential	24.426	10.652	2.347	1.006
Log-normal	25.707	12.887	4.477	1.018
Gamma	21.741	7.105	0.696	0.996
Normal	23.034	8.060	0.771	1.011

Since the goal was to have distributions with an average delay of 1 minute, the Gamma distribution comes the closest with an average disturbance that is only around 0.444% less. The average disturbance of the Log-normal distribution is around 1.806% larger, that of the Normal distribution about 1.132% and that of the Exponential distribution roughly 0.564%. Based on the deviation from the preferred average the Gamma distribution seems to be the best option. However, as the average disturbance is lower than the preferred average this might be undesirable in practice. As this might underestimate the disturbances that happen.

If we look at the other distributions that slightly overestimates the preferred average disturbance, it seems that the Normal distribution would be the best choice. The reason being that, although it has a higher deviation from the preferred average disturbance, the average delays are the shortest for all three instances of S compared to the other distributions that overestimate the preferred average disturbance.

5.2 Passenger Satisfaction

To measure passenger satisfaction we first need to generate the amount of passengers on each trip. A single trains unit has approximately 320 seats (NS 2020). Most trajectories have two units, thus on average there are about 640 seats available in a train with 10 trips. The passengers are then generated with a Weibull distribution with the scale parameter equal to 640 and shape parameter equal to 3. These are chosen in such a way that on average 90% op the seats in the train will be occupied. Choosing the distribution this way will give both scenarios of the train being over crowed and the train being almost completely empty.

The results obtained from the normal model given in Subsection 3.1 will from this point on be referenced to as the *Base Case*. The only difference with the results from Subsection 3.1 is that in the *Base Case* the level of passenger dissatisfaction will be measured when solving the model. The three instances of passenger allocation will be reference to as *Case 1*, *Case 2* and *Case 3*. These cases are composed in the way that was discussed in Subsection 3.3.

In *Case 1* most of the passengers are grouped at the last few trips. This means that in the first few trips few passengers will take the train, whereas in the last trips a lot of passengers will want to take the train. The number of passengers increases over the trips. For this computational instance it means that after the passengers are generated according to the Weibull distribution they are sorted in increasing order for each realisation. *Case 2* will have the passengers mostly located in the first few stations. This depicts the opposite of *Case 1*. Meaning, that after the passengers are generated they are sorted in decreasing order for each realisation. Then *Case 3* will generate the passengers along the trajectory at random. This means that the passengers will be distributed relatively evenly amongst the trips in all realisations. That is on average each station will have approximately the same number of passengers. Thus, after generating the passengers they will be left the same.

Figure 8 shows the results from maximizing passenger satisfaction for *Case 1* together with the results from the *Base Case*. From this figure we can see that passenger satisfaction indeed has influence over the amount of running time supplement in this case. There is a clear shift in the supplements from the first few trips to the later few. Which can be explained by the fact that trips 6 to 10 service the most passengers. The only thing that remains the same between the *Base Case* and *Case 1* is that the last trip does not get any time supplement. Intuitively this might strike one as odd. However, if we look back at the penalties that come with delays we see that trip 10 does not have the penalty of waiting time as it is the last trip on the route.

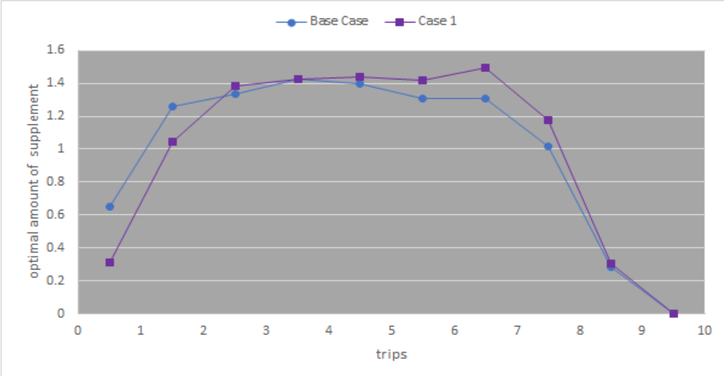


Figure 8: Comparison of the optimal allocation of running time supplements of *Case 1* and the *Base Case*

Although the supplement is allocated slightly different the general shape of the graph remains the same. Which is logical as the delay builds up over the trips. Hence, regardless of where the supplements are allocated the delay will build up nonetheless.

Next in Figure 9 the comparison of the results of *Case 2* and the *Base Case*. Here again a clear difference in allocation of the time supplements is visible. The last few trips get almost no to no run time supplement. Since most of the passengers are grouped in the beginning, more time supplement is allocated to the first few trips. This is to ensure little to no delay will occur in these few trips as to minimise the passenger dissatisfaction. From Figure 9 and 8 it seems that if passengers are distributed according to *Case 2* it has a bigger impact on the redistribution than if they are distributed according to *Case 1*.

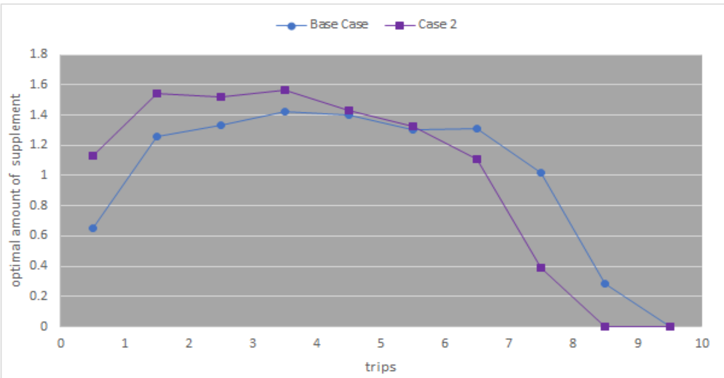


Figure 9: Comparison of the optimal allocation of running time supplements of *Case 2* and the *Base Case*

Then if we look at *Case 3* compared to the *Base Case* in Figure 10 only a slight distinction can be made. As the passengers are allocated to the stations in a random manner there is not a distinct part on the trajectory of the train where passenger satisfaction can maximized. One thing that does specifically stand out is that no time supplement is allocated to the ninth trip as well. This is probably due to the same reason as why in the *Base case* this trip gets very little time supplement. With the reason being that more improvement can be obtained from earlier trips as, like discussed for *Case 1*, delay builds up over time. In this case, minimising the delays seems to be almost the same as minimising passenger satisfaction.

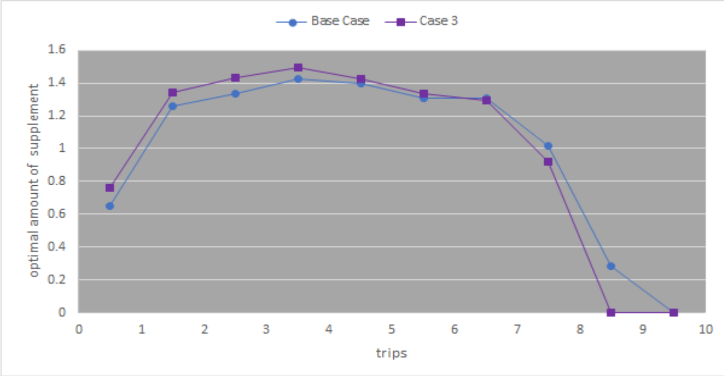


Figure 10: Comparison of the optimal allocation of running time supplements of *Case 3* and the *Base Case*

To evaluate the effect maximizing the average passenger satisfaction has we look at the results depicted in Figure 11. This figure gives the results of applying the extended model on 2 to 25 consecutive trips for all cases. Here we look at how much the passenger satisfaction relatively improves when this is maximized as opposed to the Normal model where the delay is minimized and how much the delay increases.

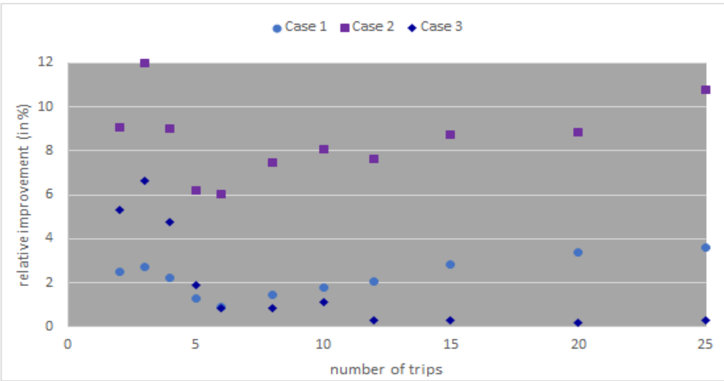


Figure 11: Comparison of the increase in average passenger satisfaction

What is interesting to see in this figure is that the average improvement of the optimal average satisfaction differs quite depending on which length the trajectory has. All points seem to reach a local maximum at 3 trips after which the relative improvement seem to decline until about 10-12 trips. After which *Case 1* and *Case 2* improve again. Another thing to note is that if the there are more than 10 trips *Case 3* has almost no improvement. Hence if a trajectory has more that 10 trips and the passengers are evenly distributed amongst the trips the maximisation of passenger satisfaction seems unnecessary. To see if the increase in passenger satisfaction is worth it one would need to look at the trade off made with the increase in delay the maximization of passenger satisfaction causes.

Table 3 shows the relative increase and decrease of the delay passengers experience and the total amount of delay when the focus is put on passenger satisfaction. From this we can see that both *Case 1* and *Case 2* show that the decrease in the delay passengers experience is bigger than the increase in total delay. As for *Case 3* both show an increase, this could be explained by the way passengers are generated for this case. In this case the passengers are generated according to a Weibull distribution and left as is, where for the other two cases the number of passengers get sorted in ascending or descending order. This causes the variation in number of passenger on a trip to be larger, as one trip could have the highest number of passengers in one realisation and the least in the next. This makes it harder to take the amount of passengers into account.

Table 3: Comparison of the increase and decrease of total delay and passenger delay (in %)

	Total Delay	**Passenger Delay**
Case 1	2.283	-3.482
Case 2	6.734	-8.155
Case 3	1.194	1.094

The results shown in Table 3 seem to further support that if passengers are distributed along the trips according to *Case 3* the maximisation of passenger satisfaction seems unnecessary. Especially since the delays passengers seem to experience do not decrease. For the other two cases, when there are 10 trips, it does seem worthwhile to consider. To see if it would be worthwhile for a different amount of trips we look at Figure 12. This figure depicts the relative change of the total and passenger delay when applying the model to 2 to 25 trips. Figure 12a depicts the results from *Case 1* and 12b those of *Case 2*.

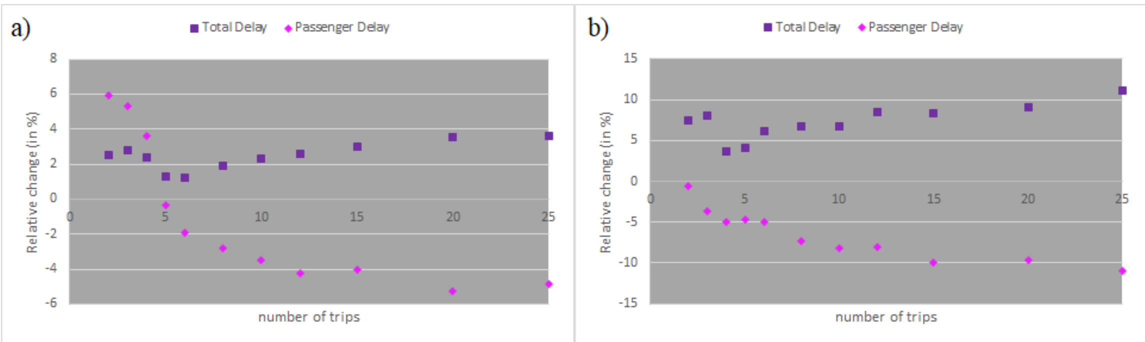


Figure 12: Comparison of the relative change of total delay and passenger delay

From the figure above we can see that for trajectories with less than 5 trips it is not worth considering *Case 1*. This could be because of the build up of the delays and the most passengers being consecrated on the last few trips. When there are few trips the number passengers that experience delay on the first couple of trips is not enough to weigh out the delay that passengers on the last couple of trips experience. If the number of trips is larger than 5 the passenger delay decreases more than the total delay increases for *Case 1*. In these instances it could be rewarding to maximise passenger satisfaction. *Case 2* does not have this problem as the passengers are grouped at the first few trips. If one where to have *Case 2* the maximisation of passenger satisfaction seems to be beneficial. Even though the total delay increases the delays for the passengers seem to decrease at least the same amount for the instances where the trajectory has more than 4 trips.

6 Conclusion and Further Research

In this thesis the following three points were looked at passenger satisfaction, delay distributions and the replicability of the research done by Kroon, Dekker, et al. (2005). Where the main focus was the following research question : "*How does passenger satisfaction affect the distribution of run time supplements?*". The main goal was to find out if passenger satisfaction could influence the way the supplements are distributed.

The first thing that was looked at in this thesis was the replicability of the research done by Kroon, Dekker, et al. (2005). The main reason for this replication was to see if said research was credible. When looking at the results discussed in Section 4 the research done by Kroon, Dekker, et al. (2005) prove to be replicable. This indicates that the research done in the paper and the outcomes are credible. Meaning it is more likely to represent a reliable claim to the fact that their way of allocating time supplements gives better results then the previous way of proportionally allocating supplements.

The second point this paper looked at was different delay distributions. Here the goal was to analyse which disturbance distribution, that according to the literature accurately depicts real-time disturbances, gives the best allocation of run time supplements. The Gamma distribution seems to give the lowest average delays and the second highest relative improvement. However, the average disturbance is a slight underestimation of the theoretical average disturbance that we work with. Which could be undesirable in practice. If a distribution that slightly overestimates the theoretical average disturbance is preferred the Normal distribution seems the best choice. This is because it has the second best average delays and has the best relative improvement. In this paper all trips are subject to the same kind of disturbances. However, in practice this need not be the case. Further research could focus on field research to find out the different kinds of disturbances that could occur. For example a rural area might have different types disturbances than a more urban area.

Lastly, for passenger satisfaction we found that if passenger satisfaction is maximised in the model that the supplements are distributed differently. We found that even though the total delay might increase the passenger delay decreases for *Case 1* and *Case 2* when there are more than 4 trips. There also was a positive improvement in the overall satisfaction of passengers. Thus, even though the maximisation of passenger satisfaction results in longer delays it also results in shorter passenger delays and an increase in satisfaction. Hence, a trade off is made between the overall punctuality and passenger satisfaction. The case that shows the most improvement of passenger satisfaction is *Case 2*. The way the time supplements were distributed for this case also differed the most from the base case. Hence, if a passengers are distributed amongst trips like *Case 2* the maximisation of passenger satisfaction is advantageous. *Case 1*, although it shows less relative improvement in satisfaction, shows a higher decrease in passenger delay than increase in total delay for more than 5 trips compared to *Case 2*. In *Case 2* this decrease in passenger delay is approximately the same as the increase in total delay. Hence, if in practice *Case 1* fits the distribution of passengers allocating the supplements according to the maximisation of passenger satisfaction could also be valuable.

However these results are limited as the number of passengers differs per line and time period. If used in practice this would need extra research on how passengers are distributed among the different stations of a specific line. Furthermore, one would need to differentiate between an outbound and an inbound train. The reason being that some stations are more crowded than others. If, for example, a train would go from a big city to a small town most passengers would board and alight at the first few stations. While if it was going in the opposite direction, most passengers would board and alight at the last few stations.

Another point that would need further research is incorporating the desired arrival and departure times in the satisfaction. In the current model dissatisfaction is measured when deviation from the schedule occurs. The time supplements might be allocated differently if deviation from the preferred arrival time is taken into account. This is because if then a delay occurs the 'new' arrival time might be closer to the preferred arrival time. Furthermore, if applied in practice one might also want to incorporate in vehicle time. As this would also affect the overall satisfaction of passengers.

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Appendix

List of Figures

List of Tables

Algorithm 1: Code for generating disturbances

```
%generate a sz1 by sz2 matrix with exponential distributed disturbances with mean mu
ex = exprnd(mu,sz1,sz2);

%generate a sz1 by sz2 matrix with log-normal distributed disturbances with mean mu and variance sigma
lognormal = lognrnd(mu1, sigma, sz1, sz2);

%generate a sz1 by sz2 matrix with gamma distributed disturbances with shape parameter a and scale parameter b
gam = gamrnd(a,b,sz1,sz2);

%generate a sz1 by sz2 matrix with normal distributed disturbances with mean mu andvariance sigma
normal = abs( normrnd(mu, sigma, sz1, sz2));
```

Algorithm 2: Code for generating passengers

```
%generate a sz1 by sz2 matrix with passengers generated from a Weibull distribution with shape parameter a and scale parameter b distribution in ascending order along the trips
r = sort(round(wblrnd(a,b,sz1,sz2)), 1, 'ascend');

%generate a sz1 by sz2 matrix with passengers generated from a Weibull distribution with shape parameter a and scale parameter b distribution in descending order along the trips
r1 = sort(round(wblrnd(a,b,sz1,sz2)), 1, 'descend');

%generate a sz1 by sz2 matrix with passengers generated from a Weibull distribution with shape parameter a and scale parameter b
r2 = round(wblrnd(a,b,sz1,sz2));
```

Algorithm 3: Replication

```
#parameters
int n = ...;
range N = 0..n;
int z = ...;
range R = 1..z;
int S = ...;

float delta[1..n][R]=...;
#float s[i in 1..n] = 1; use for proportional allocation

#variables
dvar float+ s[1..n];
dvar float+ D[N][1..z];

dexpr float Delay = sum(t in N) sum(r in R) D[t][r]/z;

#model
minimize Delay;
subject to{
   forall(r  in R){
    D[0][r] == 0.0;
   }
   forall(t in N : t !=0){
    forall(r in R){
      D[t-1][r] + delta[t][r] - s[t] <= D[t][r];
    D[t][r] >= 0.0;
    }
   }
   sum (t in N : t !=0) s[t] <= S;
}
```

Algorithm 4: Extension

```
#parameters
int n = ...;
range N = 0..n;
int z = ...;
range R = 1..z;
int S = ...;
float VOT = ...;

float delta[1..n][R]=...;
float passenger[1..n][R]=...;

#variables
dvar float+ s[1..n];
dvar float+ D[N][1..z];
dexpr float Cost[t in 1..n][r in R] = (VOT/60)*(2.5*D[t][r] + D[t][r]+0.5*abs(
    minl(D[t-1][r]+delta[t][r]-s[t],0)));
dexpr float Delay = sum(t in N) sum(r in R) D[t][r]/z;
dexpr float Satisfaction = sum(t in 1..n)sum(r in R) passenger[t][r]*Cost[t][r]/
    z - sum(r in R) (passenger[n][r]*(VOT/60)*(2.5*D[n][r]))/z;

#model
minimize Satisfaction;
subject to{
   forall(r  in R){
    D[0][r] == 0.0;
   }
   forall(t in N : t !=0){
    forall(r in R){
      D[t-1][r] + delta[t][r] - s[t] <= D[t][r];
    D[t][r] >= 0.0;
    }
   }
   sum (t in N : t !=0) s[t] <= S;
}
```