The quantile autoregressive model and idiosyncratic risk: a comparison with other volatility models in terms of modelling, forecasting and portfolio construction

Student name	***Student number***
Jackson Tabe	476441
Supervisor name	***Second assessor name***
(Daan) PA Opschoor	(Nick) NW Koning

Erasmus School of Economics

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Abstract

Research on idiosyncratic risk and quantile regression has been a growing point of focus in the last 20 years. This paper estimates and predicts the conditional variance of financial return series with a quantile regression framework. This is compared to different asymmetric models in terms of capturing the asymmetric response of volatility, the forecasting ability and performance of portfolios. The research is done for both total and idiosyncratic returns. The data we use consists of several stock indices throughout the world over the period 2000 until 2021. For both type of returns, we show that the asymmetry is indeed captured by the quantile autoregressive (QAR) model but the forecasting ability is not significantly better than the compared models. The portfolio performance is better in the total return case and the QAR portfolio shows the highest return.

Contents

1 Introduction

The stock market consists of numerous stocks and stock indices where investors throughout the whole world are invested in. Each of these stocks and indices have different characteristics and display a unique development over time. This is why one must widely diversify their portfolio in a way to minimise risk and maximise returns. To do this it is crucial for investors to understand how stock market volatility changes and is influenced. Engle (1982) introduces a model of (time-dependent) conditional variance which depends on the past returns. This model is the autoregressive conditional heteroskedasticity (ARCH) model and he shows that it improves the performance of a least squares model and obtains more realistic variance forecasts. The generalized ARCH (GARCH) model was later introduced by Bollerslev (1986) which allows for a much more flexible lag structure. However, these models do not allow for asymmetric responses of negative and positive lagged returns on the conditional variance. Consequently, the threshold- and exponential-GARCH models, which incorporate this asymmetry, were introduced by Glosten, Jagannathan, and Runkle (1993) and Nelson (1991), respectively. In this paper we use another model which captures the asymmetric response of the variance. This is the linear quantile autoregressive (QAR) model in the time-series context introduced by Koenker and Xiao (2006). We follow the procedure of Baur and Dimpfl (2019) to propose an estimator for the conditional variance of the return time series which depends on the conditional density estimated with the QAR model.

Until now, many papers have only forecasted and constructed portfolios based on the total variance or volatility. However, volatility of financial assets may mostly be characterised by the risk in the general market. For instance, in crisis periods volatility in the general market increases and most stocks and indices follow this trend. By separating the market or systematic volatility from the total volatility we can extract the part of the volatility which is unique to the stock or index. The latter is called idiosyncratic or unsystematic volatility and Malkiel and Xu (2002) state that it is a powerful measure in explaining the cross section of returns. Moreover, Bozhkov, Lee, Sivarajah, Despoudi, and Nandy (2018) find stronger correlation between idiosyncratic risk and returns during recessions. As our prediction period for the forecasting and portfolio construction contains the COVID-19 crisis, constructing portfolio weights based on forecasted idiosyncratic volatility, instead of total volatility, may result in better performing portfolios.

So in this research we compare the QAR model to other volatility models and examine how our results change if idiosyncratic returns are used instead of total returns. From this follows the central research question:

How does the quantile autoregressive (QAR) model compare to other volatility models in terms of modelling, forecasting and portfolio construction with total and idiosyncratic returns?

The data used in this research consists of return series of 20 international stock market indices over the sample period January 1, 2000 until May 4, 2021. The whole research is done for estimated idiosyncratic returns and compared with the results using the total returns. We employ asset pricing models to estimate the idiosyncratic returns.

We make use of a measure to capture the asymmetric response of volatility to positive and negative innovations. The measure is evaluated based on an empirical application with our data and we see that the asymmetry is captured by each model for both type of returns. Next, we construct forecasts of the conditional variance of the returns with the QAR model. This is then compared to the variance forecasts of other volatility models and evaluated to test which model predicts the variance most accurately. In all return cases, there is no significant evidence that the QAR model has better forecasting ability than the compared models. Finally, a weighted portfolio is constructed based on the QAR variance forecasts. The performance of the portfolio is evaluated and compared to the performance of weighted portfolios constructed with the other models. We find that the total return case outperforms the idiosyncratic cases and that the QAR portfolio results in the best return for our portfolio sample period.

The remainder of the paper is constructed as follows. Section 2 discusses some relevant literature, Section 3 describes the data used to conduct the research and Section 4 explains the estimation procedure and all the methods used to answer our research question. Section 5 then shows and discusses the obtained results which are compared to the results in the literature. Section 6 gives a discussion about the paper and possible future research. Finally, Section 7 summarizes the paper and gives concluding remarks.

2 Literature

The risk of a financial time series or portfolio is given by its volatility or variance. As stated earlier, the TGARCH and EGARCH models allow for a different impact of negative and positive lagged returns on the volatility. Black (1976) showed that this asymmetry is indeed the case for stock returns. This is not only true for stock returns as, for instance, Beaudry and Koop (1993) have argued that positive shocks to US GDP are more persistent than negative shocks. The QAR model allows for even more flexible asymmetry because the impact of lagged returns can be different on each specified quantile of returns. This means that the model is capable of altering the location, scale and shape of the conditional densities (Koenker & Xiao, 2006). Therefore, Ma and Pohlman (2008) state that quantile regression reveals more information than classical methods so we expect it to perform better.

Parmeggiani (2016) shows some important features of quantile regression in his paper. The first feature is that quantile regression is robust to outliers in the response variable. This is because the parameter values may be different for each quantile so the influence of an outlying observation is limited. Quantile regression estimates the conditional median of the target and there is still some influence of outliers but not as severe as a mean regression like ordinary least squares (OLS). A second feature is that the semi-parametric procedure to estimate the QAR model, which is explained later in Section 4.2, is more flexible than OLS. The semi-parametric technique imposes minimal distributional assumptions on the underlying data generating process (White, Kim, & Manganelli, 2015). This is the reason that quantile regression is a more flexible method compared to OLS.

Since the papers of Merton et al. (1987) and Levy (1978), idiosyncratic risk received a lot of attention in the financial literature. In this research we want to estimate and forecast idiosyncratic variance based on the idiosyncratic returns. Consequently, the question of how to quantify idiosyncratic returns arises. Malkiel and Xu (2002) state that most empirical studies estimate idiosyncratic volatility using the standard deviation of residuals from fitting a market model. We follow this methodology and simply use the residuals of an asset pricing model as our measure of idiosyncratic returns. Malkiel and Xu (2002) uses the three-factor model of Fama and French (1993) for estimating the idiosyncratic volatility. Another widely known asset pricing model is the Capital Asset Princing Model (CAPM) developed by Sharpe (1964), Lintner (1965), Treynor (1962) and Mossin (1966). Fama and French (1996) conclude that the three-factor model explains on average 93% of the variation in stock returns and the CAPM only explains 78% on average. Thereby, we opt for the three-factor model of Fama and French (1993) to estimate the idiosyncratic returns. However, the estimation of the idiosyncratic returns is very dependent on the asset pricing model chosen. In fact, Malkiel and Xu (1997) found that size and idiosyncratic risk are highly correlated which means that the size effect could be attributed to idiosyncratic risk. From this follows that the CAPM may also be a good model to extract the idiosyncratic part of the returns so we conduct the research for this model as well. The models and estimation of the idiosyncratic returns are explained more thoroughly in Section 3.

The literature on portfolio construction in a QAR framework is minimal. Moreover, this paper is, to the best of our knowledge, the first that considers portfolio construction based on the estimated conditional variance with quantile regression. Clarke, De Silva, and Thorley (2006) investigate minimum-variance portfolios in the US equity market. To do this in the QAR framework, the covariances must be estimated. This requires an estimation of all the joint densities between two time series which is out of the scope of this paper so we leave it for future research.

3 Data

The data we use for this paper is obtained from the Oxford-Man Institute's (OMI) realized library. We use a sample of 20 international stock market indices over the sample period January 1, 2000 until May 4, 2021. For each stock index, missing observations were already deleted in the library so we did not have to delete additional observations in the dataset. From this follows that the number of observations and corresponding dates are different for each stock index. We use open-to-close return time series as it is given in the dataset and were also considered by other papers like Bollerslev, Litvinova, and Tauchen (2006) and Hansen, Huang, and Shek (2012). [1] Later in this paper we use the realized variance (RV), which is also obtained from the OMI realized library, as the variance benchmark to evaluate our variance forecasts.

[1] The formula for calculating the open-to-close returns is given in Appendix A

The Fama/French three-factor model used to estimate the idiosyncratic returns is given by:

$$R_{i,t} = R_{ft} + \beta_{i,m}(R_{mt} - R_{ft}) + \beta_{i,SMB}R_{SMBt} + \beta_{i,HML}R_{HMLt} + \varepsilon_{i,t} \quad \text{for}\, i = 1, \ldots, N \,\text{and}\, t = 1, \ldots, T_i, \quad (1)$$

where $R_{i,t}$ is the return of stock or index i, R_{ft} the risk-free rate, R_{mt} the market return; N denotes the number of stock indices and T_i the number of observations of stock index i. R_{SMBt} and R_{HMLt} are the returns on portfolios formed to capture the global size and book-to-market equity effect respectively. The data for the latter two measures and the risk-free rate are collected from the Kenneth French data library and we use the Fama/French Developed Markets Factors as it captures almost all countries in our sample of international stock indices. For the market return we choose the S&P Global 1200 Index (SPG) collected from the database of the S&P Dow Jones Indices. We opt for this global index because it captures all countries considered in our sample of stock indices (except India).[2] The series is transformed such that we obtain the vector of returns. With these return series we can use OLS to estimate the β's in Equation (1). We can not use all observations in the series because we need to take the intersection of the dates to perform the linear regressions. From this we obtain series of residuals which are considered the idiosyncratic returns.

The second asset pricing model used to estimate the idiosyncratic returns is the CAPM model and it is given by:

$$R_{i,t} = R_{ft} + \beta_i(R_{mt} - R_{ft}) + \varepsilon_{it} \qquad \text{for}\, i = 1, \ldots, N \,\text{and}\, t = 1, \ldots, T_i, \quad (2)$$

where $R_{i,t}$, R_{ft} and R_{mt} and the data for these variables are exactly the same as in Equation (1). Again, the intersection of the dates of the series is taken to perform OLS and estimate the β in the model.

Descriptive statistics of the 20 stock indices and global stock market index are given in Table 1 below.[3] We see that the average return is negative for most indices. This is most likely due to the sample period chosen as it contains the dot-com bubble, the 2008 financial crisis and the recent COVID-19 crisis. The sample period for MIB starts from June 1, 2009 but still has a negative return on average. In contrast, the sample period of SPG starts from May 31, 2011 and shows a positive return on average. The returns of the indices are in general negatively skewed and leptokurtic because the kurtosis is greater than 3 for each index. This can also be seen based on the high standard deviations and minimum and maximum values far from the average return. The Ljung-Box statistics $Q_r(1)$ show that some indices exhibit no first-order autocorrelation in the returns but for some the hypothesis of independent returns is rejected. The statistics $Q_{\hat{\varepsilon}}(1)$ display high p-values for all indices which indicates that no autocorrelation is left in the residuals of the AR(1) models. However, when testing for autocorrelation in the squared residuals of the AR(1) models, $Q_{\hat{\varepsilon}^2}(5)$ show that all squared residuals reveal significant autocorrelation for a 1-week (5 trading days) horizon. From the latter follows that there is a strong indication for heteroskedasticity in the returns.

The descriptive statistics for the estimated idiosyncratic returns are given in Table 7 and 8 in Appendix

[2] We also could have used the market return in the Fama/French Developed Markets Factors but then the countries Brazil, South Korea, Mexico and India would not have been incorporated

[3] The full name of the stock indices are given in Appendix B

C. The number of observations are much lower compared to the total return case as the sample periods only start from May 31, 2011. The average daily returns in the idiosyncratic cases are all negative and much lower than the average returns in the total return case. This is mostly attributed to the second half in the idiosyncratic return series where the COVID-19 crisis is contained. The high volatility in this period causes the residuals in the asset pricing models to become large in absolute value. The rest of the statistics are similar to the total return case and the same conclusion is reached for all Ljung-Box tests.

Table 1: Descriptive statistics of the return series

Index	Obs	Avg.	Std.Dev.	Min	Max	ζ	κ	$Q_r(1)$	$Q_{\hat{\varepsilon}}(1)$	$Q_{\hat{\varepsilon}^2}(5)$
AEX	5437	-3.10	110.01	-8.42	9.24	-0.24	7.35	0.008	0.939	0.000
AORD	5387	0.22	84.68	-7.80	5.36	-0.75	6.55	0.000	0.905	0.000
BVSP	5247	-0.79	143.40	-9.80	11.92	0.02	4.67	0.000	0.926	0.000
DAX	5406	-2.83	124.84	-9.41	9.99	-0.13	5.28	0.516	0.991	0.000
DJIA	5345	1.78	110.30	-8.41	10.75	-0.07	8.44	0.000	0.791	0.000
ES50	5436	-2.07	129.60	-11.50	8.27	-0.34	6.13	0.112	0.992	0.000
FCHI	5439	-2.74	115.26	-8.12	7.28	-0.19	4.86	0.001	0.975	0.000
FTSE	5379	-0.43	116.14	-10.30	9.39	-0.36	7.23	0.004	0.901	0.000
FTSTI	3408	-3.60	81.44	-4.88	4.99	-0.02	3.39	0.021	0.980	0.000
HSI	5224	-4.16	103.64	-11.62	12.16	0.16	10.67	0.000	0.823	0.000
IBEX	5404	-5.24	120.62	-7.58	13.04	-0.06	5.96	0.530	0.998	0.000
KS11	5251	-4.59	113.97	-11.78	8.76	-0.44	7.26	0.000	0.711	0.000
MIB	3020	-3.84	117.60	-12.98	5.59	-0.76	7.15	0.343	0.987	0.000
MXX	5348	2.04	122.15	-8.26	9.95	-0.02	5.53	0.000	0.706	0.000
Nasdaq	5349	-1.39	131.77	-8.05	14.91	0.02	7.51	0.000	0.731	0.000
Nikkei	5184	-3.06	111.55	-10.56	11.66	-0.52	11.16	0.000	0.899	0.000
NSEI	5289	-5.02	117.99	-11.44	9.51	-0.56	8.26	0.129	0.948	0.000
Russel	5346	3.61	120.65	-12.38	8.06	-0.37	7.00	0.000	0.966	0.000
SP500	5348	0.76	112.25	-9.35	10.22	-0.22	8.36	0.000	0.733	0.000
SSMI	5343	-1.70	96.65	-10.13	8.68	-0.50	10.54	0.958	1.000	0.000
SPG	2610	3.45	93.50	-9.49	8.73	-0.91	15.72	0.680	0.964	0.000

Notes: Descriptive statistics of the 20 stock indices and the S&P Global 1200 index. The first column Obs is the number of observations, Avg. is the average daily return in basis points and Std.Dev. is the empirical standard deviation in basis points. Min and Max are the minimum and maximum of the return series in % respectively. ζ and κ are the empirical skewness and kurtosis respectively. The 3 last columns in the table contain p-values of Ljung-Box tests for autocorrelation. The null hypotheses are given by independence of the index returns (r), the residuals of an AR(1) model ($\hat{\varepsilon}$) and the squared residuals of an AR(1) model ($\hat{\varepsilon}^2$) up to the lag indicated in parentheses.

4 Methodology

All methods used and measures computed are done for the total return and both idiosyncratic return series. In Section 5 the results are compared and discussed.

4.1 The QAR model and the conditional moments

Quantile regression allows the dependent variable to react differently to a shock in the exogenous variable for each quantile in the distribution. It is considered an extension of linear regression and used when the conditions of linear regression are not met. Quantile regression estimates the conditional median of the target in contrast to linear regression, which estimates the conditional mean. The linear QAR(p) model from Koenker and Xiao (2006) is given by

$$Q_{y_t}(\tau|y_{t-1},\ldots,y_{t-p}) = \theta_0(\tau) + \theta_1(\tau)y_{t-1} + \cdots + \theta_p(\tau)y_{t-p}, \tag{3}$$

where $\theta_j(\cdot)$ for each $j = 1,\ldots,p$ is a function mapping $\tau \in [0,1] \to \mathbb{R}$. The lag length p in the linear model may differ for each quantile τ which is the case in the later discussed QAR($p(\tau)$) model. Koenker and Xiao (2006) rewrite the QAR(p) model in random coefficient notation to analyze its statistical properties. With the assumption $\mathbb{E}[\theta_0(U_t)] = 0$, the model becomes

$$y_t = \alpha_{1,t}y_{t-1} + \alpha_{2,t}y_{t-2} + \cdots + \alpha_{p,t}y_{t-p} + u_t, \tag{4}$$

where $u_t = \theta_0(U_t)$ and $\alpha_{j,t} = \theta_j(U_t)$ for $j = 1,\ldots,p$. The random coefficients $\alpha_{j,t}$ are functions of U_t which is an independent identically distributed (iid) standard uniform random variable. Consequently, u_t is iid by construction. In Equation (4) the lag length p can not differ because the quantile is a random variable, in contrast to Equation (3). We choose $p = 1$ as in the literature and Equation (4) becomes $y_t = \alpha_t y_{t-1} + u_t$. From this we derive the conditional mean and variance of the QAR(1) process. Koenker and Xiao (2006) show that $y_t = \alpha_t y_{t-1} + u_t$ is covariance stationary under two assumptions[4] and the conditional mean becomes

$$\mathbb{E}[y_t|y_{t-1}] = \mu_\alpha y_{t-1} \tag{5}$$

and the conditional variance results in

$$Var[y_t|y_{t-1}] = (\omega_\alpha^2 - \mu_\alpha^2)y_{t-1}^2 + 2\psi y_{t-1} + \sigma^2 \tag{6}$$

where $\psi = \mathbb{E}[\alpha_t u_t] = \mathbb{E}[\theta_0(U_t)\theta_1(U_t)]$. We stated earlier that we assume $\mathbb{E}[\theta_0(U_t)] = 0$ and hence, ψ equals the covariance between $\theta_0(U_t)$ and $\theta_1(U_t)$.

4.2 QAR variance estimator

We follow the two-step semi-parametric procedure, proposed in Baur and Dimpfl (2019), to find an estimator of the conditional variance of returns. We do this based on a QAR(1) model for simplicity and Equation (3) becomes

$$Q_{y_t}(\tau|y_{t-1}) = \theta_0(\tau) + \theta_1(\tau)y_{t-1}. \tag{7}$$

The actual estimation is done for the QAR(p) model as we want to estimate the conditional probability density function (pdf) as accurate as possible. It is more accurate with the QAR(p) than the QAR(1) model because the QAR(p) is a more flexible specification.

After estimating Equation (7) we can use the estimated model to derive an estimate of the conditional

[4]These assumptions are given in Appendix D

distribution function as

$$\hat{F}(y_t|y_{t-1}) = \sup\{\tau \in [0,1]|\hat{Q}_{y_t}(\tau|y_{t-1}) \leq y_t\} \tag{8}$$

where $\hat{Q}_{y_t}(\tau|y_{t-1})$ is the estimator of the τth conditional quantile as in Bassett and Koenker (1982) and this concludes the parametric first step.

In the second, non-parametric step we compute conditional moments of the time series y_t. To do this we need the pdf $\hat{f}(y_t|y_{t-1})$ of y_t which is calculated by taking the derivative of $\hat{F}(y_t|y_{t-1})$. From this and under the two covariance stationarity assumptions, the conditional expectation and variance of y_t are given as

$$\mathbb{E}[y_t|y_{t-1}] = \int_{-\infty}^{\infty} y_t \hat{f}(y_t|y_{t-1}) dy_t = \mu_t, \tag{9}$$

$$Var[y_t|y_{t-1}] = \int_{-\infty}^{\infty} (y_t - \mathbb{E}[y_t|y_{t-1}])^2 \hat{f}(y_t|y_{t-1}) dy_t = \sigma_t^2. \tag{10}$$

Next, I explain the empirical estimation method which follows the two-step procedure above. The variance estimate is more accurate for a more flexible model and hence, we consider the QAR($p(\tau)$) model. This is even more flexible than the QAR(p) model in Equation (3) because it allows the lag length to vary for each quantile τ. To select the lag length for each quantile we use the Schwarz Bayesian Information Criterion (SIC) in its L1 form (Hurvich & Tsai, 1990):

$$SIC(\tau) = n\, ln\, \hat{s}^2 + p\, ln\, n, \tag{11}$$

where $\hat{s}$ is the mean absolute deviation of the data from the fitted values, ln denotes the natural logarithm, n is the number of observations and p the number of AR terms or lags. As in Baur and Dimpfl (2019) we use 100 quantiles and estimate the QAR($p(\tau)$) model with the method of Koenker and Bassett (1978). This coincides with solving the problem:

$$\min_{\theta \in \mathbb{R}^{p+1}} \sum_{t=1}^{n} \rho_\tau (y_t - x_t^\top \theta), \tag{12}$$

where $\rho_\tau(u) = u(\tau - I(u < 0))$ and n is the number of observations. With the estimated model we predict $\hat{F}(y_t|y_{t-1})$ with the observations for y_{-p}, which is a p-vector of p lags of y_t. For the second step we first construct the empirical pdf from the QAR forecasts $\underline{\hat{y}}_t$ with the adaptive kernel method of Portnoy, Koenker, et al. (1989). Afterwards, the integrals in Equation (9) and (10) are numerically calculated. To calculate the expectation, we derive an approximation of $\hat{y}_t f(\hat{y}_t|y_{-p})$ and the integral is then computed using trapezoid rule integration of Kincaid and Cheney (2009). This results in an estimate $\hat{\mu}_t$ for the conditional expectation and consequently an estimate $\hat{\sigma}_t^2$ for the conditional variance is obtained.

4.3 Asymmetry in the volatility response

Baur and Dimpfl (2019) state that an asymmetric response of the return variance to lagged returns is characterised by the following two properties.

P.1 The marginal distribution of $\theta_0(U_t)$ and $\theta_1(U_t)$ are asymmetric around zero, and

P.2 $\theta_0(U_t)$ and $\theta_1(U_t)$ are negatively correlated.

Baur, Dimpfl, and Jung (2012) show empirically that $\theta_0(U_t)$ and $\theta_1(U_t)$ are monotonically increasing and decreasing respectively. This implies that the covariance between $\theta_0(U_t)$ and $\theta_1(U_t)$ is negative such that ψ is negative. All other terms in Equation (6) are positive by construction causing the conditional variance of y_t to increase for negative values of y_{t-1} and vice versa. In a more general QAR(p) model, the relation between $\theta_0(U_t)$ and $\theta_1(U_t)$ is still valid. The equation of the conditional variance then includes additional terms but the calculation of $\delta(\tau, 1-\tau)$, which is given below, based on $\hat{\theta}_1(\tau)$ is still possible.

Baur and Dimpfl (2019) propose a measure for the strength of the asymmetric response of the conditional variance. This measure is the difference between the estimates $\hat{\theta}_1(\tau)$ of extremely low and high quantiles. We use this same measure and it is given as

$$\delta(\tau, 1-\tau) = \hat{\theta}_1(\tau) - \hat{\theta}_1(1-\tau). \tag{13}$$

Baur and Dimpfl (2019) provide an explanation on what the choice of τ must be based on and that it must balance two effects. First, small values of τ are preferred as we want to consider the whole range of QAR estimates. However, the QAR parameter estimates become less reliable when small values of τ are used. Conversely, the asymmetry becomes less pronounced as τ goes to 0.5 so choosing τ too large would underestimate the asymmetry effect. We follow the proposition of Baur and Dimpfl (2019) and choose τ between 0.01 and 0.1 to calculate the asymmetry measure.

We use an empirical application to evaluate how well this measure captures the asymmetric response of volatility. This is done by calculating $\delta(\tau, 1-\tau)$ for $\tau \in \{0.01, 0.02, 0.05\}$ with our estimated QAR model and comparing it to our estimated asymmetry parameters $\hat{\gamma}$ and $\hat{\alpha}$ in the TGARCH and EGARCH model respectively.[5] The comparison is done by deriving the sample correlation coefficients[6] between the asymmetry measures and the asymmetry parameter estimates in the GARCH models. To further enhance the comparison, we also perform linear regressions of the estimated asymmetry measures δ on the estimated asymmetry parameters $\hat{\gamma}$ and $\hat{\alpha}$.

A final way to evaluate the asymmetric effect of past returns on future volatility is by considering the news impact curve (NIC). From Equation (6) the NIC becomes

$$\sigma^2_{y,t} = (\omega^2_\alpha - \mu^2_\alpha) y^2_{t-1} + 2\psi y_{t-1} + \sigma^2. \tag{14}$$

Following Baur and Dimpfl (2019), the NIC is computed by replacing expectations ω^2_α, μ_α and ψ with the

[5]The TGARCH and EGARCH models we use are given in Appendix E

[6]This represents the degree of linear association between two sample series (Taylor, 1990)

average of $\hat{\theta}_1^2(\tau)$, the average of $\hat{\theta}_1(\tau)$ and the covariance between $\hat{\theta}_0(\tau)$ and $\hat{\theta}_1(\tau)$, respectively. Lastly, σ^2 is replaced by the variance of $\hat{\theta}_0(\tau)$.

4.4 QAR-based forecasting

We estimate the QAR model based on a subsample of the data to construct pseudo out-of-sample variance forecasts. The model is newly estimated for each forecast while keeping the sample size in the estimation window constant. To evaluate the variance forecasts of the QAR model, we compare it with the corresponding variance forecasts of the TGARCH, EGARCH and the heterogeneous autoregressive (HAR) model of Corsi (2009).[7] For the latter three models, the model is also newly estimated for each forecast with a rolling window. The evaluation criterion used for the variance forecasts of each model is the root mean squared prediction error (RMSPE) criterion

$$RMSPE = \sqrt{\frac{1}{L}\sum_{i=1}^{L}(RV_{t+1,i} - \hat{\sigma}^2_{t+1|t,i})^2}, \tag{15}$$

where $RV_{t+1,i}$ and $\hat{\sigma}^2_{t+1|t,i}$ denote the realized variance and the 1-day ahead predicted variance, resulting from the ith out-of-sample forecast, respectively. L denotes the total number of observations in the prediction period which is the same for each stock index. The Diebold and Mariano (2002) test is used to compare the QAR variance forecasts to the variance forecasts of the other models. It is executed as a one-sided test where the alternative hypothesis is that the variance forecast of the QAR model is more accurate than the forecast of the compared model. The Diebold-Mariano (DM) test statistic is asymptotically standard normally distributed and the test is implemented with a squared error loss function as in the RMSPE criterion.

In the idiosyncratic case it is not possible to estimate the HAR model because realized idiosyncratic variances are not available. Nevertheless, we still need a proxy for the idiosyncratic variances in order to calculate the RMSPE values. A common proxy for the variance of returns in the literature is the squared returns. From this follows that we employ the squared residuals in the asset pricing models as a proxy for the idiosyncratic variance.

4.5 Portfolio construction

Lastly, we construct portfolios with portfolio weights based on the conditional variance forecasts from the previous section. We already explained in Section 2 why we do not investigate the minimum-variance portfolio as in Clarke et al. (2006). However, Blitz and Van Vliet (2007) show that the the volatility reduction with their portfolio construction strategy is greater than with the minimum-variance portfolio. This is evidence that additional information about the correlations between assets does not necessarily lead to better portfolio construction. Another example of this evidence is the risk parity portfolio as in Chaves, Hsu, Li, and Shakernia (2011). They conclude that the risk parity portfolio significantly outperforms allocation

[7] The HAR model and its explanation are given in Appendix F

strategies such as minimum-variance and mean-variance efficient portfolios.

From this follows that we choose the risk parity portfolio as our portfolio strategy. The weights are then calculated in the following way:

$$w_i = \frac{1/\sigma_i}{\sum_{j=1}^{N} 1/\sigma_j} \qquad \text{for } i = 1, \ldots, N, \tag{16}$$

where σ_i is the volatility of stock index i and N is equal to the total number of stock indices considered. From Equation (16) it follows that $w_i > 0$ for each i which means that each stock index must be invested in with a long position. The portfolio is rebalanced and new weights are computed for each trading day based on the corresponding 1-step ahead variance predictions.

We use this same procedure to compute portfolio weights with the TGARCH, EGARCH and HAR model. The performances of the portfolios for each model are then evaluated and compared to each other. The evaluation is done with the Sharpe ratio introduced in Sharpe (1966). A popular tool to compare Sharpe ratios is the test of Jobson and Korkie (1981) with the Memmel (2003) correction. This test, however, is not valid when the returns have tails heavier than the normal distribution or are of time series nature (Ledoit & Wolf, 2008). From Section 3, it can be seen that our series are of time series nature as the first-order autocorrelations in the returns are not negligible for some stock indices and we also concluded that there is a strong indication of heteroskedasticity in all the return series. Therefore, we opt for the (robust) inference method from Ledoit and Wolf (2008), using a studentized time series bootstrap to compare Sharpe ratios. The benchmark portfolio we use is the equally weighted or 1/N portfolio.

The portfolio construction and comparison is repeated with the forecasted idiosyncratic volatilities. The portfolio construction is not done for the HAR model because the idiosyncratic variance forecasting is not possible for this model. We examine whether the computed weights and portfolio performances are significantly different from the approach with total volatility.

5 Results

All the methods and formulas given in the previous section are implemented in the program R (2021) and the obtained results are shown and discussed in this section. All results in the idiosyncratic case with the CAPM are given in the Appendix as most literature prefer the Fama and French three-factor model to estimate the idiosyncratic component. The idiosyncratic case with the CAPM is used as a robustness check for the asset pricing model chosen to estimate the idiosyncratic returns. The main comparison is thus done for the total returns and the idiosyncratic returns from the three-factor model.

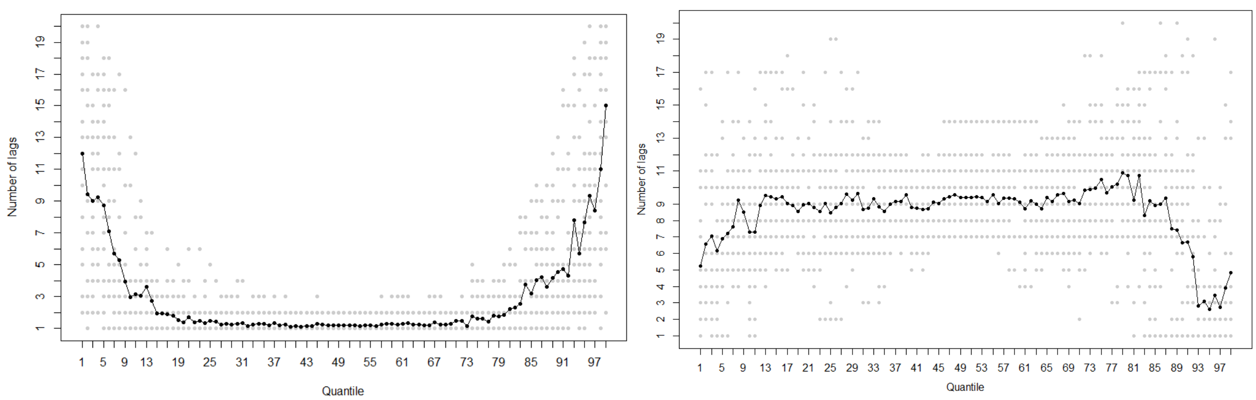
5.1 Estimation of the QAR model and asymmetry in the volatility response

Following Baur and Dimpfl (2019), we estimate the QAR($p(\tau)$) model and set the maximum lag length (p_{max}) equal to 20 (4 trading weeks). The dispersion of selected lags and average lag lengths for each quantile are given in Figure 1. For the total return case it can be seen that the extreme quantiles require a substantial number of lags. This is in contrast to the central quantiles for which 1 or 2 lags are sufficient. In other words, more historic return information is needed to estimate the tails than the central quantiles. This looks very different in the idiosyncratic case as the central quantiles require approximately 9 lags and the tails select less lags than this on average. The average lags selected in central quantiles are also more dispersed than in the total return case. The average selected lags figure in case with the CAPM is shown in Figure 7 in Appendix G. This is similar to Figure 1b and indicates that the asset pricing model chosen does not result in significantly different optimal lag selection in the QAR($p(\tau)$) model.

Figure 1: The average and dispersion of the selected lags

(a) Total return case

(b) Idiosyncratic return case

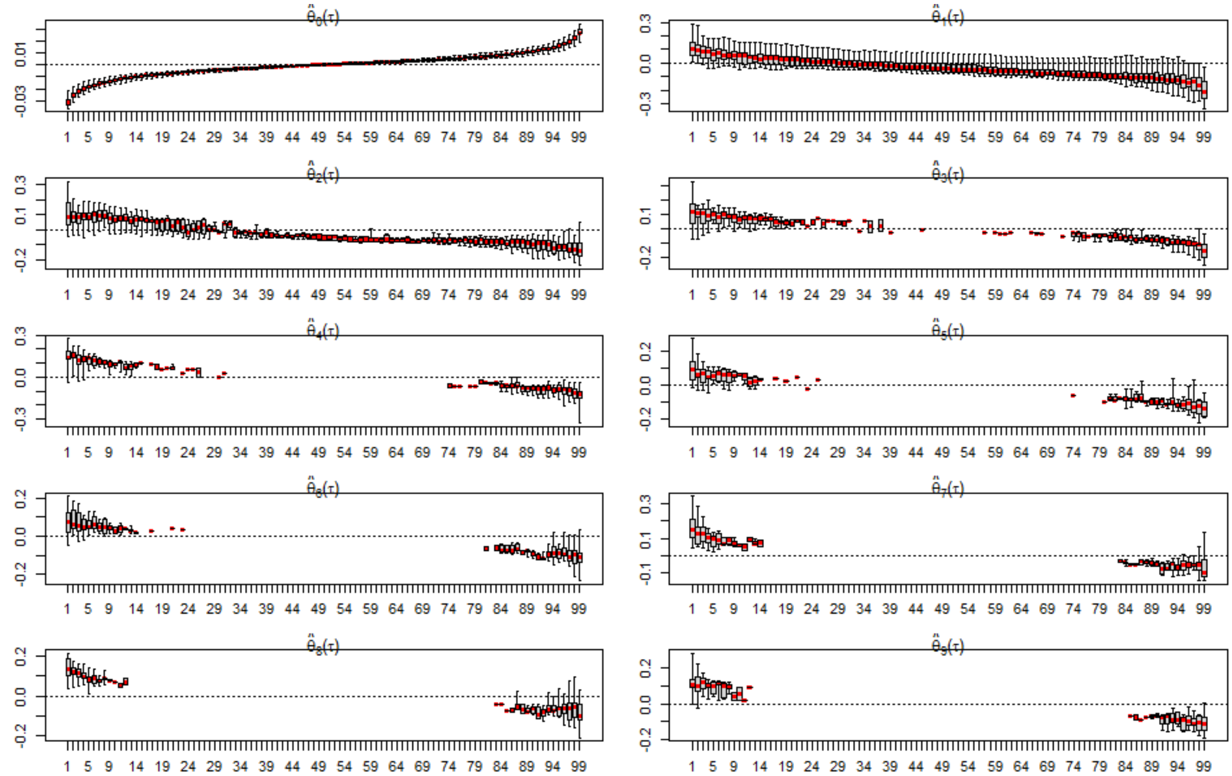


With these optimal lag lengths we estimate the QAR($p(\tau)$) model for each stock index. The QAR coefficient estimates in the total return case are shown in Figure 2, where each quantile in each coefficient is presented by a boxplot. As expected, we find very similar results as Baur and Dimpfl (2019) with a decreasing pattern of the QAR lag coefficients.[8] The intercept shows an increasing pattern which implies that low quantiles are associated with negative returns and high quantiles with positive returns. For the idiosyncratic case, the QAR coefficient estimates are given in Figure 3 and in this case the central quantiles for higher lag order coefficients are not empty.[9] This is a consequence of the higher selection of lags in the idiosyncratic case and it implies that idiosyncratic returns are more persistent in the central quantiles than the total returns. The QAR coefficients in the idiosyncratic CAPM case are given in Figure 8 and 9 in Appendix G. The differences with the main idiosyncratic case are minimal and we conclude that the estimation of the QAR($p(\tau)$) model is robust to the asset pricing model chosen to estimate the idiosyncratic returns.

Next, we discuss the asymmetric effect of volatility based on the results of the empirical application and the NICs. The estimated asymmetry measures δ and asymmetry parameters $\hat{\gamma}$ and $\hat{\alpha}$ are given in Table 2.

[8]The QAR coefficient estimates of lags 10-20 are shown in Figure 12 in Appendix H.
[9]The QAR coefficient estimates of lags 10-20 in this case are shown in Figure 13 in Appendix H.

Figure 2: Boxplots of QAR coefficient estimates



The values in parentheses are the corresponding asymmetry measures and parameters in the idiosyncratic case.

It can be seen that all asymmetry measures in the total return case are positive for each index. This is due to the decreasing pattern in the first QAR lag coefficient which is stated above. The asymmetric effect of volatility is not as strong in the idiosyncratic case and some asymmetry measures are even negative. On average, the asymmetry measures in the total (idiosyncratic) case are 0.310 (0.191) for $\tau = 0.01$, 0.268 (0.152) for $\tau = 0.02$ and 0.201 (0.117) for $\tau = 0.05$. In both cases, we see that $\hat{\gamma}$ and $\hat{\alpha}$ are positive and negative respectively for each index. This was expected because it indicates that negative innovations increase the variance more than positive innovations. In the idiosyncratic case, the asymmetry is not as pronounced again but there is no parameter with the opposite of the expected sign. On average in the total (idiosyncratic) case, the asymmetric response parameter in the TGARCH is equal to 0.117 (0.063) and in the EGARCH -0.094 (-0.041). The asymmetry values in the total return case are somewhat different from the values found in Baur and Dimpfl (2019). This was expected because we have a larger sample period and do not change the maximum lag length in the QAR($p(\tau)$) model to three for calculating the asymmetry measures. Keeping the original optimal lag lengths should give better QAR coefficient estimates and consequently, better estimation of the asymmetry measures. The estimated asymmetry measures and parameters in the idiosyncratic case with the CAPM are given in Table 9 in Appendix G. In this case there are even more negative asymmetry

Figure 3: Boxplots of QAR coefficient estimates (idiosyncratic case)

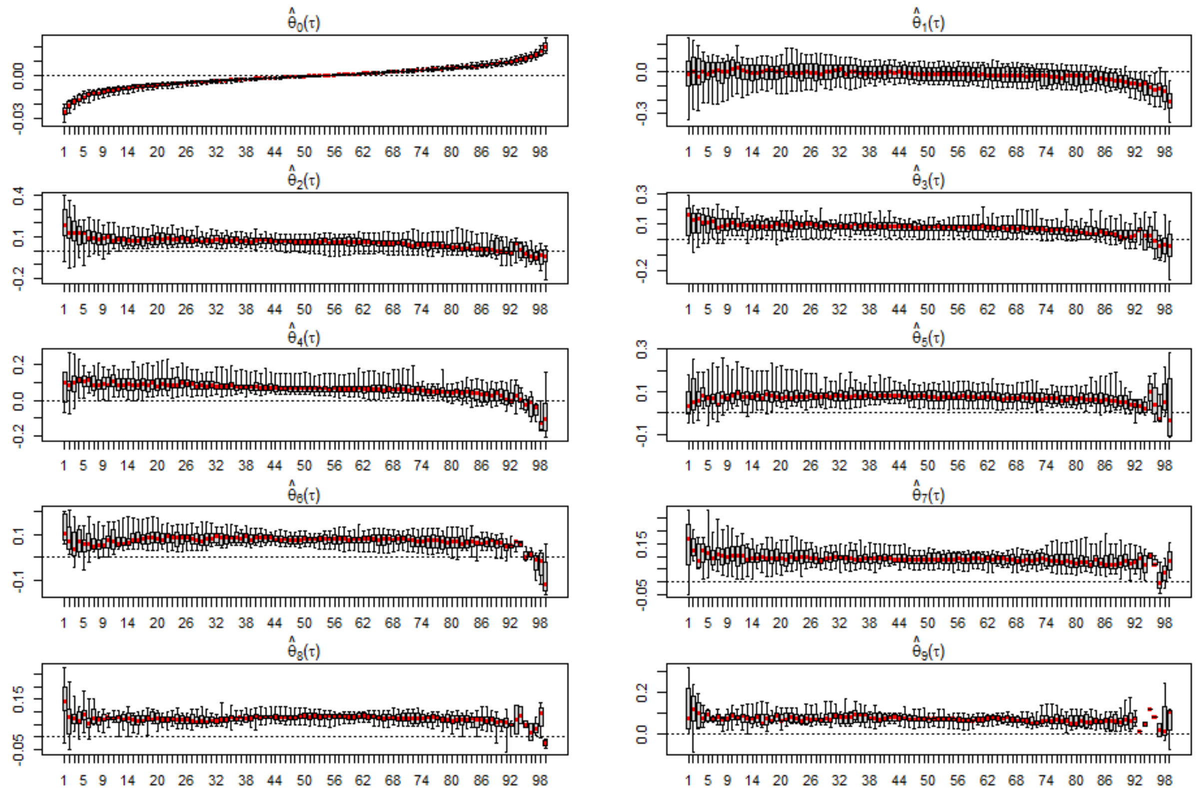


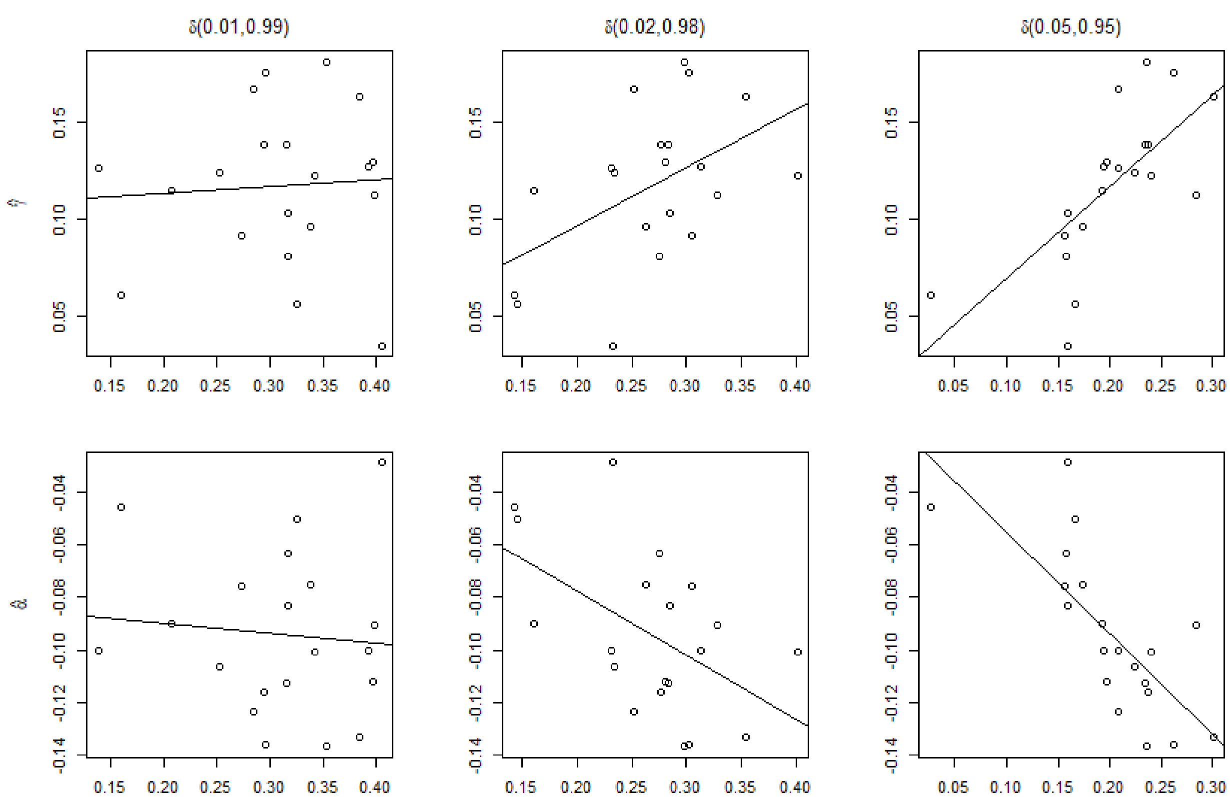
Table 2: The asymmetry measures and asymmetric parameters

Index	$\delta(0.01, 0.99)$	$\delta(0.02, 0.98)$	$\delta(0.05, 0.95)$	$\hat{\gamma}$	$\hat{\alpha}$
AEX	0.2951 (0.2456)	0.2761 (0.1865)	0.2378 (0.1769)	0.1382 (0.0553)	-0.1159 (-0.0403)
AORD	0.3426 (0.4628)	0.4018 (0.2776)	0.2399 (0.1754)	0.1222 (0.0852)	-0.1007 (-0.0623)
BVSP	0.3165 (0.1556)	0.2753 (0.0610)	0.1576 (0.0653)	0.0810 (0.0660)	-0.0631 (-0.0397)
DAX	0.1382 (0.0318)	0.2314 (-0.1151)	0.2081 (0.0165)	0.1263 (0.0270)	-0.1000 (-0.0222)
DJIA	0.3540 (0.1074)	0.2979 (0.1532)	0.2358 (0.1552)	0.1807 (0.0856)	-0.1365 (-0.0573)
ES50	0.3849 (0.4091)	0.3543 (0.3420)	0.3007 (0.2260)	0.1632 (0.0748)	-0.1330 (-0.0505)
FCHI	0.3158 (0.1262)	0.2833 (0.1589)	0.2342 (0.0668)	0.1380 (0.0542)	-0.1125 (-0.0335)
FTSE	0.2852 (0.3120)	0.2517 (0.2498)	0.2089 (0.2199)	0.1669 (0.1033)	-0.1231 (-0.0663)
FTSTI	0.1604 (-0.1194)	0.1421 (-0.1486)	0.0257 (-0.1261)	0.0609 (0.0368)	-0.0460 (-0.0170)
HSI	0.4053 (0.2679)	0.2333 (0.2203)	0.1592 (0.0711)	0.0351 (0.0381)	-0.0291 (-0.0395)
IBEX	0.2075 (0.1257)	0.1610 (0.0528)	0.1931 (0.0303)	0.1143 (0.0234)	-0.0898 (-0.0168)
KS11	0.3253 (0.2977)	0.1451 (0.0990)	0.1666 (0.1598)	0.0564 (0.0712)	-0.0503 (-0.0356)
MIB	0.2530 (0.1261)	0.2345 (0.3020)	0.2239 (0.1879)	0.1237 (0.0737)	-0.1065 (-0.0621)
MXX	0.3176 (0.0859)	0.2851 (0.1543)	0.1598 (0.1152)	0.1033 (0.0556)	-0.0834 (-0.0307)
Nasdaq	0.3985 (0.0993)	0.3280 (0.0046)	0.2838 (0.0588)	0.1120 (0.0733)	-0.0904 (-0.0516)
Nikkei	0.3380 (0.2360)	0.2625 (0.1909)	0.1731 (0.2590)	0.0960 (0.0238)	-0.0751 (-0.0059)
NSEI	0.2735 (-0.0406)	0.3053 (0.1294)	0.1560 (0.0514)	0.0917 (0.0658)	-0.0761 (-0.0453)
Russel	0.3922 (0.1228)	0.3136 (0.1237)	0.1933 (0.1566)	0.1270 (0.0993)	-0.1001 (-0.0662)
SP500	0.2953 (0.3146)	0.3017 (0.2957)	0.2615 (0.1368)	0.1749 (0.0971)	-0.1360 (-0.0608)
SSMI	0.3968 (0.4609)	0.2804 (0.3086)	0.1964 (0.1372)	0.1291 (0.0516)	-0.1120 (-0.0262)

measures and in general the asymmetry is greater in the main idiosyncratic case.

We now move to the comparison and calculate the sample correlation coefficients. The results in the total return case are discussed first. For the TGARCH we find 0.063 for $\tau = 0.01$, 0.495 for $\tau = 0.02$ and 0.707 for $\tau = 0.05$. On the other hand, the correlations for the EGARCH equal -0.092, -0.521 and -0.746 for the same respective τ's. From this we can conclude that, for our stock index sample, a larger γ or $|\alpha|$ corresponds with a larger asymmetry measure δ. Figure 4 contains scatter plots with regression lines of the estimated asymmetry measures δ and the estimated $\hat{\gamma}$ and $\hat{\alpha}$. The slope coefficients with corresponding p-value and the correlation coefficients are given in Table 13 in Appendix I. The slope coefficients show the relationship between the asymmetry measures and the asymmetric GARCH parameter estimates. The slope coefficient is most significant for δ with $\tau = 0.05$ which indicates that the asymmetric response of volatility is best captured by this measure. This corresponds with the results in Baur and Dimpfl (2019) but our results show even stronger evidence for the use of δ with $\tau = 0.05$. The results in the idiosyncratic case are similar and are given in Table 14 in Appendix I and in Figure 5. This time the slope coefficient is most significant for δ with $\tau = 0.02$. The results for the idiosyncratic returns with the CAPM are similar as well and advocate the use of δ with $\tau = 0.05$ as in the total return case. These results are shown in Figure 10 in Appendix G and in Table 15 in Appendix I.

Figure 4: Scatter plots with regression lines of the asymmetry measures and parameters



Lastly, we investigate the NICs to consider the asymmetric effect of past returns on future volatility.

Figure 5: Scatter plots with regression lines of the asymmetry measures and parameters (idiosyncratic case)

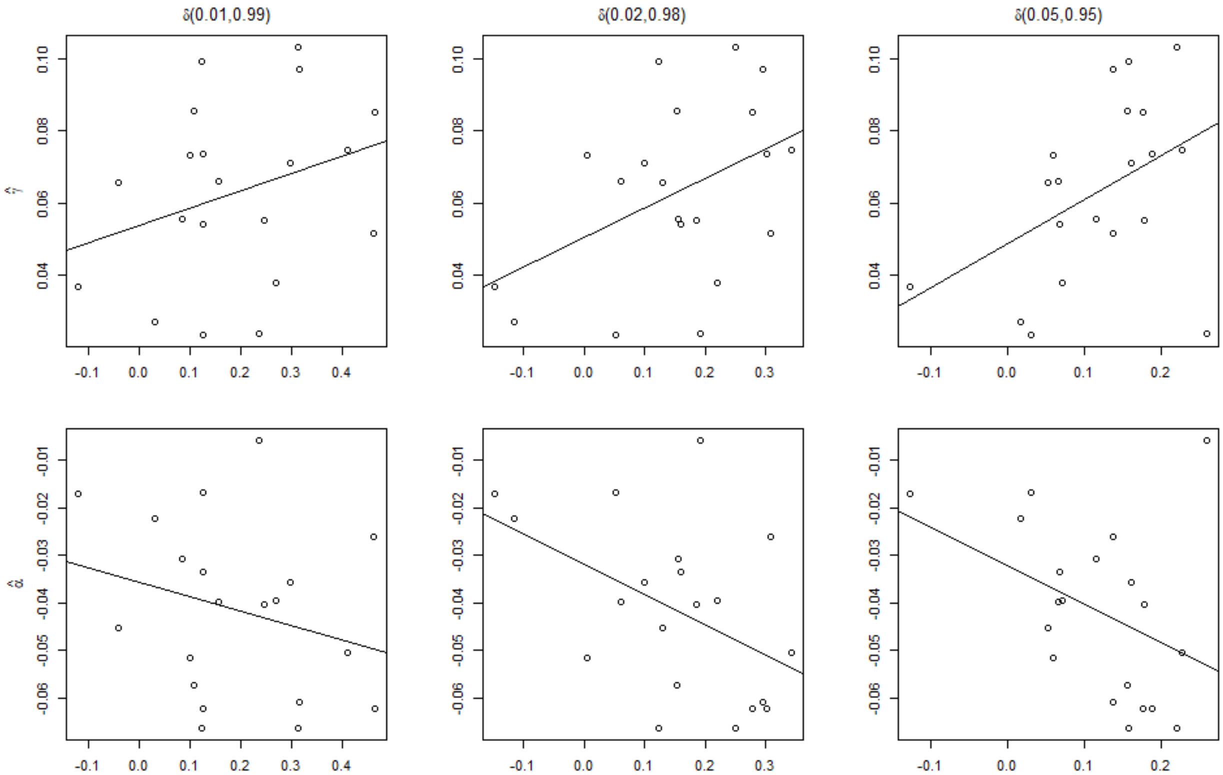


Figure 6 displays the NIC in both cases for the DAX, S&P 500, IBEX and DJIA. From this it can be seen that the asymmetry of the response of volatility is not centred around zero but shifted to the right. In the total return case the shift to the right is on average 0.148 for our sample which is in line with Baur and Dimpfl (2019). In the idiosyncratic case the shift is 0.095 on average and with the CAPM it is a shift of 0.086 on average. The plot of the NICs in the idiosyncratic case with the CAPM are shown in Figure 11 in Appendix G.

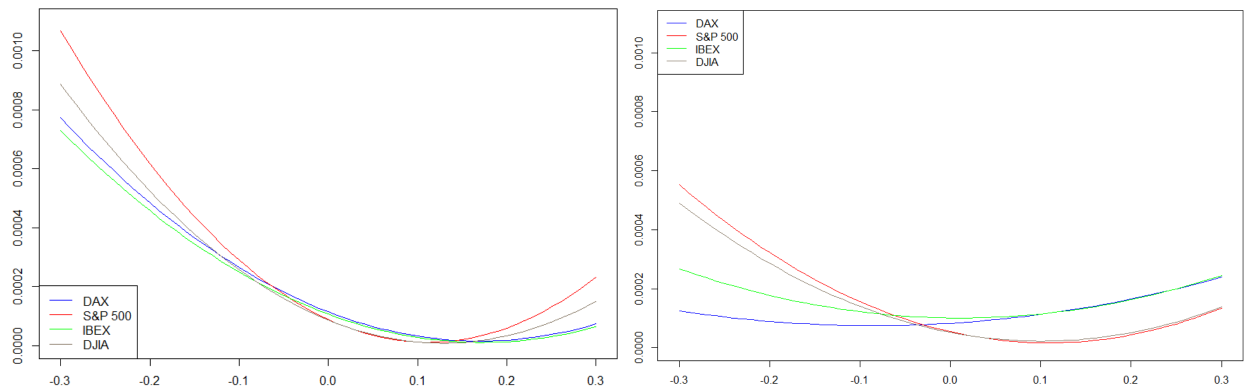
From all this we conclude that the asymmetric response of volatility is strongest in the total return case which is followed by the main idiosyncratic case and lastly the idiosyncratic case with the CAPM. However, we can still say that the asset pricing model chosen to estimate the idiosyncratic returns is robust to capturing the asymmetric effect of volatility.

5.2 QAR-based forecasting

We choose to estimate the QAR model with a subsample leaving the last 500 observations as the prediction period. This results in different estimation windows and prediction periods for each index but the same number of forecast observations. It would be optimal to select new lag lengths with Equation (11) every time the QAR model needs to be newly estimated. However, implementing this in R (2021) was too time-consuming for the time we were given to conduct this research. Instead, we newly estimate the model each

Figure 6: The news impact curves

(a) Total return case (b) Idiosyncratic return case



time with the same lag lengths as the first out-of-sample forecast.[10] There were some estimation errors for the stock indices FTSTI and NSEI in the estimation of the HAR model. Therefore, we delete these two indices from our sample and use a sample containing the other 18 stock indices for the forecasting and portfolio construction.

We first discuss our results in the total return case. The corresponding RMSPE values and p-values of the DM tests are given in Table 3. The p-values are almost all above 90% and there is no case where the QAR($p(\tau)$) has significantly better forecasting accuracy than the compared models. The RMSPE values of the QAR($p(\tau)$), EGARCH and HAR model are in general larger than the ones found in Baur and Dimpfl (2019). For the TGARCH we find mixed results and MXX is the only index for which the RMSPE values are lower in each model.[11] This is most likely due to the prediction period as it contains the COVID-19 crisis. Beyond the spread of the disease it had major economic consequences and caused the stock market to crash in March 2020. This was not like any other economic crisis before as it is due to a pandemic and this is probably the reason that the forecasting ability of the models is worse than for a stable prediction period. We also implemented the forecasting with a sample period excluding the COVID-19 period. The RMSPE values become much lower but the DM tests still do not give significant p-values. Another reason for the higher RMSPE values of the QAR($p(\tau)$) model is because the chosen lags in the model are not updated for each forecast. However, updating it every 100, 50 or 25 forecasts does not give significantly better RMSPE values for the QAR($p(\tau)$) model which is shown and explained in Appendix J.

The results in the idiosyncratic case are given in Table 4. The average RMSPE values of the QAR($p(\tau)$), TGARCH and EGARCH model are respectively 4.781, 4.511 and 4.529 which are higher than the average values in the total return case.[12] In contrast, the p-values of the DM tests are on average lower in the

[10] In the total return case, we also tried selecting new lag lengths every 25, 50 and 100 predictions. The RMSPE values in these cases are compared to the RMSPE value with the same lag lengths in Appendix J

[11] We are not able to compare the RMSPE values of KS11, Nasdaq, Nikkei and Russel as the evaluation is not done for these indices in Baur and Dimpfl (2019).

[12] In the total return case, the average RMSPE values of the QAR($p(\tau)$), TGARCH and EGARCH model are respectively

Table 3: RMSPE values and p-values of the DM tests

Index	QAR($p(\tau)$)	TGARCH	EGARCH	HAR
AEX	3.6251	3.0726 (0.997)	3.2271 (0.999)	2.5442 (0.996)
AORD	3.6577	2.7719 (0.968)	2.7651 (0.996)	3.1690 (0.999)
BVSP	4.0721	2.9776 (1.000)	3.0990 (1.000)	2.6811 (0.999)
DAX	2.6313	2.0621 (0.999)	2.1611 (1.000)	1.8450 (0.999)
DJIA	4.0210	2.7906 (1.000)	2.9764 (1.000)	2.9022 (0.999)
ES50	3.9479	2.6543 (0.981)	2.6444 (0.997)	3.2757 (0.999)
FCHI	3.5380	2.7635 (0.999)	2.9803 (0.999)	2.4614 (0.996)
FTSE	4.2485	3.3944 (0.984)	3.3235 (0.999)	3.6172 (1.000)
HSI	1.2257	1.1758 (0.982)	1.1838 (0.986)	1.1310 (1.000)
IBEX	3.0806	2.3354 (0.995)	2.4415 (0.998)	2.3313 (0.998)
KS11	1.9408	1.8047 (0.766)	1.4827 (0.994)	1.4685 (0.999)
MIB	2.2325	1.7899 (0.884)	1.3371 (0.991)	1.5765 (1.000)
MXX	0.8722	0.7466 (0.995)	0.7758 (0.966)	0.7336 (1.000)
Nasdaq	3.9776	3.3623 (0.999)	3.4871 (1.000)	2.8889 (0.997)
Nikkei	2.2507	1.8291 (0.995)	1.8044 (0.998)	1.8328 (0.999)
Russel	3.1966	2.3848 (0.956)	1.9410 (0.997)	2.3377 (0.999)
SP500	3.9809	2.8019 (0.999)	2.9822 (1.000)	2.8042 (0.999)
SSMI	4.3691	3.0535 (0.997)	3.3867 (0.999)	3.2896 (0.999)

Notes: The RMSPE values based on the 500 variance predictions are multiplied by 10,000 for readability. The p-values of the Diebold-Mariano tests are given in parentheses.

idiosyncratic case. However, they are still very high and there is no significant evidence that the QAR($p(\tau)$) model has better forecasting ability than the GARCH-type models in the idiosyncratic case. For the idiosyncratic returns based on the CAPM, all RMSPE values are higher than for the main idiosyncratic case. We conclude that the variance forecasting is more accurate in the total return case than the idiosyncratic cases. This conclusion, however, needs to be taken critically because the variance benchmark used to calculate the RMSPEs depends on the estimated idiosyncratic returns.

5.3 Portfolio construction

Lastly, we discuss the performances of our portfolios based on the variance forecasts. As the prediction periods are different for the indices, we need to find the intersection of the dates in the prediction period between all indices.[13] For each date in the intersected prediction period, portfolio weights are created with our weighting scheme. The average weight of each index for the different portfolios are given in Table 5, both for the total and idiosyncratic case.

On average in the total return case, the index with the largest weight in the QAR($p(\tau)$), TGARCH, EGARCH and HAR portfolio are respectively AORD, AEX, AEX and Nikkei. The indices with the average largest weight in the idiosyncratic case are KS11 in the QAR($p(\tau)$), SSMI in the TGARCH and again SSMI in the EGARCH model. There is no index with a very large or small weight which results in well diversified portfolios. The portfolio returns and Sharpe ratios with p-values of our Sharpe ratio test are given in Table 6. The p-value of the QAR($p(\tau)$) portfolio in the total return case comes from the Sharpe ratio test between

3.159, 2.432 and 2.444

[13]This results in 353 dates in the total return case and 355 dates in both idiosyncratic cases.

Table 4: RMSPE values and p-values of the DM tests (idiosyncratic case)

Index	QAR($p(\tau)$)	TGARCH	EGARCH
AEX	3.0079	2.8622 (0.932)	2.8671 (0.985)
AORD	6.9913	6.1014 (0.963)	6.2078 (0.996)
BVSP	4.9642	4.4721 (0.977)	4.5437 (0.988)
DAX	3.7236	3.5844 (0.859)	3.5842 (0.937)
DJIA	2.4815	2.2152 (0.974)	2.2504 (0.997)
ES50	7.4552	7.1012 (0.893)	7.0914 (0.980)
FCHI	3.6573	3.4987 (0.907)	3.5099 (0.979)
FTSE	6.5586	6.3091 (0.839)	6.3221 (0.993)
HSI	2.4094	2.2846 (0.999)	2.2900 (1.000)
IBEX	3.5671	3.4390 (0.948)	3.4656 (0.979)
KS11	5.6224	5.3289 (0.842)	5.3212 (0.948)
MIB	9.1672	9.2730 (0.311)	9.1486 (0.568)
MXX	1.6463	1.5570 (0.988)	1.5646 (0.995)
Nasdaq	3.1929	3.0187 (0.894)	3.0285 (0.974)
Nikkei	2.6308	2.3442 (0.980)	2.3642 (0.987)
Russel	9.3441	8.5405 (0.949)	8.7426 (0.987)
SP500	3.0501	2.7984 (0.935)	2.8404 (0.990)
SSMI	6.5925	6.4755 (0.677)	6.3789 (0.928)

Notes: The RMSPE values based on the 500 variance predictions are multiplied by 10,000 for readability. The p-values of the Diebold-Mariano tests are given in parentheses.

the QAR($p(\tau)$) and equally weighted portfolio.[14] None of the portfolios show a profit but also do not result in large losses. The p-values of the Sharpe ratio test indicate that the Sharpe ratio of the QAR($p(\tau)$) model is not significantly better than the Sharpe ratio of the TGARCH model in the total return case. However, the portfolio return series from the QAR($p(\tau)$) model has a better Sharpe ratio than the EGARCH and HAR model on the 10% and 5% significance level, respectively. The portfolio returns in the idiosyncratic case are somewhat lower and there is no significant evidence that the Sharpe ratio of the QAR($p(\tau)$) model is better than the Sharpe ratio of the GARCH-type models. The results in the idiosyncratic case with the CAPM are very similar but show even lower portfolio returns.

We conclude that the total return case is better in terms of portfolio construction than the idiosyncratic cases because the returns are always higher.[15] Although there is no significant evidence that the QAR($p(\tau)$) portfolio is better than the TGARCH or equally weighted portfolio, in terms of return an investor would be better off employing the QAR($p(\tau)$) portfolio during our prediction period.

6 Discussion

In this study we demonstrate that the variance forecasting and portfolio construction is generally better in the total return case than the idiosyncratic cases. As stated earlier, the estimation of the idiosyncratic returns is very dependent on the specification of the asset pricing model. A better measure for the risk-free

[14]The portfolio return and Sharpe ratio in the equally weighted portfolio are respectively -4.10% and -0.0127

[15]We are not able to implement the Sharpe ratio test between total and idiosyncratic cases because the number of return observations are not the same.

Table 5: Average portfolio weights for each portfolio

Index	QAR($p(\tau)$)	TGARCH	EGARCH	HAR
AEX	0.0589 (0.0633)	0.0679 (0.0627)	0.0710 (0.0624)	0.0546
AORD	0.0765 (0.0560)	0.0563 (0.0528)	0.0570 (0.0540)	0.0667
BVSP	0.0420 (0.0426)	0.0447 (0.0470)	0.0439 (0.0475)	0.0467
DAX	0.0503 (0.0528)	0.0551 (0.0561)	0.0545 (0.0564)	0.0532
DJIA	0.0577 (0.0667)	0.0585 (0.0596)	0.0579 (0.0603)	0.0589
ES50	0.0501 (0.0511)	0.0515 (0.0502)	0.0543 (0.0503)	0.0496
FCHI	0.0552 (0.0568)	0.0595 (0.0585)	0.0609 (0.0584)	0.0513
FTSE	0.0563 (0.0540)	0.0494 (0.0497)	0.0494 (0.0499)	0.0480
HSI	0.0600 (0.0539)	0.0572 (0.0548)	0.0551 (0.0547)	0.0575
IBEX	0.0514 (0.0493)	0.0556 (0.0568)	0.0542 (0.0573)	0.0452
KS11	0.0562 (0.0684)	0.0593 (0.0597)	0.0584 (0.0606)	0.0546
MIB	0.0500 (0.0469)	0.0519 (0.0541)	0.0522 (0.0538)	0.0550
MXX	0.0514 (0.0530)	0.0531 (0.0562)	0.0502 (0.0551)	0.0571
Nasdaq	0.0509 (0.0566)	0.0508 (0.0547)	0.0506 (0.0544)	0.0537
Nikkei	0.0577 (0.0500)	0.0638 (0.0588)	0.0656 (0.0576)	0.0689
Russel	0.0507 (0.0458)	0.0429 (0.0441)	0.0419 (0.0433)	0.0549
SP500	0.0576 (0.0676)	0.0594 (0.0615)	0.0597 (0.0611)	0.0625
SSMI	0.0671 (0.0650)	0.0632 (0.0628)	0.0633 (0.0628)	0.0619

Table 6: Portfolio returns and Sharpe ratios

	Total Return case		Idiosyncratic case	
Portfolio	Portfolio return (%)	Sharpe ratio	Portfolio return (%)	Sharpe ratio
QAR($p(\tau)$)	-2.06	-0.0050 (0.138)	-5.03	-0.0168
TGARCH	-3.54	-0.0117 (0.124)	-4.82	-0.0167 (0.992)
EGARCH	-4.05	-0.0137 (0.060)	-5.08	-0.0175 (0.922)
HAR	-4.60	-0.0162 (0.030)	/	/

rate in our asset pricing models might have been the London Interbank Offered Rate (LIBOR). This is because it is regarded as the global benchmark risk-free interest rate (Bryan & Rafferty, 2016). The risk-free rate we employ in this research is also a global rate but some countries from our sample are not incorporated. An additional robustness check would be whether the estimation of idiosyncratic returns is robust to the risk-free rate chosen in the asset pricing model.

The data we employ in this research only consists of stock market indices and no individual stocks. In reality, it is not possible to directly invest in indices but it is possible through index funds whose portfolio mirrors and tracks some stock index. A portfolio of several indices is a low-cost strategy in order to obtain a well diversified portfolio. However, applying our portfolio strategy to a wide range of stocks may give better results as there are far more portfolio construction possibilities than when only 18 indices are considered. The sample of securities contained in the portfolio should still be international as Hunter and Coggin (1990) show that international diversification can reduce investment risk by about 44% compared to national diversification. Also, trading costs are ignored when the portfolio is rebalanced each trading day. This could become very costly so in reality it might be better to only rebalance the portfolio, for instance, every one or two weeks.

The realized variance is chosen as the variance benchmark in the evaluation of the forecasts. However, Christensen, Oomen, and Podolskij (2010) state that the realized variance is limited in two ways. First, it is overly sensitive to market microstructure noise which causes the consistency of the estimator to be destroyed. Second, it is an estimator of the total variation (the sum of diffusive and jump variations) and from this follows that it cannot distinguish between these two sources of risk. Andersen, Bollerslev, and Diebold (2007) and Bollerslev, Kretschmer, Pigorsch, and Tauchen (2009) state that the ability to disentangle jumps from volatility is of key importance for the empirical modelling of asset price dynamics and forecasting of volatility. Andersen et al. (2007) propose a new quantile-based realized variance (QRV) measure. Using this measure as the variance benchmark might be better so we leave it for future research.

7 Conclusion

This paper estimates and forecasts volatility with a QAR approach based on total and idiosyncratic returns. Based on these volatility forecasts, portfolios were constructed and evaluated. For comparison, the research was also conducted for other volatility models. The data used consists of daily open-to-close returns of 20 stock indices over the sample period January 1, 2000 until May 4, 2021. The idiosyncratic returns are estimated using the residuals in asset pricing models and the sample period becomes May 31, 2011 until May 4, 2021.

First, the QAR($p(\tau)$) model is estimated from which asymmetry measures were constructed. The latter is compared to asymmetry parameters of GARCH-type models and NICs are computed with the estimated QAR coefficients. We show that the asymmetry of volatility is indeed captured by the QAR model in all cases. Next, we perform pseudo out-of-sample forecasts with a subsample of the data leaving the last 500 observations as the prediction period. We find no statistical evidence that the QAR($p(\tau)$) model has better forecasting ability than GARCH-type models in all cases. However, we do conclude that variance forecasting is more accurate in the total return case than the idiosyncratic cases. Lastly, portfolios are constructed with a weighting scheme depending on the forecasted volatilities. Based on the portfolio return, it can be seen that the total return case is always better than the idiosyncratic cases. Within the total return case, the QAR($p(\tau)$) portfolio return series has significantly better Sharpe ratio than the EGARCH and HAR portfolio return series. There is no significant evidence that the QAR($p(\tau)$) portfolio is better than the TGARCH or equally weighted portfolio but it does outperform them in terms of returns in the portfolio sample period.

All in all, we conclude that the QAR($p(\tau)$) model does a good job in capturing the asymmetric effect of volatility but not so much in forecasting volatility. The portfolio performances are better in the total return case so we conclude that having additional information about the general market return helps constructing better portfolio weights. Using the Fama/French 3-factor model or the CAPM does not result in large differences and we come to the conclusion that the estimation of idiosyncratic returns is robust to the asset pricing model considered. The research on quantile regression is still in the early stages and there is still a bright future in many different research areas ahead.

Appendix A Formula to calculate open-to-close return

The open-to-close return is calculated as follows (Kudryavtsev, 2015):

$$R_{O-C,it} = \frac{P_{C,it}}{P_{O,it}} - 1 \qquad \text{for } i = 1, \ldots, N \text{ and } t = 1, \ldots, T_i, \tag{17}$$

where $R_{O-C,it}$ is stock index i's open-to-close return on day t and $P_{C,it}$ and $P_{O,it}$ are the corresponding closing and opening price respectively. N denotes the number of stock indices and T_i the number of observations for stock index i.

Appendix B The full names of the stock indices

1. AEX = AEX index
2. AORD = All Ordinaries
3. BVSP = BVSP BOVESPA Index
4. DAX = DAX
5. DJIA = Dow Jones Industrial Average
6. ES50 = EURO STOXX 50
7. FCHI = CAC 40
8. FTSE = FTSE 100
9. FTSTI = Straits Times Index
10. HSI = HANG SENG Index
11. IBEX = IBEX 35 Index
12. KS11 = Korea Composite Stock Price Index
13. MIB = FTSE MIB
14. MXX = IPC Mexico
15. Nasdaq = Nasdaq 100
16. Nikkei = Nikkei 225
17. NSEI = NIFTY 50
18. Russel = Russel 2000
19. SP500 = S&P 500 Index
20. SSMI = Swiss Stock Market Index
21. SPG = S&P Global 1200

Appendix C Descriptive statistics in the idiosyncratic cases

Descriptive statistics for the estimated idiosyncratic returns of the 20 stock indices. The first column Obs is the number of observations, Avg. is the average daily return in basis points and Std.Dev. is the empirical standard deviation in basis points. Min and Max are the minimum and maximum of the return series in % respectively. ζ and κ are the empirical skewness and kurtosis respectively. The 3 last columns in the table contain p-values of Ljung-Box tests for autocorrelation. The null hypotheses are given by independence of the index returns (r), the residuals of an AR(1) model ($\hat{\varepsilon}$) and the squared residuals of an AR(1) model ($\hat{\varepsilon}^2$) up to the lag indicated in parentheses.

Table 7: Descriptive statistics of returns (idiosyncratic case)

Index	Obs	Avg.	Std.Dev.	Min	Max	ζ	κ	$Q_r(1)$	$Q_{\hat{\varepsilon}}(1)$	$Q_{\hat{\varepsilon}^2}(5)$
AEX	2531	-16.48	84.54	-7.15	5.10	-0.26	4.06	0.000	0.621	0.000
AORD	2510	-17.60	97.13	-9.45	5.76	-1.05	9.44	0.516	0.932	0.000
BVSP	2438	-18.56	114.77	-7.39	7.98	0.07	2.56	0.000	0.761	0.000
DAX	2507	-16.46	98.75	-7.76	5.20	-0.36	4.10	0.001	0.802	0.000
DJIA	2485	-15.10	82.27	-6.38	3.93	-0.67	4.11	0.021	0.581	0.000
ES50	2535	-16.20	105.31	-12.10	4.20	-0.90	8.94	0.001	0.757	0.000
FCHI	2534	-16.40	93.71	-7.99	5.01	-0.38	4.39	0.018	0.840	0.000
FTSE	2508	-15.88	99.72	-11.05	3.77	-0.94	8.05	0.004	0.781	0.000
FTSTI	1405	-30.08	79.89	-4.50	5.39	-0.07	4.79	0.273	0.858	0.000
HSI	2431	-22.93	90.15	-3.95	5.79	-0.15	1.69	0.187	0.841	0.000
IBEX	2530	-21.23	105.92	-8.02	4.29	-0.32	3.08	0.004	0.836	0.000
KS11	2441	-24.09	87.04	-10.51	5.23	-0.96	10.74	0.693	0.947	0.000
MIB	2507	-20.48	112.80	-13.82	5.29	-1.04	10.84	0.030	0.850	0.000
MXX	2486	-16.33	93.79	-6.92	3.12	-0.50	2.99	0.003	0.759	0.000
Nasdaq	2490	-15.98	91.26	-7.19	3.90	-0.64	3.48	0.123	0.834	0.000
Nikkei	2421	-17.85	99.95	-8.35	5.44	-0.59	6.49	0.042	0.847	0.000
NSEI	2436	-26.41	99.82	-6.72	8.14	0.09	4.78	0.002	0.843	0.000
Russel	2487	-16.25	120.20	-12.88	7.11	-0.75	9.17	0.000	0.363	0.000
SP500	2487	-15.52	84.32	-7.68	4.65	-0.84	6.46	0.384	0.835	0.000
SSMI	2478	-17.41	87.68	-11.25	4.96	-1.91	20.87	0.000	0.529	0.000

Table 8: Descriptive statistics of returns (idiosyncratic case with CAPM)

Index	Obs	Avg.	Std.Dev.	Min	Max	ζ	κ	$Q_r(1)$	$Q_{\hat{\varepsilon}}(1)$	$Q_{\hat{\varepsilon}^2}(5)$
AEX	2531	-16.84	92.53	-8.00	4.74	-0.30	4.63	0.075	0.833	0.000
AORD	2510	-17.81	97.53	-9.37	5.49	-1.13	9.66	0.637	0.950	0.000
BVSP	2438	-18.64	117.69	-7.17	9.31	0.19	3.82	0.000	0.686	0.000
DAX	2507	-16.83	107.61	-8.75	5.71	-0.33	4.52	0.010	0.818	0.000
DJIA	2485	-15.31	98.00	-6.59	6.70	-0.58	6.10	0.000	0.155	0.000
ES50	2535	-16.93	119.53	-13.54	5.97	-0.92	10.19	0.007	0.811	0.000
FCHI	2534	-16.95	105.04	-9.17	5.75	-0.37	5.59	0.001	0.785	0.000
FTSE	2508	-16.51	109.75	-12.23	5.82	-1.01	10.13	0.055	0.877	0.000
FTSTI	1405	-30.89	81.48	-5.28	5.32	-0.16	6.12	0.035	0.698	0.000
HSI	2431	-23.05	90.91	-4.09	7.25	-0.02	2.70	0.692	0.951	0.000
IBEX	2530	-22.01	117.06	-9.35	6.52	-0.35	4.62	0.122	0.897	0.000
KS11	2441	-24.26	87.63	-10.60	6.38	-0.90	11.65	0.476	0.900	0.000
MIB	2507	-21.03	123.09	-15.05	5.37	-0.89	10.35	0.003	0.761	0.000
MXX	2486	-16.72	96.97	-6.92	3.79	-0.43	2.86	0.269	0.908	0.000
Nasdaq	2490	-15.26	100.11	-8.55	4.81	-0.73	4.69	0.000	0.472	0.000
Nikkei	2421	-18.18	100.59	-8.38	5.58	-0.62	6.64	0.092	0.862	0.000
NSEI	2436	-26.74	101.31	-7.24	10.02	0.20	7.40	0.089	0.890	0.000
Russel	2487	-16.41	123.35	-14.15	6.77	-0.91	11.19	0.000	0.166	0.000
SP500	2487	-15.45	98.69	-7.89	6.56	-0.67	7.51	0.000	0.145	0.000
SSMI	2478	-17.67	93.80	-11.95	5.14	-1.80	19.76	0.082	0.844	0.000

Appendix D The assumptions for covariance stationarity in y_t

A.1 The series $\{u_t\}$ consists of iid random variables with mean 0 and finite variance σ^2. The distribution function F of u_t has a continuous density function f with $f(u) > 0$ on $U = \{u : 0 < F(u) < 1\}$.

A.2 $\mathbb{E}[\alpha_t^2] = \omega_\alpha^2 < 1$.

Appendix E The asymmetric models for the conditional variance

The first model which captures the asymmetric response of the variance is the TGARCH (GJR-GARCH) model of Glosten et al. (1993) and it is given by

$$h_t = \omega + \alpha\,\varepsilon_{t-1}^2 + \gamma\,\varepsilon_{t-1}^2 I(\varepsilon_{t-1} < 0) + \beta\,h_{t-1} \tag{18}$$

where $I(\cdot)$ is an indicator function which equals 1 if $\varepsilon_{t-1} < 0$ and 0 otherwise. The second model is the EGARCH model of Nelson (1991) and it is given by

$$ln(h_t) = \omega + \alpha\left(\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}\right) + \lambda\left[\frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}} - \sqrt{\frac{2}{\pi}}\right] + \beta\,ln(h_{t-1}). \tag{19}$$

Both models have orders $p = q = 1$ which correspond with the TGARCH(1,1) and EGARCH(1,1) models.

Appendix F The HAR model of Corsi (2009)

As in Baur and Dimpfl (2019), we also implement a HAR model with daily, weekly and monthly input variables for the natural logarithm of realized volatility:

$$ln\,\sigma_{RV,t} = c_0 + c_1\,ln\,\sigma_{RV,t-1} + c_2\,ln\,\sigma_{RV,t-1}^w + c_3\,ln\,\sigma_{RV,t-1}^m + \varepsilon_t, \tag{20}$$

where $\sigma_{RV,t} = \sqrt{RV_t}$, $\sigma_{RV,t-1}^w = \frac{1}{5}\sum_{i=1}^{5}\sigma_{RV,t-i}$ and $\sigma_{RV,t-1}^m = \frac{1}{22}\sum_{i=1}^{22}\sigma_{RV,t-i}$. Lütkepohl and Xu (2012) found that taking the log of a series to forecast it and then transforming it back with the exponential function can result in dramatic gains in forecast precision. This is when the log transformation gives a more stable variance than the original series. Andersen, Bollerslev, Diebold, and Labys (2003) state that the log of RV, in contrast to RV itself, is approximately normally distributed which means that it has a more stable variance. Consequently, we take a simple exponential function without correction terms to retransform the log of volatility which is then squared to obtain the variance.

Appendix G Results for the idiosyncratic case with the CAPM

Figure 7: The average and dispersion of the selected lags

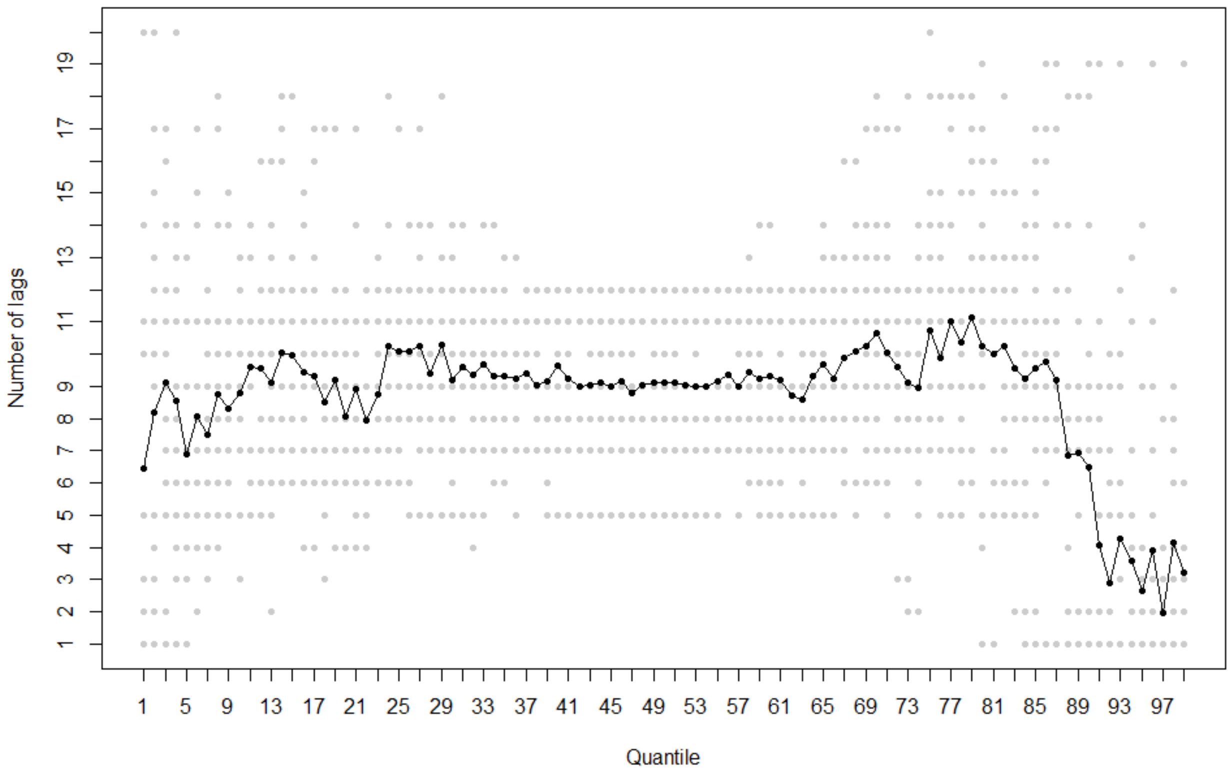


Figure 8: Boxplots of QAR coefficient estimates

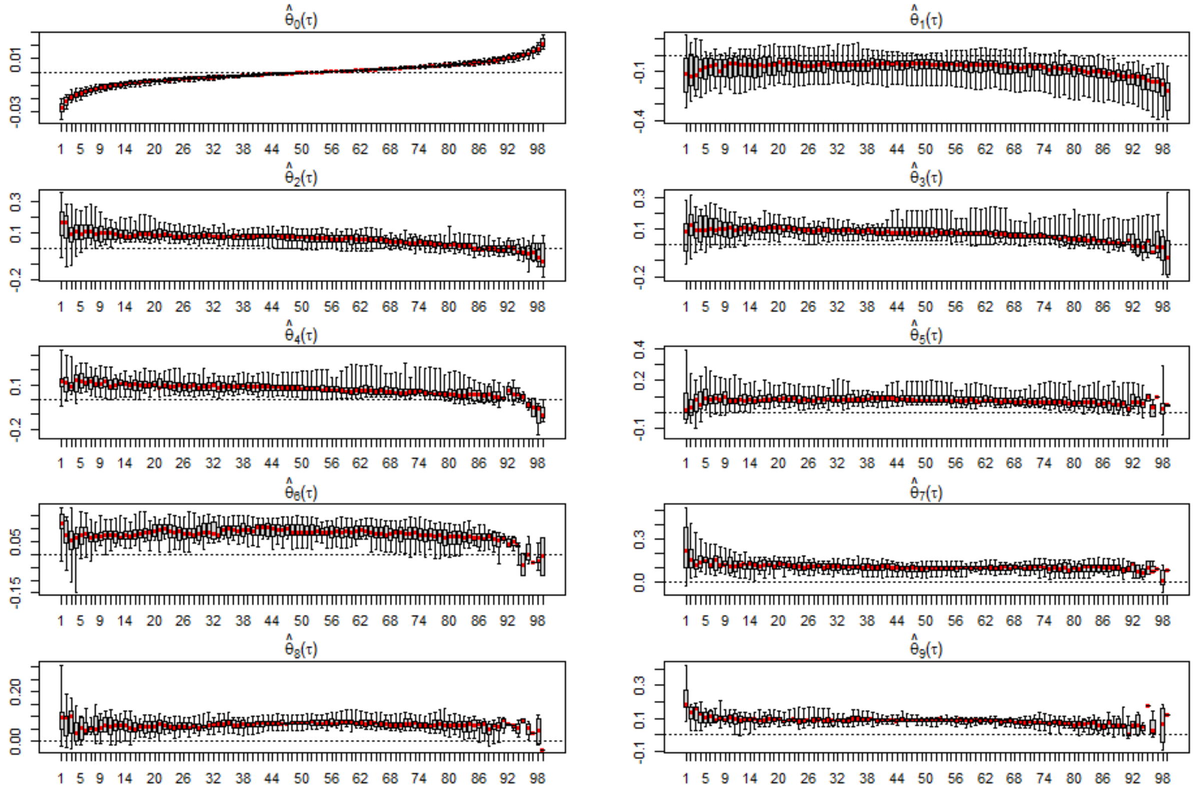


Figure 9: Boxplots of QAR coefficient estimates at lags 10 through 20

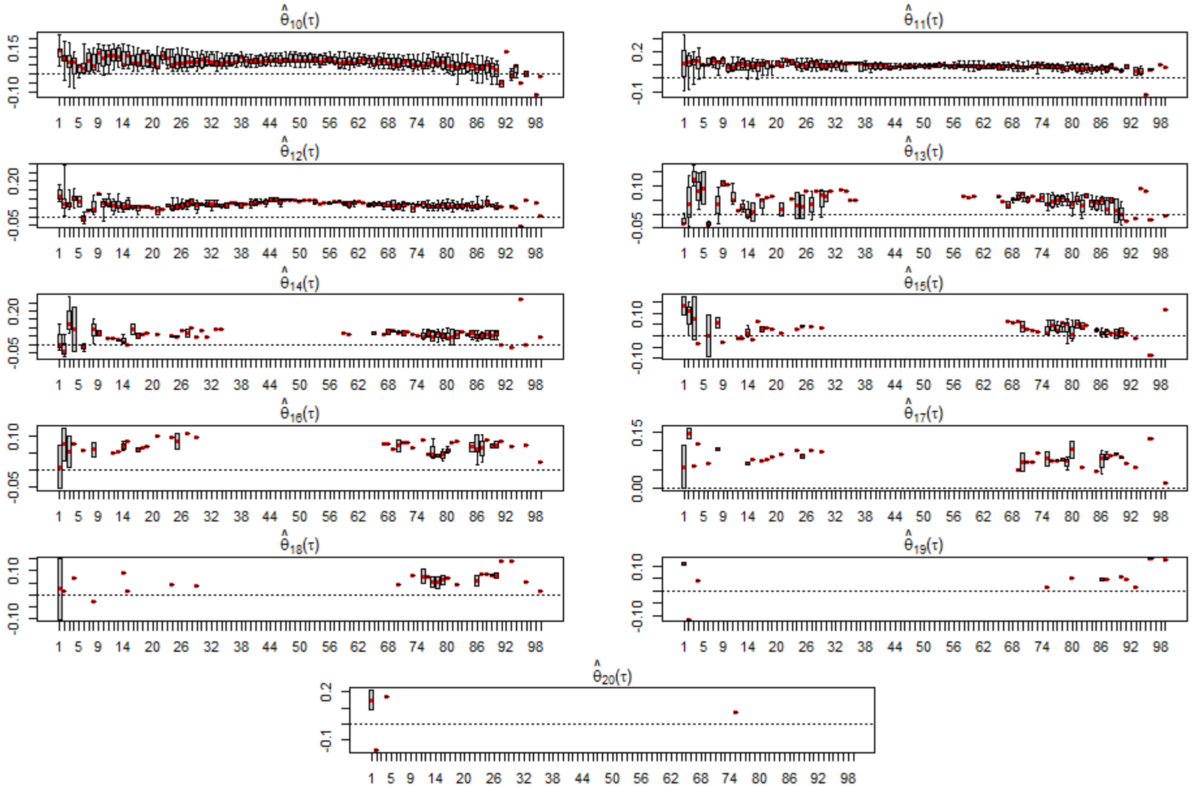


Table 9: The asymmetry measures and asymmetric parameters

Index	$\delta(0.01, 0.99)$	$\delta(0.02, 0.98)$	$\delta(0.05, 0.95)$	$\hat{\gamma}$	$\hat{\alpha}$
AEX	0.2144	0.1279	0.1908	0.0686	-0.0467
AORD	0.3930	0.2924	0.1571	0.0829	-0.0600
BVSP	0.0978	0.0535	0.0672	0.0662	-0.0409
DAX	-0.0162	-0.0607	-0.0353	0.0478	-0.0335
DJIA	0.2490	0.1477	0.1851	0.1472	-0.0974
ES50	0.2732	0.1753	0.0769	0.0976	-0.0606
FCHI	-0.1556	-0.0892	-0.0178	0.0765	-0.0404
FTSE	0.1600	0.1870	0.1687	0.1561	-0.0945
FTSTI	0.0210	-0.1286	-0.1258	0.0333	-0.0157
HSI	0.2040	0.0705	0.0391	0.0520	-0.0470
IBEX	-0.0350	-0.0872	-0.0196	0.0392	-0.0188
KS11	0.3150	0.0921	0.1604	0.0748	-0.0360
MIB	0.0747	0.1839	0.1647	0.0754	-0.0526
MXX	0.0713	0.1967	0.0643	0.0615	-0.0339
Nasdaq	0.0181	0.0721	0.0746	0.0913	-0.0697
Nikkei	0.2090	0.1535	0.1450	0.0228	-0.0069
NSEI	-0.0800	-0.0467	0.0695	0.0721	-0.0466
Russel	0.0842	0.1020	0.0963	0.1195	-0.0813
SP500	0.1932	0.2800	0.2212	0.1244	-0.0813
SSMI	0.3073	0.2412	0.1214	0.0668	-0.0444

Figure 10: Scatter plots with regression lines of the asymmetry measures and parameters

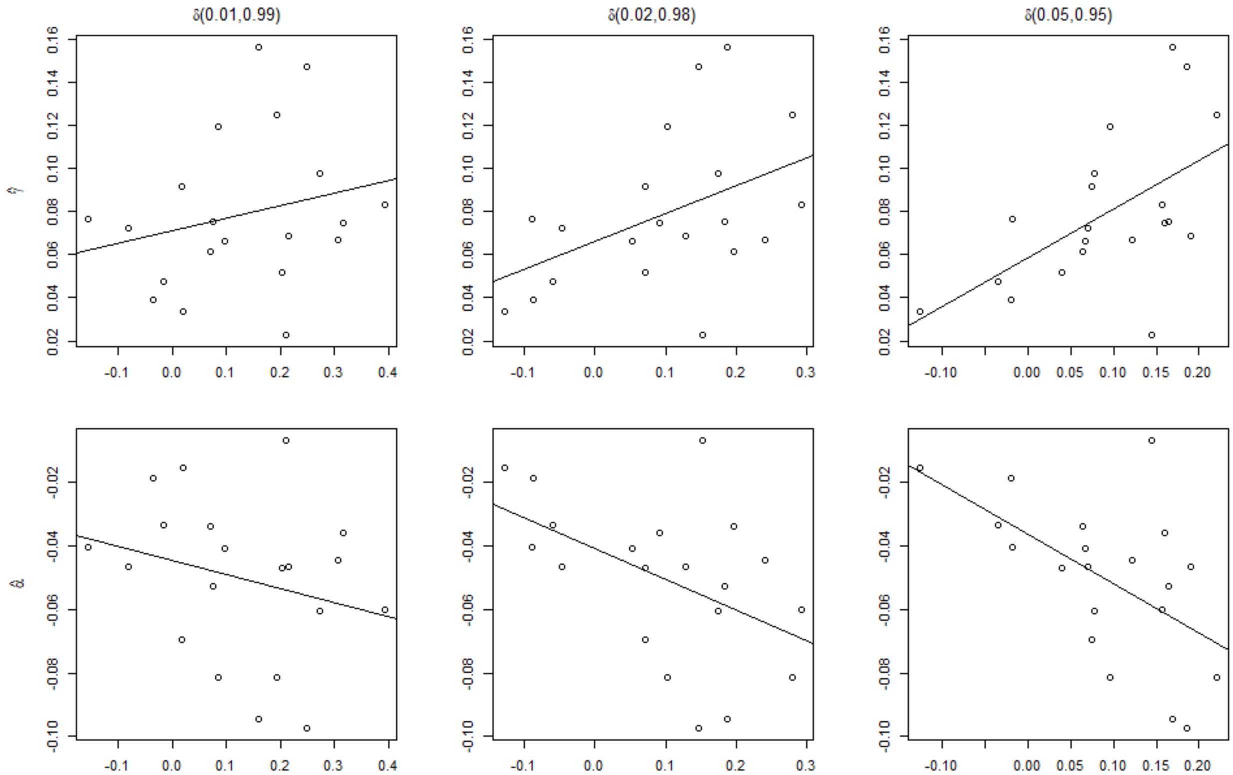


Figure 11: The news impact curves

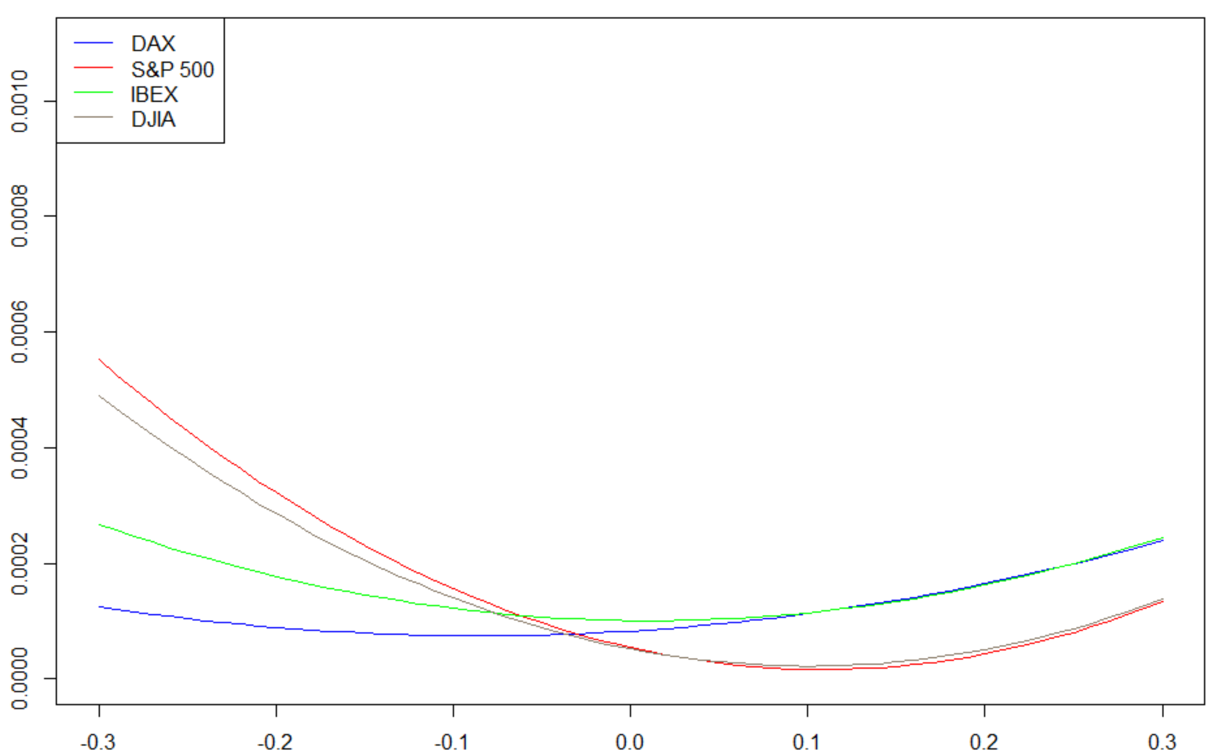


Table 10: RMSPE values and p-values of the DM tests

Index	QAR($p(\tau)$)	TGARCH	EGARCH
AEX	3.6022	3.4235 (0.910)	3.4012 (0.985)
AORD	7.1402	6.2425 (0.962)	6.3522 (0.996)
BVSP	6.2548	5.5374 (0.985)	5.6485 (0.993)
DAX	4.0488	3.9969 (0.825)	3.9616 (0.983)
DJIA	4.6743	3.6412 (0.988)	3.9028 (0.997)
ES50	9.7739	9.4940 (0.726)	9.3958 (0.952)
FCHI	4.8315	4.7146 (0.777)	4.6583 (0.964)
FTSE	8.875	8.4426 (0.788)	8.4773 (0.970)
HSI	3.0602	2.9379 (0.992)	2.9481 (0.998)
IBEX	4.9970	4.8213 (0.914)	4.8426 (0.959)
KS11	5.9387	5.5542 (0.853)	5.5553 (0.944)
MIB	10.4307	10.6065 (0.191)	10.3733 (0.713)
MXX	2.0685	1.9513 (0.978)	1.9576 (0.994)
Nasdaq	4.5613	4.1572 (0.945)	4.2158 (0.989)
Nikkei	2.6731	2.3572 (0.982)	2.3859 (0.987)
Russel	10.9841	9.8362 (0.962)	10.1619 (0.990)
SP500	5.0679	4.0815 (0.969)	4.3182 (0.992)
SSMI	7.6762	7.4334 (0.736)	7.3182 (0.953)

Table 11: Average portfolio weights for each portfolio

Index	QAR($p(\tau)$)	TGARCH	EGARCH
AEX	0.0617	0.0626	0.0627
AORD	0.0609	0.0545	0.0555
BVSP	0.0453	0.0489	0.0488
DAX	0.0508	0.0547	0.0550
DJIA	0.0610	0.0567	0.0568
ES50	0.0485	0.0500	0.0505
FCHI	0.0542	0.0569	0.0574
FTSE	0.0536	0.0493	0.0497
HSI	0.0571	0.0577	0.0571
IBEX	0.0486	0.0560	0.0568
KS11	0.0726	0.0624	0.0627
MIB	0.0454	0.0536	0.0544
MXX	0.0544	0.0566	0.0556
Nasdaq	0.0555	0.0521	0.0521
Nikkei	0.0537	0.0620	0.0601
Russel	0.0489	0.0459	0.0448
SP500	0.0622	0.0579	0.0576
SSMI	0.0656	0.0622	0.0624

Table 12: Portfolio returns and Sharpe ratios

Model	Portfolio return (%)	Sharpe ratio
QAR($p(\tau)$)	-5.78	-0.0205
TGARCH	-4.96	-0.0176 (0.716)
EGARCH	-5.27	-0.0187 (0.750)

Appendix H Boxplots of QAR coefficient estimates (lags 10-20)

Figure 12: Boxplots of QAR coefficient estimates at lags 10 through 20

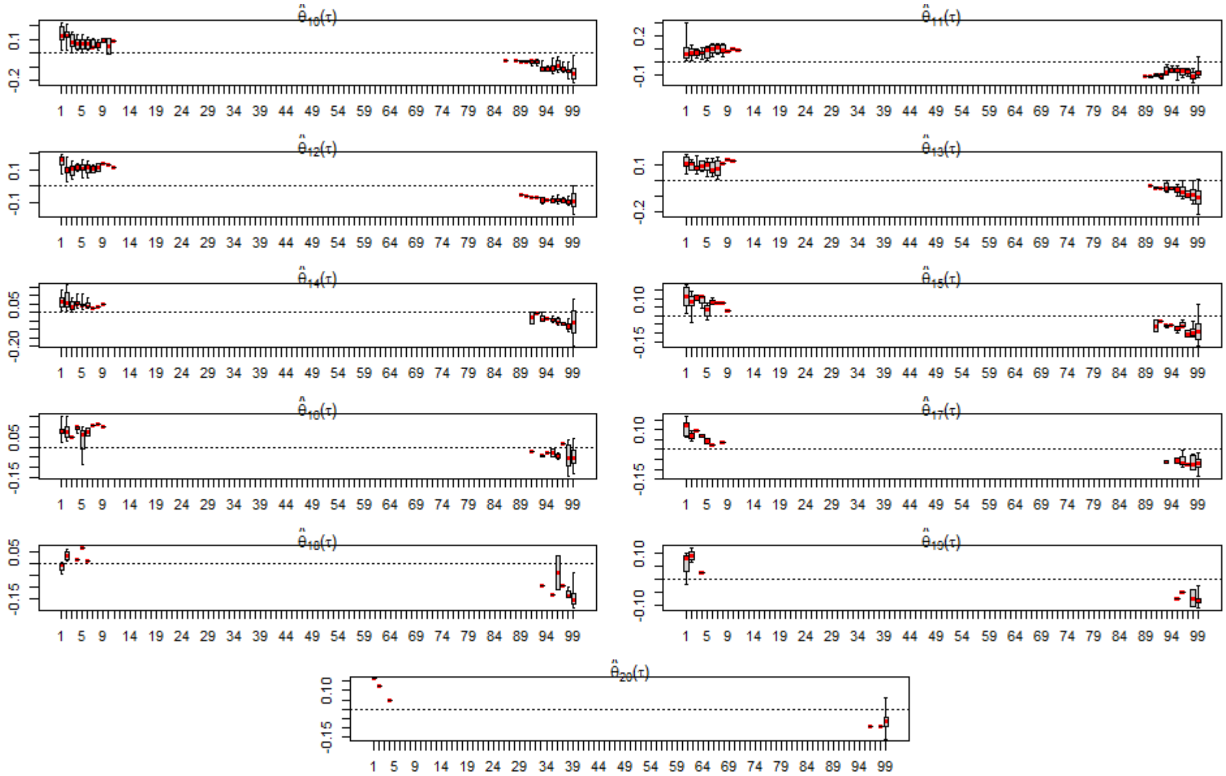
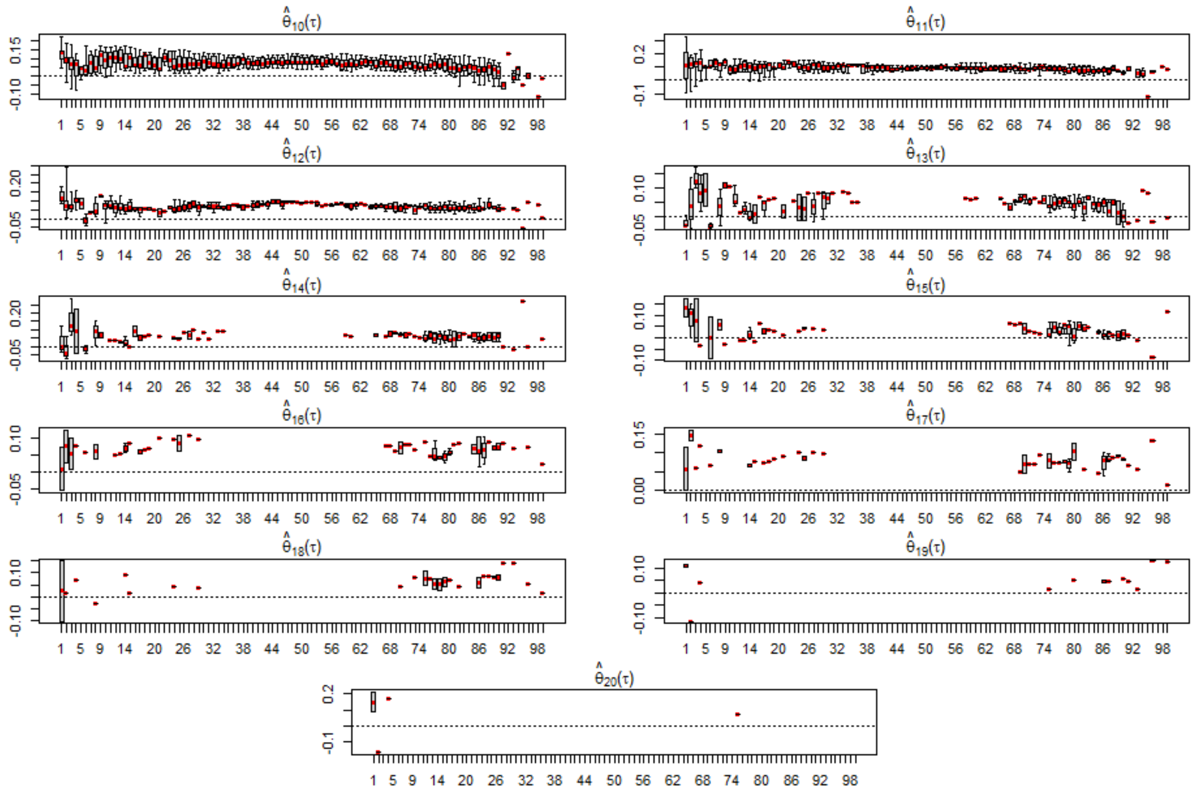


Figure 13: Boxplots of QAR coefficient estimates at lags 10 through 20 (idiosyncratic case)



Appendix I The correlation and slope coefficients

The sample correlation coefficients are given in the top two rows of the Tables below. The sample slope coefficients with corresponding p-value, which are given in parentheses, are shown in the bottom two rows of these Tables.

Table 13: The sample correlation and slope coefficients

	$\delta(0.01, 0.99)$	$\delta(0.02, 0.98)$	$\delta(0.05, 0.95)$
$\hat{\gamma}$	0.0625	0.4951	0.7073
$\hat{\alpha}$	-0.0925	-0.5207	-0.7463
$\hat{\gamma}$	0.0325 (0.7934)	0.2975 (0.0264)	0.4708 (0.0005)
$\hat{\alpha}$	-0.0371 (0.6982)	-0.2416 (0.0186)	-0.3835 (0.0002)

Table 14: The sample correlation and slope coefficients (idiosyncratic case)

	$\delta(0.01, 0.99)$	$\delta(0.02, 0.98)$	$\delta(0.05, 0.95)$
$\hat{\gamma}$	0.3047	0.4485	0.4441
$\hat{\alpha}$	-0.2656	-0.4723	-0.4004
$\hat{\gamma}$	0.0480 (0.1920)	0.0825 (0.0473)	0.1226 (0.0498)
$\hat{\alpha}$	-0.0307 (0.2580)	-0.0637 (0.0355)	-0.081002 (0.0802)

Table 15: The sample correlation and slope coefficients (idiosyncratic case CAPM)

	$\delta(0.01, 0.99)$	$\delta(0.02, 0.98)$	$\delta(0.05, 0.95)$
$\hat{\gamma}$	0.2375	0.4566	0.5634
$\hat{\alpha}$	-0.2595	-0.4850	-0.5587
$\hat{\gamma}$	0.0583 (0.3130)	0.1297 (0.0430)	0.2244 (0.0097)
$\hat{\alpha}$	-0.0444 (0.2690)	-0.0962 (0.0302)	-0.1554 (0.0105)

Appendix J Comparison of different lag selection intervals

Table 16 contains RMSPE values of the QAR model based variance forecasts, in the total return case, for different lag selection intervals. The lowest RMSPE value is achieved by seven indices in case of same lag lengths for every forecast. For new lag lengths every 100, 50 and 25 forecasts the lowest RMSPE value is attained by two, five and four indices respectively. We expected that selecting new lag lengths every 25 forecasts would give lowest RMSPE in most cases because it is the closest approximation to selecting new lag lengths every forecast. However, the change in RMSPE values for different lag selection intervals is negligible which indicates that the lag selection in the QAR model does not play an important role in the forecasting ability.

The change in RMSPE values for different lag selection intervals could be significant in the idiosyncratic case. However, for the sake of comparison, we also use the same lag lengths for every forecast in both idiosyncratic cases.

Table 16: RMSPE values of the QAR model for different lag selections

Index	same lags	every 100	every 50	every 25
AEX	3.6251	3.6352	3.6357	3.6361
AORD	3.6577	3.6581	3.6585	3.6540
BVSP	4.0721	4.0708	4.0707	4.0724
DAX	2.6313	2.6141	2.6153	2.6141
DJIA	4.0210	4.0038	4.0037	4.0047
ES50	3.9479	3.8786	3.8752	3.8845
FCHI	3.5380	3.5566	3.5564	3.5563
FTSE	4.2485	4.2490	4.2488	4.2464
HSI	1.2257	1.2273	1.2271	1.2270
IBEX	3.0806	3.1092	3.1093	3.1105
KS11	1.9408	1.9472	1.9466	1.9457
MIB	2.2325	2.2195	2.2220	2.2204
MXX	0.8722	0.8683	0.8693	0.8681
Nasdaq	3.9776	3.9779	3.9779	3.9780
Nikkei	2.2507	2.2499	2.2497	2.2527
Russel	3.1966	3.1798	3.1802	3.1855
SP500	3.9809	3.9457	3.9456	3.9511
SSMI	4.3691	4.3728	4.3729	4.3737

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