Erasmus University Rotterdam

Bachelor Thesis in Financial Econometrics

Forecasting volatility and Value-at-Risk with real-time smooth transition volatility models

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May 11, 2021

Abstract

Smooth transition approaches allow for intermediate regimes with a smooth transition function. This paper evaluates the volatility forecasting performance of these models versus the standard exponential smoothing/GARCH models, while assuming that volatility is a function of past information only or both past and current information. Previous research shows that current information or allowing for intermediate regimes with a smooth transition function for GARCH formulations improve volatility predictions substantially. This paper combines these findings by evaluating the performance of real-time smooth transition GARCH (RT-STGARCH) versus the real-time GARCH models in forecasting volatility and Value-at-risk (VaR) using daily returns from three major stock indices. We also evaluate whether real-time smooth transition exponential smoothing (RT-STES) models have better volatility predictions and are faster in the adjustment to the new unconditional volatility level than the smooth transition exponential smoothing models. Our empirical results suggest that RT-STGARCH models provide better volatility predictions than the real-time GARCH models, but have slightly worse VaR predictions. Further, although the RT-STES models have potential, our results suggest that they are equivalent to the smooth transition exponential smoothing models and have similar volatility forecasting performance.

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1 Introduction

Volatility modeling and forecasting plays a crucial role in risk management, investment, asset allocation and option pricing. Furthermore, it is an important task in financial markets as it is widely investigated by academics and practitioners for many years. Well known observation-driven volatility forecasting approaches are: the standard GARCH models (Bollerslev (1986), Glosten et al. (1993) and Zakoian (1994)), Smooth Transition GARCH (ST-GARCH) models (González-Rivera, 1998) and the smooth transition exponential smoothing (STES) models (Taylor, 2004a). The smooth transition models incorporate the asymmetric response of volatility to past returns, known as the leverage effect, with a smooth transition function. While a sharp transition function indicates two regimes, high versus low volatility (threshold specification), a smoother transition function allows for other, intermediate regimes. González-Rivera (1998) provides a clear overview of the smooth transition approach for GARCH models. A common property of the aforementioned models is that they assume that volatility is a function of past information only. This gives rise to the question whether current information should also be incorporated when modelling and forecasting volatility.

This paper builds upon the paper of Smetanina (2017) who introduces the real-time GARCH model (RT-GARCH). She assumes that the volatility of asset returns is a function of past information and current information. By also assuming current information Smetanina (2017) is able to obtain better volatility forecasts than standard GARCH-type models and to make the model faster in its adjustment to the new unconditional level of volatility.

We build upon this by proposing several real-time smooth transition models that assume that volatility of returns is a function of past and current information but also incorporates the leverage effect with a smooth transition function.

In order to check whether the aforementioned findings of Smetanina (2017) hold for our real-time smooth transition models, we investigate the performance of the real-time STES models versus the standard STES in forecasting volatility and adjustment to the new unconditional level of volatility. Further, we investigate the performance of the real-time smooth transition models versus the (real-time) GARCH models in forecasting volatility and Value-at-Risk (VaR) using daily returns. Last, we investigate whether there is a single model that provides the best volatility/VaR forecasts. We are interested in forecasting volatility and VaR 1-step ahead for three large stock indices of Europe and the United States.

In order to evaluate and compare the forecasts, we need a robust loss function, because while evaluating and comparing volatility forecasts is a well-studied problem, it becomes difficult if there is a biased proxy of volatility. Patton (2011) shows that distortions may arise when common loss functions are employed and that these distortions vary greatly with the choice of loss function. He introduces a robust parametric family of loss functions, which we also use in the forecast evaluation. We test equality of forecast accuracy with the Diebold-Mariano test, for which we use the best performing model as our benchmark.

First, we find that within our estimation approach the real-time STES models are equivalent to the standard STES models and assuming current information does not improve the volatility forecasts for these models. However, we find that if volatility is a function of past and current information, it makes the model faster in its adjustment to the unconditional level of volatility.

Further, we find that the real-time smooth transition GARCH models provide significantly better volatility predictions than the real-time GARCH models. Due to the smoothness parameter the real-time smooth transition GARCH is able to react much better to changes in volatility. However, the real-time smooth transition GARCH model have worse VaR forecasting performance and the real-time GARCH is preferred.

Last, the standard smooth transition exponential smoothing models and the smooth transition GARCH models with a logistic transition function have the best volatility forecasting performance. However, the models have similar volatility forecasting performance and we find no clear winner.

The paper proceeds as follows. Section 2 covers the literature on this topic. Section 3 provides this data diagnostics. Section 4 introduces the real-time smooth transition GARCH models and describes our approach for parameter estimation. Also, in Section 4 we introduce the real time STES models and describe our simple approach for STES parameter estimation. Section 5 presents how new information affects the volatility of returns for the STES models. In Section 6 we describe our forecast evaluation. Section 7 presents and discusses the results, and in Section 8 we conclude and suggest further research.

2 Literature Review

Researchers often assume that all volatility models can be classified as either parameter-driven or observation-driven (Shephard, 1996). One of the most popular approaches for observation-driven volatility forecasting is the GARCH model proposed by Bollerslev (1986). Often it is assumed that the error series ε_t is demeaned returns series. Previous research finds that financial markets become more volatile in response to negative shocks than to positive shocks. The standard GARCH model does not incorporate this asymmetric response of volatility to shocks, known as the leverage effect. Previous econometric modeling has lead to multiple GARCH models to represent this phenomenon. For example the GJRGARCH model proposed by Glosten et al. (1993), the treshold GARCH (TGARCH) model proposed by Zakoian (1994) and the the exponential GARCH (EGARCH) of Nelson (1991). They are widely implemented to represent the volatility in many empirical applications (e.g. Alberg et al. (2008), Hentschel (1995) and Franses and Van Dijk (1996)). These methods do not permit the existence of intermediate regimes between high-volatility and low-volatility regimes. Research into the intermediate regimes has led to multiple GARCH models. For example, the smooth transition models introduced by González-Rivera (1998), which allow a parameter to vary over time as a continuous function of a transition variable. He introduces the LSTGARCH model which uses a logistic function to dictate the smooth transition between parameters. Independently, Hagerud (1997) proposes the same LSTGARCH model. However, he also proposes the ESTGARCH model which uses an exponential function to dictate the smooth transition between parameters. Taylor (2004b) provides the more detailed model expressions. Another approach in the exponential smoothing literature, is developed by Taylor (2004a) and is called the smooth transition exponential smoothing (STES) method. This STES method uses a logistic function of a variable as adaptive time-varying smoothing parameter. Liu et al. (2020) extend the method by introducing the STES-AE model, when $|\varepsilon_{t-1}|$ is the transition variable, and the STES-E model, when ε_{t-1} is the transition variable. Liu et al. (2020) show in his analysis of eight stock indices that the STES method performs well in comparison with a range of standard and robust ES and GARCH models in forecasting volatility.

A main similarity of these univariate ES/GARCH models is that they assume that the volatility of asset returns, σ_t, is a function of past information only, that is, σ_t is $\mathcal{F}_{t-1}$-measurable, where $\mathcal{F}_{t-1}$ is the sigma algebra induced by the history of returns up to time $t-1$ (Smetanina, 2017). By only using past information, GARCH models make an inefficient use of all available information when forecasting the volatility of returns. Kim et al. (1998) show that, in comparison to stochastic volatility (SV) models, GARCH models are not adequately able to capture changing volatility, especially in periods after large shocks had occurred. Smetanina (2017) shows that it is possible to efficiently utilize all available internal information GARCH models. That is, by assuming that σ_t is $\mathcal{F}_t$-measurable. She introduces the RT-GARCH model (with leverage and feedback). The feedback effect is incorporated acknowledging the different effect of positive and negative current shocks on the conditional variance of returns. We refer to the different effects of the past positive and negative returns on conditional variance as leverage effect. (Nelson, 1991). Also, Smetanina (2017) shows that, during a crisis, the real-time GARCH model with feedback and the real-time GARCH method with leverage and feedback outperform the standard real-time GARCH model. Therefore it is reasonable to incorporate this asymmetric response of volatility to shocks and to past returns. This gives rise to the question whether this is also reasonable for real-time volatility models that permit the existence of intermediate regimes.

In this paper we propose four new real-time smooth transition models: The real-time LSTGARCH (RT LSTGARCH) model; the real-time ESTGARCH (RT ESTGARCH) model; the real-time STES-AE (RT STES-AE) model and the real-time STES-SE (RT STES-SE) model, and compare these models with the real time GARCH model (with leverage and feedback). For the purpose of comparison, the standard GARCH model and the standard smooth transition models are included as benchmark models. Section 4 provides a more detailed overview.

3 Data

We use the daily returns and the realized variance based on 5-minute intra-day returns of Euro Stoxx 50 (STOXX50), Dow Jones Industrial Average (DJI) and the S&P 500 stock index covering the sample period from 3 January 2000 to 21 May 2021. For the stock indices the data are sourced from The

Oxford-Man Institute's realised library.

The data are chosen so that it contains several volatility increasing events, like the internet bubble (2000-2002), the great financial crisis (2008) and the economic impact of the COVID 19 pandemic (2020-present day). We want to investigate volatility models in times with extreme shocks.

In order to evaluate the volatility forecast we split the data into two parts. The first part serves for the parameter estimation, and the remaining part is used for the volatility forecast evaluation with the rolling window approach. We use 10 years of data for estimation and 11 years of data for evaluation. Besides the full-sample, we evaluate the forecasts for two sub-samples. The pre-crisis period spans from 3 January 2000 to 31 July 2008 and we take the remaining sample to be the crisis and pre-crisis period. For the pre-crisis period we use 4 years of data for estimation and 4 years of data for evaluation. For the crisis and post-crisis period we use 6 years of data for estimation and 7 years of data for evaluation.

Now, we examine the properties of daily returns and the realized volatility. In Table 1 we present descriptive statistics of the daily and realized volatility for STOXX50E, DJI and S&P 500 stock index. A number of interesting features follow from these statistics. First, the daily returns for the STOXX50E index have negative mean, whereas the daily returns for DJI and S&P500 indices have positive mean. Also, the STOXX50E stock index is more skewed to the left. These findings indicate that there are more large negative returns in Europe than in the United States. The mean of the realized volatility is also higher in Europe which suggests that the large negative returns makes the market more volatile. Further, the kurtosis is less extreme which means that there are less returns around the mean than expected for the STOXX50E stock index.

The realized volatility exhibits excess kurtosis. That means that price changes in the data are too peaked and thick-tailed compared with samples from the normal distribution. This is confirmed by the Jarque-Bera test, which reject the null hypothesis of normality for the three stock indices. Taking the logarithm of realized variance instead leads to a strong reduction in skewness and kurtosis. However, the Jarque-Bera still rejects the null hypothesis of normality for the three stock indices. Finally, the Augmented Dickey-Fuller (ADF) test confirms the stationarity of the daily return, realized volatility and log realized variance series.

Table 1: Summary statistics

Realized volatility $(\times 10^4)$	STOXX50E	DJI	S&P 500
Mean	1.064	0.860	0.851
St.dev	0.674	0.622	0.617
Skewness	3.217	3.595	3.354
Kurtosis	23.109	26.056	22.544
Jarque-Bera	1.012E+05 (0.000)	1.302E+06 (0.000)	9.537E+04 (0.000)
ADF	−10.752 (0.000)	−10.300 (0.000)	−10.078 (0.000)
Daily returns $(\times 10^2)$	**STOXX50E**	**DJI**	**S&P 500**
Mean	-0.020	0.018	0.008
St.dev	1.295	1.102	1.122
Skewness	-0.344	-0.068	-0.220
Kurtosis	9.134	11.446	11.365
Jarque-Bera	8649.757 (0.000)	15928.63 (0.000)	15675.15 (0.000)
ADF	−75.444 (0.000)	−56.345 (0.000)	−57.012 (0.000)
Log realized variance $(\times 10^4)$	**STOXX50E**	**DJI**	**S&P 500**
Mean	-0.164	-0.646	-0.677
St.dev	1.051	1.120	1.145
Skewness	0.011	0.433	0.379
Kurtosis	5.715	3.517	3.395
Jarque-Bera	1673.821 (0.000)	227.412 (0.000)	163.373 (0.000)
ADF	−2.862 (0.000)	−2.862 (0.000)	−2.862 (0.000)

Note: the notation $(\times 10^2)$ or $(\times 10^4)$ means that the data has been multiplied by this factor. The realized volatility is calculated by taking the square root of the realized variance and the natural logarithm of the realized variance is taken to tranform the data to log realized variance. For the Jarque-Bera test the null hypothesis is of normality and for the Augmented Dickey-Fuller (ADF) test the null hypothesis is that the series has a unit root. The ADF test statistic has corresponding one-sided p-values (MacKinnon, 1996). For the tests, we assume a significance level of 95%. Finally, the source of data is OxfordMan.

4 Models

In this section we introduce the real-time GARCH, and the real-time STES models. Also, we give the specification of the standard volatility models.

4.1 GARCH Models

The specification of the RT-GARCH(1,1) model is given by

$$\varepsilon_t = \lambda_t z_t, \tag{1}$$

$$\lambda_t^2 = \omega + \beta\lambda_{t-1}^2 + \alpha\varepsilon_{t-1}^2 + \phi \underbrace{\frac{\varepsilon_t^2}{\lambda_t^2}}_{=z_t^2}, \qquad (\omega, \beta, \alpha, \phi) \geq 0, \tag{2}$$

where $\varepsilon_t = r_t - \mu$ and z_t follows a Student's t-distribution. The standard GARCH(1,1) model is obtained by setting $\phi = 0$.

To account for the different effect of positive and negative shocks on the conditional variance of returns we follow the work of Glosten et al. (1993). Therefore, the baseline model in equation (2) can be extended to incorporate the feedback effect as follows

$$\lambda_t^2 = \omega + \beta\lambda_{t-1}^2 + \alpha\varepsilon_{t-1}^2 + \phi_1 z_t^2 1_{(z_t>0)} + \phi_2 z_t^2 1_{(z_t\leq 0)}. \tag{3}$$

The RT-GARCH from equation (2) is obtained by setting $\phi_1 = \phi_2$ which nests GARCH with $\phi_1 = \phi_2 = 0$.

We also incorporate the different effect of positive and negative returns on the conditional volatility. Different as in Smetanina (2017), we refer to this as leverage effect. We extend the real-time GARCH model with feedback (RT-GARCH-F) from equation (3) as follows

$$\lambda_t^2 = \omega + \beta\lambda_{t-1}^2 + \alpha_1\varepsilon_{t-1}^2 1_{(\varepsilon_t>0)} + \alpha_2\varepsilon_{t-1}^2 1_{(\varepsilon_t\leq 0)} + \phi_1 z_t^2 1_{(z_t>0)} + \phi_2 z_t^2 1_{(z_t\leq 0)}. \tag{4}$$

The model nest the RT-GARCH-F model by setting $\alpha_1 = \alpha_2$, which nest the RT-GARCH model by also setting $\phi_1 = \phi_2$. The standard GARCH is obtained by also setting $\phi_1 = \phi_2 = 0$.

Contrary to the real-time GARCH with leverage and feedback (RT-GARCH-LF) given by equation (4) the RT-STGARCH(1,1) models permit intermediate regimes between high volatility and low volatility. The specification of the RT-LSTGARCH(1,1) is given by

$$\begin{aligned}\lambda_t^2 = \omega + \beta\lambda_{t-1}^2 + \alpha_1\varepsilon_{t-1}^2(1 - F(\varepsilon_{t-1},\gamma)) + \alpha_2\varepsilon_{t-1}^2 F(\varepsilon_{t-1},\gamma) + \phi_1 z_t^2(1 - F(\varepsilon_{t-1},\gamma)) \\ + \phi_2 z_t^2 F(\varepsilon_{t-1},\gamma), \qquad (\omega, \beta, \alpha_1, \alpha_2, \phi_1, \phi_2) \geq 0,\end{aligned} \tag{5}$$

where

$$F(\varepsilon_{t-1},\gamma) = \frac{1}{1 + e^{-\gamma\varepsilon_{t-1}}}, \qquad \gamma > 0, \tag{6}$$

is a transition function for which ε_{t-1} is the transition variable and γ is the smoothness parameter. The transition function $F(\varepsilon_{t-1},\gamma)$ is a logistic function which varies between 0 and 1.

The RT-ESTGARCH(1,1) model specification is given by the same equation (5), but now the transition function $F(\varepsilon_{t-1},\gamma)$ in equation (6) is replaced by the following exponential transition function

$$F(\varepsilon_{t-1},\gamma) = 1 - e^{-\gamma\varepsilon_{t-1}^2}. \tag{7}$$

When γ becomes large, the smooth transition function $F(\varepsilon_{t-1}, \gamma)$ becomes steep and the model converges to the RT-GARCH-LF model from equation (4). Also, the standard LSTGARCH and ESTGARCH are obtained by setting $\phi_1 = \phi_2 = 0$. While for the logistic function the behavior of the conditional variance depends on the size of the shock, the exponential function enables the behavior of the conditional variance to depend on the magnitude of the shock. It is symmetric with respect to the sign of the past shock. Again, for the standard models it holds that for large γ the function $F(\varepsilon_{t-1}, \gamma)$ in LSTGARCH and ESTGARCH becomes steep. In this case, the smooth-transition models converge to the TGARCH model of Zakoian (1994).

For the GARCH models we obtain the parameter estimates using Maximum Likelihood Estimation (MLE) analysis. The log-likelihood function can be written as follows

$$\hat{\theta}_{ML} = \arg\max \log L_t(\theta) = \arg\max \sum_{t=1}^{T} l_t(\theta), \tag{8}$$

where

$$l_t(\theta) = -\log \lambda_t + \log \frac{\Gamma(\frac{v+1}{2})}{\Gamma(v/2)\sqrt{\pi(v-2)}} - \frac{v+1}{2} \log\left(1 + \frac{\varepsilon_t^2}{(v-2)\lambda_t^2}\right), \qquad v > 2. \tag{9}$$

Here, θ is the vector of parameters to be estimated for each model, λ_t^2 is obtained recursively for each model, $\Gamma(v)$ is the gamma function and v is the parameter that measures the tail thickness.

4.2 STES Models

The specification of the RT STES model is given by

$$\lambda_t^2 = \alpha_{t-1}\varepsilon_{t-1}^2 + (1-\alpha_{t-1})\lambda_{t-1}^2 + \phi z_t^2, \qquad \phi \geq 0, \tag{10}$$

where $\alpha_{t-1} = \frac{1}{1+e^{\beta+\kappa V_{t-1}}}$, $(\beta, \kappa) \geq 0$, V_{t-1} is the transition variable and β and κ are constant parameter estimates. The logistic function restricts α_{t-1} to lie between 0 and 1 and behaves according to changes in the transition variable V_{t-1}. As sign and size of past shocks have been used in non-linear volatility models, we propose the use both $|\varepsilon_{t-1}|$ and ε_{t-1} as transition variable V_{t-1} in the STES model. The model is called RT STES-AE when $|\varepsilon_{t-1}|$ is the transition variable; and it is called RT STES-E when ε_{t-1} is the transition variable. Again, the standard STES-AE and STES-E models are obtained by setting $\phi = 0$.

For the STES models we estimate the parameters by the minimization of the squared prediction errors. As proposed by Taylor (2004b) we optimise by using realized volatility as a proxy for actual volatility. This proposal amounts to the parameters being derived using the following minimisation

$$\hat{\alpha} = \arg\min \sum_t (\lambda_{Rt} - \hat{\lambda}_t)^2, \tag{11}$$

where λ_{Rt} is calculated using intra-day data.

5 News Impact Curve

A measure of how new information affects the volatility of returns is the news impact curve, defined by Engle and Ng (1993). Let us denote by λ^2 the unconditional variance of ε_t. By replacing the lagged variance with the unconditional variance we find the NIC of the GARCH(1,1) model. Using the same principle, we derive the NIC for the other STES models. For the RT-STES models, the news impact curve is given by the following equation:

$$E[r_{t+1}^2|\mathcal{F}_t] = \mu^2 + \phi\kappa + b_t, \tag{12}$$

with $b_t = (1-\alpha_t)\lambda_t^2 + \alpha_t\varepsilon_t^2 + \phi z_t^2$ with α_t calculated as in Section 4.2.

In Figure 1 we compare the news impact curves for the RT-STES models for different values of ϕ. In case of $\phi = 0$, the model corresponds to the standard STES(1,1) model. We observe that for values of ϕ for the RT-STES models the volatility responds much more to shocks than the standard STES models. This effect is more extreme for larger values of ϕ. Furthermore, the difference in volatility response to shocks is larger for the RT-STES-E model than for the RT-STES-AE model, due to the smoothing parameter α_{t-1} being larger for negative shocks

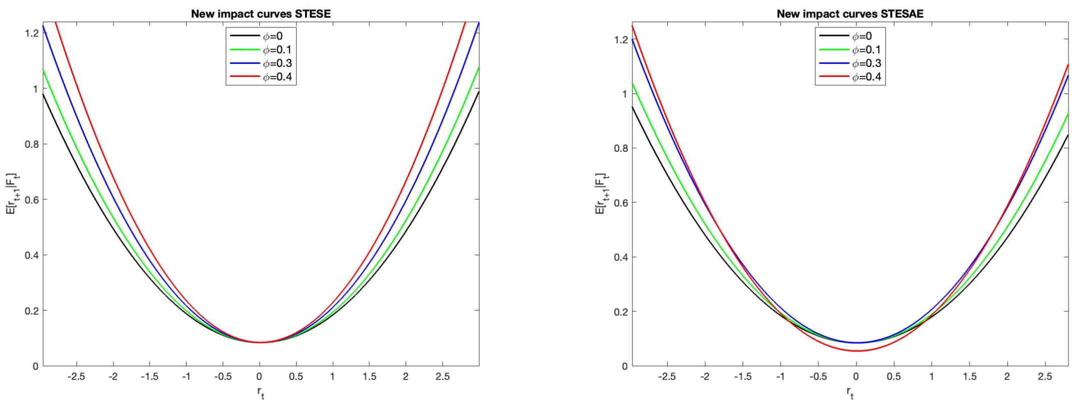

Figure 1: The news impact curves for the STES models for different values of ϕ. For both the graphs the values are obtained from the S&P500 stock return data where we set $[\beta, \kappa]$ in α_{t-1} equal to [2.220, 0.05].

6 Forecast Evaluation

In order to compare the forecast performance of the models we evaluate the volatility forecast performance in absolute and relative terms. An accurate forecast is when the prediction error is close to zero. The forecast error is defined as follows

$$\varepsilon_{T+k|T} = \lambda_{T+k}^2 - \hat{\lambda}_{T+k}^2. \tag{13}$$

In terms of the accuracy of volatility forecasts, researchers often refer to loss functions. Patton (2011) finds that among nine commonly used loss functions to compare volatility forecasts, the QLIKE and squared-error (MSE) are the only robust to noise in the volatility proxy. They also proposed a new parametric family of robust and homogeneous loss functions, which we use for the forecast evaluation. The family nests both MSE and the QLIKE loss functions and the formulation is given by

$$L(\lambda_{RV}^2, \hat{\lambda}^2; b) = \begin{cases} \frac{1}{(b+1)(b+2)}(\lambda_{RV}^{2b+4} - \hat{\lambda}^{b+2}) - \frac{1}{b+1}\hat{\lambda}^{b+1}(\lambda_{RV}^2 - \hat{\lambda}^2), & for\ b\ \notin \{-1,-2\} \\ \hat{\lambda}^2 - \lambda_{RV}^2 + \lambda_{RV}^2 \log \frac{\lambda_{RV}^2}{\hat{\lambda}^2}, & for\ b\ = -1 \\ \frac{\lambda_{RV}^2}{\hat{\lambda}^2} - \log \frac{\lambda_{RV}^2}{\hat{\lambda}^2} - 1, & for\ b\ = -2. \end{cases} \tag{14}$$

The MSE loss function is obtained when $b = 0$ and the QLIKE loss function is obtained when $b = -2$. The formulation of the two loss functions is given by

$$\underset{b=0}{MSE} : L(\lambda_{RV}^2, \hat{\lambda}^2) = (\lambda_{RV}^2 - \hat{\lambda}^2)^2, \tag{15}$$

$$\underset{b=-2}{QLIKE} : L(\lambda_{RV}^2, \hat{\lambda}^2) = log\hat{\lambda}^2 + \frac{\lambda_{RV}^2}{\hat{\lambda}^2}. \tag{16}$$

In terms of relative volatility forecasting performance, we need a measure that can determine whether forecasts are statistically different. Diebold and Mariano (2002) propose the Diebold-Mariano (DM) testing of equality of forecast accuracy. Name the k-step ahead forecasts of model i and j as $\hat{\lambda}_{i,t+k|t}$ and $\hat{\lambda}_{j,t+k|t}$ and the corresponding forecast errors as $\varepsilon_{i,t+k|t}$ and $\varepsilon_{j,t+k|t}$ both calculated by equation (13). We say that the two forecasts have equal accuracy if and only if the loss differential has zero expectation for all t. The loss differential is defined as follows

$$d_{t+k} = \varepsilon_{i,t+k|t} - \varepsilon_{j,t+k|t}. \tag{17}$$

The DM statistic is calculated as

$$DM = \frac{\overline{d}}{\sqrt{V(d_{t+k})/T}} \overset{asy}{\sim} N(0,1), \tag{18}$$

where $\overline{d}$ is the sample mean loss differential, T is the number of predictors and $V(d_{t+k})$ is the variance of d_{t+k} which is calculated as follows

$$V(d_{t+k}) = \frac{1}{T-1}\sum_{t=1}^{T}\left(d_{t+k} - \overline{d}\right)^2. \tag{19}$$

We also evaluate our forecasts with Value-at-Risk loss function. We compute 1-step VaR forecasts and the VaR is given by

$$VaR_t(1-q,1) = \hat{\mu} + t_q^{df}\hat{\lambda}_{t+1|t}\sqrt{\frac{v-2}{v}}, \tag{20}$$

where $\hat{\mu}$ is the estimated constant, $\hat{\lambda}_{t+1|t}$ is the volatility forecasts and t_q^v is the quantile of the Student's t-distribution with the optimal degree of freedom v given by the parameter estimation for each model.

In order to evaluate the VaR estimates we use the process of backtesting which determines how well our models perform. For evaluation of VaR forecasts, we define the indicator function I_{t+1} as follows

$$I_{t+1} = \begin{cases} 1 & \text{if } r_{t+1} < VAR_t(1-q,1); \\ 0 & \text{if } r_{t+1} \geq VAR_t(1-q,1); \end{cases} \tag{21}$$

I_{t+1} indicates whether the VaR forecast is violated or not. First, we compute the violation ratio (VR), which is the ratio between the number of returns that violated the VaR forecast to the number of the expected violations based on a significance level α, for which we take 5% or 1%. Smetanina (2017) finds that if the model is accurate, the VR is expected to be exactly 1. A model has good forecasts if the VR is between 0.8 and 1.2; and a model has inaccurate forecasts if the VR > 0.5 or VR < 1.5. Also, we conduct the likelihood ratio test of correct coverage of VaR estimates (Christoffersen, 1998). The likelihood ratio test is computed as

$$LR_{cc} = -2\log\left(\frac{\mathcal{L}(q; I_1, I_2, ..., I_T)}{\mathcal{L}(\hat{\Pi}_1; I_1, I_2, ..., I_T)}\right) \overset{asy}{\sim} \chi^2(2), \tag{22}$$

where $\mathcal{L}(q; I_1, I_2, ..., I_T) = (1-q)^{T_0} q^{T_1}$ corresponds to the null hypothesis of $E[I_{t+1}|I_t] = q$, for all t with $T_1 = \sum_{t=1}^{T} i_t$ the number of violations and $T_0 = T - T_1$ the number of non-violations. Also, $\mathcal{L}(\hat{\Pi}; I_1, I_2, ..., I_T) = (1-\pi_{01})^{T_{00}}(\pi_{01})^{T_{01}}(1-\pi_{11})^{T_{10}}(\pi_{11})^{T_{11}}$ corresponds to the alternative hypothesis with the estimate of the first order Markov-chain with transition probability matrix Π_1 given by $\hat{\Pi}_1 = \begin{pmatrix} \frac{T_{00}}{T_{00}+T_{01}} & \frac{T_{01}}{T_{00}+T_{01}} \\ \frac{T_{10}}{T_{10}+T_{11}} & \frac{T_{11}}{T_{10}+T_{11}} \end{pmatrix}$ where T_{ij} is the number of observations such that $I_{t+1} = j$ and $I_t = i$.

7 Results

In this section we present the in-sample and out-of-sample results. The parameter estimates are presented in Table 7 to Table 10 in the Appendix. We focus on two main in-sample results. First, the standard error of the transition parameter γ for the (RT-)LSTGARCH and (RT-)ESTGARCH models are large, resulting in small t-statistics. For large γ the standard errors become substantially larger. González-Rivera (1998) shows that this is due the behavior of the transition function $F(\varepsilon_{t-1}, \gamma)$.

Second, we observe positive values for ϕ for the real-time GARCH models which are significant different from zero for most of the stock indices. The RT-GARCH-F , RT-GARCH-LF and RT-ESTGARCH models have significantly positive ϕ_2, whereas the RT-LSTGARCH has significantly positive ϕ_1. For both the real-time STES models the real-time variable ϕ is not significantly different from zero which makes the models equivalent to the standard STES models. The first subsections shows the first-stage volatility forecasting results. In the second subsection we present the second-stage volatility forecasting results. Finally, we show the Value-at-Risk forecasting results. For simplicity, we show 1-step ahead volatility and VaR forecasts.

7.1 First-Stage Volatility Forecasts

For the out-of-sample volatility evaluation we present the full-sample results in Table 2. An important finding is that the standard STES models have equal losses as the real-time STES models. As before-mentioned the models are equivalent and equal losses are expected.

For the 1-step ahead volatility forecasts we focus on four main findings. First, when investigating which model provides the best volatility predictions, we observe that RT-LSTGARCH. (RT-)STES-AE and the (RT-)STES-E are the three best performing models, but we do not observe a clear winner. Whereas for the STOXX50E stock index the RT-LSTGARCH has the best forecasting performance, the STES models outperform the RT-LSTGARCH for the DJI and S&P500 stock indices. Because the RT-LSTGARCH is outperforming the RT-GARCH (with leverage and feedback), it is beneficial to incorporate intermediate regime approach as it is expected that the transition from a low- to a high-volatility regime is smoother during large periods with good economic conditions.

Second, when investigating the STES models (group 3), for $b < 0$, we observe that the (RT-)STES-AE are the better performing models and for $b > 0$ the (RT-)STES-E have better forecasting performance. This suggest that the (RT-)STES-AE is likely to overpredict volatility and the (RT-)STES-E is likely to underpredict volatility as underpredictions/overpredictions are heavilier penalized for negative/positive b-values.

Third, to investigate whether assuming current information provides better volatility predictions we divide the models in three subgroups. For the GARCH models (group 1 and 2), we observe that by also incorporating current information the RT-GARCH models are outperforming the standard GARCH models. In contrary to the GARCH models, the RT-STES models (group 3) have equal forecasting performance.

Fourth, we observe lower losses for models that incorporate the asymmetric response of volatility to shocks and to past returns. For all three stock indices the RT-GARCH-F and RT-GARCH-LF models outperform the RT-GARCH model. Also, the standard GARCH model is outperformed by the LSTGARCH model.

Table 2: Forecasts evaluation based on losses out of the family of robust loss functions (full sample)

1-step ahead volatility forecasts

Model		STOXX50E				DJI				S&P500			
		$b=-2$	$b=-1$	$b=0$	$b=1$	$b=-2$	$b=-1$	$b=0$	$b=1$	$b=-2$	$b=-1$	$b=0$	$b=1$
GARCH	*1*	0.273	0.426	4.291	55.548	0.393	0.446	4.210	55.579	0.394	0.401	3.313	35.766
RT-GARCH		0.243	0.374	4.020	55.332	0.316	0.384	4.016	55.037	0.321	0.335	3.048	34.748
RT-GARCH-F		0.223	0.342	3.947	56.103	0.306	0.354	3.888	54.667	0.332	0.331	3.119	35.534
RT-GARCH-LF		0.219	0.328	3.488	48.651	0.352	0.384	4.094	55.830	0.352	0.308	2.753	33.133
LSTGARCH	*2*	0.255	0.384	3.891	52.206	0.384	0.423	4.098	55.176	0.388	0.356	3.184	35.294
ESTGARCH		0.273	0.426	4.291	55.548	0.393	0.446	4.214	55.610	0.394	0.400	3.308	35.756
RT-LSTGARCH		**0.209**	0.342	**3.436**	**44.824**	**0.297**	0.328	3.548	52.227	0.312	0.300	2.747	32.808
RT-ESTGARCH		0.247	0.378	4.044	55.545	0.326	0.391	4.047	55.181	0.377	0.350	3.055	34.798
STES-E	*3*	0.262	0.350	3.492	46.371	0.323	0.315	**3.307**	**49.824**	0.316	0.277	**2.468**	**30.171**
STES-AE		0.243	**0.341**	3.851	55.604	0.300	**0.309**	3.424	51.547	**0.303**	**0.271**	2.567	31.819
RT-STES-E		0.262	0.350	3.492	46.371	0.323	0.315	**3.307**	**49.824**	0.316	0.277	**2.468**	**30.171**
RT-STES-AE		0.243	**0.341**	3,851	55.604	0.300	**0.309**	3.424	51.547	**0.303**	**0.271**	2.567	31.819

Note: $p = 0$ corresponds to the MSE loss and $p = -2$ corresponds to the QLIKE loss. We include $p = -1$ and $p = 1$ to heavier penalty either on under-prediction or over-prediction respectively. The numbers in italics divides the models in three subgroups: 1, 2 and 3. Finally, the three best performing model are given in bold. The power x above the loss value means that the two models have equal losses and are at place x, where $x = 1, 2, 3$.

For the out-of-sample (pre-crisis period) volatility forecasting evaluation we present the results in Table 13 in Appendix A.2. In comparison to the full-sample period there are two main differences and two main similarities. Similar as for the full-sample period, for the pre-crisis period the RT-LSTGARCH and the (RT-)STES-AE models have the best volatility forecasts. However, for the pre-crisis period we observe a pattern. Whereas the RT-LSTGARCH is outperformed by the STES-AE model for $b < 0$, the model is outperforming the STES-AE model for $b > 0$. Again, this suggest that the STES-AE model is likely to overpredict volatility and the RT-LSTGARCH is likely to underpredict volatility. Also, similar as for the full-sample period, it is beneficial to incorporate the intermediate regime approach as less extreme shocks are expected in the period before a crisis.

Further, similar as for the full-sample period, models that also assume current information provide better volatility predictions. In group 1 and 2 the real-time GARCH models outperform the standard GARCH models for every stock index.

In contrary to the full-sample period, for the STES models (group 3), we observe that the (RT-)STES-AE model is outperforming the (RT-)STES-E model for $b = -2, -1, 0$ and 1 for every stock index. Because the STES-E model takes large negative shocks into account it follows that it has relatively worse volatility forecasting pre-crisis as less extreme shocks occur during periods with good economic conditions.

Further, for the pre-crisis period the picture is somewhat different when evaluation whether models that incorporate the asymmetric response of volatility to shocks and past returns provide better volatility predictions. It is still beneficial to incorporate this asymmetric response with a smooth transition function, but no longer with a sharp transition function such as in the RT-GARCH-F model.

For evaluation of the out-of-sample volatility forecasts (crisis and post-crisis period) we present the results in Table 14 in Appendix A.2. The crisis and post-crisis results are similar as the full-sample period results, but we observe two differences. First, relative to the full-sample period, the RT-ESTGARCH model provides better volatility predictions. In the crisis and post-crisis period the losses are substantially closer to the RT-LSTGARCH model and it even outperforms the RT-LSTGARCH model for every stock index for $b = -2$. The RT-ESTGARCH model incorporates the large negative shocks in the period with less favourable economic conditions and this improves the volatility forecasting performance substantially.

Second, we observe a similar picture for the STES-E model. It is outperforming the STES-AE model for two out of three stock indices, which suggests that in times of a crisis it is beneficial to take these large negative shocks into account.

7.2 Second-Stage Volatility Forecasting Results

In order to investigate second-stage volatility forecasting performance, we performed the DM test two times: First, when (RT-)STES-AE is chosen as the benchmark (Table 3) and next when the RT-LSTGARCH model is chosen as the benchmark (Table 4).

Table 3 presents the relative forecasting performance results for the full-sample period. The good performing STES-AE model is chosen as a benchmark, to investigate whether there is a single model that provides the best volatility predictions. From Table 3 we observe that only the RT-LSTGARCH and the (RT-)STES-E models do not have significantly worse forecasting performance than the (RT-)STES-AE for all three stock indices. The RT-GARCH-LF model is the fourth best performing model because it has equal forecasting performance as the (RT-)STES-AE model for two out of three stock indices. This confirms that there is no clear winner that provides the best volatility predictions.

Table 3: Diebold-Mariano test statistic against the STES-AE model as a benchmark, with realized variance as a proxy for volatility (full-sample)

STES-AE as benchmark						
	STOXX50E		DJI		S&P500	
Model	$\bar{d}$	DM	$\bar{d}$	DM	$\bar{d}$	DM
GARCH	1.004	4.708*	0.834	3.331*	0.835	5.403*
RT-GARCH	0.529	1.521	0.709	3.331*	0.579	2.416*
RT-GARCH-L	0.455	1.148	0.581	2.717*	0.650	2.215*
RT-GARCH-LF	-0.004	-0.025	0.787	2.355*	0.285	1.706
LSTGARCH	0.399	1.436	0.791	2.884*	0.716	2.646*
ESTGARCH	0.798	2.604*	0.907	3.330*	0.840	3.257*
RT-LSTGARCH	-0.056	-0.509	0.242	1.830	0.279	1.817
RT-ESTGARCH	0.552	1.565	0.740	2.757*	0.586	2.458*
STES-E	0.000	1.668	0.000	1.866	0.000	-0.838

Note: $^{*}p < 0.05$; the mean loss differential is denoted as $\bar{d}$; and DM is the Diebold-Mariano test statistic.

Table 15 and Table 16 in Appendix A.3 show the relative forecasting performance results for the pre-crisis period and crisis and post-crisis period respectively. The picture is similar as for the full-sample period. However, the RT-GARCH-LF is outperformed by the STES-AE for every stock index in the pre-crisis period and for two out of three stock indices in the crisis and post-crisis period.

Notable is that the negative values for $\bar{d}$ for the STES-E model in the crisis and post-crisis period indicate that it beneficial to take these large negative shocks into account during periods with less favourable economic conditions.

Table 4 presents the full-sample relative forecasting performance. The RT-LSTGARCH model is chosen as a benchmark, to investigate if it is significantly better to incorporate the intermediate regime approach and if assuming current information improves the volatility forecasts significantly. From Table 4 it could be argued that it is beneficial to incorporate a intermediate regime switching model such as the RT-LSTGARCH within the GARCH approach, because it significantly outperforms the RT-GARCH and RT-GARCH-F models for two out of three stock indices. However, it significantly outperforms the other RT-GARCH-LF model for only one out of three stock indices. A two-regime GARCH model may be better, because it has significantly equal forecasting performance and it is the simpler model.

Further, we find that it is beneficial to incorporate current information in the smooth transition GARCH models as the RT-LSTGARCH significantly outperforms the LST-GARCH model (which only assumes past information) for the DJI and S&P500 stock indices.

Table 4: Diebold-Mariano test statistic against the RT-LSTGARCH model as a benchmark, with realized variance as a proxy for volatility (full-sample)

RT-LSTGARCH as benchmark						
	STOXX50E		DJI		S&P500	
Model	$\bar{d}$	DM	$\bar{d}$	DM	$\bar{d}$	DM
RT-GARCH	0.583	1.395	0.467	3.267*	0.301	2.908*
RT-GARCH-F	0.510	1.092	0.339	2.670*	0.372	2.412*
RT-GARCH-LF	0.051	0.215	0.546	2.778*	0.006	0.128
LSTGARCH	0.455	1.384	0.549	3.415*	0.437	3.113*

Note: $^*p < 0.05$; the mean loss differential is denoted as $\bar{d}$; and DM is the Diebold-Mariano test statistic.

Table 17 and Table 18 in Appendix A.3 show the relative forecasting performance results for the pre-crisis period and crisis and post-crisis period respectively. Still, it is beneficial to incorporate current information in the smooth transition GARCH model as the RT-LSTGARCH model provides significantly better volatility predictions than the LSTGARCH model for two out of three stock indices.

In contrary to the full-sample the RT-LSTGARCH significantly outperforms the RT-GARCH-LF model for two out of three stock indices. That suggests that a intermediate regime switching model has the preference above the two-regime GARCH model.

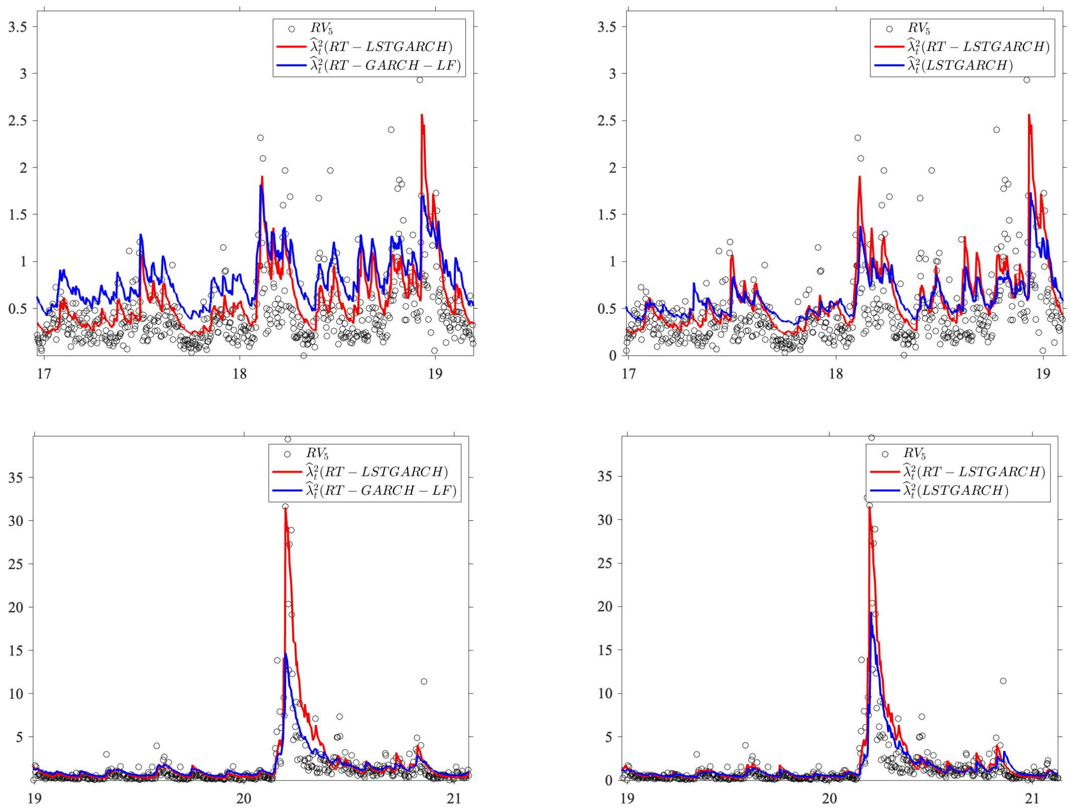

Figure 2: Out-of-sample volatility forecasts of the real-time logistic smooth transition GARCH versus the real-time GARCH with leverage and feedback zoomed in the sample period from 01/01/2017 to 01/02/2019 (left upper panel); zoomed in the sample period 01/01/2019 to 01/02/2021 (left under pannel); and real-time logistic smooth transition GARCH versus the standard logistic smooth transition GARCH for the sample period from 01/01/2017 to 01/02/2019 (right upper panel) and zoomed in for the sample period from 01/01/2019 to 01/02/2021 (right under panel) For the four graphs the forecasts are obtained from the STOXX50E stock returns data. The realized variance based on 5-minute intra-day returns is used as implied volatility.

The left upper/under panel of Figure 2 show the volatility forecasting pattern during periods of good/less favourable economic conditions of the RT-GARCH-LF against the RT-LSTGARCH model. Due to the smoothness parameter the RT-LSTGARCH model is able to adapt to the latest characteristics of the time series. Therefore, the RT-LSTGARCH models reacts much better to volatility increasing events. We observe less underpredictions in time of a crisis and less overpredictions in times of good economic conditions.

From the right panels of Figure 2 we observe that by also assuming current information the logistic smooth transition GARCH is able to react better to volatility increasing/decreasing events.

To conclude from Section 7.1 and Section 7.2, it is likely that the standard models underpredict or overpredict volatility. To minimize risk, you should depend on models that assume that volatility is a function of past- and current information, but also incorporate the asymmetry of the response of volatility to negative and positive shocks/past returns with a smooth transition function.

7.3 Value-at-Risk Forecasting Results

Table 5: Evaluation of 1-step 95% VaR forecasts

	STOXX50E		DJI		SP500	
Model	VR	LR	VR	LR	VR	LR
GARCH	0.756	5.451	0.624	5.533	0.588	5.555
RT-GARCH	0.907	5.363	0.841	5.398	0.770	5.441
RT-GARCH-F	0.832	5.404	0.638	5.524	0.609	5.542
RT-GARCH-LF	0.777	5.437	0.589	5.555	0.469	5.634
LSTGARCH	0.777	5.437	0.673	5.502	0.595	5.551
ESTGARCH	0.749	5.455	0.666	5.506	0.623	5.533
RT-LSTGARCH	0.845	5.396	0.624	5.533	0.609	5.542
RT-ESTGARCH	0.832	5.405	0.834	5.402	0.763	5.445

Note: the VR is the violation ratio calculated as (# of 95% VaR violations)/ (# of the expected violations). If $0.8 < \text{VR} < 1.2$ the model has good forecasts. LR_{cc} is statistic of the likelihood ratio test for correct coverage which has critical value of 5.99 for $p = 0.05$ and 9.21 for $p = 0.01$.

Table 6: Evaluation of 1-step 99% VaR forecasts

	STOXX50E		DJI		SP500	
Model	VR	LR	VR	LR	VR	LR
GARCH	0.997	9.010	0.701	9.053	0.560	9.094
RT-GARCH	0.910	9.028	0.771	9.020	0.910	9.029
RT-GARCH-F	1.065	8.998	0.639	9.046	0.398	9.073
RT-GARCH-LF	0.962	9.017	0.561	9.066	0.385	9.128
LSTGARCH	1.065	8.961	0.701	9.053	0.595	9.055
ESTGARCH	1.031	8.985	0.736	9.033	0.630	9.035
RT-LSTGARCH	0.997	9.010	0.561	9.059	0.358	9.100
RT-ESTGARCH	1.065	8.992	0.736	9.039	0.665	9.100

Note: the VR is the violation ratio calculated as (# of 99% VaR violations)/ (# of the expected violations). If $0.8 < \text{VR} < 1.2$ the model has good forecasts. LR_{cc} is statistic of the likelihood ratio test for correct coverage which has critical value of 5.99 for $p = 0.05$ and 9.21 for $p = 0.01$.

We also evaluate the forecasts with the 1-step VaR forecasts. Table 5 and 6 present the results. First, we observe that for all the models and stock indices the 95% VaR and 99% VaR estimates have correct conditional coverage: the fraction of VaR violations is approximately equal to the nominal probability, while also spread out over the sample.

When evaluating the 95% VaR forecasts, we observe that out of all the models only the RT-GARCH model and the RT-ESTGARCH models (for STOXX50E and DJI stock) and the RT-GARCH-F and RT-LSTGARCH models (STOXX50E stock) have good VR. This suggests that having a time-varying kursosis of returns plays a crucial role for forecasting.

However, for the 99% VaR forecasts, the picture is different. Now, the real-time smooth transition models do not produce better VaR forecasts than the standard smooth transition models. The RT-GARCH model is the only model that produces good VaR forecasts as the model has acceptable VR for two out of three stock indices.

Overall, for the real-time GARCH models incorporating the intermediate regime approach does not improve the 95% VaR and 99% VaR forecasts. As most of the models underpredict the VaR estimates, the real-time GARCH is preferred. For example, Table 5 and Figure 3 show that the RT-LSTGARCH model underpredicts the 95% VaR estimates. The risk of losing 95%/99% VaR of your set of investments with probability $q = 0.05/0.01$ is substantially decreased when estimated by the RT-GARCH model.

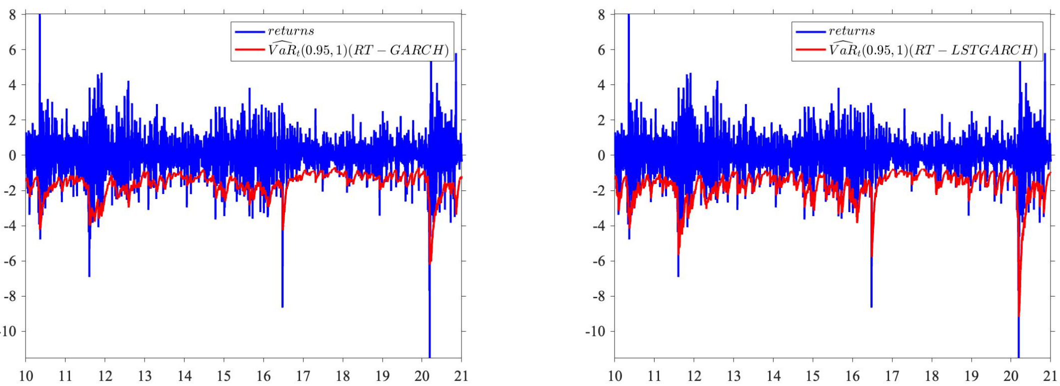

Figure 3: Out-of-sample volatility forecasts of the real-time logistic smooth transition GARCH versus the real-time GARCH for the full-sample period. For both graphs the forecasts are obtained from the STOXX50E stock returns data.

8 Discussion and Conclusion

It is notoriously difficult to predict volatility accurate based on historical returns. The standard GARCH models have been known as a parsimonious approach with good volatility forecasting results. Like most models it assumes that volatility is a function of past information only. Smetanina (2017) proposes the real-time GARCH model which also incorporates current information. The model can be extended to a two-regime model that incorporates leverage and feedback effects by differentiating between positive and negative shocks and returns. We proposes new models that allow for intermediate regimes but also incorporates current information: the smooth transition GARCH approach and the smooth transition exponential smoothing approach. In sum, we compare the new smooth transition models with the models proposed by Smetanina (2017) and find that it is beneficial to allow for intermediate regimes in real-time GARCH models as it significantly improves the out-of-sample volatility forecasts. Due to the smoothness parameter the smooth transition models are able to react much better to volatility changing events. However, the real-time smooth transition GARCH have worse Value-at-Risk forecasting performance. The real-time GARCH is preferred as it minimizes the risk of investment loss. Further, the smooth transition exponential smoothing models provide the best volatility predictions together with the real-time smooth transition logistic GARCH, but there is not a single model with the best volatility predictions. On the other hand, assuming current information for the smooth transition exponential smoothing formulation does not improve the volatility forecasts. We find that the real-time smooth transition exponential smoothing models are equivalent to the standard smooth transition exponential smoothing models with similar prediction losses. Also, by assuming current information allow the STES models to respond quicker to changes of the unconditional level of volatility.

The out performance of the smooth transition models is the largest during the pre-crisis period and the crisis and pre-crisis period. The new RT-LSTGARCH significantly outperforms the real-time GARCH models for at least two out of three stock indices, because in times of favourable/unfavourable economic conditions the RT-LSTGARCH has less overpredictions/underpredictions of implied volatility. However, for the Value-at-Risk predictions the RT-LSTGARCH underpredicts the VaR. The RT-GARCH provides much better 95%/99% VaR predictions. // Interesting may be to investigate whether other estimation approaches such as most likelihood estimation for the real-time smooth transition exponential smoothing models will provide positive values for ϕ that may result in improvement of the volatility predictions. We believe this should improve the volatility forecasts, because assuming current information allow the STES models to respond quicker to changes of the unconditional level of volatility Further, it may be possible to forecast volatility or VaR based on (the logarithm of) realized variance as it is known that using intra-day data improves the volatilty/VaR forecasts substantially. Table 1 shows that the logarithm of the realized variance is almost normally distributed. Estimating the parameters using the logarithm of the realized variance with most likelihood estimation of the normal distribution could have interesting results.

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A Appendix

A.1 Parameter Estimates

Table 7: Parameter estimates for every standard model with corresponding standard error in parentheses (full-sample)

	STOXX50E	DJI	S&P 500
GARCH			
ω	0.015	0.010	0.009
	(0.047)	(0.003)	(0.003)
β	0.900	0.916	0.922
	(0.010)	(0.009)	(0.009)
α	0.096	0.079	0.075
	(0.010)	(0.009)	(0.009)
LSTGARCH			
ω	0.025	0.013	0.015
	(0.005)	(0.006)	(0.003)
β	0.903	0.921	0.916
	(0.012)	(0.010)	(0.011)
α_1	0.095	0.077	0.081
	(0.012)	(0.012)	(0.011)
α_2	0.046	0.045	0.041
	(0.009)	(0.010)	(0.008)
γ	25.128	140.036	172.152
	(103.7)	(1050.2)	(5707.1)
ESTGARCH			
ω	0.012	0.0007	0.007
	(0.007)	(0.002)	(0.007)
β	0.911	0.926	0.924
	(0.014)	(0.010)	(0.024)
α_1	0.086	0.072	0.074
	(0.014)	(0.010)	(0.041)
α_2	0.084	0.070	0.071
	(0.011)	(0.010)	(0.041)
γ	4.062	6.975	0.264
	(0.864)	(1.608)	(0.041)
STESE			
β	1.722	1.709	1.638
	(0.077)	(0.086)	(0.109)
κ	0.000	0.017	0.004
	(0.014)	(0.013)	(0.012)

STESAE			
β	1.418	1.387	1.341
	(0.087)	(0.092)	(0.098)
κ	0.145	0.168	0.171
	(0.014)	(0.016)	(0.015)

Table 8: Parameter estimates for every standard model with corresponding standard error in parentheses (pre-crisis period)

	STOXX50E	DJI	S&P 500
GARCH			
ω	0.028	0.025	0.022
	(0.015)	(0.012)	(0.013)
β	0.904	0.897	0.908
	(0.016)	(0.017)	(0.020)
α	0.088	0.092	0.083
	(0.015)	(0.016)	(0.018)
LSTGARCH			
ω	0.055	0.046	0.074
	(0.200)	(0.157)	(0.024)
β	0.896	0.898	0.871
	(0.023)	(0.239)	(0.028)
α_1	0.097	0.093	0.119
	(0.022)	(0.237)	(0.027)
α_2	0.046	0.035	0.0200
	(0.017)	(0.089)	(0.015)
γ	34.928	107.594	67.222
	(3866.6)	(1445.6)	(1528.1)
ESTGARCH			
ω	0.027	0.0007	0.025
	(0.012)	(0.027)	(0.014)
β	0.905	0.926	0.910
	(0.017)	(0.910)	(0.022)
α_1	0.089	0.072	0.082
	(0.016)	(0.081)	(0.020)
α_2	0.083	0.070	0.074
	(0.017)	(0.073)	(0.018)
γ	0.216	6.975	1.167
	(0.602)	(5.047)	(1.261)
STESE			
β	1.741	1.975	2.117
	(0.0126)	(0.092)	(0.187)
κ	0.000	0.000	0.010
	(0.034)	(0.016)	(0.037)

STESAE			
β	1.434	1.508	1.397
	(0.142)	(0.157)	(0.194)
κ	0.192	0.274	0.371
	(0.023)	(0.057)	(0.045)

Table 9: Parameter estimates for every standard model with corresponding standard error in parentheses (crisis and post-crisis period)

	STOXX50E	DJI	S&P 500
GARCH			
ω	0.032	0.019	0.021
	(0.010)	(0.005)	(0.006)
β	0.893	0.860	0.868
	(0.017)	(0.017)	(0.017)
α	0.093	0.132	0.122
	(0.015)	(0.017)	(0.017)
LSTGARCH			
ω	0.050	0.028	0.028
	(0.015)	(0.006)	(0.007)
β	0.888	0.872	0.875
	(0.022)	(0.016)	(0.019)
α_1	0.105	0.125	0.120
	(0.021)	(0.015)	(0.018)
α_2	0.035	0.045	0.049
	(0.013)	(0.015)	(0.015)
γ	50.001	508.291	30.613
	(1.770)	(2259.0)	(251.312)
ESTGARCH			
ω	0.021	0.017	0.018
	(0.011)	(0.006)	(0.0007)
β	0.912	0.873	0.878
	(0.018)	(0.020)	(0.019)
α_1	0.077	0.122	0.116
	(0.016)	(0.019)	(0.018)
α_2	0.080	0.118	0.114
	(0.018)	(0.021)	(0.020)
γ	11.381	1.028	1.437
	(66.423)	(2.282)	(18.479)
STESE			
β	1.781	1.549	1.507
	(0.070)	(0.102)	(0.117)
κ	0.004	0.023	0.000
	(0.012)	(0.015)	(0.015)

STESAE			
β	1.488	1.214	1.210
	(0.108)	(0.119)	(0.117)
κ	0.132	0.152	0.153
	(0.016)	(0.018)	(0.016)

Table 10: Parameter estimates for every real-time model with corresponding standard error in parentheses (full-sample)

	STOXX50E	DJI	S&P 500
RT-GARCH			
ω	0.003	0.001	0.005
	(0.006)	(0.005)	(0.012)
β	0.913	0.925	0.913
	(0.014)	(0.007)	(0.023)
α	0.069	0.059	0.081
	(0.013)	(0.004)	(0.052)
ϕ	0.008*	0.006*	0.002
	(0.006)	(0.004)	(0.017)
RT-GARCH-F			
ω	0.000	0.000	0.000
	(0.827)	(0.897)	(1.227)
β	0.914	0.927	0.916
	(0.286)	(2.402)	(2.885)
α	0.053	0.045	0.053
	(0.007)	(1.367)	(0.715)
ϕ_1	0.000	0.000	0.000
	(0.251)	(1.053)	(1.184)
ϕ_2	0.048*	0.036*	0.057*
	(0.583)	(0.224)	(0.731)
RT-GARCH-LF			
ω	0.000	0.000	0.000
	(0.570)	(1.128)	(0.007)
β	0.917	0.874	0.918
	(0.794)	(2.457)	(0.024)
α_1	0.025	0.047	0.022
	(0.969)	(0.742)	(0.041)
α_2	0.096	0.118	0.093
	(0.710)	(1.555)	(0.041)
ϕ_1	0.000	0.000	0.000
	(0.999)	(1.008)	(0.041)
ϕ_2	0.038*	0.042*	0.043*
	(0.101)	(0.580)	(0.041)

RT-LSTGARCH			
ω	0.012	0.003	0.001
	(0.006)	(0.042)	(1.051)
β	0.901	0.935	0.905
	(0.028)	(0.042)	(1.621)
α_1	0.160	0.061	0.104
	(0.040)	(0.398)	(1.734)
α_2	0.000	0.000	0.000
	(0.022)	(0.453)	(0.918)
ϕ_1	0.010*	0.021*	0.019
	(0.007)	(0.171)	(0.753)
ϕ_2	0.000	0.006	0.001
	(0.010)	(0.031)	(0.303)
γ	3.471	47.472	13.7467
	(0.519)	(53.83)	(251.4)
RT-ESTGARCH			
ω	0.004	0.001	0.001
	(0.002)	(0.007)	(0.229)
β	0.915	0.928	0.932
	(0.042)	(0.033)	(0.532)
α_1	0.067	0.057	0.057
	(0.024)	(0.018)	(0.282)
α_2	0.007	0.008	0.055
	(0.008)	(0.009)	(0.119)
ϕ_1	0.006	0.004*	0.002
	(0.747)	(0.005)	(0.040)
ϕ_2	0.030*	0.010*	0.005*
	(0.350)	(0.008)	(0.014)
γ	0.000	0.001	2.3552
	(0.421)	(0.008)	(3.553)
RT-STES-E			
β	1.722	1.709	1.638
	(1.417)	(3.861)	(2.882)
κ	0.000	0.017	0.004
	(0.209)	(0.317)	(0.353)
ϕ	0.000	0.000	0.000
	(0.375)	(0.880)	(0.979)
RT-STES-AE			
β	1.418	1.387	1.341
	(2.559)	(2.544)	(0.963)
κ	0.145	0.168	0.171
	(0.292)	(0.030)	(0.098)
ϕ	0.000	0.000	0.000
	(0.576)	(0.703)	(0.119)

Note: for the real-time variable ϕ the * corresponds to $p < 0.05$.

Table 11: Parameter estimates for every real-time model with corresponding standard error in parentheses (pre-crisis period)

	STOXX50E	DJI	S&P 500
RT-GARCH			
ω	0.036	0.017	0.021
	(0.720)	(0.007)	(0.011)
β	0.896	0.913	0.910
	(0.244)	(0.013)	(0.016)
α	0.078	0.058	0.063
	(0.178)	(0.009)	(0.016)
ϕ	0.001	0.009*	0.005*
	(0.665)	(0.013)	(0.009)
RT-GARCH-F			
ω	0.001	0.001	0.000
	(0.456)	(0.030)	(0.135)
β	0.926	0.944	0.940
	(0.205)	(0.216)	(1.246)
α	0.049	0.026	0.025
	(0.355)	(0.064)	(0.314)
ϕ_1	0.000	0.000	0.000
	(1.006)	(0.614)	(1.046)
ϕ_2	0.063*	0.054*	0.070*
	(0.281)	(0.065)	(0.684)
RT-GARCH-LF			
ω	0.000	0.000	0.000
	(0.222)	(0.047)	(0.014)
β	0.932	0.953	0.945
	(0.045)	(0.018)	(0.011)
α_1	0.012	0.002	0.011
	(0.059)	(0.011)	(0.024)
α_2	0.041	0.028	0.037
	(0.027)	(0.025)	(0.029)
ϕ_1	0.000	0.000	0.000
	(0.325)	(0.036)	(0.104)
ϕ_2	0.124*	0.062*	0.059*
	(0.036)	(0.036)	(0.039)

RT-LSTGARCH			
ω	0.012	0.003	0.012
	(0.011)	(0.006)	(1.051)
β	0.902	0.885	0.892
	(0.012)	(0.009)	(0.015)
α_1	0.140	0.192	0.182
	(0.012)	(0.017)	(0.016)
α_2	0.000	0.008	0.000
	(0.008)	(0.022)	(0.010)
ϕ_1	0.023*	0.011*	0.019*
	(0.023)	(0.018)	(0.032)
ϕ_2	0.000	0.000	0.000
	(0.009)	(0.005)	(0.010)
γ	1.046	1.024	1.045
	(0.020)	(0.012)	(0.026)
RT-ESTGARCH			
ω	0.006	0.007	0.008
	(0.009)	(0.007)	(0.027)
β	0.916	0.911	0.919
	(0.014)	(0.033)	(0.025)
α_1	0.058	0.083	0.076
	(0.015)	(0.018)	(0.019)
α_2	0.066	0.044	0.052
	(0.025)	(0.009)	(0.014)
ϕ_1	0.002	0.011	0.092
	(0.747)	(0.014)	(0.242)
ϕ_2	0.029*	0.000*	0.002*
	(0.003)	(0.008)	(0.002)
γ	1.126	0.579	1.242
	(0.029)	(0.117)	(0.144)
RT-STES-E			
β	1.741	1.975	2.117
	(2.311)	(11.275)	(3.214)
κ	0.000	0.000	0.010
	(0.406)	(0.262)	(0.373)
ϕ	0.000	0.000	0.000
	(0.293)	(0.743)	(0.703)
RT-STES-AE			
β	1.434	1.508	1.397
	(3.485)	(0.224)	(0.308)
κ	0.192	0.274	0.371
	(0.730)	(0.128)	(0.157)
ϕ	0.000	0.000	0.000
	(0.814)	(0.065)	(0.036)

Note: for the real-time variable ϕ the * corresponds to $p < 0.05$.

Table 12: Parameter estimates for every real-time model with corresponding standard error in parentheses (crisis and post-crisis period)

	STOXX50E	DJI	S&P 500
RT-GARCH			
ω	0.000	0.000	0.000
	(0.018)	(0.012)	(0.011)
β	0.912	0.877	0.870
	(0.025)	(0.010)	(0.019)
α	0.057	0.090	0.088
	(0.017)	(0.017)	(0.015)
ϕ	0.026*	0.016*	0.017*
	(0.011)	(0.008)	(0.010)
RT-GARCH-F			
ω	0.001	0.000	0.000
	(0.456)	(0.030)	(0.689)
β	0.905	0.887	0.891
	(0.205)	(0.216)	(0.245)
α	0.039	0.062	0.060
	(0.355)	(0.064)	(0.094)
ϕ_1	0.000	0.000	0.000
	(1.006)	(0.614)	(0.836)
ϕ_2	0.115*	0.064*	0.062*
	(0.281)	(0.065)	(0.048)
RT-GARCH-LF			
ω	0.000	0.000	0.000
	(0.532)	(0.867)	(0.769)
β	0.859	0.861	0.859
	(0.363)	(0.547)	(0.327)
α_1	0.060	0.053	0.011
	(0.983)	(0.899)	(0.573)
α_2	0.153	0.154	0.037
	(0.104)	(0.120)	(0.093)
ϕ_1	0.004	0.004	0.000
	(0.047)	(0.036)	(0.035)
ϕ_2	0.050*	0.046*	0.059*
	(0.069)	(0.093)	(0.046)

RT-LSTGARCH			
ω	0.015	0.008	0.004
	(0.011)	(0.006)	(0.009)
β	0.889	0.886	0.892
	(0.012)	(0.009)	(0.016)
α_1	0.099	0.110	0.102
	(0.012)	(0.017)	(0.014)
α_2	0.000	0.000	0.000
	(0.008)	(0.022)	(0.017)
ϕ_1	0.063*	0.024*	0.027*
	(0.023)	(0.018)	(0.038)
ϕ_2	0.006	0.009	0.014
	(0.009)	(0.005)	(0.008)
γ	7.437	6.027	10.885
	(0.020)	(0.012)	(0.024)
RT-ESTGARCH			
ω	0.0060	0.000	0.000
	(0.015)	(0.025)	(0.046)
β	0.915	0.889	0.882
	(0.067)	(0.077)	(0.047)
α_1	0.000	0.018	0.016
	(0.036)	(0.096)	(0.047)
α_2	0.054	0.083	0.087
	(0.058)	(0.029)	(0.032)
ϕ_1	0.023	0.026	0.028
	(0.547)	(0.036)	(0.253)
ϕ_2	0.028*	0.002*	0.002*
	(0.003)	(0.006)	(0.004)
γ	4.670	0.806	1.907
	(0.106)	(0.323)	(0.683)
RT-STES-E			
β	1.862	1.539	1.507
	(1.590)	(2.367)	(3.054)
κ	0.025	0.023	0.000
	(0.123)	(0.115)	(0.036)
ϕ	0.000	0.000	0.000
	(0.379)	(0.270)	(0.507)
RT-STES-AE			
β	1.484	1.214	1.210
	(1.543)	(0.125)	(0.129)
κ	0.132	0.152	0.153
	(0.334)	(0.040)	(0.020)
ϕ	0.000	0.000	0.000
	(0.712)	(0.063)	(0.017)

Note: for the real-time variable ϕ the * corresponds to $p < 0.05$.

A.2 Forecast Losses

Table 13: Forecasts evaluation based on losses out of the family of robust loss functions (pre-crisis period)

1-step ahead volatility forecasts

Model		STOXX50E $b=-2$	STOXX50E $b=-1$	STOXX50E $b=0$	STOXX50E $b=1$	DJI $b=-2$	DJI $b=-1$	DJI $b=0$	DJI $b=1$	S&P500 $b=-2$	S&P500 $b=-1$	S&P500 $b=0$	S&P500 $b=1$
GARCH	*1*	0.306	0.327	1.801	13.361	0.266	0.197	0.684	3.020	0.284	0.204	0.638	2.228
RT-GARCH		0.243	0.246	1.447	12.160	0.234	0.178	0.659	2.998	0.255	0.182	0.599	2.169
RT-GARCH-F		0.246	0.239	1.414	12.085	0.264	0.192	0.661	2.973	0.302	0.206	0.608	**2.133**
RT-GARCH-LF		0.226	0.227	1.397	12.187	0.225	0.173	0.649	2.986	0.272	0.191	0.601	2.163
LSTGARCH	*2*	0.341	0.342	1.746	12.950	0.319	0.224	0.695	2.977	0.400	0.271	0.683	2.139
ESTGARCH		0.305	0.324	1.779	13.233	0.280	0.205	0.693	3.020	0.298	0.211	0.642	2.215
RT-LSTGARCH		0.209	0.214	**1.306**	**11.330**	0.193	0.148	**0.585**	**2.867**	0.205	0.146	**0.499**	**1.971**
RT-ESTGARCH		0.246	0.250	1.473	12.256	0.207	0.165	0.642	2.983	0.221	0.166	0.578	2.152
STES-E	*3*	0.229	0.221	1.356	12.025	0.197	0.151	0.618	2.965	0.183	**0.141**	0.548	2.151
STES-AE		**0.199**	**0.206**	1.349	11.961	**0.178**	**0.144**	0.599	2.920	**0.178**	**0.141**	0.536	2.097
RT-STES-E		0.229	0.221	1.356	12.025	0.197	0.151	0.618	2.965	0.183	**0.141**	0.548	2.151
RT-STES-AE		**0.199**	**0.206**	1.349	11.961	**0.178**	**0.144**	0.599	2.920	**0.178**	**0.141**	0.536	2.097

Note: $p = 0$ corresponds to the MSE loss and $p = -2$ corresponds to the QLIKE loss. We include $p = -1$ and $p = 1$ to heavier penalty either on under-prediction or over-prediction respectively. The numbers in italics divides the models in three subgroups: 1, 2 and 3. Finally, the three best performing model are given in bold. The power x above the loss value means that the two models have equal losses and are at place x, where $x = 1, 2, 3$.

Table 14: Forecasts evaluation based on losses out of the family of robust loss functions (crisis and post-crisis)

1-step ahead volatility forecasts													
		STOXX50E				DJI				S&P500			
Model		$b=-2$	$b=-1$	$b=0$	$b=1$	$b=-2$	$b=-1$	$b=0$	$b=1$	$b=-2$	$b=-1$	$b=0$	$b=1$
GARCH	*1*	0.319	0.459	5.124	76.692	0.401	0.441	5.137	79.545	0.429	0.387	3.718	49.971
RT-GARCH		**0.257**	0.396	5.159	81.116	**0.285**	0.366	5.073	80.465	0.340	0.303	3.251	44.511
RT-GARCH-F		0.306	0.401	5.077	81.652	0.355	0.380	5.148	81.383	0.394	0.343	3.819	50.304
RT-GARCH-LF		0.280	**0.365**	**3.981**	61.773	0.350	0.341	4.589	76.973	0.397	0.306	3.243	45.636
LSTGARCH	*2*	0.320	0.432	4.673	71.464	0.403	0.431	5.186	80.576	0.434	0.382	3.789	49.041
ESTGARCH		0.319	0.475	5.370	79.359	0.406	0.450	5.233	80.289	0.430	0.393	3.776	48.398
RT-LSTGARCH		0.272	0.392	4.227	**61.287**	0.308	**0.303**	**3.920**	**69.626**	0.337	0.273	**2.799**	**40.457**
RT-ESTGARCH		0.269	0.407	5.182	80.801	0.301	0.355	4.812	78.263	0.334	0.315	3.609	48.344
STES-E	*3*	0.283	0.390	4.342	63.701	0.291	0.307	4.172	72.148	**0.297**	**0.255**	2.808	40.539
STES-AE		**0.257**	0.378	4.877	78.778	0.291	0.311	4.295	75.306	0.303	0.261	3.049	44.088
RT-STES-E		0.283	0.390	4.342	63.701	0.291	0.307	4.172	72.148	**0.297**	**0.255**	2.808	40.539
RT-STES-AE		**0.257**	0.378	4.877	78.778	0.291	0.311	4.295	75.306	0.303	0.261	3.049	44.088

Note: $p = 0$ corresponds to the MSE loss and $p = -2$ corresponds to the QLIKE loss. We include $p = -1$ and $p = 1$ to heavier penalty either on under-prediction or over-prediction respectively. The numbers in italics divides the models in three subgroups: 1, 2 and 3. Finally, the three best performing model are given in bold. The power x above the loss value means that the two models have equal losses and are at place x, where $x = 1, 2, 3$.

A.3 Diebold Mariano

Table 15: Diebold-Mariano test statistic against the STES-AE model as a benchmark, with realized variance as a proxy for volatility (pre-crisis period)

STES-AE as benchmark						
	STOXX50E		DJI		S&P500	
Model	$\bar{d}$	DM	$\bar{d}$	DM	$\bar{d}$	DM
GARCH	0.454	2.414*	0.088	7.814*	0.103	6.072*
RT-GARCH	0.132	5.126*	0.060	2.619*	0.063	5.601*
RT-GARCH-L	0.070	2.833*	0.063	3.889*	0.086	8.160*
RT-GARCH-LF	0.098	2.516*	0.051	5.120*	0.065	2.958*
LSTGARCH	0.399	2.312*	0.099	10.261*	0.148	5.835*
ESTGARCH	0.432	2.307*	0.096	7.895*	0.107	8.140*
RT-LSTGARCH	0.410	0.863	-0.015	-0.795	-0.036	-0.996
RT-ESTGARCH	0.103	3.391*	0.043	3.079*	0.042	4.645*
STES-E	0.005	0.006	0.019	1.934	0.013	0.734

Note: $^{*}p < 0.05$; the mean loss differential is denoted as $\bar{d}$; and DM is the Diebold-Mariano test statistic.

Table 16: Diebold-Mariano test statistic against the STES-AE model as a benchmark, with realized variance as a proxy for volatility (crisis and post-crisis period)

STES-AE as benchmark						
	STOXX50E		DJI		S&P500	
Model	$\bar{d}$	DM	$\bar{d}$	DM	$\bar{d}$	DM
GARCH	0.247	0.751	0.834	3.331*	0.835	5.403*
RT-GARCH	0.282	2.368*	0.709	3.331*	0.579	2.416*
RT-GARCH-L	0.200	1.136	0.581	2.717*	0.650	2.215*
RT-GARCH-LF	-0.895	-1.572	0.194	2.488*	0.194	2.778*
LSTGARCH	-0.204	-0.521	0.791	2.884*	0.716	2.646*
ESTGARCH	0.492	1.695	0.907	3.330*	0.840	3.257*
RT-LSTGARCH	-0.191	-0.220	0.242	1.830	0.279	1.817
RT-ESTGARCH	0.316	2.505*	0.740	2.757*	0.586	2.458*
STES-E	-0.535	-0.817	-0.223	-1.571	-0.241	-1.640

Note: $^{*}p < 0.05$; the mean loss differential is denoted as $\bar{d}$; and DM is the Diebold-Mariano test statistic.

Table 17: Diebold-Mariano test statistic against the RT-LSTGARCH model as a benchmark, with realized variance as a proxy for volatility (pre-crisis period)

RT-LSTGARCH as benchmark						
	STOXX50E		DJI		S&P500	
Model	$\bar{d}$	DM	$\bar{d}$	DM	$\bar{d}$	DM
RT-GARCH	0.140	0.645	0.062	1.307	0.099	2.237*
RT-ARCH-F	0.113	0.645	0.108	4.334*	0.116	3.135*
RT-GARCH-LF	0.141	0.635	0.053	2.003*	0.101	4.493*
LSTGARCH	0.442	1.118	0.101	3.988*	0.184	11.303*

Note: $^{*}p < 0.05$; the mean loss differential is denoted as $\bar{d}$; and DM is the Diebold-Mariano test statistic.

Table 18: Diebold-Mariano test statistic against the RT-LSTGARCH model as a benchmark, with realized variance as a proxy for volatility (crisis and post-crisis period)

RT-LSTGARCH as benchmark						
	STOXX50E		DJI		S&P500	
Model	$\bar{d}$	DM	$\bar{d}$	DM	$\bar{d}$	DM
RT-GARCH	1.023	1.545	1.152	2.687*	0.808	2.640*
RT-GARCH-F	0.941	1.314	1.325	2.683*	1.029	2.646*
RT-GARCH-LF	-0.153	-1.000	0.688	2.548*	0.443	2.479*
LSTGARCH	0.537	1.392	1.266	2.700*	0.989	2.784*

Note: $^{*}p < 0.05$; the mean loss differential is denoted as $\bar{d}$; and DM is the Diebold-Mariano test statistic.