Erasmus Universiteit Rotterdam

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Volatility Forecasts from CAViaR Models in Times of Financial Crisis

Abstract

This thesis forecasts volatility based on quantile forecasts. This method has the advantageous feature of not having to assume a distribution for the returns. Five CAViaR models are used to forecast quantiles of which the Asymmetric Slope and Improved Asymmetric Slope CAViaR models give the best results. This finding advocates for the leverage effect among returns. In addition, the results of the best performing CAViaR models were compared to the results of the GJRGARCH model, which does need distributional assumptions. In general, the CAViaR models forecasts volatility better for the one-day-ahead, 10-days-ahead and 20-days-ahead holding period. Furthermore, the CAViaR models also produced better results for a data set in which the forecast periods contains times of financial crisis whereas the estimation periods does not.

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1 Introduction

Management and risk decision makers often base their decisions on predicted volatility. In order to make good decisions, volatility should be forecasted accurately. There is an extensive literature on the popular GARCH models (Engle, 1982; Bollerslev, 1986), which estimate volatility reasonably well. However, although these models may allow the conditional volatility or conditional mean to vary, they assume that the shape of the conditional distribution is fixed over time. Due to this assumption the volatility forecasts resulting from GARCH models may be prone to error.

This potential problem can be avoided by using a semi-parametric model such as the Conditional Autoregressive Value at Risk (CAViaR) models by Engle & Manganelli (2004), which require no assumptions on the distribution of returns. Subsequently, volatility forecasts can be created from these quantile forecasts produced by CAViaR models. Taylor (2005) introduces a method that will be the basis for the upcoming thesis. In his article, Taylor shows that the volatility forecasts from the CAViaR models outperform the results of the traditional GARCH models. Taylor (2005) has inspired others too. For example, Huang (2012) uses the method and extends it such that the volatility forecasts are not estimated from one single symmetric pair of quantiles but incorporates all the estimated quantiles. In addition, Taylor himself has built further on his work by generating not only volatility forecasts from quantile estimates but also expected shortfall forecasts (Taylor, 2008).

Moreover, it is found that variance forecasts of the asymmetric CAViaR model are most explanatory (Taylor, 2005). The asymmetric CAViaR model considers the leverage effect, this finding can be paralleled with the observation that GARCH models which can deal with the leverage effect also perform relatively better than those that cannot (Alberg et al., 2008; Peters, 2001). This result is sustained by Şener et al. (2012), who also perform a comparison on various GARCH and CAViaR and who argue that the performance of Value at Risk (VaR) methods does not depend entirely on whether they are parametric, non-parametric of semi-parametric, but rather on whether the methods can model the asymmetry of the underlying data effectively. Building further on this observation, Huang et al. (2009) propose a new improved asymmetric CAViaR model which is more parsimonious. Whereas Huang et al. (2009) consider oil price risk in their empirical results, in this thesis it will be investigated whether this model also performs superior volatility forecasts for financial indices. It will be shown that the parsimonious model produces forecasts slightly lower quality compared to the asymmetric CAViaR model. The second way in which the thesis will contribute to the current literature is that the models will be applied to another data set and it will be examined whether the CAViaR models also work during times of financial crisis when the parameters are estimated in times of non-crisis. It will be demonstrated that the parameter estimates are robust to such a shift since the volatility forecasts for this data set are equally or even more explanatory of the realised returns.

This thesis is organised in the following way. First, in Section 2 related literature will be discussed. Then, in Section 3, an overview of the GARCH and CAViaR models is given and the tests according to which the models will be evaluated are introduced. Thereafter, a description of the data that will be used for the empirical analysis is given in Section 4. In Section 5, the results for the estimation and forecast will be examined. This section will also compare the performance of the models. Finally, this thesis will close with a conclusion and discussion of the methods and results in Section 6.

2 Literature

There is a vast literature on volatility forecasting in stock markets. This is due to the fact that when traders are able to produce good forecasts, they can respond to the market and gain financially because of it. This makes volatility forecasting a valuable topic to investigate. There has been a lot of criticism on the methods of volatility forecasting after the financial crisis of 2008 as they were not able to predict its impact on the stock market (Tularam & Subramanian, 2013).

One way of forecasting volatility is via autoregressive methods of which the GARCH model is a widely known example (Bollerslev, 1986). Since this model was introduced, various versions have been examined such as the IGARCH and GJRGARCH model which are also investigated in this thesis and will be explicated in more detail below (Nelson, 1990; Glosten et al., 1993). One mutual characteristic of these models is that they have to make an assumption about the distribution of the returns. In general, a Student's t-distribution is assumed for financial returns due to the excess kurtosis which is often observable in data samples of stock indices (Peiro, 1994).

In contrast, the volatility forecasts in Taylor (2005) do not need any distributional assumptions. This is because the volatility forecasts are based on quantile forecasts models which were introduced by Engle & Manganelli (2004). Since this article was published, much research has been done into CAViaR models. For example, the parameters of the CAViaR models of Engle & Manganelli (2004) are constant over time. D. Huang et al. (2010) on the other hand have introduced time-varying CAViaR models for which the parameters are time-varying functions of the market index. D. Huang et al. (2010) find that as a result of this feature, their CAViaR models are able to capture different processes of tail behaviour. In addition, the time-varying models generate better forecasts when there are spillover effects from one market to the other. Spillovers are the dependence between large negative shocks among international financial markets.

Moreover, when there are spillover effects among markets and financial products of these markets are in one portfolio, a multivariate model may be more applicable. The multivariate and multi-quantile CAViaR framework of White et al. (2015) can directly examine the degree of tail interdependence among different random variables. This model is also useful when estimating the impact of a macro shock on the quantile of risk variables.

Finally, these two extensions of the original CAViaR models of Engle & Manganelli (2004) are both present in the model of Klochkov et al. (2019). They have introduced a multivariate CAViaR model which also incorporates time variation among the parameter estimates. In their results, they find that the volatility of one stock can best be estimated with varying parameters across time and in addition that negative shocks of one stock can be contagious for other stocks. Although these more extensive models are not investigated in this thesis, it would be interesting for further research to examine the volatility forecasts that these CAViaR models produce as a result of the method of Taylor (2005) which will also be explained below in Section 3.2.

3 Methodology

In this section, the methodology of the thesis is explicated. First, methods of moving average and GARCH models are given. Thereafter, the VaR methods are shown. Moreover, it is elucidated how volatility forecasts are created from the VaR quantile estimates. Finally the tests according to which the models will be compared are introduced. In order to evaluate the model later in the thesis, not only methods for the one-day-ahead forecasts are evaluated, but the 10-day and 20-day-ahead forecasts are also given.

3.1 Moving Average and GARCH Methods

The simplest way of forecasting volatility is by estimating the variance as a moving average of the squared residuals. However, it may seem arbitrary how many periods the moving window should include. This motivates the use of an exponentially weighted moving average of squared shocks:

$$\hat{\sigma}_{t+1}^2 = \alpha\epsilon_t^2 + (1-\alpha)\hat{\sigma}_t^2, \tag{1}$$

where $\hat{\sigma}_{t+1}^2$ is the one-day-ahead variance forecast, α is called the smoothing parameter, ϵ_t^2 is the squared error and $\hat{\sigma}_t^2$ is the volatility estimate at time t. For moving average and exponentially smoothing models, the multi-period variance forecast, $\hat{\sigma}_{t,k}^2$, equals the one-step-ahead forecast, $\hat{\sigma}_{t+1}^2$, multiplied by k.

Differently, the GARCH(1,1) models present the conditional variance as a linear function of one lagged squared error term and the lagged conditional variance:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \tag{2}$$

where ω, α and β are parameters. The multiperiod variance forecast for the GARCH(1,1) model equals the sum of the variance forecasts for all k periods:

$$\hat{\sigma}_{t,k}^2 = \frac{\omega k}{1-\alpha-\beta} + \left(\hat{\sigma}_{t+1}^2 - \frac{\omega}{1-\alpha-\beta}\right)\left(\frac{1-(\alpha+\beta)^k}{1-\alpha-\beta}\right), \tag{3}$$

where $\hat{\sigma}_{t+1}^2$ is the one-step-ahead forecast. In addition, the second GARCH model that will be investigated is the integrated GARCH (IGARCH) model from Nelson (1990). This model is similar to GARCH(1,1), however, it prescribes that $\beta = 1-\alpha$. The multiperiod forecast for the IGARCH model is established as:

$$\hat{\sigma}_{t,k}^2 = \frac{1}{2}k(k-1)\omega + k\hat{\sigma}_{t+1}^2. \tag{4}$$

It is observed that volatility in negative returns is often larger compared to positives returns. An asymmetric GARCH model, the GJRGARCH(1,1) model by Glosten et al. (1993), can capture this leverage effect and is given by

$$\sigma_t^2 = \omega + (1 - I[\epsilon_{t-1} > 0])\alpha\epsilon_{t-1}^2 + (I[\epsilon_{t-1} > 0])\gamma\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \tag{5}$$

where ω, α, γ and β are parameters; and $I[\cdot]$ is the indicator function. The multiperiod forecast expression of the GJRCARCH(1,1) model is as follows

$$\hat{\sigma}_{t,k}^2 = \frac{\omega k}{1-\frac{1}{2}(\alpha+\gamma)-\beta} + \left(\hat{\sigma}_{t+1}^2 - \frac{\omega}{1-\frac{1}{2}(\alpha+\gamma)-\beta}\right)\cdot\left(\frac{1-(\frac{1}{2}(\alpha+\gamma)+\beta)^k}{1-(\frac{1}{2}(\alpha+\gamma)-\beta}\right). \tag{6}$$

All parameters of the three GARCH are estimated by maximum likelihood, assuming that the standardised errors, ϵ_t/σ_t are independent and identically distributed according to a Student's t-distribution.

In addition, in order to check for the robustness regarding long-tailed situations two benchmark approaches are included. These models use a winsorised data set. The first model uses a simplistic winsorisation on the data which means that the largest 1% in-sample observations are set to the unconditional in-sample 0.99 quantile and the lowest in-sample 1% observations are set to the 0.01 unconditional in-sample quantile. The second approach bases the winsorisation on the Asymmetric Slope CAViaR quantile model for the 0.99 and 0.01 quantiles. For the in-sample observations, values that are larger than their conditional 0.99 quantile are set to this value and values that are lower than their corresponding conditional 0.01 quantile are set to the value of that quantile. The estimation of the Asymmetric Slope CAViaR quantile model is explicated in the next subsection.

3.2 VaR based methods

Pearson & Tukey (1965) have found that the relations between the standard deviation and the interval between symmetric quantiles in the tails of a distribution is surprisingly constant. This result gives motive to estimate the variance on the basis of quantile estimates. This method has the advantage that it does not need any distributional assumptions. Below the approximations for the standard deviation, $\hat{\sigma}$, in terms of quantiles from Pearson & Tukey (1965) are given for the 98%, 95% and 90% interval, respectively:

$$\hat{\sigma} = \frac{\hat{Q}(0.99) - \hat{Q}(0.01)}{4.65} \qquad \hat{\sigma} = \frac{\hat{Q}(0.975) - \hat{Q}(0.025)}{3.92} \qquad \hat{\sigma} = \frac{\hat{Q}(0.95) - \hat{Q}(0.05)}{3.25}, \tag{7}$$

where $\hat{Q}(\theta)$ is the unconditional θ quantile estimate.

Taylor (2005) investigates non-parametric and semiparametric models to estimate quantiles. A popular non-parametric method is historical simulation. By making use of a moving window of historical observations, this method does not require any assumptions on the distribution of the returns. Alternatively, the BRW method could be used which maintains that more recent observations have a larger weight in the quantile estimate (Boudoukh et al., 1998). This method allocates exponentially decreasing weights, which sum to one, to the n most recent returns,

$$\left(\frac{1-\lambda}{1-\lambda^n}\right), \left(\frac{1-\lambda}{1-\lambda^n}\right)\lambda, \left(\frac{1-\lambda}{1-\lambda^n}\right)\lambda^2, \ldots, \left(\frac{1-\lambda}{1-\lambda^n}\right)\lambda^{n-1}, \tag{8}$$

for which the parameter λ is optimised according to Equation 14. The θ quantile estimate is established by summing the weights until the value of θ is reached, the θ quantile estimate then corresponds to the return of the last weight in the summation. When θ is in between two weights linear interpolation is used to find the return for which the quantile is equal to θ.

In addition, the semiparametric methods in Taylor (2005) are the CAViaR models from Engle & Manganelli (2004). CAViaR models are auto-regressive models and do not have any distributional assumptions. Engle and Manganelli introduce four models which are presented below. In addition to these, a fifth model from Huang et al. (2009) will be considered.

Indirect GARCH(1,1) CAViaR (GARCH CAViaR):

$$Q_t(\theta) = (1 - 2I[\theta < 0.5])(\omega + \alpha Q_{t-1}(\theta)^2 + \beta\epsilon_{t-1}^2)^{\frac{1}{2}} \tag{9}$$

Adaptive CAViaR (AD CAViaR):

$$Q_t(\theta) = Q_{t-1}(\theta) + \alpha(\theta - I[\epsilon_{t-1} \leq Q_{t-1}(\theta)] \tag{10}$$

Symmetric Absolute Value CAViaR (SAV CAViaR):

$$Q_t(\theta) = \omega + \alpha Q_{t-1}(\theta) + \beta|\epsilon_{t-1}| \tag{11}$$

Asymmetric Slope CAViaR (AS CAViaR):

$$Q_t(\theta) = \omega + \alpha Q_{t-1}(\theta) + \beta_1(\epsilon_{t-1})^+ + \beta_2(\epsilon_{t-1})^- \tag{12}$$

Improved Asymmetric Slope CAViaR (IAS CAViaR):

$$Q_t(\theta) = \omega + \alpha Q_{t-1}(\theta) + (1-\alpha)\left(\frac{v}{1-\beta}I(\epsilon_{t-1} > 0) + \frac{v}{\beta}I(\epsilon_{t-1} \leq 0)\right)|\epsilon_{t-1}| \tag{13}$$

where $Q_t(\theta)$ is the conditional θ quantile. Furthermore, ω, α, β and β_i are parameters, $(x)^+ = max(x, 0)$ and $(x)^- = -min(x, 0)$. In addition, $v = \sqrt{\beta^2 + (1-\beta)^2}$, with $(0 < \beta < 1)$. The improved model in Equation 13 has the feature that it has to estimate three instead of four parameters, making it more parsimonious compared to the asymmetric slope CAViaR model of Engle & Manganelli (2004).

The parameters of the CAViaR models are estimated by making use of the quantile regression minimisation:

$$min\left(\sum_{t|y_t \geq Q_t(\theta)} \theta|y_t - Q_t(\theta)| + \sum_{t|y_t < Q_t(\theta)} (1-\theta)|y_t - Q_t(\theta)|\right), \tag{14}$$

for which $Q_t(\theta)$ equals the model for the θ quantile of the dependent variable y_t. The parameters are estimated by creating 10^4 vectors of parameters between 0 and 1 from a uniform random number generator for each model. Subsequently, all vectors are evaluated by Equation 14 and the vectors producing the lowest 20 values, will function as initial values in a quasi-Newton algorithm. The 20 resulting vectors are then evaluated by Equation 14. The vector with the lowest value is the concluding parameter vector for that model.

After the parameters of the CAViaR models are estimated, Taylor (2005) shows that the one-step-ahead forecast of the volatility can be estimated from symmetric quantiles according to the following expression:

$$\hat{\sigma}_{t+1}^2 = \alpha_1 + \beta_1(\hat{Q}_{t+1}(1-\theta) - \hat{Q}_{t+1}(\theta))^2, \tag{15}$$

with parameters α_1 and β_1, which are estimated by a least squares regression of the squared errors ϵ_{t+1}^2 on the squared symmetric interquantile range and a constant. For the multiperiod variance forecast, multiperiod realised variance are needed and given by the sum of squared errors of the upcoming k periods:

$$\sigma_{Rt,k}^2 = \sum_{i=1}^{k} \epsilon_{t+i}^2. \tag{16}$$

The multiperiod variance forecast are then produced by substituting the one-day-ahead realised variance by the multiperiod realised variance

$$\hat{\sigma}_{t,k}^2 = \alpha_k + \beta_k(\hat{Q}_{t+1}(1-\theta) - \hat{Q}_{t+1}(\theta))^2, \tag{17}$$

where α_k and β_k are parameters estimated by a least squares regression of $\sigma_{Rt,k}^2$ on the squared symmetric interquantile range and a constant.

3.3 Model Evaluation and Comparison Tests

In order to compare the best performing models in more detail, an encompassing test is executed. This test evaluates whether the forecast performance of one model is better than another model. For this test, the weighted average of the two models under evaluation is used to forecast the realised variance:

$$\sigma_{Rt,k}^2 = w\hat{\sigma}_{At,k}^2 - (1-w)\hat{\sigma}_{Bt,k}^2 + e_t, \tag{18}$$

where $\hat{\sigma}_{At,k}^2$ is the estimated variance of one model, $\hat{\sigma}_{Bt,k}^2$ is the estimated variance of the other model and e_t is the residual term. When the weight, w, of method A is zero, this method is encompassed by method B. The weight is obtained by regressing $(\sigma_{Rt,k}^2 - \hat{\sigma}_{Bt,k}^2)$ on $(\hat{\sigma}_{At,k}^2 - \hat{\sigma}_{Bt,k}^2)$. For a holding period of more than one day, non-overlapping realised variances and forecasted volatilities were used. This means for example that for the 10-days-ahead forecast only values of 1, 11, 21, ..., 481 are used.

In addition to the volatility forecasts of the best models, their one-day-ahead quantile forecasts are evaluated according to its hit percentage, dynamic quantile (DQ) test statistic and QR sum score. These test statistics are also used by Engle & Manganelli (2004). First of all, the hit percentage calculates the unconditional coverage of the quantile forecasts for some θ. Effectively the hit percentage investigates how many times the observed returns fall below its quantile forecast by summing over $I[\epsilon_t \leq \hat{Q}_t(\theta)]$ and dividing it by the total amount of out of sample observations. When a model would forecast perfectly the hit percentage would be equal to θ for which the model is forecasted. Secondly, the DQ test measures the conditional coverage. It does so by regressing the hit variable on a constant, the quantile forecasts and 5 lags of the hit variable. In this test the hit variable for some θ is defined as $I[\epsilon_t \leq \hat{Q}_t(\theta)] - \theta$. Finally the QR sum score is defined as in Equation 14. The concept can be seen as the equivalent of the mean squared error but instead of evaluating the squared difference it takes the absolute value of the difference. Furthermore, it is able to assign a higher weight to observations that fall outside of the interval, that is, below the lower quantile or above the higher quantile. For the QR sum as well as the DQ test, lower results are better than high values.

4 Data

In order to compare the models stated above, they will be tested on empirical data. This thesis uses data from a collection of stock indices, namely the French CAC 40, the German DAX 30, the Japanese Nikkei 225, and the U.S. S&P 500. The data set that Taylor (2005) uses contains daily log returns and runs from April 29, 1993, to April 28, 2003. This period contains 2608 trading days. However, due to missing values the data, the set that will be used contains less observations. The stock indices do not have missing values on the same days. Therefore the missing values are removed for each stock index individually, making the amount of observations different for each stock index. The forecasting sample is set to 500 observations for all stock indices and serves to evaluate the forecasts of the one day, 10 days and 20 days holding period. As a result, the estimation sample for each stock is different.

Moreover, in addition to the data set that Taylor (2005) uses, the models are evaluated according to how well they can forecast the volatility during periods of crisis while the models have been estimated in non-crisis times. For this extension, also a data set of 10 years is used which runs from April 28, 2000 to April 27, 2010. The most volatile period in the Financial Crisis of 2008 was between August and October in 2008, which is included in the forecast sample of the data set (Schwert, 2011; Chen et al., 2012). Missing data will be handled in the same way as it was removed for the data set used in Taylor (2005). All volatility forecasting methods were applied to the demeaned returns, $\epsilon_t = r_t - \mu$. The forecasts will be compared through values of R^2. In Table 1 the skewness and excess kurtosis are presented. From this table it is striking that the excess kurtosis is much larger for the data set that contains crisis period when compared to the data set that is used by Taylor (2005). In addition it can be seen that the values for skewness and excess kurtosis in Table 1a are slightly different from the values in Taylor (2005) indicating that the data used for this thesis differs from that of Taylor. This difference is also likely to have an impact on the estimation and forecasting results. Furthermore, in Figure 1 the returns for the in and out of sample data of the CAC40 is given. In Figure 1b the financial crisis is distinguishable by the high peaks in the last 500 observations.

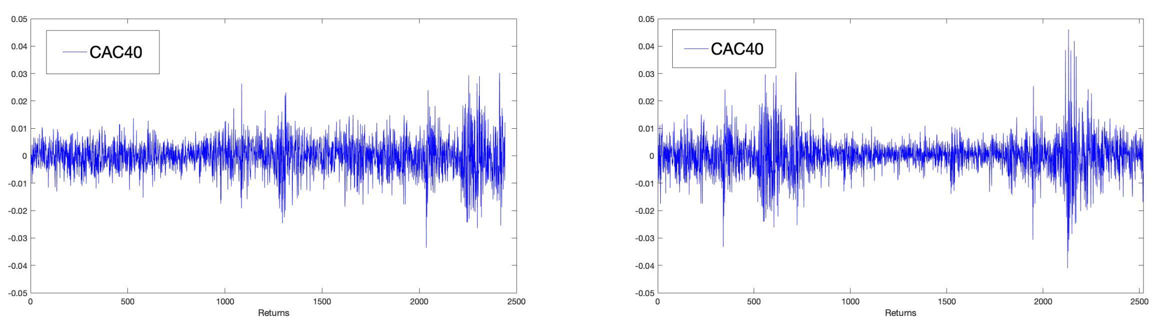
Table 1: Skewness and Excess Kurtosis for All Observations of the Four Stock Indices

(a) Original Data Set

Stock Index	Skewness	Excess Kurtosis
CAC40	-0.08	2.18
DAX30	-0.16	2.79
NIKKEI225	0.01	2.14
S&P500	-0.11	3.41

(b) Extension Data Set

Stock Index	Skewness	Excess Kurtosis
CAC40	0.03	5.22
DAX30	0.03	4.50
NIKKEI225	-0.61	5.24
S&P500	-0.06	8.38



(a) All Returns of the Original Data Set

(b) All Returns of the Extension Data Set

Figure 1: In and Out of Sample Returns of the CAC40 for Both Data Sets.

5 Results

5.1 Estimation

After the optimal parameters of Equations 9 - 13 have been estimated, the resulting time series for the quantiles are used to estimate the parameters in Equation 15 and 17. In Table 2 the parameter estimation of α_1 and β_1 in Equation 15 are shown for the AS CAViaR model as a result of a LS regression of ϵ_i^2 on the interval between symmetric quantiles for each stock index with standard errors in parentheses. The values that are found by Pearson & Tukey (1965), are presented in the last column. The relation between β_1 and these values of x is $\beta_1 = \frac{1}{x^2}$.

For all indices the constant is not significantly different from zero in 11 of the 12 LS regressions at a 5% level. In addition, it can be seen from the table above that the estimated value for β_1 is often close to the value of Pearson & Tukey (1965), but further forecasting methods will be produced using the estimated values.

Table 2: Parameters α_1 and β_1 of Equation 15 from LS Regression of the Asymmetric Slope CAViaR model

Interval	Parameter	CAC40	DAX30	NIKKEI225	S&P500	Pearson and Tukey Values
0.98	α_1 x 10^6	-8.13	-4.85	3.69	2.67	0
		(2.77)	(2.28)	(3.17)	(1.46)	
	β_1	0.0554	0.0523	0.0373	0.0324	$4.65^{-2} = 0.0462$
		(0.0037)	(0.0026)	(0.0031)	(0.0020)	
0.95	α_1 x 10^6	-1.75	-4.15	2.70	0.30	0
		(2.32)	(2.24)	(3.27)	(1.62)	
	β_1	0.0677	0.0730	0.0591	0.0625	$3.92^{-2} = 0.0651$
		(0.0043)	(0.0036)	(0.0049)	(0.0040)	
0.90	α_1 x 10^6	-0.77	-0.93	1.19	0.53	0
		(2.22)	(2.10)	(3.16)	(1.57)	
	β_1	0.0920	0.0962	0.0945	0.0911	$3.25^{-2} = 0.0947$
		(0.0057)	(0.0046)	(0.0072)	(0.0056)	

5.2 Volatility Forecast

In order to evaluate the out of sample forecast performance, the R^2 are assessed for a LS regression of the realised variances on the forecasted variances based on the models explicated in Section 3. Higher values for the R^2 are better as the measure indicates how much of the volatility in the realised variance can be explained by the forecasted variance. The first five columns in Tables 3-5 show the results for the one-day-ahead, 10-days-ahead and 20-days-ahead forecasts for the data

Table 3: R^2 Measure of the 500 One-Day-Ahead Variance Forecasts for Stock Indices

	CAC40	DAX30	NIKKEI225	S&P500	Mean	MeanX
Moving Average and GARCH Methods						
Simple Moving Average	10.3	9.8	1.0	4.5	6.4	13.7
Exponential Smoothing	13.4	14.1	2.8	10.8	10.3	18.1
GARCH	11.8	13.4	3.6	6.8	8.9	17.7
IGARCH	11.5	12.9	2.4	11.7	9.6	18.4
GJRGARCH	11.1	13.5	2.9	7.9	8.9	17.1
Simplistic Winsorised GJRGARCH	9.7	12.5	2.9	7.9	8.3	17.1
CAViaR Winsorised GJRGARCH	11.2	13.0	2.9	8.8	9.0	17.7
VaR-based Methods						
Historical Simulation 98% Interval	3.1	5.4	0.1	0.1	2.2	0.2
Historical Simulation 95% Interval	4.4	5.6	0.0	0.0	2.5	0.0
Historical Simulation 90% Interval	2.8	3.9	0.0	0.1	1.7	0.1
BRW 98% Interval	4.1	6.2	0.4	1.3	3.0	5.4
BRW 95% Interval	7.7	8.0	0.5	2.6	4.7	10.8
BRW 90% Interval	8.1	10.3	1.4	1.3	5.3	10.2
Indirect GARCH CAViaR 98% Interval	13.1	13.9	3.5	13.1	10.9	19.9
Indirect GARCH CAViaR 95% Interval	12.7	13.1	3.7	10.5	10.0	19.3
Indirect GARCH CAViaR 90% Interval	12.4	13.4	3.5	11.4	10.2	18.9
Adaptive CAViaR 98% Interval	4.1	7.3	2.0	0.4	3.4	2.6
Adaptive CAViaR 95% Interval	6.2	6.3	0.7	2.4	3.9	5.6
Adaptive CAViaR 90% Interval	8.1	8.0	1.2	4.4	5.4	5.4
Sym Abs Value CAViaR 98% Interval	13.0	12.4	2.5	10.7	9.6	19.0
Sym Abs Value CAViaR 95% Interval	12.4	12.6	2.7	9.9	9.4	18.0
Sym Abs Value CAViaR 90% Interval	11.8	12.7	2.6	9.9	9.2	17.6
Asym Slope CAViaR 98% Interval	16.4	13.8	3.7	18.6	13.1	24.0
Asym Slope CAViaR 95% Interval	15.4	14.1	3.9	**21.5**	13.7	23.6
Asym Slope CAViaR 90% Interval	**16.5**	15.5	4.7	19.4	**14.0**	**24.4**
Impr Asym Slope CAViaR 98% Interval	15.9	**16.9**	**4.9**	17.0	13.7	20.1
Impr Asym Slope CAViaR 95% Interval	15.6	15.2	4.3	14.9	12.5	21.5
Impr Asym Slope CAViaR 90% Interval	14.3	15.0	3.7	13.6	11.7	21.4

*Note that R^2 values are percentages. Moreover, the R^2 values of the best model per index are presented in bold.

sample between 1993 and 2003. In the last column of these tables, the average value of the R^2 of the indices for the extension data set is shown, named MeanX, which is assessed in more detail below. The result of the best performing model per index is presented in bold.

The tables show that the exponential smoothing model performs well compared to the GARCH models for the one-day-ahead forecasts but that the latter perform better for longer holding periods. In addition, the IGARCH model gives relatively high values for the R^2 especially in for the 10-days-ahead forecasts. This finding is also present in the 20-days-ahead forecasts of the extension data.

Table 4: R^2 Measure of the 500 10-Days-Ahead Variance Forecasts for Stock Indices

	CAC40	**DAX30**	**NIKKEI225**	**S&P500**	**Mean**	**MeanX**
Moving Average and GARCH Methods						
Simple Moving Average	33.6	27.6	2.6	8.8	18.2	33.0
Exponential Smoothing	42.4	34.8	5.9	19.5	25.7	41.7
GARCH	38.4	36.2	12.2	13.6	25.1	42.9
IGARCH	**56.0**	**51.2**	**21.1**	40.1	**42.1**	**55.8**
GJRGARCH	40.9	45.8	17.9	19.9	31.1	47.6
Simplistic Winsorised GJRGARCH	34.9	40.4	17.8	19.7	28.2	47.4
CAViaR Winsorised GJRGARCH	41.5	42.3	19.1	22.6	31.4	50.0
VaR-based Methods						
Historical Simulation 98% Interval	11.1	16.4	0.1	2.1	7.4	0.0
Historical Simulation 95% Interval	14.9	16.7	0.0	0.1	7.9	0.5
Historical Simulation 90% Interval	10.1	12.3	0.1	0.0	5.6	1.3
BRW 98% Interval	10.2	16.1	0.4	1.3	7.0	12.4
BRW 95% Interval	22.1	18.7	0.7	4.8	11.6	23.1
BRW 90% Interval	23.5	24.1	3.2	1.6	13.1	22.0
Indirect GARCH CAViaR 98% Interval	43.3	34.4	7.7	24.9	27.6	44.1
Indirect GARCH CAViaR 95% Interval	39.8	32.6	8.5	19.5	25.1	43.5
Indirect GARCH CAViaR 90% Interval	38.5	33.1	8.1	21.7	25.4	42.6
Adaptive CAViaR 98% Interval	13.0	19.8	12.9	0.0	11.4	5.5
Adaptive CAViaR 95% Interval	18.8	15.7	2.4	3.7	10.2	11.1
Adaptive CAViaR 90% Interval	24.8	20.7	2.1	7.8	13.8	12.1
Sym Abs Value CAViaR 98% Interval	41.2	32.5	7.4	20.2	25.4	41.7
Sym Abs Value CAViaR 95% Interval	39.1	33.2	8.5	18.6	24.8	40.5
Sym Abs Value CAViaR 90% Interval	36.8	33.1	7.8	18.7	24.1	39.6
Asym Slope CAViaR 98% Interval	55.5	43.9	11.7	48.1	39.8	47.7
Asym Slope CAViaR 95% Interval	54.7	37.0	12.0	**53.7**	39.4	47.0
Asym Slope CAViaR 90% Interval	53.8	42.0	14.1	48.7	39.6	47.4
Impr Asym Slope CAViaR 98% Interval	54.5	46.8	15.6	41.5	39.6	44.4
Impr Asym Slope CAViaR 95% Interval	51.8	42.9	13.6	37.1	36.3	47.0
Impr Asym Slope CAViaR 90% Interval	46.0	42.0	11.3	33.4	33.1	46.7

*Note that R^2 values are percentages. Moreover, the R^2 values of the best model per index are presented in bold.

Although the IGARCH model that is estimated in often the best performing model among the moving average and GARCH models, in the following the CAViaR models will be compared to the GJRGARCH model. This is because the latter is able to capture the leverage effect that is present among stock indiced and which is supported in the existing literature (Bouchaud et al., 2001; Alberg et al., 2008; Peters, 2001). This effect is also discernible among the CAViaR models as those that are able to incorporate a different impact of positive and negative returns have higher values for R^2. In addition, the CAViaR winsorised GJRGARCH model only perform slightly better compared to the GJRGARCH model without winsorised data whereas the simplistic winsorised model does not

Table 5: R^2 Measure of the 500 20-Days-Ahead Variance Forecasts for Stock Indices

	CAC40	**DAX30**	**NIKKEI225**	**S&P500**	**Mean**	**MeanX**
Moving Average and GARCH Methods						
Simple Moving Average	29.6	25.2	0.8	7.5	15.8	26.5
Exponential Smoothing	36.5	29.2	3.5	11.8	20.2	34.2
GARCH	33.4	31.1	8.4	7.7	20.2	35.4
IGARCH	46.7	38.8	14.4	22.4	30.6	**45.6**
GJRGARCH	34.9	36.9	11.6	11.9	23.8	38.8
Simplistic Winsorised GJRGARCH	29.4	33.9	11.5	11.5	21.6	38.6
CAViaR Winsorised GJRGARCH	35.5	35.1	12.8	13.4	24.2	40.9
VaR-based Methods						
Historical Simulation 98% Interval	11.1	19.0	0.0	4.8	8.7	0.5
Historical Simulation 95% Interval	15.4	17.6	0.1	0.4	8.4	2.0
Historical Simulation 90% Interval	10.5	13.2	0.8	0.2	6.2	3.2
BRW 98% Interval	6.8	16.1	0.1	0.2	5.8	8.5
BRW 95% Interval	17.2	17.2	0.1	3.6	9.5	16.8
BRW 90% Interval	18.4	22.6	1.3	0.8	10.8	16.9
Indirect GARCH CAViaR 98% Interval	36.7	28.9	5.5	13.5	21.1	35.6
Indirect GARCH CAViaR 95% Interval	33.9	27.8	6.5	12.0	20.0	35.3
Indirect GARCH CAViaR 90% Interval	32.9	28.2	5.9	12.9	19.9	34.7
Adaptive CAViaR 98% Interval	11.3	19.6	17.1	0.5	12.1	3.5
Adaptive CAViaR 95% Interval	16.5	13.5	2.7	1.5	8.5	7.2
Adaptive CAViaR 90% Interval	21.6	18.1	1.2	4.0	11.2	9.1
Sym Abs Value CAViaR 98% Interval	36.7	28.9	5.4	11.9	20.7	34.2
Sym Abs Value CAViaR 95% Interval	34.5	29.3	6.7	11.4	20.5	33.1
Sym Abs Value CAViaR 90% Interval	32.2	29.3	5.8	11.3	19.6	32.3
Asym Slope CAViaR 98% Interval	50.6	37.2	10.6	34.9	33.3	38.1
Asym Slope CAViaR 95% Interval	49.9	32.3	11.1	**38.1**	32.9	37.8
Asym Slope CAViaR 90% Interval	48.8	37.4	13.6	33.9	33.4	36.9
Impr Asym Slope CAViaR 98% Interval	**51.5**	**42.1**	**15.3**	31.9	**35.2**	36.6
Impr Asym Slope CAViaR 95% Interval	49.3	40.5	12.3	29.1	32.8	38.6
Impr Asym Slope CAViaR 90% Interval	42.0	39.3	9.2	26.2	29.2	38.2

*Note that R^2 values are percentages. Moreover, the R^2 values of the best model per index are presented in bold.

perform better.

Regarding the CAViaR models, the results show that these models outperform the models that estimate quantiles based on historical simulation and the BRW method. Furthermore, the tables show that for the original data set the AS CAViaR model compared to its improved version produces slightly better results. For the individual indices and in all holding periods, the AS CAViaR model performed better in 22 out of the 36 instances when models with the same interval are compared. However, the difference between the two models' performance is not very large. Moreover, the

IAS CAViaR model was only 1 out of 9 times better than its normal version. In addition, the performances of all models increase as the holding period gets longer. Either the AS or IAS CAViaR model outperformed the GJRGARCH model for all indices in all holding periods except for the NIKKEI225 for the 10-days-ahead forecasts and for the extension mean for the 20-days-ahead forecasts. Nevertheless, this finding advocates for the superiority of the volatility estimation of Taylor (2005).

The results are mostly in line with the results of Taylor (2005) since some values for the R^2 vary a little bit. However, the performance IGARCH model is higher in this thesis. In addition, Taylor (2005) finds that the forecasts of the CAViaR models for the 90% and 95% intervals produce consistently higher values for R^2 than the 98% interval does. This typical finding is not discerned in the current results.

As was introduced before, this thesis does not only offer an additional model to the methods of Taylor (2005), it also tests whether the models also forecasts well in times of high volatility. The models were also evaluated according to their R^2 values of which the entire tables can be found in Appendix A. Interestingly, apart from the VaR model that bases its estimates on historical simulations, all models have higher values for R^2 meaning that all models are able to forecast effectively when the parameters are estimated under different circumstances than the forecasts. For the 20-days-ahead forecasts the difference in R^2 values between the original data set and the extension is smaller compared to the other two holding periods.

Next the GJRGARCH model, the AS and IAS CAViaR models were investigated more closely. In order to check which model performs best, encompassing tests were executed for the GJRGARCH model and the 90% interval AS CAViaR model and for the GJRGARCH model and the 90% interval IAS CAViaR model. The results for $\hat{w}$ together with their corresponding p-values for $w = 1$ and $w = 0$ are shown in Table 6 and 7. When the hypothesis $w = 1$ is accepted, this means that the (I)AS CAViaR model encompasses the GJRGARCH model. Similarly, when the hypothesis $w = 0$ is accepted the GJRGARCH model encompasses the (I)AS CAViaR model.

From Table 6 it is discernible that the hypothesis $H_0 : w = 1$ is never rejected at a 5% significance level, indicating that the AS CAViaR model always encompasses the GJRGARCH model for all indices and all holding periods. In line with this is the result that the hypothesis $H_0 : w = 0$ is always rejected. In Table 7, the IAS CAViaR model encompasses the GJRGARCH model for all stock indices except the NIKKEI225. For this index, the results neither state that the IAS CAViaR model encompasses the GJRGARCH model nor the other way around. Similar are found for the data set containing a crisis period, for which the tables can be found in Appendix B.

Table 6: Encompassing Test Results for the GJRGARCH and Asym Slope CAViaR Model for the Original Data Set

	CAC40	**DAX30**	**NIKKEI225**	**S&P500**
One-step-ahead				
w	2.27	0.99	0.79	0.91
p-value for H0: w = 1, H1: w<1	0.97	0.48	0.20	0.24
p-value for H0: w = 0, H1: w>0	0.01	0.01	0.03	0.01
10-day holding period				
w	1.20	1.14	1.01	1.53
p-value for H0: w = 1, H1: w<1	0.92	0.84	0.55	0.97
p-value for H0: w = 0, H1: w>0	0.00	0.00	0.00	0.00
20-day holding period				
w	1.24	1.15	1.10	1.49
p-value for H0: w = 1, H1: w<1	0.88	0.79	0.79	0.93
p-value for H0: w = 0, H1: w>0	0.01	0.01	0.00	0.01

Table 7: Encompassing Test Results for the GJRGARCH and Improved Asym Slope CAViaR Model for the Original Data Set

	CAC40	**DAX30**	**NIKKEI225**	**S&P500**
One-step-ahead				
w	0.99	0.90	0.03	0.77
p-value for H0: w = 1, H1: w<1	0.49	0.30	0.00	0.13
p-value for H0: w = 0, H1: w>0	0.06	0.02	0.29	0.02
10-day holding period				
w	1.37	0.92	0.45	0.81
p-value for H0: w = 1, H1: w<1	0.96	0.21	0.01	0.08
p-value for H0: w = 0, H1: w>0	0.00	0.00	0.01	0.01
20-day holding period				
w	1.37	0.95	0.54	0.83
p-value for H0: w = 1, H1: w<1	0.92	0.34	0.02	0.13
p-value for H0: w = 0, H1: w>0	0.01	0.01	0.01	0.01

5.3 Quantile Forecast

In addition to volatility forecasts, quantile forecasts were evaluated. Quantiles were calculated for their corresponding variances forecasts of the GJRGARCH. In Tables 8 and 9, results for the hit percentage, DQ test statistics and QR sum for the GJRGARCH, AS and IAS CAViaR models are presented for the 98% and 90% intervals respectively. The results for the best model per quantile are given in bold.

The performance of the models for the hit % and the DQ test differ and as a result there is not one model that can be said to be the best performing model. However for the QR sum, the GJRGARCH model shows the best result for almost all the instances of the evaluated quantiles.

What is also interesting about these tables is that the lower quantiles of the AS and IAS CAViaR models often have a higher DQ test statistic and QR sum than the upper quantile for almost all indices. This means that the lower conditional θ quantiles differ relatively more from negative observations. In addition, the hit percentage of the lower quantile of the IAS CAViaR model is relatively high for the CAC40 and DAX30. This means that relatively more observations were

Table 8: Hit Percentage, Dynamic Quantile Test Statistic and QR Sum for One-Day-Ahead Forecasts of the 98% interval for the Original Data Set

	CAC40		**DAX30**		**NIKKEI225**		**S&P500**	
	0.01	0.99	0.01	0.99	0.01	0.99	0.01	0.99
Hit %								
GJRGARCH	**0.6**	**98.6**	**0.0**	99.8	**0.4**	99.8	0.2	99.6
AS CAViaR	**0.6**	97.6	3.6	**98.8**	2.4	99.4	2.6	**99.4**
Improved AS CAViaR	2.6	97.8	6.8	98.4	1.8	**99.0**	**0.4**	**99.4**
DQ								
GJRGARCH	**1.8**	27.2**	**2.7**	8174.7**	**2.5**	3.2	3.4	5.0
AS CAViaR	34.0**	29.5**	59.7**	**0.7**	15.5*	**0.9**	54.8**	**1.0**
Improved AS CAViaR	47.7**	**26.6****	300.7**	6.2	14.3*	38.7**	**3.2**	1.1
QR Sum								
GJRGARCH	**0.11**	**0.12**	**0.12**	0.15	**0.09**	**0.09**	0.08	0.10
AS CAViaR	0.13	0.12	0.16	0.12	0.10	0.11	0.11	**0.08**
Improved AS CAViaR	0.15	0.12	0.24	**0.12**	0.11	0.11	**0.08**	0.08

Note that the DQ test having * or ** are 5% or 1 % significant respectively according to a $\chi^2(7)$ distribution. Furthermore, values of the best performing methods are presented in bold.

Table 9: Hit Percentage, Dynamic Quantile Test Statistic and QR Sum for One-Day-Ahead Forecasts of the 90% interval for the Original Data Set

	CAC40		**DAX30**		**NIKKEI225**		**S&P500**	
	0.05	0.95	0.05	0.95	0.05	0.95	0.05	0.95
Hit %								
GJRGARCH	7.4	**94.8**	**3.4**	98	6.2	95.8	**4**	96.8
AS CAViaR	**5**	95.4	7	94.4	**5.8**	**95.2**	7.2	**94.2**
Improved AS CAViaR	13.6	95.4	7.6	**94.8**	2	**95.2**	2.2	93.8
DQ								
GJRGARCH	7.2	**3.9**	**5.1**	9.9	9.5	17.5*	**7.7**	**5.7**
AS CAViaR	**1.6**	4.2	7.8	**9.1**	**8.6**	**3.9**	20.0**	5.8
Improved AS CAViaR	115.7**	4.2	34.7**	9.2	15.4*	3.9	12.0	7.4
QR Sum								
GJRGARCH	**0.41**	0.44	**0.41**	**0.47**	**0.34**	**0.35**	**0.30**	0.33
AS CAViaR	0.42	**0.44**	0.47	0.49	0.37	0.38	0.32	**0.32**
Improved AS CAViaR	0.60	0.44	0.55	0.48	0.45	0.38	0.33	0.32

Note that the DQ test having * or ** are 5% or 1 % significant respectively according to a $\chi^2(7)$ distribution. Furthermore, values of the best performing methods are presented in bold.

below the quantile estimate than expected. When negative returns occur, a model must estimate the lower quantile effectively as larger financial losses may result when this is not the case. This observation is even more visible in test statistics for the extension data set. The tables for this data set can be found in Appendix C.

6 Conclusion

To summarise, this thesis has investigated how volatility forecasts perform that are based on quantile forecasts. In addition to a historical simulation and BRW model, five CAViaR models were introduced and their performances were compared to the forecasting ability of the famous GARCH, IGARCH and GJRGARCH models. The results for the CAViaR models are very similar to those that can be found in Taylor (2005) and also advocate for the usage of CAViaR models for volatility forecasts since these mostly outperform GARCH and GJRGARCH models. The surprisingly good performance of the IGARCH model may be due to parameter estimation and the method it uses to produced forecasts for longer holding periods. Although the improved Asymmetric Slope CAViaR model was not always the best performing model, it does forecast well and encompasses the GJR-GARCH model for most stock indices. In addition it has the feature of being more parsimonious compared to its normal version, making it a more efficient model to use as when more parameters need to be estimated, there is more noise in the estimation. Furthermore, it may make a difference in computing time when data sets are very large.

Moreover, this thesis has examined the models' forecasting performance when the parameters have been estimated in times of non crisis while the forecast are done in times of crisis. It has been demonstrated that the models are robust to this difference and that the models even generate higher R^2's for their forecasts. This implicates that no additional indicator functions or parameter estimations have to be employed when a financial crisis occurs.

Although the differences between the results of this thesis and those of Taylor are not very large, they may be due to the fact that the data sample that is used is not identical as has been identified. Moreover, the optimisation techniques to estimate and forecast may have been slightly different.

Finally, what is not investigated in this thesis but is related to it and would be interesting for further research is how the Improved Asymmetric Slope CAViaR model performs for expected shortfall forecasts compared to the other CAViaR models. When the model is also able to generate good expected shortfall estimates, this may further advocate for its usage. In addition, further research could examine the feasibility and effectiveness of time variant and/or multivariate volatility forecasts that are based on CAViaR models to improve forecasts further and compare portfolios.

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A Tables of the R^2 Values for the Extension Data Set

Table 10: R^2 Measure of the 500 One-Day-Ahead Variance Forecasts for Stock Indices for the Extension Data Set

	CAC40	**DAX30**	**NIKKEI225**	**S&P500**	**MeanX**
Moving Average and GARCH Methods					
Simple Moving Average	12.8	11.3	12.7	18.1	13.7
Exponential Smoothing	15.5	13.2	21.3	22.3	18.1
GARCH	15.5	13.2	21.6	20.7	17.7
IGARCH	15.2	12.1	26.0	20.5	18.4
GJRGARCH	15.3	12.6	21.0	19.3	17.1
Simplistic Winsorised GJRGARCH	15.2	12.6	21.1	19.3	17.1
CAViaR Winsorised GJRGARCH	15.4	12.7	22.2	20.5	17.7
VaR-based Methods					
Historical Simulation 98% Interval	0.2	0.2	0.1	0.2	0.2
Historical Simulation 95% Interval	0.0	0.0	0.1	0.0	0.0
Historical Simulation 90% Interval	0.3	0.1	0.1	0.1	0.1
BRW 98% Interval	5.8	5.4	1.6	9.0	5.4
BRW 95% Interval	10.9	6.1	13.0	13.4	10.8
BRW 90% Interval	8.3	7.5	9.6	15.4	10.2
Indirect GARCH CAViaR 98% Interval	15.7	13.4	29.1	21.3	19.9
Indirect GARCH CAViaR 95% Interval	16.0	13.4	26.9	21.1	19.3
Indirect GARCH CAViaR 90% Interval	15.9	13.4	25.2	21.1	18.9
Adaptive CAViaR 98% Interval	2.9	4.2	0.0	3.3	2.6
Adaptive CAViaR 95% Interval	6.9	4.7	5.6	5.2	5.6
Adaptive CAViaR 90% Interval	7.6	4.3	2.1	7.6	5.4
Sym Abs Value CAViaR 98% Interval	15.4	13.9	25.8	20.9	19.0
Sym Abs Value CAViaR 95% Interval	15.3	13.7	22.1	21.0	18.0
Sym Abs Value CAViaR 90% Interval	15.2	13.5	21.1	20.7	17.6
Asym Slope CAViaR 98% Interval	17.6	**19.7**	34.2	24.4	24.0
Asym Slope CAViaR 95% Interval	18.1	17.2	33.3	**25.9**	23.6
Asym Slope CAViaR 90% Interval	18.9	19.7	**35.7**	23.4	**24.4**
Impr Asym Slope CAViaR 98% Interval	18.3	18.0	21.4	22.7	20.1
Impr Asym Slope CAViaR 95% Interval	**18.9**	17.9	24.3	25.0	21.5
Impr Asym Slope CAViaR 90% Interval	17.5	16.9	25.9	25.3	21.4

*Note that R^2 values are percentages. Moreover, the R^2 values of the best model per index are presented in bold.

Table 11: R^2 Measure of the 500 10-Days-Ahead Variance Forecasts for Stock Indices for the Extension Data Set

	CAC40	**DAX30**	**NIKKEI225**	**S&P500**	**MeanX**
Moving Average and GARCH Methods					
Simple Moving Average	35.3	33.6	17.9	45.1	33.0
Exponential Smoothing	41.9	41.5	28.6	54.6	41.7
GARCH	44.0	42.9	32.9	51.9	42.9
IGARCH	**52.6**	**52.1**	**52.5**	**66.1**	**55.8**
GJRGARCH	49.5	49.1	38.2	53.7	47.6
Simplistic Winsorised GJRGARCH	48.9	48.7	38.4	53.7	47.4
CAViaR Winsorised GJRGARCH	50.3	49.6	41.1	59.1	50.0
VaR-based Methods					
Historical Simulation 98% Interval	0.0	0.1	0.0	0.0	0.0
Historical Simulation 95% Interval	0.3	0.1	1.2	0.6	0.5
Historical Simulation 90% Interval	2.0	1.0	1.1	1.0	1.3
BRW 98% Interval	13.4	15.0	0.8	20.3	12.4
BRW 95% Interval	23.6	17.4	16.6	34.8	23.1
BRW 90% Interval	19.9	21.7	12.4	33.8	22.0
Indirect GARCH CAViaR 98% Interval	42.6	42.3	40.1	51.3	44.1
Indirect GARCH CAViaR 95% Interval	43.6	42.5	36.9	51.1	43.5
Indirect GARCH CAViaR 90% Interval	43.3	42.1	34.4	50.8	42.6
Adaptive CAViaR 98% Interval	5.4	10.6	0.8	5.4	5.5
Adaptive CAViaR 95% Interval	16.0	12.9	5.7	10.0	11.1
Adaptive CAViaR 90% Interval	19.1	11.9	1.4	16.2	12.1
Sym Abs Value CAViaR 98% Interval	42.3	43.3	32.6	48.8	41.7
Sym Abs Value CAViaR 95% Interval	42.0	42.4	28.1	49.4	40.5
Sym Abs Value CAViaR 90% Interval	41.6	41.5	26.9	48.3	39.6
Asym Slope CAViaR 98% Interval	42.8	49.8	43.0	55.1	47.7
Asym Slope CAViaR 95% Interval	43.3	45.5	40.9	58.4	47.0
Asym Slope CAViaR 90% Interval	42.8	47.3	49.5	49.9	47.4
Impr Asym Slope CAViaR 98% Interval	46.2	50.1	29.6	51.7	44.4
Impr Asym Slope CAViaR 95% Interval	47.4	49.7	33.2	57.5	47.0
Impr Asym Slope CAViaR 90% Interval	45.1	47.9	35.2	58.7	46.7

*Note that R^2 values are percentages. Moreover, the R^2 values of the best model per index are presented in bold.

Table 12: R^2 Measure of the 500 20-Days-Ahead Variance Forecasts for Stock Indices for the Extension Data Set

	CAC40	**DAX30**	**NIKKEI225**	**S&P500**	**MeanX**
Moving Average and GARCH Methods					
Simple Moving Average	31.0	27.4	10.5	37.0	26.5
Exponential Smoothing	37.3	35.7	16.9	46.8	34.2
GARCH	39.6	37.7	19.8	44.3	35.4
IGARCH	**47.4**	**45.9**	**33.0**	**56.2**	**45.6**
GJRGARCH	44.1	42.6	23.5	45.1	38.8
Simplistic Winsorised GJRGARCH	43.5	42.2	23.6	45.2	38.6
CAViaR Winsorised GJRGARCH	45.0	43.2	25.4	50.1	40.9
VaR-based Methods					
Historical Simulation 98% Interval	0.5	0.2	1.0	0.4	0.5
Historical Simulation 95% Interval	1.9	0.9	3.0	2.2	2.0
Historical Simulation 90% Interval	4.7	3.0	2.6	2.8	3.2
BRW 98% Interval	9.1	10.8	0.0	14.1	8.5
BRW 95% Interval	15.3	13.6	9.3	29.0	16.8
BRW 90% Interval	15.4	17.6	6.4	28.5	16.9
Indirect GARCH CAViaR 98% Interval	38.0	36.6	24.1	43.5	35.6
Indirect GARCH CAViaR 95% Interval	39.4	36.8	22.1	42.8	35.3
Indirect GARCH CAViaR 90% Interval	38.8	36.4	20.6	42.9	34.7
Adaptive CAViaR 98% Interval	2.4	6.3	3.1	2.1	3.5
Adaptive CAViaR 95% Interval	12.0	9.3	1.7	5.7	7.2
Adaptive CAViaR 90% Interval	15.6	8.3	0.1	12.2	9.1
Sym Abs Value CAViaR 98% Interval	38.0	37.7	20.6	40.4	34.2
Sym Abs Value CAViaR 95% Interval	37.6	36.4	17.7	40.8	33.1
Sym Abs Value CAViaR 90% Interval	37.2	35.3	17.0	39.9	32.3
Asym Slope CAViaR 98% Interval	38.1	42.4	27.3	44.8	38.1
Asym Slope CAViaR 95% Interval	38.3	39.6	26.0	47.1	37.8
Asym Slope CAViaR 90% Interval	36.4	40.0	31.2	40.2	36.9
Impr Asym Slope CAViaR 98% Interval	40.8	43.8	18.9	42.7	36.6
Impr Asym Slope CAViaR 95% Interval	42.0	43.3	21.2	48.0	38.6
Impr Asym Slope CAViaR 90% Interval	39.8	41.3	22.4	49.1	38.2

*Note that R^2 values are percentages. Moreover, the R^2 values of the best model per index are presented in bold.

B Tables of the Encompassing Tests for the Extension Data Set

Table 13: Encompassing Test Results for the GJRGARCH and Asym Slope CAViaR Model for the Extension Data Set

	CAC40	**DAX30**	**NIKKEI225**	**S&P500**
One-step-ahead				
w	1.30	1.34	1.06	1.02
p-value for H0: w = 1, H1: w<1	0.81	0.87	0.63	0.53
p-value for H0: w = 0, H1: w>0	0.02	0.01	0.01	0.02
10-day holding period				
w	1.19	1.22	1.90	1.23
p-value for H0: w = 1, H1: w<1	0.90	0.84	0.97	0.86
p-value for H0: w = 0, H1: w>0	0.00	0.01	0.01	0.01
20-day holding period				
w	0.96	1.21	1.61	1.24
p-value for H0: w = 1, H1: w<1	0.42	0.81	0.82	0.77
p-value for H0: w = 0, H1: w>0	0.02	0.01	0.04	0.02

Table 14: Encompassing Test Results for the GJRGARCH and Asym Slope CAViaR Model for the Extension Data Set

	CAC40	**DAX30**	**NIKKEI225**	**S&P500**
One-step-ahead				
w	0.45	0.43	0.47	0.68
p-value for H0: w = 1, H1: w<1	0.07	0.04	0.05	0.09
p-value for H0: w = 0, H1: w>0	0.09	0.06	0.06	0.03
10-day holding period				
w	1.59	1.61	1.60	1.63
p-value for H0: w = 1, H1: w<1	0.97	0.94	0.93	0.96
p-value for H0: w = 0, H1: w>0	0.00	0.01	0.01	0.01
20-day holding period				
w	1.30	1.55	1.25	1.58
p-value for H0: w = 1, H1: w<1	0.83	0.92	0.68	0.90
p-value for H0: w = 0, H1: w>0	0.02	0.01	0.05	0.02

C Tables of the Test Statistics for the Extension Data Set

Table 15: Hit Percentage, Dynamic Quantile Test Statistic and QR Sum for One-Day-Ahead Forecasting of the 98% interval for the Extension Data Set

	CAC40		**DAX30**		**NIKKEI225**		**S&P500**	
	0.01	0.99	0.01	0.99	0.01	0.99	0.01	0.99
Hit %								
GJRGARCH	**0.6**	99.8	**0.4**	**99.8**	**0.8**	100.0	**1.2**	99.6
AS CAViaR	2.4	**98.4**	3.0	**98.2**	2.6	**99.0**	4.4	99.6
Improved AS CAViaR	2.4	**98.4**	6.6	**98.2**	3.6	99.2	8.2	**99.4**
DQ								
GJRGARCH	50.3**	**3.6**	**1.9**	**3.2**	**37.0****	N/A	**0.3**	4.0
AS CAViaR	**19.2****	15.0*	29.5**	37.4**	91.3**	43.4**	82.7**	3.6
Improved AS CAViaR	205.6**	14.3*	313.6**	34.5**	274.4**	**26.0****	572.6**	**1.5**
QR Sum								
GJRGARCH	**0.10**	**0.10**	**0.10**	**0.10**	**0.12**	0.10	**0.11**	0.10
AS CAViaR	0.14	0.12	0.13	0.12	0.17	0.10	0.17	**0.10**
Improved AS CAViaR	0.17	0.12	0.29	0.13	0.29	**0.09**	0.39	0.10

Note that the DQ test having * or ** are 5% or 1 % significant respectively according to a $\chi^2(7)$ distribution. Furthermore, values of the best performing methods are presented in bold.

Table 16: Hit Percentage, Dynamic Quantile Test Statistic and QR Sum for One-Day-Ahead Forecasting of the 90% interval for the Extension Data Set

	CAC40		**DAX30**		**NIKKEI225**		**S&P500**	
	0.05	0.95	0.05	0.95	0.05	0.95	0.05	0.95
Hit %								
GJRGARCH	**7.2**	96	**6.8**	96	**5.2**	96.2	**6.6**	96.4
AS CAViaR	8.4	92.2	7.8	**94.4**	9.2	93.4	9.0	93.8
Improved AS CAViaR	16	**94.2**	18.2	**94.4**	11.8	**94.4**	19.6	**95**
DQ								
GJRGARCH	**5.4**	9.5	**6.8**	12.9	**11.6**	5.6	**7.3**	4.6
AS CAViaR	13.8	15.8*	12.9	**6.4**	47.8**	**5.1**	28.8**	4.2
Improved AS CAViaR	145.2**	**5.7**	213.2**	8.0	78.1**	6.7	254.0**	**3.1**
QR Sum								
GJRGARCH	**0.40**	**0.40**	**0.38**	**0.39**	**0.44**	**0.38**	**0.43**	0.38
AS CAViaR	0.45	0.43	0.45	0.42	0.55	0.39	0.50	**0.38**
Improved AS CAViaR	0.69	0.44	0.69	0.43	0.68	0.39	0.82	0.38

Note that the DQ test having * or ** are 5% or 1 % significant respectively according to a $\chi^2(7)$ distribution. Furthermore, values of the best performing methods are presented in bold.