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Volatility Forecasting using Standard CAViaR Models, Threshold-CAViaR, and Smooth Transition: Evaluative Comparison

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Abstract

Standard volatility modelling methods such as GARCH models rely on an underlying assumption of a fixed conditional distribution. CAViaR models relax this assumption, so that we model conditional quantiles based on lagged time series information. These quantiles can then be used to model volatility through a least squares regression. This paper evaluates the performance of such VaR-based methods, compared to each other, and different moving average and GARCH methods for four stock indices. Previous research shows that the Asymmetric Slope CAViaR model performs well for a 95% and 90% interval compared to other models. With the introduction of two new models to the work of Taylor (2005), the *Threshold*-CAViaR and Smooth Transition CAViaR, we find that more accurate modelling of the leverage effect can lead to better volatility forecasts. The CAViaR model which performs the best differs depending on the index and holding period under consideration.

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1 Introduction

In financial markets, volatility forecasting is important for risk management, portfolio management, and asset pricing. This has given rise to many different methods to forecast such fluctuations based on time series data such as daily returns. Early models, such as the (generalized) autoregressive conditional heteroskedasticity (ARCH/GARCH) models, first proposed by Engle (1982) and Bollerslev (1986), estimate the variance by a weighted combination of previous periods shocks and variance. Campbell and Hentschel (1992) investigated how large negative returns occur more often in financial time series data than large positive returns, indicating a negatively skewed returns. This allows for the introduction of asymmetry in financial returns data, known as the leverage effect, and motivated Glosten et al. (1993) to introduce the GJRGARCH specification which takes this effect into account. These three models and other GARCH type models rely on an assumption of the shape of the conditional distribution, such as a Student's t-distribution.

If the shape of the distribution exhibits variation over time, such an assumption can lead to poor volatility forecasts. This led to the introduction of conditional autoregressive value at risk (CAViaR) models, first introduced by Engle and Manganelli (2004). These models do not require assumptions about the underlying conditional distribution and make use of regression quantiles to specify the evolution of the quantile over time using an AR process. These models were first not used to estimates volatility forecast but Pearson and Tukey (1965) find that the ratio of standard deviation to the interval between symmetric quantiles are constant for many distributions. Taylor (2005) therefore proposes a least squares regression to estimate variance forecasts from the quantile estimates.

In the empirical research of Taylor (2005), multiple models and methods are used to forecast volatility and are evaluated using different performance measures. Taylor considers a wide range of models / methods containing the simple moving average method, exponential smoothing, different GARCH models, and a variety of Value-at-Risk (VaR) based methods. The models that are the most interesting are the four CAViaR models: Indirect GARCH, Adaptive, Symmetric Absolute Value, and ASymmetric Slope. In his research he shows that the Asymmetric Slope CAViaR model performs best compared to moving average and GARCH methods, and other VaR-based methods. This model takes into account the leverage effect in the daily returns of stocks but makes no distributional assumptions, in contrast to the GARCH methods. More specifically, the performance was the best for the 95% and 90% intervals.

Gerlach et al. (2011) expands on the Asymmetric Slope CAViaR model and argues that even though the model captures the leverage effect, adding only one parameter causes the types of asymmetry captured to be limited. He therefore proposes a new *Threshold*-CAViaR (T-CAViaR) model in his work, for forecasting VaR and shows that these models can be favorable compared to other standard CAViaR models. The model is similar to the Asymmetric Slope CAViaR model but allows for separate parameters depending on the sign of the past shock. This allows the model to capture more flexible asymmetric and nonlinear responses. In later research, Chen et al. (2012), also show that the T-CAViaR model, outperforms other standard CAViaR models in terms of forecasting VaR effectively and accurately while incorporating intra-day price ranges.

In both the Asymmetric Slope CAViaR and T-CAViaR models the threshold for the regime switching is set equal to zero. While these models only consider two regimes, González-Rivera et al. (1998) introduces a

GARCH model with multiple regimes called smooth-transition GARCH (ST-GARCH). This model allows for a continuous transition between regimes based on a certain transition function with an estimated optimal smoothness parameter. The smooth transition regimes also allow for the two regime threshold specification. This is because for certain parameter values, the model collapses to the more standard threshold model. This transition is explained by the smoothness parameter, where the degree of the smoothness controls the number of regimes in the model. A steep transition function, indicates two regimes such as the threshold model, while a smoother transition function indicates more than two regimes. We are interested in extending the idea of smooth transition to create a new model, the smooth transition CAViaR model (ST-CAViaR), which allows for multiple regimes for VaR based methods of volatility forecasting. The potential superiority of the models based on different regimes (to take into account the leverage effect) leads us to our research question to investigate if more complex regime switching models perform even better:

Does the T-CAViaR model and the ST-CAViaR model perform better than the Asymmetric Slope CAViaR model for forecasting volatility based on daily returns?

Using the parameter estimates of Pearson and Tukey (1965) for the volatility forecasting from CAViaR models, we find that the ST-CAViaR model performs the best although the difference with Asymmetric Slope GARCH is minimal. In terms of out of sample forecasting performance, we find that for the one-day-ahead forecasts the T-CAViaR outperforms all other models on average for the 95% and 90% intervals. However, when the holding period if increased to 10 or 20 days, the ST-CAViaR model for the 98% interval outperforms the T-CAViaR and other models on average. Performance measures, such as the hit percentage, dynamic quantile test statistic, and quantile regression sum indicate that the T-CAViaR model performs the best compared to other models but that the differences are small and it depends on the interval and performance measure under consideration.

We will first introduce the data in Section 2 that will be used in the empirical analysis. In Section 3, we will present the methodology used in the paper of Taylor (2005) where we focus mostly on the VaR methods to generate volatility forecasts. We will also introduce the methodology for the T-CAViaR and ST-CAViaR models and the approach for parameter estimation and forecast evaluation. Next, we discuss the results of our empirical research and compare them to the results of Taylor in Section 4. Finally, we discuss our main findings and form a conclusion in Section 5.

2 Data

We use the same data that is used by Taylor (2005) in his research. We use daily log returns of four stock market indices covering four different countries for the period April 29, 1993 to April 28, 2003. The countries that are used in the research are France (Paris CAC40), Germany (Frankfurt DAX30), US (New York S&P500), and Japan (Tokyo NIKKEI225). Only trading day observations are used; therefore we have 2,513 observations for CAC 40, 2,519 for DAX 40 and S&P500, and 2464 for NIKKEI 225. The results in Table 1 show the Skewness and Excess Kurtosis (Kurtosis subtracted by three) of the indices.

Table 1: Skewness and Excess Kurtosis for the Four Stock Indices

	Skewness	Excess Kurtosis	Excess Kurtosis in Taylor	Difference in Excess Kurtosis
Stock Indices				
CAC 40	-0.06	2.15	2.36	-0.21
DAX 30	-0.13	2.80	3.14	-0.34
NIKKEI 225	0.10	2.14	2.45	-0.31
S&P500	-.11	3.46	3.69	-0.23

The log returns depict a negative skewness for all indices except for NIKKEI 225. This suggests that large positive returns occur more often than large negative returns in the Japanese stock market index, and vice versa for the other three. We also notice a substantial excess kurtosis for all the indices indicating a fat tailed distribution compared to the normal distribution due to the large amount of large negative and large positive returns. The data used in Taylor (2005) shows similar descriptive statistics. The skewness is exactly the same except for DAX 30 which is $-.13$, but in Taylor it is $-.23$. Our data also has a little less excess kurtosis.The models are built using the error terms of the log returns. The log returns are built as $logreturn_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$, where P_t is the closing price of day t. Then the mean of the in sample log returns are calculated and these are removed from the log returns. The code was written and run in Matlab R2021a. The basis of the code for the dynamic quantile test, discussed later in Section 3.7, was taken from the work by Engle and Manganelli (2004).[1]

3 Methodology

3.1 Direct Volatility Modelling

Taylor (2005) focuses on a time series-method in which estimates the conditional variance of the log returns at time t conditional on the information set I_{t-1}. These estimates are given by $\sigma_t^2 = \text{var}(r_t \mid I_{t-1})$, which can be interpreted as the variance of an error term, ε_t. This error term is defined as the demeaned return,

[1]The original code by Engle and Manganelli (2004) can be found at http://www.simonemanganelli.org/Simone/Research.html, under publications in refereed journals. The name of the code is DQtest.m in the matlab folder found by the CAViaR: Conditional Autorgressive Value at Risk by Regression Quantiles article.

$\varepsilon_t = r_t - \text{E}(r_t \mid I_{t-1})$. The most simple approach used by Taylor (2005) is the 30 day moving average approach of past squared shocks. In this method the variance is estimated as: $\hat{\sigma}_t^2 = \frac{1}{30}\sum_{i=1}^{T}\varepsilon_i^2 = \frac{1}{30}\sum_{i=1}^{T}(r_{t-i} - \bar{r})^2$. This method has a few drawbacks as the number of past periods included (in this case 30) is arbitrary. Using too few past observations leads to a large sampling error, and too many causes slow reaction to changes in volatility. This motivates the choice to give more weight to more recent observations in the exponentially weighted moving average method of past squared shocks. Taylor (2005) uses a long history of observations so that the model can be written as a simple exponential smoothing model with smoothing parameter α:

$$\hat{\sigma}_t^2 = \alpha\varepsilon_{t-1}^2 + (1-\alpha)\hat{\sigma}_{t-1}^2 \tag{1}$$

The optimization of parameters in equation (1) is done by minimizing the in-sample sum of squared deviations between the forecasts of the variance and the squared error. This minimization is given by equation (2).

$$\hat{\alpha} = \arg\min \sum_t (\varepsilon_t^2 - \hat{\sigma}_t^2)^2 \tag{2}$$

3.1.1 GARCH Models

GARCH models estimate the conditional variance as a linear function of lagged squared error terms and lagged conditional variance terms. The most standard GARCH(1,1) model is given by Bollerslev (1986),

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{3}$$

where $w > 0$, $\alpha \geq 0$, and $\beta \geq 0$. The parameter α is the model parameter for the lagged shocks, and the β is that of the lagged variance. The multiperiod variance forecast for the GARCH(1,1) model, is the sum of the variance forecasts for the k day holding period

$$\hat{\sigma}_{t,k}^2 = \frac{\omega k}{1-\alpha-\beta} + \left(\hat{\sigma}_{t+1}^2 - \frac{\omega}{1-\alpha-\beta}\right)\left(\frac{1-(\alpha+\beta)^2}{1-\alpha-\beta}\right). \tag{4}$$

Nelson (1990) shows that for empirical research we often see that $\beta \approx (1-\alpha)$. This model is referred to as the integrated GARCH (IGARCH(1,1)) model, which is similar to the exponential smoothing method from equation (1) where $\omega = 0$,

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + (1-\alpha)\sigma_{t-1}^2 \tag{5}$$

and the multiperiod variance forecast is given by

$$\hat{\sigma}_{t,k}^2 = \frac{1}{2}k(k-1)\omega + k\hat{\sigma}_{t+1}^2. \tag{6}$$

Empirical research has shown that large negative returns have a larger impact on volatility compared to large positive returns. This effect, known as the leverage effect, led to the development of asymmetric GARCH models which take into account two regimes. One of these models is the GJRGARCH(1,1) model first introduced by Glosten et al. (1993),

$$\sigma_t^2 = \omega + (1-\iota_{t-1})\alpha\varepsilon_{t-1}^2 + \iota_{t-1}\gamma\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{7}$$

where $w > 0$, $\alpha \geq 0$, $\gamma \geq 0$, $\beta \geq 0$, and ι_{t-1} is an indicator function which takes the values:

$$\iota_{t-1} = \begin{cases} 1 & \text{if } \varepsilon_{t-1} > 0; \\ 0 & \text{if } \varepsilon_{t-1} \leq 0; \end{cases} \tag{8}$$

When the past shock is negative the indicator function, ι_{t-1}, will be equal to 0 and then the model reduces to a normal GARCH model. When the past shock is positive, the model uses the γ parameter to estimate the variance. Therefore, for the GJRGARCH, the α parameter captures the effect of negative lagged shocks on the variance, and γ captures the effect of positive lagged shocks, which together captures the leverage effect. The multiperiod variance forecast of the GJRGARCH(1,1) is given by

$$\hat{\sigma}_{t,k}^2 = \frac{\omega k}{1-\frac{1}{2}(\alpha+\gamma)-\beta} + \left(\hat{\sigma}_{t+1}^2 - \frac{\omega}{1-\frac{1}{2}(\alpha+\gamma)-\beta}\right)\left(\frac{1-(\frac{1}{2}(\alpha+\gamma)+\beta)^k}{1-\frac{1}{2}(\alpha+\gamma)-\beta}\right) \tag{9}$$

The parameters of the three GARCH models are estimated using maximum likelihood estimation considering a Student's t-distribution for the shocks. We consider this distribution due to the excess kurtosis found in the returns time series.

3.2 Value-at-Risk Methods

Engle and Manganelli (2004) describe three different categories for VaR Methods. First there is the parametric approach which contains exponential smoothing and GARCH models. These approaches entail quantiles being estimated from volatility forecasts under the assumption of a certain distribution for the shocks. The second category is the nonparametric approach. This method requires no assumptions for the distribution and estimates VaR as the quantile of the empirical distribution of historical returns from a moving window of recent periods. One standard approach in this category is historical simulation. However, historical simulation can bring the same limitations as the moving window approach mentions in Section 3.1. Therefore, Boudoukh et al. (1998) propose a method similar to exponential smoothing which involves allocated exponentially decreasing weights to the n most recent returns with regards to a decay factor, π,

$$\left(\frac{1-\pi}{1-\pi^n}\right), \left(\frac{1-\pi}{1-\pi^n}\right)\pi, \left(\frac{1-\pi}{1-\pi^n}\right)\pi^2, \cdots, \left(\frac{1-\pi}{1-\pi^n}\right)\pi^{n-1}. \tag{10}$$

The decay factor, π allows more weight to be assigned to more recent observations. The sum of these weights add up to one. First, the weights are assigned and then the returns are ordered in ascending order. Then, starting from the lowest return, the corresponding weights are summed up until the value of the quantile θ is reached. This means that the θ quantile estimate is equal to the return observation for which the respective weight was the last used in the summation to reach the value θ. If the estimate of θ falls between two consecutive returns then the midpoint is selected through linear interpolation.

3.2.1 CAViaR Models

Finally, the last VaR method is the semiparametric category. These methods use extreme value theory and quantile regression. The CAViaR models introduced by Engle and Manganelli (2004) are a popular approach which involves autoregressive modelling of the conditional quantiles. The conditional quantiles of a return, Q_t, is defined as the VaR and is the quantile for which $P(r_t \leq Q_t(\theta)) = \theta$. Similar to the nonparametric approach, semiparametric approach requires no distributional assumptions. The four CAViaR models introduced by Engle and Manganelli (2004) and also used in the research of Taylor (2005) are the following:

Indirect GARCH(1,1) CAViaR:

$$Q_t(\theta) = (1 - 2I[\theta < 0.5])(\omega + \alpha Q_{t-1}(\theta)^2 + \beta \varepsilon_{t-1}^2)^{\frac{1}{2}} \tag{11}$$

Adaptive CAViaR:

$$Q_t(\theta) = Q_{t-1}(\theta) + \alpha(\theta - I[\varepsilon_{t-1} \leq Q_{t-1}(\theta)]) \tag{12}$$

Symmetric Absolute Value CAViaR:

$$Q_t(\theta) = \omega + \alpha Q_{t-1}(\theta) + \beta \mid \varepsilon_{t-1} \mid \tag{13}$$

Asymmetric Slope CAViaR:

$$Q_t(\theta) = \omega + \alpha Q_{t-1}(\theta) + \beta_1(\varepsilon_{t-1})^+ + \beta_2(\varepsilon_{t-1})^- \tag{14}$$

In the models in equations (11) to (14), $Q_t(\theta)$ represent the conditional θ quantile, $\omega, \alpha, \beta, and \beta_i (i = 1,2)$ are the parameters. For the Asymmetric Slope CAViaR model, $(\varepsilon_{t-1})^+$ and $(\varepsilon_{t-1})^-$ are the positive and negative shocks respectively. The parameters of the CAViaR models are estimated using the quantile regression minimization (QR Sum) introduced by Koenker and Basset (1978) where $Q_t(\theta)$ is the quantile estimate from the CAViaR model for the θ quantile of the dependent variable y_t, which is log returns:

$$\min\left(\sum_{t|y_t \geq Q_t(\theta)} \theta \mid y_t - Q_t(\theta) \mid + \sum_{t|y_t < Q_t(\theta)} (1-\theta) \mid y_t - Q_t(\theta) \mid \right). \tag{15}$$

3.3 VaR to Volatility Estimates

In order to produce volatility forecasts from quantile estimates received from the VaR methods mentioned in section 3.2, Taylor (2005) proposes a least squares regression of the squared shocks, ε_i^2 on the square of the interval between symmetric quantile estimates. One-step ahead variance forecasts are then given by:

$$\hat{\sigma}^2_{t+1} = \alpha_1 + \beta_1 \left(\hat{Q}_{t+1}(1-\theta) - \hat{Q}_{t+1}(\theta)\right)^2 \tag{16}$$

where $\hat{Q}_{t+1}(1-\theta)$ and $\hat{Q}_{t+1}(\theta)$ are the one-step-ahead quantile forecasts from the VaR methods (Historical simulation, BRW method, and the four CAViaR models). The analysis that Taylor considers, and which we also consider, are the 98%, 95%, and 90% intervals.

For the historical simulation method, BRW method, and the Adaptive CAViaR model, we obtain multiperiod variance forecasts by multiplying the one-step-ahead variance forecasts by the holding period, k. This method to obtain multiperiod variance forecasts is not suitable for the other three CAViaR models and Taylor (2005) proposes a separate least squares regression to obtain these estimates. Under the assumption that the conditional mean is constant over the k days holding period, and that there is no autocorrelation between successive daily shocks, Taylor proposes to regress the realized multiperiod variance $\sigma^2_{Rt,k}$ on the square of the interval between symmetric quantile estimates, $\left(\hat{Q}_{t+1}(1-\theta) - \hat{Q}_{t+1}(\theta)\right)^2$. The realized multiperiod variance is given as, $\sigma^2_{Rt,k} = \sum_{i=1}^{k} \varepsilon^2_{t+i}$. Then equation (16) becomes,

$$\hat{\sigma}^2_{t,k} = \alpha_k + \beta_k \left(\hat{Q}_{t+1}(1-\theta) - \hat{Q}_{t+1}(\theta)\right)^2, \tag{17}$$

where the parameters are estimated by a least squares regression.

3.4 Robust Approaches

Two robust benchmark approaches are also included in the study. This is to reduce the presence of outliers which cause poor estimates for both volatility and quantile estimates. The first approach entails trimming the lowest and highest 1% by replacing any return observations exceeding the 1% and 99% unconditional quantiles by the value of the unconditional quantile. The second approach estimates the CAViaR quantiles for the 1% and 99% in-sample fitted conditional quantiles. All return observations that were lower or higher than the 1% and 99% respectively were set equal to this quantile value. These approaches are known as winsorizing the data. The empirical research of Taylor (2005) found that the first approach works best for the GJRGARCH(1,1) model, and the second for the Asymmetric Slope CAViaR model followed by GJRGARCH, so we include these two. We will refer to these models as the Simplistic winsorized GJRGARCH model, and the CAViaR Winsorized GJRGARCH model. To give more insight into the second approach, we first estimate conditional quantiles for the 1% and 99% quantiles using the Asymmetric Slope CAViaR model for the in sample log returns. Next, we compare the in sample returns to their corresponding in sample fitted conditional quantile. All observations that are lower than their conditional quantile are set equal to the value of the quantile. Finally, With this new data set of in sample returns (winsorized), we run the normal GJRGARCH model.

3.5 Threshold and Smooth Transition CAViaR

Two new models are introduced into the research of Taylor (2005). These models are added into the research to see if they outperform the Asymmetric Slope CAViaR model which performs well in the paper of Taylor. We will first discuss the *Threshold*-CAViaR (T-CAViaR), first introduced by Gerlach et al. (2011):

$$Q_t(\theta) = \begin{cases} \omega_1 + \alpha_1 Q_{t-1}(\theta) + \beta_1 \mid \varepsilon_{t-1} \mid & \text{if } \varepsilon_{t-1} \leq 0 \\ \omega_2 + \alpha_2 Q_{t-1}(\theta) + \beta_2 \mid \varepsilon_{t-1} \mid & \text{if } \varepsilon_{t-1} > 0 \end{cases} \tag{18}$$

This model is very similar to the Asymmetric Slope CAViaR model except that in the T-CAViaR model you have a separate parameter for ω and α depending on the sign of the shock. According to Gerlach et al. (2011) this allows the model to capture more flexible asymmetric and nonlinear responses than the Asymmetric Slope CAViaR model.

We also introduce a smooth-transition CAViaR (ST-CAViaR) model specification to see if they outperform the Asymmetric Slope CAViaR model. The ST-CAViaR model is based on the combined research of González-Rivera et al. (1998) and Hubner and Cizek (2019) and is given by:

$$Q_t(\theta) = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-1}^2 F(\varepsilon_{t-d,\delta}) + \beta Q_{t-1}(\theta) \tag{19}$$

where

$$F(\varepsilon_{t-d,\delta}) = \frac{1}{1 + e^{\delta \varepsilon_{t-d}}} - \frac{1}{2}, \tag{20}$$

is a transition function. ε_{t-d} is the transition variable, and δ is the smoothness parameter. A large smoothness parameter indicates a steep transition function, and then the model collapses to a two regime model. On the other hand, a relatively small smoothness parameter indicates more than two regimes. This transition function is a logistic function which varies between 0 and 1 (Taylor, 2004).

3.6 Parameter Estimation

The data is split between estimation and forecasting at the date of 28th of May, 2001. Taylor (2005) uses 500 observations for out of sample forecasting, however his data set includes non-trading days with *null* observations while ours does not. Therefore, our out of sample forecasting observations are slightly less than 500, but our data split between estimation and forecasting is close to that of Taylor. For the simple moving average method, we use a window of 30 days to estimate the volatility forecasts. The parameters of the exponential smoothing method are optimized using the minimization in equation (2) which is the minimization of the in-sample sum of squared deviations between the forecasts of the variance and the squared error. Parameters of the GARCH models are estimated with maximum likelihood. Since the data shows significant excess kurtosis, we consider a Student's t-distribution for the error terms with optimized degrees of freedom.

For the VaR-based methods we use one year of observations for the moving window of the historical simulation and the BRW method, similar to Taylor. We optimize the π parameter for the BRW method using the Quantile Regression Sum of equation (15). The parameters of the six CAViaR models are estimated using a procedure similar to that of what Taylor (2005), uses, which was introduced by Engle and Manganelli (2004). First 10,000 vectors of parameters are generated using a uniform random number generator between 0 and 1. The 10 parameter vectors which result in the lowest QR Sum were used as the 10 starting values. Each of these 10 parameter vectors was then run in the quasi-Newton algorithm to optimize the parameters. The optimal parameter vector that resulted in the lowest QR Sum was taken as the optimal parameter vector for the CAViaR model. The optimal starting values for the CAViaR models (the parameter vector which eventually resulted in the lowest QR Sum) and the optimal parameters for each method/model can be found in Appendix A in Tables 13 to 18.

The quantile estimates constructed by the models using the optimal parameters for the VaR-based methods are then used to construct the interval between symmetric quantile estimates. We then regress the squared errors on these intervals to construct one day ahead volatility forecasts as depicted in equation (16) in Section 3.3. We also evaluate the volatility forecasts for the 10 and 20 day ahead forecasts. For the moving average methods, GARCH models, Historical Simulation, BRW, and the Adaptive CAViaR model, these k-day ahead volatility forecasts are calculated by multiplying the one-day ahead forecasts by the holding period, k. For the other five CAViaR models, we find the k-day ahead volatility forecasts using the realized multiperiod variance and equation (17) also discussed in Section 3.3. For all the methods/models in the VaR based methods we evaluate the volatility forecasts for the one-, 10, and 20 day ahead considering a 98%, 95%, and 90% interval.

3.7 Post-Sample Volatility Forecast Evaluation

To evaluate the performance of the different models/methods, we will look at a few performance measures and perform a series of tests. We will first evaluate the α_1 and β_1 parameter estimations from the least squares regression of ε_i^2 on the interval between symmetric quantiles from equation (16). This will be done for the Asymmetric Slope CAViaR, T-CAViaR, and ST-CAViaR models. The parameter estimates will be compared to the parameter values of Pearson and Tukey (1965) through a t-test. Pearson and Tukey (1965) find constant values for the ratio of the standard deviation to the interval between symmetric quantiles which are given by:

$$\hat{\sigma} = \frac{\hat{Q}(0.99) - \hat{Q}(0.01)}{4.65} \qquad \hat{\sigma} = \frac{\hat{Q}(0.975) - \hat{Q}(0.025)}{3.92} \qquad \hat{\sigma} = \frac{\hat{Q}(0.95) - \hat{Q}(0.05)}{3.25} \tag{21}$$

One of the main performance measures that we will consider for all methods/models is the coefficient of determination, R^2, from the least squares regression of the realized multiperiod variance on the post-sample variance forecast. This will be done for the one-day ahead forecasts, and a 10 and 20 days holding period. Next we perform an encompassing test based on the findings of Poon and Granger (2003). We perform tests to investigate whether the forecasting performance of the Asymmetric Slope CAViaR, T-

CAViaR, and ST-CAViaR models are significantly better than the GJRGARCH model. This is done by creating a combined forecast with weights given to the different models and testing if the coefficient is significantly different from zero:

$$\sigma^2_{Rt,k} = w\hat{\sigma}^2_{Ct,k} + (1-w)\hat{\sigma}^2_{Gt,k} + e_t, \tag{22}$$

where w is the weight term, $\sigma^2_{Rt,k}$ is the realized multiperiod variance, $\hat{\sigma}^2_{Ct,k}$ is the variance forecast of one of the three CAViaR models in question, $\hat{\sigma}^2_{Gt,k}$ is the variance forecast of the GJRGARCH model, and e_t is a residual term. Non overlapping data is used, meaning we take the sum of every ten observations for the ten day holding period, and twenty observations for the twenty day holding period. We only perform this test for the 90% interval since it had the best results in terms of R^2 for the Asymmetric Slope CAViaR model in the work of Taylor (2005). Our analysis, does not indicate that this interval is the best performing for the Asymmetric Slope CAViaR model but that the 95% interval is superior in terms of R^2. However, we wish to remain consistent in order to compare our results with those of Taylor.

To evaluate the quantile forecasts obtained from the different VaR methods, we look at three different measures which was first looked at by Engle and Manganelli (2004) and also used by Taylor (2005). These three measures are the hit percentage, the dynamic quantile (DQ) test statistic and the QR Sum. Hit percentage is the percentage of out of sample observations that fall below the forecasted quantile of that model, this percentage should ideally be equal to θ. The DQ test statistic tests for conditional coverage and evaluates the dynamic properties of the quantile estimator. We first defined a hit variable, like Engle and Manganelli (2004), as $Hit_t = I[\varepsilon_t \leq \hat{Q}_t(\theta)] - \theta$. We test if this is distributed i.i.d. Bernoulli with probability θ, and also if it is independent of the quantile estimator, $\hat{Q}_t(\theta)$. In the test we include five lags for the hit variable. For hit percentage, values closer to θ are better; for DQ and QR sum, lower values are better. We evaluate these three performance measures for the Asymmetric Slope CAViaR, T-CAViaR, ST-CAViaR, and the GJRGARCH models.

4 Results

4.1 CAViaR Volatility Forecasting Parameters

We will first compare the Asymmetric Slope CAViaR, T-CAViaR, and ST-CAViaR in terms of the α_1 and β_1 estimates for the least sqaures regression of the ε_i^2 on the interval between symmetric quantiles for the three different intervals. We compare the parameter estimates to the values of Pearson and Tukey (1965) in equation (21) and on this basis compare the models to each other.

Table 2: Parameters, α_1 and β_1, in Expression 16, from LS Regression of ε_i^2 on the Interval Between Symmetric Quantiles Estimated by Asymmetric Slope CAViaR Model for In-Sample Stock Index Data

Interval	Parameter	CAC 40	DAX 30	NIKKEI 225	S&P 500	Pearson and Tukey values
0.98	$\alpha_1 \times 10^6$	-37.2	-10.1	70.5	-182.5	0
		(26.3)	(21.7)	(30.5)	(24.3)	
	β_1	0.0467	0.0429	0.0307	0.0758	$4.65^{-2} = 0.0462$
		(0.0033)	(0.0021)	(0.0024)	(0.0044)	
0.95	$\alpha_1 \times 10^6$	-52.0	-47.8	45.6	-8.8	0
		(24.6)	(22.6)	(31.9)	(16.0)	
	β_1	0.0742	0.0707	0.0569	0.0664	$3.92^{-2} = 0.0651$
		(0.0046)	(0.0034)	(0.0044)	(0.0038)	
0.90	$\alpha_1 \times 10^6$	26.0	-8.8	13.3	-2.8	0
		(20.8)	(21.3)	(33.7)	(15.8)	
	β_1	0.0819	0.0959	0.0952	0.0921	$3.25^{-2} = 0.0947$
		(0.0052)	(0.0047)	(0.0073)	(0.0054)	

Note: Standard errors in parentheses.

In Table 2 the parameter estimates of the Asymmetric Slope CAViaR model are depicted, along with the standard errors given in parentheses. The Pearson and Tukey (1965) values are given in the last column. It appears that many of the estimated parameters are close to these values. However, in only four out of the 12 regression, we have that the constant is not significantly different than zero (at a 5% level). The results of Taylor (2004) indicated that 13 of the 15 (he also considers the FTSE 100 index) regressions have a constant not significantly different from zero. This difference is due to the fact that our standard errors are larger compared to the standard errors that Taylor has. Our results for the β_1 estimates are very positive. For all 12 estimates they are not significantly different from the Pearson and Tukey values, while Taylor only has this for the indices belonging to the DAX 30 index and for all the indices corresponding to the 95% interval.

Table 3: Parameters, α_1 and β_1, in Expression 16, from LS Regression of ε_i^2 on the Interval Between Symmetric Quantiles Estimated by Threshold CAViaR Model for In-Sample Stock Index Data

Interval	Parameter	CAC 40	DAX 30	NIKKEI 225	S&P 500	Pearson and Tukey values
0.98	$\alpha_1 \times 10^6$	-48.0	-18.6	-33.6	7.4	0
		(19.6)	(22.5)	(43.6)	(16.5)	
	β_1	0.0482	0.0459	0.0381	0.0347	$4.65^{-2} = 0.0462$
		(0.0032)	(0.0024)	(0.0036)	(0.0023)	
0.95	$\alpha_1 \times 10^6$	-37.5	0.8	19.4	-3.8	0
		(24.1)	(21.3)	(33.4)	(15.7)	
	β_1	0.0722	0.0612	0.0617	0.0628	$3.92^{-2} = 0.0651$
		(0.0045)	(0.0031)	(0.0048)	(0.0036)	
0.90	$\alpha_1 \times 10^6$	4.3	9.6	-39.9	-6.8	0
		(21.6)	(20.7)	(38.3)	(16.0)	
	β_1	0.0895	0.0912	0.1093	0.0970	$3.25^{-2} = 0.0947$
		(0.0055)	(0.0045)	(0.0087)	(0.0056)	

Note: Standard errors in parentheses.

Table 4: Parameters, α_1 and β_1, in Expression 16, from LS Regression of ε_i^2 on the Interval Between Symmetric Quantiles Estimated by Smooth Transition CAViaR Model for In-Sample Stock Index Data

Interval	Parameter	CAC 40	DAX 30	NIKKEI 225	S&P 500	Pearson and Tukey values
0.98	$\alpha_1 \times 10^6$	6.7	-69.8	-98.1	-108.2	0
		(22.7)	(23.4)	(41.2)	(20.9)	
	β_1	0.0412	0.0485	0.0468	0.0701	$4.65^{-2} = 0.0462$
		(0.0027)	(0.0023)	(0.0036)	(0.0041)	
0.95	$\alpha_1 \times 10^6$	-59.1	-221.6	95.6	8.8	0
		(25.7)	(30.2)	(28.7)	(15.8)	
	β_1	0.0819	0.1053	0.0460	0.0602	$3.92^{-2} = 0.0651$
		(0.0052)	(0.0052)	(0.0036)	(0.0037)	
0.90	$\alpha_1 \times 10^6$	45.3	16.8	10.5	17.9	0
		(19.9)	(20.6)	(33.7)	(15.3)	
	β_1	0.0728	0.0880	0.0975	0.0831	$3.25^{-2} = 0.0947$
		(0.0046)	(0.0044)	(0.0075)	(0.0050)	

Note: Standard errors in parentheses.

The parameter estimates of α_1 and β_1 for the LS regression for the T-CAViaR and ST-CAViaR model are shown in Table 3 and Table 4 respectively. Similar to the Asymmetric Slope CAViaR model the parameter estimates also appear to be very close to the Pearson and Tukey Values. For the T-CAViaR model the constant is not significantly different from zero at a 5% level for only one of the 12 regression, the 0.98 interval for the CAC 40 index. For the ST-CAViaR the constant is not significantly different from zero for seven regressions. Similar to the Asymmetric Slope CAViaR model, the parameter estimates for β_1 are not significantly different from the Pearson and Tukey estimates for all of the regressions in both the CAViaR models. Regarding the parameter estimates, for the one-step-ahead variance forecasts using the regression of ε_i^2 on the interval between symmetric quantiles, we conclude that the ST-CAViaR is the most preferable when compared to the Pearson and Tukey (1965) values.

4.2 Out of Sample Forecasting Performance

Table 5: R^2 Measure of Informational Content for Post-Sample One-Day-Ahead Variance Forecasts for Stock Indices

	CAC 40	DAX 30	NIKKEI 225	S&P 500	Mean
Moving Average and Garch Methods					
Simple moving average	9.6	8.3	1.4	6.4	6.4
Exponential Smoothing	10.9	13.0	3.8	13.2	10.2
GARCH	11.1	12.1	2.8	8.6	8.7
IGRACH	12.2	12.7	3.0	10.6	9.6
GJRGARCH	14.0	14.7	4.0	16.0	12.2
Simplistic winsorized GJRGARCH	12.2	14.4	3.9	15.6	11.5
CAViaR winsorized GJRGARCH	14.6	14.9	4.2	16.5	12.5
VaR-based methods					
Historical Simulation 98%	2.7	4.8	0.8	0.0	2.1
Historical Simulation 95%	4.0	5.1	0.9	0.1	2.5
Historical Simulation 90%	2.5	3.5	0.4	0.0	1.6
BRW 98% interval	3.9	4.6	1.2	2.8	3.1
BRW 95% interval	8.4	6.6	0.3	4.6	5.0

BRW 90% interval	6.2	9.0	1.1	2.6	4.7
Indirect GARCH CAViaR 98% interval	12.6	12.4	2.9	11.0	9.7
Indirect GARCH CAViaR 95% interval	12.8	12.2	3.4	10.6	9.8
Indirect GARCH CAViaR 90% interval	12.1	12.6	3.3	10.6	9.6
Adaptive CAViaR 98% Interval	3.3	5.5	0.2	2.9	3.0
Adaptive CAViaR 95% Interval	8.0	6.8	0.9	4.1	5.0
Adaptive CAViaR 90% Interval	5.4	7.7	1.4	4.9	4.9
Sym Abs Value CAViaR 98% interval	12.3	13.3	2.2	12.0	9.9
Sym Abs Value CAViaR 95% interval	12.2	12.0	2.5	10.2	9.2
Sym Abs Value CAViaR 90% interval	12.1	12.2	2.4	9.3	9.0
Asym Slope CAViaR 98% interval	12.9	15.9	4.3	19.1	13.0
Asym Slope CAViaR 95% interval	16.2	**16.1**	4.3	19.5	14.0
Asym Slope CAViaR 90% interval	15.5	16.0	4.5	19.6	13.9
T-CAViaR 98% interval	15.1	14.5	**5.4**	17.3	13.1
T-CAViaR 95% interval	**16.4**	15.4	4.4	**19.8**	14.0
T-CAViaR 90% interval	15.9	15.7	5.3	19.6	**14.2**
ST-CAViaR 98% interval	13.6	**16.1**	5.1	19.6	13.6
ST-CAViaR 95% interval	14.8	15.0	5.0	19.2	13.5
ST-CAViaR 90% interval	15.5	15.7	4.7	19.3	13.8

Note: R^2 values are percentages. The highest value per index and for the mean is given in bold.

In Table 5 we summarize the R^2 for the out of sample volatility forecasting performance, with the highest and best value given in bold. These coefficients of determination are calculated from the LS regression of realized variance on the post-sample variance forecast from the models. For the VaR-based methods, the R^2 entail the informational content in the squared interval between symmetric quantile forecasts. First focussing on the Moving Average and GARCH methods we see in Table 5 that the GJRGARCH and the CAViaR winsorized GJRGARCH outperform all the other models in this category for every index. This shows how taking the leverage effect into account, greatly improves volatility forecasts. The CAViaR Winsorized GJR-GARCH performs slightly better than GJRGARCH for every index as this model is more robust due to the winsorization of the data being based on the in sample fitted conditional quantiles.

With regards to the leverage effect we also see that this is the reason that the VaR-based Asymmetric Slope CAViaR model performs very well in comparison with all the other models used in the work of Taylor (2005). This leverage effect is also present in the newly introduced T-CAViaR and ST-CAViaR models. They both perform very well in terms of R^2, most of the time even slightly better than the Asymmetric Slope CAViaR model. For the one-day-ahead forecasts it appears that the T-CAViaR model performs best for the CAC 40, NIKKEI 225, and S&P 500 index. For the DAX 30 index the Asymmetric Slope for the 95% interval and the ST-CAViaR for the 98% interval perform equally well in terms of the coefficient of determination.

Table 6: R^2 Measure of Informational Content for Post-Sample Ten-Day-Ahead Variance Forecasts for Stock Indices

	CAC 40	DAX 30	NIKKEI 225	S&P 500	Mean
Moving Average and Garch Methods					
Simple moving average	32.4	24.5	3.2	15.5	18.9
Exponential Smoothing	41.0	36.3	7.0	29.1	28.4
GARCH	36.8	33.6	6.4	21.4	24.5
IGRACH	40.8	34.8	6.7	25.3	26.9
GJRGARCH	47.8	41.9	9.4	41.7	35.2
Simplistic winsorized GJRGARCH	41.0	41.0	9.4	40.4	33.0
CAViaR winsorized GJRGARCH	50.7	42.7	9.4	42.7	36.4
VaR-based methods					
Historical Simulation 98%	10.5	16.3	3.6	0.2	7.7
Historical Simulation 95%	14.8	17.5	3.3	0.1	8.9
Historical Simulation 90%	9.7	12.6	1.2	0.0	5.9
BRW 98% interval	11.0	12.9	1.4	6.7	8.0
BRW 95% interval	25.3	18.8	0.7	10.9	13.9
BRW 90% interval	19.9	23.9	3.3	5.6	13.2
Indirect GARCH CAViaR 98% interval	43.5	34.2	6.3	26.0	27.5
Indirect GARCH CAViaR 95% interval	43.7	33.6	7.2	25.4	27.5
Indirect GARCH CAViaR 90% interval	40.4	34.6	7.0	25.3	26.8
Adaptive CAViaR 98% Interval	10.5	15.0	0.5	7.3	8.3
Adaptive CAViaR 95% Interval	25.6	21.7	3.8	10.3	15.3
Adaptive CAViaR 90% Interval	18.5	24.4	4.3	12.2	14.8
Sym Abs Value CAViaR 98% interval	41.6	38.3	6.6	29.0	28.9
Sym Abs Value CAViaR 95% interval	41.2	34.4	7.7	25.2	27.1
Sym Abs Value CAViaR 90% interval	40.8	35.0	7.5	23.0	26.6
Asym Slope CAViaR 98% interval	50.1	45.7	11.1	44.5	37.9
Asym Slope CAViaR 95% interval	**56.2**	44.9	11.7	45.4	39.6
Asym Slope CAViaR 90% interval	52.0	45.0	13.0	45.6	38.9
T-CAViaR 98% interval	34.2	45.5	**16.4**	37.9	33.5
T-CAViaR 95% interval	53.8	45.8	12.4	47.1	39.8
T-CAViaR 90% interval	53.4	45.2	15.0	45.3	39.7
ST-CAViaR 98% interval	52.5	**46.2**	14.1	**47.5**	**40.1**
ST-CAViaR 95% interval	52.1	45.2	11.3	43.2	38.0
ST-CAViaR 90% interval	53.6	44.1	13.1	43.8	38.6

Note: R^2 values are percentages. The highest value per index and for the mean is given in bold.

Table 6 above and Table 12 in Appendix A gives the R^2 measure of the informational content for the out of sample variance forecasts for the holding period of 10 and 20 days respectively. Similar to what Taylor (2005) discusses is that the gap between the R^2 measure for the GJRGARCH model and the CAViaR Winsorized GJRGARCH and the other non VaR-based methods becomes larger for the longer holding periods compared to the one-day-ahead forecasts. Interestingly, the difference is also larger between the Asymmetric Slope CAViaR model and the GJRGARCH model. In the 10 and 20 day holding setting we see that the Asymetric Slope CAViaR 95% interval model is the best for the CAC 40 index. For the other indices it is either the T-CAViaR or the ST-CAViaR model for the 98% interval. Compared to the one-day-ahead forecasts, the T-CAViaR performs worse in comparison to the ST-CAViaR model. For the NIKKEI 225 index, T-CAViaR is still superior but for DAX 30 and S&P 500 we see that the ST-CAViaR for the 98% interval performs the best. This is also shown in the mean R^2 values for the 10 and 20 day holding periods.

When considering the informational content measure, R^2, for the post-sample variance forecasts we conclude that the T-CAViaR models, especially for the 95% index, performs the best for the one-day-ahead variance forecasts. When extending the holding period to 10 or 20 days, the ST-CAViaR model for the 98% is superior on average. An interesting note, is that the ST-CAViaR model for the 98% interval is always the best performing for the DAX 30 index, for every holding period. If we compare these results to the parameter estimates for the ST-CAViaR model found in Appendix A, Table 18, we find that the DAX 30 index has the lowest estimated δ parameter, which is the smoothing parameter. This indicates that a smooth transition between multiple regimes is more fitting for this index. Furthermore, the results suggest that the leverage effect becomes more powerful when the holding period increase as the T-CAViaR and ST-CAViaR models try to model this effect more accurately.

4.3 Encompassing Test

Table 7: Results of the Encompassing Test $\sigma^2_{Rt,k} = w\hat{\sigma}^2_{Ct,k} + (1-w)\hat{\sigma}^2_{Gt,k} + e_t$, for the Asymmetric Slope CAViaR model, for Stock Index Data

	CAC 40	FAX 30	NIKKEI 225	S&P 500
One-step-ahead				
$\hat{w}$	1.95	0.92	1.19	1.05
p-value for $H_0: w = 1$	0.98	0.43	0.65	0.59
$H_1: w < 1$				
p-value for $H_0: w = 0$	0.00	0.03	0.02	0.00
$H_1: w > 0$				
10-day holding period				
$\hat{w}$	1.63	-0.09	1.94	0.80
p-value for $H_0: w = 1$	0.83	0.00	0.97	0.24
$H_1: w < 1$				
p-value for $H_0: w = 0$	0.02	0.80	0.00	0.01
$H_1: w > 0$				
20-day holding period				
$\hat{w}$	1.03	0.43	1.39	0.79
p-value for $H_0: w = 1$	0.52	0.10	0.75	0.30
$H_1: w < 1$				
p-value for $H_0: w = 0$	0.20	0.33	0.03	0.06
$H_1: w > 0$				

Note: Tests uses multiperiod forecasts for nonoverlapping holding periods in the post-sample period. $\sigma^2_{Rt,k}$, is realized multiperiod variance; $\sigma^2_{Ct,k}$ is the VaR-based variance forecast using 90% interval estimated by the Asymmetric Slope CAViaR model; $\sigma^2_{Gt,k}$ is the GJRGARCH variance forecast; and ε_t is a residual term.

The next test we perform is the encompassing test to see if the forecasting performance of the CAViaR model significantly outperforms the GJRGARCH model. We only consider the Asymmetric Slope CAViaR, T-CAViaR, and ST-CAViaR model for the 90% interval. We consider these three because the Asymmetric Slope CAViaR model performed the best in Taylor (2005), and we are interested if the T-CAViaR and ST-CAViaR perform better than the Asymmetric Slope CAViaR model. We are interested in the estimation of the w parameter; if we reject the null hypothesis that $w = 1$ then we reject that the CAViaR method encompasses GJRGARCH, meaning it does not significantly outperform the GJRGARCH method. If we reject the null hypothesis that $w = 0$ then we reject that the GJRGARCH method encompasses the CAViaR method.

Table 7 shows the results of the test for the Asymmetric Slope CAViaR model. Similar to the results of Taylor, we find that in only one of the cases (we have 12 and Taylor had 15) rejects the null hypothesis for $w = 1$ at a 5% significance level. We reject the null hypothesis for the DAX 30 index for a 10 day holding period while for Taylor it is the 20 day holding period. Regarding the null hypothesis of $w = 0$, we can reject this for nine out of the 12 cases. This is mainly for the DAX 30 index, where we reject for the 10 and 20 day holding period, and for the CAC 40 index for the 20 day holding period. Taylor does not reject this null hypothesis as well for the DAX 30 index for the one-step-ahead, but he does reject it for the CAC 40 index for the 20 day holding period.

Table 8: Results of the Encompassing Test $\sigma^2_{Rt,k} = w\hat{\sigma}^2_{Ct,k} + (1-w)\hat{\sigma}^2_{Gt,k} + e_t$, for the ST-CAViaR model, for Stock Index Data

	CAC 40	FAX 30	NIKKEI 225	S&P 500
One-step-ahead				
$\hat{w}$	1.58	1.21	1.00	1.04
p-value for $H_0 : w = 1$	0.95	0.66	0.50	0.58
$H_1 : w < 1$				
p-value for $H_0 : w = 0$	0.00	0.02	0.00	0.00
$H_1 : w > 0$				
10-day holding period				
$\hat{w}$	1.69	-0.21	1.27	0.86
p-value for $H_0 : w = 1$	0.85	0.00	0.81	0.32
$H_1 : w < 1$				
p-value for $H_0 : w = 0$	0.01	0.60	0.00	0.01
$H_1 : w > 0$				
20-day holding period				
$\hat{w}$	0.87	0.58	0.94	0.81
p-value for $H_0 : w = 1$	0.44	0.20	0.43	0.32
$H_1 : w < 1$				
p-value for $H_0 : w = 0$	0.30	0.26	0.01	0.06
$H_1 : w > 0$				

Note: Tests uses multiperiod forecasts for nonoverlapping holding periods in the post-sample period. $\sigma^2_{Rt,k}$, is realized multiperiod variance; $\sigma^2_{Ct,k}$ is the VaR-based variance forecast using 90% interval estimated by the Threshold CAViaR model; $\sigma^2_{Gt,k}$ is the GJRGARCH variance forecast; and ε_t is a residual term.

Table 9: Results of the Encompassing Test $\sigma^2_{Rt,k} = w\hat{\sigma}^2_{Ct,k} + (1-w)\hat{\sigma}^2_{Gt,k} + e_t$, for the T-CAViaR model, for Stock Index Data

	CAC 40	FAX 30	NIKKEI 225	S&P 500
One-step-ahead				
$\hat{w}$	1.38	0.81	1.52	1.06
p-value for $H_0 : w = 1$	0.84	0.33	0.82	0.60
$H_1 : w < 1$				
p-value for $H_0 : w = 0$	0.00	0.06	0.01	0.00
$H_1 : w > 0$				
10-day holding period				
$\hat{w}$	1.25	-0.07	2.04	0.79
p-value for $H_0 : w = 1$	0.68	0.00	0.97	0.25
$H_1 : w < 1$				
p-value for $H_0 : w = 0$	0.03	0.85	0.00	0.02
$H_1 : w > 0$				
20-day holding period				
$\hat{w}$	0.73	0.47	1.52	0.83
p-value for $H_0 : w = 1$	0.34	0.12	0.81	0.35
$H_1 : w < 1$				
p-value for $H_0 : w = 0$	0.26	0.29	0.01	0.06
$H_1 : w > 0$				

Note: Tests uses multiperiod forecasts for nonoverlapping holding periods in the post-sample period. $\sigma^2_{Rt,k}$, is realized multiperiod variance; $\sigma^2_{Ct,k}$ is the VaR-based variance forecast using 90% interval estimated by the Smooth Transition CAViaR model; $\sigma^2_{Gt,k}$ is the GJRGARCH variance forecast; and ε_t is a residual term.

Tables 8 and 9 show the results of the encompassing test for the T-CAViaR and ST-CAViaR model at a 90% interval compared to the GJRGARCH respectively. The results of the tests are almost the same for both and very close to the results for the test with the Asymmetric Slope CAViaR model. Again we only reject the null hypothesis for $w = 1$ for only the DAX 30 index for the 10 day holding period. However, for the null hypothesis of $w = 0$ we only reject for eight out of the 12 cases for the T-CAViaR and seven out of the 12 cases for ST-CAViaR. We do not reject the hypothesis for the same cases as the Asymmetric Slope CAViaR model but now we also do not reject it for the S&P 500 index for a 20 day holding period for both of the other two CAViaR models. For the ST-CAViaR model we also do not reject the DAX 30 index for the one-step-ahead case. Even though the difference between the three CAViaR models is small as the extra rejections in the T-CAViaR and ST-CAViaR all have a p-value of 0.06, meaning they just do not reject, the results show that the Asymmetric Slope CAViaR model better outperforms the GJRGARCH model compared to the other two CAViaR models under investigation.

4.4 Quantile Forecast Evaluation

As the last part of our evaluation, we consider the quantile forecasts from the GJRGARCH, Asymmetric Slope CAViaR, T-CAViaR, and ST-CAViaR models. The three measures used are the hit percentage, DQ test statistic, and the QR sum, which were also used in the research of Engle and Manganelli (2004). For these tests we consider the 98% and 90% interval, meaning we consider the 0.01, 0.99, 0.05, and 0.95 quantiles. For the hit percentage, values closest to the actual θ quantile are better and given in bold in the results. For both the DQ test statistic and the QR Sum the lowest values are better and also given in bold.

Table 10: Hit Percentage, Dynamic Quantile Test Statistic, and QR Sum for One-Day-Ahead Forecasting of the 0.01 and 0.99 Quantiles for the Stock Index Data Using GJRGARCH, Asymmetric Slope CAViaR, T-CAViaR, and ST-CAViaR

	CAC 40		DAX 30		NIKKEI225		S&P500	
	0.01	0.99	0.01	0.99	0.01	0.99	0.01	0.99
Hit %								
GJRGARCH	0.6	**99.0**	0.6	**99.8**	0.2	99.6	**0.8**	99.6
Asym Slope CAViaR	0.6	97.3	**1.0**	98.1	0.4	99.6	2.1	99.6
T-CAViaR	**1.0**	99.4	1.4	98.1	**0.6**	**99.4**	**0.8**	99.8
ST-CAViaR	0.6	97.1	0.4	98.1	0.4	99.6	2.1	**99.0**
DQ								
GJRGARCH	1.0**	20.2**	**0.7****	**3.2**	3.3	1.9	**2.1***	1.8
Asym Slope CAViaR	25.5**	26.2**	**0.7**	10.0	1.7	1.7	16.5*	1.7
T-CAViaR	2.6	**0.9**	3.1	7.5	**0.9**	**1.4**	3.0	3.0
ST-CAViaR	**0.7**	29.5**	1.8	6.4	1.9	2.4	18.7**	**0.3**
QR Sum								
GJRGARCH	11.5	10.9	**12.3**	**12.0**	9.0	**10.0**	**8.5**	7.9
Asym Slope CAViaR	11.0	13.3	12.7	12.2	**8.8**	10.4	9.3	8.0
T-CAViaR	**8.7**	**7.7**	13.1	12.3	9.0	10.5	9.5	7.8
ST-CAViaR	11.2	14.2	12.6	13.2	**8.8**	10.6	9.0	**7.3**

Note: Significance at 5% and 1% levels indicated by * and **, respectively. Tests were performed on DQ but not Hit % because sample size is not sufficiently large. QR sum values have been multiplied by 100.

Table 10 summarizes the results of the three measures for the 98% interval. Similar as in the research done by Taylor (2005) GJRGARCH and Asymmetric Slope CAViaR perform about the same in terms of hit percentage. The T-CAViaR model performs better on average than all the other models, but it is close to the performance of the GJRGARCH. T-CAViaR beats the Asymmetric Slope CAViaR model in every Index for the 0.01 and 0.99 quantiles except for the 0.01 quantile of the DAX 30 index and the 0.99 quantile of the S&P500 index. ST-CAViaR performs the same as the Asymmetric Slope CAViaR or slightly worse for every case except for the 0.99 quantile of the S&P 500 index where it is the best performing. The DQ test statistic shows that the T-CAViaR and the GJRGARCH perform about the same on average. The same pattern that was seen in the hit percentage between Asymmetric Slope and T-CAViaR is present in the DQ test statistic as well. T-CAViaR is better for all cases except for DAX 30 at the 0.01 quantile and for the S&P 500 at the 0.99 quantile. The Asymmetric Slope performs better when compared to the ST-CAViaR. The QR Sum values calculated from equation (15), shows that the GJRGARCH model works the best out of the four models on average. The Asymmetric Slope performs better compared to the T-CAViaR model and about the same compared to the ST-CAViaR model. These result are not surprising, the R^2 values in Table 5 show that the Asymmetric Slope CAViaR, T-CAViaR, and ST-CAViaR are very close together, and the GJRGARCH is only slightly lower.

Table 11: Hit Percentage, Dynamic Quantile Test Statistic, and QR Sum for One-Day-Ahead Forecasting of the 0.05 and 0.95 Quantiles for the Stock Index Data Using GJRGARCH, Asymmetric Slope CAViaR, T-CAViaR, and ST-CAViaR

	CAC 40		DAX 30		NIKKEI225		S&P500	
	0.05	0.95	0.05	0.95	0.05	0.95	0.05	0.95
Hit %								
GJRGARCH	8.0	95.5	9.1	**95.0**	6.4	**94.5**	6.4	**95.0**
Asym Slope CAViaR	4.5	**95.1**	**7.6**	94.2	**5.1**	94.3	**5.0**	95.9
T-CAViaR	5.6	95.7	**7.6**	94.4	**5.1**	94.3	5.4	95.2
ST-CAViaR	**4.7**	95.3	7.9	94.4	5.7	94.3	5.2	95.7
DQ								
GJRGARCH	10.8	4.6	30.5	**5.6**	8.7	**6.6**	10.5	7.1
Asym Slope CAViaR	7.0	4.5	24.7**	13.9	**5.6**	7.4	**4.5**	4.4
T-CAViaR	6.8	3.0	**23.8****	11.4	6.6	8.6	6.4	**3.2**
ST-CAViaR	**4.1**	**1.8**	32.7**	10.9	8.1	7.4	7.8	3.8
QR Sum								
GJRGARCH	42.5	43.6	**48.3**	48.4	33.9	35.6	29.7	31.1
Asym Slope CAViaR	41.2	43.3	49.4	49.1	33.8	**35.5**	29.7	**30.8**
T-CAViaR	42.1	**42.7**	48.5	**48.3**	**33.5**	35.7	**29.3**	31.0
ST-CAViaR	**40.6**	43.8	49.8	48.8	33.8	35.7	30.0	**30.8**

Note: Significance at 5% and 1% levels indicated by * and **, respectively. Tests were performed on DQ but not Hit % because sample size is not sufficiently large. QR sum values have been multiplied by 100.

Table 11 depicts the results of the three performance measures for the 0.05 and 0.95 quantiles (90% interval). Similar as before, the results between the GJRGARCH and Asymmetric Slope reveal the same pattern as they did for Taylor. The Asymmetric Slope outperforms GJRGARCH for every performance measure considered (in terms of the number of cases). This is in line with the result from Table 5, where Asymmetric Slope CAViaR outperforms GJRGARCH for every index for the 90% interval. When considering the two other CAViaR models, we see that Asymmetric Slope still outperforms these two in terms of hit percentage. The difference in results however are very small. In terms of the DQ test statistic every model has two cases in which they perform the best compared to the other three models. If we compare two models together we find that the ST-CAViaR does perform slightly better than the Asymmetric Slope, and that the T-CAViaR outperforms the Asymmetric Slope for five of the eight cases. T-CAViaR performs better than all the other models for the QR sum performance measure.

The results of the quantile forecast evaluation indicate that for the 98% interval, it depends on which performance measure you take into account to decide which model is better. T-CAViaR is the best in terms of hit percentage, T-CAViaR and GJRGARCH perform the best in terms of the DQ test statistic, and GJR-GARCH is the best in terms of the QR Sum. For the 90% interval the T-CAViaR performs the best overall.

These results are interesting, as it shows how volatility forecasting performance is related to the quality of the quantile forecasts. Table 5 shows that when looking at the mean, the T-CAViaR 90% and 95% perform very well and this is evident for the 90% in the results of Table 11.

5 Conclusion and Discussion

Taylor (2005) shows in his research that VaR estimates can be used to produce volatility forecasts. He shows that CAViaR models, first introduced by Engle and Manganelli (2004) can be favorable to the normal GARCH approaches and other approaches such as moving average or exponential smoothing. This is because the CAViaR models do not require any distribution assumptions. In this paper we evaluate the results of Taylor in our own empirical research using similar data and also evaluated two other CAViaR models. The first was the T-CAViaR first proposed by Gerlach et al. (2011), and the second was the newly introduced ST-CAViaR which allows for a smooth transition between regimes based on the past shocks. The potential superiority of the T-CAViaR model is that it allows seperate parameters to be estimated depending on the sign of the past shock. This allows the model to capture the leverage effect better. The ST-CAViaR model extends the scope of regime switching models by allowing the model to smoothly transition between more than two regimes depending on past shocks. We answer the following research question: *Does the T-CAViaR model and the ST-CAViaR model perform better than the Asymmetric Slope CAViaR model for forecasting volatility based on daily returns?*

We first consider the parameter estimates of the least squares regression of ε_i^2 on the interval between symmetric quantiles and compared α_1 and β_1 to the Pearson and Tukey (1965) values. We evaluate these estimates for the Asymmetric Slope, T-CAViaR, and ST-CAViaR models. Compared to Taylor, our results for the α_1 estimate are worse due to the fact that our standard errors were substantially larger for the Asymmetric Slope CAViaR model, but that our performance for the β_1 estimates were significantly better than those of Taylor. When comparing the three CAViaR models with each other we find that the ST-CAViaR is the most preferable when compared to the Pearson and Tukey (1965) values for the one-day-ahead variance forecasts. Our analysis of the coefficient of determination, R^2, for the least squares regression of the realized variance on the out of sample variance forecast give different results depending on the number of holding days considered. For the one-day-ahead forecasts we conclude that the T-CAViaR model for the 95% and 90% performs the best out of all the models based on the mean value of R^2 of the indices. In the majority of cases it outperforms the Asymmetric Slope CAViaR model for these two intervals. This is not the case for the ST-CAViaR model compared to the Asymmetric Slope CAViaR model. When we increase the number of holding days, k, to 10 or 20 days, the ST-CAViaR model for the 98% interval is the best performing model. We also see from the parameter estimates for the ST-CAViaR in Appendix A, Table 18, that the smoothness parameter, δ is the lowest for the DAX 30 index, and that for this index the ST-CAViaR model for the 98% interval always performs the best in terms of R^2. This shows that sometimes using more than two regimes with a smooth transition function can improve the volatility forecasts.

When considering the encompassing test from equation (22), we find that all three the CAViaR models in our main investigation outperform the GJRGARCH. However, the results do show that the Asymmetric Slope CAViaR model better outperforms the GJRGARCH than the other two CAViaR model as it rejects the null hypothesis of $w = 0$ for more cases. The final performance measures considered is regarding the quantile forecasting of the three CAViaR models and the GJRGARCH. For the 98% interval there is no model which seemed to clearly outperform the other models when all three measures were taken into account, but T-CAViaR and GJRGARCH seemed to perform the best. T-CAViaR was the best in terms of hit percentage, T-CAViar and GJRGARCH outperform the other two models for the DQ test statistic, and GJRGARCH was the best in terms of the QR Sum. The T-CAViaR model also performed the best overall in the case of the 90% interval. Based on our results, we conclude that for the one-day-ahead variance forecasts the T-CAViaR does outperform the Asymmetric Slope CAViaR model, but that the difference with the Asymmetric Slope CAViaR model is minimal. When the holding period increases to 10 and 20, the ST-CAViar model, performs better in terms of the information content but that this is not evident in the one-day-ahead variance forecasts for the other tests and performance measures.

Even though the T-CAViaR and ST-CAViaR models seem to have the potential at times to outperform the Asymmetric Slope CAViaR model it is important to consider the drawbacks of the models. The Asymmetric Slope CAViaR model from Taylor (2005) has the most parameters for the different VaR based methods which made it more computationally expensive. The two new CAViaR models consider even more parameters, where T-CAViaR has six, and ST-CAViaR has five. This makes these two models even more computationally expensive compared to the Asymmetric Slope model. Further research is also recommended in using the T-CAViaR and ST-CAViaR model to forecast volatility using conditional quantiles through quantile regression. We perform most of our analysis for the GJRGARCH, Asymmetric Slope CAViaR, T-CAViaR, and ST-CAViaR model based on the 90% interval in order to compare our results with those of Taylor. Our own empirical research, indicates that T-CAViaR is also promising for the 95% interval and ST-CAViaR for the 98% interval. Therefore, it would be interesting to see if the results would differ if these intervals are considered as well. Furthermore, there are different ways that the smooth transition function can be modelled for the ST-CAViaR model. We use the logistic function from González-Rivera et al. (1998). Another option is to consider an exponential smooth transition function, such as in the work of Hagerud (1997) (regarding ST-GARCH), and see how this affects the volatility forecasts. Smooth transition can also be applied to other models such as GARCH models and exponential smoothing methods. Liu et al. (2020) shows that the Smooth Transition Exponential Smoothing model, in which the α parameter in equation (1) can transition over time, performs well in comparison with other GARCH models. It would be interesting to see the performance of these models compared to VaR-based methods.

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A Appendix

Table 12: R^2 Measure of Informational Content for Post-Sample Twenty-Day-Ahead Variance Forecasts for Stock Indices

	CAC 40	DAX 30	NIKKEI 225	S&P 500	Mean
Moving Average and Garch Methods					
Simple moving average	29.0	22.8	2.4	14.0	17.0
Exponential Smoothing	35.3	27.9	5.9	20.5	22.4
GARCH	31.8	28.9	5.5	17.7	21.0
IGRACH	35.6	29.2	5.9	19.7	22.6
GJRGARCH	42.5	34.8	10.2	34.7	30.6
Simplistic winsorized GJRGARCH	35.8	34.2	10.3	33.5	28.5
CAViaR winsorized GJRGARCH	45.3	35.1	10.3	35.2	31.5
VaR-based methods					
Historical Simulation 98%	10.7	19.1	3.6	1.4	8.7
Historical Simulation 95%	15.4	18.9	3.0	0.0	9.3
Historical Simulation 90%	10.3	14.4	0.4	0.2	6.4
BRW 98% interval	8.4	11.1	1.4	3.8	6.2
BRW 95% interval	20.8	19.3	0.9	8.7	12.4
BRW 90% interval	17.4	24.2	3.0	4.3	12.2
Indirect GARCH CAViaR 98% interval	37.6	29.1	5.4	19.9	23.0
Indirect GARCH CAViaR 95% interval	38.1	28.9	6.5	19.7	23.3
Indirect GARCH CAViaR 90% interval	35.3	29.2	6.2	19.4	22.5
Adaptive CAViaR 98% Interval	8.5	13.7	0.2	5.2	6.9
Adaptive CAViaR 95% Interval	21.7	23.0	6.3	8.3	14.8
Adaptive CAViaR 90% Interval	16.7	23.8	4.6	9.6	13.7
Sym Abs Value CAViaR 98% interval	37.6	31.9	5.5	20.1	23.8
Sym Abs Value CAViaR 95% interval	37.4	30.3	6.9	18.9	23.4
Sym Abs Value CAViaR 90% interval	37.0	30.4	6.6	17.5	22.9
Asym Slope CAViaR 98% interval	45.5	37.6	12.3	34.6	32.5
Asym Slope CAViaR 95% interval	**51.3**	39.0	13.0	35.4	34.7
Asym Slope CAViaR 90% interval	47.0	37.7	14.6	35.5	33.7
T-CAViaR 98% interval	24.6	36.6	**22.0**	27.4	27.7
T-CAViaR 95% interval	47.0	37.8	13.5	37.7	34.0
T-CAViaR 90% interval	47.6	38.1	17.7	35.1	34.6
ST-CAViaR 98% interval	47.4	**39.7**	17.2	**39.7**	**36.0**
ST-CAViaR 95% interval	47.0	38.2	14.0	33.1	33.1
ST-CAViaR 90% interval	48.3	36.2	15.6	33.8	33.5

Note: R^2 values are percentages. The highest value per index and for the mean is given in bold.

Table 13: Starting Parameters and Optimal Parameters for the Indirect GARCH CAViaR Model

			CAC 40		**DAX 30**	
			Starting Values	Optimal Values	Starting Values	Optimal Values
98%	0.99	ω	0.156	0.212	0.066	0.052
		α	0.784	0.702	0.823	0.876
		β	0.614	0.848	0.688	0.440
	0.01	ω	0.055	0.035	0.056	0.029
		α	0.920	0.936	0.847	0.902
		β	0.298	0.316	0.838	0.538
95%	0.975	ω	0.084	0.027	0.056	0.016
		α	0.808	0.887	0.866	0.917
		β	0.399	0.286	0.272	0.229
	0.025	ω	0.059	0.015	0.025	0.014
		α	0.860	0.921	0.871	0.916
		β	0.503	0.304	0.538	0.345
90%	0.95	ω	0.026	0.009	0.007	0.011
		α	0.862	0.917	0.881	0.908
		β	0.220	0.179	0.334	0.194
	0.05	ω	0.061	0.007	0.002	0.009
		α	0.858	0.937	0.840	0.907
		β	0.175	0.171	0.489	0.250
			NIKKEI 225		**S&P 500**	
			Starting Values	Optimal Values	Starting Values	Optimal Values
98%	0.99	ω	0.046	0.002	0.027	0.010
		α	0.933	0.920	0.851	0.895
		β	0.328	0.567	0.794	0.552
	0.01	ω	0.118	0.128	0.038	0.037
		α	0.854	0.828	0.903	0.876
		β	0.595	0.722	0.487	0.726
95%	0.975	ω	0.055	0.019	0.014	0.021
		α	0.794	0.883	0.867	0.809
		β	0.667	0.408	0.567	0.638
	0.025	ω	0.126	0.074	0.062	0.021
		α	0.767	0.849	0.833	0.857
		β	0.698	0.446	0.452	0.586
90%	0.95	ω	0.056	0.011	0.028	0.007
		α	0.765	0.917	0.755	0.873
		β	0.485	0.176	0.423	0.298
	0.05	ω	0.107	0.053	0.042	0.002
		α	0.771	0.837	0.752	0.969
		β	0.420	0.314	0.490	0.076

Table 14: Starting Parameters and Optimal Parameters for the Adaptive CAViaR Model

Adap			CAC 40		DAX 30	
			Starting Values	Optimal Values	Starting Values	Optimal Values
98%	0.99	α	0.000	-0.001	0.001	-0.004
	0.01	α	0.000	-0.003	0.001	-0.006
95%	0.975	α	0.000	-0.001	0.000	-0.003
	0.025	α	0.001	-0.004	0.000	-0.003
90%	0.95	α	0.000	-0.001	0.001	-0.001
	0.05	α	0.001	-0.001	0.001	-0.002
			NIKKEI 225		**S&P 500**	
			Starting Values	Optimal Values	Starting Values	Optimal Values
98%	0.99	α	0.000	0.000	0.000	-0.003
	0.01	α	0.000	0.001	0.001	-0.002
95%	0.975	α	0.000	-0.003	0.001	-0.002
	0.025	α	0.000	-0.005	0.001	-0.002
90%	0.95	α	0.001	-0.001	0.001	-0.001
	0.05	α	0.000	-0.002	0.001	-0.001

Table 15: Starting Parameters and Optimal Parameters for the Symmetric Absolute Value CAViaR Model

			CAC 40		DAX 30	
			Starting Values	Optimal Values	Starting Values	Optimal Values
98%	0.99	ω	0.124	0.001	0.259	0.000
		α	0.464	0.863	0.013	0.929
		β	0.404	0.270	0.928	0.158
	0.01	ω	0.094	-0.014	0.207	-0.022
		α	0.419	0.247	0.069	-0.095
		β	0.586	0.526	0.916	0.658
95%	0.975	ω	0.107	0.007	0.052	0.000
		α	0.427	0.198	0.574	0.909
		β	0.372	0.426	0.441	0.162
	0.025	ω	0.050	0.000	0.085	0.000
		α	0.352	0.923	0.066	0.931
		β	0.399	-0.171	0.355	-0.161
90%	0.95	ω	0.094	0.000	0.058	0.000
		α	0.499	0.935	0.035	0.918
		β	0.292	0.115	0.471	0.126
	0.05	ω	0.122	-0.008	0.155	-0.015
		α	0.128	-0.029	0.291	-0.598
		β	0.429	-0.222	0.501	-0.067
			NIKKEI 225		**S&P 500**	
			Starting Values	Optimal Values	Starting Values	Optimal Values
98%	0.99	ω	0.235	0.006	0.068	0.000
		α	0.094	0.479	0.254	0.891
		β	0.764	0.497	0.608	0.292
	0.01	ω	0.019	-0.001	0.089	-0.021
		α	0.189	0.789	0.559	-0.722
		β	0.441	-0.505	0.278	0.093
95%	0.975	ω	0.285	0.001	0.190	0.000
		α	0.020	0.786	0.071	0.884
		β	0.531	0.443	0.798	0.252
	0.025	ω	0.068	-0.001	0.128	-0.016
		α	0.355	0.842	0.180	-0.723
		β	0.958	-0.285	0.939	0.020

90%	0.95	ω	0.079	0.000	0.047	0.000
		α	0.543	0.917	0.126	0.897
		β	0.140	0.137	0.963	0.197
	0.05	ω	0.073	-0.001	0.058	-0.001
		α	0.085	0.783	0.110	0.762
		β	0.631	-0.302	0.970	-0.333

Table 16: Starting Parameters and Optimal Parameters for the Asymmetric Slope CAViaR Model

Asym			**CAC 40**		**DAX 30**	
			Starting Values	Optimal Values	Starting Values	Optimal Values
98%	0.99	ω	0.067	0.001	0.059	0.000
		α	0.323	0.828	0.474	0.910
		β_1	0.852	0.013	0.868	0.102
		β_2	0.561	0.335	0.683	0.218
	0.01	ω	0.138	-0.023	0.203	-0.007
		α	0.061	-0.287	0.128	0.661
		β_1	0.944	0.656	0.894	0.549
		β_2	0.626	0.316	0.946	-0.307
95%	0.975	ω	0.103	0.009	0.131	0.006
		α	0.393	-0.020	0.212	0.339
		β_1	0.554	0.513	0.049	0.183
		β_2	0.849	0.568	0.581	0.345
	0.025	ω	0.034	-0.014	0.047	0.000
		α	0.308	-0.034	0.159	0.900
		β_1	0.415	0.453	0.869	-0.049
		β_2	0.328	0.315	0.816	-0.293
90%	0.95	ω	0.097	0.001	0.084	0.000
		α	0.364	0.692	0.160	0.921
		β_1	0.776	0.328	0.863	0.078
		β_2	0.543	0.297	0.965	0.151
	0.05	ω	0.094	-0.009	0.157	-0.012
		α	0.279	-0.033	0.116	-0.279
		β_1	0.663	-0.091	0.624	-0.149
		β_2	0.845	-0.094	0.610	-0.153
			NIKKEI 225		**S&P 500**	
			Starting Values	Optimal Values	Starting Values	Optimal Values
98%	0.99	ω	0.059	0.000	0.055	0.000
		α	0.048	0.916	0.709	0.887
		β_1	0.752	0.243	0.084	0.112
		β_2	0.856	0.316	0.266	0.481
	0.01	ω	0.022	-0.022	0.091	-0.001
		α	0.193	-0.315	0.185	0.883
		β_1	0.662	0.161	0.430	0.225
		β_2	0.839	-0.152	0.667	-0.445
95%	0.975	ω	0.092	0.000	0.052	0.001
		α	0.151	0.930	0.191	0.693
		β_1	0.021	0.063	0.084	0.502
		β_2	0.739	0.222	0.795	0.834
	0.025	ω	0.045	0.000	0.039	-0.010
		α	0.012	0.884	0.402	-0.019
		β_1	0.467	-0.150	0.146	0.020
		β_2	0.562	-0.327	0.398	-0.051

90%	0.95	ω	0.123	0.000	0.125	0.004
		α	0.062	0.931	0.353	0.169
		β_1	0.818	0.046	0.498	0.448
		β_2	0.928	0.171	0.367	0.572
	0.05	ω	0.192	-0.014	0.187	-0.001
		α	0.064	-0.376	0.276	0.849
		β_1	0.827	0.069	0.715	0.103
		β_2	0.976	0.176	0.677	-0.428

Table 17: Starting Parameters and Optimal Parameters for the T-CAViaR Model

			CAC 40		**DAX 30**	
			Starting Values	Optimal Values	Starting Values	Optimal Values
98%	0.99	ω_1	0.153	0.000	0.207	0.010
		ω_2	0.230	0.000	0.118	0.011
		α_1	0.792	0.864	0.257	0.193
		α_2	0.090	0.920	0.621	0.148
		β_1	0.765	0.381	0.803	0.246
		β_2	0.816	0.126	0.561	0.196
	0.01	ω_1	0.078	-0.018	0.098	-0.015
		ω_2	0.166	-0.020	0.161	-0.002
		α_1	0.255	-0.411	0.062	-0.151
		α_2	0.217	-0.738	0.943	0.786
		β_1	0.952	0.120	0.615	-0.863
		β_2	0.518	-0.037	0.634	-0.022
95%	0.975	ω_1	0.105	0.008	0.256	0.000
		ω_2	0.323	0.005	0.116	0.002
		α_1	0.172	0.054	0.157	0.893
		α_2	0.726	0.371	0.398	0.742
		β_1	0.916	0.575	0.977	0.343
		β_2	0.605	0.280	0.425	-0.017
	0.025	ω_1	0.102	-0.006	0.323	0.000
		ω_2	0.175	-0.003	0.034	-0.002
		α_1	0.375	0.340	0.886	0.794
		α_2	0.667	0.563	0.097	0.759
		β_1	0.802	-0.666	0.836	-0.717
		β_2	0.857	-0.227	0.589	-0.025
90%	0.95	ω_1	0.077	0.006	0.278	0.000
		ω_2	0.098	0.006	0.135	0.000
		α_1	0.727	-0.012	0.230	0.900
		α_2	0.132	0.153	0.649	0.929
		β_1	0.475	0.918	0.215	0.153
		β_2	0.343	0.327	0.817	0.089
	0.05	ω_1	0.181	-0.012	0.121	-0.011
		ω_2	0.104	-0.008	0.004	-0.017
		α_1	0.576	-0.310	0.656	-0.242
		α_2	0.095	-0.078	0.027	-0.802
		β_1	0.193	-0.194	0.738	-0.226
		β_2	0.972	-0.203	0.963	-0.029

			NIKKEI 225		S&P 500	
			Starting Values	Optimal Values	Starting Values	Optimal Values
98%	0.99	ω_1	0.073	0.006	0.176	0.000
		ω_2	0.031	0.012	0.169	0.000
		α_1	0.545	0.563	0.058	0.913
		α_2	0.962	0.041	0.748	0.827
		β_1	0.819	0.264	0.257	0.481
		β_2	0.189	0.711	0.206	0.303
	0.01	ω_1	0.051	-0.028	0.086	-0.017
		ω_2	0.119	-0.021	0.106	-0.013
		α_1	0.123	-0.724	0.549	-0.250
		α_2	0.329	-0.463	0.029	-0.167
		β_1	0.893	-0.111	0.708	0.227
		β_2	0.960	-0.428	0.463	0.072
95%	0.975	ω_1	0.038	0.006	0.145	0.000
		ω_2	0.263	0.009	0.107	0.000
		α_1	0.427	0.268	0.654	0.990
		α_2	0.056	0.041	0.190	0.888
		β_1	0.796	0.809	0.498	0.233
		β_2	0.855	0.581	0.150	0.032
	0.025	ω_1	0.090	-0.011	0.088	-0.014
		ω_2	0.119	-0.004	0.150	-0.016
		α_1	0.398	-0.069	0.311	-0.450
		α_2	0.897	0.440	0.262	-0.771
		β_1	0.699	-0.553	0.755	0.028
		β_2	0.874	-0.595	0.064	-0.020
90%	0.95	ω_1	0.327	0.000	0.183	0.001
		ω_2	0.199	0.000	0.088	0.005
		α_1	0.056	0.821	0.158	0.597
		α_2	0.413	0.866	0.049	0.014
		β_1	0.428	0.399	0.751	0.556
		β_2	0.276	0.139	0.914	0.547
	0.05	ω_1	0.081	-0.011	0.101	-0.010
		ω_2	0.086	-0.007	0.099	-0.008
		α_1	0.879	-0.182	0.769	-0.289
		α_2	0.651	0.188	0.493	-0.231
		β_1	0.418	-0.415	0.804	-0.007
		β_2	0.362	-0.192	0.271	-0.167

Table 18: Starting Parameters and Optimal Parameters for the ST-CAViaR Model

STCAV			CAC 40		DAX 30	
			Starting Values	Optimal Values	Starting Values	Optimal Values
98%	0.99	ω	0.132	0.011	0.105	0.013
		α_1	0.710	-0.072	0.214	-0.098
		α_2	0.853	-1.548	0.704	0.464
		β	0.028	-0.047	0.491	0.113
		δ	297.967	273.896	297.654	148.049
	0.01	ω	0.225	-0.003	0.073	-0.008
		α_1	0.875	0.348	0.558	0.419
		α_2	0.710	0.169	0.522	0.660
		β	0.009	0.781	0.476	0.472
		δ	211.617	193.841	285.913	81.761

95%	0.975	ω	0.179	0.008	0.044	0.011
		α_1	0.257	-0.073	0.352	-0.111
		α_2	0.437	-0.172	0.118	0.491
		β	0.173	0.203	0.299	0.042
		δ	271.164	202.882	195.157	30.469
	0.025	ω	0.073	-0.017	0.125	-0.015
		α_1	0.477	0.091	0.916	0.204
		α_2	0.278	0.649	0.393	0.429
		β	0.373	-0.516	0.306	-0.197
		δ	263.851	137.384	258.361	19.665
90%	0.95	ω	0.094	0.001	0.048	0.016
		α_1	0.416	-0.035	0.140	-0.062
		α_2	0.074	-0.486	0.080	-0.105
		β	0.578	0.851	0.512	-0.799
		δ	295.703	295.703	299.377	147.524
	0.05	ω	0.071	-0.003	0.109	-0.001
		α_1	0.826	0.314	0.589	0.101
		α_2	0.275	0.734	0.602	0.684
		β	0.159	0.556	0.000	0.865
		δ	195.420	237.550	268.991	98.029

			NIKKEI 225		**S&P 500**	
			Starting Values	Optimal Values	Starting Values	Optimal Values
98%	0.99	ω	0.156	0.020	0.093	0.006
		α_1	0.105	-0.055	0.285	0.396
		α_2	0.124	0.727	0.198	0.904
		β	0.171	-0.069	0.226	0.716
		δ	139.580	115.996	35.068	226.899
	0.01	ω	0.045	-0.011	0.091	-0.001
		α_1	0.526	-0.127	0.842	0.279
		α_2	0.183	0.202	0.259	0.231
		β	0.246	0.292	0.109	0.865
		δ	235.401	266.658	193.872	193.872
95%	0.975	ω	0.116	0.012	0.093	0.000
		α_1	0.727	0.051	0.135	-0.117
		α_2	0.608	-0.823	0.146	-0.473
		β	0.045	-0.034	0.006	0.903
		δ	104.078	237.190	206.293	206.292
	0.025	ω	0.116	-0.022	0.106	-0.013
		α_1	0.943	0.170	0.449	-0.031
		α_2	0.316	0.426	0.339	0.473
		β	0.344	-0.742	0.299	-0.377
		δ	133.829	103.146	255.081	208.389
90%	0.95	ω	0.125	0.009	0.072	0.001
		α_1	0.502	-0.026	0.167	-0.082
		α_2	0.522	0.557	0.319	-2.222
		β	0.039	0.084	0.342	0.755
		δ	109.249	50.236	265.548	67.852
	0.05	ω	0.079	-0.008	0.132	-0.004
		α_1	0.922	0.075	0.943	0.365
		α_2	0.117	0.852	0.162	0.575
		β	0.033	0.147	0.410	0.439
		δ	120.960	198.836	275.848	91.292