BACHELOR THESIS

Control of closed loop systems

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Summary

We considered a closed loop system with demands that occur according to a Poisson process and where the market sojourn time of the items are Gamma distributed. There is a certain probability that an item will return. If they will return, then they will be send to the repair centre, where the items will be checked, cleaned and repaired if necessary. The repair time is exponential. We simulated the demand, return and repair times with a simulation model. We kept these times fixed for the four models that we consider: model I where we have the exponential repair times with the inventory position updated when an item returns to the repair centre, model II where we have exponential repair times with the inventory position updated when an item returns to inventory, model III has a deterministic repair time that is equal to the mean of the stochastic repair times and the inventory position is updated when an item returns to the repair centre and as last we have model IV that is equal to model III except that the inventory is not updated when an item returns to the inventory. We use two methods to optimize the models. The first one is to meet a certain fill rate of α% and the second one is based on minimizing the total average costs per item. We determine the reorder point that is associated with these optimums and the amount of necessary items to fulfill α% of the demand. We have simulated for a time horizon of 337 days. The results do not differ much between the stochastic and deterministic models. The necessary amount of items that are needed are around 2716 for models I and III, and 2658 for models II and IV. Using the first method to obtain a fill rate of 80%, we found associated costs of about € 173 for models I and III, and € 177 for models II and III with reorder points 63 and 43 respectively. When we optimize the total average costs, we find costs of more or less € 138 for models I and III and € 133.50 for models II and III with a reorder points 110 and 88 respectively. All the four models have a fill rate of 100% each. If we compare the two methods with each other, we see that using the method to optimize the costs are giving lower costs with higher fill rates. The model with the lowest costs are given by model IV, that is the deterministic model where the inventory position is updated when an item returns to the inventory, which is € 133.61 with a fill rate of 100%.

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1. Introduction

Recycling and reuse of products and materials are very common nowadays, it is already a very old practice. Environmental reasons were the cause of reusing products. However, companies realized the opportunities in collecting, recycling and reusing products and materials ((De Brito & Dekker, Reverse Logistics - a framework, 2002), (Kroon & Vrijens, 1994)). This flow from the consumer back to the producer is called *reverse logistics*, which plays an important role in inventory management. Reverse logistics concerns activities associated with the handling and management of equipment, products, components, materials or even entire technical systems to be recovered (De Brito & Dekker, Reverse Logistics - a framework, 2002). However, producers can hardly influence the quantity, quality and timing of the return flow, which leads to difficulties into the inventory management (Fleischmann, Kuik, & Dekker, Controlling inventories with stochastic item returns: A basic model, 2002).

An example of items which are being reused are instruments that are used in surgery rooms in hospitals (Glorie, 2008). These instruments belong to a sterilization cycle. After a surgery, all the instruments will be cleaned, disinfected and sterilized. Then the instruments will be checked if they are complete and function well. When the instruments are not complete or do not work properly anymore, they will be repaired, completed or ordered at a supplier. When the instruments are complete again, they are ready for use and the cycle will start over. Other situations where returns occur are in maintenance settings, products with warranties and lease products. Examples are spare parts that return from maintenance engineers, recycling of waste paper and deposit systems for bottles of soft drink, beer kegs and crates, but also items that have been rented will return after a prescribed lease period, like video tapes. These kinds of returns have been around for a long time. ((De Brito & Dekker, 2003), (Yuan & Cheung, 1998)).

It is important to know how many items will be needed to be able to fulfill the demand of the items and when the *reorder point* is. The stock on hand can then be replenished to satisfy the demand. Costs are an important aspect in this context, because we want to keep them as low as possible.

Most papers describe the inventory control by assuming that the demand and return process are homogenous (compound) Poisson processes. This will not be the case in this thesis. The demand will be a homogenous Poisson process and the return of items will be described by a renewal process. These processes are generated by a simulation model, which is described in later sections.

1.1 Problem introduction

For many items the use can be described by a cycle. The demand of the items in this thesis occur according to a homogeneous Poisson process. After some time they will return to the stock and after some cleaning the items can be used again. Suppose that every unit that is in use has its own use duration given by a known general distribution. Unfortunately, not every item will return to the stock. Some users do not return the item or it will get lost somewhere in the network, so that the number of items in use will not be constant in the network. Because these items do not return, new items have to be ordered then to replenish the inventory. New orders will also be placed when the inventory level is below the size of demand, so to say that not enough items have returned yet to satisfy the demand. We want to research different methods with which we can determine the quantity of supply that is needed in a stock where a certain service level is met to satisfy the demand. We formulate this as follows:

The purpose of this thesis is to develop and analyze methods to determine the optimal number of products floating in a given closed loop system.

In our study *optimal* is defined as:

- In relation to an objective function and or constraints.
- It also depends on the information available for decision making, especially concerning returning products.

The data that we consider for the closed loop system are cycle time distribution, return information time and the return probability of products.

We will consider the following four hypotheses that can help us analyze the methods that will be developed:

H_0 : if there is information known about the return time of products to the repair centre, then this will lead to lower costs than when there is no information known.

H_1 : if there is no information available about the return time of products to the repair centre, this will lead to lower costs than when there is information known.

H_2 : if the return time of products to the repair centre are deterministic, this will lead to lower costs than stochastic return times.

H_3 : deterministic return times of products will not lead to better results than stochastic return times.

1.2 Organization of the thesis

This thesis will proceed as follows. Section 2 describes the inventory management and gives an explanation about terms and relevant costs that are quite common used in inventory management. A review of literature concerning cases with reverse logistics will also be given in this section. In section 3 we state the problem explicitly and provide a detailed description of the model with its assumptions. Section 4 describes an analytical approach and section 5 the simulation model of the problem. Section 6 gives the results of the simulation model with some numerical values. We end with the conclusions of this thesis.

2. Terminology

This chapter starts with an introduction about inventory management and related commonly used terms. It ends with a short review of literature on reverse logistics as to present several inventory models considering return flows.

2.1 Inventory management

Inventories are stockpiles of items waiting to be processed, transported or used at a point of the supply chain. There are a number of reasons why inventories are held, such as to improve the service level, reducing the overall logistics costs, coping with randomness in customer demand and lead times. At the same time, holding an inventory can be very expensive for several reasons, e.g. warehousing costs (Ghiani, Laporte, & Musmanno, 2004).

The aim of inventory management is to determine inventory levels in order to minimize the total operating cost while satisfying customer service requirements. Inventory management amounts to deciding for each stocking point in the supply chain when to reorder and how much the order should be, so that the costs are minimized while meeting a certain service level. To know whether a stocking point is well managed we can turn to the inventory turnover ratio that is defined as the ratio between the annual sales priced at the value of the items in stock and the average inventory investment. A high ratio refers to a well managed stocking point (Ghiani, Laporte, & Musmanno, 2004). We will however not include the turnover ratio in this thesis.

There are certain important terms to inventory management. In literature, many synonyms are used for these terms. Most terms that will be used in this thesis will be explained here (Ghiani, Laporte, & Musmanno, 2004). We refer customers to anyone in the supply chain who orders a certain item.

- *Stock on hand* refers to items which are available right in the stock, as to say that they can be sold from the shelf. This is also called the *inventory*.
- All the demands that are not satisfied by stock on hand are *backordered*. These are items that the customer did not obtain, because a part of the inventory, or even all of it, required to fulfill the order is out of stock. The customer will receive the demand after the replenishment of the item.
- *Stock on order* refers to items which are not in stock, but an order will be placed for it. These are also called *outstanding replenishment orders*.

- *Outstanding order* is a demand that occurs after a replenishment order is released.
- *Inventory level* is the stock on hand plus the sum of the changes in inventory.
- *Net inventory* is stock on hand minus backorders
- *Inventory position* is defined as net inventory plus stock on order.

When a replenishment order has been placed, it will take some time before it will arrive at the stock. This time lapse between the order placement and arrival is referred to by *lead time*. The proportion of customer demand that is satisfied from the stock on hand is called the (*product) fill rate*. In this thesis we will only consider one type of product. The term n*umber in circulation* is only used when items will be written off as they stay longer than *T* time units in the market. This is because they are not expected to return after *T* time units.

2.2 Relevant costs

The costs relevant to inventory management can be classified into four broad categories (Ghiani, Laporte, & Musmanno, 2004).

- *Procurement costs*: these costs are associated with the acquisition of goods. They include *fixed costs* and *variable costs*. In this thesis we include fixed reorder costs, the cost of issuing and processing an order through the purchasing and accounting departments if the goods are bought.
- *Inventory holding costs*: we refer to these costs as holding costs that are incurred when materials are stored for a period of time. They include opportunity costs and warehousing costs.
- *Shortage costs*: these costs are paid when customer orders are not met. They can be classified as *lost sale costs* or *backorder costs*. A lost sale is likely to occur if the unavailable items can be easily obtained from a competitor. We will only use backorder costs in this thesis, assuming that no customer will order from a competitor when the item is not in stock. Backorder costs are met when goods are difficult to replace, because shortages often results in delayed sale.
- *Obsolescence costs*: when stocked items lose some of their value over time, obsolescence costs arises. This is the case for perishable items, for example, for food, newspapers and clothes. These costs will however not be included in this thesis.

2.3 Literature review

For many years there has been a lot of literature concerning the potential and actual uses of Operations Research in inventory management. Most of them discusses only a forward flow from the supplier to the customer. However, return flow has also taken his place now in the scientific literature. Reverse logistics (see section 1) is a new and emerging field of research, but there is a lot of literature concerning reverse logistics nowadays. For a review on the differences between forward and reverse logistics we refer to Tibben-Lembke & Rogers (2002). For a better understanding of the reverse logistics we refer to the framework of De Brito et al. (2002) and a review of case studies by De Brito et al. (2003). Several authors have proposed inventory control models taking the return of used items into account. Fleischmann et al. (1997) has provided a review of quantitative models for reverse logistics.

Yuan and Cheung (1998) presents a single item continuous review (s,S) inventory system with returns that are dependent of the demands. They derive essential characteristics of the system via a Markovian formulation, such as the total costs, and propose an algorithm to search for the optimal replenishment parameters. They assume that the demands follow a Poisson process, the returns have an exponential market sojourn time and the replenishment of items are instantaneous. They based their model on the sum of the stock on hand and the number of items in circulation. The difference with this thesis is that they model exponential return variables and they do not consider a purchasing lead time.

Fleischmann et al. (2002) presents a basic inventory model with Poisson demand and returns which are independent of each other, by extending the traditional single item Poisson demand inventory model. They provide a derived optimal control policy and optimal control parameters are computed. They also provided a modification of the model where the return of items are dependent of the number of items in circulation. This is described as a two-dimensional Markov process and they provide the corresponding optimal average costs. We research a different distribution for the returning items.

Glorie (2008) has researched a reverse instrument flow in care logistics (see section 0). He considered a stochastic demand and return with an unknown distribution that is different for weekdays and weekend. He therefore uses the empirical distribution of the data, that belongs to a Dutch hospital. The paper presents a model to estimate the minimal amount of instrument per instrument type that is necessary to meet a certain service level. This thesis employs the method that is proposed by Glorie to estimate the amount of items that is needed.

Widi (2009) considers a case where many types of beer bottles are returned from the German market to a brewer company. The demand is according to a Poisson process, but they assume that the demand may be seasonal dependent so that four seasons is modeled with fixed duration. The returns follows a lognormal distribution. The study is still progressing when this thesis is written, so that a method to optimize the value of the amount of bottles has not yet been found.

These are some of the case studies that have been conducted towards the closed loop system. This thesis will implement some of the methods used by these authors. The difference between their and this study is that the returns follows a different distribution.

3. Formal problem description

This section describes the cycle of the products with their associated costs and gives a mathematical formulation of the problem.

3.1 Product flows and costs

The inventory in our model is holding one type of product. Demands occur according to a Poisson process with a known intensity $\lambda > 0$. Random items will eventually return to the inventory after a time that differs for every item, following a known general distribution $G(\cdot)$. This is known as a renewal process. For a more detailed explanation of the processes, we refer to appendix A. The distribution $G(\cdot)$ can be every distribution. The case of an exponential distribution is a simple case to solve with a continuous time Markov model. When an item returns, it is sent to the repair centre. The item will be cleaned if necessary and an inspection will be held to check if the item is in a good state or that it has to be repaired before it is sent back to the inventory. When an item arrives at the repair centre, the inventory position in our model will increase because it knows that an item will return to the inventory, despite the time that it will take to arrive at the inventory in reality. However, the return of an item is independent from customer to customer. Some products will not be reused, because a customer did not return it.

Let the inventory follow a continuous review replenishment policy. This means that the inventory position is checked continuously and at some point the inventory has to be refilled, otherwise one cannot satisfy any demand anymore. This point is when the inventory level drops below the reorder point and a constant order of size Q is placed then (Ghiani, Laporte, & Musmanno, 2004). It will take L units of time then for the reorder to arrive at the inventory. A graphical interpretation of the model is given in Figure 1.

When there are backorders, i.e. demand that cannot be fulfilled by the stock on hand, the demand will be backlogged at all times that lead to backorder costs per item per time and no lost customers. Other costs are fixed costs for every replenishment order and holding costs for the inventory per item per time. In order to minimize these costs, we want to research if the costs are dependent of the reorder point or any other variable.

This leads to the following objective of the thesis:

The purpose of this thesis is to develop and analyze methods to determine the optimal number of products floating in a given closed loop system.

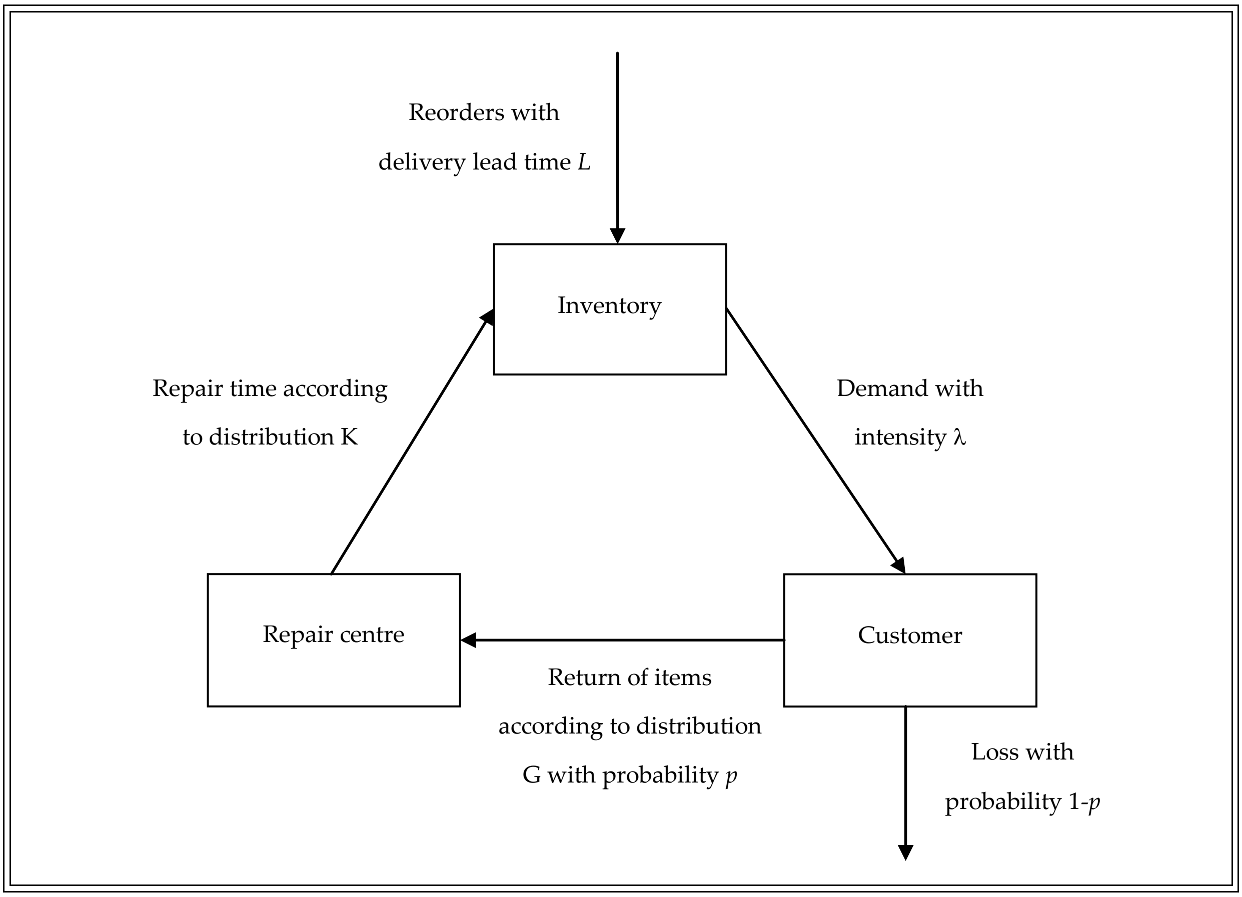

Figure 1 Inventory model with stochastic return flow

3.2 Model definition

We use the following notation:

N	=	start value pooling size
λ	=	demand intensity
D(t)	=	demand at time *t*, Poisson process with intensity λ
G(t)	=	return to repair centre at time *t*, renewal process
K(t)	=	repair at time *t*
L	=	fixed order lead time
s	=	reorder point
Q	=	reorder size
p	=	probability that an order will eventually return
c_f	=	fixed reorder costs

c_h = holding costs

c_b = backorder costs

The model has one condition: α % of the total demand must be provided from the stock on hand.

Fleischmann et al. (2002) let *C(s,Q)* denote the expected average costs per item per time for given control parameters *s* and *Q* and by using the renewal reward theorem they got their cost function. We slightly change the way the fixed order costs per item is calculated which gives us the following cost function:

$$C(s,Q) = c_f \frac{\sum_{t=1}^{T} E(D(t)) - \sum_{t=1}^{T} E(G(t))}{k \cdot Q} + c_h E[IL^+] + c_b E[IL^-]$$

Where IL^+ and IL^- denote the positive and negative part of inventory level *IL* respectively, and *k* denotes the amount of times that a reorder is placed.

3.2.1 Assumptions

We make the following assumptions for our model:

(i) Only one event occur at time *t*.
(ii) Demand and returns are dependent.
(iii) The average return is smaller than the average demand rate.
(iv) No items will be disposed of when they are returned to the repair centre.
(v) The transport time between the repair centre and the inventory is negligible.

Assumption (i) is made to simplify the model. At time *t* only one demand, return or a replenishment will take place. Assumption (ii) is based on the fact that a return of a product can only occur when the product has been demanded. When there is no demand, then no return will be expected.
Assumption (iii) is made, because otherwise the inventory gets controlled through disposal instead of additional orders (Fleishmann et al., 2002). Because of assumption (iv), all returned items can be reused. In this case, the number in circulation will only decrease when an item will not be returned to the repair centre. Assumption (v) indicates that items can be used when they are released from the repair centre.

4. Analytical approach

Here we give an analytical formulation of the stock on hand satisfying the service level. We will also give an analytical average demand and return during lead time, which we then will compare with the simulated values.

We want to know at which reorder point the stock on hand can satisfy a service level of α%.

Let us use the following notations:

L	=	Fixed order lead time
D_L	=	demand during lead time, Poisson process with intensity λ
R_L	=	returns to the inventory during lead time
ND_L	=	net demand during lead time
p	=	probability that an order will eventually return

The net demand at time *t* is given as the demand minus the returns at time *t*: $ND_L = D_L - R_L$. Denote by *F* the cumulative distribution function of ND_L. To prevent the inventory to run out of items, we have to choose the reorder point at such a level that the net demand during the lead time is not greater than the amount of items left in stock at the reorder point, which leads to a service level of 100%. To obtain a service level of α%, we have to choose the reorder point so that the following holds:

Prob(F< ROP) = α%, where ROP stands for the reorder point.

Average demand and return during lead time

The average demand during the lead time is equal to the average demand per time unit times the lead time. In this case it is λ x L. In our simulation we have $\lambda = 50$ and L = 5, which gives us 50x5 = 250 as mean demand during lead time.

The average return during the lead time is equal to the average demand during lead time times the probability that the demand will return. This is equal to λ x L x p. In our simulation we have p = 0.8, so the analytical average return during lead time is 50x5x0.8 = 200.

5. The simulation model

This chapter describes the simulation model where we present the algorithms that we have used to set up the model. We will mention the difference with the analytical approach as well. At the end of this chapter the warm up period will be determined.

5.1 Purpose of the simulation

The number of items necessary in stock to satisfy a service level of α% will be determined by the model. The input for this model are the amount of items in a pool, lead time, reorder size and costs. We want to vary the input parameter reorder point, to obtain the fill rate and costs that are associated with the chosen reorder point. Another output is the necessary amount of items.

5.2 Set up of the simulation model

The demand and return processes are implemented as a Monte Carlo simulation with continuous time. This leads to only one event occurrence per time t, either a single item demand, return or replenishment. The reorder size may be more than 1 at the same time. For each item that is purchased, a return time is generated, whether the item will return eventually or not. We also keep track of when a replenishment order has to be made and the time when it arrives. We do not implement lost sales, i.e. customer losses caused by stock outs.

To determine the quantity of stock on hand that is needed to obtain the service level α%, we will experiment with the control parameter s and increase this with a certain step size to obtain minimal average costs per item. This will be repeated for several different continuous distributions chosen for $G(\cdot)$.

5.3 Implementation

We have implemented this model in the program Matlab, version 7.0. The demand and return processes are renewal processes. For a general description of counting processes we refer to appendix A.

5.3.1 The demand times

Demands occur according to a Poisson process. A property of the Poisson process is that the interarrival times between events are independent identically distributed exponential random variables with mean 1/λ (Ross, 2007). Let U_i denote the time between the $(i-1)$st and the ith demand of the process, so that the actual time of the jth demand will equal the sum of the first j interarrival times. To generate the first T_{max} demand times, it follows that these values are $\sum_{i=1}^{j} U_i\,, j = 1, \ldots, T_{\max}$.

To generate the first T_{max} time units of a Poisson Process, we start with generating the interarrival times U_i until their sum exceeds T_{max}. When these interarrival times are added, this leads to the time of the demand. The following algorithm is used to generate all the demand times occurring in [0, T_{max}] with intensity λ, where $T(I)$ represents the time of demand I.

Algorithm for generating demand times

STEP 1. $I = 0$.

STEP 2. Generate an exponentially distributed random number Us with mean 1/λ.

STEP 3. Demand time $T(I + 1) = T(I) + U$.

STEP 4. If $T(I + 1) > T_{max}$, stop.

STEP 5. $I = I + 1$.

STEP 6. Go to step 2.

Validation of simulation

We want to test whether the simulated value differs significantly from the analytical value by using the mean test[1]. The t-value is calculated as $t = \frac{\bar{X}-\mu}{s/\sqrt{N}}$, where $\bar{X}$ is the simulated average value, μ is the analytical average, s the standard deviation of the simulation values and N is the sample size. The demand and return times that have been simulated are set fixed, see section 5.4, so that these are independent of the reorder points. In section 6 are the different models described for the simulation. Now we have randomly chosen the reorder point of 150 of model I to compare the average demand and return of the simulated data with the analytical analysis. In appendix B are the corresponding simulated total demands and returns during the lead times given for each order that has been placed, which in total is 65. The result of the mean test are given in the following Table 1.

[1] This test has been conducted in EViews 5.1.

Hypothesis Testing for average_demand		
Date: 09/23/09 Time: 22:36		
Sample: 1 65		
Included observations: 65 after adjustments		
Test of Hypothesis: Mean = 250.0000		
Sample Mean = 248.1077		
Sample Std. Dev. = 14.76495		
Method	Value	Probability
t-statistic	-1.033276	0.3054

Table 1 Mean test statistics for the average demand during lead time

The t-statistic is -1.033 with a p-value of 0.31. This indicates that the simulated average demand of 248.11 does not differ significantly from 250 with a significance level of 5%.

5.3.2 The return times

The return of items will occur according to a renewal process, because the use duration is given by distribution $G(\cdot)$ and returns are independent of each other. For every demand that occur over interval [0, t], a return time to the repair centre will be generated. We begin with generating the duration times that an item will stay in the market, which are $G(\cdot)$ distributed with chance p that the item eventually returns and with chance 1-p the item will not return, in which case the duration time is set to be infinity, stopping when the sum of the interarrival times exceeds the total number of demands m. Then we generate the repair times of distribution $K(\cdot)$ for every item demanded and that will return to the repair centre. After that the interarrival times and repair times will be added up to the demand times U_i, $i = 1,\ldots,m$, which results in the actual return times to the inventory. This can be written algorithmically as follows, where I will represent the number to which demand time the interarrival time belongs to, *S(I)* stands for the interarrival time and *Rep(I)* denotes the repair time of demand i:

Algorithm for generating return times belonging to the demand times.

STEP 1. $I = 1$.

STEP 2. Generate an uniform random number V.

STEP 3. if $V < (1-p)$, then generate *S(I)* = random number W from G(·)
and generate *Rep(I)* = random number Y from K(·),
else *S(I)* = infinity, *Rep(I)* = infinity.

STEP 4. If $I > m$, stop.

STEP 5. Return time $R(I) = S(I) + Rep(I) + U(I)$.

STEP 6. $I = I + 1$.

STEP 7. Go to step 2.

Validation of simulation

In the same way as we have tested for the demand times in section 5.3.1, we will now test whether the simulated average return value will differ significantly from the analytical value. The simulated overall average return is about 196.97. We will compare this value with the analytical value of 200 again by the means of the mean test. The results are given in Table 2.

Hypothesis Testing for average_return
Date: 09/23/09 Time: 23:20
Sample: 1 65
Included observations: 65 after adjustments
Test of Hypothesis: Mean = 200.0000

Sample Mean = 196.9692
Sample Std. Dev. = 13.53580

Method	Value	Probability
t-statistic	-1.805202	0.0757

Table 2 Mean test statistics for the average return during lead time

Again, we can see that the simulated overall average return does not differ significantly from 200 with a p-value of 0.08. Thus the difference between the average demand and return during lead time of the simulated data and the analytical approach is insignificant, so that we can assume that the simulated data is generated correctly in the next section.

5.3.3 Inventory level

When the demand and return time are known, the inventory level can be updated. When there is a demand and the stock is below the reorder point s, then a new item will be reordered. This will be delivered in L time units. The demands and returns will be simulated so that there is only one event happening per time t, either a demand or return. Let D_t denote the demand on event time t, R_t the return of an item on event time t, IL_t the inventory level, IP_t the inventory position and Q the reorder size. Let $t+1$ and $t-1$ denote the next and previous event time respectively Start with inventory level N.

Algorithm for keeping track of the inventory level.

STEP 1. $IL_{t=0} = N$.

STEP 2. When $D_t = 1$, then $IL_t = IL_{t-1} - 1$.

STEP 3. When $R_t = 1$, then $IL_t = IL_{t-1} + 1$.

STEP 4. If $IP_t < s$, then

$IP_{t+1} = IP_t + Q$

$IL_{t+L} = IL_{t+L-1} + Q$.

STEP 5. Stop when there are no more demands, else go to step 2.

5.3.4 Fill rate

To calculate the fill rate, we count the times when there is a demand, but no more items in stock: amount of shortages. We divide this through the total demand to obtain the fill rate:

fill rate = $1 - \frac{total\ shortages}{total\ demand}$

5.3.5 The total costs

Eventually, we want to run the simulation a couple of times with different inputs. To compare the results, it is better to calculate the costs per item. Following Fleishmann et al. (1998), we will include only the fixed reorder costs, holding costs and shortage costs. Let c_f denote the fixed costs, c_h the holding costs and c_b the backorder costs. The stock on hand as well as the shortages will be denoted by IL^+ and IL^- respectively and k will be the amount of times that a reorder is placed.

$$C(s,Q) = c_f \frac{\sum_{t=1}^{T} D(t) - \sum_{t=1}^{T} G(t)}{k \cdot Q} + c_h E[IL^+] + c_b E[IL^-]$$

5.3.6 Amount of items needed

To determine how many items are needed to satisfy the demand, we determine the net demand *ND* per time unit, that is the demand minus the returns per time unit. To satisfy at least α% of the demand, we take the α-th percentile of the net demand (Glorie, 2008) after we have left out the warm up period (see section 5.4.1). We do not take the amount of stock on hand into account, so that we know the net amount of items needed. Let t denote the time of event and D_t and R_t the demand and return respectively at the event time t. We use the following algorithm:

Algorithm for keeping track of the inventory level.

STEP 1. $ND_{t=0} = 0$

STEP 2. When $D_t = 1$, then $ND_t = ND_{t-1} + 1$

STEP 3. When $R_t = 1$, then $ND_t = ND_{t-1} - 1$

STEP 4. Stop when there are no more demands and go to step 5, else go to step 2.

STEP 5. Leave the warm up period out.

STEP 6. Take the α-th percentile of ND.

5.3.7 The output

All these previous algorithms leads to a matrix with:

- the times when a demand or return occurs
- the return time to the repair centre
- the repair time
- the stock on hand
- the inventory position

We are however more interested in the following scalar outputs:

- fill rate
- costs
- amount of items needed

5.4 Length of the simulation

This section reflects the amount of days that our simulation model has generated by means of a warm up period and the total demand size. For all the simulation runs, that is with different variables, we use fixed demand and return times. After generating these times we save them and use them throughout the simulation runs. This way we can compare the different outcomes of the model.

5.4.1 Warm up period

Our simulation of the demand times starts at time zero and after a certain time, that is the sojourn time in the market plus the repair time, the items return to the inventory. This means that in the beginning of our simulation, no items will return to the inventory. In addition, no replenishment is needed in the beginning days, because the inventory begins with a certain amount of items in stock. After a while, the steady state will be reached. This can be seen in Figure 2 where the reorder point is set at 40. The other input parameters are given in section 6.1 and 6.2. The first time that a replenishment is needed at this reorder point is after 28,9 days. We round this day off to 28 days and take the first 28 days as a warm up period, because we notice that the figure shows that the steady state is reached when the inventory position reaches the reorder point for the first time. When we choose a higher reorder point, this will lead to an earlier time when a replenishment is necessary. Because we run our model with the same demand and return times every time, the figure as shown in Figure 2 will stay the same for different reorder points, except the fact that the figure will shift in the direction of the reorder point. When the reorder point is set at 40, the fill rate is about 49% with the warm up period. We assume that the requested fill rate will be at least 50%. This warm up period is equal for all our models in section 6.

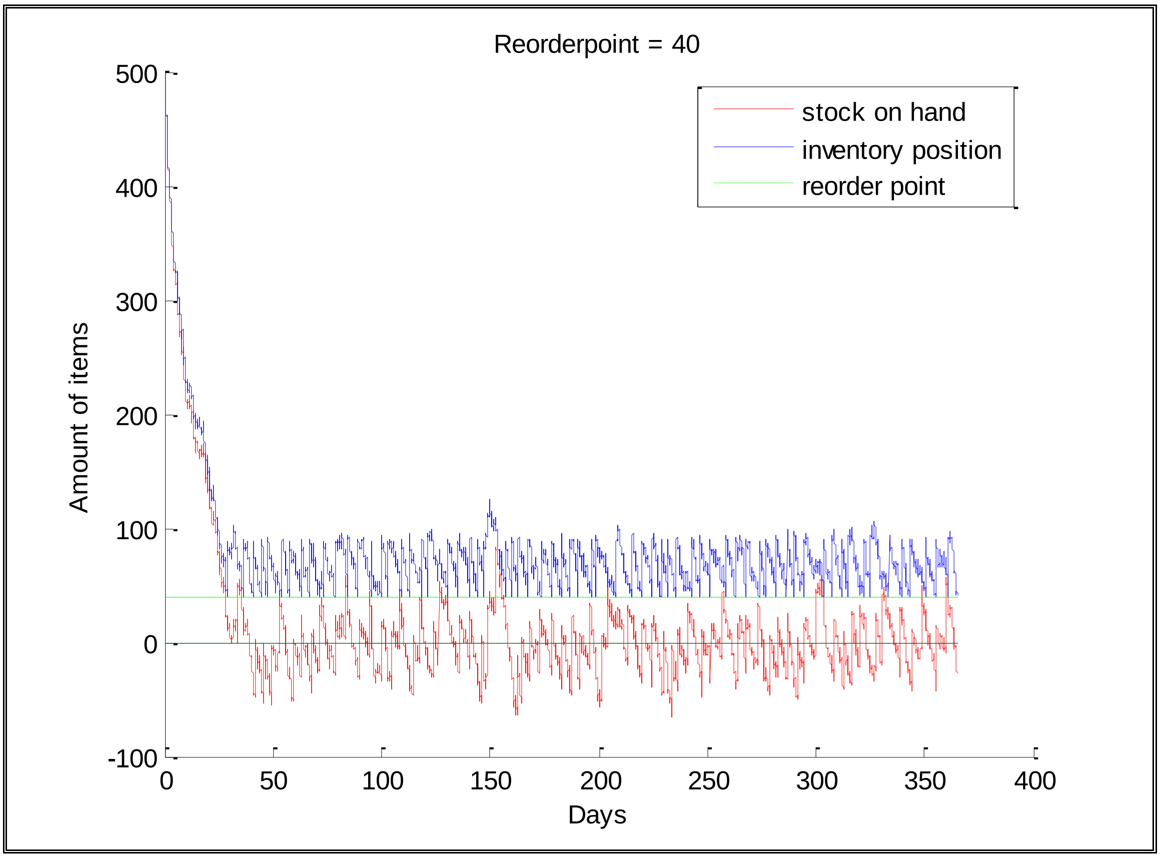

Figure 2 Illustration of the inventory level and inventory position throughout the year. The inventory level starts with 500 at time 0.

5.4.2 Amount of total demand

We generated the demand for a whole year with corresponding return times. Hereby the average daily demand is set at 50 per day. This corresponds to a total average demand of 18250 throughout the year. However, we exclude the warm up period of 28 days, which results in a total demand of 16850 in our simulation model.

6. Results

In this chapter the results will be discussed obtained by the simulation with different parameter input. We have modeled 2 kinds of demand and return processes, the first one is with a stochastic repair distribution and the second is with a deterministic repair time. So every section in this chapter has a stochastic part and a deterministic part. A short analysis of the simulated data will be given first and then the output of the data will be given with the corresponding input in days. In the last section of this chapter we will compare the optimal results obtained by the different methods for the models.

6.1 Demand and return processes

In this section we choose the input parameters for the demand and return processes and discuss the relevant statistics.

6.1.1 Stochastic repair times

The chosen parameters for the simulation of the demand and returns are shown in Table 3 for the stochastic repair time. These are chosen randomly. Cases with return distribution log normal and exponential are already researched as discussed in section 2.3 and 3.1. Therefore we choose the Gamma distribution as the return distribution with mean 4 and variance 8. As for the repair distribution, we choose another distribution with a small variance of the repair time that resulted in the exponential distribution with mean 0.5 and variance 0.25. If we add the means of these two distributions it results to a mean return of the items of 4.5 days to the inventory and the variance results in 8.25 days. We simulated for one year that consists of 365 days. However, as discussed in section 5.4.2, the warm up period takes 28 days.

Input demand and return processes (in days)		
Intensity rate	=	50
Time horizon	=	337
Mean demand time distribution	=	196.50
Return probability	=	0.80
Return time distribution	=	Gamma
Parameters	=	Shape k = 2, Scale θ = 2,

Mean return time distribution	=	4
Variance return time distribution	=	8
Repair time distribution	=	Exponential
Parameter	=	2
Mean repair time distribution	=	0.50
Variance repair time distribution	=	0.25

Table 3 Input parameters for demand and return processes.

The statistics of the simulation of the demand and return processes are given in Table 4. We see that the results are relatively the same as the analytical statistics given in Table 3. For example, the mean and variance of the simulated return time to the repair centre are 4.02 and 8.16 days respectively. In Table 3 we wanted a mean and variance of 4 and 8 days respectively. When an item is demanded, it is expected to return to the inventory after approximately 4.53 days, when the analytical calculation resulted in 4.5 days as discussed in section 6.1.1. So the differences between the analytical and simulated statistics are insignificant.

Statistics (in days)	*Demand time*	*Return time to repair centre*	*Repair time*	*Return time to inventory*
Mean	195.53	4.02	0.51	4.53
Variance	9346.70	8.16	0.25	8.39
Std. deviation	96.68	2.86	0.50	2.90

Table 4 Statistics of the demand and return processes given in days

6.1.2 Deterministic repair times

In this section we discuss the statistics of the models with deterministic repair times. We took the mean of the stochastic repair times, so that we can compare the results of the simulation model in a later section. The mean of the repair time that is stochastic is analytically 0.5 days. Thus we take for the deterministic time 0.5 days where no variance and standard deviation is present. We see that the only difference between the deterministic and stochastic models are the variance and standard deviation of the return time to the inventory. This is the cause of the warm up period. There is a difference in the amount of returns after the warm up period, because the repair times are different. The other input parameters for the processes are the same as shown in Table 3.

Statistics (in days)	*Demand time*	*Return time to repair centre*	*Repair time*	*Return time to inventory*
Mean	195.53	4.02	0.50	4.53
Variance	9346.70	8.16	--	8.19
Std. deviation	96.68	2.86	--	2.86

Table 5 Statistics of the demand and return processes given in days

6.2 Methods based on fill rate and costs

We have simulated two different models for both the stochastic and deterministic repair time which we will discuss in different sections. Thus we have a total of four models. The other difference between the models besides the different repair times, is that one model considers an item as already returned when it arrives at the repair centre, so that the inventory position is updated. The other model does not have the information when an item returns to the repair centre so that the inventory position is only updated when an reorder is placed. As said in section 5.4, we simulate with fixed demand and return times. For the first model it takes about 15 minutes to simulate, the second model takes about 5 minutes to run where the inventory position is does not have information about the repair centre.

For all the models we used the following input shown in Table 6. The backorder costs are chosen higher than the other costs, because a backorder comes with a penalty for not being able to deliver the product on time.

Input simulation model			
Pool size	=	250	items
Lead time	=	5	days
Reorder size	=	50	items
Fixed order costs	=	2	euro
Holding costs	=	2	euro
Backorder costs	=	10	euro

Table 6 Input parameters for the model

The reorder point is not included in this table, because we vary this parameter in the simulation runs. We choose a certain reorder point and run the simulation model. Then we choose another (higher) reorder point that differs 10 items and run the simulation model again. When the reorder point comes close to the optimum, we differ the step size of the reorder point with less than 10 items. We consider two kinds of optimums. One is based on the method to obtain a certain fill rate, which we set at 80%. For general results of the fill rates we consider a minimum amount of 50% service level, so that the reorder point begins at the point where it is below the 50%. The other method is to vary the reorder point to obtain the minimum total average costs per item. We stop with varying the reorder point when the fill rate is 100% and the costs will not decrease anymore, where the minimum costs are between the minimum and maximum reorder point. The results are given in the next sections.

6.2.1 Stochastic repair time

First we consider the model where the inventory position has information about the item arrivals at the repair centre, so that if the inventory position is updated when a reorder is placed or when an item returns from the market to the repair centre. The results are given in Table 7.

Model I : Inventory position is updated when item arrives at the repair centre

Reorder point	*Fill rate*	*Average fixed costs*	*Average holding costs*	*Average backorder costs*	*Total average costs*	*Amount needed*	*Standard deviation backorders*	*Std. dev. holding costs*	*Std. dev. backorder costs*
40	0.450	2.04	38.14	192.81	232.98	2370	0.8037	29.0748	131.8908
50	0.626	2.04	45.05	159.47	206.55	2370	0.6784	30.9873	115.0720
60	0.780	2.04	53.99	121.44	177.47	2370	0.5289	34.2253	87.2778
61	0.786	2.04	54.47	120.47	176.97	2370	0.5193	34.4364	87.3287
62	0.797	2.04	55.58	116.23	173.84	2370	0.5074	34.8431	84.5918
63	0.806	2.00	56.86	114.31	173.18	2370	0.4947	35.2259	83.0762
70	0.890	2.00	67.18	100.93	170.12	2370	0.3697	37.7891	79.2765
80	0.948	2.00	79.50	69.15	150.65	2370	0.2420	40.8854	51.5554
90	0.989	2.00	97.18	47.45	146.63	2370	0.1026	45.3750	39.8510
100	0.998	2.00	116.34	29.38	147.72	2370	0.0394	46.0745	18.8265
109	1	2.00	131.12	9.33	142.46	2370	0.0083	45.0322	5.7735
110	1	2.00	136.53	--	138.53	2370	--	44.8822	--
111	1	2.00	137.43	--	139.44	2370	--	44.8837	--
120	1	2.00	157.33	--	159.33	2370	--	45.0137	--
130	1	2.00	174.39	--	176.40	2370	--	45.0050	--
140	1	2.00	195.80	--	197.80	2370	--	46.6336	--

150	1	2.00	216.04	--	218.04	2370	--	46.4086	--

Table 7 Results of the different reorder points when the inventory position has information about the repair centre. The repair time is stochastic.

The minimum reorder point is 40 and the maximum is set at 150. The minimum reorder point of 40 is chosen because that has a fill rate of 45%, which is lower than 50%. From reorder point 120 to 150 it is clear that the total average costs per item will not decrease anymore. The minimum total costs are found at the reorder point of 110 with a fill rate of 100% and the reorder point which first reach the fill rate of 80% is 63. Because of the fixed demand and return times, the amount of reorders does not differ much. The higher the reorder point, less reorders are necessary to replenish the stock on hand. The amount of items necessary to meet the service level is 2370. The small differences in the amount of needed items are caused by the warm up period.

Next we consider the model where the inventory position does not have any information about when an item arrives at the repair centre. The results are given in Table 8.

Model II : Inventory position is not updated when item arrives at the repair centre

Reorder point	*Fill rate*	*Average fixed costs*	*Average holding costs*	*Average backorder costs*	*Total average costs*	*Amount needed*	*Standard deviation backorders*	*Std. dev. holding costs*	*Std. dev. backorder costs*
20	0.454	2.02	37.97	194.43	234.43	2370	1.4175	27.9339	136.3408
30	0.616	2.02	46.18	158.72	206.92	2370	1.2520	31.7686	116.0697
40	0.772	2.02	55.34	128.22	185.58	2370	0.9740	34.3975	93.1852
42	0.797	2.02	56.75	120.62	179.40	2370	0.9130	34.5677	88.1111
43	0.807	2.02	57.19	117.86	177.07	2370	0.8896	34.5946	87.4338
45	0.822	2.02	60.90	113.56	176.51	2370	0.8425	36.7714	85.0562
50	0.873	2.02	66.50	104.85	173.37	2370	0.7239	38.2773	81.4889
60	0.963	2.02	80.26	68.40	150.68	2370	0.3431	43.0929	52.3984
70	0.986	2.02	98.15	47.08	147.25	2370	0.2192	44.8868	40.0788
80	0.997	2.02	116.87	23.13	142.02	2370	0.0902	46.5055	15.3202
85	0.998	2.02	124.91	22.38	149.31	2370	0.0630	46.0430	17.8619
86	0.999	2.02	127.92	22.38	152.32	2370	0.0630	46.7943	17.8619
87	0.999	2.02	129.60	23.68	155.31	2370	0.0624	46.1027	18.3214
88	1	2.02	132.28	--	134.30	2370	--	45.9402	--

89	1	2.02	135.17	--	137.19	2370	--	46.0013	--
90	1.00	2.02	136.88	--	138.90	2370	--	45.9309	--
100	1.00	2.02	155.41	--	157.43	2370	--	46.1936	--
110	1.00	2.02	176.58	--	178.60	2370	--	45.9519	--
120	1	2.02	196.32	--	198.34	2370	--	46.5915	--
130	1	2.02	216.40	--	218.43	2370	--	47.0120	--

Table 8 Results of the different reorder points when the inventory position has no information about the repair centre. The repair time is stochastic.

The minimum reorder point is set at 20 and the maximum is 130. An 80% fill rate is found at reorder point 43 and the minimum total average costs per item is when the reorder point is set at 88 with a fill rate of 100%. When the fill rate is 100%, the standard deviations are zero for backorders. This is because there are no shortages. When we look at the average fixed costs per item, then the amount of replenishment ordered are the same for the different reorder points. The total amount of items that are needed is 2370 when we want to meet the optimums by either using the method of meeting the fill rate or minimizing the total average costs.

6.2.2 Deterministic repair time

Like in section 6.2.1, we first consider the model where the inventory position has information about the item arrivals at the repair centre, so that it the inventory position is updated when a reorder is placed or when an item returns from the market to the repair centre. The results are given in Table 8.

Model III : Inventory position is updated when item arrives at the repair centre

Reorder point	*Fill rate*	*Average fixed costs*	*Average holding costs*	*Average backorder costs*	*Total average costs*	*Amount needed*	*Standard deviation backorders*	*Std. dev. holding costs*	*Std. dev. backorder costs*
40	0.453	2.03	38.59	191.71	232.34	2722	0.7883	29.1405	133.3683
50	0.628	2.03	45.23	156.64	203.90	2722	0.6560	31.1328	112.9929
60	0.783	2.03	54.37	119.81	176.21	2722	0.5067	34.2851	85.2735
62	0.799	2.03	55.99	116.08	174.07	2722	0.4865	34.9124	83.1812
63	0.809	2.00	57.18	113.74	172.92	2722	0.4763	35.2600	81.7845
70	0.892	2.00	67.26	97.56	166.83	2722	0.3589	38.2532	76.2614
80	0.945	2.00	80.35	68.94	151.29	2722	0.2400	41.2164	52.9780
90	0.987	2.00	98.04	56.61	156.65	2722	0.1182	45.3159	43.7782

100	0.998	2.00	116.92	51.82	170.74	2722	0.0464	45.7720	34.9513
108	1	2.00	134.16	20.00	156.16	2722	0.0118	45.1489	10.0000
109	1	2.00	135.72	--	137.72	2722	--	44.9047	--
110	1	2.00	137.13	--	139.14	2722	--	44.7764	--
111	1	2.00	138.02	--	140.02	2722	--	44.8238	--
120	1	2.00	157.68	--	159.69	2722	--	45.2057	--
130	1	2.00	174. 92	--	176. 92	2722	--	45.5786	--
140	1	2.00	196.49	--	198.50	2722	--	46.7829	--
150	1	2.00	216.66	--	218.67	2722	--	46.0710	--

Table 9 Results of the different reorder points when the inventory position has information about the repair centre. The repair time is deterministic.

When we want to have a fill rate of 80%, then we need to place a replenishment when the inventory level will decrease below 63 items. The total average costs that comes with it is € 172.92. When we want to minimize the total average costs we end up with having a reorder point of 109 with costs of € 137.72, where all the demand is met from the stock on hand. The difference in the costs are caused by the backorder costs, which are expensive. The total amount of items that are needed to satisfy the 80% service level is 2722 for both methods.

In Table 10 below are the results given when the inventory position does not know when items are returning beforehand.

Model IV : Inventory position is not updated when item arrives at the repair centre

Reorder point	*Fill rate*	*Average fixed costs*	*Average holding costs*	*Average backorder costs*	*Total average costs*	*Amount needed*	*Standard deviation backorders*	*Std. dev. holding costs*	*Std. dev. backorder costs*
20	0.432	2.02	37.62	196.40	236.03	2722	1.3587	28.3922	136.8315
30	0.608	2.02	44.37	158.82	205.20	2722	1.1502	30.4830	115.7966
40	0.764	2.02	54.42	123.94	180.38	2722	0.8961	34.0680	90.3317
42	0.791	2.02	56.11	121.00	179.13	2722	0.8534	34.3240	89.3811
43	0.805	2.02	57.20	117.76	176.97	2722	0.8326	34.5455	88.0780
45	0.824	2.02	60.45	119.06	181.53	2722	0.7914	35.9220	88.1284
50	0.864	2.02	67.93	108.01	177.96	2722	0.6992	39.3247	83.4700
60	0.916	2.02	81.64	74.21	157.87	2722	0.3881	42.2663	53.9149

70	0.969	2.02	96.35	57.18	155.54	2722	0.2484	44.7084	44.6317
80	0.994	2.02	115.20	47.50	164.72	2722	0.0863	45.2947	31.5891
85	0.998	2.02	128.46	47.69	178.17	2722	0.0646	45.8732	28.0439
86	0.999	2.02	130.40	33.68	166.11	2722	0.0587	45.7903	16.4014
87	1	2.02	131.59	--	**133.61**	2722	--	45.6887	--
88	1	2.02	132.85	--	134.87	2722	--	45.6736	--
90	1	2.02	135.66	--	137.68	2722	--	45.3737	--
100	1	2.02	155.65	--	157.67	2722	--	47.8763	--
110	1	2.02	177.11	--	179.13	2722	--	45.9857	--
120	1	2.02	194.46	--	196.48	2722	--	46.4886	--
130	1	2.02	214.82	--	216.84	2722	--	47.0120	--

Table 10 Results of the different reorder points when the inventory position has no information about the repair centre. The repair time is deterministic.

A fill rate of 80% is found at a reorder point of 43, where the total average costs are € 177.97. At this reorder point, an amount of 2722 items are needed. When we use the other method, that is by minimizing the total average costs, we have a reorder point at 87 with costs of € 133.61. The quantity of items necessary is here 2722. The amount of items needed are slightly different, because of the warm up period. The amount of items returning in the beginning are different for every reorder point.

6.3 Stochastic vs. Deterministic repair times

We have summarized the results in Table 11. We have divided the table into the stochastic model and deterministic model, both with or without updating the inventory position regarding the returns to the repair centre and based on the two methods. Afterwards we will compare the two methods for both models.

Let us first discuss the optimization method based on the fill rate. We see that when the objective is based on meeting the service level of 80%, the reorder points are equal for the two models of both the stochastic as deterministic model when the inventory position with and without the information from the repair centre are compared (model I & II as III & IV). Looking at the total average costs and amount of items needed, we see that, just like the fill rates, these don't differ much between the models of inventory position with information of both the stochastic and deterministic model and the models where the inventory position is only certain of items flowing into the stock when replenishments are ordered for both the stochastic model and deterministic model. The total average costs are higher for the last mentioned models compared to the first two mentioned models, whereas the amount of needed items are higher for the models the other way around.

Optimal results	*Stochastic model*		*Deterministic model*	
Model	I	II	III	IV
	IP with information	IP without information	IP with information	IP without information
Service level				
Reorder point	63	43	63	43
Fill rate	80.6%	80.7%	80.9%	80.5%
Total average costs	173.18	177.07	172.92	176.97
Necessary amount	2370	2370	2722	2722
Total average costs				
Reorder point	110	88	109	87
Fill rate	100%	100%	100%	100%
Total average costs	138.53	134.30	137.72	133.61
Necessary amount	2370	2370	2722	2722

Table 11 Summary of the optimal results for both the stochastic and deterministic model with different inventory position updating.

When we look at the method based on minimizing the total average costs, we see the same similarity as for the models based on meeting the service level. That is that the fill rate is equal for all four models and that model I almost equals model III as do the two models without the information. If we compare these four models based on the two methods, we see that the reorder points are much higher for the method based on the costs, so that the stock is replenished earlier. However, the same amount of replenishment is ordered because of the fixed demand and return times. Also, every demand is satisfied by the stock on hand that results in a 100% service level and the total average costs are lower using this method. The quantity that is needed to satisfy the 80% service level is the same for the four models, whether based on the fill rate or costs because of the fixed demand and return times. So in fact the method based on the costs is better than the method based on the service level in this case. Whether we consider stochastic or deterministic repair times also does not differ much, because the results are almost equal for the four models.

7. Conclusions

We considered four hypotheses in section 1.1:

H_0 : if there is information known about the return time of products to the repair centre, then this will lead to lower costs than when there is no information known.

H_1 : if there is no information available about the return time of products to the repair centre, this will lead to lower costs than when there is information known.

H_2 : if the return time of products to the repair centre are deterministic, this will lead to lower costs than stochastic return times.

H_3 : deterministic return times of products will not lead to better results than stochastic return times.

We test these hypotheses by means of the four different models that we have simulated::

I. The inventory position is updated when an item arrives at the repair centre for stochastic repair times.
II. The inventory position has no information over the arrival times of items at the repair centre for the stochastic repair times.
III. The inventory position is updated when an item arrives at the repair centre for deterministic repair times.
IV. The inventory position has no information over the arrival times of items at the repair centre for the deterministic repair times.

We considered two methods, one based on the fill rate and one based on costs. The method where the service level has to be met, gives as optimums the reorder point 63 for models I and III en reorder point 43 for models II and IV. If we look at the associated costs for this method, then model III has the minimum costs of the four models with a total average cost per item of € 172.92 and the second place goes to model I with a total average costs of € 173.18 per item. So model I and III have the lowest total average costs and these models have both information about when an item will return to the inventory. We also see that the deterministic models III and IV have lower costs than the stochastic models I and II. This shows that hypotheses H_0 and H_2 are true when the optimization is based on the service level.

When optimizing the models with the method based on the costs, we find model IV with the least total average costs per item with €133.61 and every model has a fill rate of 100%. This leads to the comparison of the two methods. The models that are based on the method with the costs have lower costs than the method with the fill rate. Furthermore, all the fill rates are 100% at every model when the fill rates are just above the 80% with the other method. So when we want to find the optimum of the optimums these four models, this would be model with deterministic repair times where the inventory is updated when the items return to the stock instead of the repair centre. The associated net amount of items necessary to satisfy the demand is 2722. The total average costs of the models with information are in this case not cheaper than the models without information, so that hypothesis H_1 holds here and H_0 is rejected. When we look at the costs by comparing the deterministic against the stochastic models, we see that the costs of both the deterministic models are lower. This leads to the fact that hypothesis H_3 holds for all models, whether based on the fill rate or costs.

8. References

De Brito, M. P., & Dekker, R. (2003). Modelling product returns in inventory control - exploring the validity of general assumptions. *International Journal of Production Economics (81-82)* , 225-241.

De Brito, M. P., & Dekker, R. (2002). *Reverse Logistics - a framework.* Erasmus University, Rotterdam. The Netherlands: Econometric Institute Report EI 2002-38.

De Brito, M. P., Dekker, R., & Flapper, S. D. (2003). Reverse Logistics - a review of case studies. *ERIM Report Series Research in Management* , 29.

Fleischmann, M., Bloemhof-Ruwaard, J. M., Dekker, R., van der Laan, E., van Nunen, J. A., & van Wassenhove, L. N. (1997). Quantitative models for reverse logistics: A review. *European Journal of Operational Research 103* , 1-17.

Fleischmann, M., Kuik, R., & Dekker, R. (2002). Controlling inventories with stochastic item returns: A basic model. *European Journal of Operational Research 138* , 63-75.

Ghiani, G., Laporte, G., & Musmanno, R. (2004). Solving Inventory Management Problems. In G. Ghiani, G. Laporte, & R. Musmanno, *Introduction to Logistics Systems Planning and Control* (pp. 121-122). New York: John Wiley & Sons, Ltd.

Glorie, K. (2008). *Optimization of the instrument stock in the Erasmus MC.* Research project, Erasmus University, Rotterdam, The Netherlands (in Dutch).

Kroon, L., & Vrijens, G. (1994). Returnable containers: an example of reverse logistics. *International Journal of Physical Distribution & Logistics* , 56-68.

Ross, S. M. (2007). Renewal Theory and Its applications. In S. Ross, *Introduction to Probability Models, 3th Edition* (pp. 302-303, 308, 316-317, 417). San Diego: Academic Press.

Tibben-Lembke, R. S., & Rogers, D. S. (2002). Differences between forward and reverse logistics in a retail environment. *Supply Chain Management, Vol. 7, No. 5* , 271-282.

Widi, S. (2009). *Inventory Management of Returnable Bottles of Brewery.* Master thesis, Erasmus University Rotterdam, The Netherlands.

Yuan, X. M., & Cheung, K. L. (1998). Modeling returns of merchandise in an inventory system. *OR Spektrum 20* , 147-154.

Appendix A : Counting processes

Suppose that *M(t)* equals the total number of events that have occurred by time *t*. These events can represent the demands or returns of items. Then the stochastic process { $M(t), t \geq 0$ } is said to be a counting process. Such a process is said to possess *independent increments* if the number of demands that occur in disjoint time intervals are independent. Another possession of the counting process is that it has *stationary increments* when the number of events in the interval [$t, t + s$] has the same distribution for all *t*.

Let T_n denote the time between the (n - 1)st and the *n*th event of this process, the so called interarrival times with $n \geq 1$, then the counting process will be called a *renewal process* when the sequence { T_n, n = 1,2,.. } is independent and identically distributed with some distribution *F(t)*. When this sequence is exponentially distributed, then the renewal process is called a Poisson process with rate λ (Ross, 2007).

Appendix B : Average demand and return during lead time

The total demand and return during lead time are given per placed order in the following table. When a reorder is placed until the time when the reorder has arrived, we keep track of the amount of demand that is placed and the amount of returns to the inventory.

- **Stochastic repair times**

Model I : Inventory position is updated when item arrives at the repair centre

Mean demand during lead time	*Mean return during lead time*
242	225
240	177
259	184
262	193
245	198
231	199
258	217
252	198
243	196
235	217
249	182
227	192
257	190
237	218
221	183
264	185
241	195
246	191
243	194
242	193
231	199
264	186
259	200
234	223
269	184
274	192
276	220
245	194
243	204
261	197
270	208
271	213

261	216
260	182
249	211
218	210
228	176
227	189
270	198
269	181
275	212
268	222
246	219
265	218
241	193
240	184
234	188
242	191
255	186
239	183
256	203
251	185
260	211
244	207
234	176
248	180
251	185
244	201
253	202
252	212
217	189
240	182
242	198
226	179
231	187

Table 12 Total demand and return during lead time for the 65 orders that has been placed at reorder point of 150.