Portfolio optimisation using cryptocurrencies and the equity market

A study that assesses portfolio performance with the help of univariate and multivariate GARCH models

Author:

Camil Papa[1]

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Supervisor:

Dr. S.O.R Lönn

Second assessor:

Dr. A. Tetereva

Erasmus School of Economics

Master Thesis Quantitative Finance

[1]Student number: 546407

Abstract

In this paper, I evaluate the performance of portfolios consisting of cryptocurrencies and the equity market. This paper extends the existing literature by combining GARCH estimation with portfolio optimisation for cryptocurrencies. Based on the log-returns of the seven most capitalized cryptocurrencies and the Fama/French market factor, I predict the out-of-sample volatility of these assets using univariate and multivariate GARCH models. With the predicted volatilities, it is possible to construct conditional covariance matrices, which are used to optimise the global minimum variance portfolio allocations. I find that portfolios consisting of only cryptocurrencies outperform portfolios that are a mix of cryptocurrencies and the equity market. Moreover, taking hedging relations into account, through a covariance matrix estimated with the DCC-GARCH model, and restricting the optimisation problem to no short-selling, leads to well-performing portfolios. Finally, I conclude that diversifying an equity portfolio with cryptocurrencies improves portfolio performance. Therefore, taking the volatility, diversification possibilities and hedging relations of cryptocurrencies into account may lead to more informed decisions about investing in cryptocurrencies.

Keywords: cryptocurrencies, dynamic conditional correlation, GARCH, global minimum variance, portfolio optimisation, Sharpe ratio

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1 Introduction

Cryptocurrencies and blockchain technology have been gaining the attention of investors, regulators and academics over the past few years. Recently, also big institutional entities such as J.P. Morgan, PayPal and several hedge funds could not ignore the growing importance of these digital assets (Hajric, 2020). The cryptocurrency community expanded itself from initial adopters to the wider public, resulting in an influx of capital. This is also reflected in the price of the cryptocurrencies, as an investment of $1000 in July 2010 in Bitcoin, the most popular cryptocurrency, would have turned into $81,000,000 7 years later (Phillip et al., 2018). Unlike traditional asset classes, the value of cryptocurrencies is not based on any tangible asset, economy or firm, but instead, their value is based on the security of an algorithm (Corbet et al., 2018).

On the other hand, cryptocurrencies are sometimes viewed critically as speculative (Phillip et al., 2018). Bitcoin and other cryptocurrencies are subject to large price fluctuations. The Bitcoin crash in 2018 did not have a noticeable effect on the US economy (Taskinsoy, 2019), but Kurka (2019) concludes that market disruptions can spread from the cryptocurrency market to the traditional economy.

Several studies have shown that there exists a positive correlation between the returns of cryptocurrencies (Härdle et al., 2020; Kyriazis et al., 2019). Furthermore, Cheikh et al. (2020) mention that a positive return-volatility relationship, which traditional financial assets do not have, may imply possible hedging opportunities and safe haven properties of cryptocurrencies. Therefore, the volatility clustering of cryptocurrencies and the diversification and hedging possibilities while taking cryptocurrencies into account are addressed in this paper. The outcomes of a volatility analysis would be particularly useful for portfolio allocations and risk management and could aid others in making informed decisions regarding financial investments in cryptocurrencies (Chu et al., 2017).

In this paper, I will investigate the volatility of cryptocurrencies using various conditional volatility models. Eventually, the aim is to find the best model for predicting the volatility of an individual cryptocurrency. Furthermore, the focus will be on jointly modelling the volatility of several cryptocurrencies to come up with a conditional covariance matrix. This covariance matrix can be used to form a global minimum variance portfolio allocation and the performance of this portfolio is evaluated using the Sharpe ratio. The second goal of this research is to investigate if a portfolio consisting of only cryptocurrencies outperforms a more diversified portfolio that also includes the equity market.

This research focuses on seven of the most capitalised cryptocurrencies at this moment: Bitcoin, Ethereum, Binance Coin, Cardano, Dogecoin, Ripple and Litecoin. I will examine their prices and log returns from 1 March 2018 up until 5 June 2021. The conditional volatility models that are central to this study are GARCH models. Both univariate models and the multivariate DCC-GARCH model will be used to model the volatility of the seven cryptocurrencies. These models have been widely used to model the exchange rate of traditional assets (Chu et al., 2017). However, there exists little work on fitting GARCH models to the return series of cryptocurrencies, and most of the work that does is focused only on Bitcoin. I specifically look at GARCH models, because they are stochastic models that are well suited to modelling randomness and uncertainty, two characteristics of the volatility of financial assets returns (Cerqueti et al., 2020). Another advantage of GARCH models is that they allow for non-Gaussian adaptations for modelling purposes; also a feature of financial asset returns.

I conclude that diversifying a portfolio of stocks with cryptocurrencies results in better portfolio performance. Also, for a portfolio consisting of only cryptocurrencies, the portfolio that takes the hedging relations into account, while restricting short-selling, performs best out-of-sample. Restricting the portfolio weights will result in less parameter uncertainty which can lead to better performance. Finally, I conclude that portfolios consisting only of cryptocurrencies generally outperform the equity market and portfolios that are a combination of cryptocurrencies and the equity market. Once the equity market is introduced to the portfolio, this value-weighted asset takes on a dominant position. Since the Sharpe ratio of the equity market is smaller than most of the Sharpe ratios of the cryptocurrencies, this may explain the lower Sharpe ratio of the portfolio.

Furthermore, I evaluate the fit of several GARCH-type models to the log-returns of the cryptocurrencies. I find that the more restricted iGARCH model fits best to most of the cryptocurrencies under investigation and the unrestricted eGARCH model performs best for the log-returns of the equity market. Contrary to expectations, the asymmetric gjrGARCH model is never selected as an optimal GARCH-type model.

The remaining of this paper is structured as follows, Section 2 introduces the concept of cryptocurrencies and gives a background on the most important characteristics. Section

3 summarizes the literature and other research that have already been conducted on the relevant topics. In Section 4 the data is introduced and I clarify the use case of the cryptocurrencies under investigation. The methodology I use to conduct the analysis is presented in Section 5. Then, the obtained results are presented and discussed in Section 6. Finally, Section 7 concludes the findings of my research and Section 8 critically reflects on the methodology and results.

2 Cryptocurrencies

This section will provide a brief introduction to cryptocurrencies and the underlying algorithms. Some key features and definitions are presented and the technology behind cryptocurrencies is explained.

2.1 The blockchain

Cryptocurrencies are digital assets that use blockchain technology to enable secure transactions. Contrary to traditional currencies, cryptocurrencies have no physical representation. Digital currencies are not a new concept. What makes cryptocurrencies different from previous digital assets, however, is that they eliminate a central point of control, such as a bank (Härdle et al., 2020). The blockchain is a peer-to-peer mechanism that effectively eliminates the need for a financial institution as the middle man. This implies that every individual that participates in transactions on the blockchain, is solely responsible for her funds. This makes it even possible to securely borrow and lend cryptocurrencies directly from another individual. A digital currency using this technology was first introduced by someone under the pseudonym "Satoshi Nakamoto" and is called Bitcoin, the biggest and most popular cryptocurrency at this moment (Nakamoto, 2008). Nakamoto's original motivation behind the creation of Bitcoin was to develop a payment system that allowed electronic transactions while maintaining many of the advantageous characteristics of physical cash.

To accomplish this, Nakamoto (2008) proposes the use of the blockchain. This blockchain is a database that caries the records of all the previous Bitcoin transactions. Therefore, it is also referred to as the ledger of the system. A blockchain is divided into blocks, which are subsets of data. Every block ends with a specific line of code, the digest, that summarizes the content of the block and this digest is repeated as the first line of the next block. Changing the content of one of the historical blocks will cause the digest to change, resulting in a mismatch with the next block. When such a mismatch is detected by the network, the altered block is thrown out and replaced with the original one. This feature ensures that the historical data is immutable.

The structure of the blockchain is distributed rather than centrally managed, which means that the peers in the network have equal standing. No bank or government decides what is possible on the blockchain. It is the participants of the network who together control and verify the transactions. An advantage of this system is that it is unnecessary to know or trust other peers. Each participant owns a copy of the ledger and offers consent on which new block is added and which block is rejected as a new part of the blockchain. A new block will be added if and only if a predefined set of rules is met.

2.2 Bitcoin mining

Once the legitimacy of the transactions is confirmed, the transactions are assembled in a block candidate. If the block candidate fulfils a specific set of predetermined criteria, it will be added to the blockchain. This implies that all the other active participants in the network also add the new block to their copy of the Bitcoin blockchain. Not every individual that owns Bitcoin will actively join in the process of forming new blocks on the blockchain. However, the ones who do are called miners. Pending Bitcoin transactions are collected and verified by a miner. In general, anyone can become a miner. However, mining is expensive as the computations are highly dependent on specialized hardware and the computations for forming block candidates require large amounts of electricity. Therefore, in practice, there are only a few large mining companies that produce most of the new blocks.

The network is developed in such a way that, on average, a valid block candidate is found every 10 minutes. For every block that is added to the blockchain, the miners receive a predefined number of newly created Bitcoin units. After the addition of 210.000 new blocks, which corresponds to around every four years, the reward the miners receive is halved.

Currently, Bitcoin miners receive 6.25 Bitcoin for every newly added block to the blockchain. The money creation within the Bitcoin system is scheduled so that the maximum number of Bitcoins will converge to 21 million. According to Berentsen & Schär (2018), this limited supply causes some people to believe that Bitcoin's value will forever increase. However, since Bitcoin units have no intrinsic value, their price is solely determined by the expectations about Bitcoin's future price. If all market participants expect that Bitcoin will not be valuable in the future, then they will not be willing to pay anything for it today (Berentsen & Schär, 2018).

2.3 Market capitalisation

Calculating the market capitalisation of cryptocurrencies is not as straightforward as calculating the market cap of a traditional company. As mentioned before, the total number of Bitcoin units is still increasing every day. Therefore, to calculate the market capitalisation of a cryptocurrency, one has to complement the price of a specific cryptocurrency with information on the aggregate supply (Chimienti et al., 2019). This aggregate supply can be broken down into four supply measures: circulating supply, total supply, maximum supply and variations of inflation-adjusted supply.

The circulating supply is the best approximation for the number of units of a cryptocurrency that are actively circulating in the market. The total supply is the number of units of a cryptocurrency that exist at that specific moment in time. This means that, in addition to the circulating supply, the total supply also includes the units that are locked or removed from the public markets (burned). Some cryptocurrencies, like Bitcoin, have a maximum supply. This is a predetermined maximum amount of units that will ever exist during the lifetime of a cryptocurrency. However, not all cryptocurrencies have a maximum supply. Finally, in some cases, cryptocurrencies have an inflation-adjusted supply where a forecast of the supply for, for example, the next five years is added to the circulating supply.

2.4 Altcoins

Besides Bitcoin there exist numerous other cryptocurrencies that can be grouped under the name "altcoins". These altcoins might have the same application as Bitcoin, a transaction mechanism, but the majority was developed for different purposes such as personal identity security, supply chain monitoring and digital contracts.

Some altcoins can be grouped into the class of distributed computation tokens. The coins in this group are linked to decentralized networks for applications on which other programs, exchanges and protocols can be built. The Ethereum network is an example of such a decentralized network for applications. It is possible to launch a decentralized cryptocurrency exchange on the Ethereum network. The Ethereum network is linked to its cryptocurrency, the Ethereum token, which can be used to perform transactions on the Ethereum network. The Ethereum network can be compared to the internet. The internet provides other companies with the possibility to develop their services or platform (for example a website or a social media platform such as Facebook).

Another class of cryptocurrencies is the stable coin. These cryptocurrencies are designed to be fully correlated to fiat currencies (such as the U.S. dollar), real assets (such as gold) or other cryptocurrencies. This means that these coins will accurately follow the movement of their counterparts. Stable coins achieve price stability through the use of algorithms that buy and sell the underlying asset or its derivatives.

These were just some applications of cryptocurrencies, but there exist many more as new cryptocurrencies are launched frequently. For this research, I will focus on the following cryptocurrencies: Bitcoin, Ethereum, Binance Coin, Cardano, Dogecoin, Ripple and Litecoin. Their application will be discussed in more depth in Section 4. Figure 2.1 shows the prices of Bitcoin and Ethereum, the two largest cryptocurrencies, and the stock of Apple (AAPL) during roughly the past three years. Both Bitcoin and Ethereum experienced a strong increase in prices at the beginning of 2021. However, after this increase, the graphs also show a steep drop in the prices. More on these price variations will be discussed in the next section.

2.5 Risks

Cryptocurrencies and blockchain technology introduce some risks. According to Harvey (2014), two significant hurdles could prevent Bitcoin and other cryptocurrencies from becoming the new big technological innovation: the high volatility and the regulatory uncertainty.

First, I will discuss the high volatility of Bitcoin and other cryptocurrencies. According to coinmarketcap.com the price of Bitcoin skyrocketed from $2,000 in April 2017 to $20,089 on December 17. After this fast increase, the price quickly dropped back to $8,400 on the 8th of February 2018. Following a period of lower volatility, the price of Bitcoin

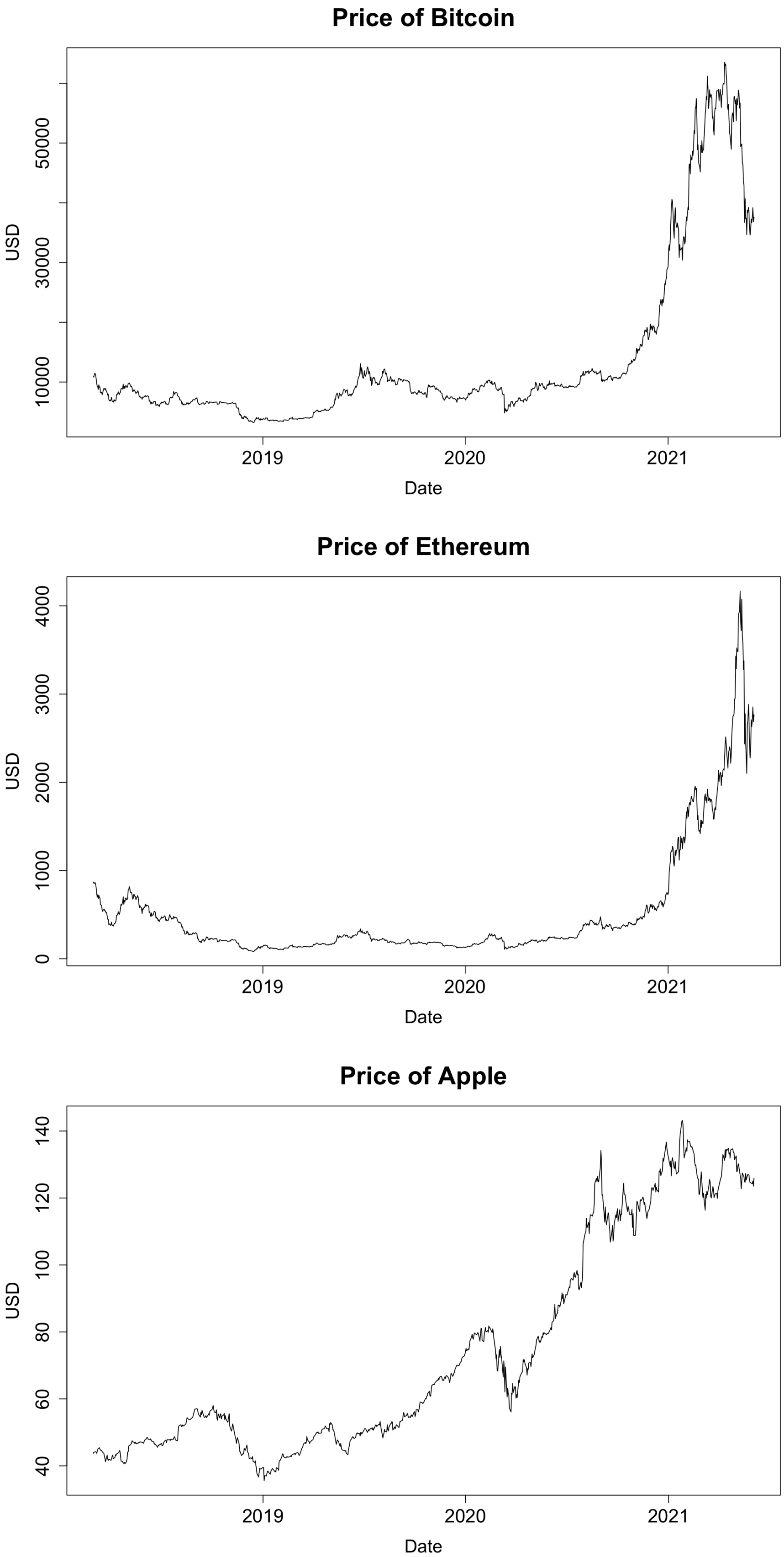

Figure 2.1: The prices of Bitcoin, Ethereum and Apple from 1 March 2018 until 5 June 2021.

shot up again from \$10,170 in September 2020 to \$63,729 on April 13 2021 as shown in Figure 2.1. Recently, another crash caused Bitcoin to lose almost 50% of its worth in less than a month. Comparing the cryptocurrencies in Figure 2.1 to the Apple stock, it can be concluded that the cryptocurrencies experienced a steeper increase in prices than the Apple stock. The price of Apple went from around \$90 to approximately \$140 in the same period from September 2020 to April 2021.

The second cycle in which Bitcoin flourished and crashed is presented in the first graph of Figure 2.1. Taskinsoy (2019) calls this phenomenon the Bitcoin mania and compares it to the Dutch tulip mania of the 17th century where a single bulb of the Semper Augustus tulip was valued at 6,219 guilders, which would have been about \$80,362. The Bitcoin mania distinguishes itself from the tulip mania by the fact that the tulip speculation disappeared after the crash in February 1637 while Bitcoin is still very popular despite multiple sudden price crashes (Taskinsoy, 2019). Due to the extreme volatility, many economists question if Bitcoin and other cryptocurrencies should be used as a payment method (Berentsen & Schär, 2018). On the other hand, Bitcoin and alternative cryptocurrencies should not be neglected as a speculative asset (Berentsen & Schär, 2018; Taskinsoy, 2019).

Second, is the risk of regulatory uncertainty. Extreme price movements due to high volatility may result in potentially large losses. Depending on the magnitude of a potential cryptocurrency crash, the losses may be passed on to the creditors of the cryptocurrency holders or other entities (Chimienti et al., 2019). This spillover from the cryptocurrency market into the traditional economy could result in implications for financial stability. Besides that, the decentralised feature of cryptocurrencies induces the absence of a formal governance structure. The anonymity of all the peers in a network in which there exist no hierarchy may result in criminal activity such as money laundering and terrorist funding (Chimienti et al., 2019).

Several countries, such as Nigeria, Vietnam and Egypt, have even banned or severely restricted the use of cryptocurrencies. At the moment there does not exist a regulatory framework for cryptocurrencies. This means that in the event of a hack or bankruptcy of a cryptocurrency exchange, all the cryptocurrency holders will lose their funds. Therefore, the ECB states that "in the light of the implications they might have for the stability and efficiency of the financial system and the economy, crypto-assets warrant continuous monitoring" (Chimienti et al., 2019, p. 18). New regulations imposed by countries or economic institutions may have a lasting effect on the rise and popularity of this technological innovation.

3 Literature review

Academic research on cryptocurrencies is still in its infancy, yet the number of studies related to this topic is increasing rapidly. The empirical results show that the research topics on cryptocurrencies are shifting from the underlying technology to the economic applications (Jiang, Li & Wang, 2021).

3.1 Financial opportunities

For some time there was no consensus on whether cryptocurrencies can be seen as an individual asset class or if there are substantial similarities to stocks, bonds, commodities or foreign exchanges. Contrary to other markets, the cryptocurrency market is open 24 hours per day, with no standardised closing time (Chimienti et al., 2019). Besides that, recent studies have shown that cryptocurrencies distinguish themselves remarkably from traditional asset classes in terms of risk and return (Ankenbrand & Bieri, 2018; Liu & Tsyvinski, 2021).

Since cryptocurrencies do not have an underlying claim, such as the right to a future cash flow or to discharge any payment obligation, it is argued that they lack any intrinsic value (Chimienti et al., 2019; Taskinsoy, 2019). However, García-Monleón et al. (2021) do not agree with this and succeed in furnishing a theoretical valuation model to measure the intrinsic value of cryptocurrencies. In this framework, they divide cryptocurrencies into three categories: initial coin offerings, single layer cryptocurrencies and multiple layer cryptocurrencies. The group of initial coin offerings includes the coins that are launched with the sole purpose of funding a specific crypto project. This group can be valued using the same valuation models that are applied to the Initial Public Offerings (IPOs) of stocks. Single-layer cryptocurrencies belong to blockchain networks that are designed exclusively for the movement of currencies within the network. These can be valued based on the utility they generate. Finally, multiple layer cryptocurrencies are those belonging to networks that allow the circulation of other kinds of information. These coins generate additional utilities and will therefore receive the value derived from the additional functionalities that the network generates on top of the value of the single-layer cryptocurrencies.

Several studies have tried to model the exposure of cryptocurrencies to common macroeconomic factors. Gregoriou (2019) concludes that the abnormal returns of cryptocurrencies cannot be explained by stock markets, by regressing the ten most capitalised cryptocurrencies to the Fama-French stock factors. Corbet et al. (2018) go a step further by stating that it is practically impossible to construct cryptocurrency factors based on the external information from other types of financial markets since the crypto market contains its own idiosyncratic risks. However, Liu et al. (2019) were able to obtain sizable and statistically significant excess returns by constructing cryptocurrency counterparts of price- and market-related factors in the stock market. They investigate 25 factors related to size, momentum, volume and volatility. Ultimately, they conclude that size and momentum factors are important in capturing the cross-section of cryptocurrency returns.

Furthermore, Lui & Tsyvinski (2021) also find that the returns of cryptocurrency can be predicted by two factors specific to its markets: momentum and investors' attention (how often a cryptocurrency is searched for on Google). Finally, Liu et al. (2020) specify three common factors unique to cryptocurrencies: the market factor and factors related to size and momentum. Contrary to the above, Grobys & Sapkota (2019) do not find conclusive evidence of a momentum factor for cryptocurrencies.

When looking at the relations among the various cryptocurrencies, Härdle et al. (2020) show that both Ripple and Ethereum are positively correlated with Bitcoin. Kyriazis et al. (2019) and Canh et al. (2019) use a multivariate GARCH model to reveal significant positive and strong correlations between cryptocurrencies. They, therefore, conclude that the cryptocurrency market lacks hedging opportunities. At this moment it is not possible to go long or short on all cryptocurrencies. One can simply compose a portfolio that consists of certain amounts of every coin. If the price of one specific cryptocurrency increases, its positively correlated counterpart will also increase and vice versa. In this way, it is not possible to hedge a long position in one coin with a short position in another coin.

Besides that, Bouri et al. (2019) show that there is evidence of co-movement in the cross-sectional returns' dispersion across cryptocurrency markets. This implies that crypto traders imitate the decisions of other investors without a reference to fundamentals, also known as herding behaviour (Bouri et al., 2019). Most crypto investors do not look at the fundamental building blocks (like revenue of the company, the business model and the management) of a certain coin, but simply perform technical analysis based on the price charts. Since these charts are the same for every trader, they tend to make the same decisions when looking at certain indicators. This herding behaviour becomes more prominent as market uncertainty increases and it causes investors to be exposed to additional risk (Bouri et al., 2019).

Furthermore, both Ankenbrand & Bieri (2018) and Petukhina et al. (2018) conclude that there will be limited benefits to hedging a traditional equity portfolio with cryptocurrencies due to their specific volatility structure. However, an interesting characteristic of cryptocurrencies is that they exhibit no or low correlation with the traditional asset classes (Ankenbrand & Bieri, 2018; Baur et al., 2018; Härdle et al., 2020). Therefore, cryptocurrencies and traditional asset classes will have asymmetric responses to the volatility of the financial market. This trait induces a diversification potential, since adding cryptocurrencies to an investment portfolio including traditional asset classes will spread the chances of success. Eventually, this leads to a more favourable risk/return profile of a portfolio that is a mix of traditional assets and cryptocurrencies (Ankenbrand & Bieri, 2018; Kurka, 2019). Vaddepalli & Antoney (2018) substantiate this claim by showing that neither of the three economic indicators financial openness, inflation and internet penetration are driving the transaction volume of Bitcoin.

Therefore, it is possible to optimise an investment portfolio containing cryptocurrencies through diversification. This diversification can be applied to solely cryptocurrencies and to cryptocurrencies and the equity market jointly. To properly diversify a portfolio, it is essential to study the volatility of the assets within it. This will be the subject of the following subsection.

3.2 Volatility modelling

Bitcoin displays many diverse stylized facts including long memory and heteroskedasticity (Phillip et al., 2018; Zhang et al., 2018). Phillip et al. (2018) also discover that the leverage effect is present for cryptocurrencies in general. They define the leverage effect as the notion of a negative correlation between one-day ahead volatility and returns. Besides that, research revealed that the returns of cryptocurrencies show a significant deviation from the normal distribution and that most cryptocurrencies exhibit heavy tails (Chan et al., 2017; Härdle et al., 2020). An important note on this finding is that there is not one specific distribution that fits well to all the cryptocurrencies (Chan et al., 2017).

Furthermore, Zhang et al. (2018) discover that the returns of cryptocurrencies display strong volatility clustering. Canh et al. (2019) explore the concept of volatility clustering in more detail and provide evidence that there are structural breaks present in the cryptocurrency market. They also discover that these structural breaks spread from cryptocurrencies with a smaller market cap to cryptocurrencies with a large market cap. In addition, Mensi et al. (2019) show that the returns of Bitcoin experience high and low volatility regimes. They, therefore, conclude that ignoring the structural breaks when modelling the volatility of cryptocurrencies will lead to volatility persistence overestimation, which hinders the prediction process.

There are various options to model and forecast the volatility of time series. Bollerslev (1986) introduced a natural generalization of the ARCH models of Engle (1982) and named them Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models. Later on, variations on the standard GARCH model were introduced, like a model with additional constraints (iGARCH) and an asymmetric model (gjrGARCH). These GARCH models are widely used in academic research on volatility modelling.

Chu et al. (2017) were the first to use GARCH models for volatility estimation of cryptocurrencies. They fitted twelve different GARCH models to the returns of the seven most popular cryptocurrencies at that moment. After testing the suitability of the specific models against various criteria, they conclude that the constrained iGARCH model and asymmetric gjrGARCH model are best suited to model the volatility of cryptocurrencies. Furthermore, Salamat et al. (2020) compare the fit of GARCH models with Normal and Student's t distributed errors and conclude that GARCH models with Student's t distributed errors provide a better fit.

Moreover, Caporale & Zekokh (2019) reveal that using standard GARCH models can produce inaccurate predictions and suggest that these predictions can be improved by using models that account for asymmetries and regime-switching. Previously, Bauwens et al (2010) had already deduced that standard GARCH models can produce biased results when a time series exhibits structural breaks. Since structural breaks are inherent to cryptocurrencies, Mensi et al. (2019) demonstrate that a GARCH model with structural break variables (fiGARCH) provides a superior forecasting accuracy compared to other GARCH models.

A different approach, in the case of the presence of structural breaks, was suggested by Ardia et al. (2018). They propose the use of Markov-Switching GARCH (MS-GARCH) models. This model allows the volatility to be estimated based on different regimes: periods of high and low volatility. Maciel (2020) applies this model to cryptocurrencies and compares the predictive performance of the MS-GARCH model with that of traditional single-regime GARCH models. His results show that MS-GARCH models do provide more accurate forecasts than their single-regime counterparts. Cheikh et al. (2020) expand the GARCH models that allow for only two possible variance regimes by experimenting with a Smooth Transition GARCH (ST-GARCH) model, where intermediate states are allowed between the two extreme volatility regimes. Based on their results, they argue that ST-GARCH models have adequate performance in terms of log-likelihood and information criteria compared to other conditional variance models.

4 Data

In this section, the data used to conduct the volatility analysis is introduced. First, a small description of the cryptocurrencies under investigation is given. Next, the descriptive statistics and log-returns of these cryptocurrencies are presented. Finally, a small analysis of the characteristics of the time series is performed.

The data used in this paper consist of the daily prices of seven cryptocurrencies. According to https://coinmarketcap.com, these cryptocurrencies can be ranked on market capitalisation. From high to low market cap, the cryptocurrencies under investigation are Bitcoin, Ethereum, Binance Coin, Cardano, Dogecoin, Ripple and Litecoin. I specifically opt for these cryptocurrencies because they cover a large part of the crypto market with their market cap and because they have been important assets for a longer period. Also, these cryptocurrencies differ widely in their applications. Table 4.1 summarizes the cryptocurrencies under investigation and provides a small description of their application. The abbreviations in between the brackets are commonly used when talking about cryptocurrencies.

Table 4.1: Description of the cryptocurrencies Bitcoin, Ethereum, Binance Coin, Cardano, Dogecoin, Ripple and Litecoin

Cryptocurrency	**Description**
Bitcoin (BTC)	Bitcoin is a peer-to-peer digital currency, meaning that all transactions occur directly between independent network participants, with no intermediary needed to enable or facilitate them.
Ethereum (ETH)	Ethereum is a decentralized blockchain system that has its own cryptocurrency, Ether. Ethereum works as a platform for numerous other cryptocurrencies but is also utilized for the realisation of decentralized smart contracts.
Binance Coin (BNB)	Binance is one of the largest cryptocurrency exchanges worldwide. Their goal is to increase the financial activity of cryptocurrency exchanges. Binance Coin is a key factor in the successful operation of many of Binance's sub-projects.
Cardano (ADA)	Cardano is also a blockchain platform and therefore a competitor of Ethereum. The ADA token is designed to ensure that owners can participate in voting for the future of the platform.
Dogecoin (DOGE)	Unlike the other cryptocurrencies, Dogecoin has no specific use. It has recently become very popular due to its association with Tesla CEO Elon Musk.
Ripple (XRP)	Ripple is a company that facilitates a digital payment platform. The cryptocurrency that runs on this platform is XRP.
Litecoin (LTC)	Litecoin is a cryptocurrency designed to make fast, secure and cheap payments by using the features of blockchain technology. This application is somewhat similar to Bitcoin.

The daily closing prices of the cryptocurrencies were retrieved from https://www.investing.com/crypto/currencies using a period of March 1, 2018, to June 5, 2021, resulting in 1193 observations. After extracting the prices, the data is converted into log-returns using the formula

$$r_t = \log \frac{P_t}{P_{t-1}} \tag{1}$$

where P_t is the price at time t and P_{t-1} the price at time $t-1$. The descriptive statistics of the log-returns of the cryptocurrencies are presented in Table 4.2. Besides the data on the cryptocurrencies, I also make use of the value-weighted market factor from the Fama/French 5 factor model, which can be found on their website. This is daily data excluding the weekends, as the traditional stock exchange is closed at that time. Therefore, for the market factor, I examine 823 observations from March 1, 2018, to June 5, 2021.

Table 4.2: Descriptive statistics of the log-returns of Bitcoin, Ethereum, Binance Coin, Cardano, Dogecoin, Ripple, Litecoin and the value-weighted market factor from 1 March 2018–to 5 June 2021

Statistic	**BTC**	**ETH**	**BNB**	**ADA**	**DOGE**	**XRP**	**LTC**	**Market**
Q1	-0.015	-0.021	-0.023	-0.029	-0.022	-0.023	-0.028	-0.004
Median	0.001	0.001	0.002	0.000	0.000	-0.000	-0.001	0.001
Mean	0.001	0.001	0.003	0.001	0.003	0.000	0.000	0.001
Q3	0.018	0.027	0.029	0.030	0.018	0.022	0.027	0.007
Volatility	0.040	0.053	0.059	0.062	0.190	0.062	0.056	0.014
Sharpe ratio	0.026	0.018	0.052	0.024	0.018	0.001	0.001	0.041
Skewness	-1.570	-1.360	-0.27	-0.280	0.420	0.120	0.850	-1.100
Kurtosis	21.870	14.960	16.760	6.540	385.300	14.410	10.050	16.010

Table 4.2 shows that indeed the volatility of cryptocurrencies is higher than the volatility of the market factor. The values for skewness and kurtosis indicate that both the cryptocurrencies and the market factor are non-normally distributed and exhibit heavy tails. Next to Table 4.2, Figure 4.1 displays the log-returns of the eight time series under investigation. Again, based on these graphs, it can be concluded that the returns of the cryptocurrencies fluctuate more than the returns of the market factor. The cryptocurrency that stands out the most, volatility wise, is Dogecoin.

According to the literature presented in the previous section, there exists a positive correlation among cryptocurrencies. To check this, Table 4.3 displays the Pearson correlation coefficients of the log-returns of the cryptocurrencies. This correlation coefficient measures the linear association between two variables.

Table 4.3: The correlation coefficients of the log-returns of Bitcoin, Ethereum, Binance Coin, Cardano, Dogecoin, Ripple and Litecoin and the value-weighted market factor from 1 March 2018–to 5 June 2021

	BTC	ETH	BNB	ADA	DOGE	XRP	LTC	Market
BTC	1.00							
ETH	0.83	1.00						
BNB	0.66	0.69	1.00					
ADA	0.71	0.79	0.62	1.00				
DOGE	0.18	0.17	0.15	0.18	1.00			
XRP	0.61	0.68	0.54	0.64	0.14	1.00		
LTC	0.79	0.83	0.66	0.74	0.18	0.67	1.00	
Market	-0.04	-0.04	-0.05	-0.05	-0.01	-0.04	-0.02	1.00

Table 4.3 shows that all the cryptocurrencies exhibit a positive correlation coefficient. For almost all the coins it holds that the correlation with other coins is moderate to high. Only Dogecoin shows a low correlation with the other cryptocurrencies and therefore behaves differently compared to the other cryptocurrencies. In Figure 4.1, we find that Dogecoin has experienced a period of extreme volatility compared to the other cryptocurrencies. During this time, Elon Musk first began tweeting about his interest in Dogecoin. After the first tweet, the price of Dogecoin skyrocketed but it also quickly retraced back to its normal price.

Overall, the findings on the correlations between cryptocurrencies are consistent with the literature. Table 4.3 demonstrates that there exists almost no correlation between the market factor and the cryptocurrencies. Another finding that stands out, is that the sign of the small correlation coefficients for the market factor is negative. These findings are also in line with the literature, as it was already mentioned in Section 3.1 that cryptocurrencies exhibit no or low correlation with the traditional asset classes.

5 Methodology

In this section, the methodology used to conduct a volatility analysis on the cryptocurrencies is presented and clarified. First, some univariate GARCH models are displayed and used to obtain the best fitting models for the cryptocurrencies. The predicted volatilities obtained from the best-fitting models will be used to explore optimal diversification strategies. Next, the multivariate DCC-GARCH model is introduced and used for forecasting covariance matrices. It is now possible to examine the hedging possibilities of cryptocurrencies with the help of these covariance matrices. Finally, I investigate the performance of various portfolios using portfolio optimisation.

5.1 Univariate GARCH models

The GARCH-type models are commonly utilized for modelling the conditional variance present in financial time series. Let r_t denote the observed log-returns of a specific cryp-

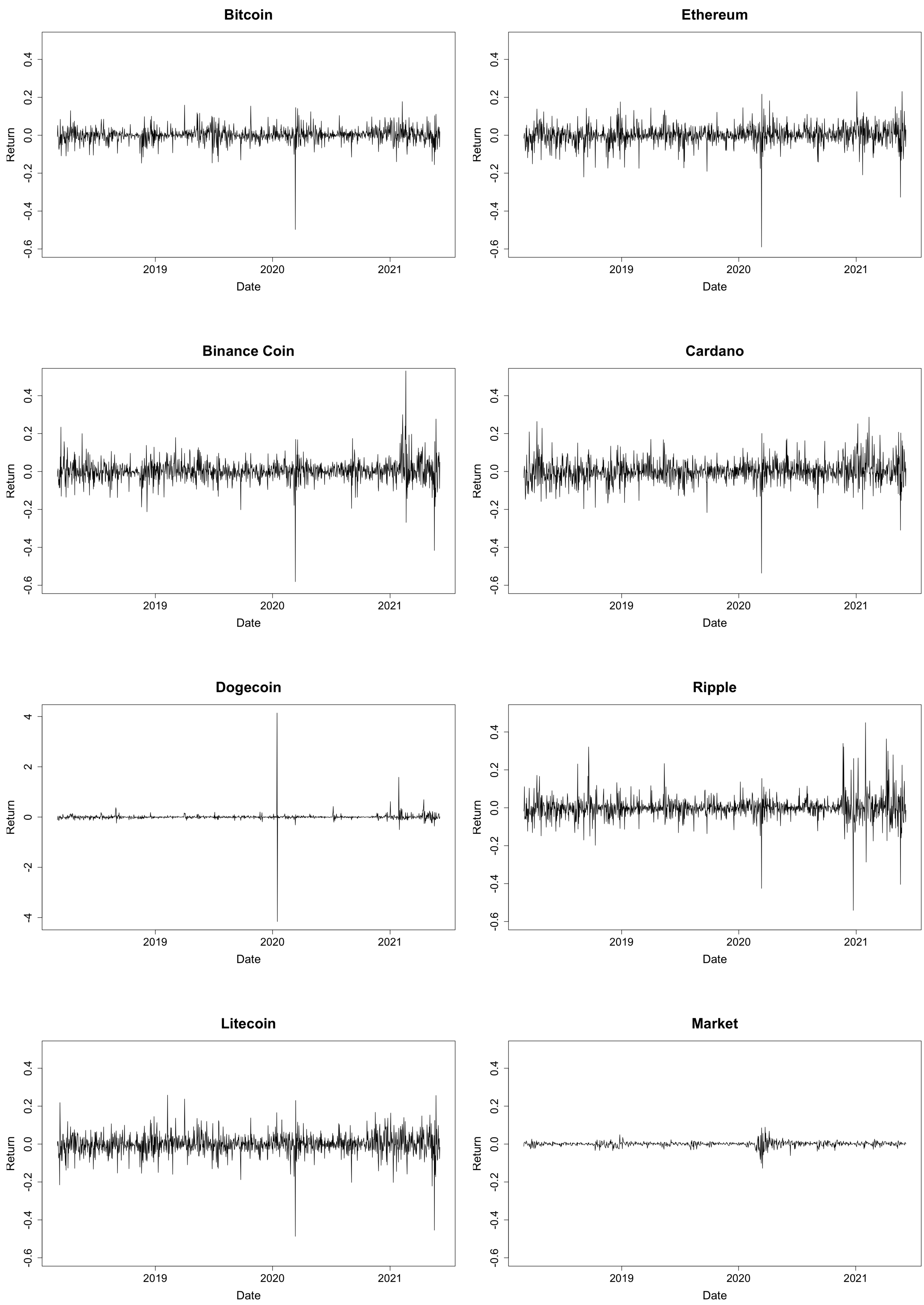

Figure 4.1: The log-returns of the cryptocurrencies under investigation and the market factor from 1 March 2018 until 5 June 2021.

tocurrency at time t. Then, this return can be specified as

$$r_t = \mu_t + \sigma_t Z_t \tag{2}$$

where μ_t is the conditional mean given the information set F_{t-1} and σ_t denotes the volatility process. All of the univariate GARCH models used in this research follow the same above specification for the return. However, they differ in their volatility process as will be elaborated in the following. Furthermore, for each GARCH-type model, the innovation Z_t is included, which is an independent and identically distributed innovation with a mean of zero and unit variance. For this research I allow Z_t to follow one of six distributions; these are the Normal distribution, the Student's t distribution, the Generalized Error distribution and their skew variants (Ghalanos, 2020). All GARCH models used will only be evaluated at the first order lags since empirical evidence suggests that higher-order models rarely perform better in out-of-sample analysis (Hansen & Lunde, 2005). The following will briefly introduce the univariate GARCH models.

Following on from Equation 2, the standard GARCH model (**sGARCH**), introduced by Bollerslev (1986), has a volatility process denoted by

$$\sigma_t^2 = \omega + \alpha_1 Z_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

where $\omega > 0$, $\alpha_1 > 0$ and $\beta_1 > 0$. These restrictions ensure that the conditional variance is always positive. Furthermore, the restriction that $\alpha_1 + \beta_1 < 1$ implies that the GARCH process is weakly stationary since the mean, variance, and autocovariance are finite and constant over time. This GARCH model forms the basis for the other models.

The sGARCH model is extended by adding an extra restriction to it, resulting in the integrated GARCH model (**iGARCH**). This model, by Engle & Bollerslev (1986), is a particular case of the sGARCH model in which $\alpha_1 + \beta_1 = 1$:

$$\sigma_t^2 = \omega + \alpha_1 Z_{t-1}^2 + (1 - \alpha_1)\sigma_{t-1}^2.$$

The fact that $\alpha_1 + \beta_1 = 1$ restricts the sGARCH model and imports a unit root into the GARCH process. This model will have a lower estimation error because it does not require β to be estimated.

Another extension of the sGARCH model is the exponential GARCH model (**eGARCH**) by Nelson (1991), with a volatility process

$$\log \sigma_t^2 = \omega + \alpha_1 Z_{t-1}^2 + \gamma_1[|Z_{t-1}| - E(|Z_{t-1}|)] + \beta_1 \log \sigma_{t-1}^2.$$

The main advantage of the eGARCH model is that no constraints on the parameters are required since the positivity of the variance is automatically satisfied due to the log transformation.

Finally, I will also examine the asymmetric **gjrGARCH** model (Glosten et al., 1993) with a conditional variance equation

$$\sigma_t^2 = \omega + \alpha_1 Z_{t-1}^2 + \gamma_1 I_{t-1} Z_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

where $\omega > 0$, $\alpha_1 > 0$, $\gamma_1 > 0$, $\beta_1 > 0$. Also, $I_{t-1} = 1$ if $Z_{t-1} \leq 0$ and $I_{t-1} = 0$ if $Z_{t-1} > 0$. This model extends the sGARCH model by accommodating positive and negative shocks. Different shocks will lead to different effects on the conditional variance. In asset pricing, it is often observed that negative shocks have a larger impact on the volatility than positive shocks (Rabemananjara & Zakoian, 1993) and this feature is included in the gjrGARCH model.

5.1.1 Best performing univariate models

The four univariate GARCH models (sGARCH, iGARCH, eGARCH and gjrGARCH) will now be utilized to conduct a volatility analysis on the cryptocurrencies. First, the most appropriate model, with the best fit to the individual time series of each cryptocurrency, is selected.

To accomplish this, the log-return data is split in half into a training- and a test set. Therefore, the estimation sample period spans the period from March 1, 2018, through October 18, 2019 (596 observations) and the out-of-sample evaluation sample spans the period from October 19, 2019, through June 5, 2021 (596 observations). Using the training set, it is possible to select the best fitting innovation distribution Z_t for every cryptocurrency. This is achieved by fitting the sGARCH model with the six different innovation distributions to the log-returns of the cryptocurrencies.

Following Chu et al. (2017), the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) are used to select the best fitting innovation distribution for each cryptocurrency. The model with the lowest AIC and BIC score is assumed to be the most appropriate model with the best fit. In case the AIC and BIC do not agree on the best fit, I will prioritise the BIC as the AIC is known to sometimes overfit. Next, the four GARCH models, with the best fitting innovation distribution per cryptocurrency, are fitted to the different cryptocurrencies. Again, the AIC and BIC are used to select the best fitting GARCH-type model for every individual time series.

While fitting the optimal GARCH models to the in-sample data points, the parameters of the most appropriate GARCH-type model (selected for each cryptocurrency) are estimated using Quasi-maximum likelihood estimation (QMLE). These parameters are fixed and then utilized to predict the out-of-sample volatility. The volatility processes presented in Section 4.1.1 are used to predict the conditional volatility σ^2_{t+1} one day ahead:

$$\hat{\sigma}^2_{t+1} = E[\sigma^2_{t+1}|I_t]. \tag{3}$$

Utilizing a rolling-window approach, the one-day-ahead predictions of the conditional variance are obtained for all the out-of-sample data points. These conditional variances are important for portfolio optimisation purposes explained later on in this section.

In this section, I discussed modelling the individual volatility of the log-returns of the cryptocurrencies. In the following, a different method for modelling the volatility of the log-returns of the cryptocurrencies combined is introduced.

5.2 Multivariate DCC-GARCH model

Since financial volatilities move closely together over time across assets and markets, it is interesting to investigate the dependence in the co-movements of asset returns. To do so, I will use a multivariate GARCH model to predict the daily covariances of the cryptocurrencies. Next to diversification purposes, calculating these covariances will introduce hedging relations between the cryptocurrencies. Therefore, taking into account this feature, while looking at a multivariate GARCH model instead of a univariate model, should lead to more relevant empirical results. An example of such a multivariate GARCH model is the Dynamic Conditional Correlation GARCH model (**DCC-GARCH**), by Engle & Sheppard (2001), which will be explained and examined below. Following the notation of

Engle & Sheppard (2001), I will now refer to the variance as h instead of σ^2 as specified in the volatility processes of Section 5.1.

For the DCC-GARCH model, denote $\mathbf{r}_t$ as a n x 1 vector of the log-returns of n assets at time t. The DCC-GARCH model can then be specified as:

$$\boldsymbol{r}_t = \boldsymbol{\mu}_t + \boldsymbol{\alpha}_t \tag{4}$$

$$\boldsymbol{\alpha}_t = \boldsymbol{H}_t^{1/2}\boldsymbol{z}_t \tag{5}$$

$$\boldsymbol{H}_t = \boldsymbol{D}_t\boldsymbol{R}_t\boldsymbol{D}_t \tag{6}$$

where $\boldsymbol{\alpha}_t$ is a n x 1 vector of the mean-corrected returns of n assets at time t, with E[$\boldsymbol{\alpha}_t$]=0 and Cov[$\boldsymbol{\alpha}_t$]=$\boldsymbol{H}_t$. Furthermore, $\boldsymbol{\mu}_t$ is a n x 1 vector of the expected value of the conditional log-returns $\mathbf{r}_t$. The conditional variances of $\boldsymbol{\alpha}_t$ are collected in the n x n matrix $\boldsymbol{H}_t$. As in the univariate GARCH models, $\boldsymbol{z}_t$ is an independent and identically distributed innovation. In the multivariate case, these $\boldsymbol{z}_t$'s are n x 1 vectors with E[$\boldsymbol{z}_t$]=0 and E[$\boldsymbol{z}_t\boldsymbol{z}_t'$]= I. In Equation 6, $\boldsymbol{D}_t$ is a n x n diagonal matrix of the conditional standard deviations of $\boldsymbol{\alpha}_t$ at time t. The elements in the diagonal matrix $\boldsymbol{D}_t$ are the standard deviations from univariate GARCH models:

$$\boldsymbol{D_t} = \begin{pmatrix} \sqrt{h_{1t}} & 0 & \dots & 0 \\ 0 & \sqrt{h_{2t}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sqrt{h_{nt}} \end{pmatrix}$$

Finally, the conditional correlation matrix of $\boldsymbol{\alpha}_t$ is written as the n x n matrix $\boldsymbol{R}_t$. This matrix $\boldsymbol{R}_t$ can also be interpreted as the conditional correlation matrix of the standardized disturbances $\boldsymbol{\epsilon_t}$, such that:

$$\boldsymbol{\epsilon_t} = \boldsymbol{D}_t^{-1}\boldsymbol{\alpha}_t \sim N(\mathbf{0}, \boldsymbol{R}_t)$$

This correlation matrix is symmetric by definition, resulting in the following elements of $\boldsymbol{H}_t = \boldsymbol{D}_t\boldsymbol{R}_t\boldsymbol{D}_t$:

$$[\boldsymbol{H}_t]_{ij} = \sqrt{h_{it}h_{jt}}\rho_{ij} \tag{7}$$

with ρ_{ij} the correlation between asset i and j and $\rho_{ii} = 1$.

Since $\boldsymbol{H}_t$ is a covariance matrix, it has to be positive definite. To ensure that this criterion holds, $\boldsymbol{R}_t$ must meet two requirements: $\boldsymbol{R}_t$ also has to be positive definite and all the elements in $\boldsymbol{R}_t$ have to be equal to or less than one by definition. The DCC-GARCH model meets both these requirements by decomposing $\boldsymbol{R}_t$ into:

$$\boldsymbol{R}_t = \boldsymbol{Q}_t^{*-1}\boldsymbol{Q}_t\boldsymbol{Q}_t^{*-1} \tag{8}$$

$$\boldsymbol{Q}_t = (1-a-b)\overline{\boldsymbol{Q}} + a\boldsymbol{\epsilon_{t-1}}\boldsymbol{\epsilon_{t-1}}' + b\boldsymbol{Q}_{t-1} \tag{9}$$

where a and b are scalars and $\boldsymbol{Q}_t^*$ is a diagonal matrix containing the square root of the diagonal elements of $\boldsymbol{Q}_t$:

$$\boldsymbol{Q}_t^* = \begin{pmatrix} \sqrt{q_{11t}} & 0 & \dots & 0 \\ 0 & \sqrt{q_{22t}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sqrt{q_{nnt}} \end{pmatrix}$$

This matrix $\boldsymbol{Q}_t^*$ ensures that the elements of $\boldsymbol{R}_t$ are smaller than or equal to one in absolute value by rescaling the elements in $\boldsymbol{Q}_t$. In order for $\boldsymbol{H}_t$ to be positive definite, the following restrictions on a and b must hold:

$$a \geq 0, \quad b \geq 0 \quad \text{and} \quad a+b<1$$

Furthermore, $\overline{\boldsymbol{Q}} = Cov[\boldsymbol{\epsilon_t}\boldsymbol{\epsilon_t}'] = E[\boldsymbol{\epsilon_t}\boldsymbol{\epsilon_t}']$ is the unconditional covariance matrix of the standardized disturbances $\boldsymbol{\epsilon_t}$. According to Engle (2002), the matrix $\overline{\boldsymbol{Q}}$ can be estimated as:

$$\bar{\boldsymbol{Q}} = \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{\epsilon_t}\boldsymbol{\epsilon_t}'$$

The specifications above are for a DCC(1,1)-GARCH model. It is possible to generalise this model into a DCC(M, N)-GARCH model, but in this thesis, I will only consider the model with first-order lags.

A neat feature of this model is that both $\boldsymbol{D}_t$ and $\boldsymbol{R}_t$ are designed to be time-varying, which makes the set-up of the model more realistic. In this way, it is possible to better model the co-movements of the cryptocurrencies. This research considers two different distributions for the standardized error $\boldsymbol{z}_t$: the multivariate Gaussian- and the multivariate Student's t-distribution. How to estimate the $\boldsymbol{D}_t$ and $\boldsymbol{R}_t$ matrices when the error terms are multivariate Gaussian or multivariate Student's t-distributed is explained in Appendices A.1 and A.2.

5.2.1 Best performing DCC-GARCH model

To select the best fitting DCC-GARCH model, I will use the same in-sample out-of-sample split as in Section 5.1.1. Furthermore, estimating the DCC-GARCH model first requires estimating univariate GARCH models. For the estimation of the DCC-GARCH model, I will make use of the best fitting univariate GARCH-type model per cryptocurrency obtained from the univariate GARCH analysis.

Utilizing the univariate GARCH models, the DCC-GARCH model is estimated considering the multivariate Gaussian- and the multivariate Student's t-distribution for the standardized error $\mathbf{z}_t$. Similar to the methodology on univariate GARCH models, the best fitting DCC-GARCH model is selected by comparing the AIC and BIC of the in-sample fit of the different error distributions.

After selecting the best fitting DCC-GARCH model, the in-sample parameters are fixed and it is possible to estimate the out-of-sample conditional covariance matrices $\mathbf{H}_t$ one day ahead. Finally, I will perform a portfolio optimisation with the estimated covariance matrices. This will be the topic of the following section.

5.3 Portfolio optimisation

In this part of my research I will look at different ways to form and optimise a portfolio considering both the cryptocurrency market and the equity market. I will utilize the Global Minimum Variance portfolio optimiser, which has the following optimisation problem:

$$\begin{aligned} \min_{\boldsymbol{w}} \quad & \boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} \\ \text{s.t.} \quad & \boldsymbol{\iota}'\boldsymbol{w} = 1 \end{aligned} \tag{10}$$

where $\boldsymbol{w}$ is a vector containing the weights assigned to the assets in the portfolio, $\boldsymbol{\Sigma}$ is the population covariance matrix and $\boldsymbol{\iota}$ is a column vector of ones with length n, the number of assets. It is possible to add a no short-selling constraint, which implies that the constraint $\boldsymbol{w} \geq 0$ is added to the minimization. The optimal portfolio weights are presented later on in this section.

Hereafter, four different layers to my analysis are highlighted. I start with analyzing a value-weighted crypto market portfolio and then extend this concept to an equally-weighted market portfolio. Next, I look at an optimally diversified portfolio and I introduce hedging relations in addition to the optimal diversification. Finally, I add short-selling restrictions to the portfolio with hedging relations.

Value-weighted crypto portfolio

As discussed earlier, Bitcoin is the largest and most popular cryptocurrency and price variations in this asset tend to affect the prices of the other crypto assets. Therefore, I will consider Bitcoin as a value-weighted portfolio of the cryptocurrency market. The performance of this portfolio is measured in the form of the Sharpe ratio, which is defined by:

$$Sharpe = \frac{\mu_p - r_f}{\sigma_p} \tag{11}$$

where μ_p is the expected portfolio return, r_f is the risk-free rate and σ_p is the standard deviation of the portfolio's excess return. For this research, I opt r_f to be equal to zero due to the current low interest rates. Thus, the Sharpe ratio of a portfolio only consisting of Bitcoin is the out-of-sample log-return of Bitcoin divided by the out-of-sample standard deviation.

Equally-weighted 1/N portfolio

Second, I will examine a portfolio that is evenly divided over the seven cryptocurrencies in this research: a 1/N portfolio allocation. While this likely introduces a bias, there is also no estimation uncertainty. Again, the Sharpe ratio is considered as the performance measure. In this case, the out-of-sample portfolio return is defined as $\boldsymbol{R}_p\boldsymbol{w}_{1/N}$ and the out-of-sample volatility of the portfolio is defined as $\sqrt{\boldsymbol{w}'_{1/N}\boldsymbol{\Sigma}\boldsymbol{w}_{1/N}}$, where $\boldsymbol{R}_p$ is a vector containing the out-of-sample returns of the cryptocurrencies, $\boldsymbol{w}_{1/N}$ is the vector of portfolio weights (in this case all entries are equal to 1/7) and $\boldsymbol{\Sigma}$ is the out-of-sample covariance matrix.

Optimally diversified portfolio

Next, I will make use of the predicted volatilities of the univariate GARCH models as mentioned in Section 5.1.1. Using the best fitting GARCH-type model for each cryptocurrency, the one-day-ahead out-of-sample conditional volatility is predicted for each cryptocurrency individually. Taking the predicted volatilities together, it is possible to create a 7 by 7 diagonal matrix with zeros on the off-diagonal elements and the estimated variances of the cryptocurrencies on the diagonal elements for every out-of-sample data point: I denote this $\hat{\boldsymbol{\Sigma}}_t$.

Subsequently, this matrix $\hat{\boldsymbol{\Sigma}}_t$ can be utilized to obtain the daily portfolio weights of the Global Minimum Variance portfolio. This GMV portfolio is the portfolio that provides the lowest possible risk for the rate of expected return and the optimal weights of this portfolio are:

$$\boldsymbol{w}_{gmv} = \frac{1}{\boldsymbol{\iota}'\hat{\boldsymbol{\Sigma}}_t^{-1}\boldsymbol{\iota}}\hat{\boldsymbol{\Sigma}}_t^{-1}\boldsymbol{\iota} \tag{12}$$

Finally, it is possible to calculate the Sharpe ratio of the portfolio using the global minimum variance weights $\boldsymbol{w}_{gmv}$:

$$SR_{gmv} = \frac{\mu_{gmv}}{\sigma_{gmv}} = \frac{\boldsymbol{\mu}'\boldsymbol{w}_{gmv}}{\sqrt{\boldsymbol{w}'_{gmv}\boldsymbol{\Sigma}_t\boldsymbol{w}_{gmv}}} \tag{13}$$

with $\boldsymbol{\mu}$ a vector of the expected log-returns of the cryptocurrencies and $\boldsymbol{\Sigma}_t$ the population covariance matrix at time t. Since $\boldsymbol{\Sigma}_t$ is unknown, I will use the volatility of the daily out-of-sample portfolio returns as a proxy for $\boldsymbol{\Sigma}_t$.

In this layer, it is assumed that we can estimate the variances but not the covariances of the assets. Estimating the covariances of the cryptocurrencies is the subject of the fourth layer and will introduce hedging relations.

Diversified portfolio with hedging relations

For the last part of the portfolio analysis, I utilize the DCC-GARCH model to estimate both the variances and the covariances of the cryptocurrencies. This implies that it is possible to construct an optimal portfolio while taking into account the hedging relations of the cryptocurrencies. The estimated $\hat{\boldsymbol{H}}_t$ matrix that results from the DCC-GARCH model is used to calculate the daily weights of the global minimum variance portfolio:

$$\boldsymbol{w}_{gmv} = \frac{1}{\boldsymbol{\iota}'\hat{\boldsymbol{H}}_t^{-1}\boldsymbol{\iota}}\hat{\boldsymbol{H}}_t^{-1}\boldsymbol{\iota} \tag{14}$$

Thereafter the Sharpe ratio is calculated following Equation 16.

For these optimisations I consider a portfolio consisting only of cryptocurrencies. Hereafter, I will expand my portfolio to include the value-weighted equity market factor and add short-selling constraints.

Additions to the portfolios

The equally-weighted portfolio, optimally diversified portfolio and diversified portfolio with hedging relations can be expanded by adding the value-weighted market factor to the portfolio and treating it as a tradable asset. As a result, it will be possible to compare the performance of a portfolio solely consisting of cryptocurrencies with a portfolio that is more diversified by incorporating the equity market. However, since stocks are only traded during the weekdays, it is necessary to alter the log-returns of the cryptocurrencies by omitting the weekends from the dataset. After this modification, the same steps are applied to the new portfolio to obtain the new 8 by 8 estimated matrices $\hat{\boldsymbol{\Sigma}}_t$ and $\hat{\boldsymbol{H}}_t$.

Furthermore, I will also look at the effect of imposing a short sale restriction on the portfolio allocations with hedging relations. This means that in this case, the portfolio weights must be larger than or equal to zero, as explained at the beginning of this section.

6 Results

This section will highlight the results of my research. I will begin with portfolio optimisation and present the Sharpe ratios of the different portfolios. Next, I will deconstruct the origin of these Sharpe ratios by investigating the global minimum variance portfolio weights. Finally, I will present the optimal univariate GARCH-type models and the optimal error distribution for the DCC-GARCH model found during this study. These optimal models are used to obtain the global minimum variance weights.

6.1 Portfolio optimisation

6.1.1 Sharpe ratios

The daily Sharpe ratios of the portfolios consisting of only cryptocurrencies are presented in Table 6.1. In this table, the various layers of my research are presented: the value-weighted crypto market portfolio, equally-weighted 1/N portfolio, optimally diversified portfolio, optimally diversified portfolio with hedging relations and the diversified portfolio with hedging relations under short-selling restrictions.

Moving from the value-weighted crypto market portfolio to the equally-weighted 1/N portfolio, I find that the Sharpe ratio and therefore the performance increases. Investing in more cryptocurrencies other than just Bitcoin will therefore improve the performance of a portfolio. With the help of the diagonal matrix $\hat{\boldsymbol{\Sigma}}_t$, it is possible to look at the performance of an optimally diversified portfolio without taking the hedging relations into account. Table 6.1 shows that an optimally diversified portfolio performs better than a naively diversified portfolio. However, when hedging relations are introduced to an optimally diversified portfolio with the help of $\hat{\boldsymbol{H}}_t$, the Sharpe ratio of the portfolio decreases. This is because it is currently not possible to properly hedge all cryptocurrencies, as explained in Section 3.1.

Finally, I investigate a portfolio that is optimally diversified and takes the hedging relations into account, while adding a short-selling restriction to the optimisation problem. Table 6.1 shows that this portfolio leads to the highest Sharpe ratio. The short-selling restriction causes the portfolio to perform better since the hedging relations are noisy estimates. Therefore, it is possible to make a positive bias-variance trade-off by imposing no short-selling. In addition, adding short-selling constraints makes the application more practical, since it is not possible to take short positions in every cryptocurrency.

Table 6.1: The out-of-sample daily Sharpe ratio and volatility of crypto portfolios, obtained using various covariance matrices.

Portfolio	BTC	1/N	Diversified	Hedging	Restricted hedging
Sharpe ratio	0.061	0.071	0.082	0.074	0.090
Volatility	0.043	0.063	0.059	0.070	0.076

NOTE: The value-weighted cryptocurrency market portfolio consists of only Bitcoin.

In addition, the Sharpe ratios of the portfolios that include the equity market are shown in Table 6.2. Comparing the Sharpe ratio of the value-weighted equity market to the other portfolios, it can be concluded that adding cryptocurrencies to an equity market portfolio does improve the performance. Therefore, the conclusion of Ankenbrand & Bieri (2018), who argue that a mix of traditional assets and cryptocurrencies leads to a more favourable risk/return profile of a portfolio, is confirmed.

Table 6.2 also shows that introducing hedging relations to a mixed portfolio improves the Sharpe ratio, but this portfolio is still outperformed by the naively diversified portfolio. However, imposing a short-selling restriction on the portfolio that takes into account hedging relations again increases the performance of the portfolio. These findings contradict both Ankenbrand & Bieri (2018) and Petukhina et al. (2018), who argue that there are limited benefits to hedging a traditional equity portfolio with cryptocurrencies. Finally, I find that for mixed portfolios, the optimally diversified portfolio performs best. This finding is consistent with Ankenbrand & Bieri (2018) and Kurka (2019).

Table 6.2: The out-of-sample daily Sharpe ratio and volatility of portfolios consisting of cryptocurrencies and the value-weighted market factor, obtained using various covariance matrices

Portfolio	Market	1/N	Diversified	Hedging	Restricted hedging
Sharpe ratio	0.055	0.069	0.077	0.067	0.076
Volatility	0.018	0.061	0.026	0.022	0.022

NOTE: The value-weighted equity market portfolio is treated as a tradable asset.

After comparing Table 6.1 to Table 6.2, it becomes clear that in all cases the Sharpe ratio of the portfolio decreases when the equity market is added to a portfolio of cryptocurrencies. This implies that there will be no diversification premium when regular stocks are added to a portfolio of cryptocurrencies. However, Table 6.2 does show that there is a diversification premium when it is the other way around. When cryptocurrencies are added to an equity market portfolio, the Sharpe ratio increases.

Tables 6.1 and 6.2 also indicate that the volatility of the portfolios drops when the equity market is added to a portfolio of cryptocurrencies. An explanation for this finding will be discussed in Section 6.1.2.

While the focus of this paper is on predicting volatilities, it is important to consider the uncertainty associated with this process. Therefore, I use the tests proposed by Ledoit & Wolf (2008) and Ledoit & Wolf (2011) to check if there is a statistically significant difference between the Sharpe ratios and between the volatilities of the portfolios. The details of these tests are explained in Appendices A.6 and A.7. I use the restricted portfolio containing the hedging relations as a benchmark and test if the Sharpe ratio and volatility of the other portfolios significantly differ from this benchmark. The fact that this portfolio takes the covariances between the cryptocurrencies into account and also restricts the weights to be positive makes it the natural benchmark. Table 6.9 presents the p-values of the performed tests for the portfolios consisting of only cryptocurrencies. Table 6.10 presents the p-values of the performed tests for the portfolios consisting of both the equity market and cryptocurrencies.

Table 6.3: The p-values of the HAC inference test by Ledoit & Wolf (2008) to test the difference between the Sharpe ratios and the volatilities of the crypto portfolios.

Portfolio	BTC	1/N	Diversified	Hedging
Δ_{Sharpe}	0.465	0.474	0.663	0.058
Δ_{vol}	0.037*	0.260	0.048*	0.004**

NOTE: The portfolio obtained by restricting the weights and incorporating the hedging relations is the benchmark portfolio; $H_0 : \Delta = 0$; $*p < 0.05$, $**p < 0.01$.

Table 6.4: The p-values of the HAC inference test by Ledoit & Wolf (2011) to test the difference between the Sharpe ratios and the portfolio volatilities of the combined portfolios.

Portfolio	Market	1/N	Diversified	Hedging
Δ_{Sharpe}	0.679	0.902	0.930	0.507
Δ_{vol}	0.498	0.001**	0.000**	0.983

NOTE: The portfolio obtained by restricting the weights and incorporating the hedging relations is the benchmark portfolio; $H_0 : \Delta = 0$; $*p < 0.05$, $**p < 0.01$.

Although the two tables above show that there is no statistically significant difference between the Sharpe ratios of the portfolios and the benchmark, the volatility of the portfolios does differ significantly in some cases.

6.1.2 Portfolio weights

Following the methodology of Section 5.3, the optimal univariate GARCH models are used to estimate the volatility of the cryptocurrencies out-of-sample. An example of this estimation is presented in Figure 6.1. This figure shows that the predicted volatility is high when the log returns are becoming noisier. Therefore, the GARCH models seem to predict the volatility accurately.

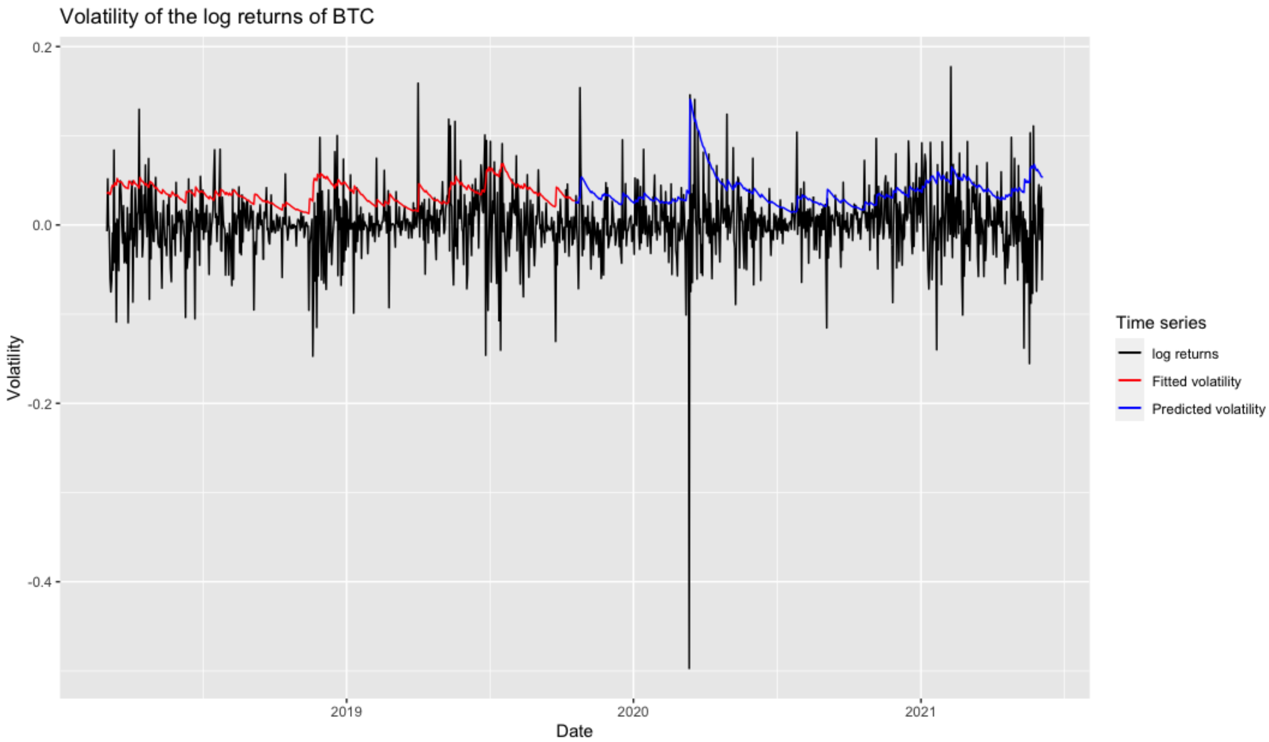

Figure 6.1: The log-returns, fitted volatility and predicted volatility of Bitcoin during the whole sample period.

Combining all the individual volatility forecasts results in the estimated diagonal covariance matrices $\hat{\boldsymbol{\Sigma}}_t$. The same optimal univariate GARCH models are used in step 1 of the out-of-sample estimation of $\hat{\boldsymbol{H}}_t$ using the best fitting DCC-GARCH model. The assigned global minimum variance weights are derived with the help of these estimated covariance matrices and some descriptive statistics of the obtained portfolio weights are presented in Table 6.5. Since $\hat{\boldsymbol{\Sigma}}_t$ is a diagonal covariance matrix, it can only give strictly positive weights. Therefore, it is not necessary to look at the effect of short-selling constraints on the portfolio that results from this allocation.

On the other hand, the global minimum variance weights obtained with $\hat{\boldsymbol{H}}_t$ can take on a negative value. Therefore, the portfolio weights assuming no short-selling are also presented in Table 6.5. It is noteworthy that on some days the optimal portfolio obtained with $\hat{\boldsymbol{H}}_t$ under short-selling restrictions consist only of Bitcoin or Litecoin, according to the maximum values. Comparing the sum of squared weights of the portfolios presented in Table 6.5 to the sum of squared weights of the 1/N portfolio ($\|w_{1/N}\|_2 = 0.143$), I find that $\|w\|_2$ is higher for all portfolios. This implies that the positions in the portfolios are more leveraged and extreme. This holds especially for the unconstrained portfolio formed with hedging relations.

Table 6.5: Descriptive statistics of the global minimum variance weights assigned to a portfolio of cryptocurrencies, calculated with different covariance matrices.

	BTC	ETH	BNB	ADA	DOGE	XRP	LTC
				$\hat{\boldsymbol{\Sigma}}_t$	$\|w\|_2 = 0.199$		
Min	0.060	0.034	0.026	0.037	<0.001	0.006	0.033
Mean	0.272	0.115	0.129	0.101	0.149	0.098	0.135
Max	0.605	0.265	0.334	0.337	0.457	0.243	0.537
				$\hat{\boldsymbol{H}}_t$	$\|w\|_2 = 1.212$		
Min	-0.299	-0.806	-0.214	-0.534	-0.077	-0.266	-0.511
Mean	0.775	-0.220	0.119	-0.013	0.228	0.082	0.030
Max	1.497	0.582	0.893	1.684	0.848	0.882	1.494
			(no shorts)	$\hat{\boldsymbol{H}}_t$	$\|w\|_2 = 0.580$		
Min	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Mean	0.536	0.005	0.080	0.007	0.272	0.030	0.071
Max	1.000	0.472	0.726	0.986	0.861	0.512	1.000

NOTE: The sum of the assigned weights is always equal to 1; imposing short sell restrictions implies that $w_{gmv} \geq 0$; $\|w\|_2$ is the average sum of squared portfolio weights.

For diversification purposes, I also investigate portfolios in which the equity market is included. As mentioned in Section 5.3, it was necessary to alter the data of the log-returns to perform the analysis. The same optimal univariate GARCH-type models are selected for forecasting purposes, but the optimal parameters vary slightly. The new estimated parameters are presented in Appendix A.5 and these GARCH models are used to estimate the new $\hat{\boldsymbol{\Sigma}}_t$ and $\hat{\boldsymbol{H}}_t$.

The results for the optimal weights, obtained with the estimated covariance matrices including the equity market, are presented in Table 6.6. The weights belonging to the equity market show that, in most cases, the largest weight is assigned to the equity market. This is because the stock market is less volatile than the crypto market and therefore a portfolio that consists primarily of stocks will have a lower variance than a portfolio consisting primarily of cryptocurrencies. Again, the positions in the portfolios are extreme compared to the 1/N portfolio.

Table 6.6: Descriptive statistics of the global minimum variance weights assigned to a portfolio of cryptocurrencies and the value-weighted market factor, calculated with different covariance matrices.

	BTC	ETH	BNB	ADA	DOGE	XRP	LTC	Market
				$\hat{\boldsymbol{\Sigma}}_t$	$\|w\|_2 = 0.614$			
Min	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
Mean	0.078	0.037	0.029	0.022	0.059	0.037	0.042	0.697
Max	0.401	0.267	0.158	0.106	0.330	0.168	0.292	1.000
				$\hat{\boldsymbol{H}}_t$	$\|w\|_2 = 0.846$			
Min	-0.193	-0.248	-0.096	-0.369	-0.024	-0.398	-0.096	0.001
Mean	0.159	0.001	-0.005	-0.044	0.068	0.032	0.027	0.762
Max	1.173	1.072	0.124	0.063	0.527	0.465	0.578	1.009
			(no shorts)	$\hat{\boldsymbol{H}}_t$	$\|w\|_2 = 0.784$			
Min	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
Mean	0.097	0.001	0.004	0.000	0.077	0.004	0.019	0.798
Max	0.755	0.064	0.090	0.015	0.591	0.150	0.578	1.000

NOTE: The covariance matrices are estimated using only the log-returns of the weekdays; the sum of the assigned weights is always equal to 1; imposing short sell restrictions implies that $w_{gmv} \geq 0$; $\|w\|_2$ is the average sum of the squared portfolio weights.

To have a closer look at the weights assigned to the equity market, the obtained daily weights are plotted in Figure 6.2. In the period of relatively high volatility in the equity market, as shown in Figure 4.1, the weight assigned to the equity market drops. After stabilizing, the weight increases and again takes on the biggest position in the portfolio. Figure 6.2 also shows that the weight of the equity market that is obtained using the estimated diagonal covariance matrix $\hat{\boldsymbol{\Sigma}}_t$ is almost always lower than the weight assigned to the equity market obtained with the estimated DCC covariance matrix $\hat{\boldsymbol{H}}_t$. This finding is in line with the expectations, as $\hat{\boldsymbol{H}}_t$ also takes the hedging relations between the assets into account. As mentioned in Section 3.1, there exist limited hedging possibilities between cryptocurrencies and other assets. Therefore, one would expect the weight assigned to the more stable equity market to increase when it is not possible to hedge properly.

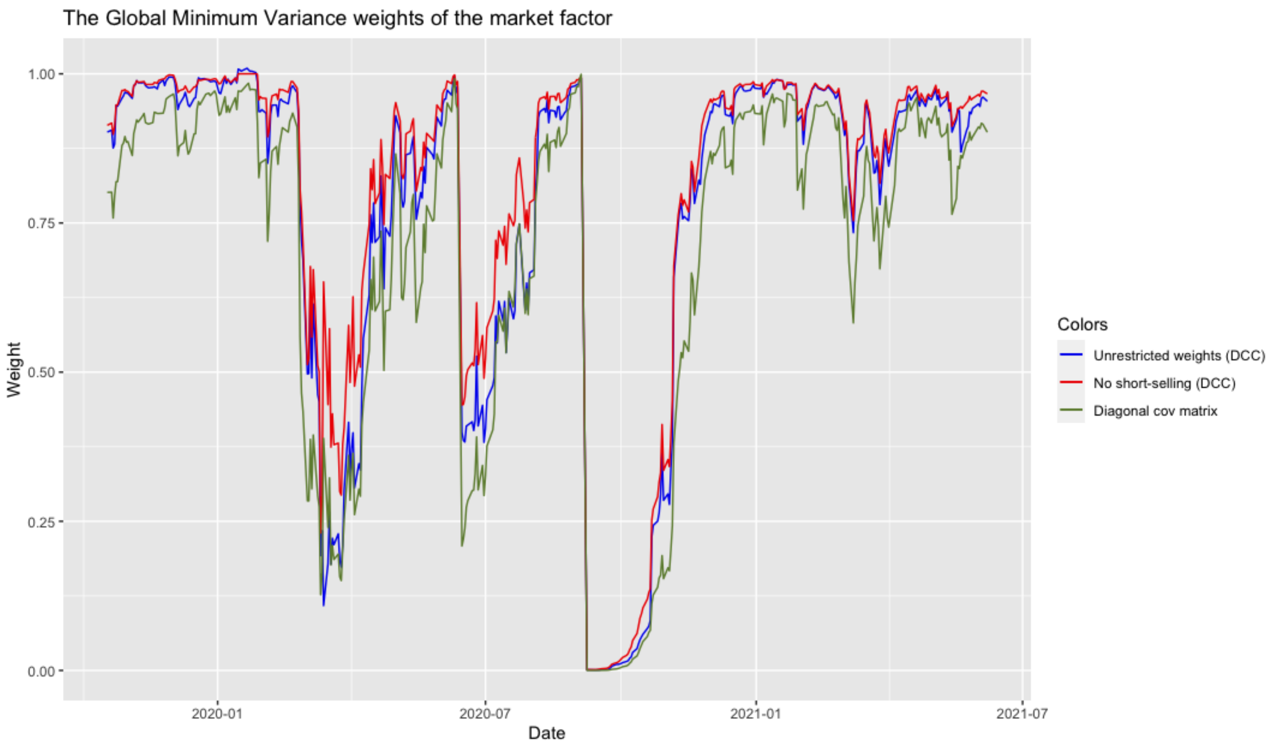

Figure 6.2: The daily weights of the equity market over the out-of-sample period obtained with the different covariance matrices ($\hat{\boldsymbol{\Sigma}}_t$, $\hat{\boldsymbol{H}}_t$ and restricted $\hat{\boldsymbol{H}}_t$).

6.2 Optimal GARCH models

In this section, I will highlight the optimal GARCH models that were used to estimate the necessary covariance matrices. After splitting the dataset into a training- and test set, the in-sample log-returns of the cryptocurrencies and the value-weighted market factor are individually fitted to the sGARCH model with six different error distributions. The fits are assessed based on the AIC and the BIC. Because the AIC is known to sometimes overfit, Table 6.7 only presents the BIC of every model. The full overview of AICs and BICs can be found in Appendix A.3.

Table 6.7 reveals that the best fitting innovation distribution for the in-sample log-

Table 6.7: The BIC for the in-sample fit of the sGARCH model with six different error distributions for each cryptocurrency and the value-weighted market factor

BIC	Norm	Std	GED	sNorm	sStd	sGED
BTC	-3.770	**-4.030**	-4.020	-3.762	-4.022	-4.014
ETH	-3.128	-3.304	**-3.333**	-3.125	-3.294	-3.326
BNB	-3.077	-3.205	**-3.224**	-3.066	-3.195	-3.213
ADA	-2.825	-2.912	**-2.913**	-2.815	-2.901	-2.903
DOGE	-3.294	-3.538	**-3.550**	-3.289	-3.528	-3.543
XRP	-3.163	-3.424	**-3.428**	-3.155	-3.413	-3.419
LTC	-3.041	-3.183	**-3.200**	-3.031	-3.173	-3.189
Market	-6.534	-6.610	-6.584	-6.587	**-6.626**	-6.616

NOTE: This table presents the BIC of the sGARCH model with different error distributions: Normal, Student's t, Generalized Error Distribution and their skewed variants; The lower the BIC, the better the fit; The lowest BIC value per asset is written in bold.

returns of Bitcoin is the regular Student's t distribution. This finding is consistent with Hansen & Lunde (2005), who conclude that, for the IBM stock, the best performing GARCH model is one with a t-distributed error term. However, for all the other cryptocurrencies, the Generalised Error Distribution fits best in-sample. Analyzing the log-returns of the value-weighted market factor, Table 6.7 shows that the skewed Student's t distribution results in the best in-sample fit for the sGARCH model. Another finding that stands out is that for the cryptocurrencies, the skewed distributions in all cases produce a worse fit than the regular distributions. The opposite is true for the market factor: here, skewed distributions yield a better fit than their regular counterparts. Therefore, it can be concluded that the log-returns of cryptocurrencies behave differently from the log-returns of traditional stocks.

Next, these optimal error distributions are utilized to obtain the best fitting GARCH-type model for each asset. I followed the same procedure to derive the in-sample fit for each univariate GARCH-type model. The BICs are presented in Table 6.8 and the rest of the analysis is included in Appendix A.4.

From Table 6.8, it can be concluded that the iGARCH model is the most appropriate model for most cryptocurrencies. Chu et al. (2017) also find the iGARCH model as the best fitting model for Bitcoin. Hansen & Lunde (2005) conclude that, for the IBM stock, the sGARCH is inferior to other models. However, looking at the BIC for the cryptocurrencies, I find that the sGARCH model performs second-best in most cases. For Litecoin, the sGARCH model even fits best. Again, the value-weighted market factor behaves differently from the cryptocurrencies. For this asset, the eGARCH model fits best to the in-sample data. This finding is consistent with the conclusions of Hansen & Lunde (2005), who find results that strongly suggest using a GARCH specification that can accommodate a leverage effect, such as the eGARCH model. Contrary to the expectations, the asymmetric gjrGARCH model does not fit well to any of the assets.

Table 6.8: The BIC for the in-sample fit of the different GARCH-type models with the optimal error distributions for each cryptocurrency and the value-weighted market factor

BIC	sGARCH	iGARCH	eGARCH	gjrGARCH
BTC	-4.030	**-4.041**	-4.038	-4.019
ETH	-3.333	**-3.333**	-3.321	-3.323
BNB	-3.224	**-3.232**	-3.216	-3.213
ADA	-2.913	**-2.920**	-2.908	-2.903
DOGE	-3.550	**-3.558**	-3.543	-3.540
XRP	-3.428	**-3.436**	-3.423	-3.418
LTC	**-3.199**	-3.198	-3.186	-3.188
Market	-6.626	-6.634	**-6.688**	-6.643

NOTE: This table presents the BIC of the different GARCH-type models with their optimal error distributions per asset; The lower the BIC, the better the fit; The lowest BIC value per asset are written in bold.

The optimal GARCH models per asset are found and summed up below. For Bitcoin the iGARCH model with Student's t distributed error terms is selected. The iGARCH model with Generalized Error distributed innovations fits best to the in-sample data of Ethereum, Binance Coin, Cardano, Dogecoin and Ripple, while the sGARCH model with Generalized Error distributed error terms is selected for Litecoin. Looking at the equity market, the eGARCH model with skewed Student's t distributed errors is assumed to be most fitting.

The parameter estimates for the most appropriate GARCH models are presented below in Table 6.9. According to the standard deviations, the constant term ω is insignificant in most of the GARCH models. On the other hand, most of the α_1 estimates are significant on at least a 5% significance level. For most assets, the β_1 parameter is not estimated, but it is possible to conclude that this estimate is not significant for the sGARCH model fitted to Litecoin. Evaluating the estimated coefficients for the value-weighted market factor, it stands out that all the estimates are significant on at least a 5% significance level.

Table 6.9: The estimated parameters of the selected optimal GARCH-type models per asset

Coefficients	BTC	ETH	BNB	ADA	DOGE	XRP	LTC	Market
ω	0.000	0.000	0.000	0.000	0.000*	0.000	0.002*	-0.349**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.008)
α_1	0.076**	0.115*	0.033	0.022	0.338**	0.201**	0.142*	-0.242**
	(0.017)	(0.053)	(0.024)	(0.049)	(0.093)	(0.077)	(0.071)	(0.031)
β_1	0.924	0.885	0.967	0.978	0.662	0.799	0.000	0.964**
	(NA)	(NA)	(NA)	(NA)	(NA)	(NA)	(0.361)	(0.000)
γ	-	-	-	-	-	-	-	0.055*
								(0.026)
$\alpha_1 + \beta_1$	1.000	1.000	1.000	1.000	1.000	1.000	0.143	0.721
Shape	3.117**	0.792**	0.945**	1.112**	0.797**	2.612**	0.981**	10.220*
	(0.214)	(0.057)	(0.072)	(0.111)	(0.053)	(0.170)	(0.076)	(4.662)

NOTE: Standard errors are in parenthesis; in the iGARCH models β_1 is set to $1 - \alpha_1$, so β_1 is not estimated and the standard errors are NA; Shape corresponds to the estimated shape parameter of the innovation distribution; $^*p < 0.05$, $^{**}p < 0.01$.

Multivariate DCC-GARCH model

Now that the optimal univariate GARCH-type models are derived per cryptocurrency, they can be implemented in the multivariate DCC-GARCH model. As illustrated in Section 5.2, the multivariate DCC-GARCH model is estimated in two steps. Step 1 requires the estimation of univariate GARCH models for the individual time series. Therefore, for the first phase of the DCC-GARCH estimation, I will use the best-fit GARCH type models derived above. This implies that the parameters estimated in this step are equal to the parameters presented in Table 6.9. Proceeding with the second step of the estimation, while using two different error distributions, resulted in the estimated parameters presented in Table 6.10.

Based on both the AIC and BIC of the in-sample fit of the two different DCC-GARCH models, the model with Multivariate Student's t-distributed errors is selected as best fitting. All the estimated coefficients are significant on at least a 1% significance level and the restrictions that $a \geq 0$, $b \geq 0$ and $a + b < 1$ are met.

Table 6.10: Results of the in-sample fit of two different DCC-GARCH models to the log-returns of the seven cryptocurrencies

Error distribution	AIC	BIC	LL	a	b	ν
Multivariate Normal	-28.521	-28.035	8565.272	0.035**	0.894**	-
				(0.003)	(0.022)	
Multivariate Student's t	-30.524	-30.030	9163.074	0.031**	0.937**	4.000**
				(0.005)	(0.018)	(0.221)

NOTE: Standard errors are in parenthesis; LL stands for the log-likelihood of the fit; $*p < 0.05$, $**p < 0.01$.

7 Conclusion

Cryptocurrencies can be treated as a new type of asset class with their own characteristics and investment strategies. Since cryptocurrencies have little or no correlation with traditional asset classes, there could be an advantage to diversifying a portfolio with cryptocurrencies. Therefore, analyzing the volatility of cryptocurrencies can be useful for portfolio allocations and risk management.

In this research, I focus on optimizing portfolios that consist of or contain cryptocurrencies. The log-returns of seven cryptocurrencies (Bitcoin, Ethereum, Binance Coin, Cardano, Dogecoin, Ripple and Litecoin) are analyzed and compared to the value-weighted market factor. To obtain the global minimum variance portfolios, I utilize univariate- and multivariate GARCH models. Employing an in-sample/out-of-sample analysis, these GARCH models are used to predict the volatility and conditional covariance matrix of the cryptocurrencies one day ahead.

I conclude that adding hedging relations to the portfolio optimisation leads to a portfolio that performs worse than a portfolio that is only optimally diversified and does not take hedging relations into account. This confirms that there are limited hedging possibilities among cryptocurrencies Canh et al. (2019). Only after introducing short-selling restrictions to the optimisation problem do I find that the portfolio containing hedging relations outperforms the optimally diversified portfolio, since restricting the optimisation results in a positive bias-variance trad-off. Given the Sharpe ratio of a portfolio consisting of only cryptocurrencies, I conclude that it is better to hold an equally-weighted 1/N portfolio than a value-weighted crypto market portfolio consisting solely of Bitcoin. This contrasts with the finding of Canh et al. (2019) that the crypto market lacks diversification opportunities.

Next, I introduced the equity market to the portfolios and compared the obtained Sharpe ratios to the Sharpe ratios of portfolios consisting only of cryptocurrencies. Based on the Sharpe ratios of the different portfolios, I find that adding cryptocurrencies to an equity market portfolio does improve the performance. This confirms that a portfolio will perform better, based on the risk/return profile, when it is a mix of traditional assets and cryptocurrencies (Ankenbrand & Bieri, 2018). Again, imposing short-selling restrictions on the portfolio obtained while taking hedging relations into account, results in a good performance. However, in the case of a mixed portfolio, the optimally diversified portfolio that does not take hedging relations into account performs best. Another important result is the fact that in all cases the Sharpe ratio of a portfolio consisting of only cryptocurrencies is higher than the Sharpe ratio of a portfolio that is a mix of cryptocurrencies and the equity market. Therefore, there exists no diversification premium when adding the equity market to a portfolio of cryptocurrencies. This finding will be discussed in more depth in the next section.

To calculate the Sharpe ratios of the portfolios, I used various covariance matrices obtained with univariate and multivariate GARCH models. I derived the best fitting in-sample univariate GARCH-type model and error distribution per cryptocurrency based on the Bayesian information criterion. For Bitcoin, the iGARCH model with Student's t distributed error terms is selected as optimal. The iGARCH model with Generalized Error distributed innovations fits best to the in-sample data of Ethereum, Binance Coin, Cardano, Dogecoin and Ripple, while the sGARCH model with Generalized Error distributed errors fits best to the log-returns of Litecoin. For the equity market, the eGARCH model with skewed Student's t distributed error terms is found to be most fitting. The parameters of these optimal models are used for the estimation of the out-of-sample volatility of the cryptocurrencies and the creation of daily diagonal covariance matrices.

The multivariate DCC-GARCH model by Engle & Sheppard (2001) was introduced to capture the hedging relations between the cryptocurrencies. The Multivariate Student's t distributed innovation terms were found the fit best to the joint log-returns of the cryptocurrencies. I obtained the parameters of the DCC-GARCH model by fitting the model to the in-sample data. Then, the out-of-sample daily conditional covariance matrices were estimated with the help of the fixed in-sample parameters.

Finally, both the daily diagonal covariance matrices and the daily conditional covariance matrices were used to obtain the global minimum variance weights of the crypto portfolios. I discovered that imposing short-selling restrictions sometimes results in daily portfolios consisting of only Bitcoin or Litecoin. In addition, I conclude that when the equity market is added to the portfolios, this asset takes on the largest weight on most days. This finding is in line with the principles of a global minimum variance allocation, as this allocation seeks to create a portfolio with the lowest possible variance.

8 Discussion

One of the main conclusions of this paper is that portfolios consisting of only cryptocurrencies outperform portfolios that are a mix of cryptocurrencies and the equity market based on the Sharpe ratio. Tables 6.5 and 6.6 show that introducing the equity market to a portfolio of cryptocurrencies results in a shift from Bitcoin to the value-weighted market factor as the largest portfolio holding. Figure 6.2 substantiates the previous by making clear that the weight assigned to the equity market tends to be close to 1 except for three dips. Furthermore, it becomes clear from Figures 6.1 and 6.2 that the out-of-sample Sharpe ratio of Bitcoin is higher (0.061) than the Sharpe ratio of the equity market (0.055). Therefore, the change in dominant weight could be the explanation for the fact that portfolios consisting of only cryptocurrencies outperform portfolios in which the equity market is introduced.

Furthermore, I predicted the volatility of the cryptocurrencies with the help of univariate and multivariate GARCH models. For now, I only focused on four different univariate GARCH models. However, in practice, there are different and more complicated models that one could use for out-of-sample volatility predictions. For example, the regime-switching GARCH models that were briefly introduced in Section 3.2.

Moreover, I selected the best fitting GARCH models based on the Bayesian information criterion (BIC). Another method that can be used for selecting the best fit, is evaluating the prediction error. Previous literature argues that the realized variance, equal to the sum of the squared returns, can be used as a proxy for the ex-post unobserved variance. However, since the data I used are daily returns and not intraday returns, this would lead to a very noisy proxy. Therefore, I preferred the BIC as the measure for the fit instead of taking a proxy for the unobserved variance.

As for the in-sample out-of-sample analysis, I fixed the obtained in-sample parameters when forecasting the volatility out-of-sample. This can be considered a static approach. Another way to perform such an analysis is to dynamically estimate the out-of-sample parameters. This implies that the parameters of the GARCH model are re-estimated every predetermined period using a rolling window. In Figure 8.1, I compare the two methods for predicting the volatility one-day-ahead out-of-sample for Bitcoin. Figure 8.1 shows that the two different methods result in the same volatility forecast for the first seven months. Thereafter, the volatility predicted with the static method is lower than the volatility of the dynamic rolling window method. However, this difference is not substantial.

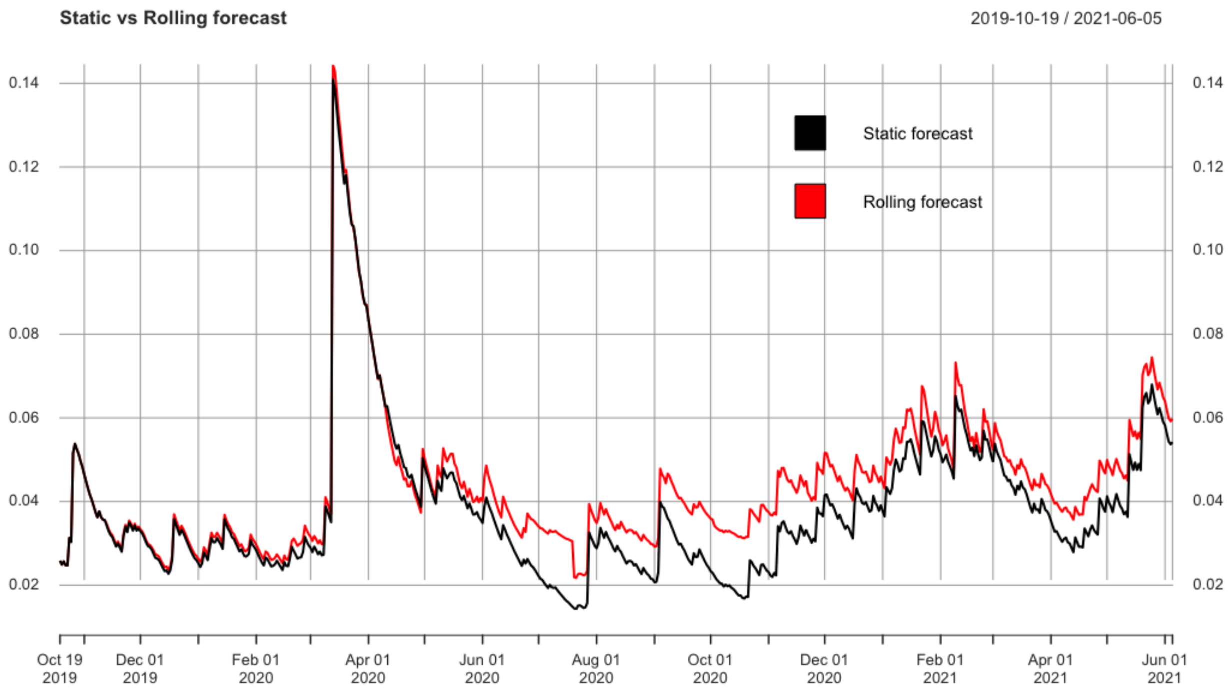

Figure 8.1: Comparison of the static volatility forecast and the dynamic volatility forecast one-day-ahead for Bitcoin. For the dynamic forecast, a 25-day rolling window is used.

For further research, I suggest expanding the asset space and imposing more restrictions on the estimated DCC-GARCH covariance matrices. There are over 2000 cryptocurrencies and they can be used to generate larger covariance matrices. The diagonal covariance matrix that is obtained with the help of the univariate GARCH models can be seen as a matrix that was shrunk extremely, leaving only the diagonal elements non-zero. However, it is also possible to impose shrinkage on the obtained conditional covariance matrices of the DCC-GARCH model. For more information on the shrinkage possibilities, I refer to Ledoit & Wolf (2017).

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A Appendix

A.1 Multivariate Gaussian distributed errors

Considering multivariate Gaussian distributed $\boldsymbol{z}_t$'s, the joint distribution of $z_1, ..., z_t$ is defined as:

$$f(\boldsymbol{z}_t) = \prod_{t=1}^{T} \frac{1}{(2\pi)^{\frac{n}{2}}} e^{-\frac{1}{2}\boldsymbol{z}_t'\boldsymbol{z}_t}$$

since E[$\boldsymbol{z}_t$]= 0 and E[$\boldsymbol{z}_t\boldsymbol{z}_t'$] = I. Here n equals the number of univariate GARCH models, one for every cryptocurrency, estimated and $t = 1, ..., T$ is the time period. Following Engle & Sheppard (2001) and taking the logarithm, the log-likelihood function for $\boldsymbol{\alpha}_t = \boldsymbol{H}_t^{1/2}\boldsymbol{z}_t$ becomes:

$$l(\boldsymbol{\theta})) = -\frac{1}{2}\sum_{t=1}^{T}\left(n\ln(2\pi) + \ln(|\boldsymbol{H}_t|) + \boldsymbol{\alpha}_t'\boldsymbol{H}_t^{-1}\boldsymbol{\alpha}_t\right) \tag{15}$$

where $\boldsymbol{\theta}$ holds all the parameters of the model. These parameters can be split into two groups: $(\boldsymbol{\phi}, \boldsymbol{\psi}) = (\phi_1, ..., \phi_n, \boldsymbol{\psi})$, where ϕ_i are the parameters for the univariate GARCH model for the i^{th} cryptocurrency (see Section 5.1) and $\boldsymbol{\psi} = (a, b)$ are the parameters of $\boldsymbol{Q}_t$ in Equation 9. By substituting $\boldsymbol{H}_t = \boldsymbol{D}_t\boldsymbol{R}_t\boldsymbol{D}_t$, the log-likelihood can be rewritten as:

$$\begin{aligned}
l(\boldsymbol{\theta})) &= -\frac{1}{2}\sum_{t=1}^{T}\left(n\ln(2\pi) + \ln(|\boldsymbol{D}_t\boldsymbol{R}_t\boldsymbol{D}_t|) + \boldsymbol{\alpha}_t'\boldsymbol{D}_t^{-1}\boldsymbol{R}_t^{-1}\boldsymbol{D}_t^{-1}\boldsymbol{\alpha}_t\right) \\
&= -\frac{1}{2}\sum_{t=1}^{T}\left(n\ln(2\pi) + 2\ln(|\boldsymbol{D}_t|) + \ln(|\boldsymbol{R}_t|) + \boldsymbol{\alpha}_t'\boldsymbol{D}_t^{-1}\boldsymbol{R}_t^{-1}\boldsymbol{D}_t^{-1}\boldsymbol{\alpha}_t\right)
\end{aligned}$$

The DCC-GARCH model is estimated in two stages: in the first step univariate GARCH models are fitted to the individual time series to obtain the residual series, then in the second step, the parameters of the dynamic correlation matrix $\boldsymbol{R}_t$ are estimated.

Model estimation: step one

In the first step, $\boldsymbol{R}_t$ is replaced by the identity matrix I_n, resulting in the quasi-likelihood function:

$$
\begin{aligned}
l_1(\boldsymbol{\phi}|\boldsymbol{r_t})) &= -\frac{1}{2}\sum_{t=1}^{T}\left(n\ln(2\pi) + 2\ln(|\boldsymbol{D}_t|) + \ln(|I_n|) + \boldsymbol{\alpha}_t'\boldsymbol{D}_t^{-1}I_n\boldsymbol{D}_t^{-1}\boldsymbol{\alpha}_t\right) \\
&= -\frac{1}{2}\sum_{t=1}^{T}\left(n\ln(2\pi) + 2\ln(|\boldsymbol{D}_t|) + \boldsymbol{\alpha}_t'\boldsymbol{D}_t^{-2}\boldsymbol{\alpha}_t\right) \\
&= -\frac{1}{2}\sum_{t=1}^{T}\left(n\ln(2\pi) + \sum_{i=1}^{n}\left(\ln(h_{it}) + \frac{r_{it}^2}{h_{it}}\right)\right) \\
&= -\frac{1}{2}\sum_{i=1}^{n}\left(T\ln(2\pi) + \sum_{t=1}^{T}\left(\ln(h_{it}) + \frac{r_{it}^2}{h_{it}}\right)\right)
\end{aligned} \tag{16}
$$

which can be recognized as the sum of the log-likelihoods of individual GARCH models for the cryptocurrencies (Engle & Sheppard, 2001). Therefore, the parameter set $\boldsymbol{\phi} = \phi_1, ..., \phi_n$ is estimated in this step. While estimating $\boldsymbol{\phi}$, the conditional variance h_{it} is also estimated for each cryptocurrency. After obtaining h_{it}, it is possible to estimate $\boldsymbol{\epsilon_t} = \boldsymbol{D}_t^{-1}\boldsymbol{\alpha}_t$ and $\overline{\boldsymbol{Q}} = E[\boldsymbol{\epsilon_t}\boldsymbol{\epsilon_t}']$ as discussed in the previous section.

Model estimation: step two

In step two, the remaining unknown parameters $\boldsymbol{\psi} = (a, b)$ will be estimated. Using the log-likelihood function the second stage is estimated conditioning on the parameters estimated in the first stage:

$$
\begin{aligned}
l_2(\boldsymbol{\psi}|\hat{\boldsymbol{\phi}}, \boldsymbol{r_t})) &= -\frac{1}{2}\sum_{t=1}^{T}\left(n\ln(2\pi) + 2\ln(|\boldsymbol{D}_t|) + \ln(|\boldsymbol{R}_t|) + \boldsymbol{\alpha}_t'\boldsymbol{D}_t^{-1}\boldsymbol{R}_t^{-1}\boldsymbol{D}_t^{-1}\boldsymbol{\alpha}_t\right) \\
&= -\frac{1}{2}\sum_{t=1}^{T}\left(n\ln(2\pi) + 2\ln(|\boldsymbol{D}_t|) + \ln(|\boldsymbol{R}_t|) + \boldsymbol{\epsilon}_t'\boldsymbol{R}_t^{-1}\boldsymbol{\epsilon}_t\right)
\end{aligned} \tag{17}
$$

in which the first two terms are constant conditional on $\hat{\boldsymbol{\phi}}$. Therefore, this DCC-GARCH model is solved by maximizing:

$$
l_2^*(\boldsymbol{\psi}|\hat{\boldsymbol{\phi}}, \boldsymbol{r_t})) = -\frac{1}{2}\sum_{t=1}^{T}\left(\ln(|\boldsymbol{R}_t|) + \boldsymbol{\epsilon}_t'\boldsymbol{R}_t^{-1}\boldsymbol{\epsilon}_t\right)
$$

A.2 Multivariate Student's t-distributed errors in DCC-GARCH model

Considering multivariate Student's t-distributed $\boldsymbol{z}_t$'s, the joint density of $z_1, ..., z_t$ can be specified as:

$$f(\boldsymbol{z}_t|\nu) = \prod_{t=1}^{T} \frac{\Gamma(\frac{\nu+n}{2})}{\Gamma(\frac{\nu}{2})(\pi(\nu-2))^{n/2}} \left[1 + \frac{\boldsymbol{z}_t'\boldsymbol{z}_t}{\nu-2}\right]^{-\frac{n+\nu}{2}}$$

where $\Gamma(\cdot)$ is the Gamma function. The likelihood function for $\boldsymbol{\alpha}_t = \boldsymbol{H}_t^{1/2}\boldsymbol{z}_t$ then becomes:

$$L(\boldsymbol{\theta})) = f(\boldsymbol{z}_t|\nu) = \prod_{t=1}^{T} \frac{\Gamma(\frac{\nu+n}{2})}{\Gamma(\frac{\nu}{2})(\pi(\nu-2))^{n/2}|\boldsymbol{H}_t|^{1/2}} \left[1 + \frac{\boldsymbol{\alpha}_t'\boldsymbol{H}_t^{-1}\boldsymbol{\alpha}_t}{\nu-2}\right]^{-\frac{n+\nu}{2}}$$

In this step, the rule for the linear transformation of variables, that Engle & Sheppard (2001) also used for the multivariate Gaussian distributed errors is utilized.

The log-likelihood function results from taking to logarithm and substituting $\boldsymbol{H}_t = \boldsymbol{D}_t\boldsymbol{R}_t\boldsymbol{D}_t$:

$$\begin{aligned} l(\boldsymbol{\theta}) = \sum_{t=1}^{T} \Bigg(&\ln\left[\Gamma(\frac{\nu+n}{2})\right] - \ln\left[\Gamma(\frac{\nu}{2})\right] - \frac{n}{2}\ln\left[\pi(\nu-2))\right] - \frac{1}{2}\ln\left[|\boldsymbol{D}_t\boldsymbol{R}_t\boldsymbol{D}_t|\right] \\ &- \frac{\nu+n}{2}\ln\left[1 + \frac{\boldsymbol{\alpha}_t'\boldsymbol{D}_t^{-1}\boldsymbol{R}_t^{-1}\boldsymbol{D}_t^{-1}\boldsymbol{\alpha}_t}{\nu-2}\right]\Bigg) \end{aligned} \tag{18}$$

Again, $\boldsymbol{\theta}$ is divided into two groups: $(\boldsymbol{\phi}, \boldsymbol{\psi}) = (\phi_1, ..., \phi_n, \boldsymbol{\psi})$, where ϕ_i are the parameters of the univariate GARCH model for the i^{th} cryptocurrency (see Section 5.1) and $\boldsymbol{\psi} = (a, b, \nu)$. Similar to the model solution using multivariate Gaussian distributed errors, the optimisation of the log-likelihood function above is obtained in two steps.

Model estimation: step one

In the first step of estimating the DCC-GARCH model with Student's t-distributed standardized errors, the parameters of the univariate GARCH models $\boldsymbol{\phi}$ are fitted using pseudo-maximum likelihood. This implies that in this step it is assumed that the errors are Gaussian distributed. Therefore, the first step results in the same quasi-likelihood function as the first step with the multivariate Gaussian distributed $\boldsymbol{z}_t$s. Thus, the remaining unknown parameters after step one are a, b and ν. These will be estimated in step two.

Model estimation: step two

Using the parameters estimated in step one and the correctly specified log-likelihood function of Equation 18, the second stage quasi-likelihood becomes:

$$\begin{aligned} l_2(\boldsymbol{\psi}|\hat{\boldsymbol{\phi}}, \boldsymbol{r_t})) &= \sum_{t=1}^{T} \Bigg(\ln \left[\Gamma(\frac{\nu+n}{2}) \right] - \ln \left[\Gamma(\frac{\nu}{2}) \right] - \frac{n}{2} \ln \left[\pi(\nu-2) \right] \\ &- \frac{1}{2} \ln \left[|\boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t| \right] - \frac{\nu+n}{2} \ln \left[1 + \frac{\boldsymbol{\alpha}_t' \boldsymbol{D}_t^{-1} \boldsymbol{R}_t^{-1} \boldsymbol{D}_t^{-1} \boldsymbol{\alpha}_t}{\nu-2} \right] \Bigg) \\ &= \sum_{t=1}^{T} \Bigg(\ln \left[\Gamma(\frac{\nu+n}{2}) \right] - \ln \left[\Gamma(\frac{\nu}{2}) \right] - \frac{n}{2} \ln \left[\pi(\nu-2) \right] - \frac{1}{2} \ln \left[|\boldsymbol{R}_t| \right] \\ &- \ln \left[|\boldsymbol{D}_t| \right] - \frac{\nu+n}{2} \ln \left[1 + \frac{\boldsymbol{\epsilon}_t' \boldsymbol{R}_t^{-1} \boldsymbol{\epsilon}_t}{\nu-2} \right] \Bigg) \end{aligned} \tag{19}$$

where $\boldsymbol{D}_t$ is a constant conditional on the parameters from step one. Therefore, the solution of the DCC-GARCH model using multivariate Student's t-distributed errors is obtained by maximizing:

$$\begin{aligned} l_2^*(\boldsymbol{\psi}|\hat{\boldsymbol{\phi}}, \boldsymbol{r_t})) &= \sum_{t=1}^{T} \Bigg(\ln \left[\Gamma(\frac{\nu+n}{2}) \right] - \ln \left[\Gamma(\frac{\nu}{2}) \right] - \frac{n}{2} \ln \left[\pi(\nu-2) \right] - \frac{1}{2} \ln \left[|\boldsymbol{R}_t| \right] \\ &- \frac{\nu+n}{2} \ln \left[1 + \frac{\boldsymbol{\epsilon}_t' \boldsymbol{R}_t^{-1} \boldsymbol{\epsilon}_t}{\nu-2} \right] \Bigg) \end{aligned}$$

A.3 Best fitting innovation distribution

Table A.1: The BIC for the in-sample fit of the sGARCH model with six different innovation terms for each cryptocurrency and the value-weighted market factor

	Norm	Std	GED	sNorm	sStd	sGED
			BTC			
AIC	-3.814	**-4.081**	-4.072	-3.814	-4.081	-4.073
BIC	-3.770	**-4.030**	-4.020	-3.762	-4.022	-4.014
Log-likelihood	1142.547	1223.269	1220.419	1143.482	1224.110	1221.623
			ETH			
AIC	-3.172	-3.356	-3.384	-3.177	-3.353	**-3.385**
BIC	-3.128	-3.304	**-3.333**	-3.125	-3.294	-3.326
Log-likelihood	951.303	1007.002	1015.521	953.623	1007.181	1016.593
			BNB			
AIC	-3.121	-3.257	**-3.275**	-3.118	-3.254	-3.272
BIC	-3.077	-3.205	**-3.224**	-3.066	-3.195	-3.213
log-likelihood	936.115	977.541	982.987	936.133	977.550	982.995
			ADA			
AIC	-2.870	-2.963	**-2.965**	-2.866	-2.960	-2.961
BIC	-2.825	-2.912	**-2.913**	-2.815	-2.901	-2.903
Log-likelihood	861.101	889.993	890.487	861.124	890.019	890.501
			DOGE			
AIC	-3.338	-3.589	-3.601	-3.341	-3.589	**-3.602**
BIC	-3.294	-3.538	**-3.550**	-3.289	-3.528	-3.543
Log-likelihood	1000.668	1076.600	1080.169	1002.470	1076.939	1081.343
			XRP			
AIC	-3.207	-3.475	**-3.480**	-3.206	-3.472	-3.477
BIC	-3.163	-3.424	**-3.428**	-3.155	-3.413	-3.419
Log-likelihood	961.781	1042.664	1044.040	962.396	1042.715	1044.268
			LTC			
AIC	-3.085	-3.235	**-3.251**	-3.083	-3.232	-3.248
BIC	-3.041	-3.183	**-3.200**	-3.031	-3.173	-3.189
Log-likelihood	925.262	970.968	975.666	925.578	971.088	975.786
			Market			
AIC	-6.593	-6.679	-6.653	-6.656	-6.704	**-6.695**
BIC	-6.534	-6.610	-6.584	-6.587	**-6.626**	-6.616
Log-likelihood	1357.604	1376.181	1370.826	1371.423	1382.340	1380.390

NOTE: This table presents the AIC, BIC and log-likelihood of the sGARCH model with different innovation terms: Normal, Student's t, Generalized Error Distribution and their skewed variants. The lower the BIC, the better the fit. The lowest values for the BIC per asset are written in bold.

A.4 Best fitting GARCH-type model

Table A.2: The BIC for the in-sample fit of the different GARCH-type models with the optimal error distributions for each cryptocurrency and the value-weighted market factor

	sGARCH	iGARCH	eGARCH	gjrGARCH
		BTC		
AIC	-4.081	-4.085	**-4.096**	-4.078
BIC	-4.0300	**-4.041**	-4.038	-4.019
Log-likelihood	1223.269	1223.417	1228.739	1223.356
		ETH		
AIC	**-3.384**	-3.378	-3.380	-3.381
BIC	-3.333	**-3.333**	-3.321	-3.323
Log-likelihood	1015.521	1012.505	1015.335	1015.652
		BNB		
AIC	-3.275	**-3.28**	-3.275	-3.272
BIC	-3.224	**-3.232**	-3.216	-3.213
Log-likelihood	982.987	982.262	983.985	983.036
		ADA		
AIC	**-2.965**	-2.964	-2.967	-2.961
BIC	-2.913	**-2.920**	-2.908	-2.903
Log-likelihood	890.487	889.220	892.061	890.496
		DOGE		
AIC	-3.601	**-3.602**	-3.602	-3.598
BIC	-3.550	**-3.558**	-3.543	-3.540
Log-likelihood	1080.169	1079.438	1081.42	1080.333
		XRP		
AIC	-3.480	-3.480	**-3.482**	-3.477
BIC	-3.428	**-3.436**	-3.423	-3.418
Log-likelihood	1044.040	1042.968	1045.478	1044.001
		LTC		
AIC	**-3.251**	-3.2418	-3.245	-3.247
BIC	**-3.200**	-3.198	-3.186	-3.188
Log-likelihood	975.666	972.015	974.928	975.534
		Market		
AIC	-6.704	-6.702	**-6.776**	-6.731
BIC	-6.626	-6.634	**-6.688**	-6.643
Log-likelihood	1382.340	1380.934	1398.034	1388.867

NOTE: This table presents the AIC, BIC and log-likelihood of the sGARCH model with different innovation terms: Normal, Student's t, Generalized Error Distribution and their skewed variants. The lower the BIC, the better the fit. The lowest values for the BIC per asset are written in bold.

A.5 GARCH models fitted to log-return data excluding weekends

Table A.3: The estimated parameters of the selected optimal GARCH-type models per asset while taking into account only the weekdays

Coefficients	BTC	ETH	BNB	ADA	DOGE	XRP	LTC	Market
ω	0.000	0.000	0.000	0.000	0.000*	0.001	0.000	-0.349**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.008)
α_1	0.120**	0.000	0.041	0.015	0.534**	0.482*	0.000	-0.242**
	(0.029)	(0.002)	(0.049)	(0.406)	(0.140)	(0.213)	(0.000)	(0.031)
β_1	0.880	1.000	0.960	0.985	0.466	0.518	0.999**	0.964**
	(NA)	(NA)	(NA)	(NA)	(NA)	(NA)	(0.000)	(0.000)
γ	-	-	-	-	-	-	-	0.055*
								(0.026)
$\alpha_1 + \beta_1$	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.721
Shape	3.008**	0.870**	1.010**	0.988**	0.838**	0.777**	0.994**	10.220*
	(0.248)	(0.065)	(0.101)	(0.611)	(0.070)	(0.065)	(0.078)	(4.662)

NOTE: Standard errors are in parenthesis; in the iGARCH models β_1 is set to $1 - \alpha_1$, so β_1 is not estimated and the standard errors are NA; Shape corresponds to the estimated shape parameter of the innovation distribution; $^*p < 0.05$, $^{**}p < 0.01$.

A.6 The HAC inference test by Ledoit & Wolf (2008) for testing Sharpe ratios

Take two daily excess portfolio returns $r_{t,i}$ and $r_{t,j}$ for $t = 1, ..., T$. Furthermore, assume that $r_{t,i}$ and $r_{t,j}$ are strictly stationary processes with

$$\mu = \begin{bmatrix} \mu_i \\ \mu_j \end{bmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} \sigma_i^2 & \sigma_{i,j} \\ \sigma_{i,j} & \sigma_j^2 \end{bmatrix}$$

Then the difference in Sharpe ratio is given by:

$$\Delta = \frac{\mu_i}{\sigma_i} - \frac{\mu_j}{\sigma_j},$$

and the estimator using sample moments is denoted by $\hat{\Delta}$.

Next, denote $E[r_{1,i}^2] = \gamma_i$ and $E[r_{1,j}^2] = \gamma_j$, with estimates $\hat{\gamma}_i$ and $\hat{\gamma}_j$, and $\nu = (\mu_i, \mu_j, \gamma_i, \gamma_j)'$ with estimate $\hat{\nu}$. This allows us to write $\Delta = f(\nu)$ and $\hat{\Delta} = f(\hat{\nu})$ with

$$f(a, b, c, d) = \frac{a}{\sqrt{c - a^2}} - \frac{b}{\sqrt{d - b^2}}$$

We assume that $\sqrt{T}(\hat{\nu} - \nu) \xrightarrow{d} \text{N}(0, \mathbf{\Psi})$, where $\mathbf{\Psi}$ is an unknown symmetric positive semi-definite matrix. Using the Delta method we further find:

$$\sqrt{T}(\hat{\Delta} - \Delta) \xrightarrow{d} N(0, \nabla' f(\nu) \mathbf{\Psi} \nabla f(\nu))$$

with $\nabla' f(a, b, c, d) = \left(\frac{c}{(c-a^2)^{1.5}}, -\frac{d}{(d-b^2)^{1.5}}, -\frac{1}{2}\frac{a}{(c-a^2)^{1.5}}, \frac{1}{2}\frac{b}{(d-b^2)^{1.5}} \right)$.

Now, if there is a consistent estimator $\hat{\mathbf{\Psi}}$ available, the standard error for $\hat{\Delta}$ is given by

$$s(\hat{\Delta}) = \sqrt{\frac{\nabla' f(\hat{\nu}) \hat{\mathbf{\Psi}} \nabla f(\hat{\nu})}{T}}$$

The limiting covariance matrix of $\mathbf{\Psi}$ is given by:

$$\mathbf{\Psi} = \lim_{T \to \infty} \frac{1}{T} \sum_{s=1}^{T} \sum_{t=1}^{T} E[y_s y_t'] \quad \text{with} \quad y_t' = (r_{t,i} - \mu_i, r_{t,j} - \mu_j, r_{t,i}^2 - \gamma_i, r_{t,j}^2 - \gamma_j)$$

By change of variable, the limit can be alternatively expressed as:

$$\mathbf{\Psi} = \lim_{T \to \infty} \mathbf{\Psi_T} \quad \text{with} \quad \mathbf{\Psi_T} = \sum_{m=-T+1}^{T+1} \Gamma_T(m), \quad \text{where}$$

$$\Gamma_T(m) = \begin{cases} \frac{1}{T} \sum_{t=m+1}^{T} E[y_t y_{t-m}'], & \text{for } m \geq 0 \\ \frac{1}{T} \sum_{t=-m+1}^{T} E[y_{t+m} y_t'], & \text{for } m < 0 \end{cases}$$

To come up with a consistent estimator we should use heteroskedasticity and autocorrelation robust (HAC) kernel estimation. Using a kernel function $k(\cdot)$ with a bandwidth S_T, the kernel estimate for $\mathbf{\Psi}$ is given by:

$$\hat{\mathbf{\Psi}} = \hat{\mathbf{\Psi_T}} = \frac{T}{T-4} \sum_{m=-T+1}^{T-1} k\left(\frac{m}{S_T}\right) \hat{\Gamma}_T(m)$$

Given a kernel $k(\cdot)$ with bandwidth S_T, the standard error $s(\hat{\Delta})$ is obtained. Finally, a two-sided p-value for the null hypothesis $H_0 : \Delta = 0$ is given by:

$$\hat{p} = 2\Phi\left(- \frac{|\hat{\Delta}|}{s(\hat{\Delta})} \right)$$

A.7 The HAC inference test by Ledoit & Wolf (2011) for testing variances

Again, take two daily excess portfolio returns $r_{t,i}$ and $r_{t,j}$ for $t = 1, ..., T$. Furthermore, assume that $r_{t,i}$ and $r_{t,j}$ are strictly stationary processes with

$$\mu = \begin{bmatrix} \mu_i \\ \mu_j \end{bmatrix} \quad \text{and} \quad \Sigma = \begin{bmatrix} \sigma_i^2 & \sigma_{i,j} \\ \sigma_{i,j} & \sigma_j^2 \end{bmatrix}$$

The ratio of the two variances is given by

$$\Theta = \frac{\sigma_i^2}{\sigma_j^2}$$

Now, define $\Delta = \log \Theta = \log \sigma_i^2 - \log \sigma_j^2$ and $H_0 : \Delta = 0$.
Using the same notation as in Appendix A.6, we find $\Delta = f(\nu)$ and $\hat{\Delta} = f(\hat{\nu})$ with

$$f(a, b, c, d) = \log(c - a^2) - \log(d - b^2)$$

Again, similar to Appendix A.6, we have $\sqrt{T}(\hat{\nu} - \nu) \xrightarrow{d} \text{N}(0, \mathbf{\Psi})$ and the Delta method implies:

$$\sqrt{T}(\hat{\Delta} - \Delta) \xrightarrow{d} N(0, \nabla' f(\nu) \mathbf{\Psi} \nabla f(\nu))$$

with

$$\nabla' f(a, b, c, d) = \left(-\frac{2a}{c - a^2}, \frac{2b}{d - b^2}, \frac{1}{c - a^2}, -\frac{1}{d - b^2} \right)$$
$$s(\hat{\Delta}) = \sqrt{\frac{\nabla' f(\hat{\nu}) \hat{\mathbf{\Psi}} \nabla f(\hat{\nu})}{T}}$$

.

Following the same HAC inference steps as in the test for Sharpe ratios, the two-sided p-value for the null hypothesis $H_0 : \Delta = 0$ is:

$$\hat{p} = 2\Phi\left(-\frac{|\hat{\Delta}|}{s(\hat{\Delta})} \right)$$