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A Bayesian Structural Time Series Approach in Exploring the Main Drivers of Bitcoin's Market Price

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Abstract

In this paper I examine the main drivers of Bitcoin's market price and to what extent they have any predictive power. For this purpose I use Bayesian Structural Time Series models and the Time-Varying Parameter model. I also use the Spike-and-slab prior, which is a shrinkage method, to select the most important predictor variables. I consider a set of 15 explanatory variables with daily data over the period 1 May 2013 until 20 May 2020. I find important relationships between Bitcoin's market price and explanatory variables, such as the S&P500, gold, the VIX index and others. The results show that Bitcoin has its own characteristics as asset class, which makes Bitcoin interesting for diversification purposes. I also conclude that I can increase the prediction power of the models in predicting Bitcoin's market price by including the explanatory variables. The best model correctly predicts the direction of Bitcoin's market price in at least 65% and up to 70% of the cases for all forecast horizons.

Keywords: Bayesian Structural Time Series, Bitcoin, Cryptocurrency, Forecasting, Spike-and-slab prior, Time-varying parameter model

Contents

1 Introduction

The last couple of years we face a new asset class. Next to stocks, bonds, commodities and so on, we are able to invest in cryptocurrencies. According to the European Central Bank (ECB), we can define this new asset class in the following way: *"A digital representation of value, not issued by a central bank, credit institution or e-money institution, which in some circumstances can be used as an alternative to money"*[1]. As of February 2022, there exist more than 17,000 cryptocurrencies with a total market capitalization of approximately \$1.79T[2]. One main characteristic of cryptocurrencies is that it is not part of a centralized financial system. Another characteristic is that the price of many cryptocurrencies has a high volatility, which is consistent with the opinion of people who argue that cryptocurrencies are only speculative assets. Nowadays there are many different digital currencies with their own characteristics, for example Bitcoin, Ethereum, Ripple and Litecoin. The most popular one in terms of volume and liquidity is Bitcoin, which is also considered as the first decentralized cryptocurrency.

Many people became aware of the investment opportunity in cryptocurrencies in the fall of 2017, when the price of several cryptocurrencies increased rapidly. However, there is still much debate whether cryptocurrencies have a fundamental value or not and to what extent the prices are predictable. For example, stocks clearly have a fundamental value based on the finances of the specific companies and exchange rates are mainly influenced by the monetary policy of specific countries. However, this does not hold for cryptocurrencies, so their market prices are primarily determined by supply and demand. The big question is whether there exist variables which capture this supply and demand side, and to what extent they have any predictive power. Due to the relative short existence of cryptocurrencies there is not much literature available about the main drivers and the predictability of cryptocurrency price levels. Hence, the main goal of my research is to examine the main drivers of a cryptocurrency's market price. In my research I only focus on the Bitcoin's market price. For this purpose I use a Bayesian Structural Time Series (BSTS) model, which is based on the methodology of Scott & Varian (2014). By using this model I create several layers, such as trends, seasonality and explanatory variables which can vary stochastically over time. This is particularly useful to apply on Bitcoin's market price, because generally the price is very volatile and it exhibits much time-variation. Due to this time-variation the impact of the explanatory variables can change over time. The BSTS model easily handles the time-varying influence of the explanatory variables. Besides, another advantage of the BSTS model compared to ARIMA models is that it does not require the time series to be stationary,

[1] Source ECB definition cryptocurrency: https://www.ecb.europa.eu/pub/pdf/other/virtualcurrencyschemesen.pdf

[2] Source: https://coinmarketcap.com/

because the BSTS model can handle structural changes in the time series. This is particularly useful in my research, because I examine time series data of Bitcoin's market price which is non-stationary, as indicated by Mudassir et al. (2020). Furthermore, it is possible to select the most important predictor variables by using the Spike-and-slab prior, which is a shrinkage method with a standard and clear interpretation. The advantage of using a shrinkage method is that it reduces over-fitting problems and it improves the forecast performance of the model. Another advantage of the BSTS model is that it is possible to set empirical priors on the parameters in a fully Bayesian treatment. In this way I can examine the dynamic behavior and relation between the Bitcoin's market price and a set of explanatory variables. Next to the BSTS model I also apply a time-varying parameter (TVP) model, based on the methodology of Fruhwirth-Schnatter & Wagner (2010) and Belmonte et al. (2014). This model contains both static and dynamic regression components. In this way I examine both the static and dynamic relationship between the explanatory variables and Bitcoin's market price. Subsequently, I evaluate the out-of-sample prediction performance of several versions of the BSTS model and the TVP model to examine the predictive power of the explanatory variables in predicting Bitcoin's market price.

Due to the relatively short existence of cryptocurrencies, the lack of literature and much uncertainty among investors, my research is relevant and interesting both for scientists and practical applications. The research gives new insights in the dynamic behavior and relation between the Bitcoin's market price and a set of explanatory variables by using the BSTS model. This model can easily incorporate the time-varying nature of Bitcoin's market price. Besides, the BSTS model has a clear interpretable structure in both observable and unobservable dynamic components, which is a big advantage compared to ARIMA and Machine Learning models. Furthermore, I examine the predictive power of the explanatory variables in predicting Bitcoin's market price, which can be a useful contribution to the general debate to what extent Bitcoin's market price is predictable. Perhaps it is possible by using a BSTS model to outperform ARIMA and Machine Learning models in predicting Bitcoin's market price, due to the flexible way how the BSTS model incorporates time-variation and the clear interpretable structure. These findings can also give answers on problems related to portfolio management and/or risk management, which can be interesting for investors and the general public who want to invest in the cryptocurrency market.

In the last couple of years some literature has found drivers and predictability of Bitcoin's market price, such as Chen et al. (2020) in which the authors use machine learning techniques to predict the price of Bitcoin at different frequencies. They use a set of high-dimension features including property and network, trading and market, attention and gold spot price to predict Bitcoin's daily price. Besides, they use basic trading features for 5-minute interval price prediction. For the daily price prediction they achieve an accuracy of 66% by using statistical methods, such as Logistic Regression and Linear Discriminant Analysis, where they outperform the machine learning algorithms. However, for the 5-minute interval price prediction they find an accuracy of 67.2% by using machine learning models, such as Random Forest, XGBoost, Quadratic Discriminant Analysis, Support Vector Machine and Long Short-term Memory, where they outperform the statistical methods.

Another article about the predictability of Bitcoin's market price by using machine learning techniques is Mangla et al. (2019). The authors use a dataset which consists of the price of Bitcoin sampled at approximately one-hour intervals between October 10, 2015 and March 01, 2019. They use four methods to predict the direction of change in Bitcoin's market price, namely Logistic Regression, Support Vector Machine, ARIMA and Recurrent Neural Networks (RNN). They find that among the four methods the ARIMA model performs best with an accuracy of 53%, especially for short-term predictions like the next day the model performs well. However, the ARIMA model performs poorly for price predictions for the next 5-7 days. The RNN model has an accuracy of 50% and performs consistently for the next 6 days.

In Othman et al. (2020) the authors predict the Bitcoin price trend based on its symmetric volatility structure by using the Rapid-Miner program based on the Artificial Neural Network (ANN) algorithm. The symmetric volatility structure can be measured through four input attributes, such as the open price, high price, low price and close price. They find that the ANN model is effective for correctly predicting Bitcoin's market price with an accuracy of 92.15%. The low price is the major promoter for the Bitcoin price trend with a percentage of 63%, followed by close price, high price and open price with percentages of 49%, 46% and 37%, respectively.

There is also literature available about the predictability of Bitcoin returns, such as Huang et al. (2019) in which the authors use a large set of Bitcoin price-based technical indicators. They find that their model, a classification tree-based model using 124 technical indicators, has strong out-of-sample predictive power for narrow ranges of daily Bitcoin returns.

The application of the BSTS model in predicting Bitcoin's market price has been done once, namely in the literature of Poyser (2019). The main findings of the author are that that the Bitcoin's price is negatively associated with the price of gold and the exchange rate between Chinese Yuan and United States Dollar, while positively correlated to the stock market index S&P500, United States Dollar to Euro exchange rate and diverse signs among the different countries' search trends. Generally, the author finds that the Bitcoin's price has the highest correlation with the attractiveness in certain countries, in particular the Unites States, Brazil and Russia. Furthermore, the exchange rates have more impact on Bitcoin's price than gold and the S&P500. The author does not find a relevant relation between Bitcoin's price and internal variables, such as the hashrate (the speed at which a computer is completing an operation in the Bitcoin code) and the volume of daily transactions. However, the researcher uses an old sample period with daily data from January 2013 until May 2017. This sample period does not contain the strong peak of cryptocurrency prices at the end of 2017, the collapse that followed and recent price developments. Therefore, there may be a reasonable chance that his main findings do not hold anymore. By using a longer and more recent sample period and more different explanatory variables in my research I examine whether the main findings of Poyser (2019) still hold. Besides, I do more research on the dynamic relation between Bitcoin's market price and the set of explanatory variables. In this way I may identify new main drivers of Bitcoin's market price and/or different relations between the price and the explanatory variables over time. Lastly, by using a BSTS and TVP model I try to outperform the models discussed in the literature review in predicting Bitcoin's market price for a daily frequency.

In my research I consider daily data over the sample period 1 May 2013 until 20 May 2020 where the dataset consists of 1760 observations per series; Bitcoin's market price as dependent variable and 15 explanatory variables. Based on the literature review I do not consider high-frequency data, because this implies high dimensions which can be better handled by using Machine Learning models. To examine the impact of the explanatory variables on the Bitcoin's market price, I consider a set of 15 variables divided into internal and external variables. I select the set of explanatory variables based on the literature review and my own insights.

Based on the main results of my research I conclude that Bitcoin has its own characteristics as asset class, which makes Bitcoin interesting for diversification purposes. For example, I find that the level of the S&P500 has a negative impact on Bitcoin's market price. Due to the negative correlation with the S&P500 Bitcoin does not behave like a regular stock, but it serves as a hedge for stocks. However, this relationship has been changing recently as the correlation becomes more positive. This means that Bitcoin is going to behave more like stocks and that it loses power in being a hedge for stocks. Besides, I find that Bitcoin has a relatively small positive correlation with gold. Both assets are scarce in terms of supply and costly to extract. Due to Bitcoin's limited supply its scarcity, measured by the stock-to-flow variable, has a positive impact on the market price. Although Bitcoin has some similar characteristics as gold, it does not behave exactly the same, because it does not serve as safe haven in times of uncertainty. This is explained by the VIX index which has a negative correlation with Bitcoin's market price according to my results. In general I find different relations between certain explanatory variables and Bitcoin's market price, both in sign and magnitude, compared to the results of Poyser (2019).

I also evaluate the prediction performance of the different BSTS models and the TVP model, where I use forecast horizons ranging from one until five trading days ahead. I find that the static and dynamic Gaussian BSTS models generally outperform the other models in predicting Bitcoin's market price for all forecast horizons. For example, the dynamic Gaussian BSTS model correctly predicts the direction of Bitcoin's market price in 65% of the cases for almost all forecast horizons. For a forecast horizon of four trading days this is even 70%. The static Gaussian BSTS model correctly predicts the direction of Bitcoin's market price in at least 50% of the cases for all forecast horizons. For a forecast horizon of one trading day this is even 70%. I conclude that I can increase the prediction power of the models in predicting Bitcoin's market price and its direction by including predictor variables and a local linear trend.

In the following section I describe the data, where my analysis is based on, and provide some descriptive statistics. In the third section I explain the econometric methods and techniques which I use to investigate the research problem. The fourth section contains my results with a detailed explanation and discussion. Lastly, the fifth section contains my conclusions, a discussion of some limitations of my research and some topics for further research.

2 Data

For my research I source data from several databases. The dependent variable in my research is the exchange rate Bitcoin with US Dollar (BTC/USD), which I collect from CoinMarketCap[3]. Due to data availability I consider daily data over the sample period 1 May 2013 until 20 May 2020. Although Bitcoin can be traded every day, I only consider US trading days in my research due to the set of explanatory variables which is not available for every day. This results in a dataset of 1760 observations per series.

Figure 1 shows how the exchange rate Bitcoin with US Dollar developed over time during my sample period. Unlike the sample period of Poyser (2019) which runs from January 2013 until May 2017, my sample period contains the strong peak during the end of 2017, the collapse that followed in 2018, the temporary peak in 2019 and the crash in 2020 due to the outbreak of COVID-19. It is clearly visible that Bitcoin's market price exhibits high volatility and different trends changing over time. This time-variation strongly motivates the use of the BSTS model due to the flexible way how the model incorporates this.

[3]Database website: https://coinmarketcap.com/

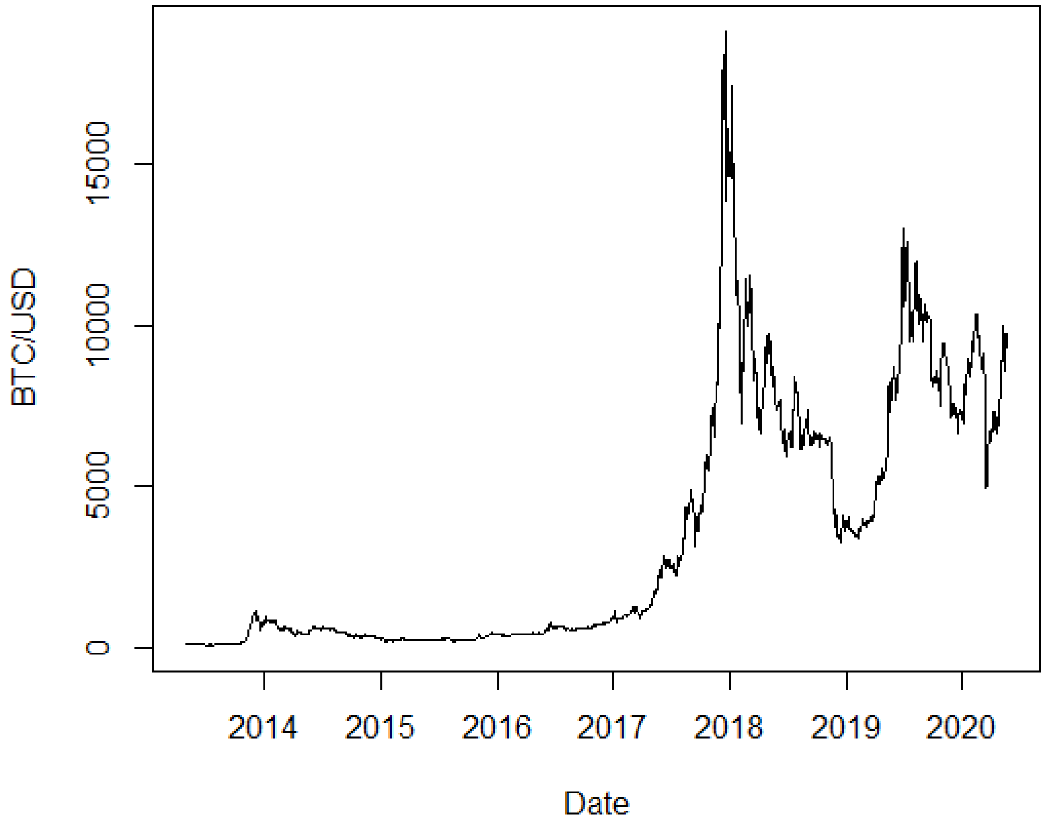

Figure 1: BTC/USD exchange rate

The set of explanatory variables consists of 15 variables in total. To examine the impact of these variables on the Bitcoin's market price, I divide the explanatory variables in a set of internal and a set of external variables. The internal variables capture the supply and demand characteristics of Bitcoin. The set of external variables consists of macro-financial variables and variables which capture the attractiveness of Bitcoin. Table 1 shows a list of these internal and external variables including a description and descriptive statistics.

2.1 Internal Variables

The internal variables may have an important impact on the Bitcoin's market price. In contrast to a stock or commodity, I assume that Bitcoin has no intrinsic value and that its price is particularly influenced by its supply and demand. The supply of Bitcoin is determined by its mining process. Bitcoin miners who successfully verify a block in the Bitcoin network, get a miners reward in the form of new Bitcoins. However, this reward is reduced by 50% every four years until the final Bitcoin has been mined. The supply of Bitcoin is constructed in such a way that it has a maximum of 21 million Bitcoins. Due to the reward halving every four years, the amount of new Bitcoins decreases over time.

Table 1 shows the internal variables which may have an impact on the Bitcoin's market price. I collect these variables from the database Blockchain via the Quandl platform[4] [5].

One of the internal variables is the stock-to-flow ratio, which measures the scarcity of Bitcoin. This ratio is defined as the Bitcoin stock on a specific day divided by its daily production. The higher the ratio, the scarcer the object is in general. For example, in comparison with other commodities the precious metal gold has a relatively high stock-to-flow ratio. Therefore, people consider gold as a scarce commodity. Some researchers, such as PlanB and Ammous (2018), argue that Bitcoin is just as scarce as gold and silver[6]. Bitcoin is costly to produce and its supply is determined by the already mentioned miners reward halving and the maximum supply of Bitcoins. They find relatively high stock-to-flow values for Bitcoin and a statistically significant relationship between stock-to-flow and Bitcoin market value. I test their findings in my research by adding the stock-to-flow to my list of internal variables.

[4] Database website: https://blockchain.com/

[5] Platform website: https://quandl.com/

[6] Website: https://medium.com/@100trillionUSD/modeling-bitcoins-value-with-scarcity-91fa0fc03e25

2.2 External Variables

The external variables are related to the perspective of the financial markets, the macro-economy and the attractiveness of Bitcoin. For example, I consider the S&P500 as external variable to examine whether Bitcoin behaves like a regular stock. There is also a group of people who claim that Bitcoin behaves as safe haven, such as gold. Therefore, I include the gold price in my research.

Table 1 shows the external variables which may have an impact on the Bitcoin's market price. I collect these variables from the database Yahoo Finance[7].

Lastly, to examine the attractiveness of Bitcoin I consider relative search volume for the word 'Bitcoin' which I collect from Google Trends. I filter this data by geographical location, which is 'Worldwide' in my research. Google Trends provides the relative search volume as a scaled range between 0 and 100, where 0 indicates the lowest relative search interest for the word 'Bitcoin' and 100 indicates the highest relative search interest over the given sample period. However, for my sample period Google Trends only provides indexed monthly data. Therefore, I need to transform this monthly data to daily data. I do this by obtaining indexed daily data for each individual month which is part of my sample period. To allow inter-month comparisons over my sample period, I multiply the daily data for each month by the corresponding monthly search weight with respect to all months which are part of my sample period.

[7]Database website: https://finance.yahoo.com/

Table 1: Descriptive statistics dependent and explanatory variables

Name	Description	Mean	St. dev.	Min.	Max.
Dependent variable					
BTC/USD	Bitcoin exchange rate with US Dollar	3,342.46	3,854.53	68.43	19,114.20
Internal variables					
BTC Hash rate	The estimated number of giga hashes per second (billions of hashes per second) the Bitcoin network is performing	21,697,400	33,138,115	71	136,264,980
BTC Miners revenue	(number of Bitcoins mined per day + transaction fees) * market price in USD	6,956,660	7,908,027	302,513	53,191,582
BTC Market capitalization	The total number of Bitcoins in circulation * market price in USD	$5.725 \cdot 10^{10}$	$6.758 \cdot 10^{10}$	$7.718 \cdot 10^{8}$	$3.231 \cdot 10^{11}$
BTC Number of transactions	Total number of unique Bitcoin transactions per day	202,068	102,750.100	34,053	490,644
BTC Average block size	The average block size in MB	0.703	0.350	0.088	1.353
BTC Cost per transaction	Miners revenue divided by the number of transactions	30.965	27.084	3.443	161.686
BTC Difficulty	Measure of how difficult it is to find a hash below a given target	$2.964 \cdot 10^{12}$	$4.547 \cdot 10^{12}$	$1.008 \cdot 10^{7}$	$1.655 \cdot 10^{13}$
BTC Number of transactions per block	The average number of transactions per block	1,338	716.914	154	2,763
BTC Stock-to-flow	Measure of the scarcity of Bitcoin	6,690	3,277.996	1,836	27,229
External variables					
S&P500	Broad American stock market index	2,334	436.009	1,573	3,386
Gold	Commodity price in USD	1,291	123.020	1,051	1,769
VIX	Chicago Board Options Exchange's Volatility Index	15.950	7.119	9.140	82.690
EUR/USD	Euro exchange rate with US Dollar	1.176	0.097	1.039	1.393
USD/CNY	US Dollar exchange rate with Chinese Yuan	6.539	0.330	6.031	7.178
Search trend 'Bitcoin'	Measure of the attractiveness of Bitcoin	6.787	7.718	0.420	100

Note: The table shows the descriptive statistics, such as mean, standard deviation, minimum and maximum, for the dependent variable, internal and external variables over the sample period 1 May 2013 - 20 May 2020.

3 Methodology

3.1 Modeling and Estimating the Bayesian Structural Time Series Model

In my research I apply a Bayesian Structural Time Series (BSTS) model to examine which variables have an impact on the Bitcoin's market price. Based on the literature of Brodersen et al. (2015), Jammalamadaka et al. (2018), Scott & Varian (2014) and Scott & Varian (2015), structural time series models are state-space models for time series data. The BSTS model is a regression model. It includes latent factors which capture Bitcoin's price time series properties, namely the general trend and seasonality. I filter these latent factors by using the Kalman filter, based on the literature of Durbin & Koopman (2001). The model also allows a set of explanatory variables to contribute to the prediction power. I select the most important predictor variables by using the Spike-and-slab prior, which I will discuss later in this section.

Firstly, I introduce the general framework of the BSTS model and motivate why I use Bayesian methods in my research. Besides, I explain how I obtain the latent factors by using the Kalman filter. Subsequently, I discuss the practical implementation of the BSTS model in my research.

Following the literature of Brodersen et al. (2015), Poyser (2019), Scott & Varian (2014) and Scott & Varian (2015), the structural time series model can be described by the following pair of equations:

$$y_t = \mathbf{Z}_t'\boldsymbol{\alpha}_t + \epsilon_t, \qquad \epsilon_t \sim N(0, H_t), \tag{1}$$

$$\boldsymbol{\alpha}_{t+1} = \mathbf{D}_t\boldsymbol{\alpha}_t + \mathbf{R}_t\boldsymbol{\eta}_t, \qquad \boldsymbol{\eta}_t \sim N(\mathbf{0}, \mathbf{Q}_t). \tag{2}$$

Equation (1) represents the observation equation where the observed data, the Bitcoin's market price, y_t is linked to a latent $m \times 1$ state vector α_t. In my research y_t is a scalar observation, the latent factors consist of a trend and seasonality factor, Z_t is a $m \times 1$ output vector and ϵ_t is an independent random error term which is normally distributed with mean 0 and variance H_t. Equation (2) represents the transition equation, which indicates how the unobserved latent state vector α_t evolves over time. D_t defines the $m \times m$ transition matrix, R_t is a $m \times r$ control matrix and η_t is a $r \times 1$ system error which is normally distributed with mean 0 and variance Q_t, where Q_t is a $r \times r$ state-diffusion matrix with $r \leq m$. In this way it is possible to include state components of less than full rank, which can be important when I want to include a seasonality component for example.

I can not solve this system by using Maximum Likelihood Estimation (MLE) methods, because the state vector α_t and system matrices Z_t, H_t, D_t and Q_t are unknown. For this reason and the fact that I can take into account some prior beliefs about the model in the form of prior distributions, I use Bayesian methods in my research to solve this system where I follow the literature of Brodersen et al. (2015), Poyser (2019), Scott & Varian (2014) and Scott & Varian (2015). An important task is to obtain future observations in the unobserved states. The primary tool to achieve this is by applying the Kalman filter. I denote $y_{1:t} = y_1, \ldots, y_t$. The Kalman filter estimates the value of the state vector α_t at time t by using the given observations up to and including time t. In this way I compute the conditional densities $\pi(\alpha_t|y_{1:t})$, $\pi(\alpha_{t+1}|y_{1:t+1})$, $\pi(\alpha_{t+2}|y_{1:t+2})$, $\ldots$, $\pi(\alpha_{t+n}|y_{1:t+n})$, where n is the length of the time series. Subsequently, when I estimate α_{t+1} it is possible to generate the observation y_{t+1}, the Bitcoin's market price at time $t+1$, and so on. I take the prior specification $\alpha_0 \sim \pi(\alpha_0)$, where I assume that the prior follows a normal distribution with mean α_0 and variance P_0, which is also independent of the error terms in Equation (1) and (2) for every time t. Starting from this prior I recursively compute the state α_t for $t = 1, 2, \ldots, n$ to obtain the predictive densities $\pi(\alpha_{t+1}|y_{1:t})$ and $\pi(y_{t+1}|y_{1:t})$.

Next to the state components like a trend or seasonality, I also include a regression component and I define the BSTS model in the following way:

$$\begin{aligned} y_t &= \mu_t + \tau_t + \beta' x_t + \epsilon_{y,t}, & \epsilon_{y,t} &\sim N(0, \sigma^2_{\epsilon_y}), \\ \mu_t &= \mu_{t-1} + \delta_{t-1} + \epsilon_{\mu,t}, & \epsilon_{\mu,t} &\sim N(0, \sigma^2_{\epsilon_\mu}), \\ \delta_t &= \delta_{t-1} + \epsilon_{\delta,t}, & \epsilon_{\delta,t} &\sim N(0, \sigma^2_{\epsilon_\delta}), \\ \tau_t &= -\sum_{s=1}^{S-1} \tau_{t-s} + \epsilon_{\tau,t}, & \epsilon_{\tau,t} &\sim N(0, \sigma^2_{\epsilon_\tau}), \end{aligned} \tag{3}$$

where the static regression coefficients β and variances $\sigma^2_{\epsilon_y}, \sigma^2_{\epsilon_\mu}, \sigma^2_{\epsilon_\delta}, \sigma^2_{\epsilon_\tau}$ are the unknown parameters which I need to estimate. The first line in Equation (3) is the observation equation like Equation (1), where the observed Bitcoin's market price is linked to the latent time series components and the regression component. The BSTS model in Equation (3) consists of a latent trend component, where μ_t represents the current level of the trend and δ_t is the current slope of the trend. The second and third line in Equation (3) are the transition equations like Equation (2) of the mean and slope of the trend component, respectively. The fourth line in Equation (3) is the transition equation of the seasonal component. I assume a local linear trend where the mean μ_t and slope δ_t of the trend follow a random walk. The seasonal component τ_t consists of a set of S dummy variables where the coefficients have an expectation equal to zero over a full cycle of S seasons. For example, if I assume four seasons per year, namely spring, summer, autumn and winter. In that case $S = 4$ and the mean of the *spring* coefficient is equal to $-1 \times (summer + autumn + winter)$. Lastly, the BSTS model has a static regression component $\beta' x_t$ where x_t is the set of explanatory variables which includes the 15 internal and external variables as discussed in Section 2. Hence, the dimension of β and x_t is a 15×1 vector.

In general Bitcoin's market price exhibits much time-variation with different trends over time. By introducing components like a trend and seasonality in the BSTS model I try to capture a part of the volatile behaviour of Bitcoin's market price. It is interesting to examine to what extent these components can explain the variability in Bitcoin's market price and whether they have some prediction power.

Next to the BSTS model with a static regression component $\beta' x_t$, I also consider a version of the BSTS model with a dynamic regression component $\beta_t' x_t$. In that case the regression coefficients β_t can vary over time. Due to the fact that Bitcoin's market price exhibits much time-variation, it is likely that the effects of the explanatory variables also vary over time. Given the explanatory variables $j = 1, \dots, J$; I assume that the coefficients change over time according to a random walk and I define this in the following way based on the literature of Brodersen et al. (2015):

$$\begin{aligned} y_t &= \mu_t + \tau_t + \beta_t' x_t + \epsilon_{y,t}, \qquad \epsilon_{y,t} \sim N(0, \sigma^2_{\epsilon_y}), \\ \beta_{j,t} &= \beta_{j,t-1} + \eta_{\beta,j,t-1}, \qquad \eta_{\beta,j,t-1} \sim N(0, \sigma^2_{\beta_j}), \end{aligned} \tag{4}$$

where $\beta_{j,t}$ is the regression coefficient for the jth explanatory variable and $\sigma^2_{\beta_j}$ is the variance of its associated random walk.

The second main component of the BSTS model is that it contains a regression component $\beta' x_t$ with the set of explanatory variables that can contribute to the prediction power. I use the Spike-and-slab prior in order to select the most important predictor variables. The method is developed by George & McCulloch (1997) and Madigan & Raftery (1994), and it is particularly useful when the number of possible predictors is larger than the number of observations. The Spike-and-slab prior is a shrinkage method with the advantages that it reduces over-fitting problems and that it improves the forecast performance of the model. The idea is that the Spike-and-slab prior is a hierarchical Bayesian model, where the spike refers to the probability of a particular coefficient in the model to be zero, while the slab is represented as the prior distribution for the regression coefficient values. By using Bayes's rule, probabilities are updated in order to generate a joint posterior distribution of the variables with the highest marginal posterior inclusion probabilities.

Based on Scott & Varian (2014) and Scott & Varian (2015), β represents the parameter vector and $\gamma = (\gamma_1, \dots, \gamma_K)$, where K represents the total number of regressors, and defines which regressors are included in the BSTS model, i.e. $\gamma_i = 1$ indicates $\beta_i \neq 0$ and $\gamma_i = 0$ indicates $\beta_i = 0$. I denote β_γ as the subset of β for which holds $\gamma_i = 1$. Besides, σ^2_ϵ is the residual variance from the regression model.

I write the Spike-and-slab prior as:

$$p(\beta, \gamma, \sigma_\epsilon^{-2}) = p(\beta_\gamma | \gamma, \sigma^2_\epsilon) p(\sigma^2_\epsilon | \gamma) p(\gamma). \tag{5}$$

The Spike-and-slab prior consists of a 'spike' part which is reflected by the marginal distribution $p(\gamma)$. It places a positive probability mass at zero. Another possibility is to use a continuous distribution for the 'spike' part. For $p(\gamma)$ I use an independent Bernoulli prior:

$$\gamma \sim \prod_{i=1}^{K} \pi_i^{\gamma_i}(1-\pi_i)^{1-\gamma_i}. \tag{6}$$

Due to the fact that I do not have detailed prior information, it is convenient to set all π_i equal to the same value π. I determine this value of the prior inclusion probability by considering the expected model size, so if I expect k nonzero coefficients out of all K coefficients, I calculate $\pi = k/K$.

The Spike-and-slab prior also consists of a 'slab' part which is reflected by the conditional priors $p(\beta_\gamma|\gamma, \sigma_\epsilon^2)$ and $p(\sigma_\epsilon^{-2}|\gamma)$. These priors represent the prior distribution of the regression nonzero coefficients, conditional on γ. I denote b as the vector of prior means, which I assume to be equal to zero. Besides, Ω^{-1} is the prior precision matrix and Ω_γ^{-1} denote the rows and columns of Ω^{-1} corresponding to $\gamma_i = 1$. I express the conditional priors as the conditionally conjugate prior:

$$\begin{aligned} \beta_\gamma|\sigma_\epsilon^2 &\sim N(b_\gamma, \sigma_\epsilon^2(\Omega_\gamma^{-1})^{-1}), \\ \frac{1}{\sigma_\epsilon^2} &\sim \Gamma(\frac{\nu}{2}, \frac{ss}{2}), \end{aligned} \tag{7}$$

where ν denotes the prior sample size and ss is the prior sum of squares. These values are based on the expected R^2 from the regression and the weight ν assigned to this guess, measured in terms of the equivalent number of observations. The following holds $ss = \nu(1-R^2)s_y^2$, where s_y^2 is the marginal standard deviation of the dependent variable. Furthermore, I assume that $\Omega^{-1} \propto \mathbf{X}^T\mathbf{X}$. I denote $\mathbf{X}$ as the design matrix which I construct in such a way that the vector of explanatory variables x_t is row t. In this case Equation (7) is known as Zellner's g-prior (Chipman et al. (2001)). I have the Fisher information matrix $\frac{\mathbf{X}^T\mathbf{X}}{\sigma^2}$, so by taking $\Omega^{-1} = \frac{\kappa(\mathbf{X}^T\mathbf{X})}{n}$ I place the average information available from κ observations as weight on the prior mean b.

To obtain forecasts of the Bitcoin's market price I apply a procedure, which is Bayesian model averaging over the space of time series regression models. By taking draws of the parameters and states of the posterior distribution I can combine them with the available data to obtain predictions of the dependent variable for those particular draws. Repeating this procedure leads to an estimate of the posterior distribution of the predictions. I smooth the predictions over a large number of potential models. In this way I prevent an arbitrary selection of predictor variables and I reduce over-fitting problems. I sample from a posterior distribution by using Markov Chain Monte Carlo (MCMC) techniques, in particular the Gibbs sampling algorithm.

3.2 Modeling and Estimating the Time-Varying Parameter Model

Next to the BSTS models with only a static regression component $\beta' x_t$ or a dynamic regression component $\beta_t' x_t$, I also examine a model that contains both a static and dynamic regression component. For this purpose I use a non-centered parameterization time-varying parameter (TVP) model, where I use the methodology of Fruhwirth-Schnatter & Wagner (2010) and Belmonte et al. (2014). For this model I also apply a Bayesian approach by using shrinkage priors on both the static and dynamic parameters. In this way I can select the most important static and dynamic predictor variables out of a set of explanatory variables, which reduces the problem of over-fitting and improves the forecast performance.

Firstly, based on Fruhwirth-Schnatter & Wagner (2010) and Belmonte et al. (2014) I define the state space form of a TVP model in the following way:

$$\begin{aligned} y_t &= \beta_t' x_t + \epsilon_t, \qquad \epsilon_t \sim N(0, \sigma^2), \\ \beta_t &= \beta_{t-1} + w_t, \qquad w_t \sim N(0, W), \end{aligned} \tag{8}$$

where y_t represents Bitcoin's market price at time t and x_t is a vector containing the set of explanatory variables at time t which includes the internal and external variables as discussed in Section 2. Besides, given the explanatory variables $j = 1, \dots, J$; I assume that $W = Diag(\theta_1, \dots, \theta_J)$ is a diagonal matrix, where the state innovations θ_j are conditionally independent. I also assume that $\beta_0 \sim N(\beta, W)$ with initial mean $\beta = (\beta_1, \dots, \beta_J)$.

Subsequently, I rewrite Equation (8) as the non-centered parameterization TVP model based on Fruhwirth-Schnatter & Wagner (2010) and Belmonte et al. (2014):

$$\begin{aligned} y_t = \beta' x_t + \tilde{\beta}_t' Diag(\sqrt{\theta_1}, \dots, \sqrt{\theta_J}) x_t + \epsilon_t, \qquad \epsilon_t \sim N(0, \sigma^2), \\ \tilde{\beta}_t = \tilde{\beta}_{t-1} + \tilde{u}_t, \qquad \tilde{u}_t \sim N(0, I_J), \end{aligned} \tag{9}$$

where I_J is the J-dimensional identity matrix and I assume that $\tilde{\beta}_0 \sim N(0, I_J)$. Fruhwirth-Schnatter & Wagner (2010) argue and present strong evidence for using a normal prior on the state innovations θ_j instead of an inverted Gamma prior, which is usually applied in Bayesian analysis. The main reason is that an inverted Gamma prior can obstruct the shrinkage process, because it is bounded away from zero. By using a normal prior it is possible to put probability mass around zero.

Shrinkage in the TVP model in Equation (9) leads to four possibilities. Firstly, the model contains a constant parameter on predictor variable j if $\sqrt{\theta_j}$ is shrunk to zero and β_j is not shrunk to zero. Secondly, the model excludes predictor variable j if both $\sqrt{\theta_j}$ and β_j are shrunk to zero. Another possibility is that the model contains a small time-varying parameter on predictor variable j if $\sqrt{\theta_j}$ is not shrunk to zero and β_j is shrunk to zero. Lastly, the model contains an unrestricted time-varying parameter on predictor variable j if both $\sqrt{\theta_j}$ and β_j are not shrunk to zero.

Based on Fruhwirth-Schnatter & Wagner (2010) I use for each predictor variable j conditionally independent normal-gamma priors for $\sqrt{\theta_j}$ and β_j and I define it in the following way:

$$\sqrt{\theta_j}|\xi_j^2 \sim N(0, \xi_j^2), \qquad \xi_j^2|a^\xi, \kappa^2 \sim \Gamma(a^\xi, \frac{a^\xi \kappa^2}{2}), \tag{10}$$

$$\beta_j|\tau_j^2 \sim N(0, \tau_j^2), \qquad \tau_j^2|a^\tau, \lambda^2 \sim \Gamma(a^\tau, \frac{a^\tau \lambda^2}{2}), \tag{11}$$

where a^ξ and a^τ are the pole parameters, because as they become smaller more mass is placed around zero. The global shrinkage parameters are κ^2 and λ^2, which determine the shrinkage effect on all parameters.

I let the pole parameters a^ξ, a^τ and the global shrinkage parameters κ^2, λ^2 learn from the data through specific prior distributions, namely independent gamma distributions:

$$\kappa^2 \sim \Gamma(d_1, d_2), \qquad \lambda^2 \sim \Gamma(e_1, e_2), \tag{12}$$

$$a^\xi \sim \Gamma(\alpha_{a^\xi}, \alpha_{a^\xi}\beta_{a^\xi}), \qquad a^\tau \sim \Gamma(\alpha_{a^\tau}, \alpha_{a^\tau}\beta_{a^\tau}), \tag{13}$$

with hyperparameters $d_1, d_2, e_1, e_2, \alpha_{a^\xi}, \beta_{a^\xi}, \alpha_{a^\tau}, \beta_{a^\tau}$.

I define the prior distribution on the volatility parameter in Equation (9) in the following way:

$$\sigma^2|C_0 \sim \Gamma^{-1}(c_0, C_0), \qquad C_0 \sim \Gamma(g_0, G_0), \tag{14}$$

with hyperparameters c_0, g_0, G_0.

Just like the BSTS model I also use the Gibbs sampling algorithm as MCMC technique to obtain draws from the posterior distribution of the TVP model parameters.

3.3 Prediction Performance

After obtaining the posterior predictive distribution I examine the out-of-sample prediction performance of several versions of the BSTS model and the TVP model. I do this for different future time horizons. For this purpose I divide the data in a training sample and a test sample. Furthermore, I use a rolling window with a fixed size of observations. To obtain the direct forecasts I draw multiple times from the posterior predictive distribution for each future time horizon. Subsequently, I summarize the draws by taking their mean. For the BSTS models I consider three versions of the local linear trend model, namely a naïve one, a version with static regression coefficients and a version with dynamic regression coefficients.

I use three metrics to evaluate the out-of-sample prediction performance of each model for each future time horizon, namely the mean squared forecast error (MSFE), mean absolute forecast error (MAFE) and the mean correct prediction (MCP).

As stated in Heij et al. (2004), the mean squared forecast error (MSFE) is defined as:

$$MSFE = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2, \tag{15}$$

which is the average squared deviations of the observed values (y_i) and the predicted values ($\hat{y}_i$), where N denotes the number of observations in the test sample.

The second metric that I use in my research is the mean absolute forecast error (MAFE). An advantage of the MAFE is that it is more robust to outliers than the MSFE, because it makes use of absolute deviations instead of squared deviations. Based on Heij et al. (2004), the mean absolute forecast error (MAFE) is defined as:

$$MAFE = \frac{1}{N}\sum_{i=1}^{N}|y_i - \hat{y}_i|, \tag{16}$$

which is the average absolute deviations of the observed values (y_i) and the predicted values ($\hat{y}_i$), where N denotes the number of observations in the test sample. The model with the lowest MSFE and MAFE value is preferred.

The last metric that I use in my research is the mean correct prediction (MCP). The MCP counts the number of times when the sign of changes is correctly predicted, divided by the number of observations in the test sample. An advantage of the MCP is that it evaluates in this case the directional movement of Bitcoin's market price, while the MSFE and MAFE consider the magnitude of the errors. Regardless of magnitude, it is interesting whether you can correctly predict Bitcoin's price movement, because this is still useful for practical investment applications. Based on the work of Chalamandaris & Tsekrekos (2010), the mean correct prediction (MCP) is defined as:

$$MCP = \frac{1}{N}\sum_{i=1}^{N}\mathbf{1}_{sign(y_i - y_{i-1}) == sign(\hat{y}_i - \hat{y}_{i-1})}, \tag{17}$$

where $\mathbf{1}$ is the indicator function when the sign of change of the observed values (y_i) is equal to the sign of change of the predicted values $(\hat{y}_i)$, and N denotes the number of observations in the test sample. The model with the highest MCP value is preferred.

4 Results

4.1 The Bayesian Structural Time Series Model

As a first step in the process before estimating the BSTS model, I check whether I need to include a seasonal component τ_t in the BSTS model in Equation (3). If I visually inspect the Bitcoin's market price in Figure 1, I do not clearly see any recurring patterns over time. This is also confirmed by doing a periodogram analysis on Bitcoin's market price. Figure 2 shows the periodogram where the highest periodic signal peak appears at a frequency equal to $5.682 \cdot 10^{-4}$. This means that there are $5.682 \cdot 10^{-4}$ cycles per day or a full cycle per 1760 days. However, my total sample period consists of 1760 days, so this result is meaningless. Therefore, I conclude that there are no seasonal patterns present in Bitcoin's market price and I exclude the seasonal component τ_t from the BSTS model in Equation (3).

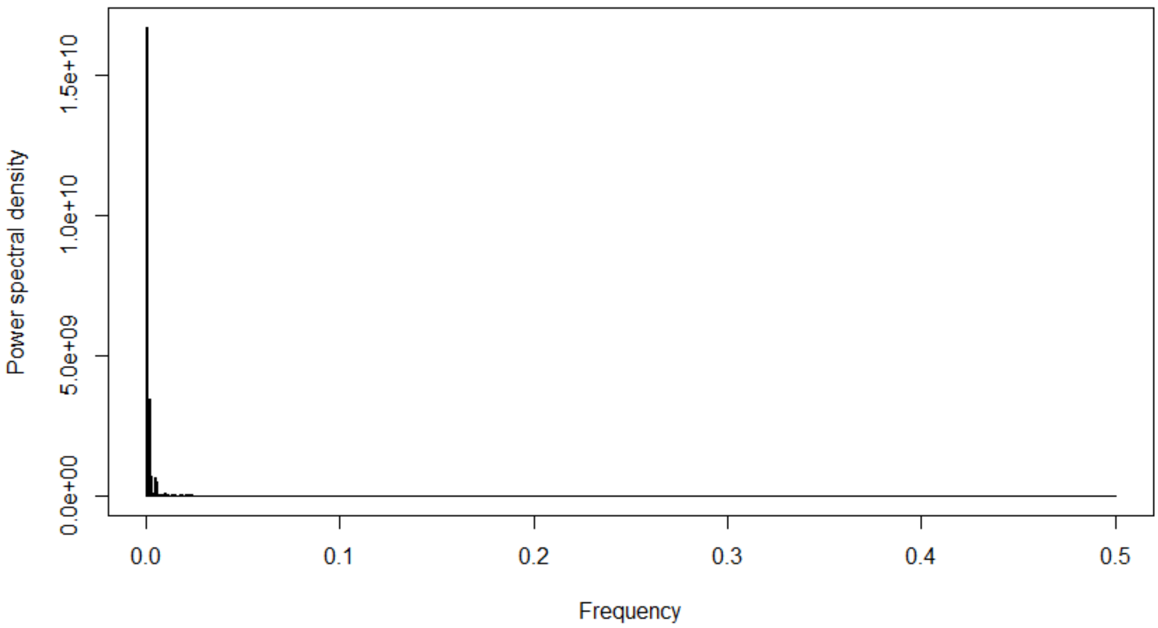

Figure 2: Periodogram Bitcoin's market price

Subsequently, I estimate the different versions of the BSTS model to examine the impact of the explanatory variables on the Bitcoin's market price. For this purpose I use the **bsts** package in R, which is based on the literature of Scott & Varian (2014). Firstly, I fit the BSTS model that contains a static regression component $\beta' x_t$ and Gaussian error terms, where I use the Spike-and-slab prior. I sample from the posterior distribution of the BSTS model to obtain the parameter estimates by using 10,000 MCMC iterations, discarded the first 3,575 iterations as burn-in sample. Secondly, I fit the BSTS model that contains a dynamic regression component $\beta_t' x_t$ and Gaussian error terms. I sample from the posterior distribution of the BSTS model to obtain the parameter estimates by using 10,000 MCMC iterations, discarded the first 347 iterations as burn-in sample.

Figure 3 and 4 show the posterior distribution of the conditional mean $Z_t' \alpha_t$ from Equation (1) given the full data of the static Gaussian BSTS model and the dynamic Gaussian BSTS model, respectively. The blue circles represent the raw observations. In general both BSTS models are correctly fitted to the raw data which is important to know for my inference. Figure 5 and 6 show the contributions of the individual state components of the static Gaussian BSTS model and the dynamic Gaussian BSTS model in explaining the variation of Bitcoin's market price, respectively. For example, a value of zero means that the specific state component does not contribute in explaining the Bitcoin's market price at that specific point in time. Figures 3, 4, 5 and 6 look all fuzzy, because they show the marginal posterior distribution at each point in time. For example, the black line in the figures represents the posterior median at each point in time, where each 1% quantile away from the posterior median is shaded a little bit lighter, so that the 1st and 99th percentiles are shaded white ultimately. Figure 5 shows that both the trend and regression component explain a substantial amount of variation in Bitcoin's market price. In particular the spike of Bitcoin's market price during the end of 2017, the collapse that followed and the rise of the price in 2019 are explained by both individual components. However, it seems that the decline of the price at the end of 2019 and the crash in March 2020 due to the outbreak of COVID-19 are better explained by the regression component. Figure 6 shows that the dynamic regression component explains almost all variation in Bitcoin's market price. The trend component explains part of the variation during the peak at the end of 2017 and beginning of 2018, and the collapse that followed. From 2019 onwards I also see that the trend component explains a small part of the variation, which steadily increases. In comparison with the static Gaussian BSTS model it is clear that if I allow the coefficients of the explanatory variables to vary over time, that most of the variation in Bitcoin's market price can be explained by the dynamic regression component.

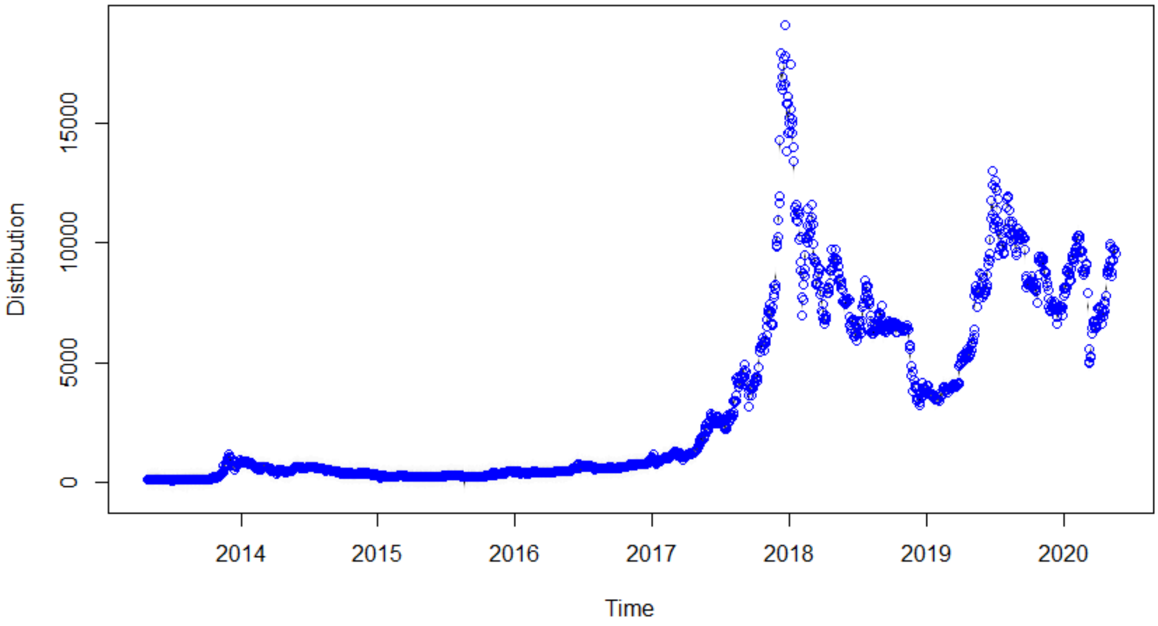

Figure 3: Posterior distribution at each point in time of the static Gaussian BSTS model state

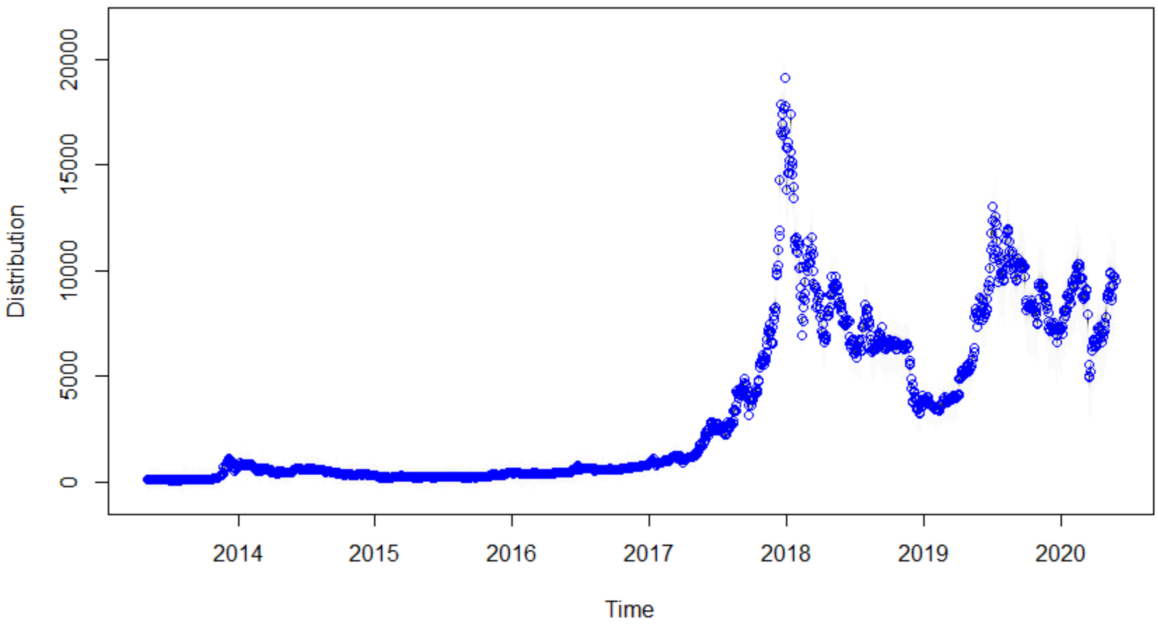

Figure 4: Posterior distribution at each point in time of the dynamic Gaussian BSTS model state

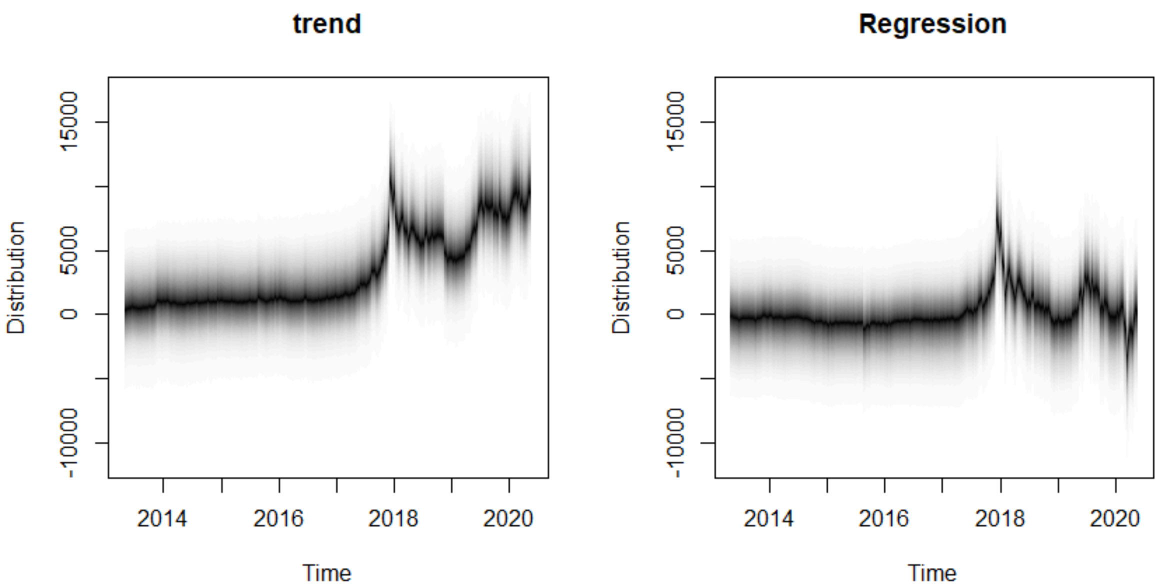

Figure 5: Contributions of the individual state components of the static Gaussian BSTS model

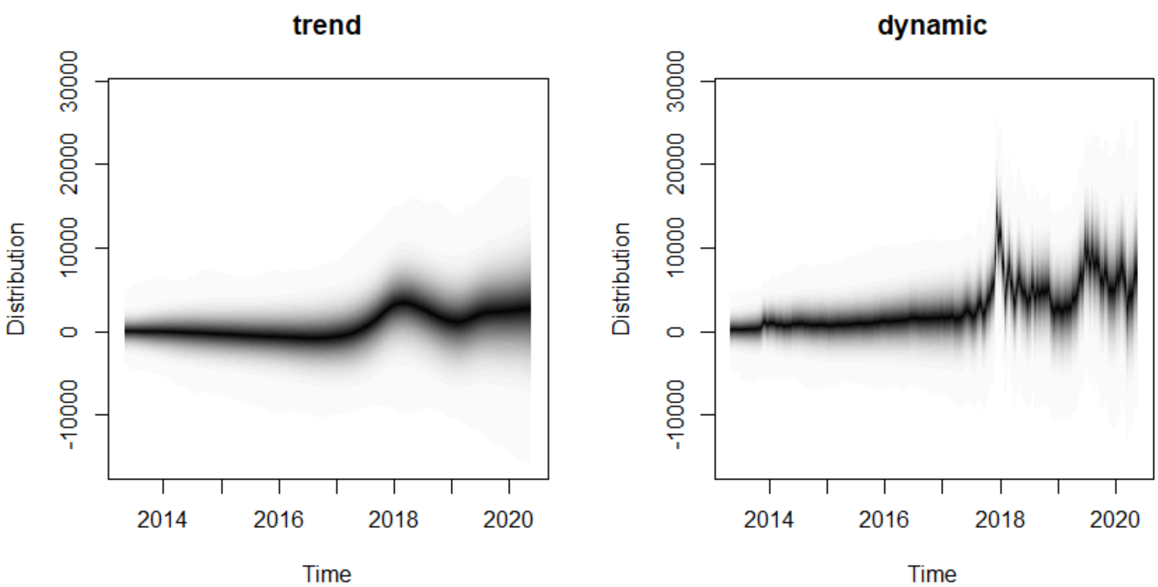

Figure 6: Contributions of the individual state components of the dynamic Gaussian BSTS model

Figure 7 shows the assigned posterior inclusion probabilities of all explanatory variables for the static Gaussian BSTS model. The bars are shaded according to the probability of a positive coefficient, where the white coloured bars represent a positive coefficient, the black coloured bars represent a negative coefficient and the gray coloured bars represent an indeterminate sign. Besides, Table 2 shows the posterior mean, the 95% highest posterior density interval (HDI) and the posterior inclusion probabilities for all coefficients of the explanatory variables. Figure 8 shows the posterior distribution of the static Gaussian BSTS model size, where the mean and median of the number of explanatory variables to include in the model are both around 11 out of the 15 explanatory variables in total. Hence, based on Table 2 I can exclude the variables Bitcoin's number of transactions, average block size, cost per transaction and its search trend from the model, because these variables have the lowest inclusion probabilities. Figure 9 shows the dynamic contribution of each explanatory variable of the dynamic Gaussian BSTS model.

Regarding the internal variables Figure 9 shows that the impact of Bitcoin's hash rate (BTC_HashRate) is stable and negative over time, but since the beginning of 2019 the effect becomes steadily less negative. When the hash rate increases more blocks in the Bitcoin network are being mined, which makes it more likely that Bitcoin miners successfully verify a block in the Bitcoin network and get a miners reward in the form of new Bitcoins. This has a positive impact on the supply side of Bitcoin and a higher supply usually leads to a lower price. Figure 7 and Table 2 also indicate that Bitcoin's hash rate has a negative impact on Bitcoin's market price with a high inclusion probability. Except for the number of transactions per block, all other internal variables have a generally stable and positive impact on Bitcoin's market price over time according to Figure 9. For Bitcoin's market capitalization (BTC_MC) the effect is relatively strong positive during the beginning of the sample period, but it steadily decreases until 2018. Thereafter the effect remains relatively stable. Table 2 also indicates that Bitcoin's market capitalization has a positive impact on Bitcoin's market price with a high inclusion probability. This also holds for Bitcoin's miners revenue (BTC_MinersRev). However, a difference between the static and dynamic Gaussian BSTS model is that Bitcoin's difficulty has a negative impact on Bitcoin's market price with a high inclusion probability according to Table 2. For the number of transactions the impact has become more positive since 2018 with a small dip in 2019. However, Table 2 shows that Bitcoin's number of transactions has a relatively low inclusion probability and Figure 8 indicates to exclude this variable from the model. This also holds for the variables Bitcoin's average block size and cost per transaction.

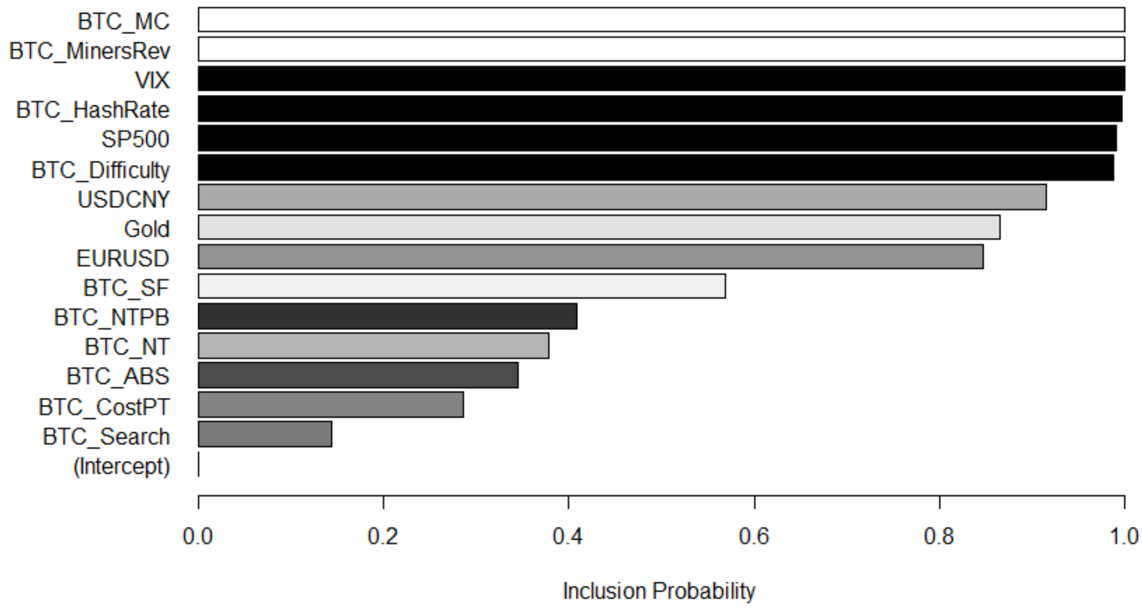

Figure 7: Posterior inclusion probabilities of all explanatory variables for the static Gaussian BSTS model

Table 2: Posterior parameter statistics of the static Gaussian BSTS model

Variable	Mean	2.5% HDI	97.5% HDI	Inclusion Probability
BTC Market capitalization	$2.387 \cdot 10^{-8}$	$1.958 \cdot 10^{-8}$	$2.876 \cdot 10^{-8}$	1.000
BTC Miners revenue	$4.242 \cdot 10^{-5}$	$2.133 \cdot 10^{-5}$	$6.179 \cdot 10^{-5}$	1.000
VIX	-35.561	-45.044	-24.954	1.000
BTC Hash rate	$-9.310 \cdot 10^{-6}$	$-1.436 \cdot 10^{-5}$	$-3.941 \cdot 10^{-6}$	0.996
S&P500	-0.825	-1.555	-0.150	0.991
BTC Difficulty	$-1.237 \cdot 10^{-10}$	$-2.232 \cdot 10^{-10}$	$-3.342 \cdot 10^{-11}$	0.987
USD/CNY	92.087	-364.257	584.683	0.915
Gold	0.483	-0.241	1.531	0.865
EUR/USD	143.218	-1210.483	1596.048	0.847
BTC Stock-to-flow	0.00765	-0.000876	0.0280	0.569
BTC Number of transactions per block	-0.0192	-0.127	0.0431	0.409
BTC Number of transactions	$9.283 \cdot 10^{-5}$	$-4.056 \cdot 10^{-4}$	$8.623 \cdot 10^{-4}$	0.379
BTC Average block size	-17.460	-182.131	94.194	0.346
BTC Cost per transaction	0.00592	-2.143	2.277	0.287
Search trend 'Bitcoin'	-0.00236	-2.304	2.168	0.145
(Intercept)	0.000	0.000	0.000	0.000

Note: The table shows the posterior parameter statistics, such as the posterior mean, the 95% highest posterior density interval (HDI) and the posterior inclusion probabilities of all explanatory variables for the static Gaussian BSTS model.

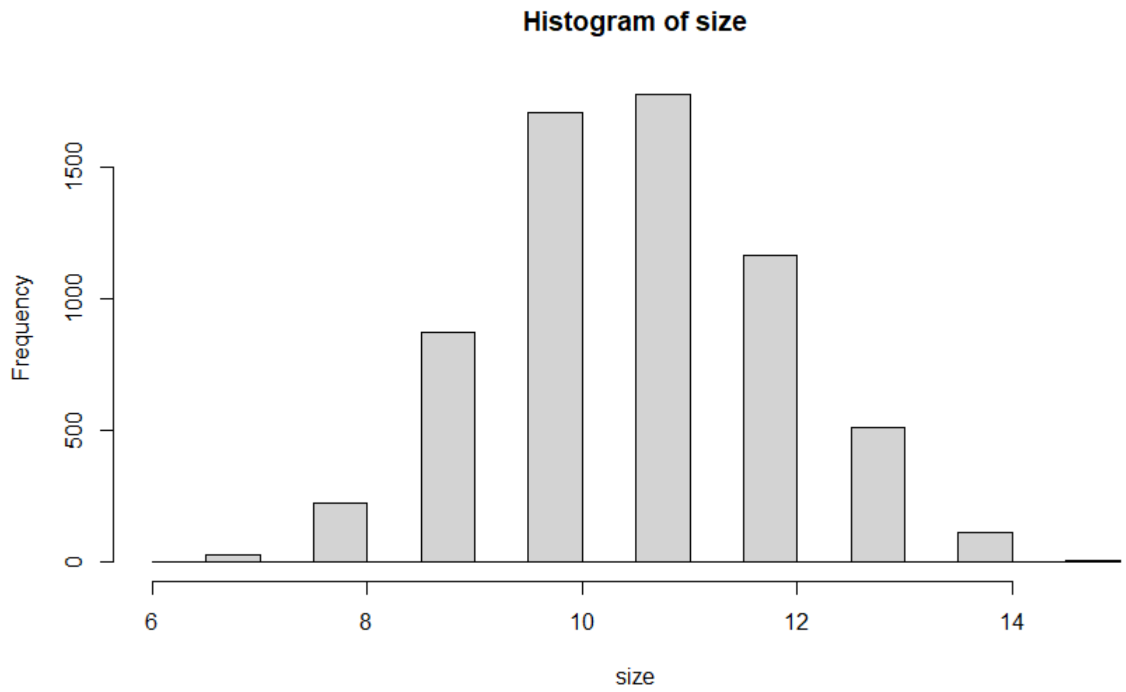

Figure 8: Posterior distribution of the static Gaussian BSTS model size

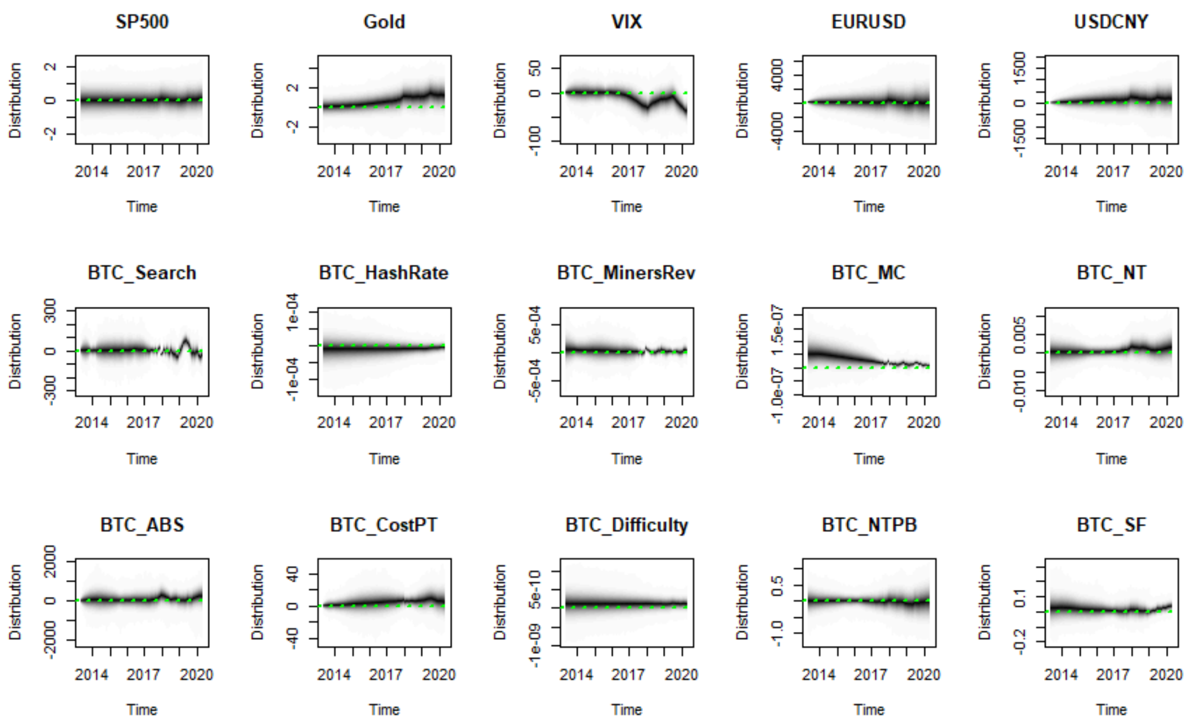

Figure 9: Dynamic contributions of the explanatory variables of the dynamic Gaussian BSTS model

Besides, the S&P500 has a negative impact on Bitcoin's market price with a high inclusion probability according to Table 2. Figure 9 indicates that the impact of the S&P500 on Bitcoin's market price is negative and relatively stable over time. However, the impact of the S&P500 becomes positive during the price peak at the end of 2017 and beginning of 2018, and since mid-year 2019 onwards. It is interesting to see that in the first couple of years the impact of the S&P500 is negative, but that it has become positive more recently. Furthermore, gold has a relatively small positive impact on Bitcoin's market price with a high inclusion probability according to Table 2. This also holds for the exchange rates USD/CNY and EUR/USD. Figure 9 shows that the impact of gold is generally positive, where the strength of the effect increases over time. The effect of the exchange rate EUR/USD is generally stable over time around zero with a relatively small positive effect during the price peak at the end of 2017 and the beginning of 2018. However, since the beginning of 2019 the impact becomes negative, albeit small, with the exception of mid-year 2019 with a relatively small positive peak. In contrast with the EUR/USD exchange rate, the USD/CNY exchange rate has a relatively small and stable positive impact over time with two small positive peaks during the price peak at the end of 2017/beginning of 2018, and mid-year 2019. However, based on Figure 7 I conclude that in general the signs are indeterminate. Based on these results it seems that Bitcoin has its own characteristics as asset class. It does not behave like a regular stock due to the negative correlation with the S&P500, but instead it serves as a hedge for stocks. Bitcoin can also serve as an interesting asset to create extra diversification in an investment portfolio consisting of stocks. In general gold shares these same characteristics and it has a well-recognized status of being a safe haven.

Some literature (Bouri et al. (2017), Dyhrberg (2016)) argues that in general many economists have compared Bitcoin to gold as they have many similarities. For example, both assets are scarce in terms of supply and costly to extract. I measure Bitcoin's scarcity by using the stock-to-flow variable, which is positively correlated with Bitcoin's market price according to Table 2. Figure 9 shows that the impact of the stock-to-flow variable is generally positive and stable over time where especially from mid-year 2019 onwards the effect is steadily increasing. However, gold only has a relatively small positive correlation with Bitcoin's market price, which is insufficient evidence to conclude that Bitcoin behaves exactly the same as gold. I also find that the VIX index, which is considered as the level of uncertainty, has a relatively strong negative correlation with Bitcoin's market price with a high inclusion probability according to Table 2. Figure 9 shows that the impact of the VIX index is generally negative, where the effect decreases steadily since mid-year 2016 until the beginning of 2018. From the beginning of 2018 until mid-year 2019 the effect of the VIX index becomes less negative, but from mid-year 2019 onwards the effect becomes more negative again to a similar level as the beginning of 2018. In general stocks are also negatively correlated with the VIX index, so Bitcoin can not serve as safe haven just like gold in times when there is much uncertainty.

Lastly, I find that the search trend for Bitcoin has a weak impact on Bitcoin's market price with a relatively low inclusion probability according to Table 2. In general the sign of the coefficient is indeterminate. This can be due to the fact that people search for Bitcoin during situations when either the price goes up or down. Another explanation can be the fact that people search for Bitcoin, but that they do not actively participate in the markets by buying or selling Bitcoin. Figure 9 shows that the impact of the search trend for Bitcoin is stable over time, where it has a relatively small positive impact during the first couple of years until the end of 2017. From the beginning of 2018 onwards the effect is generally negative, but with the exception of the first half of 2019 with a relatively strong positive impact. As already mentioned, the sign of the effect is generally indeterminate. I find evidence that the effect becomes more positive or negative when Bitcoin's market price makes relatively large movements either upward or downward. For example, when Bitcoin's market price decreases at the end of 2018 the effect of the search trend becomes more negative. This also applies on the crash of the price in March 2020. When the price increases during the first half of 2019 the effect of the search trend becomes more positive. Hence, if people search more for Bitcoin this can apply on situations when either the price goes up or down rapidly.

4.2 Sensitivity Analysis

As robustness check I consider a static and dynamic version of the BSTS model where the error terms follow a Student-t distribution instead of the normal distribution. Figure 1 shows that the exchange rate Bitcoin with US Dollar remains relatively flat during the first part of my sample period until the big rise at the end of 2017. From that date onwards the volatility of Bitcoin's market price increases much and the price exhibits many upward and downward movements. For this reason I also consider a static and dynamic version of the BSTS model where the error terms follow a Student-t distribution instead of the normal distribution. In particular, I assume that the error terms in the observation equation and in the transition equations for the trend component from Equation (3) follow a Student-t distribution, where I assume independent priors for the standard deviations and the tail thickness. In this way the BSTS model is robust against individual outliers, sudden persistent shifts in the mean of the time series and in the level or slope of the trend component. By using Student-t distributed error terms it is interesting to examine to what extent the results differ compared to the BSTS models with Gaussian error terms. However, the Kalman filter and the Spike-and-slab prior require observations and state variables to be Gaussian, so I express the Student-t distributed error terms as conditionally Gaussian by using data augmentation to solve this issue.

Firstly, I fit the BSTS model that contains a static regression component $\beta' x_t$ and Student-t distributed error terms, where I use the Spike-and-slab prior. I sample from the posterior distribution of the BSTS model to obtain the parameter estimates by using 10,000 MCMC iterations, discarded the first 1,000 iterations as burn-in sample. Secondly, I fit the BSTS model that contains a dynamic regression component $\beta_t' x_t$ and Student-t error terms. I sample from the posterior distribution of the BSTS model to obtain the parameter estimates by using 10,000 MCMC iterations, discarded the first 1,000 iterations as burn-in sample.

Figure 10 and 11 show the posterior distribution of the conditional mean $Z_t' \alpha_t$ from Equation (1) given the full data of the static Student-t BSTS model and the dynamic Student-t BSTS model, respectively. The blue circles represent the raw observations. In general both BSTS models are correctly fitted to the raw data. Figure 12 and 13 show the contributions of the individual state components of the static Student-t BSTS model and the dynamic Student-t BSTS model in explaining the variation of Bitcoin's market price, respectively.

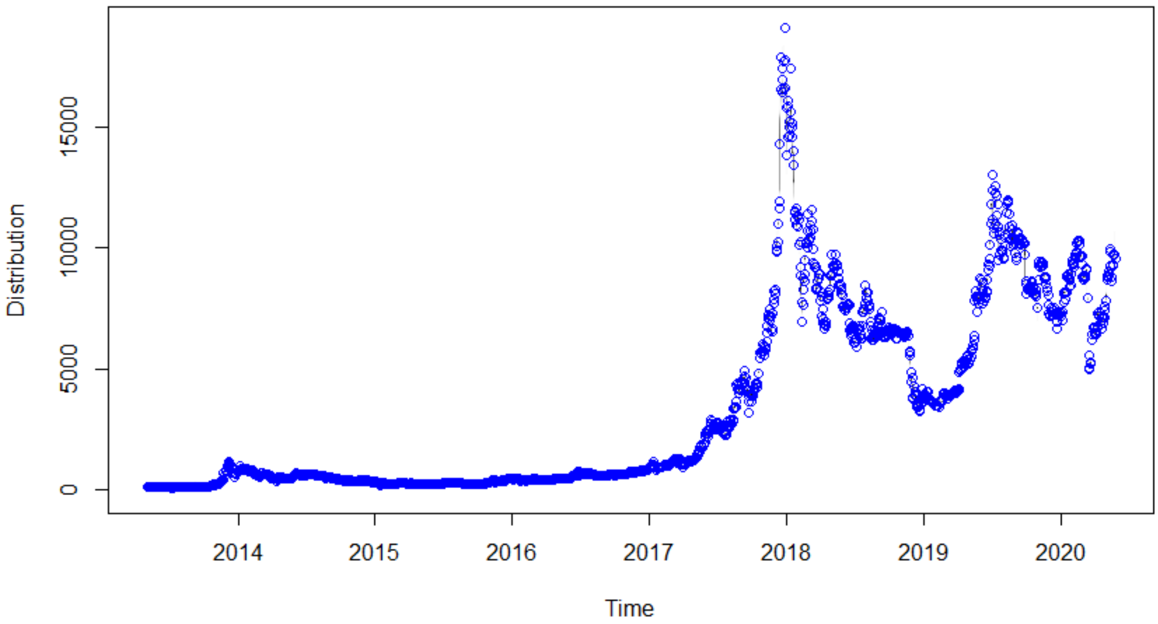

Figure 10: Posterior distribution at each point in time of the static Student-t BSTS model state

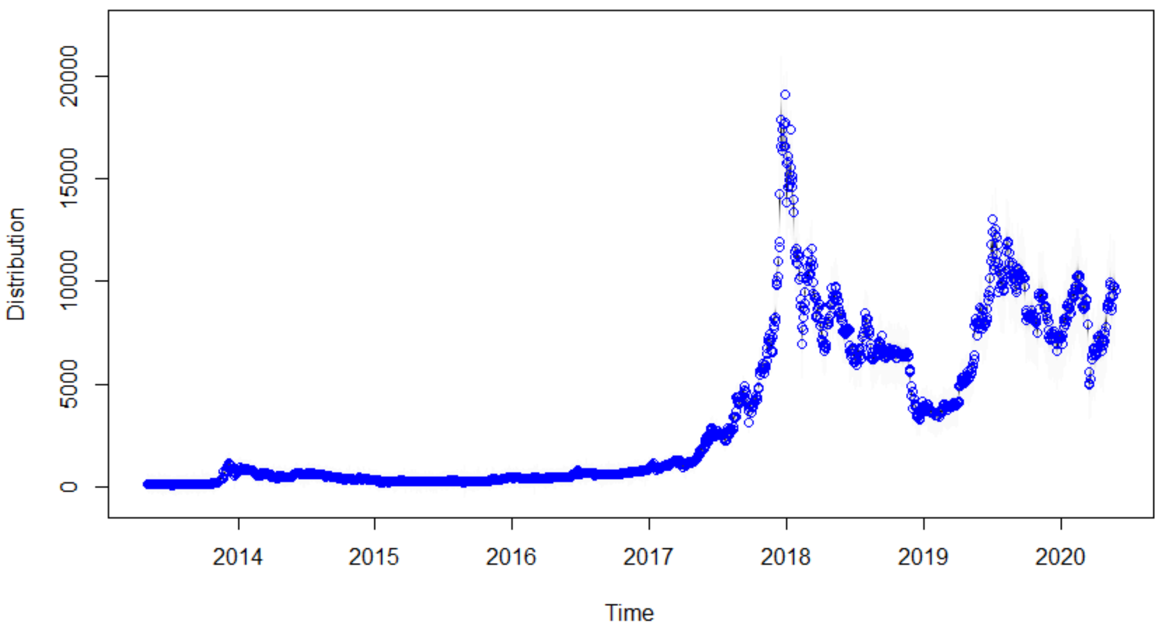

Figure 11: Posterior distribution at each point in time of the dynamic Student-t BSTS model state

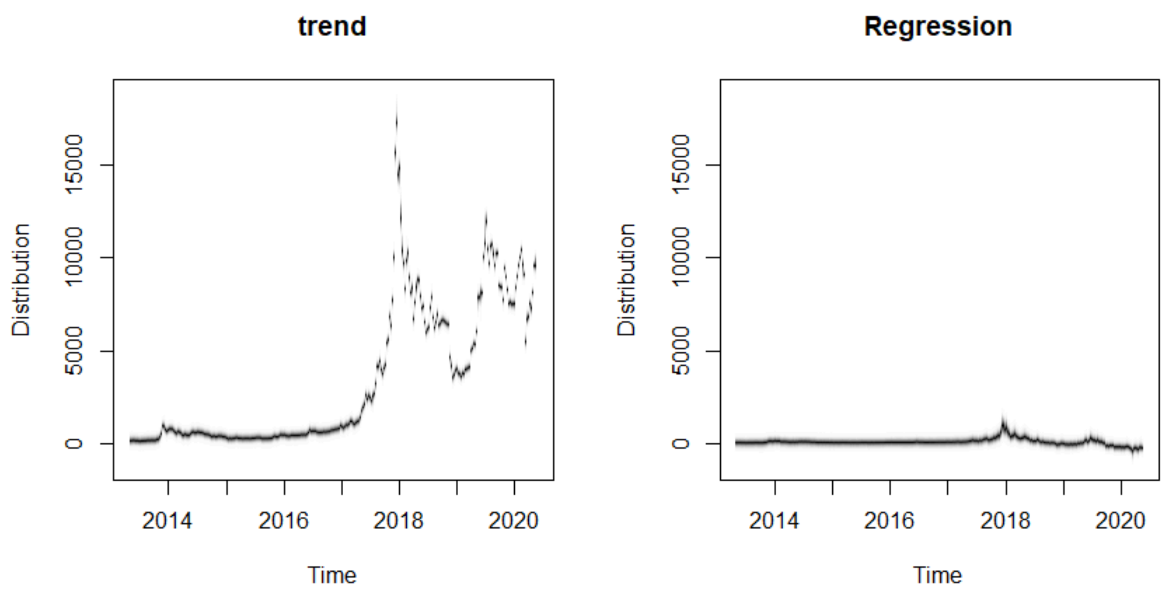

Figure 12: Contributions of the individual state components of the static Student-t BSTS model

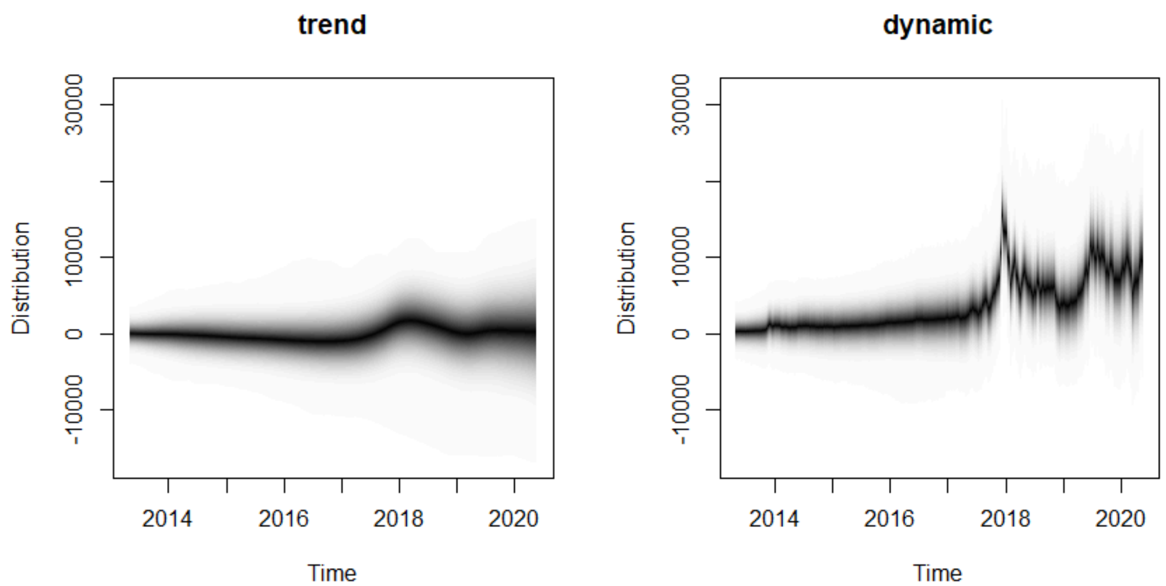

Figure 13: Contributions of the individual state components of the dynamic Student-t BSTS model

Figure 12 shows that almost all variation in Bitcoin's market price is explained by the trend component. The regression component only explains a small part of the variation during the price peak at the end of 2017/beginning of 2018, the collapse that followed and the price peak in 2019. By introducing Student-t distributed error terms the trend component becomes more robust against the volatile nature of Bitcoin's market price, especially during the second half of my sample period. Hence, the trend component captures more variation in Bitcoin's market price than the regression component. This is a big difference compared to the static Gaussian BSTS model, where both the trend and regression component capture a substantial amount of variation in Bitcoin's market price. Figure 13 shows that the dynamic regression component explains almost all variation in Bitcoin's market price. The trend component explains part of the variation during the price peak at the end of 2017 and beginning of 2018, and the collapse that followed. In comparison with the dynamic Gaussian BSTS model the results look generally similar.

Figure 14 shows the assigned posterior inclusion probabilities of all explanatory variables for the static Student-t BSTS model and Table 3 shows the posterior mean, the 95% highest posterior density interval (HDI) and the posterior inclusion probabilities for all coefficients of the explanatory variables. Figure 15 shows the posterior distribution of the static Student-t BSTS model size, where the mean and median of the number of explanatory variables to include in the model are around 12 and 11 out of the 15 explanatory variables in total, respectively.

Figure 16 shows the dynamic contribution of each explanatory variable of the dynamic Student-t BSTS model. The results are similar with the results of the dynamic Gaussian BSTS model, so in general the same interpretation holds. However, there are some small differences where for example the effect of a certain variable is relatively smaller. This holds for gold, but it is especially clear for the VIX index.

Figure 14 and Table 3 show that Bitcoin's difficulty has a negative impact with the highest inclusion probability. Bitcoin's hash rate also has a negative impact, although less strong, with a relatively high inclusion probability. Besides, Bitcoin's market capitalization and miners revenue have a positive impact on Bitcoin's market price with a relatively high inclusion probability. These results are similar compared to the static Gaussian BSTS model.

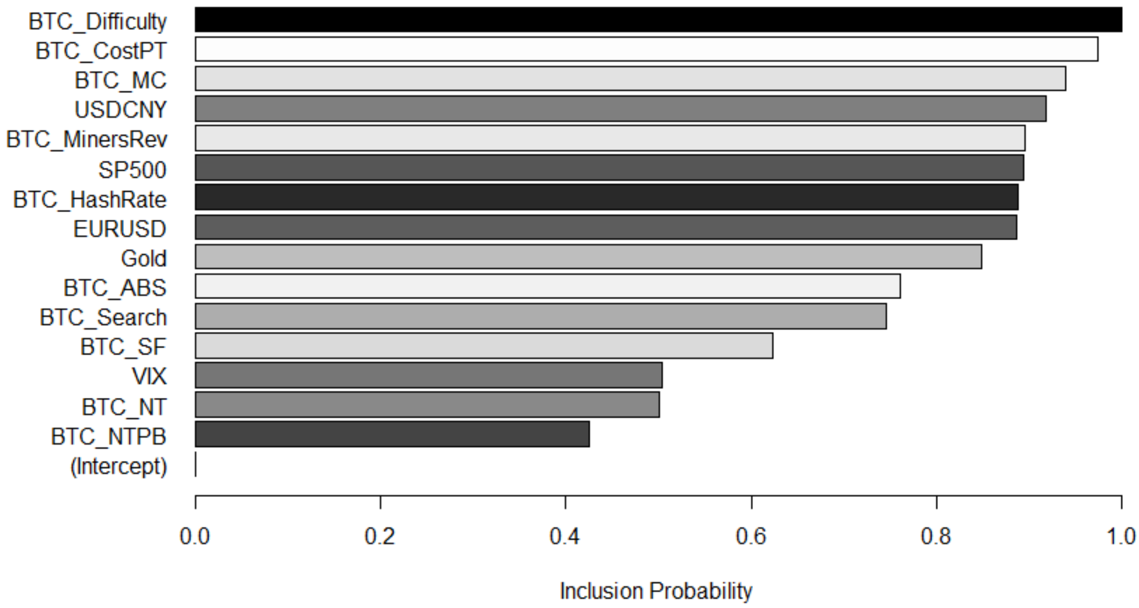

Figure 14: Posterior inclusion probabilities of all explanatory variables for the static Student-t BSTS model

Table 3: Posterior parameter statistics of the static Student-t BSTS model

Variable	Mean	2.5% HDI	97.5% HDI	Inclusion Probability
BTC Difficulty	$-3.760 \cdot 10^{-11}$	$-8.974 \cdot 10^{-11}$	$-8.084 \cdot 10^{-12}$	1.000
BTC Cost per transaction	1.016	0.000	1.755	0.974
BTC Market capitalization	$1.692 \cdot 10^{-9}$	$-6.198 \cdot 10^{-10}$	$4.923 \cdot 10^{-9}$	0.940
USD/CNY	0.182	-44.423	44.519	0.919
BTC Miners revenue	$8.366 \cdot 10^{-6}$	$-2.759 \cdot 10^{-6}$	$2.497 \cdot 10^{-5}$	0.896
S&P500	-0.0139	-0.0950	0.0651	0.894
BTC Hash rate	$-1.297 \cdot 10^{-6}$	$-3.991 \cdot 10^{-6}$	$1.163 \cdot 10^{-6}$	0.888
EUR/USD	-33.675	-238.716	139.496	0.887
Gold	0.0313	-0.0515	0.152	0.848
BTC Average block size	18.596	-3.825	52.822	0.760
Search trend 'Bitcoin'	1.019	-2.630	5.367	0.745
BTC Stock-to-flow	0.00131	-0.00113	0.00560	0.624
VIX	0.000324	-1.356	1.148	0.505
BTC Number of transactions	$2.538 \cdot 10^{-6}$	$-9.106 \cdot 10^{-5}$	$1.055 \cdot 10^{-4}$	0.501
BTC Number of transactions per block	-0.00149	-0.0117	0.00549	0.426
(Intercept)	0.000	0.000	0.000	0.000

Note: The table shows the posterior parameter statistics, such as the posterior mean, the 95% highest posterior density interval (HDI) and the posterior inclusion probabilities of all explanatory variables for the static Student-t BSTS model.

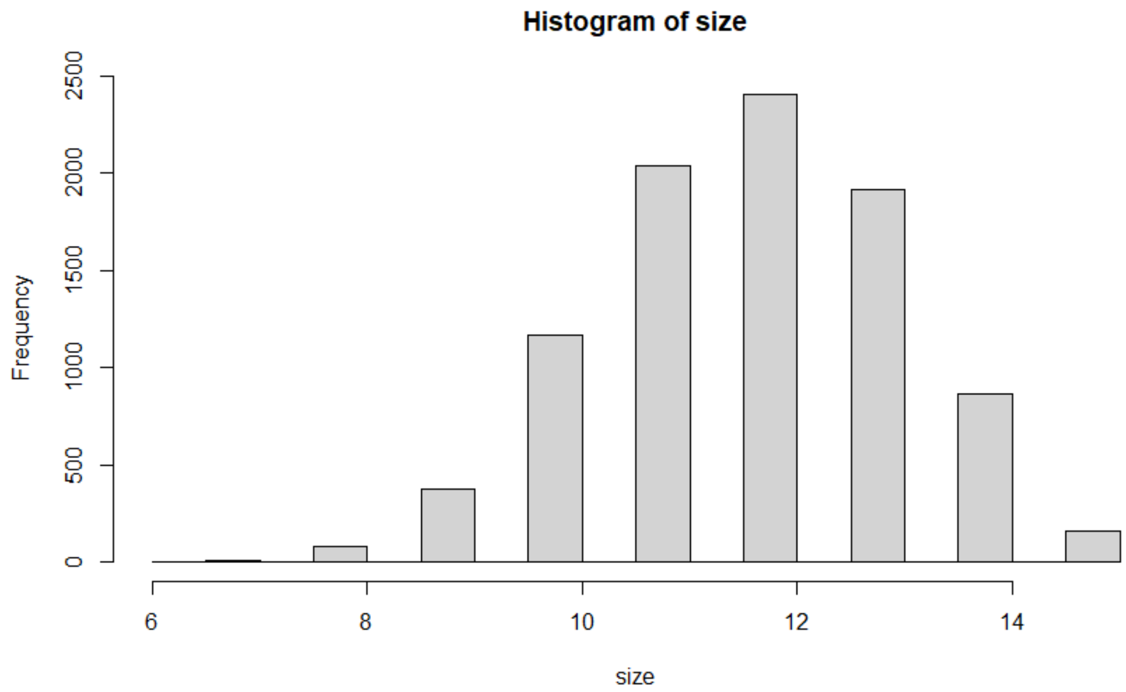

Figure 15: Posterior distribution of the static Student-t BSTS model size

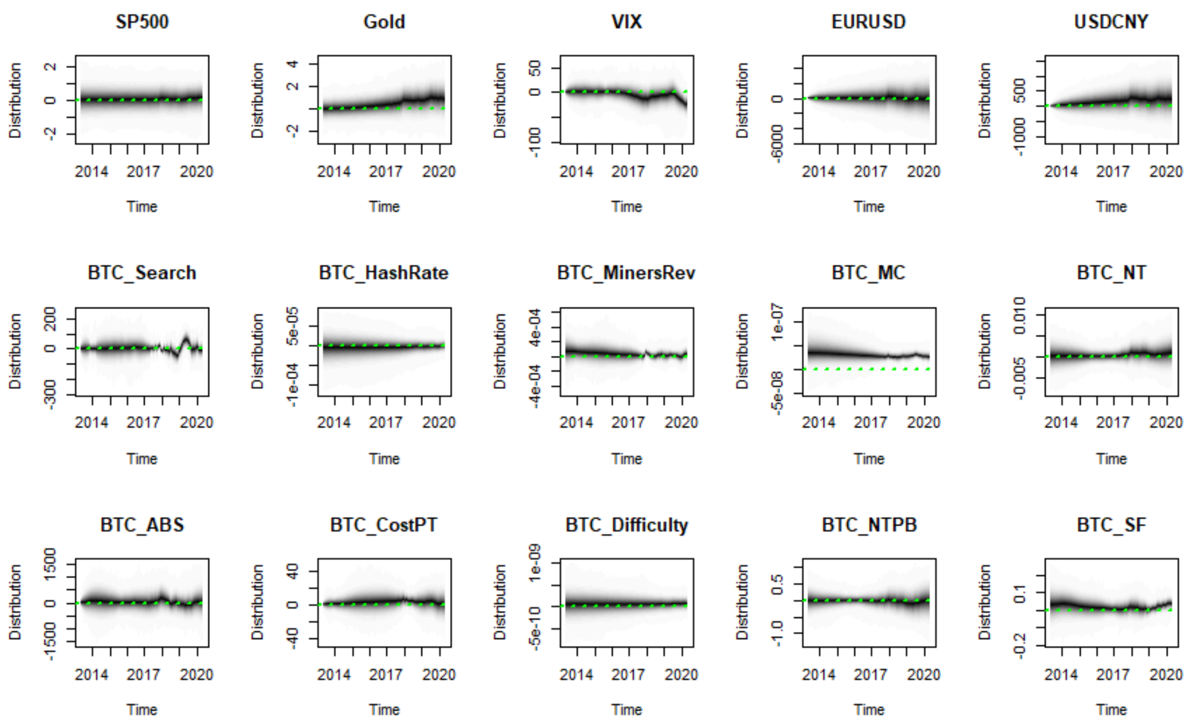

Figure 16: Dynamic contributions of the explanatory variables of the dynamic Student-t BSTS model

If I compare other variables with the results of the static Gaussian BSTS model, the effect of the S&P500 is negative and the effect of gold is positive, although less strong. The impact of the exchange rates is also less strong, where the effect of the EUR/USD exchange rate is even negative for the static Student-t BSTS model. However, the sign of the impact of the exchange rates remains generally indeterminate. For Bitcoin's stock-to-flow variable the impact is relatively weak positive and similar to the other model. However, there are also some differences compared to the static Gaussian BSTS model. For example, Bitcoin's cost per transaction has a relatively low inclusion probability in the static Gaussian BSTS model, but in the Student-t version it has a relatively high inclusion probability where the impact is positive. The variable Bitcoin's average block size is also part of the Student-t version, where the effect is positive. Furthermore, the search trend for Bitcoin has a higher inclusion probability in the Student-t version, where the effect is positive, but not very strong. According to its highest posterior density interval the sign of the effect is generally indeterminate which is also in line with the findings of the static Gaussian BSTS model.

Lastly, the VIX index has a lower inclusion probability in the Student-t version than in the Gaussian version. It indicates that there is no correlation between the VIX index and Bitcoin's market price which is a big difference compared to the Gaussian BSTS models in which the effect of the VIX index is strongly negative. In the dynamic case the impact is less negative during the end of 2017/beginning of 2018 and 2020 in the Student-t BSTS model than in the Gaussian version. By introducing Student-t distributed error terms, the BSTS model becomes more robust against sudden persistent moves in Bitcoin's market price. As a consequence, the VIX index loses part of its explanation power in the variation of Bitcoin's market price.

4.3 The Time-Varying Parameter Model

To examine the static and dynamic impact of the explanatory variables on Bitcoin's market price I estimate the TVP model. For this purpose I use the **shrinkTVP** package in R. I sample from the posterior distribution of the TVP model to obtain the parameter estimates by using 60,000 MCMC iterations with a thinning value of 10, discarded the first 10,000 iterations as burn-in sample. To check whether the Markov chain has converged, I use Geweke's convergence diagnostic (Geweke (1992)). This test is based on the equality of the means of the first 10% and last 50% of the iterations in the Markov chain. Figure 20 in Appendix A shows Geweke's convergence diagnostic for nine explanatory variables in the TVP model. For each variable I see the development of Geweke's Z-score when I discard larger numbers of iterations from the beginning of the Markov chain. I do not discard more than 50% of the iterations of the Markov chain to preserve the asymptotic conditions. For all variables almost all Geweke's Z-scores do not exceed the critical values, which are indicated by the dashed lines in Figure 20. Based on these results I conclude that the Markov chain has converged.

Table 4 shows the posterior mean, the 95% highest posterior density interval (HDI) and the posterior inclusion probabilities for all static coefficients of the explanatory variables in the TVP model. Figures 17-19 show the dynamic contributions of all explanatory variables in the TVP model. The black line in the figures represents the posterior median at each point in time. Furthermore, the shaded areas represent the pointwise 95% and 50% posterior credible intervals.

Based on Figures 17-19 the S&P500, Bitcoin's search trend, market capitalization and difficulty have a clear dynamic impact on Bitcoin's market price. The S&P500 has a positive impact over time, where the effect gradually increases until the end of 2017/beginning of 2018. After this period the positive impact of the S&P500 remains relatively constant over time. This is different compared to the effect of the S&P500 in the BSTS model, where it is generally negative and stable over time. However, in the BSTS model the impact of the S&P500 becomes positive more recently, namely since mid-year 2019 onwards. For Bitcoin's search trend the effect fluctuates around zero, although it is generally small positive over time. The effect becomes a little bit more positive during Bitcoin's price peak at the end of 2017/beginning of 2018 and mid-year 2019. In general this effect is comparable to the effect in the BSTS model. The impact of Bitcoin's market capitalization is positive over time, where the effect gradually decreases until mid-year 2017. In the second half of 2017 the impact increases again during Bitcoin's price surge in this period, but it collapses thereafter. Only during Bitcoin's price peak in 2019 the effect increases a little bit again, but in general the impact remains relatively constant on a positive level. This effect is in line with the findings of the BSTS model. Bitcoin's difficulty also has a positive impact over time, where the effect gradually increases until the end of 2017/beginning of 2018 followed by a decrease until the end of 2018. From the beginning of 2019 onwards the positive effect remains relatively constant with a small surge during Bitcoin's price peak at mid-year 2019. The dynamic effect of Bitcoin's difficulty in the BSTS model is also positive over time, although it remains relatively constant over time.

Table 4 shows that the other explanatory variables have a constant effect on Bitcoin's market price over time with high inclusion probabilities. Gold has a small positive impact on Bitcoin's market price. The effect is even smaller than the impact of gold in the BSTS model. For the exchange rates there is a clear difference compared to the BSTS model. In the TVP model the exchange rate USD/CNY has a relatively small negative impact, while this effect is positive in the BSTS model. Besides, the exchange rate EUR/USD has a positive impact in the TVP model, while the effect is negative in the BSTS model. However, in general the sign of the effects of the exchange rates is indeterminate in the TVP model and this is also the case for the BSTS model. The VIX index has a relatively small negative impact, where there is almost no correlation between the VIX index and Bitcoin's market price. This finding is comparable with the BSTS model.

Table 4: Posterior statistics of the static parameters in the TVP model

Variable	Mean	2.5% HDI	97.5% HDI	Inclusion Probability
BTC Difficulty	$4.200 \cdot 10^{-11}$	$-6.000 \cdot 10^{-11}$	$6.290 \cdot 10^{-10}$	1.000
BTC Cost per transaction	0.244	-0.137	2.225	1.000
BTC Market capitalization	$6.165 \cdot 10^{-8}$	$3.043 \cdot 10^{-8}$	$9.683 \cdot 10^{-8}$	1.000
USD/CNY	-0.0427	-0.0488	0.0472	1.000
BTC Miners revenue	$3.458 \cdot 10^{-7}$	$-4.879 \cdot 10^{-7}$	$9.180 \cdot 10^{-6}$	1.000
S&P500	0.00540	-0.00174	0.0320	1.000
BTC Hash rate	$1.853 \cdot 10^{-8}$	$-1.655 \cdot 10^{-7}$	$1.001 \cdot 10^{-7}$	1.000
EUR/USD	0.238	-0.0603	0.0590	1.000
Gold	0.00110	-0.00164	0.0315	1.000
BTC Average block size	10.095	-4.412	15.672	1.000
Search trend 'Bitcoin'	0.307	-0.401	1.340	1.000
BTC Stock-to-flow	0.000205	-0.000169	0.000839	1.000
VIX	-0.0743	-0.0221	0.0248	1.000
BTC Number of transactions	$1.086 \cdot 10^{-6}$	$-1.500 \cdot 10^{-5}$	$5.556 \cdot 10^{-6}$	1.000
BTC Number of transactions per block	0.000351	-0.00246	0.00170	1.000
(Intercept)	0.352	-1.202	2.349	1.000

Note: The table shows the posterior statistics, such as the posterior mean, the 95% highest posterior density interval (HDI) and the posterior inclusion probabilities of all explanatory variables for the static parameters in the TVP model.

The internal variables Bitcoin's cost per transaction, miners revenue, average block size, stock-to-flow and number of transactions have a positive impact on Bitcoin's market price. The effects are comparable with those in the BSTS model. However, in the TVP model Bitcoin's hash rate has a positive impact, while the effect is negative in the BSTS model. Lastly, the TVP model does not contain a trend component as in the BSTS model, so there is a positive intercept term in this case while the BSTS model does not contain an intercept term.

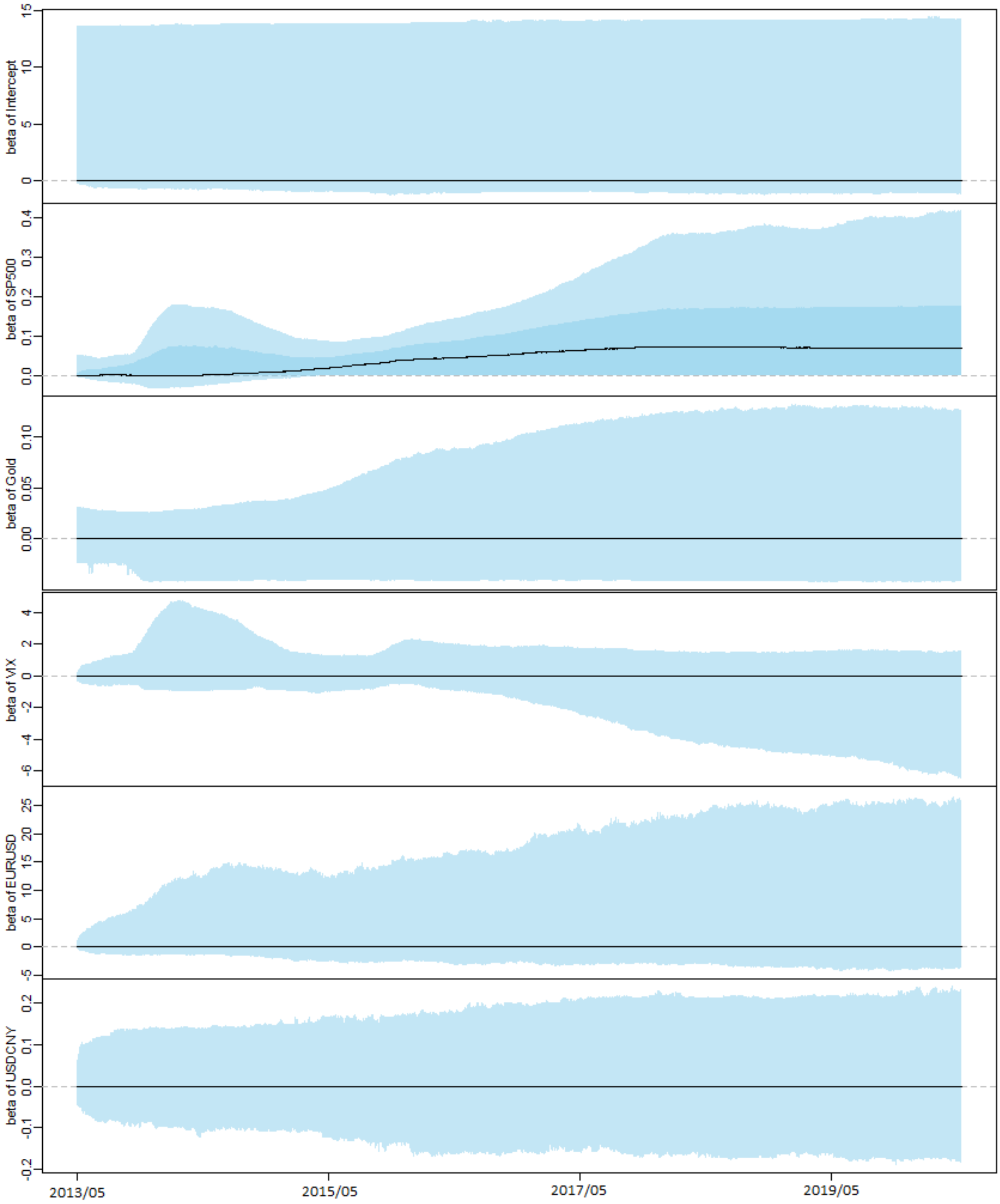

Figure 17: Dynamic contributions of the intercept, S&P500, Gold, VIX index and the exchange rates EUR/USD, USD/CNY of the TVP model

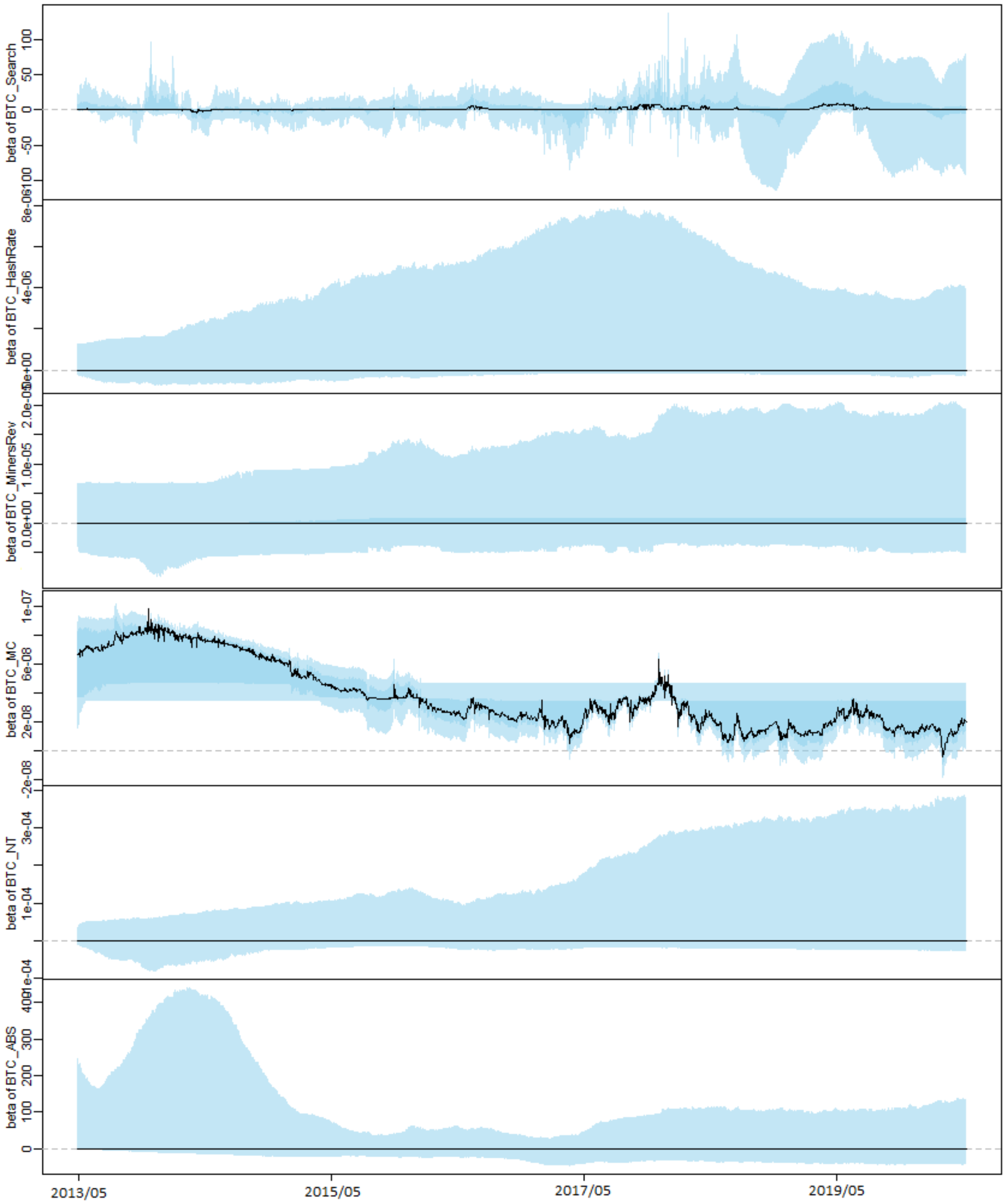

Figure 18: Dynamic contributions of Bitcoin's search trend, hash rate, miners revenue, market capitalization, number of transactions and average block size of the TVP model

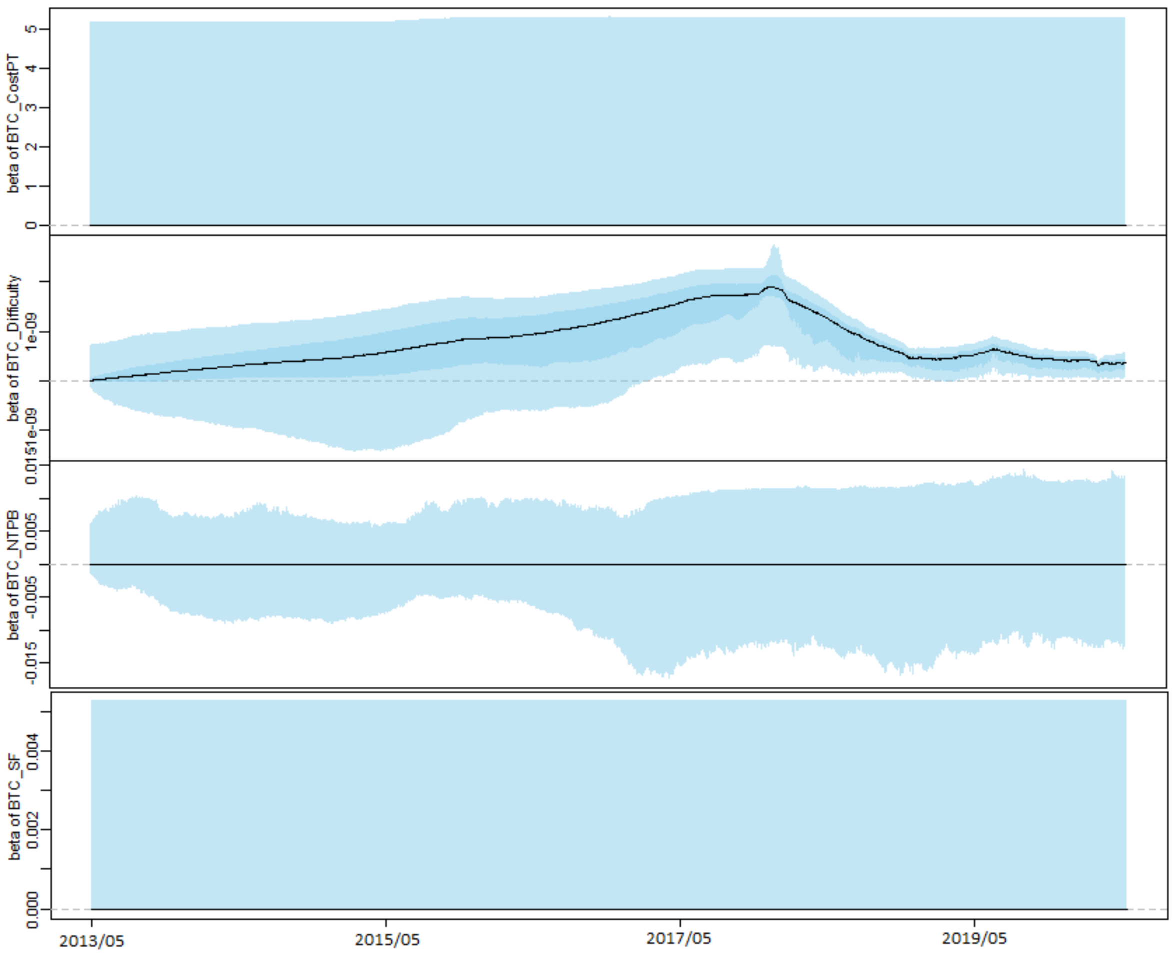

Figure 19: Dynamic contributions of Bitcoin's cost per transaction, difficulty, number of transactions per block and stock-to-flow of the TVP model

4.4 Prediction Performance

In this subsection I evaluate the out-of-sample prediction performance of several versions of the BSTS model. I consider BSTS models with a local linear trend (LLT) which contain a static or dynamic regression component, but also naive versions which do not contain a regression component. In this way I can examine the predictive power of the explanatory variables in predicting Bitcoin's market price. Furthermore, I consider BSTS models where the error terms follow a normal distribution or a Student-t distribution. Lastly, I examine the out-of-sample prediction performance of the TVP model which contains both static and dynamic regression components.

To evaluate the out-of-sample prediction performance I divide the data in a training sample and a test sample. The training sample runs from May 2013 until March 2020. The test sample contains the month April 2020. Furthermore, I use a rolling window with a size of 1725 observations to get the predictions for the forecast horizons $h = 1, \ldots, 5$. Subsequently, I use the three metrics in Equations (15) - (17), namely the mean squared forecast error (MSFE), mean absolute forecast error (MAFE) and the mean correct prediction (MCP), respectively, to evaluate the prediction performance of all models for each forecast horizon. For the MSFE and MAFE values I report the relative measures of the models compared to a benchmark model, which is the LLT Gaussian static model[8]. Subsequently, I use the Diebold-Mariano test to evaluate whether the differences in prediction performance of the models compared to the benchmark model are statistically significant. Table 5 shows the values of these metrics of the different BSTS and TVP models for each forecast horizon $h = 1, \ldots, 5$. Regarding prediction performance, the model with the lowest MSFE/MAFE value and the highest MCP value is preferred. For the relative measures this means that a model has a better prediction performance than the benchmark when the relative MSFE/MAFE value is smaller than 1. When the relative MSFE/MAFE value is bigger than 1 the benchmark model outperforms the other model in terms of prediction performance.

Based on the relative MSFE and MAFE values in Table 5 the BSTS model that contains a static regression component and Gaussian error terms outperforms all other models in predicting Bitcoin's market price for all forecast horizons, although the outperformance is not statistically significant in all cases. It is clear that by including predictor variables to the BSTS model I can increase the precision in predicting Bitcoin's market price. For example, the relative MSFE and MAFE values of the naive versions of the BSTS models increase relatively fast when the forecast horizon increases. The differences in prediction performance between the naive versions of the

[8] In a pre-analysis I found that the LLT Gaussian static model performs best in terms of MSFE and MAFE, which is the reason why I use this model as benchmark.

BSTS models and the benchmark are also statistically significant in general, except for predicting Bitcoin's market price one trading day ahead. However, in terms of the relative MSFE value the difference in performance of the LLT Gaussian naive model is not statistically significant compared to the benchmark model for predicting the price five trading days ahead. This is also the case for the LLT Student-t naive model for predicting Bitcoin's market price two and three trading days ahead. Besides, the local linear trend component in the BSTS model adds prediction power. For example, the TVP model does not contain a trend component, but it only contains a static and dynamic regression component. In general, based on the relative MSFE and MAFE values the TVP model does not belong to the best models in predicting Bitcoin's market price. Furthermore, the difference in prediction performance between the TVP model and the benchmark model is statistically significant, except for predicting Bitcoin's market price one and five trading days ahead. After the LLT Gaussian static BSTS model, the BSTS model that contains a dynamic regression component and Student-t distributed error terms generally performs best for most forecast horizons in terms of relative MSFE and MAFE values. The difference in prediction performance between this model and the benchmark model is also not statistically significant, except for predicting Bitcoin's market price two trading days ahead in terms of relative MAFE. The LLT Gaussian dynamic model also performs well in general, but the benchmark model has a statistically significant outperformance in predicting Bitcoin's market price two, four and five trading days ahead in terms of relative MAFE. Lastly, the LLT Student-t static model has a relatively weak performance in terms of relative MSFE and MAFE values, where the benchmark model has a statistically significant outperformance in predicting Bitcoin's market price for almost all forecast horizons.

Table 5: Relative MSFE, relative MAFE and MCP values of the models for 5 forecast horizons

Panel A: Relative MSFE	Forecast horizon				
	h = 1	h = 2	h = 3	h = 4	h = 5
LLT Gaussian Naive	1.20	2.07***	2.84**	3.79*	4.06
LLT Gaussian Dynamic	1.33	2.18	2.42	2.74	2.66
LLT Student-t Naive	1.10	1.84	2.10	2.84**	3.41***
LLT Student-t Static	1.36	2.11*	1.98***	2.81**	2.47**
LLT Student-t Dynamic	1.07	1.24	1.33	1.39	1.30
TVP	1.13	1.78*	2.40**	2.86**	2.73
Panel B: Relative MAFE	**Forecast horizon**				
	h = 1	h = 2	h = 3	h = 4	h = 5
LLT Gaussian Naive	1.11	1.81***	1.82***	2.11**	2.11*
LLT Gaussian Dynamic	1.14	1.59**	1.44	1.60**	1.68**
LLT Student-t Naive	1.25	1.73***	1.54**	1.79**	1.93***
LLT Student-t Static	1.36*	1.79***	1.55***	1.70**	1.59**
LLT Student-t Dynamic	1.14	1.22*	1.12	1.14	1.19
TVP	1.07	1.55**	1.65***	1.82**	1.70
Panel C: MCP	**Forecast horizon**				
	h = 1	h = 2	h = 3	h = 4	h = 5
LLT Gaussian Naive	0.35	0.60	0.50	0.45	0.45
LLT Gaussian Static	**0.70**	0.55	0.50	0.55	0.55
LLT Gaussian Dynamic	0.65	**0.65**	**0.65**	**0.70**	**0.65**
LLT Student-t Naive	0.40	0.40	0.40	0.55	0.30
LLT Student-t Static	0.45	0.35	**0.65**	0.55	0.55
LLT Student-t Dynamic	0.60	0.60	0.55	0.60	0.60
TVP	0.50	0.55	0.40	0.60	0.45

Note: The table shows the relative mean squared forecast error (relative MSFE; Panel A) and relative mean absolute forecast error (relative MAFE; Panel B), where I use the LLT Gaussian Static model as benchmark. For significance I use the following notation: ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$. It also shows the mean correct prediction (MCP; Panel C), where the highest value is represented in bold for each forecast horizon $h = 1, \ldots, 5$.

Table 5 shows that based on the MCP values the BSTS model that contains a dynamic regression component and Gaussian error terms outperforms all other models for almost all forecast horizons. Only for predicting one trading day ahead the LLT Gaussian static BSTS model performs better. For almost all forecast horizons the LLT Gaussian dynamic BSTS model correctly predicts the direction of Bitcoin's price movement in 65% of the cases. For predicting the direction of Bitcoin's market price four trading days ahead this is even 70%. The LLT Gaussian static BSTS model, which was best in terms of MSFE and MAFE values, correctly predicts the direction of the price movement for all forecast horizons for at least 50% of the cases. It outperforms the other models in predicting one trading day ahead, where it correctly predicts the direction in 70% of the cases. The naive versions of the BSTS models also perform relatively poorly in terms of MCP values. It is clear that I can increase the prediction power of the BSTS models in predicting Bitcoin's market price and its direction by including predictor variables to the models. Besides, the BSTS models that contain Gaussian distributed error terms generally perform better in predicting the direction of Bitcoin's price movement than BSTS models with Student-t distributed error terms. Lastly, the local linear trend component in the BSTS models also increases the prediction power in predicting the direction of Bitcoin's price movement. For example, the TVP model does not contain a trend component, but only a static and dynamic regression component. Despite the fact that the TVP model allows for both static and dynamic predictor variables, it does not belong to the best models in predicting Bitcoin's market price. In short, the combination of a local linear trend with the inclusion of predictor variables increases the prediction power in predicting Bitcoin's market price.

5 Conclusion

In this paper I examine the main drivers of Bitcoin's market price and to what extent they have any predictive power. For this purpose I construct several BSTS models. I consider BSTS models which contain a static regression component or a dynamic regression component with Gaussian distributed error terms. As robustness check I also use BSTS models where the error terms follow a Student-t distribution. Lastly, I use a TVP model which contains both static and dynamic regression components.

To examine the main drivers of Bitcoin's market price I consider a set of 15 variables with data over the sample period 1 May 2013 until 20 May 2020. The set of variables is divided into internal and external variables.

Firstly, the internal variables capture the supply and demand characteristics of Bitcoin. In all BSTS models Bitcoin's hash rate has a relatively strong negative impact on Bitcoin's market price, but in the TVP model this effect is positive. Bitcoin's miners revenue and market capitalization have a positive impact on the price with a relatively high inclusion probability in all BSTS models and the TVP model. The effect of Bitcoin's market capitalization on the price is also very dynamic over time, where the effect is relatively strong positive during the beginning of the sample period, but steadily decreases thereafter to remain positive and stable in the second half of the sample period. For Bitcoin's number of transactions the inclusion probabilities are relatively low in all BSTS models, so I exclude this variable from the models. This variable does not have a meaningful contribution in explaining Bitcoin's market price. This also holds for Bitcoin's average block size and cost per transaction in the Gaussian BSTS models. However, for the TVP and Student-t BSTS models these variables have relatively high inclusion probabilities with a positive impact on Bitcoin's market price. In the static BSTS models Bitcoin's difficulty has a negative impact on Bitcoin's market price with a high inclusion probability. However, this effect is positive and stable over time in the dynamic BSTS models. In the TVP model the effect of Bitcoin's difficulty is very dynamic, where it remains positive over time. Bitcoin's number of transactions per block has a negative impact on Bitcoin's market price in all BSTS models, but the inclusion probabilities in these models are relatively low. Like Bitcoin's number of transactions also this variable has little power in explaining Bitcoin's market price. Bitcoin's stock-to-flow variable has a positive and relatively stable impact over time in all BSTS models and the TVP model.

Secondly, the set of external variables consists of macro-financial variables and variables which capture the attractiveness of Bitcoin. In all BSTS models the S&P500 has a negative impact on Bitcoin's market price with a high inclusion probability. However, the dynamic BSTS models show that the effect of the S&P500 is negative in the first couple of years of the sample period and that it becomes positive during the most recent years. In the TVP model the effect of the S&P500 is very dynamic. Over time the effect is positive which is different compared to the BSTS models where the effect is generally negative. However, also in the TVP model the effect becomes more positive in the most recent years of the sample period. The precious metal gold has a relatively small positive impact on Bitcoin's market price with a high inclusion probability in all BSTS models and the TVP model. The dynamic BSTS models indicate that the effect becomes more positive over time. In the Gaussian BSTS models the VIX index has a relatively strong negative correlation with Bitcoin's market price with a high inclusion probability. However, in the Student-t BSTS models the effect of the VIX index is much smaller and has a lower inclusion probability. Due to the introduction of Student-t distributed error terms the BSTS model becomes more robust against sudden persistent moves in Bitcoin's market price. Therefore, the VIX index loses part of its explanation power. Besides, in the TVP model the VIX index has a relatively small negative impact on Bitcoin's market price. The effect of the exchange rates EUR/USD, USD/CNY and Bitcoin's search trend is generally indeterminate in all BSTS models and the TVP model.

Based on these results I conclude that Bitcoin has its own characteristics as asset class, which makes Bitcoin interesting for diversification purposes. Due to the negative correlation with the S&P500 Bitcoin does not behave like a regular stock, but it serves as a hedge for stocks. However, this relationship has been changing recently as the correlation becomes more positive. This means that Bitcoin is going to behave more like stocks and that it loses power in being a hedge for stocks. Besides, Bitcoin has a relatively small positive correlation with gold. Both assets are scarce in terms of supply and costly to extract. Due to Bitcoin's limited supply its scarcity, measured by the stock-to-flow variable, has a positive impact on the market price. Although Bitcoin has some similar characteristics as gold, it does not behave exactly the same, because it does not serve as safe haven in times of uncertainty. This is explained by the VIX index which has a negative correlation with Bitcoin's market price.

I also evaluate the prediction performance of the different BSTS models and the TVP model. In terms of the relative MSFE and MAFE values the static Gaussian BSTS model outperforms all other models in predicting Bitcoin's market price for all forecast horizons. However, the outperformance is not statistically significant in all cases. In terms of the MCP values the dynamic Gaussian BSTS model outperforms all other models for almost all forecast horizons. The model correctly predicts the direction of Bitcoin's market price in 65% of the cases for almost all forecast horizons. For a forecast horizon of four trading days this is even 70%. The static

Gaussian BSTS model correctly predicts the direction of Bitcoin's market price in at least 50% of the cases for all forecast horizons. For a forecast horizon of one trading day this is even 70%. I conclude that I can increase the prediction power of the models in predicting Bitcoin's market price and its direction by including predictor variables and a local linear trend.

An important limitation of my research is that I only consider homoscedastic error terms in the observation equation of both the BSTS and TVP models. As a topic for further research it would be interesting to also consider heteroscedastic error terms in the observation equation of both the BSTS and TVP models. It is possible to model the heteroscedasticity by using a stochastic volatility specification. The stochastic volatility captures part of the variation in the error terms which may lead to better calculations of the model parameters and forecasts. Due to the volatile nature of Bitcoin's market price over time, it would be interesting to compare the results of the stochastic volatility specification with my current results. Besides, it is possible to repeat my research with an extended sample period. Bitcoin's market price exhibits much time-variation, so the impact of certain explanatory variables on the price can change over time. For example, I find that the effect of the S&P500 becomes more positive during the most recent years, so it would be interesting to know whether that continues to hold. Lastly, it is possible to include other explanatory variables in the analysis trying to detect other relevant relationships. For example, I find that the effect of Bitcoin's search trend is generally indeterminate. If people search more for Bitcoin this can apply on situations when either the price goes up or down rapidly. Therefore, it may be interesting to replace Bitcoin's search trend variable by two search trend variables focused on either an upward or downward trend.

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Appendix A Markov Chain Convergence Diagnostics

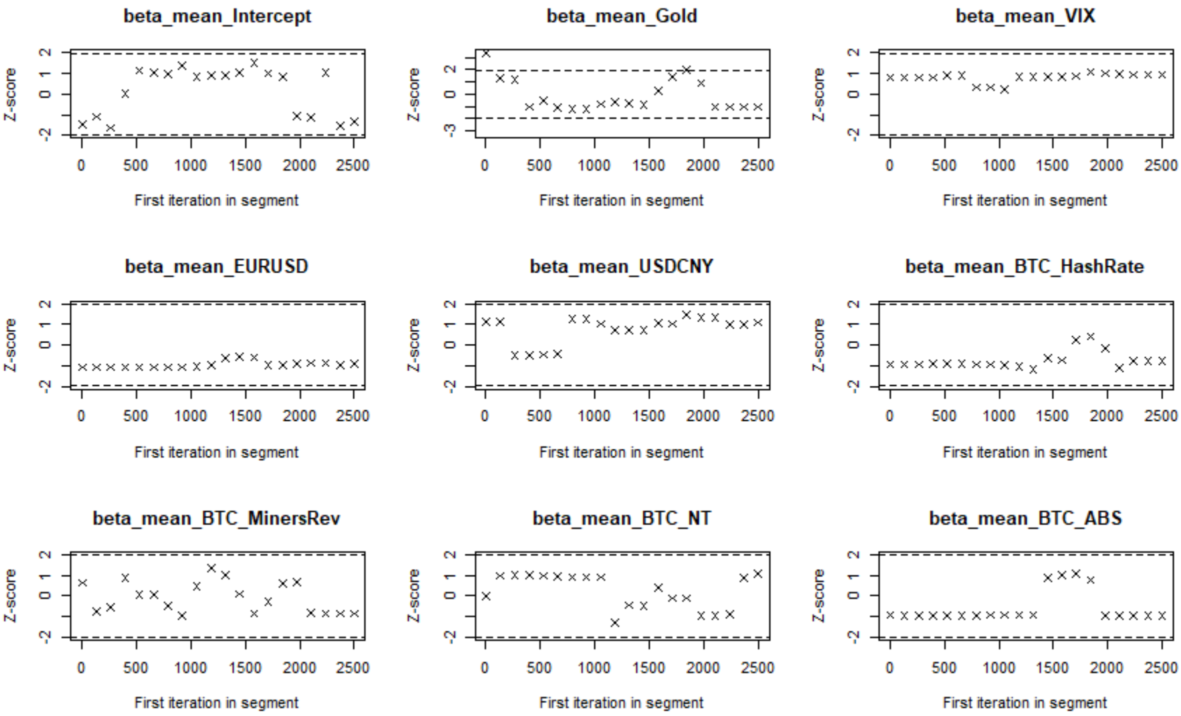

Figure 20: Geweke's convergence diagnostic for nine variables in the TVP model