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Master Thesis [Quantitative Finance]

Using macroeconomic information to forecast probability of default with regime-switching methods

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Abstract

In this paper, we study the relevance of macroeconomic information in probability of default modelling. As previous research states, probability of default estimates are likely to be dependent on the macro economy. Furthermore, under regulation companies must include macroeconomic information in their models. We show that using different combinations of macroeconomic variables can lead to a significant improvement in forecasting the probability of default with regime-switching methods, namely the Markov-Switching and Smooth Transition model.

Contents

Acronyms

AIC Akaike information criterion

BIC Bayesian information criterion

CPI consumer price index

EAD exposure at default

ECL expected credit loss

EPU economic policy uncertainty

ESG environmental, social and governance

FASB Financial Accounting Standards Board

GDP gross domestic product

IASB International Accounting Standards Board

IFRS 9 International Financial Reporting Standard 9

LGD loss given default

LLP loan loss provision

MS Markov switching

PCA Principal Components Analysis

PD probability of default

RMSE Root Mean Squared Error

STAR Smooth transition autoregressive

VIX Volatility Index

1 Introduction

In credit risk, probability of default (PD) is an important tool used by lenders to assess the risk of an obligor not paying back certain debt. For an obligor, a high risk of default can lead to high-interest rates for example. For the lender, a higher risk of default leads to needing more provisions for possible defaults. Modelling the PD provides both computational and regulatory challenges as many pieces of information need to be taken into account. Usually, lenders use company-specific qualitative and quantitative assessments to assign a credit rating to a loan that determines its quality. To model the PD from a credit rating, other factors are taken into account as well like the state of the economy and environmental, social and governance (ESG) factors.

While PD modelling is based on many aspects, the focus of this research lies in the incorporation of macroeconomic information. The reason for this specific focus is that research like Koopman and Lucas (2005) conclude that company defaults may well be dependent on the state of the economy. Gavalas and Syriopoulos (2014) also suggest that shifts in the economy have a direct impact on the transition rates, which directly translates to the probability of default (see Section 3.2). Furthermore, Nickell, Perraudin, and Varotto (2000) research the dependence between transition matrices and business cycles. They conclude that the default probabilities depend strongly on the stage of the business cycle. Next to the literature, in the past few years, new accounting standards were introduced. Under these regulations, companies are required to take macroeconomic information into account. Even though these sources imply that the macro economy is a valuable source of information for PD modelling, there is no research into which specific macroeconomic variables can lead to the most accurate forecasts in company defaults.

In this paper, we investigate if the usage of different macroeconomic variables in regime-switching methods can lead to significant differences in forecasting the probability of default. And if so, can we conclude which variables are best to use. In these methods, the macroeconomic variables influence the output through both a parameter estimate and by influencing the regime at each point in time. We aim to obtain the best performing model with the lowest number of variables as possible as we do not want to overflow the models with information and the chosen methods do not perform well on many exogenous variables. We aim to draw conclusions by analyzing the in-sample fit, comparing the forecast performance and by analyzing the best performing models. We strive for a method that can predict company defaults as accurately as possible while following regulation and is applicable in practice. While this is a fine line to walk, this research tries to find this balance and tries to improve on simple and easy to use methods that are used often in practice and described in academic literature.

To learn the relevance of improving PD models, we can take a look at its main application which is in the calculation of expected credit loss (ECL). It tells how much money a lender is expected to lose in its portfolios because obligors could go into default. Going into default means that the obligor is unable to meet certain financial obligations. The PD indicates the likelihood of going into default over a certain time horizon. The PD is, next to exposure at default (EAD) and loss given default (LGD), one of the three main variables in ECL calculation. The EAD tells the value of the outstanding loan and the LGD gives the percentage of the exposure which is lost when a default happens. This research solely focuses on the PD as this variable has the most impact on the ECL and provides the most challenges as the EAD is usually the outstanding debt that shrinks each year according to payments and the LGD depends mostly on the type of debt and collateral which requires specific information for each loan which is out of the scope of this research. The International Accounting Standards Board (IASB) introduced the International Financial Reporting Standard 9 (IFRS 9) in 2018 and the Financial Accounting Standards Board (FASB) introduced an update of the United States Generally Accepted Accounting Principles (US GAAP) in 2021. These new accounting standards include the new expected-loss impairment framework. These accounting standards are largely relevant to a company as the provisions hold for ECL, the loan loss provision (LLP), are stated on the balance sheet and are compulsory to report under regulation. Changes on the balance sheet logically impact a company substantially. Gea-Carrasco (2015) provides an overview of the impact on the provisions and bank statements of a company. The main driver behind the new regulations was the global financial crisis where the previous framework did not hold up as provisions were only made as losses incurred. Therefore, the main focus points in these new regulations are that the calculations need to be forward-looking and to take the economic cycle into account. The regulations are principle-based and therefore do not specify what methods to use to incorporate these focus points.

The data used in this research consists of non-retail company default transition rates and macroeconomic variables reaching from 1981 until 2020. We choose to include a variety of macroeconomic variables to cover commonly used variables, indices that track a certain part of the economy and financial conditions. Some of these variables are chosen because of previous research like Gavalas and Syriopoulos (2014) where they research the link between rating transitions and macroeconomic conditions. They use GDP and VIX as main macroeconomic indicators. Next to these variables, we use Principal Components Analysis (PCA) to take the key aspects of the ten variables and use the first two principal components as input variables in the models. Furthermore, we construct a macro factor using the Vasicek model.

We propose to use the following regime-switching methods to model the PD: the Markov switching (MS) and Smooth transition autoregressive (STAR). The first main consideration in choosing these methods is that the macroeconomic variables are multipurpose. They influence the output of the model both through an estimated parameter and influence the regimes at each point in time in these models. Therefore using different variables can lead to a different interpretation of regimes or different timing of the regimes. For different interpretations, you could think of expansions or contractions of the economy or of a low- and high volatility regime. With timing of the regimes, we mean that for one variable an expansion could start half a year earlier than for another variable.

Secondly, we strive to keep the models transparent. In practice, regulators will only approve of models for which the relation between input and output is visible and can be explained. Therefore, machine learning methods with possibly higher performance are not applicable as these methods are seen as 'black box' methodologies. In the MS and STAR model, parameter estimation and interpretation of regimes lead to transparent and understandable models.

Thirdly, one of the main drivers behind the new accounting standards was the global financial crisis. Regime-switching methods can account for financial crises by modelling a regime to these extreme financial conditions. Usually, defaults are more likely to happen in financial crises such as the dot-com bubble (1998-2002) and the global financial crises (2007-2009). The data supports this intuition as defaults spike in these periods (see Section 2, Figure 1).

Lastly, these methods also build upon previous research like Bangia, Diebold, Kronimus, Schagen, and Schuermann (2002) and Abad and Suarez (2018) as they use an MS model to split up the average credit transition matrix into an average for contractions and expansions of the economy. Then they use the average transition matrix corresponding to the current regime to predict the next year PD (See Section 3.2 for the relation between transition matrix and PD). At the time of their research, IFRS 9 did not exist and predicting more than one year-ahead PDs was not necessary. This framework was therefore useful then, however not since 2018. Companies are required to use methods that can predict the PD multiple years ahead. Also, the used method in their research is more simplistic. In this research, the goal is to use the MS model directly to forecast the PD. Furthermore, Jacobs Jr (2019) researches the use of regime-switching models to generate macroeconomic scenarios which can then be used in ECL modelling where the PD is one of the main variables. This research was conducted closely after the introduction of the new regulations and focuses on how to be forward-looking and how it compares to previous regulations. He concludes that regime-switching models work best for scenario generation compared to an autoregressive model. A shortfall of the research is that only a small part of ECL is investigated, namely the scenario generation. As they indicate future research can be done on PD models which could impact ECL more.

The MS model is first introduced by Hamilton (1989). Since then, this framework is used extensively in academic literature. Guidolin (2011) gives an overview of development in MS models in empirical finance and risk management. MS models are well suited to model the behaviour of business cycles or financial crises. An advantage of the MS model is that the interpretation of the regimes can be done after estimating the model and not on beforehand which leads to less misclassification. Also, the model has great interpretability afterwards where we can see what year accounts for which regime and what the probability of a regime switch is for either regime. These transition probabilities are assumed to be constant over time.

Teräsvirta (1994) introduced the smooth transition autoregressive (STAR) model. This model also has had, since then, many extensions and developments. See Hubrich and Teräsvirta (2013) for the most recent developments in STAR models. The most noticeable difference to the Markov models is that the regimes need to be defined beforehand. Next to that, a transition variable needs to be assigned that can capture the state of the economy best. Using the transition variable in the transition function, the model runs smoothly between the regimes. For example, GDP is a commonly used indicator for the state of the economy. Also, the type of transition function needs to be assigned. On the upside, by fine-tuning these specifications, we can arrive at the most suited model to the data set. However on the downside, when these choices are not made correctly, this can lead to misclassification.

We find that each credit rating has its own time series with different characteristics. Therefore, we can not draw conclusions over all the ratings together. For most credit ratings, we can find a preferred combination of variables with a method. For credit ratings BBB and B, the inclusion of the 3-month interest rate leads to the best performing models. For these ratings, the best performing models have significantly lower Root Mean Squared Error (RMSE) values compared to other combinations of variables. For credit rating BB, we can not find one model which performs on multiple forecast horizons best. For credit rating CCC, we find that including the unemployment rate leads to most low RMSE values. Although these differences are not significant, we can see a pattern of low RMSE values when this variable is included.

What these results implicate, is that we can find significant differences in forecasting accuracy by using different macroeconomic variables. This shows the relevance of researching the best combination of variables for each implemented model. As previous research (Koopman and Lucas (2005), Gavalas and Syriopoulos (2014) and Nickell et al. (2000)) suggests that transition rates and PD may depend on the state of the macro economy is something we can support with this research. Unfortunately, we can not conclude that one set of variables is best across all rating and forecasting time horizons, however, we can conclude that certain variables like the 3-month interest rate and the unemployment rate can lead to the best performing models.

This paper is outlined as follows. Section 2 describes the data used in this research and the data sources. In Section 3, we provide background to the main application of PD in ECL and explain all the steps required to move from information to ECL. Section 4 specifies model building procedures for both the MS and STAR model including how forecasts can be obtained from them. Section 5 displays the results on the in-sample fit and insights, the out-of-sample performance comparison between all candidate models and an analysis of the best performing models. Lastly, Section 6 concludes and discusses possible future research.

2 Data

2.1 Defaults

For company defaults, data is extracted from the Standard & Poor's (S&P) Global Rating Default Study of 2020[1]. S&P is widely known as a credible source for company credit ratings and defaults. The data-set contains annual default rates in percentages on the history of 21,693 companies that S&P Global Ratings rated with their first rating between 1980 and 2020. These are global companies with a variety of industrials, utilities and financial institutions. Structured finance vehicles, public-sector issuers, and sovereign issuers were excluded from the study. The available data from this study only contains the transition percentage of defaults per credit rating category and not individual defaults of companies. Therefore, this research forecasts the default rate based on a credit rating and not on the information of individual companies. The advantage of doing so is that the results are applicable to portfolios of assets with similar characteristics and an equal credit rating. In practice, lenders already have their own models to assign a credit rating to an obligor. Therefore, this process is not included in this research. Also, this approach requires less data on individual companies which is not always available. The credit rating categories used from best to worst credible are AAA, AA, A, BBB, BB, B and CCC/C. As the study describes a default means either the date a debt payment was missed, the date a distressed exchange offer was announced or the date the debtor filed for or was forced into, bankruptcy. In the study, if an issuer defaults, they end its rating history at 'D'.

2.1.1 Annual frequency

Figure 1 shows the annual default rate per credit rating category reaching from 1981 until 2020. The default rate is displayed as a percentage with respect to the number of issuers in each credit rating category. What stands out is that defaults in the lowest credit rating category (CCC/C) are very volatile over the years with the highest peak of 49.46% in 2009 and lowest value in 1983 with a default rate of 6.67%. We can identify 3 crises periods in the defaults. The dot-com bubble (1998-2002), the global financial crises (2007-2009) and the start of the COVID-19 pandemic (2020). Defaults spike in these periods and drop quickly in the following years. From 1989 to 1992, defaults were also quite high during these four years, this could be due to the early 1990s recession. In

[1] https://www.spglobal.com/ratings/en/research/articles/210407-default-transition-and-recovery-2020-annual-global-corporate-default-and-rating-transition-study-11900573

2016, defaults also started to rise. The Standard & Poor's (S&P) Global Rating Default Study of 2016 indicate that this was due to high stress in global energy markets[2]. The middle credit rating categories (BBB, BB and B) follow some patterns of the CCC/C category with less variation over the years. High credit rating categories (AAA, AA and A) do not display much difference over the years with default rates equal to or close to 0%. Table 3 supports these statements when we look at the descriptive statistics. Notable is that the correlations between credit rating categories are no higher than 0.637. This shows that these are different time series and do not move simultaneously all the time. Next to that, in almost all years a lower credit rating leads to a higher percentage of defaults. The exceptions are in 1981 where only defaults were registered for category B. Moreover, in 1982 more defaults were observed in BB than in B, in 1994 and 2004 defaults were registered in A and not in BBB, in 1997 and 2019 more defaults took place in BBB than in BB. For these exceptions, differences are small and therefore seem possible to occur.

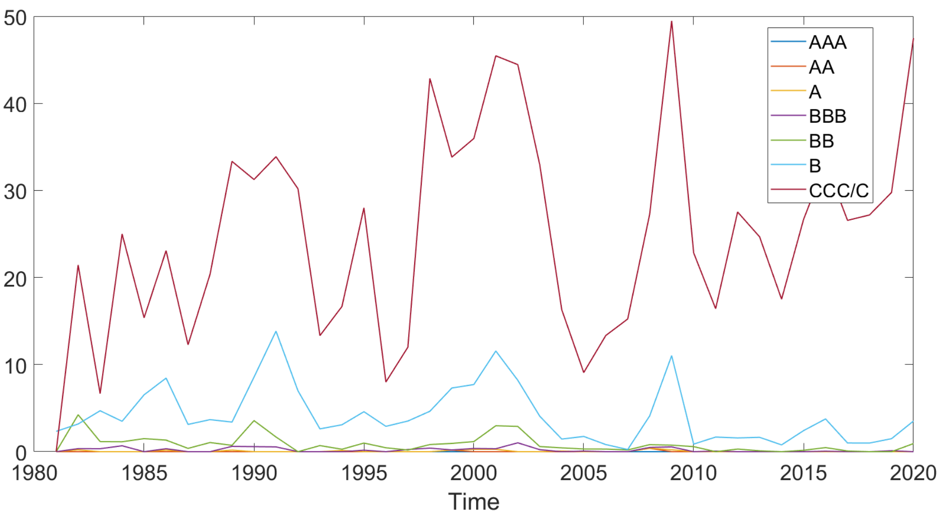

Figure 1: Yearly default rates per credit rating category from 1981 until 2020

Table 1 shows the one-year transition matrix averaged over 40 years. A transition matrix includes both the transition rates between credit ratings and the transition rate to default. To clarify, Figure 1 relates to column D in Table 1 over time instead of the average. In Table 1, we can see that in higher credit ratings companies are more likely to remain in the same category compared to lower rating categories. Furthermore, companies tend to receive more downgrades than upgrades in credit rating, except for the lowest credit rating. On the other hand, in the lowest

[2]https://www.spglobal.com/en/research-insights/articles/2016-annual-global-corporate-default-study-and-rating-transitions

credit rating, a company is on average more likely to go into default than receive an upgrade. Table 21, in Appendix C, also shows that on average 8.34% of companies upgrade and 11.85% downgrade in credit rating over time. This shows that companies are more likely to switch to lower credit ratings which consequently have a higher chance of going into default. When a company goes into default, we stop its trail there which is why the last row is absorbing which means it can not switch back to a credit rating.

Table 1: Average one-year transition matrix (%)

From/to	AAA	AA	A	BBB	BB	B	CCC/C	D
AAA	89.85	9.35	0.55	0.05	0.11	0.03	0.05	0.00
AA	0.50	90.77	8.09	0.49	0.05	0.06	0.02	0.02
A	0.03	1.67	92.61	5.23	0.27	0.12	0.02	0.05
BBB	0.00	0.10	3.45	91.93	3.78	0.46	0.11	0.17
BB	0.01	0.03	0.12	5.03	85.99	7.51	0.61	0.70
B	0.00	0.02	0.08	0.17	5.18	85.08	5.66	3.81
CCC/C	0.00	0.00	0.12	0.20	0.65	14.72	50.90	33.41
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00

Read the table from left to right, for example, the average default rate (D) when a company has credit rating A is 0.05%. The percentages in each row add up to 100%.

2.1.2 Quarterly frequency

The data set provides annual default rates for different ratings over time. However, to estimate the regime-switching models and to generate a significant number of forecasts, more data points are required. A data set at an annual rate from 1981 to 2020 includes 40 data points. A more preferable frequency would be quarterly data as this increases to 160 data points. Next to estimation and forecasting purposes, quarterly default rates are in practice also often used. This frequency is preferable because a lower frequency runs into the mentioned issues and a higher frequency can lead to many data points without any defaults. When a time series has too many zeros, the estimation algorithms for these models do not work. For this reason, we exclude the ratings AAA, AA and A from this study as the models can not deal with all the zeros in those time series.

To obtain a quarterly data set, we can follow a simple example with the knowledge that in the S&P study they work with a static pool methodology. Assuming that in year 1 20% defaults from rating category CCC/C for a group of 100 companies. This means that 20 defaulted in one year. By applying this to quarterly data, we can assume that in each quarter of the year on average 5% has defaulted. By calculating the average quarterly defaults for each year and inserting the value into the middle of the year (second quarter), we now have inserted 40 data points of 160 total. Next, we apply spline interpolation to fill in the other 3 quarters of the year. This gives a smooth graph connecting all these data points which are taken from the real data set. The average is inserted into the second quarter to balance out the total defaults. This method is applied to all credit rating categories. Figure 2 shows the resulting time series. The results of this paper are based on the resulting quarterly data set. As you could argue that this graph runs too smoothly to represent an actual graph, we test the robustness of the obtained results by simulating extra variance into this data-set (Section 5.5).

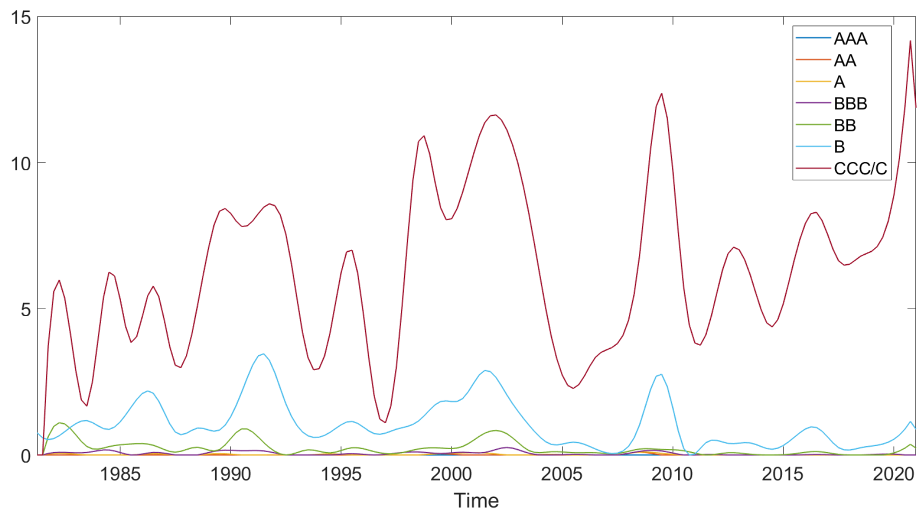

Figure 2: Quarterly default rates from 1981 Q1 until 2020 Q4

2.1.3 Simulated data

As in reality, the data set may possibly run not as smooth as Figure 2 displays, we run the results on the data set with simulated added variance and test the robustness of the outcomes. As this research includes many models on different credit ratings, we focus on the credit rating with the highest volatility, the rating CCC/C, and assume that the results also hold for other ratings. To do so, we first need a distribution for the default transition rates for this rating category. A logical option is to use the Normal distribution. To see whether the data is normally distributed we can apply multiple normality tests: the Anderson-Darling test, the Chi-square goodness-of-fit test and the Jarque-Bera test. Table 2 shows the test statistics, p-values and whether the null-hypothesis of normality is rejected for this credit rating. For all three tests, normality is significantly not rejected for a 5% significance level.

Table 2: Normality tests defaults rating CCC/C

	test statistic	p-value	reject normality
AD	0.272	0.675	No
chi2	2.150	0.542	No
JB	0.588	>0.5	No

With the test outcomes, we assume Normality for the defaults in category CCC/C. Figure 3 shows the histogram comparing the CCC/C defaults with a normal distribution. We can see that overall this fits the distribution well to some extent.

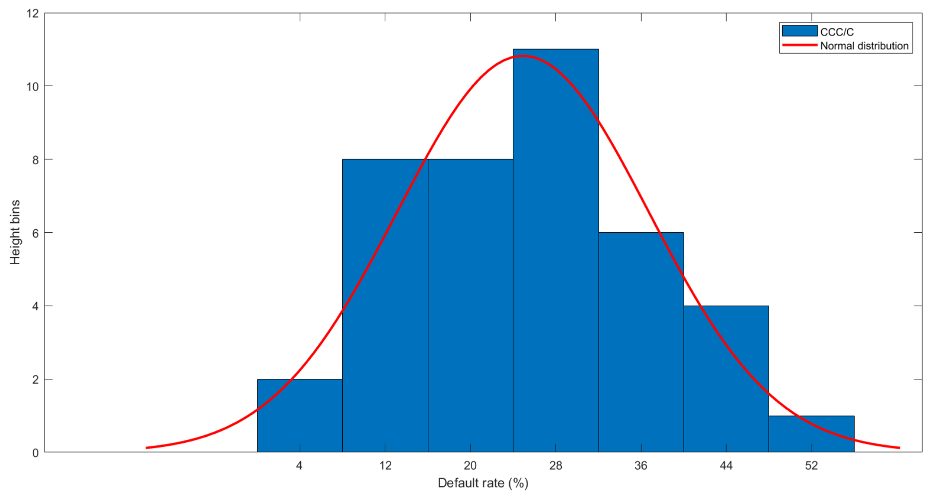

Figure 3: Normal distribution vs defaults in rating CCC/C

To simulate variance, we can draw from the Normal distribution with mean 0 and variance σ^2. We chose to draw the variance from the inverse gamma distribution with shape parameter $\alpha = 4$ and scale parameter $\beta = 1$ and use the resulting σ^2 to draw from the Normal distribution. The drawn value from the Normal distribution is added to the interpolated curve. Figure 4 shows both the interpolated curve with and without added simulated variances for one simulation.

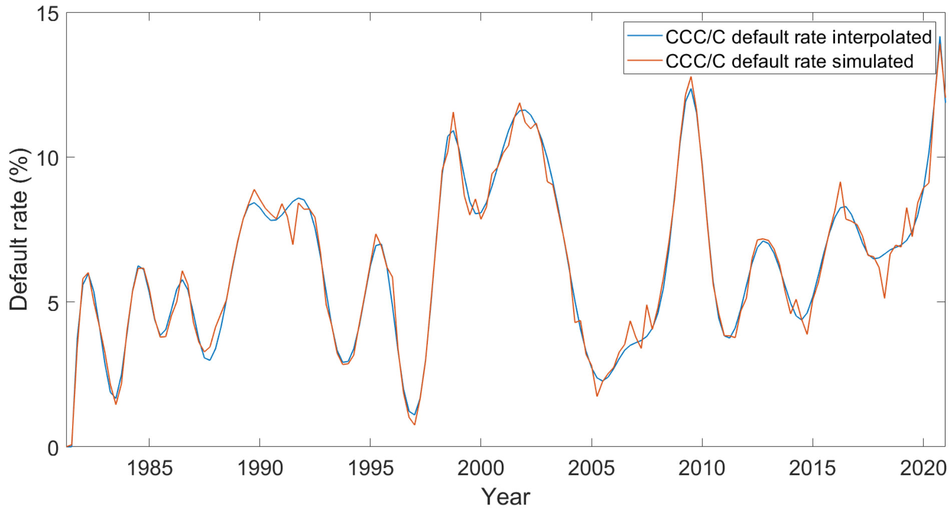

Figure 4: Defaults rating CCC/C with and without added simulated variance

2.2 Macro economy

To get a range of different macroeconomic variables, we can look at various types of variables. Firstly, real macroeconomic variables as these are commonly used and provide nice insights into the economy. These variables also have a large track record in academic literature. Secondly, we look at indices that track a certain aspect of the economy. These indices try to capture aspects of the economy which are not directly observable. Therefore, they could provide added value. Lastly, we consider interest rates. Interest rates do not display the same patterns as the aforementioned variables, although could influence the state of the economy. Therefore, they are also considered valid candidates. All variables are extracted from the Federal Reserve Economic Data (FRED) [3] which is a commonly used source for research on economic time series. These variables are extracted at a quarterly frequency which is accomplished by a built-in function by the FRED which averages data over the quarter. We use the quarterly data to match the quarterly default from the defaults. The data is based on the United States (US) since the annual default study by the S&P indicates that most-watched companies are based in the US, also most defaulted companies are from the US. The variables we use are gross domestic product (GDP), consumer price index (CPI), volume of imports, volume of exports, unemployment rate, the Volatility Index (VIX) the index of economic policy uncertainty (EPU), the Chicago Fed National Financial Conditions Index (NFCI), the 3-Month Treasury Constant Maturity Rate and the 10-Year Treasury Constant Maturity Rate. The

[3] https://fred.stlouisfed.org/

CPI is a way of tracking inflation measured by how expensive goods and services are for consumers. The VIX, first introduced in 1985, tracks the volatility of the market and economy and is, for example, used in Cerboni Baiardi et al. (2020) in a Markov switching method. The EPU index is based on economic policy uncertainty from newspapers. It is introduced by Baker, Bloom, and Davis (2016), where they also describe the methodology to obtain the index. The NCFI tracks financial conditions and is composed by the Federal Reserve Bank of Chicago. In Table 3, we can see the descriptive statistics and correlation for all these variables together with the defaults transition rates. What stands out is the high standard deviation value for the EPU index and the CPI has a relatively low standard deviation. We can see that imports- and exports of goods and services are closely correlated with a value of 0.822, they also correlate to GDP with values of 0.702 and 0.742. The two interest rates are quite closely correlated with a value of 0.943, this also questions whether it is useful to include both variables in a model. While these variables are highly correlated they might still provide different information to a model. Therefore both variables are included for comparison. The interest rates mostly have a low correlation to other variables, only the CPI and NFCI correlate somewhat to them.

Table 3: Descriptive statistics in panel A and correlation matrix in panel B

Panel A: Descriptive statistics

	AAA	AA	A	BBB	BB	B	CCC	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
min	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-9.329	3.600	-2.290	-19.832	-24.216	10.308	-1.034	43.718	0.015	0.651
median	0.000	0.000	0.000	0.023	0.142	0.851	6.228	1.242	5.700	0.725	1.942	1.638	17.391	-0.433	94.853	3.863	5.093
mean	0.000	0.004	0.015	0.050	0.221	1.049	6.303	1.249	6.223	0.701	1.535	1.343	19.475	-0.195	106.802	3.958	5.767
max	0.000	0.095	0.101	0.255	1.103	3.460	14.168	8.528	13.067	2.785	19.539	15.028	58.588	3.285	451.504	15.051	14.838
st. dev	0.000	0.015	0.024	0.059	0.242	0.789	2.812	1.279	1.784	0.565	3.947	3.599	7.238	0.784	51.941	3.391	3.286

Panel B: correlation matrix

	AAA	AA	A	BBB	BB	B	CCC	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
AAA	1.000																
AA	0.000	1.000															
A	0.000	0.606	1.000														
BBB	0.000	0.206	0.387	1.000													
BB	0.000	-0.027	0.370	0.633	1.000												
B	0.000	0.095	0.405	0.637	0.563	1.000											
CCC/C	0.000	0.129	0.308	0.489	0.310	0.552	1.000										
gdp	0.000	-0.138	-0.148	-0.052	-0.038	-0.096	-0.257	1.000									
unemp	0.000	-0.144	-0.068	0.021	0.165	0.041	-0.127	-0.116	1.000								
cpi	0.000	-0.024	0.025	0.069	0.226	-0.025	-0.286	0.455	0.000	1.000							
imports	0.000	-0.050	-0.162	-0.085	-0.127	-0.150	-0.211	0.702	-0.056	0.497	1.000						
exports	0.000	-0.030	-0.153	-0.125	-0.188	-0.161	-0.242	0.742	-0.154	0.444	0.822	1.000					
vix	0.000	0.362	0.469	0.588	0.388	0.489	0.598	-0.341	0.226	-0.352	-0.371	-0.426	1.000				
nfci	0.000	0.236	0.441	0.323	0.470	0.141	0.008	-0.048	0.358	0.297	-0.252	-0.240	0.771	1.000			
unc	0.000	0.016	-0.005	0.041	0.138	0.159	0.394	-0.398	0.619	-0.257	-0.304	-0.402	0.493	0.248	1.000		
3month	0.000	-0.104	0.159	0.300	0.482	0.200	-0.312	0.295	0.068	0.564	0.083	0.060	-0.089	0.529	-0.305	1.000	
10year	0.000	-0.122	0.139	0.329	0.511	0.251	-0.345	0.300	0.245	0.525	0.114	0.071	-0.073	0.524	-0.250	0.943	1.000

Variable x_1: GDP, x_2: Unemployment rate, x_3: CPI, x_4: Imports of goods and services, x_5: exports of goods and services, x_6: VIX, x_7: NFCI, x_8: Uncertainty ind x, x_9: 3-month interest rate, x_{10}: 10-year interest rate.

3 Probability of default

The Probability of Default (PD) describes the probability that an obligor defaults on an obligation for a loan payment. The PD is a key variable in credit risk, especially for expected credit loss (ECL) calculations. This section therefore first explains how the PD is used in its main application to show the relevance of PD models. Thereafter, the section shows how the obtained data in this study links to the PD.

3.1 Application in expected credit loss

In expected credit loss (ECL) modelling, focus points are calculation, staging assessment and incorporating macroeconomic information. The calculation of ECL relies on the variables probability of default (PD), exposure at default (EAD), loss given default (LGD) and the discount rate (D). The PD indicates the probability that a company defaults. An important distinction is marginal versus cumulative PD, marginal indicates the probability that a default occurs at a certain point in time while the cumulative indicates the total probability up until the time period. From here on, the cumulative PD is used. The EAD is the exposure of the loan, so it says how much money is left to be paid. The LGD indicates how much money is lost when an obligor defaults, it is given as a percentage of the EAD. The LGD usually links to what assets can be recovered and sold off to pay off parts of the loan. Usually, there is some sort of collateral in the contract. Committee (2016) shows an in-depth review on ECL calculations, considerations and challenges. To calculate ECL we multiply these variables

$$ECL_{h|i} = PD_{h|i} \cdot EAD_h \cdot LGD_h \cdot D_h \quad \text{for } i = AAA, AA, \ldots, CCC/C, \tag{1}$$

where h stands for the timer periods ahead in the future and the discount factor equals $D_h = \left(\frac{1}{1+r}\right)^h$ with interest rate r. The EAD and LGD also depend on the time period as parts of the loan could be paid off in the future and collateral can depreciate over time. The calculation depends on how many periods we look ahead in the future (h). h equals either one year for stage 1 assets or equals the remaining maturity of the asset in the case of stage 2 and 3 assets. These stages relate to the staging assessment. IFRS 9 makes a distinction between counterparties by looking at the deterioration of the credit quality since initial recognition. Companies that have not deteriorated are stage 1 assets ($j = 1$). For these assets, a 1-year ECL must be reported. Companies that have some deterioration are stage 2 ($j = 2$) assets and the ones that are significantly impaired and went into default are stage 3 ($j = 3$) assets. For these last two categories, a lifetime ECL needs to be reported for the asset. The deterioration is usually measured by a company's credit rating over the years from initial recognition. If a credit rating has moved by a certain set amount of categories it will become a stage 2 asset. Since the lender has this information at hand, this research does not focus on the staging assessment. However, we do focus on forecasting multiple periods ahead of time. We can see in Equation 1 that small changes in PD impact ECL a lot due to multiplication with the EAD and LGD, especially when these have large values. When losses can add up to millions of dollars, one percent difference in PD changes the ECL greatly. Since the provision for ECL must be reported on the balance sheet of a company, these differences can have a large impact. The incorporation of macroeconomic information into ECL can be done in many ways. Out of the four main variables, macroeconomic information is usually taken into account through the PD and not so much in the LGD or EAD.

3.2 From information to expected credit loss

Figure 5 shows the entire process from information to ECL and where this research exactly fits into that. The data used are the transition rates to default for a given credit rating at a certain point in time (Figure 1, Section 2) together with macroeconomic information. As Figure 5 shows, we use the credit rating and macroeconomic information as input and we forecast transition rates to default. These default rates are a part of the transition matrix. The transition matrix describes the probability of a company either moving to a different credit rating or to default. Table 1 (Section 2.1.1) shows such a credit transition matrix in a table. For one year ahead, we can easily read the chance to default as these are stated in the default column. However, when calculating multiple years ahead PDs, we need to multiply the entire forecasted credit transition matrices with each other as we need to take into account that a company can switch credit ratings after each year. With the obtained PD together with the EAD, LGD and discount factor, we can calculate the ECL (Equation 1).

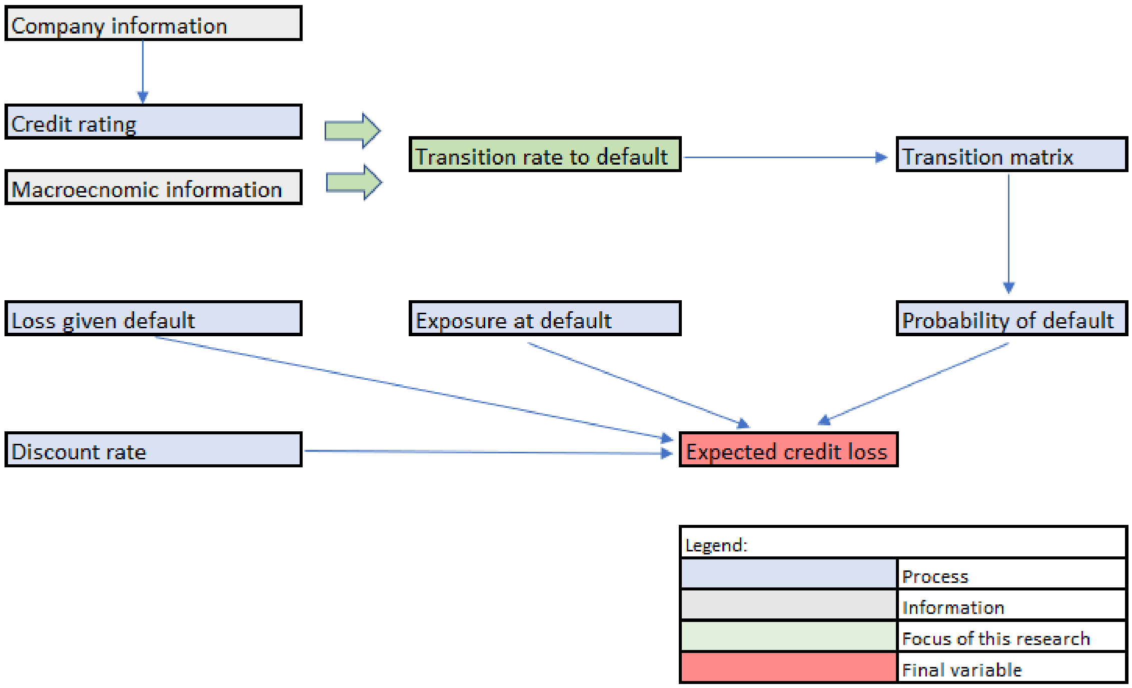

Figure 5: Flowchart from information to ECL

4 Methods

This section describes the model-building procedure for both the Markov Switching (MS) and Smooth transition autoregressive (STAR) framework. Furthermore, the forecasting methods are also discussed for both frameworks. Thereafter, we show the calculation to create two new variables from the data, namely with principal component analysis (PCA) and with some aspects of the Vasicek model. Lastly, we discuss model evaluation.

4.1 Markov Switching model

As introduced by Hamilton (1989), the Markov Switching (MS) model provides a way of modelling shifts in regimes where the parameters are dependent on a discrete-state Markov process. The shifts between regimes are not observed directly however can be analysed afterwards by making an inference on the probability of being in a certain regime. y_{it} refers to the transition rate to default from credit ratings $i = AAA, AA, \ldots, CCC/C$, for simplicity, in the equations below we use y_t. The relation between this variable and an intercept (a_{S_t}), autoregressive lags (y_{t-p}), exogenous variables ($x_t = (x_{1t}, \ldots, x_{kt})$) and an error term ($\varepsilon_{t,\sigma_{S_t}}$) are described by

$$\begin{aligned} y_t =& a_{S_t} + \beta_{S_t} x_t' + (\phi_{1,S_t} y_{t-1} + \cdots + \phi_{p,S_t} y_{t-p}) + \varepsilon_{t,\sigma_{S_t}} \\ =& \theta_{S_t} u_t' + \varepsilon_{t,\sigma_{S_t}}, \text{for } t = 1, \ldots, T \end{aligned} \tag{2}$$

where $u_t = (1, x_t, y_{t-1}, \ldots, y_{t-p})$ and the parameters are captured in $\theta_{S_t} = (a_{S_t}, \beta_{S_t}, \phi_{1,S_t}, \ldots, \phi_{p,S_t})$ and $\varepsilon_t \sim N\left(0, \sigma_{S_t}^2\right)$ which depend on the regimes $S_t = 1 \ldots k$. The regimes are latent and can only be interpreted afterwards. We assume that the probabilities of switching regimes are constant over time. These probabilities are estimated together with all parameters. For parameter estimation, the expectation maximization (EM) algorithm is used. Appendix section A.1 shows how this algorithm works. Furthermore, we need to determine the number of regimes we use in our models. As explained in Zhang and Stine (2001) this can be done by minimizing the Akaike information criterion (AIC) and Bayesian information criterion (BIC) criteria.

4.1.1 Forecasting

With the following recursion

$$\begin{aligned} \boldsymbol{\xi}_{t|t} &= \frac{1}{\boldsymbol{\xi}_{t|t-1}' \boldsymbol{f}_t} \boldsymbol{\xi}_{t|t-1} \odot \boldsymbol{f}_t \\ \boldsymbol{\xi}_{t+1|t} &= \boldsymbol{P} \boldsymbol{\xi}_{t|t}, \end{aligned} \tag{3}$$

we can get to the predictive state, where $\widehat{\boldsymbol{\xi}}_{T|T}$ is the smoothed probability obtained with the Kim smoother, f_t is the likelihood function and $P = \begin{bmatrix} \widehat{p}_{11} & 1 - \widehat{p}_{22} \\ 1 - \widehat{p}_{11} & \widehat{p}_{22} \end{bmatrix}$ is the transition matrix which contains the probabilities of regime switches. These equations are called the Hamilton filter (Hamilton (1994)). This leads to the one-step-ahead forecast

$$\begin{aligned} \hat{y}_{T+1|T} &= \mathrm{E}\left[\xi_{T+1} \mid \xi_T\right] \mathrm{E}\left[u_{T+1} \mid y_T\right] \\ &= P \widehat{\boldsymbol{\xi}}_{T|T} \left(\hat{\boldsymbol{\alpha}} + \hat{\boldsymbol{\beta}} x_T + \hat{\boldsymbol{\phi}}_{\mathbf{1}} y_t + \cdots + \hat{\boldsymbol{\phi}}_{\boldsymbol{p}} y_{T+1-p}\right), \end{aligned} \tag{4}$$

where we use a reduces version of the Hamilton filter

The recursive relationship allows the h-step-ahead forecasts to be obtained

$$\begin{aligned} \hat{y}_{T+h|T} &= \mathrm{E}\left[\xi_{T+h} \mid \xi_T\right] \mathrm{E}\left[u_{T+h} \mid y_T\right] \\ &= P^h \widehat{\boldsymbol{\xi}}_{T|T} \left(\hat{\boldsymbol{\alpha}} + \hat{\boldsymbol{\beta}} \hat{x}_{T+h} + \hat{\boldsymbol{\phi}}_{\mathbf{1}} \hat{y}_{T+h-1|T+h-2} + \cdots + \hat{\boldsymbol{\phi}}_{\boldsymbol{p}} \hat{y}_{T+h-p|T+h-p-1}\right), \end{aligned} \tag{5}$$

where we set future values of the exogenous variables to the last known value at time T, $\hat{x}_{T+h} = x_T$. The advantage of doing so is to not put forecasts into a model that makes a forecasts. This can lead to uncertainty of the model and the performance of the models could be dependent on the obtained forecasts for the input. A disadvantage is that we keep these variables constant in an out-of-sample forecast which could lead to extreme values when the starting point is at a all-time low or high. An alternative could be to set out scenarios for the exogenous variables and put loadings on each scenario, however this is not in the scope of this research.

4.2 Smooth Transition Autoregression model

The specification of the STAR model follows van Dijk (1999). The general representation of a STAR model is

$$y_t = \theta_1 u_t' \left(1 - G\left(s_t; \gamma, c\right)\right) + \theta_2 u_t' G\left(s_t; \gamma, c\right) + \varepsilon_t, \quad t = 1, \ldots, T, \tag{6}$$

where s_t is the transition variable. The parameters are captured in $\theta_i = (a_i, \beta_i, \phi_{1,i}, \ldots, \phi_{p,i})$, for $i = 1, 2$. Just as van Dijk (1999), we assume that ε_t are a martingale difference sequence which is captured in $\Omega_{t-1} = \left\{y_{t-1}, y_{t-2}, \ldots, y_{1-(p-1)}, y_{1-p}\right\}$. Mathematically this translates to $\mathrm{E}\left[\varepsilon_t \mid \Omega_{t-1}\right] = 0$. Also, we assume that the conditional variance of ε_t is constant, $\mathrm{E}\left[\varepsilon_t^2 \mid \Omega_{t-1}\right] = \sigma^2$. The transition function $G\left(s_t; \gamma, c\right)$ is a continuous function and is bounded between 0 and 1. The transition function depends on the transition variable s_t which is the variable that determines the regimes, γ which determines the smoothness of the function and c which stands for the threshold between regimes.

The transition function is set to the logistic transition function

$$G\left(s_t; \gamma, c\right) = \frac{1}{1 + \exp\left\{-\gamma\left(s_t - c\right)\right\}}, \quad \gamma > 0 \tag{7}$$

as this function is a safe choice and often used. For the transition variable, we opt for the exogenous variable. This way the macroeconomic variable can set the regime at each point in time.

The parameter estimation for Equation 6 can be found in Appendix Section A.2.

4.2.1 Forecasting

The one-step-ahead forecast is

$$
\begin{aligned}
\hat{y}_{T+1|T} &= \mathrm{E}\left[y_{T+1} \mid \Omega_T\right] = F\left(\hat{u}_{T+1};\theta\right) \\
&= \phi_1' \hat{u}_{T+1}\left(1 - G\left(\hat{s}_{T+1};\gamma,c\right)\right) + \phi_2' \hat{u}_{T+1} G\left(\hat{s}_{T+1};\gamma,c\right),
\end{aligned} \tag{8}
$$

where $\hat{u}_{T+1} = (1, \hat{x}_{1,T+1}, \ldots, \hat{x}_{k,T+1}, y_T, \ldots, y_{T-p+1})'$ and using that $\mathrm{E}\left[\varepsilon_{T+1} \mid \Omega_T\right] = 0$. The value of the exogenous variable for time $T+1$ is not known at time T, therefore we estimate $\hat{x}_{k,T+1} = x_{k,T}$. Since we use an exogenous variable as the transition variable as well, we also estimate $\hat{s}_{T+1} = s_T$. The autoregressive values are all known at this point in time.

For the two-step-ahead forecast

$$
\hat{y}_{T+2|T} = \mathrm{E}\left[y_{T+2} \mid \Omega_T\right] = \mathrm{E}\left[F\left(\hat{u}_{T+2};\theta\right) \mid \Omega_T\right], \tag{9}
$$

as van Dijk (1999) indicates, we can not interchange the two functions $\mathrm{E}[F(\cdot)] \neq F(\mathrm{E}[\cdot])$. Therefore in the lagged values of y, we need to use a bootstrap method to estimate future residuals as $\mathrm{E}\left[\varepsilon_{T+2} \mid \Omega_T\right] \neq 0$. The estimated residuals are based on residuals of the in-sample data and the out-of-sample autoregressive components in $\hat{u}_{T+2}$ change from $\hat{y}_{T+1}, \ldots, \hat{y}_{T+2-p}$ to $((\hat{y}_{T+1|T} + \hat{\varepsilon}_i), \ldots, \hat{y}_{T+2-p}$. In this case we estimate two steps ahead, therefore only the first lag is out-of-sample.

This leads to the 2-step-ahead bootstrapped forecast

$$
\begin{aligned}
\hat{y}^{(b)}_{T+2|T} &= \frac{1}{T}\sum_{i=1}^{T} F\left(\hat{u}_{T+2};\hat{\varepsilon}_i,\theta\right), \\
&= \frac{1}{T}\sum_{i=1}^{T} \big(\phi_1' \hat{u}_{T+2}\left(1 - G\left(\hat{s}_{T+2};\gamma,c\right)\right) + \phi_2' \hat{u}_{T+2} G\left(\hat{s}_{T+2};\gamma,c\right)\big),
\end{aligned} \tag{10}
$$

with $\hat{u}_{T+2} = \big(1, \hat{x}_{1,T}, \ldots, \phi_k \hat{x}_{k,T}, (\hat{y}_{T+1|T} + \hat{\varepsilon}_i), \ldots, y_{T+2-p}\big)'$ where we also estimate the exogenous variables to its last known value and $\hat{s}_{T+2} = s_T$.

Recursively we can compute the h-step-ahead forecast

$$
\hat{y}^{(b)}_{T+h|T} = \frac{1}{T}\sum_{i=1}^{T} F\left(\hat{u}_{T+h|T};\hat{\varepsilon}_i,\theta\right), \tag{11}
$$

where $\hat{u}_{T+h} = \big(1, \hat{x}_{1,T+h}, \ldots, \hat{x}_{k,T+h}, (\hat{y}_{T+h-1|T} + \hat{\varepsilon}_i), \ldots, (\hat{y}_{T+h-p|T} + \hat{\varepsilon}_i\big)$.

4.3 Autoregressive lags

In both models, we need to select the number of autoregressive lags. To do so, we first specify a linear model as

$$y_t = \phi' x_t + \varepsilon_t, \quad t = 1, \ldots, T, \tag{12}$$

where $\phi = (\phi_0, \phi_1, \ldots, \phi_m)'$ and x_t are the parameters and variables which are explained in the following sections and ε_t follows a normal distribution. The optimal number of lags are then selected by minimizing Akaike's Information Criterion (AIC) and Bayesian Information Criterion (BIC). A maximum of 4 lags is set, as adding more lags would increase the amount of parameters estimated and mean more data need to be trimmed of the first few years to estimate the model which would lead to a larger loss of information. The criteria are calculated as follows

$$\begin{aligned} \text{AIC} &= T \ln \hat{\sigma}^2 + 2k, \\ \text{BIC} &= T \ln \hat{\sigma}^2 + k \ln T, \end{aligned} \tag{13}$$

where k equals the number of parameters estimated.

4.4 Principal component analysis and macro-factor

Next to the more conventional macroeconomic variables, we also estimate the models with two generated variables.

First, we use PCA to reduce the dimensions of the ten macroeconomic variables into a few components which can capture a large amount of the total variance. PCA uses singular value decomposition to create these principal components (PCs). Table 4 shows the loadings for each of the PCs and Figure 6 shows how the first three components over time. We see that especially the first PC displays dynamics around crisis periods. Therefore this PC could be the volatility of the variables as they move most in these periods. The loading on this PC seems to be primarily on GDP, imports and exports. The second PC seems to follow some of the patterns of the interest rates which is due to the loading on these variables.

Table 4: Principal component loadings

Variable loading	Principal component: 1	2	3	4	5	6	7	8	9	10
GDP	0.197	-0.001	0.198	-0.561	0.734	0.214	-0.020	0.141	0.043	-0.008
unemp	-0.004	-0.018	-0.004	0.141	0.027	0.116	-0.101	-0.053	0.975	0.048
CPI	0.058	0.043	-0.047	0.045	-0.169	0.035	-0.194	0.960	0.026	0.032
imports	0.711	-0.126	-0.682	-0.009	-0.006	0.016	0.097	-0.052	0.005	-0.010
exports	0.658	-0.057	0.691	0.172	-0.215	-0.015	-0.071	-0.063	-0.024	0.011
VIX	-0.005	0.000	-0.006	0.011	0.004	0.090	0.077	-0.019	-0.054	0.991
NFCI	-0.034	0.000	0.082	0.131	-0.080	0.555	0.795	0.130	0.012	-0.110
EPU	-0.038	-0.049	-0.082	0.171	-0.040	0.777	-0.547	-0.147	-0.181	-0.043
3-m	0.084	0.739	-0.030	-0.495	-0.413	0.119	-0.021	-0.099	0.075	0.001
10-y	0.093	0.656	-0.040	0.584	0.455	-0.073	0.005	0.021	-0.074	-0.006

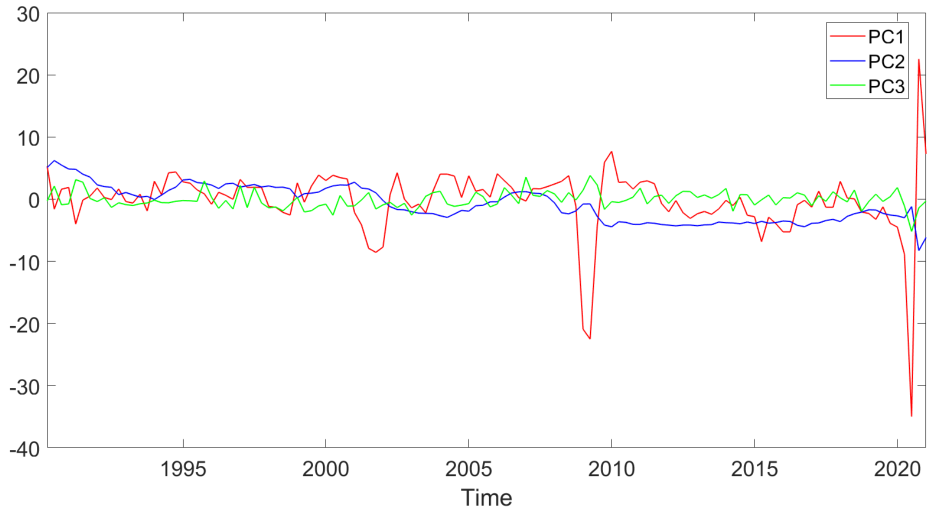

Figure 6: The first three principal components

Second, we use, as described in Magnou (2018), some structure of the Vasicek model to decompose the default rate time series ($PD_{pit,t}$) into a constant default rate (PD_{ttc}) together with a macro-factor (Z_t). We have

$$PD_{\text{pit, }t} = N\left(\frac{N^{-1}\left(PD_{TTC}\right) + \sqrt{\rho}\left(Z_t\right)}{\sqrt{1-\rho}}\right), \tag{14}$$

where $Z_t \sim N(0,1)$, $N()^{-1}$ is the inverse of the standard normal cumulative distribution function and correlation ρ . We can rewrite this to

$$N^{-1}\left(PD_{\text{pit },t}\right)=\frac{N^{-1}\left(PD_{TTC}\right)+\sqrt{\rho}\left(Z_{t}\right)}{\sqrt{1-\rho}}. \tag{15}$$

By taking the expected value we get

$$m=E\left[N^{-1}\left(PD_{pit,t}\right)\right]=E\left[\frac{N^{-1}\left(PD_{TTC}\right)+\sqrt{\rho}\left(Z_{t}\right)}{\sqrt{1-\rho}}\right]=\frac{N^{-1}\left(PD_{TTC}\right)}{\sqrt{1-\rho}}, \tag{16}$$

the last step holds since $E(Z_t)=0$ and $N^{-1}(PD_{TTC})$ is a constant, just as ρ. We rewrite this to

$$PD_{TTC}=N\left(E\left[N^{-1}\left(PD_{\text{pit },t}\right)\right]\sqrt{1-\rho}\right)=N(m\cdot\sqrt{1-\rho}). \tag{17}$$

By taking the variance, we can write this is terms of ρ

$$s^{2}=V\left[N^{-1}\left(PD_{\text{pit },t}\right)\right]=V\left[\frac{N^{-1}\left(PD_{TTC}\right)+\sqrt{\rho}\left(Z_{t}\right)}{\sqrt{1-\rho}}\right]=V\left(\frac{\sqrt{\rho}}{\sqrt{1-\rho}}\right)V\left(Z_{t}\right)=\frac{\rho}{1-\rho}. \tag{18}$$

The relation between ρ and the variance is then as follows

$$\rho=\frac{s^{2}}{1+s^{2}}. \tag{19}$$

Now, we can calculate the constant PD

$$PD_{TTC}=N\left(\frac{m}{\sqrt{1+s^{2}}}\right), \tag{20}$$

with this result, we can calculate Z_t as well by filling in the previous formulas. For rating CCC/C, This leads to the following graph in Figure 7. We can see that the macro-factor is similar to the default rate, however with some differences, especially in the lower area of the graph.

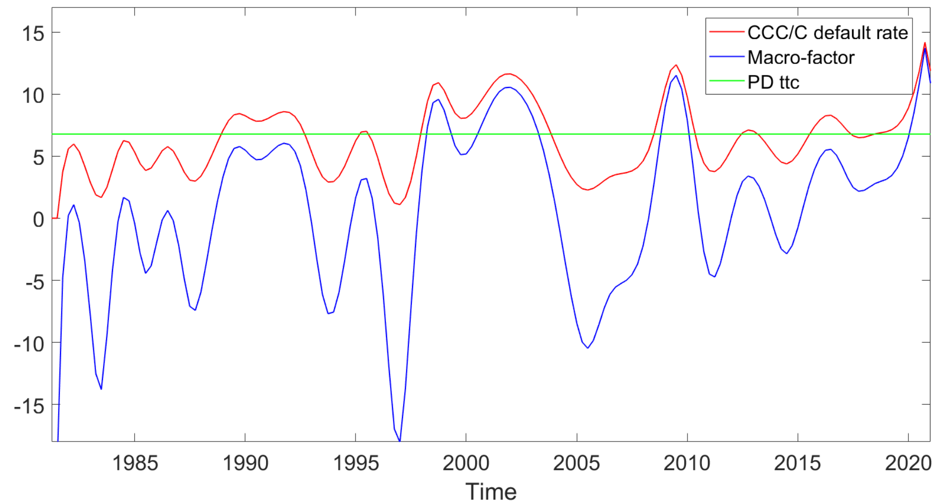

Figure 7: Macro-factor over time with the TTC PD for rating CCC/C

4.5 Model evaluation

To compare these methods with multiple combinations of variables, it is important to set out how we can evaluate these models. To do so, a division between in-sample and out-of-sample is made. In-sample is set from 1981 Q4 until 2012 Q4 and out-of-sample from 2013 Q1 until 2020 Q4. We can start the in-sample data in Q4 as we need the first three quarters to make sure all the autoregressive lags have a value from the start. For out-sample forecasting, we use an expanding window. As Diebold (2015) discusses, to compare forecasting models to each other we should use forecasting measures as well as in-sample evaluation measures. Even though the goal of the model is to provide the most accurate forecasts, for a full model comparison we should also look into the in-sample fit of the model. For the in-sample, fit we can look at the AIC and BIC of different models. In Equation 13, the equations for these criteria are set out.

To measure the out-of-sample performance of a model, we use the forecast evaluation measure Root Mean Square Error (RMSE)

$$e^{\text{RMSE}} = \sqrt{\frac{\sum_{t=T}^{T+N} \left(\hat{y}_{t+h|T} - y_{t+h}\right)^2}{N}}, \tag{21}$$

where N equals the number of forecasts, $\hat{y}_{t+h|t}$ is the h-step-ahead forecasted value and y_{t+h} is the observed value.

This measure is often used and easy to interpret. The goal is to minimize this measurement, the closer the forecasted value to the actual value the smaller the measure will be. We use the Diebold-Mariano (DM) (Diebold and Mariano (2002)) test statistics to test whether an evaluation measure is significantly different to the other under 5% significance. We test under the null hypothesis that the forecasting errors are equal. We use a modification to this test introduced by Harvey, Leybourne, and Newbold (1997). Appendix B shows the math behind the extension of the DM test. Because the DM-test collapses when comparing nested models, we only use the test to compare models with the same amount of macroeconomic variables as these will not include nested models.

5 Results

In this section, we first show insights into the Markov Switching (MS) and Smooth Transition Autoregression model (STAR) models by looking into the parameter estimates, how the regimes behave over time and the in-sample fit. Thereafter, we compare all models based on out-of-sample RMSE values and see which variables and models perform best. By testing for a significant difference in forecast errors between models, we can conclude which models perform best for each credit rating. The credit ratings BB, BB, B and CCC each have their own default rate time series and display their own dynamics, therefore we can not analyze them all together. Furthermore, we can compare the regime-switching methods to each other. Lastly, we analyze the best performing models to explain why these models yield the best forecasts. We do so by analyzing the dynamics of these models, especially in regime changes.

5.1 Model specifications

Table 5 we can see that, with a maximum of four regimes, the AIC and BIC are both minimized by choosing two regimes for the MS model. This seems appropriate due to the not so large sample of 160 observations. By increasing the regimes, the amount of data points per regime drops fast and parameter estimation will not be as accurate and the estimation algorithm is more prone to unconvergence. For these reasons, we use two regimes for both the MS and STAR models.

Table 5: AIC and BIC values for each amount of regimes k in MS model for $k = 1, \ldots, 3$

k =	1	2	3
AIC	291.2	291.8	299.4
BIC	308.9	322.4	346.1

Table 6 shows the AIC and BIC for lags $p = 1, \ldots, 4$ for the composed linear model (Equation 12, Section 4.3). For both criteria, the minimum lies at three lags ($p = 3$). Therefore, three lags are included in both the MS and STAR models.

Table 6: AIC and BIC values for each AR(p) model for $p = 1, \ldots, 4$

p =	1	2	3	4
AIC	305.6	305.3	300.8	301.5
BIC	307.3	308.6	305.8	308.2

5.2 Parameter estimates, insights into regimes and in-sample fit

5.2.1 Markov Switching model

Table 7 shows the parameter estimates for the MS model with one macroeconomic variable included. We can see that the dynamics of the cycles are modelled through the autoregressive lags where the first lag is positive, the second lag is negative and the third lag is positive again. This pattern shows for all different variables as we can see that these models are estimated similarly. The level estimate ($\hat{a}_{S_t}$) differs over models. When the level is estimated differently, the regimes could be set at different points. Overall, in regimes with high default rates, the level estimate is also higher. Logically the estimates for $\hat{\beta}_{S_t}$ differ as different macroeconomic variables correlate differently to the default rate with different magnitudes. The estimates for $\hat{\sigma}_{S_t}$ indicate the volatility in each regime. We can see for most models that the volatility in both regimes is close to each other. However, for the VIX and interest rates, the volatility in each regime differs which means that in these regimes the movement of the curve differs. This translates to a larger difference in the loading of the autoregressive lags in each regime as they mostly determine the shape of the curve. More curvature translates to more volatility.

Table 7: Parameter estimates MS model with one macroeconomic variable for credit rating CCC/C

	GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
$\hat{p}_{11}$	0.869	0.912	0.826	0.858	0.863	0.882	0.754	0.974	0.825	0.943
$\hat{p}_{22}$	0.890	0.964	0.909	0.881	0.883	0.830	0.953	0.852	0.926	0.881
$\hat{a}_1$	0.291	0.030	0.151	0.235	0.232	-0.045	-0.148	0.074	0.211	0.263
$\hat{\beta}_1$	-0.072	0.037	0.036	-0.019	-0.027	0.017	0.135	-0.062	-0.036	-0.008
$\hat{\phi}_{1,1}$	2.441	2.297	2.342	2.427	2.437	2.566	2.688	2.663	2.702	2.416
$\hat{\phi}_{2,1}$	-2.190	-1.995	-2.019	-2.169	-2.188	-2.339	-2.713	-2.533	-2.635	-2.129
$\hat{\phi}_{3,1}$	0.698	0.628	0.623	0.692	0.703	0.741	1.022	0.865	0.893	0.667
$\hat{a}_2$	0.258	0.416	0.172	0.237	0.258	-0.068	0.201	0.408	0.127	0.844
$\hat{\beta}_2$	0.005	-0.044	0.038	0.004	-0.002	-0.001	0.148	-0.013	0.014	-0.036
$\hat{\phi}_{1,2}$	2.563	2.642	2.605	2.588	2.571	2.932	2.612	2.639	2.583	2.403
$\hat{\phi}_{2,2}$	-2.393	-2.514	-2.467	-2.437	-2.410	-3.051	-2.471	-2.631	-2.393	-2.204
$\hat{\phi}_{3,2}$	0.804	0.854	0.843	0.825	0.814	1.118	0.841	0.961	0.790	0.740
$\hat{\sigma}_1$	0.102	0.128	0.104	0.099	0.098	0.077	0.026	0.095	0.052	0.114
$\hat{\sigma}_2$	0.100	0.090	0.101	0.098	0.099	0.044	0.104	0.120	0.111	0.076

Figures 8 and 9 show how the regime changes for rating CCC/C when either GDP or the 3-month interest rate is included in the MS model. We can see that including different variables lead to different regimes over time. Due to the inclusion of autoregressive lags, the regime at a certain point in time is determined by both the macroeconomic variable and past default rates. We can see for GDP that regime 1 mostly occurs during times when GDP growth is low or negative with high default rates. Especially during the first ten years, the regime changes a lot which can be due to the fluctuations in default rates. We can recognise that regime 1 occurs mostly in times of crisis as this regime occurs around 2000, 2008 and 2020. When the 3-month interest rate is included, the graph shows some differences in regimes. Especially in the first half of the data set. We can also see that after the global financial crisis (2010) the timing of a regime switch differs. When GDP is included, the regime switches back about a year earlier.

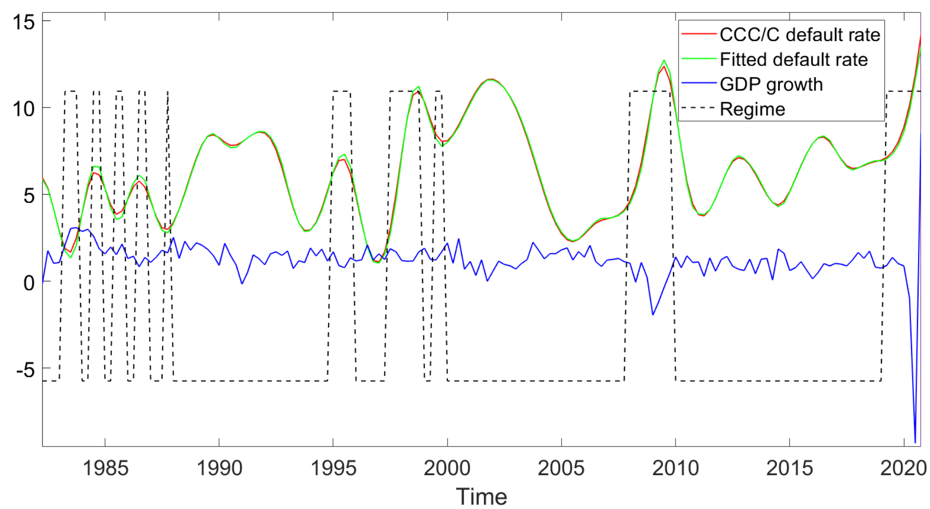

Figure 8: Regime changes MS model with GDP

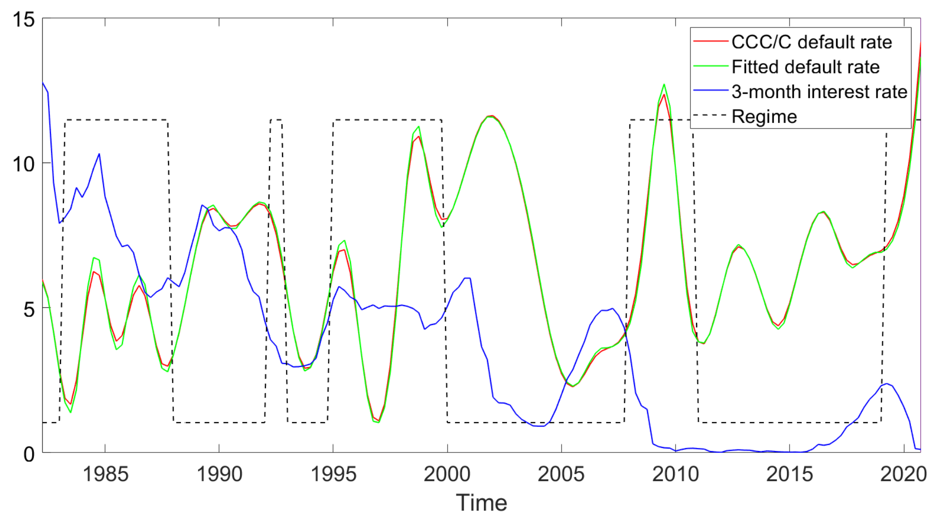

Figure 9: Regime changes MS model with 3-month interest rates

5.2.2 Smooth Transition Autoregression model

Table 8 shows the parameter estimates for the STAR model when one macroeconomic variable is included. Similarly to the MS model, we can see that the dynamics of the cycles are modelled through the autoregressive lags. However, we do see larger differences between models compared to the MS models where estimates were more persistent. We can see large differences in $\hat{c}$ and $\hat{\gamma}$. For $\hat{c}$, this is logical as this value usually lies somewhere around the mean of the transition variable. $\hat{\gamma}$ indicates the curvature of the transition function. A high gamma indicates that the model has abrupt changes in regimes and does not run smoothly between them. Figures 10 and 11 show the difference between a $\hat{\gamma}$ of 0.575 and 99.859. For imports, we can see that the transition function can have values everywhere between 0 and 1, while the transition function for exports changes abruptly from 0 to 1 and vice versa. Even though you could argue that abrupt changes in regimes are preferable, it does take away from the purpose of the model to run smoothly between regimes. The level estimates $\hat{a}_{S_t}$ differ quite a bit, especially in the second regime where GDP, NFCI and 10-year interest rate have rather larger estimates. This, in most cases, goes together with a larger $\hat{\beta}_{S_t}$ which compensates for the higher level.

Table 8: Parameter estimates STAR model with one macroeconomic variable for credit rating CCC/C

	GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
$\hat{\gamma}$	1.879	98.604	85.965	0.575	99.859	11.946	49.165	50.610	1.939	6.763
$\hat{c}$	4.414	8.614	0.777	0.000	3.761	19.967	1.960	90.305	5.769	10.737
$\hat{a}_1$	0.158	0.293	0.045	0.349	0.143	0.011	0.112	0.162	0.000	0.037
$\hat{\beta}_1$	-0.016	-0.038	-0.009	0.001	-0.013	0.007	0.076	-0.002	0.012	0.004
$\hat{\phi}_{1,1}$	2.567	2.649	2.690	2.523	2.540	2.644	2.637	2.725	2.746	2.689
$\hat{\phi}_{2,1}$	-2.353	-2.532	-2.624	-2.278	-2.337	-2.494	-2.533	-2.650	-2.688	-2.596
$\hat{\phi}_{3,1}$	0.763	0.874	0.929	0.713	0.778	0.825	0.883	0.922	0.937	0.899
$\hat{a}_2$	2.694	1.256	0.000	0.000	0.109	0.382	16.957	0.164	0.000	2.558
$\hat{\beta}_2$	-1.612	-0.090	0.183	0.012	0.017	-0.002	-3.379	-0.001	0.025	-0.115
$\hat{\phi}_{1,2}$	3.298	2.271	2.506	2.634	2.691	2.591	2.877	2.678	2.390	1.796
$\hat{\phi}_{2,2}$	-7.667	-1.929	-2.261	-2.524	-2.569	-2.473	-5.389	-2.588	-2.148	-1.329
$\hat{\phi}_{3,2}$	5.790	0.588	0.726	0.880	0.848	0.847	2.573	0.902	0.735	0.273

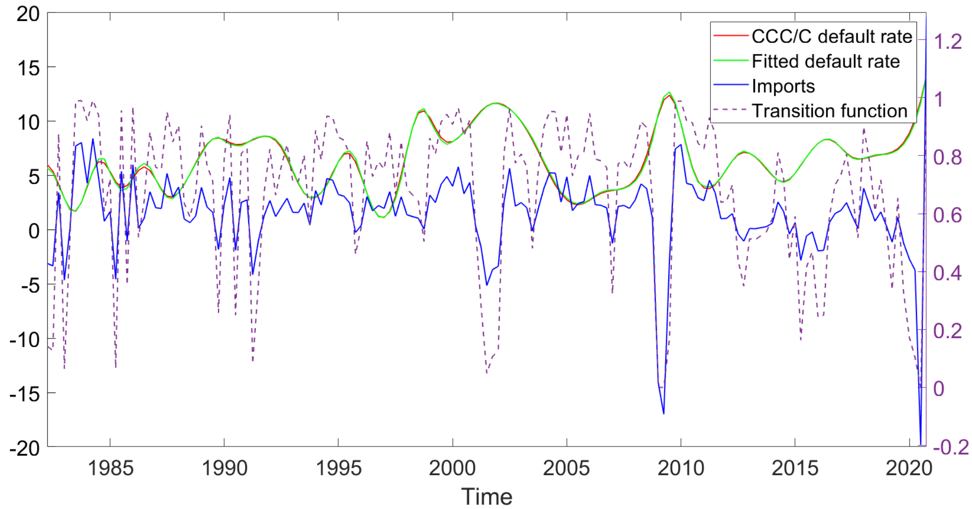

Figure 10: Transition function STAR model for Imports

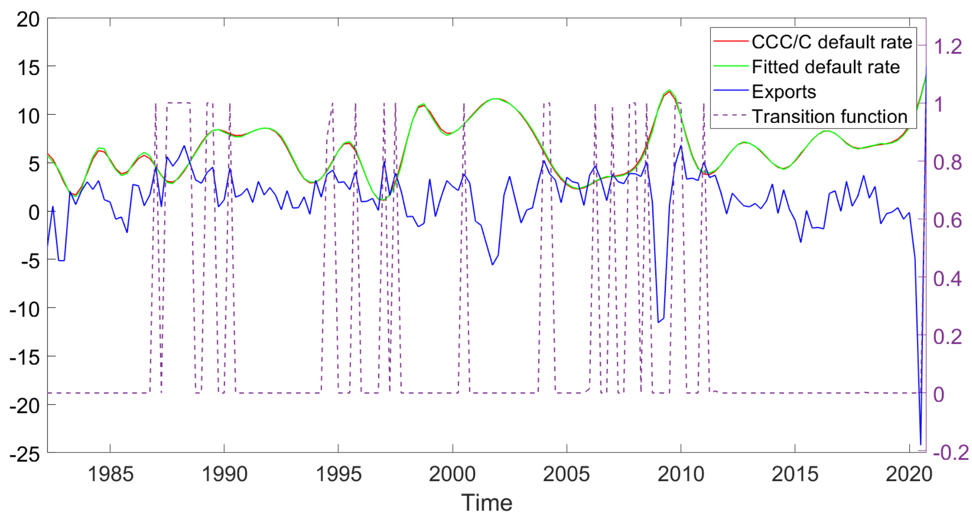

Figure 11: Transition function STAR model for Exports

5.2.3 In-sample fit models

Table 9 shows the AIC and BIC values for both the MS and STAR model when one macroeconomic variable is included. These values show the in-sample fit based on the sample from 1981 Q4 until 2012 Q4. Firstly, it is notable that both criteria find the minimal value for the same model each time. When we run by each variable, we can see that most criteria values are comparable. Only the VIX and the EPU have notably less favourable values for each model and credit rating. The NFCI has the best in-sample fit for the credit rating BBB and BB, the unemployment rate for rating B and GDP for rating CCC. We can see that the VIX and EPU seem like less desirable variables to include in either model as these values seem notably larger compared to the other models.

Table 9: AIC and BIC for one macroeconomic variable ($*10^2$)

		Macroeconomic variable included									
Rating	Model	GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
		AIC									
BBB	MS	-6.35	-6.90	-6.31	-6.01	-6.08	-3.53	-7.12	-2.32	-6.53	-6.30
	STAR	-5.95	-5.95	-5.68	-5.96	-6.01	-2.31	-5.97	-2.31	-5.99	-5.99
BB	MS	-5.52	-5.57	-5.53	-5.50	-5.54	-3.43	-5.77	-2.29	-5.73	-4.60
	STAR	-4.47	-4.93	-4.56	-4.55	-4.68	-2.20	-4.43	-2.20	-4.50	-4.51
B	MS	-2.90	-3.10	-3.04	-2.96	-3.02	-2.46	-2.90	-1.84	-2.97	-2.34
	STAR	-1.53	-1.59	-1.76	-1.66	-1.66	-1.07	-1.54	-1.10	-1.61	-1.55
CCC	MS	0.15	0.69	0.34	0.27	0.27	0.23	0.43	0.50	0.51	0.54
	STAR	1.40	1.16	1.40	1.44	1.41	1.03	1.21	1.03	1.10	1.08
		BIC									
BBB	MS	-5.90	-6.45	-5.86	-5.55	-5.62	-3.09	-6.67	-1.89	-6.08	-5.85
	STAR	-5.61	-5.62	-5.34	-5.63	-5.67	-1.98	-5.63	-1.99	-5.65	-5.65
BB	MS	-5.07	-5.12	-5.08	-5.05	-5.09	-3.00	-5.32	-1.86	-5.28	-4.15
	STAR	-4.13	-4.59	-4.22	-4.21	-4.34	-1.88	-4.09	-1.88	-4.16	-4.17
B	MS	-2.45	-2.65	-2.59	-2.51	-2.57	-2.03	-2.45	-1.41	-2.52	-1.89
	STAR	-1.19	-1.25	-1.42	-1.32	-1.33	-0.74	-1.20	-0.77	-1.27	-1.21
CCC	MS	0.60	1.14	0.79	0.72	0.72	0.67	0.89	0.93	0.96	0.99
	STAR	1.74	1.50	1.73	1.78	1.75	1.36	1.55	1.36	1.44	1.42

All values are multiplied by 10^{-2}.
A green cell defines the lowest value for a rating category.

5.3 Out-of-sample performance comparison

5.3.1 One macroeconomic variable

This subsection discusses the models where one macroeconomic variable is included in either the MS or STAR model, this comes to a total of twenty models.

Tables 10 and 11 presents the RMSE values for the 1 quarter-, 1 year-, 2 year- and 4 year-step-ahead forecast. When we take a first glance at the table and look at the lowest (highlighted in green) RMSE values, we can see that the GDP, unemployment rate and both interest rates deliver some of the best forecasts for mostly the MS model and sometimes the STAR model. When we take a closer look we can see that the only significant differences in forecasting performance are produced by either the 3-month or 10-year interest rate. Even though these variables do not have the best in-sample fit, they do provide significantly better forecasts with the Diebold-Mariano (DB) test statistic on multiple occurrences.

Further noticeable is that unemployment rate has the lowest RMSE values for rating CCC/C. Although not significant, it does perform best on almost all forecast horizons all in the MS model. Strangely, unemployment rate has, contrary to the MS model, large values for the STAR model. Next to the unemployment rate, the EPU index specifically has large RMSE values in the STAR framework, especially when the forecast horizon goes up. This could be due to the high variance in these variables which apparently does not translate well into a transition variable for forecasting. GDP performs well on the 1-quarter forecasts in the MS model, however, these differences in the short term do not provide significant value. Table 9 showed that the NFCI has the best in-sample fit for ratings BBB and BB. For these ratings, this does not translate into the best performing forecasts.

To summarize, we see that the 3-month and 10-year interest rates lead to some significant differences in forecasts. Furthermore, the unemployment rate seems a valid candidate, mostly in the CCC/C rating category.

Table 10: RMSE values for both MS and STAR model with one macroeconomic variable on 1-quarter and 1-year horizons

		Macroeconomic variable included:									
Rating	Model	GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
		1q ahead RMSE									
BBB	MS	0.004	0.005	0.004	0.005	0.005	0.007	0.005	0.007	0.004	0.005
	STAR	0.004	0.005	0.008	0.004	0.005	0.008	0.005	0.005	0.004	0.049
BB	MS	0.049	0.055	0.050	0.047	0.048	0.044	0.049	0.044	0.046	0.047
	STAR	0.042	0.046	0.048	0.048	0.047	0.048	0.047	0.046	0.045	0.051
B	MS	0.112	0.098	0.103	0.110	0.110	0.093	0.102	0.093	0.099	0.099
	STAR	2.262	0.096	0.102	0.090	0.081	0.090	0.096	0.091	0.095	0.095
CCC	MS	0.757	0.797	0.796	0.796	0.770	0.798	0.784	0.794	0.781	0.778
	STAR	1.254	0.743	0.795	0.839	0.856	0.802	0.796	0.807	0.815	0.817
		1y ahead RMSE									
BBB	MS	0.029	0.038	0.022	0.034	0.040	0.041	0.036	0.041	0.011^{+}	0.019
BBB	STAR	0.040	0.044	0.084	0.045	0.056	0.050	0.032	0.033	0.016^{+}	0.079
BB	MS	0.075	0.074	0.074	0.075	0.078	0.073	0.058	0.073	0.057	0.069
BB	STAR	0.092	0.164	0.091	0.097	0.093	0.128	0.092	0.105	0.085	0.120
B	MS	0.202	0.221	0.208	0.207	0.205	0.210	0.202	0.210	0.196	0.241
B	STAR	0.304	0.391	0.311	0.300	0.368	0.316	0.305	0.310	0.313	0.297
CCC	MS	1.260	1.028	1.335	1.298	1.236	1.391	1.445	1.235	1.429	1.556
CCC	STAR	1.853	1.328	1.911	1.843	1.811	1.768	2.241	1.771	2.185	2.041

A green cell defines the lowest RMSE value for a forecast horizon and rating category.
A * indicates that a single model has the significant lowest RMSE value according to the DM test compared to all other models in the same forecast horizon and rating category.
Multiple $^{+}$'s indicate that multiple models have significant lower RMSE values to all other models, however, do not differ from each other significantly.

Table 11: RMSE values for both MS and STAR model with one macroeconomic variable on 2-year and 4-year horizons

		Macroeconomic variable included:									
Rating	Model	GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
		2y ahead RMSE									
BBB	MS	0.053	0.072	0.046	0.060	0.071	0.077	0.087	0.077	0.016^{+}	0.027
BBB	STAR	0.061	0.063	0.077	0.066	0.081	0.072	0.043	0.052	0.012^{+}	0.385
BB	MS	0.141	0.152	0.138	0.144	0.159	0.108	0.113	0.108	0.081	0.094
BB	STAR	0.151	0.282	0.117	0.190	0.136	0.222	0.127	0.140	0.095	0.223
B	MS	0.433	0.529	0.464	0.458	0.461	0.512	0.413	0.512	0.426	0.469
B	STAR	0.489	0.870	0.485	0.444	0.850	0.582	0.486	0.636	0.538	0.399
CCC	MS	2.457	1.932	2.534	2.502	2.361	3.158	2.982	2.373	2.694	2.537
CCC	STAR	2.547	2.542	2.860	2.752	2.464	2.476	3.563	2.504	4.330	3.546
		4y ahead RMSE									
BBB	MS	0.075	0.075	0.064	0.059	0.072	0.078	0.133	0.078	0.021^{+}	0.027^{+}
BBB	STAR	0.053	0.045	0.054	0.059	0.069	0.068	0.034	0.049	0.011^{+}	48.726
BB	MS	0.181	0.195	0.174	0.187	0.203	0.137	0.131	0.137	0.106	0.117
BB	STAR	0.140	0.161	0.139	0.138	0.211	0.562	0.096	0.164	0.143	0.309
B	MS	0.509	0.573	0.472	0.497	0.550	0.897	0.598	0.897	0.402^{+}	0.362^{+}
B	STAR	0.470	0.479	0.461	0.389	0.691	0.600	0.375	0.679	0.605	0.361^{+}
CCC	MS	2.745	3.493	3.200	2.981	2.897	3.046	3.129	2.615	2.543	2.424
CCC	STAR	4.312	4.435	4.803	4.251	4.048	4.347	5.230	4.420	7.019	5.540

A green cell defines the lowest RMSE value for a forecast horizon and rating category.
A * indicates that a single model has the significant lowest RMSE value according to the DM test compared to all other models in the same forecast horizon and rating category.
Multiple $^{+}$'s indicate that multiple models have significant lower RMSE values to all other models, however, do not differ from each other significantly.

5.3.2 Two macroeconomic variables

This section presents results for the models where combinations of two macroeconomic variables are included in both the MS and STAR model. This leads to a total of 135 models, 45 MS models and 90 STAR models. The STAR models are doubled of the MS models since, in this framework, we need to select either one of the variables to be the transition variable. The AIC and BIC estimates are stated in Tables 24, 23, 22 and 25 in Appendix Section C.1.1. The RMSE values are computed for the 135 models for the credit ratings B, BB, BBB and CCC on the forecast horizons one-quarter- ($h = 1$), one-year- ($h = 4$), two-year- ($h = 8$) and four-year-ahead ($h = 16$) forecasts. All these Estimates are presented in Tables 26, 27, 28, 29, 30, 31, 32 and 33 stated in Appendix Section

C.1.2. Next to the ten macroeconomic variables, tables 34, 36, 35 and 37 in Appendix C.1.2 show the results for using the Z-factor and the first principal component in the models combined with one macroeconomic variable. In this section, we highlight the most relevant results for each credit rating.

Credit rating category BBB shows interestingly that not one combination of variables significantly outperforms all the others. What it does show for the 2-year and 4-year forecasts is that almost all MS and STAR models which include the 3-month interest rate do significantly outperform all other models however not each other. The Z-factor in the STAR model together with the unemployment rate or CPI index also belong to this group of well-performing models. Table 12 highlights these results as we can see that significance lies over most of the row of the 3-month interest rate for the 2-year-ahead forecast. This shows the impact of the inclusion of the 3-month interest rate in either model for this credit rating.

Table 12: RMSE for 2-year-ahead forecasts with 2 variables for rating BBB

model	1st variable	2nd variable GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
MS	GDP										
	unemp	0.072									
	CPI	0.043	0.065								
	imp	0.056	0.060	0.039							
	exp	0.070	0.086	0.058	0.080						
	VIX	0.034	0.039	0.035	0.037	0.055					
	NFCI	0.046	0.063	0.075	0.036	0.072	0.053				
	EPU	0.092	0.063	0.037	0.066	0.084	0.141	0.060			
	3-m	0.029	0.018^{+}	0.012^{+}	0.017^{+}	0.030	0.022^{+}	0.022^{+}	0.019^{+}	NA	
	10-y	0.032	NA	0.043	0.032	NA	0.507	0.015^{+}	NA	0.184	
STAR	GDP		0.070	0.051	0.065	0.079	0.041	0.047	0.090	0.027	0.030
	unemp	0.128		0.104	0.173	0.206	0.050	0.057	0.072	0.183	0.225
	CPI	0.051	0.055		0.049	0.064	0.033	0.035	0.047	0.012^{+}	0.019
	imports	0.065	0.104	0.049		0.081	0.043	0.053	0.083	0.022^{+}	0.031
	exports	0.079	0.092	0.064	0.081		0.061	0.071	0.105	0.039	0.050
	VIX	0.621	0.048	0.033	0.049	0.061		0.030	0.037	0.069	0.120
	NFCI	0.047	0.057	0.035	0.053	0.071	0.030		0.047	0.012^{+}	0.022^{+}
	EPU	0.091	0.072	0.044	0.083	0.106	0.045	0.047		0.101	0.059
	3-m	0.027	0.023^{+}	0.01^{+}2	0.022^{+}	0.039	0.016^{+}	0.015^{+}	0.013^{+}		0.020^{+}
	10-y	0.027	0.210	0.085	0.031	0.049	0.069	0.033	0.236	0.020^{+}	
	Z	0.030	0.014^{+}	0.014^{+}	0.029	0.031	0.032	0.028	0.030	0.030	0.030

A green cell defines the lowest RMSE value for a forecast horizon and rating category.
A * indicates that a single model has the significant lowest RMSE value according to the DM test compared to all other models in the same forecast horizon and rating category.
Multiple $^{+}$'s indicate that multiple models have significant lower RMSE values to all other models, however, do not differ from each other significantly.
N.A. stands for not available since the estimation algorithm was not able to process this combination of credit rating and variables.

Rating BB shows that only a significant result is realized in the 4-year-ahead forecasts for the STAR model with the NFCI as transition variable together with the unemployment rate. In Table 13, we can see that the RMSE for this particular combination is quite lower than the others. On the other forecast horizons, no particular model stands out. The difference between some models is small and without significance. For this rating category, the PC and Z-factor do not yield any useful results.

Table 13: RMSE for 4-year-ahead forecasts with 2 variables for rating BB

	1st	2nd variable									
model	variable	GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
MS	GDP										
	unemp	0.209									
	CPI	0.168	0.185								
	imp	0.211	0.242	0.166							
	exp	0.186	0.209	0.127	0.170						
	VIX	0.107	0.084	0.113	0.092	0.260					
	NFCI	0.121	0.144	0.140	0.141	0.165	0.115				
	EPU	0.252	0.105	0.128	0.205	0.239	10.212	0.115			
	3-m	0.256	0.242	0.235	0.139	0.150	0.399	0.099	0.553		
	10-y	0.186	0.181	0.195	0.192	0.142	0.316	0.115	0.170	0.078	
STAR	GDP		0.139	0.137	0.176	0.176	0.098	0.114	0.330	0.177	0.113
	unemp	0.109		0.244	0.094	0.081	0.143	0.193	0.151	0.349	0.348
	CPI	0.137	0.117		0.145	0.141	0.099	0.171	0.274	0.142	0.108
	imports	0.176	0.163	0.135		0.184	0.094	0.150	0.271	0.126	0.097
	exports	0.176	0.182	0.141	0.187		0.090	1.126	0.329	0.109	0.087
	VIX	0.098	0.072	0.098	0.094	0.090		0.099	0.125	0.136	0.102
	NFCI	0.114	0.052*	0.110	0.094	0.101	0.099		0.196	0.171	0.122
	EPU	0.245	0.511	0.152	0.271	0.315	0.141	0.273		0.227	0.148
	3-m	0.177	0.158	0.143	0.127	0.110	0.136	0.170	0.230		0.172
	10-y	0.113	0.093	0.324	0.097	0.087	0.102	0.122	0.148	0.173	

A green cell defines the lowest RMSE value for a forecast horizon and rating category.
A * indicates that a single model has the significant lowest RMSE value according to the DM test compared to all other models in the same forecast horizon and rating category.
Multiple $^+$'s indicate that multiple models have significant lower RMSE values to all other models, however, do not differ from each other significantly.

Rating B only shows significant results for the 1-year forecast horizon as highlighted in Table 14. The MS model with the 3-month interest rate and the CPI index performs significantly best. As CPI tracks inflation for consumers, the combination between inflation and interest rate is somewhat surprising as they usually have an inverse relationship. What we mean by that is when interest rates go down, inflation usually goes up. The CPI has the lowest correlation out of all variables to the default rate of this credit rating with a value of -0.025 (Section 2.2, Table 3) and can therefore provide extra useful information. To show the difference in forecasting performance between models, we can take a closer look at the two models. Figure 12 shows the comparison between one-year-ahead forecasts of two MS models where the 3-month interest rate and either the CPI or imports are taken into the model. The model with CPI significantly outperforms the other with a test statistic of 2.134 and a p-value equal to 0.0418. We can also see that the point forecasts indicated by an x for model 2 clearly lies closer in most cases to the observed value. For the 2-year ahead forecasts, the lowest RMSE value lies in the MS model with the first PC and exports although not significantly different from models with close RMSE values.

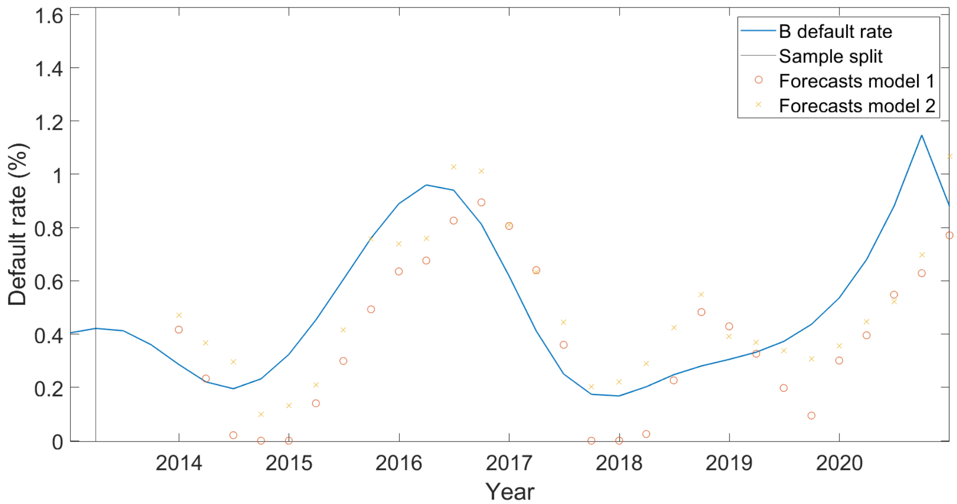

Figure 12: 1-year ahead prediction on rating B with two MS models

Model 1 includes variables 3-month interest rate and CPI. Model 2 includes 3-month interest rate and imports.

Table 14: RMSE for 1-year-ahead forecasts with 2 variables for rating B

model	1st variable	2nd variable GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
MS	GDP										
	unemp	0.217									
	CPI	0.209	0.222								
	imp	0.212	0.254	0.209							
	exp	0.196	0.226	0.214	0.220						
	VIX	0.206	0.226	0.205	0.220	0.214					
	NFCI	0.202	0.229	0.227	0.252	0.204	0.214				
	EPU	0.215	0.217	0.209	0.244	0.209	0.550	0.214			
	3-m	0.239	0.222	0.189*	0.241	0.203	0.242	0.291	0.242		
	10-y	0.309	0.245	0.224	0.198	0.216	0.278	0.292	0.232	0.272	
STAR	GDP		0.363	0.314	0.299	0.303	0.293	0.322	0.330	0.343	0.297
	unemp	0.363		0.358	0.437	0.383	0.502	0.418	0.476	0.477	0.411
	CPI	0.330	0.358		0.311	0.310	0.299	0.304	0.318	0.361	0.316
	imports	0.308	0.358	0.318		0.334	0.306	0.341	0.330	0.319	0.308
	exports	0.302	0.549	0.338	0.348		0.305	0.412	0.450	0.311	0.312
	VIX	0.293	0.440	0.302	0.306	0.312		0.390	0.355	0.395	0.556
	NFCI	0.322	0.418	0.334	0.344	0.322	0.303		0.372	0.345	0.315
	EPU	0.335	0.881	0.476	0.387	0.350	0.368	0.395		0.766	0.478
	3-m	0.343	0.699	0.312	0.319	0.311	0.430	0.345	0.355		0.359
	10-y	0.297	3.682	0.440	0.309	0.299	0.768	0.628	0.316	0.359	

A green cell defines the lowest RMSE value for a forecast horizon and rating category.
A * indicates that a single model has the significant lowest RMSE value according to the DM test compared to all other models in the same forecast horizon and rating category.
Multiple $^+$'s indicate that multiple models have significant lower RMSE values to all other models, however, do not differ from each other significantly.

Rating CCC shows no significant results and therefore we can not draw conclusions based on significance for this credit rating category. However, we can see a pattern of low RMSE values when unemployment rate is included for both the MS and STAR model as we see in Table 15. Noticeably, when one macroeconomic variable is included unemployment rate also leads to the lowest RMSE value. Including one extra variable does lead to better forecasts as the RMSE value on the 2-year forecasts with only unemployment rate equals 1.932 (Table 11, Section 5.3.1) compared to 1.837 when the CPI index is included as well. On the 1-year forecast horizon, the lowest RMSE value also includes the unemployment rate for the MS model together with the first PC. The lack of significance could be due to high volatility over time in defaults for this rating. This could also be the reason that some RMSE values on the 4-year forecast horizon are very high (Section C.1.2, Table 33).

Table 15: RMSE for 2-year-ahead forecasts with 2 variables for rating CCC

model	1st variable	2nd variable GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
MS	GDP										
	unemp	3.181									
	CPI	2.490	1.837								
	imp	2.442	1.885	2.704							
	exp	2.098	1.886	2.366	2.212						
	VIX	2.875	2.639	3.111	4.331	4.193					
	NFCI	2.798	2.237	2.788	3.098	2.904	3.004				
	EPU	2.355	2.373	3.498	2.909	3.033	1.912	3.004			
	3-m	2.735	3.025	2.688	2.573	2.536	3.477	2.751	3.477		
	10-y	2.589	2.694	2.540	2.419	2.333	2.595	2.374	2.595	2.712	
STAR	GDP		1.905	3.881	2.562	2.320	6.166	3.257	14.269	4.581	4.167
	unemp	2.824		2.930	2.663	2.519	3.079	2.829	2.632	2.371	2.487
	CPI	3.104	2.160		2.850	2.590	4.017	3.084	3.115	5.058	4.614
	imports	2.782	2.174	3.470		2.834	4.278	3.118	2.848	4.067	4.996
	exports	2.784	1.894	2.882	2.538		4.102	3.039	3.521	3.894	4.405
	VIX	4.667	3.430	5.118	4.718	3.882		3.698	4.880	4.034	3.214
	NFCI	3.686	3.063	3.534	3.141	2.954	4.300		3.431	5.350	4.783
	EPU	2.663	2.140	3.303	2.461	2.403	5.371	3.172		3.870	4.140
	3-m	4.449	2.971	4.334	4.311	4.472	5.181	5.634	4.835		3.570
	10-y	3.514	2.240	3.708	3.596	3.510	4.308	3.541	2.650	3.219	

A green cell defines the lowest RMSE value for a forecast horizon and rating category.
A * indicates that a single model has the significant lowest RMSE value according to the DM test compared to all other models in the same forecast horizon and rating category.
Multiple +'s indicate that multiple models have significant lower RMSE values to all other models, however, do not differ from each other significantly.

When we compare the RMSE with two variables to the models with one variable, we notice that for each credit rating including a second variable leads to better RMSE values. This shows the added effect of adding another variable to these models. A downside for including two variables is that the interpretation of regimes becomes more challenging as the regimes are determined by the past default rates and 2 sources of exogenous information.

Comparing the MS to the STAR method, the MS model clearly performs best for most credit ratings and forecast horizons. Not only performs the MS model better, but also the estimation of seems persistent while for the STAR model, estimates of $\hat{c}$ and $\hat{\gamma}$ can 'shut down' a model. What we mean is when $\hat{c}$ is estimated above 20, for example, the function does not run smoothly between regimes. Moreover, when $\hat{\gamma}$ is estimated below the minimum or above the maximum value of what the transition variable will ever reach, the model collapses to a linear equation as one of the two regimes will never occur. For these reasons, we prefer the MS model over the STAR model.

To summarize, we see that for higher credit rating categories like BBB, including an interest rate (mostly the 3-month interest rate) leads to a significant difference in forecasting accuracy. Also for rating B, the inclusion of the 3-month interest rate leads to a significant difference. Furthermore, we see that the unemployment rate is a valid candidate to include in models. Rating BB shows significance with the STAR model where unemployment rate is included. Also for rating CCC, we can see that the best performing models include the unemployment rate. Even though we can not provide significant results for rating CCC, we can see a pattern.

5.4 Best performing models

In this section, we take a closer look at the three best performing models and analyze why these models have the lowest RMSE values. For credit rating BBB, the best performing models are a multitude of MS models and one STAR model which all include the 3-month interest rate. Also for credit rating B, The only significant result lies in the usage of the MS model with the 3-month interest rate and CPI index. Since the same model yields the best result for two ratings we focus on just analyzing one of the ratings, namely the BBB rating. Table 16 shows the parameter estimation of the MS model for rating BBB with the CPI index and 3-month interest rate. We can see large differences in $\hat{\sigma}_i$ for $i = 1, 2$. This means that regime 1 has low variance while regime 2 has a higher variance. Also, the shape of the curve is different in each regime as especially the second autoregressive lag parameter differs a lot over the 2 regimes. Also, the relation between the macroeconomic variables and the output is inverted in the regimes as the sign changes from negative to positive. In Figure 14, we can see what happens with the forecasts when the regime changes that are indicated by the black vertical line. The forecasts starting before the regime change are rather flat while the starting after the regime change have more curvature. Furthermore, we can see that the predicted increase in defaults from 2018 do hold however the increase does start half about a year later than predicted.

Table 16: Parameter estimates MS model with CPI and 3-month interest for credit rating CCC/C

$\hat{p}_{11}$	$\hat{a}_1$	$\hat{\phi}_{1,1}$	$\hat{\phi}_{2,1}$	$\hat{\phi}_{3,1}$	$\hat{\beta}_{1,1}$	$\hat{\beta}_{2,1}$	$\hat{\sigma}_1$
0.758	0.004	2.700	-2.704	0.967	-0.002	-0.002	0.001
$\hat{p}_{22}$	$\hat{a}_2$	$\hat{\phi}_{1,1}$	$\hat{\phi}_{2,1}$	$\hat{\phi}_{3,1}$	$\hat{\beta}_{1,2}$	$\hat{\beta}_{2,2}$	$\hat{\sigma}_2$
0.966	-0.001	2.296	-1.984	0.638	0.001	0.002	0.008

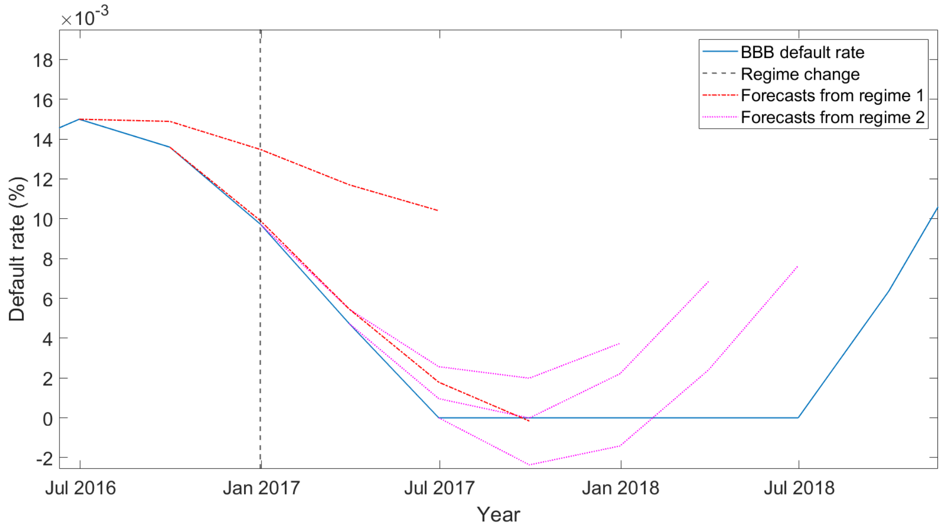

Figure 14: Forecasts with MS model CPI and 3-month interest rate for rating BBB during regime changes

For credit rating BB, there is no one model which is clearly the best performing on all forecast horizons. Only on the 4-year horizon, the STAR model with the NFCI and unemployment rate provide significantly better forecasting over the other combinations of variables. When we take a look at the parameter estimates of this model, we see that $\hat{c} = 35.379$ while the transition variable never reaches over 3.068. When the transition variable never reaches the threshold between regimes, the model will only be in one regime over the entire sample. Consequently, the STAR model collapses to an AR model as the transition function will always equal 0 or close to it. This is a disadvantage of the STAR model, where parameter estimation can shut down the purpose of the model. As Section 5.5 also shows that on the 4-year forecast horizon, results are not robust and therefore it seems that this particular result seems a coincidence and not relevant.

For credit rating CCC, we take a look at the MS model with the first PC and unemployment rate as this model performs best on the 1- and 2-year forecast horizon. In Figure 15, we can see that the regimes only switch in crises periods, namely the dot-com bubble, global financial crises and the Covid-19 pandemic.

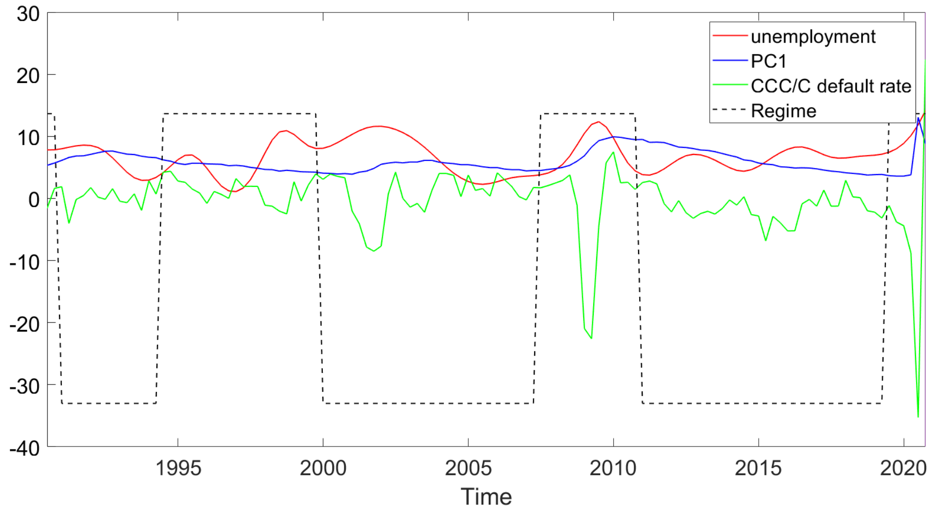

Figure 15: Regime changes MS model with unemployment rate and the first principal component for rating CCC/C

Since the start of the COVID-19 pandemic is the only crisis included in the out-of-sample space, we take a look at the forecasts around this period in Figure 16. Here we see that it predicts the rising defaults actually sooner than they happened in 2017 Q3. After a correction at the end of 2018, it starts predicting high defaults again with a small delay on the actual defaults. Even though these forecasts do not seem incredible accurate at first sight, predicting defaults before or during crisis periods is a challenge. That this model can be this close to the actual defaults in this period is a large advantage over other models.

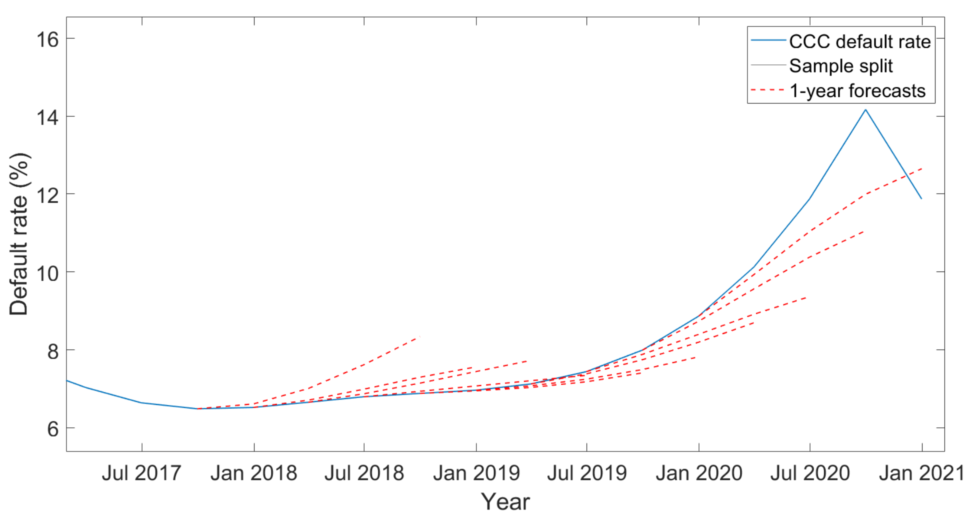

Figure 16: Zoom-in on Forecasts 1-year-ahead MS model with unemployment rate and the first principal component for rating CCC/C

From these best performing models, we can see that the transition rates do depend on the state of the economy. This supports findings in literature as Koopman and Lucas (2005), Gavalas and Syriopoulos (2014) and Nickell et al. (2000) all suggest that shifts in the economy impact transition rates. From the best performing models, we can see that how the impact is defined can differ depending on how the regimes are modelled. Examples are that defaults spike in crisis periods or that the curvature of the default rate differs across regimes. We have also seen that macroeconomic variables can impact the default rate positively in one regime and negatively in the other. Furthermore, we show that significant differences can be made by choosing the right variables for each credit rating category. Together, this shows the dependence between the macro economy and transition rates to default and therefore the PD.

5.5 Robustness analysis

To test the robustness of the obtained results, we analyze the outcomes when simulated variance is added to the data as described in Section 2.1.3. We want to see whether the best performing model remains the best when the data does not run as smoothly. Therefore, we add variance to the interpolated default curve as described in Section 2.1.2 and analyze whether this impacts the results.

Tables 17 and 18 show the RMSE values for the MS and STAR model with one macroeconomic variable included for the 1- and 2-year-ahead forecasts. We can see, for almost all simulations, that the best performing model does not change. This shows that the findings in this research are robust to variation in the data set on these horizons. However, on the 1-quarter and 4-year ahead forecasts, we find that results are not as robust at all as Tables 19 and 20 show. This is not surprising as we can also not come to any significant conclusions on the 1-quarter horizon and only some on the 4-year horizon in Section 5.3.

Table 17: RMSE for different simulations for the MS model, 1-year and 2-year ahead

MS model	Macroeconomic variable included: GDP	unemp	CPI	imports	exports	VIX	NFCI	EPU	3-m	10-y
	1y ahead RMSE									
No sim	1.260	1.028	1.335	1.298	1.236	1.391	1.445	1.235	1.429	1.556
Sim #1	1.959	1.568	1.926	1.892	2.381	2.454	2.052	2.454	2.068	2.050
Sim #2	2.392	1.754	2.167	2.596	2.695	2.586	2.304	2.586	2.159	1.834
Sim #3	2.061	1.799	2.168	2.126	2.087	1.998	2.703	1.998	2.628	2.395
Sim #4	2.074	1.885	2.085	2.238	2.286	2.651	2.298	2.651	2.154	2.168
Sim #5	2.338	2.039	2.559	2.578	2.563	2.407	2.274	2.407	2.280	2.322
Sim #6	1.982	1.505	2.426	1.956	2.078	2.491	2.543	2.491	1.833	2.042
Sim #7	2.077	1.907	2.131	2.045	2.050	2.317	2.388	2.317	1.897	2.058
Sim #8	2.117	1.669	2.071	2.066	1.997	2.433	2.281	2.433	2.131	2.143
Sim #9	2.354	1.910	2.478	2.692	2.291	2.547	2.355	2.547	2.479	2.312
	2y ahead RMSE									
No sim	2.457	1.932	2.534	2.502	2.361	3.158	2.982	2.373	2.694	2.537
Sim #1	2.562	1.928	2.589	2.521	3.108	3.203	2.772	3.203	2.466	2.572
Sim #2	3.381	1.974	2.544	3.675	3.501	3.874	3.251	3.874	2.820	2.191
Sim #3	2.771	1.641	2.714	2.872	2.822	2.763	3.623	2.763	2.485	2.256
Sim #4	2.691	2.174	2.587	2.760	2.880	3.402	2.938	3.402	2.539	2.490
Sim #5	2.655	2.110	3.208	2.818	2.916	2.862	2.530	2.862	2.448	2.583
Sim #6	2.710	2.051	2.827	2.830	2.901	3.079	NaN	3.079	2.390	2.322
Sim #7	2.582	2.476	2.751	2.636	2.704	3.124	3.271	3.124	2.396	2.221
Sim #8	2.794	2.025	2.706	2.788	2.550	3.022	3.079	3.022	2.512	2.378
Sim #9	3.054	2.056	2.614	2.806	2.648	2.759	2.821	2.759	2.472	2.637

A green cell defines the lowest RMSE value.

Table 18: RMSE for different simulations for the STAR model, 1-year and 2-year ahead

STAR model	Macroeconomic variable included: GDP	unemp	CPI	imports	exports	VIX	NFCI	EPU	3-m	10-y
	1y ahead RMSE									
No sim	1.853	1.328	1.911	1.843	1.811	1.768	2.241	1.771	2.185	2.041
Sim #1	3.231	1.497	3.356	2.321	2.830	2.210	2.517	2.247	2.100	2.073
Sim #2	2.624	1.793	2.273	2.599	2.956	2.280	2.586	2.452	2.178	2.292
Sim #3	2.511	1.836	5.505	2.813	3.143	2.470	2.745	2.441	2.372	2.340
Sim #4	2.752	1.891	2.568	2.844	2.843	2.493	2.691	2.458	2.469	2.464
Sim #5	2.628	2.087	2.806	2.944	3.283	2.436	2.610	2.583	2.523	2.525
Sim #6	2.752	1.456	2.454	2.658	3.293	2.257	2.663	2.473	2.220	2.326
Sim #7	2.533	1.709	2.523	2.542	2.878	2.258	2.589	2.684	2.254	2.292
Sim #8	2.599	1.661	2.246	2.562	3.149	2.257	2.559	2.397	2.273	2.396
Sim #9	2.683	1.981	3.106	3.209	3.166	2.576	2.747	2.939	2.558	2.499
	2y ahead RMSE									
No sim	2.547	2.464	2.860	2.752	2.542	2.476	3.563	2.504	4.330	3.546
Sim #1	4.690	1.400	8.186	3.094	3.923	2.714	3.262	2.798	2.510	2.489
Sim #2	3.237	1.908	3.019	3.576	3.618	2.871	3.415	3.067	2.605	2.838
Sim #3	3.164	2.085	24.633	3.489	4.561	3.026	3.413	2.969	2.771	2.714
Sim #4	3.243	1.793	3.128	3.994	4.309	2.875	3.248	2.778	2.701	2.701
Sim #5	3.259	1.905	3.868	4.138	4.799	2.704	3.177	2.950	2.872	2.881
Sim #6	3.621	1.465	3.078	3.353	5.442	2.785	3.435	3.094	2.668	2.965
Sim #7	3.225	1.989	3.438	3.923	4.161	2.697	3.295	3.129	2.675	2.792
Sim #8	3.229	1.778	3.795	3.350	4.798	2.693	3.240	3.092	2.748	2.986
Sim #9	3.108	1.687	4.768	3.803	3.648	2.965	3.142	3.750	2.671	2.678

A green cell defines the lowest RMSE value.

Table 19: RMSE for different simulations for the MS model, 1-quarter and 4-year ahead

MS model	Macroeconomic variable included:									
	GDP	unemp	CPI	imports	exports	VIX	NFCI	EPU	3-m	10-y
	1q ahead RMSE									
No sim	0.757	0.797	0.796	0.796	0.770	0.798	0.784	0.794	0.781	0.778
Sim #1	0.764	0.967	0.974	0.916	0.917	1.049	0.942	1.049	0.855	0.858
Sim #2	1.001	0.942	1.019	1.057	1.002	0.938	0.933	0.938	0.910	0.924
Sim #3	0.875	0.799	0.834	0.829	0.856	0.835	0.899	0.835	0.851	0.852
Sim #4	0.781	0.891	0.845	0.841	0.859	0.966	0.887	0.966	0.906	0.908
Sim #5	1.220	1.290	1.233	1.323	1.189	1.234	1.235	1.234	1.214	1.239
Sim #6	0.766	0.879	0.792	0.673	0.838	0.893	NaN	0.893	0.808	0.791
Sim #7	0.668	0.810	0.802	0.731	0.699	0.820	0.821	0.820	0.772	0.772
Sim #8	0.762	0.890	0.854	0.788	0.832	0.892	0.881	0.892	0.831	0.834
Sim #9	1.069	1.082	1.039	1.103	0.989	1.041	1.019	1.041	1.016	1.030
	4y ahead RMSE									
No sim	2.745	3.493	3.200	2.981	2.897	3.046	3.129	2.615	2.543	2.424
Sim #1	4.766	4.580	4.688	4.721	4.811	4.575	4.764	4.575	3.970	3.971
Sim #2	4.727	4.401	4.386	4.996	4.953	4.523	4.674	4.523	4.304	4.233
Sim #3	4.767	3.949	4.582	4.908	4.958	4.960	5.163	4.960	4.262	4.240
Sim #4	4.225	4.408	4.275	4.588	4.386	4.686	4.714	4.686	3.979	3.963
Sim #5	4.191	4.031	4.508	4.536	4.395	4.383	4.247	4.383	3.854	3.830
Sim #6	4.552	4.537	4.344	4.750	4.822	4.489	NaN	4.489	3.981	3.945
Sim #7	4.383	4.643	4.570	4.633	4.381	4.478	4.954	4.478	4.024	3.985
Sim #8	4.541	4.402	4.338	4.497	4.593	4.507	4.726	4.507	3.878	3.861
Sim #9	4.598	4.551	4.326	4.478	4.433	4.539	4.535	4.539	3.978	3.910

A green cell defines the lowest RMSE value.

Table 20: RMSE for different simulations for the STAR model, 1-quarter and 4-year ahead

MS model	Macroeconomic variable included:									
	GDP	unemp	CPI	imports	exports	VIX	NFCI	EPU	3-m	10-y
	1q ahead RMSE									
No sim	1.254	0.743	0.795	0.839	0.856	0.802	0.796	0.807	0.815	0.817
Sim #1	1.480	1.161	0.839	0.904	0.599	0.997	0.915	0.937	0.941	0.940
Sim #2	1.216	1.326	0.928	1.022	0.888	0.946	0.941	1.577	0.929	0.932
Sim #3	1.821	0.788	1.010	0.892	0.935	0.869	0.856	3.481	0.833	0.830
Sim #4	1.191	0.850	0.863	0.767	0.794	0.965	0.893	2.216	0.915	0.914
Sim #5	1.453	1.628	1.454	1.318	1.120	1.240	1.236	1.210	1.266	1.269
Sim #6	0.844	1.147	0.861	0.992	0.684	0.923	0.860	0.876	0.855	0.865
Sim #7	1.029	0.789	0.837	0.838	0.723	0.797	0.810	0.866	0.805	0.809
Sim #8	1.653	1.115	0.913	1.557	0.762	0.940	0.855	5.174	0.876	0.883
Sim #9	2.460	1.435	1.013	1.052	0.989	1.055	1.029	1.093	1.046	1.040
	4y ahead RMSE									
No sim	4.312	4.435	4.803	4.251	4.048	4.347	5.230	4.420	7.019	5.540
Sim #1	9.274	4.523	13.960	4.643	6.354	4.560	4.699	4.559	4.385	4.407
Sim #2	4.552	4.546	4.308	4.957	5.239	4.527	4.682	4.512	4.326	4.501
Sim #3	4.642	4.557	996.948	6.048	6.306	4.757	4.906	4.784	4.607	4.601
Sim #4	4.519	4.465	4.520	5.120	6.770	4.595	4.669	4.637	4.447	4.472
Sim #5	4.250	4.394	4.696	6.498	6.522	4.259	4.463	4.202	4.311	4.311
Sim #6	4.760	4.587	4.509	4.367	8.084	4.559	4.813	4.546	4.373	4.632
Sim #7	4.585	4.435	4.837	4.565	6.146	4.543	4.740	4.143	4.492	4.575
Sim #8	4.657	4.473	636.433	5.083	7.587	4.448	4.621	317.590	4.478	4.568
Sim #9	4.498	4.503	6.631	6.054	4.928	4.577	4.665	4.636	4.211	4.451

A green cell defines the lowest RMSE value.

6 Conclusion

In this research, we try to determine if we can significantly improve on forecasting the Probability of Default (PD) by varying the input of macroeconomic variables. And if so, which macroeconomic variables are best to use. To do so, we use data on transition rates to default per credit rating category and build regime-switching models to forecast them. We find that we can not draw conclusions over all credit ratings together since each credit rating has its own time series with different characteristics. For most credit ratings, we can find a preferred combination of variables and methods. For credit ratings BBB and B, the inclusion of the 3-month interest rate leads to the best performing models.. For credit rating BB, we can not find one model which performs on multiple forecast horizons best. For credit rating CCC, we find that including the unemployment rate leads to the lowest RMSE values. Although these differences are not significant, we can see a pattern of low RMSE values when this variable is included. What these results implicate, is that we can find significant differences in forecasting accuracy by using different macroeconomic variables in most cases. This shows the relevance of researching the best combination of variables for each implemented model. As previous research (Koopman and Lucas (2005), Gavalas and Syriopoulos (2014) and Nickell et al. (2000)) suggests that transition rates (and therefore the PD) may depend on the state of the macro economy is something we can support with this research. Unfortunately, we can not conclude that one set of variables is best across all rating and forecasting time horizons, however, we can conclude that varying over these variables can lead to significantly better results. Furthermore, we can say that the 3-month interest rate and the unemployment rate are the most promising variables to include in a PD model and can lead to the best performance. The results are supported by showing that the obtained results are robust to added variation in the data set. We suggest to researchers and companies to research into their own models and see if the optimal sources of macroeconomic information are in use as we show that changing them can lead to a significant outperformance in forecasting accuracy.

Suggestions for future research are to see whether macroeconomic variables influence existing PD models as much as the models used in this research. Next to that, the used models in this research could be explored more as there are several extensions on these models that could lead to even better forecasting performance. Next to PD, the influence of the macro economy on loss given default and exposure at default could be researched. Together with the PD, you could see what the total influence on the expected credit loss is.

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A Parameter estimation methods

This section explains how the Expectation–maximization (EM) algorithm works for the MS framework and how the parameters are estimated in the STAR framework with nonlinear least squares (NLS).

A.1 Expectation–maximization algorithm

The Expectation–maximization algorithm (EM) algorithm consists of initialization, expectation (E-step) and maximization (M-step). After the initialization the algorithm iterates through the steps. Usually, parameter convergence should happen efficiently and no more then 100 iterations are required.

In the E-step, the aim is to take the expectation of the log-likelihood function ($\mathrm{E}_{\mathbf{S}_T|\mathcal{I}_T}\left[\ln f\left(y_T, \mathbf{S}_T; \boldsymbol{\theta}\right)\right]$). To accomplish this we start with the likelihood function which depends on the time series and its parameters

$$f\left(y_T; \boldsymbol{\theta}\right) = \prod_{t=1}^{T} f\left(y_t; \boldsymbol{\theta}\right) = \prod_{t=1}^{T} \left(p_1 \phi\left(y_t; u_t, \theta_1, \sigma_1\right) + \left(1 - p_1\right) \phi\left(y_t; u_t, \theta_2, \sigma_2\right)\right), \tag{22}$$

where the pdf of y_t equals

$$\phi\left(y_t; u_t, \theta_{S_t}, \sigma_{S_t}\right) = (2\pi)^{-\frac{1}{2}} \sigma_{S_t}^{-\frac{1}{2}} \exp\left\{-\frac{1}{2} \frac{\left(y_t - \theta_{S_t} u_t\right)^2}{\sigma_{S_t}^2}\right\}. \tag{23}$$

Then we take the natural logarithm to compute to the log-likelihood

$$\begin{aligned} \ln f\left(y_T, \mathbf{S}_T; \boldsymbol{\theta}\right) = \sum_{t=1}^{T} & \left(\mathrm{I}\left[S_t = 1\right] \ln p_1 + \mathrm{I}\left[S_t = 2\right] \ln\left(1 - p_1\right)\right. \\ & \left. + \mathrm{I}\left[S_t = 1\right] \ln \phi\left(y_t; u_t, \theta_1, \sigma_1\right) + \mathrm{I}\left[S_t = 2\right] \ln \phi\left(y_t; u_t, \theta_2, \sigma_2\right)\right), \end{aligned} \tag{24}$$

where we split up the two regimes with indicator functions $\mathrm{I}\left[S_t = k\right]$. Now, we can take the expectation of the log-likelihood

$$\begin{aligned} \mathrm{E}_{\mathbf{S}_T|\mathcal{I}_T}\left[\ln f\left(y_T, \mathbf{S}_T; \boldsymbol{\theta}\right)\right] = \sum_{t=1}^{T} & [p_{1t}^* \ln p_1 + \left(1 - p_{1t}^*\right) \ln\left(1 - p_1\right) \\ & + p_{1t}^* \ln \phi\left(y_t; u_t, \theta_1, \sigma_1\right) + \left(1 - p_{1t}^*\right) \ln \phi\left(y_t; u_t, \theta_2, \sigma_2\right)], \end{aligned} \tag{25}$$

where the expectation of the indicator function is defined as p^*_{kt}. the derivation for $p^*_{kt}$is as follows.

$$\begin{aligned} P\left[S_t=1 \mid \mathcal{I}_T, \boldsymbol{\theta}\right] &= \frac{f\left(y_t, S_t=1; \boldsymbol{\theta}\right)}{f\left(y_t; \boldsymbol{\theta}\right)} \\ &= \frac{f\left(y_t \mid S_t=1; \boldsymbol{\theta}\right) P\left[S_t=1\right]}{\sum_{s_t} f\left(y_t \mid s_t; \boldsymbol{\theta}\right) P\left[S_t=s_t\right]} \\ &= \frac{p_1 \phi\left(y_t; u_t, \theta_1, \sigma_1\right)}{p_1 \phi\left(y_t; u_t, \theta_1, \sigma_1\right)+\left(1-p_1\right) \phi\left(y_t; u_t, \theta_2, \sigma_2\right)} \equiv p^*_{1t} \end{aligned} \tag{26}$$

To draw inference about the probabilities of the regimes over time we can use the Kim smoother which uses all information until time T. For computational issues, a four state representation is used from now on with:

- State 1: $S_t = 1$ and $S_{t-1} = 1$
- State 2: $S_t = 1$ and $S_{t-1} = 2$
- State 3: $S_t = 2$ and $S_{t-1} = 1$
- State 4: $S_t = 2$ and $S_{t-1} = 2$

The Kim smoother then captures the probabilities of being in either one of the four states.

$$\widehat{\boldsymbol{\xi}}^*_{t|T} = \begin{pmatrix} p^*_{11}(t) \\ p^*_{12}(t) \\ p^*_{21}(t) \\ p^*_{22}(t) \end{pmatrix}. \tag{27}$$

Derivations can be found in Kim (1994). We can then link the probabilities of being in one of the four states to being in one of the two original states we initialised

$$\begin{aligned} p^*_1(t) &:= P\left(s_t = 1 \mid \mathcal{I}_T\right) = p^*_{11}(t) + p^*_{12}(t-1) \\ p^*_2(t) &:= P\left(s_t = 2 \mid \mathcal{I}_T\right) = p^*_{21}(t) + p^*_{22}(t-1) \end{aligned}, \tag{28}$$

where the

$$p^*_{ij}(t) := P\left(S_t = i, S_{t-1} = j \mid \widehat{\boldsymbol{\theta}}_0, \mathcal{I}_T\right), fori, j = 1, 2. \tag{29}$$

For forecasting purposes, we use the smoothed probabilities from the original two state representation

$$\widehat{\boldsymbol{\xi}}_{t|T} = \begin{pmatrix} p^*_1(t) \\ p^*_2(t) \end{pmatrix}. \tag{30}$$

In the M-step, we maximize the obtained function in the E-step (Equation 25) by taking the derivative to each of the parameters. This leads to the following estimates

$$\begin{aligned}
\widehat{p}_{11} &= \frac{\sum_{t=2}^{T} p_{11}^{*}(t)}{\sum_{t=2}^{T} p_{1}^{*}(t-1)}, \\
\widehat{p}_{22} &= \frac{\sum_{t=2}^{T} p_{22}^{*}(t)}{\sum_{t=2}^{T} p_{2}^{*}(t-1)}, \\
\widehat{\theta}_{i} &= \frac{\sum_{t=1}^{T} p_{i}^{*}(t) y_t u_t}{\sum_{t=1}^{T} p_{i}^{*}(t) u_t^2}, \\
\hat{\sigma}_{i}^{2} &= \frac{\sum_{t=1}^{T} p_{i}^{*}(t) \left(y_t - \widehat{\theta}_{S_t} u_t\right)^2}{\sum_{t=1}^{T} p_{i}^{*}(t)}.
\end{aligned} \tag{31}$$

We initialise the algorithm by setting the parameters in the M-step to a certain starting point. $p11$ and $p22$ are set at random between 0 and 1. In $\theta_{S_t} = (a_{S_t}, \beta_{S_t}, \phi_{1,S_t}, \ldots, \phi_{p,S_t})$, we chose to set α_{S_t} to the mean of y_t. The other parameters $\beta_{S_t}, \phi_{1,S_t}, \ldots, \phi_{p,S_t}$, which are linked to the exogenous variables and the autoregressive lags, can be picked at random and are therefore set at 0.1. σ_1 and σ_2 are set close to the standard deviation of y_t, for computational issues these are not set equal at initialisation. By plugging these starting values into the E-step, the algorithm can start. At the end of each iteration all above mentioned parameters are updated and used in the next iteration.

A.2 Smooth transition model

To estimate the parameters $\theta = (\phi_1', \phi_2', \gamma, c)'$, nonlinear least squares (NLS) is used. This boils down to the following minimization

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}}\, Q_T(\theta) = \underset{\theta}{\operatorname{argmin}} \sum_{t=1}^{T} \left(y_t - F\left(x_t; \theta\right)\right)^2, \tag{32}$$

where $F\left(x_t; \theta\right)$ equals

$$F\left(x_t; \theta\right) = \phi_1' x_t \left(1 - G\left(s_t; \gamma, c\right)\right) + \phi_2' x_t G\left(s_t; \gamma, c\right). \tag{33}$$

To optimize the minimization algorithm, it helps to pick good starting values. To achieve this, we can estimate the parameters $\phi = (\phi_1', \phi_2')'$, conditional on γ and c, with ordinary least squares

(OLS). This leads to

$$\hat{\phi}(\gamma, c) = \left(\sum_{t=1}^{T} x_t(\gamma, c) x_t(\gamma, c)'\right)^{-1} \left(\sum_{t=1}^{T} x_t(\gamma, c) y_t\right), \tag{34}$$

where $x_t(\gamma, c) = \left(x_t' \left(1 - G\left(s_t; \gamma, c\right)\right), x_t' G\left(s_t; \gamma, c\right)\right)'$. The residuals equal $\hat{\varepsilon}_t = y_t - \hat{\phi}(\gamma, c)' x_t(\gamma, c)$ with variance $\hat{\sigma}^2(\gamma, c) = T^{-1} \sum_{t=1}^{T} \hat{\varepsilon}_t^2(\gamma, c)$. Then, with a grid search around the mean of the transition variable and γ values between 1 and 5, sensible starting values can be chosen for γ and c by allocating the lowest residual variance.

B Diebold Mariano extension

Harvey et al. (1997) extend on the standard Diebold-Mariano test statistic. They define a loss differential as $d_t = \left[g\left(e_{it}\right) - g\left(e_{jt}\right)\right]$, which under the null-hypothesis equals zero. They assume that $\bar{d}_t$ has an asymptotic normal distribution

$$\sqrt{T}(\bar{d} - \mu) \xrightarrow{d} N\left(0, 2\pi V(\bar{d})\right), \tag{35}$$

with

$$\bar{d} = \frac{1}{T} \sum_{t=1}^{T} \left[g\left(e_{it}\right) - g\left(e_{jt}\right)\right] \tag{36}$$

and

$$V(\bar{d}) = n^{-1}\left[\gamma_0^* + 2n^{-1} \sum_{k=1}^{h-1} (n-k) \gamma_k^*\right] \tag{37}$$

where γ_k is the k^{th} autocovariance of d_t and n equals the amount of forecasts made. This autocovariance can be calculated as

$$\begin{aligned} \hat{\gamma}_k^* &= (n-k)^{-1} \sum_{t=k+1}^{n} \left(d_t - \bar{d}\right)\left(d_{t-k} - \bar{d}\right) \\ &= (n-k)^{-1} n \hat{\gamma}_k \end{aligned} \tag{38}$$

The test statistic then equals

$$S = [\hat{V}(\bar{d})]^{-1/2} \bar{d}. \tag{39}$$

The test modification also suggest to compare the test statistic to the Student-t distribution with

$(n-1)$ degrees of freedom.

C Figures and tables

Table 21: Up- and downgrades in credit rating over the years

Year	Issuers as of Jan. 1	Upgrades	Downgrades§	Defaults	Downgrade/upgrade ratio
1981	1,349	9.86	13.27	0.15	1.35
1982	1,398	5.65	12.73	1.22	2.25
1983	1,420	7.18	11.97	0.77	1.67
1984	1,510	11.06	10.13	0.93	0.92
1985	1,598	7.76	13.70	1.13	1.77
1986	1,835	7.25	15.59	1.74	2.15
1987	1,991	7.18	12.00	0.95	1.67
1988	2,081	8.84	11.87	1.39	1.34
1989	2,122	9.71	11.07	1.79	1.14
1990	2,117	6.19	15.30	2.74	2.47
1991	2,053	6.09	14.27	3.26	2.34
1992	2,137	9.59	11.51	1.50	1.20
1993	2,321	8.57	9.26	0.60	1.08
1994	2,553	7.09	9.36	0.63	1.32
1995	2,862	9.08	9.89	1.05	1.09
1996	3,117	9.69	7.83	0.51	0.81
1997	3,478	9.23	7.96	0.63	0.86
1998	4,068	7.55	11.63	1.28	1.54
1999	4,518	5.95	12.04	2.15	2.02
2000	4,670	6.90	12.68	2.48	1.84
2001	4,745	5.96	16.65	3.79	2.79
2002	4,780	5.23	19.14	3.60	3.66
2003	4,777	6.49	14.61	1.93	2.25
2004	5,011	8.78	7.60	0.78	0.87
2005	5,301	12.88	9.22	0.60	0.72
2006	5,460	12.33	8.70	0.48	0.71
2007	5,648	13.54	9.31	0.37	0.69
2008	5,723	7.92	15.99	1.80	2.02
2009	5,607	4.82	19.12	4.19	3.97
2010	5,305	11.88	8.73	1.21	0.73
2011	5,621	12.22	11.99	0.80	0.98
2012	5,803	8.36	12.22	1.14	1.46
2013	6,036	11.45	9.34	1.06	0.82
2014	6,478	9.14	8.41	0.69	0.92
2015	6,895	7.35	11.81	1.36	1.61
2016	6,902	7.87	12.17	2.09	1.55
2017	6,877	8.91	8.71	1.21	0.98
2018	6,966	9.00	8.76	1.03	0.97
2019	7,234	6.32	9.03	1.30	1.43
2020	7,222	2.78	18.47	2.74	6.64
Average		8.34	11.85	1.48	1.66
Median		8.14	11.84	1.21	1.39
Standard deviation		2.36	3.14	0.98	1.11
Minimum		2.78	7.60	0.15	0.69
Maximum		13.54	19.14	4.19	6.64

C.1 Two macroeconomic variables

C.1.1 AIC & BIC

Table 22: AIC and BIC rating BBB with two macroeconomic variables ($*10^2$)

AIC		2nd variable									
model	1st variable	GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
MS	GDP										
	unemp	-6.48									
	CPI	-6.25	-6.36								
	imp	-5.92	-6.21	-6.23							
	exp	-6.21	-6.33	-6.03	-5.96						
	VIX	-5.52	-6.19	-5.59	-5.56	-5.59					
	NFCI	-6.41	-6.22	-7.14	-6.07	-6.79	-6.17				
	EPU	-5.53	-5.43	-5.59	-5.48	-5.53	-4.71	-6.20			
	3-m	-6.06	-6.05	-6.01	-5.96	-6.62	-5.98	-6.01	-5.35		
	10-y	-6.92		-5.91	-6.57			-5.93		-5.93	
STAR	GDP		-5.92	-5.92	-5.92	-5.97	-5.31	-5.93	-5.29	-5.99	-5.96
	unemp	-6.02		-5.94	-5.68	-5.67	-5.29	-5.97	-5.32	-5.91	-5.71
	CPI	-5.92	-5.92		-5.96	-6.02	-5.25	-5.93	-5.25	-5.95	-5.93
	imp	-5.92	-6.04	-5.96		-5.97	-5.32	-5.94	-5.31	-5.98	-5.96
	exp	-5.97	-5.99	-6.02	-5.97		-5.35	-5.97	-5.34	-6.02	-6.00
	VIX	-5.14	-5.22	-5.11	-5.24	-5.34		-5.26	-4.89	-5.15	-5.27
	NFCI	-5.93	-5.97	-5.93	-5.94	-5.97	-5.29		-5.29	-5.96	-5.94
	EPU	-5.31	-5.31	-5.29	-5.32	-5.35	-5.26	-5.29		-5.19	-5.23
	3-m	-5.99	-5.96	-5.95	-5.98	-6.02	-5.29	-6.10	-5.29		-5.96
	10-y	-5.96	-6.08	-5.97	-5.96	-6.00	-5.28	-6.00	-5.32	-5.96	
BIC		2nd variable									
model	1st variable	GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
MS	GDP										
	unemp	-6.03									
	CPI	-5.80	-5.91								
	imp	-5.47	-5.76	-5.78							
	exp	-5.76	-5.88	-5.58	-5.51						
	VIX	-5.08	-5.76	-5.15	-5.12	-5.16					
	NFCI	-5.96	-5.77	-6.69	-5.62	-6.34	-5.73				
	EPU	-5.09	-4.99	-5.16	-5.04	-5.10	-4.28	-5.76			
	3-m	-5.61	-5.60	-5.56	-5.51	-6.17	-5.54	-5.56	-4.92		
	10-y	-6.47		-5.46	-6.12			-5.48		-5.48	
STAR	GDP		-5.53	-5.53	-5.53	-5.58	-4.93	-5.53	-4.91	-5.60	-5.56
	unemp	-5.63		-5.54	-5.29	-5.27	-4.91	-5.58	-4.94	-5.52	-5.32
	CPI	-5.53	-5.52		-5.57	-5.63	-4.87	-5.54	-4.87	-5.55	-5.53
	imp	-5.53	-5.65	-5.57		-5.57	-4.94	-5.54	-4.93	-5.58	-5.56
	exp	-5.58	-5.60	-5.63	-5.57		-4.97	-5.58	-4.96	-5.63	-5.60
	VIX	-4.76	-4.84	-4.73	-4.86	-4.97		-4.88	-4.51	-4.77	-4.89
	NFCI	-5.53	-5.58	-5.54	-5.54	-5.58	-4.91		-4.91	-5.57	-5.55
	EPU	-4.93	-4.93	-4.91	-4.94	-4.97	-4.88	-4.92		-4.81	-4.85
	3-m	-5.60	-5.56	-5.55	-5.58	-5.63	-4.91	-5.70	-4.91		-5.57
	10-y	-5.56	-5.69	-5.57	-5.56	-5.60	-4.90	-5.61	-4.94	-5.57	

Table 23: AIC and BIC rating BB with two macroeconomic variables ($*10^2$)

AIC		2nd variable									
model	1st variable	GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
MS	GDP										
	unemp	-5.56									
	CPI	-5.73	-5.57								
	imp	-5.23	-5.38	-5.52							
	exp	-5.33	-4.63	-4.68	-4.62						
	VIX	-4.50	-5.36	-5.36	-4.50	-5.32					
	NFCI	-5.87	-6.00	-5.89	-5.80	-5.82	-5.35				
	EPU	-5.46	-5.42	-5.42	-4.41	-5.37	1.85	-5.41			
	3-m	-4.79	-4.59	-4.75	-5.88	-5.09	-5.02	-4.79	-5.08		
	10-y	-5.13	-4.48	-4.96	-5.73	-4.95	-4.83	-5.34	-4.31	-5.74	
STAR	GDP		-4.31	-4.31	-4.32	-4.34	-4.36	-4.42	-4.34	-4.49	-4.39
	unemp	-4.76		-4.82	-4.74	-4.69	-4.45	-4.89	-4.42	-4.78	-4.78
	CPI	-4.31	-4.52		-4.34	-4.36	-4.23	-4.61	-4.41	-4.46	-4.37
	imp	-4.32	-4.30	-4.38		-4.52	-4.25	-4.62	-4.25	-4.48	-4.40
	exp	-4.34	-4.31	-4.36	-4.31		-4.41	-4.81	-4.38	-4.50	-4.41
	VIX	-4.28	-4.35	-4.26	-4.31	-4.31		-4.35	-4.27	-4.37	-4.24
	NFCI	-4.42	-4.43	-4.41	-4.39	-4.39	-4.31		-4.31	-4.54	-4.45
	EPU	-4.25	-4.36	-4.24	-4.29	-4.29	-3.88	-4.45		-4.31	-4.25
	3-m	-4.49	-4.46	-4.46	-4.48	-4.50	-4.31	-4.54	-4.32		-4.50
	10-y	-4.39	-4.58	-4.39	-4.40	-4.41	-4.25	-4.45	-4.25	-4.50	
BIC		2nd variable									
model	1st variable	GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
MS	GDP										
	unemp	-5.11									
	CPI	-5.28	-5.12								
	imp	-4.77	-4.93	-5.07							
	exp	-4.88	-4.18	-4.23	-4.17						
	VIX	-4.07	-4.93	-4.93	-4.07	-4.88					
	NFCI	-5.42	-5.55	-5.44	-5.35	-5.37	-4.92				
	EPU	-5.03	-4.98	-4.98	-3.98	-4.93	2.28	-4.97			
	3-m	-4.34	-4.14	-4.30	-5.43	-4.64	-4.58	-4.34	-4.65		
	10-y	-4.68	-4.03	-4.51	-5.28	-4.50	-4.40	-4.89	-3.88	-5.29	
STAR	GDP		-3.91	-3.92	-3.93	-3.94	-3.98	-4.02	-3.96	-4.10	-3.99
	unemp	-4.36		-4.42	-4.34	-4.29	-4.07	-4.49	-4.04	-4.38	-4.38
	CPI	-3.92	-4.12		-3.95	-3.96	-3.85	-4.21	-4.03	-4.06	-3.98
	imp	-3.93	-3.91	-3.98		-4.12	-3.87	-4.23	-3.87	-4.09	-4.00
	exp	-3.94	-3.92	-3.96	-3.91		-4.03	-4.41	-4.01	-4.11	-4.02
	VIX	-3.90	-3.97	-3.88	-3.93	-3.93		-3.97	-3.89	-3.99	-3.86
	NFCI	-4.02	-4.03	-4.02	-4.00	-4.00	-3.93		-3.93	-4.14	-4.06
	EPU	-3.87	-3.98	-3.86	-3.91	-3.91	-3.50	-4.07		-3.93	-3.87
	3-m	-4.10	-4.07	-4.06	-4.09	-4.11	-3.93	-4.14	-3.94		-4.10
	10-y	-3.99	-4.19	-3.99	-4.00	-4.02	-3.87	-4.06	-3.87	-4.10	

Table 24: AIC and BIC rating B with two macroeconomic variables ($*10^2$)

AIC		2nd variable									
model	1st variable	GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
MS	GDP										
	unemp	-3.19									
	CPI	-3.04	-3.18								
	imp	-2.69	-3.27	-3.04							
	exp	-3.02	-3.13	-3.07	-2.73						
	VIX	-2.84	-2.85	-2.84	-2.46	-2.86					
	NFCI	-2.93	-3.14	-3.04	-2.71	-3.03	-2.66				
	EPU	-2.65	-2.64	-2.63	-2.31	-2.65	-0.66	-2.62			
	3-m	-3.11	-3.28	-3.13	-2.44	-2.51	-2.75	-2.65	-2.64		
	10-y	-2.92	-3.61	-3.08	-2.96	-3.07	-2.09	-2.62	-2.51	-1.95	
STAR	GDP		-1.55	-1.51	-1.66	-1.67	-1.08	-1.52	-1.10	-1.57	-1.51
	unemp	-1.55		-1.57	-1.88	-2.04	-1.84	-1.66	-1.64	-1.96	-2.05
	CPI	-1.76	-1.57		-1.72	-1.51	-1.22	-1.67	-1.36	-1.81	-1.74
	imp	-1.50	-1.70	-1.50		-1.66	-1.25	-1.65	-1.24	-1.57	-1.52
	exp	-1.49	-1.79	-1.67	-1.64		-1.26	-1.76	-1.30	-1.57	-1.65
	VIX	-1.17	-1.22	-1.18	-1.17	-1.17		-1.24	-1.17	-1.23	-1.18
	NFCI	-1.52	-1.66	-1.53	-1.53	-1.51	-1.16		-1.16	-1.58	-1.52
	EPU	-1.28	-1.31	-1.27	-1.20	-1.21	-1.19	-1.38		-1.27	-1.18
	3-m	-1.57	-1.90	-1.57	-1.57	-1.57	-1.24	-1.58	-1.37		-1.66
	10-y	-1.51	-2.00	-2.07	-1.52	-1.51	-1.10	-2.17	-1.33	-1.66	
BIC		2nd variable									
model	1st variable	GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
MS	GDP										
	unemp	-2.74									
	CPI	-2.59	-2.73								
	imp	-2.24	-2.82	-2.59							
	exp	-2.57	-2.68	-2.62	-2.28						
	VIX	-2.41	-2.42	-2.41	-2.03	-2.43					
	NFCI	-2.47	-2.69	-2.59	-2.26	-2.58	-2.22				
	EPU	-2.22	-2.21	-2.20	-1.88	-2.22	-0.23	-2.19			
	3-m	-2.65	-2.83	-2.68	-1.99	-2.06	-2.31	-2.20	-2.21		
	10-y	-2.47	-3.16	-2.63	-2.50	-2.62	-1.66	-2.17	-2.07	-1.50	
STAR	GDP		-1.16	-1.11	-1.27	-1.27	-0.70	-1.13	-0.72	-1.18	-1.11
	unemp	-1.16		-1.18	-1.49	-1.65	-1.46	-1.27	-1.26	-1.56	-1.65
	CPI	-1.36	-1.18		-1.32	-1.11	-0.84	-1.28	-0.98	-1.42	-1.35
	imp	-1.10	-1.30	-1.11		-1.26	-0.87	-1.26	-0.86	-1.18	-1.12
	exp	-1.09	-1.40	-1.28	-1.24		-0.88	-1.37	-0.92	-1.17	-1.26
	VIX	-0.79	-0.84	-0.80	-0.79	-0.79		-0.86	-0.79	-0.85	-0.80
	NFCI	-1.13	-1.27	-1.13	-1.14	-1.11	-0.78		-0.78	-1.18	-1.12
	EPU	-0.90	-0.93	-0.89	-0.82	-0.83	-0.81	-1.00		-0.89	-0.80
	3-m	-1.18	-1.50	-1.17	-1.18	-1.17	-0.86	-1.18	-0.99		-1.26
	10-y	-1.11	-1.60	-1.68	-1.12	-1.11	-0.72	-1.78	-0.95	-1.26	

Table 25: AIC and BIC rating CCC/C with two macroeconomic variables ($*10^2$)

AIC		2nd variable									
model	1st variable	GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
MS	GDP										
	unemp	0.48									
	CPI	0.32	0.47								
	imp	0.25	0.44	0.18							
	exp	0.39	0.40	0.25	0.01						
	VIX	0.52	0.24	0.47	0.02	0.74					
	NFCI	0.31	0.59	0.34	0.54	0.43	0.33				
	EPU	0.51	0.21	0.45	-0.01	0.72	1.17	0.29			
	3-m	0.48	0.08	0.52	0.50	0.50	0.48	0.37	0.45		
	10-y	0.46	0.03	0.49	0.50	0.60	0.51	0.34	0.49	1.05	
STAR	GDP		1.39	1.42	1.44	1.53	1.06	1.45	0.92	1.33	1.39
	unemp	1.15		1.16	1.17	1.17	0.86	0.98	0.83	1.19	1.18
	CPI	1.41	1.40		1.40	1.41	0.95	1.39	0.88	1.28	1.36
	imp	1.47	1.40	1.36		1.42	0.95	1.36	0.99	1.36	1.43
	exp	1.40	1.35	1.38	1.42		1.05	1.28	0.95	1.38	1.36
	VIX	1.11	1.04	1.07	1.09	1.07		0.94	1.11	1.03	1.11
	NFCI	1.11	1.19	1.32	1.33	1.28	0.69		0.68	0.97	0.97
	EPU	1.00	0.99	1.01	1.01	1.04	1.02	0.95		1.03	0.99
	3-m	1.14	0.96	1.22	1.14	1.26	0.91	1.02	0.91		1.07
	10-y	1.11	0.96	1.10	1.12	1.11	0.93	1.27	0.80	1.02	

BIC		2nd variable									
model	1st variable	GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
MS	GDP										
	unemp	0.93									
	CPI	0.77	0.92								
	imp	0.70	0.89	0.63							
	exp	0.84	0.85	0.70	0.46						
	VIX	0.96	0.67	0.90	0.46	1.17					
	NFCI	0.76	1.04	0.80	0.99	0.88	0.76				
	EPU	0.94	0.65	0.89	0.43	1.15	1.60	0.72			
	3-m	0.93	0.53	0.97	0.95	0.96	0.91	0.82	0.89		
	10-y	0.91	0.49	0.94	0.95	1.05	0.95	0.80	0.93	1.51	
STAR	GDP		1.79	1.81	1.84	1.93	1.44	1.85	1.30	1.72	1.78
	unemp	1.55		1.56	1.56	1.56	1.23	1.38	1.21	1.59	1.58
	CPI	1.81	1.79		1.80	1.81	1.33	1.78	1.26	1.67	1.75
	imp	1.87	1.79	1.75		1.81	1.33	1.76	1.37	1.75	1.82
	exp	1.79	1.75	1.78	1.82		1.43	1.68	1.33	1.78	1.75
	VIX	1.49	1.42	1.45	1.47	1.45		1.32	1.48	1.41	1.49
	NFCI	1.50	1.58	1.71	1.73	1.67	1.07		1.06	1.36	1.36
	EPU	1.38	1.37	1.39	1.39	1.42	1.40	1.33		1.40	1.37
	3-m	1.54	1.35	1.61	1.54	1.66	1.29	1.42	1.29		1.46
	10-y	1.51	1.36	1.49	1.51	1.51	1.31	1.66	1.18	1.42	

C.1.2 RMSE

Table 26: RMSE 1-quarter and 1-year ahead with two macroeconomic variables for rating BBB

model	1st variable	2nd variable GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
		1q ahead RMSE									
MS	GDP										
	unemp	0.007									
	CPI	0.003	0.006								
	imp	0.004	0.005	0.003							
	exp	0.005	0.005	0.004	0.006						
	VIX	0.004	0.006	0.005	0.004	0.005					
	NFCI	0.005	0.006	0.005	0.005	0.005	0.005				
	EPU	0.007	0.006	0.005	0.005	0.006	0.012	0.007			
	3-m	0.004	0.005	0.005	0.004	0.003	0.006	0.006	0.004		
	10-y	0.003		0.005	0.004		0.005	0.004		0.007	
STAR	GDP		0.005	0.004	0.004	0.005	0.003	0.004	0.005	0.003	0.003
	unemp	0.053		0.010	0.014	0.024	0.006	0.006	0.006	0.050	0.056
	CPI	0.004	0.005		0.004	0.004	0.005	0.005	0.005	0.004	0.004
	imports	0.004	0.007	0.004		0.005	0.004	0.004	0.005	0.003	0.003
	exports	0.005	0.005	0.004	0.005		0.004	0.004	0.005	0.003	0.003
	VIX	0.083	0.006	0.005	0.004	0.004	0.000	0.005	0.005	0.006	0.007
	NFCI	0.004	0.006	0.005	0.004	0.004	0.005		0.006	0.004	0.004
	EPU	0.006	1.541	0.077	0.005	0.005	0.008	0.006		0.008	1.247
	3-m	0.003	0.005	0.004	0.003	0.003	0.005	0.004	0.005		0.005
	10-y	0.357	0.011	0.022	0.003	0.003	0.019	0.010	1.674	0.005	
		1y ahead RMSE									
MS	GDP										
	unemp	0.051									
	CPI	0.024	0.037								
	imp	0.032	0.043	0.026							
	exp	0.038	0.049	0.034	0.041						
	VIX	0.020	0.029	0.020	0.021	0.033					
	NFCI	0.023	0.036	0.031	0.020	0.040	0.028				
	EPU	0.048	0.041	0.021	0.035	0.050	0.071	0.036			
	3-m	0.020	0.014	0.011	0.017	0.018	0.014	0.022	0.011		
	10-y	0.019		0.017	0.017		0.137	0.013		0.081	
STAR	GDP		0.048	0.036	0.045	0.054	0.029	0.034	0.057	0.023	0.024
	unemp	0.103		0.081	0.155	0.172	0.042	0.045	0.055	0.134	0.174
	CPI	0.036	0.039		0.038	0.048	0.026	0.027	0.031	0.016	0.018
	imports	0.045	0.080	0.038		0.055	0.031	0.039	0.054	0.022	0.025
	exports	0.054	0.066	0.048	0.055		0.041	0.050	0.067	0.032	0.037
	VIX	0.287	0.040	0.026	0.036	0.041		0.023	0.025	0.048	0.071
	NFCI	0.034	0.045	0.027	0.039	0.050	0.023		0.031	0.016	0.019
	EPU	0.058	0.055	0.031	0.054	0.067	0.036	0.031		0.052	0.037
	3-m	0.023	0.023	0.016	0.023	0.032	0.019	0.016	0.015		0.024
	10-y	0.022	0.131	0.064	0.025	0.036	0.061	0.045	0.130	0.024	

Table 27: RMSE 2-year and 4-year ahead with two macroeconomic variables for rating BBB

model	1st variable	2nd variable GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
		2y ahead RMSE									
MS	GDP										
	unemp	0.072									
	CPI	0.043	0.065								
	imp	0.056	0.060	0.039							
	exp	0.070	0.086	0.058	0.080						
	VIX	0.034	0.039	0.035	0.037	0.055					
	NFCI	0.046	0.063	0.075	0.036	0.072	0.053				
	EPU	0.092	0.063	0.037	0.066	0.084	0.141	0.060			
	3-m	0.029	0.018^{+}	0.012^{+}	0.017^{+}	0.030	0.022^{+}	0.022^{+}	0.019^{+}	NA	
	10-y	0.032	NA	0.043	0.032	NA	0.507	0.015^{+}	NA	0.184	
STAR	GDP		0.070	0.051	0.065	0.079	0.041	0.047	0.090	0.027	0.030
	unemp	0.128		0.104	0.173	0.206	0.050	0.057	0.072	0.183	0.225
	CPI	0.051	0.055		0.049	0.064	0.033	0.035	0.047	0.012^{+}	0.019
	imports	0.065	0.104	0.049		0.081	0.043	0.053	0.083	0.022^{+}	0.031
	exports	0.079	0.092	0.064	0.081		0.061	0.071	0.105	0.039	0.050
	VIX	0.621	0.048	0.033	0.049	0.061		0.030	0.037	0.069	0.120
	NFCI	0.047	0.057	0.035	0.053	0.071	0.030		0.047	0.012^{+}	0.022^{+}
	EPU	0.091	0.072	0.044	0.083	0.106	0.045	0.047		0.101	0.059
	3-m	0.027	0.023^{+}	$0.01^{+}2$	0.022^{+}	0.039	0.016^{+}	0.015^{+}	0.013^{+}		0.020^{+}
	10-y	0.027	0.210	0.085	0.031	0.049	0.069	0.033	0.236	0.020^{+}	
		4y ahead RMSE									
MS	GDP										
	unemp	0.074									
	CPI	0.047	0.076								
	imp	0.059	0.070	0.044							
	exp	0.066	0.076	0.057	0.088						
	VIX	0.029	0.024	0.031	0.035	0.049					
	NFCI	0.047	0.057	0.118	0.040	0.072	0.059				
	EPU	0.108	0.076	0.048	0.088	0.095	0.167	0.063			
	3-m	0.025	0.016^{+}	0.019^{+}	0.011^{+}	0.020^{+}	0.027	0.019^{+}	0.025		
	10-y	0.033		0.042	0.043		0.064	0.029		0.048	
STAR	GDP		0.054	0.044	0.058	0.067	0.034	0.038	0.093	0.029	0.027
	unemp	0.068		0.036	0.073	0.063	0.027	0.031	0.036	0.046	0.065
	CPI	0.044	0.037		0.044	0.052	0.024	0.027	0.043	0.011^{+}	0.016
	imports	0.058	0.066	0.044		0.069	0.035	0.045	0.086	0.018	0.027
	exports	0.067	0.072	0.052	0.069		0.052	0.059	0.101	0.031	0.041
	VIX	1.357	0.027	0.025	0.041	0.052		0.022	0.028	0.048	0.103
	NFCI	0.038	0.031	0.027	0.045	0.059	0.022		0.039	0.013	0.017
	EPU	0.094	0.036	0.040	0.086	0.102	0.036	0.039		0.097	0.058
	3-m	0.029	0.013^{+}	0.011^{+}	0.019^{+}	0.031	0.012^{+}	0.013^{+}	0.013^{+}		0.016^{+}
	10-y	0.025	0.196	0.118	0.027	0.041	0.075	0.035	0.034	0.016	

Table 28: RMSE 1-quarter and 1-year ahead with two macroeconomic variables for rating BB

	1st	2nd variable									
model	variable	GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
		1q ahead RMSE									
MS	GDP										
	unemp	0.048									
	CPI	0.054	0.055								
	imp	0.047	0.042	0.044							
	exp	0.047	0.044	0.046	0.047						
	VIX	0.047	0.044	0.049	0.043	0.051					
	NFCI	0.047	0.046	0.047	0.045	0.043	0.045				
	EPU	0.042	0.045	0.044	0.038	0.042	0.300	0.045			
	3-m	0.033	0.043	0.042	0.044	0.041	0.047	0.047	0.051		
	10-y	0.040	0.045	0.045	0.044	0.043	0.051	0.044	0.052	0.047	
STAR	GDP		0.050	0.049	0.050	9.584	0.045	0.051	1.869	0.040	0.042
	unemp	0.078		0.049	0.088	0.079	0.059	0.048	0.064	0.053	0.077
	CPI	0.049	0.047		0.044	0.044	0.047	0.047	0.047	0.045	0.046
	imports	0.050	0.046	0.009		0.048	0.046	0.048	0.042	0.043	0.044
	exports	0.049	0.045	0.044	0.045		0.046	0.049	0.032	0.042	0.043
	VIX	0.046	0.046	0.047	0.046	0.046		0.047	0.045	0.046	0.047
	NFCI	0.051	0.046	0.047	0.047	0.046	0.047		0.045	0.045	0.046
	EPU	0.041	0.118	0.046	0.042	0.041	0.236	0.243		0.045	0.045
	3-m	0.040	0.045	0.045	0.043	0.042	0.046	0.045	0.045		0.046
	10-y	0.097	0.045	0.039	0.044	0.043	0.047	0.046	0.045	0.046	
		1y ahead RMSE									
MS	GDP										
	unemp	0.075									
	CPI	0.083	0.072								
	imp	0.066	0.077	0.073							
	exp	0.066	0.076	0.066	0.067						
	VIX	0.051	0.057	0.056	0.074	0.085					
	NFCI	0.062	0.074	0.064	0.071	0.070	0.073				
	EPU	0.082	0.062	0.073	0.072	0.074	2.846	0.073			
	3-m	0.095	0.067	0.090	0.061	0.065	0.109	0.054	0.159		
	10-y	0.100	0.078	0.089	0.082	0.084	0.107	0.072	0.080	0.054	
STAR	GDP		0.091	0.090	0.091	0.095	0.091	0.090	0.131	0.102	0.097
	unemp	0.182		0.243	0.242	0.210	0.160	0.197	0.165	0.225	0.221
	CPI	0.090	0.089		0.091	0.094	0.090	0.103	0.117	0.086	0.089
	imports	0.091	0.097	0.100		0.112	0.088	0.101	0.116	0.080	0.086
	exports	0.095	0.103	0.094	0.099		0.088	0.114	0.122	0.079	0.085
	VIX	0.091	0.089	0.090	0.088	0.088		0.093	0.108	0.089	0.090
	NFCI	0.090	0.097	0.093	0.092	0.092	0.093		0.120	0.102	0.101
	EPU	0.117	0.106	0.104	0.116	0.124	0.119	0.134		0.111	0.107
	3-m	0.102	0.091	0.087	0.080	0.079	0.089	0.102	0.111		0.080
	10-y	0.097	0.120	0.182	0.085	0.085	0.090	0.101	0.108	0.081	

Table 29: RMSE 2-year and 4-year ahead with two macroeconomic variables for rating BB

model	1st variable	2nd variable GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
		2y ahead RMSE									
MS	GDP										
	unemp	0.161									
	CPI	0.126	0.145								
	imp	0.152	0.191	0.131							
	exp	0.148	0.161	0.109	0.133						
	VIX	0.089	0.105	0.092	0.121	0.157					
	NFCI	0.114	0.153	0.125	0.121	0.136	0.110				
	EPU	0.147	0.114	0.106	0.131	0.135	7.531	0.110			
	3-m	0.197	0.131	0.161	0.099	0.091	0.211	0.090	0.431		
	10-y	0.183	0.131	0.162	0.145	0.120	0.225	0.110	0.117	0.087	
STAR	GDP		0.145	0.126	0.160	0.177	0.097	0.112	0.213	0.151	0.097
	unemp	0.338		0.420	0.402	0.358	0.308	0.301	0.293	0.387	0.385
	CPI	0.126	0.118		0.152	0.165	0.102	0.147	0.186	0.098	0.077
	imports	0.160	0.179	0.174		0.246	0.099	0.134	0.212	0.095	0.084
	exports	0.177	0.204	0.165	0.189		0.097	0.290	0.241	0.088	0.086
	VIX	0.097	0.134	0.095	0.099	0.097		0.102	0.134	0.096	0.093
	NFCI	0.112	0.167	0.119	0.122	0.135	0.102		0.170	0.142	0.107
	EPU	0.194	0.174	0.134	0.212	0.240	0.155	0.199		0.156	0.118
	3-m	0.151	0.114	0.099	0.096	0.089	0.096	0.141	0.158		0.117
	10-y	0.097	0.207	0.166	0.084	0.086	0.093	0.107	0.118	0.118	
		4y ahead RMSE									
MS	GDP										
	unemp	0.209									
	CPI	0.168	0.185								
	imp	0.211	0.242	0.166							
	exp	0.186	0.209	0.127	0.170						
	VIX	0.107	0.084	0.113	0.092	0.260					
	NFCI	0.121	0.144	0.140	0.141	0.165	0.115				
	EPU	0.252	0.105	0.128	0.205	0.239	10.212	0.115			
	3-m	0.256	0.242	0.235	0.139	0.150	0.399	0.099	0.553		
	10-y	0.186	0.181	0.195	0.192	0.142	0.316	0.115	0.170	0.078	
STAR	GDP		0.139	0.137	0.176	0.176	0.098	0.114	0.330	0.177	0.113
	unemp	0.109		0.244	0.094	0.081	0.143	0.193	0.151	0.349	0.348
	CPI	0.137	0.117		0.145	0.141	0.099	0.171	0.274	0.142	0.108
	imports	0.176	0.163	0.135		0.184	0.094	0.150	0.271	0.126	0.097
	exports	0.176	0.182	0.141	0.187		0.090	1.126	0.329	0.109	0.087
	VIX	0.098	0.072	0.098	0.094	0.090		0.099	0.125	0.136	0.102
	NFCI	0.114	0.052*	0.110	0.094	0.101	0.099		0.196	0.171	0.122
	EPU	0.245	0.511	0.152	0.271	0.315	0.141	0.273		0.227	0.148
	3-m	0.177	0.158	0.143	0.127	0.110	0.136	0.170	0.230		0.172
	10-y	0.113	0.093	0.324	0.097	0.087	0.102	0.122	0.148	0.173	

Table 30: RMSE 1-quarter and 1-year ahead with two macroeconomic variables for rating B

model	1st variable	2nd variable GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
		1q ahead RMSE									
MS	GDP										
	unemp	0.108									
	CPI	0.102	0.100								
	imp	0.107	0.107	0.104							
	exp	0.113	0.107	0.106	0.108						
	VIX	0.112	0.101	0.104	0.108	0.111					
	NFCI	0.112	0.099	0.101	0.111	0.111	0.093				
	EPU	0.101	0.094	0.097	0.107	0.102	0.097	0.093			
	3-m	0.124	0.098	0.102	0.104	0.110	0.093	0.095	0.093		
	10-y	0.122	0.097	0.103	0.108	0.112	0.094	0.094	0.094	0.102	
STAR	GDP		0.104	0.103	0.222	0.244	7.162	0.112	6.304	0.087	0.096
	unemp	0.104		0.097	0.092	0.111	0.336	0.093	0.357	0.093	0.146
	CPI	0.110	0.097		0.099	0.096	0.098	0.099	0.075	0.099	0.101
	imports	0.109	0.097	0.101		0.087	0.106	0.099	8.591	0.098	0.100
	exports	0.108	0.077	0.076	0.089		0.102	0.069	0.075	0.094	0.076
	VIX	0.105	0.099	0.099	0.106	0.099		0.095	0.092	0.092	0.096
	NFCI	0.112	0.093	0.098	0.106	0.101	0.094		0.090	0.094	0.095
	EPU	0.399	0.099	0.378	0.419	0.362	0.997	0.681		0.095	1.051
	3-m	0.087	0.099	0.095	0.098	0.094	0.088	0.094	0.092		0.097
	10-y	0.096	0.333	0.078	0.100	0.096	0.111	0.097	0.092	0.097	
		1y ahead RMSE									
MS	GDP										
	unemp	0.217									
	CPI	0.209	0.222								
	imp	0.212	0.254	0.209							
	exp	0.196	0.226	0.214	0.220						
	VIX	0.206	0.226	0.205	0.220	0.214					
	NFCI	0.202	0.229	0.227	0.252	0.204	0.214				
	EPU	0.215	0.217	0.209	0.244	0.209	0.550	0.214			
	3-m	0.239	0.222	0.189*	0.241	0.203	0.242	0.291	0.242		
	10-y	0.309	0.245	0.224	0.198	0.216	0.278	0.292	0.232	0.272	
STAR	GDP		0.363	0.314	0.299	0.303	0.293	0.322	0.330	0.343	0.297
	unemp	0.363		0.358	0.437	0.383	0.502	0.418	0.476	0.477	0.411
	CPI	0.330	0.358		0.311	0.310	0.299	0.304	0.318	0.361	0.316
	imports	0.308	0.358	0.318		0.334	0.306	0.341	0.330	0.319	0.308
	exports	0.302	0.549	0.338	0.348		0.305	0.412	0.450	0.311	0.312
	VIX	0.293	0.440	0.302	0.306	0.312		0.390	0.355	0.395	0.556
	NFCI	0.322	0.418	0.334	0.344	0.322	0.303		0.372	0.345	0.315
	EPU	0.335	0.881	0.476	0.387	0.350	0.368	0.395		0.766	0.478
	3-m	0.343	0.699	0.312	0.319	0.311	0.430	0.345	0.355		0.359
	10-y	0.297	3.682	0.440	0.309	0.299	0.768	0.628	0.316	0.359	

Table 31: RMSE 2-year and 4-year ahead with two macroeconomic variables for rating B

model	1st variable	2nd variable GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
		2y ahead RMSE									
MS	GDP										
	unemp	0.554									
	CPI	0.465	0.596								
	imp	0.421	0.643	0.468							
	exp	0.426	0.563	0.499	0.418						
	VIX	0.364	0.541	0.468	0.336	0.375					
	NFCI	0.427	0.566	0.486	0.525	0.454	0.528				
	EPU	0.486	0.533	0.516	0.538	0.481	1.509	0.528			
	3-m	0.539	0.497	0.406	0.591	0.400	0.507	0.699	0.507		
	10-y	0.579	0.629	0.371	0.355	0.409	0.722	0.673	0.500	0.529	
STAR	GDP		0.800	0.504	0.622	0.659	0.422	0.524	0.651	0.634	0.400
	unemp	0.800		0.756	1.296	1.009	1.300	0.905	1.267	1.271	1.117
	CPI	0.564	0.756		0.489	0.505	0.432	0.453	0.548	0.629	0.447
	imports	0.472	0.788	0.510		0.687	0.416	0.739	0.575	0.548	0.418
	exports	0.489	1.374	0.712	0.782		0.436	0.995	1.217	0.533	0.578
	VIX	0.422	1.028	0.461	0.414	0.521		0.891	0.690	0.862	1.573
	NFCI	0.524	0.905	0.557	0.536	0.503	0.442		0.732	0.631	0.464
	EPU	0.640	2.098	1.174	0.704	0.607	0.720	0.746		2.424	1.024
	3-m	0.634	1.309	0.533	0.548	0.533	0.575	0.631	0.487		0.787
	10-y	0.400	5.642	0.585	0.420	0.406	0.924	0.550	0.480	0.787	
		4y ahead RMSE									
MS	GDP										
	unemp	0.542									
	CPI	0.473	0.565								
	imp	0.571	0.568	0.477							
	exp	0.520	0.645	0.573	0.434						
	VIX	0.607	0.411	0.532	0.583	0.620					
	NFCI	0.496	0.573	0.348	0.230	0.557	0.884				
	EPU	0.818	0.754	0.730	0.591	0.731	2.059	0.884			
	3-m	0.505	0.476	0.403	0.686	0.411	0.634	0.765	0.634		
	10-y	0.568	0.488	0.383	0.402	0.400	1.311	0.756	0.517	0.345	
STAR	GDP		0.394	0.488	2.005	2.359	0.373	0.508	0.749	0.727	0.363
	unemp	0.394		0.320	0.252	0.512	0.369	0.386	0.530	0.639	0.241
	CPI	0.760	0.320		0.468	0.479	0.384	0.373	0.597	0.672	0.435
	imports	0.428	0.295	0.493		0.487	0.393	1.138	0.570	0.636	0.414
	exports	0.475	0.766	0.523	0.568		0.380	0.858	1.014	0.592	0.414
	VIX	0.373	0.977	0.498	0.390	0.537		0.785	0.732	0.758	1.788
	NFCI	0.508	0.386	0.557	0.523	0.401	0.311		0.807	0.727	0.409
	EPU	0.625	4.082	1.817	0.806	0.534	0.592	0.609		6.048	1.248
	3-m	0.727	0.863	0.603	0.636	0.592	0.381	0.727	0.462		1.132
	10-y	0.363	7.777	0.560	0.417	0.372	0.777	0.473	0.462	1.132	

Table 32: RMSE 1-quarter and 1-year ahead with two macroeconomic variables for rating CCC/C

model	1st variable	2nd variable GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
		1q ahead RMSE									
MS	GDP										
	unemp	0.683									
	CPI	0.761	0.763								
	imp	0.771	0.741	0.785							
	exp	0.786	0.723	0.766	0.792						
	VIX	0.933	0.766	0.773	0.877	0.866					
	NFCI	0.827	0.780	0.799	0.850	0.757	0.782				
	EPU	0.904	0.784	0.802	0.913	0.785	0.816	0.782			
	3-m	0.759	0.790	0.788	0.785	0.774	0.800	0.779	0.800		
	10-y	0.743	0.807	0.786	0.767	0.712	0.790	0.779	0.790	0.816	
STAR	GDP		2.645	0.816	3.042	0.772	0.760	0.803	2.053	3.211	1.165
	unemp	0.852		0.725	0.727	0.677	1.043	0.749	1.168	0.732	0.734
	CPI	0.802	0.783		0.751	0.779	0.856	0.792	0.894	0.825	0.837
	imports	0.828	0.875	0.653		0.812	0.742	0.757	0.785	0.200	0.836
	exports	0.813	0.812	0.817	0.892		0.753	0.785	0.825	0.843	0.838
	VIX	0.822	0.787	0.813	0.795	0.817		0.795	0.791	0.791	0.784
	NFCI	0.823	0.783	0.803	0.788	0.763	0.808		0.787	0.797	0.797
	EPU	0.580	0.806	1.633	0.763	0.780	0.787	0.789		0.809	1.442
	3-m	0.803	0.824	0.825	0.814	0.833	0.813	0.809	0.828		0.831
	10-y	0.832	0.808	0.827	0.825	0.813	0.738	0.711	0.825	0.850	
		1y ahead RMSE									
MS	GDP										
	unemp	1.593									
	CPI	1.288	1.185								
	imp	1.295	1.179	1.294							
	exp	1.240	1.202	1.244	1.248						
	VIX	1.390	1.243	1.549	1.937	1.862					
	NFCI	1.437	1.123	1.438	1.557	1.399	1.348				
	EPU	1.169	1.084	1.436	1.113	1.276	1.167	1.348			
	3-m	1.432	1.392	1.448	1.362	1.356	1.455	1.407	1.455		
	10-y	1.534	1.255	1.762	1.482	1.595	1.286	1.712	1.286	1.167	
STAR	GDP		1.515	2.186	1.833	1.761	2.484	2.158	2.050	2.379	2.389
	unemp	1.427		1.462	1.357	1.326	1.636	1.535	1.344	1.324	1.391
	CPI	2.072	1.604		1.876	1.841	2.397	2.015	1.979	2.650	2.610
	imports	1.823	1.482	1.999		1.882	1.960	1.946	1.776	2.214	2.604
	exports	2.043	1.498	1.875	1.844		2.093	1.881	2.116	2.223	2.743
	VIX	2.465	1.879	2.749	2.454	2.232		2.219	2.670	2.367	2.011
	NFCI	2.268	1.962	2.209	2.002	1.921	2.433		2.173	2.803	2.684
	EPU	1.814	1.397	2.032	1.696	1.719	2.724	2.112		2.088	2.254
	3-m	2.219	1.516	2.021	2.181	2.044	2.458	2.672	2.202		1.669
	10-y	2.017	1.384	2.029	2.057	2.035	3.370	3.473	1.694	1.344	

Table 33: RMSE 2-year and 4-year ahead with two macroeconomic variables for rating CCC/C

model	1st variable	2nd variable GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
		2y ahead RMSE									
MS	GDP										
	unemp	3.181									
	CPI	2.490	1.837								
	imp	2.442	1.885	2.704							
	exp	2.098	1.886	2.366	2.212						
	VIX	2.875	2.639	3.111	4.331	4.193					
	NFCI	2.798	2.237	2.788	3.098	2.904	3.004				
	EPU	2.355	2.373	3.498	2.909	3.033	1.912	3.004			
	3-m	2.735	3.025	2.688	2.573	2.536	3.477	2.751	3.477		
	10-y	2.589	2.694	2.540	2.419	2.333	2.595	2.374	2.595	2.712	
STAR	GDP		1.905	3.881	2.562	2.320	6.166	3.257	14.269	4.581	4.167
	unemp	2.824		2.930	2.663	2.519	3.079	2.829	2.632	2.371	2.487
	CPI	3.104	2.160		2.850	2.590	4.017	3.084	3.115	5.058	4.614
	imports	2.782	2.174	3.470		2.834	4.278	3.118	2.848	4.067	4.996
	exports	2.784	1.894	2.882	2.538		4.102	3.039	3.521	3.894	4.405
	VIX	4.667	3.430	5.118	4.718	3.882		3.698	4.880	4.034	3.214
	NFCI	3.686	3.063	3.534	3.141	2.954	4.300		3.431	5.350	4.783
	EPU	2.663	2.140	3.303	2.461	2.403	5.371	3.172		3.870	4.140
	3-m	4.449	2.971	4.334	4.311	4.472	5.181	5.634	4.835		3.570
	10-y	3.514	2.240	3.708	3.596	3.510	4.308	3.541	2.650	3.219	
		4y ahead RMSE									
MS	GDP										
	unemp	4.520									
	CPI	4.452	4.612								
	imp	4.377	4.598	4.555							
	exp	4.091	4.401	4.281	4.073						
	VIX	4.677	4.623	4.665	5.795	5.879					
	NFCI	5.064	5.106	5.084	5.525	4.670	4.941				
	EPU	4.085	4.540	5.303	4.373	5.914	4.555	4.556			
	3-m	4.181	3.902	4.244	4.106	4.095	5.474	4.236	5.837		
	10-y	4.113	4.509	4.132	4.065	4.026	5.430	4.077	4.289	4.496	
STAR	GDP		4.100	5.246	4.299	4.072	>10	5.066	>10	6.496	5.593
	unemp	4.135		5.548	4.014	4.183	5.393	5.594	4.723	5.032	5.364
	CPI	4.753	4.316		4.734	4.295	5.365	5.173	5.196	7.230	6.201
	imports	4.291	4.111	5.073		3.866	5.890	4.567	6.088	5.683	6.572
	exports	4.506	3.961	4.517	4.105		5.288	4.387	5.118	5.810	6.260
	VIX	5.166	5.673	5.184	4.942	5.334		5.245	5.174	4.968	4.659
	NFCI	5.482	5.347	5.540	4.941	4.506	5.564		5.155	8.126	6.801
	EPU	4.912	4.539	5.463	4.946	4.207	6.526	5.061		6.514	6.213
	3-m	7.213	6.497	6.859	6.981	6.977	6.566	8.915	6.403		7.409
	10-y	5.573	4.442	5.981	5.660	5.461	5.137	4.896	4.275	7.198	

Table 34: RMSE 1-year and 2-year ahead with PC1 or macro-factor and one macroeconomic variable for rating BBB

		2nd variable									
model	1st var	GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
		1y ahead RMSE									
MS	Z	0.033	0.026	0.029	0.028	0.035	0.026	0.027	0.020	0.026	N.A.
STAR1	Z	0.044	0.075	0.042	0.064	0.049	0.038	0.041	0.038	0.040	0.039
STAR2	Z	0.044	0.014	0.014	0.046	0.049	0.038	0.041	0.038	0.040	0.039
MS	PC1	0.035	0.051	0.033	0.043	0.026	0.023	0.015	0.048	0.018	0.033
STAR1	PC1	0.044	0.062	0.042	0.049	0.045	0.036	0.042	0.055	0.027	0.031
STAR2	PC1	0.044	0.062	0.042	0.049	0.045	0.036	0.041	0.053	0.027	0.031
		2y ahead RMSE									
MS	Z	0.033	0.027	0.041	0.032	0.033	0.023	0.022	0.020	0.030	N.A.
STAR1	Z	0.030	0.050	0.029	0.016	0.031	0.032	0.028	0.030	0.030	0.030
STAR2	Z	0.030	0.014	0.014	0.029	0.031	0.032	0.028	0.030	0.030	0.030
MS	PC1	0.055	0.077	0.053	0.081	0.042	0.036	0.018	0.080	0.043	0.059
STAR1	PC1	0.066	0.084	0.059	0.077	0.070	0.052	0.061	0.089	0.032	0.043
STAR2	PC1	0.066	0.084	0.059	0.077	0.070	0.052	0.062	0.087	0.032	0.043

Table 35: RMSE 1-year and 2-year ahead with PC1 or macro-factor and one macroeconomic variable for rating BB

		2nd variable									
model	1st var	GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
		1y ahead RMSE									
MS	Z	0.096	0.082	0.098	0.097	0.096	0.068	0.092	0.068	0.069	0.107
STAR1	Z	0.089	0.347	0.097	0.417	0.085	0.111	0.092	0.241	0.084	0.097
STAR2	Z	0.100	0.100	0.100	0.100	0.100	0.110	0.100	0.100	0.100	0.100
MS	PC1	0.071	0.068	0.079	0.068	0.072	0.055	0.075	0.072	0.050	0.077
STAR1	PC1	0.087	0.204	0.089	0.088	0.088	0.088	0.092	0.104	0.083	0.096
STAR2	PC1	0.087	0.099	0.087	0.088	0.088	0.088	0.092	0.104	0.083	0.088
		2y ahead RMSE									
MS	Z	0.226	0.203	0.230	0.223	0.239	0.115	0.210	0.115	0.086	0.266
STAR1	Z	0.119	0.452	0.144	0.370	0.172	0.175	0.127	0.441	0.089	0.114
STAR2	Z	0.124	0.123	0.124	0.124	0.124	0.173	0.125	0.123	0.123	0.124
MS	PC1	0.113	0.127	0.118	0.106	0.113	0.092	0.106	0.116	0.079	0.130
STAR1	PC1	0.109	0.339	0.106	0.110	0.107	0.098	0.104	0.175	0.090	0.174
STAR2	PC1	0.109	0.176	0.109	0.110	0.107	0.098	0.104	0.175	0.090	0.095

Table 36: RMSE 1-year and 2-year ahead with PC1 or macro-factor and one macroeconomic variable for rating B

		2nd variable									
model	1st var	GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
		1y ahead RMSE									
MS	Z	0.244	0.326	0.231	0.266	0.238	0.229	0.219	0.229	0.235	0.325
STAR1	Z	0.300	0.372	0.317	0.346	0.435	0.296	0.311	0.380	0.286	0.306
STAR2	Z	0.371	0.371	0.371	0.371	0.371	0.370	0.371	0.370	0.371	0.370
MS	PC1	0.200	0.209	0.198	0.219	0.210	0.201	0.196	0.215	0.222	0.217
STAR1	PC1	0.320	0.428	0.327	0.309	0.308	0.302	0.442	0.523	0.318	0.307
STAR2	PC1	0.320	0.368	0.321	0.315	0.307	0.321	0.442	0.312	0.318	0.314
		2y ahead RMSE									
MS	Z	0.599	0.623	0.493	0.701	0.583	0.500	0.430	0.500	0.637	0.866
STAR1	Z	0.490	0.784	0.533	0.649	0.857	0.465	0.536	0.843	0.407	0.454
STAR2	Z	0.509	0.509	0.509	0.509	0.509	0.508	0.509	0.508	0.508	0.508
MS	PC1	0.384	0.486	0.412	0.335	0.326	0.386	0.360	0.383	0.586	0.497
STAR1	PC1	0.489	1.292	0.520	0.458	0.548	0.402	0.813	1.456	0.589	0.409
STAR2	PC1	0.489	0.834	0.485	0.460	0.437	0.468	0.813	0.499	0.589	0.443

Table 37: RMSE 1-year and 2-year ahead with PC1 or macro-factor and one macroeconomic variable for rating CCC/C

		2nd variable									
model	1st var	GDP	unemp	CPI	imp	exp	VIX	NFCI	EPU	3-m	10-y
		1y ahead RMSE									
MS	Z	1.790	1.628	1.777	1.779	1.901	2.008	2.256	2.008	1.913	1.953
STAR1	Z	2.061	1.570	2.086	1.932	1.889	2.871	2.225	2.142	1.952	2.583
STAR2	Z	2.142	2.148	2.145	2.142	2.142	2.144	2.145	2.142	2.145	2.141
MS	PC1	1.270	1.010	1.219	1.248	1.247	2.054	1.366	1.222	1.391	1.470
STAR1	PC1	1.846	1.350	2.010	1.722	1.851	2.247	2.034	1.916	2.070	2.135
STAR2	PC1	1.846	1.457	1.838	1.807	1.852	1.847	2.243	1.771	2.131	1.964
		2y ahead RMSE									
MS	Z	3.009	2.889	3.023	3.010	2.815	3.124	2.926	3.124	2.775	2.786
STAR1	Z	2.687	2.276	2.798	2.727	2.631	3.243	3.178	2.594	3.212	2.984
STAR2	Z	2.804	2.818	2.806	2.810	2.804	2.805	2.807	2.805	2.808	2.813
MS	PC1	3.471	1.908	2.550	3.390	3.377	4.656	2.933	2.483	3.374	3.496
STAR1	PC1	3.059	2.625	3.365	2.992	3.042	4.168	3.360	3.733	4.651	3.870
STAR2	PC1	3.059	2.359	3.092	2.873	2.988	3.768	4.000	2.735	4.359	3.479

D Overview of supplied coding files

D.1 Scripts

1. Markov_Switching: This script runs the Markov Switching model.
2. Smooth_Transition: this script runs the Smooth transition model.
3. data_simvariance: This script makes the data with added simulated variance.
4. macrofactor: This script makes the macro-factor.

D.2 Functions

1. EM_stepT: This function goes trough one iteration of the EM algorithm.
2. Hamilton_filterT: This function runs the Hamilton filter.
3. Hamilton_smootherT: This function runs the Hamilton smoother.
4. LogLikelihood: This function calculates the log likelihood.
5. my_pdfT: This function returns the pdf of normal distribution which is assumed for the error term.
6. optimization: This function runs the optimization for the STAR model.
7. transition_logistic: This function returns the value of the logistic transiton function .
8. dmtest_modified: Package by Jaime Trujillo (2022). dmtest_modified(e1, e2, h) [4], MATLAB Central File Exchange.

[4] https://www.mathworks.com/matlabcentral/fileexchange/58787-dmtest_modified-e1-e2-h