Erasmus School of Economics

Master Thesis Econometrics and Management Science,
Quantitative Finance

January 23, 2022

Hedging Investor's Downside Risk Using (Sparse) Vine Copula Models

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Abstract

Establishing an optimal hedging strategy is a well-visited topic in the financial literature. An adequate hedging strategy can prevent investors from suffering significant losses. This paper proposes a hedging framework that limits investors' downside risk in a single- and multi-product framework. The joint distribution of asset returns has to be estimated to minimize downside risk. This research takes a copula-based approach to estimate the joint distribution of asset returns. We introduce vine copula models in the multi-product setting. The advantage over standard multivariate copula models is that vine copulas allow heterogeneous dependence patterns across return pairs. The complexity and risk of overfitting increase with the dimension in vine copula models. Therefore, we include sparse vine copula models as a hedging strategy. We assess and compare the in-sample fit and out-of-sample hedging performance against standard copula models and the traditional non-parametric model. The results show that several copula models perform competitive to the non-parametric benchmark approach.

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1 Introduction

Determining an optimal hedging strategy has become an important area in risk management. A hedging strategy aims to minimize or limit downside risks resulting from unfavourable financial products or commodities movements. A steep increase or decrease in the price of assets can cause severe losses. For investors, financial institutions or companies highly exposed to the price of commodities, it is vital to hedge against unfavourable price moments. Whereas establishing an optimal hedging strategy is essential to various market participants, this research focuses on investors in the stock market. However, the results could easily be extended to other markets, such as the commodity market.

To hedge against downside losses, one has to estimate the optimal hedge ratio (OHR). The OHR is defined as the size of a position an investor should take in the futures market in order to hedge a position in the spot market (Barbi & Romagnoli, 2014). Futures are derivatives that allow investors to transfer the risk of a price change to the speculators.

This research proposes a hedging framework that limits the downside risk of investors. The joint distribution of asset prices is required to minimize risk and estimated by a copula-based approach. The advantage of copulas is that they can capture the most important characteristics of asset returns by splitting up the joint distribution in marginal distributions of the individual returns and a copula characterizing the dependence structure. The OHR minimizing the risk is derived from the joint distribution of asset returns. Investors commonly have extended portfolios consisting of multiple assets. The hedging framework is accordingly tested for both single- and multi-product portfolios.

This paper introduces vine copula models in the multi-product setting. Vine copula models are flexible multivariate copula models that consist of numerous bivariate copula pairs (Joe (1996); Bedford & Cooke (2002)). A different copula family and parameter can characterize each variable pair. Therefore, it allows heterogeneous dependence patterns across variable pairs. In contrast, standard multivariate copulas assume similar dependence patterns between pairs of variables. In the context of an investor, this would mean that the dependence between assets is identical across all assets pairs. This assumption is too restrictive and can be relaxed using vine copula models. The type of the vine describes how the bivariate copula pairs are linked to each other. This research employs two classes of vine copula models, namely the canonical (C-) and drawable (D-) vine copula model (Kurowicka & Cooke, 2004).

The disadvantages of vine copula models are that the complexity, computing time, and risk of overfitting increase with the dimension (Nagler et al., 2019). Therefore, we additionally include sparse vine models as a hedging strategy. Conditional bivariate pairs with a conditioning set higher than a pre-set truncation level are replaced by the independence copulas in the sparse vine models (Brechmann et al., 2012). A reduced computing time may be beneficial for investors that want to respond quickly to changes in the market.

The models are evaluated in terms of hedging effectiveness, defined as the risk reduction relative to the unhedged portfolio. We compare the performance against parametric benchmark models (Gaussian, Student's t, Clayton, and Gumbel copula model) and a non-parametric benchmark model. The parametric copula models have been widely applied in financial literature and demonstrate characteristics that make them suitable for modelling asset returns (Sukcharoen & Leatham, 2017). The non-parametric benchmark model relies on historical simulation (HS) and is, therefore, easy to estimate and flexible.

Few have employed copulas in estimating the OHR since standard practices rely on non-parametric methods. For example, Harris & Shen (2006) use the Value-at-Risk (VaR) and Expected Shortfall (ES) as risk-measures to estimate the OHR non-parametrically by historical simulation. The HS model heavily relies on historical data. This often yields inaccurate estimates of extreme quantiles (McNeil & Frey, 2000). In later research, Cao et al. (2010) also include a semi-parametric approach to estimate the OHR. Their approach to the problem is based on the Cornish and Fisher expansion (Cornish & Fisher, 1937) of the quantile of the hedged portfolio return distribution and find that this semi-parametric approach is superior to the non-parametric approach.

Barbi & Romagnoli (2014) present a single-product hedging framework based on standard bivariate Archimedean copulas. They estimate the OHR by minimizing quantile risk measures. The model proposed by Barbi & Romagnoli (2014) outperforms the non-parametric model by Harris & Shen (2006). The plausible reason for the superior performance is the ability to capture most important characteristics of asset returns, including skewness and kurtosis, by separately modelling the individual marginals and the dependence structure.

Previous research has mainly focused on deriving the OHR in a single-product setting. Sukcharoen & Leatham (2017) contribute to the literature by estimating the OHR in a multi-product setting and introducing vine copula models in the context of hedging. They minimize the downside risk of an oil refinery using various risk measures and compare the performance of the vine copula models against non-parametric and parametric benchmark methods. The vine copula model outperforms both parametric and non-parametric approaches.

This paper contributes to the existing literature by proposing a multi-product hedging strategy based on vine and sparse vine copula models. While Sukcharoen & Leatham (2017) propose a multi-product vine copula-based hedging framework for minimizing downside risk for oil refineries, it has not been adopted yet in asset markets. Hence, to the best of our knowledge, this is the first paper to estimate multi-product OHRs in asset markets adopting vine copula models. Additionally, this paper introduces sparse vine models as a hedging strategy. Brechmann et al. (2012) propose the sparse vine models to estimate the dependence in the market portfolio of a large Norwegian institution. However, to the best of our knowledge, it had not been adopted yet in the context of hedging.

The data consist of daily prices of ten global stock market indices and their index futures, resulting in twenty indices in total. The prices are converted to log-returns and cover the period 2000-2020, resulting in 5,218 observations. This equals twenty years of daily trading data, excluding weekends and holidays. We evaluate the hedging strategy for ten single-product and one multi-product portfolio. The single-product portfolios are formed by taking a long position in each index and a short position in its index future. We estimate the size of the short position as OHR. Indices have to be adequately correlated to hedge a position effectively (Barbi & Romagnoli, 2014). Therefore, we form the multi-product portfolio based on highly correlating indices, reducing to the US multi-product portfolio based on US indices.

The results show that several copula models perform competitive to the non-parametric benchmark model. The D-vine copula model yields the highest average fit to the data. However, the fit is comparable to the Student's t copula model, explained by the fact that the Student's t copula model can capture dependence in both normal and extreme market circumstances. The Student's t copula model achieves the most considerable risk reduction among all copula models. The D-vine performance lags behind since there are signs that the model is overfitting the in-sample data.

The dependence in high-order trees in the C-vine copula model is high. Therefore, the C-vine model is not truncated in most estimation windows, and the hedging effectiveness equals the effectiveness of the non-sparse C-vine model. Truncation is, however, effective for the D-vine copula model. This comes at the cost of lower hedging effectiveness but a reduced computing time. Therefore, the investor should make a trade-off between hedging effectiveness and computing time.

The differences in risk reduction between the hedging strategies in multi-product and single-product settings are relatively small. The hedging effectiveness is a positive function of dependency. The single-product portfolios show that a strong dependence between assets is more important than selecting the right hedging model. In both settings, the Student's t copula model achieves a competitive risk reduction, has a smaller risk of overfitting than both vine copula models, and is efficient to estimate. Therefore, it is considered adequate to use as a hedging model.

The structure of this paper is as follows. Section 2 describes the data. The different copula models and the non-parametric benchmark model are introduced in Section 3. Section 4 elaborates on the results. Finally, Section 5 concludes this research.

2 Data

The data consist of spot and futures prices of ten global major stock market indices, which result in twenty series in total. The data is retrieved daily from Datastream and ranges from 01/01/2000 to 01/01/2020. We take closing prices for both spot and futures indices. The number of observations per series equals 5,218, corresponding to twenty years of daily trading data, excluding weekends and holidays. We convert the price indices to daily log returns. An overview of the stock indices included in this research, together with their descriptive statistics, is displayed in Table 1. The research is more comprehensive by employing indices as assets as hundreds of stocks underlie these individual indices.

2.1 Data Characteristics

Table 1 confirms that all stylized facts of asset returns, such as excess kurtosis, skewness and significant Jarque-Bera (JB) statistics, are present in the data. This implies that the returns are fat-tailed, asymmetric, and deviate from normality on all significance levels. The Ljung-Box (LB) statistic indicates some evidence for serial correlation in the individual asset returns. The LB statistic for squared returns suggests strong serial correlation in the squared returns, implying volatility clustering. This phenomenon can be observed in graphical representations of the returns, displayed in Appendix Figure 8 to 11 in Section A.1. One can observe here adjacent periods with high volatility. This is, for instance, visible in the years 2008-2010 in the $S\&P$ 500 series. The augmented Dickey–Fuller (ADF) test indicates that the returns are stationary, so there is no need for first differencing the asset returns.

Table 1: Descriptive Statistics

	Mean	**SD**	**Kurtosis**	**Skewness**	**Min**	**Max**	**JB statistic**	**LB statistic**	**LB^2 statistic**	**ADF statistic**
$S\&P$ 500_s	0.02	1.17	9.07	-0.23	-9.47	10.96	17934.87 (0.00)	49.23 (0.00)	517.95 (0.00)	-17.19 (0.00)
$FTSE$ 100_s	0.00	1.14	6.80	-0.17	-9.27	9.38	10064.66 (0.00)	73.79 (0.00)	299.49 (0.00)	-13.81 (0.00)
DAX_s	0.01	1.44	4.88	-0.06	-8.88	10.80	5177.04 (0.00)	32.27 (0.12)	288.96 (0.00)	-34.20 (0.00)
$Nikkei$ 225_s	0.00	1.44	7.08	-0.42	-12.11	13.24	11052.75 (0.00)	13.95 (0.95)	896.69 (0.00)	-43.55 (0.00)
$NASDAQ_s$	0.02	1.72	7.47	0.15	-11.12	17.20	12161.89 (0.00)	53.27 (0.01)	248.44 (0.00)	-11.82 (0.00)
CAC 40_s	0.00	1.39	5.44	-0.05	-9.47	10.60	6433.08 (0.00)	53.18 (0.00)	154.84 (0.00)	-24.83 (0.00)
$Dow\ Jones_s$	0.02	1.10	8.73	-0.13	-8.20	10.51	16591.08 (0.00)	55.03 (0.00)	518.43 (0.00)	-17.38 (0.00)
$Hang\ Seng_s$	0.01	1.42	8.74	-0.11	-13.58	13.41	16619.98 (0.00)	52.04 (0.00)	425.91 (0.00)	-11.89 (0.00)
$Euro\ Stoxx$ 50_s	-0.01	1.41	5.06	-0.06	-9.01	10.44	5565.62 (0.00)	51.02 (0.01)	120.02 (0.00)	-12.56 (0.00)
AEX_s	-0.00	1.37	7.12	-0.11	-9.59	10.03	11024.41 (0.00)	76.47 (0.00)	415.25 (0.00)	-23.08 (0.00)
$S\&P$ 500_f	0.02	1.18	11.35	-0.11	-10.40	13.20	28005.65 (0.00)	47.06 (0.00)	783.84 (0.00)	-25.95 (0.00)
$FTSE$ 100_f	0.00	1.14	7.00	-0.19	-9.70	9.58	10692.32 (0.00)	77.50 (0.00)	321.70 (0.00)	-13.85 (0.00)
DAX_f	0.01	1.43	7.04	-0.19	-14.82	12.08	10817.92 (0.00)	51.81 (0.00)	181.86 (0.00)	-34.37 (0.00)
$Nikkei$ 225_f	0.00	1.49	12.74	-0.22	-14.30	19.27	35317.87 (0.00)	46.14 (0.00)	1739.82 (0.00)	-20.63 (0.00)
$NASDAQ_f$	0.02	1.70	7.78	0.03	-10.82	15.45	13175.70 (0.00)	55.16 (0.00)	187.19 (0.00)	-11.67 (0.00)
CAC 40_f	0.00	1.39	5.26	-0.08	-8.82	10.29	6009.02 (0.00)	51.80 (0.00)	129.40 (0.00)	-24.93 (0.00)
$Dow\ Jones_f$	0.02	1.12	12.30	0.12	-9.61	12.75	32846.96 (0.00)	60.35 (0.00)	823.28 (0.00)	-25.50 (0.00)
$Hang\ Seng_f$	0.01	1.50	6.18	-0.08	-11.63	11.34	8313.55 (0.00)	45.29 (0.05)	313.02 (0.00)	-20.40 (0.00)
$Euro\ Stoxx$ 50_f	-0.01	1.47	5.69	-0.06	-9.41	11.38	7041.18 (0.00)	68.75 (0.00)	90.89 (0.00)	-17.34 (0.00)
AEX_f	-0.00	1.37	8.02	-0.26	-12.06	11.15	14045.86 (0.00)	74.82 (0.00)	465.87 (0.00)	-23.15 (0.00)

Notes: This table reports descriptive statistics of the different spot indices and their futures, where s denotes a spot index and f an index futures. JB stands for the Jarque Bera test for normality. The LB statistic is the Ljung-Box statistic up to 24th order serial correlations in returns and squared returns. ADF statistic denotes the augmented Dickey-Fuller statistic which tests for an unit root in the indices. The values in parentheses display the p-value of the different statistics respectively.

2.2 Modelling Marginals

The LB statistic confirms serial correlation in most returns and squared returns. Therefore, we model the conditional mean by an AR(1) model to capture the serial correlation in returns. The conditional variance is modelled by the GARCH(1,1) model (Bollerslev, 1986) to account for the serial correlation in the squared returns. Furthermore, most indices are negatively skewed. Hence, we model the residuals by the skewed Student's t distribution. The skewed Student's t distribution has an advantage over both the Gaussian and Student's t distribution, as it can account for excess kurtosis and skewness. The skewness and kurtosis in the skewed Student's t distribution are captured by the kurtosis parameter (ν) and skewness parameter (τ) respectively (Hansen, 1994). To check whether the performance is not sensitive to the residuals following the skewed Student's t distribution, we also evaluate the model fit and performance based on Student's t and Gaussian residuals. Furthermore, we assess the performance based on non-parametrically modelled marginals using the empirical distribution function.

Each stock index $j = 1, \cdots, n$, can be described according to the AR(1)-GARCH(1,1) model:

$$\begin{aligned}
R_{t,j} &= \mu_j + \psi_j R_{t-1,j} + \varepsilon_{t,j} \\
\varepsilon_{t,j} &= \sigma_{t,j} Z_{t,j} \\
\sigma^2_{t,j} &= \alpha_{0j} + \alpha_{1j}\varepsilon^2_{t-1,j} + \beta_j \sigma^2_{t-1,j} \\
Z_{t,j} &\sim skewedt(Z_j|\nu, \tau),
\end{aligned}$$

where $Z_{t,j}$ present innovations that are assumed to be independently and identically distributed following the skewed Student's t distribution. We inspect the (squared) standardized residuals to check whether the model successfully captures the serial correlation in the conditional mean and variance. The LB statistics of the (squared) standardized returns can be found in Table 18 in the Appendix in Section A.2. The statistic indicates that the null hypothesis of no serial correlation cannot be rejected for most standardized residuals, confirming that the AR(1) model successfully captures the serial correlation in the returns. The null hypothesis is rejected for eight squared standardized residual series, while we cannot reject the null hypothesis for the remaining squared standardized residuals. This means that the GARCH(1,1) model successfully models the conditional variance in most returns. The standardized residuals from the AR(1)-GARCH(1,1) model are subsequently transformed into uniform variables using the parametric distribution function. For robustness of the results, we evaluate the hedging performance using a semi-parametric (S-P) approach proposed by Genest et al. (1995). This approach transforms the standardized residuals to copula data using the empirical distribution function. An complete overview of the model specification robustness checks included in this research can be found in Table 19 in Appendix Section A.3.

2.3 Portfolios

We take a close look at the dependence characteristics of the data to construct portfolios in which the indices are highly correlated. In total, we construct ten single-product and one multi-product portfolio.

2.3.1 Multi-Product

The multi-product portfolio is based on highly correlated indices since indices have to be sufficiently correlated to hedge a position effectively (Barbi & Romagnoli, 2014). Figure 1 and 22 display scatter plots and empirical Kendall's tau estimates of the transformed copula data. Compared to the dependence relation between spot and futures indices, the dependence across spot indices is relatively weak. This holds as well for the dependence between index futures illustrated in Figure 22 in Appendix Section A.4. The dependence is somewhat stronger within the US indices (*S&P* 500, *NASDAQ*, *Dow Jones*) and European indices (*DAX*, *CAC* 40, *Euro Stoxx* 50, *AEX*), whereas the dependence in the Asian indices is fairly weak (*Nikkei* 225, *Hang Seng*). This leads to the US multi-product portfolio that includes the *S&P* 500, *NASDAQ*, and *Dow Jones* index. To check for the robustness of the results, we also assess the hedging effectiveness on the portfolio based on highly correlated EU indices: *DAX*, *CAC* 40, *Euro Stoxx* 50, and *AEX*. Each spot index receives a weight of one in both portfolios. The position in the index futures is estimated as OHR.

Dependencies data We take a close look at the data characteristics in the US portfolio to obtain an idea of which copula model might fit the data well. The EU portfolio is solely included as a robustness check, and therefore, the dependence characteristics are not mentioned in this section. However, Section A.6 in the Appendix provides insight on the dependence within the EU portfolio.

Table 2 displays the empirical estimates of Kendall's tau. The empirical Kendall's tau estimates are based on the transformed standardized residuals from the AR(1)-GARCH(1,1) model. The average Kendall's tau estimates of the US portfolio range between 0.53 and 0.87. The most substantial dependency is found between the *S&P* 500 spot and future index, which is expected as spot and future series are closely tied by arbitrage forces. Looking at the cross-index dependency, the *S&P* 500 spot and *Dow Jones* index future exhibit the highest correlation in terms of Kendall's tau of 0.82. The weakest dependency appears between the *NASDAQ* spot and *Dow Jones* future index of 0.53. The *NASDAQ* is, as opposed to the *S&P* 500 and *Dow Jones*, a technology index mainly covering technology stocks. This can explain the relatively weak correlation of the *NASDAQ* index with the remainder of the indices.

Table 3 displays the average empirical estimates of the tail dependence for the US portfolio, based on the transformed standardized residuals from the AR(1)-GARCH(1,1) model. Most index pairs show significant tail dependence. The left window presents the lower tail dependence, whereas the right window displays the upper tail dependence. The lower tail dependence ranges between 0.52 and 0.85, while the upper tail dependence ranges between 0.48 and 0.82. The dependence structure is relatively symmetric for most index pairs. The *NASDAQ* spot and the *Dow Jones* index future display the most considerable difference between upper-and lower tail dependence, equal to 0.06. Although the differences between lower- and upper-tail dependence are relatively comparable across index pairs, the level of tail dependence varies per pair.

The dependence relation varies across indices within the US portfolio. Hence, modelling the portfolio by vine structures that allow dependence specifications to differ per index pair is helpful in this portfolio. However, standard multivariate copulas would imply identical dependence relations across index pairs, resulting in a poor fit because of the varying relations across indices.

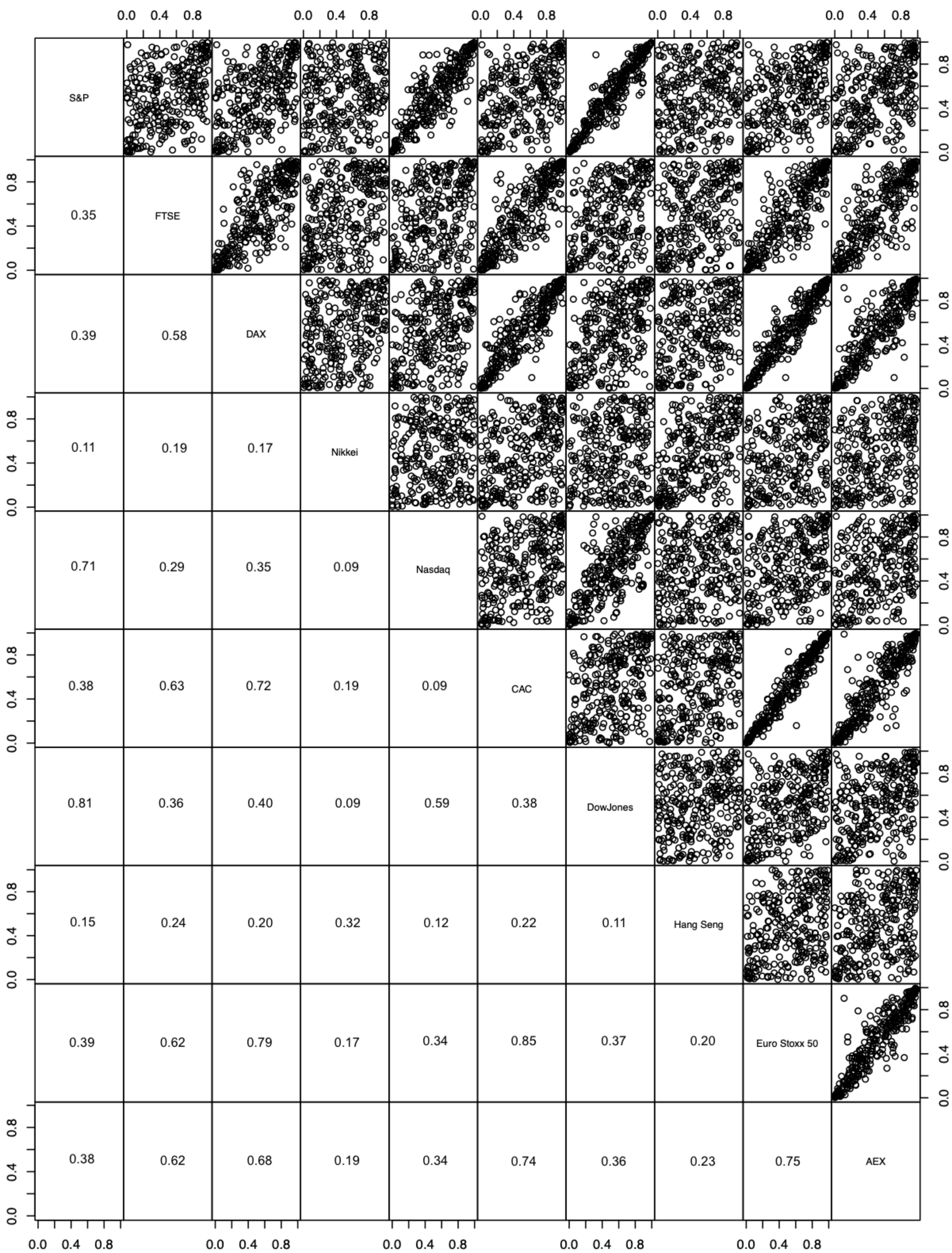

Figure 1: Pair plot with the empirical Kendall's tau estimates of the transformed standardized residuals of spot indices in the lower triangle and the scatter plot of the transformed standardized residuals in the upper triangle.

Table 2: Empirical Kendall's tau estimates US portfolio

	$S\&P500_s$	$NASDAQ_s$	$Dow\ Jones_s$	$S\&P500_f$	$NASDAQ_f$	$Dow\ Jones_f$
$S\&P500_s$	1.00 (0.00)	0.71 (0.02)	0.82 (0.03)	0.87 (0.01)	0.68 (0.02)	0.67 (0.25)
$NASDAQ_s$	0.71 (0.02)	1.00 (0.00)	0.62 (0.06)	0.68 (0.02)	0.87 (0.01)	0.53 (0.19)
$Dow\ Jones_s$	0.82 (0.03)	0.62 (0.05)	1.00 (0.00)	0.78 (0.03)	0.56 (0.06)	0.75 (0.28)
$S\&P500_f$	0.87 (0.01)	0.68 (0.02)	0.78 (0.03)	1.00 (0.00)	0.70 (0.02)	0.71 (0.26)
$NASDAQ_f$	0.68 (0.02)	0.87 (0.01)	0.60 (0.06)	0.70 (0.02)	1.00 (0.00)	0.54 (0.20)
$Dow\ Jones_f$	0.67 (0.25)	0.53 (0.19)	0.75 (0.28)	0.71 (0.26)	0.54 (0.20)	1.00 (0.00)

Notes: This table reports the average empirical estimates of Kendall's tau for the US portfolio across the different estimation windows.

Table 3: Empirical lower and upper tail dependence of the US portfolio

	$S\&P\ 500$	$NASDAQ_s$	$Dow\ Jones_s$	$S\&P\ 500_f$	$NASDAQ_f$	$Dow\ Jones_f$	$S\&P\ 500_s$	$NASDAQ_s$	$Dow\ Jones_s$	$S\&P\ 500_f$	$NASDAQ_f$	$Dow\ Jones_f$
$S\&P\ 500_s$	1.00 (0.00)	0.65 (0.18)	0.81 (0.07)	0.85 (0.04)	0.60 (0.18)	0.66 (0.24)	1.00 (0.00)	0.65 (0.07)	0.78 (0.04)	0.82 (0.07)	0.61 (0.06)	0.64 (0.23)
$NASDAQ_s$	0.65 (0.18)	1.00 (0.00)	0.56 (0.19)	0.59 (0.13)	0.83 (0.07)	0.52 (0.20)	0.65 (0.07)	1.00 (0.00)	0.55 (0.09)	0.64 (0.05)	0.82 (0.06)	0.46 (0.17)
$Dow\ Jones_s$	0.81 (0.07)	0.56 (0.19)	1.00 (0.00)	0.77 (0.06)	0.52 (0.18)	0.72 (0.27)	0.78 (0.04)	0.55 (0.09)	1.00 (0.00)	0.72 (0.06)	0.51 (0.07)	0.68 (0.25)
$S\&P\ 500_f$	0.85 (0.04)	0.59 (0.13)	0.77 (0.06)	1.00 (0.00)	0.59 (0.14)	0.73 (0.28)	0.82 (0.07)	0.64 (0.05)	0.72 (0.06)	1.00 (0.00)	0.66 (0.05)	0.69 (0.24)
$NASDAQ_f$	0.60 (0.18)	0.83 (0.07)	0.52 (0.18)	0.59 (0.14)	1.00 (0.00)	0.51 (0.20)	0.61 (0.06)	0.82 (0.06)	0.51 (0.07)	0.66 (0.05)	1.00 (0.00)	0.48 (0.18)
$Dow\ Jones_f$	0.66 (0.24)	0.52 (0.20)	0.72 (0.27)	0.73 (0.28)	0.51 (0.20)	1.00 (0.00)	0.64 (0.23)	0.46 (0.17)	0.68 (0.25)	0.69 (0.24)	0.48 (0.18)	1.00 (0.00)

Notes: This table reports the average empirical estimates of the upper and lower tail dependence of the US portfolio. The left panel displays the lower tail dependence, whereas the right panel displays the upper tail dependence.

2.3.2 Single-Product

The single-product portfolios are based on a long position in each spot index, and a short position is the corresponding index future, respectively. We base the portfolios on direct hedging, which means taking a position in the futures market to hedge a position in the spot market. The number of futures to hold to hedge the spot position is the OHR, and is estimated accordingly. This results in ten single-product portfolios.

Pair plots of the transformed standardized residuals and estimated Kendall's tau of the single-product portfolios are displayed in Figure 12 to 21 in Appendix Section A.4. Both estimates are based on the transformed standardized residuals of the AR(1)-GARCH(1,1) model.

The spot and futures indices in most portfolios show a clear linear relation, explained by the fact that futures are tied to spot prices by arbitrage forces. Table 4 displays the empirical estimates of Kendall's tau and tail dependence of the ten single-product portfolios. On average, the dependence is relatively strong, and the average Kendall's tau estimates for the different portfolios range between 0.75 and 0.94. The CAC 40 and AEX spot and future index show the strongest dependence, with Kendall's tau estimates equal to 0.94 and 0.93, respectively. On the other hand, the $Nikkei$ 225 and $Dow\ Jones$ portfolios show a relatively weak correlation where Kendall's tau equals 0.76 and 0.75, respectively.

All portfolios demonstrate both upper and lower tail dependence. The dependence is rather symmetric for most portfolios as the difference between upper and lower tail dependence is small. The $Nikkei$ 225 and

Dow Jones portfolios show the largest difference between the upper and lower tail dependence of 0.04 and 0.05, respectively. The lower tail dependence is stronger than the upper tail dependence in both cases. The *DAX* portfolio has a perfectly symmetric dependence structure since the upper tail dependence equals the lower tail dependence. The difference is positive for 5 out of 10 portfolios, indicating a stronger lower tail dependence than upper tail dependence. The *CAC* 40, *Hang Seng*, *Euro Stoxx* 50 and *AEX* portfolio exhibit a stronger upper tail dependence than lower tail dependence.

The upper and lower tail dependence are quite significant for all portfolios. This tells us that a copula that displays both upper and lower tail dependence possibly fits the data well. However, there is some asymmetry in tail dependence in some of the single-product portfolios. Asymmetric copulas could capture this particular asymmetry.

Table 4: Empirical Kendall's tau and tail dependence estimates of the single-products portfolios

	$\hat{\lambda}_l$	$\hat{\lambda}_u$	$\hat{\tau}$	$\hat{\lambda}_l$-$\hat{\lambda}_u$
S&P 500	0.85 (0.04)	0.82 (0.07)	0.87 (0.01)	0.03
FTSE	0.83 (0.06)	0.81 (0.05)	0.87 (0.02)	0.02
DAX	0.83 (0.13)	0.83 (0.10)	0.87 (0.07)	0.00
Nikkei 225	0.69 (0.24)	0.65 (0.22)	0.76 (0.18)	0.04
NASDAQ	0.83 (0.07)	0.82 (0.06)	0.87 (0.01)	0.01
CAC 40	0.91 (0.07)	0.93 (0.05)	0.94 (0.02)	-0.01
Dow Jones	0.72 (0.27)	0.68 (0.25)	0.75 (0.28)	0.05
Hang Seng	0.78 (0.06)	0.79 (0.06)	0.83 (0.02)	-0.02
Euro Stoxx 50	0.81 (0.10)	0.82 (0.08)	0.86 (0.08)	-0.01
AEX	0.90 (0.04)	0.91 (0.04)	0.93 (0.02)	-0.01

Notes: This table reports the average empirical estimates of the tail dependence and Kendall's tau of the 10 single-product portfolios. The values in parentheses indicate the standard deviation of the estimates.

3 Models

This section presents the models that are used throughout the research. The section starts by defining the OHR and subsequently introduces copula models. Moreover, we discuss the traditional benchmark methods against which the performance of the copula models are compared. Finally, the estimation procedure is explained.

3.1 Optimal Hedge Ratio

The Optimal Hedge Ratio (OHR) is defined as the size of an investor's position in the futures market to hedge a position in the spot market. For example, in the multi-product case with n assets, the per-period return on the hedged portfolio is equal to:

$$R_t^p(\mathbf{h}) = R_{1,t}^S + R_{2,t}^S + \cdots + R_{n,t}^S - h_1 R_{1,t}^F - h_2 R_{2,t}^F - \cdots - h_n R_{n,t}^F, \tag{1}$$

where R_t^p denotes the return on the hedged portfolio, $R_{i,t}^S$, $R_{i,t}^F$ the return on the spot and future indices respectively, and $\mathbf{h} = \{h_1, h_2, \cdots, h_n\}$ the hedge ratios. The aim of this research is to estimate optimal hedge ratios $\mathbf{h}^*$ such that it minimizes the downsize risk of the portfolio:

$$\mathbf{h}^* = \underset{\mathbf{h}}{\operatorname{argmin}}\, Risk(R_t^p(\mathbf{h})). \tag{2}$$

3.2 Copula Models

Most risk measures depend on the joint distribution of the returns. Therefore, this research adopts a copula-based approach for estimating the joint distribution of asset returns. Subsequently, we can minimize Equation 2 and obtain the OHR.

The copula approach is based on Sklar's theorem (Sklar, 1959). Sklar's theorem describes that any multivariate distribution can be decomposed into a copula function and individual marginal distributions of variables. The copula function describes the dependence structure between variables. Sklar's theorem is formally defined as follows:

$$F(x_1, \cdots, x_n) = C(F_1(x_1), \cdots, F_n(x_n)), \tag{3}$$

where F denotes the joint distribution of variables $x_1, \cdots, x_n$ with marginal distributions $F_i = F_i(x_i)$ for $i = 1, \cdots, n$, and where $C : [0,1]^n \to [0,1]$ is a copula function. The joint density function of the variables, assuming that F_i and C are differentiable, is defined as:

$$f(x_1, \cdots, x_n) = c(F_1(x_1), \cdots, F_n(x_n))f_1(x_1) \cdots f_n(x_n), \tag{4}$$

where $f_i(x_i)$ for $i = 1, \cdots, n$ represents the density of F_i and c the density of the copula. The equation follows from the chain rule.

The most attractive implication of this decomposition is that the dependence structure can be independently modelled from the marginal distributions. The marginals can either be modelled parametrically or non-parametrically. Since there are many possibilities for modelling the marginals, stylized facts of asset returns such as skewness and excess kurtosis can be captured. There is also a wide variety in modelling the dependence structure by copulas.

3.2.1 Vine Copula Models

We first look at vine copula models, which are a particular class of multivariate copula models (Joe (1996); Bedford & Cooke (2002)). A vine copula model is a multivariate copula model that consists of standard conditional and unconditional bivariate copulas and marginal distributions. Each bivariate copula describes the dependence pattern of a pair of variables and can differ across variable pairs. Accordingly, vine copula models allow heterogeneous dependence patterns between variable pairs. To illustrate the working of vine copula models, we focus on a three-dimensional case that can be extended to higher dimensions. The joint density of 3 variables (x_1, x_2, x_2) can be expressed as the product of both conditional and unconditional densities:

$$f(x_1, x_2, x_3) = f_1(x_1) f_{2|1}(x_2|x_1) f_{3|1,2}(x_3|x_1, x_2). \tag{5}$$

By applying Sklar's Theorem (Equation 4), we can present the conditional density $f_{2|1}(x_2|x_1)$ as:

$$f_{2|1}(x_2|x_1) = \frac{f(x_1, x_2)}{f_1(x_1)} = c_{1,2}(F_1(x_1), F_2(x_2)) f_2(x_2), \tag{6}$$

where $c_{1,2}$ denotes the bivariate copula function between x_1 and x_2. The last conditional density in Equation 5 can be written accordingly:

$$\begin{aligned} f_{3|1,2}(x_3|x_1, x_2) &= \frac{f_{2,3|1}(x_2, x_3|x_1)}{f_{2|1}(x_2|x_1)} \\ &= c_{2,3|1}(F_{2|1}(x_2|x_1), F_{3|1}(x_3|x_1)) f_{3|1}(x_3|x_1) \\ &= c_{2,3|1}(F_{2|1}(x_2|x_1), F_{3|1}(x_3|x_1)) c_{1,3}(F_1(x_1), F_3(x_3)) f_3(x_3), \end{aligned}$$

using $f_{3|1}(x_3|x_1) = c_{1,3}(F_1(x_1), F_3(x_3)) f_3(x_3)$. Combining the above equations leads to the following multivariate density of x_1, x_2 and x_3:

$$f(x_1, x_2, x_3) = f_1(x_1) f_2(x_2) f_3(x_3) c_{1,2}(F_1(x_1), F_2(x_2)) c_{1,3}((F_1(x_1), F_3(x_3))) c_{2,3|1}(F_{2|1}(x_2|x_1), F_{3|1}(x_3|x_1)). \tag{7}$$

This approach involves the marginal conditional distribution $F_{i|j}(x_i|x_j)$. Joe (1996) shows that this conditional marginal distribution function can be written, for every v_j in the vector v in $F(x|\mathbf{v})$, as:

$$F(x|\mathbf{v}) = \frac{\partial C_{x,v_j|\mathbf{v_{-j}}}(F(x|\mathbf{v_{-j}}), F(v_j|\mathbf{v_{-j}}))}{\partial F(v_j|\mathbf{v_{-j}})}, \tag{8}$$

where $C_{x,v_j|\mathbf{v_{-j}}}$ is a conditional bivariate copula, and $\mathbf{v_{-j}}$ is the vector $\mathbf{v}$ excluding component v_j.

The joint density in Equation 7 can be expressed in terms of individual marginal distributions and a group of conditional and unconditional bivariate copulas because of Equation 8.

There is not a unique decomposition of Equation 7. We can use both x_1, x_2 and x_3 as conditioning variable. The number of possible vine copula constructions is large, and we must specify the conditioning variables. For that purpose, we exploit two common-used vine copula structures, namely drawable (D) and canonical (C) vines (Kurowicka & Cooke, 2004).

Canonical Vine Models Figure 2 displays the structure of a six-dimensional C-vine. An n-dimensional C-vine copula consists of $n-1$ trees, where n denotes the dimension of the variables. The number of bivariate conditional and unconditional copula pairs is equal to $(n \times (n-1))/2$. All trees in the C-vine copula structure include one root variable connected to all other variables. The root node in each tree T_i has a degree of $n-i$, where the degree is defined as the number of edges connected to the node.

The variable x_1 is the root variable in the first tree T_1 in Figure 2. This tree suggests that the dependence between x_1 and the remaining variables x_2, x_3, x_4, x_5, and x_6 can be modelled by five unconditional bivariate copulas. Subsequently, tree T_2 characterizes the dependence pattern of the x_2 with x_3, x_4, x_5, x_6 by conditioning on x_1. Generalizing, tree T_i displays the relation of variable x_i with all variables between variable x_i and the last variable x_n conditioning on the variables before x_i $(x_1, \cdots, x_{i-1})$. The order of the variables determines the root nodes in the trees and the conditioning variables. The C-vine copula model is thus highly dependent on the variable order. We explain the order determination in the estimation section. The joint density is the product of all edges (pair copulas) and is as follows defined in the six-dimensional case:

$$f(x_1, \cdots, x_6) = f_1 \cdots f_6 c_{1,2} c_{1,3} c_{1,4} c_{1,5} c_{1,6} c_{2,3|1} c_{2,4|1} c_{2,5|1} c_{2,6|1} c_{3,4|1,2} c_{3,5|1,2} c_{3,6|1,2} c_{4,5|1,2,3} c_{4,6|1,2,3} c_{5,6|1,2,3,4}. \tag{9}$$

which can be generalized to:

$$f(x_1, \cdots, x_n) = \prod_{i=1}^{n} f(x_i) \times \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{j,j+i|1,\cdots,j-1}, \tag{10}$$

where $f(x_i)$ denote the density functions of the variables, and $c_{i,j}$ the bivariate copula pair linking variables i and j.

Drawable Vine Models Figure 3 shows the specification corresponding to a six-dimensional D-vine. A n dimensional D-vine copula constitutes of $n-1$ trees. The number of bivariate conditional and unconditional copula pairs is equal to $(n \times (n-1))/2$. Each tree T_i consists of $(n+1)-i$ nodes and $n-i$ edges where each edge represents a bivariate copula density and has a degree of maximum two. In order words, each variable is linked to a maximum of two other variables. The edges in tree T_i become nodes in tree T_{i+1}.

Tree T_1 in Figure 3 indicates that the dependence pattern between following variable pairs can be modelled by five unconditional bivariate pair-copulas (x_1 and x_2; x_2 and x_3; x_3 and x_4; x_4 and x_5; x_5 and x_6). Tree T_2 suggests to model the dependence between x_1 and x_3, x_2 and x_4, x_3 and x_5, and x_4 and x_6 by conditional bivariate copulas, conditioning on the variable in between the pair. Generalizing, the dependence pattern between any two variables x_i and x_j can be modelled conditional on the variables that are positioned in between the variables in tree T_1. The order in the D-vine structure is crucial as it points out the variables that have to be used as conditioning variables. We elaborate on the order determination later on in this paper. The joint density function of the six-dimensional D-vine copula model is defined as:

$$f(x_1, \cdots, x_6) = f_1 \cdots f_6 c_{1,2} c_{2,3} c_{3,4} c_{4,5} c_{5,6} c_{1,3|2} c_{2,4|3} c_{3,5|4} c_{4,6|5} c_{1,4|2,3} c_{2,5|3,4} c_{3,6|4,5} c_{1,5|2,3,4} c_{2,6|3,4,5} c_{1,6|2,3,4,5}, \tag{11}$$

which can be generalized to:

$$f(x_1, \cdots, x_n) = \prod_{i=1}^{n} f(x_i) \times \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{i,i+j|i+1,\cdots,i+j-1}. \tag{12}$$

where $f(x_i)$ denote the density functions of the variables, and $c_{i,j}$ the bivariate copula pair linking variables i and j.

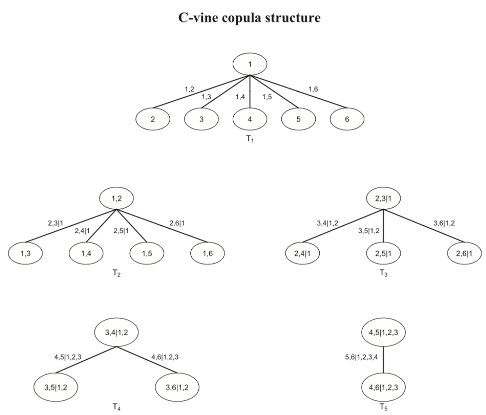

Figure 2: C-vine copula structure (Sukcharoen & Leatham, 2017)

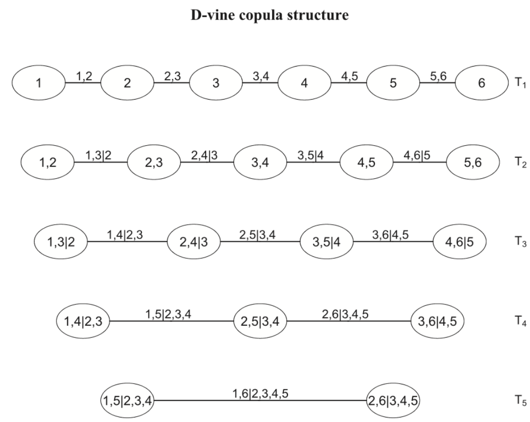

Figure 3: D-vine copula structure (Sukcharoen & Leatham, 2017)

Each tree T_i has a unique node (root) node with a degree of $n - i$ in the C-vine copula model. On the contrary, a D-vine copula model is a structure where each tree T_i has a degree of most two. Hence, in the context of our data, the C-vine copula model always connects one root variable (index) to the remaining indices conditional on the indices that were root nodes in the previous trees. In contrast, the D-vine copula model links two indices at most resulting in one straight line, conditional on the indices in between the indices in the previous trees.

3.2.2 Sparse Vine Copula Models

The number of parameters in both vine copula models grows exponentially with the dimension. This increases the risk of overfitting the copula model and brings an extra burden by estimating many parameters. The number of bivariate copulas to estimate is equal to $\frac{n\times(n-1)}{2}$, where each bivariate copula can have multiple parameters. Hence, computing the optimal hedge ratio and assessing hedging effectiveness may be very demanding in the case of high dimensional data. Therefore, we also estimate sparse vine copula models truncated at a certain level.

When a vine copula model is truncated at a level K, all bivariate copulas with a conditioning set larger or equal to K are modelled as independent copulas. It extracts a subvine from the vine copula model with fewer trees since the number of conditioning variables increases per tree. The sparse vine model consists of K trees instead of $n - 1$ trees. The independence copula models the bivariate relations in the remaining $n - 1 - K$ trees. If K is equal to 1, the vine copula reduces to a single tree consisting of only unconditional bivariate copulas (Brechmann et al., 2012). The density of the sparse vine copula model truncated at level K can be formulated as:

$$c_{V(K)}(u|\eta_T(K)) = \prod_{i=1}^{K} \prod_{e\in E_i} c_{j(e),k(e)|D(e)}, \tag{13}$$

where $c_{V(K)}$ denotes the sparse vine copula model truncated at level K, $e \in E_i, i = 1, \cdots, K$ represent the edges in the vine copula model, and $\theta_{j(e),k(e)|D(e)}$ the parameters of the copula model $c_{j(e),k(e)|D(e)}$.

The truncation level K can be selected according to several statistical criteria. Nagler et al. (2019) show that the modified Bayesian information criteria (mBICv) should be preferred over more standard statistical criteria such as the Aikaike information criterion (AIC) or Bayesian information criteria (BIC) if there is a preference to keep the model sparse. The advantage of the mBICv criteria is that it selects models with more independent copulas than the AIC and BIC criteria. It does so by assigning a prior probability $\psi_0^{T_i}$ to each copula pair of not being independent. This probability is fixed for every tree T_i. As the number of trees grows, this probability decreases and the chance for selecting the independence copula grows. Nagler et al. (2019) argue that this is a fair assumption as vine copula model estimation procedures try to capture the strongest relations in the first few trees. Furthermore, vine copula models are often estimated sequentially, resulting in accumulating estimation errors. Hence, it is plausible to assume that the probability of having an independent bivariate copula should increase per tree. If $\psi_0^{T_i} > 0.5$, the tree is more inclined to select a non-independence copula. The number of variables in the multi-portfolio is equal to 6. Hence, the total number of trees constructed is 5 $(n-1)$. As we would like a sparse model, ψ_0 is set to 0.8. This causes that trees 4 and 5 have a prior probability of selecting an independent copula larger than 0.5. We can calculate the criteria as follows (Nagler et al., 2019):

$$mBICv = -2loglik(\eta) + \nu log(n) - 2\sum_{i=1}^{n-1}(q_{T_i} log(\psi_0^{T_i}) - (n - T_i - q_{T_i}) log(1 - \psi_0^{T_i})), \tag{14}$$

where $loglik(\eta) = \sum_{i=1}^{n} c^{\eta}(u_i)$ is the copula log-likelihood, ν is the number of parameters in the model, q_{T_i} the number of non-independence copulas in tree T_i, ψ_0 the prior probability of having an non-independent copula, and n the dimension of the data.

3.3 Traditional Copula Models

This research exploits the performance of traditional copulas models in the context of hedging downside risk in addition to the multi-variate vine copula models. The traditional copula models include the Gaussian, Student's t, Clayton and Gumbel copula. These copulas show different dependence patterns, which we highlight shortly. The bivariate densities and tail dependence characteristics of the copula models are presented in Appendix Section B in Figure 33 and Table 23.

The Gaussian and Student's t copula models are elliptical copulas of normal mixture distributions. Both copula models exhibit a symmetric tail dependence structure. Tail dependence defines the limiting probability of a joint extreme event. The Gaussian copula model does not exhibit tail dependence in both tails. Therefore, the disadvantage of this copula is that it could underestimate the probability of a joint crash. The advantage of the Student's t copula model over the Gaussian is that it demonstrates tail dependence. However, the tail dependence is symmetric, assuming the dependence in expansions to be identical to the dependence during recessions. The difference in tail dependence between both copulas becomes evident from the first and second row in Figure 33 in Appendix Section B. Whereas there is a strong dependence in the Student's t copula model in the right and left tail in the surface plot, the surface plot of the Gaussian copula model is rather flat. Both copula models are characterized by correlation matrix ρ as parameters, dependent on the number of variables. In addition, the Student's t copula model has a parameter defining the degrees of freedom which controls the tail dependence for all variable pairs. The number of parameters is therefore equal to $(n \times (n-1))/2$ for the Gaussian copula model and $(n \times (n-1))/2 + 1$ for the Student's t copula model.

This research employs two Archimedean copula models next to the discussed elliptical copulas, the Gumbel and Clayton copula. Both copula models can capture asymmetric and nonlinear relations between variables. The copulas are asymmetric because the lower and upper tail dependence are not equal. The Clayton copula only exhibits lower-tail dependence, whereas the Gumbel copula only exhibits upper tail dependence. This can be observed in Figure 33 in the third and fourth row, where the Clayton copula has a clear peak in the left tail and the Gumbel copula in the right tail. The dependence structure in both normal and extreme conditions is characterized by one parameter θ, which is a very restrictive assumption in the case of many variables.

3.4 Benchmark Model

To evaluate the performance of the copula hedging models, we compare the performance of the different copula models against the non-parametric benchmark model frequently used in literature. The traditional model included is the non-parametric (NP) approach proposed by Harris & Shen (2006). Harris & Shen (2006) show that the NP approach effectively reduces the Value at Risk and conditional Value at Risk in the hedged portfolio. The NP approach relies on historical simulation in estimating the OHR, and minimizes the HS estimate of the different risk measures. The NP approach is very flexible and easily adaptable in multi-product hedging. Therefore, the NP method has become the standard in risk management literature. For that purpose, we use the non-parametric approach as a benchmark.

3.5 Downside Risk Measures

The OHR is estimated for four different downside risk measures, on which we elaborate shortly. The risk measures follow the research of Sukcharoen & Leatham (2017).

Value at Risk The Value at Risk (VaR) defines the largest possible loss for a given confidence level. The confidence level determines the probability that the loss exceeds the VaR. Generally, given a confidence level q, the VaR is defined as:

$$VaR_q = F^{-1}(q), \tag{15}$$

where F is the joint distribution of the losses. The data employed in this research consists of returns instead of losses. To calculate the largest possible loss, we convert the returns to losses by multiplying by -1. The resulting VaR estimate demonstrates the level of losses that is only exceeded by a chance of $(1-q)$ and coincides with the estimate of the level of returns exceeded by q. The VaR is calculated for three confidence levels: namely $q = \{0.90, 0.95, 0, 99\}$. Hence, in this way, we obtain the largest losses.

Expected Shortfall The VaR does reveal the magnitude of the losses beyond the VaR. In contrast, the Expected Shortfall (ES) defines the average loss an investor can expect, given that the loss exceeds the VaR.

$$ES_q = E[L_t | L_t \geq VaR_q], \tag{16}$$

where L_t denotes the loss of the portfolio $(-R_t)$. The ES is calculated for the same confidence levels as the VaR.

Semivariance The Semivariance (SV) estimates the variability of the returns below a target level. In contrast to variance as a risk measure, it focuses on losses and does not take returns above a certain target level into account. It is defined as follows:

$$SV = \int_{x=-\infty}^{c} (c - R_t)^2 dF(R_t), \tag{17}$$

where R_t is the portfolio's return, F the return's distribution, and c the target level. c is equal to 0 in this research since the goal with hedging is to avoid any losses.

Lower partial moment The lower partial moment(LPM) corresponds to any moment of the return below a target level and is a generalization of Equation 18:

$$SV = \int_{x=-\infty}^{c} (c - R_t)^n dF(R_t), \tag{18}$$

where n denotes the order of the partial moment. Fishburn (1977) shows that the larger n becomes, the more risk-averse the behaviour of the hedger. To focus on a risk-averse hedger, n is equal to 3 in this research. The level of losses c is again 0.

3.6 Estimation Procedure

In this section, we present the estimation procedure of the OHR by first elaborating on the estimation of the vine copula models. Subsequently, we discuss the estimation of the copula-GARCH model. Finally, we explain the optimization process of the OHR and assessment of hedging effectiveness.

3.6.1 Vine Estimation

The estimation of vine copulas proceeds in multiple steps. The C- and D-vine structure specifications highly depend on the order of the variables. Therefore, the order of the variables has to be determined first. For the C-vine structure, we follow the approach of Czado et al. (2012) where the root variable is chosen as the variable that has the most substantial dependence with the remainder of the variables in terms of absolute pairwise Kendall's tau estimates. That is, maximizing the following Equation:

$$\hat{S}_{\tau}^{i} = \sum_{j=1, i \neq j}^{n} |\tau_{i,j}|. \tag{19}$$

The D-vine structure is estimated using the approach of Dissman et al. (2013). In Dissman et al. (2013)'s approach, the variables are ordered such that the sum of absolute pairwise Kendall's tau estimates is maximized. Hence, we look at all possible orders in the first tree with a node degree of maximum two. We choose the order that has the highest sum of absolute pairwise correlation, maximizing the below Equation:

$$\hat{S}_{\tau} = \sum_{j=1}^{n-1} |\tau_{j,j+1}|. \tag{20}$$

Second, one has to decide on conditional and unconditional pairwise copula models. This process involves sequential estimation, proposed by Aas et al. (2007). Sequential estimation is necessary because the conditional copulas in the tree m of C and D vines depend on the copula models and parameters estimated in trees $j = 1, \cdots, m-1$ through Equation 8. Hence, bivariate copulas in a given tree are selected and estimated before proceeding to the next tree. The copula parameters are estimated through maximum likelihood. According to the Akaike Information Criteria (AIC), 31 different bivariate copulas are considered and selected. An overview of the copulas together with their dependence characteristics considered in the vine-structure are displayed in Table 23 and 24 in Section B. Figure 4 present a schematic overview of the estimation process of the vine copula models.

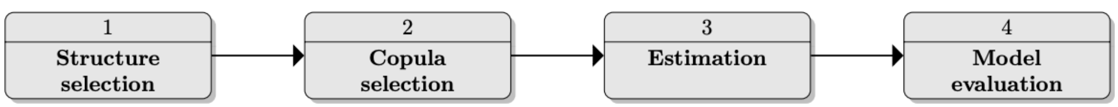

Figure 4: Vine copula model estimation process (Brechmann & Schepsmeier, 2013)

3.6.2 Copula GARCH Estimation

The conditional mean and variance of each index are modelled by the AR(1)-GARCH(1,1) model. The joint distribution of the standardized residuals can be described by the following Equation using Sklar (1959)'s theorem:

$$F_{\mathbf{z}}(z_1, \cdots, z_n; \nu_1, \cdots \nu_n, \tau_1, \cdots, \tau_n, \omega) = C(F_1(z_1; \nu_1, \tau_1), \cdots, F_n(z_n; \nu_n, \tau_n); \omega), \tag{21}$$

where $(z_1 \cdots z_n)$ represent the i.i.d. standardized skewed Student's t residuals, ω the copula parameter vector, and $(F_1 \cdots \mathrm{F}_n)$ the skewed Student's t distributions with parameters $(\nu_1 \cdots \nu_n)$ and $(\tau_1 \cdots \tau_n)$. We define $\eta_j = (\nu_j, \tau_j, \mu_j, \psi_j, \alpha_{0j}, \alpha_{1j}, \beta_j)$ as the parameters estimated in the conditional mean and variance model. Equation 21 results in the following joint density function of the standardized residuals:

$$f_{\mathbf{z}}(z_1, \cdots, z_n; \nu_1, \cdots \nu_n, \tau_1, \cdots, \tau_n, \omega) = c(F_1(z_1; \nu_1, \tau_1), \cdots, F_n(z_n; \nu_n, \tau_n); \omega)(f_1(z_1; \nu_1, \tau_1) \cdots f_n(z_n; \nu_n, \tau_n)). \tag{22}$$

The joint log likelihood of the is defined as follows:

$$
\begin{aligned}
L(\eta_1, \cdots, \eta_j, \omega) &= \sum_{t=1}^{T} f_{\mathbf{z}}(z_{t,1}, \cdots, z_{t,n}; \eta_1, \cdots, \eta_n, \omega) \\
&= \sum_{t=1}^{T} \{\log c(F_1[R_{t,1} - \mu_1 - \psi_1 R_{t-1,1}]/s_{t1}; \nu_1, \tau_1), \cdots, F_n([R_{t,n} - \mu_n - \psi_n R_{t-1,n}]/s_{t,n}; \nu_n, \tau_n); \omega) + \cdots \\
&\quad \sum_{j=1}^{n} \log[s_{t,j}^{-1} f_j([R_{t,j} - \mu_j - \psi_j R_{t-1,j}]/s_{t,j}; \nu_j, \tau_j)]\},
\end{aligned}
$$

with $s_{tj} = \sqrt{\alpha_{0j} + \alpha_{1j}\varepsilon_{t-1,j}^2 + \beta_j s_{t-1,j}^2}$, and $c(.;\omega)$ the copula density for $C(.;)$.

For the estimation of the model, we rely on the Inference Function of Margins (IFM) method by Joe & Xu (1996), involving a two-step approach. In the first step, the AR(1)-GARCH(1,1) model is estimated which result in in the parameters $\hat{\eta}_j = (\hat{\nu}_j, \hat{\tau}_j, \hat{\mu}_j, \hat{\psi}_j \hat{\alpha}_{0j}, \hat{\alpha}_{1j}, \hat{\beta}_j)$. Equation 3.6.2 can be reduced to the first part relating to the copula log-likelihood since the parameters from the first step are fixed. Subsequently, the joint log-likelihood in the above Equation is maximized over the copula parameter vector ω with the $\hat{\eta}_j$ parameters fixed at the estimated values from the first step.

After estimating the marginal distribution and copula parameters, the four downside risk measures are computed using the Monte Carlo simulation method. First, we draw 1,000 samples of standard uniform variables from each copula model $\{u_{1,s}, \cdots, u_{n,s}{}_{s=1}^{s=1,000}\}$. The uniform draws are then converted back to standardized residuals using the inverse of the skewed Student's t distribution. Subsequently, we aggregate the residuals to returns using the parameter estimates from the first step in the IFM method. The OHRs is after that derived for each hedging objective by minimizing Equation 2 numerically using the Nelder-Mead optimization procedure.

The models are evaluated on their out-of-sample hedging effectiveness which is measured as:

$$
HE = (1 - \frac{Risk(R_t(\mathbf{h}*))}{Risk(R_t(0))}) \times 100, \tag{23}
$$

where $Risk(*)$ denotes the risk measure to be minimized, R_t the return of the hedged portfolio, and $R_t(0)$ the return of the unhedged portfolio, where the hedge ratios equal 0.

We adopt the approach of both Barbi & Romagnoli (2014) and Sukcharoen & Leatham (2017) who use a fixed-length moving window for the out-of-sample evaluation. First, the AR(1)-GARCH(1,1) model and copula parameters are estimated using the first three years of data, corresponding to 780 observations. With these estimated coefficients, 1,000 samples from the standard uniform variables are simulated from the copula models and are converted back to returns. We subsequently derive the minimum hedge ratios from the simulated returns. The portfolio returns for the following year (260 observations) are calculated with the derived hedge ratios. Finally, we assess the constructed portfolio returns in terms of hedging effectiveness, and the window is moved forward by one and a half years, corresponding to 390 observations. This procedure results in eleven test windows in which the final performance is the average of the performances in the different test windows. A graphical representation of the approach is displayed in Figure 5. Sukcharoen & Leatham

(2017) re-estimate the model weekly since they assume that the dependence structure between asset returns frequently varies over time. Our correlation and tail dependence estimates suggest that the dependence in our data set varies frequently. However, we can not re-estimate the model weekly due to computational constraints. We, therefore, choose to set the re-estimation time to one and a half years, half of the size of the in-sample estimation window. In this way, there is overlap between the estimation windows and reduces the sensitivity to the choice of estimation window. An complete overview of the years corresponding to each estimation window is presented in Figure 25 in Appendix Section B. The US portfolio is also re-estimated by shifting all windows one year, where the window size is fixed, to test whether the results are robust to different estimation periods (Table 19).

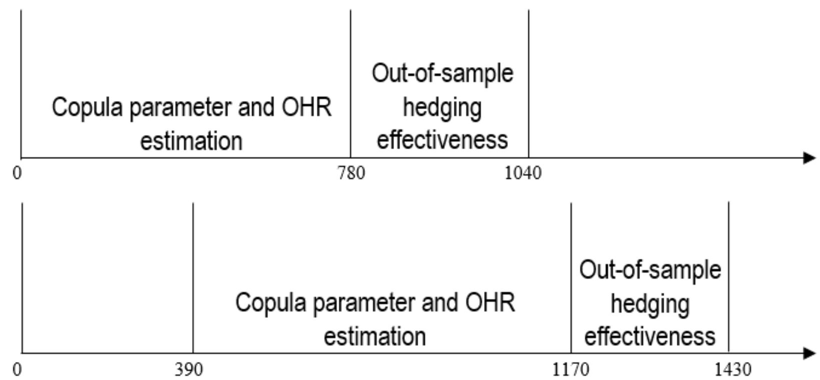

Figure 5: Estimation procedure for the copula parameters, hedge ratio and evaluation of out-of-sample effectiveness.

4 Results

This section presents the hedging effectiveness of the single- and multi-product portfolios. First, evidence is provided on the fit of the different copula models. Subsequently, we take a look at the computed hedge ratios and assess whether the use of copulas can reduce the risk of an investor's portfolio. The section starts with presenting the results on the US multi-product portfolio and thereafter presents the results of the single-product portfolios.

4.1 Copula Fit

Table 5 shows the average log-likelihood of the different multivariate copula models for the US portfolio.

Table 5: Average copula fit of the multivariate copula models for the US portfolio.

	Gumbel	Clayton	Student's t	Gaussian	C-vine	D-vine
Log-likelihood	3347.51 (1009.85)	3049.19 (922.95)	5910.38 (909.25)	5668.01 (841.86)	5847.81 (847.05)	**5937.89 (873.77)**

Notes: This table presents the average log-likelihood of the different copula models for the US portfolio across the different estimation windows. The standard deviation of the copula fit is displayed in parentheses.

The D-vine copula realizes the highest average fit to the data. On the contrary, the Clayton copula results in the poorest average fit. Table 26 and Figure 34 in Appendix Section C.1 shows that this pattern is consistent over time. Additionally, we observe a gradual increase in copula fit over time, explained by an increase in dependency between asset returns over time.

The copulas that provide the poorest fit are the Gumbel and Clayton copula. Both copulas are Archimedean copulas in which the dependence structure is characterized by one parameter, assuming the dependence structure to be identical in both extreme and normal market conditions. Moreover, both copulas show asymmetric tail-dependencies. The Gumbel copula has no lower tail dependence, whereas the Clayton copula has no upper tail dependence. The poor fit is partly caused by the restrictive assumption of identical dependence structures across indices and partially because both copula models are characterized by only one parameter. Additionally, the data show no clear sign of asymmetric tail dependence relationships suggested by Figure 1 and Table 3 which can result in a poor fit of both copulas.

The Gaussian and Student's t copula yield a better fit to the data compared to both Archimedean copulas. The parameters that have to be estimated in both copulas are the coefficients of the correlation matrix. Hence, each return pair has a different parameter characterizing the dependence in normal market circumstances. This is the most evident advantage over both Archimedean copulas and is likely causing the superior performance of both elliptical copulas compared to the Archimedean copulas. The Gaussian copula lacks tail dependence and could underestimate the probability of a joint crash. In addition to the correlation matrix, the Student's t copula has the degree of freedom parameter characterizing the tail dependence relation in each return pair, resulting in a higher average fit. This tail dependence is symmetric across different return pairs. From Table 3 we can observe symmetric tail dependence relations across different variable pairs resulting in high average fit of the Student's t copula model. The difference in the fit of the Student's t copula compared to both vine copulas is not statistically significant.

Both vine copulas provide a high average fit to the data. The D-vine obtains a higher average fit than the C-vine copula, although this difference is not statistically significant. As the US portfolio consists of 6 variables, the number of trees to construct per estimation window is equal to $n - 1 = 5$, and the number of bivariate copulas to estimate is equal to $\frac{n*(n-1)}{2} = 15$, where each bivariate copula can originate from a different copula family. We take a close look at the structure of both vine copulas to obtain an idea of the dependence structure of the US portfolio and possibly explain their high average fit to the data.

C-vine The C-vine copula model is structured such that it maximizes the dependence between the (un)conditional return pairs as this maximizes the log-likelihood of the copula model. Therefore, the root node is chosen as the variable that maximizes Equation 19. The order of the C-vine copula across different estimation windows is displayed in the left panel in Table 6. The order is relatively stable across the estimation windows, with $S\&P\ 500_s$ being the root node in every estimation window. This tells us that $S\&P\ 500_s$ has the strongest dependency in terms of Kendall's tau with the remainder of the variables, confirmed by Table 2.

Table 6: C-vine and D-vine order

1	$S\&P_s$	$S\&P_f$	$Nasdaq_s$	$DowJones_s$	$Nasdaq_f$	$DowJones_f$	$DowJones_f$	$Nasdaq_s$	$Nasdaq_f$	$S\&P_f$	$S\&P_s$	$DowJones_s$
2	$S\&P_s$	$S\&P_f$	$Nasdaq_s$	$DowJones_s$	$Nasdaq_f$	$DowJones_f$	$Nasdaq_s$	$Nasdaq_f$	$S\&P_f$	$S\&P_s$	$DowJones_s$	$DowJones_f$
3	$S\&P_s$	$S\&P_f$	$DowJones_s$	$Nasdaq_f$	$Nasdaq_s$	$DowJones_f$	$Nasdaq_s$	$Nasdaq_f$	$S\&P_f$	$S\&P_s$	$DowJones_s$	$DowJones_f$
4	$S\&P_s$	$S\&P_f$	$DowJones_s$	$Nasdaq_f$	$Nasdaq_s$	$DowJones_f$	$DowJones_s$	$DowJones_f$	$S\&P_f$	$S\&P_s$	$Nasdaq_s$	$Nasdaq_f$
5	$S\&P_s$	$S\&P_f$	$DowJones_s$	$Nasdaq_f$	$Nasdaq_s$	$DowJones_f$	$DowJones_s$	$DowJones_f$	$S\&P_f$	$S\&P_s$	$Nasdaq_s$	$Nasdaq_f$
6	$S\&P_s$	$S\&P_f$	$DowJones_s$	$Nasdaq_f$ f	$Nasdaq_s$	$DowJones_f$	$DowJones_f$	$DowJones_s$	$S\&P_s$	$S\&P_f$	$Nasdaq_f$	$Nasdaq_s$
7	$S\&P_s$	$S\&P_f$	$DowJones_s$	$Nasdaq_f$	$Nasdaq_s$	$DowJones_f$	$DowJones_s$	$DowJones_f$	$S\&P_f$	$S\&P_s$	$Nasdaq_s$	$Nasdaq_f$
8	$S\&P_s$	$S\&P_f$	$Nasdaq_f$	$DowJones_s$	$Nasdaq_s$	$DowJones_f$	$DowJones_s$	$DowJones_f$	$S\&P_f$	$S\&P_s$	$Nasdaq_s$	$Nasdaq_f$
9	$S\&P_s$	$S\&P_f$	$Nasdaq_s$	$DowJones_f$	$DowJones_s$	$Nasdaq_f$	$DowJones_s$	$DowJones_f$	$S\&P_f$	$S\&P_s$	$Nasdaq_s$	$Nasdaq_f$
10	$S\&P_s$	$S\&P_f$	$DowJones_s$	$Nasdaq_f$	$Nasdaq_s$	$DowJones_f$	$Nasdaq_f$	$Nasdaq_s$	$S\&P_s$	$S\&P_f$	$DowJones_f$	$DowJones_s$
11	$S\&P_s$	$S\&P_f$	$DowJones_s$	$Nasdaq_f$ f	$Nasdaq_s$	$DowJones_f$	$DowJones_s$	$DowJones_f$	$S\&P_f$	$S\&P_s$	$Nasdaq_s$	$Nasdaq_f$

Notes: This table reports the C- and D-vine order across the different estimation window. The subscript s denotes a spot index, whereas a f denotes a future index. The left panel displays the C-vine order and the right panel the D-vine order.

The C-vine tree structure is re-estimated in each estimation window. The first tree in the tenth estimation window is displayed in Figure 6. That is the estimation window covering the years 2013 to 2017. The remainder of the trees in the tenth estimation window is displayed in Figure 35 in Appendix Section C.1 with an explanation on the interpretation of the different trees.

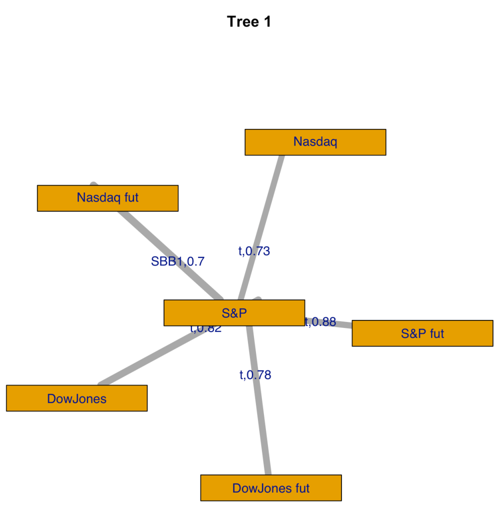

Figure 6: This figure displays the C-vine copula structure of the first tree in the tenth estimation window.

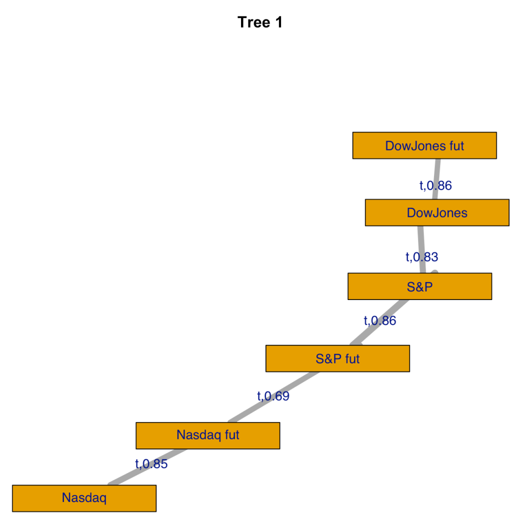

Figure 7: This figure displays the D-vine copula structure of the first tree in the third estimation window.

The $S\&P\ 500_s$ index is the root node in the tenth estimation window. Hence, the first tree consists of five unconditional bivariate copulas, each describing the relation with $S\&P\ 500_s$, shown in Figure 6. We model the dependence in each pair by a copula selected according to the AIC criteria. Except for the pair $S\&P\ 500_s$ and $NASDAQ_f$, the dependence in each pair is modelled by the Student's t copula. This copula demonstrates symmetric tail dependence patterns. The tail dependence symmetry can be confirmed by Table 3. We characterize the dependence in the pair $S\&P\ 500_s$ and $NASDAQ_f$ by the rotated Joe-Clayton (BB1) copula. In contrast to the standard Clayton copula, this copula includes two parameters. Each parameter separately controls the level of upper and lower tail dependence, observable in Table 23. Hence, the copula knows a non-zero asymmetric tail dependence relation. The upper tail dependence between the pair $S\&P\ 500_s$ and $NASDAQ_f$ in this window particularly is equal to 0.56, whereas the lower tail dependence is equal to 0.78. This explains the high fit of the Joe-Clayton copula for this pair.

D-vine The order of the variables in the D-vine is crucial in estimating the D-vine structure. We choose the D-vine order such that the sum of pairwise Kendall's tau coefficients is maximized (Equation 20). We find the strongest dependence is between spot and future series (see Table 2). Therefore, the indices are ordered such that each spot index is linked to its corresponding index future. The D-vine order is displayed in the right panel of Table 6. The D-vine order knows a higher variability than the C-vine order.

We look at the D-vine copula structure to gain insight into the dependency structure of the US portfolio. Figure 7 displays the first tree in the third estimation window, which covers the years 2003-2006. The remainder of the trees along with an description of the interpretation is presented in Figure 36 in Appendix Section C.1.

The first D-vine tree gives a clear idea of the D-vine dependence structure. Each spot index is linked to its corresponding future index. Additionally, the *S&P* 500 indices serve as a link between both the *Dow Jones* and *NASDAQ* spot and future index. This indicates that both *Dow Jones* and *NASDAQ* have the strongest relation (in terms of Kendall's tau) with *S&P* 500, confirmed by Table 2. The spot and future indices in the first tree are linked to each other by five bivariate Student's t copulas, implying a symmetric tail dependence structure between the bivariate return pairs. We can verify the presence of symmetric tail dependence in Table 3. In all estimation windows, most unconditional bivariate copulas in the first tree are characterized by the Student's t copula. This suggests that the Student's t copula is well able to describe the dependence between spot and future returns.

Concluding, the C-vine tree captures the dependencies of one root index with the remainder of the indices. On the contrary, the D-vine links each index to the index with whom it knows the most substantial dependency. Therefore, each spot index is linked to its corresponding future index. Only the high dependence between the spot and future index of the *S&P* 500 series is captured in the C-vine structure. This could explain the slightly higher fit of the D-copula model. This result is in line with Sukcharoen & Leatham (2017), who find a higher likelihood for the D-vine model compared to the C-vine model in a setting where the joint distribution of spot and future returns is estimated. The average upper and lower tail dependence is relatively symmetric across the different indices (Table 3), resulting in a copula-fit comparable to both vine copula models. The Student's t copula model fit is hardly lower than the D-vine copula model. The only deficiency in the Student's t copula model is that it assumes the tail dependence to be identical across index pairs. Although the tail dependence is symmetric, the level of tail dependence differs per pair.

4.2 Hedge Ratios

The average hedge ratios for the US future indices, hedging objectives, and hedging models are displayed in Table 7.

The hedge ratios of the different models generally agree on the sign and magnitude of the hedge. Given different models and hedging objectives, the average hedge ratios lie between 0.61 and 1.49.

The standard deviation of the hedge ratios is displayed in parentheses in Table 7. We find that extreme quantile hedge ratios are more volatile than lower quantiles. Moreover, the models tend to disagree more on the magnitude of the hedge ratio when estimating more extreme confidence levels. This can be explained by the increased difficulty in estimating extreme quantiles of the portfolio's return distribution. The fact that hedge ratios for extreme quantiles are highly volatile is in line with earlier literature of Sukcharoen & Leatham (2017) and Barbi & Romagnoli (2014).

Additionally, the hedge ratios for the C- and D-vine copula are more volatile than other hedging models. A possible explanation for this is that the copula structure and parameters are more sensitive for new data information than the other multivariate copula. Whereas the "standard" multivariate copula models have a limited number of parameters that may change, the C- and D-vine consist of more parameters. Both vine copula models allow the entire structure to change: the bivariate copula families, the parameters, and the order.

Table 7: Hedge ratios US portfolio

$S\&P\ 500_f$	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	0.91 (0.20)	0.99 (0.11)	1.02 (0.20)	1.04 (0.24)	1.06 (0.32)	1.01 (0.44)	0.77 (0.47)	0.83 (0.56)
Clayton	0.88 (0.14)	1.03 (0.22)	0.95 (0.31)	0.98 (0.26)	0.97 (0.20)	0.94 (0.34)	0.88 (0.24)	0.84 (0.29)
Student's t	0.98 (0.07)	0.93 (0.16)	0.94 (0.31)	0.83 (0.38)	1.15 (0.40)	1.12 (0.39)	1.29 (0.57)	1.25 (0.62)
Gaussian	0.92 (0.19)	1.04 (0.20)	0.97 (0.11)	1.03 (0.33)	1.04 (0.35)	1.07 (0.41)	1.22 (0.58)	1.19 (0.64)
C-vine	1.12 (0.27)	1.09 (0.33)	1.19 (0.39)	1.28 (0.32)	1.26 (0.35)	1.14 (0.39)	1.32 (0.45)	1.30 (0.48)
D-vine	0.90 (0.52)	0.99 (0.41)	0.96 (0.52)	0.66 (0.73)	0.65 (0.75)	0.70 (0.73)	0.81 (0.77)	0.78 (0.80)
HS	1.06 (0.14)	1.19 (0.40)	1.13 (0.43)	1.41 (0.47)	1.29 (0.67)	1.12 (0.88)	1.41 (0.51)	1.37 (0.62)
$NASDAQ_f$	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	0.87 (0.11)	0.90 (0.17)	0.99 (0.33)	0.93 (0.25)	0.98 (0.28)	1.18 (0.38)	1.02 (0.35)	1.11 (0.45)
Clayton	0.76 (0.31)	0.86 (0.32)	0.83 (0.30)	0.86 (0.35)	0.92 (0.47)	0.88 (0.57)	0.82 (0.25)	0.82 (0.24)
Student's t	0.93 (0.10)	0.98 (0.15)	0.97 (0.20)	0.92 (0.19)	0.93 (0.24)	1.00 (0.34)	0.95 (0.24)	0.95 (0.29)
Gaussian	0.97 (0.18)	0.87 (0.13)	1.03 (0.16)	0.82 (0.43)	0.86 (0.47)	0.93 (0.41)	1.03 (0.21)	1.06 (0.21)
C-vine	0.93 (0.27)	0.86 (0.27)	0.83 (0.62)	0.72 (0.41)	0.71 (0.50)	0.86 (0.53)	0.74 (0.60)	0.92 (0.62)
D-vine	0.98 (0.52)	0.91 (0.44)	0.73 (0.48)	0.94 (0.62)	0.92 (0.67)	0.90 (0.61)	0.86 (0.59)	0.92 (0.62)
HS	1.02 (0.10)	0.98 (0.06)	1.04 (0.17)	0.92 (0.10)	0.97 (0.11)	0.93 (0.26)	0.91 (0.09)	0.87 (0.14)
$Dow\ Jones_f$	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	0.97 (0.10)	1.03 (0.16)	1.08 (0.21)	1.05 (0.23)	1.03 (0.26)	0.95 (0.34)	1.12 (0.35)	1.11 (0.40)
Clayton	0.97 (0.19)	0.83 (0.30)	0.96 (0.35)	0.83 (0.22)	0.84 (0.23)	0.87 (0.24)	0.91 (0.28)	0.92 (0.30)
Student's t	1.04 (0.17)	1.00 (0.11)	0.98 (0.18)	0.92 (0.43)	0.89 (0.36)	0.61 (0.68)	0.71 (0.45)	0.77 (0.49)
Gaussian	1.01 (0.27)	0.98 (0.20)	0.97 (0.22)	1.18 (0.37)	1.18 (0.44)	1.15 (0.51)	0.77 (0.48)	0.77 (0.51)
C-vine	0.96 (0.41)	0.97 (0.49)	0.92 (0.63)	0.99 (0.52)	1.03 (0.56)	1.06 (0.60)	0.91 (0.59)	0.89 (0.63)
D-vine	1.25 (0.55)	1.24 (0.59)	1.34 (0.59)	1.52 (0.61)	1.53 (0.62)	1.47 (0.59)	1.49 (0.61)	1.48 (0.60)
HS	0.97 (0.21)	0.85 (0.39)	0.76 (0.47)	0.65 (0.49)	0.69 (0.62)	0.81 (0.61)	0.63 (0.46)	0.68 (0.53)

Notes: This table reports the average hedge ratios of the US portfolio and the standard deviation in parentheses.

4.3 Out-of-Sample Hedging Effectiveness

Table 8 presents the average out-of-sample hedging performance of the different hedging models applied to the 8 risk measures for the US portfolio. The best performing model per hedging objective is highlighted in bold. On average, all hedging models achieve a competitive risk reduction varying between 67.47% and 93.01%. The total risk is presented in Table 31 in Section C.2 in the Appendix. Additionally, the unhedged risk of the US portfolio is shown in Table 33 in Section C.2 in the Appendix.

Among the copula models, the Student's t copula model yields the best performance and achieves the greatest risk reduction on VaR90, VaR99, ES90, ES95, SV and LPM objectives. The Student's t copula knows a high average fit (Table 5) to the data and is likely causing its superior performance. The fit is lower compared to the D-vine copula model. However, the D-vine copula model never provides a higher average risk reduction than the Student's t copula model. The D-vine copula out-of-sample risk reduction is also lower than the C-vine copula model.

These findings show that superior in-sample fit does not necessarily translate to a superior out-of-sample performance because the risk exists that the model is over-fitting the in-sample (Hsu & Tseng, 2008). Both vine copula models consist of more parameters than the other copula models. Hence, the risk of over-fitting is more evident for vine copula models.

In particular, the low risk reduction is mainly caused by the poor performance of the D-vine copula model in the first, second and fifth estimation window. The dependence between the *Dow Jones* spot and future index rapidly changes, as can be observed in Figure 29. The D-vine copula model is the one that directly links each spot to its corresponding future index, leading to poor out-of-sample performance if the dependence between spot and futures changes rapidly. Moreover, the D-vine structure (order) changes more frequently than the C-vine structure. Additionally, the D-vine hedge ratios know a higher variance than other copula models. This suggests that the D-vine structure might over-fit the in-sample, resulting in poor out-of-sample performance. Therefore, a copula model with fewer parameters with a reasonable fit to the data (e.g. Student's t) should be preferred for out-of-sample hedging over more advanced copula models if the data display symmetric tail dependence patterns. The results of the hedging performance in-sample, demonstrated in Table 34 in Appendix Section C.2, are more in line with the copula fit. The D-vine copula model achieves the best performance on 4 out of 8 objectives among the copula models.

The Clayton copula model realizes a poor hedging performance. The Clayton copula is the copula model with the lowest average fit caused by its restrictive parameter assumptions and its inability to capture upper-tail dependence. On the contrary, the Gumbel copula accomplishes a competitive risk reduction compared to the remaining copula models. The Gaussian model achieves a risk reduction comparable to the Gumbel copula model and is competitive with the remaining copula models.

Similarly to the copula fit, the hedging performance increases over time. The hedging effectiveness is positively correlated with the dependence between spot and future indices. This holds regardless of the considered risk measure and is explained because the basis risk (unhedgable risk) is higher when indices are less linearly correlated. Additionally, the hedging performance decreases with the increase of confidence level. This confirms that it is more difficult to hedge extreme risk. However, the hedging performance for these extreme quantiles is still high. Both results are in line with the research of Sukcharoen & Leatham (2017) and Barbi & Romagnoli (2014).

Table 7 shows us that the hedge ratios generally agree on the size and magnitude of the hedge. This leads to relatively similar model performance across different hedging models and suggests that all hedging models can effectively reduce the risk of investors' portfolios.

Although all copula models achieve a good hedging performance, the non-parametric benchmark model of Harris & Shen (2006) is difficult to outperform. For 6 out of 8 hedging objectives, the HS benchmark that relies on historical simulation realizes the greatest risk reduction. The difference compared to the other copula models is however most of the time not statistically significant (Table 32 in Appendix Section C.2). A possible explanation for this finding is due to computational limitations. Only 1,000 returns are simulated from the copula model, which is relatively low for Monte-Carlo simulation and could result in more inaccurate estimates of the hedge ratios. One of the major pitfalls of the HS model is the strong dependence on historical data. In this research, the HS estimates rely on 780 observations comparable to the number of simulations from the copula models. Therefore, the advantage of the copula models over the HS model is negligible and does not result in superior performance.

Another critical note is that the copula models and hedge ratios are only estimated every one and half years due to computational limitations. However, the dependence structure likely varies more over time, increasing the need to re-estimate the dependence structure more often. There are only eleven test windows in the current setting, making the performance likely highly sensitive to the choice of in-and out-sample window. This effect is more gradual in a setting where the structure is more often re-estimated (and the in-sample windows overlap).

Table 8: US hedging effectiveness

	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	78.35 (4.97)	79.62 (4.80)	75.68 (9.33)	79.20 (3.21)	77.96 (3.91)	73.03 (10.13)	92.85 (7.39)	97.15 (9.62)
Clayton	74.44 (9.09)	75.09 (6.55)	71.58 (8.88)	71.91 (7.06)	71.38 (8.18)	67.47 (11.41)	91.24 (9.93)	96.59 (19.04)
Student's t	80.85 (2.80)	78.67 (4.74)	76.82 (6.01)	79.58 (2.88)	77.98 (4.02)	74.43 (7.94)	**96.39 (2.60)**	98.71 (3.47)
Gaussian	79.29 (3.36)	79.14 (4.39)	75.64 (4.83)	76.59 (4.94)	75.45 (6.47)	72.53 (9.01)	95.04 (3.79)	98.47 (4.41)
C-vine	79.99 (3.05)	79.26 (4.06)	76.26 (6.34)	78.12 (3.39)	77.65 (3.35)	**75.57 (8.69)**	93.21 (7.04)	97.85 (7.29)
D-vine	77.77 (6.77)	78.64 (5.59)	76.25 (6.86)	74.87 (7.28)	73.72 (8.11)	70.68 (9.40)	95.69 (13.26)	97.91 (16.65)
HS	**80.98 (3.04)**	**81.23 (2.87)**	**77.71 (7.09)**	**79.95 (3.19)**	**78.96 (4.04)**	75.43 (9.37)	96.04 (5.41)	**99.04 (2.69)**

Notes: This table reports the average hedging effectiveness of the different hedging models for the US portfolio. The standard deviation of the hedging performance is denoted in parentheses. Additionally, the best performing hedging model per hedging objective is highlighted in bold.

4.3.1 Robustness

We examine the copula model fit and performance for different model specifications to check for the robustness of our results. The sensitivity specifications are displayed with a short description in Table 19 in Appendix Section A.3. The copula fit for the different model specifications, along with an explanation, is given in Appendix Section C.1.1. Section C.2.1 in the Appendix presents the hedging effectiveness of the different model specifications. The most prominent findings are highlighted here shortly.

Copula fit The different model specifications lead to differing levels of copula fit. However, the D-vine copula model yields the highest fit in each specification.

The copula model fit is insensitive to the distribution of the residuals for all copula models. The Gaussian distribution obtains the highest fit for all copula models, but this difference is insignificant to the remaining residuals specifications. The constant mean model provides a lower average copula fit than the base model, which is significant for most copula models. Both alternatives of a non-parametric transformation of the standardized residuals to copula data using the empirical distribution function and the non-parametrically modelling of marginals lead to a significantly better copula fit in most copula models.

Moreover, the shift in estimation periods significantly improves the copula model fit, indicating that the copula fit and model performance is sensitive for the estimation period. The in-sample periods in the model in which the in-sample periods are shifted by a year are more concentrated around clear regimes instead of covering estimation periods that consist of different regimes. The fit of the model specifications where the window sizes are decreased and where the model is re-estimated more often can not be compared directly with the base model, as the estimation windows and number of estimation windows do not match.

Both vine copula models provide a superior fit in the EU portfolio. However, the difference is insignificant to the Student's t copula model, explained by the fact that the EU portfolio demonstrates symmetric tail dependence patterns (Appendix Section A.6).

Hedging effectiveness Except for the shifted model, the different model specifications do not seem to improve the out-of-sample hedging effectiveness nor to differ the best performing hedging model. The hedging effectiveness is comparable across the different model specifications employed. The HS model achieves the highest performance for most hedging objectives, followed by the Student's t copula model. However, its performance does not significantly differ from the copula models. This leads to the conclusion that a significant better copula fit, which is found for the models where the marginals are modelled non-parametrically and where the residuals are transformed to copula data by the empirical distribution function, does not necessarily lead to better hedging effectiveness.

The shifted model does lead to better hedging effectiveness than the base model. We can explain a part of this by the increased copula fit of the shifted model compared to the base model. However, as the most considerable difference in hedging effectiveness is found in the window where the difference in copula fit is the smallest, this is not the sole reason. Next to copula fit, the effectiveness is highly sensitive to the choice of estimation window.

Both the model where the number of re-estimations is increased and the model where the window sizes are decreased lead to a lower overall hedging performance than to the base model. In the first model, the lower performance is mainly caused by the increased number of out-of-sample windows that result in a poor hedging performance and lower average effectiveness. However, as the overlap in estimation windows is greater, the model is less sensitive to the choice of estimation periods. The difference for the latter model is mainly caused by a poor performance in estimation windows that cover recessions. The smaller estimation windows capture more temporary regimes in the copula parameters, leading a lower overall hedging performance. The difference in performance between the hedging models becomes more evident in both cases.

The copula model performance increases when we simulate more observations in the Monte Carlo simulation. The HS benchmark is outperformed by the Gumbel and Student's t copula model on most hedging objectives. The performance of The D-vine copula model lags behind, caused by the fact that the model overfits the in-sample.

In this research, it was not feasible to re-estimate the models more often and increase the number of simulated observations. However, the robustness checks show that both estimating the model more often and increasing the number of observations leads to the desired effect as it enhances the copula model performance and makes the results more robust to the choice of estimation window.

Finally, the EU portfolio's hedging effectiveness is higher than the US portfolio, which is explained by the stronger EU portfolio dependence than the US portfolio since the hedging effectiveness is a positive function of the correlation. Moreover, both vine copula models achieve the greatest risk reduction and suffer less from overfitting than in the US portfolio. A likely explanation for this is that the dependence structure changes less frequently which can be observed in the tail-dependency graphs in Figure 25 to Figure 32 in Appendix Section A.4. The Student's t copula model achieves a risk reduction comparable to the D-vine copula model and has a smaller risk of overfitting when the dependence structure changes rapidly. However, in a setting where the dependence is relatively stable, an investor may benefit from employing vine copula models to increase the hedging effectiveness.

4.4 Sparse Vine Copula Model

In this section, we present the result of the sparse vine copula model. First, we look at the truncation levels for the C- and D-vine copula models. Subsequently, we evaluate the difference in computing times. Finally, we assess the sparse copula model fit and hedging effectiveness, compare it with the non-sparse C- and D-vine copula model, and conclude whether an investor can benefit from employing a sparse vine copula model.

Table 9 presents the truncation levels that are selected according the mBICv. Even though the prior probability of having an independent copula exceeds 0.5 after tree 3, in 10 out of 11 estimation windows, the C-vine copula model is not truncated. It is only truncated in the first estimation window after the third tree. This means that both the relation $Dow\ Jones_s$ with $NASDAQ_f$ and $Dow\ Jones_f$ and the relation $NASDAQ_f$ with $Dow\ Jones_f$ are modelled by the independence copula (Table 6). This is not surprising, as, in this particular estimation window, the correlation between these pairs is close to 0. The C-vine order structure can justify the evidence that the C-vine is almost never truncated. The indices in the C-vine copula model are ordered sequentially where the sequence starts with the index with the highest correlation with the other indices. Subsequently, the index with the highest correlation with the remaining indices (excluding the first index) is ordered second. Since the root variables in the C-vine structure are chosen based on this strong dependency, most bivariate copula pairs (which always consist of a relation with the root variable) know an acceptable correlation.

In contrast, the D-vine copula model is truncated often earlier. In the majority of the estimation windows, the D-vine copula model is truncated after the first tree, meaning that five unconditional bivariate copulas model the joint relation. The relation is truncated after the first tree and can be explained by how the D-vine copula model is structured. First, the D-vine order is chosen by looking at all possible orders where each index is connected to mostly two other indices, resulting in a straight line (Figure 7). Then, the order leading to the highest pair correlation sum is selected. This causes the variables connected in the first tree know a high correlation. However, this does not necessarily mean that the indices linked in the second tree know a strong correlation since the second tree links index 1 to index 3 in the second tree. This correlation is not taken into account when calculating the order. The D-vine copula model is truncated at level 1 in the first, second, third, fourth, sixth and seventh estimation window. In contrast, the D-vine is not truncated at the last two estimation windows (tenth and eleventh). The conceivable explanation for this is that the dependence between indices has increased over time, resulting in an acceptable correlation between most index pairs.

Table 9: Truncation Level vine copulas

K	C-vine	D-vine
1	0	5
2	0	0
3	1	4
4	0	0
5	10	2

Notes: This table reports truncation levels of the sparse C- and D-vine copula model.

The average computing time to fit the different vine copula models is displayed in Table 10. The sparse C-vine copula model is only truncated once at the third estimation window. The sparse C-vine copula model in the remaining estimation windows is equal to the non-sparse C-vine copula model. Therefore, the computing times do not differ. There is, however, a gain in the sparse D-vine copula model compared to the non-sparse one. This difference is, however, not statistically significant. The computing benefit of employing the sparse D-vine copula model over the non-sparse one is in this research 7.25 minutes (number of estimation windows $\times$ estimation time). If the performance can be considered comparable, it may be beneficial to estimate the dependence structure using the sparse D-vine copula model.

Table 10: Computing time

	C-vine	D-vine	Sparse C-vine	Sparse D-vine	C-difference	D-difference
time	4.00 (1.21)	3.40 (0.96)	4.00 (1.25)	3.01 (1.19)	0.00 (1.00)	0.39 (0.59)

Notes: This table reports the average estimation time of the different vine copula models. The standard deviation of the computing time is displayed in parentheses. Additionally, the difference between the both sparse and non-sparse vine copula models are displayed where the p-value is denoted in parentheses.

Table 11 presents the log-likelihood of the (sparse) C-and D-vine copula model. The difference and p-value between the sparse and non-sparse models are also presented.

Similar to the computing time, the difference in copula fit between the sparse, and non-sparse vine is negligible. The difference between the sparse and non-sparse D-vine copula model is more evident. However, this difference is not statistically significant. The sparse D-vine copula model is more efficient than the non-sparse one, whilst the truncation does not lead to a significantly worse fit. Additionally, the standard deviation of the fit of both sparse vine copula models is lower than the non-sparse ones.

Table 11: Copula fit

	C-vine	D-vine	Sparse C-vine	Sparse D-vine	C-difference	D-difference
Log Likelihood	5847.81 (847.05)	5937.89 (873.77)	5847.44 (842.05)	5633.79 (852.17)	0.37(1.00)	304.10 (0.59)

Notes: This table reports the average copula fit in terms of log-likelihood for the vine copula models. Additionally, the difference between the both truncated and non-truncated vine copula models are displayed where the p-value is denoted in parentheses.

Finally, we compare the hedging effectiveness between the sparse and non-sparse vine copula models. Table 12 presents the hedging effectiveness of both sparse and non-sparse vine copula models and their difference, respectively. The p-value of the difference is displayed in parentheses.

The performance of the sparse and non-sparse C-vine copula model is comparable as expected. The difference is relatively small and always insignificant since the sparse C-vine copula model is almost similar to the non-sparse one. Nagler et al. (2019) argue that a vine copula model with less than seven is already considered sparse. Therefore, it is not surprising that the vine copula model is not truncated if it only consists of six tees.

The sparse D-vine copula model results in poorer performance than the non-sparse one. This difference is significant for the majority of the risk measures. Truncating the D-vine copula model thus reduces the hedging effectiveness, although it still produces a competitive fit. Therefore, the investor should make a trade-off between computing time and hedging performance when computing the OHR.

Table 12: Hedging effectiveness

	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
C-vine	**79.99 (3.05)**	**79.26 (4.06)**	**76.26 (6.34)**	**78.12 (3.40)**	**77.65 (3.35)**	**75.57 (8.69)**	93.21 (7.04)	97.85 (7.29)
D-vine	77.77 (6.77)	78.64 (5.59)	76.25 (6.86)	74.87 (7.28)	73.72 (8.11)	70.68 (9.40)	**95.69 (13.26)**	**97.91 (16.65)**
Sparse C-vine	78.47 (2.99)	78.19 (4.22)	75.12 (5.99)	76.56 (3.41)	75.32 (3.52)	75.53 (9.01)	92.28 (6.81)	95.89 (3.56)
Sparse D-vine	69.23 (5.90)	70.48 (5.01)	65.25 (6.90)	70.42 (6.98)	64.52 (7.92)	62.39 (8.94)	86.89 (12.77)	92.74 (16.40)
C-difference	-0.48 (0.71)	1.07 (0.55)	1.14 (0.67)	1.56 (0.29)	2.33 (0.13)	0.04 (0.99)	0.93 (0.76)	1.96 (0.43)
D-difference	8.54 (0.01)	8.16 (0.00)	11.01 (0.00)	4.45 (0.16)	9.22 (0.01)	8.32 (0.05)	8.82 (0.13)	5.17 (0.47)

Notes: This table reports the average hedging effectiveness of the vine copula models. Additionally, the difference between the both truncated and non-truncated vine copula models are displayed where the p-value is denoted in parentheses

4.5 Single-Product Portfolios

This section presents the results of the hedging effectiveness of the different single product portfolios. It starts by presenting the copula fit. Subsequently, the optimal hedge ratios and hedging performance are discussed.

4.5.1 Copula fit

Table 13 presents the average fit of the bivariate copula models of the 10 single product portfolios.

Across all portfolios, the Student's t copula model yields the highest fit in terms of log-likelihood. In contrast to the remaining copula hedging models, the Student's t copula model has an additional parameter controlling the level of tail dependence. The Student's t copula exhibits a symmetric tail dependence relation. The 10 single product portfolios show an approximately symmetric tail dependence pattern explaining the superior fit of the Student's t copula model (Table 4).

The *CAC* 40, *Hang Seng*, *Euro Stoxx* 50, and *AEX* are the portfolios where the upper tail dependence exceeds the lower tail dependence (Table 4). Since the difference is rather small, the Student's t copula achieves the highest average fit. Except for the *Hang Seng* portfolio, the Gumbel copula model achieves the second-highest fit after the Student's t copula model. This shows that in portfolios where the upper tail dependence is stronger than the lower tail dependence, the Gumbel copula (which demonstrates upper tail dependence) should be preferred over the symmetric Gaussian copula.

In the remaining portfolios, the Gaussian copula model provides realizes a higher fit than both Archimedian copulas, which is consistent with Hsu & Tseng (2008). Spot returns are tied closely to future returns by no-arbitrage assumptions (Hsu & Tseng, 2008) which causes an approximate linear relation between spot and futures indices (see Figure 12 to 21 in Appendix Section A.4).The Gaussian copula is symmetric does not demonstrate tail dependence. Hence, its symmetry likely causes its superior performance to both Archimedian copulas in cases where the upper tail dependence is weaker than the lower tail dependence.

The Gumbel and Clayton copula yield the poorest fit to the data in the portfolios that have stronger lower tail dependence than upper tail dependence. Therefore, the fit of the Gumbel copula is worse than portfolios where the upper tail dependence exceeds lower tail dependence, and is comparable with the Clayton copula. Both copulas exhibit asymmetric tail dependence relations between spot and future returns. The Gumbel copula lacks lower tail dependence, whereas the Clayton copula lacks upper tail dependence. The tail dependence is relatively symmetric resulting in a poor fit of both Archimedean copulas. The copula fit of the Gumbel copula is in most portfolios higher compared to the Clayton copula, which is in line with Barbi & Romagnoli (2014).

We obtain the highest average fit for the *CAC* 40 portfolio, explained by the fact that it provides the highest average Kendall's tau between spot and future returns (0.94), which can be observed in Table 4. The *AEX* portfolio obtains the second-highest fit and also knows a strong dependence relation with an average Kendall's tau of 0.93. The lowest average fit is for the *Nikkei* 225 portfolio, followed by the *Hang Seng*. Both portfolios have Kendall's tau estimates of 0.76 and 0.83, respectively. These findings confirm that the copula fit is a positive function of the correlation.

Table 13: Average copula fit of the bivariate copula models

	Gumbel	Clayton	Student's t	Gaussian
S&P 500	1196.73 (89.49)	1049.02 (82.16)	**1260.58 (83.32)**	1220.18 (93.73)
FTSE 100	1207.22 (94.02)	1073.38 (102.47)	**1280.13 (96.29)**	1232.70 (114.56)
DAX	1271.67 (332.33)	1116.25 (312.67)	**1335.92 (340.71)**	1286.84 (337.66)
Nikkei 225	870.76 (431.66)	753.69 (368.37)	**945.87 (448.39)**	822.13 (418.21)
NASDAQ	1155.60 (71.76)	1009.94 (66.41)	**1220.66 (69.88)**	1172.02 (63.37)
CAC 40	1757.49 (198.30)	1557.06 (150.18)	**1930.74 (227.43)**	1511.53 (103.80)
Dow Jones	996.70 (462.85)	877.27 (415.79)	**1060.07 (478.68)**	1001.60 (475.80)
Hang Seng	999.23 (70.14)	854.33 (54.60)	**1046.57 (70.22)**	1026.45 (72.12)
Euro Stoxx 50	1198.59 (353.15)	1010.87 (303.73)	**1264.85 (365.00)**	1134.51 (304.91)
AEX	1629.59 (182.82)	1453.24 (183.82)	**1789.96 (227.82)**	1473.45 (136.53)

Notes: This table reports the average copula fit in terms of log-likelihood for the different copula models.

4.5.2 Hedge Ratios

The average hedge ratios of the the 10 single-product portfolios are presented in Figure 14 and 15 respectively.

The hedging models for the different portfolios tend to agree on the sign and magnitude of the average hedge ratios. The average hedge ratios for the VaR and ES objective for the different confidence levels range between 0.57 and 1.15. The HS model achieves the lowest standard deviation in hedge ratios for most portfolios. A conceivable explanation for this is that the HS model calculates the hedge ratios based on the true returns. Since there is overlap in the estimation windows, the HS model party calculates the hedge ratios based on the same data. This is different from copula models, where the dependence structure is re-estimated in each estimation window, and new data is simulated. For most portfolios, the standard deviation increases with the confidence level. Additionally, the hedge ratios for more extreme quantiles are generally more dispersed than less extreme quantiles, caused by the increased difficulty in estimating these quantiles.

The hedge ratios tend to get closer to 1.00 as the dependency increases. This can be observed in the hedge ratios of the *CAC* 40 and *AEX* portfolio. Both portfolios know strong dependency and hedge ratios that converge to 1.00. This result is in line with earlier research of Barbi & Romagnoli (2014).

Table 14: Average hedge ratios for the different single-product portfolios

S&P 500	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	1.07 (0.33)	1.06 (0.22)	1.14 (0.29)	1.10 (0.23)	1.14 (0.26)	1.15 (0.24)	1.09 (0.12)	1.13 (0.11)
Clayton	0.94 (0.25)	0.92 (0.20)	0.80 (0.33)	0.84 (0.28)	0.82 (0.31)	0.78 (0.37)	0.95 (0.15)	0.97 (0.17)
Student's t	1.02 (0.13)	1.00 (0.13)	1.07 (0.20)	1.03 (0.17)	1.05 (0.19)	1.05 (0.22)	1.04 (0.09)	1.01 (0.10)
Gaussian	0.95 (0.11)	1.04 (0.19)	1.04 (0.20)	1.01 (0.19)	1.01 (0.18)	1.02 (0.17)	1.00 (0.09)	1.02 (0.09)
HS	1.01 (0.03)	0.98 (0.03)	0.97 (0.06)	1.00 (0.02)	0.99 (0.03)	0.97 (0.04)	1.01 (0.01)	0.99 (0.02)
FTSE 100	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	0.79 (0.34)	0.78 (0.32)	0.78 (0.43)	0.74 (0.44)	0.75 (0.45)	0.76 (0.48)	0.80 (0.22)	0.81 (0.23)
Clayton	0.66 (0.31)	0.64 (0.32)	0.65 (0.37)	0.58 (0.31)	0.57 (0.31)	0.59 (0.33)	0.63 (0.16)	0.67 (0.16)
Student's t	0.82 (0.29)	0.79 (0.35)	0.76 (0.41)	0.71 (0.40)	0.70 (0.41)	0.69 (0.41)	0.80 (0.20)	0.76 (0.20)
Gaussian	0.79 (0.31)	0.75 (0.37)	0.81 (0.42)	0.74 (0.39)	0.74 (0.41)	0.77 (0.44)	0.81 (0.20)	0.72 (0.21)
HS	0.99 (0.04)	0.99 (0.02)	0.87 (0.26)	0.84 (0.20)	0.81 (0.24)	0.71 (0.27)	0.98 (0.09)	0.93 (0.14)
DAX	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	0.95 (0.07)	0.98 (0.07)	1.03 (0.06)	0.98 (0.07)	1.00 (0.06)	1.04 (0.08)	0.94 (0.03)	0.99 (0.03)
Clayton	0.88 (0.13)	0.90 (0.12)	0.92 (0.13)	0.84 (0.11)	0.85 (0.14)	0.83 (0.17)	0.92 (0.06)	0.89 (0.07)
Student's t	0.97 (0.06)	0.98 (0.06)	0.97 (0.09)	0.96 (0.07)	0.96 (0.08)	0.93 (0.11)	0.96 (0.04)	0.95 (0.05)
Gaussian	0.95 (0.09)	0.93 (0.12)	0.96 (0.14)	0.95 (0.07)	0.95 (0.08)	0.96 (0.09)	0.94 (0.04)	0.96 (0.03)
HS	0.99 (0.04)	0.98 (0.05)	1.00 (0.06)	0.99 (0.04)	0.99 (0.05)	0.98 (0.07)	0.99 (0.02)	0.97 (0.03)
Nikkei 225	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	1.01 (0.02)	1.01 (0.05)	1.07 (0.107)	1.03 (0.05)	1.05 (0.05)	1.08 (0.08)	1.02 (0.02)	1.03 (0.03)
Clayton	0.93 (0.10)	0.89 (0.09)	0.86 (0.11)	0.84 (0.06)	0.83 (0.07)	0.80 (0.08)	0.90 (0.03)	0.90 (0.03)
Student's t	1.00 (0.04)	1.00 (0.03)	1.00 (0.04)	0.99 (0.03)	0.99 (0.03)	0.98 (0.06)	1.02 (0.01)	0.98 (0.02)
Gaussian	0.99 (0.02)	1.01 (0.04)	0.99 (0.04)	0.99 (0.04)	0.98 (0.04)	0.98 (0.06)	0.98 (0.02)	0.96 (0.02)
HS	1.00 (0.01)	1.00 (0.02)	0.99 (0.03)	0.99 (0.02)	0.98 (0.02)	0.97 (0.04)	1.01 (0.01)	1.04 (0.01)
NASDAQ	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	0.98 (0.06)	0.98 (0.08)	1.02 (0.08)	0.99 (0.07)	1.00 (0.07)	1.02 (0.13)	0.97 (0.04)	1.01 (0.05)
Clayton	0.91 (0.07)	0.88 (0.11)	0.79 (0.16)	0.79 (0.09)	0.77 (0.11)	0.75 (0.15)	0.92 (0.05)	0.85 (0.06)
Student's t	0.99 (0.06)	0.97 (0.09)	0.96 (0.07)	0.96 (0.06)	0.95 (0.07)	0.94 (0.09)	0.99 (0.03)	0.98 (0.04)
Gaussian	0.98 (0.04)	0.98 (0.04)	0.98 (0.04)	0.97 (0.042)	0.97 (0.05)	0.97 (0.04)	0.96 (0.02)	0.99 (0.02)
HS	0.99 (0.05)	0.99 (0.02)	0.97 (0.05)	0.98 (0.02)	0.97 (0.03)	0.94 (0.08)	0.98 (0.01)	0.98 (0.02)

Notes: This table reports the average hedge ratios of the different single product portfolios with the standard deviation in parentheses.

Table 15: Average hedge ratios for the different single-product portfolios

CAC 40	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	1.01 (0.03)	1.02 (0.05)	1.03 (0.06)	1.02 (0.04)	1.03 (0.04)	1.03 (0.04)	1.01 (0.02)	1.03 (0.02)
Clayton	0.95 (0.08)	0.94 (0.08)	0.85 (0.10)	0.89 (0.07)	0.87 (0.07)	0.84 (0.07)	0.97 (0.03)	0.98 (0.05)
Student's t	1.00 (0.03)	1.00 (0.02)	0.99 (0.04)	1.00 (0.02)	1.00 (0.02)	1.00 (0.02)	0.99 (0.01)	1.01 (0.01)
Gaussian	0.99 (0.02)	1.00 (0.03)	1.03 (0.06)	1.01 (0.05)	1.02 (0.07)	1.02 (0.08)	1.00 (0.03)	1.01 (0.04)
HS	1.00 (0.01)	1.00 (0.01)	1.01 (0.03)	1.00 (0.01)	0.99 (0.02)	0.99 (0.04)	1.01 (0.01)	1.00 (0.02)
Dow Jones	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	0.81 (0.50)	0.92 (0.17)	0.85 (0.35)	0.85 (0.31)	0.86 (0.29)	0.87 (0.35)	0.87 (0.16)	0.84 (0.16)
Clayton	0.78 (0.31)	0.83 (0.28)	0.78 (0.27)	0.70 (0.27)	0.68 (0.28)	0.65 (0.19)	0.74 (0.13)	0.81 (0.12)
Student's t	0.91 (0.29)	0.90 (0.14)	0.84 (0.29)	0.83 (0.31)	0.81 (0.31)	0.78 (0.31)	0.89 (0.15)	0.90 (0.16)
Gaussian	0.79 (0.46)	0.82 (0.39)	0.80 (0.34)	0.80 (0.33)	0.81 (0.30)	0.78 (0.34)	0.81 (0.15)	0.79 (0.15)
HS	0.86 (0.35)	0.84 (0.37)	0.87 (0.31)	0.84 (0.36)	0.83 (0.36)	0.80 (0.37)	0.84 (0.17)	0.82 (0.17)
Hang Seng	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	0.84 (0.11)	0.94 (0.07)	0.93 (0.09)	0.90 (0.07)	0.91 (0.09)	0.96 (0.12)	0.87 (0.04)	0.89 (0.06)
Clayton	0.85 (0.10)	0.75 (0.14)	0.63 (0.08)	0.67 (0.06)	0.65 (0.07)	0.60 (0.08)	0.64 (0.05)	0.65 (0.06)
Student's t	0.92 (0.10)	0.94 (0.08)	0.93 (0.07)	0.90 (0.05)	0.90 (0.05)	0.88 (0.10)	0.93 (0.02)	0.90 (0.03)
Gaussian	0.86 (0.08)	0.88 (0.13)	0.89 (0.12)	0.88 (0.07)	0.89 (0.08)	0.89 (0.10)	0.86 (0.04)	0.87 (0.05)
HS	0.93 (0.06)	0.94 (0.09)	0.97 (0.06)	0.92 (0.04)	0.92 (0.05)	0.93 (0.08)	0.95 (0.02)	0.92 (0.02)
Euro Stoxx 50	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	0.97 (0.08)	0.98 (0.07)	0.96 (0.15)	0.97 (0.12)	0.98 (0.14)	1.00 (0.16)	0.99 (0.06)	0.95 (0.07)
Clayton	0.93 (0.11)	0.79 (0.14)	0.88 (0.40)	0.81 (0.12)	0.79 (0.16)	0.83 (0.39)	0.82 (0.08)	0.86 (0.14)
Student's t	0.92 (0.11)	0.93 (0.11)	0.89 (0.13)	0.93 (0.09)	0.92 (0.12)	0.91 (0.14)	0.91 (0.05)	0.89 (0.06)
Gaussian	0.91 (0.09)	0.96 (0.07)	0.94 (0.11)	0.92 (0.09)	0.92 (0.09)	0.93 (0.09)	0.95 (0.04)	0.91 (0.04)
HS	0.96 (0.07)	0.95 (0.07)	0.98 (0.06)	0.95 (0.07)	0.94 (0.08)	0.94 (0.07)	0.92 (0.03)	0.94 (0.03)
AEX	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	1.00 (0.03)	1.00 (0.05)	1.01 (0.10)	1.01 (0.07)	1.02 (0.07)	1.05 (0.09)	1.01 (0.03)	0.99 (0.04)
Clayton	0.94 (0.07)	0.89 (0.08)	0.87 (0.12)	0.87 (0.07)	0.85 (0.06)	0.78 (0.08)	0.89 (0.03)	0.91 (0.03)
Student's t	1.00 (0.03)	1.01 (0.03)	1.01 (0.05)	1.01 (0.05)	1.01 (0.05)	1.01 (0.08)	1.00 (0.03)	0.99 (0.04)
Gaussian	0.99 (0.02)	0.98 (0.04)	1.01 (0.05)	0.99 (0.05)	0.99 (0.06)	1.01 (0.08)	0.97 (0.03)	1.00 (0.03)
HS	1.00 (0.01)	0.99 (0.02)	0.99 (0.02)	0.99 (0.01)	1.00 (0.02)	0.99 (0.03)	1.01 (0.01)	0.98 (0.02)

Notes: This table reports the average hedge ratios of the different single product portfolios with the standard deviation in parentheses.

4.5.3 Out-of-Sample Hedging Effectiveness

Table 16 and 17 present the average out-of-sample hedging effectiveness of the 10 single product portfolios for the different hedging models and objectives. Table 41 and 42 in Appendix Section C.3 present the difference in performance between the HS model and different copula models.

The average performance for the different portfolios and hedging objectives varies between 48% and 99%. The highest hedging effectiveness is achieved for portfolios that show the most substantial dependency (e.g. *CAC* 40, *AEX*), and the lowest hedging effectiveness is produced by portfolios that exhibit the weakest dependency (e.g. *Nikkei*, *Dow Jones*). This illustrates that hedging effectiveness is positively correlated with dependency. The strong dependency translates to higher effectiveness in 2 ways. First, the copula fit is higher for portfolios with a stronger dependency. This fit can lead to higher hedging effectiveness. Second, the basis risk for weaker correlated indices is considerable. This basis risk is the unhedgable risk caused by the fact that the spot and futures indices are not perfectly correlated. The lower the correlation, the higher this basis risk resulting from price movements that do not perfectly match each other, leading to lower hedging effectiveness. Barbi & Romagnoli (2014) confirm this finding by testing the effectiveness of direct hedging (hedging the spot index directly with its index future) and cross-hedging (hedging the spot index with a different index future). In their setting, the hedging effectiveness is considerably higher for direct hedging than cross-hedging. The difference in dependency between portfolios leads to a more considerable difference in hedging performance than the difference is effectiveness caused by using different hedging models. This leads to the finding that having an adequate hedging asset (e.g. a highly correlated asset) is regarded more important than the choice of hedging model.

For most single product portfolios and hedging models, the hedging effectiveness decreases with the increase of confidence level. Moreover, the standard deviation of the effectiveness increases, showing the difficulty of estimating extreme quantiles.

No clear conclusion can be drawn on the best performing copula models. In terms of copula fit, the Student's t copula provides the highest log-likelihood for all portfolios, but this does not translate to superior effectiveness. It performs well for most portfolios, but its performance can be compared to the Gaussian and Gumbel copula models. The HS model provides the greatest risk reduction for most portfolios for most hedging objectives. When the HS model is outperformed by a copula model, this difference is never statistically significant (Table 41 and 42). This suggests that the HS model is well-able to estimate and hedge extreme quantiles. The caveat often found in this model is that it produces inaccurate estimates of extreme quantiles when relying on a small data set. The comparative advantage of the copula models over the HS model in the number of data points in this research is small, making it difficult to compete with the performance of the HS model.

A more detailed explanation of hedging effectiveness per portfolio can be found in Section C.3.

Table 16: Average hedging effectiveness of the single product portfolios

S&P 500	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	**81.45 (3.47)**	80.47 (2.65)	77.31 (12.45)	79.37 (4.20)	78.73 (5.81)	74.73 (11.43)	95.07 (2.34)	98.39 (1.77)
Clayton	78.98 (5.11)	76.37 (5.39)	72.07 (9.22)	72.57 (6.62)	70.64 (7.45)	68.08 (9.88)	91.48 (3.90)	96.81 (2.01)
Student's t	80.89 (3.17)	80.64 (2.74)	79.08 (9.07)	80.00 (3.79)	79.30 (5.26)	76.09 (9.56)	95.33 (2.00)	98.60 (1.40)
Gaussian	81.13 (2.68)	80.94 (3.75)	79.01 (8.27)	79.86 (3.74)	79.16 (5.18)	**76.87 (9.67)**	95.29 (1.98)	98.63 (1.42)
HS	81.35 (3.14)	**81.29 (3.32)**	**79.85 (6.80)**	**80.23 (3.82)**	**79.59 (5.11)**	75.79 (9.60)	**95.41 (2.08)**	**98.64 (1.39)**
FTSE 100	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	80.24 (4.58)	79.61 (4.94)	77.96 (6.48)	79.52 (4.59)	79.28 (5.07)	78.04 (5.17)	95.52 (1.70)	98.94 (0.51)
Clayton	77.48 (6.18)	76.23 (4.21)	75.76 (5.63)	75.43 (3.02)	75.28 (3.77)	73.64 (4.59)	93.85 (1.38)	98.11 (0.73)
Student's t	79.86 (4.65)	**80.39 (4.78)**	79.77 (4.85)	80.47 (3.87)	80.71 (3.99)	80.51 (4.23)	95.96 (1.33)	99.20 (0.39)
Gaussian	79.88 (4.48)	79.79 (4.76)	**80.94 (4.34)**	80.60 (3.551)	**80.92 (3.88)**	80.78 (4.13)	96.02 (1.30)	99.211 (0.41)
HS	**80.37 (4.50)**	80.36 (4.58)	80.82 (4.11)	**80.68 (3.67)**	80.81 (3.67)	**81.17 (3.47)**	**96.05 (1.30)**	**99.24 (0.34)**
DAX	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	79.74 (14.18)	79.84 (13.76)	**77.76 (12.14)**	79.77 (12.80)	**80.28 (12.07)**	78.26 (10.60)	94.50 (8.62)	98.03 (4.58)
Clayton	75.25 (15.52)	76.41 (13.46)	76.15 (11.08)	74.45 (13.11)	73.83 (12.23)	71.07 (11.23)	91.73 (8.44)	96.63 (4.30)
Student's t	81.12 (14.29)	**80.98 (14.09)**	78.63 (10.97)	79.93 (12.55)	79.81 (11.83)	78.30 (9.71)	94.48 (8.44)	98.04 (4.30)
Gaussian	78.51 (13.77)	77.97 (15.40)	76.61 (14.66)	79.56 (13.00)	79.60 (12.49)	**78.87 (11.58)**	94.25 (8.65)	98.02 (4.58)
HS	**81.24 (13.93)**	80.79 (14.15)	77.45 (13.09)	**80.49 (13.23)**	79.89 (12.83)	78.63 (11.59)	**94.54 (9.17)**	**98.10 (4.45)**
Nikkei 225	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	63.02 (29.46)	63.15 (28.95)	59.33 (30.70)	57.12 (33.44)	54.92 (32.86)	48.33 (30.98)	66.00 (39.11)	63.99 (43.22)
Clayton	56.72 (27.50)	56.26 (27.43)	55.23 (28.81)	51.80 (28.07)	50.21 (28.33)	47.64 (28.59)	65.41 (34.48)	65.12 (38.55)
Student's t	65.96 (25.19)	63.48 (28.75)	60.02 (29.92)	58.24 (32.45)	55.94 (32.27)	50.42 (31.59)	67.21 (37.50)	65.10 (41.23)
Gaussian	62.145 (30.13)	61.68 (31.72)	59.06 (29.77)	58.53 (31.41)	56.30 (31.50)	50.66 (30.05)	68.94 (34.46)	68.35 (37.59)
HS	**70.40 (23.70)**	**71.03 (22.15)**	**65.45 (22.57)**	**66.72 (20.77)**	**63.84 (22.85)**	**55.24 (23.92)**	**80.41 (21.07)**	**78.69 (28.60)**
NASDAQ	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	71.34 (21.68)	74.29 (11.13)	67.42 (21.12)	71.04 (12.06)	68.07 (15.33)	66.55 (13.29)	89.21 (11.51)	94.65 (9.48)
Clayton	71.36 (12.13)	72.73 (8.84)	60.71 (17.56)	65.85 (11.90)	62.87 (14.31)	57.91 (17.85)	85.33 (12.75)	91.39 (10.71)
Student's t	76.77 (6.20)	77.21 (5.28)	72.92 (9.91)	74.25 (6.91)	72.79 (7.59)	68.36 (10.55)	91.96 (5.09)	96.96 (3.13)
Gaussian	77.59 (6.67)	74.85 (8.14)	73.62 (10.18)	74.71 (9.709)	73.68 (9.34)	71.07 (9.05)	92.42 (6.39)	97.55 (2.86)
HS	**78.77 (1.892)**	**78.75 (2.26)**	**76.14 (6.34)**	**77.49 (2.99)**	**76.22 (4.21)**	**72.30 (7.45)**	**94.51 (1.55)**	**98.37 (1.01)**

Notes: This table reports the average hedging effectiveness of the different single product portfolios.

Table 17: Average hedging effectiveness of the single product portfolios

CAC 40	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	91.90 (3.04)	90.94 (4.26)	88.74 (1.83)	90.15 (2.92)	89.38 (2.83)	87.42 (4.58)	98.93 (0.63)	99.81 (0.19)
Clayton	88.72 (4.77)	88.74 (6.83)	81.23 (6.07)	85.23 (5.80)	83.81 (6.59)	81.33 (5.85)	97.21 (2.61)	98.98 (1.62)
Student's t	91.83 (2.77)	92.07 (2.90)	89.55 (2.52)	90.85 (2.51)	90.11 (2.43)	87.69 (3.90)	99.04 (0.55)	**99.84 (0.14)**
Normal	91.66 (2.64)	91.67 (3.09)	88.74 (3.71)	89.82 (3.70)	88.75 (4.26)	85.86 (6.04)	98.73 (0.98)	99.73 (0.33)
HS	**92.30 (2.84)**	**92.42 (2.64)**	**90.15 (3.01)**	**91.04 (2.65)**	**90.18 (2.56)**	**87.91 (3.82)**	**99.08 (0.57)**	99.84 (0.15)
Dow Jones	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	66.66 (43.33)	76.40 (11.26)	67.60 (24.93)	**70.26 (24.45)**	**70.91 (21.81)**	**66.31 (25.09)**	**85.17 (28.55)**	**89.38 (28.53)**
Clayton	66.73 (24.79)	69.74 (22.66)	68.37 (19.45)	63.53 (23.54)	62.33 (24.74)	60.81 (15.75)	81.16 (25.91)	87.14 (26.21)
Student's t	**73.48 (23.41)**	**76.05 (9.41)**	**71.25 (22.97)**	69.92 (25.04)	68.96 (25.68)	65.85 (25.86)	84.80 (28.22)	87.33 (31.04)
Gaussian	64.95 (38.72)	68.22 (32.32)	67.30 (27.49)	68.26 (27.03)	68.81 (24.82)	66.22 (28.94)	84.03 (28.45)	89.00 (27.77)
HS	69.82 (27.69)	68.83 (29.84)	70.72 (24.25)	68.15 (28.39)	67.30 (28.65)	64.20 (29.76)	83.52 (28.20)	86.75 (27.37)
Hang Seng	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	71.42 (5.54)	72.14 (5.15)	72.78 (5.22)	73.04 (3.52)	72.87 (3.40)	71.35 (4.58)	92.41 (1.75)	97.55 (0.79)
Clayton	72.31 (3.89)	67.58 (8.89)	61.74 (6.15)	65.69 (5.07)	63.02 (6.75)	58.31 (7.89)	86.35 (6.65)	91.94 (8.18)
Student's t	71.70 (6.27)	**72.93 (5.57)**	72.37 (5.16)	**73.46 (4.51)**	**73.69 (4.09)**	**72.83 (5.73)**	**92.73 (2.14)**	**97.96 (0.91)**
Gaussian	72.10 (5.13)	70.92 (5.87)	71.89 (5.77)	73.20 (3.92)	73.31 (4.19)	72.18 (4.49)	92.45 (2.11)	97.80 (0.93)
HS	**73.14 (4.76)**	72.28 (5.42)	**73.34 (5.38)**	73.54 (4.03)	73.53 (4.15)	72.14 (4.62)	92.59 (2.17)	97.88 (0.91)
Euro Stoxx 50	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	75.25 (19.02)	75.55 (18.74)	77.56 (12.71)	76.81 (15.41)	76.50 (15.50)	76.11 (15.30)	92.37 (10.58)	96.87 (6.26)
Clayton	74.50 (18.11)	68.15 (18.02)	63.21 (27.19)	69.55 (13.68)	67.28 (14.37)	60.70 (27.82)	87.55 (13.32)	88.79 (23.71)
Student's t	75.48 (19.34)	74.48 (19.96)	76.97 (11.97)	77.62 (14.95)	77.69 (13.46)	77.90 (12.70)	93.12 (9.85)	97.36 (5.04)
Gaussian	74.94 (18.08)	**77.03 (18.65)**	79.21 (10.96)	77.15 (14.67)	77.78 (13.14)	**79.17 (12.04)**	92.96 (9.52)	**97.51 (4.98**
HS	**75.86 (19.04)**	76.62 (18.44)	**79.67 (11.31)**	**77.77 (14.73)**	**78.06 (13.08)**	78.72 (12.73)	**93.27 (9.83)**	97.50 (5.21)
AEX	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	90.32 (3.64)	89.50 (3.76)	86.24 (5.46)	87.79 (4.32)	86.72 (5.26)	84.30 (8.62)	98.00 (1.91)	98.98 (2.55)
Clayton	88.56 (5.38)	85.13 (6.06)	82.23 (7.42)	83.16 (6.00)	81.42 (6.09)	75.78 (10.25)	96.29 (2.52)	98.32 (1.85)
Student's t	90.56 (3.52)	90.15 (3.47)	87.22 (3.93)	**88.65 (3.92)**	87.37 (4.65)	85.44 (8.62)	98.23 (1.84)	99.13 (2.12)
Gaussian	**90.77 (3.24)**	89.76 (3.69)	87.19 (3.84)	88.34 (3.93)	87.14 (4.36)	85.24 (7.95)	98.23 (1.79)	99.12 (2.22)
HS	90.42 (3.72)	**90.57 (3.64)**	**88.27 (3.84)**	89.57 (4.02)	**88.36 (4.75)**	**86.77 (8.16)**	**98.47 (1.87)**	**99.14 (2.36)**

Notes: This table reports the average hedging effectiveness of the different single product portfolios.

5 Conclusion

In this study, a copula-based single- and multi-product hedging strategy is proposed to reduce investors' downside risk. We compare the copula hedging strategies with the traditional non-parametric HS approach introduced by Harris & Shen (2006). In the multi-product setting, vine copula models are introduced, which are flexible multi-variate copula models consisting of a group of unconditional and unconditional bivariate copula models (Joe, 1996). Additionally, this paper assesses sparse vine copula models, which reduce the computing time and the risk of overfitting with respect to non-sparse vine copula models. The models are evaluated in terms of hedging effectiveness, defined as the risk reduction relative to the unhedged portfolio. We compare the performance against parametric benchmark models (Gaussian, Student's t, Clayton, and Gumbel copula model) and a non-parametric benchmark model.

We assess the performance of the hedging strategy for major world stock indices. The multi-product portfolio consists of three US indices (*S&P* 500, *NASDAQ*, *Dow Jones*) with their index futures, and the position in the index future is estimated as OHR. Additionally, we construct ten single-product portfolios, where each spot index is directly hedged by its index future. We estimate the position in the index future as OHR.

The main findings are as follows. The D-vine copula model provides the highest average fit to the data in the multi-product setting. The D-vine copula model is ordered such that each spot index directly to its index future, leading to a high model fit. A model with a similar high fit is the Student's t copula model, caused by its ability to capture dependencies in both normal and extreme market conditions.

Focusing on the hedging effectiveness, the Student's t copula model yields the most considerable risk reduction. As the number of parameters in both vine copula models is higher than in the Student's t copula model, the vine copula models have a higher risk of overfitting. There are signs that the D-vine copula model is overfitting the in-sample, eventually resulting in poorer out-of-sample performance than the Student's t copula model. Although all copula models perform competitive to the HS benchmark, no copula model can out perform this benchmark.

The best performing copula hedging strategy is different for the EU portfolio, included as a robustness check. The D-vine copula model yields the greatest risk reduction among all copula models on most hedging objectives. The EU portfolio's dependence structure is more stable than the US portfolio's dependence structure. Therefore, the D-vine copula model does not overfit the in-sample.

The dependence in high-order trees in the C-vine copula model is substantial. Therefore, the C-vine copula model is not truncated in most estimation windows, and the performance equals non-sparse C-vine performance. In contrast, the D-vine model is truncated in most estimation windows due to its structure. As a result, the sparse D-vine model's computing time is lower than the non-sparse model. Truncating the D-vine model reduces the hedging effectiveness, although it still yields a competitive fit. Therefore, the investor should make a trade-off between computing time and hedging effectiveness.

Generally, the difference in performance between hedging models is relatively small. The hedging effectiveness is a positive function of the dependence and is highest for highly correlated assets. The single-product portfolios show that a strong correlation between the index and index future is more important than selecting the right hedging model. The Student's t copula model consistently achieves a competitive risk reduction in both single- and multi-product setting. The results suggest that the Student's t copula model is a safe choice as a hedging model as it has a smaller risk of overfitting than the vine copula models, is efficient to estimate and produces a competitive risk reduction.

A critical note is that the model is only re-estimated eleven times due to computational constraints. The initial goal was to replicate the estimation procedure of Sukcharoen & Leatham (2017) who re-estimate the copula structure every week, resulting in 1,123 test windows. Additionally, the number of simulated observations in the HS model is only 1,000, which is small compared to Sukcharoen & Leatham (2017), where 10,000 observations are simulated. Re-estimating the model more often and increasing the number of simulated observations make the results more robust to the estimation window choice and enhance the copula model performance relative to the HS model.

Future researches could examine different approaches towards sparse vine models. In the current setting, the bivariate pairs in trees higher than a truncation level K are modelled by the independence copula. However, our research shows that the Student's t copula model often provides a good fit. Therefore, it might be interesting to see the hedging effectiveness if the Student's t copula jointly models all high order trees. Additionally, one can decrease the prior probability of having a non-independent copula model to enforce sparser vine copula models. Finally, we suggest conducting a copula model fit based on the tail mean squared error since the copulas have to yield a good fit in the lower tail of the distribution.

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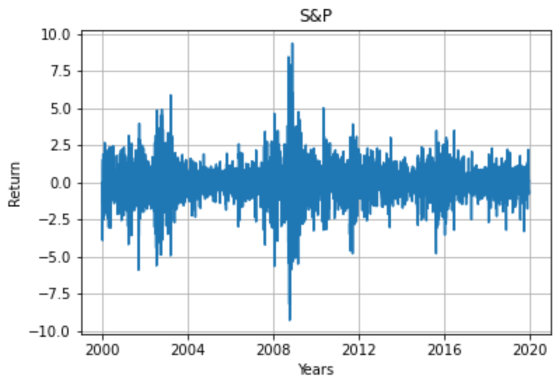
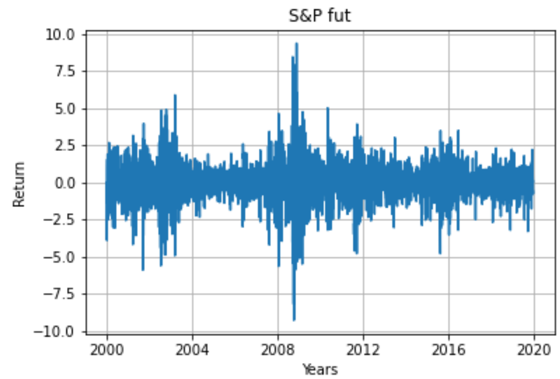
Appendix

A Data

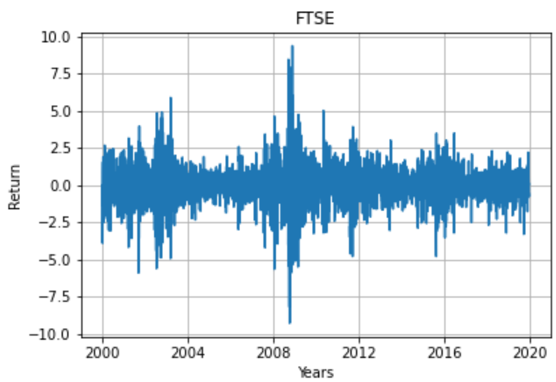
A.1 Spot and Future Indices



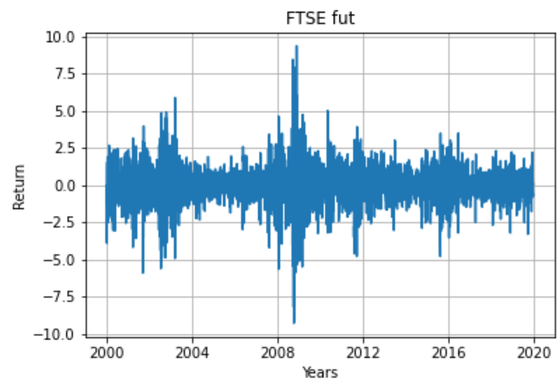
(a) S&P 500



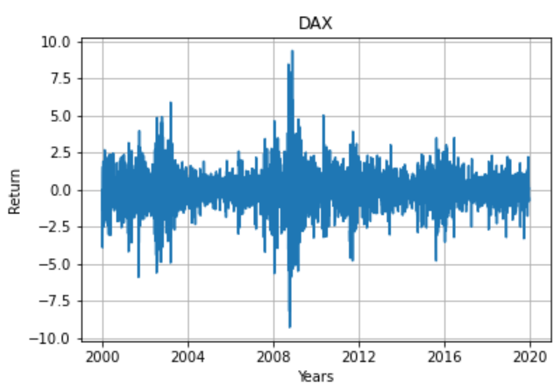
(b) S&P 500 Futures



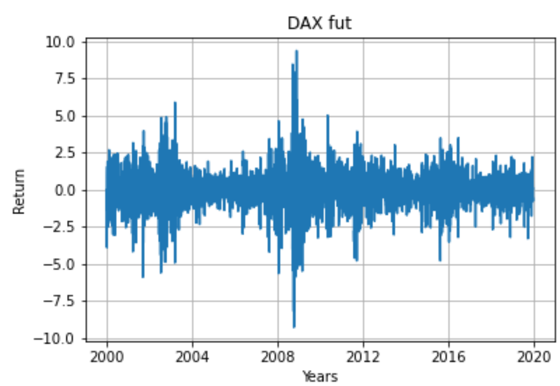
(c) FTSE 100



(d) FTSE 100 Futures

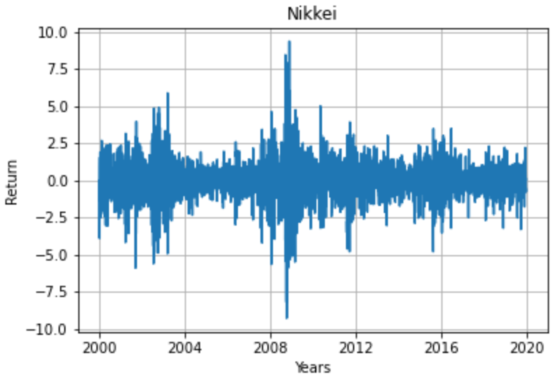


(e) DAX

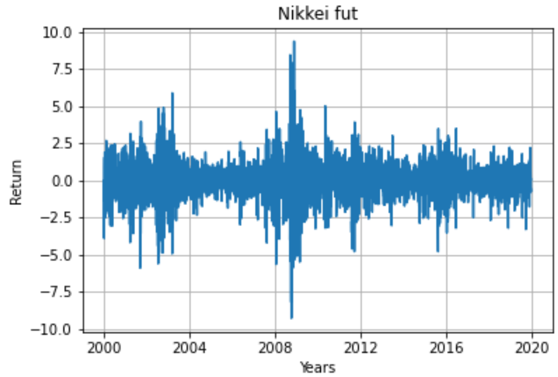


(f) DAX Futures

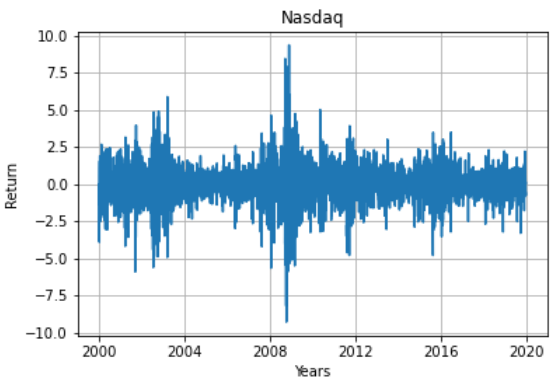
Figure 8: Time series plot of the spot and future indices ranging from 2000-2020.



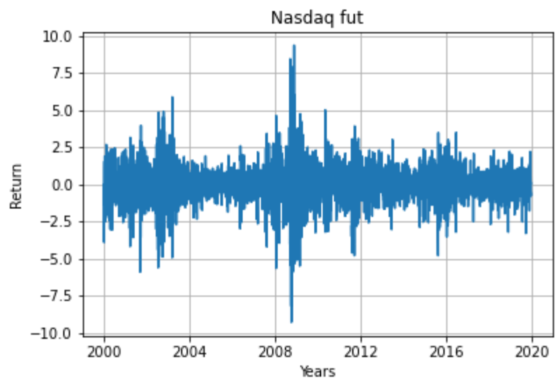
(a) Nikkei 225



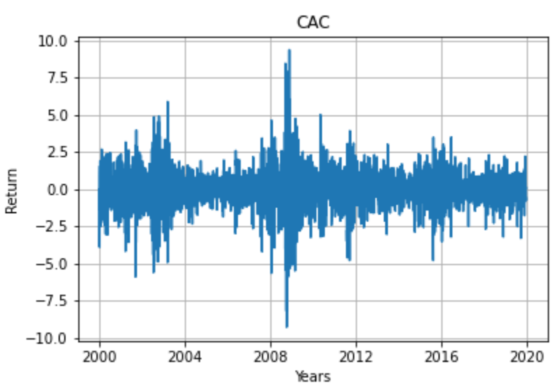
(b) Nikkei 225 Futures



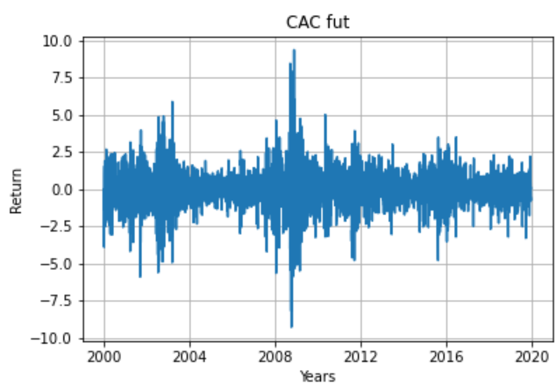
(c) NASDAQ



(d) NASDAQ Futures

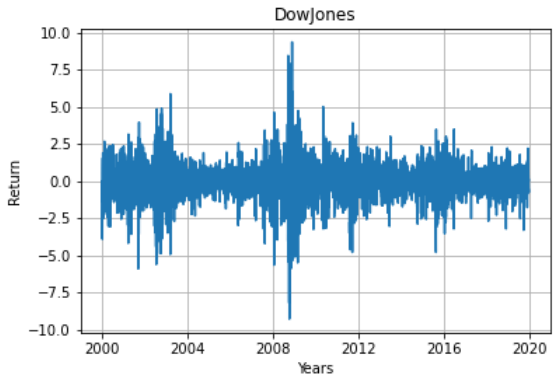


(e) CAC 40

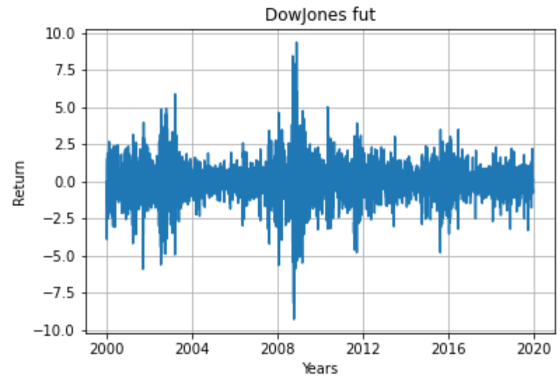


(f) CAC 40 Futures

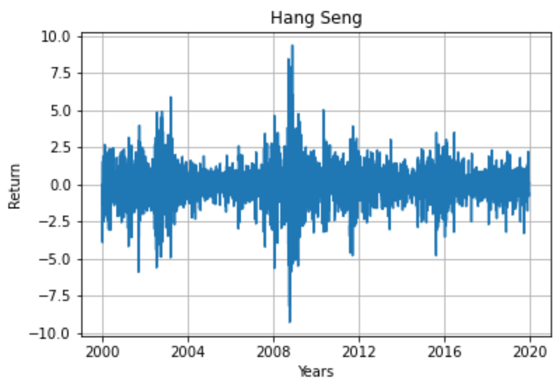
Figure 9: Time series plot of the spot and future indices ranging from 2000-2020.



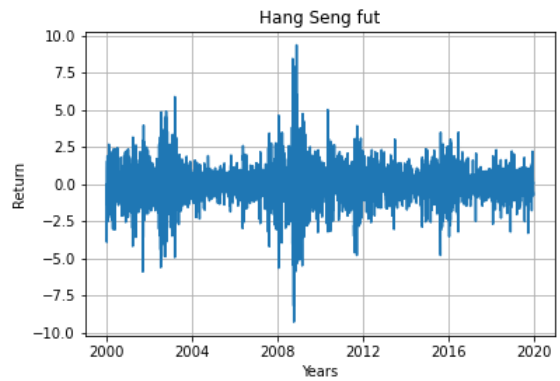
(a) Dow Jones



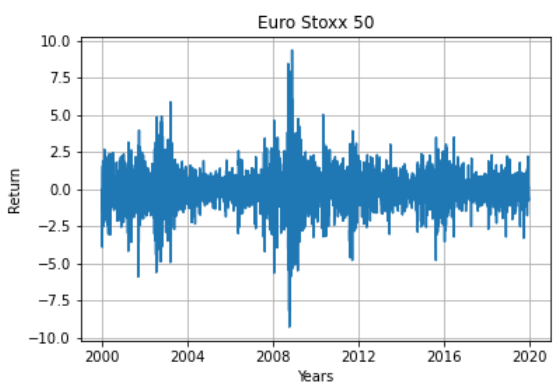
(b) Dow Jones Futures



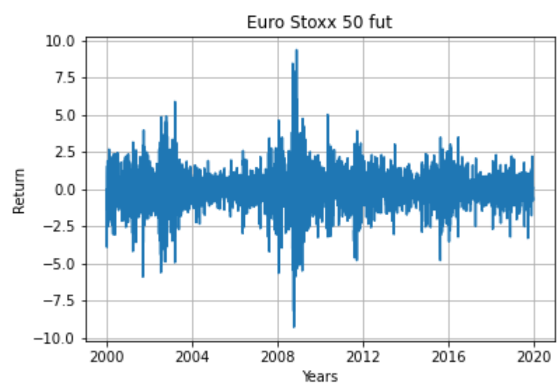
(c) Hang Seng



(d) Hang Seng Futures

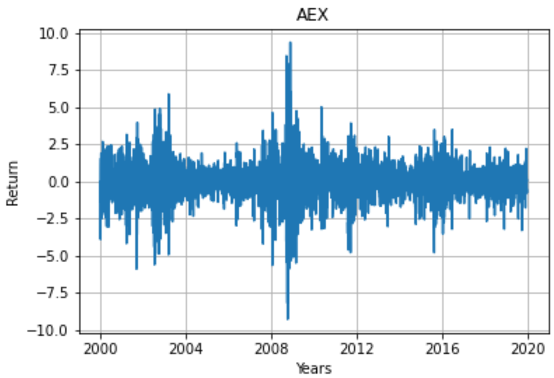


(e) Euro Stoxx 50

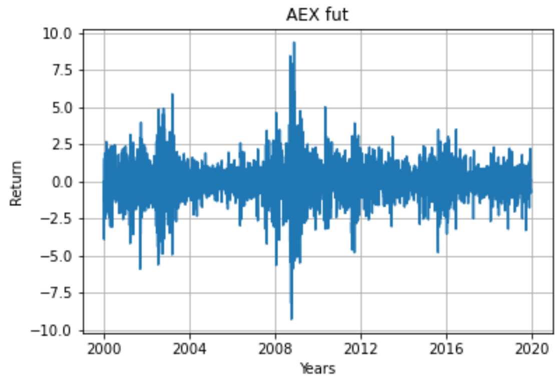


(f) Euro Stoxx 50 Futures

Figure 10: Time series plot of the spot and future indices ranging from 2000-2020.



(a) AEX



(b) AEX Futures

Figure 11: Time series plot of the spot and future indices ranging from 2000-2020.

A.2 Modelling Marginals

Table 18: Residual statistics

	LB statistic	LB^2 *statistic*
$S\&P$ 500_s	24.72 (0.42)	25.10 (0.40)
$FTSE$ 100_s	24.26 (0.45)	35.16 (0.07)
DAX_s	22.35 (0.56)	34.74 (0.07)
$Nikkei$ 225_s	20.04 (0.69)	25.02 (0.40)
$NASDAQ_s$	21.15 (0.63)	29.35 (0.21)
CAC 40_s	24.46 (0.44)	33.92 (0.09)
$Dow\ Jones_s$	23.45 (0.49)	21.38 (0.62)
$Hang\ Seng_s$	38.50 (0.03)	25.12 (0.40)
$Euro\ Stoxx$ 50_s	25.87 (0.36)	34.96 (0.07)
AEX_s	21.17 (0.63)	37.56 (0.04)
$S\&P$ 500_f	23.67 (0.48)	23.53 (0.49)
$FTSE$ 100_f	22.35 (0.56)	36.25 (0.05)
DAX_f	24.67 (0.42)	31.89 (0.13)
$Nikkei$ 225_f	17.88 (0.81)	43.14 (0.01)
$NASDAQ_f$	19.46 (0.73)	28.32 (0.25)
CAC 40_f	24.23 (0.45)	32.91 (0.11)
$Dow\ Jones_f$	23.97 (0.46)	23.98 (0.46)
$Hang\ Seng_f$	30.52 (0.17)	13.32 (0.96)
$Euro\ Stoxx$ 50_f	27.26 (0.29)	29.02 (0.22)
AEX_f	22.00 (0.58)	35.05 (0.07)

Notes: This table reports the LB statistic of the standardized AR(1)-GARCH(1,1) residuals, where s denotes a spot index and f a futures index. The LB statistic is the Ljung-Box statistic up to 24th order serial correlations in returns and squared returns. The values in parentheses display the p-value of the different statistics respectively.

A.3 Robustness Checks

Table 19: Robustness checks

Model Specification	Explanation
Gaussian residuals	The residuals of the AR(1)-GARCH(1,1) are modelled by the Gaussian distribution instead of by the skewed Student's t distribution.
Student's t residuals	The residuals of the AR(1)-GARCH(1,1) are modelled by the Student's t distribution instead of the skewed Student's t distribution.
Constant mean model	The AR(1) specification of the base model is replaced by the AR(0) specification, resulting in a AR(0)-GARCH(1,1) model (constant mean).
Semi-Parametric (SP) approach	The standardized residuals from the AR(1)-GARCH(1,1) model are transformed to copula data by the empirical distribution function instead of using the skewed Student's t distribution.
Non-Parametric (NP) marginals	The marginals of the copulas are constructed by using the empirical distribution function of the data instead of employing the copula-GARCH specification in the base model.
Shifted model	All estimation periods are shifted by 1 year compared to the base model. This leads to the windows presented in Table 25.
Increased re-estimations	The models are re-estimated every half year instead of every one and a half year, resulting in more estimation and out-of-sample windows.
Decreased window sizes	The in- and out-of-sample window sizes are smaller than in the base model. The in- and out-sample periods are equal to 2 and 0.5 years, respectively.
Increased simulations	We simulate more observations from the copula models relative to the base model.
EU portfolio	The base model is re-estimated for a portfolio that is based on the EU indices (*DAX, CAC 40, Euro Stoxx 50, AEX)*.

Notes: This table describes the robustness checks included in the research.

A.4 Pair Plots

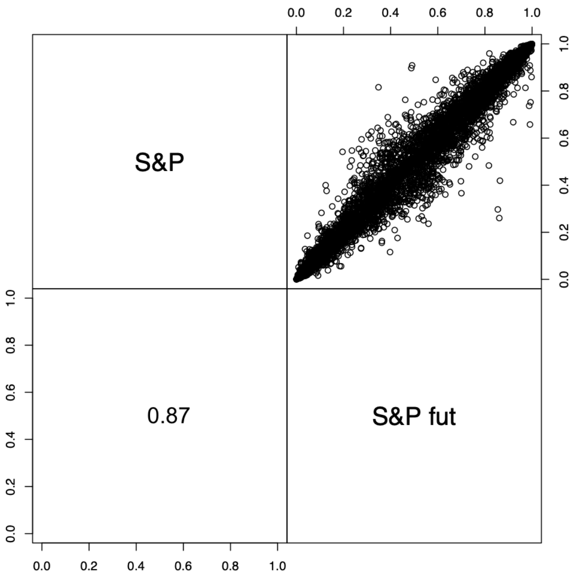


Figure 12: Pair plots of transformed standardized residuals and the corresponding estimated Kendall's τ of the S&P 500 spot and future index

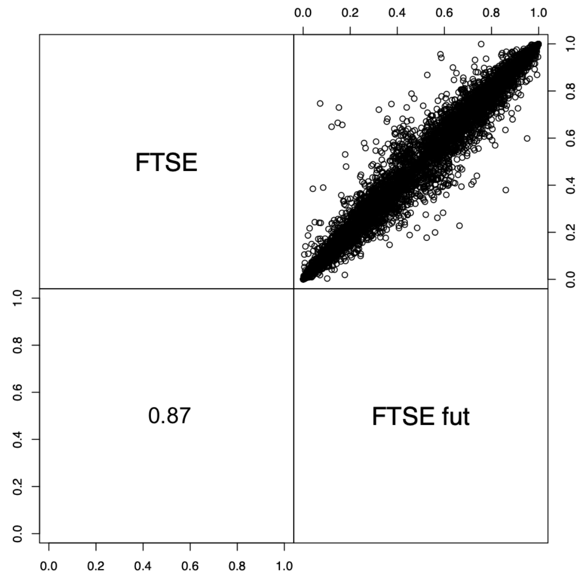


Figure 13: Pair plots of transformed standardized residuals and the corresponding estimated Kendall's τ of the FTSE 100 spot and future index

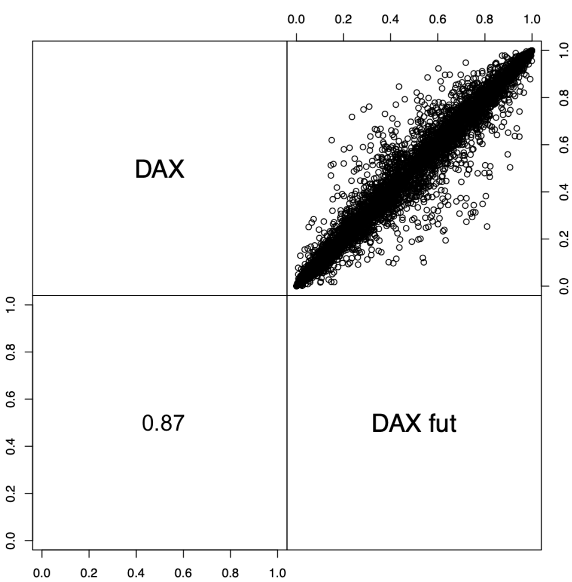


Figure 14: Pair plots of transformed standardized residuals and the corresponding estimated Kendall's τ of the DAX spot and future index

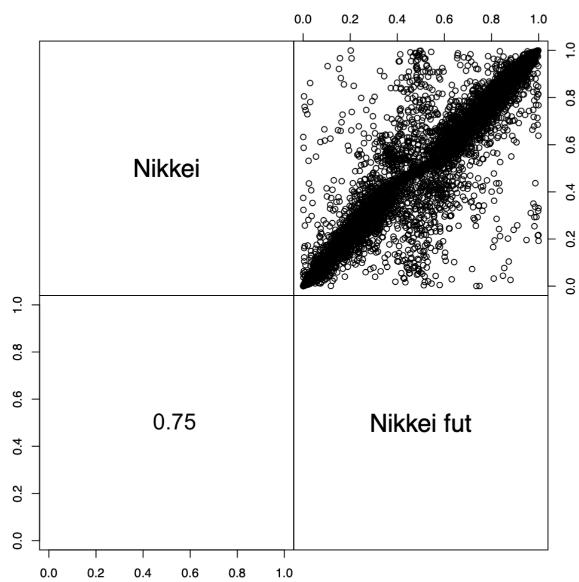


Figure 15: Pair plots of transformed standardized residuals and the corresponding estimated Kendall's τ of the Nikkei 225 spot and future index

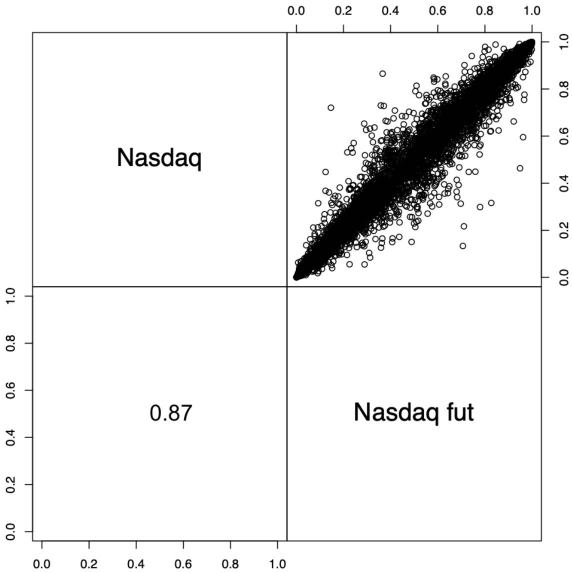


Figure 16: Pair plots of transformed standardized residuals and the corresponding estimated Kendall's τ of the NASDAQ spot and future index

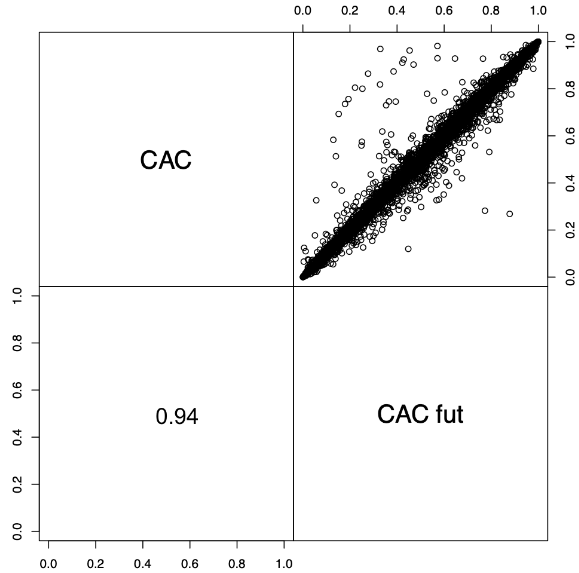


Figure 17: Pair plots of transformed standardized residuals and the corresponding estimated Kendall's τ of the CAC 40 spot and future index

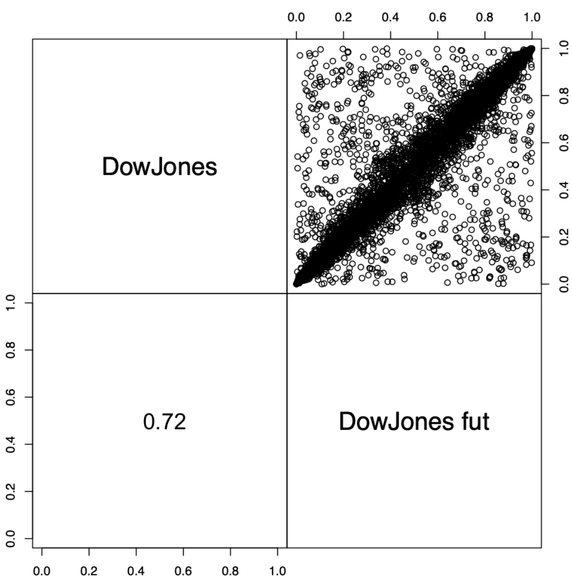


Figure 18: Pair plots of transformed standardized residuals and the corresponding estimated Kendall's τ of the Dow Jones spot and future index

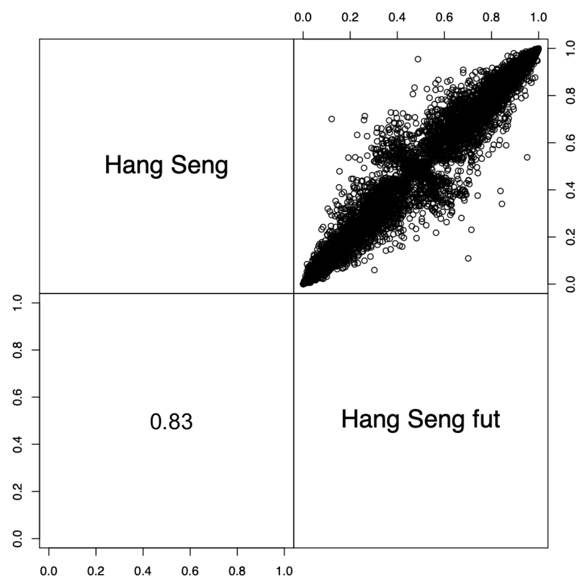


Figure 19: Pair plots of transformed standardized residuals and the corresponding estimated Kendall's τ of the Hang Seng spot and future index

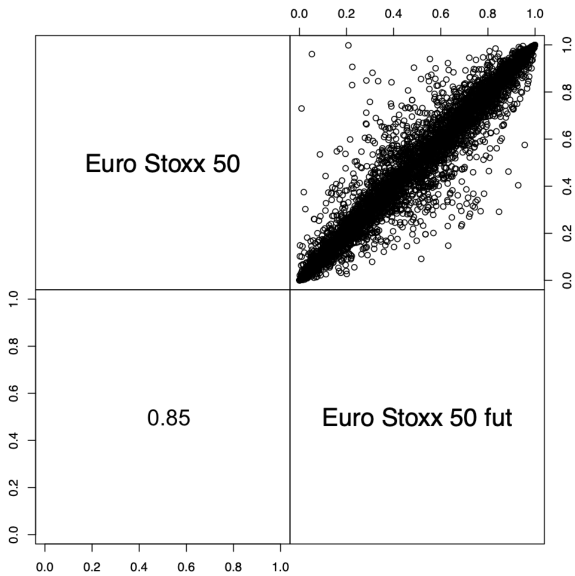


Figure 20: Pair plots of transformed standardized residuals and the corresponding estimated Kendall's τ of the Euro Stoxx 50 spot and future index

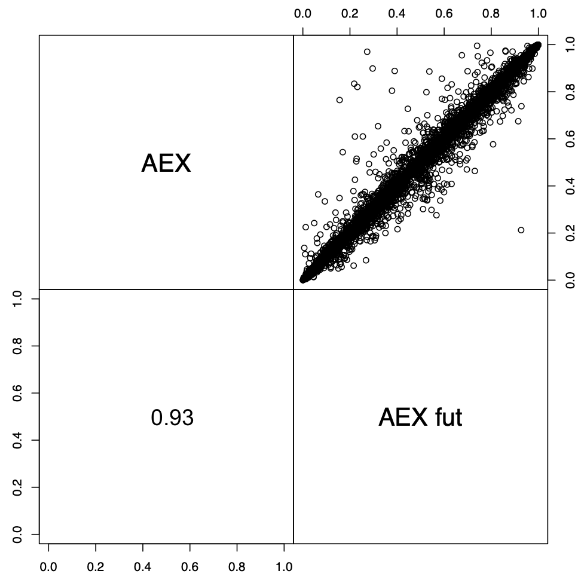


Figure 21: Pair plots of transformed standardized residuals and the corresponding estimated Kendall's τ of the AEX spot and future index.

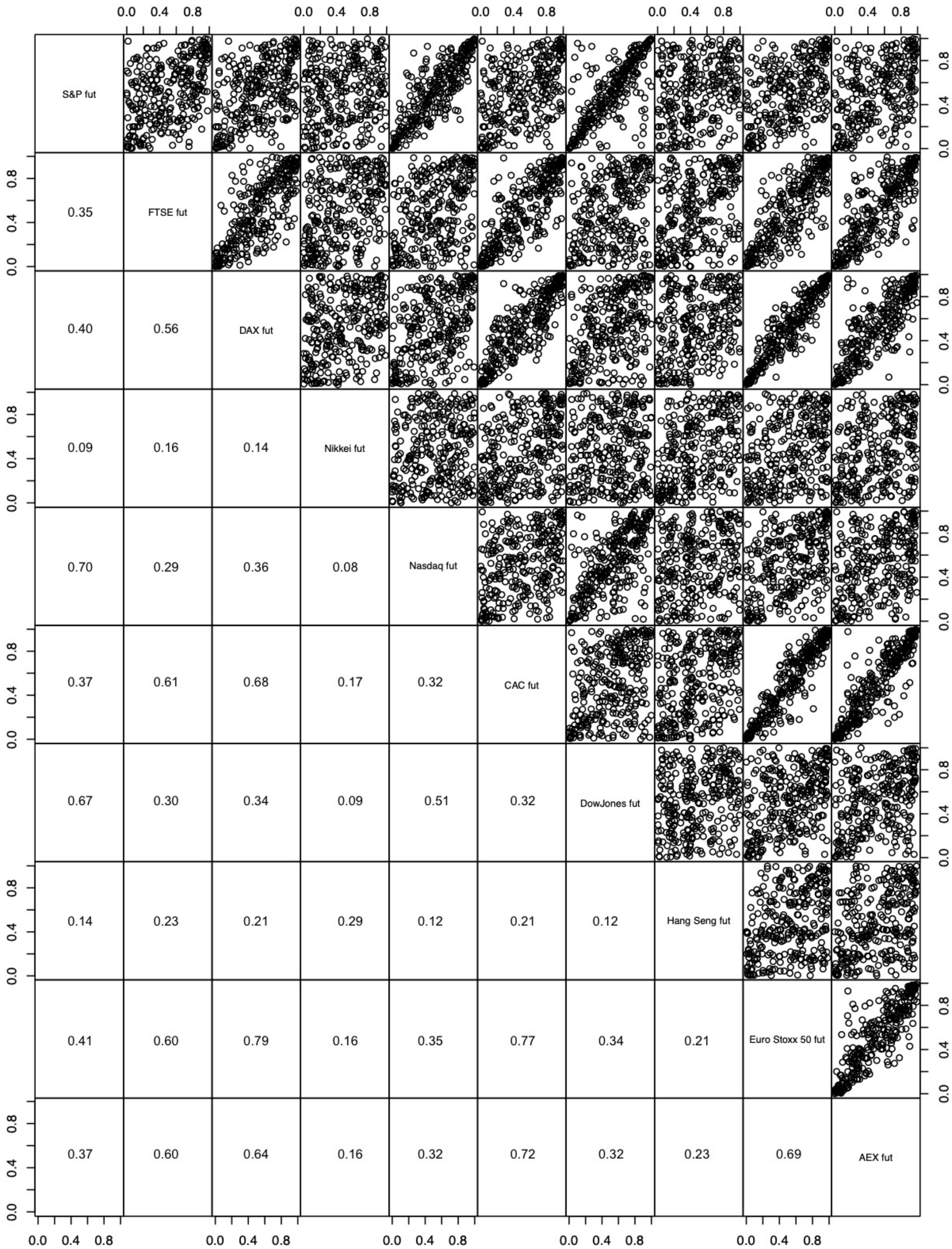


Figure 22: Pair plot with the empirical Kendall's tau estimates of the transformed standardized residuals of index futures in the lower triangle and the scatter plot of the transformed standardized residuals of the index futures in the upper triangle.

A.5 Tail Dependence over Time

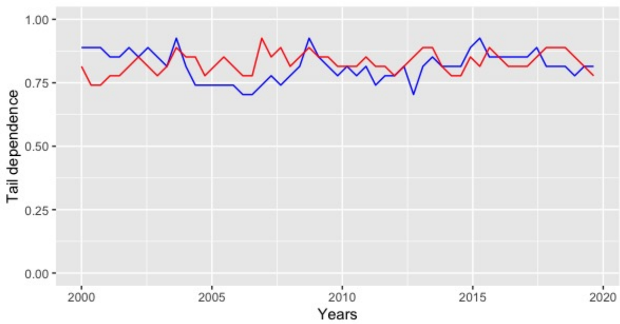


Figure 23: Tail dependence between the *S&P* 500 spot and future index, where the blue line presents the upper tail dependence and red line lower tail dependence.

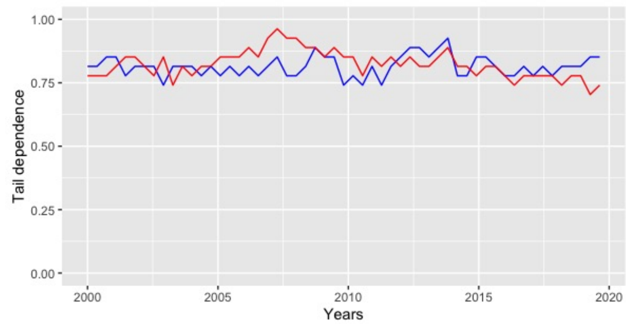


Figure 24: Tail dependence between the *FTSE* 100 spot and future index, where the blue line presents the upper tail dependence and red line lower tail dependence.

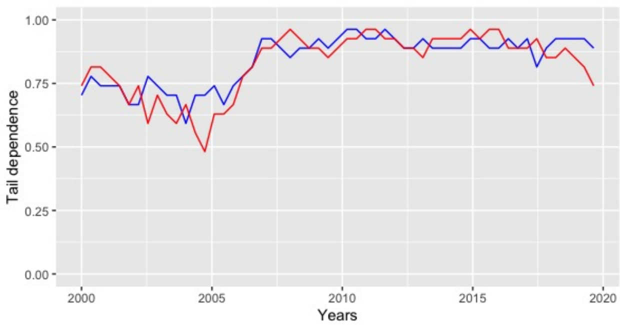


Figure 25: Tail dependence between the *DAX* spot and future index, where the blue line presents the upper tail dependence and red line lower tail dependence.

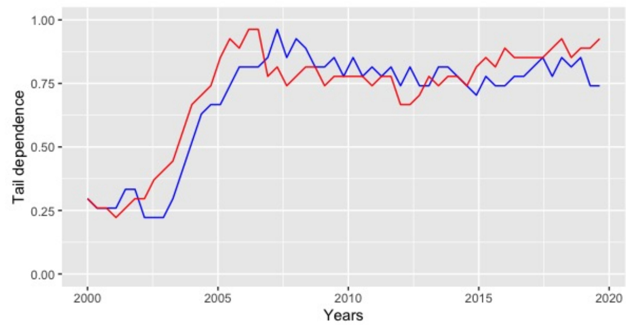


Figure 26: Tail dependence between the *Nikkei* 225 spot and future index, where the blue line presents the upper tail dependence and red line lower tail dependence.

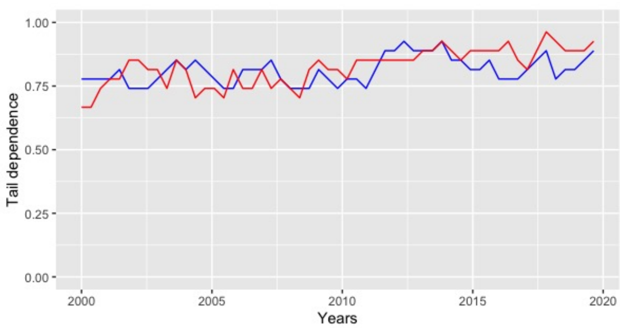


Figure 27: Tail dependence between the *NASDAQ* spot and future index, where the blue line presents the upper tail dependence and red line lower tail dependence.

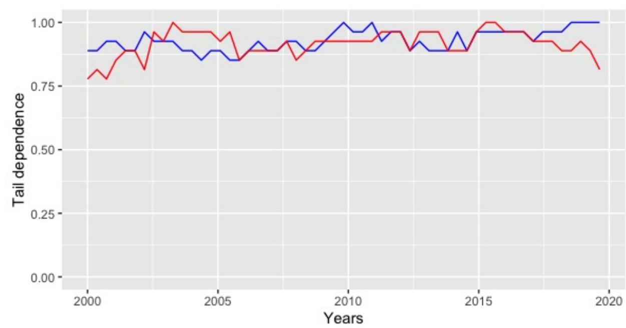


Figure 28: Tail dependence between the *CAC* 40 spot and future index, where the blue line presents the upper tail dependence and red line lower tail dependence.

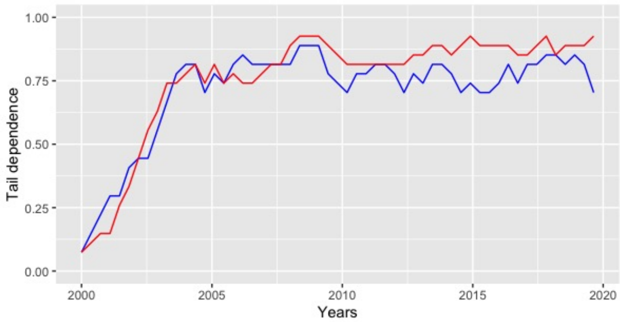


Figure 29: Tail dependence between the *Dow Jones* spot and future index, where the blue line presents the upper tail dependence and red line lower tail dependence.

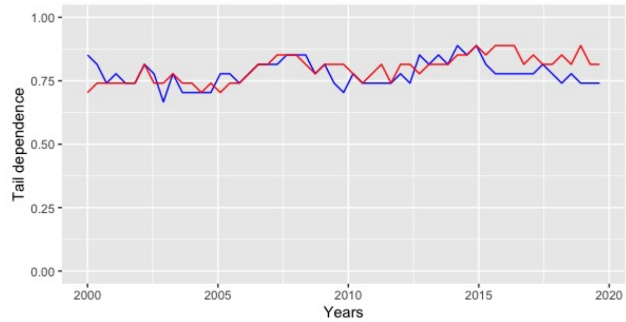


Figure 30: Tail dependence between the *Hang Seng* spot and future index, where the blue line presents the upper tail dependence and red line lower tail dependence.

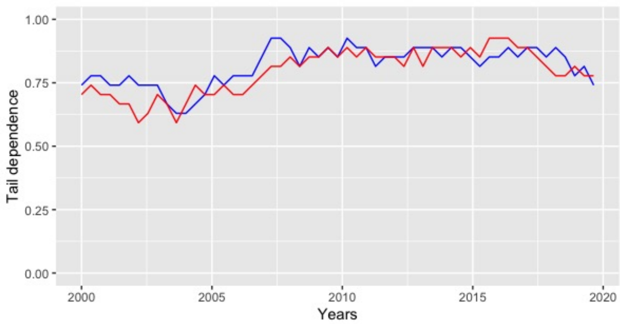


Figure 31: Tail dependence between the *Euro Stoxx* 50 spot and future index, where the blue line presents the upper tail dependence and red line lower tail dependence.

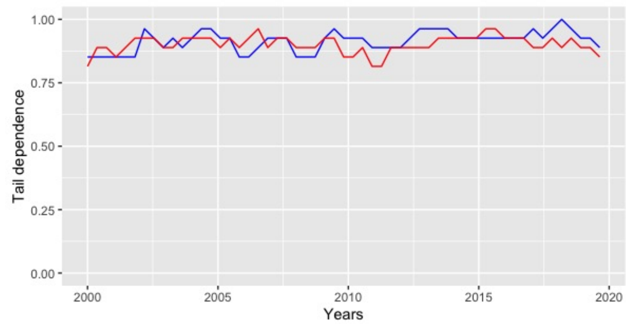


Figure 32: Tail dependence between the *AEX* spot and future index, where the blue line presents the upper tail dependence and red line lower tail dependence.

A.6 Data Characteristics EU Portfolio

The Kendall's tau estimates of the EU portfolio are displayed in Table 20. The average empirical estimates of Kendall's tau range between 0.66 and 0.94. The strongest correlation is found between the *CAC* 40 spot and the future index of 0.94. The highest cross-index correlation equals 0.86, corresponding to the Kendall's tau between the *Euro Stoxx* 50 and *CAC* 40 spot index. The weakest dependency can be observed between the *DAX* and *AEX* index futures with an average Kendall's tau of 0.66. The Kendall's tau estimates are on higher for the EU portfolio than the US portfolio indicating a stronger dependency between EU indices than US indices.

Table 21 and 22 show the lower and upper tail dependence respectively between the individual indices in the EU portfolio. The average lower and upper tail dependence range between 0.57 and 0.91 and 0.56 and 0.93 ,respectively. The highest lower and upper tail dependence is found between the *CAC* 40 spot and future index. Conversely, the lowest tail dependence can be observed between the *DAX* and *AEX* future indices.

The tail dependence shows a relatively symmetric pattern. The most considerable absolute difference between upper and lower tail dependence is equal to 0.05, corresponding to the pair CAC 40 and AEX spot. The most symmetric relation is presented between *Euro Stoxx* 50 spot and CAC 40 future series with the difference close to 0.

The varying dependence structures across different indices raise the need to explore vine copulas as they allow different dependence patterns. On average, the indices know a relatively high tail dependence. This suggests modelling the pairs with copulas that can capture tail dependence. This dependence is not symmetric across all indices, giving the opportunity to explore copulas that exhibit asymmetric dependence patterns.

Table 20: Kendall's tau estimates EU portfolio

	DAX_s	$CAC\ 40_s$	$Euro\ Stoxx\ 50_s$	AEX_s	DAX_f	$CAC\ 40_f$	$Euro\ Stoxx\ 50_f$	AEX_f
DAX_s	1.00 (0.00)	0.74 (0.05)	0.80 (0.04)	0.69 (0.04)	0.87 (0.07)	0.73 (0.05)	0.76 (0.05)	0.69 (0.05)
$CAC\ 40_s$	0.74 (0.05)	1.00 (0.00)	0.86 (0.02)	0.75 (0.03)	0.70 (0.07)	0.94 (0.02)	0.79 (0.07)	0.74 (0.04)
$EuroStoxx\ 50_s$	0.80 (0.04)	0.86 (0.02)	1.00 (0.00)	0.76 (0.02)	0.75 (0.06)	0.85 (0.02)	0.86 (0.08)	0.75 (0.02)
AEX_s	0.69 (0.04)	0.75 (0.03)	0.76 (0.02)	1.00 (0.00)	0.66 (0.06)	0.74 (0.03)	0.71 (0.05)	0.93 (0.02)
DAX_f	0.87 (0.07)	0.70 (0.07)	0.75 (0.06)	0.66 (0.06)	1.00 (0.00)	0.69 (0.06)	0.79 (0.04)	0.66 (0.06)
$CAC\ 40_f$	0.73 (0.05)	0.94 (0.02)	0.85 (0.02)	0.74 (0.03)	0.69 (0.06)	1.00 (0.00)	0.78 (0.07)	0.74 (0.04)
$Euro\ Stoxx\ 50_f$	0.76 (0.05)	0.79 (0.07)	0.86 (0.08)	0.71 (0.05)	0.79 (0.04)	0.78 (0.07)	1.00 (0.00)	0.71 (0.06)
AEX_f	0.69 (0.05)	0.74 (0.04)	0.75 (0.02)	0.93 (0.02)	0.66 (0.06)	0.74 (0.03)	0.71 (0.06)	1.00 (0.00)

Notes: This table reports the average empirical estimates of Kendall's tau for the EU portfolio across the different estimation windows. The standard deviation of the estimates are displayed in parentheses.

Table 21: Lower tail dependence EU portfolio

	DAX_s	$CAC\ 40_s$	$Euro\ Stoxx\ 50_s$	AEX_s	DAX_f	$CAC\ 40_f$	$Euro\ Stoxx\ 50_f$	AEX_f
DAX_s	1.00 (0.00)	0.69 (0.09)	0.73 (0.08)	0.61 (0.10)	0.83 (0.13)	0.67 (0.14)	0.68 (0.07)	0.59 (0.12)
$CAC\ 40_s$	0.69 (0.09)	1.00 (0.00)	0.83 (0.05)	0.72 (0.09)	0.64 (0.11)	0.91 (0.06)	0.73 (0.08)	0.69 (0.08)
$EuroStoxx\ 50_s$	0.73 (0.08)	0.83 (0.05)	1.00 (0.00)	0.72 (0.06)	0.68 (0.11)	0.82 (0.09)	0.81 (0.10)	0.70 (0.08)
AEX_s	0.61 (0.12)	0.72 (0.09)	0.72 (0.06)	1.00 (0.00)	0.59 (0.09)	0.71 (0.14)	0.66 (0.08)	0.91 (0.04)
DAX_f	0.83 (0.13)	0.64 (0.11)	0.68 (0.11)	0.59 (0.09)	1.00 (0.00)	0.64 (0.13)	0.71 (0.08)	0.57 (0.11)
$CAC\ 40_f$	0.67 (0.14)	0.91 (0.06)	0.82 (0.09)	0.71 (0.13)	0.64 (0.13)	1.00 (0.00)	0.73 (0.09)	0.69 (0.09)
$Euro\ Stoxx\ 50_f$	0.68 (0.07)	0.73 (0.08)	0.81 (0.10)	0.66 (0.08)	0.71 (0.08)	0.73 (0.09)	1.00 (0.00)	0.63 (0.09)
AEX_f	0.59 (0.12)	0.69 (0.08)	0.71 (0.08)	0.92 (0.04)	0.57 (0.11)	0.69 (0.09)	0.63 (0.09)	1.00 (0.00)

Notes: This table reports the empirical average estimates of the lower tail dependence of the EU portfolio. The standard deviation of the estimates are displayed in parentheses.

Table 22: Upper tail dependence EU portfolio

	DAX_s	$CAC\ 40_s$	$Euro\ Stoxx\ 50_s$	AEX_s	DAX_f	$CAC\ 40_f$	$Euro\ Stoxx\ 50_f$	AEX_f
DAX_s	1.00 (0.00)	0.65 (0.11)	0.72 (0.12)	0.61 (0.12)	0.83 (0.10)	0.64 (0.12)	0.73 (0.12)	0.61 (0.12)
$CAC\ 40_s$	0.65 (0.11)	1.00 (0.00)	0.81 (0.05)	0.68 (0.06)	0.61 (0.12)	0.93 (0.05)	0.73 (0.12)	0.66 (0.07)
$Euro\ Stoxx\ 50_s$	0.71 (0.12)	0.81 (0.05)	1.00 (0.00)	0.71 (0.04)	0.65 (0.12)	0.81 (0.04)	0.82 (0.08)	0.70 (0.05)
AEX_s	0.62 (0.11)	0.68 (0.06)	0.71 (0.04)	1.00 (0.00)	0.57 (0.11)	0.69 (0.06)	0.66 (0.06)	0.91 (0.03)
DAX_f	0.83 (0.10)	0.61 (0.12)	0.65 (0.12)	0.57 (0.11)	1.00 (0.00)	0.61 (0.12)	0.71 (0.09)	0.56 (0.10)
$CAC\ 40_f$	0.64 (0.12)	0.93 (0.05)	0.81 (0.04)	0.69 (0.06)	0.61 (0.12)	1.00 (0.00)	0.76 (0.12)	0.68 (0.05)
$Euro\ Stoxx\ 50_f$	0.71 (0.12)	0.73 (0.12)	0.82 (0.08)	0.66 (0.06)	0.70 (0.09)	0.76 (0.11)	1.00 (0.00)	0.64 (0.05)
AEX_f	0.60 (0.11)	0.66 (0.07)	0.72 (0.05)	0.91 (0.03)	0.56 (0.10)	0.68 (0.05)	0.64 (0.05)	1.00 (0.00)

Notes: This table reports the empirical average estimates of the upper tail dependence of the EU portfolio. The standard deviation of the estimates are displayed in parentheses.

B Models

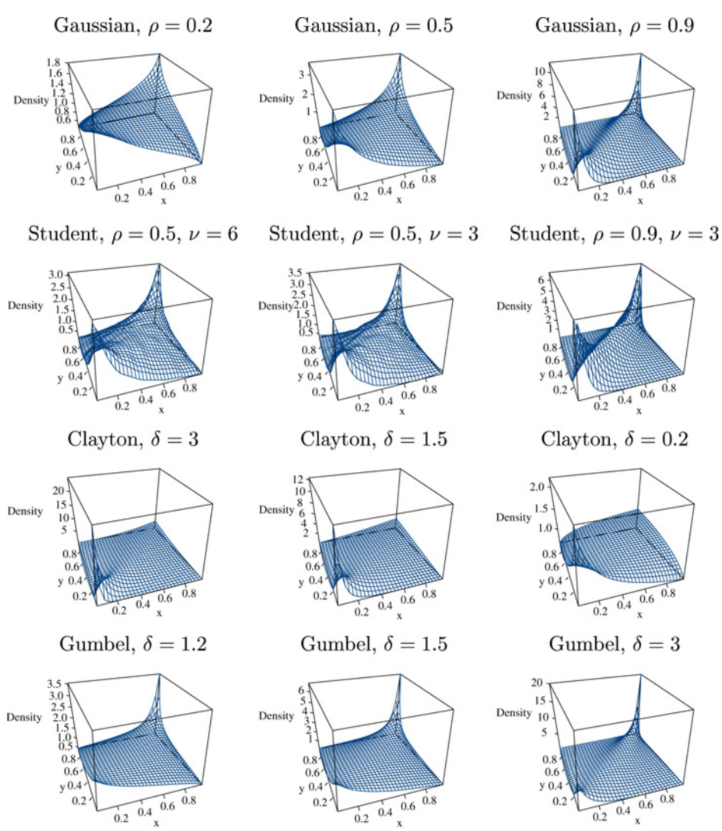


Figure 33: Surface plot of different copula models employed in this research with different parameter settings (Aas et al., 2007).

Table 23: Properties of bivariate elliptical and Archimedean copula families

Name	Parameter range	Kendall's τ	Tail dependence ($_{l},\ _{u}$)
Gaussian	$\rho \in (-1,1)$	$\frac{2}{\pi} arcsin(\rho)$	0
Student's t	$\rho \in (-1,1), \nu > 2$	$\frac{2}{\pi} arcsin(\rho)$	$2\mathrm{t}_{\nu+1}(-\sqrt{\nu+1}\sqrt{\frac{1-\rho}{1+\rho}})$
Clayton	$\theta > 0$	$\frac{\theta}{\theta+2}$	$(2^{-\frac{1}{\theta}}, 0)$
Gumbel	$\theta \geq 1$	$1 - \frac{1}{\theta}$	$(0, 2 - 2^{\frac{1}{\theta}})$
Frank	$\theta \in \mathbb{R} - \{0\}$	$1 - \frac{4}{\theta} + 4\frac{D_1(\theta)}{\theta}$	$(0, 0)$
Joe	$\theta > 1$	$1 + \frac{4}{\theta^2} \int_0^1 tlog(t)(1-t)^{2(1-\theta)/\theta} dt$	(0,2-$2^{\frac{1}{\theta}}$)
BB1	$\theta > 0, \delta \geq 1$	$1 - \frac{2}{\delta(\theta+2)}$	$(2^{-\frac{1}{\delta\theta}}, 2 - 2^{\frac{1}{\delta}})$
BB6	$\theta \geq 1, \delta \geq 1$	$1 + \frac{4}{\theta\delta} \int_0^1 (-\log\big(1 - (1-t)^{\theta}\big) \times (1-t)(1-(1-t)^{-\theta}))dt$	$(0, 2 - 2^{\frac{1}{\delta\theta}})$
BB7	$\theta \geq 1, \delta > 0$	$1 + \frac{4}{\theta\delta} \int_0^1 (-(1-(1-t)^{\theta})^{\delta+1} \times \frac{(1-(1-t)^{\theta})^{\delta}-1}{(1-t)^{\theta-1}})dt$	$(2^{-\frac{1}{\delta}}, 2 - 2^{\frac{1}{\delta}})$
BB8	$\theta \geq 1, \delta \in (0,1]$	$1 + \frac{4}{\theta\delta} \int_0^1 (-log(\frac{(1-t\delta)^{\theta})-1}{(1-\delta)^{\theta}-1}) \times (1-t\delta)(1-(1-t\delta)^{-\theta}))dt$	(0,0)

Notes: This table presents the different copula models considered in the estimation of the vine copula models with the parametric estimates of Kendall's tau and tail dependence.

Table 24: Copula model overview

1	Gaussian copula
2	Students' t copula
3	Clayton copula
4	Gumbel copula
5	Frank copula
6	Joe copula
7	BB1 copula
8	BB6 copula
9	BB7 copula
10	BB8 copula
11	Rotated Clayton 180 degrees
12	Rotated Gumbel 180 degrees
13	Rotated Joe 180 degrees
14	Rotated BB1 copula 180 degrees
15	Rotated BB6 copula 180 degrees
16	Rotated BB7 copula 180 degrees
17	Rotated BB8 copula 180 degrees
18	Rotated Clayton 90 degrees
19	Rotated Gumbel 90 degrees
20	Rotated Joe 90 degrees
21	Rotated BB1 copula 90 degrees
22	Rotated BB6 copula 90 degrees
23	Rotated BB7 copula 90 degrees
24	Rotated BB8 copula 90 degrees
25	Rotated Clayton 270 degrees
26	Rotated Gumbel 270 degrees
27	Rotated Joe 270 degrees
28	Rotated BB1 copula 270 degrees
29	Rotated BB6 copula 270 degrees
30	Rotated BB7 copula 270 degrees
31	Rotated BB8 copula 270 degrees

Notes: This table presents the different copula models that are considered in the estimation of the vine copula models.

Table 25: Estimation periods base and shifted model

	base model		shifted model	
	in	out	in	out
1	2000-2003	2003-2005	2001-2004	2004-2005
2	2001Q3-2004Q2	2004Q3-2005Q2	2002Q3-2005Q2	2005Q3-2006Q2
3	2003-2006	2006-2007	2004-2007	2007-2008
4	2004Q3-2007Q2	2007Q3-2008Q2	2005Q3-2008Q2	2008Q3-2009Q2
5	2006-2009	2009-2010	2007-2010	2010-2011
6	2007Q3-2010Q2	2010Q3-2011Q2	2008Q3-2011Q2	2011Q3-2012Q2
7	2009-2012	2012-2013	2010-2013	2013-2014
8	2010Q3-2013Q2	2013Q3-2014Q2	2011Q3-2014Q2	2014Q3-2015Q2
9	2012-2015	2015-2016	2013-2016	2016-2017
10	2013Q3-2016Q2	2016Q3-2017Q2	2014Q3-2017Q2	2015Q3-2018Q2
11	2015-2018	2018-2019	2016-2019	2019-2020

Notes: This table presents the years corresponding to the different estimation periods in the base and shifted model.

C Results

C.1 Copula Fit

Table 26: Copula model fit over time

	Gumbel	Clayton	T	Norm	Cvine	Dvine
2000-2003	908.10	948.01	3853.61	3795.86	3894.98	3938.78
2001Q3-2004Q2	2009.24	1652.11	4570.77	4515.38	4628.73	4681.00
2003-2006	3578.25	3006.66	5859.64	5482.65	5844.68	5909.70
2004Q3-2007Q2	3344.02	3103.85	5687.97	5482.93	5706.59	5719.62
2006-2009	3720.42	3514.44	6178.74	5880.17	6106.72	6171.01
2007Q3-2010Q2	4298.24	3867.75	6739.69	6605.89	6657.67	6731.71
2009-2012	4213.19	3885.94	6609.34	6426.77	6436.86	6593.97
2010Q3-2013Q2	3988.49	3615.09	6430.12	6250.39	6370.56	6439.54
2012-2015	3555.20	3239.64	6274.83	6059.92	6166.71	6296.95
2013Q3-2016Q2	3716.49	3448.95	6563.43	6098.55	6452.32	6583.00
2015-2018	3491.01	3258.64	6246.06	5749.64	6060.09	6251.53

Notes: This table presents the copula model log-likelihood over time for the US portfolio.

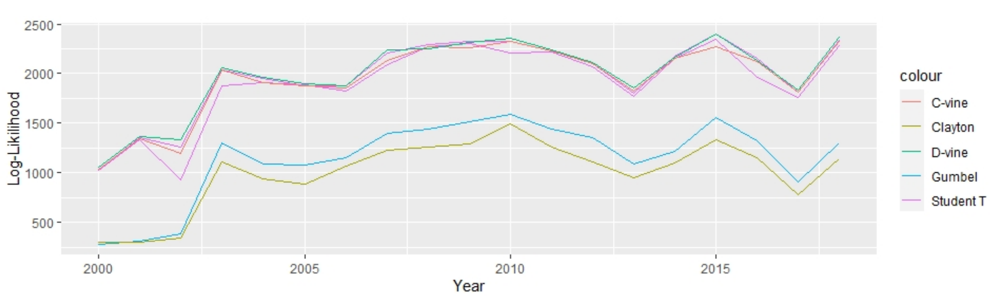


Figure 34: Log-likelihood of the different copula models over time for the US portfolio.

C-Vine Figure 35 presents the C-vine copula model trees in the tenth estimation window.

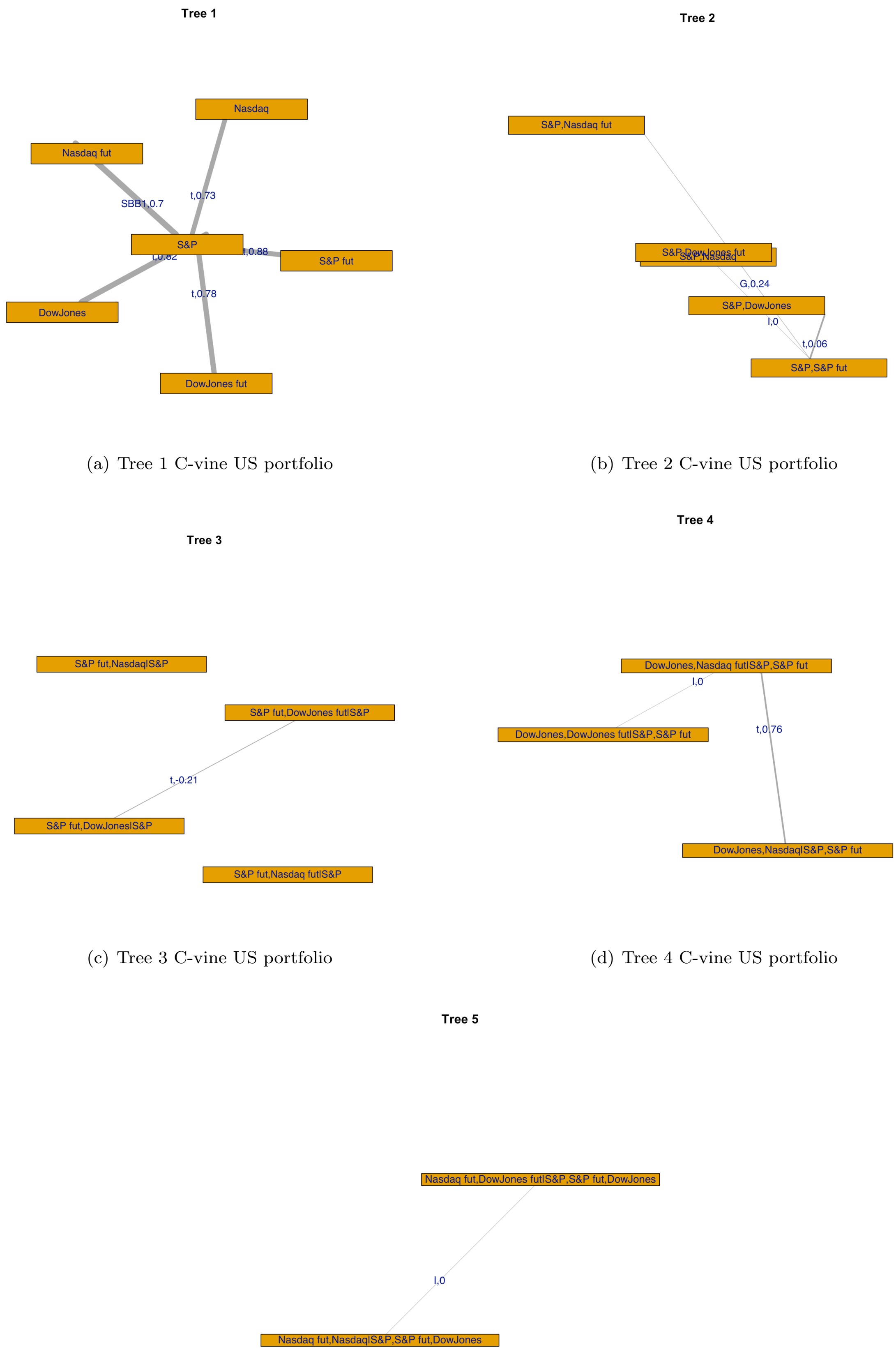
The first tree of the C-vine presents the dependence between the $S\&P\ 500_s$ and the other US indices and is modelled by five unconditional bivariate copulas. The dependence is in 4 out of 5 bivariate pairs modelled by the Student's t copula, indicating a symmetric tail dependence relation. Only the relation between the $NASDAQ_f$ and the $S\&P\ 500_s$ deviates as the rotated BB1 copula captures this relation. Characteristic for this copula is an asymmetric dependence structure with the upper and lower tail dependence being equal to $(2-2^{\frac{1}{\delta}}, 2^{\frac{1}{\delta}})$.

The second tree shows the dependence pattern between $S\&P\ 500_f$ and the remaining indices (without $S\&P\ 500_s$), described by conditional pair copulas where $S\&P\ 500_s$ is the conditioning variable. In the displayed estimation window, these relations are characterized by the Student's t ($S\&P\ 500_f$ and *Dow Jones*$_s$), Gumbel ($S\&P\ 500_f$ and $NASDAQ_f$), independence ($S\&P\ 500_f$ and $NASDAQ_s$), and rotated Gaussian ($S\&P\ 500_f$ and *Dow Jones*$_f$) copula.

The third tree suggests modelling the relation between *Dow Jones*$_s$ and $NASDAQ_s$, $NASDAQ_f$ and *Dow Jones*$_f$ by a Student's t copula model conditional on $S\&P\ 500_s$ and $S\&P\ 500_f$.

The fourth tree displays the relation between $NASDAQ_s$ and *Dow Jones*$_s$ conditional on $S\&P\ 500_f$, $S\&P\ 500_s$ and *Dow Jones*$_s$. In addition, the dependence of $NASDAQ_f$ and *Dow Jones*$_f$ is modelled conditional on $S\&P\ 500_f$, $S\&P\ 500_s$ and *Dow Jones*$_s$. The Frank copula describes the former relation and the independence copula the latter.

Finally, the last tree shows us the relation between the $NASDAQ_s$ and *Dow Jones*$_f$ conditional on all other US indices. This relation is captured by the independence copula.



(a) Tree 1 C-vine US portfolio

(b) Tree 2 C-vine US portfolio

(c) Tree 3 C-vine US portfolio

(d) Tree 4 C-vine US portfolio

(e) Tree 5 C-vine US portfolio

Figure 35: C-vine structure for the US portfolio the tenth estimation window.

D-Vine Figure 36 presents the D-vine trees in the third estimation window.

The first tree links the US indices with five unconditional bivariate copula pairs. The indices are linked such that the sum of Kendall's tau estimates is maximized, linking all spot indices to the corresponding index futures. Furthermore, each pair is modelled by a bivariate Students 't copula. This implies that the bivariate dependence relations are symmetric, with the upper tail dependence being equal to the lower tail dependence.

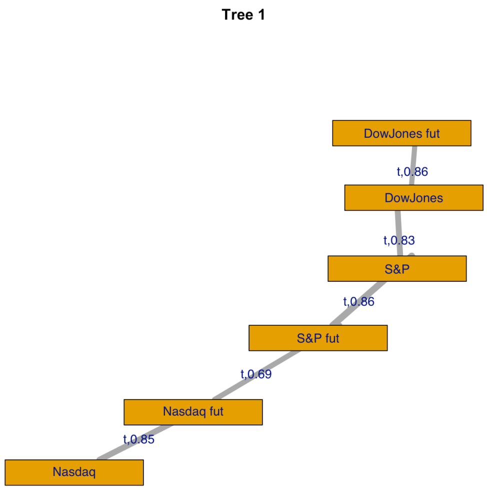
The second tree suggests to model the dependence structure $NASDAQ_s$ and $S\&P\ 500_f$ conditional on $NASDAQ_f$. The Frank copula describes this conditional bivariate copula pair. The relation $NASDAQ_f$ and $S\&P\ 500_s$ is estimated with a Gumbel copula, while both $S\&P\ 500_f$,$DowJones_s$ and $S\&P\ 500_s$, $DowJones_f$ are described by the independence copula.

The third three shows the relation between the conditional copula pair $NASDAQ_s$ and $S\&P\ 500_s$ conditional on $NASDAQ_f$ and $S\&P\ 500_f$, which is described by the Student's t copula. The relation between $NASDAQ_f$ and $DowJones_s$ conditional on $S\&P\ 500_s$ and $S\&P\ 500_f$ can be modelled through the Frank copula. Additionally, the tree indicates modelling $S\&P\ 500_f$ and $DowJones_f$ conditional on $S\&P\ 500_s$ and $DowJones_s$ by the Student's t copula.

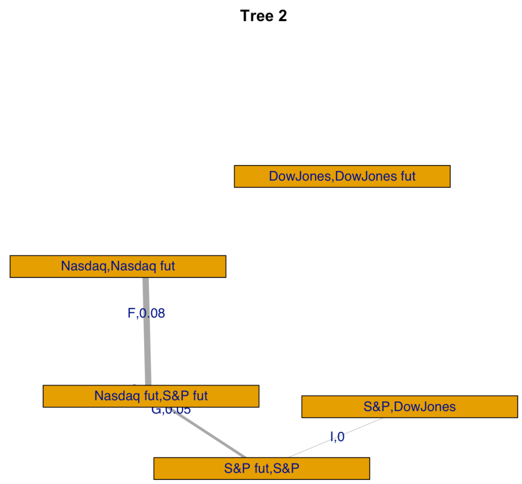
Tree 4 shows that the conditional relation $S\&P_f$, $DowJones_f$ and $NASDAQ_f, DowJones_s$ conditional on $S\&P_f$, $S\&P_s$ and $DowJones_f$ can be described by the independence copula while the relation $DowJones_s$, $NASDAQ_f$ conditional on $S\&P_f$, $S\&P_s$ and $NASDAQ_f$ is described by the Frank copula.

The final tree shows the relation between the variables on the edges of the first tree: $DowJones_s$ and $NASDAQ_f$ conditional on the remainder of the indices, which is modelled by the independence copula.

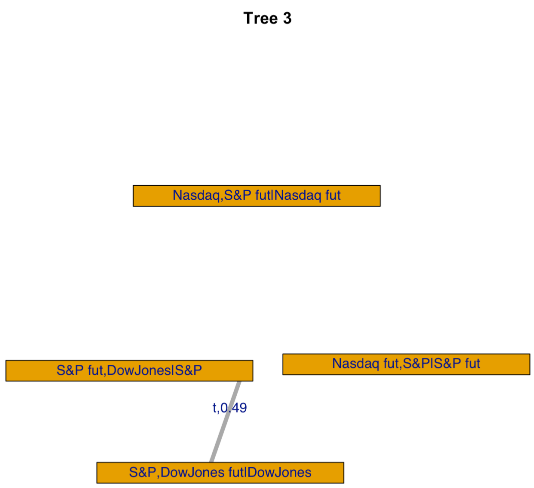
By choosing the conditional variables that exhibit the strongest dependence relation with the bivariate pair copula, the dependence is maximized and highest likelihood is obtained.



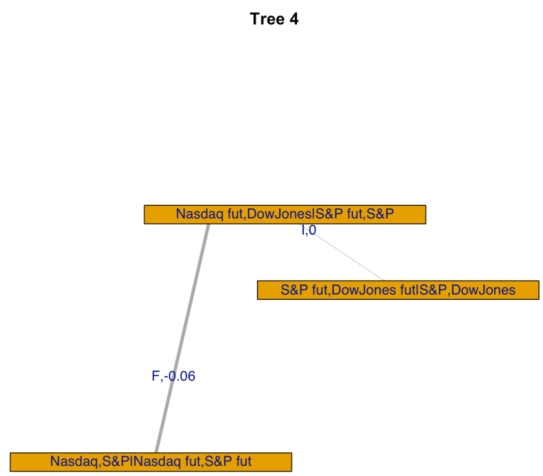
(a) Tree 1 D-vine US portfolio



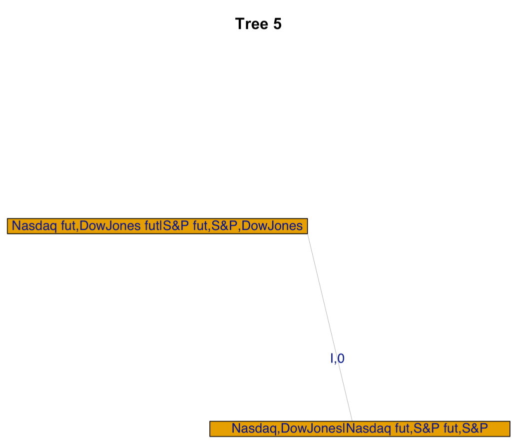
(b) Tree 2 D-vine US portfolio



(c) Tree 3 D-vine US portfolio



(d) Tree 4 D-vine US portfolio



(e) Tree 5 D-vine US portfolio

C.1.1 Robustness

We examine different model specifications to check for the robustness of the results. The different model specifications are summarized in Table 19 in Appendix Section A.3. The average log-likelihood for the different model specifications and copula models respectively is presented in Table 27.

The different model specifications lead to differing levels of copula fit. However, the D-vine copula model achieves the highest average fit in each specification.

Table 27: Average copula model fit for the different model specifications

	Gumbel	Clayton	Student's t	Gaussian	C-vine	D-vine
Base model	3347.51 (1009.85)	3049.19 (922.95)	5910.38 (090.25)	5668.01 (841.86)	5847.81 (847.05)	**5937.89 (873.77)**
Gaussian residuals	3360.88 (1015.53)	3056.15 (926.31)	5929.15 (911.46)	5693.21 (846.21)	5877.16 (859.09)	**5957.67 (877.08)**
Student's residuals	3347.85 (1009.18)	3045.10 (919.53)	5904.97 (904.87)	5662.26 (835.80)	5842.60 (844.86)	**5933.81 (871.40)**
Constant mean	3340.14 (1005.65)	3040.46 (918.28)	5880.22 (897.26)	5636.05 (823.93)	5826.36 (840.21)	**5909.99 (862.18)**
SP approach	3563.23 (1112.98)	3077.97 (941.39)	6111.66 (1045.70)	5857.51 (961.10)	6079.89 (988.30)	**6129.29 (1006.96)**
NP-marginals	3485.94 (1160.48)	3161.60 (1037.01)	6149.23 (973.74)	5750.07 (868.02)	6041.34 (942.15)	**6169.31 (963.18)**
Shifted model	3711.21 (851.03)	3193.20 (739.78)	6224.06 (824.33)	5985.47 (797.40)	6200.95 (753.59)	**6282.04 (760.75)**
Increased re-estimations	3609.05 (963.62)	3117.97 (820.96)	6141.06 (929.44)	5912.91 (873.91)	6112.39 (868.03)	**6189.24 (887.87)**
Decreased window sizes	2370.36 (705.72)	2051.16 (596.70)	4063.20 (645.79)	3925.07 (601.81)	4033.10 (614.35)	**4090.38 (622.78)**
Increased simulations	3531.34 (1104.91)	3067.37 (938.54)	6075.29 (1025.19)	5820.89 (946.73)	6028.61 (960.84)	**6077.87 (978.09)**
EU portfolio	5797.60 (809.27)	5216.70 (649.74)	9507.61 (1185.86)	8559.56 (893.02)	9559.30 (1184.40)	**9681.30 (2307.18)**

Notes: This table presents the average copula model log-likelihood of the different model specifications.

Gaussian and Student's t residuals First, we look at the different distributions for the GARCH residuals. In the base model, the residuals are modelled by the skewed Student's t distribution which is presented in the first row of Table 27. The other model specifications considered to check for robustness include the Gaussian and Student's t distribution for the residuals. The Gaussian residuals obtain a higher average likelihood than the skewed Student's t residuals. This difference is, however, not statistically significant on 1% significance level, which can be observed in Table 28. The skewed Student's t residual model provides a higher average likelihood than the model specification with Gaussian residuals. Again, this difference is not statistically significant.

Constant mean model The conditional mean in the base model is modelled by an AR(1) model. This conditional mean model is also modified to check whether the results are robust to the conditional mean model choice. The average fit of the different copula models for the AR(0)-GARCH(1,1) model is displayed in the fourth row of Table 27. The AR(1)-GARCH(1,1) specification obtains a higher average likelihood for every copula model compared to the AR(0)-GARCH(1,1) model. This difference is significant for the Student's t copula model, while the difference is not significant for the remainder of the copula models.

SP approach and NP marginals Two alternatives for modelling the marginals are tested to check whether the results can be improved using different marginal specifications. The first includes the transformation of GARCH filtered residuals using the empirical distribution function to transform to copula data which is the semi-parametric approach. Moreover, the residuals are modelled non-parametrically as in Sukcharoen & Leatham (2017), employing the empirical distribution function. Both specifications lead to a better copula fit relative to the base model. This difference is significant for the Student's t, Gaussian, C-vine and D-vine copula models. Additionally, the difference is significant for the Gumbel model for the specification that uses the empirical distribution function to transform to copula data.

Shifted model The copula models are re-estimated using different starting points for the estimation windows. The window sizes are equal to the window sizes for the base model, but the estimation periods are delayed by one year. This results in the in and out of sample windows presented in Table 25. The shifted estimation periods lead to a superior copula fit to the base model for each copula model. Except for the Clayton copula model, this difference is statistically significant. This result suggests that the results are sensitive to the estimation periods. Table 29 displays the difference in copula fit over the different estimation periods, from which we can assess where the significant differences are caused. The difference in copula log-likelihood between the different estimation periods is mainly caused by the large difference in the first, second, fourth and fifth estimation windows. The in-sample periods for the base model coincide with the years 2000-2003, 2001Q3-2004Q2, 2004Q3-2007Q2, and 2006-2009. The model in which the in- and out-of-sample periods are shifted by a year covers 2001-2004, 2002Q3-2005Q2, 2005Q3-2008Q2 and 2007-2010. Characterizing for the latter are the periods of high volatility, which can be observed in Figure 8 to Figure 10. The first high volatility period in which a different regime can be observed is 2001-2004. Additionally, a different regime with high volatility can be observed around 2006-2010. Therefore, the base model covers in those four estimation windows two different regimes. In contrast, the estimation periods of the shifted model are better in line with the different regimes of the data, as most of the time, the in-sample estimation window falls in one regime.

Table 28: T-test p-values robustness results

	Gumbel	Clayton	Student 's t	Gaussian	C-vine	D-vine
Gaussian residuals	0.01	0.23	0.08	0.03	0.13	0.07
Student's t residuals	0.82	0.06	0.07	0.08	0.08	0.11
Constant mean	0.24	0.06	0.01	0.01	0.11	0.02
Semi-parametric approach	0.00	0.06	0.00	0.00	0.00	0.00
NP marginals	0.05	0.03	0.00	0.00	0.00	0.00
Shifted model	0.00	0.13	0.00	0.00	0.00	0.00

Notes: This table presents the p-values of the test whether the base model fit is significant different from the specific model specification model fit.

Table 29: Difference in copula fit

	Gumbel	Clayton	Student's t	Gaussian	C-vine	D-vine
Period 1	722.82	406.28	519.12	545.00	503.03	497.22
Period 2	822.98	803.41	494.80	237.74	674.26	769.99
Period 3	-159.00	-106.91	13.87	252.76	73.02	-0.72
Period 4	494.24	250.91	465.00	401.10	488.97	484.53
Period 5	719.43	300.14	778.69	918.37	808.45	767.75
Period 6	187.01	-18.33	104.61	78.96	40.47	72.79
Period 7	266.77	-66.16	156.21	144.06	293.06	215.79
Period 8	32.05	-174.91	306.20	258.77	313.74	307.30
Period 9	346.91	182.30	435.97	403.84	495.11	428.31
Period 10	421.64	78.95	67.72	121.56	-21.25	47.03
Period 11	145.83	-71.35	108.28	129.84	215.74	195.68

Notes: This table presents the per-period difference in copula fit between the base model and shifftd model.

Increased number of re-estimations Moreover, the re-estimation time is decreased to see whether the hedging performance increases when the model is re-estimated more often and to reduce the sensitivity to the choice of estimation window. The model is re-estimated every half year instead of one and a half years. Since this is computationally intensive, we only run the model once and see whether it leads to better results. The in-and out-of-sample window length remains fixed. The D-vine copula results in the highest average fit, whilst the Clayton copula model yields the lowest average fit, in line with the base model results. We do not test for a statistically different model fit with the shorter estimation window as the number of in-sample windows is larger.

Decreased window sizes Additionally, the copula models are estimated using shorter in-and out-sample periods. Since the in-sample period contains fewer observations, the log-likelihood, the sum of all observations, is lower than in the remainder of the model specifications. Therefore, the model fit is not compared with the base model fit in Table 28. The copula model that achieves the highest fit is the D-vine copula model, consistent with the base model. Moreover, the C-vine and Student's t copula achieve a comparable fit to the D-vine copula.

Increased number of simulations We increase the number of simulated observations in the Monte-Carlo simulation to see if the performance of the copula models enhances compared to the base model. Since this is computationally intensive, we run the model once using the increased number of observations and assess whether it leads to the desired effect. The models' fit with the increased number of simulations is larger the base model's fit since the number of observations on which the likelihood is calculated is higher. The copula model with the highest fit, the D-vine copula model, is in line with the base model.

EU Portfolio The final robustness check included is the estimation of the base model on a different portfolio. The base model is re-estimated for the EU portfolio, consisting of the indices DAX, CAC 40, *Euro Stoxx* 50, and *AEX*.

Table 27 displays the average log-likelihood of the EU portfolio for the different copula models, respectively. Additionally, Table 30 presents the log-likelihood of the different copula models over the different estimation windows. The D-vine copula model yields the highest average fit whilst the Clayton copula model yields the poorest fit. This pattern can be observed over the different estimation windows and is consistent with the copula fit of the US portfolio.

Both vine copulas provide a superior fit than the other multivariate copula models. However, the difference between the C- and D-vine copula models and the Student's t-copula model is insignificant. The copula models that generate the poorest fit to the data are both Archimedean copulas, namely the Clayton and Gumbel copula. Comparable to the US portfolio, this is likely caused by the fact that the dependence structure is characterized by one parameter and the lack of asymmetric tail dependence patterns in the data. The Gumbel copula model yields a higher fit in each estimation window than the Clayton copula, suggesting that modelling dependence in the highest returns is a better alternative than modelling dependence in the lowest returns.

The Student's t and Gaussian copula models provide a higher average fit to the data than both Archimedean copulas. The Gaussian copula estimates the multivariate dependence structure by 28 correlation parameters and does not exhibit tail dependence, meaning it fails to capture dependency in extreme events (high or low returns). On the contrary, the Student's t copula displays a symmetric tail dependence structure controlled by the extra degrees of freedom parameter and the 28 correlation parameters. The dependence characteristics of the data match the Student's t characteristics as most indices show considerable upper and lower tail dependence shown in Table 22 and 21. This could explain the comparable fit of the Student-t copula with both vine copulas.

The presented results above align with the US portfolio, suggesting that both vine copula models and the Student's t copula model are well-qualified to model the dependence between stock indices. However, the results are not tested on their statistical difference compared to the base model, as the number of indices is higher for the EU portfolio and translates to a higher fit.

Table 30: Copula model fit over time EU portfolio

	Gumbel	Clayton	Student's t	Gaussian	C-vine	D-vine
2000-2003	4403.39	4059.59	7424.58	7035.33	7454.00	7515.14
2001Q3-2004Q2	5162.93	4730.62	8422.08	7963.36	8513.68	8567.87
2003-2006	4813.74	4413.46	8076.01	7409.36	8145.84	8255.89
2004Q3-2007Q2	5302.72	4830.48	8395.14	7703.21	8463.23	8580.72
2006-2009	6278.73	5768.33	9942.11	8817.36	9923.99	10127.71
2007Q3-2010Q2	6767.08	6159.54	10605.58	9439.45	10648.69	10727.00
2009-2012	6996.31	5879.99	10726.39	9700.04	10803.48	10872.30
2010Q3-2013Q2	6236.37	5386.91	10263.08	9089.14	10308.46	10403.60
2012-2015	5701.05	5205.76	9898.60	8707.00	9899.86	10080.89
2013Q3-2016Q2	6208.13	5643.15	10478.33	9379.54	10547.34	10730.35
2015-2018	5902.68	5305.94	10351.75	8911.38	10443.59	10632.44

Notes: This table presents the copula mode log-likelihood over time for the EU portfolio.

C.2 Hedging Performance

VaR Objective The HS model accomplishes the most significant VaR reduction on the 90%, 95%, and 99% confidence levels. Table 32 shows that the VaR reduction is not statistically significant compared to the Gumbel, Student's t, Gaussian, C- and D-vine copula models. The best performing copula model for the 90% and 99 % confidence level is the Student's t copula with an average reduction in VaR of 80.85% and 76.82%, respectively. The Gumbel model is the copula model that achieves the most considerable VaR95 reduction. The hedge ratios for higher confidence levels know a higher variance (Table 7), causing a higher variance in hedging effectiveness at these confidence levels. Table 8 shows lower hedging effectiveness at higher confidence levels, which suggests that it is more difficult to hedge extreme risk.

We can not establish a clear trend of the performance of the different copula models if we look at the different out-of-sample estimation windows. For example, the HS model always generates a competitive risk reduction but is only the best performer on the 90% and 99% confidence level in 2 (out of 11) out-of-sample periods. On the contrary, the D-vine achieves the greatest risk reduction in 3 and 4 out-of-sample periods. However, it performs relatively poorly than the HS model in the first and second estimation window.

ES Objective The HS model achieves the most considerable ES reduction on 90% and 95% confidence levels. On a 99% confidence level, the C-vine provides the greatest risk reduction. Again, the ES hedging effectiveness decreases with the increase of the confidence level. Only the Clayton copula yields a significantly different performance on a 1% significance level for the ES90 objective. On 95% confidence level, the copula models achieve an ES reduction that is not statistically different from the HS model. Although the Student's t and D-vine copula achieve the highest in-sample log-likelihood, the best performing copula model is the Gumbel for the 90% and 95% confidence level. In contrast, the C-vine performs best on a 99% confidence level. This suggests that the effectiveness depends not solely on the in-sample fit.

SV and LPM objective The models achieve a risk reduction of at least 91% for the SV and LPM objective. The Student's t copula model achieves the greatest average risk reduction in terms of SV, whilst the HS model yields the greatest risk reduction in terms of LPM. This risk reduction is insignificant compared to all other copula models except for the Clayton copula model. Again there is no clear visible pattern of superior models over time. The hedging effectiveness for the LPM objective is relatively high compared to other hedging objectives. Characteristic for this objective is that it knows a high unhedged risk as displayed in Table 33. Therefore, a substantial reduction in LPM results in high effectiveness.

Table 31: Total risk US portfolio

	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	0.96 (0.32)	0.91 (0.33)	1.72 (0.65)	1.07 (0.32)	1.39 (0.41)	2.33 (0.82)	0.69 (1.11)	2.99 (5.53)
Clayton	1.10 (0.39)	1.12 (0.46)	2.02 (0.69)	1.42 (0.41)	1.77 (0.50)	2.82 (0.98)	0.77 (1.06)	2.87 (5.11)
Student's t	0.86 (0.27)	0.97 (0.41)	1.69 (0.67)	1.11 (0.38)	1.42 (0.53)	2.30 (0.98)	**0.33 (0.27)**	0.94 (1.21)
Gaussian	0.92 (0.32)	0.91 (0.25)	1.73 (0.51)	1.24 (0.61)	1.62 (0.93)	2.54 (1.47)	0.39 (0.35)	1.43 (2.27)
C-vine	0.90 (0.30)	0.94 (0.36)	1.74 (0.73)	1.13 (0.36)	1.42 (0.47)	**2.11 (0.70)**	0.47 (0.44)	1.43 (2.03)
D-vine	1.01 (0.48)	0.96 (0.41)	1.75 (0.83)	1.27 (0.46)	1.63 (0.55)	2.62 (1.12)	0.99 (1.97)	1.40 (10.72)
HS	**0.85 (0.28)**	**0.84 (0.27)**	**1.58 (0.54)**	**1.03 (0.31)**	**1.34 (0.44)**	2.19 (1.00)	0.34 (0.26)	**0.68 (0.95)**

Notes: This table presents the average risk of the US portfolio for the different copula models and hedging objectives respectively.

Table 32: T-test US portfolio

	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	-2.64 (0.06)	-1.61 (0.22)	-2.03 (0.30)	-0.76 (0.21)	-1.01 (0.28)	-2.41 (0.39)	-3.19 (0.31)	-1.89 (0.21)
Clayton	-6.54 (0.02)	-6.14 (0.00)	-6.14 (0.06)	-8.04 (0.00)	-7.59 (0.01)	-7.96 (0.06)	-4.80 (0.98)	-2.45 (0.58)
Student's t	-0.14 (0.85)	-2.56 (0.05)	-0.90 (0.43)	-1.38 (0.05)	-1.04 (0.26)	-1.01 (0.66)	0.65 (0.26)	-1.33 (0.21)
Gaussian	-1.69 (0.08)	-2.08 (0.10)	-2.08 (0.29)	-3.37 (0.03)	-3.51 (0.11)	-2.90 (0.31)	-1.00 (0.33)	-0.57 (0.25)
C-vine	-0.99 (0.11)	-1.97 (0.08)	-1.45 (0.52)	-1.83 (0.01)	-1.31 (0.16)	0.14 (0.96)	-2.83 (0.30)	-1.19 (0.25)
D-vine	-3.21 (0.04)	-2.56 (0.05)	-1.47 (0.46)	-5.08 (0.02)	-5.25 (0.05)	-4.75 (0.17)	-0.35 (0.28)	-1.13 (0.19)

Notes: This table presents the difference in performance between the HS model and different copula models. The p-values of the test whether the differences are significant are denoted in parentheses.

Table 33: Unhedged risk US portfolio

in sample	2000-2003	2001Q3-2004Q2	2003-2006	2004Q3-2007Q2	2006-2009	2007Q3-2010Q2	2009-2012	2010Q3-2013Q2	2012-2015	2013Q3-2016Q2	2015-2018
VaR90	7.09	5.92	3.25	2.60	4.78	6.17	4.83	3.34	2.50	3.01	2.54
VaR95	8.49	7.04	4.21	3.48	7.09	8.65	6.80	4.87	3.72	4.35	3.80
VaR99	12.46	10.13	6.41	5.13	15.84	15.84	11.80	8.26	6.04	7.05	7.22
ES90	9.54	7.87	4.66	3.86	9.06	10.34	7.8	5.85	4.04	4.83	4.52
ES95	11.25	9.16	5.54	4.64	12.21	13.32	9.87	7.55	5.03	6.05	5.90
ES99	15.86	12.65	8.06	6.52	21.07	21.07	14.23	12.33	6.85	8.75	9.25
SV	16.11	9.76	3.36	2.26	12.37	15.88	8.47	4.96	2.24	3.30	2.86
LPM	147.28	76.24	16.03	9.25	173.89	208.95	75.47	39.74	9.63	17.97	16.91
out sample	2003-2005	2004Q3-2005Q2	2006-2007	2007Q3-2008Q2	2009-2010	2010Q3-2011Q2	2012-2013	2013Q3-2014Q2	2015-2016	2016Q3-2017Q2	2018-2019
VaR90	4.06	3.13	2.17	5.41	5.55	2.91	2.56	2.27	3.15	1.11	3.29
VaR95	4.95	3.86	3.45	7.01	7.83	4.50	3.58	3.25	4.41	1.93	5.82
VaR99	7.48	4.97	5.13	8.46	12.64	6.69	5.63	6.11	7.45	5.33	10.32
ES90	5.83	4.03	3.68	7.02	8.69	4.98	4.16	3.87	5.34	2.95	6.66
ES95	7.05	4.58	4.55	7.93	10.65	6.16	5.10	4.89	6.75	4.08	8.64
ES99	9.80	5.64	6.08	9.23	13.84	8.82	7.18	6.98	10.44	8.20	12.10
SV	5.23	2.66	2.07	7.67	10.43	3.45	2.48	2.09	4.32	1.52	6.07
LPM	31.14	9.75	7.76	48.31	95.20	19.15	10.93	9.24	26.94	9.73	48.44

Notes: This table presents the unhedged in-and out-of-sample risk of the US portfolio.

Table 34: In-sample hedging effectiveness US portfolio

	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	77.24 (9.55)	78.13 (8.56)	73.26 (9.13)	75.11 (9.42)	73.52 (9.66)	67.88 (14.52)	94.40 (2.58)	98.21 (1.43)
Clayton	72.59 (12.30)	66.95 (14.90)	72.14 (13.45)	70.81 (12.49)	69.77 (13.02)	68.64 (9.30)	93.54 (2.58)	98.01 (1.13)
Student's t	80.78 (6.14)	81.25 (2.61)	79.89 (5.68)	79.55 (3.53)	79.41 (4.49)	74.28 (7.50)	95.19 (1.85)	**98.73 (0.87)**
Gaussian	80.72 (2.04)	81.16 (3.06)	78.62 (5.23)	79.57 (3.49)	78.28 (4.36)	74.91 (6.80)	94.75 (1.84)	98.55 (0.96)
C-vine	79.69 (5.50)	79.64 (5.35)	79.48 (7.86)	78.69 (5.51)	77.38 (5.59)	74.74 (7.79)	94.50 (1.86)	98.50 (0.97)
D-vine	80.59 (3.81)	81.37 (2.82)	80.16 (5.81)	79.45 (2.98)	79.04 (4.39)	74.99 (6.27)	95.09 (3.64)	98.21 (3.15)
HS	**80.97 (5.41)**	**82.07 (2.69)**	**80.81 (5.12)**	**80.76 (3.33)**	**80.18 (4.20)**	**79.11 (5.44)**	**95.48 (1.65)**	98.72 (1.16)

Notes: This table presents the in-sample hedging effectiveness of the US portfolio.

C.2.1 Robustness

Table 37 and 38 present the hedging effectiveness of the different model specifications for the US portfolio.

Gaussian and Student's t residuals The fit of the different copula models is insensitive to the distribution of the residuals as proven in Section A.3. We can not observe a clear difference in hedging effectiveness among the different residuals specifications when we assess the hedging effectiveness of the different copula models. The HS model still provides the highest performance for the different specifications. Hence, we can conclude that the hedge effectiveness is insensitive to different residual distributions.

Constant mean model The AR(0)-GARCH(1,1) produces hedging effectiveness that is comparable with the base model's effectiveness. The HS model performs best on most risk criteria which is in line with the base model.

Semi-parametric approach and NP marginals Both the approach of the non-parametric transformation of the standardized residuals to copula data and the non-parametric modelling of marginals lead to a significant better copula fit as shown in Table 27 and Table 28. However, the hedging effectiveness is comparable to the effectiveness of the base model and does not seem to be improved. Even with the models that obtain a significantly higher fit than the base model, it is difficult to achieve a better performance than the HS model on all risk measures. However, the differences between the best copula models and the HS model are relatively small.

Shifted model In the shifted model, we observed an increase in copula fit in specific estimation windows because the in-sample windows are better able to cover different regimes. The out-of-sample hedging performance is also higher than in the base model. The increased copula fit could cause this, but as earlier discussed, this does not necessarily lead to a better hedging performance. To investigate the reason for the increased hedging performance over time, we look at the difference in copula fit and hedging performance across the difference hedging objectives.

Table 29 represents the difference in copula fit between the base and shifted model (Table 25). Table 35 presents the difference in hedging effectiveness of the different risk measures over the estimation period. The risk measure's hedging effectiveness is calculated as the average of the different copula models' effectiveness. A positive coefficient indicates a better performance of the shifted model than the base model. The largest difference in hedging effectiveness is found in the third estimation window. However, in Table 29 we can observe that the copula fit of the shifted model in this estimation window is comparable to the copula fit of the base model, suggesting that an increased copula fit does not solely drive the difference in hedging effectiveness. Next to the increased copula fit, the effectiveness is highly driven by choice of out-of-sample period. The third estimation window corresponds in the shifted model to the years 2007-2008. From Table 36 one can observe that the in-sample risk, on which the copula is fitted and hedge ratio is derived, is broadly in line with the out-of-sample risk which, leading to more accurate estimates of the hedge ratio. The second-largest difference in hedging effectiveness between the two alternatives for the estimation period is found in the fifth estimation window. This estimation window corresponds to the largest difference in copula fit, suggesting that an increased copula fit partially drives the hedging effectiveness.

The HS model achieves the best hedging effectiveness in 4 out 8 risk measures. The best performing copula model, the Gumbel copula, produces the best hedging effectiveness in 3 out of 8 risk measures. However, the performance is rather comparable across the different hedging models, so no clear conclusion on a superior model can be drawn and shows that the best performing hedging models are broadly in line with the base model.

Table 35: Difference in hedging effectiveness shifted and base model

	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Period 1	1.08	1.88	0.53	-1.56	-4.14	-7.71	-7.91	1.38
Period 2	3.92	2.57	-3.36	-1.52	-2.65	-3.92	17.85	16.62
Period 3	7.37	8.18	8.65	7.63	6.60	8.84	13.80	7.35
Period 4	-1.45	-1.80	-2.92	-1.85	-1.03	0.38	-5.02	-4.62
Period 5	2.76	0.43	7.42	5.14	7.71	11.62	-0.04	0.23
Period 6	1.93	-0.14	4.78	1.15	1.04	-1.52	2.76	1.29
Period 7	-5.63	0.11	3.63	1.79	3.37	-4.97	0.27	0.65
Period 8	3.54	6.59	2.01	3.27	2.33	-3.38	6.04	2.82
Period 9	1.28	-0.94	6.80	3.65	5.93	8.95	-1.01	-0.23
Period 10	2.18	7.06	-3.69	0.48	1.42	-8.15	0.28	0.74
Period 11	-1.45	-1.13	4.09	1.00	0.20	3.89	2.46	0.92

Notes: This table presents difference in average hedging effectiveness between the base and shifted model in the US portfolio.

Table 36: Unhedged risk shifted model

in sample	2001-2004	2002Q3-2005Q2	2004-2007	2005Q3-2008Q2	2007-2010	2008Q3-2011Q2	2010-2013	2011Q3-2014Q2	2013-2016	2014Q3-2017Q2	2016-2019
VaR90	6.54	4.22	2.84	3.21	6.00	5.25	3.74	3.05	2.74	2.69	2.39
VaR95	7.99	6.16	3.63	4.84	8.42	8.42	5.41	4.74	4.06	4.17	3.90
VaR99	11.64	9.23	5.02	7.58	15.84	15.84	9.89	7.75	6.44	7.22	7.98
ES90	8.73	6.62	3.89	5.32	10.16	10.06	6.37	5.54	4.49	4.73	4.94
ES95	10.28	8.19	4.54	6.64	13.23	13.30	8.18	7.27	5.64	6.08	6.78
ES99	14.19	10.84	5.69	8.91	21.07	21.07	12.78	12.04	8.35	9.25	10.57
SV	12.50	6.49	2.30	3.93	15.17	14.47	5.79	4.50	2.85	3.11	3.44
LPM	105.95	43.30	8.20	22.13	203.12	199.21	47.15	36.42	14.94	18.14	24.08
out sample	2004-2005	2005Q3-2006Q2	2007-2008	2008Q3-2009Q2	2010-2011	2011Q3-2012Q2	2013-2014	2014Q3-2015Q2	2016-2017	2015Q3-2018Q2	2019-2020
VaR90	3.17	2.21	3.53	9.70	3.79	4.83	2.04	2.60	2.74	1.85	2.77
VaR95	4.05	2.95	5.32	13.23	4.99	6.58	3.25	4.17	4.03	3.40	4.71
VaR99	5.02	5.13	7.89	19.80	9.83	13.39	4.74	5.50	7.74	7.12	8.37
ES90	4.21	3.53	5.84	15.19	6.15	7.91	3.66	4.20	4.92	4.88	5.41
ES95	4.72	4.48	7.13	18.39	7.80	10.04	4.51	5.05	6.37	6.81	7.14
ES99	5.45	6.08	9.22	26.71	10.55	15.43	6.09	5.85	9.16	11.10	9.21
SV	2.74	2.06	4.65	34.46	5.39	9.12	1.83	2.46	3.45	3.50	4.01
LPM	10.34	7.45	28.63	539.64	37.59	90.37	7.25	10.02	20.89	27.42	25.56

Notes: This table presents the unhedged risk of the shifted portfolio in the in-and out sample estimation windows.

Increased number of re-estimations Since the models are highly sensitive to the choice of estimation window, the model is re-estimated more often to reduce this sensitivity. Poor performance in a specific window due to regime switches will be present regardless of the estimation period as the model is re-estimated more often. As a result, the model's performance is lower than the base model. We can observe specific periods with poor performance (the beginning of the sample), resulting in a lower average effectiveness. The difference in performance between the hedging models has increased compared to the base model. The HS model achieves the highest performance on all hedging objectives. The Student's t and Gaussian copula models are the best performing copula models, although the difference is small with the C- and D-vine copula models.

Decreased window sizes The in-and out-of-sample window sizes are decreased to assess the effect of smaller estimation and test windows. We look at the base model with in-sample and out-of-sample periods equal to two and a half years, respectively. The average hedging effectiveness is generally lower than the models with fewer estimation windows. Compared to the base model, there are more out-of-sample windows in which all hedging models produce poor hedging effectiveness. This is often found during years where the out-of-sample period covers a recession, whereas the in-sample period covers an expansion. This results in more unsatisfactory average performance. Compared to the base model, the window sizes are smaller, enabling the model to capture more temporary trends in the estimation of copula parameters, producing more inaccurate estimates. The HS model accomplishes the greatest risk reduction on most hedging objectives, but the difference among copula models is small.

Increased number of simulations The performance of most copula models for most hedging objectives increases with the increased number of simulated observations. The HS model achieves the best hedging performance on 2 out of 8 objectives, whilst the copula models perform best on the remaining hedging objectives. Especially the Gumbel and Student's t copula yield high hedging effectiveness and perform best on 4 out of 8 hedging objectives. The performance of both vine copula models lacks behind, suggesting that the model is still overfitting the in-sample.

The results suggest that increasing the number of observations enhances the performance of the copula models since it produces more accurate estimates of the hedge ratios.

EU Portfolio The EU portfolio's hedging effectiveness is higher than the US portfolio. The correlation in terms of Kendall's tau of the EU portfolio is higher than the US portfolio, resulting in a higher average hedging performance. As seen before in the US, the hedging effectiveness is a positive function of the correlation, explaining the higher hedging effectiveness for the EU portfolio. Moreover, the performance of the D-vine copula model in the EU portfolio is better compared to the US portfolio. While the performance D-vine copula model in the US portfolio lacks behind the other copula models in the first and second estimation window, the D-vine copula model achieves a high performance in the first and second estimation window in the EU portfolio resulting in a higher average performance. The vine copula models suffer less from overfitting the in-sample in the EU portfolio case. A likely explanation for this is that the dependence structure changes less frequently which can be observed in the tail-dependency graphs in Figure 25 to Figure 32 in Appendix Section 2. Part of the EU portfolios is the CAC 40 and AEX index, which know the strongest stable dependency, leading to a better performance of the vine copula models. Therefore, it is important to obtain a clear idea of the dependence structure before proceeding with a copula model. The Student's t copula model always achieves a competitive risk reduction and has a smaller risk of overfitting when the dependence structure changes rapidly. However, in a setting where the dependence is relatively stable, an investor may benefit from employing vine copula models to increase the hedging effectiveness.

Comparable to the US portfolio, the models produce fairly similar hedging effectiveness for all hedging objectives. The HS model achieves the greatest reduction for most hedging objectives but this difference is not significant relative to the D-vine model.

Table 37: Hedging effectiveness of the different model specifications

Base model	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	78.35 (4.97)	79.62 (4.80)	75.68 (9.33)	79.20 (3.21)	77.96 (3.91)	73.03 (10.13)	92.85 (7.39)	97.15 (9.62)
Clayton	74.44 (9.09)	75.09 (6.55)	71.58 (8.88)	71.91 (7.06)	71.38 (8.18)	67.47 (11.41)	91.24 (9.93)	96.59 (19.04)
Student's t	80.85 (2.80)	78.67 (4.74)	76.82 (6.01)	78.58 (2.88)	77.93 (4.02)	74.43 (7.94)	**96.39 (2.60)**	98.71 (3.47)
Gaussian	79.29 (3.36)	79.14 (4.39)	75.64 (4.83)	76.59 (4.94)	75.45 (6.47)	72.53 (9.01)	95.04 (3.79)	98.47 (4.41)
C-vine	79.99 (3.05)	79.26 (4.06)	76.26 (6.34)	78.12 (3.39)	77.65 (3.35)	**75.57 (8.69)**	93.21 (7.04)	97.85 (7.29)
D-vine	77.77 (6.77)	78.64 (5.59)	76.25 (6.86)	74.87 (7.28)	73.72 (8.11)	70.68 (9.40)	95.69 (13.26)	97.91 (16.65)
HS	**80.98 (3.04)**	**81.23 (2.87)**	**77.71 (7.09)**	**79.95 (3.19)**	**78.96 (4.04)**	75.43 (9.37)	96.04 (5.41)	**99.04 (2.69)**
Gaussian residuals	**VaR90**	**VaR95**	**VaR99**	**ES90**	**ES95**	**ES99**	**SV**	**LPM**
Gumbel	78.41 (5.17)	80.00 (3.53)	73.09 (10.18)	76.47 (8.66)	74.77 (9.99)	67.86 (13.99)	83.90 (8.72)	92.70 (7.30)
Clayton	75.67 (5.74)	77.19 (6.01)	75.97 (5.45)	75.31 (4.83)	74.70 (5.50)	73.91 (8.23)	80.32 (9.43)	90.46 (7.21)
Student's t	78.43 (5.26)	79.14 (4.24)	73.26 (6.64)	75.22 (8.62)	73.67 (9.58)	67.29 (13.39)	84.49 (6.71)	92.29 (6.87)
Gaussian	78.48 (5.69)	79.73 (3.89)	78.54 (4.63)	77.10 (8.19)	76.61 (9.48)	73.43 (11.82)	**85.74 (7.16)**	**93.76 (5.91)**
C-vine	77.73 (4.07)	79.66 (3.08)	72.58 (16.75)	76.85 (6.95)	74.96 (10.75)	72.05 (13.06)	74.65 (31.70)	79.55 (37.52)
D-vine	75.07 (7.60)	76.59 (8.03)	74.05 (10.27)	74.97 (9.77)	73.70 (11.75)	69.46 (15.48)	84.86 (8.39)	90.01 (14.83)
HS	**79.17 (4.80)**	**81.23 (2.87)**	**77.71 (7.09)**	**79.95 (3.19)**	**78.96 (4.04)**	**75.43 (9.37)**	78.12 (23.96)	84.63 (25.71)
Student's t residuals	**VaR90**	**VaR95**	**VaR99**	**ES90**	**ES95**	**ES99**	**SV**	**LPM**
Gumbel	77.05 (5.40)	80.06 (3.84)	72.40 (8.04)	78.60 (3.59)	76.81 (4.21)	69.74 (11.06)	85.78 (7.61)	93.62 (5.83)
Clayton	77.29 (6.55)	74.76 (5.15)	72.79 (5.44)	74.96 (4.53)	74.00 (5.02)	70.47 (7.57)	81.55 (6.62)	90.73 (4.90)
Student's t	79.13 (4.95)	80.65 (2.99)	77.67 (5.49)	78.94 (3.41)	77.77 (4.32)	72.83 (7.34)	**86.91 (5.53)**	**94.08 (5.24)**
Gaussian	79.02 (4.54)	81.02 (2.57)	75.64 (9.50)	78.89 (4.00)	78.00 (4.83)	74.41 (10.01)	86.10 (6.03)	94.00 (5.03)
C-vine	76.58 (6.48)	76.48 (6.66)	76.45 (9.44)	73.15 (10.74)	72.303 (11.21)	70.52 (7.56)	79.42 (24.74)	86.10 (26.53)
D-vine	77.79 (5.68)	77.72 (5.21)	75.71 (11.27)	75.95 (6.12)	75.67 (6.37)	73.54 (10.00)	80.38 (21.20)	88.01 (19.65)
HS	**79.17 (4.80)**	**81.23 (2.87)**	**77.71 (7.09)**	**79.95 (3.19)**	**78.96 (4.04)**	75.43 (9.37)	78.12 (23.96)	84.63 (25.71)
Constant mean model	**VaR90**	**VaR95**	**VaR99**	**ES90**	**ES95**	**ES99**	**SV**	**LPM**
Gumbel	77.65 (7.19)	80.22 (3.05)	75.28 (8.86)	78.96 (3.38)	78.11 (4.05)	71.77 (7.90)	83.89 (9.71)	**92.64 (7.02)**
Clayton	71.16 (5.25)	72.54 (5.82)	73.68 (7.74)	72.94 (5.04)	72.73 (6.32)	71.35 (7.22)	76.83 (12.09)	87.44 (8.39)
Student's t	78.82 (5.75)	79.14 (4.59)	75.50 (6.23)	78.40 (4.35)	78.14 (4.18)	**76.79 (5.51)**	78.12 (19.88)	87.42 (18.83)
Gaussian	76.67 (6.84)	79.06 (4.04)	76.62 (6.95)	78.98 (4.19)	78.80 (4.47)	76.05 (7.22)	77.90 (23.71)	87.67 (21.56)
C-vine	78.23 (7.67)	78.49 (4.88)	75.93 (8.00)	76.27 (5.30)	75.90 (5.06)	71.93 (8.25)	78.69 (23.60)	86.30 (22.81)
D-vine	74.97 (7.13)	77.10 (6.91)	76.44 (5.72)	76.24 (5.89)	75.79 (6.76)	72.73 (9.43)	**84.83 (9.05)**	90.49 (12.53)
HS	**79.17 (4.80)**	**81.23 (2.87)**	**77.71 (7.09)**	**79.95 (3.19)**	**78.96 (4.04)**	75.43 (9.37)	78.12 (23.96)	84.63 (25.71)
Semi-parametric approach	**VaR90**	**VaR95**	**VaR99**	**ES90**	**ES95**	**ES99**	**SV**	**LPM**
Gumbel	75.40 (10.07)	78.50 (6.71)	76.42 (6.17)	77.59 (4.09)	75.82 (4.83)	70.52 (9.83)	**81.31 (10.77)**	**90.59 (8.02)**
Clayton	75.43 (5.62)	74.25 (8.60)	73.69 (9.49)	73.21 (6.82)	73.58 (7.33)	73.15 (8.96)	76.15 (12.55)	85.54 (11.96)
Student's t	77.44 (6.01)	79.61 (3.76)	77.41 (6.00)	79.17 (3.32)	78.38 (3.77)	**75.59 (8.00)**	77.97 (25.23)	85.55 (26.71)
Gaussian	78.90 (4.91)	79.84 (4.24)	76.58 (7.23)	78.79 (3.94)	78.51 (4.04)	75.87 (7.73)	79.08 (23.17)	87.57 (22.29)
C-vine	79.02 (4.90)	79.87 (3.58)	77.43 (6.21)	77.29 (4.36)	77.61 (4.57)	74.56 (9.01)	75.64 (26.60)	80.53 (32.89)
D-vine	76.21 (7.39)	75.94 (8.70)	76.56 (8.26)	76.47 (6.88)	76.58 (7.94)	72.14 (11.01)	83.59 (9.30)	88.28 (17.01)
HS	**79.17 (4.80)**	**81.23 (2.87)**	**77.71 (7.09)**	**79.95 (3.19)**	**78.96 (4.04)**	75.43 (9.37)	78.12 (23.96)	84.63 (25.70)

Notes: This table presents the out-of-sample hedging effectiveness of the different model specifications. The values in parentheses indicate the standard deviation in hedging effectiveness.

Table 38: Hedging effectiveness of the different model specifications

NP marginals	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	77.81 (6.02)	78.40 (5.87)	71.41 (12.85)	77.33 (4.90)	75.29 (5.67)	71.03 (7.66)	85.02 (7.54)	93.27 (5.63)
Clayton	76.67 (6.61)	73.55 (7.61)	73.94 (7.02)	74.27 (4.93)	73.17 (6.12)	71.39 (7.68)	81.28 (6.86)	90.78 (5.35)
Student's t	**79.82 (4.31)**	79.73 (4.53)	**77.90 (6.70)**	78.95 (4.26)	78.14 (5.40)	73.64 (8.53)	**87.15 (5.60)**	**94.54 (4.93)**
Gaussian	78.44 (5.44)	80.64 (3.90)	77.31 (7.05)	79.11 (3.58)	78.06 (4.70)	**75.50 (8.19)**	86.76 (5.91)	94.28 (5.06)
C-vine	76.23 (7.07)	78.67 (4.39)	76.29 (9.29)	77.07 (3.89)	76.70 (4.35)	74.42 (8.72)	80.08 (23.54)	87.94 (22.94)
D-vine	78.27 (5.03)	78.95 (4.18)	75.77 (8.91)	76.19 (6.16)	75.92 (6.18)	73.12 (10.16)	79.77 (24.36)	86.62 (26.75)
HS	79.17 (4.80)	**81.23 (2.87)**	77.71 (7.09)	**79.95 (3.19)**	78.96 (4.04)	75.43 (9.37)	78.12 (23.96)	84.63 (25.71)
Shifted model	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	80.92 (3.26)	**82.46 (3.29)**	78.57 (7.87)	79.81 (4.96)	78.78 (6.24)	71.55 (11.33)	**87.43 (6.19)**	**94.80 (4.90)**
Clayton	77.74 (5.99)	78.89 (5.66)	75.47 (5.72)	76.41 (3.35)	75.69 (3.92)	71.61 (8.93)	83.04 (8.12)	92.49 (4.87)
Student's t	80.92 (4.43)	81.91 (3.30)	**79.85 (7.47)**	80.58 (4.58)	80.24 (5.34)	74.08 (9.09)	85.84 (11.48)	92.86 (11.47)
Gaussian	80.55 (3.42)	80.84 (4.91)	78.30 (7.67)	79.15 (4.80)	77.73 (6.51)	71.56 (12.05)	84.92 (13.68)	93.37 (9.19)
C-Vine	80.13 (3.43)	78.96 (7.75)	78.48 (5.45)	76.39 (5.81)	76.44 (4.88)	73.21 (4.95)	84.98 (10.00)	92.07 (8.26)
D-vine	80.144 (4.34)	80.88 (4.63)	77.26 (8.82)	78.13 (5.99)	77.02 (7.71)	72.74 (12.87)	85.09 (12.31)	92.31 (11.68)
HS	**81.16 (3.11)**	82.23 (3.55)	79.78 (8.70)	80.93 (4.58)	**80.35 (5.58)**	**77.17 (7.88)**	83.28 (17.84)	88.50 (20.03)
Increased re-estimations	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	77.70 (8.90)	76.68 (10.01)	71.96 (13.90)	74.78 (10.26)	73.12 (11.67)	66.73 (14.96)	70.39 (37.01)	69.23 (54.63)
Clayton	72.38 (12.00)	72.53 (11.57)	69.44 (14.03)	70.77 (10.37)	70.22 (10.77)	68.48 (10.46)	67.03 (33.05)	71.06 (41.61)
Student's t t	80.69 (3.01)	80.76 (4.35)	78.10 (7.38)	79.81 (4.08)	79.05 (4.62)	75.53 (7.47)	72.62 (32.98)	74.76 (44.32)
Gaussian	81.21 (2.09)	80.50 (3.68)	78.13 (5.49)	79.62 (3.03)	78.80 (3.68)	74.82 (5.74)	72.46 (32.40)	74.68 (43.64)
C-vine	80.69 (2.84)	79.64 (6.07)	77.05 (8.46)	77.31 (7.39)	76.59 (8.80)	73.77 (9.51)	72.12 (35.05)	73.04 (49.34)
D-vine	80.98 (4.44)	79.47 (5.50)	77.64 (6.25)	77.60 (5.66)	77.20 (5.31)	73.77 (8.07)	72.62 (35.28)	73.11 (51.71)
HS	**81.69 (3.99)**	**81.53 (4.92)**	**81.18 (4.82)**	**81.10 (3.03)**	**80.50 (3.91)**	**78.82 (5.49)**	**74.57 (32.35)**	**79.73 (36.15)**
Decreased window sizes	VaR90	VaR95	VaR99	ES90	ES95	ES99	SVM	LPM
Gumbel	77.14 (17.11)	77.04 (10.28)	75.51 (26.44)	75.51 (26.83)	**74.38 (26.52)**	67.65 (26.48)	75.38 (26.65)	81.58 (32.94)
Clayton	72.52 (18.93)	69.85 (29.20)	71.63 (24.47)	69.44 (28.42)	63.23 (28.87)	60.16 (29.04)	69.62 (27.33)	74.64 (39.59)
Student's t	78.81 (8.09)	77.60 (20.25)	77.70 (6.68)	77.71 (4.80)	73.25 (18.41)	69.11 (18.39)	74.27 (24.93)	82.00 (25.27)
Gaussian	77.38 (7.79)	77.59 (7.52)	77.45 (7.39)	77.32 (6.02)	73.64 (18.40)	70.78 (17.94)	74.77 (24.10)	**82.82 (25.03)**
C-vine	79.24 (5.50)	77.32 (6.05)	**77.64 (5.75)**	77.34 (5.23)	74.16 (6.03)	71.87 (9.37)	**77.19 (22.55)**	81.88 (25.65)
D-vine	78.25 (5.57)	77.54 (4.08)	77.45 (4.84)	78.52 (4.27)	73.69 (6.08)	72.62 (10.46)	75.77 (22.66)	82.74 (24.85)
HS	**79.70 (4.06)**	**77.66 (4.25)**	76.26 (5.56)	**79.42 (3.45)**	72.09 (4.59)	**73.24 (9.38)**	73.59 (26.04)	79.24 (27.72)
Increased simulations	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	79.45 (5.06)	**81.97 (2.42)**	**80.09 (9.71)**	**79.82 (3.59)**	79.37 (4.47)	76.12 (8.98)	92.85 (1.78)	97.15 (1.01)
Clayton	75.72 (5.44)	78.66 (7.26)	76.85 (6.6)	76.58 (5.70)	75.78 (6.75)	75.02 (8.86)	91.24 (2.83)	96.59 (1.93)
Student's t	79.11 (4.44)	79.44 (3.20)	76.57 (9.26)	79.18 (3.69)	**79.53 (3.79)**	76.68 (6.43)	**96.39 (1.72)**	98.71 (0.85)
Gaussian	78.08 (5.43)	79.21 (3.72)	74.28 (9.02)	79.74 (3.85)	80.73 (3.93)	**79.11 (5.93)**	95.04 (1.96)	98.47 (0.89)
C-vine	77.46 (9.31)	76.87 (10.17)	78.89 (12.95)	77.06 (5.43)	76.67 (5.88)	75.53 (9.29)	93.21 (3.30)	97.85 (1.29)
D-vine	78.3 (5.30)	78.54 (6.85)	78.54 (7.37)	74.61 (7.17)	73.44 (8.30)	71.39 (11.76)	95.69 (4.17)	97.01 (3.28)
HS	**80.98 (3.39)**	81.23 (2.59)	77.71 (7.75)	79.95 (3.45)	78.96 (4.37)	75.43 (5.41)	96.04 (2.69)	**99.04 (3.69)**
EU portfolio	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	87.59 (7.58)	87.54 (7.14)	86.18 (7.35)	87.46 (5.75)	86.79 (6.95)	83.73 (8.09)	95.52 (2.42)	97.97 (1.02)
Clayton	83.56 (9.71)	82.31 (9.18)	81.15 (10.39)	81.85 (7.40)	82.53 (7.36)	83.24 (8.55)	95.53 (5.35)	98.30 (3.84)
Student's t	87.58 (7.29)	88.35 (7.08)	87.67 (5.47)	87.15 (10.11)	87.12 (8.96)	87.13 (7.76)	97.60 (2.62)	98.76 (1.18)
Gaussian	87.30 (8.95)	86.07 (11.59)	85.51 (12.08)	86.51 (10.42)	86.23 (8.95)	85.88 (8.07)	97.62 (11.73)	98.74 (9.11)
C-vine	86.73 (9.38)	87.10 (7.93)	88.03 (6.52)	86.96 (8.50)	87.13 (8.45)	86.54 (9.49)	96.02 (2.00)	**99.17 (0.64)**
D-vine	87.68 (8.56)	88.13 (7.41)	88.35 (6.04)	88.36 (6.83)	88.34 (6.73)	87.80 (6.95)	**97.81 (2.90)**	98.94 (1.27)
HS	**88.00 (6.27)**	**88.46 (6.33)**	**89.29 (4.79)**	**89.93 (4.25)**	**89.91 (4.21)**	**89.32 (4.09)**	98.78 (2.05)	98.84 (0.70)

Notes: This table presents the out-of-sample hedging effectiveness of the different model specifications. The values in parentheses indicate the standard deviation in hedging effectiveness.

C.3 Single Product

S&P 500 The hedging effectiveness of the *S&P* 500 portfolio ranges between 68.08% and 98.64%. The highest hedging effectiveness is achieved for both SV and LPM objectives. The HS model achieves the greatest risk reduction on 6 out of 8 hedging objectives. However, compared to the Gumbel, Student's t and Gaussian copula model, this difference is not statistically significant for all different hedging objectives. The Clayton copula model yields the lowest risk reduction likely caused by its poor fit to the data.

FTSE 100 The hedging models for the *FTSE* 100 portfolio generate a risk reduction varying between 73.64% and 99.24%. The HS model is the best performing model on 5 out of 8 hedging objectives. However, this difference is not statistically significant to the Gaussian, and Student's t copula on all hedging objectives, coinciding with the best performing copula's in terms of copula fit.

DAX The hedging effectiveness of the *DAX* portfolio ranges between 71.07% and 98.10%. The highest risk reduction is achieved for the SV and LPM objective. There is no clear distinction between the best performing hedging models. The HS achieves the greatest risk reduction on 4 out of 8 hedging objectives while the Gumbel, Gaussian and Student's t hedging models perform best on the remainder of hedging objectives. The differences in risk reduction are not significant compared to the HS hedging model. The different copula models all achieve a high average fit (Table 13), causing all copula models to perform relatively similar.

Nikkei 225 The hedging effectiveness of the *Nikkei* 225 portfolio is relatively low compared to other single-product portfolios. The hedging effectiveness ranges between 47.64% (found in the most extreme quantiles) and 80.41%. The HS model achieves the highest risk reduction in each hedging objective. Unlike the previously mentioned portfolios, this difference is significant with the best performing copula model (Student's t) on most hedging objectives. This statistically significant difference may be caused by the poor copula fit of the *Nikkei* 225 portfolio compared to other portfolios. Moreover, the likely reason for the poor average performance of all different hedging models is the weak dependence between the Nikkei spot and future index, making it harder to hedge the risk. The Nikkei portfolio also knows a high standard deviation in hedging effectiveness, caused by the high standard deviation in the dependency structure (Kendall's tau) and copula fit.

NASDAQ The HS model achieves the most significant risk reduction for every hedging objective for the *NASDAQ* portfolio. The performance difference is negligible compared to the Student's t and Gaussian copula model. The average hedging effectiveness ranges between 57.91% and 98.37%, where the lowest risk reductions are produced on extreme risk levels. The best performing copula model is the Gaussian copula model for all different hedging objectives.

CAC 40 The average hedging effectiveness for the *CAC* 40 portfolio varies between 83.26% and 99.84%, which is the highest effectiveness across all portfolios that is achieved. The *CAC* 40 is the portfolio that has the highest correlation in terms of Kendall's tau between spot and future returns. Therefore, the highest risk reduction can be achieved when taking positions in the *CAC* 40 spot and future index. While all copula models achieve a high average fit to the *CAC* 40 portfolio, the HS model yields the highest risk reduction on every hedging objective. This difference is not statistically significant to the Gumbel, Student's t and Gaussian copula model.

Dow Jones The *Dow Jones* portfolio yields relatively low hedging effectiveness and high standard deviation of effectiveness. The average hedging effectiveness ranges between 60.81% and 89.38%. Comparable to the *Nikkei* 225 portfolio, the *Dow Jones* portfolio has a high standard deviation in Kendall's tau over time and a considerable low average Kendall's tau resulting in a poor copula fit and a relatively poor hedging performance. The Gumbel copula model achieves the greatest risk reduction on 5 out of 8 hedging objectives, whereas the Student's t copula achieves the most significant risk reduction on the remainder of the objectives. This difference is not statistically significant compared to other hedging models as the standard deviation in effectiveness is high. The standard deviation in hedging effectiveness is seemingly large because of the significant standard deviation in dependency structure.

Hang Seng The average hedging effectiveness for the *Hang Seng* portfolio ranges between 58.31% and 97.96%. The hedging effectiveness is somewhat poor for the extreme quantiles. The best performing hedging model is the Student's t copula on most hedging objectives (5 out of 7), with the difference being statistically insignificant compared to other hedging objectives. The HS model achieves the greatest risk reduction on the remainder of the hedging objectives.

Euro Stoxx 50 For the *Euro Stoxx* 50 portfolio, the average hedging effectiveness varies between 60.70% and 97.51%. The hedging effectiveness has a relatively high standard deviation presumable caused by the high variation in dependency structure. The HS model yields the most significant risk reduction on most hedging objectives (5 out of 8). The Gumbel, Gaussian and Student's t copula's performance is not significantly different caused by the high standard deviation in hedge ratios. The best performing copula model is the Gaussian copula model.

AEX The *AEX* portfolio is among the portfolios that achieve the greatest hedging performance with the effectiveness ranging between 82.23% and 99.14%. The average Kendall's tau for this portfolio is 0.93, which causes a high hedging performance. Most copula models provide a high fit to the data. Nonetheless, the HS model achieves the greatest risk reduction for most hedging objectives. The performance of the HS model is not statistically significant compared to both Gaussian and Student's t copula models.

Table 39: Average risk of the different single product portfolios

S&P 500	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	**0.27 (0.09)**	0.28 (0.11)	0.53 (0.241)	0.34 (0.11)	0.43 (0.15)	0.71 (0.30)	0.02 (0.01)	0.01 (0.01)
Clayton	0.30 (0.14)	0.34 (0.13)	0.72 (0.47)	0.47 (0.24)	0.63 (0.34)	0.98 (0.58)	0.05 (0.05)	0.05 (0.07)
Student's t	0.28 (0.11)	0.28 (0.10)	0.49 (0.18)	0.33 (0.10)	0.42 (0.14)	0.68 (0.29)	0.02 (0.01)	0.01 (0.01)
Gaussian	0.27 (0.09)	0.27 (0.09)	0.51 (0.23)	0.34 (0.14)	0.43 (0.20)	**0.67 (0.33)**	0.02 (0.02)	0.01 (0.02)
HS	0.27 (0.09)	**0.27 (0.09)**	**0.48 (0.17)**	**0.32 (0.10)**	**0.41 (0.14)**	0.70 (0.32)	**0.02 (0.01)**	**0.01 (0.01)**
FTSE 100	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	0.27 (0.05)	0.28 (0.05)	0.50 (0.14)	0.33 (0.06)	0.42 (0.09)	0.66 (0.23)	0.02 (0.01)	0.01 (0.01)
Clayton	0.33 (0.13)	0.34 (0.09)	0.56 (0.16)	0.42 (0.14)	0.52 (0.18)	0.80 (0.27)	0.03 (0.02)	0.02 (0.02)
Student's t	0.28 (0.05)	**0.27 (0.05)**	0.46 (0.11)	0.32 (0.07)	0.39 (0.09)	0.58 (0.18)	0.02 (0.01)	0.01 (0.00)
Gaussian	0.28 (0.06)	0.28 (0.06)	**0.43 (0.10)**	0.32 (0.07)	0.39 (0.08)	0.56 (0.14)	0.02 (0.01)	0.01 (0.00)
HS	**0.27 (0.05)**	0.27 (0.05)	0.44 (0.09)	**0.32 (0.07)**	0.39 (0.09)	**0.56 (0.17)**	**0.02 (0.01)**	**0.01 (0.01)**
DAX	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	0.36 (0.19)	0.36 (0.19)	**0.62 (0.31)**	0.41 (0.20)	**0.49 (0.23)**	0.78 (0.30)	0.03 (0.03)	0.02 (0.03)
Clayton	0.45 (0.23)	0.42 (0.18)	0.70 (0.38)	0.54 (0.27)	0.68 (0.33)	1.10 (0.60)	0.06 (0.05)	0.06 (0.07)
Student's t	0.34 (0.20)	**0.34 (0.20)**	0.61 (0.32)	0.41 (0.20)	0.51 (0.23)	0.81 (0.39)	0.03 (0.03)	0.02 (0.03)
Gaussian	0.38 (0.19)	0.39 (0.21)	0.65 (0.34)	0.42 (0.21)	0.51 (0.24)	**0.76 (0.32)**	0.03 (0.03)	0.02 (0.03)
HS	**0.34 (0.19)**	0.34 (0.20)	0.65 (0.37)	**0.40 (0.21)**	0.51 (0.25)	0.78 (0.35)	**0.03 (0.03)**	**0.02 (0.03)**
Nikkei 225	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	0.71 (0.58)	0.71 (0.57)	1.25 (0.96)	0.96 (0.73)	1.24 (0.86)	2.22 (1.28)	0.26 (0.31)	0.93 (1.21)
Clayton	0.84 (0.55)	0.86 (0.55)	1.39 (0.88)	1.11 (0.61)	1.40 (0.73)	2.26 (1.18)	0.28 (0.26)	0.95 (1.19)
Student's t	0.66 (0.51)	0.71 (0.59)	1.23 (0.94)	0.94 (0.72)	1.22 (0.85)	2.15 (1.33)	0.26 (0.31)	0.94 (1.23)
Gaussian	0.72 (0.56)	0.74 (0.64)	1.25 (0.91)	0.93 (0.68)	1.21 (0.83)	2.14 (1.29)	0.24 (0.27)	0.85 (1.13)
HS	**0.59 (0.57)**	**0.58 (0.53)**	**1.09 (0.80)**	**0.76 (0.50)**	**1.02 (0.66)**	**1.99 (1.18)**	**0.17 (0.20)**	**0.72 (1.12)**
NASDAQ	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	0.58 (0.66)	0.49 (0.36)	1.01 (0.89)	0.63 (0.41)	0.85 (0.61)	1.18 (0.70)	0.09 (0.14)	0.14 (0.35)
Clayton	0.56 (0.40)	0.51 (0.26)	1.20 (0.74)	0.74 (0.41)	0.98 (0.58)	1.47 (0.83)	0.12 (0.16)	0.19 (0.38)
Student's t	0.43 (0.21)	0.42 (0.16)	0.82 (0.43)	0.55 (0.24)	0.71 (0.32)	1.10 (0.53)	0.06 (0.06)	0.06 (0.10)
Gaussian	0.41 (0.17)	0.48 (0.27)	0.79 (0.42)	0.55 (0.32)	0.69 (0.37)	1.03 (0.46)	0.06 (0.08)	0.05 (0.10)
HS	**0.39 (0.14)**	**0.40 (0.16)**	**0.69 (0.21)**	**0.47 (0.15)**	**0.61 (0.19)**	**0.94 (0.30)**	**0.04 (0.02)**	**0.03 (0.02)**

Notes: This table presents average hedged risk of the different single product portfolios.

Table 40: Average Risk of the different single product portfolios

CAC 40	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	0.14 (0.06)	0.16 (0.11)	0.33 (0.10)	0.20 (0.08)	0.27 (0.10)	0.44 (0.13)	0.01 (0.07)	0.00 (0.00)
Clayton	0.19 (0.08)	0.19 (0.07)	0.55 (0.23)	0.29 (0.08)	0.40 (0.11)	0.68 (0.23)	0.02 (0.01)	0.01 (0.01)
Student's t	0.14 (0.06)	0.14 (0.05)	0.30 (0.08)	0.19 (0.05)	0.25 (0.06)	0.44 (0.13)	0.01 (0.00)	**0.00 (0.00)**
Gaussian	0.14 (0.04)	0.15 (0.07)	0.32 (0.14)	0.21 (0.10)	0.29 (0.15)	0.52 (0.25)	0.01 (0.01)	0.01 (0.01)
HS	**0.13 (0.04)**	**0.13 (0.04)**	**0.28 (0.09)**	**0.18 (0.05)**	**0.25 (0.06)**	**0.43 (0.12)**	**0.01 (0.00)**	0.00 (0.00)

Dow Jones	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	0.49 (0.71)	0.32 (0.17)	0.71 (0.55)	**0.49 (0.46)**	**0.58 (0.49)**	**0.97 (0.80)**	**0.07 (0.13)**	**0.09 (0.23)**
Clayton	0.46 (0.39)	0.43 (0.39)	0.66 (0.35)	0.57 (0.41)	0.73 (0.52)	1.08 (0.52)	0.07 (0.11)	0.09 (0.21)
Student's t	**0.37 (0.39)**	**0.34 (0.23)**	**0.64 (0.52)**	0.49 (0.46)	0.62 (0.56)	0.96 (0.79)	0.07 (0.13)	0.12 (0.27)
Gaussian	0.49 (0.61)	0.44 (0.52)	0.70 (0.57)	0.50 (0.46)	0.60 (0.51)	0.95 (0.88)	0.06 (0.12)	0.08 (0.22)
HS	0.41 (0.43)	0.43 (0.46)	0.61 (0.39)	0.49 (0.44)	0.61 (0.50)	0.95 (0.75)	0.06 (0.12)	0.08 (0.19)

Hang Seng	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	0.53 (0.27)	0.51 (0.22)	0.76 (0.31)	0.57 (0.23)	0.69 (0.27)	1.02 (0.36)	0.06 (0.04)	0.05 (0.05)
Clayton	0.51 (0.23)	0.59 (0.24)	1.10 (0.55)	0.73 (0.32)	0.96 (0.45)	1.51 (0.71)	0.12 (0.15)	0.24 (0.49)
Student's t	0.52 (0.23)	0.49 (0.20)	0.79 (0.41)	**0.56 (0.24)**	**0.68 (0.29)**	**0.97 (0.37)**	**0.05 (0.05)**	**0.04 (0.05)**
Gaussian	0.51 (0.22)	**0.53 (0.23)**	0.79 (0.36)	0.56 (0.22)	0.68 (0.27)	0.99 (0.37)	0.05 (0.05)	0.04 (0.05)
HS	**0.49 (0.20)**	0.51 (0.24)	**0.75 (0.32)**	0.56 (0.24)	0.68 (0.30)	1.00 (0.40)	0.06 (0.05)	0.05 (0.07)

Euro Stoxx 50	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	0.41 (0.29)	0.41 (0.28)	0.65 (0.36)	0.46 (0.27)	0.58 (0.34)	0.86 (0.48)	0.04 (0.06)	0.04 (0.06)
Clayton	0.41 (0.23)	0.54 (0.27)	1.08 (0.67)	0.63 (0.31)	0.84 (0.42)	1.41 (0.79)	0.08 (0.09)	0.14 (0.19)
Student's t	0.40 (0.25)	0.42 (0.30)	0.68 (0.41)	0.45 (0.28)	0.56 (0.34)	0.85 (0.62)	0.04 (0.06)	0.04 (0.08)
Gaussian	0.41 (0.24)	**0.38 (0.25)**	0.60 (0.34)	0.45 (0.26)	0.55 (0.31)	**0.75 (0.40)**	0.04 (0.06)	**0.03 (0.06)**
HS	**0.40 (0.27)**	0.38 (0.25)	**0.60 (0.38)**	**0.44 (0.27)**	**0.55 (0.31)**	0.78 (0.46)	**0.04 (0.06)**	0.03 (0.07)

AEX	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	0.17 (0.09)	0.19 (0.12)	0.41 (0.31)	0.26 (0.17)	0.34 (0.23)	0.60 (0.47)	0.02 (0.03)	0.03 (0.08)
Clayton	0.21 (0.16)	0.27 (0.20)	0.54 (0.39)	0.36 (0.24)	0.49 (0.31)	0.90 (0.57)	0.03 (0.04)	0.05 (0.08)
Student's t	0.16 (0.08)	0.17 (0.08)	0.37 (0.19)	**0.23 (0.13)**	0.33 (0.19)	0.57 (0.48)	0.02 (0.02)	0.03 (0.07)
Gaussian	**0.16 (0.07)**	0.18 (0.09)	0.37 (0.19)	0.24 (0.12)	0.33 (0.17)	0.56 (0.43)	0.01 (0.02)	0.02 (0.07)
HS	0.16 (0.07)	**0.16 (0.06)**	**0.33 (0.11)**	0.21 (0.11)	**0.29 (0.17)**	**0.50 (0.43)**	**0.01 (0.02)**	**0.02 (0.07)**

Notes: This table presents average hedged risk of the different single product portfolios.

Table 41: Difference in performance single product portfolios

S&P 500	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	0.10 (0.73)	-0.82 (0.34)	-2.54 (0.22)	-0.86 (0.151)	-0.86 (0.18)	-1.06 (0.54)	-0.33 (0.19)	-0.25 (0.16)
Clayton	-2.37 (0.04)	-4.92 (0.01)	-7.77 (0.02)	-7.66 (0.01)	-8.96 (0.00)	-7.71 (0.01)	-3.93 (0.01)	-1.83 (0.01)
Student's t	-0.46 (0.29)	-0.65 (0.37)	-0.76 (0.43)	-0.23 (0.14)	-0.30 (0.31)	0.30 (0.70)	-0.07 (0.39)	-0.04 (0.40)
Gaussian	-0.22 (0.67)	-0.35 (0.30)	-0.84 (0.32)	-0.36 (0.48)	-0.44 (0.55)	1.07 (0.35)	-0.12 (0.58)	-0.01 (0.89)
FTSE 100	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	-0.13 (0.51)	-0.76 (0.10)	-2.86 (0.06)	-1.16 (0.02)	-1.54 (0.04)	-3.13 (0.05)	-0.53 (0.03)	-0.30 (0.01)
Clayton	-2.89 (0.13)	-4.13 (0.01)	-5.07 (0.01)	-5.26 (0.00)	-5.53 (0.00)	-7.52 (0.001)	-2.19 (0.00)	-1.12 (0.00)
Student's t	-0.51 (0.15)	0.03 (0.92)	-1.05 (0.06)	-0.21 (0.14)	-0.10 (0.65)	-0.66 (0.30)	-0.09 (0.36)	-0.04 (0.47)
Gaussian	-0.49 (0.13)	-0.57 (0.13)	0.12 (0.58)	-0.08 (0.50)	0.11 (0.58)	-0.39 (0.38)	-0.02 (0.69)	-0.03 (0.46)
DAX	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	-1.51 (0.09)	-0.96 (0.47)	0.31 (0.78)	-0.72 (0.46)	0.39 (0.67)	-0.37 (0.81)	-0.04 (0.89)	-0.06 (0.47)
Clayton	-6.00 (0.06)	-4.38 (0.06)	-1.30 (0.34)	-6.04 (0.02)	-6.06 (0.02)	-7.56 (0.02)	-2.81 (0.05)	-1.47 (0.03)
Student's t	-0.12 (0.91)	0.19 (0.85)	1.19 (0.27)	-0.56 (0.35)	-0.09 (0.91)	-0.33 (0.78)	-0.06 (0.84)	-0.06 (0.59)
Gaussian	-2.74 (0.08)	-2.82 (0.34)	-0.84 (0.66)	-0.93 (0.45)	-0.29 (0.81)	0.24 (0.80)	-0.29 (0.58)	-0.08 (0.53)
Nikkei 225	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	-7.38 (0.25)	-7.88 (0.21)	-6.12 (0.08)	-9.60 (0.07)	-8.92 (0.05)	-6.92 (0.10)	-14.41 (0.08)	-14.70 (0.10)
Clayton	-13.68 (0.06)	-14.77 (0.04)	-10.22 (0.03)	-14.92 (0.01)	-13.63 (0.01)	-7.61 (0.04)	-15.00 (0.05)	-13.58 (0.09)
Student's t	-4.44 (0.23)	-7.55 (0.11)	-5.44 (0.10)	-8.48 (0.09)	-7.90 (0.07)	-4.83 (0.15)	-13.20 (0.09)	-13.60 (0.09)
Gaussian	-8.25 (0.23)	-9.35 (0.17)	-6.39 (0.11)	-8.19 (0.10)	-7.54 (0.09)	-4.59 (0.19)	-11.47 (0.10)	-10.35 (0.12)
NASDAQ	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	-7.40 (0.27)	-4.45 (0.18)	-8.71 (0.18)	-6.45 (0.07)	-8.15 (0.08)	-5.75 (0.14)	-5.30 (0.13)	-3.73 (0.21)
Clayton	-7.40 (0.07)	-6.01 (0.04)	-15.42 (0.01)	-11.64 (0.00)	-13.35 (0.00)	-14.39 (0.01)	-9.19 (0.03)	-6.99 (0.05)
Student's t	-2.00 (0.27)	-1.54 (0.35)	-3.22 (0.20)	-3.24 (0.05)	-3.43 (0.05)	-3.94 (0.13)	-2.55 (0.08)	-1.42 (0.12)
Gaussian	-1.18 (0.56)	-3.89 (0.11)	-2.52 (0.27)	-2.78 (0.24)	-2.55 (0.21)	-1.23 (0.43)	-2.09 (0.21)	-0.82 (0.26)

Notes: This table the difference in performance between the HS model and the different copula models. The p-value is denoted in parentheses.

Table 42: Difference in performance single product portfolios

CAC 40	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	-0.40 (0.47)	-1.48 (0.22)	-1.41 (0.05)	-0.89 (0.18)	-0.80 (0.16)	-0.49 (0.18)	-0.15 (0.18)	-0.03 (0.16)
Clayton	-3.59 (0.04)	-3.68 (0.03)	-8.92 (0.00)	-5.82 (0.00)	-6.36 (0.00)	-6.59 (0.00)	-1.87 (0.02)	-0.86 (0.09)
Student's t	-0.47 (0.28)	-0.35 (0.39)	-0.60 (0.02)	-0.19 (0.29)	-0.07 (0.53)	-0.22 (0.29)	-0.04 (0.20)	0.00 (0.62)
Gaussian	-0.65 (0.07)	-0.74 (0.23)	-1.41 (0.07)	-1.23 (0.17)	-1.43 (0.20)	-2.05 (0.20)	-0.35 (0.19)	-0.12 (0.25)
Dow Jones	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	-3.16 (0.67)	7.58 (0.28)	-3.12 (0.54)	2.11 (0.65)	3.61 (0.48)	2.11 (0.73)	1.65 (0.67)	2.63 (0.58)
Clayton	-3.10 (0.16)	0.91 (0.87)	-2.35 (0.54)	-4.62 (0.18)	-4.96 (0.28)	-3.39 (0.58)	-2.36 (0.31)	0.39 (0.91)
Student's t	3.66 (0.32)	7.22 (0.35)	0.53 (0.95)	1.77 (0.70)	1.67 (0.72)	1.64 (0.64)	1.28 (0.74)	0.58 (0.90)
Gaussian	-4.88 (0.19)	-0.60 (0.75)	-3.43 (0.60)	0.11 (0.97)	1.51 (0.69)	2.01 (0.70)	0.51 (0.79)	2.25 (0.54)
Hang Seng	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	-1.72 (0.29)	-0.14 (0.89)	-0.56 (0.66)	-0.51 (0.37)	-0.66 (0.39)	-0.80 (0.54)	-0.18 (0.53)	-0.33 (0.27)
Clayton	-0.83 (0.39)	-4.70 (0.16)	-11.60 (0.00)	-7.86 (0.00)	-10.51 (0.00)	-13.83 (0.00)	-6.24 (0.01)	-5.94 (0.03)
Student's t	-1.44 (0.18)	0.65 (0.49)	-0.97 (0.30)	-0.09 (0.86)	0.16 (0.61)	0.69 (0.46)	0.14 (0.33)	0.08 (0.55)
Gaussian	-1.06 (0.45)	-1.35 (0.29)	-1.45 (0.27)	-0.35 (0.56)	-0.21 (0.77)	0.03 (0.97)	-0.14 (0.71)	-0.08 (0.69)
Euro Stoxx 50	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	-0.62 (0.19)	-1.07 (0.30)	-2.10 (0.23)	-0.96 (0.43)	-1.56 (0.43)	-2.61 (0.26)	-0.90 (0.28)	-0.64 (0.22)
Clayton	-1.36 (0.32)	-8.47 (0.01)	-16.46 (0.02)	-8.21 (0.01)	-10.78 (0.00)	-18.01 (0.01)	-5.72 (0.01)	-8.72 (0.16)
Student's t	-0.38 (0.54)	-2.14 (0.25)	-2.70 (0.14)	-0.15 (0.51)	-0.36 (0.52)	-0.82 (0.66)	-0.15 (0.33)	-0.15 (0.54)
Gaussian	-0.92 (0.26)	0.41 (0.48)	-0.46 (0.63)	-0.61 (0.18)	-0.28 (0.61)	0.46 (0.58)	-0.31 (0.24)	0.01 (0.95)
AEX	VaR90	VaR95	VaR99	ES90	ES95	ES99	SV	LPM
Gumbel	-0.10 (0.85)	-1.07 (0.22)	-2.04 (0.17)	-1.79 (0.05)	-1.64 (0.03)	-2.47 (0.04)	-0.47 (0.05)	-0.16 (0.05)
Clayton	-1.87 (0.13)	-5.44 (0.00)	-6.05 (0.02)	-6.41 (0.00)	-6.94 (0.00)	-11.00 (0.00)	-2.18 (0.00)	-0.82 (0.04)
Student's t	0.13 (0.70)	-0.42 (0.37)	-1.05 (0.21)	-0.92 (0.22)	-0.99 (0.30)	-1.32 (0.38)	-0.24 (0.25)	-0.01 (0.92)
Gaussian	0.35 (0.15)	-0.81 (0.14)	-1.08 (0.14)	-1.23 (0.04)	-1.22 (0.11)	-1.52 (0.09)	-0.24 (0.06)	-0.01 (0.83)

Notes: This table the difference in performance between the HS model and the different copula models. The p-value is denoted in parentheses.

Table 43: Single product unhedged risk

S&P 500	2003-2004	2004Q3-2005Q2	2006-2007	2007Q3-2008Q2	2009-2010	2010Q3-2011Q2	2012-2013	2013Q3-2014Q2	2015-2016	2016Q3-2017Q2	2018-2019
VaR90	1.84	1.54	0.98	0.79	1.44	2.14	1.68	1.13	0.81	0.95	0.83
VaR95	2.38	1.99	1.32	1.04	2.38	3.01	2.41	1.68	1.23	1.40	1.25
VaR99	3.46	3.16	1.78	1.60	5.41	5.43	4.37	2.92	2.09	2.18	2.18
ES90	2.56	2.24	1.39	1.16	3.09	3.64	2.76	2.01	1.34	1.56	1.46
ES95	3.05	2.74	1.67	1.40	4.27	4.71	3.48	2.61	1.67	1.94	1.89
ES99	4.32	3.80	2.45	2.01	7.68	7.68	5.10	4.25	2.32	2.83	2.99
SV	1.07	0.77	0.30	0.20	1.47	1.96	1.07	0.59	0.25	0.35	0.30
LPM	2.69	1.76	0.44	0.26	7.59	9.31	3.39	1.68	0.36	0.61	0.57
FTSE 100	2003-2004	2004Q3-2005Q2	2006-2007	2007Q3-2008Q2	2009-2010	2010Q3-2011Q2	2012-2013	2013Q3-2014Q2	2015-2016	2016Q3-2017Q2	2018-2019
VaR90	1.23	0.87	0.67	1.62	2.03	1.02	0.82	0.71	1.04	0.38	0.86
VaR95	1.47	1.08	0.96	2.36	2.73	1.59	1.26	1.08	1.43	0.66	1.75
VaR99	2.52	1.46	1.60	2.98	4.66	2.31	1.72	2.11	2.60	1.83	3.29
ES90	1.74	1.16	1.08	2.37	3.14	1.69	1.37	1.24	1.74	0.97	2.04
ES95	2.08	1.32	1.32	2.75	3.89	2.07	1.69	1.58	2.17	1.379	2.67
ES99	3.06	1.63	1.78	3.13	5.08	2.98	2.38	2.32	3.42	2.66	3.78
SV	0.48	0.20	0.17	0.85	1.39	0.40	0.27	0.22	0.46	0.17	0.57
LPM	0.86	0.23	0.19	1.84	4.60	0.73	0.39	0.32	0.94	0.35	1.43
DAX	2003-2004	2004Q3-2005Q2	2006-2007	2007Q3-2008Q2	2009-2010	2010Q3-2011Q2	2012-2013	2013Q3-2014Q2	2015-2016	2016Q3-2017Q2	2018-2019
VaR90	2.15	0.92	1.19	1.50	1.87	1.17	1.28	0.90	1.91	0.76	1.40
VaR95	3.09	1.22	1.62	1.85	2.74	1.71	1.98	1.49	2.47	1.22	1.60
VaR99	4.53	2.06	2.31	3.42	4.74	2.13	3.23	2.38	3.63	1.83	2.35
ES90	3.44	1.40	1.80	2.36	3.12	1.79	2.15	1.65	2.68	1.56	1.85
ES95	4.19	1.697	2.17	2.98	3.89	2.14	2.59	2.16	3.21	2.05	2.17
ES99	5.21	2.45	2.91	5.57	5.09	2.95	3.45	3.11	4.11	4.12	2.90
SV	1.72	0.29	0.47	0.99	1.47	0.47	0.65	0.39	1.06	0.44	0.55
LPM	5.89	0.44	0.88	3.42	4.67	0.86	1.43	0.78	2.79	1.73	0.97
Nikkei 225	2003-2004	2004Q3-2005Q2	2006-2007	2007Q3-2008Q2	2009-2010	2010Q3-2011Q2	2012-2013	2013Q3-2014Q2	2015-2016	2016Q3-2017Q2	2018-2019
VaR90	1.66	1.06	1.56	2.11	2.34	1.57	1.14	1.64	1.44	1.14	1.140
VaR95	2.41	1.31	1.921	2.510	2.701	1.889	1.499	2.194	2.12	1.45	1.88
VaR99	3.78	2.13	2.88	4.59	4.01	2.90	2.11	3.35	3.91	2.15	3.79
ES90	2.68	1.53	2.23	3.18	3.10	2.53	1.69	2.51	2.50	1.95	2.19
ES95	3.23	1.89	2.66	3.98	3.65	3.26	2.02	3.01	3.14	2.55	2.87
ES99	4.46	2.85	3.443	5.40	4.86	7.05	2.73	3.91	4.30	5.62	4.48
SV	1.03	0.38	0.764	1.58	1.39	1.26	0.44	0.924	0.88	0.72	0.71
LPM	2.89	0.67	1.67	5.18	4.22	7.49	0.72	2.29	2.42	3.31	2.00

Notes: This table presents the average unhedged risk over the different out-of-sample windows of the single product portfolios.

Table 44: Single product unhedged risk

NASDAQ	2003-2004	2004Q3-2005Q2	2006-2007	2007Q3-2008Q2	2009-2010	2010Q3-2011Q2	2012-2013	2013Q3-2014Q2	2015-2016	2016Q3-2017Q2	2018-2019
VaR90	1.87	1.48	1.09	2.09	1.98	1.20	1.09	0.90	1.19	0.60	1.45
VaR95	2.41	1.83	1.54	2.51	2.77	1.61	1.40	1.30	1.65	0.88	2.26
VaR99	3.60	2.43	2.22	3.07	3.80	2.88	2.50	2.34	2.91	2.47	3.99
ES90	2.67	1.94	1.68	2.67	2.97	1.89	1.61	1.54	2.02	1.31	2.66
ES95	3.15	2.19	2.03	2.97	3.49	2.30	1.98	1.97	2.55	1.83	3.39
ES99	4.08	2.61	2.54	3.69	4.52	3.28	2.60	2.76	3.79	3.09	4.52
SV	1.05	0.58	0.46	1.13	1.18	0.50	0.40	0.33	0.60	0.27	0.97
LPM	2.70	1.01	0.74	2.73	3.53	1.06	0.66	0.59	1.41	0.61	2.92
CAC 40	2003-2004	2004Q3-2005Q2	2006-2007	2007Q3-2008Q2	2009-2010	2010Q3-2011Q2	2012-2013	2013Q3-2014Q2	2015-2016	2016Q3-2017Q2	2018-2019
VaR90	1.69	0.75	1.08	1.71	1.79	1.30	1.57	0.92	1.69	0.83	1.11
VaR95	2.54	1.05	1.44	2.13	2.45	1.91	2.23	1.25	2.30	1.12	1.48
VaR99	3.74	1.94	2.45	3.57	4.34	2.67	2.87	2.68	3.48	1.89	2.14
ES90	2.82	1.25	1.69	2.61	2.89	2.05	2.31	1.53	2.60	1.57	1.67
ES95	3.46	1.58	2.10	3.25	3.66	2.54	2.73	1.94	3.21	2.15	1.97
ES99	4.82	2.28	2.96	5.15	4.54	3.30	3.24	3.08	4.32	4.57	2.72
SV	1.17	0.25	0.43	1.17	1.27	0.66	0.77	0.37	1.00	0.53	0.41
LPM	3.49	0.34	0.79	3.64	3.74	1.35	1.70	0.71	2.75	2.65	0.69
Dow Jones	2003-2004	2004Q3-2005Q2	2006-2007	2007Q3-2008Q2	2009-2010	2010Q3-2011Q2	2012-2013	2013Q3-2014Q2	2015-2016	2016Q3-2017Q2	2018-2019
VaR90	1.13	0.90	0.65	1.64	1.67	0.95	0.79	0.82	1.07	0.38	1.03
VaR95	1.58	1.08	0.94	2.06	2.36	1.40	1.10	1.01	1.55	0.58	1.75
VaR99	2.16	1.59	1.65	2.68	4.09	2.25	1.84	1.64	2.10	1.51	3.14
ES90	1.71	1.18	1.07	2.19	2.72	1.52	1.26	1.19	1.72	0.09	2.10
ES95	2.05	1.35	1.34	2.52	3.41	1.84	1.56	1.44	2.11	1.27	2.74
ES99	2.94	1.711	1.89	3.01	4.41	2.59	2.20	2.15	3.23	2.47	4.05
SV	0.45	0.23	0.17	0.74	1.06	0.32	0.24	0.21	0.45	0.14	0.62
LPM	0.80	0.25	0.20	1.48	3.06	0.52	0.32	0.27	0.84	0.28	1.64
Hang Seng	2003-2004	2004Q3-2005Q2	2006-2007	2007Q3-2008Q2	2009-2010	2010Q3-2011Q2	2012-2013	2013Q3-2014Q2	2015-2016	2016Q3-2017Q2	2018-2019
VaR90	1.08	0.77	1.01	2.71	2.52	1.34	1.18	1.11	1.32	0.83	1.42
VaR95	1.53	1.17	1.32	3.67	2.95	1.56	1.39	1.54	1.79	1.38	2.11
VaR99	2.16	1.62	2.44	5.52	3.93	2.13	2.44	2.27	3.14	2.12	3.13
ES90	1.72	1.28	1.60	4.13	3.27	1.76	1.71	1.75	2.28	1.52	2.30
ES95	2.12	1.51	1.98	4.96	3.74	2.07	2.10	2.10	2.95	1.89	2.84
ES99	3.05	1.92	2.89	6.75	4.91	2.65	2.97	2.79	4.85	2.85	4.00
SV	0.43	0.23	0.37	2.55	1.73	0.47	0.46	0.43	0.85	0.33	0.84
LPM	0.86	0.28	0.68	11.10	5.20	0.79	0.82	0.76	2.55	0.59	2.06

Notes: This table presents the average unhedged risk over the different out-of-sample windows of the single product portfolios.

Table 45: Single product unhedged risk

Euro Stoxx 50	2003-2004	2004Q3-2005Q2	2006-2007	2007Q3-2008Q2	2009-2010	2010Q3-2011Q2	2012-2013	2013Q3-2014Q2	2015-2016	2016Q3-2017Q2	2018-2019
VaR90	1.81	0.81	1.11	1.63	1.79	1.28	1.55	0.94	1.78	0.80	1.10
VaR95	2.54	0.96	1.40	1.93	2.58	1.76	2.19	1.39	2.40	1.18	1.49
VaR99	4.40	1.97	2.39	3.85	4.81	2.48	2.92	2.59	3.68	1.99	2.21
ES90	2.97	1.24	1.67	2.48	3.06	2.03	2.31	1.64	2.71	1.63	1.63
ES95	3.64	1.57	2.07	3.12	3.88	2.44	2.73	2.03	3.30	2.20	1.93
ES99	4.91	2.30	2.90	5.45	5.03	3.22	3.18	3.22	4.53	4.65	2.69
SV	1.31	0.24	0.42	1.08	1.44	0.64	0.77	0.39	1.07	0.57	0.42
LPM	4.02	0.34	0.76	3.68	4.60	1.29	1.67	0.78	3.04	3.18	0.69
AEX	2003-2004	2004Q3-2005Q2	2006-2007	2007Q3-2008Q2	2009-2010	2010Q3-2011Q2	2012-2013	2013Q3-2014Q2	2015-2016	2016Q3-2017Q2	2018-2019
VaR90	2.22	0.85	0.97	1.62	1.89	1.02	1.17	0.74	1.50	0.85	1.11
VaR95	3.19	1.16	1.18	2.11	2.55	1.60	1.63	1.07	2.17	1.15	1.49
VaR99	4.99	1.60	2.52	3.80	4.91	2.34	2.52	2.34	3.57	1.93	2.07
ES90	3.49	1.30	1.55	2.51	3.09	1.74	1.83	1.29	2.48	1.44	1.64
ES95	4.37	1.60	1.99	3.13	3.93	2.08	2.22	1.67	3.05	1.85	1.99
ES99	5.82	2.48	2.92	4.67	5.25	2.95	2.69	2.60	4.24	3.68	2.97
SV	1.86	0.26	0.37	1.04	1.44	0.46	0.49	0.26	0.91	0.37	0.40
LPM	6.69	0.39	0.68	2.99	4.78	0.81	0.87	0.41	2.44	1.12	0.71

Notes: This table presents the average unhedged risk over the different out-of-sample windows of the single product portfolios.