

Proportional representation: fair to the voter?

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The most important criterion for a good voting procedure is that it be *fair*; and a voting procedure is fair if it reflects as accurately as possible the preferences of the voters. (Dummett [1984](#), 29)

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Introduction

This thesis discusses the question in how far proportional representation is fair to the voters. I will first show that this discussion needs more than plainly using formal tools that analyse the properties of an election rule. Instead we need some conceptual base of what fair representation as such is. Therefore, second, I will present and discuss three different suggestions for an electoral system that yields proportional representation. While doing so I elicit their underlying conceptual arguments. Finally, using the aforementioned formal tools I will be able to show to what extent and in which respect those proposals for proportional representation can be said to be fair to the voters.

Motivation

Focusing on the voters when analysing the fairness of an electoral system for representative democracies seems to be obvious. Voters demand this over and over again: be it for instance the yellow vest movement in France, or the demonstrations around the fall of the Berlin Wall 1989, where the protesters chanted: “*We* are the people!” The people in the German Democratic Republic lived in a non-democratic state and demanded fair representation. By contrast, in France as a democratic state the people protested because they did not see their democratic elected government to be a fair representation of themselves. Thus, there are two reasons why fair representation is an important topic: First, fair representation is an important value *in itself* that is demanded and fought for. Second, democracies—in particular *representative* democracies—are aimed at realising representation, and most people would at

least vaguely agree that this representation ought to be fair.

The yellow vest movement as well as many other protests and populist movements worldwide are indications that representative democracies are in crisis: people feel alienated, disillusioned by their elected politicians, reject political participation or support populist parties (Valgarðsson 2019; Dahl et al. 2018; Evans, Stoker, and Nasir 2013). It is obvious that many voters in democracies do not believe that they are represented fairly.

Comparing this situation with the philosophical literature on democracy and its justification we find many discussions on how well democracies perform in contrast to other systems or on the intrinsic values of democracy (Christiano and Bajaj 2022). The former are mainly aimed at the good laws and policies democratic institutions produce (Mill 1861; Acemoglu et al. 2019; Sen 1999), or refer to the good effects of cognitive diversity and the rule of the many (Landemore 2012). The latter focus on values as liberty (Gould 1995) and especially equality (Singer 1973; Waldron 2004).

While it is the *peoples' fair representation* that matters, there is much less discussion in the philosophical literature of the aspect that democracy ought to be fair, and that representation in a democracy ought to be fair. With this thesis I aim to take the demand of the Berlin Wall and yellow vest protesters, and all other voters center-stage: it is their fair representation that should be achieved in democratic elections of representatives.

Research question

There are broadly speaking two different electoral systems in use to elect representatives in democracies: (1) single-member district elections as for instance the General Elections in the United Kingdom for the Parliament, and (2) electoral systems that aim for proportional representation. While the first is often regarded as leading to stable governments, proponents of the second claim that proportional representation is the fairer system (Christiano and Bajaj 2022; Blau 2004).

What exactly fairness means in the context of political representation and in particular in proportional representation is usually left vague. Blau

2004 criticizes this vagueness in the context of electoral systems and requests: “Conceptual clarity requires that protagonists identify explicitly which ideas of fairness they favour, or preferably, that they simply avoid the misleading and overly rhetorical language of fairness.” (Blau 2004, 165) I am not going to follow Blau’s lead here and avoid the language of fairness but rather contrarily focus on the fairness of proportional representation. However, I agree with him that we have to be clear about the concept of fair representation that is applied. I will mainly do so by zeroing in on the issue of *whom* proportional representation is fair *to*.

Relevance and contributions

My thesis analyses three competing answers to this very question who is treated fairly with proportional representation. The first will be Michael Dummett’s proposal that protects *groups of minority opinions* (Dummett 1984). This will be followed by the discussion of the account of John R. Chamberlin and Paul N. Courant who derive from the ideal of direct democracy *for each individual voter a claim to be represented as well as possible* (Chamberlin and Courant 1983). Finally, I introduce and question Eliora van der Hout and Anthony J. McGann’s approach that is based on a libertarian understanding of fairness and aims to *treat all individual voters equally* in the process of electing representatives (van der Hout and McGann 2009b, 2009a).

While I aim for analysing fairness to the voters (be it to the individual voters or groups of voters), this distinction between fairness to the voters and fairness to parties often gets blurred in discussions about which rule should be applied to achieve a fair representation. This happens because the arguments are not clear on what is meant by fairness and are rather based on intuitive reasoning (Blau 2004). Even more, Lagerspetz 2016 criticises the advocates of proportional representation for not considering the social choice literature. The three accounts I am going to present and discuss in this thesis are the very few exceptions who do exactly this. Dummett, Chamberlin and Courant as well as van der Hout and McGann approach the topic of representative

elections by (1) making use of the formal models belonging to social choice theory, committee selection and/ or fair allocation problems, but also (2) combine it with a conceptual foundation of fair representation.

I explore what these authors view to be *fair representation of the voters* and discuss to what extent the three different proposals of proportional representation can be said to be fair to the voters. Even though the proposals all agree on proportional representation, their underlying and partially implicit views on fairness to the voters are very different. This thesis thus makes three contributions: (1) it clarifies what it means to treat an individual voter fairly in contrast to treating groups of voters or parties fairly, (2) it compares three hitherto unconnected accounts of proportional representation as fair to the voters, (3) it elucidates possible different aims of fair representation as for instance representation of minorities, representation in deliberation and/or decision making, and therefore also differentiates between the underlying conceptual ideas of fair representation.

More specifically, this thesis uses formal tools from the literature on social choice theory, committee selection and allocation problems to approach the question in which way the authors' proposals of proportional representation are fair to the voter in a rigorous way, just as Blau [2004](#) requests. These tools will enable me to pinpoint the weaknesses and strengths of the different electoral systems. On the one hand, Dummett and Chamberlin and Courant's suggestions rely on social choice theory and to a large extent on considerations about committee selection. Van der Hout and McGann, on the other hand, apply results from social choice theory to the problem of seat allocation.

My analysis shows the importance of the question which framework is used and which specific further assumptions and adaptations of them are applied in each proposed account of proportional representation. Thus, we need to first understand the differences and peculiarities of each framework to appreciate the results of the analysis.

Furthermore, I will evaluate the results the literature on social choice theory, committee selection and allocation problems has to offer by itself to analyse the question to which extent proportional representation is fair to

the voter. I show that these findings are not enough to answer my question whether proportional representation is fair to the voters. Until now, there has been few research on proportionality and representative elections in general in social choice theory or committee selection. Using the literature on allocation problems seems to assume proportionality is a good in itself and mainly treats the candidates and parties fair.

While the review of the formal literature shows that there are two ways in which representation can be said to be fair (voters or parties), the discussion of the three proposals of proportional representation will lead to a more precise distinction: (1) fairness to the parties or candidates, (2) fairness to groups of voters, and (3) fairness to the individual voters.

Chapter summaries

To present the discussion and achievements of my research this thesis is structured as follows.

Part I

In the first part I will show how the axiomatic approaches social choice theory, committee selection rules, allocation rules and apportionment theory can be used to analyze elections in general and more specifically proportional representation. These formal tools all have their disadvantages if we want to set the focus on the fair treatment of the voters.

Chapter 1 deals with elections of a single winner. It introduces the basic and main notation that will be used throughout the thesis. While doing so, I also introduce basic axioms and aggregation rules, and important theorems and their results that will reappear later on.

Chapter 2 deals with elections where more than one candidate is chosen. It shows the differences between selecting several best candidates and electing representative committees. While doing so, I problematize the missing research on proportionality but also representativeness in general.

Chapter 3 deals with allocating the seats of a committee to the candidates or parties. I first show how seats can be allocated such as to be as proportional as possible to the received vote shares, assuming that proportionality is desirable. Even with this assumption there are several questions how to approximate proportionality without bias to small or large parties. I will then go on and introduce the more general theory of claims problems, some axioms and allocation rules. I thereby show that the intuitive justification of proportionality is based on fairness to parties and only derivatively on fairness to voters (fairness is understood as proportional satisfaction of claims; parties have claims to seats in virtue of the votes they received).

Part II

Part II introduces the the proposals from Dummett, Chamberlin and Courant, and van der Hout and McGann. I discuss to what extent their models of proportional representation treat the voters fairly. First I describe each account's framework, elucidate the background assumptions what fair representation means and how each account proposes to model those assumptions into an electoral system of proportional representation. Further, I evaluate and discuss those proposals. This discussion will elucidate different concepts of what fair representation means and to what extent each proposal is fair to the voter.

Chapter 4 describes and analyzes Dummett's requirement for an electoral system to be fair: to protect minorities. This means that he views it as necessary to not only represent the majority but also other prevailing opinions. To achieve this aim Dummett proposes one property of an election rule: the solid coalitions property.

Chapter 5 introduces and discusses how Chamberlin and Courant want to satisfy each individual voter's claim for being represented as well as possible. To this aim, they develop a modified version of the Borda rule. Each individual is represented by one candidate as well as possible according to the individual's ranking of the candidates.

Chapter 6 finally shows and analyzes van der Hout and McGann's pro-

posal of proportional representation which treats each individual voter equally. They aim to show that interpreting the election as an allocation problem that allocates seats to the parties while satisfying certain axioms that guarantee the fair treatment of the individual voters implies proportional representation.

For now, let me start with the introduction and discussion of the literature on social choice, committee selection and allocation problems.

Part I

Elections: preference aggregation and fair allocation

Chapter 1

Electing the best candidate

This chapter deals with elections of one single winner. Imagine there are seven voters and three candidates. The individual voters have each a preference ranking over the candidates from the one they favour the most to the candidate they see least likely to fit the position. Our task is to find out who is the collectively most preferred candidate according to the individual preference rankings.

To do so, I use the axiomatic approach of social choice theory. This means that I focus on *rules* that derive the collectively most preferred alternative from the given input. Such rules have different specific properties (for instance anonymity, the Pareto criterion or that there is a dictator who is decisive, or rather not a dictator). Those properties are more or less desirable in certain contexts. Thus, social choice theory deals a lot with those properties: how to formulate the properties as axioms, finding rules that satisfy a set of certain axioms, or developing theorems which axioms cannot be satisfied at the same time by any rule.

Another question could be how exactly the input has to be like: are the individuals' complete rankings of all alternatives taken into account or can they state only their most preferred option? I will present different ways how to model elections where these differences will play an important role.

To discuss those questions, it is helpful to begin with a reformulation of the input into a clear structure. Let me illustrate a first possible reformula-

tion with the example of the election of a president.

1.1 President election

Take the seven voters from above. Their preferences over the three candidates can be presented as follows:

$$N = \{v_1, v_2, \dots, v_7\}, A = \{c_1, c_2, c_3\}$$

$v_1 : c_1, c_2, c_3,$

$v_2 : c_1, c_2, c_3,$

$v_3 : c_2, c_1, c_3,$

$v_4 : c_2, c_1, c_3,$

$v_5 : c_3, c_2, c_1,$

$v_6 : c_3, c_2, c_1,$

$v_7 : c_3, c_2, c_1.$

In this election we have seven voters v_1 to v_7 forming the set N and three candidates c_1 , c_2 and c_3 forming the set A . Each voter was asked for a ranking over all candidates. The candidates are then just listed from the first to the last rank. Voter 1 thus ranks candidate 1 the highest, candidate 2 second and most disfavours candidate 3 for the position.

Which of the three candidates should be the winner of the election? Candidate c_3 maybe? c_3 is ranked first more often than any of the other candidates. Although, all the other voters most dislike c_3 , which seems to outweigh the three other voters. While c_1 and c_2 both are most liked by two voters each, c_1 is most disliked by three voters, while c_2 is never ranked lower than second.

I started this chapter with the task to find a way to elect the collectively most preferred candidate. Rules which compute a set of those candidates to which no other candidate is preferred to, are often called *social choice functions*, while a rule that always computes exactly one winning alternative

are called *social decision functions*.¹

But sometimes we do not only want to know the most preferred alternative but the whole collective preference order. Probably not in an election of a president, but think of an open position in a company. In such a case it might be good to have a full ranking, in case the first candidate decides to take the offer of a different company. Therefore, it is sometimes desirable to have a complete preference order and then choose the highest ranked option that is still available. Those rules that produce a complete collective ranking are often called *social welfare functions*. Social welfare functions will not play an important role in this discussion here, but they will serve as an example for what we do not want when dealing with representativeness later on.²

1.1.1 Some aggregation rules and axioms

To illustrate some axioms and one social choice function which will reappear in several ways in this thesis let me introduce *May's theorem* (May 1952). The axioms and the theorem will also play an important role when discussing van der Hout and McGann's approach in Chapter 6. The theorem proves that in situations with exactly two alternatives any social choice function that satisfies the three axioms of *anonymity*, *neutrality* and *positive responsiveness* is the *simple majority rule*.

To illustrate what exactly this means let me begin with explaining the property of anonymity.

Definition 1. (Anonymity) The social choice function depends only on the individuals' rankings independent of the respective individuals' names,

¹According to a social choice function there can be several alternatives which all qualify in the same way for being the best alternative: take for instance the rule that collectively ranks alternatives according to the number of individuals who rank them first. If there are ten individuals of which five prefer alternative *a* to alternative *b*, and five prefer *b* to *a* and there are no other alternatives, then this rule will rank both alternatives as the collectively most preferred one.

²There remains another issue: If we take away the availability of one alternative, there are welfare functions that will lead to a different ranking than if this alternative were available. The property a rule has to satisfy to avoid this is the *independence of irrelevant alternatives*.

i.e. if the names of the individuals are swapped around and everything else stays the same, the outcome does not change.

In formal terms this does not necessarily mean that the names of the individuals are not known, but rather that it does not matter which individual has which preference order. It should not matter which voter prefers one candidate over another.

In the same way as the names of the individual voters are ignored if anonymity is satisfied, neutrality ignores the names of the candidates:

Definition 2. (Neutrality) The social choice function does not favour an alternative, i.e. if the names of the alternatives are swapped around and everything else stays the same, the outcome does only change according to the change of the alternatives' names.

The third axiom—positive responsiveness—requires a rule to acknowledge if an alternative raises in the favour of the individuals. Imagine that the rule selects candidate c_1 and c_2 as the two possible winners. Now voter 5 recognizes that there is a mistake in her preference order and she actually prefers c_2 over c_3 over c_1 . Positive responsiveness requires that now c_2 is the only winner.

Definition 3. (Positive responsiveness) If the social choice function selects alternative x as the or one of the best alternatives for preference profile \mathbf{P} , then the same social choice function selects x as the only best alternative for preference profile \mathbf{P}' , if $\mathbf{P} = \mathbf{P}'$ with the exception that x is ranked better at least once and everything else remains the same.

May shows that there is exactly one social choice function that can satisfy all three axioms. It is the simple majority rule which can be defined as follows.

Definition 4. (Simple majority rule) The simple majority rule is the social choice function that selects from two alternatives x and y the alternative as the best one that is ranked first by the majority of the individuals. If both alternatives are ranked first by half of the individuals, both are selected as the best alternative.

An important issue is that the simple majority rule only works for situations where two alternatives are given but not for more. If we generalized the rule to more alternatives, there are possible preference profiles for which there exists no majority of individuals who rank one alternative first. In such situations the rule would leave us with an empty set of best alternatives.

This is almost characteristic for social choice theory: there are many impossibility results. Beginning with Arrow's theorem that there exists no social welfare function that satisfies a set of four desirable axioms (Arrow 1951), but also that all the common social choice rules, have always some undesirable properties.³ Usually the decision for one or another social choice rule for a specific context will involve a trade-off between those properties.

One such rule is the Borda rule. It does not have the property of independence of irrelevant alternatives (for the idea of this property see Footnote 2) and allows for strategic voting and manipulation. However, it has also many desirable properties and in some contexts the missing properties are not a problem or are the necessary cost for having those other desirable properties.

The definition of the Borda rule is the following:

Definition 5. (Borda rule) The Borda rule is the social choice function that assigns scores to each alternative depending on the rankings of the individuals. Given that there are m alternatives, an alternative gets $m + 1 - r$ points from each voter, where r is the rank assigned by the voter (with $r = 1$ being the most preferred alternative). The alternatives with the highest sum of points are the best alternatives.

While this might sound very technical, it actually just means that for every first rank an alternative gets as many points as there are candidates, and one less for second rank, another one less for third rank and so on until 1 point for the last rank. For President Election the Borda rule will assign 3 points for each first rank, 2 points for second rank, and 1 point for the last rank. Therefore, candidate c_1 would receive $2 \times 3 + 2 \times 2 + 3 \times 1 = 13$

³Arrow's axioms are unrestricted domain, weak Pareto, independence of irrelevant alternatives and non-dictatorship. One of the most striking undesirable properties most rules have is that of allowing for strategic voting (Gibbard 1973; Satterthwaite 1975).

points, c_2 receives $2 \times 3 + 5 \times 2 + 0 \times 1 = 16$ points. Candidate c_3 receives $3 \times 3 + 0 \times 2 + 4 \times 1 = 13$ points. Thus, the Borda winner is candidate c_2 .

The Borda rule satisfies among other axioms anonymity and neutrality as shown by Young 1974. Young also presented a proof for the characterization of the Borda rule. Characterization means that the given set of axioms are satisfied by the Borda rule but also that any rule that satisfies these properties is the Borda rule. We will come back to this in the discussion of Chamberlin and Courant's proposal in Chapter 5.

1.2 President: single and tallied-vote election

In many elections voters are not asked for a complete ranking of all candidates but rather only mark the box next to their most favoured candidate on the ballot. This makes it less cumbersome for the voters instead of ranking all candidates.

President – single-vote election

$$N = \{v_1, v_2, \dots, v_7\}, A = \{c_1, c_2, c_3\}$$

$$v_1 : c_1$$

$$v_2 : c_1$$

$$v_3 : c_2$$

$$v_4 : c_2$$

$$v_5 : c_3$$

$$v_6 : c_3$$

$$v_7 : c_3$$

Furthermore, usually anonymity is assumed and the social choice rule that is used is the *plurality rule* which selects the winner in the following way:

Definition 6. (Plurality rule) The plurality rule is the social choice function which selects as the best alternative the one that gets the most first ranks.

This rule makes it especially much easier for the tellers who now just have to make piles of ballots for each candidate and then count them. Such elections can be called tallied-vote elections (Wintein and Heilmann 2022, see). The example of the election of the president will then be translated in the following way:

President – tallied-vote election

$$N = \{v_1, v_2, \dots, v_7\}, A = \{c_1, c_2, c_3\}$$

$$c_1 : 2$$

$$c_2 : 2$$

$$c_3 : 3$$

We still have the set of individuals who are the voters, and the set of alternatives who are the candidates running for office. Instead of a list stating each voters' most favourite candidate only a vector is given. The first number denotes the number of ballots stating the first candidate, the second number the second candidate, and so on.

It is easy to see who is the winner when we use the plurality rule: it is candidate c_3 who receives three votes and is the plurality winner of the election. Which is a different candidate than before where we used the Borda rule.

If rules would not select different winners for the same profiles it might not be interesting to study the field of social choice theory. But, it is also one reason why some people—most famously Riker—proposed that democracy seems meaningless. This is especially the case if we also add the result of the Gibbard-Satterthwaite theorem: there exists no social choice function that is not dictatorial and at the same time strategy-proof (Gibbard 1973;

Satterthwaite 1975). This means that every social choice function which does not have as the output the preference ranking of one individual (the dictator), offers for at least one individual in some case a better result if this individual does not state the true preference ranking but a strategically amended ranking. Thus, almost every rule offers individuals reasons to not vote faithfully. Riker concluded therefore “that the outcomes of voting are not necessarily fair and true amalgamations of voters’ values, that these outcomes may be meaningless” (Riker 1988, 233).

However, this thesis does not deal with strategyproofness or the true amalgamation of voters values, but rather with the conceptual meaning of fair representation of the voters by an elected parliament. I will assume for the discussion that the voters state their true preferences. What exactly a fair representation of the voters means is the main issue of the upcoming discussion.

Chapter 2

Electing several candidates: best vs representative

Lagerspetz [2016](#) refers to two functions of an election. As described above we can hold an election to select the winner as for example a president from a set of candidates. However, especially in parliamentary democracies, an election often is hold to select a representative parliament. According to Lagerspetz there has not been much connection between social choice theory and representative elections. There have been a few exceptions, but social choice theorists mostly focused on the axiomatic approach for single-winner elections and the literature dealing with parliamentary elections was much less studied from such a formal perspective (Lagerspetz [2016](#), 129; see also Chamberlin and Courant [1983](#), 718; Dummett [1984](#), 6; van der Hout and McGann [2009a](#), 736).

This chapter will introduce how the social choice approach can be used to analyse representative elections, reasons for the extension to the committee selection literature, and the limits of this literature.

Let me begin with an example:

Assume there are three parties A, B, C who each sends two candidates for election: $a_1, a_2, b_1, b_2, c_1, c_2$. Of the ten voters, there are three who are in favour of party A , four who want party B to win, and three who like party C the most (as can be seen that those voters ranked both candidates

of their most preferred party on top of their ranking in the matrix below). In total there are four seats in the parliament, and thus, a committee of four candidates shall be selected. The following is the formal description of the exemplary parliamentary election:

Parliamentary election

$$N = \{v_1, \dots, v_{10}\}, A = \{a_1, a_2, b_1, b_2, c_1, c_2\}, k = 4,$$

- $v_1 - v_3 : a_1, a_2, b_1, b_2, c_2, c_1$
- $v_4 - v_5 : b_1, b_2, c_1, c_2, a_1, a_2$
- $v_6 - v_7 : b_1, b_2, c_2, c_1, a_1, a_2$
- $v_8 : c_1, c_2, b_1, b_2, a_1, a_2$
- $v_9 - v_{10} : c_2, c_1, b_1, b_2, a_1, a_2$

Using the Borda rule as a social welfare function and just selecting the four candidates who get the most points leads to the following selection of a committee that has all four candidates as members who are from party *B* and *C* but not even one candidate from party *A*.

Borda points for Parliamentary election:

- a_1 32
- a_2 22
- b_1 44
- b_2 42
- c_1 36
- c_2 34

Though it seems intuitively not fair that seven voters for parties *B* and *C* are each represented by their most favoured candidate, while three voters are not, the question remains which committee would be a fair representation for all ten voters.

Felsenthal and Maoz [1992](#) do discuss some desirable properties of committee selection functions, but claim that “the fair representation criterion does

not have a formal representation that is commonly acceptable” (Felsenthal and Maoz 1992, 125). In Part II of this thesis I will come back to this and elicit three proposals for such fair representation criteria. However, besides the three proposals I discuss, I don’t know of any research about which committee selection function leads to a fair representation of the voters. Even more, this can be said not only about fairness criteria but also more general: Elkind et al. 2017, 627 write in their article *The properties of multiwinner voting rules* that “[t]he literature on the properties of committee selection rules is still somewhat sparse”. Fortunately, the axiomatic approaches currently try to close this gap as can be seen by a call for papers for a special issue of *Social Choice and Welfare* on “Fair public decision making” that aims to connect collective decision-making and fair division.¹

¹See <https://www.springer.com/journal/355/updates/19966564>.

Chapter 3

Allocating seats to candidates

Another way to describe the election of a parliament is by deciding which candidate should get a seat or how many seats each party should get. The seat share that the candidates or parties receive should be according to the given votes, but how exactly? So, the problem to solve is: how do we translate the votes into seats? And how do we do so if we want to have a fair representation of the voters?

3.1 Apportionment theory

One way to deal with this problem, is by claiming that the amount of seats one party gets should be proportional to the votes this party got.

The second part of this thesis will deal with arguments why proportionality is desirable. But for a moment let us assume we just want proportionality between the votes a party got and the seats they get, for whatever reason. Further, we assume that the votes are tallied votes, i.e. that the voters can only state their most favoured candidate.

There is still one problem involved: seats in a parliament are usually not divisible, but rather there can be exactly one candidate per seat appointed. How to deal with such allocation problems where the good to allocate is not divisible but we want to approximate a proportional allocation as well as possible, is the topic of apportionment theory. See for instance the following

example:

Assume we want to allocate 100 indivisible seats to three parties, who got votes from 202, 303 and 404 voters respectively:

$$C := 100, \{a_1, a_2, a_3\}, \begin{pmatrix} 202 \\ 303 \\ 404 \end{pmatrix}.$$

Perfectly proportional would be to give them $22.\overline{22}$ ¹, $33.\overline{33}$ and $44.\overline{44}$ units respectively. However, let's see what happens if we cannot divide the good, like seats in a committee which are not divisible.

If we go on using the common rounding rule (round down until .4 and round up from .5 on), we would have to give the three parties 22, 33, and 44 seats. Thereby allocating 99 seats and leaving one seat empty. Thus, a different rounding rule would be needed if we want to allocate all seats. Those rules will usually be biased in some way. For instance there are rules that favour parties that got more votes and give them slightly more seats proportionally, while other rules do exactly that to smaller parties. Thus, each of those rules will have advantages and disadvantages itself. Apportionment theory deals with them and especially Balinski and Young 2001 give a very detailed overview on this topic.

However, I set out to focus on fair representation of the voters, and apportionment theory as I introduced it here focuses on the fair allocation to the parties. Riedwyl and Steiner 1995 criticize this one sided perspective and propose two distinct views on proportionality regarding allocating seats. From the parties' perspective "high proportionality means that the percentages of seats do not deviate much from the percentages of votes" while proportionality from the voters' perspective means "that about the same number of voters stand behind each seat" (368). Though it might seem that these are just two sides of one coin they show that both definitions of proportionality pull in opposite directions and evaluate different rounding rules as either more proportional for the parties or the voters. However, also their discussion of proportionality from the voters' perspective is built on a vague if not

¹ $\frac{100}{202+303+404} \times 202 = 22.\overline{22}$.

even questionable criterion. As we will see in Part II the idea that the same number of voters stand behind each seat is not that clear and not the only criterion that makes a representation fair to the voters.

Furthermore, I aim to analyse justifications of proportional representation as a means of fair representation of the voters. Thus, I want to elucidate reasons why and in how far proportional representation is fair to the voter, and not just assume proportionality to be something good in itself.

3.2 Claims problems

While apportionment theory assumes proportionality to be desirable, the literature on claims problems takes a step back and analyses situations where there is some amount of a good to be allocated to several agents. Those agents might want to have as much of the good as possible but the issue in a claims problem is, that they have an objective given claim to a specific amount of the good. The question to solve is: How much of the good should each agent get given their claim for the good? If all claims can be satisfied because the total sum of all claims does not exceed the amount of the good available, each agent can get as much as they have a claim for. However, how much should each agent get, if there is not enough to allocate to everyone?

A claims problem is defined in the following way: There is an amount e of some good, a set of agents A and a vector \mathbf{c} which specifies the amount of the good that each agent has a claim to.²

Take another tallied-vote election of a parliament. The four parties A, B, C, D got 202, 202, 303 and 404 votes respectively. There are 100 seats to allocate. Assume that the votes the parties got are the foundation for their claims to seats, i.e. if they got one vote they have a claim for one seat, two votes make a claim for two seats and so on.³ Thus, we now have the

²There is a second approach to divide a good to several agents: the fair division literature takes the preferences of the agents and aims to satisfy those preferences in a fair way. I do not want to get deeper into the satisfaction of preferences since I take it that elections are not about satisfying the subjective preferences of parties or candidates. The amount of seats they get should rather depend on the objective votes they got. Therefore, I will concentrate on fair allocation as treated in claims problems.

³In the literature on claims problems, the units of the claims and the good to divide

following claims problem to solve, searching for an answer, how many seats each party should get?

$$C := 100, \{A, B, C, D\}, \begin{pmatrix} 202 \\ 202 \\ 303 \\ 404 \end{pmatrix}.$$

Such questions were discussed already in ancient times—as for example in the Talmud—but in an axiomatic way O’Neill O’Neill 1982 was one of the first to do that. Thomson is now one of the leading figures in this field of research and Thomson 2019 gives an extensive overview of the current state of the art. He offers a detailed overview of possible axioms, allocation rules and theorems regarding both of them. While he acknowledges that there is something in the idea of proportional satisfaction of claims he also shows that this is not the only sensible way to solve claims problems.

To illustrate this, let me define some of the possible allocation rules and their properties.

3.2.1 Axioms

The first property I want to mention is the property of *balance*. It is not about fairness but about efficiency. The idea is, that the whole amount of the good has to be allocated. This means that the sum of what all individuals get has to be equal to the amount of the good that is available. There can be reasons to not require this, but in the most cases it seems to be a reasonable requirement, also in situations of representative elections.

Another property is the requirement that an allocation rule *treats equal claims equally*. It seems to be ad hoc if two agents have the exact same claim but would end up with a different amount of the good in the end. Parties *A* and *B* both got 202 votes. So, they should get the same amount of seats.

are the same. Thus, it is formally not the same since the claims correspond here to votes and the good are seats and not votes. However, also Thomson 2019, 393 takes the election of a parliament as an example of claims problems, and the formal results of the literature on claims problems can be adopted to seat allocations.

However, imagine we have five seats and each of two parties has a claim for four of them. Since we assume that seats are not divisible, and we also want to allocate all of the seats (see the property of balance), we have to give one of the parties more seats than to the other one. In such cases a different requirement could be applied, for instance that equal claims do not necessarily have to be satisfied equally but that they at most differ by one unit. Thus, one party would get three seats, while the other one gets only two.

A property which might not seem obvious first here, but will be in the later discussion, is the property of *no-advantageous transfer*: A group of agents should not gain by transferring claims among them. Imagine party *A* only got 200 votes and party *B* therefore 204. If a rule allocated 18 seats to each of them in the original case where both got 202 votes, this rule should also allocate in total 36 seats to both of them in the case where party *A* only got 200 votes and party *B* therefore 204.

There are many more axioms, and the question for research in allocation problems is which axioms are desirable but also which axioms are possible to combine, that is to find allocation rules that satisfy a specific set of properties. As a first overview let me introduce two such allocation rules.

3.2.2 Allocation rules

The *proportional rule* multiplies each agents claim by the same number λ , where λ equals the whole amount that is going to be allocated divided by the total sum of all claims.

A different idea for a rule lets all agents loose the same amount of the good. An allocation rule which does exactly this is the *constrained equal losses rule*. According to this rule all agents get their whole claim minus the same amount λ , where λ is chosen such that the whole estate is allocated. If this would lead for someone to get less than zero (i.e. that someone would have to give instead of receive something) this person merely does not get anything.

So, why would we want a specific rule to apply? To answer that, it

is not enough to study the formal possibilities of combining axioms, or to calculate different results of the rules. Rather we need to define the concepts underlying the allocation problem.

Also the question why proportionality is desirable is a conceptual question and cannot just be answered by the formal framework of allocation problems itself. However, the framework helps to raise further questions that will be helpful when I deal with those conceptual ideas of fair representation in the next part.

Shouldn't it be the voters who have a claim to something? Not exactly a seat but for being represented? I set out to be fair to the voter, and not in the first place to the parties and candidates. However, exactly this is the case if we interpret representative elections as seat allocation in the way I did above. The voters are mainly treated fairly in a derivative way, when determining what the claims of the parties are.

Furthermore, this model *prima facie* allows only to take the first votes of the voters into account. However, this excludes any other preferences right from the beginning. Or, it requires that the whole ranking of the voters are translated somehow into a claim. Leaving us with the task, to justify how.

Part II

Proportional representation is fair
to the voter: How and why?

Introduction Part II

While I showed in Part I that axiomatic approaches are helpful to discuss the question how fairness to the voter can be achieved in representative elections, I argued that further analysis needs a conceptual foundation what exactly we understand as fairness to the voter. This second part offers three proposals for such a foundation. I will show how their proponents plan to model them, and in which framework they do so. This will allow me to discuss their differences, assumptions and conditions more systematically.

Each proposed concept of fair representation of the voter will be presented and discussed in its' own chapter. Each chapter is structured in the same way: (1) I introduce the assumed framework how elections of representatives are understood and analysed, then (2) I present the proposed concept of fair representation of the voter and arguments for this view, (3) I explain how the concept is interpreted in axiomatic terms and how the authors model proportional representation, finally (4) I reflect and discuss the suggested approach. Thus, I will evaluate for each proposal to which extent it can be said to be fair to the voters.

Chapter 4

The property of solid coalitions: minorities are represented

Dummett is concerned with protecting minority opinions by having them represented. This chapter will show how he argues that proportional representation achieves this representation of minorities and how this is a fair representation of the voters. He does so by using the framework of committee selection functions to model the election and suggests and argues for a specific property a committee selection function has to satisfy to be called to be fair to the voters.

Section 4.1 will introduce Dummett's framework in which he formulates his idea of a fair representative election. I will contrast this framework with two others, which are rejected by Dummett because he sees them not suitable for the purpose of representation. Section 4.2 elucidates Dummett's concept of a fair representation. (1) I will show Dummett's argumentation why a representative election needs to adhere to different criteria than an election of a single best winners but also different than an election of several best winners. A representation fair to the individual voters, is rather about representation of groups. However, (2) those groups should only be constituted according to the individuals' preferences, they are neither preconceived (as for instance according to party lines) nor do they have to organize themselves in any way. (3) Who is going to represent these groups is again based only

on the preferences of the voters. Section 4.3 introduces Dummett's property of *solid coalitions*. This axiomatizes his idea of groups of like-minded people who are going to be represented if they form a sufficiently large group. I will further illustrate how this property leads to a proportional representation of the individuals' preferences. Section 4.4 will reflect and raise questions on Dummett's proposal of proportional representation to achieve a fair representation of the voter. Section 4.5 finally will summarize this chapter on Dummett's approach.

4.1 Framework: committee selection

Dummett writes his book *Voting Procedures* to be able to discuss in a systematic and precise way questions regarding elections, more specific with representative elections, and arguments about proportional representation. He criticizes the discussions of those topics as relying on intuition and therefore falling prey of unproven claims. They are thus not able to determine "whether or not a collective decision is fair, in the light of the preferences of those participating in it." (Dummett 1984, 8)

The main part of the book focuses on decisions where one single best alternative has to be selected according to the preferences of the individuals' in the same way as I illustrated in Chapter 1.

As I hinted at in Part I it seems wrong to understand an representative election with several winners as plainly taking the first k ranked alternatives of a social welfare function that seems the best for a single winner election. Dummett discusses this point more specifically and concludes that it is groups that should get represented (256). I will show his argument for this in more detail in the next section.

He then goes on to discuss decisions where not only one but several best alternatives have to be chosen. In this case he argues that the best model to determine such a set of best alternatives is by asking the voters to rank all the possible combinations of winning alternatives and not the individual alternatives as I illustrated in Chapter 2. This again results in an election in the standard framework of social choice theory. The rule he favours for such

decisions is the Borda rule (Dummett 1984, 247f).

However, for elections of representatives he takes a different stance. Dummett does not explicitly describe one way to frame the question which electoral procedure is the fairest in his opinion, even though in the end it is the framework used for committee selection rules in the later emerging committee selection literature. However, Dummett’s vagueness about a specific framework might be due to the fact, that there has not been much formal discussions about electoral systems as such at the time when he was writing. As I mentioned in Part I, Felsenthal and Maoz 1992 started their endeavour into committee selection in the early nineties. Thus, Dummett plainly extends the social choice methods on the go according to electoral procedures that already exist. He then discusses the advantages and disadvantages of those procedures. Further, in all his examples for representative elections it is candidates the voters have preferences about and who are supposed to represent them. The procedure that is searched for has to select a predetermined number of candidates according to the voters’ preferences (Dummett 1984, 255f.).

He sees it also as important that the voters should not only be able to cast a vote that states their most preferred candidate but to rank as many candidates as they wish. This will guarantee that it is the voters’ preferences that determine the outcome and not any bias implied by the model (166).¹

Finally, in none of his discussions are seats divisible, but a “fixed number of candidates, greater than one” is elected by some rule. Such rules and especially one property of them is what Dummett focuses on (256).

4.2 Dummett’s proposal for fair representation: minority protection

In this section I will elucidate three distinct issues important to understand Dummett’s view on fair representation. In 4.2.1 I will show Dummett’s argumentation in how far a representative election is different than choosing the

¹Dummett 1984 discusses this in much more detail in Chapter 8.

best k candidates, but rather that groups are getting represented. I will illustrate in 4.2.2 that those groups should only be constituted according to the individuals' preferences: they are neither preconceived (as for instance according to party lines) nor should they have to organize themselves. Finally, who is going to represent these groups is again based only on the preferences of the voters, as I will show in 4.2.3.

4.2.1 Groups should get represented

Let me illustrate Dummett's argumentation why groups should get represented according to his distinction between *fairness to the outcome* and *fairness to the voters*.

With fairness to the outcome Dummett refers to the aim of selecting the best alternative according to the rankings of the individuals.² This means we should take as much information from the voters' preferences into consideration as possible. Which means that it might be important to take into account voters' first preferences but also consider where they might most dislike another alternative. Let me illustrate this with the following example where a committee of nine people has to decide who of five candidates should be selected for three positions:

3 positions, 9 committee members, 5 candidates: $\{c_1, c_2, c_3, c_4, c_5\}$

1–6: c_1, c_2, c_3, c_4, c_5

7–9: c_5, c_1, c_2, c_3, c_4

Take for example the Borda rule which would select—according to the Borda counts given below— c_1, c_2 and c_3 . Candidate c_5 would not be chosen, even though it is the most favourite candidate of one third of the committee members.

Borda counts:

c_1 : $6 \cdot 5 + 3 \cdot 4 = 42$

c_2 : $6 \cdot 4 + 3 \cdot 3 = 33$

²For a detailed discussion on this see Dummett 1984, Chapter 6.

$$c_3: 6*3+3*2=24$$

$$c_4: 6*2+3*1=15$$

$$c_5: 6*1+3*5=21$$

One reason to not give c_5 a position could be that six of the committee members think that this person does not fit this position at all and that this information is so important that we should take it into account when searching for the very best candidates.³ This is what makes it fair to the outcome, taking as much information as possible from the given preferences.

In contrast to this it is important to be fair to the voter when the aim is to select the most representative committee. In this case the criteria to choose the winners of the election are different. Here it does not matter much that the majority most dislikes a candidate, as long as the candidate would be representative for a sufficiently large group of other voters (Dummett 1984, 256). See the very similar example to the one above but this time with 99 voters selecting three representatives out of five candidates:

3 seats, 99 voters, 5 candidates: $\{c_1, c_2, c_3, c_4, c_5\}$

1–66: c_1, c_2, c_3, c_4, c_5

67–99: c_5, c_1, c_2, c_3, c_4

Even if c_1 is the second most preferred candidate for the group of 33 voters, there seems to be no reason why not representing them by candidate c_5 , even though the other two third of the electorate most dislike this candidate. It is the group of the 33 who seem large enough⁴ to get represented without having the other group disregarding their preferences. Candidate c_5 only has to be representative of the smaller group and not of the other voters. Dummett takes it as widely accepted that in representation there are

³Even though ideally, we should have asked the individuals not for their preferences over the alternatives but rather over the different possible compositions of three candidates together. This would not only take the information about the individual candidates into account but further information about the conditional preferences the individuals have. This means what they think how the three candidates would work together for instance, see Dummett 1984, Chapter 14.

⁴I will in Section 4.3 specify what Dummett thinks to be large enough.

special rights for majorities as for instance the group of 66 voters above. But also for minorities there is such a right for being represented. He sees the reason for this in the very general and vague idea of the role of a parliament that “should be to some extent representative of public opinion” (Dummett 1984, 255). Thus, Dummett concludes that large enough groups have the special right to be represented. Majorities anyway, but also minorities can be such large enough groups. This turn away from a majority principle and instead to a broader consideration of minorities is what makes a decision fair to the voters instead of fair to the outcome. This turn will finally lead to a parliament that represents not just the majority but the public opinion.

What makes a group large enough will be discussed in Section 4.3. Who are these minorities? I will say more about this next.

4.2.2 Preferences constitute groups

Since Dummett requires it as being fair in representation that minorities are represented (256) it is necessary to find a more precise definition of what a minority is. Dummett does so in the following way:

“Minorities [...] should not be thought of only as those composed of supporters of relatively unpopular political parties; they include all groups who feel their interests to differ from those of the majority, whether they are distinguished by religious allegiance, racial origin, or occupation. [...] [W]e are not concerned with electoral systems that give an explicit role to political parties as such, but only with voting procedures that help any group that feels itself to have special interests to obtain representation, whether or not it has a formal organization.” (259)

This quote includes at least two important dimensions to take account of when talking about minority representation: (1) a minority is not defined by their affiliation with a party but rather by their more general interests they feel; and (2) those minorities are not necessarily organized. However, if they are not organized how then can such a group find together? All that is

available in Dummett's framework are the preference rankings of all voters over the candidates. Elsewhere he uses the notion of "feel" in connection with preferences: "any preference any voter may feel for any outcome over any other", or "any preference he may feel" (Dummett 1984, 152). Thus, it seems to make sense that "who feel their interests to differ from those of the majority" means that their preference ranking is different than that of the majority. And we will encounter how this reflects in Dummett's criterion for a fair representation later on. Exactly these preferences are what constitutes groups. Although, according to Dummett it does not have to be the case that everybody in this group shares exactly the same ranking over all or at least some candidates or even only their most preferred candidate. Imagine a group that wants to change the current climate policies and proposes better sustainability measures, but otherwise disagrees over which party they prefer. There are two candidates that claim to support exactly the same measures this group wants to achieve but they are from two different parties. Some of the group will have one of the candidates listed first and the other candidate second, while the other voters of their group ranks them the other way around. Thus, they agree on their two most favoured candidates just in different orders. Dummett introduces a measure that shows how large a minority must be to guarantee themselves to have one of their favourite candidates be elected, as long as they vote in the best manner (261f). If the mentioned minority supporting the two candidates for better sustainability measures could agree on one of the candidates to list first, they could enhance their chance to get this candidate a seat, but by even coordinating better they maybe could get both candidates into parliament (264). However, this way of coordinating is usually not feasible for many minorities, even if they seem to be large enough to be represented as a group that shares the same interests, but could just not coordinate on the order they put the candidates on the ballot. The main reason for this disability to organize has also to do with the issue that these groups are not preconceived. They are formed maybe even without the voters knowing themselves of each other's shared preferences over a small or also larger set of candidates.

Thus, even if those groups might not be organized at all a committee

selection rule should allow those sufficiently large groups to be represented without the need to get organized. It is based alone on the preferences. This means for Dummett that voters who share the same candidates as their favourites but possibly in a different order, they form such a group and if large enough they should get represented (Dummett 1984, 265).

4.2.3 Preferences determine who represents those groups

Another point to take into consideration is who of the several candidates the group is committed to, should be the representative of the group. Dummett discusses two options (287f): (1) either the preferences of the group alone should determine the selected candidates, or (2) the preferences of all voters over the candidates the group is committed to, should be used. He prefers the second but agrees that “the matter is a delicate one” (288) not giving further reasons for either option.

Dummett assumes now that one or several of those candidates over which a group agrees that they are all ranked on top should represent this group. This seems also intuitively right. However, it is not always easy to satisfy and when broadening Dummett’s requirement to weak preferences (i.e. to allow voters to be indifferent between different candidates), e.g. Aziz and Lee 2020 propose an axiom that only requires that at least one voter of the group ranks the eventually selected representative candidate of the group highest. Thus, it seems important to find further arguments why exactly one of the candidates over which the group is in agreement should be chosen. But Dummett does not give further reasons.

However, again it is clear that it is voters’ preferences who should determine the selected representatives and nothing else according to Dummett’s view on minority protection. And also the voters themselves choose what characteristics of the candidates are important for them. It does not have to be their party affiliation, but it could. It could also be any other characteristic. Though it is always limited to the given candidates.

Thus, it is groups of similar minded voters that should be represented. The representative is chosen according to the preferences of the voters. This

way of choosing will lead to a parliament that represents the public opinion and is thereby fair to the voters.

The next section shows Dummett's proposal how to implement this conceptual ideas as an axiom for committee selection rules.

4.3 Dummett's model of proportional representation: From minority protection to solid coalitions

After having elucidated Dummett's view on the conceptual idea of representation, I will introduce his suggestion for one specific property for a rule that selects representatives.

4.3.1 Solid coalitions

A solid coalition is a group of voters who all commit to a set of candidates. That means, that all voters in this group rank each of these candidates higher than any other candidate not in the set. Thus, they do not have to agree on the order of these specific candidates just that all of them represent their interest better than any other candidate.

Take again the example above, but this time the 66 voters favouring c_1, c_2, c_3, c_4 and c_5 in this order are split into three groups of which each favours c_1, c_2 and c_3 on top (just in different orders, and c_4 and c_5 are for each group the least favoured candidates):

3 seats, 5 candidates and 99 voters:

voters 1–22: c_1, c_2, c_3, c_4, c_5

voters 23–44: c_2, c_1, c_3, c_4, c_5

voters 45–66: c_3, c_2, c_1, c_4, c_5

voters 67–99: c_5, c_1, c_2, c_3, c_4

Thus, 44 voters favour c_1 and c_2 , 66 voters do favour the candidates c_1, c_2

and c_3 , and 33 favour c_5 the most. These groups of voters are some of the solid coalitions which I will denote as follows:

$$S_1 = \{1 - 44\} \text{ for } \{c_1, c_2\},$$

$$S_2 = \{1 - 66\} \text{ for } \{c_1, c_2, c_3\},$$

$$S_3 = \{67 - 99\} \text{ for } \{c_5\}.$$

Not just any group of so solidly committed voters get represented, since there is a limited number of representatives to be elected. Dummett therefore chooses to use the *Droop quota* q with q being the next integer larger than $n/(k+1)$. This quota yields the smallest number that ensures that not more candidates are elected as representatives than there are k seats (Dummett 1984, 269). Dummett does not give further reasons why he chooses this quota but it allows the smallest minorities as possible to gain representation and therefore seems to be a pragmatic choice of Dummett, since seats are not divisible.

Take the above $n = 99$ voters, and $k = 3$ seats. $\frac{99}{3+1} = 24.75$, and thus the Droop quota is 25 (the smallest whole number larger than 24.75). Thus, all the solid coalitions in the example are sufficiently large and require representation.

As described above the conceptual idea of a group that has to be represented of which the individual voters are solidly committed to a set of candidates is nothing that usually would be preconceived as in contrast for instance would be the case with groups of voters of a specific party.⁵ Contrary, the partition in solid coalitions is solely built on the given preference ranking of each individual voter. There would not even be any need of having the candidates organized in parties, but rather individual independent candidates can be elected. Therefore, Dummett asks the voters for their complete preference ranking over all candidates.

Putting all these points together leads to the following definition of the property of solid coalitions:

⁵See Chapter 6 of this thesis on van der Hout and McGann who explicitly argue against the use of such groups of voters for a specific party because this is a random partition of the voters and not fair for the individual voter.

Definition. Solid coalitions property The solid coalitions property is satisfied if a committee selection rule allows all voters to give their complete preference ranking and selects for each solid coalition of a size larger than hq at least h candidates of the set of candidates they solidly commit to. Where h is an integer smaller than or equal to the number of representatives that are selected and q is the Droop quota q , such that q is the next larger integer than $n/(k + 1)$.

Therefore, in the example above either c_1 or c_2 can represent the first two sets of 22 voters, c_1, c_2 or c_3 can represent the three sets of each 22 voters together, and c_5 can represent the last 33 voters. The solid coalitions property on itself does not say more about which of these candidates should represent the groups accordingly. Thus, the solid coalitions are also not exclusive: a voter can belong to several solid coalitions. The idea is to guarantee each of these groups representation as long as they are sufficiently large enough.

Dummett makes further suggestions for a specific selection rule. However, he is quite lenient with those specifications, as long as the solid coalitions property is satisfied. One such rule is the *single-transferable vote* (STV).⁶ The next subsection introduces STV and shows that it satisfies the solid coalitions property. Dummett however finds STV much too complicated and based on the woolly notion of wasted votes. Still, he would choose STV if there were no other rule to choose that satisfies the solid coalitions property (Dummett 1984, 284, 292).

4.3.2 Single-transferable vote

In this section I will introduce the single-transferable voting rule (STV), which is widely used and satisfies the solid coalitions property. The aim is to represent as many voters as possible, even if not everyone can be represented by her first preference. Therefore, voters are asked to rank all or as many candidates as they wish to rank. Those votes which either support a candidate who gets more votes than necessary or a candidate who is eliminated

⁶See Dummett 1984, 282 for a demonstration that STV satisfies the solid coalitions property.

because she is the one with the least votes, will get transferred to the next less preferred candidate. To get an idea of this system let me go through the example I used above to introduce the solid coalition property:

3 seats, 5 candidates and 99 voters:

voters 1–22: c_1, c_2, c_3, c_4, c_5

voters 23–44: c_2, c_1, c_3, c_4, c_5

voters 45–66: c_3, c_2, c_1, c_4, c_5

voters 67–99: c_5, c_1, c_2, c_3, c_4

We first need to define the quota, which in STV is usually the Droop quota. As elaborated above it is 25 in this election. In STV the quota is the number of votes a candidate needs to get to be given a seat.

Thus, candidate c_5 gets selected in the first telling round with 33 votes. This means there are 8 excess votes which need to be transferred, while candidate c_5 is eliminated from the voters' rankings. Therefore, in the second telling round we have now the following:

voters 1–22: c_1, c_2, c_3, c_4

voters 23–44: c_2, c_1, c_3, c_4

voters 45–66: c_3, c_2, c_1, c_4

8 excess votes: c_1, c_2, c_3, c_4

Now, c_1 is elected with 22+8 votes. The 5 excess votes will again be transferred and candidate c_1 eliminated from the voters' rankings:

voters 23–44: c_2, c_3, c_4

voters 45–66: c_3, c_2, c_4

5 excess votes: c_2, c_3, c_4

This time, c_2 is elected with 22+5 votes. Since all three seats are filled now the 2 excess votes c_2 had are not transferred anymore.⁷

⁷This example also shows nicely, that even with the transferred excess votes none of the remaining candidates would have enough votes to be elected: c_3 would have 24 votes and c_4 22. Both less than the quota of 25.

Therefore, candidates c_5, c_1 and c_2 are elected. This also satisfies the solid coalitions property. As I elaborated above it requires in this example that c_5 is elected, and further that either c_1 or c_2 , and c_1 or c_2 or c_3 are elected.

4.3.3 From solid coalitions to proportional representation

The given definition of solid coalition above leads to committees in which larger solid coalitions get represented to a higher degree and smaller coalitions to a smaller degree (and only such groups that are at least as large as the quota get represented). Also the whole electorate gets represented by k candidates, since the whole electorate is the trivial solid coalition of size n which is larger than $k \times q$ and therefore requires k representatives. All these points make it that the property of solid coalitions leads to proportional representation.

Assume for example that 60% of the voters strongly support some specific climate goal measure but otherwise have very different preferences about the parties that they want to get elected. The candidates who also support this measure are widely scattered among the parties. If the voters who support this measure now put all those candidates first on their ballots, in any order (i.e. they can for instance order them as they favour the parties they are from), then an election mechanism satisfying the solid coalition property, will select approximately 60% of the members of parliament who support this measure.

Take the following example. The candidates a_1, a_2, a_3, a_4 and a_5 are those supporting the measure, while candidates b, c, d are other candidates not doing so.⁸

Example. $n = 100, k = 5, |A| = 8, q = \lceil \frac{n}{k+1} \rceil = 17$

⁸The number of those other candidates does actually not change anything, and assuming that the other 40% of the voters rank those other candidates all before a_1, a_2, a_3, a_4 and a_5 makes the point even clearer, that those 60% still should and indeed get represented with 60% of their candidates.

1 – 20:	$a_1, a_2, a_3, a_4, a_5, b, c, d$
21 – 40:	$a_2, a_4, a_5, a_3, a_1, c, b, d$
41 – 60:	$a_5, a_3, a_2, a_1, a_4, d, b, c$
61 – 100:	$b, d, c, a_1, a_2, a_3, a_4, a_5$

Thus, we have at least one solid coalition of voters 1–60, who are a group that is more than three times larger than the required quota of 17. Thus, at least three of $\{a_1, a_2, a_3, a_4, a_5\}$ should be elected, which gives them three out of five seats. Therefore, 60% of the seats go to candidates who support the measure that is also supported by 60% of the voters.

4.4 Reflection

4.4.1 Solid coalitions: who is represented?

Dummett’s main point is that the opinions of the voters should be represented in the parliament. This can be achieved by having those candidates in parliament that represent sufficiently large groups of voters having these opinions. Therefore, it is important for him to get rid of principles and election rules that take only account of the majority’s opinion.

The first part of his solution is to have per district not only one representative elected but three or more. This allows for more variety of opinions, but in itself is not enough, as he shows by discussing several election rules (Dummett 1984, 256–265). All these rules make it necessary for the minorities to be organized and coordinate their votes with each other. In contrast, he shows that the solid coalitions property allows to represent a specific kind of minorities: groups of like minded voters. Which might be used to defend the claim that proportionality is at least fairer to the voters than an electoral system that only includes representation of majorities. He kind of raises the question what it means to be fair to the voter by distinguishing it clearly to fairness to the outcome.

However, it seems there is still something more missing as the following example shows.

3 seats, 99 voters, 6 candidates: {a, b, c, d, e, f}, $q=25$

voters 1–24: a b c d e f

voter 25: c b a f e d

voters 26–99: f e d c b a

While the group of voters 1 to 24 is one person too less for a solid coalition to require representation, voter 25 makes this group sufficiently large to do so. To satisfy the solid coalitions property it would be fine to have candidate *c* elected, which might seem unfair to the other 24 voters of this solid coalition.

However, *c* is still better ranked by everyone in this solid coalition than the three favourite candidates of the majority. Thus, even if *c* is elected, this seems a fairer representation according to the aim to have sufficiently large groups represented than having *d*, *e* and *f* elected, as applying the Borda rule and selecting the three candidates with the highest Borda points would do.

Dummett's minimal requirement therefore opens the door for having not only the majority's candidates elected but also some of the other candidates which are favoured by a minority. This is especially the case, since the solid coalitions property is one property an election rule should have according to Dummett. This means, even if this property allowed to have *c* as the representative of this solid coalition, it does not say it has to be *c*. Candidates *a* and *b* are also eligible, and therefore the election rule applied in the end might also choose them.

Eventually, it is indeed the group as such who is represented and not the individual voter. Dummett's approach is about fairness to groups. But importantly, the groups are determined by the preferences of the individual voters and not preconceived groups.

4.4.2 Decision making

Dummett himself does not write about the procedure that should take place in the parliament to make decisions. Therefore, it is interesting to explore what happens when the often used simple majority rule is applied in a parliament composed such that it satisfies the solid coalitions property.

Take the last example again. According to the solid coalitions property f and e have to be elected and also one of a , b and c . Assume that there is a decision to be made for which representative f would vote “yes” and everyone else “no”. Thus, there are two representatives voting “no” and only one “yes”. However, the 74 of 99 voters preferring candidate f the most, can be assumed to be mostly of the same opinion as candidate f . And therefore, the majority’s will might have been “yes”. Thus, even if this is an extreme example, still an electoral system satisfying the solid coalitions property does not necessarily lead to the situation that the majority’s will always prevails.

A further limitation for having the majority’s will prevail in decisions made in the parliament can be due to the fact, that Dummett models the election as happening in small districts with only a few representatives. The next subsection elaborates more on this.

4.4.3 Limitations due to small districts

Dummett himself is only interested in the election of the representatives of a given constituency. This means that usually three to five candidates have to be elected that then represent this constituency in the parliament (Dummett 1984, 286).⁹

Even if those multiple-member district elections allow for more variety than single-member districts, they still limit the inclusion of more diverse opinions. For instance Lagerspetz 2016 refers to the issue that the more seats are available the more proportional can the representation be. This is even independently of the specific election rule. For the case of the solid coalitions property see the following example.

Assume there are three different kinds of candidates: those who represent the opinion A (candidates a_1, a_2, a_3), those who represent opinion B (candidates b_1, b_2, b_3), and those who represent opinion C (candidates c_1, c_2, c_3). The country is divided in three districts in which the voters show exactly the same pattern of opinions: 220 voters prefer A over B over C, and 780

⁹This seems to be a legitimate constraint, since Dummett contributes not specifically but in view of the discussion of electoral reform in the UK in the 1980’s: The UK had and still has a single-member district electoral system.

prefer B over C over A. There are for each district 2 seats and 3 candidates run for office (for each opinion one candidate). So, for each district holds the following:

2 seats, 3 candidates, 1000 voters, $q = 334$:

voters 1–220: A B C

voters 221–1000: B C A

This will lead to the following solid coalitions which are sufficiently large to gain representation:

$$S_1 = \{221 - 1000\} \text{ for } \{B\},$$

$$S_2 = \{221 - 1000\} \text{ for } \{B, C\},$$

$$S_3 = \{1 - 1000\} \text{ for } \{A, B, C\},$$

Thus, for each district there will be one candidate of opinion B and one of opinion C. Thus, for the whole country we have candidates $b_1, b_2, b_3, c_1, c_2, c_3$ as representatives.

The solid coalitions property is also possible to apply to elections in larger districts with more seats. Thus, imagine, that the whole country is one district:

6 seats, 9 candidates, 3000 voters, $q = 429$:

voters $1_1 - 220_1, 1_2 - 220_2, 1_3 - 220_3 : a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3$

voters $221_1 - 1000_1, 221_2 - 1000_2, 221_3 - 1000_3 : b_1, b_2, b_3, c_1, c_2, c_3, a_1, a_2, a_3$

This leads to the following solid coalitions:

$$S_1 = \{1_1 - 220_1, 1_2 - 220_2, 1_3 - 220_3\} \text{ for } \{a_1\}$$

$$S_2 = \{1_1 - 220_1, 1_2 - 220_2, 1_3 - 220_3\} \text{ for } \{a_1, a_2\}$$

$$S_3 = \{1_1 - 220_1, 1_2 - 220_2, 1_3 - 220_3\} \text{ for } \{a_1, a_2, a_3\}$$

...

$$\begin{aligned}
S_4 &= \{221 - 1000\} \text{ for } \{b_1\}, \\
S_5 &= \{221 - 1000\} \text{ for } \{b_1, b_2\}, \\
S_5 &= \{221 - 1000\} \text{ for } \{b_1, b_2, b_3\}, \\
S_5 &= \{221 - 1000\} \text{ for } \{b_1, b_2, b_3, c_1\}, \\
&\dots
\end{aligned}$$

Candidates a_1, a_2, b_1, b_2, b_3 and c_1 are the elected representatives. And thereby the minority who favours opinion A would be represented.

Furthermore, it is feasible that a majority of voters is distributed over the whole country. It might be that they never or seldom gain representation in their district because in each district they are a minority slightly too small. This could for instance happen if voters of other opinions are gathered together in specific districts, where they get a representative. But over the whole country they are rather few voters. Thus, the majority's will might also not prevail in decisions made in parliament.

4.5 Summary

For Dummett fairness to the voters is when everyone's (and only their) preference rankings over the candidates are taken into account when we determine sufficiently large groups that should be represented, and when we determine who is (are) the representative(s) of them. This requirement is axiomatized by Dummett in the solid coalitions property.

Any election rule that satisfies the solid coalitions property should lead to a proportional representation and thereby to a representation of the majority's interest but—equally important—also to the representation of sufficiently large minorities' interests.

The representation of majority and minority interests makes the parliament representative of the public opinion, according to Dummett.

Dummett sees it as necessary that the voters are allowed to state as many preferences over the candidates as possible. Thus, there needs also to

be the realistic opportunity to rank as many candidates as the voter wishes. Therefore, any rule satisfying the solid coalitions property seems to be either only applicable to smaller sized elections, or elections where voters have the incentive and motivation to take the effort of ranking “enough” candidates, or where there is no necessity to rank many candidates, because there are sufficient seats to only form solid coalitions over a small set of candidates.

Dummett’s minimal request of representing sufficiently large groups of minorities seems to be a very general request that many will support. The solid coalitions property on its’ own, however, does not always lead directly to the intuitive fair representation. It still limits the eligible sets of winning candidates, and leaves the final details to the eventually selected election rule.

While the solid coalitions property leads to the inclusion of minority views by representing them, it is not enough on its own to exclude situations where the majority’s will might not prevail. Also this will depend on the eventually selected election rule.

Chapter 5

The modified Borda rule: every voter is represented as well as possible

In this chapter I am going to introduce and discuss Chamberlin and Courant's argumentation how to model in the social choice framework proportional representation and why this is fair to the voters. The whole argument is based on the ideal of direct democracy. Thus, so they argue, the aim is the satisfaction of each individual's claim to be represented as well as possible. In contrast to Dummett, who argues for representation of minorities, Chamberlin and Courant argue for full representation of all voters in all parts of the political process. (Chamberlin and Courant 1983, 719). They further distinguish throughout their whole article between representation in deliberation and representation in decision making.

What political representation should achieve can be nicely summarized in a quote from Gilpin that Chamberlin and Courant use as their starting point and call the *representation problem*:

Whether there can be a legislative assembly elected, so as to represent the respective interests of the community *in its deliberations*, and to allow the control of the majority *in its decision* to which it is entitled. (Thomas Gilpin (1844) as cited by Cham-

berlin and Courant 1983, 719)

Their solution is a modified Borda rule which aims for representation in the deliberations of the voters according to their conceptual idea of fair representation. To maintain the control of the majority in its decisions, they add a weighted voting to the decision making in the parliament.

Section 5.1 will introduce Chamberlin and Courant’s framework of how they understand representative elections and in which they formulate their ideas of a fair electoral system. It resembles mainly the classic social choice framework with some small—though different than in Dummett’s approach—amendments. Section 5.2 elucidates Chamberlin and Courant’s concept of a fair representation. Section 5.3 introduces Chamberlin and Courant’s election function and it’s relation to the Borda rule. Section 5.4 will reflect and raise questions on Chamberlin and Courant’s proposal of proportional representation to achieve a fair representation of the voter. Section 5.5 finally will summarize this chapter on Chamberlin and Courant’s approach.

5.1 Framework: social choice

The framework Chamberlin and Courant are using is that of standard social choice theory for single-winner elections where the single winner will be a committee. The agents are the individual voters and the alternatives are all the possible committees with a fixed number of candidates. Crucial in their approach however is that the individuals have preferences over the candidates, and they are required to state the complete ranking in their vote, which is the same as in the committee selection literature introduced in Part I. However, as we will see later these preferences over candidates are translated into “preferences” over the committees, and the election rule they search for is a social decision function.¹ In doing so, Chamberlin and Courant are able to distinguish between two possible questions: “How well does this committee represent you?” vs “How well does your representative

¹By translating the preferences over candidates into preferences over committees, Chamberlin and Courant violate the usual assumption of universal domain (Chamberlin and Courant 1983, 725).

on this committee represent you?” (Chamberlin and Courant 1983, 722). It is the answer to the second question they want to elicit as I will show next.

5.2 From the ideal of direct democracy to a fair representation of the voters

Chamberlin and Courant assume in their argumentation that direct democracy is the ideal that should be approximated in representative democracies. Thus, I will first elicit how they derive from this that representation does not only include representation in decision making but equally important representation in deliberations. In a second step I show how they derive from the ideal of direct representation their main component for representative democracy: each individual voters’ claim to be represented as well as possible. Third, I describe how Chamberlin and Courant argue how the satisfaction of these claims should be done, and that this leads to the election of the most representative parliament.

5.2.1 Representation for deliberation and decisions

There are mainly three reasons Chamberlin and Courant mention why representation is not only about the final decisions made in parliament: (1) legitimacy, (2) decision making is intertwined with deliberations and (3) deliberation is part of direct democracy. Let me say more about each of these points.

First, Chamberlin and Courant use an argument made by Sterne (1869) that the idea of individuals’ sovereignty demands that each individual voter is immediately or mediately present in all stages of the political process. Therefore, it would diminish the legitimacy of decisions if in the deliberations not all minority points or views are included.

Second, this becomes even clearer when looking at their view how deliberation and decision making are intertwined. The important connection between deliberation and decision making can be understood in a way, that

the representatives “learn from the deliberations that give rise to final preferences among alternatives” before the final votes are cast. Thus, it is not just the final decision that the representative should make in the sense that the represented individual would make it but also all steps before the decision (Chamberlin and Courant 1983, 721).²

Third, since Chamberlin and Courant take direct democracy as the ideal, it is also noteworthy that deliberation is part of direct democracy. Ideal direct democracy works such that in the deliberations everyone can state their view. Those who’s views are already stated will keep silent and wait for the voting. After all possible views, restatements and arguments are heard, everyone will vote (720).

Therefore, any true representation needs to include representation in deliberation and decision making. This means that (1) each voter’s view, values and interests get an airing in the deliberations, but (2) also that her representative would be similarly sensitive to other views that come up in the deliberations. This (3) finally leads to votes in the decision making close to how the represented individuals would vote themselves (721).

5.2.2 Each voter’s equal claim to one as satisfactory representative as possible

From the ideal of direct democracy follows, they argue, that each voter has an equal claim for a representative as well as possible (722). It is however important to note that it is a claim to exactly *one* representative as well as possible, and not to several, or to the most preferred committee overall. Let me elaborate on this point.

For the airing in deliberations it is important that Chamberlin and Courant don’t see it necessary that the frequency of views uttered is proportional to the amount of voters who have this view. As they described for direct democ-

²The voters often also do not know which decisions are going to be made at the moment when the representatives are elected. Therefore, they rank the candidates in the way they think they would bring their arguments to any deliberation, learn from the points others make in the way they would learn themselves, and therefore develop similar preferences that lead to their final decision.

racy, it is only important that every view is given the chance to be voiced and argued for (Chamberlin and Courant 1983, 723).

However, if the voters were asked to rank all possible committees instead of candidates this would plainly lead to have the majority choose their representative parliament that then makes decisions according to the majority's will. While the latter is not a problem, since the majority's will should prevail in the decisions made by the parliament, it would take away the representation of minority opinions (722).

Further, to evaluate the representativeness of a committee for an individual voter, it is important to only evaluate the representativeness of the representing candidate in this committee. The latter is done according to the voter's ranking of the candidates. The more favourable the rank the more representative is the candidate. Finally, the representative of a voter is her highest ranked candidate in a committee (723).

Thus, the voters are asked to rank the candidates, but then the different possible committees are compared according to how good they are in representing the individuals overall. The remaining question now is, how the equal claims of all voters for a representative as well as possible should be satisfied. Usually it will not be possible to represent all individuals by their most representative candidate. Therefore, also a measure for representativeness on the group level is needed. Chamberlin and Courant take it that this has to be "a set of candidates who satisfy to the greatest possible extent the claims of individuals to be represented on the committee." (722)

5.2.3 Decision making

So far I only focused on the representation in deliberation, following Chamberlin and Courant, because representation in deliberation should not be subordinated to representation in decision making.

The main aim in decision making in a representative system is according to Chamberlin and Courant that the majority's will should prevail. They interpret this so that the representatives of the majority of the voters should also have a majority when voting in the parliament. However, each repre-

representative can have a different amount of voters she represents. Thus, applying the simple majority rule that gives each representative the same weight would not guarantee that the majority's will would prevail. Chamberlin and Courant therefore suggest a weighted voting system according to the number of voters each representative represents (Chamberlin and Courant 1983, 722f).

5.3 Chamberlin and Courant's model of proportional representation: modified Borda rule and weighted voting

5.3.1 Representation axiom: individual voters' ranking of the committees

As explained above, Chamberlin and Courant apply the individual's ranking over the candidates as the measure how well an individual is represented in a committee: The higher a candidate ranks the better she represents the individual. These rankings are then not directly applied but in a second step are used to rank all possible committees for each voter. They formalize this step via the representation axiom (724):

Representation Axiom. Committee c is more representative for i than committee c' if and only if there exists a $j \in c$ such that i is better represented by j than by any member in c' .

This step allows to separate representation in deliberation and representation in decision making: any committee in which an individual is represented by the same representative gives this individual the same airing of her views, values and interests according to Chamberlin and Courant. Therefore, all those committees with the same representative for an individual have the same representativeness in deliberation for this individual (723). Representation in decision making will be dealt with only later after the deliberative representative parliament is determined.

Take again the example I discussed in the Dummett section before.

2 seats, 3 candidates, 1000 voters

voters 1–220: a b c

voters 221–1000: b c a

Voters 1–220 rank candidate a first. Therefore, candidate a would be the most representative candidate for these voters. Candidate b the second best and candidate c the third best. Thus, any committee with candidate a would be more representative for these voters than any other committee. Furthermore, any committee with candidate b would be better than any other committee without candidate a . More specific, there are the following three different committees possible:

$\{a, b\}$

$\{a, c\}$

$\{b, c\}$

The first two have the same degree of representativeness for the voters 1–220. According to Chamberlin and Courant's committee selection rule it does not matter if the other elected candidate is b or c , because being represented as well as possible means according to Chamberlin and Courant to be represented by one representative who is as representative as possible. The other voters are best represented by the first and third committees, since they include candidate b .

Thus, the rankings over the committees are the following:

voters 1–220: $\{a, b\} \simeq \{a, c\} \succ \{b, c\}$

voters 221–1000: $\{a, b\} \simeq \{b, c\} \succ \{a, c\}$

The next subsection will illustrate how Chamberlin and Courant propose to aggregate these individual rankings and select thereby the most representative parliament.

5.3.2 From the individual's claim for representation as well as possible to the most representative parliament

The rankings of the individual voters over all possible parliaments are then aggregated to find the most representative parliament overall. Chamberlin and Courant suggest that the sum of all individual satisfactions of all the individual claims to be represented as well as possible should be maximized. This means that each jump between two adjacent ranks has to be treated equally. Thus, we apply a modified Borda rule similar to the original Borda rule I introduced in Section 1.1.1. This time each committee gets the highest points equal to the number of candidates that are the highest ranked candidate of a voter. A committee including not the first ranked but the second ranked candidate yields one point less, and so forth for all possible compositions of the committee. We do this for every voter and for every possible parliament composition. Finally, the committee is chosen for which the sum of all these points is the highest.

For the above example this gives the following Borda points:

Borda points:

$$\begin{aligned}\{a, b\} & 220 \times 3 + 780 \times 3 = 3000 \\ \{a, c\} & 220 \times 3 + 780 \times 2 = 2220 \\ \{b, c\} & 220 \times 2 + 780 \times 3 = 2780\end{aligned}$$

According to Chamberlin and Courant's modified Borda rule the parliament consisting of candidates a and b is the most representative parliament.

5.3.3 From the most representative parliament to proportional representation: weighted voting

The elected candidates to parliament are not representative in the sense of the number of voters favouring them. In the example above representatives a and b have both one seat, while a represents 220 voters but representative b represents 780 voters.

However, Chamberlin and Courant want the majority's will to pertain in the decisions. If now the elected candidates used the simple majority rule to decide between two bills, the result would not necessarily respect the majority's will. In the example a tie between the votes of a and b could be possible if we use the simple majority rule.

Thus, they suggest to apply a weighted voting rule. More specific, they give every representative a weight according to the number of voters they represent. Representative a gets a weight of 220 and representative b a weight of 780. Thus, representative b will be decisive in any decision made in the parliament, letting the majority's will prevail in all decisions (Chamberlin and Courant 1983, 731).

5.4 Reflection

5.4.1 Violation of solid coalitions property

The modified Borda rule does violate the solid coalitions property. This can easily be shown with the example I used already earlier:

2 seats, 3 candidates, 1000 voters, $q = 334$:

voters 1–220: a b c

voters 221–1000: b c a

According to the solid coalitions property b and c have to be elected, because the group of voters who constitute a solid coalition over b, c are more than twice the quota, and therefore should be represented by two representatives. However, if we calculate the modified Borda scores of the three possible compositions a and b will be the elected representatives.

Therefore, the modified Borda rule does violate the solid coalitions property. It also might be criticised that 220 voters get as much representation as 780 voters.

Perfectly proportional would be to give the 220 voters a share of $\frac{220}{1000}$ of the two seats, resulting in 0.44 seats. The 780 voters should get a seat share

of $\frac{780}{1000}$ of the two seats, resulting in 1.56 seats. According to the Webster rule (see apportionment theory in Part I) this would lead to two seats for the majority and zero seats for the minority. Thus, the modified Borda rule does also not necessarily lead to proportional representation according to apportionment theory. However, Chamberlin and Courant's rule leads to a higher degree of variety of the represented individual voters in the composition of the parliament. They also do not aim for proportionality in the deliberations, while the solid coalitions property aims exactly for that.

For the decision part Chamberlin and Courant however do aim for a proportional reflection of the voters' preferences since they want the majority's will to prevail.

According to their suggested weighted voting system, representative b would gain a weight of 780, while representative a only of 220 (according to the number of voters they represent). Further, when voting in the parliament these weights should be applied in a simple majority vote system. Thus, representative's b vote would always have the decisive vote, in accordance with their requirement that the majority's will should prevail in the decisions.

5.4.2 District size

Similar to Dummett, Chamberlin and Courant propose their modified Borda rule for elections on the district level and not for the whole parliament.

However, while Dummett requires that voters can rank as many candidates as they wish, the modified Borda rule requires that they indeed give a full ranking. The reason is, that Chamberlin and Courant's rule requires to evaluate the representativeness of all possible compositions of the parliament. And therefore a complete ranking over all alternatives is necessary. Therefore, it is easier to find a rule that satisfies the solid coalitions property and is applicable for larger districts, while this is not the case for the modified Borda rule.

Also note, that a solid coalition is often already composed over very few candidates, especially if the number of seats is large. The reason is that then the quota becomes relatively smaller, and fewer voters constitute a solid

coalition that gains representation.

Thus, while Chamberlin and Courant's election rule leads to higher variety of voters being represented even in smaller districts, Dummett's solid coalition property is easier to apply in larger districts.

Furthermore, the modified Borda rule is hard to compute (Elkind et al. 2017, 605f). The more candidates or the more seats the more different compositions of the candidates are possible. Since for each of them the representativeness has to be calculated, this leads quickly to hard computational problems to solve.

5.4.3 Election of parties

To circumvent the problem with small sized districts, Chamberlin and Courant also suggest to apply their rule not to elections of candidates but to parties (Chamberlin and Courant 1983, 721). Thus, it would also be feasible to use it also in larger districts or even on the national level. Dummett is clearly against the election of parties, because he aims for representation of groups that feel themselves "to have special interests to obtain representation, whether or not it has a formal organization." (Dummett 1984, 259) However, the way Chamberlin and Courant argue what representation of a voter means does also fit to parties.³

5.5 Summary

For Chamberlin and Courant fairness to the voters requires that each voter is represented in the parliament as well as possible and that the majority's will prevails in the decisions made by the parliament. These two requirements are based on the ideal of direct democracy, which so they argue, results in a claim for each voter to be represented as well as possible. Their modified Borda rule leads to a composition of the parliament that represents all

³See Chamberlin and Courant 1983, 721 where they write about three criteria: (1) making the voter's interests visible during deliberation, (2) being influenced by the arguments in deliberation similar to the voter, and (3) vote in the decisions made in parliament as the voter would.

interests as well as possible. This “as well as possible” is measured by the ranks each individuals’ representative takes in their ranking. The higher the better is the satisfaction of the individual’s claim. The sum of all individual’s satisfaction is thereby maximized. For decisions made in the parliament are made by the representatives via weighted majority voting. The more voters are represented by a representative the more weight this representative has.

The combination of the modified Borda rule and the weighted voting leads to a proportional relation between the number of voters and their strength in decisions.

While Chamberlin and Courant’s modified Borda rule are most suitable for smaller sized elections similar to Dummett’s proposal, there are still differences. In contrast to Dummett, Chamberlin and Courant allow to substitute candidates with parties in case of larger sized elections. At the same time, for their rule it is important that voters rank all alternatives, while for the solid coalitions property this is not necessary.

The conceptual idea of taking ideal democracy as the foundation for their argument that the individual voter has a claim for a representative as well as possible seems convincing. However, it is less convincing that exactly one representative can do the job of representing all the different interests of a voter as well as possible. For this, it might be more convincing to indeed elect parties.

Chapter 6

Proportional representation: every voter is treated equally

As I mention in the introduction, Blau [2004](#) is quite critical about the term fairness. Van der Hout and McGann acknowledge the ambiguities Blau mentions, but see them caused by a fairness conception that is based on groups and not individuals. They, therefore, propose an individual account of fairness, because—so they argue—such an account can avoid those ambiguities. This individual fairness conception leads them to their main requirement for a fair representation, which is the equal treatment of all voters (van der Hout and McGann [2009a](#), 739f). The second requirement is popular sovereignty, that is the outcome of the election is based on the voters will alone (van der Hout and McGann [2009b](#), 618).

The outcome of an election of representatives is the candidates' seat share they get, but also the seat share of the coalitions of candidates that will be formed. Since representatives then make decisions on behalf of the voters by use of majority voting, it is notable that eventually one of those coalitions which comprises more than half of the seat share will be formed and has the power to make those decisions.

Thus, van der Hout and McGann's proposal will use the model of seat allocation and is based on an idea of procedural fairness that focuses on the individual voter.

In Section 6.1 I will introduce the framework van der Hout and McGann are using. Section 6.2 will illustrate the arguments van der Hout and McGann give for their requirements of political liberal equality and popular sovereignty. Section 6.3 will show how they model their conceptual idea of liberal equality and popular sovereignty formally as list proportional representation. Here I will come back to May's axioms (see Chapter 1). Section 6.4 will raise several questions regarding their approach.

6.1 Framework: seat allocation

Van der Hout and McGann model the election of representatives as seat allocation. They ask voters to vote for exactly one party or abstain from election.¹ From the individuals' votes the total number of votes for each party is calculated. Using this total vote a seat allocation rule outputs a seat share for each party, assuming that seats are divisible (van der Hout and McGann 2009b, 620). Their argument for divisible seats is due to their approach of justifying proportional representation instead of arguing how it can be implemented in the best way (619).

Importantly, after the election the government is formed and legislation needs to be passed. Most such cases are decided by majority voting and therefore a majority of seats is needed. Since usually one party on its own is not able to have the majority of seats the parties have to form coalitions (van der Hout and McGann 2009a, 618).

This means that all the input we get from an election is actually the total vote for each party (i.e. how many voters voted for each party). The allocation rule that maps this ballot profile into a seat share for each party (i.e. how many seats each party gets) will have to satisfy the requirements of liberal equality and popular sovereignty. The next section will illustrate van der Hout and McGann's reasoning for these two requirements.

¹They do not argue against the possibility to use a complete ranking over all alternative parties, but it would be more complicated and they show how some existing electoral systems that use multiple-votes violate their requirement of liberal equality (van der Hout and McGann 2009a, 747f.).

6.2 An argument for liberal equality and popular sovereignty

Van der Hout and McGann make an explicit difference between fairness to parties, fairness to social groups and fairness to the individual voters and criticize that “[a]ttempts to justify PR [i.e. proportional representation] have tended to be based either on its intuitive fairness to political parties, or on its being fair to social groups.” (van der Hout and McGann 2009b, 617f) They “however, do not view parties as ‘claimants’ with rights to certain seat shares, but rather require that the electoral system respect the equality and sovereignty of the voters.” (620) This equality and sovereignty of the voters is what makes the representation fair according to van der Hout and McGann.

I will begin with van der Hout and McGann’s argument why liberal equality is desirable if we want to treat the individual voter fair, and how this is different to treating groups of voters fair. Accounts that treat groups of voters fairly sometimes might look or pretend to be based on individuals but are not fair to the individual voter per se as van der Hout and McGann argue.

The first step in their argumentation is to show that individual fairness and impartiality imply liberal equality. *Fairness* means that each individual voter is treated the way each individual voter should be treated. According to *equality* all voters should be treated the same, independent of any possible reasons for treating them differently. And finally, *impartiality* allows for treating individual voters differently, but only if the reasons to do so are not based on inappropriate factors as for instance personal friendship, race or gender.

Van der Hout and McGann argue that fairness implies equality: If fairness required a different treatment of the voters, there must be some “mutually acceptable procedure” (van der Hout and McGann 2009a, 739) to find reasons why one voter should be treated differently than another voter. This is practically impossible according to van der Hout and McGann and, therefore, only if everyone is treated equally fairness can be guaranteed.

They also say that impartiality implies equality. They claim that the only impartial way of treating voters differently would be a random selection of the

voters. Since this seems impossible in an election, only an equal treatment of all the voters can be impartial. Thus, an equal treatment of all individual voters is the only way to guarantee a fair and impartial electoral system.

However, the same is not true if fairness is based on groups instead of individuals. If groups of voters have a different size it might be fair that those groups be treated differently. However, it is not clear that there are impartial reasons for treating them differently. Think of the following example.

Let us assume that it is fair if a larger group of voters is represented by more members of the parliament than a smaller group of voters. However, how do we define those groups? Should everyone who voted for the same party be in one group? Then we could ask “why should this be impartial?” because we could also have partitioned the voters into groups of voters who voted for the same two parties, and then let the party which was more often first mentioned, be the one who represents these voters. Or, both those parties represent them. And so on. None of those reasons seems to be impartial. And to just pick one of them, seems arbitrary. Therefore, if we want to stick to a group based account a justification for one specific partition would be necessary. Or even a partition regarding other properties.

The remaining problem—after determining the partition of all voters—is to determine a fairness criteria how to treat those different groups. Blau 2004 offers many different conceptions of fairness and argues, that depending on which one is used, the derived fair election system might be a different one. Van der Hout and McGann acknowledge this and claim that the main problem is the fact that all those conceptions do not take the individual voters as the relevant entity but rather preconceived groups of voters that have to be treated fairly. This focus on groups of voters leads to many ambiguities: It makes for example a difference if we ask who should be the representative of the majority of people of one specific district or if we ask who should be the representative of the majority of people of the whole nation.² A similar

²Here even the same concept of fairness can come to different conclusions which result is fair: the winner takes it all conception would claim that in each single district the candidate with the highest number of votes should be the elected representative of this district. However, with the same conception of the winner takes it all, but looking at the whole nation, we should conclude that the party that got the most votes in total should

problem arises if we claim that at least salient groups should be represented (see for example Pitkin 1967, Chapter 4): To determine the representatives of those salient groups, we first have to determine which groups are salient. And depending on which groups are defined as salient, different representatives might be chosen.³ This again, leads to many different possibilities of a fair election for a representative parliament.

Thus, van der Hout and McGann claim that fairness to the individual voter is what should be strived for in representative elections if we aim for a libertarian justification of the allocation rule for the representative elections. This requires equal treatment.

Following the libertarian view, van der Hout and McGann claim further that the outcome of an election solely has to depend on the voters preferences, and therefore they require that the election of representations has to adhere to the principle of popular sovereignty.

The next section will illustrate van der Hout and McGann's suggestion how to formalize and implement these requirements.

6.3 Van der Hout and McGann's model of proportional representation

In this section I first show how van der Hout formalize their requirements of liberal equality and popular sovereignty in the axioms of anonymity, neutrality and positive responsiveness. They claim that list proportional representation satisfies these axioms. Thus, second, I explain what they mean with list proportional representation.

be the sole winner of the election which is often not the case if the district winners are selected by a winner-takes-it-all election and then sent to parliament (Blau 2004, 172).

³A partition of the voters could for instance be regarding their gender, their race, their occupations, their most favoured party, their most favoured candidate regarding one specific topic or regarding another topic, their interest in plants, animals, economy, climate change, their hobbies and so on.

6.3.1 Axioms

The requirements van der Hout and McGann have for the allocation rule are derived from their idea of liberal equality and popular sovereignty. They argue that the properties of anonymity, neutrality and positive responsiveness are the minimal requirements we can have for an allocation rule. If they were violated, liberal equality and popular sovereignty would also be violated. These three axioms are exactly the same as May used to characterize the simple majority rule as I have shown in Chapter 1.

Anonymity is the requirement that it should not matter if one voter votes for candidate two while another voter votes for candidate one, or the other way around. The outcome has to be the same. Since all voters should be treated equally, the outcome cannot be based on their identity, and thus has to satisfy anonymity (van der Hout and McGann 2009a, 741).

Van der Hout and McGann also claim that liberal equality requires neutrality, because otherwise it were not the voters decision who becomes elected but a bias in the allocation rule. This bias would treat voters of one party more favourable than those of the other parties. Thus, the voters are not treated equally if neutrality were violated (741).

Positive responsiveness guarantees that if a party wins additional votes while everything else stays equal, this party also gains an additional seat share. If this would not be the case, the allocation rule would violate the idea that the voters alone decide the outcome (van der Hout and McGann 2009b, 623).

These three properties have been used by May to characterize the simple majority rule (see Chapter 1), but van der Hout and McGann extend these axioms further to coalitions. The idea is that it is not a single party who finally makes decisions in the parliament, but coalitions of multiple parties. Therefore it is not just the relative size of the individual seat shares of parties that is important but the relative size of winning coalitions that can be formed (621).

6.3.2 List proportional representation

Van der Hout and McGann suggest to model fair representative elections by applying list proportional representation. This can be “defined as a seat allocation rule that assigns seat share to alternatives in proportion of their vote totals” (van der Hout and McGann 2009a, 744).

Modelling elections like this, means that the seats are treated as divisible goods. They refer to apportionment theory and Balinski and Young 2001 and suggest Webster as “the closest approximation to pure PR” (van der Hout and McGann 2009a, 743). However, they admit that any approximation can lead to very disproportional outcomes. Thus, they suggest that if not an approximation rule were used but a probabilistic distribution of the remaining seats “so that the *expected* seat share of each alternative equals its vote share” (744), we could reach the highest proportionality in representation.

However, for their argumentation for proportional representation as being fair to the voter they rely on the idealized notion of list proportional representation with divisible seats. They defend this by claiming that “[t]his allows us us to separate the problem what seat share allocation is required by our axioms from the practical problem of how this can be best approximated by a finite seat voting rule” (van der Hout and McGann 2009b, 619).

Van der Hout and McGann claim that any seat allocation rule that satisfies anonymity, neutrality and positive responsiveness must yield the same results as list proportional representation. Therefore, they argue that the results of list proportional representation are the only that respect the principles of liberal equality and popular sovereignty. Thus, proportional representation is justified if we want to treat all voters equally, and in doing so fairly (617).

6.4 Reflection

6.4.1 Fairness to groups of voters vs individual voters

Van der Hout and McGann criticize arguments for proportional representation that are based on fairness to groups. They also include groups conceived

as those individuals that all voted for the same party in this critique (van der Hout and McGann 2009a, 735). However, if we take the solid coalitions Dummett introduces as the formal representations of minorities that require representation in the parliament, does van der Hout and McGann's critique still hold?

Dummett's solid coalitions are not preconceived in the sense van der Hout and McGann criticize, but they are still kind of based on their preferences over the candidates. This is similar to being voters of the same party, and that is what van der Hout and McGann criticize. However, they criticize this mainly on grounds of being a voter of one party is just one of many salient features we can use to form groups. Dummett, on the other hand, sees the concept of solid coalitions just as not relying on a salient feature, but on the preferences of the individual voters who do not need to organize or cooperate to be represented.

Van der Hout and McGann themselves require that a group based fairness account needs to "justify why fairness should be seen in terms of a particular partition of the voters into groups. This involves both justifying why fairness should be seen in terms of these groups rather than in terms of individuals, and justifying why these groups are more appropriate than other possible sets of groups. Secondly, it is necessary to justify a particular conception of fairness between groups." (740) Thus, Dummett could (1) argue that the partition of the voters which the solid coalitions property makes is justified in the way he introduces them: a solid coalition is a group of similar-minded voters, who together should be represented according to their preferences. And (2) the justification why these groups should be represented according to their size and the Droop quota can be based on the requirement that the parliament should represent the public opinion. This is a pragmatic justification, since Dummett assumes seats not to be divisible, and that not only the largest group of voters should be represented, but also minorities. In the best case all minorities should be represented, but since seats are indivisible and limited he has to find a way how to guarantee the largest of those minorities next to the majority to be represented.

Furthermore, while van der Hout and McGann will have to justify some

approximation, Dummett justifies why these solid coalitions should be represented and to which degree. He kind of starts solving the problem from the other end: While van der Hout and McGann assume to represent every individual by their most favoured candidate, they later need to find a way to deal with the case in reality how to approximate. Dummett starts from the assumption that not everyone can get their most favoured candidate, and deals with this first. He suggests the pragmatic choice of a quota applied to those solid coalitions. If he assumed seats to be divisible and the quota to be 1 (which would mean that every single individual is a group sufficiently large to gain representation), we would yield the perfect proportionality van der Hout and McGann aim to justify, too.

Finally, Dummett talks about fairness in being represented, while van der Hout and McGann talk about fair treatment in the process of the election, but not about a fair result as such. This becomes even clearer when comparing them next with Chamberlin and Courant, who also have an account of individual fairness.

6.4.2 Individual fairness: equal treatment vs equal satisfaction of claims

Chamberlin and Courant propose that each individual voter has an equal claim to be represented as well as possible. Van der Hout and McGann, on the other side, propose that each individual voter has to be treated equally. While it indeed seems desirable to treat all voters fair in the sense of equally, there also seems to be more in representative democracy: the representation should be fair to the voters. Eventually, it is the result of the election that is important.

Van der Hout and McGann use the framework of seat allocation, and refer to the literature of claims problems. They, however, respond to this that they “do not view parties as ‘claimants’ with rights to certain seat shares, but rather require that the electoral system respect the equality and sovereignty of the voters.” (van der Hout and McGann 2009b, 620) They do not discuss the possibility that the voters have a claim for being represented, but there

might be more potential in the fair allocation literature if we treated the voters as having such a claim. Chamberlin and Courant who do so, however, do model proportional representation as an aggregation problem solely in the realm of social choice theory.

6.4.3 Tallied vote vs complete ranking

Van der Hout and McGann use tallied votes though they grant that allowing voters to state their full ranking would be desirable. Dummett, however, states clearly that using tallied votes in itself makes any election rule biased (Dummett 1984, 166). Also Chamberlin and Courant need the full ranking over all candidates from each voter to calculate the representativeness of each possible composition of the parliament. On the other hand, both Dummett, and Chamberlin and Courant assume seats to not be divisible. If they were divisible each voter in both their electoral systems could be represented solely by their most favoured candidate.

When assuming seats as not divisible, using tallied votes, becomes more problematic. We will have to use some approximation rule, while further information about the preferences beyond the voters' most favoured candidate or party is not available. However, approximation in just one district for the whole country will usually not be far from the perfect proportionality van der Hout and McGann aim to argue for (van der Hout and McGann 2009a, 743). At the same time using tallied votes makes it possible to apply an election rule to this one district instead of having to divide the country in several districts.

6.5 Summary

For van der Hout and McGann fairness to the voters means that only the voters themselves decide about the allocation of seat shares of the parliament while all voters are treated equally. These requirements are axiomatized as anonymity, neutrality and positive responsiveness.

The focus on fair treatment of individuals seems convincing if seats are

divisible and each individual is represented by their most favoured alternative eventually. However, usually there will have to be some approximation, and then the focus seems to be better on the representation as such. Though, in their favour speaks that the tallied-vote seat allocation is easily applicable to large scale elections. But, if there are that many seats to be filled there could also be a rule applied that satisfies the solid coalitions property and elects candidates instead of parties. This would circumvent Dummett's request of not having parties, i.e. pre-organized groups of candidates, elected.

Conclusion

Summary

In Part I of this thesis I showed how the axiomatic approaches of social choice theory, committee selection and fair allocation problems can be used to analyse elections in general and representative elections more specifically. However, while the social choice and committee selection literature offers much too less research on proportionality and representative elections, seat allocation is *prima facie* fair to parties and only derivatively fair to the voter. Still, all axiomatic approaches served helpful in Part II to show the extent in which proportional representation is fair to the voter.

In Part II I zeroed in on the issue of *whom* proportional representation is fair *to*. Therefore, I analysed three competing answers to this very question: (1) Dummett's account of protecting groups of minority opinion, (2) Chamberlin and Courant's request to satisfy each individual voter's claim for being represented as well as possible, and (3) van der Hout and McGann's approach to treat all individual voters equally.

The differences between these accounts can be summarised in respect of formal, conceptual and normative differences, and further the limitations each model has.

Formal differences

All authors assume that there is a predetermined number of seats. However, the other assumptions differ.

Dummett requests that all voters are allowed to rank as many candidates

as they wish. He then searches for a property of a committee selection function that—if satisfied—guarantees that the voters are treated fairly. This rule should be applicable in small districts in which three to five representatives have to be elected.

Chamberlin and Courant’s modified Borda rule is also aimed to be applied in small multi-member districts. They, however, model proportional representation in the framework of social choice theory. Therefore, they require the voters to state their complete preference ranking over candidates, but they then derive from these rankings individual preference rankings over all the possible compositions of the parliament. With this step they are able to use a modified Borda rule similar to the Borda rule in classic social choice theory. Thereby they can use results regarding the Borda rule and its axioms to discuss their election rule.

Van der Hout and McGann in contrast model proportional representation as seat allocation to the parties. They ask the voters only for their most favoured party. Then they derive from these tallied votes the number of voters standing behind each party, and search for an allocation rule that translates these vote shares into seat shares according to their conceptual idea of fairness in representation.

Conceptual differences

The three discussed proposals build all on different conceptual ideas regarding what representation is about.

The vague idea of *representation of the public opinion* is what Dummett takes as the main important issue of political representation. From this he derives that minority opinions need to be represented as well as the majority opinion.

Chamberlin and Courant are very precise what they take as the ideal of democracy: direct democracy. Political representation has to aim for this ideal to be legitimate. They argue that it is therefore necessary to represent each individual voter as well as possible in the deliberations and decisions in parliament. Each voter has the claim for exactly one representative to do so.

Van der Hout and McGann do not regard deliberations as that important but rather focus on the decision making. They elucidate that the decisions are not just made by individual representatives when voting in parliament. It is rather a coalition of several candidates or parties that form a majority. Since those coalitions can be formed in several ways, the possible results of those formations are what is important. The formation should eventually only depend on the voters' will. Therefore, the seat allocation rule has to be responsive only to the votes and not some bias included in the rule itself. Furthermore, they view it as most important that all voters are treated equally in the process of the election.

Normative differences

The question to which extent an electoral system yielding proportional representation is fair to the voters depends on the understanding how political representation works and what the underlying ideals are.

For Dummett an electoral system is fair to the voters if it reflects their preferences as accurately as possible and thereby leads to the representation of the public opinion. Therefore, any principles that only take the majority opinion into account can not be fair to the voters. Thus, he designs the property of solid coalitions which is aimed at the representation of groups of minority opinions.

The claim Chamberlin and Courant derive from the ideal of direct democracy and which every individual voter has for being represented as well as possible needs now to be satisfied in a fair manner. In their framework of social choice theory this is achieved via the modified Borda rule for the representation in deliberation. For a fair representation in decision making they supplement this rule with weighted voting. Thus, the majority's will eventually prevails in the decisions.

Van der Hout and McGann offer a detailed argument why equal treatment in the election of representatives is fair to the voters. Their interpretation how political representation works eventually allows them to derive axioms for an allocation rule to achieve this fair treatment.

Limitations

However, all approaches also have their limitations, especially in the application.

Dummett offers a property that seems to be widely accepted, but is not always easy to be implemented. Asking voters for their full preference ranking over all alternatives can be very tedious, for the voters as well as the tellers. Therefore, it might be more applicable for smaller districts. Although, I argued that in case of larger or just one single district with sufficient seats there might be an electoral system that offers the voters a satisfying representation with ranking only a few candidates.

This latter solution is not applicable for the modified Borda rule Chamberlin and Courant propose. There the voters have to give their full ranking. Instead, their model is applicable to elect parties instead of candidates, and then can be applied to larger districts. If there are too many alternatives and seats to fill, however, their rule becomes quickly hard to compute.

This computation of the outcome is much easier in the model of proportional representation van der Hout and McGann propose, because the voters can only vote for their most preferred alternative. However, their approach is also limited in the sense that they focus on the fair treatment in the process of the election. Fair treatment is an important part, but it is questionable if the voters only care about this. It seems to be the outcome of an election that is what matters in the end and what makes an election fair for the voters. They want to be represented since they are the people, and they want to have their preferences reflected in the decisions made in the parliament on their behalf.

Achievements

In this thesis I aimed to set the voters who are going to be represented center-stage. I did so by analysing three accounts of proportional representation regarding the extent they can be said to be fair to the voters. They themselves do not connect with each other on the aspect of fairness to the

voters, even though van der Hout and McGann refer to both Dummett, and Chamberlin and Courant on other aspects. Each of them applies a different framework and comes to different conclusions how to model proportional representation. Thus, my first contribution is the comparison of the few and hitherto unconnected accounts of proportional representation as fair to the voter.

Furthermore, I focused on the different perspectives to whom proportional representation is supposed to be fair to. I contrasted treating individual voters fairly with treating groups of voters or parties fairly. The literature regarding political representation neglects so far the discussion of fairness to the voters or only does so incidentally and based on intuition. Therefore, my second contribution is the careful distinction between these different perspectives.

The third contribution is the careful elucidation of the contrasting conceptual ideas of political representation and how they are related with the specific models of proportional representation.

Outlook

How does this careful elucidation of contrasting frameworks, models, concepts of political representation and each their limitations help us when considering the voters request for being represented fairly? Especially now, where many people are dissatisfied with their representation as I illustrated in the introduction.

I have shown that there obviously is more discussion of the fairness claim necessary but also that there are fruitful ways to do so. Each of the formal frameworks helped to elicit when and how exactly who is treated fairly. Furthermore, van der Hout and McGann for instance nicely illustrate further opportunities: they interpret elections of representatives as seat allocation, however do not exploit the notion of a claim or the literature on allocation problems. Chamberlin and Courant on the other hand, give an argument why each individual voter has a claim for being represented as well as possible, but then apply the social choice framework. While Wintein and Heilmann

2022 use the results from the literature on allocation problems to prove that van der Hout and McGann's proof is flawed, there might be more potential to use this literature to analyse seat allocation, and how to satisfy the voters' claims for being represented.

Bibliography

- Acemoglu, Daron, Suresh Naidu, Pascual Restrepo, and James A. Robinson. 2019. “Democracy Does Cause Growth.” *Journal of Political Economy* 127 (1): 47–100.
- Arrow, Kenneth J. 1951. *Social Choice and Individual Values*. New York, NY: John Wiley and Sons.
- Aziz, Haris, and Barton E. Lee. 2020. “The Expanding Approvals Rule: Improving Proportional Representation and Monotonicity.” *Social Choice and Welfare* 54 (1): 1–45.
- Balinski, Michel L., and H. Peyton Young. 2001. *Fair Representation: Meeting the Ideal of One Man, One Vote*. 2nd ed. Washington, DC: Brookings Institution Press.
- Blau, Adrian. 2004. “Fairness and Electoral Reform.” *The British Journal of Politics & International Relations* 6 (2): 165–181.
- Chamberlin, John R., and Paul N. Courant. 1983. “Representative Deliberations and Representative Decisions: Proportional Representation and the Borda Rule.” *The American Political Science Review* 77 (3): 718–733.
- Christiano, Tom, and Sameer Bajaj. 2022. “Democracy.” In *The Stanford Encyclopedia of Philosophy*, Spring 2022, edited by Edward N. Zalta. Metaphysics Research Lab, Stanford University.

- Dahl, Viktor, Erik Amnå, Shakuntala Banaji, Monique Landberg, Jan Šerek, Norberto Ribeiro, Mai Beilmann, Vassilis Pavlopoulos, and Bruna Zani. 2018. “Apathy or Alienation? Political Passivity among Youths across Eight European Union Countries.” *European Journal of Developmental Psychology* 15 (3): 284–301.
- Dummett, Michael. 1984. *Voting Procedures*. Oxford: Oxford University Press.
- Elkind, Edith, Piotr Faliszewski, Piotr Skowron, and Arkadii Slinko. 2017. “Properties of Multiwinner Voting Rules.” *Social Choice and Welfare* 48 (3): 599–632.
- Evans, Mark, Gerry Stoker, and Jamal Nasir. 2013. *How Do Australians Imagine Their Democracy?* Canberra: ANZSOG Institute for Governance at the University of Canberra.
- Felsenthal, Dan S., and Zeev Maoz. 1992. “Normative Properties of Four Single-Stage Multi-Winner Electoral Procedures.” *Behavioral Science* 37 (2): 109–127.
- Gibbard, Allan. 1973. “Manipulation of Voting Schemes: A General Result.” *Econometrica* 41 (4): 587–601.
- Gould, Carol C. 1995. *Rethinking Democracy: Freedom and Social Cooperation in Politics, Economy, and Society*. Cambridge: Cambridge University Press.
- Lagerspetz, Eerik. 2016. *Social Choice and Democratic Values*. Cham: Springer International Publishing.
- Landemore, Hélène. 2012. *Democratic Reason: Politics, Collective Intelligence, and the Rule of the Many*. Princeton University Press.
- May, Kenneth O. 1952. “A Set of Independent Necessary and Sufficient Conditions for Simple Majority Decision.” *Econometrica* 20 (4): 680–684.
- Mill, John Stuart. 1861. *Considerations on Representative Government*. London: Parker, Son, and Bourn.

- O'Neill, Barry. 1982. "A Problem of Rights Arbitration from the Talmud." *Mathematical Social Sciences* 2 (4): 345–371.
- Pitkin, Hanna. 1967. *The Concept of Representation*. Berkeley, California: University of California Press.
- Riedwyl, Hans, and Jürg Steiner. 1995. "What Is Proportionality Anyway?" Edited by Bernard Grofman, Arend Lijphart, Rein Taagepera, Matthew Soberg Shugart, and Don Aitkin. *Comparative Politics* 27 (3): 357–369.
- Riker, William H. 1988. *Liberalism against Populism: A Confrontation between the Theory of Democracy and the Theory of Social Choice*. Long Grove, Illinois: Waveland Press.
- Satterthwaite, Mark Allen. 1975. "Strategy-Proofness and Arrow's Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions." *Journal of Economic Theory* 10 (2): 187–217.
- Sen, Amartya. 1999. *Development as Freedom*. New York: Knopf.
- Singer, Peter. 1973. *Democracy and Disobedience*. Oxford: Clarendon Press.
- Thomson, William. 2019. *How to Divide When There Isn't Enough: From Aristotle, the Talmud, and Maimonides to the Axiomatics of Resource Allocation*. Cambridge, United Kingdom; New York, NY: Cambridge University Press.
- Valgarðsson, Viktor Orri. 2019. "Differential Turnout Decline in Norway and Sweden: A Generation of Apathy or Alienation?" *Scandinavian Political Studies* 42 (3-4): 270–295.
- Van der Hout, Eliora, and Anthony J. McGann. 2009a. "Liberal Political Equality Implies Proportional Representation." *Social Choice and Welfare* 33 (4): 617–627.
- . 2009b. "Proportional Representation Within the Limits of Liberalism Alone." *British Journal of Political Science* 39 (4): 735–754.

- Waldron, Jeremy. 2004. *Law and Disagreement*. Oxford: Oxford University Press.
- Wintein, Stefan, and Conrad Heilmann. 2022. “Liberal Political Equality Does Not Imply Proportional Representation.” *Social Choice and Welfare*.
- Young, H. Peyton. 1974. “An Axiomatization of Borda’s Rule.” *Journal of Economic Theory* 9 (1): 43–52.