

ERASMUS UNIVERSITEIT ROTTERDAM

MASTER THESIS

QUANTITATIVE MARKETING

**Modelling competition between
and within genres of consumer
goods; an application to
charitable giving**

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Abstract

The main goal of this paper is to develop a model that can be used to get insight in the competition between and within genres of consumer goods. A genre of consumer goods can be defined as a group of products that operate in the same product category and have the same objective. The model we propose is a model that is based on a two-level constant elasticity of substitution function. With this function we can look at the competition between genres and within genres. With the combination of this utility function and latent class modelling we can distinguish individuals with different types of behavior. This model will be applied on data about charitable giving. We have data of eleven different charities that operate in three different genres. In this analysis we find that we can say that individuals differ in their behavior in donating to charities, because our model with four latent classes has more explaining power than the model with no latent classes or two or three classes. We found that there are at least four different types of behavior. There are two classes that are very big and they look almost the same, so there are a lot of individuals in the same class. In these two classes the genre healthcare is more important for individuals, in the third class the genre healthcare is less important. In the fourth class people donate more to charities, than in the other classes, because they care less about the money they have left for other things. The model we developed in this paper can be used to distinguish different types of behavior.

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Chapter 1

Introduction

It is well known that there is competition between consumer goods. Because consumers cannot buy everything, they have to choose which product to buy and which not to buy. In this paper we want to investigate what the competition between genres of consumer goods looks like. To get insight in the competition we develop a model, which is based on a constant elasticity of substitution function. This function can be used to investigate whether consumer goods are complements or substitutes. We will expand the basic function to look at the competition between and within genres. A genre of consumer goods can be defined as a group of products that operate in the same product category and have the same objective. This model will be tested on data about eleven charities in three different genres (healthcare, development aid and community and culture). This model can be used in all different groups of consumer goods to look at the competitiveness of specific genres of consumer goods. On the other hand this paper is useful to get insight in the competition between genres of charities, something which is never done before.

We will describe the literature that is available about charitable giving. A crucial activity for charity organizations is raising money to be able to achieve the goals of the organization. Each year charities raise a lot of money in different ways (for example door-to-door collection, street collection, church collection, sponsorfund raising event, advertising appeal, television appeal etc.). There are different charities and they all want as many donations as possible. But for an individual it is almost impossible to give to all the different charities because there are too many charities. Therefore an individual has to decide whether to give and to which charity/charities to give. About how people decide to which charities they give is only little known. In this paper we want to investigate how people divide their donations between different charities.

In earlier research it was explored which characteristics of individual donors differ between various fundraising appeals (Schlegelmilch et al. (1997)). But the most personal way of fundraising is probably via direct mailing. With direct mailing charities can decide which people to send and which people not to send a direct mailing. This is more individual specific than for example door-to-door collection and they receive a lot of donations with those mailings. Therefore we will look at donations that are a reaction on direct mailings of different charities

in the Netherlands in this paper. Charities can raise money by sending direct mailings to the right people. If they send a direct mailing to individuals of whom it is known that the probability of reacting is very small then the direct mailing may not be valuable. If they send a direct mailing only to the people that have the highest probability of reacting with a donation, they can get the highest response rate and therefore the mailing will be the most profitable.

There are several researchers that indicate which people have the highest probability of reacting on a direct mailing (Diamantopoulos et al. (1993); Jones and Posnett (1991); Kitchen and Dalton (1990); Lee and Chang (2007); Nichols (1995)) The main goal of these researches was to look how donors and non-donors can be distinguished using demographic, socio-economic and psychographic variables. With this information charities can decide to which individuals they must send a direct mailing. These papers only look at which people donate and how much those people donate overall, not to a single specific charity. An other paper Bekkers and Wiepking (2007) give an overview of the different characteristics that influence donating behavior and some other research concerning charitable giving. van Heusden (2007) looked at different charities and the characteristics of individuals donating to the different charities; it indicates that different charities have different target groups. This indicates that the most effective mailing strategy is different for each charity.

Bennett (2002) looked to donations to genres of charities (healthcare, environmental care, international care, etc.) and links this with personal values. He investigated that personal values have the potential to influence the specific genre of charity that an individual might choose to assist. He also found that individuals with certain personal values support charities with the same values. From this paper we can conclude that demographic, socio-economic and psychographic variables and personal values do not only influence whether someone donates to charities but also influences which charity/charities to support. Reinsteijn (2008) found that households that give more to one type of charity tend to give more to others, but does not distinguish different genres of charities.

To look at the competitiveness of genres of consumer goods, and in specific charities, we use an extension of a constant elasticity of substitution function. This function was first derived by Arrow et al. (1961). This function is often used in consumer demand theory or as a production function. In other researches which looked at competition between consumer goods, the price has been an important topic (Shaked and Sutton (1982)). But in charitable giving an individuals can decide by themselves how much money to spend, so price does not matter in this context.

With this research to investigate how people divide their money between genres of charity organizations, or whether one gives it to different charities in the same genre. For some individuals is healthcare very important but for other people healthcare is as equal as important as development aid. Therefore we propose that there are different groups of individuals. Across these groups individuals differ in their donating behavior. For example a group can contain individuals that find one single genre of charities very important and gives only money to charities in the same genre. Another example can be a group that contains individuals that spread their donations over different genres of charities because they find everything important. We want to investigate whether there is heterogeneity in the donating behavior of individuals.

In the next section the theoretical framework will be described, explaining

how our economic model is build up. In Section 3 one can read about the data on which we will test our model. In Section 4 the estimation methods will be explained. After that in Section 5 the results of our proposed model will be shown. In the final section the conclusions will be drawn.

Chapter 2

Theoretical framework

As described in the introduction our main goal is to develop an economic model that can be used to look at the competition between genres of consumer goods where price does not matter. We want to look how individuals divide their money between different genres of consumer goods. In this chapter we will explain how our model is build up. This model is based on an utility function, which will be described in the first section of this chapter. In the second and third section we will explain the type of utility function that we will be using.

2.1 Utility function

Our proposed model is based on an utility function. In consumer theory an utility function is build up as in (2.1), where the utility U is produced by J inputs (F_1, \dots, F_J) .

$$U = F(F_1, \dots, F_J) \tag{2.1}$$

This function F can be of all different types. The function we use is a two-level constant elasticity of substitution function described as in Sato (1967). This function is based on several constant elasticity of substitution (ces) functions. In Section 2.2 the basic ces function with some characteristics is described. After that it will be expanded to the two-level ces function in Section 2.3.

2.2 Constant elasticity of substitution utility function

A ces function is often used as production function (Arrow et al. (1961)) or as utility function (Avinash and Stiglitz (1977)). In this paper it will be used as an utility function. When using a ces utlity function, the function F in (2.1) has the specification as in (2.2).

$$U = \left[\sum_{j=1}^J a_j^{\frac{1}{s}} F_j^{\frac{(s-1)}{s}} \right]^{\frac{s}{(s-1)}} \quad (2.2)$$

In this specification a_j are the share parameters, F_j the consumer goods, J the number of consumer goods and s the elasticity of substitution. This function can indicate whether the consumer goods are substitutes or complements. This will be important because it indicates whether it is better for individuals to divide their money between consumer goods or not.

When s approaches infinity the consumer goods are perfect substitutes, and when s equals zero they are perfect complements. These properties are derived in Arrow et al. (1961), where the ces function takes the form of special forms if $s \rightarrow 0$, $s = 1$ and $s \rightarrow \infty$. If $s \rightarrow \infty$ the ces function takes the linear form, which means that the goods are perfect substitutes. For $s \rightarrow 1$ the function goes to the Cobb-Douglas function. Someone with a Cobb-Douglas utility function spends a fixed fraction of his income on each consumer good. So there is a balance between substitution and complementarity. When $s \rightarrow 0$ the ces function takes the form of a Leontief function, which includes that the goods are perfect complements.

In the next section we will expand this ces function to be able to get insight in the division between genres of consumer goods.

2.3 Two-level constant elasticity of substitution function

If we wanted to know whether the consumer goods are substitutes or complements we could use the basic ces function, but we want to look at different genres of consumer goods. To be able to look how the competition between and within genres looks like, we expand the basic ces utility function to a two-level ces utility function as in Sato (1967).

The consumer goods can be partitioned into R subsets, which are a priori known, S_1, S_2, \dots, S_R and corresponding F , which are the different consumer goods, into R bundles $F^{(1)}, F^{(2)}, \dots, F^{(R)}$ so that $F_j \in F^{(r)}$ if $j \in S_r$. These R bundles correspond to the different genres of consumer goods. Then the utility function can be written as in (2.3).

$$U = F(\theta_1(F^{(1)}), \dots, \theta_R(F^{(R)})) \quad (2.3)$$

Where $\theta_1, \dots, \theta_R$ are functions of $F^{(1)}, \dots, F^{(R)}$, the different subsets. In the case of this two-level constant elasticity of substitution function the function F in (2.1) is a ces function and the functions $\theta_1, \dots, \theta_R$ are also ces functions.

We write the different subsets, which represents the utility of the different genres of consumer goods as S_r . Then the first level of the function can be written as in (2.4).

$$U = \left[\sum_{r=1}^R \alpha_r^{\left(\frac{1}{s}\right)} S_r^{\frac{(s-1)}{s}} \right]^{\frac{s}{(s-1)}} \quad (2.4)$$

The second level consist of different ces utility functions, one for each genre of consumer goods. The general specification of such a function can be seen in (2.5).

$$S_r = \left[\sum_{F_j \in F^{(r)}} \gamma_j^{\left(\frac{1}{p_r}\right)} F_j^{\frac{(p_r-1)}{p_r}} \right]^{\frac{p_r}{(p_r-1)}} \quad (2.5)$$

In this specifications the different consumer goods are represented by F_1, \dots, F_J (J is the number of consumer goods) where $F^{(r)}$ are the consumer goods in genre r and p_r the elasticity of substitution which is potentially different for each genre. Looking at this function the utility increases when someone donates more. The most optimal solution is that an individual spends an unlimited amount of money to all the consumer goods, we have to correct for this. Therefore the utility function does not only depend on the money spend on the consumer goods, but it contains also a term that depends on the budget spend on other things. Then the specification of the utility is as in (2.6), where I is income and D ($D = \sum_{j=1}^J F_j$) represents the total amount of money spend on the different consumer goods.

$$U(I, F) = \left[\sum_{r=1}^R \alpha_r^{\left(\frac{1}{s}\right)} S_r^{\frac{(s-1)}{s}} \right]^{\frac{s}{(s-1)}} + \lambda [I - D] \quad (2.6)$$

As can be seen in this expression the utility for an individual exists of two parts. The first part is a two-level ces function, it is the utility for donating money to charities; the second part is the utility for money that an individual has left for other things. The utility funtions for spending money on the consumer goods that are considered is increasing, the more you spend on these goods the higher the utility becomes. The utility for the money for other things is decreasing, the total expenditures on the goods considered. The more money you spend on the consumer goods, the less money you have left to buy and do other things you need and like. The derivative of the second part equals $-\lambda$, so the more you donate the less money you have for other things and the utility is lower when donating more.

With this specification for the utility, we are able to get insight in the competition between genres of consumer goods and in the competition within genres of consumer goods. We will shortly describe the meaning of all the different parameters. The α parameters represent the relative attractiveness of each genre, and the γ parameters the relative attractiveness of each specific consumer good within a genre. The parameter s is the elasticity of substitution between the genres of charities and p_r is the elasticity of substitution within the genres. When λ is higher someone cares more about their own income.

We will shortly describe when particular consumer goods are complements or substitutes. As explained before, if $s \rightarrow \infty$ the genres of consumer goods are perfect substitutes. In that case someone has to spend his money in the genre which has the highest α coefficient to maximize his utility function. If two genres both have a high α coefficient, higher than the α coefficients of the other genres, these two genres does not have to be substitutes of eachother. If they have around the same coefficient it does not matter in which of the two genres an individuals spends his budget, so it does not matter in which genre

someone spends his budget. If $s \rightarrow 0$ the genres of consumer goods are perfect complements, the α 's indicate how an individual has to divide his budget across the genres of consumer goods.

We can also look at the level of the consumer goods in stead of genres. If two consumer goods are in the same genre (r) and $p_r \rightarrow \infty$ the consumer goods are perfect substitutes. Here holds the same as between the genres, someone has to spend his money to the consumer goods with the highest γ_j coefficient to maximize his utility function. If two consumer goods both have around the same coefficient which is the highest in that genre, it does not matter to which of the consumer goods the money will be spend. So these consumer goods then are not substitutes, but are both substitutes of the other consumer goods in that genre. If $p_r \rightarrow 0$ the consumer goods are perfect complements, then the γ_j coefficients indicate how an individuals has to divide his budget across the consumer goods within that genre.

If two consumer goods are in another genre it depends on different parameters wheter the consumer goods are complements or substitutes. When $s \rightarrow \infty$ the genres of consumer goods are substitutes. If two genres both have a high α coefficient, higher than the α coefficients of the other genres, these two genres does not have to be substitutes of eachother. In this case the consumer goods from these two different genres are all complements if $p_r \rightarrow 0$ of both genres. If $p_r \rightarrow \infty$ of both genres the two consumer goods with the highest γ_j coefficient can be complements of each other. If When $s \rightarrow 0$ the genres of consumer goods are complements. But if $p_r \rightarrow \infty$ of two genres, one consumer good in both genres is a substitute of the other genres. That consumer goods then are complements of eachother, but the other goods in those genres still be substitutes of eachother. This are only a few examples, but it shows that whether consumer goods are complements or substitutes depends on different parameters.

Chapter 3

Data

To test our economic model we use data about charitable giving. The data is collected across eleven charities in The Netherlands. It is collected by the charities themselves. When someone once donates to a charity, that person enters their database. To raise money the charities send direct mailings to clients in their database. They send more mailings to individuals that respond more often to raise as much money as possible. In previous research (Diamantopoulos et al. (1993), Jones and Posnett (1991), Kitchen and Dalton (1990), Lee and Chang (2007), Nichols (1995)) it is indicated which individuals are more likely to give to charity, so the charities know which mailings they send to which individuals. For every single charity all the donations are known, including donations that are monthly/quarterly/yearly authorized. For each individual they register the amount of money donated and the number of donations.

In this paper we will consider the total amount donated to a charity in one year (2007), so we can compare the amounts of money given to different charities. The database for each charity does not only contain how much and how often people donate, but also their name and address. Given this the databases of different charities can be linked. Therefore we know for each individual to which charities they donated and the amount they gave to the different charities in the year 2007.

We use data of eleven charities that can be divided into three different genres. The genres that will be used in this paper are:

- Healthcare
- Community and culture
- Development aid

Some characteristics of the different genres, like for example the number of charities in that genre, the mean donation to a charity in that genre and the number of active donors, can be seen in Table 3.1.

Only individuals donating to more than one charity will be included in the analysis, because we want to investigate how people divide their money between

Genre	Healthcare	Community and culture	Development aid
Number of charities	6	2	3
Mean donation in euro per genre	56.39	16.24	52.61
Mean donation in euro per charity	9.40	8.14	17.54
Donors to at least one charity in this category	538139	218764	312740

Table 3.1: Characteristics of different genres of charities

different charities. Therefore it will not be useful to include individuals donating to only one charity, because we can not say anything about their division. There are a lot of individuals donating to one charity, therefore if we would include them the model would possibly underestimate some effects. If we look at individuals donating to more than one charity, the mean amount of money donating to charities in one year is €125.24. How this budget on average is divided across genres of charities can be seen in Table 3.1. We can see that the most is donated to the genre healthcare and the least to charities of the genre community and culture. The amount of money per charity is the highest in the category development aid.

In Table 3.2 the total donations to the different charities in the different genres can be seen.

Genre	Charity	Total donations in 1000€
Healthcare	1	13088
	2	9227
	3	521
	4	1915
	5	6615
	6	1702
Community and culture	7	5464
	8	4059
Development aid	9	11854
	10	11924
	11	7077

Table 3.2: Total donations to the different charities

Table 3.3 contains the percentage of the individuals that donate to different numbers of charities.

Here can be seen that most of the individuals in the dataset donate to two

Number of charities	Percentage of individuals
2	55.5
3	21.8
4	10.6
5	5.8
6	3.3
7 or more	3

Table 3.3: Percentage of people donating money to different numbers of charities

charities. Only less than 3% donates to seven or more charities. On average people donate to 2.91 charities, of which 1.77 to charities concerning healthcare, 0.44 to charities of the genre community and culture and 0.70 to charities in the sector development aid.

In Table 3.4 one can see how many of the individuals donating to a particular genre also donate to a charity/charities in another genre. The numbers on the diagonal gives the number of people donating to that genre. The off diagonal elements represent the combinations of genres, for example there are 269083 individuals that donate to one or more charities in the genre healthcare and to one or more charities in the sector development aid. This number also includes the individuals that donate to at least one charity in every genre. The number of individuals that donate in all the three different genre is 93044.

	Healthcare	Community and culture	Development aid
Healthcare	538139	185616	269083
Community and culture		218764	121539
Development aid			312740

Table 3.4: Individuals donating to different charities

Chapter 4

Econometric Analysis

Until so far we explained the utility function. In this chapter we will compute the optimal division of an individuals budget between different genres of consumer goods and we will discuss how we will estimate the parameters of the utility function. In this chapter will also be described how individuals with different behavior will be distinguished.

4.1 Optimal donating behavior

In this section the optimal division F_{ji} will be computed. The utility function is maximized if the marginal utility to all the different consumer goods (F_j for $j = 1, \dots, 11$) equals zero. To compute the marginal utilities it is useful to write out the expression of the utility function(2.6), this can be seen in (4.1).

$$U(I, F) = \left[\sum_{r=1}^R \alpha_r^{\frac{1}{s}} \left(\sum_{f \in F^{(r)}} \gamma_f^{\frac{1}{p_r-1}} F_f^{\frac{p_r-1}{p_r}} \right)^{\frac{p_r(s-1)}{s(p_r-1)}} \right]^{\frac{s}{(s-1)}} + \lambda [I - D] \quad (4.1)$$

In this expression R is the total number genres and $F^{(r)}$ are the consumer goods in genre r . This expression contains parameters that vary between the consumer goods (γ_f) and there are parameters that vary across genres (p_r). With this expression for the utility, the marginal utility of the consumer goods can be written as in (4.2).

$$\begin{aligned} \frac{\delta U(I, F)}{\delta F_j} = & \frac{s}{(s-1)} \left[\sum_{r=1}^R \alpha_r^{\frac{1}{s}} \left(\sum_{f \in F^g} \gamma_f^{\frac{1}{p_r-1}} F_f^{\frac{p_r-1}{p_r}} \right)^{\frac{p_r(s-1)}{s(p_r-1)}} \right]^{\frac{1}{(s-1)}} \alpha_{r_j}^{\frac{1}{s}} \frac{p_{r_j}(s-1)}{s(p_{r_j}-1)} \\ & \left(\sum_{f \in F^{r_j}} \gamma_f^{\frac{1}{p_{r_j}-1}} F_f^{\frac{(p_{r_j}-1)}{p_{r_j}}} \right)^{\frac{s-p_{r_j}}{s(p_{r_j}-1)}} \gamma_j^{\frac{1}{p_{r_j}-1}} \frac{(p_{r_j}-1)}{p_{r_j}} F_j^{\frac{-1}{p_{r_j}}} - \lambda \end{aligned} \quad (4.2)$$

In this expression r_j represents the genre of consumer good j . When all marginal utilities are equal to zero, then you have the optimal division of your budget across the different consumer goods.

Setting the marginal utilities equal to zero, the consumer goods can be written as a function of one consumer good in that genre. Then we get the following expression for the goods in terms of one good in that genre (4.3).

$$F_j = \frac{\gamma_j}{\gamma_{r_1}} F_{r_1} \text{ where } F_j \in F^{(r)} \quad (4.3)$$

Here we compute the money spend on consumer goods with respect to the money spend on the first good in the same genre. The money spend on the first good in a particular genre is represented by F_{r_1} and has coefficient γ_{r_1} .

Looking at this equation it looks like the division does not depend on the elasticity of substitution but only on the γ 's. This is not true, because γ is influenced by p_r because we estimate $\gamma^{\frac{1}{p_r}}$. So the value of the γ 's is influenced by the elasticity of substitution. The γ 's can be interpreted as the relative attractiveness of a consumer good in comparison with another consumer good.

These expressions for the charities can be filled out in the utility function that an individual wants to maximize. Then the utility changes in (4.4).

$$U(I, F) = \left[\sum_{r=1}^R \alpha_r^{\frac{1}{s}} \left(\left(\sum_{F_f \in F^{(r)}} \gamma_f \gamma_{r_1}^{\frac{(1-p_r)}{p_r}} \right) F_{r_1}^{\frac{(p_r-1)}{p_r}} \right)^{\frac{p_r(s-1)}{s(p_r-1)}} \right]^{\frac{s}{(s-1)}} + \lambda [I - D] \quad (4.4)$$

Now the utility function only contains one consumer good for each genre, so we only have to compute those marginal utilities and set these equal to zero to find the optimal division of an individuals budget. We can compute the marginal utilities and combine them to get expressions in terms of F_1 . Then all consumer goods can be written as a function of F_1 . The expressions for the first consumer goods for each genre F_{r_1} can be seen in (4.5).

$$F_{r_1} = \left(\frac{\alpha_1^{\frac{1}{s}} \frac{p_1 s - \delta_1 - s + 1}{p_1 s - s} \left(\sum_{F_f \in F^{(1)}} \gamma_f \gamma_{1_1}^{\frac{(1-p_r)}{p_r}} \right)^{\frac{p_1(s-1)}{s(p_1-1)}}}{\alpha_r^{\frac{1}{s}} \frac{p_r s - p_r - s + 1}{s(p_r-1)} \left(\sum_{F_f \in F^{(r)}} \gamma_f \gamma_{r_1}^{\frac{(1-p_r)}{p_r}} \right)^{\frac{p_r(s-1)}{s(p_r-1)}}} \right)^{-s} F_1 \quad (4.5)$$

With the expressions for F_{r_1} , all the eleven charities can be written as a function of F_1 . By filling out these expressions in the utility function (4.4) the utility function is also only a function of F_1 . Now the utility function looks like:

$$\begin{aligned}
U(I, F) = & \sum_{r=1}^R \alpha_r^{\frac{1}{s}} \left(\left(\sum_{F_f \in F^r} \gamma_f \gamma_{r1}^{\frac{(1-p_r)}{p_r}} \right) \right. \\
& \left. \left(\frac{\alpha_1^{\frac{1}{s}} \frac{p_1 s - p_1 - s + 1}{p_1 s - s} \left(\sum_{F_f \in F^{(1)}} \gamma_f \gamma_{11}^{\frac{(1-p_r)}{p_r}} \right)^{\frac{p_1(s-1)}{s(p_1-1)}}}{\alpha_r^{\frac{1}{s}} \frac{p_r s - p_r - s + 1}{s(p_r-1)} \left(\sum_{F_f \in F^{(r)}} \gamma_f \gamma_{r1}^{\frac{(1-p_r)}{p_r}} \right)^{\frac{p_r(s-1)}{s(p_r-1)}}} \right)^{-s} F_1^{\frac{(p_r-1)}{p_r}} \right)^{\frac{p_r(s-1)}{s(p_r-1)}} \\
& + \lambda [I - D] \tag{4.6}
\end{aligned}$$

To find the optimal division of an individuals budget over the charitable organizations we have to compute the marginal utility for F_1 and find the point where it equals zero. With the expressions for the other consumer goods ((4.3), (4.5) we can compute for each charity the optimal division.

The coefficients of the utility function will be estimated using maximum likelihood, (4.7).

$$\beta_{ML} = \operatorname{argmax}_{\beta, \sigma_\epsilon^2} L(F_{ji} | \beta, \sigma_\epsilon^2) \tag{4.7}$$

In this expression β represent all the parameters and F_{ji} contains the amount of money spend on consumer good j by individual i . This amount is a function of the most optimal division of money between the organizations as can be seen in expression (4.8).

$$F_{ji} = F_{ji}^* + \epsilon_{ji} \text{ where } \epsilon_{ji} \sim N(0, \sigma^2) \tag{4.8}$$

To be able to accomplish the maximum likelihood approach we need a likelihood function. The likelihood function we use is based on (4.8). This expression results in the likelihood function as in (4.9).

$$L(F_{ji} | \beta, \sigma_\epsilon^2) = \prod_i \prod_j \frac{1}{\sigma} \varphi \left(\frac{F_{ji} - F_{ji}^*(\beta)}{\sigma} \right) \tag{4.9}$$

This is the likelihood function we want to maximize. This function contains the optimal division F_{ji}^* , this a function of the parameters (β) that we want to estimate. Previously in this section is described how the optimal division (F_{ji}^*) can be computed.

4.2 Latent class analysis

We propose that individuals differ in their behavior with respect to buying consumer goods, so we assume that there is heterogeneity in the behavior of individuals. To capture the differences in donating behavior across individuals latent class modeling will be used. With latent class modeling individuals can be divided into different groups, which are called latent classes. These latent classes are a priori unobservable. In a particular latent class individuals are more or less the same with respect to the parameters of their utility function.

Therefore we can say that individuals in the same latent class show the same type of behavior. We will estimate these latent classes as done by Wedel and DeSarbo (1995). In the next sections we will shortly describe this method. In 4.2.1 the model will be described and in 4.2.2 the maximization procedure is explained.

4.2.1 Model

In this section we will describe the model that is used in the latent class analysis. The total population consists of a finite number (K) subpopulations. The subpopulations have proportions π_1, \dots, π_K of the total population, which are a priori unknown. We only have a constraint on these proportions, which can be seen in (4.10).

$$\sum_{k=1}^K \pi_k = 1 \quad , \quad \pi_k \geq 0 \quad , \quad k = 1, \dots, K \quad (4.10)$$

We also have an expression for the conditional probability density function of F_{ji} , this follows from (4.8) and can be seen in (4.11).

$$f_{ji|k}(F_{ji}|\beta_k, \sigma_k) = \frac{1}{\sigma_k \sqrt{2\pi}} \exp\left(-\frac{1}{2} \left(\frac{F_{ji} - F_{ji}^*(\beta_k)}{\sigma_k}\right)^2\right) \quad (4.11)$$

Looking at this expression we can see that it is almost equal to the likelihood function that is described in Section 3.1.3. With this expression we can write out the unconditional probability density function of an observation vector F_i . It can be written as a finite mixture form like as in (4.12).

$$f_i(F_i|\pi, \beta, \sigma) = \sum_{k=1}^K \pi_k \prod_{j=1}^J f_{ji|k}(F_{ji}|\beta_k, \sigma_k) \quad (4.12)$$

The parameters we want to estimate are π, β, σ . Then we know the sizes of the K different classes and we have for each class the β parameters and the variance. We now have a new likelihood that we want to maximize, because we have to include that there are different classes. This likelihood is based on the unconditional probability density function as in (4.12). The likelihood can be seen in (4.13).

$$L(\pi, \beta, \sigma; F) = \prod_{i=1}^n f_i(F_i|\pi, \beta, \sigma) \quad (4.13)$$

An estimate of π, β and σ can be obtained by maximizing this likelihood with respect to π, β and σ , thereby one has to take into account the constraint on π_k (4.10). This likelihood can be maximized using the EM-algorithm, this algorithm will be described in the next section. Once an estimate of the parameters has been obtained, one can calculate the posterior probability (α_{ki}) that observation i comes from latent class k . With use of Bayes' Theorem this posterior probability is given by (4.14).

$$\alpha_{ki}(F_i, \pi, \beta, \sigma) = \frac{\pi_k \prod_{j=1}^J f_{ji|k}(F_{ji}|\beta_k, \sigma_k)}{\sum_{l=1}^L \pi_l \prod_{j=1}^J f_{ji|l}(F_{ji}|\beta_l, \sigma_l)} \quad (4.14)$$

4.2.2 The EM algorithm

We use the EM algorithm to maximize the likelihood function. We use this because it is often used to maximize the likelihood of finite mixtures Titterington (1990) and has been very successful in those problems. In this subsection we will describe the optimization procedure, which is the EM algorithm.

We introduce non-observed data to derive the EM algorithm, this data is z_{ki} which represent whether individual i belongs to latent class k ; $z_{ki} = 1$ if individual i belongs to latent class k , $z_{ki} = 0$ if not. The following holds for this non-observed data (4.15).

$$f(z_i|\pi) = \prod_{k=1}^K \pi_k^{z_{ki}} \quad (4.15)$$

Further we assume that F_{ji} given z_i has the density as in (4.16).

$$f(F_{ji}|z_i) = \prod_{k=1}^K f_{ji|k}(F_{ji}|\beta_k, \sigma_k)^{z_{ki}} \quad (4.16)$$

The data z_{ki} is missing data, because it is not observed. Given the equations (4.15) and (4.16) we can write the log-likelihood function as in (4.17).

$$\ln L(\Theta; y, Z) = \sum_{i=1}^n \sum_{j=1}^J \sum_{k=1}^K z_{ki} \ln f_{ji|k}(F_{ji}|\beta_k, \lambda_k) + \sum_{i=1}^n \sum_{k=1}^K z_{ki} \ln \pi_k \quad (4.17)$$

This is the likelihood function that will be maximized using the iterative EM-algorithm. This algorithm exists of two steps, the E-step and the M-step. In the E-step the log-likelihood is replaced by its expectation, this expectation is calculated with provisional estimates of Θ . In the second step, the M-step, the expectation of $\ln L$ is maximized with respect to Θ , to get new provisional estimates. The E- and M-step are alternated until no further improvement in the likelihood is possible. Beneath we will describe the two steps in more detail.

E-step

In this step the expectation of the log-likelihood will be calculated with respect to the conditional distribution of the non-observed data Z , given the observed data F and provisional estimates of Θ . In calculating $E(\ln L(\Theta; y, Z))$ the unobserved data z_{ki} can be replaced by their expected values, $E(z_{ki}|y, \Theta)$. This expectation is equal to the posterior probability ($\hat{\alpha}_{ki}(F_i, \Theta)$) as in (4.14), where β 's and σ 's are the current estimates of β and σ .

M-step

To maximize the expectation of the log-likelihood with respect to Θ in the M-step, the non-observed data Z in equation (4.17) is replaced by their current expectations $\hat{\alpha}_{ki}$. Then the expectation of the log-likelihood, which we want to maximize in this step, has the following representation (4.18).

$$E(\ln L(\Theta; y, Z)) = \sum_{i=1}^n \sum_{j=1}^J \sum_{k=1}^K \hat{\alpha}_{ki} \ln f_{ji|k}(F_{ij}|\beta_i, \lambda_i) + \sum_{i=1}^n \sum_{k=1}^K \hat{\alpha}_{ki} \ln \pi_k \quad (4.18)$$

This equation exists of two parts which can be maximized seperately. The second part contains the term π_k on which we have the constraint (4.10). So the maximum of (4.18) with respect to π is obtained by maximizing the function:

$$\sum_{i=1}^n \sum_{j=1}^J \sum_{k=1}^K \hat{\alpha}_{ki} \ln \pi_k - \mu \left(\sum_{k=1}^K \pi_k - 1 \right) \quad (4.19)$$

In this equation μ is the Lagrangian multiplier. If we derive (4.19) with respect to π , set this equal to zero and solve it for π we get:

$$\hat{\pi}_k = \frac{\sum_{i=1}^n \hat{\alpha}_{ki}}{n} \quad (4.20)$$

The first part of equation (4.18) is the other part that has to be maximized. Maximizing this part is the same as maximizing (4.21) for each of the K expressions.

$$L_k^* = \sum_{i=1}^n \sum_{j=1}^J \hat{\alpha}_{ki} \ln f_{ji|k}(F_{ji} | \beta_k, \sigma_k) \quad (4.21)$$

This equation can be maximized as described in Section 3.2.1, with maximum likelihood, now it only also contains weights.

4.2.3 Number of latent classes

The exact number of different classes is a priori unkown. Therefore we have to decide how many latent classes to choose. There are different ways to decide how many classes to use. The standard generalized likelihood ratio test can not be used when there is a mixture model (Aitkin and Ruben (1995)). Although there are other procedures proposed to determine the number of classes, an overview of these methods is described in McLachlan and Basford (1988). The procedure we use to determine the number of latent classes is the Akaike's Information criterion (AIC) which was proposed by Bozdogan and Sclove (1984) and Sclove (1987). We use this criterion because it is computationally easy.

The AIC is defined as in 4.22, when applying it to our model.

$$AIC = -2\ln L + 2(P * K + K - 1) \quad (4.22)$$

In this equation P represents the number of parameters per class and K the number of classes. Although this criterion still relies on the same asymptotic properties that the likelihood ratio test uses, therefore we have to correct for this. The criterion we will use is the consistent AIC, also called CAIC. This criterion developed by Bozdogan (1987) corrects the AIC with the number of observations (n). The formula for the CAIC can be seen in (4.23).

$$CAIC = -2\ln L + (P * K + K - 1)(\ln(n) + 1) \quad (4.23)$$

We will use this criterion to select the number of latent classes. We will choose the number of latent classes which minimalizes the CAIC.

4.2.4 Parameters

In Section 3 we described the utility function. The utility function contains different parameters. The α 's represent the relative attractiveness of the different genres, the γ 's representing the relative attractiveness of the different charities in a particular genre of charities, λ which is in the term that represents the utility for money not spend on charities and we have four elasticities of substitution (s, p_1, p_2 and p_3). With latent class modelling one can form latent classes on basis of the difference in the parameters. Because it would take a long time if we vary all the parameters in the analysis we decided to vary a subset of the parameters. We want to look at the rate of substitution between the genres, therefore we want to vary the elasticities of substitution. The other parameters we allow to vary across classes are λ to capture income differences and the α parameters so people can differ in the genres they choose. So our subset of parameters that can vary across classes contain the following parameters: $s, p_1, p_2, p_3, \lambda, \alpha$. The vector β in (4.18) now only contains the subset of parameters. The values of the other parameters are set equal in all the classes and are estimated on the total dataset. These values will also be estimated in each iteration of the EM algorithm. After the values of the parameters that vary across the classes are estimated, we estimate the parameters that do not vary across the classes. The estimates of these parameters will be used in the next step of the EM algorithm to estimate the parameters that do vary across the classes. The parameters for $\alpha_1, \gamma_1, \gamma_7$ and γ_9 are set equal to one because of identification issues.

Chapter 5

Results

In this chapter the results will be described. First we will describe the results of the two-level ces function without allowing for heterogeneity in Section 4.1. In Section 4.2 the results of the model with allowing for heterogeneity will be discussed.

5.1 Results of the two-level constant elasticity of substitution function

In Section 3.1 the utility function that can be used to indicate whether genres of consumer goods are substitutes or complements was described. This function is a two-level constant elasticity of substitution function. In this section the results of this function will be described.

In Table 5.1 the estimates of the coefficients of the two-level constant elasticity of substitution function can be seen.

These estimates are based on the total data, without allowing for heterogeneity in the behavior of individuals. In this table the estimates of α_1 , γ_1 , γ_7 and γ_9 are not included, as explained earlier these parameters are set equal to one because of identification. The α parameters give the relative attractiveness of the different genres of the charities and the γ parameters the relative attractiveness of the charities in a particular genre. The elasticity of substitution between the genres is s and the elasticity of substitution between the charities in a genre are p_1 , p_2 and p_3 . The parameter λ is of the term for money that you have left for other things.

Looking at Table 5.1 one can see that genre three which is development aid is the most attractive genre for individuals, because it has the highest coefficient (the coefficient for the first genre was set equal to one). Although this coefficient is not significantly different from one. The least attractive genre seems to be community and culture, which is genre two, this has the lowest coefficient, although it does not significantly differ from the first genre. The coefficients for the genres community and culture and development aid do significantly differ from each other, so we can say that the genre development aid is more

Parameter	Coefficient	Standard deviation
α_2	0.5899	0.3377
α_3	1.2259	0.4357
s	1.0803	0.2699
γ_2	0.7050	0.2970
γ_3	0.0398	0.9562
γ_4	0.1463	0.8393
γ_5	0.5054	0.5871
γ_6	0.1300	0.8414
γ_8	0.7429	0.2608
γ_{10}	1.0059	0.2116
γ_{11}	0.5970	0.3749
δ_1	1.2266	0.4661
δ_2	4.5204	2.7978
δ_3	4.2702	2.4725
λ	2.6297	1.3154
σ	322.54	

Table 5.1: Estimates of the two-level ces function

attractive than the genre community and culture. The elasticity of substitution between genres s , is slightly bigger than one, but does not significantly differ from one. This indicates that there is a balance between complementarity and substitution effects. The elasticities of substitution in the different genres (δ_s) are bigger than one, but are not significantly different from one so we can not say anything about the competition within genres.

These estimates are based upon the total dataset without allowing different choices for individuals. With latent class analysis we want to allow for different behavior between individuals. The results of this analysis will be described in Section 5.2.

5.2 Latent class analysis

To allow for different behavior between individuals we perform a latent class analysis. How this analysis works was described in Section 4.2. The parameters on which we perform the analysis are, as described in 4.2.4, the four different elasticities of substitution, the α 's, λ and σ . In this section we will describe the results of the analysis.

First we looked at the model without latent classes and the models with two, three or four latent classes. We estimated the models and computed the CAIC (as explained in Section 4.2.3), because the number of latent classes will be chosen on basis of the CAIC. In Table 5.2 the CAIC of the different models can

Number of latent classes	$\ln(L)$	CAIC
1	$-4.3024 * 10^{15}$	$8.6048 * 10^{15}$
2	$-2.1516 * 10^{15}$	$4.3032 * 10^{15}$
3	$-1.1643 * 10^{15}$	$2.3286 * 10^{15}$
4	$-1.0553 * 10^{15}$	$2.1106 * 10^{15}$

Table 5.2: Loglikelihood CAIC of the model with different number of classes

be seen.

In this table we can see that the model becomes better by adding classes, as the CAIC should be minimized. The model with four latent classes explains more than the models with less latent classes. Because it takes a long time to compute the coefficients with latent classes we do not estimate the model with more classes.

Parameter	Coefficients	Standard devi-	Coefficients	Standard devi-
	Class 1	ation Class 1	Class 2	ation Class 2
α_2	0.7742	0.1281	0.7491	0.1281
α_3	1.0792	0.1863	1.0371	0.1862
s	1.1067	0.1419	1.1155	0.1420
p_1	1.1476	0.3044	1.1691	0.3042
p_2	13.344	6.6522	14.162	6.6515
p_3	10.908	5.2080	11.193	5.2075
λ	2.6683	0.7733	2.4579	0.7734
σ	80.4416		80.4408	
Class size	562785		14635	

Table 5.3: Coefficients of the first two latent classes

In Table 5.3 and 5.4 one can see the coefficients of the four different classes from the parameters that vary across the classes. In Table 5.5 one can see the coefficients of the parameters that are constant over the classes. In this table one can see the differences in the parameters, we will shortly describe the differences. One can easily see that the coefficients of the first class and the second class are almost the same. These classes contain the most individuals. In these classes the elasticity of substitution is around one, which means that there is a balance between substitution and complementary effects. The other two classes are smaller and differ more from the other classes. The third class for example the first genre, which is healthcare, is less attractive than the other genres this is different from the first two classes. In the fourth class the elasticity of substitution in the second genre is very small, which means that the charities in that genre are complements. The lambda parameter in the last class is smaller than in the first three classes, this means that people value donating to

Parameter	Coefficients	Standard devia-	Coefficients	Standard devia-
	Class 3	tion Class 3	Class 4	tion Class 4
α_2	1.7315	0.1569	0.5506	0.0328
α_3	1.6241	0.2283	0.2758	0.0494
s	2.8443	0.1744	1.0084	0.0417
p_1	3.2341	0.3742	3.1235	0.0902
p_2	153.43	8.1325	0.0843	1.5363
p_3	17.074	6.3669	1.0062	1.2011
λ	2.8921	0.9454	2.1094	0.1785
σ	108.4822		113.3065	
Class size	457		8572	

Table 5.4: Coefficients of the third and fourth latent classes

Parameter	Coefficient	Standard deviation
γ_2	0.67761	0.3197
γ_3	0.0462	0.9500
γ_4	0.1487	0.8370
γ_5	0.4578	0.6410
γ_6	0.1396	0.8320
γ_8	0.7643	0.2462
γ_{10}	1.5079	0.6365
γ_{11}	0.9412	0.1983

Table 5.5: Coefficients of the parameters that are constant over the classes

charities more than the other individuals.

In Table 5.6 one can see what the optimal donation for each charity in each class is. This is the expected behavior of someone in a particular class. Here one can see that the individuals in class four spend a lot more money on donating to charities than the individuals in other classes, which was expected on basis of the λ coefficients of the different classes. We can see that the division within the genres is the same for all the classes, this is the result of the fact that some parameters are constant over the classes. One can also see that in the fourth class the elasticity of substitution between genres is close to zero which means that the genres of charities are complements. This can also be seen in the Table 5.6, the division of the money is in the fourth class different from the other classes.

Genre	Charity	Class 1	Class 2	Class 3	Class 4
Healthcare	1	21.12	21.14	20.90	181.69
	2	14.31	14.33	14.57	123.11
	3	0.98	0.98	0.54	8.39
	4	3.14	3.14	2.75	27.02
	5	9.67	9.68	10.38	83.17
	6	2.95	2.95	2.39	25.36
Community and culture	7	8.25	8.26	8.82	98.88
	8	6.30	6.31	6.27	75.58
Development aid	9	12.45	12.46	18.73	265.39
	10	18.78	18.80	18.59	400.18
	11	11.72	11.73	9.98	249.78

Table 5.6: The optimal donation strategy for an individual in a particular class

Chapter 6

Conclusions and Discussion

In this paper we developed a model that can be used to look at competition between and within different genres of consumer goods. Because we think consumers make choices in different ways. Therefore we developed a model that to our thoughts can be used to investigate the behavior of individuals. The model we developed is based on a two-level CES utility function. With this model and latent class modelling we are able to divide individuals into different groups. We tested this model on data about charitable giving.

When testing our proposed model on the data it turned out that the model with four latent classes explains more than the model that does not allow heterogeneity (the model without latent classes). The model with four latent classes is also better than the models with two or three latent classes. Therefore we can say that there are at least four different types of behavior that individuals perform.

There are some things we can conclude from the results of the model with four latent classes. There are two classes that are very big and they look almost the same, so there are a lot of individuals in the same class. In these two classes the genre healthcare is more important for individuals, in the third class the genre healthcare is less important. In the fourth class people donate more to charities, than in the other classes, because they care less about the money they have left for other things. This can be the case if for example it contains individuals with a higher income. In one of the classes the genres of consumer goods are complements, in this class the division of someones budget is different from the other classes.

Because of the computation time we only computed the model with four or less classes. With this we can conclude the model becomes better by adding classes, so our model explains different types of behavior. With the latent classes we can say something about the differences in behavior of individuals. It only takes a long time to estimate the parameters with a lot of latent classes. In further research can be searched for a way which takes less time to estimate.

Another topic that can be used in further research is the number of parameters that can be varied. When all parameters are allowed to vary in the latent class analysis, you may get different classes because individuals also differ on that parameters. Now some effects are not clear because only a part of the

parameters is varied.

The data we use, the data of donations to charities, contains a lot of zero's because most of the people do not donate to all the charities. This is something that does not directly suit our model. This is another point that can be used in further research, by using for example a tobit model.

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