Erasmus University Rotterdam

Erasmus School of Economics
Master Thesis Quantitative Finance

Finding the Drivers of Delta-Hedged Equity and ETF Option Returns Using Instrumented Principal Component Analysis

Author:
Gerben van der Schaaf
416661

Supervisor:
Dr. M. Grith
Second assessor:
Dr. G. Freire

May 1, 2022

Abstract

In this paper, we apply a conditional latent factor model to analyze the notoriously complex cross-section of delta-hedged option returns. Specifically, the drivers of single-name equity and equity exchange-traded fund (ETF) near-the-money call options are examined through Instrumented Principal Component Analysis (IPCA). Opposing to former models, IPCA incorporates observable pricing-relevant characteristics via time-varying loadings that instrument for unobservable dynamics. We utilize an unbalanced panel data set of 1007 companies and 278 ETFs that ranges from 2006 to 2018 and consists of 109,168 monthly delta-hedged option returns. Furthermore, this research categorizes over the eleven GICS industry sectors and tackles the richly parameterized IPCA model by implementing regularization to counter overfitting. We find that a constant and implied volatility are vital drivers of all delta-hedged returns. Additional drivers include operating leverage, market capitalization, and total assets for the equity options and average daily bid-ask spread of the underlying, gamma, and theta for the ETF options. By identifying compensation for risk exposures, IPCA is able to generate annual out-of-sample Sharpe ratios above two and shows potential for profitable trading strategies.

Keywords: delta-hedged option returns, IPCA, equity options, ETF options, regularization

Contents

1 Introduction

Since the early developments of the modern capital market in the 17^{th} century in Amsterdam (Petram and Richards, 2014) traders have spent countless hours analyzing the movements of financial securities. The quest of understanding the risk-reward trade-off expanded vigorously after the introduction of the common factor analysis model of Spearman (1904) and its successors. For more than a hundred years, the research area of analyzing returns through factor analysis flourished with respect to stocks. In terms of options, this research field remained modest. According to Christoffersen et al. (2013), this results from decades of option research where the primary focus fixated on the pricing component using valuations based on no-arbitrage, leading to the acclaimed Black and Scholes (1973), Merton (1973), Cox et al. (1979), Hull and White (1987) and Heston (1993). This follows the traditional view that options are merely leveraged positions in the underlying with no other dynamics or purpose. For decades a stochastic process is assumed regarding the underlying asset value, completely ignoring possible cross-sectional relations of the underlying through common factors.

To empirically analyze the factor structure, decisions need to be made about the unobservable nature of factors and its loadings. There are two common approaches to overcome this obstacle. One can pre-specify factors, as for instance in the renowned Fama and French (1993) and Fama and French (2015). This approach implements established knowledge about the cross-section and therefore deems factors observable. However, this is often the main interest of a research. The other approach is to treat risk factors as latent. In that environment, one can apply principal component analysis (PCA). This technique is purely statistically originated and does not require any ex ante knowledge of the data structure. The downside of this approach are the difficult to interpret latent factors, its inadequacy in a dynamic setting, and its incapability in incorporating external data that could increase performance.

In this paper, we assume risk factors to be latent and implement characteristics as conditioning information for time-varying betas to describe empirically observed option returns. We implement the Instrumented Principal Component Analysis (IPCA) approach of Kelly et al. (2017), which has been empirically applied to stock returns in Kelly et al. (2019) and option returns in Büchner and Kelly (2022). IPCA combines the benefits of common factor models and PCA, while avoiding most of their weaknesses. By forcing factor loading to partially depend on periodically observable and reported asset characteristics, IPCA implements conditional information through instrumental variables for the latent conditional loadings. The theoretical background behind instrumental variables in an asset pricing setting is extensively described in Hansen (1982) and Cochrane (2009). The main motivation behind IPCA is for a model to suggest that characteristics proxy for loadings on common risk factors. This motivation is in line with dynamic equilibrium asset pricing theories. For instance, Santos and Veronesi (2004) argue that conditional betas vary over time due to a combination of economic conditions and firm characteristics.

Several papers did analyze a potential factor structure in options, yet limited their focus to bond options (Black et al., 1990) and interest rate options (Chen and Scott, 1992). Their findings raised several questions regarding other prominent derivatives that are inextricably intertwined with the stock exchange. Recently, Christoffersen et al. (2018) and Büchner and Kelly (2022) find that equity options and index options exhibit strong factor structures that explain a substantial portion of the cross-sectional variation.

During the period 1995-2012, the largest stock markets in the U.S. increased by roughly 300%, while the notional amount of financial derivatives held by the 25 largest U.S. bank holding companies grew around 1800% (Abdel-khalik and Chen, 2015). In comparison, the U.S. GDP merely doubled over the same time period, which indicates a vast growth of the derivatives market. Combining these developments with modern modeling approaches in the field of factor analysis, the question arises whether it is possible to shed light upon the factor structure in the cross-section of single-name equity option returns. Moreover, one might wonder if this factor structure changes when the underlying is grouped together in the form of an equity exchange-traded fund (from now on *ETF* in contrast to *equity ETF*). This analysis might offer a better understanding of the drivers of option returns, which can lead to new research areas and profitable trading strategies.

The framework of the IPCA offers the model previous knowledge about the structure, which improves the estimation of factors and loadings while still bypassing the need to determine factors a priori. This last attribute combined with the dynamic loading is crucial to the success of IPCA according to Kelly et al. (2019). We argue that these features are theoretically well suited for options, due to their rapidly evolving risks at the individual level. Another benefit of IPCA is its ability to impose a dimension reduction by selecting a small number of linear combinations of characteristics that are most informative about the cross-section.

Furthermore, IPCA does not enforce no-arbitrage computations, which increases the flexibility of the model when fitting the cross-section. Büchner and Kelly (2022) argue that while no-arbitrage restrictions offer economical meaning and consistent pricing across moneyness and maturities, they eventually lead to sacrificing realism for mathematical elegance as they tend to be unsuccessful in describing empirically observed patterns. For instance, the empirical evidence of a volatility skew proves the Black-Scholes model of Black and Scholes (1973) to be flawed. This skew follows from a vast increase in risk aversion after the Black Monday crash of 1987 (Hull, 2003), (Benzoni et al., 2011). One after-effect is the realization of traders that extreme events need to be better factored into option pricing, as implied volatility adjusts when options move deeper in-the-money or out-of-the-money. Additionally, empirical research as Ofek and Richardson (2003) and Ofek et al. (2004) have established that arbitrage opportunities occur in the option market. Israelov and Kelly (2017) further discuss the inadequacy of no-arbitrage models to capture the empirical behaviour of option returns. Hence, we avoid these pricing issues by resorting to IPCA.

Since stocks are well-known financial assets, there has been decades of research with regards to its underlying structure. Unfortunately, stock market risk factors have little to no explanatory power with respect to the cross-section of option returns, following Horenstein et al. (2020), Büchner and Kelly (2022), and Zhan et al. (2022). This is not a surprise considering the complex structure of options. As for instance, options are contracts that have a duration, whereas stocks are securities that represent partial ownership. The short lives of options combined with fluctuating risk attributes as moneyness make it problematic to estimate betas with standard time series regressions. Furthermore, for a comprehensive analysis the IPCA framework requires insightful and up-to-date data, which in our research focuses on numerous characteristics of publicly traded companies. While obtaining the necessary data could prove troublesome in the past, by virtue of an increase in transparency and a continuing development of specialised databases, many research ventures have been created.

According to Büchner and Kelly (2022), the most vital driver of option returns is the variation in the price of the underlying. We apply a strategy to our near-the-money options that is known as delta-hedging, which generates a return in excess of the variation in the underlying and is crucial in finding relevant drivers. Therefore, this strategy is the common choice in the literature. In a frictionless market the net investment of a delta-hedged portfolio earns the risk-free rate. However, we find negative average returns approaching 1% per month, which is in line with similar research as Coval and Shumway (2001), Cao and Han (2013), and Zhan et al. (2022).

To compare the benefits of the dynamic IPCA, we implement the static PCA as a benchmark. Furthermore, a common problem over the last decades regarding factor models is overfitting the cross-section. In the context of the IPCA model, it is probable that there are several redundant characteristics and including these parameters may lead to suboptimal out-of-sample performance.

Therefore, we aim to enhance the IPCA model by applying well-known shrinkage estimators and obtain a more parsimonious model. We incorporate ridge regression as proposed by Tikhonov and Arsenin (1977), the least absolute shrinkage and selection operator (Lasso) as popularised in Tibshirani (1996), and a convex combination of these two methods introduced in Zou and Hastie (2005), which is generally known as the elastic net. These three techniques operate through a regression with a penalty term that restricts the optimisation function. This restriction ensures the model to counter overparameterization by shrinking or even discarding the least informative parameters.

We find that an model with four latent factors explains more than 26% of the variation in a panel of near-the-money equity option returns. For the ETF option returns, this increases up to 40%. These numbers decrease by around one-half out-of-sample, suggesting excellent model fit. Moreover, IPCA identifies a constant and implied volatility as vital drivers of delta-hedged returns. Additional drivers include operating leverage, market capitalization, and total assets for the equity options and average daily bid-ask spread of the underlying, gamma, and theta for the ETF options. Additionally, we observe that regularization improves the model fit, yet it does not outperform basic IPCA in terms of Sharpe ratios. This suggests that there is less overfitting than expected. While basic IPCA generates annual out-of-sample Sharpe ratios above two, it remains a question whether this is a viable trading strategy due to the absence of transaction costs and considerable turnover.

The rest of the paper is organised as follows. We give a deeper exploration of the research field and further elaborate on its relation to our paper in Section 2. The framework of the IPCA, the corresponding regularization extensions, the significance tests of drivers and the evaluation criteria are described in Section 3. Next, the constructed data sets and decisions that we made regarding its features are discussed in Section 4. Then, the findings are discussed in Section 5. And lastly, we conclude our research in Section 6 and put our findings into perspective in Section 7.

2 Literature Review

The essence of factor analysis revolves around obtaining a small number of common factors that drive the variation of a large cross-section (Mulaik, 2009). The creation of hypothetical variables additionally ensures a desirable dimension reduction. After the introduction of the Arbitrage Pricing Theory (APT) of Ross (1976), PCA became popular in the field of analyzing returns at the hand of Chamberlain and Rothschild (1983) and Connor and Korajczyk (1986, 1988). Moreover, PCA is closely related to the aforementioned common factor analysis, which is also known as principal factor analysis (PFA). When put in conceptual terms, PCA analyzes variance and PFA analyzes covariance. This implies that PCA is favoured when the goal is to discover patterns in the data, while PFA is preferred when there are strong beliefs about the relationship among variables (Rao, 1964), (Schneeweiss and Mathes, 1995), (Jolliffe, 2002) and (Brown, 2009). Our research favors a PCA based approach as we do not want to make assumptions regarding the underlying structure.

In hindsight, many researches in finance following Spearman (1904) are regarded as blind factor analysis due to invalid motivations and misplaced applications (Mulaik, 2009). It is not until after the introduction of the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965), which build on the revolutionary ideas on diversification of Markowitz (1952), that factor analysis solidified its place in the world of financial analysis. In this model, the return of a security depends on its exposure to market risk, which is expressed by a single factor.

A few years later, Fama and MacBeth (1973) introduced the idea of analyzing the relationship between characteristics and returns. This inspired Basu (1977) to examine whether price-earning ratios indicate future performance. For decades, researchers came across factors that have explanatory power in the cross-section of stock returns, such as the size factor of Banz (1981), the reversal factor of Jegadeesh (1990), and the value factor of Chan et al. (1991). Thereafter, Fama and French (1992) analyze the effect of the size and value factors on the return of a asset, which was followed by an extension and improvement of the CAPM in Fama and French (1993). Next, Jegadeesh and Titman (1993) created a momentum factor that lead to Carhart (1997), after which Fama and French (2015) extended their former work by including a profitability and investment factor. Eventually, Harvey et al. (2016) found that using classical tests, at least 316 factors significantly explain the cross-section of stock returns.

This raises the question when to stop creating factors. Since there is no known truth about which factors are correct and which are acting as proxies for other risks, Cochrane (2011) speaks of a zoo of factors. This alleged surplus of factors is further discussed in Fama and French (2018). They elaborate on a fear for a dark age of data dredging, where empirically robust factors lack theoretical motivation. Nonetheless, all these factors share the idea that there is essential information behind returns hidden in asset and firm characteristics, which could lead to new perspectives on price fluctuations in the market.

In a static factor loading setting, Stock and Watson (2002), Bai and Ng (2002) and Bai (2003) tackle the high-dimensional framework and find that under certain assumptions principal components are consistent estimators of the factor space. Fan et al. (2016) take a large step by bridging the gap between static latent factor models and characteristic-based models through the introduction of projected principal component analysis (P-PCA). This technique implements PCA to a matrix of returns that is projected onto a linear space which is spanned by covariates that are deemed relevant. P-PCA includes similar to IPCA covariates that are tied to factors loadings to make estimation more efficient. However, the P-PCA framework only allows loadings and covariates that are constant over time. Hence, it can be viewed as a mix of PCA and IPCA. The time-variance restriction of static factor models lead Forni et al. (2000) to extend PCA by allowing for dynamic components, creating the generalized dynamic factor model (Barhoumi et al., 2013).

Almost two decades after the introduction of dynamic factors, Kelly et al. (2017) propose IPCA to obtain accurate estimates of latent factor models when loadings are time-varying. By conditioning on beta to instrument for the factor loadings, IPCA incorporates external information in the model and brings structure to factor loadings. Due to the advent of big data, Kelly et al. (2017) claim that exposures to fluctuations are ripe for harvesting through IPCA. Kelly et al. (2019) extend their former work with an asset pricing application for the cross-section of stock returns. They conclude that IPCA outperforms PCA, Fama and French (2015), and other well-known observable factor models in describing systematic risk and risk compensation. Additionally, IPCA estimates roughly 95% fewer parameters than the other models when five factors are selected.

Büchner and Kelly (2022) is the first paper to extend the IPCA framework to option returns. Since options are more complex securities than stocks, they are prone to be linked to more characteristics, making IPCA potentiality well-suited to shed light upon a hidden factor structure in option returns. Büchner and Kelly (2022) analyze a panel of monthly S&P 500 option returns and find that the characteristics moneyness, implied volatility, and gamma are responsible for most of the variation. And now our research further continues on this path by seeking the drivers of delta-hedged equity and ETF option returns using IPCA. Büchner and Kelly (2022) argue that a single latent factor is able to describe around 73% of the variation in their delta-hedged returns. This increases up to 91% for a four-factor model. Through a correlation analysis with relevant financial time series, Büchner and Kelly (2022) aim to interpret the generated latent risk factors. They find that the IPCA factors correspond to common option factors discussed in Karakaya (2014). Similar to Kelly et al. (2019), Büchner and Kelly (2022) claim that IPCA outperforms pre-specified factor models. They conclude that for index options, IPCA generates the most accurate explanation of the risk-return trade-off.

Other literature handle the complex nature of options by implementing pre-specified factors. One direction is by constructing portfolios sorts, such as Coval and Shumway (2001), Bakshi and Kapadia (2003), Goyal and Saretto (2009), Frazzini and Pedersen (2012), Cao and Han (2013), Vasquez (2017) and Cao et al. (2016), which generally leads to mediocre performances. Cao et al. (2016) discovers various characteristics that have positive or negative relations with the cross-section of delta-hedged equity option returns. Despite finding profitable option portfolio strategies after transaction costs, they emphasize the complexity of option return predictability. A different direction is through recognising the limitations of no-arbitrage option pricing models. Jones (2006) estimates non-linear factor models for short-term deep out-of-the-money S&P 500 options and finds that volatility and jump risk premia contribute substantially to the expected returns. Brooks et al. (2018) analyze a large set of characteristics to predict the expected returns of individual equity options. Similar to our motivation of including regularization, they find profitable option portfolio strategies by removing characteristics using the adaptive Lasso.

Christoffersen et al. (2018) and Horenstein et al. (2020) estimate latent factors through PCA and play an important role in our research motivation. The former constructs an equity option valuation model and finds a strong factor structure in single-name equity options. They argue that the first principal components of equity volatility, skews and term structures are highly correlated with the S&P 500 index option volatility, skew and term structure. The latter focuses on the factor structure of delta-hedged equity option returns by analyzing eleven characteristics and two market factors. Horenstein et al. (2020) claim that a four-factor model consisting of the market volatility risk factor and three characteristic-based factors captures the cross-section. Additionally, they find that stock return factors have little to no explanatory power in explaining delta-hedged equity option returns.

The recent rise of machine learning made it possible to evaluate enormous sets of parameters and apply algorithms that diminish the effects of uninformative variables for the sake of efficiency. Specifically, regularization in the form of ridge regression, Lasso and elastic net has been an increasingly popular technique in the asset pricing literature to tackle the challenges of a high-dimensional cross-section. For instance, Kozak et al. (2020) implement adaptations of ridge regression and elastic net to integrate numerous explanatory variables in a robust stochastic discount factor (SDF) to capture the cross-section of stock returns. Other research, such as Han et al. (2018), Rapach and Zhou (2020) and Freyberger et al. (2020) solely focus on predicting asset returns using various machine learning tools. We resemble these papers by applying machine learning techniques to find the most relevant drivers through modeling and shrinkage.

3 Methodology

In this section we discuss the framework of the constructed models. We start with the basic PCA model, then we elaborate on the IPCA structure, describe asset pricing tests and outline the implementation of regularization. Subsequently, we discuss the selected evaluation criteria and some general aspects of the implementation process. Additionally, we express the total number of distinct option issuers by N, the number of distinct option returns present at time t by N_t, the number of characteristics by L, the number of factors by K and the size of the sample period by T.

For the last 50 years, asset pricing has mostly been focused on the question why different assets earn different average returns. The general consensus has always been that higher returns reflect compensation for additional risks, despite the lack of an empirical sound model. The empirical search follows from the Euler equation for investment returns. This equation assumes no-arbitrage and leads to the existence of an SDF m_{t+1}, which for any excess return $r_{i,t+1}$ satisfies the equation $\mathbb{E}_t\left[m_{t+1}r_{i,t+1}\right]=0$, and leads to:

$$\mathbb{E}_t\left[r_{i,t+1}\right]=\underbrace{\frac{\text{Cov}_t(m_{t+1},r_{i,t+1})}{\text{Var}_t(m_{t+1})}}_{\beta_{i,t}}\underbrace{\left(-\frac{\text{Var}_t(m_{t+1})}{\mathbb{E}_t\left[m_{t+1}\right]}\right)}_{\lambda_t}, \tag{1}$$

where $\beta_{i,t}$ represents the loadings that can be interpreted as the exposures to systematic risk factors, and λ_t denotes the price of risk associated with the factors. Following Ross (1976) and Hansen and Richard (1987), when the SDF is linear in factors f_{t+1}, a factor model for excess returns can be constructed of the following form:

$$r_{i,t+1}=\alpha_{i,t}+\beta_{i,t}f_{t+1}+\epsilon_{i,t+1}, \tag{2}$$

with $\mathbb{E}_t\left[\epsilon_{i,t+1}\right]=0, \mathbb{E}_t\left[\epsilon_{i,t+1}f_{t+1}\right]=\mathbf{0}_{K\times 1}$, $\text{E}_t(f_{t+1})=\lambda_t$, and most importantly $\alpha_{i,t}=0$ for all assets i and time periods t. This framework is essential for a general analysis of expected returns across assets.

3.1 PCA

PCA is the most common factor analytic technique to discover latent factors. Two important features of this method are that it requires no ex ante knowledge about the return structure and that it has does not have time-varying loadings. To obtain the static PCA estimator, we drop $\alpha_{i,t}$ from Equation (2), which implies the assumption that the model captures all risk and that anomalies do not exist. Next, we remove the time-varying feature of the loading β and define the objective function by minimizing the error terms, which looks as follows:

$$r_t = \beta f_t + \epsilon_t, \tag{3}$$

$$\min_{\beta, F} \sum_{t=1}^{T} (r_t - \beta f_t)'(r_t - \beta f_t), \tag{4}$$

where r_t is an $N \times 1$ vector that contains the delta-hedged option returns of the N firms at time t, β is an $N \times K$ matrix that denotes the loadings, f_t is a $K \times 1$ vector that represent the orthogonal factors, ϵ_t is an $N_t \times 1$ matrix that expresses the errors and F is an $N \times K$ matrix that stacks all vectors f_t over time. This implies that we consider one return for company i at time t. The unbalanced data panel this creates is controlled through an approach discussed in Section 3.7.1. We continue Equation (4) by taking the first-order condition (FOC) with respect to f_t, which leads to:

$$\beta'(r_t - \beta f_t) = \mathbf{0}_{K\times 1} \iff \hat{f}_t = (\beta'\beta)^{-1}\beta' r_t, \tag{5}$$

which is reminiscent of an OLS estimator. Next, we substitute the optimal value of f_t in the objective function of Equation (4) and obtain the following objective function for β:

$$\max_{\beta} \operatorname{tr}\left(\sum_{t=1}^{T} (\beta'\beta)^{-1}\beta' r_t r_t' \beta\right). \tag{6}$$

This function aims to maximise a sum of Rayleigh quotients. In this particular case, the PCA solution is given by the first K eigenvectors of the sample second moment matrix of delta-hedged returns, namely $\sum_t r_t r_t'$. The PCA estimator implements the singular value decomposition (SVD) to the panel of delta-hedged returns.[1]

In terms of the characteristic-managed portfolios that are introduced in Section 3.3, the PCA solution is given by the sample second moment matrix of portfolio returns $\sum_t x_t x_t'$. Kelly et al. (2019) claim that PCA is very well suited for these portfolios, which is in line with the findings of Kozak et al. (2020). However, they also find inferior results in other areas. This instability in performance further substantiate the claim that PCA is prone for misspecification. Nevertheless, it remains a relevant benchmark for the IPCA model.

3.2 IPCA framework

The IPCA framework implements observable characteristics that instrument for the latent conditional loadings. The mapping between loadings and characteristics generate an environment where

[1] SVD is a factorisation that generalises the eigendecomposition of an $m \times n$ matrix M as follows: $\mathbf{M} = \mathbf{U\Sigma V^T}$, where $\mathbf{U}$ and $\mathbf{V}$ are square unitary orthogonal matrices that form an orthonormal eigenbasis of $\mathbf{M}$.

characteristics are linked to expected returns, while retaining the general consensus that risk premia are exclusively linked to risk exposures. This approach includes external information, yet avoids defining risk factors a priori. The central motivation lies in the assumption that the characteristics proxy for risk exposures and are therefore linked to compensation.

We continue with Equation (2) and focus on the dynamic factor loadings $\beta_{i,t}$. The key feature of IPCA is the next step, which is anchoring the loadings to observable instruments. According to Kelly et al. (2017), this not only provides an economical interpretation, it increases the overall estimation efficiency as well. We formulate the IPCA framework of Kelly et al. (2019) for an excess return as follows:

$$r_{i,t+1} = \alpha_{i,t} + \beta_{i,t} f_{t+1} + \epsilon_{i,t+1}, \tag{7}$$

$$\alpha_{i,t} = z'_{i,t}\Gamma_\alpha + \nu_{\alpha,i,t},, \tag{8}$$

$$\beta_{i,t} = z'_{i,t}\Gamma_\beta + \nu_{\beta,i,t},, \tag{9}$$

where $z_{i,t}$ is an $L \times 1$ instrument vector that contains the characteristics, Γ_α is an $L \times 1$ vector that denotes the weights of the instruments for the anomaly intercept, Γ_β is an $L \times K$ matrix that defines the mapping from a large number of characteristics to a relatively small number of risk exposures, and $\nu_{\alpha,i,t}$ and $\nu_{\beta,i,t}$ stand for the error terms with the former being a scalar and the latter being a $1 \times K$ vector. The relation between the dynamic loadings and the option characteristics is summarised in the $L \times (K + 1)$ matrix $\Gamma = [\Gamma_\alpha, \Gamma_\beta]$. This matrix is time-invariant, making it equal for every option return.

The dimensions reduction feature of IPCA lies in the matrix Γ_β. The estimation of this matrix comes down to finding a number of linear combinations of characteristics that describe the latent factor structure with the highest accuracy. This approach reduces the characteristic space. For instance, when a number of characteristics contain informative but noisy signals, the aggregation of the characteristics in linear combinations tends to average out the noise and isolate the signal to reveal true risk exposures. That is why a relatively large number of characteristics is preferred in the IPCA environment. Through the error terms $\nu_{\alpha,i,t}$ and $\nu_{\beta,i,t}$, the model accepts the possibility that characteristics cannot capture all existing risk exposures. While $\epsilon_{i,t+1}$ contains the idiosyncratic mispricing that is neither associated with the characteristics nor the systematic risk exposures.

Kelly et al. (2017) describes the $N \times K$ number of estimated parameters for PCA as an unnecessary excess. An important aspect of Γ_β is that its size does not increase when the panel data either increases in N_t or T. According to Kelly et al. (2017), this results in that for fixed K, L and N, T

approaching infinity, the IPCA estimator for $\Gamma = [\Gamma_\alpha, \Gamma_\beta]$ convergence rate is $\sqrt{NT}$, while the PCA convergence rate of β is only $\sqrt{T}$.

Before the implications of the Γ_α vector are discussed, we need to state the assumptions that are required for the IPCA model to achieve reliable estimations. Following the work of Kelly et al. (2017), we create a new variable that denotes the compound errors, namely $e_{i,t+1} = \epsilon_{i,t+1} + \nu_{\alpha,i,t} + \nu_{\beta,i,t} f_{t+1}$. Next, we define six assumptions in total that need to hold. Regarding consistency, the following three assumptions are required.

- **Assumption A.** The instruments must be orthogonal to the error terms, which means $\mathbb{E}\left[z_{i,t} e_{i,t}\right] = \mathbf{0}_{L\times 1}$. In the context of Equation (7) - (9), this can be written as $\mathbb{E}\left[z_{i,t}\epsilon_{i,t}\right] = \mathbb{E}\left[z_{i,t}\nu_{\alpha,i,t}\right] = \mathbb{E}\left[z_{i,t}\nu_{\beta,i,t} f_{t+1}\right] = \mathbf{0}_{L\times 1}$.

- **Assumption B.** The following moments must exist: (1) $\mathbb{E}\left(||f_t f_t'||^2\right)$, (2) $\mathbb{E}\left(||z_{i,t}' e_{i,t}||^2\right)$, (3) $\mathbb{E}\left(||z_{i,t} z_{i,t}'||^2\right)$, (4) $\mathbb{E}\left(||z_{i,t} z_{i,t}'||^2 ||f_t||^2\right)$.

- **Assumption C.** The parameter space of Γ is compact and away from rank deficient. This means that $\det\left(\Gamma'\Gamma\right) > \epsilon$ for some $\epsilon > 0$. Almost surely, $z_{i,t}$ is bounded. Next, we define $\Omega_t^{z,z} = \mathbb{E}\left[z_{i,t} z_{i,t}'\right]$, then almost surely $\det\left(\Omega_t^{z,z}\right) > \epsilon$ for some $\epsilon > 0$.

The orthogonality condition between instruments and errors can be seen as the adaption of IPCA of the exclusion restriction in common instrumental variable regression. Assumptions B and C are regularity conditions for consistency. Specifically, assumption B lists the requirements for the panel Law of Large Numbers to hold, while assumption C ensures that matrix $\Gamma' Z_t' Z_t \Gamma$, which will be implemented frequently in later sections, remains nonsingular. The matrix Z_t is an $N_t \times L$ matrix that denotes all options characteristics at time t. For asymptotic normality and to obtain the asymptotic variance, we impose the following three assumptions.

- **Assumption D.** (1) As $N, T \longrightarrow \infty$, $\frac{1}{\sqrt{NT}} \sum_{i,t} \text{vec}\left(z_{i,t} e_{i,t} f_t'\right) \xrightarrow{d} \text{Normal}\left(0, \Omega^{z,e,f}\right)$. (2) For any t, as $N \longrightarrow \infty$, $\frac{1}{\sqrt{N}} Z_t' e_t \xrightarrow{d} \text{Normal}\left(\mathbf{0}_{L\times 1}, \Omega_t^{z,e}\right)$. (3) As $N, T \longrightarrow \infty$, $\frac{1}{\sqrt{T}} \sum_t \text{vecb}\left(f_t f_t' - V^{f,f}\right) \xrightarrow{d}$ $\text{Normal}\left(\mathbf{0}_{K\times 1}, \mathbb{V}^{[3]}\right)$, where $V^{f,f} = \mathbb{E}\left[f_t f_t'\right]$.[2]

- **Assumption E.** There exists an $M < \infty$, such that $\forall N, T$, $\frac{1}{NT}\sum_{i,j,t,s} ||\tau_{ij,ts}|| \leq M$, where $\tau_{ij,ts} = \mathbb{E}\left[z_{i,t} e_{i,t} e_{j,s} z_{j,s}'\right]$.

[2] vec(M) is an operator that transforms a matrix M to a column vector by stacking the rows in order vertically. vecb(Q) vectorizes the upper triangle entries of a square matrix Q in a similar manner to vec. However, vecb does not include the diagonal.

- **Assumption F.** $\Omega_t^{z,z}$ is constant at $\Omega^{c,c}$.

Assumption D consists of a panel-wise central limit theorem and a cross-sectional central limit theorem. Assumptions E and F limit the cross-sectional dependency of $z_{i,t}e_{i,t}$ and restrict the variation of the cross-sectional second moment of $z_{i,t}$ to have concise expressions. These assumptions are similar to the ones discussed in Bai (2003).

3.2.1 Restricted IPCA ($\Gamma_\alpha = \mathbf{0}_{L\times 1}$)

Equation (3) applies a common choice in asset pricing, which is releasing the anomaly intercept $\alpha_{i,t}$. In the context of IPCA, this implies that conditional expected returns have no intercept that depends on characteristics, which leads to a restriction in the form of $\Gamma_\alpha = \mathbf{0}_{L\times 1}$. Restricting all $\alpha_{i,t}$ to zero is equivalent to the underlying belief that the characteristics are fully capable of describing the risk exposures. This discards the idea that characteristics also represent anomaly intercepts. Similar to Kelly et al. (2019), we derive from Equation (7) and (9) the following function:

$$r_{i,t+1} = z'_{i,t}\Gamma_\beta f_{t+1} + \epsilon^*_{i,t+1}, \tag{10}$$

where $\epsilon^*_{i,t+1} = \epsilon_{i,t+1} + \nu_{\alpha,i,t} + \nu_{\beta,i,t} f_{t+1}$ is a composite error. When this equation is transformed into vector form, we obtain the following formula:

$$r_{t+1} = Z_t\Gamma_\beta f_{t+1} + \epsilon^*_{t+1}, \tag{11}$$

where Z_t is an $N_t \times L$ matrix that denotes all options characteristics at time t and ϵ^*_{t+1} is an $N_t \times 1$ vector that represents composite errors. This leads to the following objective function that minimizes the sum of squared composite model errors:

$$\min_{\Gamma_\beta, F} \sum_{t=1}^{T-1} \left(r_{t+1} - Z_t\Gamma_\beta f_{t+1}\right)' \left(r_{t+1} - Z_t\Gamma_\beta f_{t+1}\right), \tag{12}$$

which is quite similar to Equation (4). However, in this environment we need to find values for f_{t+1} and Γ_β due to the anchoring of loading to observable characteristics, as described in Equation (9). Since applying the FOC requires only one non-constant variable, we need to consider either f_{t+1} or Γ_β constant when optimizing the IPCA objective function. Taking into account that the characteristic matrix Z_t and return vector r_{t+1} are known, the FOC with respect to f_{t+1} generates similar to Equation (5) the following optimal value:

$$\hat{f}_{t+1} = \left(\hat{\Gamma}'_\beta Z'_t Z_t \hat{\Gamma}_\beta\right)^{-1} \hat{\Gamma}'_\beta Z'_t r_{t+1}, \tag{13}$$

which holds for a given value of $\hat{\Gamma}_\beta$ and for all values of t. The same logic applies to obtaining $\hat{\Gamma}_\beta$. However, the matrices Z_t and f_{t+1} do not have the same dimensions. Therefore, we implement the Kronecker product.[3] Through this operator, Equation (12) can, for a given value of $\hat{f}_{t+1}$, be rewritten as follows:

$$\min_{\Gamma_\beta} \sum_{t=1}^{T-1} \left(r_{t+1} - Z_t \otimes \hat{f}'_{t+1} \text{vec}\left(\Gamma'_\beta\right)\right)' \left(r_{t+1} - Z_t \otimes \hat{f}'_{t+1} \text{vec}\left(\Gamma'_\beta\right)\right). \tag{14}$$

Next, we take the FOC with respect to vec $\left(\Gamma'_\beta\right)$ and retrieve the following equation:

$$\text{vec}\left(\hat{\Gamma}'_\beta\right) = \left(\sum_{t=1}^{T-1} Z'_t Z_t \otimes \hat{f}_{t+1} \hat{f}'_{t+1}\right)^{-1} \left(\sum_{t=1}^{T-1} \left[Z_t \otimes \hat{f}'_{t+1}\right]' r_{t+1}\right). \tag{15}$$

According to Kelly et al. (2019), Equation (13) and (15) have no closed-form solution and require numerical optimization. Additionally, one might wonder how $\hat{\Gamma}_\beta$ and $\hat{f}_{t+1}$ are sequentially considered as given values. The answer lies in the estimation method that is discussed in Section 3.7.1.

A common issue in latent factor models is the rotational unidentification problem of estimators. Without additional assumptions, any set of solutions can be transformed and generate equivalent results by implementing a non-singular $K \times K$ matrix R. For instance, we observe $\left(\hat{\Gamma}_\beta R^{-1}\right)\left(R \hat{f}_{t+1}\right) = \hat{\Gamma}_\beta I_K \hat{f}_{t+1} = \hat{\Gamma}_\beta \hat{f}_{t+1}$ where I_K is the K-dimensional identity matrix. Therefore, to realize a unique solution we impose the following three restrictions following Kelly et al. (2017): (1) $\Gamma'_\beta \Gamma_\beta = I_K$ ensuring that Γ_β is orthonormal, (2) $\mathbb{E}\left[f_t\right] \geq 0$ and (3) $\frac{1}{T}\sum_{t=1}^{T} f_t f_t = \text{diag}\left(a\right)$ with $a_i > a_{i+1}$ $\forall i$. According to Kelly et al. (2019), these three assumptions serve as a dynamic counterpart to the static PCA identification, as described in Stock and Watson (2002). Furthermore, the three assumptions do not interfere with the outcome of the IPCA model nor the economical interpretation of the results.

3.2.2 Unrestricted IPCA ($\Gamma_\alpha \neq \mathbf{0}_{L\times1}$)

In contrast, releasing the restriction ($\Gamma_\alpha = \mathbf{0}_{L\times1}$) accepts the possibility that characteristics represent anomaly intercepts. Or in other words, that characteristics describe expected returns in a manner that is not fully explained by risk exposures. Hence, when characteristics align differently with returns than they align with factor loadings, IPCA identifies compensation for holding risk that is not incorporated in the systematic risk exposures and estimates a nonzero Γ_α. Similar to Equation

[3]The Kronecker product is an operation on two matrices of arbitrary size that generates a block matrix. For instance, if A is an $m \times n$ matrix and B is a $p \times q$ matrix, then $A \otimes B$ is an $mp \times nq$ block matrix.

(10), we derive with the help of Equation (8) the following equation:

$$r_{i,t+1} = z'_{i,t}\Gamma_\alpha + z'_{i,t}\Gamma_\beta f_{t+1} + \epsilon^*_{i,t+1}, \tag{16}$$

which can be rewritten in vector form:

$$r_{t+1} = Z_t\Gamma_\alpha + Z_t\Gamma_\beta f_{t+1} + \epsilon^*_{t+1} = Z_t\Gamma\tilde{f}_{t+1} + \epsilon^*_{t+1}, \tag{17}$$

where we have $\Gamma = [\Gamma_\alpha, \Gamma_\beta]$ and $\tilde{f}_{t+1} = \left[1, f'_{t+1}\right]'$ to have an analogous representation as Equation (11). This leads to the following optimal equations when applying the FOC:

$$\tilde{f}^*_{t+1} = \left(\hat{\Gamma}'_\beta Z'_t Z_t \hat{\Gamma}_\beta\right)^{-1} \hat{\Gamma}'_\beta Z'_t \left(r_{t+1} - Z_t\Gamma_\alpha\right), \tag{18}$$

$$\text{vec}\left(\hat{\Gamma}'\right) = \left(\sum_{t=1}^{T-1} Z'_t Z_t \otimes \tilde{f}^*_{t+1}\tilde{f}^{*\prime}_{t+1}\right)^{-1} \left(\sum_{t=1}^{T-1} \left[Z_t \otimes \tilde{f}^{*\prime}_{t+1}\right]' r_{t+1}\right). \tag{19}$$

Equation (18) displays the unrestricted estimator that optimally allocates the panel variation in returns to the anomaly and the risk exposures. Despite the three identification assumptions of the restricted IPCA model, we are now faced with another identification problem due to the additional estimation of Γ_α. For example, for non-singular Γ_α and Γ_β we find $Z_t\left(\Gamma_\alpha + \Gamma_\beta\zeta\right) + Z_t\Gamma_\beta\left(f_{t+1} - \zeta\right) = Z_t\Gamma_\alpha + Z_t\Gamma_\beta f_{t+1}$, which holds for any constant $K \times 1$ vector ζ. To counter the identification problem, we introduce another assumption on top of the three assumptions discussed in Section 3.2.1. We impose that Γ_α and Γ_β are orthogonal, hence $\Gamma'_\alpha\Gamma_\beta = \mathbf{0}_{1\times K}$. This is realized by the following steps. We start by generating estimates of f_{t+1} and Γ through Equations (18) and (19). Then, we apply $\hat{\Gamma}'_\beta\hat{\Gamma}_\alpha = \xi$, where ξ is a $K \times 1$ vector. We follow this step by computing new values $\hat{f}^*_{t+1} = \hat{f}_{t+1} + \xi$ and $\hat{\Gamma}^*_\alpha = \left(I_L - \hat{\Gamma}_\beta\hat{\Gamma}'_\beta\right)\hat{\Gamma}_\alpha$. Lastly, a sign adjustment will be applied to the new optimal estimates $\hat{f}^*_{t+1}$ and $\hat{\Gamma}^* = \left[\hat{\Gamma}^*_\alpha, \hat{\Gamma}_\beta\right]$ based on the sign of the mean per row of F.

3.3 Characteristic-managed portfolios

In this research we also examine the returns of portfolios that are managed by the characteristics. This perspective offers great insight into the data and its corresponding portfolio performance. We construct these portfolios following Kelly et al. (2019) by interacting the option returns with the instruments as follows:

$$x_{t+1} = \frac{Z'_t r_{t+1}}{N_{t+1}}, \tag{20}$$

where x_{t+1} is an $L \times 1$ vector that expresses the return on a characteristic-managed portfolio at time t + 1. This means that we obtain a weighted average of option returns for the l-th element of x_{t+1}.

The weights are driven by the value of the l-th characteristic at time t and then normalised by the number of option returns at time $t+1$. Through normalising, the portfolio is not affected by a lack of availability. By stacking the time series, we obtain the $T \times L$ matrix $X = [x1, ..., x_T]'$, where each column represents a return series of a portfolio based on the l-th characteristic. This procedure is more convenient than sorting portfolios, as for example in Fama and French (2015). In our research, double sorting would lead to an exorbitant number of portfolios.

Furthermore, restating the cross-section trough characteristic-managed portfolios allows us to analyze the data in much lower dimensions ($T \times L$) than using options returns ($T \times N_t$), which reduces computational cost. Kelly et al. (2019) suggest that IPCA is a generalisation of period-by-period cross-section regression as introduced in Fama and MacBeth (1973). For instance, when $K = L$ the f_{t+1} estimates are the characteristic-managed portfolios and therefore equivalent to the Fama-MacBeth regression coefficients. Another perspective presents itself when we rewrite Equation (6) with the dynamic betas of Equation (9). This generates for the restricted IPCA model the following objective function:

$$\max_{\Gamma_\beta} \operatorname{tr}\left(\sum_{t=1}^{T-1} \left(\Gamma_\beta' Z_t' Z_t \Gamma_\beta\right)^{-1} \Gamma_\beta' Z_t' r_{t+1} r_{t+1}' Z_t \Gamma_\beta \right). \tag{21}$$

This function aims to maximise a sum of Rayleigh quotients where it is impossible to retrieve the first K eigenvectors due to the complexity of the time-variant Z_t. Nevertheless, the dynamic problem of Equation (21) is strongly related to the static problem, which is why Kelly et al. (2019) claim that the IPCA problem can be approximated by implementing SVD to characteristic-managed portfolios.

If $Z_t' Z_t$ is replaced by their time series average $\frac{1}{T}\sum_{t=1}^{T} Z_t' Z_t$, the solution to Equation (21) for Γ_β is the first K eigenvectors of the sample second moment matrix of portfolio returns $X'X = \sum_t x_t x_t'$, which is identical to the PCA solution for β for characteristic-managed portfolios as discussed in Section 3.1. This results in that the first K principal components of the characteristic-managed portfolio panel are the estimates of f_{t+1}. Hence, conditional that $Z_t' Z_t$ is not too volatile, this solution is a useful initialization for the optimization process.

Perhaps the most significant benefit from implementing characteristic-managed portfolios is that they avoid the problem of the unbalanced panel that torments individual contract data. To construct x_{t+1}, the inner product of r_{t+1} and Z_t is obtained using only the coincident non-missing elements. Following Kelly et al. (2019), this leads to the following computations of two imperative matrix multiplications:

$$Z_t' Z_t = \sum_{i \in \mathcal{N}_{t+1}} z_{i,t} z_{i,t}' \;, \qquad Z_t' r_{t+1} = \sum_{i \in \mathcal{N}_{t+1}} z_{i,t} r_{i,t+1} \;, \tag{22}$$

where $\mathcal{N}_{t+1}$ denotes the set of options that have no missing elements in terms of characteristics at time t and returns at time $t+1$. This process revolves around the belief that the estimator considers the L-dimensional $Z_t'Z_t$ and $Z_t'r_{t+1}$ to contain sufficient information for the N-dimensional option data. The fact that these portfolios can be constructed with or without missing data is crucial because of the inevitable features of our data set, which are further discussed in Section 4.3. Further implications of characteristic-managed portfolios are discussed in the next sections.

3.4 Asset pricing tests

A growing problem in the asset pricing literature is the choice of relevant test assets that need to be priced before a model is generally approved (Lewellen et al., 2010), (Daniel and Titman, 2012). IPCA tests offer two perspectives (Kelly et al., 2019). The first being a set of test assets that contains the best resolution, which are raw delta-hedged returns. The second are characteristic-managed portfolios, as they have relatively low dimensions and average out a vast part of idiosyncratic risk.

In this section, we elaborate on the required tests to determine which characteristics are statistically significant when describing the cross-section of the delta-hedged returns. It is also of importance to determine whether these characteristics are related to the risk exposures or if they contribute via an intercept. This directly calls for a test that focuses on Γ_α and test the hypothesis if it is significantly different from zero. Testing the anomaly in the IPCA framework generalises alpha-based tests as the GRS-test of Gibbons et al. (1989). The distinction between the two tests lies in a subtle difference in their motivations. Where the former searches for latent risk factors to explain the anomaly, the latter asks the same question yet searches in a pre-specified factor space. Lastly, we analyze ceteris paribus the incremental significance of a characteristic in the IPCA model.

3.4.1 Testing the anomaly ($\Gamma_\alpha = \mathbf{0}_{L\times 1}$)

The key feature of the dynamic loadings in the IPCA framework is that the estimator decides how the insightful information of characteristics is distributed over the parameters. When the characteristic proxy for exposure to risk factors, the characteristic information is attributed to Γ_β. This implies that if IPCA is unable to identify the latent risk factors that compensate for exposures, the model concludes that the characteristic effect is riskless compensation and allocates it to Γ_α. In this case, the compensation raises several questions, among which if the model is correctly specified.

We define the null hypothesis $H_0 : \Gamma_\alpha = \mathbf{0}_{L\times 1}$, making the alternative hypothesis $H_1 : \Gamma_\alpha \neq \mathbf{0}_{L\times 1}$. The reason that we are not testing whether $\alpha_{i,t}$ is equal to zero stems from that IPCA does not care about mispricing, as long as it is entirely idiosyncratic and unrelated to the characteristics. Hence, the alternative is not focused on values of $\alpha_{i,t}$. Instead, it is interested in the characteristics that cause the rejection of the null hypothesis and the corresponding economical meaning.

The test implements a Wald-type statistic that measures the distance between zero and the unrestricted anomaly estimates. This captures the increase in model fit from releasing the no-anomaly restriction. This research applies a 'residual' bootstrap procedure following Kelly et al. (2019). Bootstrapping is known to be reliable in finite samples and valid under weak assumptions on residual distributions. We start by obtaining estimates of parameters $\Gamma_\alpha, \Gamma_\beta$ and $\{f_t\}_{t=1}^{T}$ using the unrestricted model of Section 3.2.2. The aforementioned distance is acquired by $W_\alpha = \hat{\Gamma}_\alpha' \hat{\Gamma}_\alpha$. In the bootstrapping procedure, we choose to implement characteristic-managed portfolio residuals as they involve lower dimensions and avoid missing data issues. Moreover, the objective function of Equation (21) will unintentionally and inevitably be rewritten in terms of x_t when the normalisation is dropped. We combine Equation (17) and (20) and obtain:

$$x_{t+1} = \left(Z_t' Z_t\right) \Gamma_\alpha + \left(Z_t' Z_t\right) \Gamma_\beta f_{t+1} + Z_t' \epsilon_{t+1}^{*}. \tag{23}$$

Next, we define the portfolio residuals as $d_{t+1} = Z_t' \epsilon_{t+1}^{*}$, which is an $(L \times 1)$ vector that we store in a set $\{\hat{d}_t\}_{t=1}^{T}$. Then, we create the b-th bootstrap sample of returns as follows:

$$\tilde{x}_{t+1}^{(b)} = \left(Z_t' Z_t\right) \hat{\Gamma}_\beta \hat{f}_{t+1} + \tilde{d}_{t+1}^{(b)}\ , \qquad \tilde{d}_{t+1}^{(b)} = q_{t+1}^{(b)} \hat{d}_{h_{t+1}^{(b)}}\ , \tag{24}$$

where $b = 1, ..., 1000$, $h_{t+1}^{(b)}$ represents a random time index drawn uniformly without replacement from the available periods 1 to $T-1$ and $q_{t+1}^{(b)}$ is a random variable that follows a Student t-distribution with five degrees of freedom and unit variance. The motivation for the Student t-distribution stems from the consensus that asset returns tend to suffer from heteroskedasticity and that it increases efficiency of the bootstrap inference (Fama, 1965), (Lamoureux and Lastrapes, 1990), (Nelson, 1991), (Goncalves and Kilian, 2004).

Using Equation (24), the unrestricted IPCA model of Equation (23) is re-estimated and generates 1000 estimations of Γ_α. Then, we compute the test statistics as $\tilde{W}_\alpha^{(b)} = \tilde{\Gamma}_\alpha^{(b)\prime} \tilde{\Gamma}_\alpha^{(b)}$, which summarises the amount of sampling variation under the null hypothesis. For the last step, we count the number of times that the bootstrapped test statistic exceeds the test statistic from the real data. In statistical terms, this comes down to a p-value of $\frac{1}{1000} \sum_{b=1}^{1000} \mathbb{I}\left(\tilde{W}_\alpha^{(b)} > W_\alpha\right)$, where $\mathbb{I}_A$ denotes an indicator function that equals one when the inner statement is true and zero otherwise. Through the p-value, we can determine whether $\Gamma_\alpha = \mathbf{0}_{L\times 1}$ holds and if not, we can simply detect the characteristics that cause the rejection by looking at the individual magnitudes in Γ_α.

3.4.2 Testing the characteristic ($\gamma_{\beta,l} = \mathbf{0}_{K\times 1}$)

The second test that we implement focuses on the impact characteristics. This process starts at the partition of the loading matrix Γ_β. We define for L characteristics $\Gamma_\beta = [\gamma_{\beta,1}, ..., \gamma_{\beta,L}]'$, where each $\gamma_{\beta,l}$ is an $K \times 1$ vector that connects characteristic l to the K factors. Hence, $\gamma_{\beta,l,k}$ is a scalar that represents the loading of characteristic l on factor k. For this test, we discard the idea of an existing anomaly, which means that we assume $\Gamma_\alpha = \mathbf{0}_{L\times 1}$ and estimate the restricted IPCA model.

The null hypothesis is $H_0 : \Gamma_\beta = [\gamma_{\beta,1}, ..., \gamma_{\beta,l-1}, \mathbf{0}_{K\times 1}, \gamma_{\beta,l+1}, ...\gamma_{\beta,L}]'$ and the alternative hypothesis is constructed as $H_1 : \Gamma_\beta = [\gamma_{\beta,1}, ..., \gamma_{\beta,L}]'$ with $\gamma_{\beta,l} \neq \mathbf{0}_{K\times 1}$. These hypothesis stem from the idea that the characteristic of interest does not influence the K factors in any way. Following the same residual bootstrap concept as the anomaly test, we obtain the following sets of parameters $\{\hat{\gamma}_{\beta,l}\}_{l=1}^{L}, \{\hat{f}_t\}_{t=1}^{T}$ and $\{\hat{d}_t\}_{t=1}^{T}$ by estimating Equation (11) under the alternative hypothesis with $\Gamma_\alpha = \mathbf{0}_{L\times 1}$. We measure the distance between the two hypothesis using a Wald-type statistic, which is computed as $W_{\beta,l} = \hat{\gamma}'_{\beta,l}\hat{\gamma}_{\beta,l}$. In the next step, we define $\tilde{\Gamma}_\beta = [\hat{\gamma}_{\beta,1}, ..., \hat{\gamma}_{\beta,l-1}, \mathbf{0}_{K\times 1}, \hat{\gamma}_{\beta,l+1}, ...\hat{\gamma}_{\beta,L}]'$ and incorporate this estimation in the b-th bootstrap sample of returns under the null hypothesis for $b = 1, ..., 1000$. This is constructed as $\tilde{x}_{t+1}^{(b)} = (Z_t' Z_t)\,\tilde{\Gamma}_\beta \hat{f}_{t+1} + \tilde{d}_{t+1}^{(b)}$, where $\tilde{d}_{t+1}^{(b)}$ is obtained similar to to Equation (24). This action is followed by re-estimating the restricted IPCA model under the alternative hypothesis for each sample b, which generates the test statistic $\tilde{W}_{\beta,l}^{(b)}$. Lastly, the p-value of the test is computed as $\frac{1}{1000}\sum_{b=1}^{1000}\mathbb{I}\left(\tilde{W}_{\beta,l}^{(b)} > W_{\beta,l}\right)$. In case the interest shifts to the joint significance of characteristics, we can easily extend the framework through an addition in the computation of the test statistic. Similarly, if one element in $\gamma_{\beta,l}$ is notable due to its size, we can restrict this parameter to zero and perform a similar bootstrap to analyze its significance.

3.5 Regularization

Kelly et al. (2019) claim that the IPCA method has excellent performance compared to well-known factor models. Not only is it capable in describing systematic risks and risk compensation, IPCA proves to be more computational efficient due to its dimension reduction feature. Nevertheless, an IPCA model with a limited number of characteristics, factors, and observations still contains a substantial number of parameters. This increases the chance of overfitting what generally reduces out-of-sample performance tremendously. Following recent literature, we utilize the opportunities given by regularization machine learning approaches to counter overfitting and improve model performance in high-dimensional data. In this section, we elaborate on the process of implementing ridge regression, Lasso and elastic net to enhance the IPCA framework.

There are a few distinctions to be made before we discuss the techniques. The first is that we do not regularize the factors f_t. The reason being that our interest lies in the weights of the characteristics and not the pricing of systematic risks. Hence, we solely focus on the Γ matrix. Secondly, the input data needs to be standardised before the regularization can be applied, which consists of two parts. The first is removing the need for an intercept by centering the data through a demeaning process. And the second is scaling that results in all characteristics having equal variance, which is required to interpret and compare the generated coefficients in a coherent manner. Lastly, the assumptions to counter the rotational unidentification problems of Section 3.2.1 and 3.2.2 do not change when we regularize the loading matrix Γ.

3.5.1 Ridge regression

The concepts of ridge regression are first introduced as Tikhonov regularization in Tikhonov and Arsenin (1977), making it one of the earliest implementations of regularization. Similar to the IPCA objective function, ridge regression aims to minimize the sum of squared errors (SSE). Only in this environment, there is a regularization term that penalises the square magnitudes of all coefficients in Γ, which is known as L^2 regularization or the Euclidean norm. For the restricted IPCA model, we adjust Equation (12) and obtain the following objective function:

$$\min_{\Gamma_\beta, F} \sum_{t=1}^{T-1} \left(r_{t+1} - Z_t \Gamma_\beta f_{t+1}\right)' \left(r_{t+1} - Z_t \Gamma_\beta f_{t+1}\right) + \lambda \sum_{l=1}^{L} \sum_{k=1}^{K} \gamma_{\beta,l,k}^2, \tag{25}$$

where λ denotes the penalty parameter with a non-negative value that determines the effect of the penalty term. For $\lambda \to 0$, we obtain identical estimates as for the restricted IPCA model. And when $\lambda \to \infty$, we observe that all estimates move towards zero. It is important to note that due to the construction of the ridge estimator the $\gamma_{\beta,l,k}$ coefficients can approach zero, yet never reach it. Next, we implement the optimal value for f_{t+1}^{rdg} from Equation (13) where we replace $\hat{\Gamma}_\beta$ with $\hat{\Gamma}_\beta^{\text{rdg}}$ and then take the FOC with respect to $\Gamma_\beta^{\text{rdg}}$, which leads to the following IPCA ridge estimator:

$$\text{vec}\left(\hat{\Gamma}_\beta^{\text{rdg}\,\prime}\right) = \left(\sum_{t=1}^{T-1} Z_t' Z_t \otimes \hat{f}_{t+1}^{\text{rdg}} \hat{f}_{t+1}^{\text{rdg}\,\prime} + \lambda\, I_{LK}\right)^{-1} \left(\sum_{t=1}^{T-1} \left[Z_t \otimes \hat{f}_{t+1}^{\text{rdg}\,\prime}\right]' r_{t+1}\right), \tag{26}$$

where an increase in λ directly results in a decrease in Γ_β. Analogously, we obtain the IPCA ridge estimator for the unrestricted IPCA model:

$$\text{vec}\left(\hat{\Gamma}^{\text{rdg}\,\prime}\right) = \left(\sum_{t=1}^{T-1} Z_t' Z_t \otimes \tilde{f}_{t+1}^{\text{rdg}\,*} \tilde{f}_{t+1}^{\text{rdg}\,*\prime} + \lambda\, I_{LK}\right)^{-1} \left(\sum_{t=1}^{T-1} \left[Z_t \otimes \tilde{f}_{t+1}^{\text{rdg}\,*\prime}\right]' r_{t+1}\right), \tag{27}$$

where we obtain $\tilde{f}_{t+1}^{\text{rdg}\,*}$ in a similar manner as f_{t+1}^{rdg}.

3.5.2 Lasso

The Lasso is popularised in Tibshirani (1996) and functions as an regression technique that simultaneously performs regularization and variable selection. This last feature generates sparsity and is the most important difference with ridge regression. The Lasso is able to set coefficients equal to zero by utilising an absolute value in the penalty term, which is known as the L^1 norm. Including an absolute value function leads to jumps in the derivative, opposing to ridge where the derivative is smooth. This allows the Lasso to remove uninformative variables in its entirety, making the model more parsimonious and interpretable. We refer to Hastie et al. (2009) for a deeper understanding of the distinctions between Lasso and ridge. By adjusting the penalty function of Equation (25) and adding a scalar for convenience, we obtain for restricted IPCA the following objective function:

$$\min_{\Gamma_\beta, F} \frac{1}{2} \sum_{t=1}^{T-1} \left(r_{t+1} - Z_t \Gamma_\beta f_{t+1}\right)' \left(r_{t+1} - Z_t \Gamma_\beta f_{t+1}\right) + \lambda \sum_{l=1}^{L} \sum_{k=1}^{K} |\gamma_{\beta,l,k}| \ , \tag{28}$$

where we regard $\frac{1}{2}$ in the penalty term to be subsumed by λ. Due to the absolute value function, this objective function is not differentiable at zero. Nevertheless, since the function is continuous for all values that are not zero, we can take the derivative and consider zero as a special case. We start by rewriting Equation (28):

$$\min_{\Gamma_\beta, F} \frac{1}{2} \sum_{t=1}^{T-1} \left(r_{t+1} - \left(Z_t \otimes f'_{t+1}\right) \left(\gamma_{\beta,1}, ..., \gamma_{\beta,LK}\right)'\right)' \left(r_{t+1} - \left(Z_t \otimes f'_{t+1}\right) \left(\gamma_{\beta,1}, ..., \gamma_{\beta,LK}\right)'\right) + \lambda \sum_{i=1}^{LK} |\gamma_{\beta,i}| \ , \tag{29}$$

where we vectorised Γ_β and summarised the elements of $\gamma_{\beta,l,k}$ in a vector to obtain a single summation in the penalty term. To simplify the function, we introduce $G_t = Z_t \otimes f'_{t+1}$ which is an $N_t \times LK$ matrix that can also be expressed as LK column vectors $g_{t,lk}$. Further simplification leads to the following function:

$$\min_{\Gamma_\beta, F} \frac{1}{2} \sum_{t=1}^{T-1} \left(r_{t+1} - \sum_{i=1}^{LK} \left(g_{t,i} \gamma_{\beta,i}\right)\right)' \left(r_{t+1} - \sum_{i=1}^{LK} \left(g_{t,i} \gamma_{\beta,i}\right)\right) + \lambda \sum_{i=1}^{LK} |\gamma_{\beta,i}| \ . \tag{30}$$

Next, we implement an adaptation of Equation (13) and denote the corresponding objective function with $\mathcal{Q}\left(\gamma_{\beta,i}, \lambda\right)$. This action is followed by taking the derivative with respect to $\gamma_{\beta,j}$, which produces:

$$\frac{\partial \mathcal{Q}\left(\gamma_{\beta,i}, \lambda\right)}{\partial \gamma_{\beta,j}} = - \sum_{t=1}^{T-1} g'_{t,j} \left(r_{t+1} - \sum_{i=1}^{LK} \left(g_{t,i} \gamma_{\beta,i}\right)\right) + \frac{\partial \lambda |\gamma_{\beta,j}|}{\partial \gamma_{\beta,j}} \ , \tag{31}$$

where the summation of $\gamma_{\beta,i}$ can be rewritten to single out $\gamma_{\beta,j}$. This leads to the following FOC:

$$\frac{\partial \mathcal{Q}\left(\gamma_{\beta,i}, \lambda\right)}{\partial \gamma_{\beta,j}} = - \sum_{t=1}^{T-1} g'_{t,j} \left(r_{t+1} - \sum_{i \neq j} \left(g_{t,i} \gamma_{\beta,i}\right)\right) + \sum_{t=1}^{T-1} g'_{t,lk} g_{t,lk} \gamma_{\beta,j} + \lambda \frac{\partial |\gamma_{\beta,j}|}{\partial \gamma_{\beta,j}} = 0 \ . \tag{32}$$

The problem that we face is that the absolute value term is not differentiable in the origin. Nonetheless, due to its convexity we are able to analyze its subderivative. According to Wriggers and Panagiotopoulos (1999), the subdifferential at the origin is the interval $[-1, 1]$. We apply this subdifferential using Equation (32) with $\gamma_{\beta,j} = 0$ and obtain:

$$\left[-\sum_{t=1}^{T-1} g'_{t,j} \left(r_{t+1} - \sum_{i \neq j} (g_{t,i}\gamma_{\beta,i}) \right) - \lambda \ , \ -\sum_{t=1}^{T-1} g'_{t,j} \left(r_{t+1} - \sum_{i \neq j} (g_{t,i}\gamma_{\beta,i}) \right) + \lambda \right] \ .$$

If we want the FOC to provide a global minimum, this interval must contain zero. Combining this information with the FOC generates three scenarios for the regularized $\gamma_{\beta,j}$: (1) when this interval contains zero, we set $\hat{\gamma}^{las}_{\beta,j}$ equal to zero, (2) the left-hand side of the interval exceeds zero $\iff \gamma_{\beta,j} < 0$, and we compute $\hat{\gamma}^{las}_{\beta,j}$ by solving Equation (32), (3) the right-hand side of the interval subceeds zero $\iff \gamma_{\beta,j} > 0$, and we compute $\hat{\gamma}^{las}_{\beta,j}$ by again solving Equation (32). In a similar manner, we derive the FOC for unrestricted IPCA and obtain a procedure for $\hat{\gamma}^{las}_{j}$. The optimization approach of the Lasso is further discussed in Section 3.7.

3.5.3 Elastic net

Theoretically, one would be inclined to prefer Lasso over ridge because of its sparsity feature. However, Zou and Hastie (2005) finds three major flaws in the Lasso framework. First, in an environment where the number of predictors exceeds the number of observation, also known as $p \gg n$, the Lasso is unable to select more than n parameters regardless of relevance. Second, the Lasso has an inconsistent selection procedure, as it is prone to select only one parameter of a group of variables with high pairwise correlations. And third, ridge generally outperforms Lasso when the predictors are highly correlated. Especially this last flaw inspired Zou and Hastie (2005) to create a technique that would possess the best of both worlds. They introduce a convex combination of Lasso and ridge, which is known as the elastic net. Including the L^1 and L^2 norm in the penalty term generates for the restricted IPCA model the following objective function:

$$\min_{\Gamma_\beta, F} \frac{1}{2} \sum_{t=1}^{T-1} \left(r_{t+1} - Z_t\Gamma_\beta f_{t+1}\right)' \left(r_{t+1} - Z_t\Gamma_\beta f_{t+1}\right) + \rho\lambda \sum_{l=1}^{L}\sum_{k=1}^{K} |\gamma_{\beta,l,k}| + \frac{1}{2}(1-\rho)\lambda \sum_{l=1}^{L}\sum_{k=1}^{K} \gamma^2_{\beta,l,k} \ , \quad (33)$$

where $\rho \in [0, 1]$ regulates the ratio of Lasso and ridge regression. Additionally, two scalars are added for convenience, as is common in the literature. By applying identical steps from the Lasso, we obtain the following FOC for the elastic net:

$$\frac{\partial \mathcal{Q}\left(\gamma_{\beta,i}, \rho, \lambda\right)}{\partial \gamma_{\beta,j}} = -\sum_{t=1}^{T-1} g'_{t,j} \left(r_{t+1} - \sum_{i \neq j} (g_{t,i}\gamma_{\beta,i}) \right) + \sum_{t=1}^{T-1} g'_{t,lk} g_{t,lk} \gamma_{\beta,j} + \rho\lambda \frac{\partial |\gamma_{\beta,j}|}{\partial \gamma_{\beta,j}} + (1-\rho)\lambda\gamma_{\beta,j} = 0 \ . \quad (34)$$

The addition of the ridge penalty in this framework does not lead to new adversities. Therefore, we apply the subdifferential of the absolute value at the origin using Equation (34) with $\gamma_{\beta,j} = 0$ and obtain:

$$\left[-\sum_{t=1}^{T-1} g'_{t,j} \left(r_{t+1} - \sum_{i \neq j} (g_{t,i}\gamma_{\beta,i}) \right) - \frac{1}{2}\rho\lambda \ , \ -\sum_{t=1}^{T-1} g'_{t,j} \left(r_{t+1} - \sum_{i \neq j} (g_{t,i}\gamma_{\beta,i}) \right) + \frac{1}{2}\rho\lambda \right] .$$

Similar to the Lasso, the generated interval leads to three scenarios for the regularized $\gamma_{\beta,j}$: (1) when this interval contains zero, we set $\hat{\gamma}^{enet}_{\beta,j}$ equal to zero, (2) the left-hand side of the interval exceeds zero $\iff \gamma_{\beta,j} < 0$, and we compute $\hat{\gamma}^{enet}_{\beta,j}$ by solving Equation (34), (3) the right-hand side of the interval subceeds zero $\iff \gamma_{\beta,j} > 0$, and we compute $\hat{\gamma}^{enet}_{\beta,j}$ by again solving Equation (34). In a similar manner, we derive the FOC for unrestricted IPCA and obtain a procedure for $\hat{\gamma}^{enet}_{j}$. The optimization approach of the elastic net is further discussed in Section 3.7.

3.6 Evaluation criteria

In addition to the significance analysis of the anomaly and characteristics in Section 3.4, this paper researches if IPCA suits the cross-section delta-hedged options. To assess the model performance in an accurate manner, we incorporate three criteria. The first is the well-known R^2 statistic, which will be referred to as total R^2. This measure indicates how well a model describes the common variation in returns, which are the systematic risk exposures. The second criteria is an adaptation of the first and is called predictive R^2. This criteria indicates the ability of a model to describe differences in average returns across assets, also known as the ability to describe risk compensation. The third criteria is the Sharpe ratio of Sharpe (1966) and is a prominent measure for the performance of an investment. We define Sharpe ratio as the expected excess return divided by its standard deviation.

A general weakness of total R^2 is its framework tends to reward the addition of predictors. This means that the measure increases or remains unchanged when more variables are included, regardless of relevancy. To counter this issue, predictive R^2 reflects the amount of random noise in a model. By excluding data points and then predicting the missing data, predictive R^2 measures the accuracy of model-implied conditional expected returns. Poor R^2 performance indicates overfitting and can be used as motivation to reduce the number of parameters. This is especially interesting for regularized IPCA that is constructed to reduce unnecessary information and prevent overfitting.

3.6.1 In-sample

Following Kelly et al. (2019), the in-sample total R^2 and predictive R^2 for the option returns r_t are defined as follows:

$$R^2_{r,tot} = 1 - \frac{\sum_{i,t}\left(r_{i,t+1} - z'_{i,t}\left(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{f}_{t+1}\right)\right)^2}{\sum_{i,t} r^2_{i,t+1}} \ , \quad R^2_{r,pred} = 1 - \frac{\sum_{i,t}\left(r_{i,t+1} - z'_{i,t}\left(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{\lambda}\right)\right)^2}{\sum_{i,t} r^2_{i,t+1}} \ , \quad (35)$$

where $\hat{\lambda}$ is a $K \times 1$ vector that denotes the unconditional mean estimates of factors f and can be seen as the price of risk. Moreover, to not overcomplicate the model structure, we do not incorporate risk price dynamics in the IPCA framework. The computations of the R^2 statistics for the IPCA characteristic-managed portfolios x_t and the benchmark PCA model can be found in Appendix C.

3.6.2 Out-of-sample

The in-sample performance of a model quantifies the capability of capturing dynamics in the fitted data. For our research, this represents an understanding of historical drivers of delta-hedged returns. Nonetheless, an understanding in this context is truly beneficial when its extends to generating decent forecasts. Furthermore, great performance in all sample periods diminishes the probability that one views successful results as statistical overfit. Following Kelly et al. (2019), we apply recursive backward-looking estimation. This approach involves using all available data up to and including time t. For the restricted IPCA model, we estimate the backward-looking parameter $\Gamma_{\beta|t}$ and use this estimate to compute the out-of-sample realized factor return $\hat{f}_{t+1|t} = \left(\hat{\Gamma}'_{\beta|t} Z'_t Z_t \hat{\Gamma}_{\beta|t}\right)^{-1} \hat{\Gamma}'_{\beta|t} Z'_t r_{t+1}$. We incorporate a rolling window approach as proposed in Tashman (2000), which is visually displayed in Appendix D.

Since we now evaluate out-of-sample R^2, an alteration of Equation (35) is required. For out-of-sample total R^2, we compare r_{t+1} to $Z_t \hat{\Gamma}_{\beta|t} \hat{f}_{t+1|t}$. This applies as well to out-of-sample predictive R^2, where we replace $\hat{f}_{t+1|t}$ with $\hat{\lambda}_t$, which is the factor mean over time t. Analogously, we obtain the out-of-sample performance for characteristic-managed portfolios, PCA, unrestricted IPCA and regularized IPCA through similar adaptations per approach.

Sharpe ratio

To explore whether exceptional sample fits lead to profitable investments, we analyze unconditional annualized out-of-sample Sharpe ratios. As higher-order principal components tend to suffer from in-sample overfit that generate unreasonably high Sharpe ratios, we will not consider in-sample performance (Kozak et al., 2020). Similar to Kelly et al. (2019), we consider three investment strategies.

First, we report the unconditional efficiency of individual factors. Second, we describe multivariate efficiency for a set of factors through ex ante unconditional tangency portfolios. The corresponding tangency weights follow from Brandt (2010) and incorporate the mean and covariance matrix of estimated factors through the rolling window. And third, we implement pure-alpha portfolios with weights $w_{t-1} = Z_{t-1}\left(Z'_{t-1}Z_{t-1}\right)^{-1}\Gamma_{\alpha}$.

This last investment is also known as anomaly or "arbitrage" portfolios and contain the risks that are not captured by the factor exposures. Moreover, the pure-alpha portfolio selects assets based on their conditional expected returns in excess of risk-based compensation, making the portfolio conditionally factor neutral. Hence, it exploits the mispricing through the IPCA intercept. Lastly, we re-sign portfolios to ensure positive means and assume a volatility target of 10% per year.

3.7 Implementation

In this section, we elaborate on decisions regarding model construction, estimation techniques, robustness analyses and other implementations.

3.7.1 Alternating Least Squares

As described under Equation (12), there can only be one non-fixed variable when applying the FOC to minimize the SSE. Therefore, we implement an adaptation of Alternating Least Squares (ALS).[4] This technique allows us to use analytical optimization, such as Equation (13) and (15), to generate numerical solutions to the objective function of IPCA. The ALS algorithm works as follows: we start by initializing using the aforementioned approximations discussed in Section 3.3. In the next step, the model iterates between optimal values of $\hat{\Gamma}$ and $\hat{f}_t$ until the SSE convergences to a value under a predetermined tolerance.[5] Kelly et al. (2017) proves the notably fast convergence of ALS through various simulations. Without this quick convergence, the bootstrap tests of Section 3.4 become infeasible to perform.

Furthermore, due to the substantial size of our data set in combination with the complexity of delta-hedged options, it is almost unavoidable to not have missing data. Nonetheless, since the ALS framework consists of straightforward regressions, it can be rewritten in a manner that excludes the return and characteristics of company i at time t when one or more data points are missing. This

[4] We implement the IPCA adaptation in MATLAB of Kelly et al. (2019) for model fit and bootstrapping Γ_{α}, and apply the Python adaptation of Büchner and Kelly (2022) for bootstrapping Γ_{β} and regularization. The code is available at https://sethpruitt.net/2019/12/01/characteristics-are-covariances/ and https://github.com/bkelly-lab/ipca

[5] The PCA for r_t and x_t is obtained using MATLAB's *pca.m* function with the flag 'Algorithm' set to 'als'.

leads to roughly 20% of the initially required data to be classified as either unavailable or discarded. Additionally, the introduction of characteristic-managed portfolios allows us to handle the returns in constant L-dimensional objects, opposing to the time varying N_t. This feature side steps the unbalanced panel data by construction. Furthermore, we realize the regularized IPCA models using the conditions described in Sections 3.5.2 and 3.5.3 and the coordinate descent solver of Friedman et al. (2010).

3.7.2 Number of common factors K

In contrast to Kelly et al. (2019) where the choice for including six factors seems arbitrary, we implement an analytical approach in determining the required number of factors. This is an important trade-off as a parsimonious factor model is preferred, yet the exclusion of essential factors diminishes performance. We implement the first two identification methods described in Horenstein et al. (2020) to characteristic-managed portfolios, as they capture the cross-section in lower dimensions. We define the Eigenvalue Ratio (ER) and the Growth Ratio (GR) estimators of Ahn and Horenstein (2013) as follows:

$$\tilde{k}_{ER} = \max_{1\leq k\leq kmax} \mathrm{ER}(k) = \max_{1\leq k\leq kmax} \frac{\tilde{\mu}_{LT,k}}{\tilde{\mu}_{LT,k+1}}, \quad \tilde{k}_{GR} = \max_{1\leq k\leq kmax} \mathrm{GR}(k) = \max_{1\leq k\leq kmax} \frac{\ln\left(1+\tilde{\mu}^*_{LT,k}\right)}{\ln\left(1+\tilde{\mu}^*_{LT,k+1}\right)}, \quad (36)$$

for $k = 1, 2,...,kmax$, where $\tilde{\mu}_{LT,k} = \Psi_k\left[X'X/(LT)\right]$, $\Psi_k(A)$ denotes the k-th largest eigenvalue of a positive semidefinite matrix A, and $\tilde{\mu}^*_{LT,k} = \tilde{\mu}_{LT,k}/V(k)$ with $V(k) = \sum_{j=k+1}^{m} \tilde{\mu}_{LT,j}$ and $m = \min(L,T)$. In addition, we generate the scree plot of Cattell (1966) for extra insight in the $L \times L$ matrix $X'X$. The corresponding figures can be found in Appendix E.

The ER and GR ratios suggest that one factor is optimal for the equity data and two factors is sufficient for the ETF data. While our findings substantiate this claim by frequently obtaining maximum predictive R^2 for these number of factors, they are insufficient for a coherent analysis. Moreover, similar research such as Jones (2006) notes that mispricing is reduced by increasing the number of factors. This is confirmed by Büchner and Kelly (2022), as they argue that pricing the cross-section of index option returns requires at least three factors. By increasing K, they find that the difference between the restricted and unrestricted IPCA model disappears and leads to the insignificance of Γ_α. Combining this information with the ER and GR ratios that display a substantial drop after the fourth factor, we increase the number of factors and set $K = 4$.

3.7.3 Robustness

It is important to emphasize the risk of data dependence in a research of this nature. Therefore, we apply several robustness checks to analyze whether our findings are consistent for various data periods. If this is the case, it will strengthen the validity of our findings and corresponding conclusions. Our overall sample period consists of 143 months. We assign the first 100 observations to the in-sample period and the last 43 observations to the out-of-sample period, which results in a 70-30 split of the data set. For example, month 102 is the second month in the out-of-sample period and is estimated via a rolling window that consists of months 1 to 101. To ensure a good margin, we re-estimate certain models with an in-sample period of 80 observations for robustness analyses, which are reported in Appendix J.

4 Data

In this section we elaborate on the obtained and constructed data sets that we implement in this paper. First, we discuss the equity options. Second, we review the ETF options. Third, we explain the data-filtering and the delta-hedging process. Fourth, we specify the chosen and computed characteristics that function as instrumental variables in the IPCA framework. And lastly, we comment on the descriptive statistics of the delta-hedged returns.

Regarding the sample period, we refer to Cochrane (2011), who openly requests for the repetition of Fama and French's anomaly digestion in high dimensions. Green et al. (2017) accepts this challenge and analyzes 94 firm characteristics. They find that return predictability sharply fell in 2003, which holds for hedged portfolio returns that exploit characteristics-based predictability. They advise future research to favour post-2003 data. However, Han et al. (2018) attribute earlier conclusions due to overfitting. Nevertheless, we avoid this issue by selecting a sample period of June 2006 to May 2018, with October 2014 closing the in-sample period. These years include several crises and bull markets that should offer a balanced data set in economical terms. Similar to the works of Fama and French (1993) and Freyberger et al. (2020), we estimate the returns from months June to May using balance sheet data from the previous fiscal year. This timing convention should avoid the look-ahead bias, which is important to evade for drawing realistic conclusions.

Furthermore, we analyze over the eleven economic sectors defined by the Global Industry Classification Standard (GICS), also known as the Standard & Poor's (S&P) sectors. This method assigns companies to a sector that best defines its business operations. The GICS defines the following eleven sectors that we ordered on size at the time of this paper: Information Technology, Financials, Health Care, Consumer Discretionary, Industrials, Communication Services, Consumer Staples, Energy, Materials, Real Estate, and lastly Utilities. We merge Energy and Utilities, as the companies in this sector are extensively intertwined.

4.1 Equity options

We obtain 92,596 monthly delta-hedged call single-name equity option returns of 1007 companies. The option data and the corresponding stock data are retrieved from the OptionMetrics IvyDB US database, which covers NYSE, NASDAQ and NYSE American. Our focus is on near-the-money American call options with time to maturity that ranges from 40 to 50 days. We choose call options as they are most common in the literature. We avoid survivorship biases in our data set by including companies that merged with another firm, had their initial public offering (IPO), went bankrupt or got acquired during our sample period. Additionally, to ensure that our analysis over the GICS industry sectors is balanced, we assign 100 companies with all corresponding option data to each of the ten industry groups. In rare occasions when a company is significantly present in multiple sectors, we assign it to each individual industry. However, the number of times this occurred is relatively negligible.

4.2 ETF options

To analyze whether the factor structure for single-name equity options changes when the underlying is grouped together, we resort to index trackers. Specifically, an ETF that strives to replicate the performance of an index or sector. Our interest in these products originates from the remarkable expansion after its introduction in 1993. At the start of our sample period in 2006 there were around 750 ETFs traded on U.S. exchanges, while in 2018 there were approximately 6,500 ETFs. Moreover, Ben-David et al. (2017) state that in 2016 the trading volume of ETFs exceeded 30% of total trades on U.S. exchanges, while only consisting of 10% of total market capitalization. Lettau and Madhavan (2018) argue that ETFs are one of the most important financial innovations in decades and that its performance greatly benefits from the diversification feature of indices. ETFs are also attractive as passive investment vehicles for all types of investors due to its accessibility, index tracking ability, high liquidity and low transaction costs. This flexibility over index options results in a substantially higher trading volume.

The features and retrieval of the data for the ETF options and underlying are identical to that of the the equity data. Overall, we find 448 ETFs of which 278 contain options that match our requirements and generate a total of 16,572 monthly delta-hedged call option returns. Of these ETFs, 188 are focused on our ten industry sectors and 90 vary across markets.[6] This last group contains for instance several S&P 500 trackers.

4.3 Delta-hedging

This research focuses on the drivers of equity and ETF option returns beyond the price fluctuations of the underlying. Therefore, we implement delta-hedged returns that are theoretically insensitive to changes in stock prices, which is in line with the literature (Cao and Han, 2013), (Horenstein et al., 2020), (Büchner and Kelly, 2022). Before the returns are computed, we apply similar filters as Horenstein et al. (2020) to the option data. First, we exclude illiquid options where the bid quote is zero, the trading volume is zero, the bid quote is smaller than the ask quote, or when the average of the bid and ask price is lower than 0.125 dollars. Second, we discard American options whose underlying stock pays a dividend to shareholders during the life of the option. This filter aims to remove the early exercise premium of American options, which allows us to regard them as European style. Third, we eliminate options that are not between 0.8 and 1.2 in terms of moneyness.

We obtain a delta-hedged position by going long in a call option and hedge this position by going short in a delta number of the underlying stock. Since option contracts represents 100 shares, an option with a delta of 0.8 is delta-hedged when the investor shorts 80 shares. Following Bakshi and Kapadia (2003), the delta-hedged option gain for some time period τ is constructed as follows:

$$\Pi_{t,t+\tau} = O_{t+\tau} - O_t - \int_t^{t+\tau} \Delta_u dS_u - \int_t^{t+\tau} r_u^f \left(O_u - \Delta_u S_u\right) du, \tag{37}$$

where O_t is the price of the option at time t, $\Delta_u = \frac{\partial C_u}{\partial S_t}$ is the delta of an option at time u with C_t denoting the price of an European call option at time t, S_u is the daily close price of the underlying stock and r_u^f represents the annualised risk-free rate at time u, which is the 10-year U.S. bond yield retrieved from Federal Reserve Economic Data (FRED). We can distinguish three parts of Equation (37). The first part indicates the price change of the option, the second part represents the adjustments from delta-hedging the position, and the last term computes the cost of funding the delta-hedged position using the risk-free rate, which ensures we generate excess returns.

Since this study focuses on an empirical analysis and not a simulation study, we cannot work in a continuous time framework. Therefore, we transform Equation (37) to a discrete form, which is

[6] The information regarding the various ETFs and its holdings are retrieved from ETF Database, which is owned by the nonprofit organization Mitre Media.

defined as follows:

$$\Pi_{t,t+\tau} = O_{t+\tau} - O_t - \sum_{n=0}^{N-1} \Delta_{t_n} \left[S_{t_{n+1}} - S_{t_n}\right] - \sum_{n=0}^{N-1} \frac{a_n r^f_{t_n}}{365} \left(O_{t_n} - \Delta_{t_n} S_{t_n}\right), \tag{38}$$

where a_n expresses the number of calendar days between t_n and t_{n+1}. This means that we hedge N times in a discrete manner over the time period $[t, t+\tau]$, which can be seen as rebalancing at each traded day t_n for $n = 0, 1, ..., N-1$. Since we are interested in returns and not gains, we scale the delta-hedged gains by the investment price. Hence, we obtain the delta-hedged return as follows:

$$r_{i,t+1} = \frac{\Pi_{t,t+\tau}}{\Delta_t S_t - O_t} . \tag{39}$$

After applying the data filters, we find the option in month t that is closest to being at-the-money. If we find several options with the same moneyness, we select the earliest observation. In the next step, we gather all filtered options that are traded up to a month after the selected observation. Next, we create a subgroup that contains traded options in a three to five week range from the initial observation. We then select the option in this window that is closest to being at-the-money. This generates a path of near-the-money options over which we delta-hedge. If there are no options available in a month that meet the data filters or requirements, we leave the return as undefined. As this procedure starts with buying an option in month t and ends roughly four weeks later, we assign the return to month $t+1$.

The combination of the aforementioned data filters and actions to avoid survivorship biases create a data set where companies start or cease to exist during the sample period. This leads to an unbalanced data panel that is handled via the ALS algorithm discussed in Section 3.7.1.

4.4 Characteristics

To find the drivers of the delta-hedged returns, we require a substantial set of informative characteristics that will function as instrumental variables in the IPCA framework. These variables will be a combination of firm and option characteristics and are obtained from OptionMetrics, Standard and Poor's Compustat database and the Center for Research in Security Prices (CRSP). Following Kelly et al. (2019) and Kim et al. (2021), we implement the set of firm characteristics discussed in Freyberger et al. (2020). Where Kelly et al. (2019) include an older version of the paper that considers 36 characteristics, we include a more recent version. Additionally, we add several option characteristics, such as implied volatility, moneyness, trading volume and the Greeks from Black and Scholes (1973). Unlike the balance sheet data, the delay in option and stock characteristics is only one month as it contains the features of the bought option O_t in Equation (38). The delta, gamma, vega, theta and the implied volatility are obtained from OptionMetrics and computed using the binomial tree of Cox et al. (1979).

In terms of delta-hedged option returns, Cao and Han (2013) observe that the return decreases monotonically when the idiosyncratic volatility of the underlying stock increases. This is due to the additional costs that are required to continuously hedge the volatile security. Furthermore, Cao et al. (2016) claim that delta-hedged calls are positively correlated with the cash flow variance, cash holding and the change in shares outstanding. They are on the other hand negatively correlated with stock price and the profitability of a firm. These characteristics are either subsumed in the variable set of Freyberger et al. (2020), or specifically added by us.

We categorise the characteristics for equity options similar to Freyberger et al. (2020): (1) Past returns, (2) Investment, (3) Profitability, (4) Intangibles, (5) Value, (6) Trading frictions and add a new category (7) Option features. An overview of the set of equity characteristics that are implemented in this research can be found in Appendix A. Since the holdings of ETFs are complex and consist of numerous companies, balance sheet data cannot be applied. Therefore, we can only consider a subset of the equity characteristics for the ETFs, which are listed in Appendix B. Additionally, we include a constant in Z_t to absorb shared variation that does not involve characteristics. Including this constant, we obtain a total number of 70 equity characteristics and 30 ETF characteristics, which is denoted as L. This leads to 8,866,082 characteristic observations for the equity data and 898,617 characteristic observations for the ETF data. The corresponding descriptive statistics are listed in Appendix A.1 and B.1.

In order to limit the impact of outliers and to aid the interpretation of the IPCA results, we re-scale all characteristics. This is done by cross-sectionally transforming the characteristics period-by-period. Specifically, we rank the firms per characteristic, subtract one, then divide by the number of non-missing observations at that particular period in time minus one. Next, we subtract 0.5, which maps all characteristics in the [-0.5, 0.5] interval. This means that the IPCA framework only values the ordering of characteristics at a certain time, regardless of dispersion.

4.5 Descriptive statistics

Before the IPCA models are applied, we analyze our data sets to get a better understanding of its features. The descriptive statistics of the delta-hedged option returns are listed in Table 1.

Table 1: Descriptive statistics of the monthly delta-hedged call option returns.

		1^{st}-P	Mean	Median	99^{th}-P	Std.
All	Equity	-0.343	-0.006	-0.005	0.316	0.130
	ETF	-0.320	-0.010	-0.005	0.296	0.254
Equity Sectors	Information Technology	-0.350	-0.009	-0.008	0.312	0.124
	Financials	-0.303	-0.003	-0.004	0.307	0.180
	Health Care	-0.325	-0.010	-0.008	0.298	0.119
	Consumer Discretionary	-0.374	-0.009	-0.007	0.320	0.128
	Industrials	-0.297	-0.006	-0.006	0.309	0.112
	Energy & Utilities	-0.365	-0.002	-0.002	0.351	0.130
	Communication Services	-0.332	-0.006	-0.005	0.321	0.120
	Consumer Staples	-0.308	-0.004	-0.004	0.275	0.105
	Materials	-0.412	-0.005	0.000	0.352	0.148
	Real Estate	-0.301	-0.005	-0.004	0.266	0.098

Note. This table displays the descriptive statistics of the equity and ETF monthly delta-hedged call option returns. The equity sectors each contain 100 companies and are ordered on size at the time of this paper. The sample period runs from July 2006 to May 2018. Moreover, there are 92,596 equity option returns of 1007 companies and 16,572 ETF returns from 278 ETFs. Hence, there are 109,168 returns in total. The columns represent the 1^{st} percentile, mean, median, 99^{th} percentile and the standard deviation of the corresponding returns.

The first thing that draws our attention in Table 1 is its negative mean for all data sets. Taken into account that these are monthly returns, the annualized loss across all data sets is on average two to twelve percent a year. This can be explained by the fact that delta-hedging is by construction a defensive strategy, as an investor strives to offset the risk of price changes in the underlying instead of looking for the most profitable assets. Moreover, avoiding risk should theoretically never be rewarded. The poor performance of delta-hedging is the consequence of the negative volatility risk premium (Coval and Shumway, 2001), (Bakshi and Kapadia, 2003), (Cao and Han, 2013), (Zhan et al., 2022). These papers find that delta-hedged call portfolios statistically underperform zero and that the losses are the largest for at-the-money options. The negative mean also further proves the flaws of the Black-Scholes model, as under the assumptions of Black and Scholes (1973), delta-hedged option returns have a symmetric distribution with zero mean. Additionally, the process of delta-hedging is accompanied with considerable transaction costs due to the constant rebalancing, which will decrease the returns of investors even more.

As can be observed in the last column of Table 1, the returns are quite volatile. This suggests that theoretically avoiding risk does not automatically translate well in the real world. Instead of listing the minima and maxima, we prefer the 1^{st} and 99^{th} percentile of the returns as these offer a better representation of the distribution. For instance, the two largest losses of the equity and ETF returns are roughly 1300%, which suggests a terrible short position as long position losses are always capped at 100%. Furthermore, losses beyond 100% are approximately 0.2% of all returns, while only 0.02% of the returns contain profits that exceed 100%, with a maximum of 140%. Combining this information with the kurtosis and skewness measures, we conclude that the returns are negatively skewed and have fat tails, which is in line with the general consensus in the asset pricing literature. Moreover, the returns of all data sets reject the null hypothesis of normality at the 0.1% significance level via the Jarque-Bera test of Jarque and Bera (1987).

5 Results

In this section, we discuss our findings when applying IPCA models to the constructed data sets. We divide the results into four major sections. First, we analyze the in-sample fit, anomaly existence, characteristic significance, out-of-sample fit and sector performance of the delta-hedged equity options. Second, we examine similar components for the delta-hedged ETF options and compare the findings of the two data sets. Third, we evaluate regularization in the IPCA framework by inspecting its shrinkage feature on the coefficient matrix Γ_β and its out-of-sample performance. And fourth, we explore the potential of a trading strategy.

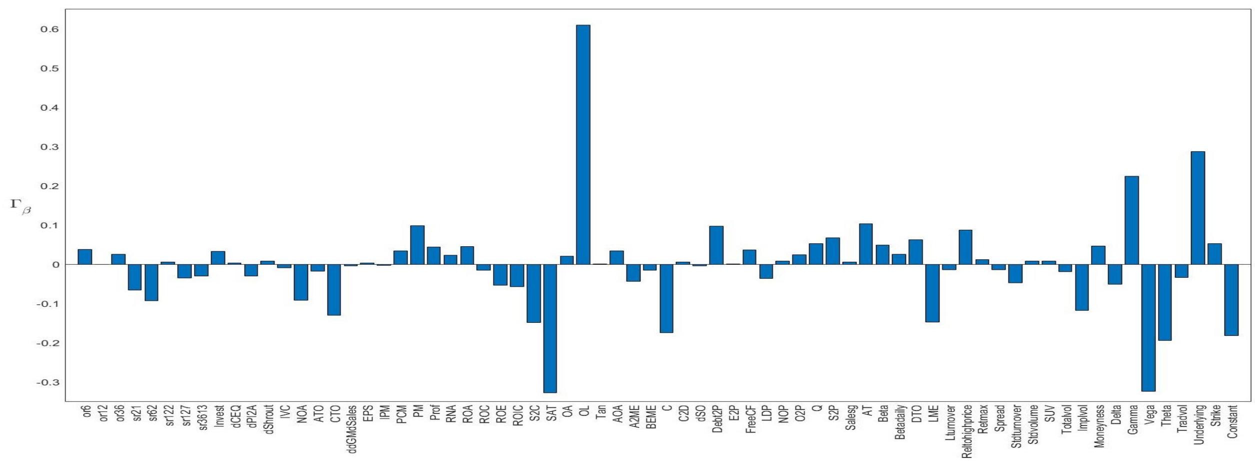
5.1 Equity options

To illustrate the IPCA framework, we start by visualizing the factor loadings Γ_β and Γ_α. Therefore, we estimate the unrestricted four-factor IPCA model of Equation (17). The k-th column of the Γ_β matrix indicates how the characteristics map into the beta of a company on the k-th factor. The analysis of this mapping produces insight into the estimated IPCA risk factors. Figure 1 shows the third column of Γ_β, while the other estimates of Γ are displayed in Figure 5 - 8 in Appendix F.1.

We observe in Figure 5 that loadings on Factor 1 are dominated by two characteristics, assets-to-market ($a2me$) and Tobin's q (q). Since we enforced a non-negative mean restriction on f_t in Section 3.2.1, all factors have positive expected returns. This means that companies with larger total assets to size and Tobin's Q have higher betas on Factor 1 and earn higher average returns.

The loadings on Factor 2 in Figure 6 are almost exclusively determined by the stock price (*underlying*) and the strike price (*strike*), which are among the fundamental features of an option contract. These characteristics are similar in magnitude and have opposing signs. This means that all else equal, options with higher stock price relative to strike price have higher betas on Factor 2. In other words, when call options are in-the-money, their average return increases. However, moneyness has a very small loading. This indicates that the proportion of stock and strike price may not be relevant. Additionally, since stock and strike price are strongly correlated for near-the-money options, we conclude that the effects of these two loadings counter each other in Factor 2. Nevertheless, the relevancy of stock price in delta-hedged returns is in line with the findings of Zhan et al. (2022).

Figure 1: Estimates of Γ_β for Factor 3 in the unrestricted $K = 4$ IPCA specification.



Note. This figure displays the third column of Γ_β in the unrestricted IPCA model with $K = 4$ and $\Gamma_\alpha \neq \boldsymbol{0}$. The data contains all equity data for the in-sample period July 2006 to October 2014. The variables are the 70 equity characteristics of Table 11 in Appendix A

As can be observed in Figure 1, the loadings that correspond to Factor 3 are much more diverse than the other factors. We find that operating leverage (ol), stock price ($underlying$), gamma ($gamma$), sales over total assets (sat), and vega ($vega$) primarily determine the loadings on Factor 3. Where the former three variables have a positive loading, the opposite is true for the latter two variables. This means that larger values of ol, stock price, and gamma will result in higher average returns and larger values of sales over total assets and vega result in lower average returns.

Exposure to Factor 4 in Figure 7 is primarily determined by the constant ($constant$), which means that all options share a common baseline exposure to this factor via the constant. Moreover, the constant has at all times the maximum value of 0.5 in the transformed characteristic matrix Z_t, opposed to other variables that vary between -0.5 and 0.5. This means that the constant has an immense impact on the model through the fourth factor, and as this loading is negative, this factor partially captures the negative average return.

Lastly, we examine Γ_α, which represents intercepts that depend on characteristics and describe the expected returns beyond the systematic risk. The corresponding loadings are an indicator which characteristics contribute to the anomaly effect. We see in Figure 8 that there are several variables with relatively large loadings. However, the loading values are quite small compared to Γ_β. This is due to the unrestricted IPCA framework of Equation (17), where the periodic return is equal to one for Γ_α. We find large positive magnitudes for *totalvol*, *dshrout* and *underlying*, which indicates that large values for the volatility in stock prices, the change in number of outstanding shares, and the stock price result in relatively higher anomaly returns. In contrast, earnings per share (*eps*), the ratio of market value of equity plus long-term debt minus total assets over cash and short-term investments (*roc*), and delta (*delta*) have large negative magnitudes and large values lead to lower anomaly returns.

5.1.1 In-sample fit

Before we further extend the analysis that examines the drivers of delta-hedged equity option returns, we need to make sure that IPCA is the suited model choice for the option data. Therefore, we need to compare the IPCA performance to its predecessor, which is the PCA of Section 3.1. The in-sample performance of IPCA and PCA can be observed in Table 2.

Table 2: IPCA in-sample performance of equity options.

		K			
		1	2	3	4
Panel A: Individual returns (r_t)					
Total R^2	$\Gamma_\alpha = \mathbf{0}$	20.0	22.7	24.3	25.7
	$\Gamma_\alpha \neq \mathbf{0}$	20.8	23.5	25.1	26.2
	PCA	24.5	41.8	45.8	48.2
Predictive R^2	$\Gamma_\alpha = \mathbf{0}$	0.30	0.28	0.53	0.70
	$\Gamma_\alpha \neq \mathbf{0}$	0.97	0.93	0.92	0.88
	PCA	< 0	< 0	< 0	< 0
Panel B: Managed portfolios (x_t)					
Total R^2	$\Gamma_\alpha = \mathbf{0}$	72.9	78.4	83.7	87.2
	$\Gamma_\alpha \neq \mathbf{0}$	74.3	79.6	84.5	88.1
	PCA	63.2	68.8	73.3	80.4
Predictive R^2	$\Gamma_\alpha = \mathbf{0}$	0.91	0.80	1.82	1.48
	$\Gamma_\alpha \neq \mathbf{0}$	2.12	1.87	1.72	1.56
	PCA	1.98	2.02	2.02	2.02
Panel C: Testing the anomaly ($H_0 : \Gamma_\alpha = \mathbf{0}$)					
W_α *p*-value		0.216	0.322	0.511	0.881

Note. This table displays the performance of the IPCA model with K factors using the equity data for the in-sample period July 2006 to October 2014. Panel A and B report the total and predictive R^2 in percentages for the restricted and unrestricted IPCA model with 70 lagged characteristics. Panel C presents the bootstrapped p-values for the anomaly test as described in Section 3.4.1.

Table 2 contains several interesting findings. For instance, Panel A reports the individual monthly returns and shows that while PCA outperforms IPCA in terms of total R^2 up to 48.2%, it is unable to generate positive predictive R^2. This means that PCA has no explanatory power for differences in individual average delta-hedged equity option returns. Moreover, for $K = 4$ the PCA model estimates $NK + KT = 4428$ parameters, while IPCA only estimates $LK + KT = 680$ parameters. Due to estimating more than six times the number of parameters, we deem PCA computationally inferior to IPCA. Additionally, as total R^2 generally increases with the number of predictors, we consider the PCA total R^2 performance for individual returns as statistical overfit from over-parameterization.

When characteristic-managed portfolios are implemented, the variables N and L are equal, and so are the number of estimated parameters. In this case, we observe in Panel B that IPCA explains up to 88.1% of the total variation in delta-hedged equity option returns. While IPCA consistently outperforms PCA in terms of total R^2, the predictive R^2 performance of PCA is very competitive for all values of K. Therefore, we need to evaluate out-of-sample predictions before we can determine the true potential of IPCA. Nonetheless, including dynamic factor exposures that depend on characteristics produce promising results.

Next, we observe in Panel A and B that as K increases the R^2 of the restricted ($\Gamma_\alpha = \mathbf{0}$) and unrestricted ($\Gamma_\alpha \neq \mathbf{0}$) IPCA models slowly converge, which is in line with the findings of Büchner and Kelly (2022). The reason for this is the relatively poor performance of restricted IPCA for lower values of K. Specifically for one or two factors, where restricted IPCA captures less than one-half of the return predictability compared to its unrestricted counterpart. Nevertheless, this difference does not expand to statistically significant Γ_α, as can be observed in Panel C. This implies that IPCA is able to describe risk compensation solely through exposures to systematic risk in Γ_β and that the anomaly intercepts are never statistically significant at the 10% level. In other words, for one to four factors Γ_α does not contribute significantly to the IPCA performance.

For robustness analysis, we re-estimate Table 2 with 80 months instead of 100 months and display the results in Table 30 in Appendix J. We argue that while overall fit slightly increases, all aforementioned claims still apply. However, for one or two factors the predictive R^2 for unrestricted IPCA is now more than four times as high as for restricted IPCA. Remarkably, this does still not lead to significant Γ_α as the p-values of Panel C barely change. It is also quite intriguing to see the resemblance in developments around K in Panel A and B with Table 1 in Kelly et al. (2019). This suggests that there are similarities in the cross-section of stock returns and delta-hedged equity option returns when comparing IPCA and PCA performance.

5.1.2 Characteristic significance

To determine which characteristics are most important for capturing the cross-section in the IPCA framework, we apply the bootstrap procedure for Γ_β of Section 3.4.2. Since Table 2 argues that Γ_α is not significant, we apply the restricted IPCA Γ_β in this section. The corresponding p-values are listed in Table 3.

Table 3: p-values of the equity characteristic coefficients in Γ_β.

Characteristic \ K	1	2	3	4	Characteristic \ K	1	2	3	4
Past returns:					**Value:**				
or_6	0.79	0.82	0.31	0.50	A2ME	**0.01**	0.98	1.00	1.00
or_{12}	0.47	0.32	0.66	0.86	BEME	0.44	0.32	0.58	0.36
or_{36}	0.50	0.97	0.97	0.29	C	0.16	0.13	0.12	0.37
sr_{2-1}	0.43	0.54	0.48	0.80	C2D	0.18	0.22	0.36	0.52
sr_{6-2}	0.74	0.54	0.15	0.18	ΔSO	0.56	0.81	0.97	0.94
sr_{12-2}	0.90	0.29	0.77	0.85	Debt2P	0.27	0.66	0.94	**0.03**
sr_{12-7}	0.98	0.46	0.41	0.38	E2P	0.38	0.14	0.40	0.76
sr_{36-13}	0.66	0.82	0.86	0.70	Free CF	0.60	0.94	1.00	0.66
					LDP	0.94	0.27	0.77	0.70
Investment:					NOP	0.69	0.34	0.60	0.79
Invest	0.49	**0.04**	0.29	0.44	O2P	0.86	0.34	0.50	0.66
ΔCEQ	0.93	0.16	0.44	0.79	Q	**0.05**	0.99	1.00	1.00
ΔPI2A	0.98	0.61	0.80	0.58	S2P	0.55	0.82	0.98	0.78
ΔShrout	0.69	0.31	0.68	0.71	Sales_g	0.70	0.34	0.78	0.76
IVC	0.59	0.33	0.70	0.80					
NOA	0.75	0.65	0.44	0.11	**Trading frictions:**				
					AT	0.22	**0.01**	**0.07**	**0.03**
Profitability:					Beta	0.18	0.90	0.18	**0.01**
ATO	0.19	0.59	0.21	0.28	Beta daily	0.17	0.26	0.27	0.12
CTO	0.73	0.74	0.95	0.73	DTO	0.41	0.38	0.40	0.36
Δ(ΔGM-ΔSales)	0.88	0.34	0.68	0.65	LME	0.36	**0.02**	**0.08**	**0.04**
EPS	0.45	0.44	0.59	0.55	Lturnover	0.86	0.57	0.55	0.62
IPM	0.26	0.76	0.71	0.93	Rel_to_high_price	0.15	0.82	**0.10**	0.17
PCM	0.62	0.50	0.45	0.65	Ret_max	0.69	0.70	0.91	1.00
PM	0.56	**0.08**	0.19	0.30	Spread	0.24	0.22	0.33	0.33
Prof	0.57	0.66	0.54	0.57	Std turnover	0.54	0.38	0.59	0.69
RNA	0.26	0.58	0.28	0.37	Std volume	0.99	0.41	0.26	0.28
ROA	0.26	0.19	0.20	0.76	SUV	0.70	0.46	0.72	0.79
ROC	0.90	0.83	0.96	0.70	Total vol	0.77	0.91	0.74	0.68
ROE	0.25	0.63	0.26	0.54					
ROIC	0.34	0.43	**0.04**	**0.08**	**Option features:**				
S2C	0.42	0.31	0.26	0.44	Impl vol	0.25	**0.10**	0.23	**0.08**
SAT	0.72	0.74	0.51	0.34	Moneyness	0.26	0.80	0.77	0.35
					Delta	0.42	0.87	0.96	0.94
Intangibles:					Gamma	0.82	0.49	0.64	0.38
OA	0.85	0.63	0.92	0.54	Vega	0.50	0.95	0.57	0.70
OL	0.93	**0.09**	**0.05**	**0.02**	Theta	0.79	0.36	0.86	0.66
Tan	0.20	0.32	0.32	0.27	Trad volume	0.34	**0.08**	**0.06**	0.12
AOA	0.85	0.95	0.80	0.28	Underlying	0.50	**0.08**	0.25	0.58
					Strike	0.90	**0.02**	0.19	0.53
					Constant	**0.00**	**0.02**	**0.00**	**0.00**

Note. This table displays the p-values of the equity characteristics in Γ_β of the restricted IPCA model with K factors. The corresponding data contains all available equity options for the in-sample period July 2006 to October 2014. Furthermore, values in bold are significant at the 10% level, following the bootstrap test of Section 3.4.2. A description of the listed characteristics can be found in Appendix A.

We observe in Table 3 that the following characteristics are statistically significant at the 10% level for at least two of the four IPCA models: return on invested capital (*roic*), operating leverage (*ol*), total assets (*at*), market capitalization (*lme*), implied volatility (*implvol*), trading volume of the option (*tradvolume*), and the constant (*constant*). The only characteristic that is significant for all IPCA models, of which 3 at the 1% level, is the constant. This suggests that all delta-hedged equity option returns share a significant common baseline exposure. Table 3 further shows significant values for *at* and *lme* when $K = 2$ or larger. For $K = 1$, we observe the importance of these two characteristics as their ratio *a2me* is significant at the 1% level. Additionally, one might wonder why the significance of a particular characteristic seems to vary with the number of factors. This is because when a factor is added or removed, the factor loadings alter and former explanatory power can disappear or be absorbed by another characteristic.

Analyzing over the different categories, we see that Trading frictions and Option features contain two times as many significant characteristics as the other categories. This is surprising with respect to Profitability as Zhan et al. (2022) argue a significant relation between the returns of delta-hedged calls and firm profitability. Some profitability characteristics are significant in Table 3, yet they are more scarce than one would expect. Furthermore, we see that Past returns has no significant characteristics at all, which suggests two things. First, delta-hedged option returns do not exhibit strong mean reversion for time periods under three years. Second, the momentum and reversal factors of the underlying do not significantly contribute to describing the cross-section. This implies that the delta-hedging process of being invariant to price fluctuations of the underlying has been successful.

Despite the insignificance of Γ_α in the four-factor IPCA model, we cannot directly compare the results of Table 3 with the figures in Appendix F.1 as they are different models. Nonetheless, it is evident that larger magnitudes of Γ_β for factor K are often followed by relatively low p-values in Table 3. Specifically, regarding the dominance of (*a2me*) and (q) for Factor 1 and (*underlying*) and (*strike*) for Factor 2.

5.1.3 Out-of-sample performance

In order to test the true potential of IPCA, we extend the analysis from in-sample to out-of-sample. This offers insight into the ability of the model to make accurate predictions that may lead to profitable trading strategies. The out-of-sample fit is reported in Table 4.

We observe in Table 4 that the IPCA total R^2 of individual returns is around one-half of the in-sample fit. This drop is less apparent for predictive R^2, which further supports the explanatory abilities of IPCA. Especially, the increase in out-of-sample performance from the second to the third factor is relatively substantial. In terms of managed portfolios, the out-of-sample total R^2 decreases as well, but with a lower amount. One of the most interesting findings is the unrestricted predictive R^2 performance with values up to 2.09% for $K = 3$. This even exceeds the in-sample performance of the same model.

Table 4: IPCA out-of-sample performance of equity options.

	Individual returns (r_t)				Managed portfolios (x_t)			
	Total R^2		Predictive R^2		Total R^2		Predictive R^2	
K	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$
1	8.80	9.16	0.26	0.63	43.6	43.9	0.92	1.70
2	9.54	9.95	0.24	0.66	49.2	49.8	0.60	1.84
3	10.7	11.1	0.05	0.67	58.8	59.2	< 0	2.09
4	11.3	12.4	0.20	0.65	59.3	69.6	< 0	1.48

Note. This table displays the performance of the IPCA model with K factors using the equity data for the out-of-sample period November 2014 to May 2018. The total and predictive R^2 are reported in percentages for the restricted and unrestricted IPCA model with 70 lagged characteristics.

Despite insignificant in-sample Γ_α, we see that unrestricted IPCA seems to outperform restricted IPCA, especially in terms of predictive R^2. Where unrestricted IPCA excels the most, the restricted model is unable to produce positive predictive R^2. This indicates that restricted IPCA fits the equity data with three or four factors rather poorly. Additionally, Table 4 does not include out-of-sample PCA performance as it generates negative R^2, which implies that its predictions tend to be less accurate than the average value of the out-of-sample returns over time. This confirms misspecification in the static latent factor model and further substantiates the need for a dynamic model as IPCA.

5.1.4 Sector analysis

At this point, we analyzed equity options over a large set of well-diversified companies and their corresponding characteristics. However, these characteristics may differ substantially across the GICS economic sectors and impact the IPCA model in various manners. Therefore, we examine the in- and out-of-sample performance of the ten sectors described in Section 4, with each group containing exactly 100 companies for a balanced analysis. Table 5 displays the sector with the most intriguing findings. The other sectors are reported in Table 16 to Table 24 in Appendix G.

In terms of in-sample fit, we observe that the Information Technology sector slightly outperforms all equity data for individual returns, yet slightly underperforms for managed portfolios. The intriguing difference occurs when we compare the out-of-sample performance. Both the total and predictive R^2 of individual returns are almost twice as high for the Information Technology sector.

And most importantly, where the equity data fails to produce positive out-of-sample predictive R^2 for the managed portfolios, the Information Technology sector generates R^2 up to 1.26%. Moreover, it is quite remarkable how the out-of-sample predictive R^2 of this sector is higher than its in-sample counterpart for almost all models, which underlines its outstanding out-of-sample fit.

Table 5: IPCA performance of equity options in the Information Technology sector.

	Individual returns (r_t)				Managed portfolios (x_t)			
	Total R^2		Predictive R^2		Total R^2		Predictive R^2	
K	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$
Panel A: In-sample								
1	31.5	37.8	0.29	1.22	57.8	58.8	0.65	1.18
2	34.9	36.0	0.60	1.16	65.9	65.9	1.08	0.97
3	38.8	39.9	0.57	1.10	72.4	72.6	1.14	0.90
4	42.4	43.3	0.51	1.09	74.5	75.4	1.19	0.78
Panel B: Out-of-sample								
1	14.7	14.5	1.16	0.94	33.3	33.4	2.35	2.63
2	15.9	16.1	0.88	0.83	36.7	37.2	1.72	2.72
3	17.7	17.5	0.84	1.00	39.0	45.0	1.26	3.17
4	19.8	19.8	0.73	1.13	45.3	46.8	1.02	3.19

Note. This table displays the performance of the IPCA model with K factors using the equity data exclusive to the Information Technology sector. The data contains the in-sample period July 2006 to October 2014 and the out-of-sample period November 2014 to May 2018. The total and predictive R^2 are reported in percentages for the restricted and unrestricted IPCA model with 70 lagged characteristics.

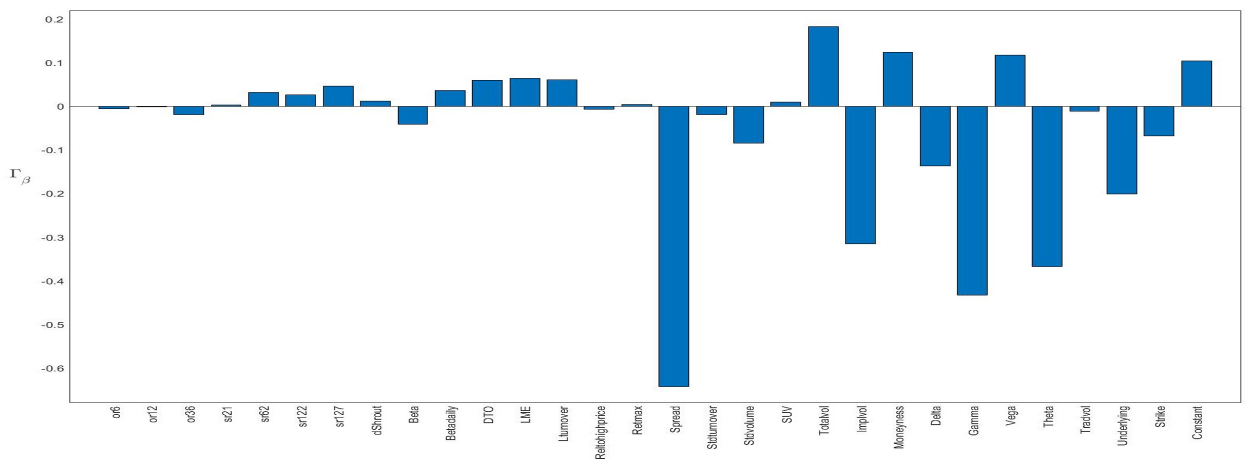
Compared to the other sectors, we find that Information Technology is unrivaled in its performance. To be more specific, the Financials, Health Care, Industrials, Energy & Utilities, Consumer Staples, and Real Estate sectors are unable to consistently generate positive predictive R^2, especially out-of-sample. While Consumer Discretionary, Communication Services, and Materials do substantially better, they do not come close to the Information Technology sector. Nonetheless, we see that all sectors produce similar or higher total R^2 than the entire equity data set. These findings are most likely the result of severe overfitting, which is due to maintaining the same number of predictors while the cross-section for individual returns shrinks in N from 1007 to 100. However, Table 24 in Appendix G contains on occasion a positive predictive R^2 for higher values of K, suggesting that inferior sector performance could simply be the outcome of egregious fit.

5.2 ETF options

In this subsection, we study if the drivers of delta-hedged returns change when the underlying is combined in the form of ETFs. This change in data is followed by reducing the number of characteristics from 70 to 30. Similar to before, we start by estimating the unrestricted four-factor

IPCA model of Equation 17 to illustrate the Γ loadings. Figure 2 displays the second column of Γ_β, while the other estimates of Γ can be observed in Figure 9 - 12 in Appendix F.2.

Figure 2: Estimates of Γ_β for Factor 2 in the unrestricted $K = 4$ IPCA specification.



Note. This figure displays the second column of Γ_β *in the unrestricted IPCA model with* $K = 4$ *and* $\Gamma_\alpha \neq \mathbf{0}$*. The data contains all ETF data for the in-sample period July 2006 to October 2014. The variables are the 30 equity characteristics of Table 13 in Appendix B*

For Factor 1 we observe that it is dominated by the ETF price (*underlying*) and the strike price (*strike*), which are similar in magnitude and have opposing signs. This result is remarkably reminiscent of the second factor for the equity data in Figure 6. Hence, it holds for equity and ETF options that when all things equal, higher underlying price relative to strike price leads to higher average returns. In this environment, it means that in-the-money options correspond with higher betas on Factor 1.

Factor 2 displays the largest loadings in Figure 2 for the average daily bid-ask spread (*spread*), gamma (*gamma*), theta (*theta*), and implied volatility (*implvol*). Where larger bid-ask spreads imply less liquid markets of the underlying, the other three characteristics all involve option sensitivity. Since these four loadings are all negative, we argue that less liquid ETF markets or higher values in gamma, theta or implied volatility lead to lower average returns.

Factor 3 is well diversified in its loadings. The largest notable loadings are the negative market capitalization (*lme*) and the positive standard deviation of daily ETF trading volume (*stdvolume*). As higher trading volume results in more liquid markets and lower spreads, this finding strongly agrees with the negative loading on average daily bid-ask spread for Factor 2.

Exposure to Factor 4 is primarily determined by vega (*vega*), and the ETF momentum from 12 to 2 months before prediction (sr_{12-2}). It appears that similar to stock returns, momentum has a positive effect on the returns of delta-hedged ETF options. In terms of vega, we observe that higher values lead to lower betas on Factor 4, which implies that options on ETFs that are sensitive to volatility earn lower average returns. This could be due to the delta-hedging process becoming unstable for unusually volatile ETFs.

Lastly, we examine Γ_α where the loadings suggest which characteristics contribute to the anomaly effect. We find relatively large positive values for *totalvol* and *reltohighprice*, which implies that large values for the volatility in ETF prices and the ETF price over its 250 trading day maximum price lead to higher anomaly returns. In contrast, short-term reversal (sr_{12-2}) from Jegadeesh (1990) has a large negative loading. This implies that relatively large reversal effects generate lower anomaly returns.

5.2.1 In-sample fit

We report the in-sample fit of the delta-hedged ETF options in Table 6.

Table 6: IPCA in-sample model performance of ETF options.

		K			
		1	2	3	4
Panel A: Individual returns (r_t)					
Total R^2	$\Gamma_\alpha = \mathbf{0}$	22.3	29.8	35.1	39.2
	$\Gamma_\alpha \neq \mathbf{0}$	23.4	30.8	36.0	40.0
	PCA	24.7	47.7	58.2	72.3
Predictive R^2	$\Gamma_\alpha = \mathbf{0}$	2.76	2.72	2.83	2.90
	$\Gamma_\alpha \neq \mathbf{0}$	3.93	3.88	3.75	3.73
	PCA	< 0	< 0	< 0	< 0
Panel B: Managed portfolios (x_t)					
Total R^2	$\Gamma_\alpha = \mathbf{0}$	54.1	79.8	83.1	84.9
	$\Gamma_\alpha \neq \mathbf{0}$	56.5	81.7	84.5	86.0
	PCA	54.8	74.2	81.9	82.7
Predictive R^2	$\Gamma_\alpha = \mathbf{0}$	5.39	5.43	5.62	5.98
	$\Gamma_\alpha \neq \mathbf{0}$	8.12	7.60	7.23	7.22
	PCA	3.89	3.89	3.89	3.89
Panel C: Testing the anomaly ($H_0 : \Gamma_\alpha = \mathbf{0}$)					
W_α p-value		0.375	0.038	0.441	0.312

Note. This table displays the performance of the IPCA model with K factors using the ETF data for the in-sample period July 2006 to October 2014. Panel A and B report the total and predictive R^2 in percentages for the restricted and unrestricted IPCA model with 30 lagged characteristics. Panel C presents the bootstrapped p-values for the anomaly test as described in Section 3.4.1.

Table 6 shows that IPCA is capable to describe the cross-section of delta-hedged ETF option returns with total R^2 up to 40.0% and predictive R^2 up to 3,93%. Similar to before, we see that PCA overfits individual returns due to an excessive number of parameters. Although this leads to high total R^2, the predictive R^2 is negative for all values of K. In terms of managed portfolios, we argue that IPCA outperforms PCA for all specifications. In terms of predictive R^2, IPCA is able to produce roughly two times the value of PCA, with a maximum of 8.12%.

As K increases, the restricted and unrestricted IPCA models converge relatively slowly in performance when compared to the equity options of Table 2. However, this still does not lead to significant Γ_α in Panel C, with the exception of $K = 2$. This means that for our ETF data the IPCA model is capable to describe risk compensation solely through exposures to systematic risk. For IPCA with two factors, it is interesting to see whether the significant in-sample $Gamma_\alpha$ translates to favorable pure-alpha portfolios.

The robustness analysis with an in-sample period of 80 months instead of 100 months is displayed in Table 31 in Appendix J. We find that the same arguments hold for Panel A and B. Therefore, we claim that in-sample the IPCA framework is superior to PCA. The anomaly test finds similar p-values for one to three factor specifications. However, for four-factor IPCA, Γ_α becomes significant at the 1% level. This is surprising as the relative difference in performance between restricted and unrestricted IPCA does not increase when we expand the model from three to four factors. Hence, significant Γ_α does not directly result in better R^2 performance.

When we compare Table 2 to the in-sample equity performance of Table 2, we observe that the same relations hold. However, it is evident that for individual returns the IPCA model describes the risk factors remarkably better for ETFs than equity firms. The reason for this could be that reducing N from 1007 to 278 makes it easier for a model to capture the cross-section. Additionally, the most compelling difference between the two tables is the predictive R^2, which is roughly four times as high for ETF options. This means that IPCA is more accurate at describing risk compensation for ETF options and raises the question whether this converts to superior out-of-sample Sharpe ratios.

5.2.2 Characteristic significance

To compare the drivers of delta-hedged equity and ETF returns, we estimate the same restricted IPCA model of Section 5.1.2 using the ETF data. The corresponding p-values are listed in Table 7.

According to Table 7, the following characteristics are statistically significant at the 10% level for at least two of the four IPCA models: the ETF momentum from 12 to 2 months before prediction (sr_{12-2}), the CAPM beta (*beta*), the average daily bid-ask spread (*spread*), the implied volatility (*implvol*), gamma (*gamma*), theta (*theta*), and the constant (*constant*). We observe that (*spread*) and (*implvol*) are significant for all IPCA models at the 5% level. Moreover, (*spread*) and (*constant*) are significant at the 1% level for three of the four specifications. Including the information from the Γ loading figures, we argue that the average daily bid-ask spread and implied volatility are the main drivers of delta-hedged ETF option returns.

Table 7: p-values of the ETF characteristic coefficients in Γ_β.

Characteristic \ K	1	2	3	4	Characteristic \ K	1	2	3	4
Past returns:									
or_6	0.34	**0.09**	0.22	0.46	Spread	**0.01**	**0.03**	**0.00**	**0.00**
or_{12}	0.32	0.11	0.44	0.71	Std turnover	0.63	0.38	0.67	0.82
or_{36}	0.24	0.36	0.47	0.62	Std volume	0.13	0.42	0.22	**0.02**
sr_{2-1}	0.58	**0.08**	0.54	0.49	SUV	0.69	0.13	0.31	0.48
sr_{6-2}	0.51	**0.05**	0.15	0.27	Total vol	0.18	0.54	0.36	0.31
sr_{12-2}	0.63	**0.05**	**0.06**	0.13					
sr_{12-7}	0.37	0.11	0.14	0.39	**Option features:**				
					Impl vol	**0.01**	**0.03**	**0.02**	**0.04**
Investment:					Moneyness	0.50	0.90	0.52	0.25
ΔShrout	0.26	0.34	0.73	**0.04**	Delta	0.27	0.79	0.48	0.21
					Gamma	**0.05**	0.24	**0.06**	**0.04**
Trading frictions:					Vega	0.54	0.16	**0.09**	0.17
Beta	**0.08**	**0.07**	0.15	0.22	Theta	**0.02**	**0.06**	**0.04**	0.11
Beta daily	0.65	0.98	0.82	0.80	Trad volume	0.91	**0.10**	0.21	0.63
DTO	0.14	0.41	0.36	0.20	Underlying	0.13	0.59	0.51	0.69
LME	0.32	0.48	0.29	**0.02**	Strike	**0.01**	0.27	0.34	0.56
Lturnover	0.16	0.43	0.62	0.46					
Rel_to_high_price	0.34	0.18	0.41	0.63	Constant	0.64	**0.00**	**0.00**	**0.00**
Ret_max	0.97	0.12	0.48	0.58					

Note. This table displays the p-values of the ETF characteristics in Γ_β of the restricted IPCA model with K factors. The corresponding data contains all available ETF options for the in-sample period July 2006 to October 2014. Furthermore, values in bold are significant at the 10% level, following the bootstrap test of Section 3.4.2. A description of the listed characteristics can be found in Appendix B

In contrast to the equity data, we see that there are various significant values in the Past returns category. This implies that the price movements of ETFs influence the delta-hedged option return, which by construction should not be possible. Hence, we have to assume that either we have exposures due to inconsistent rebalancing or the deltas of the ETF options occasionally suffer from estimation errors. Additionally, we notice that Table 7 contains relatively and absolutely more significant characteristics than Table 3. This could be due to an excessive number of predictors for the equity data, or the predictors are simply a better fit for the ETF data, which can be exemplified by the disparity in relevance of gamma and theta.

5.2.3 Out-of-sample performance

Table 8 displays the out-of-sample performance of the ETF options. Similar to before, we find inconsistent out-of-sample PCA performance, which we leave undisclosed. When comparing the sample period, we observe that the IPCA total R^2 drops by one-half for the individual returns and barely decreases for managed portfolios. The most interesting finding is the notably high out-of-sample predictive R^2. Specifically, the managed portfolios generate predictive R^2 that exceed the in-sample fit with values up to 10.7%. This is further evidence of the capabilities of IPCA. Moreover, it also indicates that the model captures the ETF data much better than the equity data. However, our ETF options findings are not close in any capacity to the IPCA R^2 performance of index options from Büchner and Kelly (2022).

Table 8: IPCA out-of-sample performance of ETF options.

	Individual returns (r_t)				Managed portfolios (x_t)			
	Total R^2		Predictive R^2		Total R^2		Predictive R^2	
K	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$
1	9.35	9.99	1.64	2.23	59.8	61.9	8.08	10.7
2	12.0	12.7	1.51	2.22	69.4	71.8	7.36	10.6
3	13.3	14.0	1.54	2.19	71.5	72.9	7.36	10.4
4	18.6	19.7	1.20	2.14	72.2	76.7	6.96	10.4

Note. This table displays the performance of the IPCA model with K factors using the ETF data for the out-of-sample period November 2014 to May 2018. The total and predictive R^2 are reported in percentages for the restricted and unrestricted IPCA model with 30 lagged characteristics.

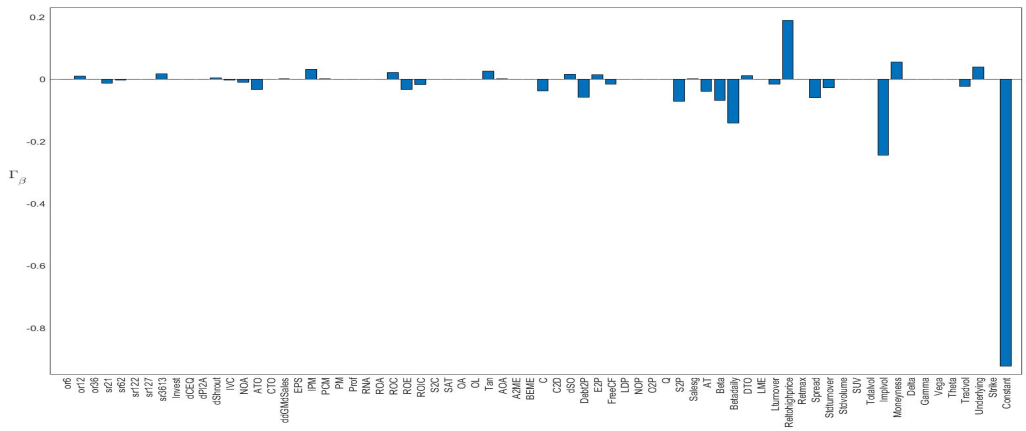
5.3 Regularized IPCA

The IPCA framework contains a substantial number of parameters for larger K, making it prone to overfit the cross-section. By removing redundant information from the model, IPCA could potentially improve out-of-sample performance. We resort to three well-known regularization techniques, ridge, Lasso, and the elastic net. Where the first shrinks the noise in Γ, the second and third technique also function as variable selection operators that can make the model sparse and easier to interpret. Figure 3 displays an example of the effects of the Lasso on Γ_β.

As this section is merely an exploration of regularized IPCA, we only focus on the equity data due to its larger number of predictors and we do not optimize over the penalty parameter λ. Hence, we choose λ such that the Lasso removes around one-half of the characteristics, ridge re-assigns the weights sufficiently, and elastic net offers an acceptable balance of the two techniques. Moreover, we find that generally λ values over 10^{-3} lead to a non-positive definite matrix, which makes it impossible to impose the IPCA restriction of orthonormalization on Γ_β. To counter this, one could replace this restriction with orthogonalization and transfer the orthonormalization restriction to the factors f_t to ensure a unique solution. However, this implies that we cannot compare factors across models, which is necessary to examine a potential trading strategy.

We observe in Figure 3 that Lasso assigns the largest loadings to the constant, the implied volatility, the stock price over its highest 250 trading days price, and the daily CAPM beta. Where the former two predictors are established as important drivers, the latter two are not, which is important if the Lasso performs well out-of-sample. Furthermore, we find that the Lasso mostly deems the Past returns, Investment, Profitability, and Intangibles catergories as unimportant relative to Value, Trading frictions and Option features.

Figure 3: Lasso estimates of Γ_β for Factor 1 in the restricted $K = 1$ IPCA specification.



Note. This figure displays the Lasso regularized Γ_β in the restricted IPCA model with $K = 1$ and $\Gamma_\alpha = \mathbf{0}$. The data contains all equity data for the in-sample period July 2006 to October 2014. The variables are the 70 equity characteristics of Table 11 in Appendix A and λ is set at 10^{-5}

Since regularization aims to prevent overfitting, we solely focus on the out-of-sample performance. Table 9 reports the out-of-sample fit of the elastic net. The performance of the Lasso and ridge are listed in Appendix H. When compared to the basic IPCA model of Table 4, we observe that the out-of-sample total R^2 are all comparable with a slight edge to the regularized models. The most remarkable finding is the predictive R^2. While sporadically competitive, regularized IPCA performs well for all values of K in contrast to basic IPCA.

Table 9: Elastic net IPCA out-of-sample fit of equity options.

	Individual returns (r_t)		Managed portfolios (x_t)	
K	Total R^2	Predictive R^2	Total R^2	Predictive R^2
1	8.76	0.21	42.8	0.85
2	10.1	0.20	60.0	0.79
3	11.1	0.21	67.8	0.90
4	11.6	0.22	72.2	0.90

Note. This table displays the performance of restricted IPCA with K factors using elastic net regularization where ρ is equal to 0.03 and λ is set at 10^{-3}. We implement the equity data with 70 lagged characteristics for the out-of-sample period November 2014 to May 2018. The total and predictive R^2 are reported in percentages.

Of the three regularization techniques, we find that ridge generates the best results. This implies that while shrinkage increases performance, it is not beneficial to reduce the model by removing predictors. Closer inspection of regularized IPCA tells us that this success stems from the distribution of the loadings. More specifically, the initially large weight for the constant is consistently amplified due to the orthonormalization of Γ_β in each ALS iteration. Overall, these findings suggest that regularization proves beneficial for the IPCA framework. However, the question arises whether this converts to profitable investments.

5.4 Trading strategy

The annualized out-of-sample Sharpe ratios are shown in Table 10 to describe the mean-variance efficiency of the IPCA factors. The K-th column denotes the Sharpe ratios for univariate factor K, the tangency allocation based on factors 1 through K, and the pure-alpha portfolio for a K factor IPCA specification.

Table 10: Out-of-sample Sharpe ratios.

	K			
	1	2	3	4
Panel A: Equity options				
Univariate	0.62	0.27	0.17	0.43
Tangency	0.62	0.82	0.05	2.38
Pure-alpha	0.22	0.17	0.15	0.20
Panel B: Ridge regularization				
Univariate	0.58	0.20	0.67	0.31
Tangency	0.58	0.24	0.72	0.79
Panel C: ETF options				
Univariate	1.68	0.21	1.28	1.14
Tangency	1.68	1.78	1.66	1.69
Pure-alpha	1.01	0.74	0.26	0.77
Panel D: Information Technology sector				
Univariate	1.15	1.07	0.86	0.93
Tangency	1.15	1.21	1.06	0.57
Pure-alpha	0.61	0.48	0.95	0.45

Note. This table displays annualized Sharpe ratios of the IPCA model with K factors using the equity and ETF data for the out-of-sample period November 2014 to May 2018. The rows represent individual factors ("univariate") and mean-variance efficient portfolio of factors in each model ("tangency") that are all based on the restricted IPCA specification. Lastly, the pure-alpha portfolios are based on Γ_α of unrestricted IPCA.

The first IPCA factor of the equity options produces a Sharp ratio of 0.62, as opposed to 1.68 for the ETF options. Adding factors increases the Sharpe ratios further to 2.38 and 1.69, respectively. To add perspective, the Sharpe ratio is 1.02 for the stock market over the same time period. Notably, this was an exceptionally good time for this market, as the Sharpe ratio is only 0.67 over our full sample period. Nevertheless, these findings imply that there is potential for profitable investments for both equity and ETF options.

The pure-alpha Sharpe ratios of Table 10 range from 0.15 to 1.01, and are substantially smaller than the factor risk premium portfolios. This performance does not make pure-alphas portfolios attractive investments in terms of mean-variance efficiency, regardless of factor neutrality. Furthermore, we report additional Sharpe ratios of unrestricted IPCA, Lasso and elastic net, and the other nine industry sectors in Appendix I. While regularization generates better out-of-sample fits, it does not transfer to Sharpe ratios as we observe that its performance remains modest. The same argument holds for the industry sectors, as seven of the ten sectors do not exceed a Sharp ratio above one. The other three sectors are Information Technology, Health Care, and Consumer Discretionary and generate maximum Sharpe ratios of 1.21, 1.29, and 2.15, respectively. Furthermore, we find that unrestricted IPCA produces tangency Sharpe ratios that are more consistent and occasionally superior. Hence, despite insignificant values for Γ_α, the unrestricted IPCA seems favorable.

The relatively high tangency Sharpe ratios suggest that IPCA is successful in capturing the comovement among delta-hedged options while simultaneously aligning their factor loadings with differences in average returns. However, it is nearly impossible for an investor to obtain these Sharp ratios due to two reasons. First, the tangency allocation comes with high turnover, which partially originates from the fast-moving characteristics, as can be observed in Appendix A.1 and B.1. Second, the delta-hedging process requires nearly daily rebalancing. This leads to substantial implementation costs that will considerably reduce profits. While research as Zhan et al. (2022) find Sharpe ratios above two for delta-hedged equity options using common option factors, the question remains whether this holds for IPCA.

6 Conclusion

In this paper, we apply a conditional latent factor model to analyze the drivers of delta-hedged returns for equity and ETF call options. Through IPCA, the model incorporates observable pricing-relevant characteristics via time-varying loadings that instrument for unobservable dynamics. This originates from the assumption that characteristics proxy for systematic risk exposures and are therefore linked to compensation. We consider 70 firm and option-related characteristics for the equity options and 30 for the ETF options. Additionally, the IPCA framework is extended through regularization to counter the risk of overfitting the cross-section. The models are then evaluated based on sample fit and Sharpe ratio, where the latter examines whether IPCA offers potential for profitable trading strategies.

Our findings show that for equity and ETF options the largest and most significant predictor of delta-hedged returns is the constant. This implies that all options share a common baseline exposure that cannot be described by time-varying characteristics. As its corresponding loadings are generally negative, the constant captures a substantial amount of the negative average delta-hedged return. Moreover, we find that implied volatility is a vital driver for all options and has a negative relation with the returns.

Focusing on the equity options, we find that operating leverage, market capitalization, and total assets further describe the delta-hedged returns. Where the former two are positively related to the returns, the latter exhibits a negative relation. This suggests that companies with a considerable amount of intangible assets that are reflected in the stock price generally produce higher delta-hedged returns. Kelly et al. (2019) argue that market capitalization and total assets are the most significant drivers of stock returns. However, they find the opposite relation to returns as our research, which can be explained by our delta-hedged position where we short the underlying stock. Overall, this implies that market capitalization and total assets are fundamental drivers across multiple asset classes.

In terms of ETF options, we find the average daily bid-ask spread of the underlying, gamma, and theta as additional drivers of delta-hedged returns. This strong significance of multiple option features is in line with Büchner and Kelly (2022). Similar to implied volatility, the three characteristics exhibit a negative relation to the returns. This suggests that less liquid ETF markets and more sensitive options in terms of gamma and theta lead to lower average returns. Therefore, we argue that when stocks are combined in the form of ETFs, the corresponding delta-hedged option returns generally have different drivers.

An IPCA model with four latent factors generates up to 26.2% in-sample and 12.4% out-of-sample total R^2 for the equity options. For the ETF options, these R^2 are 40.0% and 19.7%, respectively. In terms of predictive R^2, both data sets show consistent results for both sample periods that range from 0.7% to 3.9%. Moreover, IPCA generates tangency Sharpe ratios above two. However, these are obtained without taking transaction costs into account. We observe that separating the data on industry sectors can lead to improved out-of-sample fits. Unfortunately, this does not convert to favorable Sharpe ratios. The same observation is made for regularization, which indicates that there is less overfitting than expected. Furthermore, despite frequent insignificant values for Γ_α, we find that unrestricted IPCA consistently produces favorable results for all criteria when compared to restricted IPCA.

Additionally, the IPCA R^2 performance of stock returns in Kelly et al. (2019) is remarkably similar to our equity options. However, their Sharpe ratios are far superior to ours. A similar argument holds when we compare the ETF options performance to the delta-hedged index options of Büchner and Kelly (2022). Therefore, we argue that in terms of profit, one should favor stocks over delta-hedged equity options. And if one prefers to invest in diversified products, they should favor delta-hedged index options over delta-hedged ETF options.

7 Discussion

The core idea of IPCA, which is that characteristics proxy for risk exposures, has not been generally accepted. Hornuf and Fieberg (2020) criticise Kelly et al. (2019) and question whether characteristics are capable of resembling covariances. They claim that the explanatory power of unrestricted IPCA originates from a return phenomenon that is unrelated to risk. Moreover, their findings argue that characteristics act as characteristics in describing stock returns, and characteristics function as covariances in describing risk. Despite of Hornuf and Fieberg (2020) view that some of the conclusions of IPCA might be premature, they still praise and recommend IPCA as an insightful benchmark in the asset pricing literature.

A closer look at the data of this paper reveals several limitations. For instance, our delta-hedged returns are rebalanced on a near-daily basis. However, the underlying is often volatile and prices change every second. Hence, insufficient rebalancing might lead to exposures. The reduction from 70 to 30 characteristics could also play an important role in determining the drivers. Since it is not improbable that some characteristics suffer from too much noise in the model, the question arises how comparable the significance statistics are for the two sets of characteristics. Furthermore, an asymptotic property of IPCA relies on a large number of time periods. It is debatable if our in-sample period of 100 months suffices. Especially, since the sample period of Kelly et al. (2019) contains decades of data. It could prove beneficial to either experiment with daily returns or extend the sample period similar to Büchner and Kelly (2022). This could also lead to an analysis of out-of-sample IPCA performance during economic recessions. However, it must be noted that ETF data is relatively limited before 2010.

While regularization did not have the anticipated performance, we are still intrigued by a more parsimonious IPCA model. Specifically, if we would expand the set of characteristics by for instance, incorporating time-invariant asset-specific instruments or second-order Greeks. Kelly et al. (2019) find that their ten most significant characteristics are responsible for nearly 100% of the model's performance. It would be interesting to see if this holds for our paper. Moreover, one can also explore the significance of characteristics before initializing IPCA by performing Fama and MacBeth (1973) cross-sectional regressions (Zhan et al., 2022), or applying the adaptive Lasso (Brooks et al., 2018). In addition, one might apply the Γ_β significance test to the various industry sectors, or apply it to regularized IPCA, which could also benefit from cross-validation in determining the optimal shrinkage parameters.

Furthermore, adding a transaction cost framework, such as Zhan et al. (2022), offers a more realistic view of the potential of trading strategies using IPCA. However, one has to take into account that implementing transaction costs can alter the significant characteristics (DeMiguel et al., 2020). Additionally, it could be insightful for investors to apply Jobson and Korkie (1981) and test whether our Sharpe ratios are significantly different from one another.

A different direction could be to extend the IPCA framework with pre-specified observable factors. It could be interesting to explore the explanatory power of macroeconomic trends as the CBOE Volatility Index or common factors from option markets. Several known option factors, such as the embedded leverage factor and the straddle factor, are discussed in Coval and Shumway (2001), Frazzini and Pedersen (2012), Karakaya (2014), and Zhan et al. (2022). To this end, one can also perform interpretation analyses to explore the relation between estimated IPCA factors and common option factors, which leads to a better understanding of the behavior of the IPCA model.

Finally, Gu et al. (2021) propose a conditional latent factor model similar to IPCA. The difference is where IPCA enforces a linear relation, Gu et al. (2021) allow the loadings to be a non-parametric function of characteristics. This flexibility is realized through intensive machine learning in the form of autoencoder neural networks. Future research could examine whether this non-linearity property better suits the cross-section of delta-hedged options. Overall, we find promising results using the innovative IPCA, yet there are still a lot of questions regarding the puzzling cross-section of delta-hedged option returns.

References

Abdel-khalik, A. and Chen, P. (2015). "Growth in financial derivatives: The public policy and accounting incentives". *Journal of Accounting and Public Policy*, 34(3): pp. 291–318.

Ahn, S. and Horenstein, A. (2013). "Eigenvalue Ratio Test for the Number of Factors". *Econometrica*, 81(3): pp. 1203–1227.

Bai, J. (2003). "Inferential Theory for Factor Models of Large Dimensions". *Econometrica*, 71(1): pp. 135–171.

Bai, J. and Ng, S. (2002). "Determining the Number of Factors in Approximate Factor Models". *Econometrica*, 70(1): pp. 191–221.

Bakshi, G. and Kapadia, N. (2003). "Delta-Hedged Gains and the Negative Market Volatility Risk Premium". *The Review of Financial Studies*, 16(2): pp. 527–566.

Banz, R. (1981). "The relationship between return and market value of common stocks". *Journal of financial economics*, 9(1): pp. 3–18.

Barhoumi, K., Darné, O., and Ferrara, L. (2013). "Dynamic Factor Models: A Review of the Literature". *Journal of Business Cycle Measurement and Analysis*, 73(2): pp. 73–107.

Basu, S. (1977). "Investment performance of common stocks in relation to their price-earnings ratios: A test of the efficient market hypothesis". *The Journal of Finance*, 32(3): pp. 663–682.

Ben-David, I., Franzoni, F., and Moussawi, R. (2017). "Exchange-Traded Funds'. *Annual Review of Financial Economics*, 9: pp. 169–189.

Benzoni, L., Collin-Dufresne, P., and Goldstein, R. (2011). "Explaining Asset Pricing Puzzles Associated with the 1987 Market Crash". *Journal of Financial Economics*, 101(3): pp. 552–573.

Black, F., Derman, E., and Toy, W. (1990). "A one-factor model of interest rates and its application to treasury bond options". *Financial Analysts Journal*, 46(1): pp. 33–39.

Black, F. and Scholes, M. (1973). "The Pricing of Options and Corporate Liabilities". *Journal of Political Economy*, 81(3): pp. 637–654.

Brandt, M. (2010). *"Handbook of Financial Econometrics: Tools and Techniques"*. §5, Elsevier.

Brooks, R., Chance, D., and Shafaati, M. (2018). "The Cross-Section of Individual Equity Option Returns". *Working paper.*

Brown, J. (2009). "Principal components analysis and exploratory factor analysis - Definitions, differences, and choices". *Shiken: JALT Testing and Evaluation SIG Newsletter*, 13(1): pp. 26–30.

Büchner, M. and Kelly, B. (2022). "A Factor Model for Option Returns". *Journal of Financial Economics*, 143(3): pp. 1140–1161.

Cao, J. and Han, B. (2013). "Cross section of option returns and idiosyncratic stock volatility". *Journal of Financial Economics*, 108(1): pp. 231–249.

Cao, J., Han, B., Zhan, X., and Tong, Q. (2016). "Option Return Predictability". *Working paper. Rotman School of Management.*

Carhart, M. (1997). "On persistence in mutual fund performance". *The Journal of finance*, 52(1): pp. 57–82.

Cattell, R. (1966). "The Scree Test for the Number of Factors". *Multivariate Behavioral Research*, 1(2): pp. 245–276.

Chamberlain, G. and Rothschild, M. (1983). "Arbitrage, Factor Structure, and Mean-Variance Analysis on Large Asset Markets". *The Econometric Society*, 51(5): pp. 1281–1304.

Chan, L., Hamao, Y., and Lakonishok, J. (1991). "Fundamentals and Stock Returns in Japan". *The Journal of Finance*, 46(5): pp. 1739–1764.

Chen, R. and Scott, L. (1992). "Pricing interest rate options in a two-factor Cox–Ingersoll–Ross model of the term structure". *The Review of Financial Studies*, 5(4): pp. 613–636.

Christoffersen, P., Fournier, M., and Jacobs, K. (2013). "The Factor Structure in Equity Options". *Working paper. Center for Research in Econometric Analysis of Time Series.*

Christoffersen, P., Fournier, M., and Jacobs, K. (2018). "The Factor Structure in Equity Options". *The Review of Financial Studies*, 31(2): pp. 595–637.

Cochrane, J. (2009). *"Asset pricing: Revised edition"*. Princeton University Press.

Cochrane, J. (2011). "Presidential address: Discount rates". *The Journal of finance*, 66(4): pp. 1047–1108.

Connor, G. and Korajczyk, R. (1986). "Performance Measurement with the Arbitrage Pricing Theory: A New Framework for Analysis". *Journal of Financial Economics*, 15(3): pp. 373–394.

Connor, G. and Korajczyk, R. (1988). "Risk and return in an equilibrium APT: Application of a new test methodology". *Journal of Financial Economics*, 21(2): pp 255–289.

Coval, J. and Shumway, T. (2001). "Expected Option Returns". *The Journal of Finance*, 56(3): pp. 983–1009.

Cox, J., Ross, S., and Rubinstein, M. (1979). "Option pricing: A simplified approach". *Journal of Financial Economics*, 7(3): pp. 229–263.

Daniel, K. and Titman, S. (2012). "Testing Factor-Model Explanations of Market Anomalies". *Critical Finance Review*, 1(1): pp. 103–139.

DeMiguel, V., Martin-Utrera, A., Nogales, F., and Uppal, R. (2020). "A Transaction-cost Perspective on the Multitude of Firm Characteristics". *The Review of Financial Studies*, 33(5): pp. 2180–2222.

Fama, E. (1965). "The Behavior of Stock-Market Prices". *The Journal of Business*, 38(1): pp. 34–105.

Fama, E. and French, K. (1992). "The Cross-Section of Expected Stock Returns". *The Journal of Finance*, 47(2): pp. 427–465.

Fama, E. and French, K. (1993). "Common Risk Factors in the Returns on Stocks and Bonds". *Journal of Financial Economics*, 33(1): pp. 3–56.

Fama, E. and French, K. (2015). "A Five-Factor Asset Pricing Model". *Journal of Financial Economics*, 116(1): pp. 1–22.

Fama, E. and French, K. (2018). Choosing factors. *Journal of financial economics*, 128(2): pp. 234–252.

Fama, E. and MacBeth, J. (1973). Risk, return, and equilibrium: Empirical tests. *Journal of Political Economy*, 81(3): pp. 607–636.

Fan, J., Liao, Y., and Wang, W. (2016). "Projected principal component analysis in factor models". *The Annals of Statistics*, 44(1): pp. 219–254.

Forni, M., Hallin, M., Lippi, M., and Reichlin, L. (2000). "The Generalized Dynamic-Factor Model: Identification and Estimation ". *Review of Economics and Statistics*, 82(4): pp. 540–554.

Frazzini, A. and Pedersen, L. (2012). "Embedded Leverage". *Working paper. National Bureau of Economic Research (NBER).*

Freyberger, J., Neuhierl, A., and Weber, M. (2020). Dissecting characteristics nonparametrically. *The Review of Financial Studies*, 33(5): pp. 2326–2377.

Friedman, J., Hastie, T., and Tibshirani, R. (2010). "Regularization Paths for Generalized Linear Models via Coordinate Descent". *Journal of Statistical Software*, 33(1): pp. 1–22.

Gibbons, M., Ross, S., and Shanken, J. (1989). "A Test of the Efficiency of a Given Portfolio". *Econometrica*, 57(5): pp. 1121–1152.

Goncalves, S. and Kilian, L. (2004). "Bootstrapping autoregressions with conditional heteroskedasticity of unknown form". *Journal of Econometrics*, 123(1): pp. 89–120.

Goyal, A. and Saretto, A. (2009). "Cross-section of option returns and volatility". *Journal of Financial Economics*, 94(2): pp. 310–326.

Green, J., Hand, J., and Zhang, X. (2017). "The Characteristics that Provide Independent Information about Average U.S. Monthly Stock Returns". *The Review of Financial Studies*, 30(12): pp. 4389–4436.

Gu, S., Kelly, B., and Xiu, D. (2021). "Autoencoder Asset Pricing Models". *Journal of Econometrics*, 222(1): pp. 429–450.

Han, Y., He, A., Rapach, D., and Zhou, G. (2018). "What Firm Characteristics Drive U.S. Stock Returns". *Working paper.*

Hansen, L. (1982). "Large sample properties of generalized method of moments estimators". *Econometrica*, 50(4): pp. 1029–1054.

Hansen, L. and Richard, S. (1987). "The Role of Conditioning Information in Deducing Testable Restrictions Implied by Dynamic Asset Pricing Models". *Econometrica*, pages pp. 587–613.

Harvey, C., Liu, Y., and Zhu, H. (2016). "... and the Cross-Section of Expected Returns". *The Review of Financial Studies*, 29(1): pp. 5–68.

Hastie, T., Tibshirani, R., and Friedman, J. (2009). *"The Elements of Statistical Learning. Data Mining, Inference, and Prediction"*. §3.4, §3.6, Springer.

Heston, S. (1993). "A closed-form solution for options with stochastic volatility with applications to bond and currency options". *The Review of Financial Studies*, 6(2): pp. 327–343.

Horenstein, A., Vasquez, A., and Xiao, X. (2020). "Common Factors in Equity Option Returns". *Working paper. Canadian Derivatives Institute.*

Hornuf, L. and Fieberg, C. (2020). "Are Characteristics Covariances or Characteristics?". *Working paper. CESifo Institute, No. 8377.*

Hull, J. (2003). *"Options, Futures and Other Derivatives"*. Pearson Education India.

Hull, J. and White, A. (1987). "The pricing of options on assets with stochastic volatilities". *The Journal of Finance*, 42(2): pp. 281–300.

Israelov, R. and Kelly, B. (2017). "Forecasting the Distribution of Option Returns". *Working paper. National Bureau of Economic Research (NBER).*

Jarque, C. and Bera, A. (1987). "A Test for Normality of Observations and Regression Residuals". *International Statistical Review*, 55(2): pp. 163–172.

Jegadeesh, N. (1990). "Evidence of predictable behavior of security returns". *The Journal of finance*, 45(3): pp. 881–898.

Jegadeesh, N. and Titman, S. (1993). "Returns to buying winners and selling losers: Implications for stock market efficiency". *The Journal of finance*, 48(1): pp. 65–91.

Jobson, J. and Korkie, B. (1981). "Performance Hypothesis Testing with the Sharpe and Treynor Measures". *Journal of Finance*, pages pp. 889–908.

Jolliffe, I. (2002). *"Principal Component Analysis"*. §1, §7, Springer.

Jones, C. (2006). "A Nonlinear Factor Analysis of S&P 500 Index Option Returns". *The Journal of Finance*, 61(5): pp. 2325–2363.

Karakaya, M. (2014). "Characteristics and Expected Returns in Individual Equity Options". *Working paper.*

Kelly, B., Pruitt, S., and Su, Y. (2017). "Instrumented Principal Component Analysis".

Kelly, B., Pruitt, S., and Su, Y. (2019). "Characteristics are covariances: A unified model of risk and return". *Journal of Financial Economics*, 134(3): pp. 501–524.

Kim, S., Korajczyk, R., and Neuhierl, A. (2021). "Arbitrage Portfolios". *The Review of Financial Studies*, 34(6): pp. 2813–2856.

Kozak, S., Nagel, S., and Santosh, S. (2020). "Shrinking the cross-section". *Journal of Financial Economics*, 135(2): pp. 271–292.

Lamoureux, C. and Lastrapes, W. (1990). "Heteroskedasticity in Stock Return Data: Volume versus GARCH Effects". *The Journal of Finance*, 45(1): pp. 221–229.

Lettau, M. and Madhavan, A. (2018). "Exchange-Traded Funds 101 for Economists". *Journal of Economic Perspectives*, 32(1): pp. 135–154.

Lewellen, J., Nagel, S., and Shanken, J. (2010). "A skeptical appraisal of asset pricing tests". *Journal of Financial Economics*, 96(2): pp. 175–194.

Lintner, J. (1965). "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets". *The Review of Economics and Statistics*, 47(1): pp. 13–37.

Markowitz, H. (1952). "Portfolio Selection". *The Journal of Finance*, 7(1): pp. 77–91.

Merton, R. (1973). "Theory of Rational Option Pricing". *The Bell Journal of Economics and Management Science*, 4(1): pp. 141–183.

Mulaik, S. (2009). *"Foundations of Factor Analysis"*. §1, §8, §9, CRC Press.

Nelson, D. (1991). "Conditional Heteroskedasticity in Asset Returns: A New Approach". *Econometrica: Journal of the Econometric Society*, pages pp. 347–370.

Ofek, E. and Richardson, M. (2003). "DotCom Mania: The Rise and Fall of Internet Stock Prices". *The Journal of Finance*, 58(3): pp. 1113–1137.

Ofek, E., Richardson, M., and Whitelaw, R. (2004). "Limited arbitrage and short sales restrictions: Evidence from the options markets". *Journal of Financial Economics*, 74(2): pp. 305–342.

Petram, L. and Richards, L. (2014). *"The world's first stock exchange: how the Amsterdam market for Dutch East India Company shares became a modern securities market, 1602-1700"*. §1, Columbia Business School Publishing.

Rao, C. (1964). "The use and interpretation of principal component analysis in applied research". *The Indian Journal of Statistics, Series A*, 26(4): pp. 329–358.

Rapach, D. and Zhou, G. (2020). "Time-series and Cross-sectional Stock Return Forecasting: New Machine Learning Methods". *Machine Learning for Asset Management: New Developments and Financial Applications*, pages pp. 1–33.

Ross, S. (1976). "The Arbitrage Theory of Capital Asset Pricing". *Journal of Economic Theory*, 13(3): pp 341–360.

Santos, T. and Veronesi, P. (2004). "Conditional Betas". *Working paper. National Bureau of Economic Research (NBER).*

Schneeweiss, H. and Mathes, H. (1995). "Factor Analysis and Principal Components". *Journal of Multivariate Analysis*, 55(1): pp. 105–124.

Sharpe, W. (1964). "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk". *The Journal of Finance*, 19(3): pp. 425–442.

Sharpe, W. (1966). "Mutual Fund Performance". *The Journal of Business*, 39(1): pp. 119–138.

Spearman, C. (1904). "General intelligence, objectively determined and measured". *The American Journal of Psychology*, 15(2): pp. 201–292.

Stock, J. and Watson, M. (2002). "Forecasting Using Principal Components From a Large Number of Predictors". *Journal of the American Statistical Association*, 97(460): pp. 1167–1179.

Tashman, L. (2000). "Out-of-sample Tests of Forecasting Accuracy: an Analysis and Review". *International Journal of Forecasting*, 16(4): pp. 437–450.

Tibshirani, R. (1996). "Regression shrinkage and selection via the Lasso". *Journal of the Royal Statistical Society*, 58(1): pp. 267–288.

Tikhonov, A. and Arsenin, V. (1977). *"Solutions of ill-posed problems"*. Winston.

Vasquez, A. (2017). "Equity Volatility Term Structures and the Cross Section of Option Returns". *Journal of Financial and Quantitative Analysis*, 52(6): pp. 2727–2754.

Wriggers, P. and Panagiotopoulos, P. (1999). *"New Developments in Contact Problems"*. §8.6, Springer.

Zhan, X., Han, B., Cao, J., and Tong, Q. (2022). "Option Return Predictability". *The Review of Financial Studies*, 35(3): pp. 1394–1442.

Zou, H. and Hastie, T. (2005). "Regularization and variable selection via the elastic net". *Journal of the Royal Statistical Society. Series B (Statistical Methodology)*, 67(2): pp. 301–320.

Appendices

A Equity Characteristics by Category

Table 11: Equity characteristics sorted by category.

	Past returns:	
(1)	or_6	DHO return average over the last 6 months
(2)	or_{12}	DHO return average over the last 12 months
(3)	or_{36}	DHO return average over the last 36 months
(4)	sr_{2-1}	Stock return 1 month before prediction
(5)	sr_{6-2}	Stock return from 6 to 2 months before prediction
(6)	sr_{12-2}	Stock return from 12 to 2 months before prediction
(7)	sr_{12-7}	Stock return from 12 to 7 months before prediction
(8)	sr_{36-13}	Stock return from 36 to 13 months before prediction
	Investment:	
(9)	Invest	% change in AT
(10)	ΔCEQ	% change in BE
(11)	ΔPI2A	Change in PP&E and inventory over lagged AT
(12)	ΔShrout	% change in shares outstanding
(13)	IVC	Change in inventory over average AT
(14)	NOA	Net-operating assets over lagged AT
	Profitability:	
(15)	ATO	Sales over lagged net operating assets
(16)	CTO	Sales over lagged total assets
(17)	Δ(ΔGM-ΔSales)	Δ(% change in gross margin and % change in sales)
(18)	EPS	Earnings per share
(19)	IPM	Pretax income over sales
(20)	PCM	Sales minus costs of goods sold over sales
(21)	PM	OI after depreciation over sales
(22)	Prof	Gross profitability over BE
(23)	RNA	OI after depreciation over lagged net operating assets
(24)	ROA	Income before extraordinary items over lagged AT
(25)	ROC	Size + long-term debt - total assets over cash
(26)	ROE	Income before extraordinary items over lagged BE
(27)	ROIC	Return on invested capital
(28)	S2C	Sales over cash
(29)	SAT	Sales over total assets
	Intangibles:	
(30)	OA	Operating accruals
(31)	OL	Costs of goods sold + SG&A over total assets
(32)	Tan	Tangibility
(33)	AOA	Absolute value of operating accruals
	Value:	
(34)	A2ME	Assets-to-market
(35)	BEME	Book-to-market ratio
(36)	C	Cash to AT
(37)	C2D	Cash flow over total liabilities
(38)	ΔSO	Log change in split-adjusted shares outstanding
(39)	Debt2P	Total debt over size
(40)	E2P	Income before extraordinary items over size
(41)	Free CF	Free cash flow over BE
(42)	LDP	Trailing 12-months dividends over price
(43)	NOP	Net payouts over size
(44)	O2P	Operating payouts over market cap
(45)	Q	Tobin's Q
(46)	S2P	Sales over price
(47)	Sales_g	Sales growth
	Trading frictions:	
(48)	AT	Total assets
(49)	Beta	Correlation × ratio of vols
(50)	Beta daily	CAPM beta using daily returns
(51)	DTO	De-trended Turnover
(52)	LME	Price × shares outstanding
(53)	Lturnover	Last month's volume over shares outstanding
(54)	Rel_to_high_price	Price over 250 trading days high price
(55)	Ret_max	Maximum daily return
(56)	Spread	Average daily bid-ask spread
(57)	Std turnover	Standard deviation of daily turnover
(58)	Std volume	Standard deviation of daily volume
(59)	SUV	Standard unexplained volume
(60)	Total vol	Standard deviation of daily returns
	Option features:	
(61)	Impl vol	Implied volatility
(62)	Moneyness	Underlying price over strike price
(63)	Delta	Delta
(64)	Gamma	Gamma
(65)	Vega	Vega
(66)	Theta	Theta
(67)	Trad volume	Trading volume of the option
(68)	Underlying	Underlying price
(69)	Strike	Strike price
(70)	Constant	Constant

Note. This table displays the equity characteristics that we analyze by category. The computation of each variable is either discussed in Section 4.4 or extensively described in the online appendix of Freyberger et al. (2020). Lastly, delta-hedged option is abbreviated as DHO and Δ stands for difference in.

A.1 Descriptive statistics

Table 12: Descriptive statistics of the equity characteristics.

	Mean	Median	Std.	Freq		Mean	Median	Std.	Freq
Past returns:					**Value:**				
or_6	-0.01	-0.01	(0.06)	m	A2ME	2.56	1.02	(11.84)	y
or_{12}	-0.01	-0.01	(0.05)	m	BEME	0.63	0.43	(3.61)	y
or_{36}	-0.01	-0.01	(0.04)	m	C	0.14	0.08	(0.17)	y
sr_{2-1}	0.01	0.01	(0.20)	m	C2D	0.18	0.14	(0.59)	y
sr_{6-2}	0.05	0.03	(0.48)	m	ΔSO	0.02	0.00	(0.13)	y
sr_{12-2}	0.13	0.08	(0.84)	m	Debt2P	0.76	0.28	(3.44)	y
sr_{12-7}	0.06	0.04	(0.54)	m	E2P	0.01	0.05	(0.60)	y
sr_{36-13}	0.27	0.13	(1.28)	m	Free CF	0.08	0.10	(8.95)	y
					LDP	17.88	3.70	(73.62)	m
Investment:					NOP	0.02	0.02	(0.26)	y
Invest	0.13	0.06	(0.46)	y	O2P	0.04	0.03	(0.29)	y
ΔCEQ	0.12	0.07	(2.58)	y	Q	1.66	0.97	(5.19)	y
ΔPI2A	0.05	0.03	(0.19)	y	S2P	1.28	0.58	(5.89)	y
ΔShrout	0.78	0.00	(176.82)	m	Sales_g	0.55	0.067	(36.36)	y
IVC	0.04	0.04	(10.18)	y					
NOA	0.48	0.54	(0.41)	y	**Trading frictions:**				
					AT	4.7e+04	5.3e+03	(2.2e+05)	y
Profitability:					Beta	0.98	0.87	(0.95)	m
ATO	2.68	1.44	(72.73)	y	Beta daily	1.03	0.99	(1.12)	m
CTO	0.90	0.68	(0.87)	y	DTO	0.00	0.00	(0.67)	m
Δ(ΔGM-ΔSales)	1.71	-0.89	(358.67)	y	LME	2.6e+10	5.3e+09	(9.9e+10)	m
EPS	2.61	1.75	(18.90)	y	Lturnover	0.05	0.01	(13.99)	m
IPM	-1.07	0.11	(39.28)	m	Rel_to_high_price	0.82	0.88	(0.18)	m
PCM	-0.43	0.38	(30.10)	y	Ret_max	0.05	0.03	(0.05)	m
PM	-1.00	0.14	(39.14)	y	Spread	1.28	0.89	(12.78)	m
Prof	0.62	0.50	(41.09)	y	Std turnover	0.01	0.00	(3.05)	m
RNA	0.28	0.18	(24.63)	y	Std volume	1.5e+06	5.1e+05	(5.7e+06)	m
ROA	0.05	0.05	(0.15)	y	SUV	-0.13	-0.14	(0.14)	m
ROC	253.84	132.55	(7.3e+06)	m	Total vol	0.02	0.02	(0.02)	m
ROE	0.05	0.12	(5.67)	y					
ROIC	0.08	0.08	(0.13)	y	**Option features:**				
S2C	113.20	7.40	(2.8e+03)	y	Impl vol	0.34	0.30	(0.16)	m
SAT	0.82	0.62	(0.78)	y	Moneyness	1.00	1.00	(0.03)	m
					Delta	0.51	0.52	(0.08)	m
Intangibles:					Gamma	0.12	0.09	(0.10))	m
OA	-0.08	-0.07	(0.19)	y	Vega	7.74	5.71	(10.26)	m
OL	0.71	0.49	(0.75)	y	Theta	-9.95	-7.39	(12.40)	m
Tan	0.42	0.44	(0.18)	y	Trad volume	224.07	20.00	(1.4e+03)	m
AOA	0.14	0.10	(0.14)	y	Underlying	57.17	42.99	(74.59)	m
					Strike	57.31	43.00	(74.67)	m
					Constant	1.00	1.00	(0.00)	

Note. This table displays the descriptive statistics of the 8,866,082 characteristic observations for all equity options. The columns represent means, medians, standard deviations and the frequency of variation, which is either monthly or yearly. The characteristics are organized by category and range from July 2006 to May 2018. The computation of each variable is either discussed in Section 4.4 or extensively described in the online appendix of Freyberger et al. (2020).

B ETF Characteristics by Category

Table 13: ETF characteristics sorted by category.

	Past returns:				
(1)	or_6	DHO return average over the last 6 months	(16)	Spread	Average daily bid-ask spread
(2)	or_{12}	DHO return average over the last 12 months	(17)	Std turnover	Standard deviation of daily turnover
(3)	or_{36}	DHO return average over the last 36 months	(18)	Std volume	Standard deviation of daily volume
(4)	sr_{2-1}	Security return 1 month before prediction	(19)	SUV	Standard unexplained volume
(5)	sr_{6-2}	Security return from 6 to 2 months before prediction	(20)	Total vol	Standard deviation of daily
(6)	sr_{12-2}	Security return from 12 to 2 months before prediction			
(7)	sr_{12-7}	Security return from 12 to 7 months before prediction		**Option features:**	
			(21)	Impl vol	Implied volatility
	Investment:		(22)	Moneyness	Underlying price over strike price
(8)	ΔShrout	% change in shares outstanding	(23)	Delta	Delta
			(24)	Gamma	Gamma
	Trading frictions:		(25)	Vega	Vega
(9)	Beta	Correlation × ratio of vols	(26)	Theta	Theta
(10)	Beta daily	CAPM beta using daily returns	(27)	Trad volume	Trading volume of the option
(11)	DTO	De-trended Turnover	(28)	Underlying	Underlying price
(12)	LME	Price × shares outstanding	(29)	Strike	Strike price
(13)	Lturnover	Last month's volume over shares outstanding			
(14)	Rel_to_high_price	Price over 250 trading days high price	(30)	Constant	Constant
(15)	Ret_max	Maximum daily return			

Note. This table displays the ETF characteristics that we analyze by category. The computation of each variable is either discussed in Section 4.4 or extensively described in the online appendix of Freyberger et al. (2020). Lastly, delta-hedged option is abbreviated as DHO and Δ stands for difference in.

B.1 Descriptive statistics

Table 14: Descriptive statistics of the ETF characteristics.

	Mean	Median	Std.	Freq		Mean	Median	Std.	Freq
Past returns:									
or_6	-0.01	-0.01	(0.12)	m	Spread	0.98	0.70	(1.09)	m
or_{12}	-0.01	-0.01	(0.08)	m	Std turnover	49.93	10.88	(1.1e+03)	m
or_{36}	-0.01	0.00	(0.05)	m	Std volume	1.5e+06	1.9e+05	(5.6e+06)	m
sr_{2-1}	0.01	0.01	(0.29)	m	SUV	-0.18	-0.20	(0.14)	m
sr_{6-2}	0.04	0.03	(0.45)	m	Total vol	0.02	0.01	(0.01)	m
sr_{12-2}	0.09	0.07	(0.74)	m					
sr_{12-7}	0.05	0.04	(0.50)	m	**Option features:**				
					Impl vol	0.29	0.23	(0.18)	m
Investment:					Moneyness	1.00	1.00	(0.02)	m
ΔShrout	0.07	0.01	(2.23)	m	Delta	0.51	0.52	(0.08)	m
					Gamma	0.12	0.09	(0.10)	m
Trading frictions:					Vega	8.32	6.76	(6.03)	m
Beta	0.32	0.07	(0.91)	m	Theta	-9.19	-7.23	(7.41)	m
Beta daily	0.60	0.80	(1.03)	m	Trad volume	351.67	10.00	(2.5e+03)	m
DTO	11.91	2.24	(235.57)	m	Underlying	61.89	50.33	(44.23)	m
LME	4.2e+06	9.2e+05	(1.2e+07)	m	Strike	62.02	50.00	(44.27)	m
Lturnover	82.11	14.62	(473.13)	m					
Rel_to_high_price	0.89	0.94	(0.16)	m	Constant	1.00	1.00	(0.00)	
Ret_max	0.03	0.02	(0.03)	m					

Note. This table displays the descriptive statistics of the 898,617 characteristic observations for all ETF options. The columns represent means, medians, standard deviations and the frequency of variation, which is either monthly or yearly. The characteristics are organized by category and range from July 2006 to May 2018. The computation of each variable is either discussed in Section 4.4 or extensively described in the online appendix of Freyberger et al. (2020).

C $\mathbf{R^2}$ Statistics Computations

This section features the computations of the total R^2 and predictive R^2 of the IPCA characteristic-managed portfolios and the benchmark PCA model.

IPCA Characteristic-managed portfolios

Following Kelly et al. (2019), the total R^2 and predictive R^2 for the characteristic-managed portfolio returns x_t in an IPCA model are defined as follows:

$$R^2_{x,tot} = 1 - \frac{\sum_t \left(x_{t+1} - Z_t' Z_t \left(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{f}_{t+1}\right)\right)' \left(x_{t+1} - Z_t' Z_t \left(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{f}_{t+1}\right)\right)}{\sum_t x_{t+1}' x_{t+1}} , \tag{40}$$

$$R^2_{x,pred} = 1 - \frac{\sum_t \left(x_{t+1} - Z_t' Z_t \left(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{\lambda}\right)\right)' \left(x_{t+1} - Z_t' Z_t \left(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{\lambda}\right)\right)}{\sum_t x_{t+1}' x_{t+1}} . \tag{41}$$

PCA

Regarding the PCA model, we compute the total R^2 and predictive R^2 for the option returns r_t as follows:

$$R^2_{r,tot} = 1 - \frac{\sum_{i,t} \left(r_{i,t} - \hat{\beta}_i \hat{f}_t\right)^2}{\sum_{i,t} r_{i,t}^2} , \quad R^2_{r,pred} = 1 - \frac{\sum_{i,t} \left(r_{i,t} - \hat{\beta}_i \hat{\lambda}\right)^2}{\sum_{i,t} r_{i,t}^2} , \tag{42}$$

where the static β_i is a $1 \times K$ vector which is the i-th row of the static matrix β. Additionally, the total R^2 and predictive R^2 for characteristic-managed portfolios x_t are obtained in a similar manner:

$$R^2_{x,tot} = 1 - \frac{\sum_t \left(x_t - \hat{\beta} \hat{f}_t\right)' \left(x_t - \hat{\beta} \hat{f}_t\right)}{\sum_t x_t' x_t} , \quad R^2_{x,pred} = 1 - \frac{\sum_t \left(x_t - \hat{\beta} \hat{\lambda}\right)' \left(x_t - \hat{\beta} \hat{\lambda}\right)}{\sum_t x_t' x_t} , \tag{43}$$

where the static β is an $L \times K$ matrix.

D Rolling Window

A visualisation of the rolling window approach to generate the out-of-sample forecasts is displayed in Table 15.

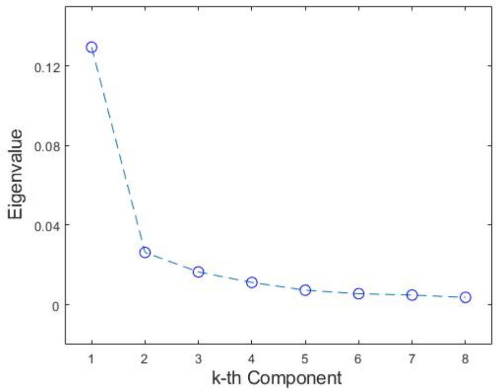
Table 15: The rolling window approach for the one month ahead out-of-sample forecasts.

	1	2	3	...	99	100	101	102	103	104	...	141	142	143
Origin = 100														
Origin = 101														
Origin = 102														
Origin = 103														
Origin = ...														
Origin = 140														
Origin = 141														
Origin = 142														

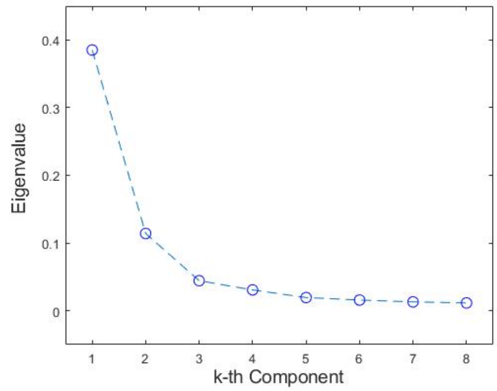
Note. White cells correspond to the in-sample period, while the grey cells correspond to the out-of-sample estimate. The forecast computation follows 3.6.2. The numbers correspond to the months in our sample period starting at July 2006. The first out-of-sample month is November 2014 and the last is month 143, which is May 2018.

E Number of Common Factors

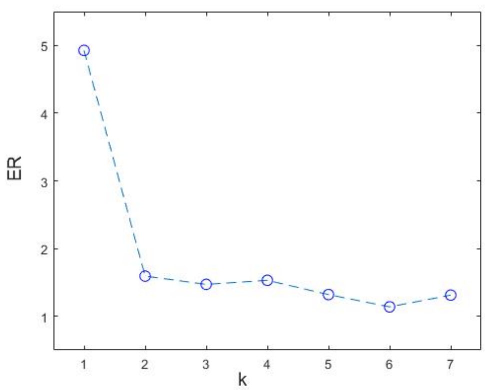
Figure 4: Scree plot, ER ratio and GR ratio for delta-hedged equity options (a)-(c) and delta-hedged ETF options (d)-(f).



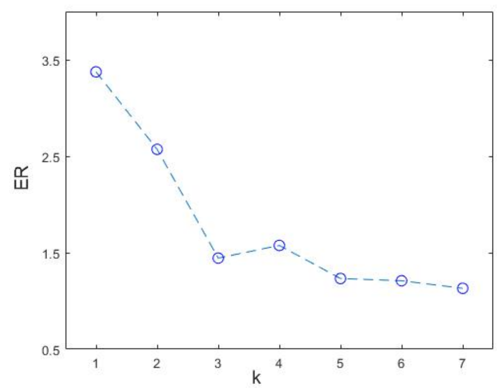
(a) Scree plot - Equity.



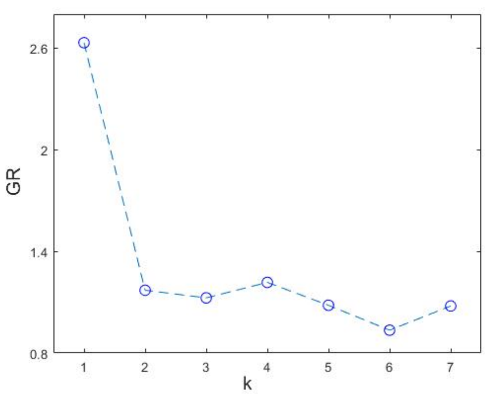
(d) Scree plot - ETF.



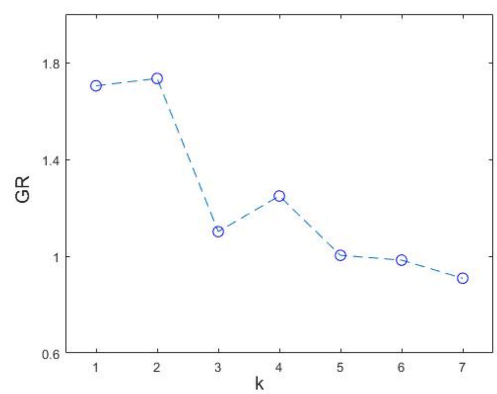
(b) ER ratio - Equity.



(e) ER ratio - ETF.



(c) GR ratio - Equity.



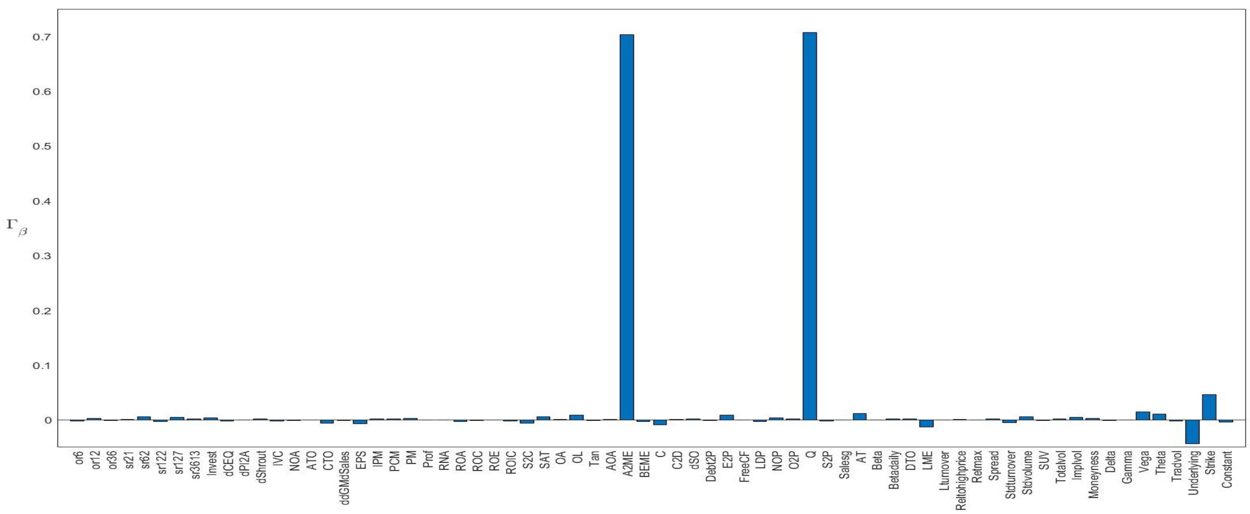
(f) GR ratio - ETF.

Note. The computations for the ER and GR ratios are discussed in Section 3.7.2. The data for these calculations ranges from the full sample period July 2006 - May 2018.

F IPCA Figures

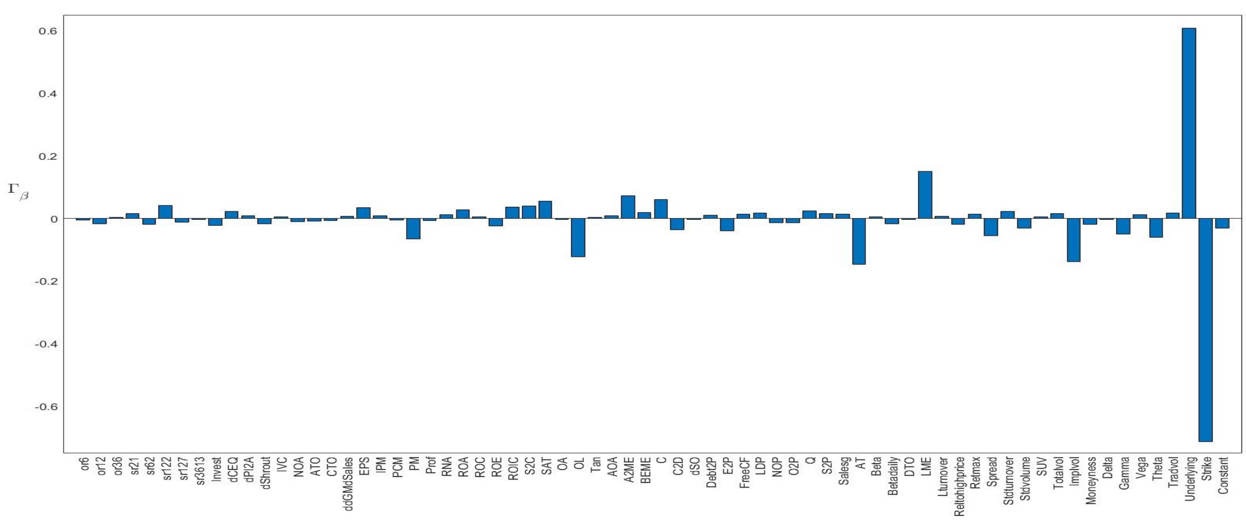
F.1 Equity options

Figure 5: Estimates of Γ_β for Factor 1 in the unrestricted $K = 4$ IPCA specification.



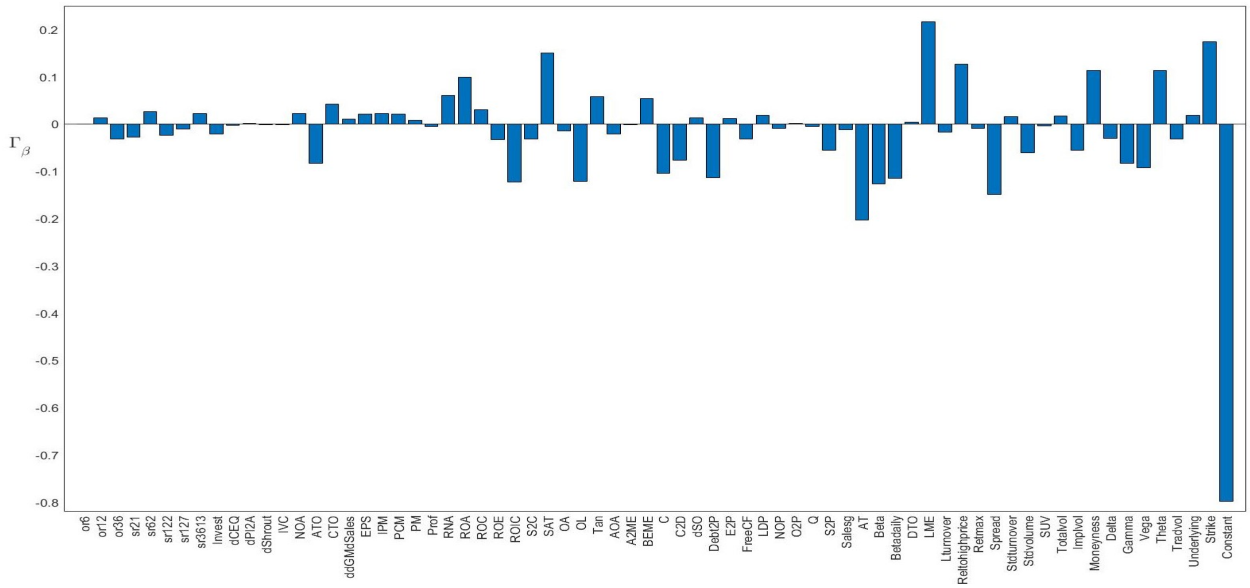
Note. This figure displays the first column of Γ_β in the unrestricted IPCA model with $K = 4$. The data contains all equity data for the in-sample period July 2006 to October 2014. The variables are the 70 equity characteristics of Table 11 in Appendix A

Figure 6: Estimates of Γ_β for Factor 2 in the unrestricted $K = 4$ IPCA specification.



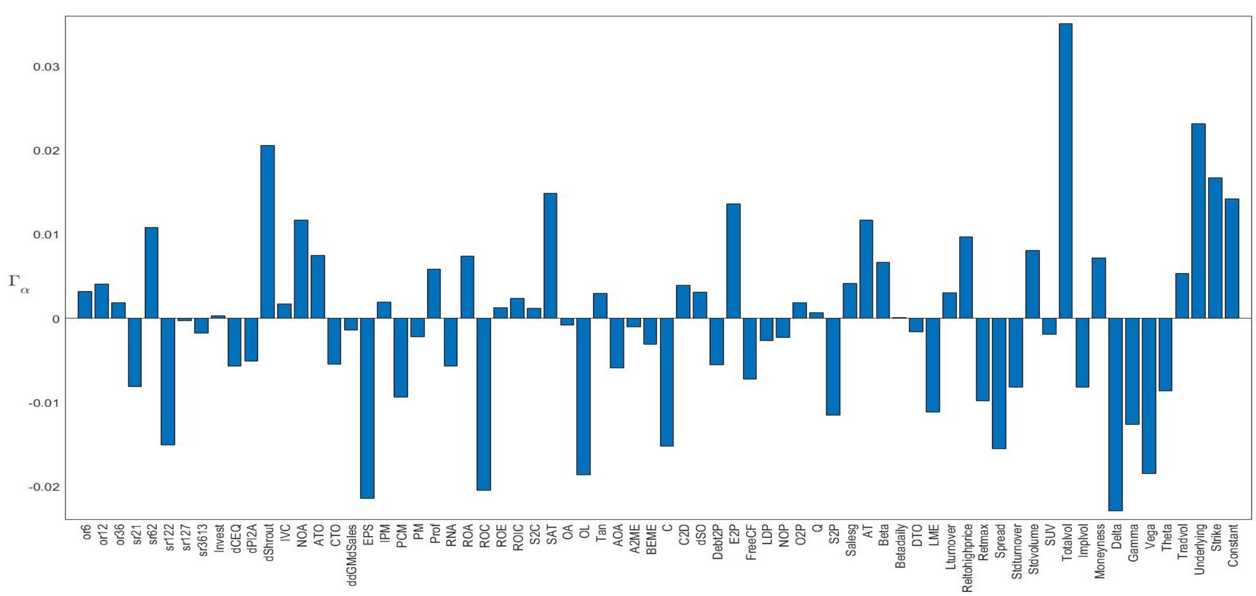
Note. This figure displays the second column of Γ_β in the unrestricted IPCA model with $K = 4$. The data contains all equity data for the in-sample period July 2006 to October 2014. The variables are the 70 equity characteristics of Table 11 in Appendix A

Figure 7: Estimates of Γ_β for Factor 4 in the unrestricted $K = 4$ IPCA specification.



Note. This figure displays the fourth column of Γ_β *in the unrestricted IPCA model with* $K = 4$*. The data contains all equity data for the in-sample period July 2006 to October 2014. The variables are the 70 equity characteristics of Table 11 in Appendix A*

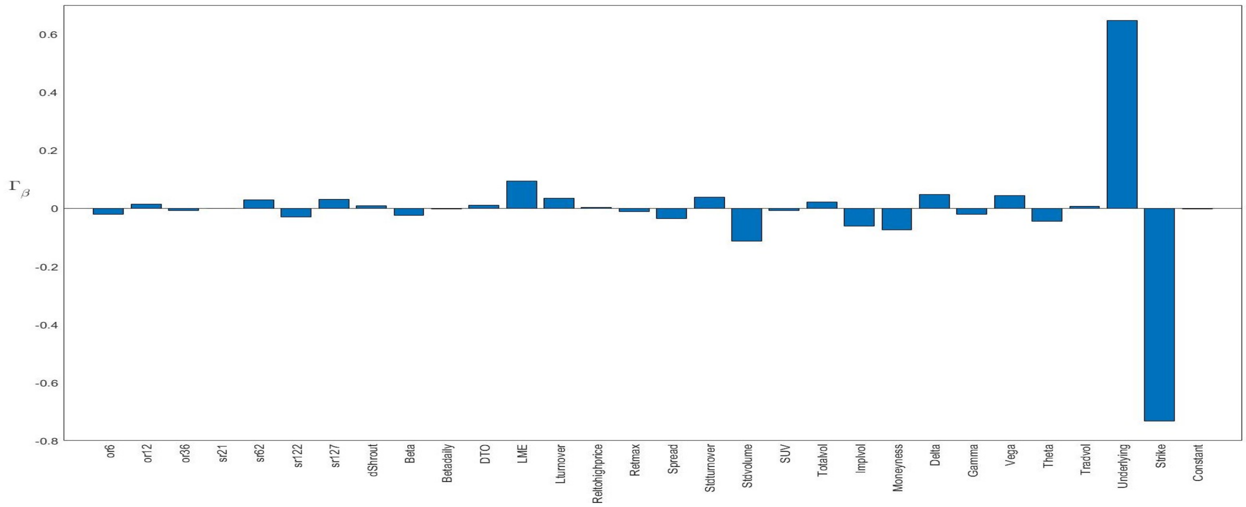
Figure 8: Estimates of Γ_α in the unrestricted $K = 4$ IPCA specification.



Note. This figure displays Γ_α *in the unrestricted IPCA model with* $K = 4$*. The data contains all equity data for the in-sample period July 2006 to October 2014. The variables are the 70 equity characteristics of Table 11 in Appendix A*

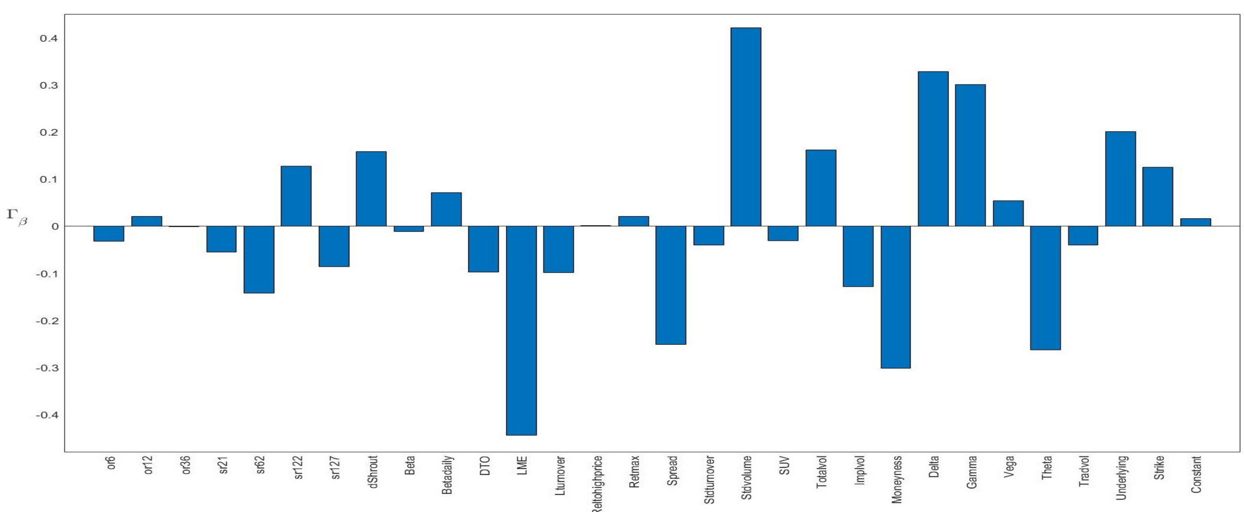
F.2 ETF options

Figure 9: Estimates of Γ_β for Factor 1 in the unrestricted $K = 4$ IPCA specification.



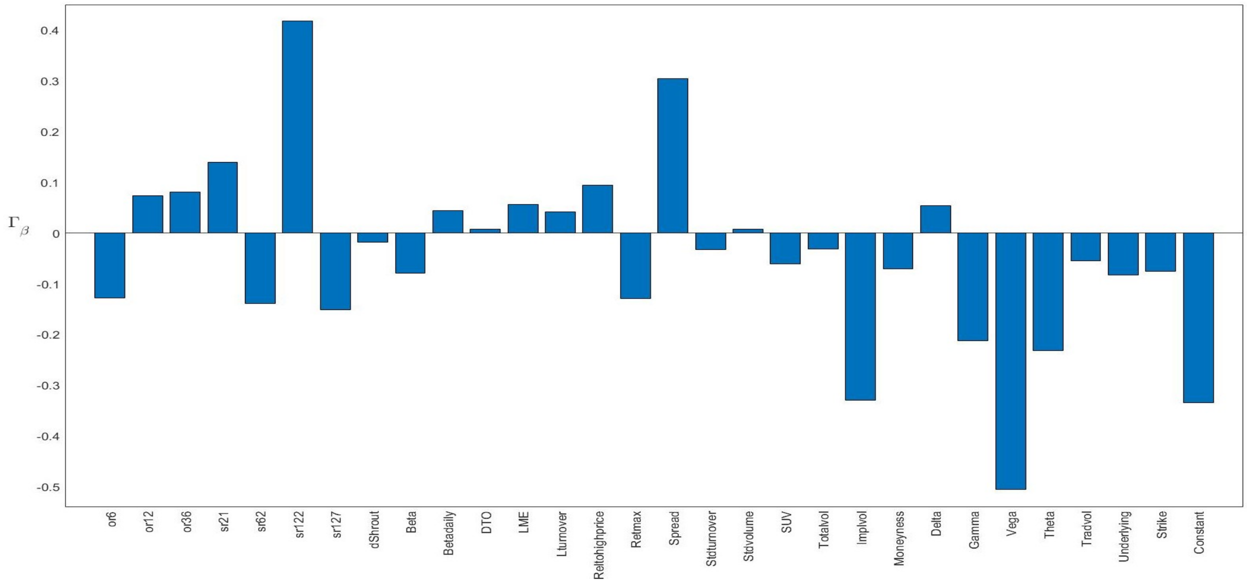
Note. This figure displays the first column of Γ_β in the unrestricted IPCA model with $K = 4$ and $\Gamma_\alpha \neq \mathbf{0}$. The data contains all ETF data for the in-sample period July 2006 to October 2014. The variables are the 30 ETF characteristics of Table 13 in Appendix B

Figure 10: Estimates of Γ_β for Factor 3 in the unrestricted $K = 4$ IPCA specification.



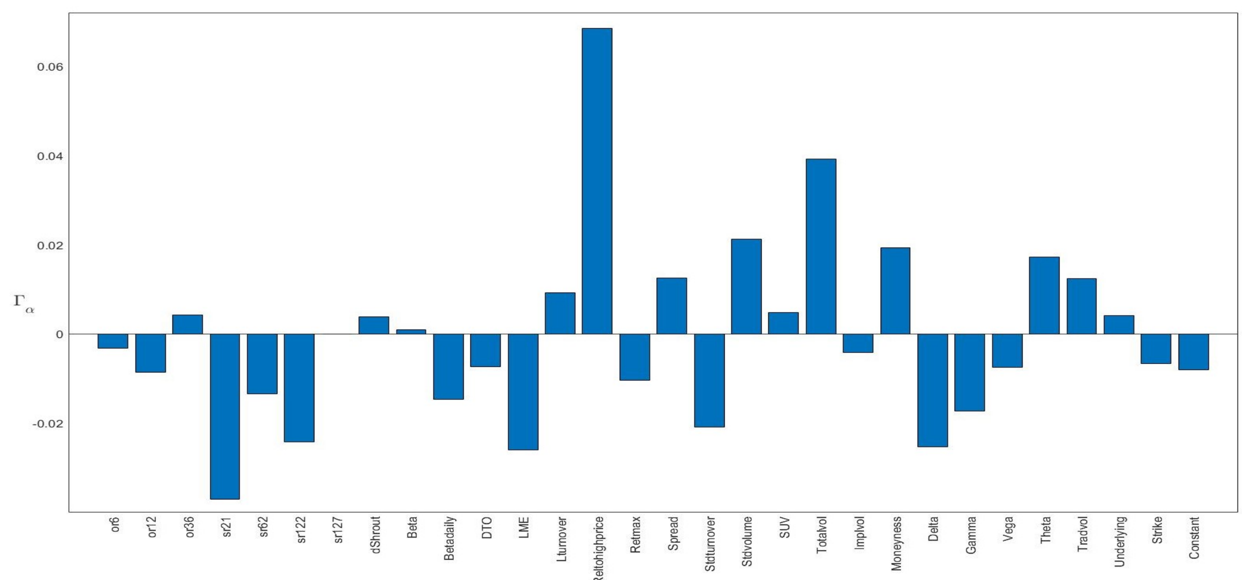
Note. This figure displays the third column of Γ_β in the unrestricted IPCA model with $K = 4$ and $\Gamma_\alpha \neq \mathbf{0}$. The data contains all ETF data for the in-sample period July 2006 to October 2014. The variables are the 30 ETF characteristics of Table 13 in Appendix B

Figure 11: Estimates of Γ_β for Factor 4 in the unrestricted $K = 4$ IPCA specification.



Note. This figure displays the fourth column of Γ_β in the unrestricted IPCA model with $K = 4$ and $\Gamma_\alpha \neq \mathbf{0}$. The data contains all ETF data for the in-sample period July 2006 to October 2014. The variables are the 30 ETF characteristics of Table 13 in Appendix B

Figure 12: Estimates of Γ_α in the unrestricted $K = 4$ IPCA specification.



Note. This figure displays Γ_α in the unrestricted IPCA model with $K = 4$. The data contains all ETF data for the in-sample period July 2006 to October 2014. The variables are the 30 ETF characteristics of Table 13 in Appendix B

G Equity Sector Fit

Table 16: IPCA performance of equity options in the Financials sector.

	Individual returns (r_t)				Managed portfolios (x_t)			
	Total R^2		Predictive R^2		Total R^2		Predictive R^2	
K	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$
Panel A: In-sample								
1	72.9	73.4	< 0	< 0	60.5	62.1	< 0	< 0
2	82.8	83.2	< 0	< 0	83.6	84.3	< 0	< 0
3	84.7	85.2	< 0	< 0	89.3	89.9	< 0	< 0
4	86.1	86.5	< 0	< 0	91.1	91.8	< 0	< 0
Panel B: Out-of-sample								
1	3.94	4.76	< 0	< 0	4.79	4.51	< 0	< 0
2	32.6	33.2	< 0	< 0	64.3	65.0	< 0	< 0
3	33.4	33.7	< 0	< 0	67.2	67.4	< 0	< 0
4	35.2	35.6	< 0	< 0	70.5	72.3	< 0	< 0

Note. This table displays the performance of the IPCA model with K factors using the equity data exclusive to the Financials sector. The data contains the in-sample period July 2006 to October 2014 and the out-of-sample period November 2014 to May 2018. The total and predictive R^2 are reported in percentages for the restricted and unrestricted IPCA model with 70 lagged characteristics.

Table 17: IPCA performance of equity options in the Health Care sector.

	Individual returns (r_t)				Managed portfolios (x_t)			
	Total R^2		Predictive R^2		Total R^2		Predictive R^2	
K	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$
Panel A: In-sample								
1	19.6	21.5	0.59	2.31	39.7	42.5	1.01	4.05
2	32.6	34.0	< 0	< 0	52.9	55.2	< 0	< 0
3	38.1	39.2	< 0	< 0	63.7	66.0	< 0	< 0
4	43.0	44.0	< 0	< 0	71.1	72.1	< 0	< 0
Panel B: Out-of-sample								
1	14.0	14.1	1.33	0.89	35.3	36.2	4.79	6.25
2	17.5	18.2	< 0	< 0	41.5	42.3	1.03	4.54
3	19.7	20.5	< 0	< 0	49.6	51.4	< 0	< 0
4	20.9	21.7	< 0	< 0	52.2	52.5	< 0	< 0

Note. This table displays the performance of the IPCA model with K factors using the equity data exclusive to the Health Care sector. The data contains the in-sample period July 2006 to October 2014 and the out-of-sample period November 2014 to May 2018. The total and predictive R^2 are reported in percentages for the restricted and unrestricted IPCA model with 70 lagged characteristics.

Table 18: IPCA performance of equity options in the Consumer Discretionary sector.

	Individual returns (r_t)				Managed portfolios (x_t)			
	Total R^2		Predictive R^2		Total R^2		Predictive R^2	
K	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$
Panel A: In-sample								
1	34.3	35.5	0.87	2.14	61.1	62.4	1.28	2.56
2	39.0	40.1	0.88	2.09	68.2	69.1	1.30	2.36
3	42.5	43.5	1.30	2.03	71.9	73.4	0.88	2.23
4	45.8	46.6	1.28	2.05	77.5	78.3	1.02	2.27
Panel B: Out-of-sample								
1	10.9	10.7	0.05	< 0	26.0	26.2	< 0	< 0
2	12.6	12.3	0.11	< 0	32.0	32.2	< 0	< 0
3	14.3	14.5	0.12	< 0	33.5	38.9	< 0	< 0
4	17.2	15.8	0.39	< 0	43.5	42.1	0.90	0.11

Note. This table displays the performance of the IPCA model with K factors using the equity data exclusive to the Consumer Discretionary sector. The data contains the in-sample period July 2006 to October 2014 and the out-of-sample period November 2014 to May 2018. The total and predictive R^2 are reported in percentages for the restricted and unrestricted IPCA model with 70 lagged characteristics.

Table 19: IPCA performance of equity options in the Industrials sector.

	Individual returns (r_t)				Managed portfolios (x_t)			
	Total R^2		Predictive R^2		Total R^2		Predictive R^2	
K	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$
Panel A: In-sample								
1	34.7	36.5	0.20	2.11	62.2	64.3	0.55	3.33
2	44.4	45.6	< 0	< 0	69.2	70.6	< 0	< 0
3	48.1	49.4	< 0	< 0	74.2	74.5	< 0	< 0
4	51.5	52.7	< 0	0.09	78.1	78.9	< 0	0.33
Panel B: Out-of-sample								
1	20.8	19.7	0.82	< 0	41.8	39.5	1.27	< 0
2	22.4	21.7	< 0	< 0	43.9	41.9	< 0	< 0
3	24.7	23.4	< 0	< 0	48.1	45.9	< 0	< 0
4	26.2	26.7	< 0	< 0	50.8	51.7	< 0	< 0

Note. This table displays the performance of the IPCA model with K factors using the equity data exclusive to the Industrials sector. The data contains the in-sample period July 2006 to October 2014 and the out-of-sample period November 2014 to May 2018. The total and predictive R^2 are reported in percentages for the restricted and unrestricted IPCA model with 70 lagged characteristics.

Table 20: IPCA performance of equity options in the Energy & Utilities sector.

	Individual returns (r_t)				Managed portfolios (x_t)			
	Total R^2		Predictive R^2		Total R^2		Predictive R^2	
K	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$
Panel A: In-sample								
1	32.9	34.7	0.21	1.97	62.5	64.9	0.42	2.40
2	43.3	44.9	< 0	1.30	74.4	76.2	< 0	0.43
3	48.9	50.1	< 0	< 0	77.3	78.7	< 0	< 0
4	52.6	53.7	< 0	< 0	81.3	82.3	< 0	< 0
Panel B: Out-of-sample								
1	26.3	26.0	< 0	< 0	60.1	59.3	< 0	< 0
2	29.7	29.3	< 0	< 0	72.3	71.3	< 0	< 0
3	32.1	31.7	< 0	< 0	72.4	72.1	< 0	< 0
4	33.4	33.3	< 0	< 0	73.8	73.6	< 0	< 0

Note. This table displays the performance of the IPCA model with K factors using the equity data exclusive to the Energy & Utilities sector. The data contains the in-sample period July 2006 to October 2014 and the out-of-sample period November 2014 to May 2018. The total and predictive R^2 are reported in percentages for the restricted and unrestricted IPCA model with 70 lagged characteristics.

Table 21: IPCA performance of equity options in the Communication Services sector.

	Individual returns (r_t)				Managed portfolios (x_t)			
	Total R^2		Predictive R^2		Total R^2		Predictive R^2	
K	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$
Panel A: In-sample								
1	28.5	30.5	0.27	2.14	53.9	57.0	0.04	2.95
2	35.7	37.4	0.00	1.55	63.8	65.7	0.04	1.94
3	42.0	43.6	< 0	1.18	69.5	71.7	< 0	1.73
4	46.9	48.6	0.33	0.18	76.0	77.8	< 0	0.48
Panel B: Out-of-sample								
1	12.3	12.8	0.28	0.83	28.3	30.4	0.66	2.96
2	14.6	14.9	< 0	0.27	35.1	38.1	< 0	2.34
3	17.3	17.5	< 0	< 0	44.7	45.3	< 0	1.09
4	18.6	17.7	< 0	< 0	49.3	48.2	< 0	< 0

Note. This table displays the performance of the IPCA model with K factors using the equity data exclusive to the Communication Services sector. The data contains the in-sample period July 2006 to October 2014 and the out-of-sample period November 2014 to May 2018. The total and predictive R^2 are reported in percentages for the restricted and unrestricted IPCA model with 70 lagged characteristics.

Table 22: IPCA performance of equity options in the Consumer Staples sector.

	Individual returns (r_t)				Managed portfolios (x_t)			
	Total R^2		Predictive R^2		Total R^2		Predictive R^2	
K	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$
Panel A: In-sample								
1	21.1	22.5	0.62	1.94	44.3	46.8	1.23	3.95
2	27.4	28.8	0.60	1.80	51.0	52.9	1.22	3.69
3	33.3	34.3	0.88	1.79	61.6	62.5	2.17	3.29
4	38.8	40.1	0.61	< 0	65.0	65.7	1.95	< 0
Panel B: Out-of-sample								
1	6.80	6.26	< 0	< 0	17.6	16.8	< 0	< 0
2	8.56	8.46	< 0	< 0	21.6	19.6	< 0	< 0
3	10.7	10.2	< 0	< 0	25.4	24.5	< 0	< 0
4	13.0	11.9	< 0	< 0	30.2	25.9	0.31	< 0

Note. This table displays the performance of the IPCA model with K factors using the equity data exclusive to the Consumer Staples sector. The data contains the in-sample period July 2006 to October 2014 and the out-of-sample period November 2014 to May 2018. The total and predictive R^2 are reported in percentages for the restricted and unrestricted IPCA model with 70 lagged characteristics.

Table 23: IPCA performance of equity options in the Materials sector.

	Individual returns (r_t)				Managed portfolios (x_t)			
	Total R^2		Predictive R^2		Total R^2		Predictive R^2	
K	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$
Panel A: In-sample								
1	31.4	33.0	0.03	1.88	51.2	53.1	< 0	2.77
2	40.0	41.4	0.22	1.67	77.2	78.3	0.50	1.26
3	44.4	45.7	0.64	1.45	82.0	83.0	0.22	0.41
4	48.0	49.6	0.64	1.44	85.0	85.9	0.23	0.72
Panel B: Out-of-sample								
1	14.6	14.3	0.15	< 0	41.5	39.3	< 0	< 0
2	20.8	20.7	0.15	< 0	63.4	64.4	< 0	< 0
3	21.6	22.3	0.23	< 0	66.3	67.4	< 0	< 0
4	23.5	24.1	< 0	< 0	68.7	70.8	< 0	< 0

Note. This table displays the performance of the IPCA model with K factors using the equity data exclusive to the Materials sector. The data contains the in-sample period July 2006 to October 2014 and the out-of-sample period November 2014 to May 2018. The total and predictive R^2 are reported in percentages for the restricted and unrestricted IPCA model with 70 lagged characteristics.

Table 24: IPCA performance of equity options in the Real Estate sector.

	Individual returns (r_t)				Managed portfolios (x_t)			
	Total R^2		Predictive R^2		Total R^2		Predictive R^2	
K	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$	$\Gamma_\alpha = \mathbf{0}$	$\Gamma_\alpha \neq \mathbf{0}$
Panel A: In-sample								
1	44.3	46.6	0.39	2.53	64.3	65.7	0.29	0.90
2	53.4	55.4	0.53	2.29	71.9	73.2	0.38	0.80
3	60.1	61.7	< 0	1.51	80.9	81.7	< 0	0.00
4	66.3	67.3	< 0	< 0	84.3	84.7	< 0	< 0
Panel B: Out-of-sample								
1	17.2	14.0	< 0	< 0	31.5	30.0	< 0	< 0
2	22.3	19.2	< 0	< 0	39.7	37.5	< 0	< 0
3	25.4	22.5	< 0	< 0	43.6	40.7	0.07	< 0
4	26.5	25.0	< 0	< 0	50.9	48.9	< 0	< 0

Note. This table displays the performance of the IPCA model with K factors using the equity data exclusive to the Real Estate sector. The data contains the in-sample period July 2006 to October 2014 and the out-of-sample period November 2014 to May 2018. The total and predictive R^2 are reported in percentages for the restricted and unrestricted IPCA model with 70 lagged characteristics.

H Regularization Out-of-sample Fit

Table 25: Lasso IPCA out-of-sample fit of equity options.

	Individual returns (r_t)		Managed portfolios (x_t)	
K	Total R^2	Predictive R^2	Total R^2	Predictive R^2
1	8.82	0.25	43.5	0.92
2	10.5	0.21	61.3	0.56
3	11.4	0.15	68.7	0.64
4	12.1	0.18	69.9	0.47

Note. This table displays the performance of restricted IPCA with K factors using Lasso regularization where λ is set at 10^{-5}. We implement the equity data with 70 lagged characteristics for the out-of-sample period November 2014 to May 2018. The total and predictive R^2 are reported in percentages.

Table 26: Ridge IPCA out-of-sample fit of equity options.

	Individual returns (r_t)		Managed portfolios (x_t)	
K	Total R^2	Predictive R^2	Total R^2	Predictive R^2
1	8.72	0.21	42.7	0.87
2	10.2	0.19	63.8	0.80
3	11.1	0.22	71.3	0.95
4	11.8	0.22	77.1	1.00

Note. This table displays the performance of restricted IPCA with K factors using ridge regularization where λ is set at 10^{-3}. We implement the equity data with 70 lagged characteristics for the out-of-sample period November 2014 to May 2018. The total and predictive R^2 are reported in percentages.

I Sharpe Ratios

Table 27: Out-of-sample Sharpe ratios for unrestricted IPCA.

	K			
	1	2	3	4
Panel A: Equity options				
Univariate	1.00	1.29	0.76	1.32
Tangency	1.00	1.69	2.33	1.89
Panel C: ETF options				
Univariate	1.49	0.44	1.02	0.89
Tangency	1.49	1.37	1.70	1.76
Panel D: Technology sector				
Univariate	1.10	1.19	0.64	1.70
Tangency	1.10	0.61	2.01	1.30

Note. This table displays annualized Sharpe ratios of the IPCA model with K factors using the equity and ETF data for the out-of-sample period November 2014 to May 2018. The rows represent individual factors ("univariate") and mean-variance efficient portfolio of factors in each model ("tangency") that are based on unrestricted IPCA.

Table 28: Elastic net and Lasso out-of-sample Sharpe ratios.

	K			
	1	2	3	4
Panel A: Lasso				
Univariate	0.63	0.61	0.67	0.63
Tangency	0.63	0.22	0.41	0.90
Panel B: Elastic net				
Univariate	0.59	0.22	1.04	1.40
Tangency	0.59	0.20	0.85	1.45

Note. This table displays annualized Sharpe ratios of restricted IPCA with K factors using Lasso and elastic net regularization. For the former we set λ at 10^{-5} and for the latter we set ρ equal to 0.03 and λ is set at 10^{-3}.
We implement the equity data for the out-of-sample period November 2014 to May 2018. The rows represent individual factors ("univariate") and mean-variance efficient portfolio of factors in each model ("tangency") that are based on the unrestricted IPCA specification.

Table 29: Out-of-sample Sharpe ratios for the equity sectors.

	K			
	1	2	3	4
Panel A: Financials				
Univariate	0.50	0.24	0.38	0.42
Tangency	0.50	0.10	0.22	0.51
Panel B: Health Care				
Univariate	1.29	1.09	1.07	0.06
Tangency	1.29	1.11	1.05	1.12
Panel C: Consumer Discretionary				
Univariate	0.62	0.82	0.23	0.86
Tangency	0.62	0.79	1.63	2.15
Panel D: Industrials				
Univariate	0.72	0.20	1.13	0.44
Tangency	0.72	0.24	0.27	0.53
Panel E: Communication Services				
Univariate	0.71	0.24	0.33	0.60
Tangency	0.71	0.13	0.48	0.65
Panel F: Consumer Staples				
Univariate	0.26	0.32	0.36	0.35
Tangency	0.26	0.26	0.24	0.42
Panel G: Energy & Utilities				
Univariate	0.14	0.55	0.53	0.30
Tangency	0.14	0.03	0.07	0.14
Panel H: Materials				
Univariate	0.20	1.00	0.37	0.51
Tangency	0.20	0.15	0.81	0.96
Panel I: Real Estate				
Univariate	0.15	0.43	0.30	0.12
Tangency	0.15	0.62	0.39	0.01

Note. This table displays annualized Sharpe ratios of the IPCA model with K factors using the equity sectors data for the out-of-sample period November 2014 to May 2018. The rows represent individual factors ("univariate") and mean-variance efficient portfolio of factors in each model ("tangency") that are all based on the restricted IPCA specification.

J Robustness

J.1 Equity options

Table 30: IPCA robustness performance of equity options for 80 months.

		K			
		1	2	3	4
Panel A: Individual returns (r_t)					
Total R^2	$\Gamma_\alpha = \mathbf{0}$	21.1	24.1	25.9	27.4
	$\Gamma_\alpha \neq \mathbf{0}$	22.0	25.0	26.7	27.7
	PCA	28.1	45.9	50.0	52.4
Predictive R^2	$\Gamma_\alpha = \mathbf{0}$	0.18	0.17	0.52	0.71
	$\Gamma_\alpha \neq \mathbf{0}$	0.99	0.95	0.92	0.88
	PCA	< 0	< 0	< 0	< 0
Panel B: Managed portfolios (x_t)					
Total R^2	$\Gamma_\alpha = \mathbf{0}$	74.6	80.4	86.0	89.4
	$\Gamma_\alpha \neq \mathbf{0}$	76.3	81.7	86.7	89.8
	PCA	65.0	65.3	74.9	81.9
Predictive R^2	$\Gamma_\alpha = \mathbf{0}$	0.54	0.44	1.81	1.48
	$\Gamma_\alpha \neq \mathbf{0}$	2.23	1.97	1.78	1.58
	PCA	1.99	1.99	1.99	1.99
Panel C: Testing the anomaly ($H_0 : \Gamma_\alpha = \mathbf{0}$)					
W_α p-value		0.171	0.322	0.445	0.953

Note. This table displays the performance of the IPCA model with K factors using the equity data for the sample period July 2006 to February 2013. Panel A and B report the total and predictive R^2 in percentages for the restricted and unrestricted IPCA model with 70 lagged characteristics. Panel C presents the bootstrapped p-values for the anomaly test as described in Section 3.4.1.

J.2 ETF options

Table 31: IPCA robustness performance of ETF options for 80 months.

		K			
		1	2	3	4
Panel A: Individual returns (r_t)					
Total R^2	$\Gamma_\alpha = \mathbf{0}$	23.7	33.1	39.7	43.3
	$\Gamma_\alpha \neq \mathbf{0}$	24.7	34.1	40.7	44.2
	PCA	30.3	56.0	70.3	80.1
Predictive R^2	$\Gamma_\alpha = \mathbf{0}$	2.72	2.62	2.66	2.70
	$\Gamma_\alpha \neq \mathbf{0}$	3.80	3.71	3.54	3.57
	PCA	< 0	< 0	< 0	< 0
Panel B: Managed portfolios (x_t)					
Total R^2	$\Gamma_\alpha = \mathbf{0}$	49.8	79.1	82.4	88.5
	$\Gamma_\alpha \neq \mathbf{0}$	52.2	80.7	84.1	89.7
	PCA	52.4	74.0	74.2	83.8
Predictive R^2	$\Gamma_\alpha = \mathbf{0}$	4.73	4.81	4.94	5.31
	$\Gamma_\alpha \neq \mathbf{0}$	7.53	6.98	6.58	6.85
	PCA	3.42	3.42	3.42	3.42
Panel C: Testing the anomaly ($H_0 : \Gamma_\alpha = \mathbf{0}$)					
W_α p-value		0.382	0.036	0.188	0.008

Note. This table displays the performance of the IPCA model with K factors using the ETF data for the sample period July 2006 to February 2013. Panel A and B report the total and predictive R^2 in percentages for the restricted and unrestricted IPCA model with 70 lagged characteristics. Panel C presents the bootstrapped p-values for the anomaly test as described in Section 3.4.1.