Erasmus University Rotterdam

Explaining delta-hedged equity option returns using conditional latent factor models

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Abstract

This thesis applies two conditional latent factor methods to a delta-hedged equity option returns panel: Instrumented principal component analysis (IPCA) and projected Principal component analysis (PPCA). IPCA estimates latent factors and a linear mapping from additional characteristics to time-varying factor loadings. PPCA estimates latent factors with static factor loadings but allows for nonlinear relations of characteristics and factor loadings and for part of the factor loadings to be unexplained by the characteristics. The data consist of options corresponding to 318 companies from the S&P500 and a large set of 69 characteristics of the options, underlying stocks and affiliated companies. PPCA is superior in explaining realized returns using the dynamic factors. IPCA outperforms PPCA in describing the variance of returns by conditional expected returns due to the time-varying loadings. The linear mapping from characteristics to factors of IPCA enables straightforward economic interpretation of the factors, even if all 69 characteristics are included. For PPCA, it is infeasible to assess the relation of all 69 characteristics with the factors. This thesis extends the IPCA testing procedure for instrument significance of Kelly et al. (2019) to PPCA. This test enables selecting the significant characteristics out of a large set and creating interpretable models. This testing procedure is not exclusively for option returns but could be applied to any PPCA model.

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1 Introduction

The first option-like contracts were used in ancient Greek history on harvests of agricultural products. The first regulated 'modern' options on stocks were created in 1973 with the establishment of the Chicago Board of Options exchange. Since 2019 the notional value of the US options market has been higher than the stock market. This evolution resembles the importance of research on option returns. Most asset pricing research focuses on stock returns. Therefore, this thesis focuses on the explanation of returns on equity options.

The most common approach to modelling option prices is to use no-arbitrage pricing models. An advantage of these models is the rigorous mathematical formulation. However, these models are highly parametric and require knowledge of the underlying distribution of option returns. These distributions are not always known. Therefore the models are likely to be incorrect. This often results in a failure to explain the variance in option returns. This thesis uses a factor model approach, which is common in the asset pricing literature on stocks. Garleanu et al. (2008) find that option returns are affected by time-varying pressures of demand, which less-parametric models could better capture.

The theoretical motivation for using factor models is identical to the motivation for stocks or other assets. In essence, options are just traded contracts with risky returns. The use of factor models for option returns depends on the Euler equation for asset returns and the no-arbitrage assumption. These two combined give rise to the existence of a stochastic discount factor m_{t+1}, which is related to returns such that $E[m_{t+1}r_{i,t+1}] = 0$[Ross (1978)]. This restriction implies the following structure for asset returns

$$E[r_{t+1}] = -\frac{Cov_t(m_{t+1}, r_{i,t+1})}{Var_t(m_{t+1})}\frac{Var_t(m_{t+1})}{E[m_{t+1}]} = \beta_{i,t}\lambda_t \tag{1}$$

The loadings $\beta_{i,t}$ represent the exposure of options to common risk factors. λ_t could be explained as the price of the risk factors. The additional assumption that the factors have a linear relation to m_{t+1}, implies the existence of the following factor representation,

$$r_{i,t+1} = \alpha_{i,t} + \beta_{i,t}f_{t+1} + e_{i,t+1} \tag{2}$$

where the error term $e_{i,t+1}$ is assumed to have mean zero and to be orthogonal to the factors f_{t+1}. In addition, the assumption $E_t[f_{t+1}] = \lambda_t$ is imposed for the expected factor mean using all information up to time t.

An essential benefit of factor models is that knowledge of the distribution of returns is not required. The most common method in asset pricing to create factor models is to use pre-specified factors, such as portfolios sorted on a characteristic. The factor loadings of the assets can be estimated by regressions. This approach is typical to model stock returns. A disadvantage is that it requires determining the relevant characteristics beforehand to create the factors. Finding ad-hoc factors is even more difficult for option returns since the amount of research on option factors is more limited than for stock returns.

This study adopts latent factor methods to estimate the common risk factors of equity option returns. Principal component analysis (PCA) is the most well-known among the latent factor methods. PCA finds purely statistical factors taking only the returns panel itself as input. PCA does not require an ad-hoc idea about the factors. However, it does result in factors that are difficult to interpret. Making latent factor models depend on additional characteristics of the options, underlying stocks, and companies could solve this difficulty. In this way, latent factors are linked to characteristics and could be interpreted economically. Furthermore, it is possible to include a large set of characteristics +foregoing the need to have ad-hoc knowledge about the relevant characteristics. This thesis applies two conditional latent factor models, which employ additional characteristics.

The first is Instrumented Principal Components Analysis (IPCA), introduced by Kelly et al. (2020). IPCA allows for time-varying factor loadings, while the number of parameters in the model does not explode. An IPCA model does not have a parameter for each factor loading. The factor loadings are entirely defined by a matrix that maps the characteristics onto the factor loadings. This matrix is static across time and the cross-section of the panel. This makes the number of estimated parameters in the model in the context of this thesis substantially smaller than for the static PCA. The resulting factor loadings are dynamic due to the time-varying characteristics. The time-varying loadings make the model more flexible for the time-varying nature of options. This is a key advantage over other latent factor methods. The linear mapping makes the interpretation of the model much more accessible. On the other hand, it limits the model's flexibility in estimating the factor loadings. IPCA allows for the use of a large set of characteristics. In this way, IPCA can statistically determine the relevant characteristics for explaining returns. Kelly et al. (2019) found that correlated or noisy characteristics are not harming the explanatory performance of IPCA. In contrast to most latent factor models, IPCA allows for direct interaction of the characteristics with returns if common risk factors cannot explain part of the variation. This direct interaction could help investors find anomalies in option returns related to the characteristics.

In addition to IPCA, the semi-parametric method projected principal component analysis (PPCA) of Fan et al. (2016) is applied to the panel of option returns. This method also uses additional covariates to estimate the factors and factor loadings. The factor loadings of PPCA are static and are estimated individually for each cross-sectional element. The static factor loadings are more restrictive than the dynamic loadings in IPCA. Also, these are a bit less representative for the general factor model in equation (2). Next to the factor loadings, the included characteristics in PPCA need to be static over time. In contrast to IPCA, the estimation procedure of PPCA does not allow for an intercept factor to relate alphas to the characteristics. However, PPCA allows part of the loadings to be unrelated to the characteristics, implying the need to estimate that part for each individual. This results in a much higher number of parameters than for IPCA. On the other hand, it allows the model to be more flexible when relevant characteristics are missing. The loading functions from characteristics to the factors are non-parametrically estimated and mostly nonlinear. From that perspective, PPCA is more flexible than IPCA. This thesis proposes a new testing procedure to identify characteristics significantly related to the systematic risk factors estimated by PPCA. Using this procedure, PPCA could be used to find the most relevant across a large set of characteristics.

PPCA and IPCA are both empirically applied to a panel of monthly option returns. The returns on an option largely depend on the movements of the underlying stock. This study uses delta-hedged option returns to exclude this effect as much as possible. The panel consists of options on 318 companies present in the S&P500 index throughout the whole sample period from January 2010 to December 2020. All options are in their last month before maturity and close to the money. In addition, 69 characteristics related to the options, the underlying stocks and the related companies are constructed. These have to do with the company's value, profitability, trading frictions and more. The models are assessed on two central performance measures. The first is total R^2, the fraction of variance in realised returns described by the factor loadings and time-varying factor realisations. This measure could be interpreted as the model's ability to describe risk. The second measure is predictive R^2, which measures the capability of the model to explain returns in terms of conditional expected returns. It could be interpreted as the description of risk compensation.

In short, the most critical differences between IPCA and PPCA are as follows. IPCA allows for time-varying characteristics, while PPCA can only handle static characteristics. Also, the factor loadings of IPCA are dynamic as opposed to static loadings in PPCA. PPCA estimates loadings for each characteristic separately and allows for part of the loadings to be unexplained by the characteristics. IPCA estimates a mapping from characteristics to factor loadings fixed over time and the cross-section, not allowing for a part orthogonal to the characteristics. Nevertheless, it allows for direct interaction of the characteristics with the factors, which is impossible in PPCA. An advantage of PPCA is the estimation by a closed-form formula, while IPCA needs an ALS algorithm to estimate the model. The factor loadings of IPCA are linear in the characteristics, while PPCA allows for a nonlinear mapping. This thesis assesses what these differences imply in terms of explanatory performance, interpretability of the models, and similarity of the time series of factor realisations.

The results of IPCA show a difference between the restricted and unrestricted models for small numbers of factors. For models with five factors and more, this difference decreases. This observation implies that the variation associated with the characteristics is captured in the risk factors for the larger models. The five-factor IPCA model scores best on the predictive R^2 measure. This finding is in line with the total R^2 of the out-of-sample analysis of the 5-factor models, suggesting that five factors suffice for this data. Also, the out-of-sample analysis shows that the good fit of IPCA is not caused by overfitting the model to the data. A testing procedure indicates that 19 out of the 69 characteristics are significantly related to the common risk factors for at least one of the models ranging from 1 to 6 factors. The most significant characteristics include the Vega of the option, the bid-ask spread of the option's underlying stock, last month's turnover and the standard deviation of turnover of the underlying stock and the free cash flow of the related company.

A specification test for PPCA indicates that the characteristics are relevant for the factor loadings. This result motivates the use of PPCA for delta-hedged option returns using these characteristics. Moreover, a test shows highly significant that the characteristics do not fully explain the factor loadings, such that PPCA attributes part of the loadings to another component.

PPCA is superior to IPCA in describing in-sample variation in realised returns by dynamic factors. On the other hand, IPCA outperforms PPCA in predictive R^2, meaning it better describes risk compensation. The IPCA models are less complicated to interpret economically due to the linear mapping from characteristics to factors. This mapping makes interpretation straightforward even when all 69 characteristics are included. For PPCA, interpreting the nonlinear relations of all characteristics with the factors is infeasible. Therefore, this thesis proposes a testing procedure to find which characteristics significantly contribute to the factor loadings. This test allows for re-estimation of the models with only the significant characteristics. This more parsimonious model is easier to give economic meaning. Nonetheless, the number of parameters is still higher than for IPCA. In-sample tangency portfolios of IPCA have higher Sharpe ratios than PPCA, implying that IPCA factors are more mean-variance efficient unconditionally. For PPCA, the test shows that 9 out of the 69 characteristics are significant in at least one of the models. Since PPCA can only handle static characteristics, the mean of the characteristics over time for each underlying stock is used. The most significant characteristics include the operating leverage and the average maximum monthly return of the affiliated companies, and the underlying stock's volatility. The characteristics that significantly contribute to the PPCA models differ significantly from the IPCA models.

The remainder of this thesis is organised as follows. Section 2 summarizes related literature and places this study into perspective. Section 3 carefully explains the PPCA and IPCA models and the estimation procedures. Section 4 defines the ways to assess the performance of both models. Section 6 presents and analyses all results . Followed by a conclusion is section 7.

2 Literature review

The main goal of this thesis is to model the factor structure in the cross-section of equity options. There are different methods available to apply to option returns. In this thesis, two conditional latent factor models that include additional characteristics are applied and compared to each other and to traditional PCA.

The traditional and most common methods to explain option prices rely on no-arbitrage pricing. This field of research builds mainly on the well-known paper of Black and Scholes (1973). They determine a closed-form pricing formula for standard call and put options based on no-arbitrage enforcement and a set of additional assumptions. Many studies extend the Black-Scholes model. For instance, by including stochastic volatility [Heston (1993)] or developing models for more exotic options. Carr and Wu (2004) recognize that the Black-Scholes model is often too simplistic. They extend it to include stochastic volatility and correlation in the volatility of different options. These types of methods benefit from rigorous mathematical formulas and enforcing no-arbitrage. A drawback of no-arbitrage models is that they are heavily parametric and require knowledge of the entire distribution of the underlying assets beforehand. These distributions are usually unknown in practice, and the large number of parameters makes these models prone to misspecification.

Factor models forego this need to specify the underlying distribution of returns. For risk managers, it is crucial to understand the exposure of their portfolios to common risk factors. The most common approach to factor models is to use pre-specified factors established by portfolios sorted on a characteristic. The loadings of options on the factors are estimated mainly by regressions. Cao et al. (2021) create factors for delta-hedged option returns by sorted portfolios on a set of characteristics such as profit margin and cash flow variance. Karakaya (2014) develops a three-factor model for the cross-section of delta-hedged and leverage-adjusted option portfolio returns consisting of a level slope and value factor for the delta-hedged returns. Goyal and Saretto (2009) find that a sorted portfolio factor based on the difference in historical and implied volatility is related to equity option returns. Pre-specified factor models have a clear economic interpretation. However, researchers need to determine the relevant characteristics for the factors beforehand, and it is not feasible to test a large set of characteristics.

Another approach is to estimate latent factors using a purely statistical approach. The most common method is principal components analysis (PCA), which estimates latent factors and factor loadings using only the returns data itself. The latent PCA factors are difficult to interpret. Therefore Horenstein et al. (2020) relate the latent factors to a set of observed factors. They find a 4-factor model that explains a large part of the variance in portfolios of delta-hedged options. Other papers study some different types of combinations of latent and pre-specified factors. Jones (2006) use non-latent factor models to show that priced observed factors are not always sufficient to explain option returns. Christoffersen et al. (2018a) develop a stochastic volatility model for equity option prices, based on the first Principal Components of the option volatility levels skew and term structure. Brooks et al. (2018) use LASSO to find relevant characteristics for explaining delta-hedged equity option returns.

This thesis uses two conditional latent factor methods, which allow considering a large set of potentially relevant characteristics for option returns. The first method is instrumented principal component analysis, proposed by Kelly et al. (2020). Kelly et al. (2019) apply IPCA to stock returns. They find a superior performance over other factor models. In addition, they discover a set of 10 out of 36 characteristics that are significantly related to stock returns. Büchner and Kelly (2022) apply IPCA to index option returns. They find that a 3-factor IPCA model explains over 85% of a panel of monthly index returns.

The other applied method is the semi-parametric Projected PCA, introduced by Fan et al. (2016). This paper argues that PPCA is especially useful in a background with a large cross-section and smaller number of time periods. This is precisely the setting of the data in this thesis. They also encounter that characteristics have significant explanatory power over factor loadings.

This thesis extends the existing literature in the following ways. Firstly, in addition to the application of Büchner and Kelly (2022) on index options returns, this thesis applies IPCA to model the more challenging equity option returns. This study is the first to apply PPCA to a panel of equity options. Also, this thesis introduces a bootstrap testing procedure for the significance of characteristics used in PPCA, based on the IPCA testing approach of Kelly et al. (2019). This test is not specifically for option pricing but could be used for any PPCA model. Lastly, this thesis is the first to compare the two latent factor methods on explanatory performance and interpretability.

3 Methodology

This study applies two conditional factor models to a panel of delta-hedged option returns, Instrumented PCA and Projected PCA. This section introduces the models themselves and highlights their main features. Furthermore, the estimation of both models is carefully described. In addition, some testing procedures are explained for the models. For comparison and to provide background on latent factor models, traditional PCA is described shortly in the first section.

3.1 Principal Component Analysis (PCA)

Principal Component Analysis is the most well-known latent factor method. It is mainly used for dimension reduction. Pearson (1901) firstly introduced this technique. The central idea is to describe most of the variation in a dataset by a few common factors, which are less than the number of variables in the dataset. The common factors are called Principal Components (PC) and are unobserved. The Principal Components are linear functions of the data's original (observed) variables. For panel data on option returns, PCA could find common time-varying factors explaining the different options' returns. The options are mapped to the factors by their factor loadings, which are estimated for options on each corresponding company and static over time. The Principal Components must be orthogonal to each other. The latent factors are ordered such that the first factor explains the largest portion of the total variance in the panel. The aim is to find a number K Principal Components that is much smaller than the number of cross-sectional units p and still describes a large part of the variance of the whole dataset. In the context of option data, this could be explained as finding common latent risk factors. An eigenvalue-eigenvector decomposition of the correlation or covariance matrix of the panel can be used to estimate the common factors and the factor loadings. Note that the only input of PCA is the panel of option returns itself.

Let $\mathbf{R}$ denote the T x N matrix of panel data. The elements $r_{i,t}$ of $\mathbf{R}$ correspond to the value for the option on company i at time t. The matrix could also be written as $\mathbf{R} = \{\mathbf{r_1}, ..., \mathbf{r_p}\}$, where $\mathbf{r_p}$ is a t-dimensional vector. PCA finds linear combinations of the original variables $\boldsymbol{f}_j = \sum_{i=1}^{p} \phi_{ij}\mathbf{x_i}$. Here $\boldsymbol{f}_j$ is the j-th PC and ϕ_{ij} is the factor loading of the i-th individual on the j-th Principal Component. The Principal Components have the maximum variance possible and must be orthogonal. If variables $\mathbf{r_i}$ are standardized before estimation to have mean zero, the variance of each Principal Component is as follows:

$$Var(\boldsymbol{f}_j) = \frac{1}{N}\phi_{\mathbf{j}}{}'\mathbf{R}'\mathbf{R}\phi_{\mathbf{j}} \tag{3}$$

The first PC maximizes the variance $Var(\boldsymbol{f}_j)$ subject to the normalization constraint $\phi_1'\phi_1 = 1$. The k-th Principal Component should also maximizes the variance subject to $\phi_k'\phi_k = 1$ (for PC $k = 2, ..., p$). Furthermore, it should satisfy $\phi_k'\phi_j = 0$ for $j = 1, ..., k-1$ to assure that the PCs are uncorrelated. This is repeated for K PC's.

For brevity, the PCA method is only summarized shortly in this section. It is not the main method employed in this thesis.

3.2 Instrumented Principal Component Analysis (IPCA)

Instrumented Principal Component analysis is an extension of PCA that was firstly introduced by Kelly et al. (2020). IPCA estimates common latent factors of two-dimensional panel data. Additional data in the form of characteristics is included to be used as instruments for the latent factor loadings. The predicted returns of the model are conditional on the characteristics. The characteristics are linked to each cross-sectional observation and can be time-varying. IPCA exploits the time-varying characteristics to allow for dynamic factor loadings. Since options contracts are changing over time and the characteristics of the underlying stock are also time-varying, it could be beneficial to model option returns using time-varying factor loadings. The factor loadings do not depend on an option contract's static 'identity'. They are fully dependent on the time-varying characteristics of the option and the corresponding company. Latent factors do not have an ad-hoc economic meaning, making interpretation more difficult. Including additional characteristics improves the economic interpretability of the latent factor model. An advantage over pre-specified factor models is that IPCA does not require a complete understanding of the factor structure before estimating the model. The factor-loadings automatically move along the changing characteristics of the options. IPCA smartly handles time-varying factor loadings such that the number of parameters does not explode. The number of estimated parameters related to the factor loadings does not depend on the size of the cross-section or the number of time-series observations. The idea is to make the unique identity of options corresponding to a stock irrelevant and let the factor loadings only depend on a set of characteristics of each individual. This approach results in a more parsimonious model and could prevent overfitting.

The general IPCA model with K latent factors can be written as follows:

$$r_{i,t+1} = \boldsymbol{\beta}_{it} f_{t+1} + \mu_{i,t+1} \tag{4}$$

$$\beta_{it} = \mathbf{c}'_{it}\Gamma + \eta_{it} \tag{5}$$

where that $i = 1,..,N$ represents the index of the cross-section and $t = 1,...,T$ is the index of the time periods. Equation (4) represents a standard factor model that defines returns in terms of common factors and time-varying loadings β_{it}, which is a K x 1 vector. The factors are represented by the K x 1 vector f_{t+1} (for each t) which is variable over time, but constant over the cross-section. The scalar idiosyncratic error is given by μ_{it}.

Equation (5) defines the dynamics of the factor loadings $\boldsymbol{\beta}_{it}$, this is specific for IPCA. This equation maps the characteristics to the factor loadings. Following equation (5), IPCA does not estimate the factor loadings themselves. All factor loadings are determined by the characteristics of the option and the L x K mapping matrix Γ. The number of estimates in Γ only depends on the number of factors K and the number of characteristics L. The number of parameters in Γ does not increase with the panel dimensions T and N. This is especially suitable for high dimensional datasets since the number of parameters to be estimated is stable across time and individuals. The L instrumental variables for option i are contained in the L x 1 vector $\boldsymbol{c}_{it}$. In the approach used in this study, the returns are related to the characteristics of the previous period. This enables to predict returns one period ahead, which can be beneficial for investors. The number of parameters estimated in the factors is K x T. In contrast, regular PCA has to estimate factor loadings for each individual, which boils down to N x K factor loadings. It can become computationally hard if the cross-section is large. Furthermore, IPCA is more flexible due to the time-varying factor loadings, whereas PCA only allows for static factor loadings.

IPCA does not require identifying a few characteristics and creating risk factors based on those as pre-specified factor models. IPCA allows to include a large set of characteristics and maps their relation to latent factors. It could be described as the best of both worlds of pre-specified and latent factor models. It does not require factors to be determined beforehand, and it shows how characteristics are related to common risk factors of option returns. As described further in this section, IPCA also allows for an intercept factor relating characteristics directly to the returns. If these values are significant, they could be interpreted as 'anomaly' alphas related to the characteristics.

Including a large set of characteristics should not result in any problems. Kelly et al. (2020) found that IPCA finds characteristics that are most related to the risk factors. Correlated or noisy characteristics are not problematic, as these are averaged out presumably.

In short, the four main reasons to apply IPCA to equity option returns are the enhanced economic interpretability, the parsimony of the model, the time-varying factor models and the absence of the necessity to have ad-hoc knowledge on the common risk factors.

3.2.1 Restricted and unrestricted models

In essence, an option pricing model is an asset pricing model. Kelly et al. (2019) apply IPCA to stock returns. In asset pricing factor models, it is common to include an intercept alpha that is not dependent on the risk factors. This allows for direct interaction of the characteristics with returns; if this cannot be attributed to common risk factors. The resulting unrestricted model is defined as follows:

$$r_{i,t+1} = \alpha_{i,t} + \beta_{i,t} f_{t+1} + \epsilon_{i,t+1} \tag{6}$$

$$\alpha_{i,t} = c'_{i,t}\Gamma_\alpha + \mu_{\alpha,i,t} \tag{7}$$

$$\beta_{i,t} = c'_{i,t}\Gamma_\beta + \mu_{\beta,i,t} \tag{8}$$

This boils down to restricting the first factor to be a vector of ones. The matrix Γ of equation (5) is split into Γ_α mapping the characteristics directly to returns and Γ_β to determine the mapping from the characteristics to common risk factors. A testing procedure is applied to determine whether the values in Γ_α are significant. This procedure is described in section 3.2.4. The null hypothesis corresponds to a restricted model, whereas the alternative hypothesis implies an unrestricted model. The restricted model does not include an intercept factor. Both restricted and unrestricted IPCA models are estimated and analyzed in this study. It has to be noted that $\Gamma_\alpha = \mathbf{0}$ does not rule out the existence of alphas entirely. In theory, alphas could still exist through the $\mu_{\alpha,i,t}$. These alphas can be viewed as mispricing unrelated to the characteristics. For the restricted model ($\Gamma_\alpha = \mathbf{0}$), the instruments only influence the returns through a set of common factors.

3.2.2 Estimation of IPCA

This section describes the estimation procedure of IPCA. The estimation procedure is introduced by Kelly et al. (2020). Firstly, the estimation of the restricted model with $\Gamma_\alpha = 0$ is explained. The second part elaborates on the estimation of the unrestricted model, which requires some slight modifications in the procedure.

All parameters that need to be estimated rely in the p x K matrix Γ_β and the K-dimensional factors f_t for each $t = 1, .., T$. The $\mathbf{F}$ is defined as the matrix containing all factor realizations for all time periods. A total of $p * K + K * T$ parameters have to be estimated. For the estimation, the restricted model is rewritten to an equivalent representation in equation (9) and (10). Equation (10) is the compounded error term of the returns and the factor loadings.

$$r_{i,t+1} = c_{i,t}\Gamma_\beta f_{t+1} + e_{i,t+1} \tag{9}$$

$$e_{i,t+1} := \eta_{i,t} f_{t+1} + \mu_{i,t+1} \tag{10}$$

Equation (9) can be vectorized as follows,

$$r_{t+1} = C_t \Gamma_\beta f_{t+1} + e_{t+1} \tag{11}$$

where r_{t+1} is the vector of option returns for each of the N companies corresponding to the options, C_t is the p x K matrix of characteristics for options on each underlying, and $\boldsymbol{e}_{t+1}$ is the N-vector of compounded error terms. Estimating this equation (11) can be formulated as a least-squares problem. The objective function of the least-squares problem is as follows,

$$\min_{\Gamma, \boldsymbol{F}} \sum_{t=1}^{T-1} (r_{t+1} - C_t \Gamma_\beta f_{t+1})'(r_{t+1} - C_t \Gamma_\beta f_{t+1}) \tag{12}$$

where, $\boldsymbol{F}$ is the matrix containing all factors in all time-periods. The mapping matrix Γ_β and the factors should be estimated simultaneously. Hence, there does not exist a closed-form solution for this problem. An alternating least-squares (ALS) algorithm is be used to alternate between the optimization over Γ_β and $\boldsymbol{F}$, while keeping the other constant. The two subproblems can be solved by ordinary least-squares (OLS). The partial optimization over f_{t+1} has an analytical solution for each t. This can be found by assuming $\hat{\Gamma_\beta}$ is known and solving the first order condition

$$\hat{\boldsymbol{f}}_{t+1} = (\hat{\Gamma_\beta}' C_t' C_t \hat{\Gamma_\beta})^{-1} \hat{\Gamma_\beta}' C_t' r_{t+1} \tag{13}$$

for each $t = 1, ..., T$, where $\hat{\Gamma}$ is the estimate of Γ. Expression (13) is found by the first order conditions w.r.t. f_{t+1} of problem (12).

To find a solution for $\hat{\Gamma_\beta}$, the sequence $\{\hat{f}_{t+1}\}$, will be assumed to be known.

$$\text{vec}(\hat{\Gamma}_\beta) = \Big(\sum_{t=1}^{T-1}(C_t \otimes \hat{f}'_{t+1})' C_t \otimes \hat{f}'_{t+1}\Big)^{-1}\Big(\sum_{t=1}^{T-1}(C_t \otimes \hat{f}'_{t+1})' r_{t+1}\Big) \tag{14}$$

This expression looks very similar to the form of an OLS sum of squared errors objective function, where vec(Γ) is a vector of parameters to be estimated and $C_t \otimes \hat{f}_{t+1}$ can be considered as known data. The solution of the first order condition w.r.t Γ can be rewritten to

$$\text{vec}(\hat{\Gamma_\beta}) = \Big(\sum_{t=1}^{T-1} C'_t C_t \otimes \hat{f}_{t+1}\hat{f}'_{t+1}\Big)^{-1}\Big(\sum_{t=1}^{T-1}(C_t \otimes \hat{f}'_{t+1})' r_{t+1}\Big) \tag{15}$$

This yields a matrix of regression coefficients of the returns regressed on the factors combined with the matrix of characteristics. ALS be used to alternate between calculating (13) and (15). Kelly et al. (2019) show empirically that this ALS algorithm converges fast. The ALS algorithm needs to start with some 'guessed' initial values for both the factors and the mapping matrix and continues alternating until the parameter values converge.

Γ_β and f_{t+1} are not uniquely identified. The least-squares problem can be solved by equivalent rotations $\Gamma_\beta M^{-1}$ and $M f_{t+1}$. This thesis imposes additional identifying restrictions to pin down a unique solution. These are suggested by Kelly et al. (2020). Therefore, Γ_β is restricted such that $\Gamma'_\beta \Gamma_\beta = I_K$, all factors have a non-negative mean $\mu_{f_t} \geq 0$ and the unconditional second moment matrix of f_t is diagonal with each diagonal entry smaller than the one before. The last restriction also implies that the factors are uncorrelated. The identifying assumptions do not affect the explanatory performance of the model.

The unrestricted model includes an intercept factor of ones in the model. The ALS procedure helps to estimate the loadings to this factor. The estimation procedure is based on the study of Kelly et al. (2019). To explain the estimation, the model is written as follows,

$$r_{i,t+1} = c'_{i,t}\Gamma_\alpha + c'_{i,t}\Gamma_\beta f_{t+1} + \epsilon^*_{i,t} \tag{16}$$

where Γ_α is a Kx1 vector mapping the characteristics directly to returns, $\epsilon^*_{i,t}$ is a compound error term of the returns and the factor loadings. To create a least-squares problem this equation is rewritten as

$$r_{i,t+1} = c'_{i,t}\tilde{\Gamma}\tilde{f}_{t+1} + \epsilon^*_{i,t} \tag{17}$$

where $\tilde{\Gamma}_\beta = (\Gamma_\beta, \Gamma_\alpha)$ and $\tilde{f}_{t+1} = (1, f_{t+1})$. To take the direct effect of the characteristics into account in the estimation of the latent factors, the partial OLS solution for the factors is changed to

$$\hat{f}_{t+1} = (\hat{\Gamma_\beta}' C'_t C_t \hat{\Gamma_\beta})^{-1}\hat{\Gamma_\beta}' C'_t (r_{t+1} - C_t \hat{\Gamma}_\alpha) \tag{18}$$

Furthermore, the only change in the partial OLS solution for $\tilde{\Gamma}$ is that the matrix Γ_β is replaced by $\tilde{\Gamma}$ and the factors f_{t+1} by $\tilde{f}_{t+1}$, Without additional restrictions, Γ_α is not uniquely identified. Therefore, the restriction $\Gamma'_\alpha \Gamma_\beta = 0_k$ is imposed. This assumption is in line with the approach of Kelly et al. (2019). In practice, this restriction is imposed by regressing Γ_α on Γ_β after a pair of those is found. The residuals of this regression are used as $\hat{\Gamma}_\alpha$. This approach attributes as much variation in option returns as possible to the common risk factors. Kelly et al. (2020) state some theoretical assumptions necessary for estimation of IPCA. These are listed in section 8.1 in the appendix.

3.2.3 Characteristic managed portfolios

The most obvious way to test the performance of the IPCA model on option returns is by using the monthly delta-hedged returns on individual equity options as test assets. In the rule, individual option returns are driven for a significant part by idiosyncratic volatility. It is interesting to test how IPCA performs on portfolios of options, which could diversify part of this variance away. In previous literature, it could be difficult to determine the test assets for different methods, as studied by Lewellen et al. (2010). A natural approach in this thesis is to use the characteristics to construct managed portfolios. The returns of the managed portfolios are defined as follows

$$x_{t+1} = \frac{C_t' r_{t+1}}{n_{t+1}} \tag{19}$$

, where n_{t+1} is the number of observations in the sample at time $t+1$. x_{t+1} is a vector containing L elements of which the i-th element corresponds to the delta-hedged return of a portfolio of option returns weighted by the realizations of characteristic i at the previous period. Kelly et al. (2019) show mathematically that the IPCA factors are approximately equal to the first K principal components of the managed portfolios. In addition, they show that the matrix Γ_β can be approximated by the first K eigenvectors of the sample second moment matrix of characteristic managed portfolio returns. Thes solutions are not exact. Their accuracy depends on the data. Since these values are fast to compute by a closed-form formula, they are used as initial values to decrease the number of iterations needed for convergence of the ALS algorithm for IPCA estimation. In addition, Kelly et al. (2019) also find that applying tests described in the next sections to managed portfolios yields equivalent results.

3.2.4 Testing for direct interaction of characteristics and factors

This section explains the benefits and the calculation of a significance test for Γ_α. Kelly et al. (2019) propose this test in their study of stock returns. The usefulness of testing whether $\Gamma_\alpha = 0$ based on the data is twofold. Firstly it allows choosing between the restricted and unrestricted models. If the hypothesis $\Gamma_\alpha = 0$ is true, the restricted model fits the data best. Furthermore, it also has an economic interpretation. If the characteristics are not significantly explaining option returns via the intercept, it means that all influence of the characteristics on the returns is associated with systematic risk factors among the options. Therefore the returns could be explained as compensation for exposure to systematic risk. If $\Gamma_\alpha \neq \mathbf{0}$, there is a direct relation of characteristics on returns without the exposure to common risk factors estimated by IPCA. In this case, Γ_alpha captures the direct influence of the characteristics on returns. In theory, there could still exist alphas if $\Gamma_\alpha = 0$. These eventual alphas have no relation to any of the characteristics in the data and are captured in the error term $\mu_{\alpha,i,t}$ as in equation (7). Note that the identifying restriction in the unrestricted models is such that the variation related to characteristics is only attributed to alpha if it cannot be related to the common risk factors in the model.

The null hypothesis $\Gamma_\alpha = 0$ is tested against $\Gamma_\alpha \neq 0$. This testing procedure of Kelly et al. (2019) applies bootstrap. Firstly, the unrestricted model is estimated. This provides estimates for $\hat{\Gamma}_\alpha$, $\hat{\Gamma}_\beta$ and $\hat{\boldsymbol{f}}_t$ (for all $t = 1, ..., T$). This allows to compute the test statistic $W_\alpha = \hat{\Gamma}'_\alpha \hat{\Gamma}_\alpha$. As shown by Kelly et al. (2019) characteristic managed portfolios as in equation (19) can equivalently be used in the testing procedure. These will be used to apply a residual bootstrap procedure. Returns on the managed portfolios are constructed as follows,

$$x_{t+1} = C_t' r_{t+1} = (C_t' C_t)\Gamma_\alpha + (C_t' C_t)\Gamma_{t+1+C_t \epsilon^*_{t+1}} \tag{20}$$

The number of characteristics is in general smaller than the number of different companies underlying the options, this results in a faster procedure than to use the option returns directly. At this point, a bootstrap procedure will be used to on the residuals of the managed portfolios to generate managed portfolio returns. Let the managed portfolio residuals denote $e_{t+1} = C_t' \epsilon^*_{t+1}$. The bootstrap returns will be generated by

$$\tilde{x}^b_{t+1} = (C_t' C_t)\hat{\Gamma}_\beta \hat{f}_{t+1} + \tilde{e}^b_{t+1} \tag{21}$$

for $b = 1, ..., B$, where B is the number of bootstrap samples. The bootstrap residuals $\tilde{e}^b_{t+1}$ are generated as follows,

$$\tilde{e}^b_{t+1} = d^b_{t+1} \hat{e}_{j^b_{t+1}} \tag{22}$$

Where d^b_{t+1} is a random number from a t-distribution with five degrees of freedom. j^b_{t+1} is a random time index from the dates in the original sample. So it chooses a random residual and matches it with the predicted return by the model. The main point here is that Γ_α is left out when creating the bootstrap samples. For each bootstrap sample, an unrestricted IPCA model is estimated and the test statistic $\tilde{W}^b_\alpha = \tilde{\Gamma}^b_\alpha{}' \tilde{\Gamma}^b_\alpha{}'$ is computed. The p-value is the fraction of the bootstrap test statistics larger than W_α computed by the IPCA Γ_α estimated on the original data.

Multiplication of the residuals with the random t distributed variable is often referred to as 'wild' bootstrap. Wu (1986) proposed this method to make the method more robust to heteroskedastic data. An advantage of this bootstrap approach is that it does not require strong assumptions on the underlying distribution of the residuals.

3.2.5 Testing the significance of Instruments

It is helpful to know which instruments significantly contribute to the factor loadings of the IPCA models. The testing procedure described in this section tests the significance of instruments in the restricted model. The test results show whether a characteristic is significantly related to the systematic risk factors estimated by IPCA. Therefore, a test of each characteristic's significance for the loadings on the factors jointly is performed. Under the null hypothesis, the mapping from the i-th characteristic to the factors is zero. This test is an example for the i-th characteristic.

$$H_0: \quad \Gamma_\beta = [\gamma_{\beta,1}, ..., \gamma_{\beta,i-1}, \mathbf{0}_{Kx1}, \gamma_{\beta,i+1}, ..., \gamma_{\beta,L}] \tag{23}$$

$$H_A: \quad \Gamma_\beta = [\gamma_{\beta,1}, ..., , \gamma_{\beta,L}] \tag{24}$$

Here $\gamma_{\beta,i}$ is a Kx1 vector mapping the i-th characteristic to the K factors.

The testing procedure follows the bootstrap approach introduced by Kelly et al. (2019) and is very similar to the testing procedure for Γ_α. Firstly, the IPCA model is estimated under the alternative hypothesis (24) without restricting $\gamma_{\beta,i}$ to zero. This way, the IPCA estimates for $\hat{\Gamma}_\beta$, $\{\hat{(f)}\}_{t=1}^T$ and the residuals $\{\hat{\epsilon}_t\}_{t=1}^T$ are obtained. The Wald type test statistic is given by

$$W_{\beta,i} = \hat{\gamma}'_{\beta,i}\hat{\gamma}_{\beta,i}$$

This is calculated by the restricted model fitted on the original data.

The next step is to conduct a bootstrap procedure. The residuals of managed portfolios are used to construct bootstrap samples similar to the test for Γ_α. The construction of the residuals follows the same approach as the test on Γ_α described in the previous subsection. For each $b = 1, ..., B$ bootstrap sample, the returns are generated under the null hypothesis, so by restricting $\hat{\gamma}_{\beta,i}$ such that $\hat{\Gamma}_\beta = [\hat{\gamma}_{\beta,1}, ..., \hat{\gamma}_{\beta,i-1}, \mathbf{0}_{Kx1}, \hat{\gamma}_{\beta,i+1}, ..., \hat{\gamma}_{\beta,p}]$. Hence the estimated contribution from the i-th characteristic is left out of the bootstrap sample. The bootstrap returns are calculated as follows:

$$\tilde{x}^b_{t+1} = C_t \tilde{\Gamma}_{\beta t+1} + \tilde{\epsilon}^b_{t+1} \tag{25}$$

$$\tilde{\epsilon}^b_{t+1} = d^b_{t+1}\hat{e}_{j^b_{t+1}} \tag{26}$$

Where d^b_t is a random draw from a t-distribution with five degrees of freedom and a variance of one, and $\hat{e}_{j^b_t}$ is a residual with a random index from the residuals of the estimated model.

The IPCA model will be fitted again under the alternative hypothesis for each bootstrap sample. For each bootstrap sample the test statistic $\tilde{W}^b_\beta = \tilde{\gamma}^b_{\beta,i}{}'\tilde{\gamma}^b_{\beta,i}$ is calculated. The p-value is the number of bootstrapped test statistics that exceed the test statistic calculated by the real data. This procedure needs to be repeated for each characteristic to find the significance of all characteristics.

3.3 Projected Principal Component Analysis (PPCA)

The second of the two conditional latent factor methods in this study is Projected Principal Component Analysis (PPCA). PPCA is introduced by Fan et al. (2016). This semi-parametric method identifies latent factors and loadings by incorporating additional characteristics in the model. The central idea is to project the panel of interest onto a linear space spanned by the characteristics. Subsequently, PCA is applied to the projected data matrix to find latent factors. Fan et al. (2016) find that PPCA estimates the factors more efficiently than PCA for sample panels with a cross-section that is larger than the number of time periods. This is the setting in this thesis, as shown in section 5. Other than IPCA, PPCA is a static factor model. It takes non-time-varying characteristics to estimate static factor loadings.

The form of the model that is estimated by PPCA is given as follows:

$$r_{it} = \sum_{k=1}^{K} \lambda_{ik} f_{tk} + u_{it} + u_{it} \tag{27}$$

$$\lambda_{ik} = g_k(\mathrm{X_i}) + \gamma_{ik} \tag{28}$$

Equation (27) represents a standard factor model with static factor loadings for the dependent variable r_{it}, where $i = 1, ..., N$ is the number of cross-sectional observations and $t = 1, ..., T$ is the number of time periods. Furthermore, $\{f_{t1}, ..., f_{tK}\}$ are the K time-varying factors and $\{\lambda_{ik}, ..., \lambda_{ik}\}$ are the static factor loadings mapping each option to each of the K factors. The part of the variance that the latent factors cannot explain is contained in the term u_{it}. Equation (28) determines the factor loadings. The functions $g_k(X_i)$ map the characteristics of options on company i to factor k for all $k = 1, ..., K$ and $i = 1, ..., n$. The error term γ_{ik} captures the part of the factor loading unrelated to the characteristics. The error terms $\{\gamma_{ik}\}$ and $\{\mu_{it}\}$ are assumed to have a mean of zero. The non-parametric loading function $g_k(\cdot)$ is allowed to be nonlinear. It is more general than the linear mapping of IPCA, which makes PPCA more flexible from this perspective. In addition, PPCA estimates a part of the factor loadings unrelated to the characteristics, in contrast to IPCA. On the other hand, IPCA allows for time-varying factor loadings, where PPCA restricts the loadings to be static over time. We can write the full model in one equation using matrix notation:

$$\mathrm{R} = \{\mathrm{G(X)} + \Gamma\}\mathrm{F}' + \mathrm{U} = \Lambda\mathrm{F}' + \mathrm{U} \tag{29}$$

Where $\mathbf{R}$ is a N x T matrix of returns data, $\mathbf{F}$ is the T x K matrix of factors f_{tk}, $\mathbf{G(X)}$ is the N x K matrix of functions $g_k(X_i)$ and Γ is the N x K matrix of γ_{ik} and U is the n x T matrix of the unexplained variance u_{it}.

As stated previously, the characteristics are mapped to the factor loadings via non-parametric functions $g_k(X_i)$. These functions consists of components for each characteristic that are additive, such that

$$g_k(X_i) = \sum_{l=1}^{p} g_{kl}(X_{il}) \quad k = 1, ..., K, i = 1, ..., N \tag{30}$$

Where the functions $g_{kj}(X_{ij})$ determines the influence of the j-th characteristic of options on company i to the loadings on factor k. The functions $g_{kj}(Xij)$ are estimated by the method of sieves[Grenander (1981)]. A set of basis functions $\{\psi_l\}$ for $l = 1, ..., L$ is used to estimate the components of $g_k(\cdot)$ in the following form.

$$g_{kl}(X_{il}) = \sum_{j=1}^{J} b_{jkl}\psi_l(X_{il}) + Q_{kl}(X_{il}) \tag{31}$$

For each $k = 1, ..., K$, $i = 1, ..., N$ and $l = 1, ..., p$. The numbers b_{lkj} are the sieve coefficients. The estimation of the basis functions is the non-parametric part. Different types of basis functions could be used within PPCA. In this thesis, the approach in the empirical analysis of Fan et al. (2016) is followed, and cubic splines are employed as basis functions. Cubic splines smooth the data by estimating functions for intervals in the data. This method could be described as a piecewise function of third-order polynomials. The goal of these basis functions is to smooth the characteristic data, removing part of the noise in the characteristics. An extensive description of cubic splines and basis functions is beyond the scope of this thesis. The term $Qkj(Xij)$ represents the error of the sieve approximation. Combining equation (30) and (31), the loadings functions can be written as follows,

$$g_k(X_i) = \boldsymbol{\psi}(X_i)' b_k + \sum_{l=1}^{p} Q_{kl}(X_{il}) \tag{32}$$

Where $\mathrm{b}_k = (b_{1k1}, ..., b_{Lk1}, ..., b_{1kp}, ..., b_{Lkp})'$ and $\boldsymbol{\psi}(X_i) = (\psi_1(X_{i1}), ..., \psi_L(X_{i1}),$ $..., \psi_1(X_{ip}), ...\psi_L(X_{ip}))$. An important note is that the functions b_{lkj} are not dependent on underlying company of the options. This drastically reduces the number of parameters of the model. The loading functions can be gathered in matrix notation as follows,

$$\mathrm{G(X)} = \Psi(\mathrm{X})\mathrm{B} + \mathrm{Q(X)} \tag{33}$$

Where $\mathrm{B} = (\mathrm{b}_1, ..., \mathrm{b_k})$, $\Psi(\mathrm{X}) = (\psi(X_1), ..., \psi(X_n))'$ and Q(X) is a matrix with $\sum_{j=1}^{p} Q_{kj}(X_{ij})$ as its i, k-th element. The complete PPCA model for option returns can now be written as,

$$\mathrm{R} = [\Psi(\mathrm{X})\mathrm{B} + \mathrm{Q(X)} + \Gamma]\mathrm{F}' + \mathrm{U} \tag{34}$$

3.3.1 Estimation of PPCA

Both the factors in F and the elements of G(X) and Γ for the loading functions need to be estimated. The estimation is performed in two steps. Firstly, the panel data R is projected onto a space spanned by the characteristics, which will be explained further in this section. The second step is to apply PCA to the projected data. The first step is to project the panel of interest R on a space spanned by the characteristics X_i $(i = 1, ..., n)$. As described before, the characteristics are static and smoothed by the Sieves method. The projected values are denoted as $\hat{\mathrm{R}} = \mathrm{PR}$, where P is a projection matrix onto the space spanned by the characteristics. This specific projection matrix is defined as follows.

$$\mathrm{P} = \Psi(\mathrm{X})[\Psi(\mathrm{X})'\Psi(\mathrm{X})]^{-1}\Psi(\mathrm{X})' \tag{35}$$

where Ψ(X) is the matrix of basis functions of the characteristics. In practice the basis functions are estimated by fitting a cubic spline model of the characteristics to the loadings of the first factor of a PCA model applied to the original panel R.

The projection matrix is idempotent (PP = P), and PU is approximately zero since the space spanned by X is approximately orthogonal to the error U. The last property shows that **U** does not play a prominent role in the projected panel data $\hat{\mathrm{U}}$. Since the cubic splines reduce the amount of noise in the model. Fan et al. (2016) impose the following normalization conditions on the factors and factor loadings for identification.

$$\mathrm{F}'\mathrm{F}/T = I_k \tag{36}$$

$$\Lambda'\mathrm{P}\Lambda \quad \text{is diagonal with distinct entries} \tag{37}$$

These restrictions allow to derive the following relations

$$\hat{\mathrm{R}}'\hat{\mathrm{R}}/T = \mathrm{R}'\mathrm{P}\mathrm{R}/T \approx \mathrm{F}\Lambda'\mathrm{P}\Lambda\mathrm{F}'/T \tag{38}$$

$$\mathrm{R}'\mathrm{P}\mathrm{R}\mathrm{F}/T \approx \mathrm{F}\Lambda'\mathrm{P}\Lambda \tag{39}$$

(39) shows that the columns of F correspond to the first K eigenvectors of the matrix $R'PR$, multiplied by $\frac{1}{\sqrt{T}}$. Hence the PPCA factor estimator is given by

$$\hat{\mathrm{F}} = \mathrm{R}'\mathrm{P}\mathrm{R}/T \tag{40}$$

For intuition, the estimated factors are the K first principal components of the projected panel of option returns. Fan et al. (2016) derive the approximate relationship $\mathrm{P}\Lambda \approx \mathrm{PRF}/T$ for the projected loading matrix. Hence, the estimator for the factor loadings based on the estimated factors is given by $\hat{\Lambda} = \mathrm{R}\hat{\mathrm{F}}$. This is an approximate relationship for the factor loadings with room for error. The next step is to estimate the different parts of the loadings $\Lambda = \Psi(\mathrm{X})\mathrm{B} + \mathrm{R}(\mathrm{X})$

The estimator of the loading functions from characteristics to factors $G(X)$ is defined by Fan et al. (2016) as $\hat{\mathrm{G}}(\mathrm{X}) = \mathrm{PR}\hat{\mathrm{F}}/T$. In practice, another equivalent approach is taken in thesis. The projected model for the projected data can be written as follows

$$\mathrm{PR} = \Psi(\mathrm{X})\mathrm{B}\mathrm{F}' + \tilde{\mathrm{E}} \tag{41}$$

$$\tilde{\mathrm{E}} = \mathrm{P}\Gamma' + \mathrm{PQ}(\mathrm{X})\mathrm{F}' + \mathrm{PU} \tag{42}$$

Where the compounded error term $\tilde{\mathrm{E}}$ is small because of the approximate orthogonality of U and Γ to the projection matrix. Equation (41) shows that B can be estimated by OLS, which defines the estimator as follows,

$$\hat{\mathrm{B}} = \left(\Psi(\mathrm{X})'\Psi(\mathrm{X})\right)^{-1}\Psi(\mathrm{X})'\mathrm{Y}\hat{\mathrm{F}} \tag{43}$$

Naturally, the estimator of G(X) is defined as $\hat{\mathrm{G}}(\mathrm{X}) = \Psi(\mathrm{X})\hat{\mathrm{B}}$. Now the parts of the factor loadings unrelated to the characteristics Γ are estimated by $\hat{\Gamma} = \hat{\Lambda} - \hat{\mathrm{G}}(\mathrm{X})$.

Fan et al. (2016) define several critical theoretical assumptions for the estimation of PPCA. These are listed in section 8.2 in the appendix.

3.3.2 Do the characteristics matter?

To asses whether the characteristics combined have significant explanatory power on the factor loadings, a test on G(X) is performed. The hypotheses of this test are

$$H_0 : \mathrm{G(X)} = 0, \quad H_A : \mathrm{G(X)} \neq 0 \tag{44}$$

If the null hypothesis is rejected, the characteristics have significant explanatory power on the factor loadings. It is valuable to use PPCA and include the characteristics in the latent factor model in this case. The test proposed by Fan et al. (2016) is employed for these hypotheses. Since Γ and Λ are assumed to be approximately orthogonal, we can write $\mathrm{P}\Lambda \approx \mathrm{G(X)} = 0$ and the null hypothesis can be rewritten as $H_0 : \mathrm{P}\Lambda = 0$. The test statistic is as follows

$$S_G = \frac{1}{p} tr(\mathrm{W}_1 \tilde{\Lambda}' \mathrm{P} \tilde{\Lambda}) \tag{45}$$

Where $\tilde{\Lambda} = \mathrm{R}\tilde{\mathrm{F}}/\mathrm{T}$ is the estimator of the factor loadings, $\tilde{\mathrm{F}}$ is the traditional PCA estimator of the latent factors and $\mathrm{W}_1 = (\frac{1}{p}\tilde{\Lambda}'\tilde{\Lambda})^{-1}$ is a weighting matrix .

Fan et al. (2016) prove that the asymptotic distribution of the test statistic under some standard assumptions for PPCA is

$$\frac{NS_G - JdK}{\sqrt{2JdK}} \xrightarrow{d} N(0,1) \tag{46}$$

where N is the number of underlying stocks in the cross-section, J is the number of Sieve dimensions, d is the number of characteristics and K is the number of factors. In practice, the for small L, the p-value of this test is computed using the upper quantile of a Chi-squared distribution with LdK degrees of freedom for NS_G.

3.3.3 Which characteristics matter?

To find which characteristics matter in the PPCA model, their significance in explaining the option returns is computed. The goal of the test developed in this section is to find which characteristics are important in explaining the returns. The benefit of this testing procedure is twofold. Firstly, it adds economic interpretability to the model. It informs the researcher which characteristics matter for describing option returns. Secondly, it allows selecting the significant characteristics from a large set to include in a new, more parsimonious model. This thesis extends and adjusts the bootstrap procedure for IPCA of Kelly et al. (2019) for use for PPCA. The approach follows roughly the same steps as the IPCA testing procedure. The difference relies on the hypotheses, the test statistic, and the variables restricted to zero.

The l^*-th characteristic influences the predicted returns for the options on the i-t underlying company through factor k observation in the cross-section viam the function

$$g_{kl^*}(X_{il^*}) = \sum_{j=1}^{J} b_{jkl^*}\psi_l(X_{il^*}) + R_{kl^*}(X_{il^*}) \tag{47}$$

based on the above, the null hypothesis is defined as follows,

$$H_0 : g_{kl^*}(X_{il^*}) = 0 \quad \forall i = 1, ..., N, \quad k = 1, ..., K \tag{48}$$

$$\tag{49}$$

The alternative hypothesis is that at least one of the terms is not equal to zero. Note that $g_{kl^*} = \sum_{l=1}^{L} b_{l,kl^*} * \psi_l(X_{il^*}) + R_{kl^*}(X_{il^*})$. Where $\sum_{l=1}^{L} b_{l,kl^*}\psi_l(X_{il^*})$ is the sum of the product of the basis functions and their characteristics and a $R_{kl^*}(X_{il^*})$ is the sieve approximation error which is assumed to have mean zero. Assuming the sieve errors to be orthogonal, the above hypothesis can be stated equivalently as

$$\begin{aligned} H_0 : \quad \mathrm{b_k} =& (b_{1,k1}, ..., b_{J,k,1}, ..., b_{1,k,l^*-1}, ..., b_{J,k,l^*-1}, 0, ..., 0, & (50)\\ & b_{1,k,l^*+1}, ..., b_{J,k,l^*+1}, ..., b_{1,k,d}, ..., b_{J,k,d})' \quad \forall k = 1, ...K & (51) \end{aligned}$$

The exact testing procedure is described under here.

Firstly the PPCA model is estimated under the alternative hypothesis. This gives the estimates on all the parameters in $\hat{\Gamma}$, $\hat{\mathrm{G}}(\mathrm{X})$ and $\hat{\mathrm{F}}$. The Wals-type test-statistic is computed as $W_{l^*} = \sum_{k=1}^{K} \hat{\mathrm{b}}_{\mathrm{k}}^{\mathrm{l}^*\prime}\hat{\mathrm{b}}_{\mathrm{k}}^{\mathrm{l}^*}$, where $\hat{\mathbf{b}}_{\mathbf{k}}^{\mathbf{l}^*} = (\hat{b}_{1,l,l^*}, ..., \hat{b}_{J,k,l^*})'$.

Then, the in-sample returns of each option-month combination is computed by $\hat{r}_{i,t} = \sum_{k=1}^{K}(\hat{g}_k(X_i) + \hat{\gamma}_{ik})\hat{f}_{tk}$. The residuals of the model are found by $\hat{\epsilon}_{it} = r_{i,t} - \hat{r}_{i,t}$. These residuals are a combination of the Sieve approximation error and the idiosyncratic error and could be written as $\hat{\epsilon}_{it} = [\sum_{l=1}^{p} R_{kl}(X_{il})]f_{tk} + u_{it}$.

Now bootstrap samples are calculated using a similar approach as Kelly et al. (2019). These samples are constructed under the null hypothesis. For each bootstrap sample $m = 1, ..., M$, the returns are calculated as

$$\tilde{r}_{i,t}^{m} = \sum_{k=1}^{K}(\tilde{g}_k(\mathrm{X}_i) + \hat{\gamma}_{ik})\hat{f}_{tk} + \tilde{\epsilon}_{it}^{m} \tag{52}$$

Where $\tilde{g}_k(X_i)$ is such that the influence of the l^*-th characteristic is set to zero, corresponding to the null hypothesis.This is achieved by equating the sieve coefficients corresponding to characteristic j^* to zero. The fitted factor loadings are given by $\hat{g}_k(\mathrm{X_i}) = \hat{\psi}(\mathrm{X_i})'\hat{\mathrm{b}}_{\mathrm{k}} + \sum_{j=1}^{d} R_{kl}(X_{il})$, where the last term is an error and $\hat{\mathrm{b}}_{\mathrm{k}} = (\hat{b}_{1,k,1}, ..., \hat{b}_{J,k,1}, ..., \hat{b}_{1,k,p}, ..., \hat{b}_{J,k,p})'$ for $k = 1, ..., K$. To simulate under the null hypothesis, all the sieve coefficients corresponding to the j^*-th characteristic are set to zero, which results in

$$\tilde{\mathrm{b}}_{\mathrm{k}} = (\hat{b}_{1,k1}, ..., \hat{b}_{J,k,1}, ..., \hat{b}_{1,k,l^*-1}, ..., \hat{b}_{J,k,l^*-1}, 0, ..., 0, \hat{b}_{1,k,j^*+1}, ..., \hat{b}_{J,k,j^*+1}, ..., \hat{b}_{1,k,p}, ...\hat{b}_{J,k,p})'$$

for each $k = 1, ...K$

The bootstrap residuals $\tilde{\epsilon}_{it}^m$ for each of the M bootstrap samples are generated by,

$$\tilde{\epsilon}_{it}^m = z_{it}\hat{\epsilon}_{i,h_{it}^m} \quad \forall i = 1, ..., p, \quad , t = 1, ..., T \tag{53}$$

where z_{it} is a random draw of a t-distribution with five degrees of freedom and a variance of 1 and h_{it}^m is a random time-index drawn from the set of time-indices of the panel.

A PPCA model is fitted to each bootstrap sample under the alternative hypothesis. The estimated vectors $\tilde{\mathrm{b}}_k^m = (\tilde{b}_{1,k,1}^m, ..., \tilde{b}_{J,k,1}^m, ..., \tilde{b}_{1,k,p}^m, ..., \tilde{b}_{J,k,p}^m)'$ are obtained for each m. The test statistic for each bootstrap sample is calculated as $\tilde{W}_{l^*}^m = \sum_{k=1}^K \tilde{\mathrm{b}}_{\mathrm{k}}^{\mathrm{l}^*\prime}\tilde{\mathrm{b}}_{\mathrm{k}}^{\mathrm{l}^*}$, where $\tilde{\mathrm{b}}_{\mathrm{k}}^{\mathrm{l}^*} = (\tilde{b}_{1,l,l^*}, ..., \tilde{b}_{J,k,l^*})'$. The p-value of the j^* characteristic is the fraction of bootstrap samples of which the test statistic is greater than W_{l^*}, computed by the model fitted to the original data. If the p-value is low, the null hypothesis is rejected, implying that characteristic l^* has significant explanatory power on the factor loadings. This procedure must be repeated for each characteristic to find significance levels for all characteristics.

The testing procedure explained above is not solely designed for option returns but could be used to determine the significance of characteristics in any PPCA model.

3.3.4 Testing if the characteristics fully explain loadings

To test whether the characteristics explain the factor loadings fully, Fan et al. (2016) proposes another specification test. The null hypothesis that is tested is defined as follows,

$$H_0 : \Gamma = 0 \tag{54}$$

If this hypothesis is not rejected, it suggests that characteristics can fully explain the factor loadings. In this case, the PPCA model could be simplified by excluding Γ since all information on the factor loadings is captured in G(X). The null hypothesis above can equivalently be written a H_0 : $\mathrm{P}\Lambda = \Lambda$. This is possible since $\mathrm{G(X)} \approx \mathrm{P}\Lambda$, so $\Lambda \approx P\Lambda + \Gamma$. Fan et al. (2016) propose the following test statistic,

$$S_\Gamma = tr(\hat{\Lambda}'(\mathrm{I} - \mathrm{P})\hat{\Sigma}_{\mathrm{u}}^{-1}(\mathrm{I} - \mathrm{P})\hat{\Lambda}) \tag{55}$$

where $\hat{\Lambda} = \mathrm{Y}\hat{\mathrm{F}}$ is the estimated matrix of factor loadings and $\hat{\mathrm{F}}$ is the matrix of estimated PPCA factors. The estimated covariance matrix $\hat{\Sigma}_u$ of U is used as weighting matrix and is calculated by

$$\hat{\Sigma}_{\mathrm{u}} = diag(\mathrm{R}(\mathrm{I} - \hat{\mathrm{F}}\hat{\mathrm{F}}'/T)\mathrm{R}, .')/T \tag{56}$$

Fan et al. (2016) derive the following distribution of the a normalized version of the test statistic under the null hypothesis,

$$\frac{TS_\Gamma - pK}{\sqrt{2pK}} \xrightarrow{d} N(0,1) \tag{57}$$

4 Model performance

Performance measures need to be defined to assess and compare the performance of different methods and different models. The main goal of the methods employed in this thesis is to explain the variation in returns. Therefore, R^2 is the main goodness of fit measure. This measures the fraction of the variance in delta-hedged option returns that the model explains. The most standard measure is the total R^2. This measures how well the factors of the model explain realized returns. The second measure is predictive R^2, which describes how much variation in return the model explains by expected factor returns. Using these two measures, the approaches of Kelly et al. (2019) and Büchner and Kelly (2022) are followed. In the remainder of this section, the performance measures are defined carefully for both ICPA and PPCA.

For the unrestricted IPCA model, the total R^2 is defined by the formula,

$$R^2_{\text{IPCA,tot}} = 1 - \frac{\sum_{i,t} \left(r_{i,t+1} - \boldsymbol{c}_{i,t}\hat{\Gamma}_\alpha - \boldsymbol{c}_{i,t}\hat{\Gamma}_\beta \hat{\boldsymbol{f}}_{t+1}\right)^2}{\sum_{i,t} r_{i,t+1}^2} \tag{58}$$

This measures the model's explanatory power using the factor realizations and time-varying conditional factor loadings. It shows the fraction of the option returns that could be explained by latent systematic risk factors estimated by IPCA and direct interaction with the characteristics via Γ_α in the case of unrestricted model. The measure for the restricted model is found by equating Γ_α to zero. It is only possible to calculate the factor loadings of the next period if the returns of that period are availabe, which not the case in practice. Therefore, the predictive R^2 is used for evaluating the models. The time-varying factor realizations are replaced by the unconditional factor means denoted by $\hat{\lambda}$. The predictive R^2 for unrestricted IPCA is defined as follows,

$$R^2_{\text{IPCA, pred}} = 1 - \frac{\sum_{i,t} \left(r_{i,t+1} - \boldsymbol{c}_{i,t}\hat{\Gamma}_\alpha - \boldsymbol{c}_{i,t}\hat{\Gamma}_\beta \hat{\boldsymbol{\lambda}}\right)^2}{\sum_{i,t} r_{i,t+1}^2} \tag{59}$$

This measures the part of variation in option returns that is explained by conditional expected returns. The factor means could be interpreted as the prices of risk, implied by the model. The time-varying nature of the factors is ignored, but the factor loadings are still time-varying conditional on the characteristics. An extension could be to develop a model for the factors to predict future factor realizations. The most standard approach would be a VAR model. Developing a model for the factor dynamics is beyond the scope of this thesis. The total and predictive R^2 are equivalently defined for PPCA as,

$$R^2_{\text{PPCA,tot}} = 1 - \frac{\sum_{i,t} \left(r_{i,t} - \sum_{k=1}^{K}([\hat{g}_k(X_i) + \gamma_{ik}]f_{tk})\right)^2}{\sum_{i,t} r_{i,t+1}^2} \tag{60}$$

$$R^2_{\text{PPCA,pred}} = 1 - \frac{\sum_{i,t} \left(r_{i,t} - \sum_{k=1}^{K}([\hat{g}_k(X_i) + \gamma_{ik}])\hat{\lambda}\right)^2}{\sum_{i,t} r_{i,t+1}^2} \tag{61}$$

The explanation of the two performance measures is equivalent to the IPCA variants. An important difference is that the factor loadings of PPCA are static. Predicting returns using the factor means, predicts the same return for each period for options on a specific company. Intuitively, PPCA is therefore expected to have lower values of predictive R^2.

The predictive R^2 measures the predictive power, but it is an in-sample measure. The next section elaborates on an out-of-sample testing procedure for IPCA.

4.1 Out-of-sample performance

Almost all machine learning and econometric models are potentially prone to overfitting. Over-fitting occurs when the model is fitted too close to the data used to estimate the model. This phenomenon could result in a very high R^2 on the data used for estimation but a low performance on unseen data. The out-of-sample fit of both methods is tested to find if the results describe the underlying patterns of the data.

To assess the out-of-sample performance for IPCA, a procedure similar to the backward-looking estimation of Kelly et al. (2019) will be employed. The idea is to use the data until half of the sample period to estimate the IPCA model, which entails also the mapping matrix $\hat{\Gamma}_{\beta,t}$ and$\hat{\Gamma}_{\alpha,t}$ for the unrestricted model. The IPCA specification as in this thesis relates characteristics to the factor loadings of one period ahead. Using the estimated model up to t, the factor realization one period ahead is calculated as

$$\hat{f}_{t+1} = (\hat{\Gamma}_t C_t' C_t \hat{\Gamma}_t')^{-1} \hat{\Gamma}_t' C_t' r_{t+1} \tag{62}$$

similar to equation (13). Where C_t is a matrix with the values for all characteristics up to time t. Equation (62) uses only information up to time t apart from r_{t+1}. The window of the estimation will be expanded for each $T/2 \leq t \leq T-1$. So for each $T/2tT-1$, the and IPCA model is estimated and the one-period-ahead factor return is calculated. The out-of-sample predicted option return is calculated as $\hat{r}^{\text{oos}}_{t+1} = C_t \hat{\Gamma}_t \hat{f}_t$. The out-of-sample R^2 is defined as:

$$R^2_{\text{oos}} = 1 - \frac{\sum_i \sum_{t=T/2}^{T-1} \left(r_{i,t+1} - \hat{r}^{\text{oos}}_{t+1} \right)}{\sum_{t=T/2}^{T-1} r^2_{i,t+1}} \tag{63}$$

The next period's returns have to be known to calculate the out of sample total R^2, t In practice, it is impossible to know the returns of subsequent periods when predicting that same return for a period in the future. Using the time-series mean of the factor realizations overcomes this issue. An out-of-sample predictive R^2 can be computed using the same idea as for the in-sample counterpart. Instead of the factor returns for the next period $\hat{f}_{t+1}$, the mean of the factor realizations up to time t ($\hat{\lambda}_t = \sum_{p=1}^{t} \hat{f}_p$) is used to calculate the predicted returns as $\hat{r}^{\text{oos}}_{t+1,\text{pred}} = C_t \hat{\Gamma}_t \hat{\lambda}_t$. By equation (62). Only returns up to period t are used to estimate the option returns of period $t+1$. The out-of-sample predictive R^2 is calculated by replacing $\hat{r}^{\text{oos}}_{t+1}$ by $\hat{r}^{\text{oos}}_{t+1,\text{pred}}$ in equation (63).

PPCA is less straightforward to estimate out-of-sample. The characteristics are static, making it more difficult to predict one period ahead. One approach uses the same out-of-sample approach as for IPCA. The mean is calculated up to the last period and estimated one period ahead. Theoretically, a problem is the component of the factor loadings that is not dependent on the characteristics, which is already fitted to the next period. This makes it not a fair comparison. Another approach is to estimate the models for each t, but leave Γ out when predicting the next period return. Both methods are examined in this thesis.

4.1.1 Out-of-sample Tangency portfolios of factors

In addition to the in-sample goodness of fit measures and its out-of-sample counterparts, it is interesting to learn about the mean-variance efficiency of the factor models. For IPCA, the comparison of restricted and unrestricted models informs on the mean-variance efficiency conditional on the characteristics. A tangency portfolio of factors is constructed. The Sharpe Ratio of this portfolio measures its unconditional mean-variance efficiency. The tangency portfolio is the most efficient portfolio in which all of the available wealth is invested in the factors. This measure of model performance is interpretable and comparable more economically. Kelly et al. (2019) calculate out-of-sample tangency portfolios of IPCA factors for stock return data. Their IPCA models outperform observed factor models in terms of Sharpe Ratios of the tangency portfolio. Büchner and Kelly (2022) analyze their IPCA model applied to index options in terms of tangency portfolio out-of-sample Sharpe ratios. They also find that IPCA results in a better performance.

The tangency portfolios are calculated using the same out-of-sample design as the backward-looking estimation as in subsection 4.1. The tangency weights π_t^{tan} are calculated using the estimated factors up to a certain period. Tangency portfolios weights for each period t are calculated as follows

$$\pi_t^{tan} = \frac{\hat{\Sigma}_{\hat{f}_{t|t}}^{-1} \lambda_{\hat{f}_{t|t}}}{\mathbf{1}'\hat{\Sigma}_{\hat{f}_{t|t}}^{-1} \lambda_{\hat{f}t|t}} \tag{64}$$

Note that $\hat{f}_{t|t}$ consists of the factor realizations up to time t estimated with data up to time t. $\hat{\Sigma}_{\hat{f}_{t|t}}$ is the estimated sample covariance matrix of the factors with information up to time t and $\hat{\lambda}_{t|t}$ are the factor means up to period t. The return on the tangency portfolio is calculated for each time period in the out-of-sample design as $r_{t+1}^{tan} = \pi_t^{tan} \hat{f}_{t+1|t}$

The most common measure in literature to assess mean-variance efficiency is the Sharpe ratio. The Sharpe ratio is the ratio of the mean of the returns on the tangency portfolio divided by standard deviation of the tangency portfolio returns.[Sharpe (1966)].

$$Sh(r^{tan}) = \mu_{r^{tan}} / \sigma_{r^{tan}} \tag{65}$$

where $\mu_{r^{tan}}$ and $\sigma(r^{tan}$ are the mean and standard deviation out-of-sample tangency portfolio returns. These returns consist of the second half of the sample, as explained in subsection 4.1.

In addition to the tangency portfolio returns, the Sharpe ratios of out-of-sample factor realizations themselves are calculated. These are similar to equation (65), replacing the tangency portfolio returns with the factor returns.

Similarly to IPCA, the out-of-sample Sharpe Ratios of the factors are computed using the out-of-sample design described in section 4.1. The out-of-sample estimation for PPCA differs slightly from the

IPCA procedure. However, the weights and returns and Sharpe ratios of the tangency portfolios and the individual factors are computed using the same approach.

4.2 In-sample tangency portfolio of factors

In addition to the Sharpe ratios that follow from tangency portfolios formed by the out-of-sample design of section 4.1.1, in-sample returns of tangency portfolios are computed. These enables comparison of the two models on unconditional mean-variance efficiency of the factors in-sample. The portfolios are evaluated in terms of Sharpe ratios as. The tangency portfolio weights are calculated using the factors estimated on the whole sample.

$$\pi^{tan} = \frac{\hat{\Sigma}_{\hat{f}}^{-1} \mu_{\hat{f}}}{\mathbf{\iota}' \hat{\Sigma}_{\hat{f}}^{-1} \mu_{\hat{f}}} \tag{66}$$

where $\hat{\Sigma}_{\hat{f}}$ and $\hat{f}$ are the covariance matrix and the vector of factor means over the whole sample period. The in-sample returns of the tangency portfolios are $r_{i,t+1} = \pi^{tan} f_{t+1}$. Finally the Sharpe ratios are computed similarly to equation (65).

4.3 Comparison of the factors of the two models

Besides the performance of the two methods, the time series of the factors themselves are compared. Canonical correlations are used to compare the sets of IPCA factors. This method is employed for two main reasons. Firstly, canonical correlations allow comparing the whole sets of factors, instead of one factor to another using 'normal' correlations. Secondly, a unique combination of factors and loadings is determined by identifying restrictions for both IPCA and PPCA. Equivalent models could be found by rotating the factor values with the matrix of factor loadings Γ. Therefore the Canonical correlations are a valuable measure to compare the different factors. It is possible to find the degree to which the two models' factors span the same spaces by canonical correlation.

The canonical correlations are used as a measure of similarity of the factors of the two models. This type of correlation tests whether the factors of one model are a conditional rotation of the factors of the other model. In other words, if the sets of factors span the same space. Gagliardini and Ma (2019) use a similar approach to compare factors of a latent factor model to observed factors.

Denote the matrix of factors of an IPCA model by $\boldsymbol{F}^{\text{IPCA}}$ and the factors of a PPCA model by $\boldsymbol{F}^{\text{PPCA}}$. Then we want to test the following

$$\text{F}^{\text{IPCA}} = M_t \text{F}^{\text{PPCA}} \tag{67}$$

Where M_t is a K_1 x K_2 matrix that is measurable at time t. K_1 and K_2 are the numbers of factors of the IPCA and the PPCA model. Equation (67) implies that the factors of IPCA are a rotation of the PPCA factors. If equation (67) holds, this means that all canonical correlations are equal to one.

Canonical variables are linear combinations of the factors needed to calculate canonical correlations.

The canonical variables will be of the following form.

$$X_i^{\text{IPCA}} = \mathbf{a}_i' \text{F}^{\text{IPCA}}, \quad i = 1, ..., K^*$$
$$X_i^{\text{PPCA}} = \mathbf{b}_i' \text{F}^{\text{PPCA}}, \quad i = 1, ..., K^*$$

Where $a_i \in R^{K_1}$, $\mathbf{b}_i \in R^{K_2}$ for $i = 1, ..., K^*$ and $K^* = \max(K_1, K_2)$. $(X_i^{\text{IPCA}}, X_i^{\text{PPCA}})$ is the i-th canonical pair. The vectors a_1 and b_1 of the first canonical pair should be such that the first canonical correlation is maximized, subject to some conditions. This is formalized as follows,

$$\rho_1 = max_{a_1,b_1} cov_t(X_1^{\text{IPCA}}, X_1^{\text{PPCA}})$$
$$\text{s.t.} \quad V_t(X_1^{\text{IPCA}}) = 1, V_t(X_1^{\text{PPCA}}) = 1$$

The other canonical pairs also maximize the canonical correlation such that that pairs are uncorrelated with all pairs before. For $i = 1, ..., K^*$

$$\rho_i = max_{a_i,b_i} cov_t(X_i^{\text{IPCA}}, X_{i,t+1}^{\text{PPCA}})$$
$$\text{s.t.} \quad V_t(X_{i,t+1}^{\text{IPCA}}) = 1, V_t(X_i^{\text{PPCA}}) = 1$$
$$cov_t(X_i^{\text{IPCA}}, X_j^{\text{IPCA}}) = 0, cov_t(X_i^{\text{PPCA}}, X_j^{\text{PPCA}}), \quad j = 1, ..., i-1$$

The canonical correlations of the two models are equivalent to the eigenvalues of the matrix

$$V_{11}^{-1} V_{12} V_{22}^{-1} V_{21} \tag{68}$$

Where $V_{11} = V(\text{F}^{\text{IPCA}})$, $V_{22} = V(\text{F}^{\text{PPCA}})$, $V_{12} = cov(\text{F}^{\text{IPCA}}, \text{F}^{\text{PPCA}})$ and $V_{21} = cov(\text{F}^{\text{PPCA}}, \text{F}^{\text{IPCA}})$. $V(\cdot)$ is the variance and $cov(\cdot)$ is the covariance.

5 Data

5.1 Option data

Option data is retrieved from the OptionMetrics IV database. This is the largest available database for American options. Most other literature uses this database for option prices and other option characteristics [Horenstein et al. (2020), Christoffersen et al. (2018a)]. The sample period is from January 2010 up to December 2020, a total of 132 months. For PPCA, the panel of option returns needs to be balanced. Therefore, options on underlying stocks present in the SP500 during the whole sample period are selected. The sample consists of options on 318 underlying stocks on companies in the SP500. Important characteristics from OptionMetrics for constructing delta-hedged returns include the bid and ask prices, expiration date, delta and the strike prices of the options. The panel consists exclusively of call options. Since there can be considered only one option per month per underlying stock, a choice between puts and calls has to be made. The methods applied in this thesis could only handle panels of two dimensions. The choice of call options instead of puts is more common in academic literature [Horenstein et al. (2020)].

Option prices depend heavily on the return of the underlying stock. Delta-hedged returns are considered in this thesis to isolate the effects of the option itself as much as possible. The panel of interest consists of monthly delta-hedged returns. The delta-hedge is re-balanced daily. Therefore daily option data is retrieved from OptionMetrics. The subsequent subsection consists of a more detailed description of the construction of these delta-hedged returns.

Prices and more characteristics of the underlying stocks of the considered options are retrieved from the Center of Research in Security Prices (CRSP) and linked to the option data from OptionMetrics. These prices enable the calculation of delta-hedged option returns as described in section 5.2.

The raw options and stock data can contain variables not desirable in our analysis. A filtering procedure is used to delete those data points. The procedure is mainly based on the papers of Horenstein et al. (2020) and Brooks et al. (2018). Firstly, options that are likely to be illiquid are deleted from the sample. Therefore, options with a higher bid price than the ask price are not considered, and only options with a positive trading volume are used. Also, observations with a negative or zero bid price and observations with an average bid and ask lower than 0.125 $ are deleted from the sample. Observations with missing ask or bid prices or deltas are also excluded from the sample.

Information on the underlying stock is needed to calculate delta-hedged returns. Therefore, the option data is merged with daily CRSP data on the underlying stocks Secid numbers from OptionMetrics are linked to PERMNO numbers of the CRSP database of stock data in combination with the dates of the observations in both databases. Observations for which the bid or ask price of the underlying stock is missing cannot be used to construct delta-hedged returns and are discarded from the sample. All options in the sample are American style. To exclude the effect of early exercising of the options, options for which the underlying pays dividend during the remaining lifetime of the option contract are removed. The remaining options can be treated as European style options.

Since the panel can only consist of one option for each month and each underlying, they are selected on moneyness and time to maturity. Firstly, only options in the last month before maturity are kept. All observations with more than 47 or less than 17 days until their expiration date are discarded. Secondly, the option with moneyness closest to the money (moneyness of 1) is selected for each unique month and underlying stock combination. Moneyness is calculated as the average of the bid and ask price of the underlying stock divided by the strike price of the corresponding option. The mean of the moneyness for each month is used to select the options. If there are two or more options with exactly equal moneyness for a month-stock combination, one of those is chosen at random. The following subsection describes the construction of the delta-hedged returns.

5.2 Delta-hedged option returns

Option returns depend largely on fluctuations in the prices of the underlying stock. In this thesis, the aim is to exclude the effect of the underlying stock price as much as possible. Therefore, the returns of delta-hedge option portfolios are considered. A delta-hedged option portfolio consists of a long position in an option and a short position of delta shares in the underlying stock. Delta is the sensitivity of the option price to the price of the underlying stock ($\Delta_t = \frac{\partial C_t}{\partial S_t}$). Where C_t is the option price at time t and S_t is the price of the underlying stock at time t. A disadvantage of delta-hedging is that it does not take higher moment sensitivities of option returns to the underlying stock prices into account, but this difference is likely to be small. The choice of delta-hedged option returns is common in academic literature. Examples are the studies of Horenstein et al. (2020) and Cao and Han (2013). Bakshi and Kapadia (2003) define the gains on a delta-hedged options portfolio as follows:

$$\Pi_{t,t+\tau} = C_{t+\tau} - C_t - \int_t^{t+\tau} \Delta_u dS_u - \int_t^{t+\tau} r_u^f (C_u - \Delta_u S_u) du \tag{69}$$

where C_t is the price of an option at time t, S_t is the price of the underlying stock at time t, $\Delta_t = \frac{\partial C_t}{S_t}$ is the sensitivity of the price of the option to the price of the underlying stock, r_t^f is the annualized risk-free rate and τ the time-span over which the gains are calculated. Equation (69) has 3 components. The first represents the gain of an option without hedging, the second is an integral that describes the influence of the delta hedge that is rebalanced continuously and the third component is an integral that represents the cost of financing this portfolio against the risk-free rate. To be able to compute these gains based on real data, Bakshi and Kapadia (2003) derive a discretized version of the delta-hedged option gain formula, where the hedge is re-balanced N times over the period from t to $t + \tau$.

$$\pi_{t,t+\tau} = C_{t+\tau} - C_t - \sum_{n=0}^{n-1} \Delta_{t_n} (S_{t_{n+1}} - S_{t_n}) - \sum_{n=0}^{n-1} \frac{q_n r_{t_n}^f}{365} (C_t - \Delta_{t_n} S_{t_n}) \tag{70}$$

Where Δ_{t_n} is the option's delta at period t_n, q_n is the number of days between t_n and t_{n+1} and $r_{t_n}^f$ is the annual risk-free rate at time t_n. The risk-free rate data is retrieved from the zero-coupon bond yield curve of OptionMetrics. The yearly rates are from zero-coupon bonds with a time to maturity of 300 up to 400 days and are corrected for the number of days, such that they represent annual rates.

In this study, formula (70) is used to construct monthly delta-hedged option gains from real data. Monthly delta-hedged gains are constructed for each option using daily option and stock data for re-balancing. Since option deltas are continuously changing, the hedge is re-balanced daily. Partly due to our filtering procedure, not all options contain an observation for each working day. In formula (70), there is accounted for this. Only options with more than two trading days in the month can be used, but most option-month observations contain more daily observations. To make the returns more comparable, options of which the first available observations are after the 13th day of the month are excluded. The average of the bid and ask price is used as the option price to calculate gains in formula (70). A similar approach is conducted for the prices of the underlying stock. In the end, the panel of interest consists of monthly returns of options on different underlying stocks. The gains of equation (70) are scaled by the absolute value of the delta-hedged portfolio. The delta-hedged returns are defined as $\Pi_{t,t+1} = \frac{\pi_{t,t+1}}{\Delta_t S_t - C_t}$. This step makes the returns comparable across different securities.

The resulting panel of delta-hedged option returns contains missing values for a small part of the combinations of dates and underlying stocks. This results in a slightly unbalanced pane panel. Although, 0.9491% of the observations is non-missing. The missing observations are caused by the filtering described in the previous subsection. This is likely due to the illiquidity of options on particular stocks in some months in the sample period. Since PPCA can not handle unbalanced panels, removal and imputation are invoked to construct a balanced panel. The sample contains options on two different stocks of the company Viacom. One of them has only 50 out of 132 monthly observations. Options on that stock are deleted from the sample. The panel consists of only 63 and 61 observations for options on stocks of the companies Westrock Co. and MeadWestvaco. These options are also removed. The resulting sample contains options on a cross-section of 318 underlying stocks. After removing the companies mentioned above, the panel still misses 4.20% of the observations. Therefore the missing imputations are imputed using K-nearest-neighbour (K-NN) imputation. This method compares the missing values to the K 'nearest' other observations. The distance of the variables is the distance of the other features in the dataset that are non-missing. K-NN imputation is non-parametric and does not require any assumptions on the underlying distribution of the panel. Fix and Hodges (1989) came up with the first non-parametric concept related to K-NN. Batista et al. (2002) successfully applied K-NN to impute data for use in machine learning algorithms. In this specific problem, a whole row of data is missing. Therefore, finding the nearest neighbours in the original form is not easy. Since the data is in panel form, this problem can be solved. Firstly, the data is converted from 'long' to 'wide' format. In the long format, all observations are ordered as rows. In the wide format, rows are determined by the dates and each underlying stock has its column. In this way, the K-NN algorithm uses the panel structure of the data to find the nearest neighbours. The result is a balanced panel. The imputed panel consists of $43,692$ option-month observations. Summary statistics on the resulting returns panel are presented in table 1

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Hedged return	43,692	−0.0024	0.0423	−0.9432	−0.0148	0.0049	1.5663

Table 1: Summary statistics on the delta-hedged returns panel

Table 1 shows that the average return on the delta-hedged option portfolios in the sample is negative. This result is similar to the studies of Bakshi and Kapadia (2003), Horenstein et al. (2020) and Cao and Han (2013).

5.3 Characteristics

An essential question for this thesis is which characteristics are relevantly related to delta-hedged option returns. This is mainly an empirical question. A large set of characterstics is tested. Testing procedures described earlier in this thesis must find the relevant charachteristics. Furthermore, it is interesting to find whether the same characteristics are suitable for the factors in the two models IPCA and PPCA. Moreover, Kelly et al. (2019) argue that a large number of characteristics is beneficial to the performance of IPCA models, even if some of them are highly correlated or noisy. Dimension reduction enables IPCA to handle many characteristics without an enormous number of parameters. PPCA models are less parsimonious than IPCA models on average since they need to estimate unique loading components for options on each underlying stock.

In previous literature, multiple characteristics are found that could be relevant in either a PPCA or IPCA model. The characteristics used by Freyberger et al. (2020) are taken as a starting point. The categorization of that study is adopted in this thesis. These are all characteristics of the underlying stocks of the options and the companies belonging to those stocks. Their characteristics are appealing because they are large in number and spread over different categories. Freyberger et al. (2020) use these characteristics to explain the cross-section of stock returns. Kim et al. (2021) also utilize these characteristics to form arbitrage portfolios of stocks. In this thesis, the focus is on option returns. Therefore the characteristics of Freyberger et al. (2020) are extended by a few option specific characteristics. Kelly et al. (2019) use 36 characteristics to build well-performing IPCA models for stock returns. The majority of those characteristics are also included in this research. Table 19 provides a short explanation of all characteristics used in this research.

Cao and Han (2013) find that delta-hedged equity option returns decrease monotonically with an increase in idiosyncratic volatility of the underlying stock. They find that this cannot be explained solely by common risk factors. Therefore it is interesting to use this variable in the conditional latent factor models and find if and how it influences option returns via conditional latent factor models. Horenstein et al. (2020) develop a characteristic based four-factor model for equity option returns. Next to Idiosyncratic volatility, they also find that historical volatility, implied volatility and size have explanatory power over equity option returns. Therefore, such characteristics are included in our analysis. Cao et al. (2021) find that expected returns are negatively correlated with stock prices, profit margin and profitability. We include past returns in our characteristics and several characteristics on the profitability of the companies, such as Earnings-per-share (EPS) and Gross profitability over book equity value(Prof).

Besides the characteristics of the underlying stock and company as in Freyberger et al. (2020), option-specific characteristics are included in the sample. These characteristics are gathered and constructed from OptionMetrics data. It includes the option Greeks Delta, Theta and Vega. Furthermore, the characteristics sample includes implied volatility, trading volume and open interest. Christoffersen et al. (2018b) find that the bid-ask spread of options has a considerable influence on delta-hedged returns, therefore this variable is also examined in this study.

The primary sources for the characteristics data are the Center for Research in Security Prices (CRSP) and COMPUSTAT. Since COMPUSTAT consists mainly of yearly financial statements of large companies, most of this data is yearly. Our panel consists of monthly returns; consequently, the COMPUSTAT data has the same value for each month of the year. From CRSP, both daily and monthly data are used, depending on the characteristic. The monthly data is already in the proper format for calculating monthly variables such as the percentage change in outstanding shares (SHROUT). Some monthly CRSP data is also aggregated to yearly numbers to use in combination with COMPUSTAT variables; an example is the total market value of equity. Daily CRSP data is used to calculate monthly statistics, such as the standard deviation of the residuals of a monthly regression, which represents Idiosyncratic volatility. This volatility results from a regression of excess returns of the underlying stocks on the 3-factor model of Fama and French (1993) (market excess return, SMB and HML). The realizations of the three factors are obtained from the website of Kenneth French. The 48 industry portfolio classification of Fama and

French (1997) is used to adjust some characteristics by their industry mean. An example is the adjusted ratio of the book value and the market value of equity.

For IPCA, a standardization of the characteristics similar to Kelly et al. (2019) is applied. Firstly, the characteristics are ranked from small to large. Subsequently, the ranks of each observation of each characteristic are divided by the number of month-option observations. Afterwards, 0.5 is subtracted to centre the characteristics around zero. All standardized characteristics observations are in the range [-0.5,0.5]. After this standardization, there can be no outliers in the data that largely influence the model. After testing, the descriptive performance of IPCA is much better with the standardization of the coefficients than without standardization.

All characteristics that are described in table 19 are time-varying. The projected PCA method of Fan et al. (2016) has static factor loadings and can only handle static characteristics. Therefore, the mean over time of the characteristics of each cross-sectional observation is used in the PPCA models.

6 Empirical results

This section presents the results of the described methods in section 3. This section is organized as follows, firstly the results of IPCA on the equity options panel are presented, and then the PPCA results are shown and analyzed. The last subsection compares the results of IPCA and PPCA and places them in contrast to the more traditional PCA approach of modelling a panel with latent factors.

6.1 Instrumented PCA results

6.1.1 In-sample explanatory performance

IPCA models are estimated on the delta-hedged call option panel described in section 5.2 using all standardized characteristics as described in in section 5.3. Table 2 shows performance measures of IPCA models for various numbers of factors K. Both restricted and unrestricted models are estimated. The intercept of the unrestricted model factor is not counted in the number of factors K. The unrestricted models consist of $K + 1$ factors, of which K are latent IPCA factors, and one is the intercept factor. The total R^2 represents the fraction of the total variance in the panel of option returns that the IPCA model describes. The IPCA factors can be considered systematic risk factors. The total R^2 represents how much of the return variation can be described by K common risk factors under the assets for the restricted model. It represents the ability of the IPCA model to describe risk. For the unrestricted model, total R^2 could also contain the results of direct interaction of the characteristics and factors.

A restricted IPCA model with one factor captures 9.85% of variation in the options returns. Adding more factors improves the fit of the model. At least up to 13 factors, the total R^2 is monotonically increasing with the number of factors. This observation makes sense since more time-varying factors are added to the model, and the number of parameters of the L x K matrix Γ_β increases with K. The model that explains most variance of the tested models contains 13 factors. The improvement in the model's total R^2 comes at the price of a less parsimonious model. The increase in R^2 is the largest for

k	Total R^2		Predictive R^2	
	Restricted	Unrestricted	Restricted	Unrestricted
1	0.0985	0.1151	-0.0030	**0.0111**
2	0.1427	0.1513	0.0045	0.0103
3	0.1672	0.1750	**0.0074**	0.0099
4	0.1884	0.1939	**0.0073**	0.0092
5	0.2051	0.2101	**0.0073**	0.0086
6	0.2188	0.2229	0.0061	0.0076
7	0.2317	0.2351	0.0055	0.0074
8	0.2421	0.2455	0.0063	0.0073
9	0.2519	0.2551	0.0060	0.0075
10	0.2612	0.2636	0.0059	0.0073
11	0.2691	0.2717	0.0062	0.0073
12	0.2764	0.2788	0.0057	0.0074
13	**0.2830**	**0.2851**	0.0052	0.0068

Table 2: Explanatory Performance of IPCA on individual option returns (all characteristics included) . The best results are highlighted.

the smallest number of factors. From $K = 1$ to $K = 2$, the total R^2 increases by 45%, while this is only 3% by increasing the number of latent factors from 10 to 11. One of the identifying assumptions explains the increase of total R^2 at a decreasing rate. The covariance matrix of the factors is imposed to be diagonal with decreasing elements. This condition implies that the first factor explains the most significant part of the variance, then the second and so on.

The unrestricted model performs better than the restricted model in terms of total R^2 for each value of K. This advantage could is no0t surprising since the unrestricted model consists of one more factor and L extra parameters in Γ_α mapping the characteristics onto the intercept factor. The total R^2 increases by 17% by allowing an intercept factor for the one-factor model. This result suggests that one risk factor does not suffice to describe the variation in realized returns conditionally. Therefore, part of the variation is attributed to conditional alphas related to the characteristics. This effect decreases when the number of factors is larger. The intercept improves the fit only by 1.9% for a six-factor model. This phenomenon suggests that most of the variation in returns could be explained by adding more latent risk factors, implying that the IPCA factors of the larger models are mean-variance efficient conditional on the characteristics. Therefore direct interaction of the characteristics with the returns becomes redundant if more factors are added to the model. Table 3 shows the results of the bootstrap test for $\Gamma_\alpha = 0$, described in section 3.2.4. It provides evidence for the significance of Γ_α only for the 4- and 5-factor models at the 5% level. Unintuitively, the improvements in total R^2 by allowing for $\Gamma_\alpha \neq 0$ are only 2.9% and 2.4%, which is much smaller than for the smaller numbers of factors. For the predictive R^2 on the right side of table 2, a similar pattern as for total R^2 can be observed, so this does also not explain the significance of Γ_α for the 4- and 5-factor models. It has to be noted that the test statistic $W_\alpha = \hat{\Gamma}'_\alpha \hat{\Gamma}_\alpha$ is not based on one of the R^2 measures but uses the difference in squared parameters in Γ_α, for the restricted and unrestricted model. The bootstrap approach takes much time and computing power. Therefore, the test for Γ_α is performed for models from 1 up to 6 factors.

The predictive R^2 values are much lower than the total R^2. This measure ignores all time-varying

	K					
	1	2	3	4	5	6
p-value	0.44	0.43	0.105	0.02	0.025	0.255

Table 3: Results IPCA bootstrap test for $\Gamma_\alpha = 0$

information in the factors. Furthermore, the ALS algorithm of IPCA solves two least-squares problems to minimize the sum of residuals. In this way, the estimation's objective is to optimize the total R^2 metric. R^2 is not a direct target of the estimation. The predictive R^2 for IPCA is somewhat dynamic because the time-varying characteristics determine dynamic factor loading through Γ. The predictive R^2 for the one-factor restricted model is negative, suggesting a failure to explain the conditional variation in option returns. However, the unrestricted 1-factor model performs particularly well in terms of predictive R^2. This model even has the highest value of all estimated models, suggesting that the one latent factor can not accurately describe variation in the delta-hedged returns in terms of conditional expected returns. However, the characteristics can describe returns quite well directly. Expanding the model with more factors makes the difference in predictive R^2 between restricted and unrestricted models smaller. Nonetheless, the unrestricted model performs always slightly better. Interestingly, the predictive R^2 of the unrestricted models decreases monotonically with the number of factors, even though the total R^2 increases. This could indicate that the larger models are over-fitted to the data. For the restricted model, the predictive R^2 has a peak at 3-, 4- and 5-factor models. Adding more factors decreases the fit again. Indicating that the models with 3,4 and 5 factors are describing the underlying risk factors most correctly, and the increase in Total R^2 for larger models could maybe be a consequence of overfitting to the data.

	Total R^2		Predictive R^2	
k	Restricted	Unrestricted	Restricted	Unrestricted
1	0.7286	0.7378	0.0083	-0.0041
2	0.8286	0.8422	0.0171	0.0065
3	0.8723	0.8808	-0.0017	0.0023
4	0.8983	0.8984	0.0146	-0.0061
5	0.9152	0.9181	0.0089	-0.0048
6	0.9287	0.9309	-0.0144	-0.0204
7	0.9387	0.9409	-0.0118	0.0081
8	0.9466	0.9478	0.0023	-0.0063
9	0.9539	0.9554	-0.0025	-0.0120
10	0.9599	0.9604	-0.0051	0.0059
11	0.9641	0.9655	0.0082	-0.0081
12	0.9684	0.9696	-0.0014	-0.0123
13	0.9719	0.9722	-0.0030	-0.0097

Table 4: Performance statistic for characteristic-managed portfolios (all characteristics are included)

As described in section 3.2.3 the set-up of this thesis provides a natural way to form portfolios to be used as test assets. All 69 characteristics are used to form 69 characteristic-managed portfolios. The standardization of the characteristics, as described in section 5.3, is applied before forming the portfolios. By combining the options into portfolios, a large part of the option's idiosyncratic risk is likely to be diversified away. Therefore it can be expected that the fit of the IPCA model for these portfolios is better than for individual assets. Table 4 shows the IPCA performance measures on managed portfolios, which confirm the hypothesis. The total R^2 of the restricted IPCA model is already 73% with only one latent factor, increasing to 94% for six latent factors. The total R^2 for the unrestricted model is better than for each k. However, the increase in R^2 seems so small that it is not worth giving up parsimony for an increase in model fit. The fit improves for the smallest three models by adding an intercept, but for models with four or more factors, most variation in returns seems covered by the latent risk factors. This means that risk factors can explain almost all variation in returns related to the 69 characteristics, and there are no alphas. The predictive R^2 is mostly lower than for the individual option returns. Also, it seems random that this measure is negative for part of the models and positive for another part of the models. No pattern is present in this result for both the restricted and unrestricted models. The most intuitive explanation would be that the model entirely fails in explaining return variation for managed portfolios by conditional expected returns, that the results are positive or negative at random.

6.1.2 Out-of-sample fit

A useful property of the IPCA model as defined in section 3.2 is that it uses characteristics of period t to determine the returns and factor realizations of period $t + 1$. This section presents the results of the out-of-sample procedure described in section 4.1. The backward-looking algorithm starts by using all data up to time $t = 66$, corresponding to June 2015. The model is estimated for an expanding window using data up to time t (for $66 \leq t \leq 131$). The out-of-sample predicted returns are saved and compared to the real returns to calculate the total and predictive out-of-sample R^2. Only the models from 1 up to 6 factors are assessed due to the large time and computing power intensity. Incorporating more factors increases the running time fastly. The models are estimated 66 times for each K, and the computation time also increases for larger models.

	Total OOS R^2		Predictive OOS R^2	
k	Restricted	Unrestricted	Restricted	Unrestricted
1	0.0587	0.0940	-0.0035	0.0066
2	0.0976	0.1130	0.0048	0.0074
3	0.1181	0.1411	0.0076	0.0076
4	0.1480	0.1614	**0.0085**	**0.0096**
5	**0.1669**	**0.1658**	0.0084	0.0075
6	0.1659	0.1635	0.0082	0.0082

Table 5: IPCA Out-of-sample performance for individual option returns

Table 5 shows The out-of-sample performance measures for individual returns. The out-of-sample total R^2 is a bit lower than the in-sample total R^2 as in table 2 for each number of factors. For the restricted 1-factor model, the out-of-sample total R^2 is 40% lower than the in-sample R^2. For the restricted 5-factor model, this is only 19%. Up to 5 factors, this gap decreases with the number of factors. These results suggest that the in-sample performance of IPCA is not a result of overfitting. Kelly et al. (2019) find also find a slight reduction in total R^2 for stock returns and Büchner and Kelly (2022) find this for index option returns.

Interestingly, the total R^2 decreases when adding a sixth factor, both for the restricted and unrestricted model. The in-sample R^2 increases when adding a sixth factor, which could be a result of overfitting. This result suggests that the 5-factor IPCA model best describes the delta-hedged equity option returns. The same phenomenon is observed for the out-of-sample predictive R^2. These results align with the finding that the in-sample predictive R^2 is highest in both the 5-factor models. In contrast to the in-sample performance measures, the out-of-sample fit of the restricted model is better than for the unrestricted model in total R^2. This could indicate that no significant alphas are associated with the characteristics, and the common risk factors are conditionally mean-variance efficient. In combination with the results in table 2, it may well be that adding an intercept to a 5- or 6-factor model results in overfitting to the data. The out-of-sample predictive R^2 is higher than its in-sample counterpart for the 2- up to 6-factor restricted models. The same yields for unrestricted models with 4 and 6 factors. The highest predictive R^2 is for $K = 4$ both for the restricted and unrestricted model. Adding more factors does not result in a better prediction of out-of-sample returns by conditional expected returns.

The out-of-sample total R^2 is not representative of predicting returns in real life. The returns of period $t + 1$ r_{t+1} are needed to calculate the factor realizations of time $t + 1$. Of course, the future returns of the future are not known in a real-world application. However, it does say something about the fact that the performance of the IPCA models is not purely dependent on the in-sample test data. The predictive R^2 could be implemented in practice since there is no need to calculate the factor realizations of period $t + 1$.

	Total OOS R^2		Predictive OOS R^2	
k	Restricted	Unrestricted	Restricted	Unrestricted
1	0.6356	0.5907	-0.0531	-0.0425
2	0.7645	0.7705	-0.0397	-0.0342
3	0.7870	0.8118	-0.0103	-0.0299
4	0.8127	0.8320	-0.0239	-0.0156
5	0.8626	0.7988	-0.0332	0.0004
6	**0.8805**	**0.8853**	-0.0319	-0.0207

Table 6: IPCA out-of-sample performance for managed portfolio returns

Table 6 reports the out-of-sample performance measures for IPCA applied to managed portfolios or the options panel. The Total R^2 for the restricted model is increasing with K. For most K, the unrestricted model performs better than the restricted except for $K = 5$ and $K = 6$, where the gap is minor. This observation suggests that there are no alphas out-of-sample related to the portfolios, and all variation in returns that relate to the characteristics is captured by the 5 or 6 common risk factors. The predictive R^2 is negative for almost all models. The model fails to describe return variation by expected returns based on the characteristics, similar to the in-sample predictive R^2 on these test assets.

6.1.3 Mean-variance efficiency

This section studies the mean-variance efficiency of the IPCA factors in terms of Sharpe ratios. This supports placing the performance in an economic context. Since the gap in total and predictive R^2

between the restricted and unrestricted model is not large(especially for models with 3 or more factors), large alphas are not expected. This suggests that the IPCA factors are mean-variance efficient conditional on expected returns. The Sharpe ratios of tangency portfolios must inform whether the factors are also unconditionally mean-variance efficient.

	K						
Factors	1	2	3	4	5	6	Tan
1	0.50						0.50
2	0.52	0.27					0.58
3	0.31	0.34	0.36				0.74
4	0.06	0.19	0.19	0.24			0.80
5	-0.05	0.27	0.35	0.29	0.47		0.88
6	-0.07	0.28	0.39	0.19	0.39	0.48	1.02

Table 7: Out-of-sample Sharpe ratios for IPCA factors and the tangency portfolio of factors of IPCA. For the restricted model estimated on individual option returns

A similar backward-looking approach as for the out-of-sample R^2 is employed, described in section 4.1.1. The out-of-sample factor realizations are calculated. Moreover, weights of tangency portfolios of the factors are computed using only information up to time t, as well as the returns of these portfolios. As in section 6.1.2, the first estimation uses all information up to June 2015, which is five and a half years. Table 7 presents out-of-sample Sharpe ratios of the individual factors and the tangency portfolios for models with one up to six factors. The restricted model is used to form these portfolios since it does not make sense to evaluate the mean-variance efficiency of a constant factor of ones.

The 1-factor model has a Sharpe Ratio of 0.50, which is by definition equal to the Sharpe Ratio of the tangency portfolio. The mean-variance efficiency of the tangency portfolio increases if more factors are added. This is the result of the fact that there are more factors to combine into a mean-variance efficient portfolio. The first factors of the larger models have lower Sharpe ratios than the first factors of the smallest two models. The identification assumptions of IPCA ensure that the first factor describes most variation in option returns, but this does not imply that it has the highest mean-variance efficiency. For comparison, the Sharpe ratios for IPCA applied to managed portfolios can be found in table 20 in the appendix.

6.1.4 Characteristic significance

One of the major advantages that IPCA has over latent factor methods such as regular PCA is economic interpretability. Thus far, many characteristics (69) have been included in the models. Since IPCA reduces the number of parameters in the factor model by estimating Γ instead of all factor loadings, it is not a problem to incorporate many characteristics. Kelly et al. (2019) expect that it is not a problem for IPCA performance if characteristics are correlated, noisy or even spurious. According to their paper, the matrix Γ can find a few characteristics to be most informative about the factor loadings and filter out noise by aggregating characteristics into linear combinations. Nevertheless, it is valuable to assess which characteristics are most crucial in explaining the option returns. For investors, it is useful to know which characteristics are related to delta-hedged option returns via common risk factors. They could use this

	K							K					
Char	1	2	3	4	5	6	Char	1	2	3	4	5	6
A2ME	0.82	0.66	0.97	0.97	0.91	0.98	**PM**	0.39	0.06	0.12	0.57	**0.03**	**0.04**
AOA	0.79	0.55	0.66	0.94	0.24	**0.01**	PM_adj	0.98	0.73	0.41	0.77	0.87	0.39
ATO	0.65	0.77	0.76	0.11	**0.1**	0.21	Prof	0.44	0.25	0.35	0.67	0.34	0.34
Spread_O	0.95	**0.08**	0.14	0.43	0.62	0.61	Q	0.43	0.15	0.26	0.14	0.25	0.33
BEME	0.72	0.65	0.76	0.47	0.55	0.01	RNA	0.58	0.84	0.66	0.2	0.23	0.3
BEME_adj	0.8	0.28	0.3	0.12	0.2	0.22	ROA	0.98	0.96	0.31	0.48	0.26	0.76
Beta_daily	0.69	0.65	0.41	0.38	0.52	0.17	ROC	0.89	0.76	0.48	0.58	0.72	0.71
C	0.35	0.29	0.2	0.2	0.25	0.39	ROE	0.91	0.52	0.87	0.91	0.96	0.95
C2D	0.17	0.13	0.21	0.4	0.43	0.52	ROIC	0.53	0.57	0.49	0.46	0.34	0.41
CTO	0.66	0.61	0.57	0.62	0.57	0.68	Rel_to_High	0.33	0.41	0.26	0.31	0.33	0.33
DGm_DSale	0.72	0.44	0.98	0.81	0.75	0.85	**Ret_max**	0.85	0.75	0.2	0.14	0.07	**0.05**
DTO	0.84	0.23	**0.1**	0.21	0.24	0.16	S2C	0.39	0.41	0.27	0.23	0.31	0.56
Debt2P	0.24	0.5	0.37	0.85	0.56	0.32	SAT	0.61	0.75	0.58	0.64	0.67	0.51
ΔCEQ	0.4	0.54	0.59	0.9	0.94	0.84	SAT_adj	0.91	0.8	0.51	0.43	0.59	0.54
ΔPI2A	0.53	0.34	0.18	0.37	0.27	0.3	**SGA2S**	0.28	0.47	0.16	**0.06**	**0.03**	0.11
ΔSHROUT	0.93	0.2	0.44	0.57	0.73	0.18	**SPREAD**	**0.0**	**0.02**	**0.0**	**0.0**	**0.0**	**0.01**
ΔSO	0.51	0.48	0.39	0.4	0.54	0.5	**SUV**	0.85	0.96	0.46	0.14	0.16	**0.03**
E2P	0.66	**0.04**	0.21	0.37	0.63	**0.03**	Sales_g	0.83	0.31	0.36	0.53	0.55	**0.85**
EPS	0.74	0.3	0.29	0.44	0.16	0.07	**Std_turn**	0.62	**0.08**	**0.05**	**0.0**	**0.0**	**0.0**
FreeCF	**0.06**	**0.07**	**0.06**	**0.01**	**0.0**	**0.03**	Std_vol	0.72	0.54	0.53	0.38	0.24	0.19
IPM	0.25	0.43	0.25	0.16	0.16	0.17	**Tan**	0.14	0.22	0.12	**0.02**	**0.01**	**0.02**
IVC	1.0	0.15	0.47	0.63	0.72	0.13	At	0.48	0.93	0.26	0.18	0.31	0.33
Idio_vol	0.32	0.35	0.43	0.75	0.81	0.58	Total_vol	0.9	0.4	0.3	0.52	0.6	0.96
Investment	0.98	0.56	0.39	0.43	0.29	0.51	**Delta**	0.24	0.35	0.46	**0.0**	**0.0**	**0.05**
LDP	0.98	0.96	0.68	0.73	0.39	0.32	**Impl_vol**	0.33	1.0	**0.05**	0.12	0.23	**0.02**
LME	0.17	0.24	0.41	0.35	0.53	0.03	**open_int**	0.35	0.39	0.31	0.76	**0.06**	0.12
LME_adj	0.47	0.12	0.14	0.32	0.16	**0.1**	r_{12-2}	0.52	0.37	0.42	0.59	0.74	0.36
LTurnover	0.75	**0.04**	**0.0**	**0.0**	**0.0**	**0.0**	r_{12-7}	0.83	0.55	0.92	0.84	0.91	0.54
Lev	0.88	0.24	0.27	0.53	0.23	0.21	r_{2-1}	0.93	0.93	0.83	0.74	0.69	0.48
NOA	**0.02**	0.12	**0.03**	**0.06**	**0.01**	**0.08**	r_{36-13}	0.84	0.57	0.88	0.97	0.78	0.86
NOP	0.35	0.88	0.95	0.91	0.74	0.81	r_{6-2}	0.64	0.52	0.56	0.63	0.87	0.8
O2P	0.42	0.83	0.86	0.85	0.84	0.72	theta	0.52	0.26	0.55	0.37	0.37	0.56
OA	0.86	0.57	0.72	0.91	0.72	0.81	**Vega**	**0.0**	**0.01**	**0.02**	**0.02**	**0.01**	0.15
Ol	0.39	0.37	0.41	0.34	0.24	0.19	Volume	0.97	0.98	0.86	0.7	0.61	0.73
PCM	0.43	0.81	0.57	0.48	0.44	0.48							

Table 8: Significance of each characteristic for the IPCA model. The numbers are calculated using the bootstrap testing approach as described in section 3.2.5 The bold numbers are significant at a 10% or lower level. The characteristics for which the name is bold are significant in at least one of the 6 models .

knowledge in their investment strategy. Section 3.2.5 describes the testing procedure for characteristic significance. It tests which rows of Γ_β are significantly different from zero. The p-values for this test are reported in table 8 for IPCA models from 1 up to 6 factors. The number of bootstrap samples for each characteristic is 200.

Table 8 presents the results of the instrument significance test. The significant characteristics at the 10% level are made bold, and names of the significant characteristics for at least one of the factor models are also made bold. 19 out of the 69 characteristics are significant for at least one model. Ten characteristics are significant for two models or more. Two variables are significant in all models: the ratio of Free Cash flow to book equity and the average daily bid-ask spread of the underlying stock. Free cash flow is the cash that remains in a company after operating and capital expenses. This shows how much cash a company generates to utilize for dividends or growth investments for example. Therefore this can be an important indicator for investors in the company's equity, which also influences the delta-hedged option returns following this result. Book equity is a scaling factor in this characteristic. Intuitively, one would expect that a large free cash flow would increase the return on call options. Table 8 does not contain information about the sign of this relation. Christoffersen et al. (2018b) find that the bid-ask spread of the options largely influences the delta-hedged returns. Table 8 shows that those are only significant for the 2-factor IPCA model. However, the bid-ask spread of the underlying stock is relevant for each model important in explaining returns. Vasquez (2017) find that returns on straddle portfolios of options (consisting of a put and call option on the same underlying) are positively related to the implied volatility term structure slope. Implied volatility is also a significant contributor in two of the IPCA models. The underlying stock's net operating assets (NOA) are significant in five out of the six models. Papanastasopoulos et al. (2011) find that NOA is a strong negative predictor for future stock returns. This strong relation also translates to equity option returns via common risk factors as shown in table 8.

Coming back to the categorization of Freyberger et al. (2020) shown in table 19, six of the significant characteristics have to do with trading frictions. Two of them are relevant in five or more models. These variables are about trading frictions of the underlying stock. The option-specific category characteristics are important, which is the category specifically for this study. Five out of the eight option specific characteristics are significant in at least one of the models. These characteristics are most directly related to the options. Vega is significant in five of the models. Vega is the option price sensitivity to the change in implied volatility on the underlying asset. Since all options are in their last month before maturity, one would expect a high Vega to harm the option returns. Büchner and Kelly (2022) find that Vega is one of the most important predictors in an IPCA model for index options. None of the characteristics in the category of past stock is significant. Although momentum and reversal effects are often found for stock returns[Fama and French (1998), Jegadeesh and Titman (1993)], there is no evidence that these effects on the stock influence delta-hedged option returns.

Only a part of the characteristics is relevant for the factor loadings. A natural next step is to estimate IPCA models, including only the significant characteristics. Table 9 shows the total and predictive R^2 for the restricted model when only the significant characteristics from table 8 are included. The last row shows that the number of significant characteristics increases with the number of factors. Although these models include all significant characteristics, the performance is worse than the 'full' model, both in total and predictive R^2. The 'small' 1-factor model's total R^2 is 48% lower than the full model. For the 6-factor model, this is 38%. Hence, the characteristics that are not significant in the 'full' models have some explanatory power for the delta-hedged option returns that cannot be ignored completely. If the main goal is to maximize the variance of delta-hedged option returns explained by the model, the best choice is to include all characteristics in the model.

6.1.5 Interpretation of the IPCA models

A major advantage of the IPCA method is that it estimates a mapping of the characteristics to the factors instead of individual factor loadings, as shown in equation (5). The models are more parsimonious and easier interpretable than regular PCA, where options on each underlying have a unique loading.

K	1	2	3	4	5	6
Total R^2	0.0504	0.0769	0.1132	0.1228	0.1367	0.1354
Predictive R^2	0.0001	-0.0004	0.0049	0.0028	0.0026	0.0005
p	4	6	7	9	12	15

Table 9: In-sample performance measures for IPCA modes without intercept where only the significant characteristics are included. p is the number of characteristics included in the model

Furthermore, PCA uses no additional information that can be interpreted in the form of characteristics. In this section, the IPCA 4-factor model is used as an example to show how the interpretability of IPCA models. Of course, the meaning of the factors could differ for different numbers of factors. The ALS optimization leads to different factors for the different models. The first factor of a 2-factor model is not the same as the first factor of a 3-factor model on the same data. Therefore the conclusions drawn in this section are only valid for this specific 4-factor model. Figure 1 shows the mapping from characteristics to factors for the first factor of the 4-factor model. It represents the first column of the matrix $\hat{\Gamma}_\beta$. Given the identification restriction, the first factor is essential since it explains the largest part variation in option returns.

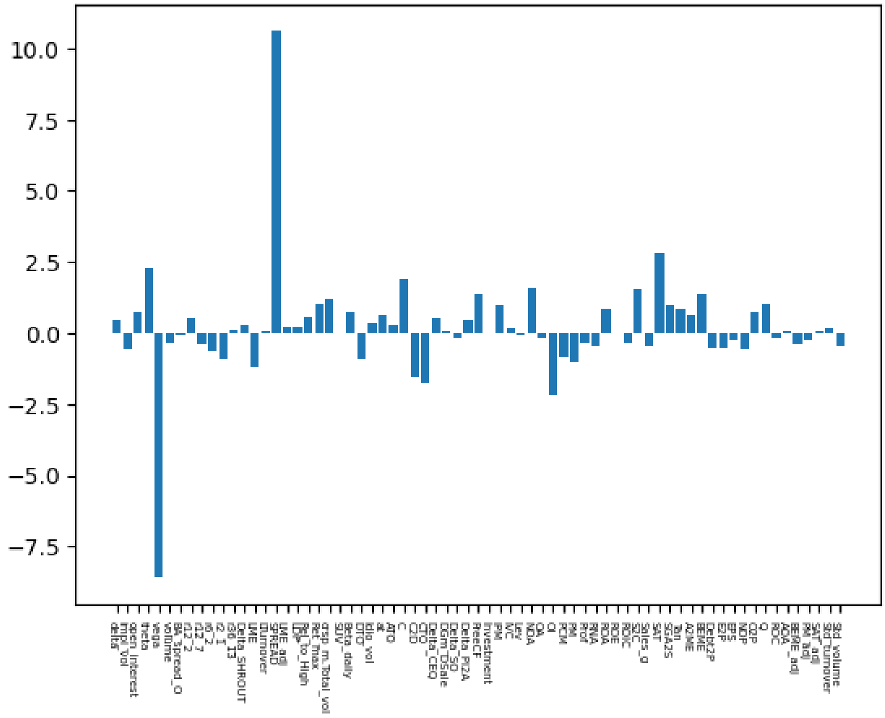

Figure 1: Coefficients of $\hat{\Gamma}_\beta$ relating to the first factor of a 4-factor IPCA model including all 69 characteristics

Figure 1 shows that the loadings on the first factor mainly depend on the options Vega and the bid-ask spread of the underlying stock. Table 8 shows that both characteristics are significant for the 4-factor model. The mapping of Vega on the first factor is negative, and the mapping of the bid-ask spread is positive. In combination with the fact that the mean of the first factor is positive (0.72), it can be concluded that Vega is negatively related to the expected returns through the first factor. An identification restriction used in this thesis imposes that the means of all IPCA factors are positive. Vega is the sensitivity of the option price to implied volatility. High sensitivity to volatility implies more insecurity. Given that all options are in the last month before maturity, it makes sense that this results in lower returns. The bid-ask spread is positively related to the option returns through the first factor. A large bid-ask spread of the stock indicates low liquidity. This means that it is more difficult to buy a certain stock. A call option gives the investor the possibility to buy a stock at the strike price. Therefore it can be beneficial to have a call option when the bid-ask spread of the underlying stock is high, which explains the positive relation of the underlying stock's bid-ask spread to the option returns.

The mappings of characteristics to the other factors of this model are reported in figures 8, 9 and, 10 in the appendix. 8 shows that the loadings on the second factor are mostly determined by the standard deviation of daily turnover (std_turnover) (negatively) and the standard deviation of daily volume (std_volume) (positively). The turnover characteristic is significant at the 1% level for the 4-factor model; the standard deviation of daily volume(std_volume) is not significant. The absolute value of the mapping coefficient of std_turnover is twice as high as that of the volume characteristic. Therefore it can be assumed that the std_turnover characteristic mostly influences the second factor. To be more specific, std_turnover is the standard deviation of a year of the daily turnover of the underlying stock. A high standard deviation can be caused by high insecurity about the turnover or because the turnover itself is high, such that a relatively 'normal' standard deviation is already high. Therefore, the cause of the positive influence on the option returns through the second factor could rely upon a high turnover. The third factor is more difficult to interpret, as can be observed in figure 9. Many characteristics influence this factor slightly. The loadings of the fourth factor in figure 10 are mostly determined by the options Delta and also quite a bit by the implied volatility. These are two option-specific characteristics. Delta is significant at the 1% level, and the implied volatility has a p-value of 0.12, which is also not very high. The significant positive relation of Delta to the fourth factor, which has a slightly positive mean of 0.03, is remarkable since the options are delta-hedged. It appears that the Delta of the option is still related to the option returns even though it is used to determine the hedge with the underlying stock.

As described in section 6.1.4, a 4-factor model is estimated, including only the characteristics found to be significant at the 10% level by the bootstrap test. The explanatory power of this model is worse than the 'full' 4-factor IPCA model. Figure 2 displays the mappings from the characteristics to each of the four factors. The bar charts correspond to the K-th row of the estimated mapping matrix $\hat{\Gamma}_\beta$. The model includes a total of 9 characteristics.

The mappings on factor 1 show a similar pattern to those of the 'full' model. They depend mainly on the Vega of the option and the bid-ask ratio of the underlying stock. Also, the signs of both characteristics are the same as in the full 4-factor model. Delta is the most influential characteristic in the second factor , which was the case in the fourth factor of the 'full' model. Last month's turnover has half the absolute value of the delta coefficient. The coefficients of the third factor are more fragmented.. However, Delta has the largest absolute value again. Last month's turnover dominates the fourth-factor's mappings. A high turnover negatively relates to the factor and thus to returns. A high turnover means that the underlying stock is actively traded. Hence a high turnover corresponds to high liquidity. For a low liquidity stock,

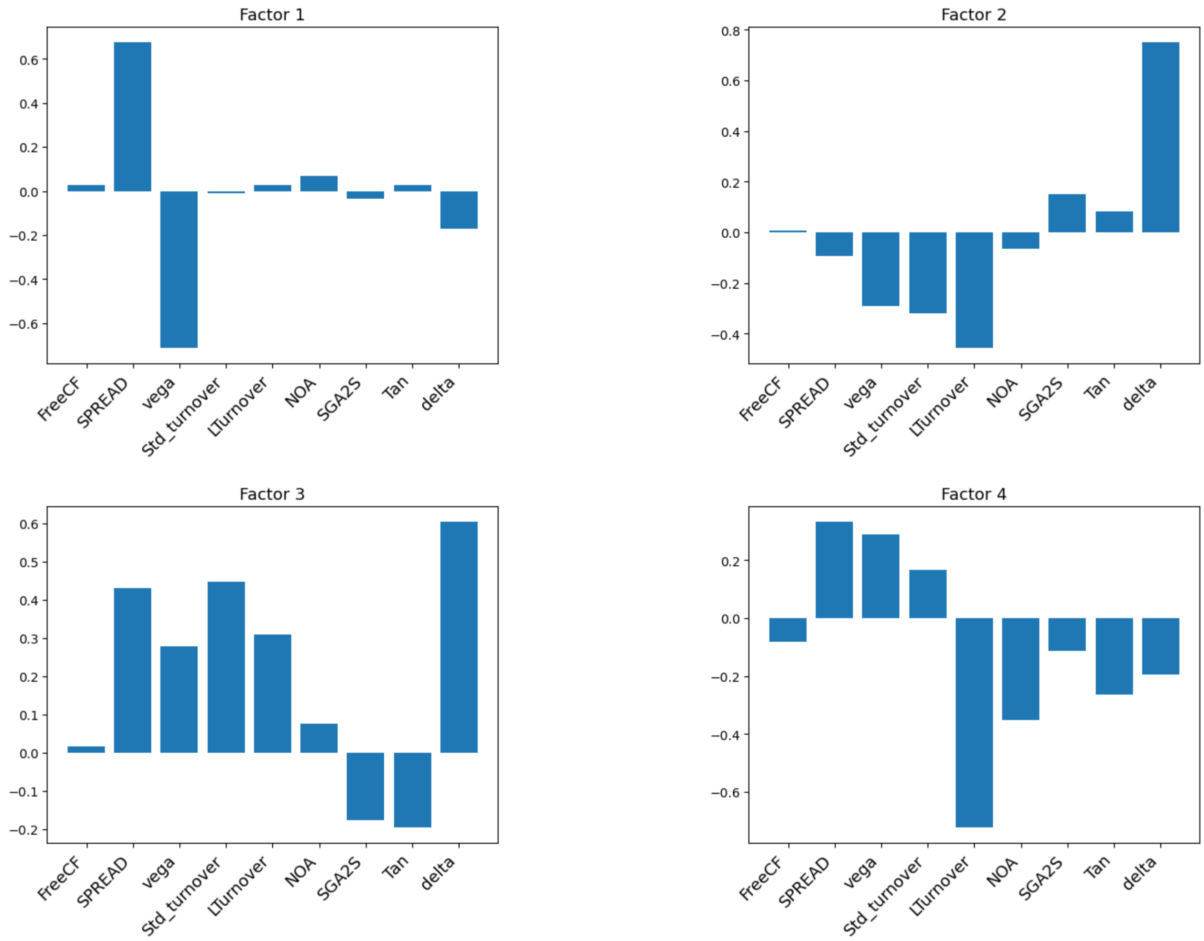

Figure 2: Mapping of characteristics onto the factors of a restricted 4-factor IPCA model including only the significant characteristics found in table 8

it can be an advantage to have a call option since it gives the right to buy a stock at the strike price. This could result in a negative relationship between liquidity and call option returns, as also last month's turnover and call option returns. Surprisingly, the Free Cash flow to book equity ratio coefficients are relatively small for all four factors. This is not expected since the coefficient is highly significant (p-value is 0.01) in the 'full' model. It could be foreseen that this characteristic is not significant if this test is performed for the small 4-factor model. To conclude on the 'small' model, it appears that the interpretation of factors is not much easier than in the 'full' model. It has fewer characteristics, but in the full model, mostly one or two characteristics stand out. Therefore the benefit of a smaller number of characteristics is not that great. A slight advantage of the 'small' model is that the loadings are easier to represent graphically. Also, there are fewer parameters to estimate. The matrix Γ_β consists of 276 parameters in the 'full' restricted 4-factor model and only 36 in the model containing only the significant characteristics. Nevertheless, given the large difference in performance found in section 6.1.4, the 'full' model would be preferred in most situations.

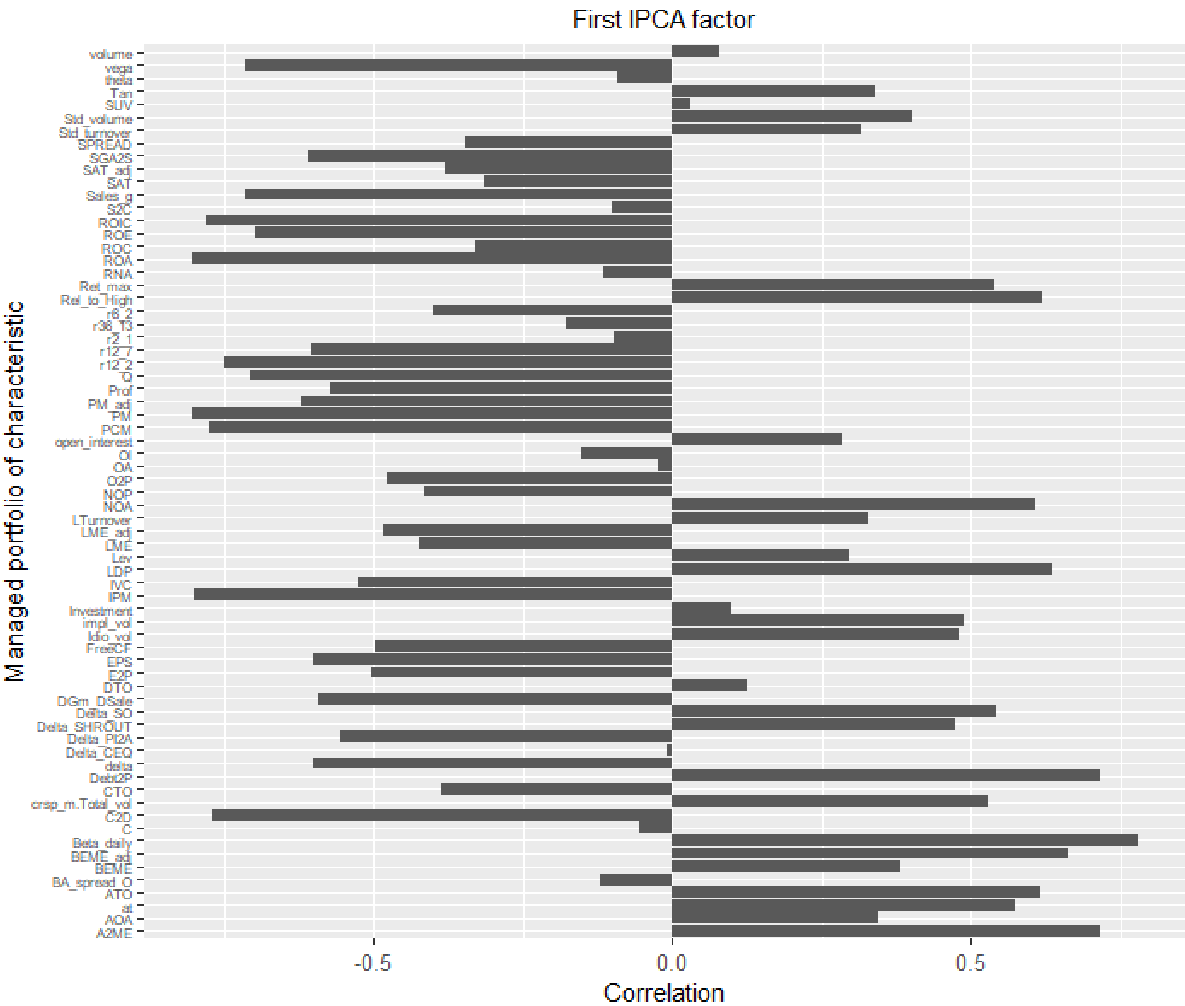

Figure 3: Correlations of the first IPCA factor of a four-factor model with managed portfolios constructed by the characteristics

Another way to interpret the factors is to link the time-series of factor realizations to the time-series of characteristic managed portfolios. Table 4 shows the correlations of the first IPCA factor of the four-factor model with managed portfolios on each of the characteristics. The managed portfolios are formed as explained in section 3.2.3. In contrast to the mapping matrix Γ_β, there are not a few characteristics that stand out. Many portfolios formed on characteristics correlate around 0.5 or -0.5 with the first IPCA factor. Not all portfolios formed on characteristics found to be significant in table 8 have strong correlations with the first factor. This makes sense since their significance could also be related to another factor. The sign of the two characteristics that dominate the first column of the matrix $\hat{\Gamma}_\beta$ as in figure 1 (bid-ask spread of the underlying stock and Vega) is the same as in figure 4. To conclude, the characteristics that mostly influence loadings on the first factor of the IPCA model are not the only characteristics correlated with that factor. Therefore it is more difficult to pin down in economic meaning of the factors using this approach. The correlations of the other factors of the four-factor model with the managed portfolios are presented in tables 14, 15 and 16 in the appendix.

6.2 Projected PCA Results

Projected PCA models are estimated on the options return panel and characteristics described in section 5. The non-parametric sieve approximations make PPCA a semi-parametric method. The approach of Fan et al. (2016) is followed and the 69 x K components $g_{kj}(X_{ij})$ are estimated by cubic splines. The sieve dimension is varied from 2 up to 4. A sieve dimension larger than four is impossible when all 69 characteristics are included in the model. The cross-section consists of options on 318 underlying stocks. A sieve dimension of $J = 5$ results in 345 sieve coefficients to be estimated, resulting in an over-identified model. Since PPCA estimates static loadings and uses static characteristics, the mean over time of the characteristics is used for options corresponding to each company. Theoretically, one could argue that returns are predicted by using information from the future. The characteristic means over the whole sample period are used.

6.2.1 In-sample explanatory performance

The same measures as for IPCA are used to assess the in-sample explanatory performance of PPCA. Table 10 shows the total and predictive R^2. The total R^2 increases both with the number of factors and the sieve dimension. The rate of this increase decreases with the number of factors. The factors are ordered such that the first factor explains the largest part of the variance in expected returns by definition. Other than for IPCA, PPCA is not estimated by an ALS algorithm. The result of the estimation is that the first factor of the 1-factor model is equal to the first factor of the 2-factor model. Similar to PCA, the larger factor models contain the smaller factor models. This yields for the factors as well as for the factor loadings. The total R^2 increases with the sieve dimension J for all considered numbers of K. This increase in total R^2 is rather small. The increase in R^2 is if the sieve dimension goes from 2 to 4 is only 3.8% for the 1-factor model and 4.5% for six factors. One could ask whether this small increase is worth doubling the number of parameters of $\mathbf{\Psi}(\mathbf{X})$ and $\mathbf{B}$ from 138 to 276. If the model is enlarged up to 13 factors 51% of the realized returns can be explained by K common risk factors of the options. However, this model is non-parsimonious and latent factor models of 13 factors are rarely found in the literature.

The predictive R^2 for the 1-factor model is relatively small for all values of J. It increases with the number of factors. For more than five factors, the predictive R^2 does increase only a little for each added factor. Starting from 5 factors, adding more sieve terms than two results for most K in a lower predictive R^2. This could result from overfitting the model to the data when adding more parameters. It seems sufficient to have five factors and a sieve dimension of two for describing realized delta-hedged option returns in this panel conditional on the 69 characteristics.

	Total R^2			Predictive R^2		
K/J	2	3	4	2	3	4
1	0.1149	0.1182	0.1193	0.0003	0.0003	0.0003
2	0.1604	0.1755	0.1770	0.0009	0.0016	0.0016
3	0.2155	0.2211	0.2236	0.0020	0.0026	0.0027
4	0.2598	0.2648	0.2709	0.0025	0.0026	0.0027
5	0.2967	0.3061	0.3138	0.0072	0.0038	0.0028
6	0.3370	0.3455	0.3523	0.0072	0.0049	0.0060
7	0.3637	0.3860	0.3915	0.0074	0.0065	0.0061
8	0.3996	0.4255	0.4313	0.0075	0.0077	0.0070
9	0.4279	0.4494	0.4593	0.0083	0.0082	0.0075
10	0.4536	0.4759	0.4860	0.0083	0.0083	0.0080
11	0.4749	0.5012	0.5101	0.0083	0.0085	0.0081
12	0.4930	0.5214	0.5334	0.0098	0.0092	0.0088
13	0.5130	0.5440	**0.5539**	**0.0099**	0.0092	0.0088

Table 10: Performance statistics for PPCA models including all 69 characteristics. K is the number of factors and J is the number of sieve coefficients for each characteristic. The best results of the two measures are highlighted.

6.2.2 Model specification tests

To motivate the use of projected PCA and incorporating these specific additional characteristics into the models, tests on $G(X)$ and Γ are performed. These are described in section 3.3.4. The first tests the hypothesis $G(X) = 0$ against the alternative $G(X) \neq 0$. It tests whether the characteristics combined have any explanatory power on the factor loadings. The second test finds whether the characteristics fully explain the factor loadings. Both tests are introduced by Fan et al. (2016). The results of both specifications tests are reported in table 11. The sieve dimensions considered in this study are relatively small. Therefore, the approach of Fan et al. (2016) is followed. The second test is compared to a chi-squared distribution with JpK degrees of freedom.

For $J = 2$ and $J = 3$ the tests rejects the null hypothesis on $G(X)$ very strongly as shown in panel A of table 11. This means that the characteristics have explanatory power for the factor loadings. Therefore, the use of PPCA on this returns panel using these characteristics is strongly supported. The loadings from options on a certain underlying stock to the factors are related to at least some of the characteristics. If $J = 4$ the p-value rises. The null hypothesis is not rejected for 2- and 4-factor models, although the p-values are still quite small. The hypothesis is significantly rejected for other numbers of factors. The relation between factor loadings and characteristics is slightly less strong for the higher sieve dimension. This is remarkable since a higher sieve dimension makes the model more flexible, and the matrix $\Psi(X)$ could better fit the data. However, the models with $J = 4$ score high on total R^2 for all numbers of

K	1	2	3	4	5	6
	A: Test for $G(X) = 0$ Chi-Squared distribution					
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0006	0.0001	0.0011	0.0000	0.0000	0.0000
4	0.1152	0.0782	0.1540	0.0793	0.0686	0.0542
J	B: Test for $\Gamma = 0$					
2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 11: p-values for specification tests on the explanatory power of the coefficients

factors.

Panel B of table 11 presents the results for the test on Γ. The null hypothesis is rejected highly significantly for all included values of K and J. This implies that Γ explains part of the factor loadings in all models. The 69 characteristics cannot fully explain the factor loadings of the PPCA models. Therefore the model that we used cannot be replaced with the more restrictive model that does not include Γ, such that $G(X) = B\Psi(X)$.

6.2.3 Out-of-sample analysis

As described in section 4.1, the out-of-sample performance of PPCA is calculated. This out-of-sample procedure does not allow for predicting real-life out-of-sample returns, as explained in section 4.1. However, it is useful to assess whether the performance of PPCA is not an in-sample result of overfitting. The results are reported in tables 29 (including Γ) and 30 (without Γ). The tables show that both approaches result in a much higher total R^2 than the in-sample measures. This could be explained by the fact that the time-varying nature of the characteristics is better incorporated into these approaches. The means of the characteristics are adjusted each time period. This makes the procedure more dynamic. Even when Γ is fully ignored, the total R^2 is much higher than the in-sample R^2. This confirms that the characteristics have an important value in the PPCA models. These results raise whether this would not be a better approach for PPCA estimation. It generates a higher total R^2, but this approach has two issues. Firstly, as explained in section 4.1, $T/2$ unique PPCA models have to be estimated, which is disastrous for the interpretability. It also takes much more time to estimate all models. Secondly, this procedure calculates the total R^2 only for the second half of the sample period. There is hardly any data to estimate the model for the first observations. An approach that solves this second problem is to estimate PPCA in smaller time windows. The first problem is not solved, but if interpretability is less of an issue, this could benefit the PPCA performance.

Table 27 displays the out-of-sample Sharpe ratios of tangency portfolios formed by the PPCA factors for different numbers of factors K and different Sieve dimensions J. If all available wealth is invested in the factors, tangency portfolios are the most-mean variance efficient portfolios. The first PPCA factors do not change if more factors are added to the model. Intuitively, it could be expected that adding more factors increases the Sharpe ratio of the tangency portfolios. For the in-sample tangency portfolios this intuition is confirmed in table 17. Nevertheless, table 27 shows that this does not hold for the out-of-sample Sharpe ratios. The out-of-sample predicted returns of the tangency portfolios are not always higher and sometimes even lower for models with more factors. In addition, the Sharpe ratios are relatively low and sometimes negative. In contrast to IPCA, PPCA does not have identifying assumptions that restrict the factor means to be positive.

6.2.4 Significance of individual characteristics

Section 6.2.2 concludes that the 69 characteristics combined have significant explanatory power over the factor loadings on the delta-hedged option panel. To gain more insights into the drivers of this explanatory power, it helps to know which characteristics significantly contribute. This study develops a bootstrap procedure to find which characteristics are significant in the PPCA models. Section 3.3.3 describes the exact testing procedure. The idea of this test is based on the bootstrap testing procedure for IPCA of Kelly et al. (2019). The results of the testing procedure for different numbers of factors are reported in table 12. The sieve dimension is 4 for all values in the table. The number of bootstrap samples for each characteristic is 200.

The p-values that correspond to a 10% significance level or even lower are in bold, as well as the names of the variables that are significant for at least one of the models. Remarkably, only 9 out of 69 characteristics are significant in one model or more. This number is less than half the number of significant characteristics of IPCA. Only the Operating Leverage characteristic is significant in all models. It corresponds to the sum of the cost of goods sold and selling general administrative expenses, divided by the total assets of the underlying company. Novy-Marx (2011) find that operating leverage could be a predictor of stock returns in the cross-section. This test result suggests that it is also related to delta-hedged option returns. Furthermore, the maximum return (Ret_max) is significant in the first five of the six models and has a low p-value of 0.12 in the 6-factor model. Ret_max is the maximum daily return of the underlying stock in the previous month. Bali et al. (2011) find a negative relation between the maximum daily return in the previous month and the expected returns of stocks. Note that the mean of the characteristics is used. This characteristic is the mean of maximum daily returns of each month. The p-value is lowest for the 1-factor model and increases slightly with each added factor. Since the first factor is equal for all PPCA models, the test suggests that Ret_max is mainly related to the first factor. The significance in the whole model diminishes a little if more factors are added to which Ret_max is less related. Total volatility is also significant in four of the six models. This characteristic represents the annual volatility of daily returns. High volatility implies insecurity about the value of the underlying stock. All options in the panel are in their last month before expiration and are all quite close to the money. High volatility brings insecurity about whether the option will end in- or out-of-the-money, which is highly undesirable at the end of the option's lifetime. Therefore, it can be expected that the influence of this variable on returns is negative. However, this cannot be concluded based on this table. Following the p-values, total volatility is likely to be strongly related to the first factor and to the fifth and sixth factor, where the p-values are smaller again. The significance of the total volatility is in line with the results of Cao and Han (2013). They find that delta-hedged equity option returns decrease monotonically with an

	K							K					
Char	1	2	3	4	5	6	Char	1	2	3	4	5	6
delta	0.95	0.76	0.87	0.89	0.51	0.48	FreeCF	0.6	0.78	0.87	0.82	0.81	0.44
impl_vol	0.33	0.41	0.61	0.64	0.33	0.33	Investment	0.5	0.68	0.49	0.42	0.36	0.2
open_interest	0.49	0.31	0.48	0.51	0.47	0.49	IPM	0.22	0.5	0.56	0.47	0.3	0.28
theta	0.14	0.44	0.45	0.51	0.34	0.35	IVC	0.9	0.9	0.9	0.92	0.79	0.77
vega	0.81	0.79	0.55	0.61	0.59	0.51	Lev	0.53	0.39	0.48	0.27	0.21	0.15
volume	0.33	0.35	0.41	0.44	0.35	0.34	NOA	0.51	0.75	0.81	0.58	0.48	0.12
BA_spread_O	0.32	0.16	0.18	0.23	0.21	0.17	OA	0.39	0.49	0.58	0.58	0.55	0.28
r12_2	**0.02**	0.38	0.44	0.49	0.54	0.58	**OL**	**0.01**	**0.02**	**0.03**	**0.04**	**0.02**	**0**
r12_7	0.25	0.52	0.66	0.6	0.61	0.65	PCM	0.38	0.45	0.53	0.49	0.41	0.42
r6_2	0.16	0.44	0.41	0.39	0.38	0.35	PM	0.53	0.68	0.73	0.83	0.7	0.6
r2_1	0.66	0.69	0.81	0.75	0.72	0.58	Prof	0.95	0.98	0.95	0.8	0.63	0.52
r36_13	0.59	0.62	0.52	0.36	0.32	0.37	RNA	0.56	0.62	0.65	0.64	0.55	0.33
ΔSHROUT	0.57	0.61	0.53	0.52	0.38	0.25	ROA	0.79	0.91	0.98	0.91	0.87	0.79
LME	0.87	0.85	0.86	0.88	0.76	0.72	ROE	0.51	0.64	0.73	0.65	0.46	0.32
LTurnover	0.45	0.33	0.38	0.19	**0.07**	**0.06**	ROIC	0.65	0.4	0.4	0.34	0.32	0.27
SPREAD	0.36	0.71	0.41	0.44	0.35	0.4	S2C	0.86	0.48	0.61	0.39	0.18	0.08
LME_adj	0.61	0.66	0.75	0.83	0.62	0.5	Sales_g	0.75	0.84	0.82	0.86	0.55	0.44
LDP	0.72	0.95	0.88	0.82	0.8	0.32	**SAT**	**0.01**	0.27	0.33	0.35	0.23	0.16
Rel_to_High	0.83	0.4	0.56	0.62	0.49	0.4	SGA2S	0.31	0.34	0.39	0.42	0.39	0.27
Ret_max	**0.01**	**0.07**	**0.06**	**0.07**	**0.09**	0.12	Tan	0.63	0.59	0.6	0.64	0.49	0.25
Total_vol	**0.01**	**0.09**	0.12	0.13	**0.06**	**0.05**	A2ME	0.15	0.24	0.36	0.27	0.12	0.15
SUV	0.59	0.45	0.54	0.63	0.47	0.43	BEME	0.53	0.77	0.79	0.82	0.72	0.57
Beta_daily	0.82	0.96	0.78	0.8	0.82	0.69	**Debt2P**	0.51	0.35	0.33	0.11	**0.07**	**0.08**
DTO	0.26	0.23	0.31	0.32	0.23	0.02	E2P	0.69	0.84	0.86	0.85	0.82	0.65
Idio_vol	0.57	0.62	0.69	0.68	0.46	0.32	EPS	0.46	0.64	0.73	0.79	0.76	0.72
at	0.23	0.27	0.4	0.36	0.42	0.51	NOP	0.86	0.84	0.92	0.74	0.7	0.54
ATO	0.53	0.47	0.42	0.37	0.21	0.17	O2P	0.82	0.69	0.74	0.66	0.65	0.58
C	0.49	0.36	0.28	0.28	0.25	**0.06**	Q	0.59	0.83	0.84	0.62	0.25	0.22
C2D	0.68	0.92	0.96	0.83	0.71	0.6	**ROC**	0.18	0.32	0.41	0.44	0.17	**0.09**
CTO	**0.04**	0.46	0.62	0.61	0.43	0.24	AOA	0.63	0.39	0.46	0.51	0.51	0.32
Delta_CEQ	0.57	0.68	0.65	0.67	0.51	0.37	BEME_adj	0.26	0.26	0.36	0.36	0.29	0.24
DGm_DSale	0.39	0.54	0.61	0.56	0.51	0.55	PM_adj	0.7	0.84	0.58	0.58	0.52	0.5
Delta_SO	0.51	0.66	0.72	0.72	0.7	0.4	SAT_adj	0.6	0.67	0.71	0.7	0.65	0.5
ΔPI2A	0.56	0.46	0.5	0.47	0.42	0.45	Std_turn	0.67	0.64	0.89	0.76	0.6	0.28
							Std_vol	0.69	0.83	0.9	0.66	0.57	0.18

Table 12: Significance levels of each characteristic in a PPCA model with K factors. The bold numbers are significant at a 10% or lower level. The characteristics for which the name is bold are significant in at least one of the 6 models. These models all use a sieve dimension of 4 ($J = 4$)
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increase in the underlying stock's volatility. The variables of last month's turnover of the underlying stock (Lturnover) and its debt to price ratio (Debt2P) are significant for the 5- and 6-factor models. These are likely to be related to the fifth and possibly also to the sixth factor. In the category trading frictions of Freyberger et al. (2020) are the most significant characteristics. Remarkably none of the option specific characteristics is significant in one of the models. These are most directly related to the options. The option specific variables are highly time-varying. PPCA only uses the means; it could be that those do not consist of relevant information.

The bootstrap testing procedure is repeated for models with $J = 3$ for the robustness of the analysis. The results are reported in table 28 in the appendix. These models have less significant variables. All significant characteristics are also significant in the models with a sieve dimension of 4.
Since only 9 out of 69 of the characteristics are significant for any of the models, it is possible to create more parsimonious models consisting of the significant characteristics only. Models are created using the characteristics found to be significant for the corresponding number of factors in table 12. The Sieve dimension of these models is four. Panel A of table 13 presents the performance in terms of total and predictive R^2 and the number of characteristics. In terms of total R^2, the models perform slightly worse than the 'full model'. For from 4 to 5 factors, there is a large jump in total R^2. The 5-factor model is closer to the 'full' model with the same number of factors. The same yields for the 6-factor model. A large part of the variation that could is explained using 69 characteristics could also be explained using only five or six characteristics. The predictive R^2 of the four, five and six-factor models are even higher than those of the 'full' model. The worse predictive R of the 'full' model could result from overfitting too many characteristics to the returns data in the 'full' model. The model with 5 and 6 significant characteristics in the 5- and 6-factor model performs almost as good as the 'full' model. In addition, the smaller model has considerable advantages in terms of parsimony. The number of parameters in B is 24 instead of 276 in the 'full' model. The smaller 5- and 6-factor models would be the better choice for this situation. The jump in performance from the four- to the five-factor model is likely due to the higher number of characteristics of the latter. The characteristics Ret_max and Total_vol are significant in 5 and 4 of the models and do not have high p-values for the other models. Therefore, these are added to the models where these are not significant. The characteristics that are only significant in the models with more factors are not included in the smaller models since these most likely relate to the last factors. Total_vol is added to the three and 4-factor models and Ret_max to the 6-factor model. Panel B in table 13 gives the results. The performance of the three and four-factor modes increases significantly by adding the extra characteristics. The jump in performance from 4 to five factors disappears. In terms of predictive R^2, the 3- and 4-factor models improve by adding Total_vol to the models, which makes it higher than the 'full' model. The panel managed portfolios described in section 3.2.3 consist of only 69 options in the cross-section. A PPCA model including all 69 characteristics is not feasible for this panel. However, PPCA could be applied using only the significant characteristics, including ret_max and total_vol in all models. The results are presented in table 14. The performance in total R^2 is better than on the individual equity option returns. Diversification of idiosyncratic risk is probably the cause of this difference.

K	1	2	3	4	5	6
A: Only the significant characteristics						
Total R^2	0.1074	0.1474	0.1686	0.1910	0.2595	0.2809
Predictive R^2	0.0002	0.0003	-0.0011	0.0034	0.0062	0.0061
p	6	3	2	2	5	6
B: Including two characteristics in all models						
Total R^2	0.1074	0.1474	0.1883	0.2147	0.2595	0.2950
Predictive R^2	0.0002	0.0003	0.0052	0.0068	0.0062	0.0062
p	6	3	3	3	5	7

Table 13: Performance measures for PPCA models only including the characteristics that are significant at the 10% level as shown in table 12. p is the number of characteristics included in each model. The sieve dimension of the models is 4, which is the same as in table 12

K	1	2	3	4	5	6
Total R^2	0.4738	0.5749	0.6591	0.7156	0.7523	0.7752
Predictive R^2	0.0048	0.0065	0.0119	0.0120	0.0123	0.0131

Table 14: PPCA applied to managed portfolios, using only the significant characteristics as in table 12 and a sieve dimension of 4

To conclude this section, this study introduces a new approach for finding strong PPCA models. Firstly, a new bootstrap procedure is introduced to identify the relevant characteristics. This enables the user to start with a large set of characteristics to narrow it down to a few significant ones. The results indicate that the parsimonious models containing only the significant characteristics do not perform significantly worse than the 'full' models and should therefore be preferred.

6.2.5 Interpretation of the PPCA models

The interpretability of the factors is an essential aspect of factor models. This section shows how the PPCA models could be interpreted. The 4-factor PPCA model with a Sieve dimension of 4 is used as an example. Interpreting the PPCA factors is a bit more involved than the IPCA factors. The loadings from characteristics to factors are often nonlinear. Therefore it is not possible to map the relation of all characteristics and factors in a neat picture. Also, the factor loadings in PPCA are only partly determined by the characteristics. Part of the loadings is specific for each cross-sectional unit and unrelated to the characteristics. This part is captured in Γ. The specification test in section 6.2.2 proves the existence of highly significant Gammas in all considered models. This shows that that part of the factor loadings cannot be ignored.

Figure 17 in the appendix displays the loadings of options of each underlying stock on each of the four factors. Since the panel consists of options corresponding to 318 different companies, a large number of loadings needs to be estimated. This makes the pictures a bit blurry and the interpretation more difficult. It does stand out that almost all loadings on the first factor are negative, while the mean of the first factor is positive (0.0053). The first factor explains most of the variation by definition and the mean of the delta-hedged returns is negative as shown in table 1. This could explain the negative loadings on the positive factor. The options on one of the companies have a loading that is much larger than the others.

This negative loading corresponds to CBS Corporate, which is a media conglomerate. The factor mean of the third factor is negative, so the returns explained by this factor will be positive for CBS. There are also three or four companies with large loadings on the other two factors. However, this does not easily explain the economic meaning of the factors.

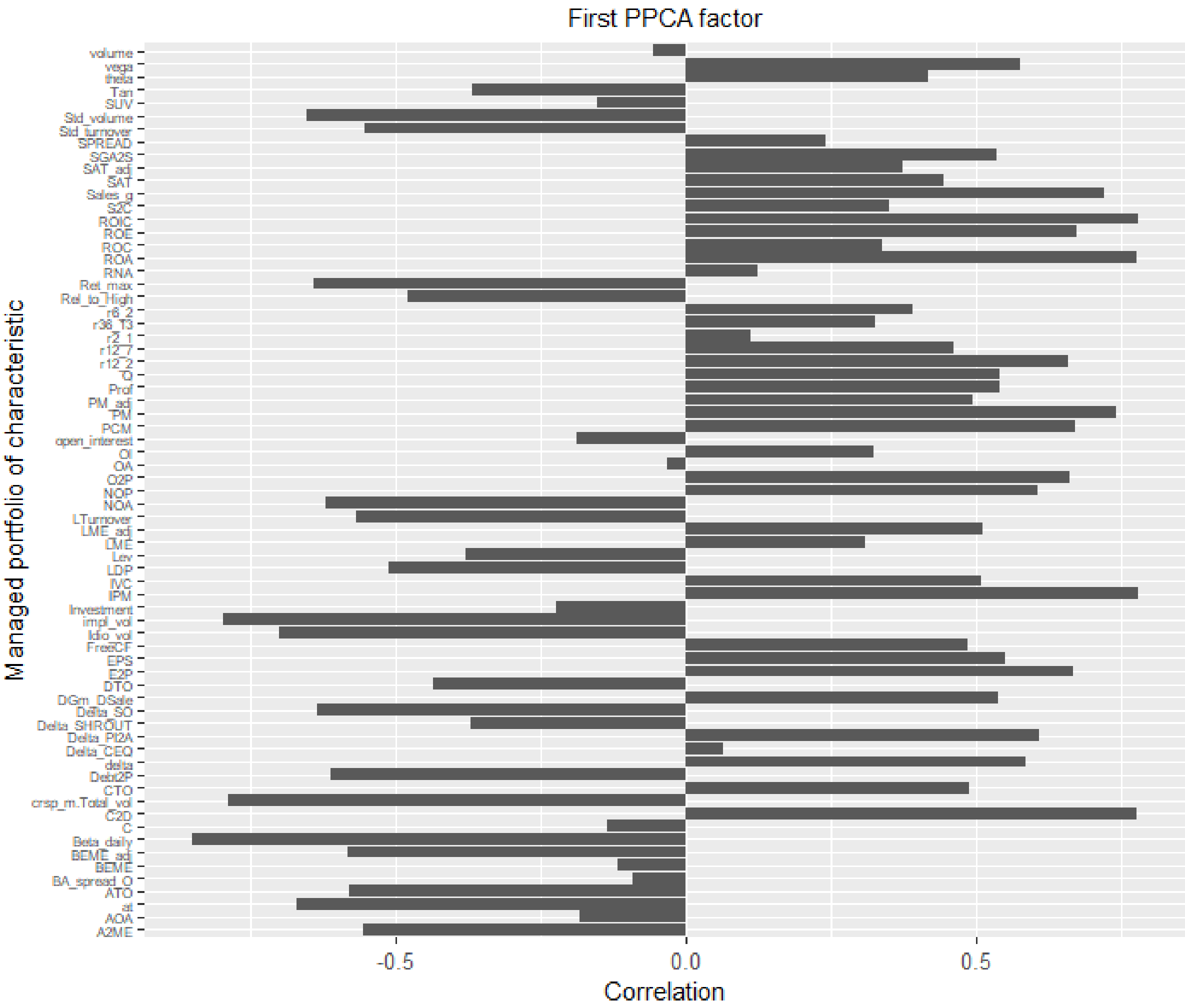


Figure 4: Correlations of the first factor of four-factor PPCA model with managed portfolios constructed by the characteristics

Therefore, the time-series of factor realizations is linked to the returns on characteristic-managed portfolios. The construction of those portfolios is described in section 3.2.3. Table 4 presents the correlations of all managed portfolios with the first factor of a four-factor PPCA model. There are not a few portfolios that stand out directly. The CAPM Beta (Beta_daily) portfolio calculated using daily returns has the strongest correlation of -0.85 with the first factor. This portfolio has a negative relation to the first factor. Nevertheless, table 12 shows that this characteristic is not significantly related to the factor loadings. Beta_daily is not influencing the loadings on the first factor, but it is strongly negatively related to the time-series variation in the factor. The mean of the first factor is slightly positive (0.0053). Therefore, options on stocks with a higher CAPM beta are expected to have a bit lower returns than options with a lower CAPM beta, ceteris paribus. Many more characteristics are correlated to the first factor positively or negatively. The correlations of the managed portfolios to the other factors of this model are displayed in tables 18, 19 and 20 in the Appendix. The absolute values of the characteristics are smaller for each higher factor.

The combination of a large number of characteristics and a nonlinear mapping from characteristics to the factors makes it infeasible to analyze the relation of all 69 characteristics to the factor loadings. Therefore, the 4-factor model, including the significant characteristics and the total volatility, is used as an example. Figure 6 shows the loading functions that relate the three characteristics to the factors. Most of the loading functions are fairly nonlinear. As an exception, the loading of the maximum return is negatively and close to linearly related to the first factor. The mean of the first factor of this model is negative (-0.018). PPCA uses the mean over time of the maximum daily return of each month. This means that options on stocks that have a higher maximum daily returns in the month on average have higher returns ceteris paribus. This suggests that the first factor is strongly related to the maximum return on the underlying stock. Operating leverage is clearly negatively related to the fourth factor, which has a negative mean. Although the mean of the fourth factor is not small, the difference in values of the loading functions of operating leverage is a lot smaller than for Max_return. Operating leverage is about the division of fixed and variable costs of a company. High operating leverage companies have relatively high fixed costs, which is considered riskier. The sensitivity of the company's profit on sales revenue is higher for high OL companies. This suggests that the fourth factor is negatively related to this type of risk. The loading functions on the other factors are considerably nonlinear and positive and negative for different values of operating leverage. Last month's turnover could better be described as the average monthly turnover in the PPCA case. It has a clear negative relation to the second factor. Turnover shows how actively the stock is traded, implying high liquidity. This suggests that the second factor is negatively related to liquidity. All three characteristics have a clear relation with one of the factors, the maximum return is related to the first factor, operation leverage to the fourth factor and turnover to the second factor. The loading functions on the third factor are all nonlinear. This suggests that none of the characteristics strongly relates to this factor, and the factor is related to something else, possibly mainly explained by Γ.

The latent factors of PPCA are not easy to interpret. The static factor loadings of unique cross-sectional units are not intuitive. Characteristics could provide additional information about the factors, but the combination of 69 characteristics and nonlinear loading functions makes the complete interpretation of these infeasible. In addition, correlations of the factor with portfolios convey no clear meaning of the factors. This emphasizes the usefulness of the bootstrap procedure for instrument significance introduced in this study. This enables to choose the most relevant out of a large set of characteristics. Subsequently, the models can be interpreted better via the loading functions of the characteristics to the factors as showcased in the above example. In addition, the performance of the more parsimonious models does not decline much by removing insignificant coefficients.

The latent factors of PPCA are not easy interpret. The static factor loadings of each cross-sectional unit are not intuitive. Characteristics could provide additional information about the factors, but the combination of 69 characteristics and nonlinear loading functions makes the complete interpretation of these infeasible. This emphasizes the usefulness of the bootstrap procedure for instrument significance

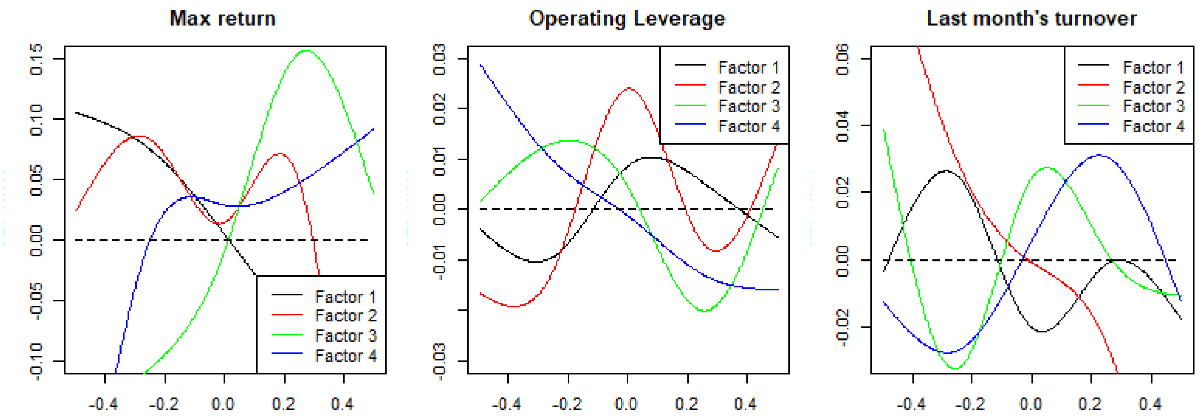


Figure 5: Loading functions g_{kj} showing the relation of the characteristics to the factors of the 4-factor PPCA model, including only these 3 significant characteristics. The horizontal axis represents the values of the characteristics. The vertical axis shows value of the loading functions

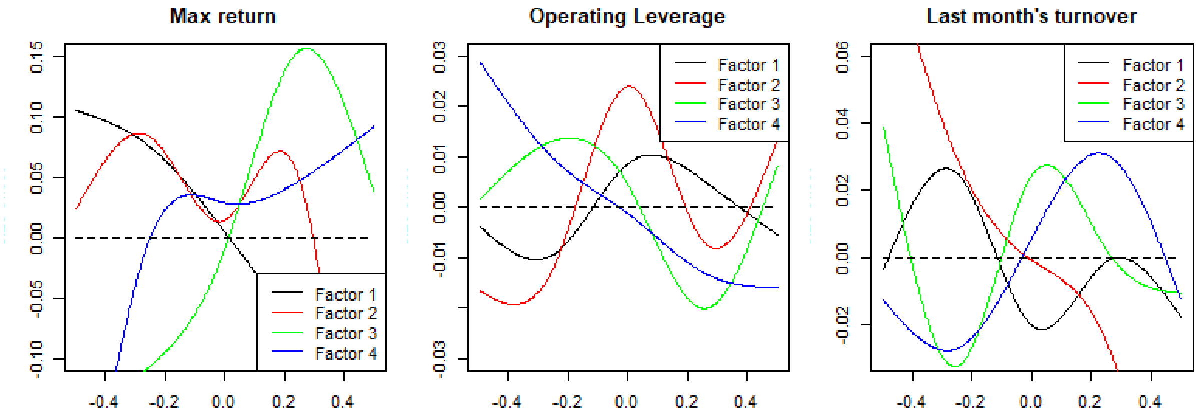


Figure 6: Loading functions g_{kj} showing the relation of the characteristics to the factors of the 4-factor PPCA model, including only these 3 significant characteristics. The horizontal axis represents the values of the characteristics. The vertical axis shows value of the loading functions

A:	Total R^2				Predictive R^2			
	IPCA		PPCA	PCA	IPCA		PPCA	PCA
K	Restricted	Unrestricted			Restricted	Unrestricted		
1	0.0985	0.1151	0.1193	0.1198	-0.0030	0.0111	0.0003	0.0002
2	0.1427	0.1513	0.1770	0.1778	0.0045	0.0103	0.0016	0.0018
3	0.1672	0.1750	0.2236	0.2260	0.0074	0.0099	0.0027	0.0020
4	0.1884	0.1939	0.2709	0.2722	0.0073	0.0092	0.0027	0.0028
5	0.2051	0.2101	0.3138	0.3156	0.0073	0.0086	0.0028	0.0028
6	0.2188	0.2229	0.3523	0.3569	0.0061	0.0086	0.0060	0.0032
B:	Number of parameters of each model							
K	'Full' model				'Small' model			
1	201	270	1002	450	136	140	498	450
2	402	471	1728	900	276	284	926	900
3	603	672	2454	1350	417	424	1398	1350
4	804	873	3180	1800	555	564	1860	1800
5	1005	1074	3924	2250	720	732	2370	2250
6	1206	1275	4632	2700	882	897	3214	2700

Table 15: Comparison of performance and numbers of parameters of three latent factor methods applied to delta-hedged equity options. For PPCA the statistics correspond to models with a Sieve dimension of four

introduced in this study. This enables to choose the most relevant out of a large set of characteristics. Subsequently, the models can be interpreted better via the loading functions of the characteristics to the factors as showcased in the above example. In addition, the performance of the more parsimonious models does not decline much by removing insignificant coefficients.

6.3 Comparison of IPCA, PPCA and regular PCA results

One of this thesis's main goals is to compare the two methods to explain delta-hedged equity option returns. This section compares the performance of the conditional latent factor methods PPCA and IPCA. In addition, traditional PCA is applied to the same dataset. This allows comparison with a latent factor model that only uses returns as input. Next to the performance, parsimony and interpretability are often essential criteria in the model choice.

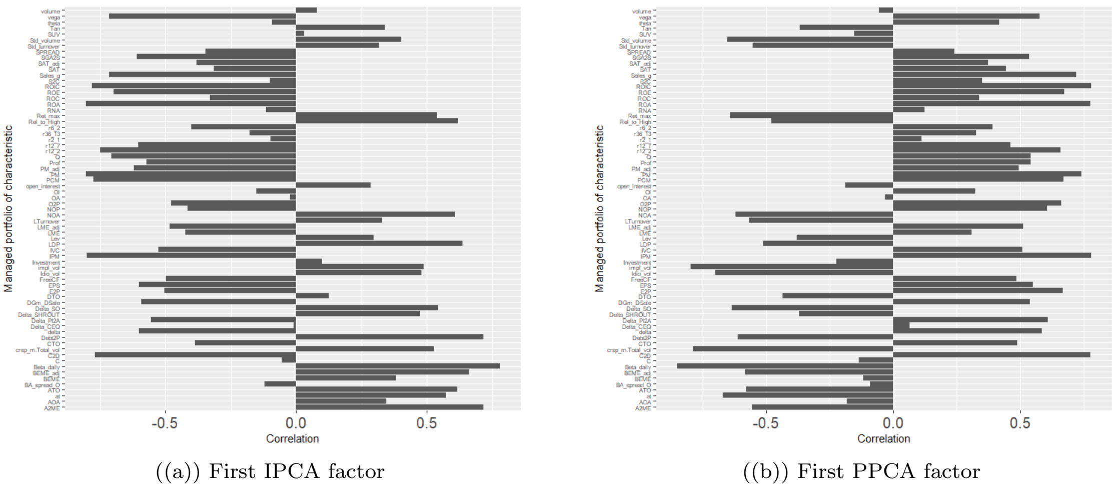
Panel A of table 15 reports the in-sample performance measures for IPCA, PPCA and PCA. Since the performance of PPCA is best for $J = 4$, these results are included in the table. PCA scores best on total R^2 for all numbers of factors, although the gap with PPCA is almost negligible. IPCA scores significantly lower on this statistic, especially for the higher numbers of factors. This result indicates that PPCA and PCA outperform IPCA in describing in-sample returns. This lower score of IPCA could be caused by the fact that PPCA estimates separate loadings for options on each stock and allows for a component of the factor loadings unrelated to the characteristics. Kelly et al. (2019) find that PCA outperforms IPCA on a stock returns panel, which is in line with the results for equity option returns in this thesis. The closeness of PPCA and IPCA on total R^2 is not unexpected since the estimation of the two methods does not differ a lot. PPCA boils down to applying PCA on the returns panel projected onto a space spanned by the (smoothed) characteristics. The estimation of IPCA is a combination of two OLS problems optimized using ALS. The estimation of the models is an important dissimilarity. The estimation of PPCA and PCA is much faster since it uses a closed-form formula. However, the estimation of IPCA is fast enough, and the speed of convergence of the ALS algorithm does not pose a problem for this dataset. Panel B of table 15 shows that the higher total R^2 of PPCA and PCA comes at the cost of much more parameters. As described earlier, the parsimony of IPCA is a considerable advantage over most other latent factor methods. The use of additional characteristics leads PPCA to have more parameters to be estimated than PCA. In PPCA, part of the factor loadings is unrelated to the characteristics. With only these parts, the number of characteristics is equal to PCA. Since PPCA contains extra parameters for related to the characteristics, the number of parameters is higher than for PCA. This illustrates the value of the test for instrument significance again. It enables the user to estimate a model with only the significant characteristics. The right side of panel B shows that the gap between PPCA and PCA in the number of parameters is much smaller if only the significant characteristics are included. Also in this setting, the 'smaller' IPCA models have substantially fewer parameters.

IPCA outperforms the other two methods in predictive R^2 for all K, except for the restricted 1-factor model. The difference between PPCA and PCA is relatively small. In the predictive R^2, realized returns are explained by the factor means. Since PPCA and PCA have static factor loadings, the predicted returns are equal for each time period. The loadings of IPCA are dynamic and depend on time-varying characteristics. As a result, the predicted returns by factor means are also dynamic. Hence, the higher predictive R^2 of IPCA points toward the existence of dynamic factor loadings, although the findings on total R^2 do not confirm this. Since the factor means could be interpreted as prices of risk, it can be concluded that IPCA does better describe risk compensation for these option returns than PPCA and PCA. This finding is based on the values for the restricted IPCA model, where returns are solely described by systematic risk factors.

Next to the explanatory performance, another important criterion is the interpretability of the models. The model's number of parameters is n indicator of interpretability. A less parsimonious model is usually more difficult to interpret. IPCA has a large advantage over the other two models on this measure. PPCA models contain the most parameters. This does not necessarily mean that the model is less interpretable than the PCA model. The latent factors do not have an ad-hoc economic meaning. To interpret PCA, the only aspects to consider are the loadings from the options on different companies (318) to the factors and the factor realizations themselves (132 for each factor). The former is shown for PPCA in figure 17. It is not easy to give economic meaning to the factors via these factor loadings. Similar figures could be produced for PCA. One way to understand the PCA factors would be to find observed factors correlated with the latent factors. Horenstein et al. (2020) find a 4-factor model for equity option returns using this approach.

PPCA incorporates additional information from characteristics in the model, which support the interpretation of factors. As described before, the nonlinearity of the characteristics makes the interpretation more difficult if all 69 are included. Therefore, significant characteristics are identified, and a new model is estimated using only these characteristics. This approach results in a smaller number of characteristics. As shown in an example in section 6.2.5, the characteristics could be linked to the factors using plots of the nonlinear loading functions. The nonlinear relation of characteristics is more flexible than the linear mapping of IPCA. This could result in a better fit of the data. IPCA does only estimate a mapping from characteristics to factor loadings. This allows IPCA to incorporate dynamic factor loadings without creating an explosion of the number of parameters. The mapping from characteristics to factors is linear and does not change through time and the cross-section. The linear mapping shows quickly how the characteristics are related to the factors. This clarifies the economic meaning of the factors, as shown in an example in section 6.1.5. However, the linear mapping from factor loadings of IPCA could be a limitation if a more flexible nonlinear approach would fit the data better. This could be an exciting extension to IPCA for future research. The ALS estimation of IPCA allows for including an intercept factor in the model. It enables the user to find whether characteristics relate to the returns directly or only through the risk factors. It could help investors identify anomalies related to characteristics that generate significant alphas. To conclude, IPCA is much easier interpretable than the other models due to its parsimony and linear mapping from characteristics to factors. The interpretability is also no problem if all 69 characteristics are used.

IPCA can include many characteristics into the model. PPCA is more limited due to the sieve approximations. This is shown when applying PPCA to returns of characteristic-managed portfolios. Since the cross-section is then limited to the number of characteristics (69), it is impossible to include all characteristics in the model. A solution is to include only the significant characteristics found in the study for individual option returns. Table 4 and 14 show that IPCA performs much better on the panel of managed portfolio returns in terms of total R^2. This could be caused by the fact that IPCA could include much more characteristics. Another reason could be the dynamic nature of the characteristics in IPCA. The time-variation in characteristics is also used for the construction of the portfolios. Figure 7



((a)) First IPCA factor ((b)) First PPCA factor

Figure 7: Correlations of the first factors of four-factor IPCA and PPCA models to characteristic managed portfolios

displays the correlations of the first IPCA and PPCA factors of a four-factor model to managed portfolios based on the characteristics. It is striking that most correlations of the two methods are almost equal in absolute value and have opposite signs. This suggests that the first IPCA factor is almost a rotation of the first PPCA factor. However, correlations are not perfectly the negative to the other model. The difference in sign could result from the different identifying assumptions of the two methods. This finding does not apply to the other three factors of these two models, as shown in the appendix.

#cor/K	1	2	3	4	5	6
1	0.77	0.82	0.84	0.82	0.84	0.89
2		0.00	0.20	0.34	0.52	0.69
3			0.00	0.15	0.36	0.53
4				0.06	0.18	0.37
5					0.00	0.02
6						0.00

Table 16: Canonical correlations of IPCA and PPCA factors with equal number of factors. The first up to the sixth canonical correlation are displayed on the vertical axis. All PPCA factors are estimated using a PPCA model with a Sieve dimension of 4.

The similarity in correlations shown in figure 7 enlarges the curiosity to find if the sets to what extent the sets of IPCA factors and PPCA factors are similar. In this thesis, canonical correlations are used to understand this relationship. The calculation of the canonical correlations is explained in section 4.3. Table 16 presents the canonical correlations of the sets of factors in IPCA and PPCA modes with an equal number of factors K. The first canonical correlation is high for all six models. This is the correlation between the sets of factors when the canonical variates are combinations of the factors, maximizing the correlation between the two canonical variates. The other covariates maximize the correlation of the other pairs of canonical variates that are orthogonal to the earlier pairs. By construction, a canonical correlation is equal to or smaller than the preceding canonical correlation. For models with more factors, the canonical correlations become higher. Adding more factors makes the factor space between the two models more similar. Note that it is also possible to compare the factors of two models with different numbers of factors using canonical correlations. This approach is chosen for the sake of brevity.

The sets of characteristics that are significant in at least one of the models of IPCA and PPCA do not overlap a lot. For example, in IPCA, five out of the eight option-specific characteristics are significant in at least one of the models, while none of those is significant in the PPCA models. The same characteristics are used in both methods, but the difference is that IPCA uses the time-varying nature of the characteristics, and PPCA uses the means of the characteristics over time. The option-specific characteristics are highly time-varying and could therefore be more valuable in the IPCA models. In addition, the difference could also be because IPCA estimates a linear relationship of characteristics and factors, while PPCA is more flexible and allows for nonlinear relationships. In IPCA, the number of significant characteristics is much higher than for PPCA. This could be because PPCA allows for parts of the factor loadings to be independent of the characteristics, while all factor loadings are related to the characteristics in IPCA. For PPCA, the smaller number of significant characteristics is beneficial for the interpretability of the estimated models with fewer characteristics. If insignificant characteristics are removed, the total R^2 does not diminish much for PPCA. If insignificant characteristics are removed from the IPCA models, the total R^2 diminishes significantly. This could have to do with the fact that all factor loadings depend on the characteristics of IPCA.

	Instrumented PCA							Projected PCA						
K	1	2	3	4	5	6	tan	1	2	3	4	5	6	tan
1	0.07						0.07	0.06						0.06
2	0.25	0.50					0.59	0.06	0.15					0.16
3	0.41	0.22	0.62				0.93	0.06	0.15	-0.16				0.23
4	0.42	0.03	0.51	0.42			1.01	0.06	0.15	-0.16	-0.00			0.23
5	0.42	0.02	0.41	0.30	0.36		0.98	0.06	0.15	-0.16	-0.00	0.04		0.23
6	0.47	0.04	0.41	0.27	0.27	0.31	1.10	0.06	0.15	-0.16	-0.00	0.04	-0.31	0.23

Table 17: In-sample Sharpe ratios for tangency portfolios of IPCA and PPCA factors and Sharpe ratios for the individual factors. For PPCA a sieve dimension of 4 was used. This dimension results in the highest Sharpe ratios for the tangency portfolio

	Regular PCA						
K	1	2	3	4	5	6	tan
1	0.04						0.04
2	0.04	0.16					0.17
3	0.04	0.16	0.07				0.18
4	0.04	0.16	0.07	-0.13			0.23
5	0.04	0.16	0.07	-0.13	0.02		0.23
6	0.04	0.16	0.07	-0.13	0.02	0.11	0.26

Table 18: In-sample share ratios of PCA factors and the tangency portfolio of PCA factors

Tables 17 and 18 report in-sample Sharpe ratios of the individual factors and the tangency portfolios of factors. The Sharpe ratios of the tangency portfolios of IPCA outperform those of PPCA and PCA for each number of factors. Hence, the risk factors found by IPCA are most mean-variance efficient. This is likely caused by the identifying restriction on IPCA that all factor means should be positive. PPCA and PCA have other restrictions in this study, as could be inferred by the negative Sharpe ratios of some factors. Changing the identifying assumptions of PPCA and PCA, such that each factor is restricted to be positive, could solve this problem. Different assumptions will imply different formulas for the estimation of PPCA and PCA. This should be possible since the factors could be rotated with the factor loadings. The hypothesis would be that this results in higher Sharpe ratios. The application of this idea is left for further research. These tables clearly show that the first factors of the smaller models if PPCA and PCA are nested in the larger models. The Sharpe ratios of those factors are equal for each model. The ALS estimation of IPCA causes the factors to differ for each model. Comparing tables 7 and 27 informs that the out-of-sample Sharpe ratios of tangency portfolios of IPCA are also higher than those of PPCA.

IPCA does not depend on each company's unique identity to which the options belong but only on a set of time-varying characteristics belonging to that observation. Therefore, it is possible to estimate the returns of an option corresponding to a company that was not in the sample used for estimating the IPCA model. Next to out-of-sample predictions of the next period, IPCA also allows applying the model out-of-sample in the cross-section. PPCA does not work this way since it estimates a unique factor loading of the options on each company on each factor. The part explained by the characteristics could be generalized to more companies in the cross-section. However, a significant part of the factor loadings in PPCA is explained by Γ, as proved by the specification test. This part is specific for options on each underlying stock. Therefore, it cannot be expected that PPCA can explain returns correctly out-of-sample in the cross-section. Out-of-sample prediction in this way is not possible using PCA since the factor loadings are only linked to the options on specific companies

7 Conclusion

This study applies and compares two conditional latent factor methods to a panel of delta-hedged equity option returns. One of the main reasons for employing latent factor models is the interpretability added by additional characteristics. For investors, it is helpful to know which characteristics drive the risk factors to optimize their investment strategy. An important advantage of the models applied in this thesis is the possibility to include a large set of characteristics, such that the model statistically finds which ones are important. This allows the data to speak for itself.

In theory, some advantages of IPCA are the dynamic factor loadings and the mapping matrix from characteristics to covariates foregoing the need to estimate factor loadings individually. However, this matrix can be restrictive since it only allows for linear relations. Also, it does not allow part of the factor loadings to be unrelated to the characteristics. PPCA is more flexible in that perspective and allows for nonlinear functions of characteristics to factor loadings and for part of the characteristics to be unrelated to the factors. On the other hand, PPCA could only account for static factor loadings and static characteristics. IPCA allows for a direct relation of characteristics and option returns if part of the variance cannot be attributed to the common risk factors. PPCA does not allow for this direct interaction.

Both methods are applied to a panel of options on 318 stocks present in the S&P500 from 2010 to 2020. An extensive set of 69 characteristics of the options, underlying stocks and affiliated companies are included in the models. The models are assessed on two main performance measures. Total R^2 measures the variation of realized returns described by estimated returns of the models. This captures the model's ability to describe risk. The other measure is predictive R^2, which uses the factor means and captures the variation of returns explained by conditional expected returns. This second measure could be interpreted as the model's ability to describe risk compensation.

For IPCA, the results show that five latent factors suffice for explaining option returns. Adding more factors increases the total R^2 a slightly but decreases the predictive R^2. The 5-factor unrestricted models also score highest on out-of-sample total R^2. This suggests that models with more factors can predict more accurately in-sample but do not improve the ability to describe the underlying patterns of the data. Direct interaction of the factors with returns does not add significantly to the total R^2 of this model. This implies that the 5-factor model is mean-variance efficient conditional on the characteristics. However, out-of-sample Sharpe ratios of the tangency portfolios show that the unconditional mean-variance efficiency of the model could be improved by adding a sixth factor.

The specifications tests for PPCA indicate that the characteristics have explanatory power on the factor loadings. This finding motivates the use of PPCA over traditional PCA. Another test indicates highly significant that the characteristics do not fully explain the factor loadings. This result advocates estimating a parameter of each factor loading unrelated to the characteristics. All PPCA models throughout this thesis include such parameters.

PPCA substantially outperforms IPCA in terms of in-sample total R^2, indicating that it describes risk more accurately in-sample. This could be caused by the fact that PPCA allows for part of the factor loadings to be unrelated to the characteristics and for nonlinear loading functions from characteristics to factors. However, IPCA scores much higher than PPCA on predictive R^2, indicating that it better describes risk compensation. Even though the factor means are used in this measure, IPCA can still predict different returns for each time period through its dynamic factor loadings. For PPCA, the predicted returns by factor means only differ in the cross-section due to the limitation of static factor loadings.

One of the primary motivations for employing conditional latent factor models over PCA is economic interpretability. IPCA is the better of the two methods in this aspect. Firstly, the dimension reduction in the mapping matrix makes IPCA much more parsimonious. The number of factors to be estimated is much lower than PPCA. Also, the linear mapping from characteristics to factors shows clear relations of the factors to the characteristics. Even when all 69 characteristics are included in a model, IPCA picks a few relevant ones that determine the factor loadings. Excluding insignificant characteristics hurts the performance of IPCA and does not improve the interpretability much. PPCA factors are much more complex to interpret due to the nonlinear loading functions. If all characteristics are included, too many nonlinear loadings functions need to be checked to find clear relations of the factors to the characteristics. Therefore, this thesis extends the bootstrap IPCA testing procedure for the characteristic significance of Kelly et al. (2019) is PPCA. This test allows for a new procedure for estimating PPCA models. Firstly, the model with all characteristics is estimated, and the test is performed. Then characteristics that significantly contribute to the factor loadings are identified and included in a more parsimonious model. The empirical analysis in this thesis demonstrates that this enables the researcher to find a clear link between the characteristics and the factors. The explanatory performance remains almost as good by excluding insignificant characteristics.

The significant characteristics are strikingly different between the two methods. This discrepancy likely relies upon the fact that PPCA uses the mean of the characteristics, whereas IPCA uses all time-varying information. For example, most of the highly time-varying option-specific characteristics, such as Gamma and Vega, are significant in at least one of the IPCA models, whereas none of them is significant in any of the PPCA models.

In addition, correlations of the time-series realizations of the factors and characteristic managed portfolios are computed for the four-factor models of both methods. The correlations of the first factor of the two methods have almost equal absolute values and are opposite in sign. These factors and related factor loadings are close to being a rotation of each other. This result does not hold for the other factors. Also, canonical correlations show that the sets of factors of both models span part of the same space but not entirely.

To summarize, PPCA is the best choice if the main goal is to describe as much as possible in-sample variation in delta-hedged option returns. IPCA is much more parsimonious and easier to interpret. In addition, it is easier to use out-of-sample and does better describe returns in terms of conditional expected returns.

The research of this thesis could be extended in a few directions. Firstly, it could be insightful to compare the two methods on a panel of index options. If options on one index are used, it is possible to include options with varying moneyness and time to maturity. These characteristics are highly time-varying, and it would be interesting to include those in the set of characteristics for estimating the models. In this study, moneyness and time to maturity are not interesting to include as characteristics. The time-to-maturity is almost fixed, and only close to the money options are included in the panel. Another more methodological extension would be to use a combination of both methods. Extend IPCA to allow for nonlinear loading functions or extend PPCA such that it can incorporate time-varying factor loadings. Note that these are not equivalent, given the different estimation methodologies of the two methods and other fundamental differences. Another approach to creating more parsimonious models could be to combine PPCA with LASSO. This could be a statistical alternative to the characteristic-selection method used in this study. Finally, the application of IPCA in this thesis could be extended by a model for the time-series of the factors. A Vector Auto-regressive model is the most common of these kind of models. Such a model combined with IPCA would enable the user to predict returns out-of-sample in a real-world application.

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8 Appendix

8.1 Theoretical assumptions IPCA

For estimation of an IPCA model, some theoretical assumptions are taken. The assumptions are described carefully by Kelly et al. (2020). This section provides a short overview of these. The following requirements should be met,

- The characteristics should be orthogonal to the compounded error term as in equation (10), i.e. $E[c_{i,t}e^*_{i,t+1}] = 0$

- The following moments must exist: $E||f_t f_t'||^2$, $E||c_{i,t}e^*_{i,t+1}||^2$, $E||c'_{i,t}c_{i,t}||^2$ and $E[||c'_{i,t}c_{i,t}||^2||f_{t+1}||^2]$. These moments are necessary for applying the law of large numbers and are therefore needed for consistency of IPCA estimation

- The parameter space of Γ_β in the restricted is compact and not rank deficient, i.e. $det(\Gamma'_\beta \Gamma_\beta) \geq \epsilon$, where $\epsilon \geq 0$. Furthermore, $c_{i,t}$ is bounded almost surely and $det(E[c'_{i,t}c_{i,t}]) \geq \epsilon$. These conditions guarantee that the inverse of the matrix $\Gamma'_\beta C'_t C_t \Gamma_\beta$ exists, which is used in estimation in equations (13) and (18).

In addition, some identifying assumptions are necessary to determine a unique set of factors and factor loadings. The assumptions that are imposed in this study are described in section 3.2.2.

8.2 Theoretical assumptions PPCA

According to Fan et al. (2016), some theoretical assumptions should be satisfied for PPCA. The key conditions are as follows:

- $c_{min} < \lambda_{min}\Lambda P \Lambda < \lambda_{max} < c_{max}$: where c_{min} and c_{min} are positive constants such that $c_{max} \geq c_{min}$ and λ_{min} and λ_{max} are the smallest and largest eigenvalue of $\Lambda P \Lambda$. This condition makes sure that the characteristics have explanatory power on the option returns. The case where the characteristics are completely unrelated to the returns does not satisfy this assumptions. Note that this assumption can only be satisfied if $Jd \geq K$, by the dimensions of $\Psi(X)$ and Λ.

- It should be that $F'F/T = I_k$ almost surely and that $\Lambda P \Lambda$ is a diagonal matrix with all distinct entries. This assumption is necessary for identification and is an assumption common for latent factor models

- There exist positive constants $0 < d_{min} < d_{max}$ such that $d_{min} < \lambda_{min}(p^{-1}\Psi(X)'\Psi(X)) < \lambda_{max}(p^{-1}\Psi(X)'\Psi(X)) < d_{max}$ with probability approaching 1 if $p \to \infty$. This assumption on the basis functions is satisfied by the cubic splines according to Fan et al. (2016).

the following assumptions assumptions are about the data-generating process.

- $\{\boldsymbol{u}_t, \boldsymbol{f}_t\}_{0 \leq t \leq T}$ should be strictly stationary, $E[u_{i,t}] = 0$ for all $0 \leq i \leq N$ and $1 \leq t \leq T$. In addition, $\{\boldsymbol{u}_t\}_{0 \leq t \leq T}$ should be independent of $\{X_i, \boldsymbol{f}_t\}_{0 \leq t \leq T, i \leq N}$.

- $\mathcal{F}^0_{-\infty}$ and $\mathcal{F}^0_{-\infty}$ are defined as σ-algebras of $\{\boldsymbol{f}_t, \boldsymbol{u}_t : t \leq 0\}$ and $\{\boldsymbol{f}_t, \boldsymbol{u}_t : t \geq T\}$. And a mixing coefficient is defined as follows $a(T) = sup_{A \in \mathcal{F}^0_{-\infty}, B \in \mathcal{F}^0_{-\infty}} |P(A)P(B) - P(AB)|$. Then it must be that there exists constants $r_1, C_1 > 0$ such that $a(T) < exp(-C_1 T^{r_1})$

- There exists a C_2 such that (i) $max_{j \leq N} \sum_{i=1}^N |E(u_{i,t}u_{j,t})| < C_2$,
  (ii) $(PT)^{-1} \sum_{i=1}^N \sum_{j=1}^N \sum_{t=1}^T \sum_{s=1}^T |E(u_{i,t}u_{j,s})| < C_2$ and
  (iii) $max_{i \leq N}(PT)^{-1} \sum_{k=1}^N \sum_{j=1}^N \sum_{t=1}^T \sum_{s=1}^T |cov(u_{i,t}u_{k,t}, u_{i,s}u_{m,s})| < 0$

- There exist constants $r_2, r_3 > 0$ that satisfy the equation $r_1^{-1} + r_2^{-1} + r_3^{-1} > 1$ and constants $b_1, b_2 > 0$. Then, for each $s > 0$, $i \neq N$ and $k \leq K$, it should be that $P(|u_{i,t}| > s) \neq exp(-(s/b_1)^{r_2})$ and $P(|f_{k,t}| > 0) \neq exp(-(s/b_1)^{r_3 d})$

8.3 Overview of characteristics

Past returns
(1) r_{2-1}: Return one month before prediction
(2) r_{6-2}: Return from 6 to 2 months before
(3) r_{12-7}: Return from 12 to 7 months before
(4) r_{12-2}: Return from 12 to 2 months before
(5) r_{36-13}: Return from 36 to 12 months before

Investment
(6) Investment: % change in total assets
(7) ΔCEQ: % change in book equity
(8) ΔPI2A: Change in PP&E and inventory over lagged total assets
(9) ΔShrout: % Change in shares outstanding
(10) IVC: Change in inventory over average total assets
(11) NOA: Net-operating assets over lagged total assets

Profitability
(12) ATO: Sales to lagged net operating assets
(13) CTO: sales to lagged total assets(AT)
(14) Δ(ΔGM-ΔSales): Δ (% change in gross margin and % changes in sales)
(15) EPS: Earnings per share
(16) IPM: Pre-tax income over sales
(17) PCM: Sales minus costs of goods sold to sales
(18) PM: Operating Income(OI) after depreciation over sales
(19) PM_adj: Profit margin - mean PM in Fama-French 48 industry
(20) Prof: Gross profitability over book equity
(21) RNA: OI after depreciation to lagged net operating assets
(22) ROA: Income before extraordinary items to lagged AT
(23) ROC: Size + longterm debt - total assets to cash
(24) ROE: Income before extraordinary items to lagged book equity
(25) ROIC: Return on invested capital
(26) S2C: Sales to cash
(27) SAT: Sales to total assets
(28) SAT_adj: SAT - mean SAT of F&F 48 industry classification

Intangibles
(29) AOA: Absolute value of operating accruals
(30) OL: Costs of goods solds + SG&A to total assets
(31) Tan: Tangibility
(32) OA: Operating Accruals

Value
(33) A2ME: Total assets to Size
(34) BEME: Book to market Ratio
(35) BEME_adj: BEME - mean BEME of F&F 48 industry portfolios
(36) C: Cash to total assets
(37): C2D: Cash flow to total liabilities
(38) ΔSO: Log change ins split-adjusted shares outstanding
(39) Debt2P: Total debt to size
(40) E2P: Income before extraordinary items to Size
(41) Free CF: Free cash flow to BE
(42) LDP: Trailing 12-months dividends to price
(43) NOP: Net payouts to size
(44) O2P: Operating payouts to market cap
(45) Q: Tobin's Q
(46) SGA2S: SG&A to sales
(47) Sales_g: Sales Growth

Trading Frictions
(48) AT: Total assets
(49) SUV: Standard unexplained volume
(50) Beta_daily: CAPM beta using daily returns
(51) DTO: De-trended Turnover - market Turnover
(52): Idio vol: Idio Vol of F&F 3-factor model
(53) LME: Size: Price x shares outstanding
(54) LME_adj: Size - mean size in FF 48 industry
(55) Lturnover: Last month's volume to shrout
(56) Rel_to_high_prc: Price to 1 year high price
(57) Ret_max: Maximum daily return
(58) Spread: Average daily bid-ask spread
(59) Std_turnover: Standard deviation of daily turnover
(60) Std_ volume: Standard deviation of daily volume
(61) Total_vol: Standard deviation of daily returns

Option Specific
(62) Delta: sensitivity to the underlying stock
(63) Vega: Vega of the option
(64): Theta: Theta of the option
(65) Impl_vol = Implied volatility
(66) Open_interest: open$_{i}nterestoftheoption$
(67) Vol: Trading volume of the option
(68) Gamma: Gamma of the option
(69) Spread_O: Bid-ask spread of th option

Table 19: Short description of the characteristics. This table is mainly based on the characteristics used in the study of Freyberger et al. (2020) and complemented with option-specific characteristics

8.4 IPCA results

	K						
Factors	1	2	3	4	5	6	Tan
1	0.50						0.50
2	0.42	0.30					0.59
3	0.57	0.28	0.13				0.67
4	0.31	0.44	0.18	0.29			0.69
5	0.11	0.25	0.16	0.23	0.48		0.68
6	-0.05	0.25	-0.04	0.10	0.39	0.37	0.72

Table 20: Out-of-sample Sharpe ratios of the factors of IPCA applied to managed portfolios and tangency portfolios of the factors, the Returns are calculated using an out-of-sample design described in section 4.1. These values are for the restricted model without intercept estimated on managed portfolio returns

Plots Γ_β IPCA 4-factor model

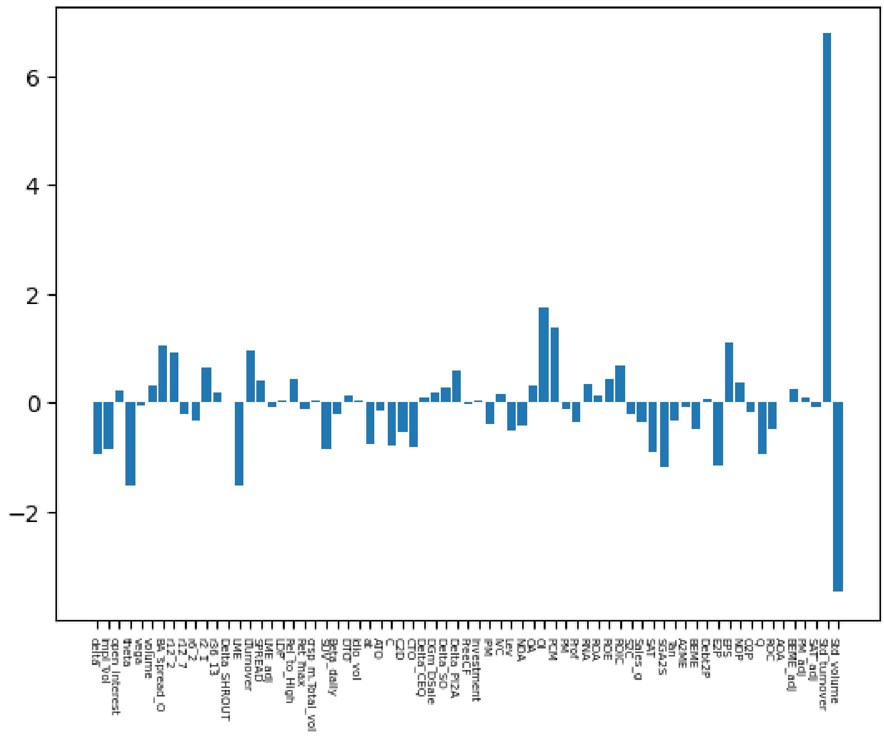

Figure 8: Coefficients of $\hat{\Gamma}_\beta$ relating to the second factor of a 4-factor IPCA model including all 69 characteristics

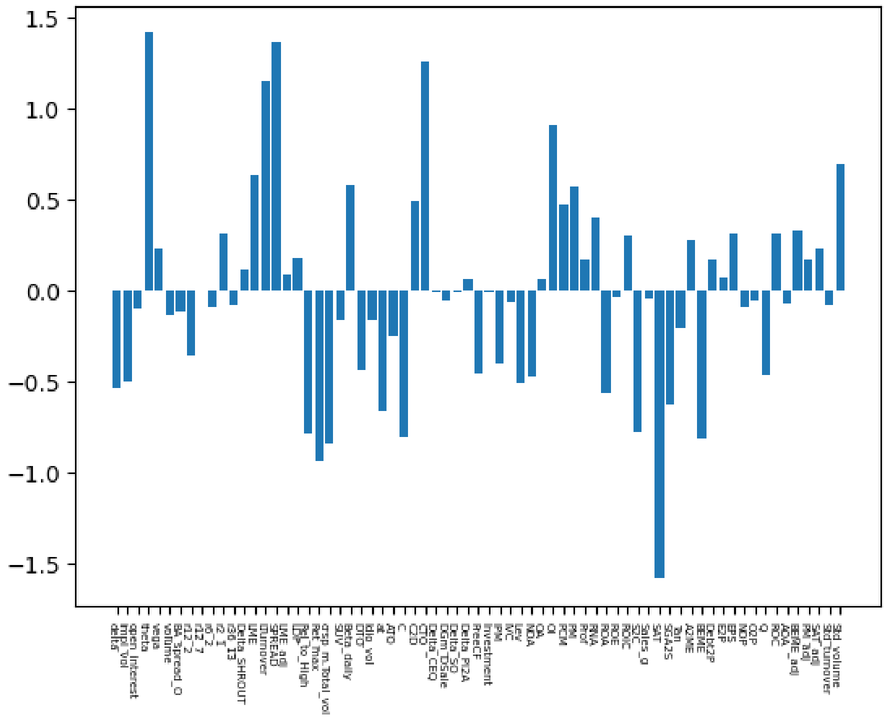

Figure 9: Coefficients of $\hat{\Gamma}_\beta$ relating to the third factor of a 4-factor IPCA model including all 69 characteristics

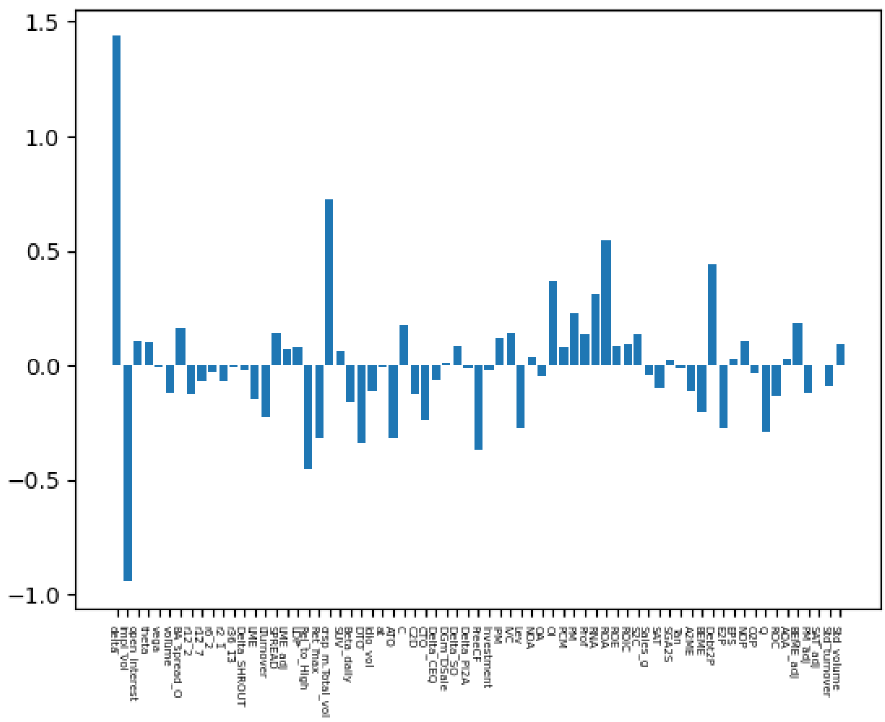

Figure 10: Coefficients of $\hat{\Gamma}_\beta$ relating to the fourth factor of a 4-factor IPCA model including all 69 characteristics

Estimated $\hat{\Gamma}_\beta$ for IPCA models consisting of only significant characteristics

Char	*K* 1
FreeCF	0.012
NOA	0.055
SPREAD	0.668
vega	-0.742

Table 21: Gamma coefficients for IPCA mode with one factor including only the significant characteristics

	K	
Char	1	2
FreeCF	0.016	-0.051
SPREAD	0.631	0.071
vega	-0.768	0.166
BA_spread_O	0.081	0.805
E2P	-0.064	-0.179
Std_turnover	0.013	0.534

Table 22: Gamma coefficients for IPCA mode with 2 factors including only the significant characteristics

	K		
Char	1	2	3
FreeCF	0.026	-0.124	0.012
SPREAD	0.670	0.232	0.448
vega	-0.736	0.110	0.458
Std_turnover	0.004	0.087	0.626
LTurnover	0.040	0.049	0.373
NOA	0.040	-0.109	0.076
impl_vol	0.073	-0.947	0.230

Table 23: Gamma coefficients for IPCA mode with 3 factors including only the significant characteristics

	K			
Char	1	2	3	4
FreeCF	0.029	0.006	0.017	-0.082
SPREAD	0.674	-0.095	0.431	0.333
vega	-0.713	-0.290	0.278	0.289
Std_turnover	-0.010	-0.319	0.448	0.168
LTurnover	0.027	-0.458	0.309	-0.724
NOA	0.070	-0.066	0.076	-0.354
SGA2S	-0.033	0.151	-0.175	-0.112
Tan	0.026	0.081	-0.196	-0.265
delta	-0.169	0.750	0.604	-0.196

Table 24: Gamma coefficients for IPCA mode with 4 factors including only the significant characteristics

Char	K				
	1	2	3	4	5
FreeCF	0.042	0.007	-0.031	-0.138	-0.030
SPREAD	0.678	-0.122	0.400	0.188	-0.020
vega	-0.707	-0.288	0.215	0.126	-0.024
Std_turnover	-0.007	-0.320	0.371	0.164	-0.202
LTurnover	0.015	-0.412	0.164	-0.532	-0.520
NOA	0.080	0.004	-0.127	-0.142	-0.437
SGA2S	-0.020	0.208	-0.297	-0.041	-0.117
Tan	0.029	0.123	-0.223	-0.048	-0.200
delta	-0.161	0.750	0.535	-0.055	-0.324
ATO	0.003	-0.058	0.044	0.049	0.072
PM	-0.063	-0.058	0.383	0.144	0.283
open_interest	0.015	0.051	0.197	-0.755	0.503

Table 25: Gamma coefficients for IPCA mode with 5 factors including only the significant characteristics

Char	K					
	1	2	3	4	5	6
FreeCF	0.058	-0.094	-0.015	-0.245	-0.316	0.027
SPREAD	0.258	0.380	-0.171	0.379	-0.102	-0.179
Std_turnover	0.258	0.336	-0.192	0.347	-0.158	0.007
LTurnover	0.388	0.322	-0.028	-0.442	-0.243	-0.432
NOA	0.074	-0.262	0.094	0.166	-0.330	-0.293
Tan	-0.018	-0.160	0.053	0.275	0.104	-0.303
delta	-0.793	0.438	-0.002	0.024	-0.377	-0.102
PM	-0.060	0.310	-0.313	-0.029	0.430	0.052
AOA	0.152	-0.034	0.113	0.203	-0.129	0.142
LME_adj	0.024	0.004	-0.025	-0.469	-0.026	-0.127
Ret_max	0.000	-0.133	0.027	-0.109	-0.362	0.042
S2C	0.081	0.247	-0.226	-0.199	0.121	0.021
SUV	-0.199	-0.194	-0.105	0.008	0.337	-0.703
impl_vol	-0.070	-0.358	-0.864	-0.011	-0.208	0.091
E2P	-0.023	0.055	0.036	-0.253	0.201	0.220

Table 26: Gamma coefficients for IPCA mode with 6 factors including only the significant characteristics

Plots of γ_β of a 3-factor IPCA model that only includes only the significant characteristics

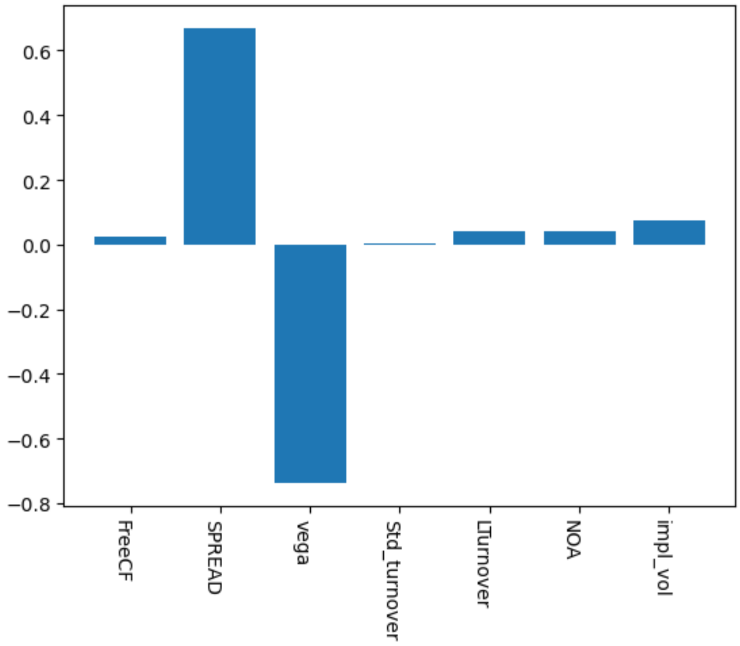

Figure 11: Coefficients of Gamma relating to the first factor of a 3-factor IPCA model including only the significant characteristics

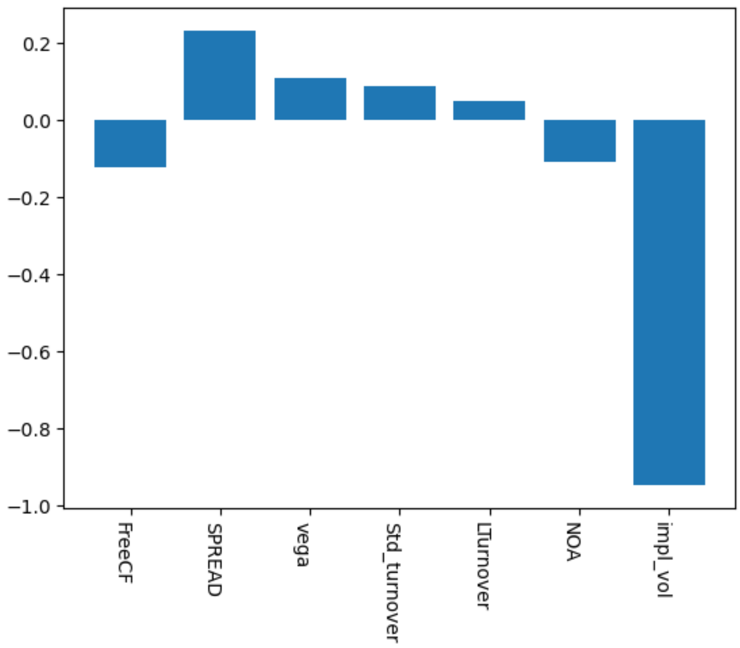

Figure 12: Coefficients of Gamma relating to the second factor of a 3-factor IPCA model including only the significant characteristics

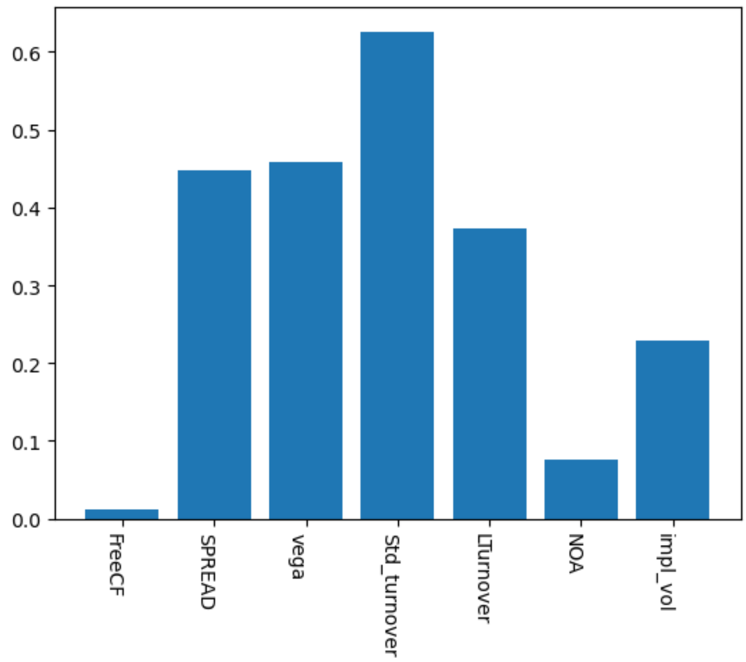

Figure 13: Coefficients of Gamma relating to the third factor of a 3-factor IPCA model including only the significant characteristics

Correlations of PPCA factors with managed portfolios of the characteristics

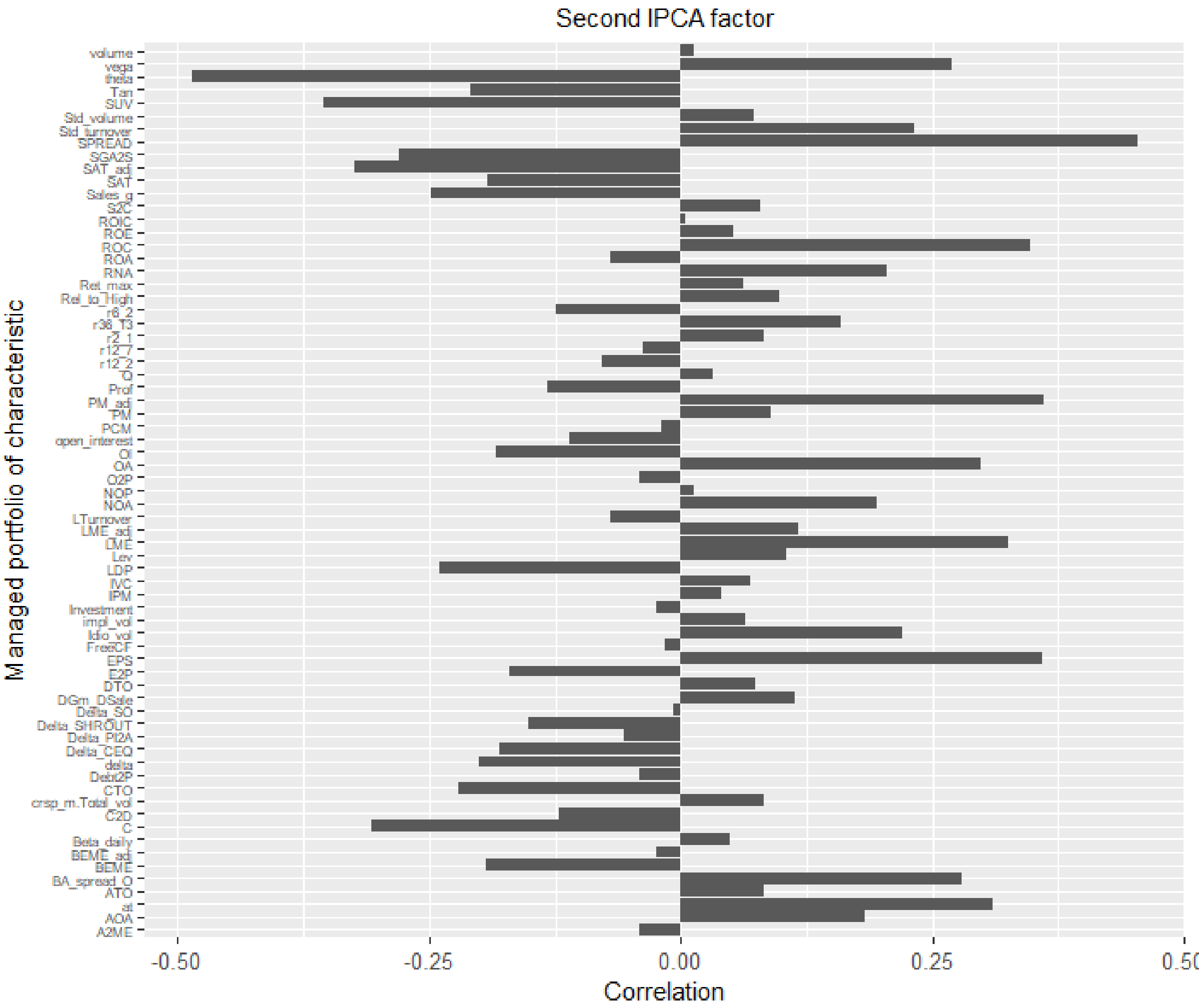

Figure 14: Correlations of the second IPCA factor with managed portfolios constructed by the characteristics

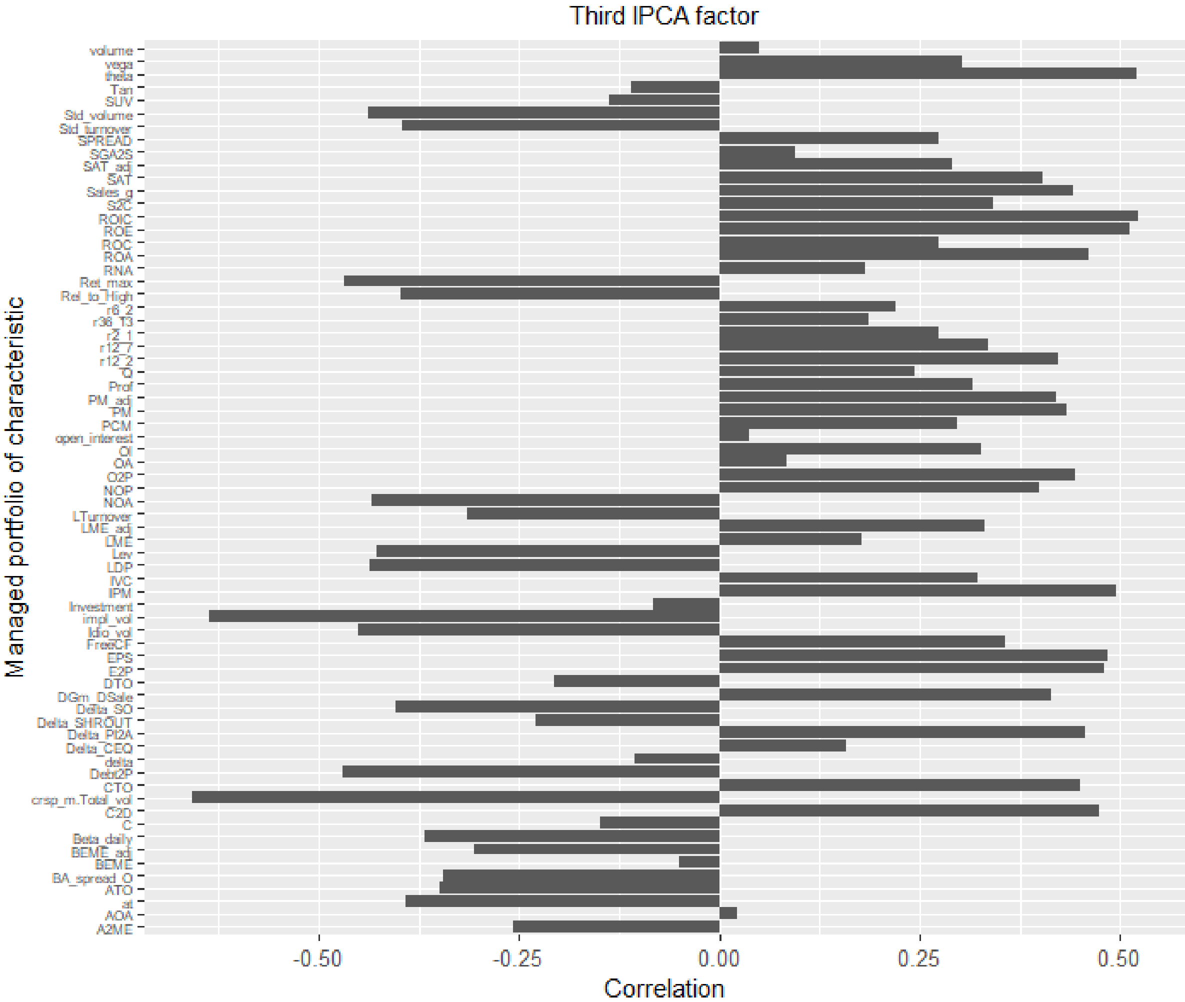

Figure 15: Correlations of the third IPCA factor with managed portfolios constructed by the characteristics

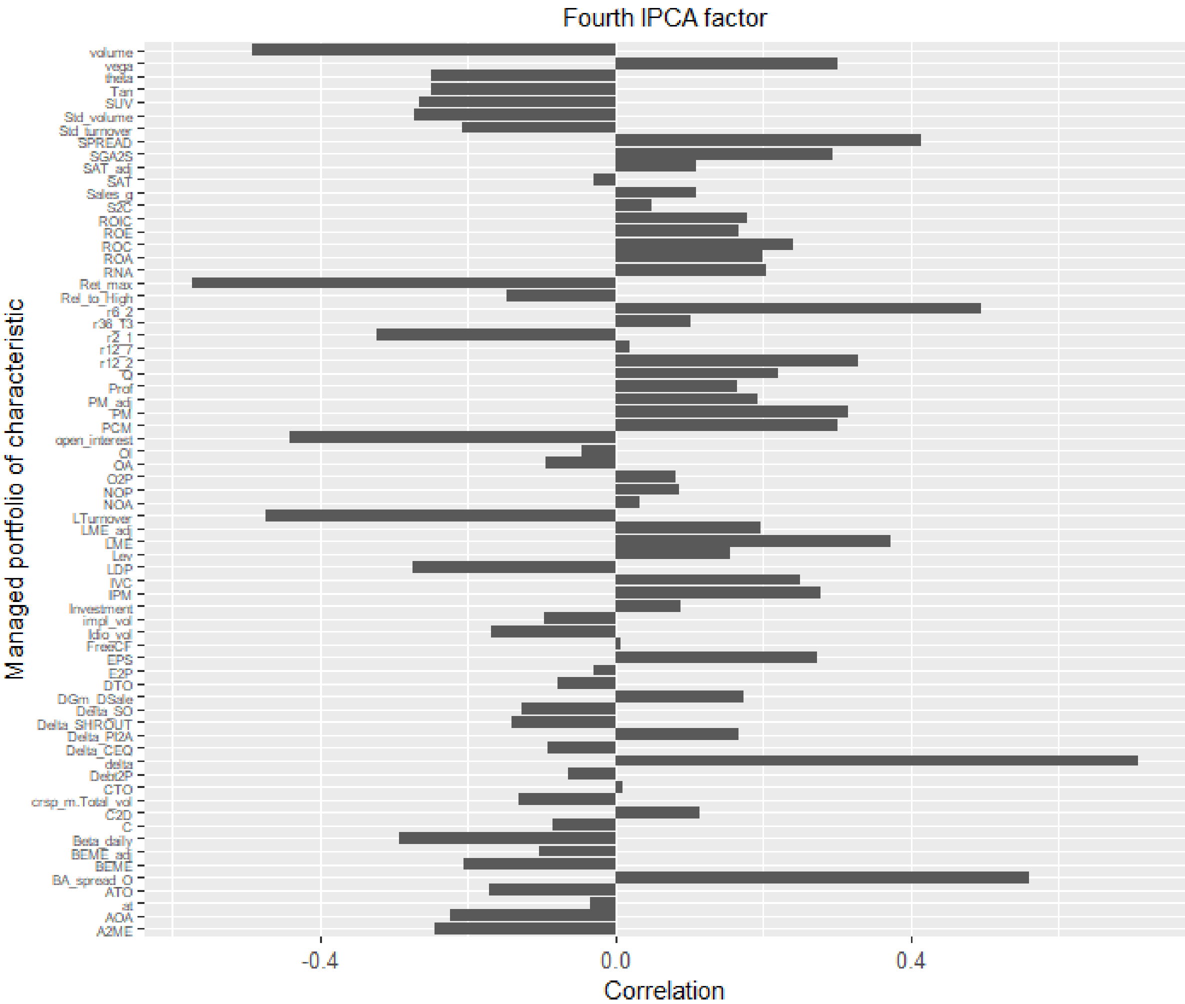

Figure 16: Correlations of the fourth IPCA factor with managed portfolios constructed by the characteristics

8.5 PPCA results

J/K	1	2	3	4	5	6
2	0.009	-0.046	0.159	0.175	0.120	0.012
3	-0.008	0.235	0.157	0.106	-0.011	-0.054
4	-0.076	0.184	0.112	-0.122	-0.049	0.247

Table 27: Out-of-sample Sharpe ratios tangency portfolios for PPCA models including all characteristics

	K							K					
Char	1	2	3	4	5	6	Char	1	2	3	4	5	6
delta	0.42	0.53	0.65	0.59	0.5	0.52	Investment	0.6	0.72	0.83	0.71	0.46	0.24
impl_vol	0.5	0.56	0.7	0.63	0.48	0.48	IPM	0.55	0.63	0.66	0.65	0.63	0.63
open_interest	0.69	0.68	0.82	0.85	0.87	0.9	IVC	0.99	0.62	0.67	0.68	0.71	0.72
theta	0.69	0.45	0.51	0.58	0.4	0.36	Lev	0.55	0.62	0.8	0.4	0.39	0.4
vega	0.77	0.95	0.75	0.78	0.4	0.52	NOA	0.21	0.31	0.38	0.38	0.35	0.27
volume	0.35	0.34	0.45	0.51	0.48	0.51	OA	0.6	0.61	0.71	0.71	0.64	0.52
BA_spread_O	0.14	0.17	0.27	0.3	0.27	0.3	**Ol**	**0.05**	0.27	0.37	0.3	0.14	0.11
r12_2	0.43	0.78	0.41	0.38	0.42	0.5	PCM	0.54	0.67	0.74	0.68	0.42	0.45
r12_7	0.33	0.76	0.39	0.42	0.46	0.38	PM	0.54	0.52	0.54	0.56	0.59	0.59
r6_2	0.58	0.53	0.55	0.42	0.44	0.44	Prof	0.54	0.56	0.72	0.72	0.54	0.36
r2_1	0.41	0.22	0.19	0.22	0.25	0.18	RNA	0.48	0.42	0.44	0.44	0.27	0.2
r36_13	0.46	0.59	0.55	0.46	0.31	0.26	ROA	0.35	0.52	0.76	0.78	0.59	0.53
ΔSHROUT	0.37	0.33	0.46	0.39	0.36	0.36	ROE	0.92	0.99	0.98	0.91	0.8	0.42
LME	0.98	0.89	0.64	0.71	0.68	0.56	ROIC	0.35	0.36	0.53	0.46	0.41	0.4
LTurnover	0.21	**0.06**	**0.09**	**0.04**	**0.01**	**0.00**	S2C	0.38	0.15	0.22	0.19	0.12	0.11
SPREAD	0.57	0.76	0.58	0.62	0.53	0.57	Sales_g	0.5	0.51	0.79	0.43	0.39	0.39
LME_adj	0.62	0.51	0.56	0.38	0.33	0.34	**SAT**	**0.01**	0.45	0.46	0.44	0.48	0.47
LDP	0.67	0.81	0.82	0.79	0.71	0.5	SGA2S	0.38	0.62	0.78	0.67	0.38	0.41
Rel_to_High	0.69	0.46	0.56	0.59	0.4	0.32	Tan	0.47	0.56	0.67	0.52	0.46	0.41
Ret_max	**0.08**	0.37	0.46	0.46	0.38	0.28	A2ME	0.82	0.36	0.55	0.61	0.48	0.5
Total_vol	0.2	0.61	0.55	0.44	0.34	0.12	BEME	0.32	0.56	0.67	0.63	0.53	0.48
SUV	0.46	0.63	0.6	0.58	0.53	0.35	Debt2P	0.9	0.81	0.85	0.34	0.23	0.25
Beta_daily	0.76	0.72	0.6	0.44	0.46	0.32	E2P	0.37	0.45	0.58	0.49	0.43	0.37
DTO	0.48	0.35	0.48	0.56	0.39	0.43	EPS	0.77	0.9	0.98	0.97	0.95	0.57
Idio_vol	0.55	0.34	0.46	0.35	0.17	0.14	NOP	0.45	0.65	0.71	0.53	0.59	0.56
at	0.81	0.18	0.27	0.26	0.3	0.27	O2P	0.66	0.8	0.88	0.76	0.67	0.5
ATO	0.5	0.41	0.39	0.36	0.32	0.23	Q	0.67	0.86	0.95	0.86	0.43	0.37
C	0.62	0.4	0.51	0.51	0.5	0.53	ROC	0.22	0.44	0.59	0.47	0.21	0.25
C2D	0.63	0.6	0.75	0.74	0.71	0.66	AOA	0.65	0.8	0.71	0.62	0.61	0.7
CTO	0.29	0.8	0.72	0.76	0.82	0.64	BEME_adj	0.28	0.24	0.39	0.34	0.28	0.29
ΔCEQ	0.57	0.8	0.91	0.86	0.51	0.4	PM_adj	0.68	0.82	0.92	0.88	0.85	0.88
ΔGm_ΔSale	0.75	0.85	0.81	0.79	0.58	0.63	SAT_adj	0.66	0.86	0.91	0.84	0.83	0.82
ΔSO	0.38	0.48	0.62	0.63	0.52	0.55	Std_turnover	0.59	0.72	0.82	0.66	0.72	0.74
ΔPI2A	0.58	0.72	0.85	0.63	0.52	0.56	Std_volume	0.46	0.59	0.69	0.55	0.51	0.65
FreeCF	0.59	0.78	0.91	0.79	0.61	0.65							

Table 28: Significance of characteristics in PPCA models with 1 up to 6 factors. In this table the sieve dimension is (L=3). This is the only difference with the models evaluated in table 12

	Total R^2			Predictive R^2		
K/J	2	3	4	2	3	4
1	0.2148	0.2377	0.2489	-0.0024	-0.0017	-0.0012
2	0.2451	0.2992	0.3349	-0.0014	0.0010	0.0015
3	0.2952	0.3467	0.3447	0.0004	0.0014	0.0010
4	0.3597	0.3760	0.3721	0.0025	0.0032	0.0015
5	0.4180	0.4000	0.4152	0.0067	0.0045	0.0027
6	0.4866	0.4562	0.4493	0.0072	0.0072	0.0056

Table 29: 'Out-of-sample' performance measure for PPCA including Γ

	Total R^2			Predictive R^2		
K/J	2	3	4	2	3	4
1	0.1533	0.1955	0.2295	-0.0010	-0.0011	-0.0010
2	0.1784	0.2446	0.3076	0.0003	0.0012	0.0014
3	0.2074	0.2841	0.3172	0.0014	0.0016	0.0010
4	0.2483	0.3080	0.3412	0.0031	0.0033	0.0017
5	0.2776	0.3268	0.3801	0.0054	0.0044	0.0029
6	0.3202	0.3661	0.4121	0.0058	0.0065	0.0056

Table 30: 'Out-of-sample' performance measure for PPCA excluding Γ

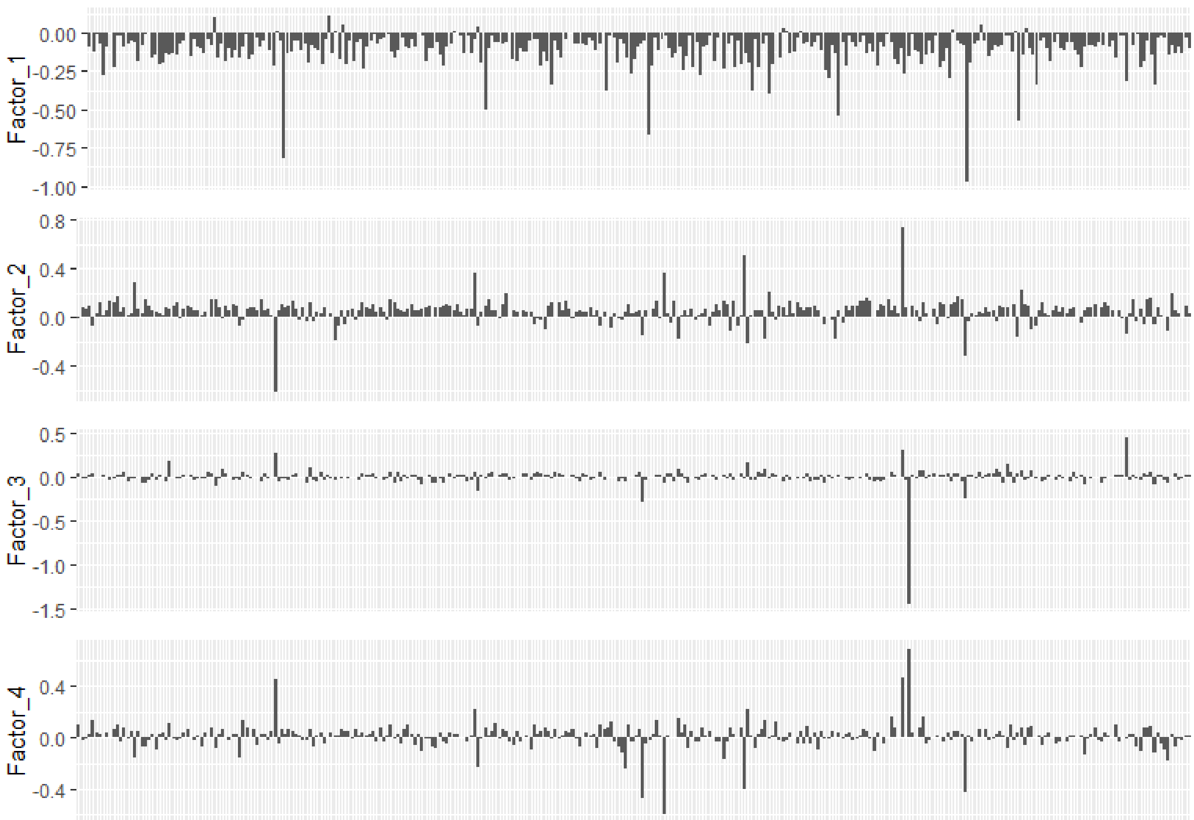

Figure 17: Loadings of each cross-sectional unit (underlying stocks of the options) on each factor of a 4-factor PPCA model including all 69 characteristics

Correlations of PPCA factors of a four-factor model with managed portfolios of the characteristics

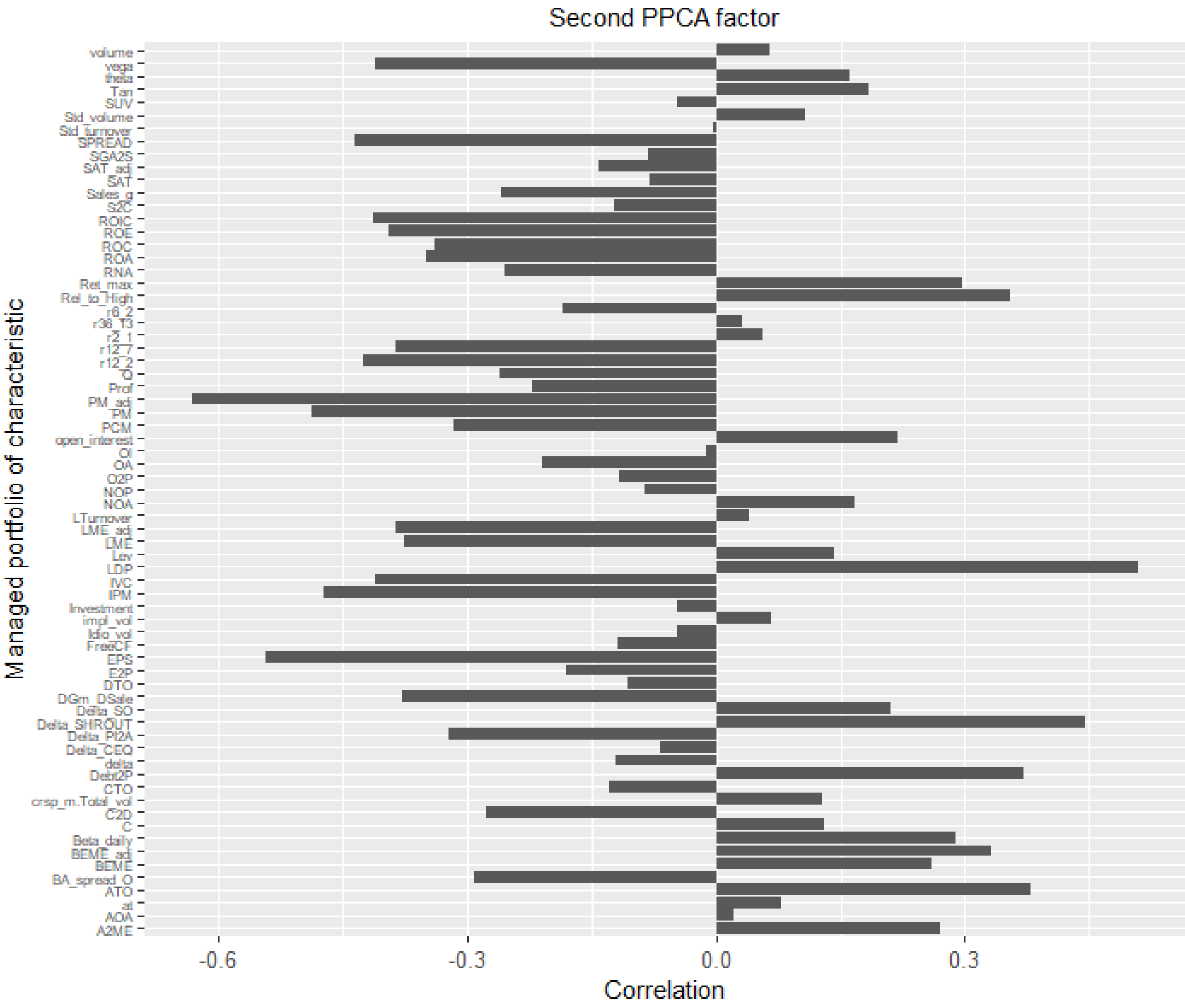

Figure 18: Correlations of the second PPCA factor of a four-factor model with managed portfolios constructed by the characteristics

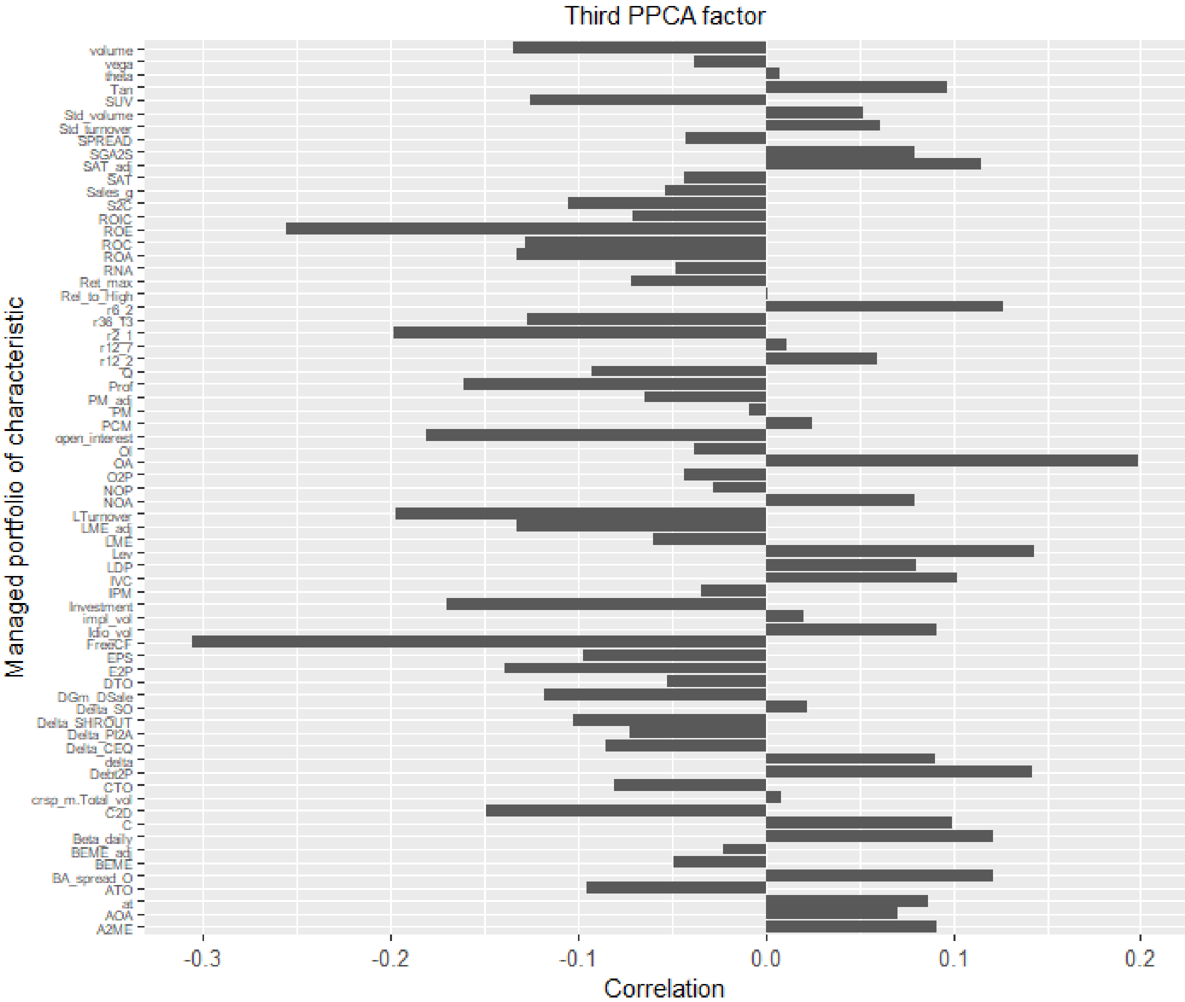

Figure 19: Correlations of the third PPCA factor of a four factor model with managed portfolios constructed by the characteristics

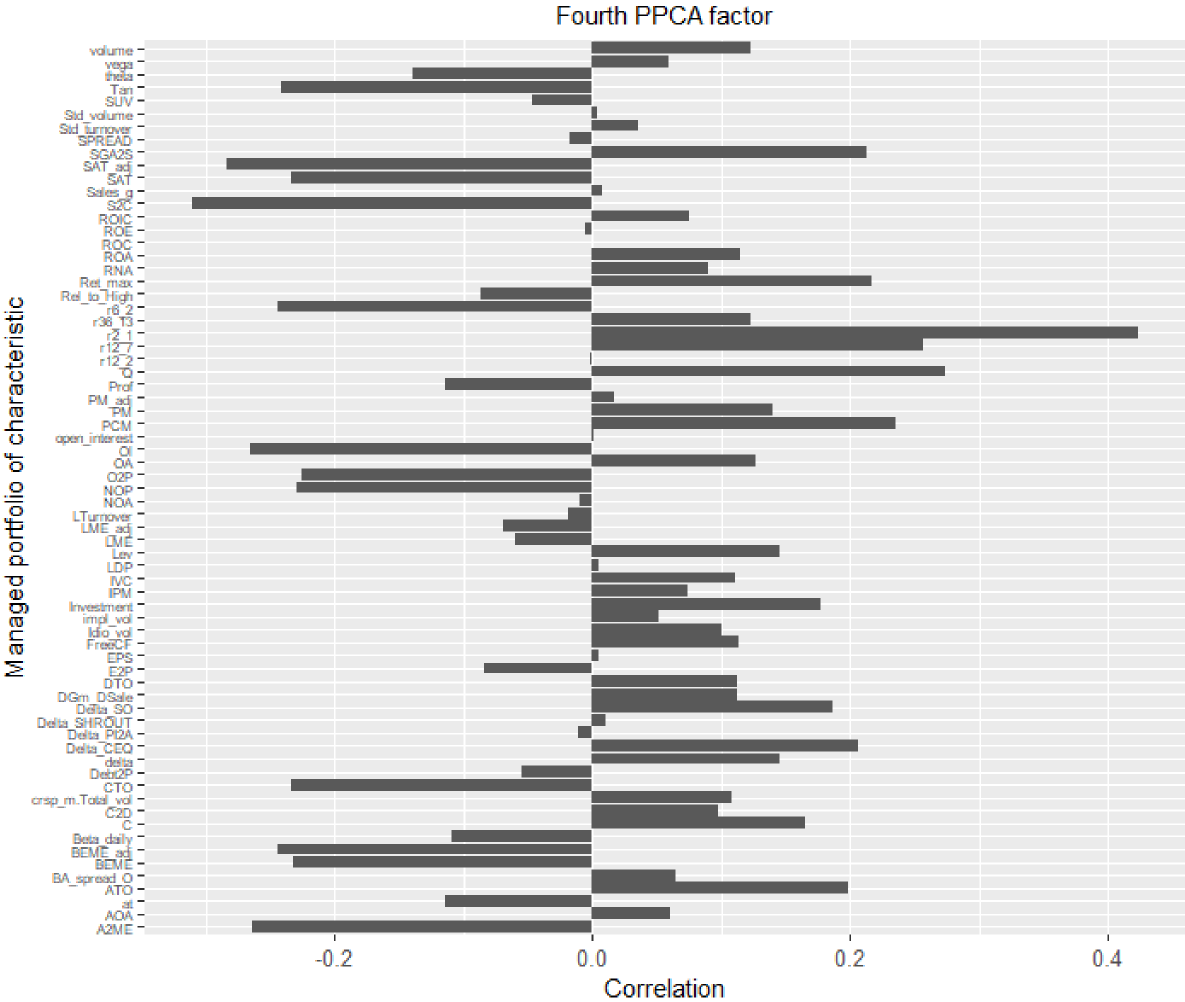

Figure 20: Correlations of the fourth PPCA factor of a four-factor model with managed portfolios constructed by the characteristics

8.6 Overview of programming code

The code files used for this thesis are handed in via a separate file. The different files could be categorized in three categories, which are data-preparation, IPCA and PPCA. All code regarding the data preparation and PPCA is written in R. The IPCA code is in Python. An overview of the files with short descriptions is listed under here.

Data preparation

- Find_S$P_500 constituents.R: Identifies the stocks that were present in the S&P500 during the whole sample period from 2010 to 2020
- Options_data_cleaning_and_constructing_delta_hedged_returns.R: Cleans the option data from OptionMetrics and corresponding stock data from CRSP. Afterwards, a panel of monthly delta-hedged option returns is constructed.
- Construct_characteristics.R: Constructs and merges all 69 characteristics with data from COMPUSTAT, CRSP and OptionMetrics
- Merge_returns_characteristics.R: This script merges the characteristics with the delta-hedged options returns panel

IPCA

- IPCA_first_correct_data.py: This script computes the performance measures for IPCA models using individual option returns and all characteristics. Also, it exports the factors for later comparison with PPCA factors in R. Furthermore, the significance tests for Γ_α and for all characterstics are performed in this script
- IPCA_cor_data_man_portfolios.py: Applies IPCA to characteristic managed portfolios and computes performance measures
- Loop_tangency_and_OOS.py: Out-of-sample procedure for IPCA. Computes out-of-sample performance measures, Sharpe ratios of tangency portfolios of factors and Sharpe ratios of the factors
- Export portfolios.py: Exports constructed managed portfolios, for use in R
- IPCA_in_sample_tan_port.py: Computes in-sample tangency portfolios of IPCA factors and their Sharpe Ratios
- IPCA_only_sign_chars.py: Computes the performance measures of IPCA models including only the significant characteristics found by the bootstrap test
- OOS man port.py: Computes out-of-sample statistics for IPCA applied to characteristic managed portfolios

PPCA

- PPCA_loop_performance measures.R: Computes performance measure for PPCA for different numbers of factors and Sieve dimensions
- PPCA_bootstrap_loop_k_cor.R: Applies to bootstrap testing procedure to find which characteristics are significant in the PPCA models
- PPCA_out_of_sample.R: Computes out-of-sample performance measures of PPCA models including different numbers of factors and sieve dimensions
- PPCA_sharpe_OOS.R: Computes out-of-sample tangency portfolios of PPCA factors and the corresponding Sharpe ratios
- PPCA_tan_in_sample_cor.R: Computes in-sample tangency portfolios of PPCA factors and the corresponding Sharpe Ratios
- PPCA_only_sign_char_j4: Computes PPCA models and performance measures including only the characteristics found to be relevant by the bootstrap testing procedure.
- PPCA_man_port.R: Computes performance measures for PPCA applied to characteristic managed portfolios for different numbers of factors and Sieve dimensions. Only the characteristics found to be significant in the bootstrap test are included in the models

Both methods

- Canonical correlations.R: Computes the canonical correlations between the sets of IPCA and PPCA factors
- Compute the correlations of the time-series realizations of IPCA and PPCA factors and characteristic managed portfolios

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