Erasmus University Rotterdam

Erasmus School of Economics

Thesis Econometrics and Management Science

Business Analytics and Quantitative Marketing

Joint model for multiple brand health KPIs: A comparison between the univariate and multivariate model

---

Author:
Raveena Dharmdasani (468497)

Academic supervisor:
Prof. dr. PHBF. Franses

Second assessor:
Prof. N.W. Koning

External supervisor:
A. Yazdiha

April 22, 2022

Abstract

This study analyzes the implementation of a multivariate Probit model in the modeling of multivariate funnel KPIs such as Awareness, Favorability, Consideration and Purchase Intent in the context of assessing the performance of a marketing campaign. This research contributes to the existing literature by incorporating the transitive correlation structure between funnel KPIs through the multivariate model and comparing that to the baseline, univariate model. The data is from the Facebook Portal 2020 US campaign provided by Nielsen. We find that the multivariate model succeeds in maintaining the existing transitive behavior from the univariate model in the base effects from the and it manages to impose the transitivity in the base effects between the funnel KPIs for which it is not present in the univariate model.

Contents

Chapter 1

Introduction

One of Nielsen's products is called the TMR (Total Media Resonance) in which the effect of a media campaign is evaluated on various KPI (Key Performance Indicator) variables such as awareness and consideration of a product, or an evaluation such as "value for money". TMR allows brands to measure more than sales by analyzing how audiences can be reached and influenced in the most impactful way. Different media channels are evaluated in order to identify the optimal investment level in each channel in order to achieve brand objectives, which are translated through KPI variables.

There are many KPI variables that are of interest for media owners and advertisers. We can consider KPIs in the marketing funnel in which awareness is at the top, followed by favorability, consideration and intent to purchase. To model these KPI variables Nielsen currently uses a respondent-level Bayesian model per KPI with explanatory variables that comprise of socio-demographic, media consumption and control variables. This setup does not use the relation between KPI variables, as the KPI variables are modeled separately. It may well be the case that several KPI variables are related and correlated to each other. For example, someone who considers a product to be of good value for money is more likely to also consider buying it. While the respondent-level models perform well and give interpretable results, it may be of interest to exploit the relation between KPI variables to attempt to obtain a better model. It is of interest to create a multivariate model, specifically a multivariate Probit model, in which we can jointly model KPI variables.

By incorporating the relation between multiple KPI in the model, we are able to compare a multivariate model for multiple KPI to a univariate model for separate KPI variables. We will be able to analyze how (dis)similarity of KPI variables and/or ordering of KPI variables in the marketing funnel affects the performance of a multivariate model. From this analysis, we can make decisions regarding in which cases it is best to use a multivariate or univariate model. This leads us to our research questions: Firstly, *how can KPI variables in the marketing funnel be modeled in a multivariate manner*? Secondly, *how does a joint multivariate model for multiple KPI compare to a default univariate model for individual KPI*?

This research is relevant to Nielsen, as establishing a joint model for multiple KPI allows the company to deliver additional interesting results to the client. Moreover, this research is also relevant to academic research, as there is limited research on the comparison of multivariate and univariate models in the context of marketing campaigns. The fact that we are researching brand health metric KPI variables in this context instead of regular sales KPI variables makes this research even more interesting, as there has not been much research done in this field.

The data used in this paper is regarding the Portal from Facebook 2020 campaign in the US. This campaign evaluates the promotion of one of Facebook's products called the Portal from Facebook, which is a video calling device. We have survey data obtained from Nielsen with a sample size of 6160 respondents and several type of variables along with media data obtained from Facebook for different media.

We find that the multivariate Probit model is suitable for modeling the funnel KPIs in a multivariate manner. Upon implementation and comparison with the baseline, univariate model we find that the multivariate model maintains the existing transitive behavior between funnel KPIs from the univariate model and manages to impose transitivity between the funnel KPIs which is not present in the univariate model. Moreover, the simulation study shows that despite the magnitude of correlation that exists between the funnel KPIs, the multivariate model succeeds in imposing transitivity in the base effects between the funnel KPIs.

The rest of the paper is organized as follows. Chapter 2 consists of the review of relevant literature. Chapter 3 describes the data, followed by the methodology introduced in Chapter 4. Chapter 5 discusses the empirical and simulation study results. Finally, the paper is concluded in Chapter 6.

Chapter 2

Literature Review

In this section I will review several studies and past literature about the use and importance of multivariate KPIs.

2.1 Multivariate KPI

Key Performance Indicators (KPIs) are variables that companies use to measure how well they perform in a way that is comparable over time, products and markets. The modeling of KPIs and making inferences from them can aid a company in planning and decision-making. KPI modeling has continually shown to be beneficial in providing companies with insights into operations and ROI (Return on Investment) in different fields such as manufacturing (White, 1996), the drug supply chain within hospitals (Mezouar et al., 2016), innovation implementation (Sawang, 2011), the growth of shareholders welfare and the satisfaction of the interests of other stakeholders in corporate finance (Strelnik, Usanova, & Khairullin, 2015), among many.

While there are many different types of KPIs that can be of interest in many different fields, it is of interest to concentrate on brand health metrics in the field of media, marketing and advertising. Oliveira and Figueira, 2017 demonstrate which widely known social media KPI are worth pursuing in terms of gathering audience's interactions and aiding companies in better designing their content strategies to obtain the best ROI. Along similar lines, (Ahmad, Musa, & Harun, 2016) investigate the role of social media content marketing in increasing the brand health score. Moreover, Berg, Matthews, and O'Hare, 2007 provide a statistically reliable set of brand health metrics for companies to use as KPIs of sales, risk and potential.

When evaluating brand health metrics, we would like to zoom into KPIs in the marketing funnel specifically. Colicev, Kumar, and O'Connor, 2019 investigate how the similarities and differences between user generated content and firm generated content influence the marketing funnel stages of awareness, consideration, purchase intent and satisfaction. Interestingly, Johnson, Lewis, and Nubbemeyer, 2017 relate the marketing funnel to the incremental effect of ads on the consumer purchase process. Relating to the aforementioned papers, in this research we will be investigating the KPIs in the marketing funnel using different media, including social media.

From the literature, we see that there is a demand for the facilitation of multiple KPI analysis. Kornas et al., 2019 introduce a novel KPI approach, which is able to identify interdependencies in production systems in the context of quality assurance in lithium-ion-battery (LIB) production. They state that their LIB KPI system structure is hierarchical and is based on the correlated interdependencies in the process charcteristics. Not only do they include the correlation structure, it is also based on a cascade principe, which is comparable to the marketing funnel. Although the context and implementation of Kornas et al., 2019's research on the introduction of a multivariate KPI-based method for the cause-effect-relationships in the manufacturing process of LIB is different from this study's direction of multivariate KPIs in the marketing funnel, it shows that there is demand for and gain from incorporating correlation structures in KPI modeling.

Additionally, Corsini et al., 2016 construct a new multivariate KPI for efficient monitoring of the energy performance of cooling systems in the food industry. Albeit the fact that Corsini et al., 2016 do not necessarily employ a multivariate model in the analysis, they do see the need for there to be a multivariate KPI that takes into account the correlation between energy consumption and the different process variables. Consequently, multivariate statistical analysis is used in the development of this new multivariate KPI. The multivariate KPI was compared to the standard KPI and they concluded that the multivariate KPI was able to incorporate the correlation between energy consumption and different process variables, while the single KPI only evaluated the energy performance. Hence, the multivariate KPI delivers a more complete interpretation of energy behavior.

Moreover, Ouyang and Fallah, 2010 implement multivariate statistical analysis to KPIs within the context of Universal Mobile Telecommunications System (UMTS). With this they aim to assist mobile operators with keeping up with the expansion of data services provided by the mobile industry. Besides this, they also aim to assist mobile operators in developing effective operational models to control for the different mixes of audio, data and video traffic on a network. They also take into account several methods in their multivariate statistical analysis, namely, correlations and factor analysis.

Hence, literature shows that there is a need for multivariate analysis of KPIs, whether that is achieved by constructing a single multivariate KPI based on multivariate analysis, or whether that is achieved by implementing a multivariate model in which KPIs are modelled jointly. Therefore, in this study it is of interest to elaborate on the latter approach of implementing a multivariate model for multiple KPI. It is also of interest to examine how this effects brand health metrics KPIs in the field of marketing research, which is the context of this study.

Chapter 3

Data

The data used in this research is regarding the Portal from Facebook from the 2020 campaign in the United States. The data consists of survey data and media data. The survey data is obtained from survey respondents through Nielsen, while the media data is obtained from Facebook. The focus of this research is to take advantage of the correlation structure between KPI variables in the marketing funnel in a multivariate manner compared to the regular univariate model.

Survey data

The survey data is obtained from respondents residing in the US with a total sample size of $n = 6160$ respondents. The survey ran from October 12$^{\text{th}}$ 2020 until January 4$^{\text{th}}$ 2021. The survey data contains multiple types of variables, namely, socio-demographic, identifier, KPI, control and media consumption variables. The survey data is assumed to be clean, since only eligible respondents are included, while incomplete surveys are excluded.

The identifier variables include variables that aid in identifying respondents and the survey itself, such as respondent ID and survey date. The socio-demographic variables available consist of age, gender, ethnicity, education level, employment status, household income, marital status, number of children, and the state the respondent resides in. The control variables are those that further define the respondent and give us insights into the behavior of respondents. These include the frequency with which the respondent makes video calls, whether they believe they stay informed about the latest technology products even when they are not planning a purchase, whether they view smart devices as secure, whether they often feel intimidated by new technologies, whether it is important for them to stay in touch with loved ones when they are traveling, whether it is important for them to share special moments with their family and close friends, several Facebook usage variables and several competitor usage variables. The competitors include Instagram, WhatsApp, and Amazon Alexa. Another control variable is regarding whether they use an ad blocker in the devices that they own.

Moreover, questions regarding media consumption patterns across different media and vehicles of interest are asked in the survey, which are eventually combined with the media data in order to estimate the exposure to the advertising. In the Facebook Portal 2020 US campaign we have 5 different media. A summary of the available media and the corresponding vehicles is depicted in Table 3.1

Table 3.1: List of all the available media and corresponding vehicles asked in the Facebook Portal 2020 survey.

Medium	**Vehicle**
TV	ABC, NBC, FOX, etc.
Video-on-Demand	ABC GO, FOX NOW, etc.
Social Network	Facebook, Instagram, Twitter, etc.
Online Music Streaming Platforms	Spotify, TuneIn, Pandora, Deezer
Other digital	Several specific websites, and categories of websites, such as news, sports, etc.

The KPI variable set consists of 6 KPI variables that were specified by the client. We can distinguish between 2 perception KPI variables and 4 KPI variable that belong in the marketing funnel. The KPIs in the marketing funnel are depicted in Figure 3.1.

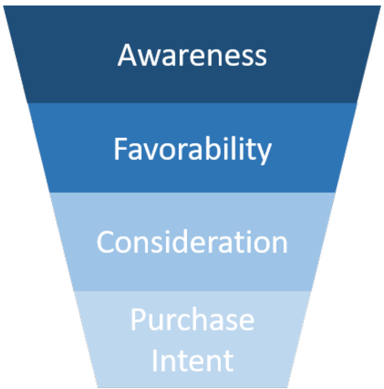


Figure 3.1: The ordering of KPIs in the marketing funnel.

In the present data, we find that the funnel KPIs are indeed correlated in a transitive manner. The correlation between *awareness* and *favorability* is 0.63. The correlation between *favorability* and *consideration* is 0.75 and the correlation between *consideration* and *purchase intent* is 0.78. Here we focus on the correlations between consecutive KPIs in the marketing funnel due to the routing mechanism used in the survey. If a respondent indicates that they are not aware of the product, then they are not shown the following questions regarding favorability, consideration and purchase intent. In a similar manner, if a respondent indicates that they are aware, but do not have a favorable opinion on a product, then they are not shown the questions regarding consideration and purchase intent. Because of this type of routing used in the survey it is of interest to focus on the transitivity in correlations between consecutive KPIs. A list of the KPIs and the corresponding survey questions and response options can be found in Table 3.2.

Table 3.2: Description of different types of KPIs and the corresponding survey questions and response options.

Perception KPI	**Survey Questions**	**Response Options**
Perception	Which of these devices do you think: "Allows me to connect with and feel closer to the most important people in my life"?	Portal from Facebook, Amazon Echo & Fire TV, Google Home & Nest Hub, Apple HomePod & TV, None of these devices
Perception Thoughtful Gift	Which of these devices do you think: "Is a thoughtful gift for loved ones"?	Portal from Facebook, Amazon Echo & Fire TV, Google Home & Nest Hub, Apple HomePod & TV, None of these devices
Funnel KPI	**Survey Questions**	**Answer Options**
Awareness	Which of the following Voice Activated Home Devices have you heard of before today?	Portal from Facebook, Amazon Echo & Fire TV, Google Home & Nest Hub, Apple HomePod & TV, None of these devices
Favorability	What is your overall opinion of these interactive smart home device brands?	Very favorable, Somewhat favorable, Neutral, Somewhat unfavorable, Very unfavorable
Consideration	If you were purchasing an interactive smart home device, how likely would you be to consider purchasing one of these devices?	It would be my first choice, I would seriously consider it, I might consider it, I would not consider it
Purchase Intent	How likely would you be to purchase each of these devices in the next 6 months?	Very likely, Somewhat likely, Neither likely nor unlikely, Somewhat unlikely, Very unlikely

Media data

The media data obtained from Facebook consists of data on 5 different media, namely, TV, FEP (Full Episode Player), On-Platform Paid Social, Performance Digital and Youtube. The marketing campaign ran from mid-October up to December 31$^{\text{th}}$ 2020, with the exact starting date varying per medium. A summary of the available media and the corresponding vehicles is depicted in Table 3.3

Table 3.3: List of all the available media and corresponding vehicles obtained by Facebook

Medium	**Vehicle**
TV	National, Local
FEP (Full Episode Player)	CBS, Hulu, NBC, ABC
On-Platform Paid Social	Facebook & Instagram
Performance Digital	SpotX, Pandora, Spotify, Kargo, Open Exchange, Amazon, Verizon, Outbrain, Target, Amazon Over The Top
Youtube	Preferred & TrueView

Chapter 4

Methodology

Given survey and media data, we can model campaign effectiveness through the following steps: contact estimation, variable selection through machine learning and Bayesian modeling. Each step is explained below along with an in-depth explanation of the Bayesian model used.

4.1 Modeling Campaign Effectiveness

4.1.1 Contact Estimation

In this step the media data is combined with the media consumption data calculated from the survey data in the data processing stage in order to estimate the amount of contacts every respondent has during the campaign period for all media. These contacts are a measure for the exposure to the marketing campaign for each respondent.

Next, weights are assigned to respondents to make certain respondents in our survey more influential in certain steps of the modeling process than other respondents. Through the weights we can make the survey sample more representative of the population we are reporting on. Some people may be overrepresented in the sample, while others may be underrepresented due to selection bias. The assigned weights are a measure for how much the influence of respondents in the sample should be adjusted to account for the misrepresentation.

Moreover, a retention parameter is included when estimating contacts. This parameter considers the weekly decay of the effects from exposure to the advertising. The default retention rate is set to 0.7, which has been shown to give reliable results in past projects done by Nielsen.

Furtherore, the contacts are unified with the rest of the survey data in order to produce one large data set that contains all relevant socio-demographics, control variables and estimated contacts for all respondents.

4.1.2 Variable Selection

In this step the GBM (Gradient Boosting Machine) is used as an exploratory step in order to get a first view on the relation between the independent variables and the KPI variables. Firstly, the data is manipulated such that missing values are filled in and relevant dummies are created, among others. Next, a k-fold cross validation is performed. However, the prediction of values obtained from from the GBM is not its main takeaway. The aim of using a GBM in this step is to acquire some initial ideas on potential variables or groupings of variables that may of interest in our model along with an initial idea of the role the different media channels play in the model through the prediction performance.

4.1.3 Bayesian Modeling

Univariate Model

The default model utilized by Nielsen uses a Bayesian model in order to predict a KPI level on a respondent level given their socio-demographic variables, control variables and media variables. The variables follow certain definitions and prior distributions in the Bayesian model. The default univariate model follows a normal distribution and is formulated as follows:

$$y_i \sim Normal\Big(\alpha + C_i + M_i, \frac{y^{scl}}{\sqrt{w_i}}\Big) \tag{4.1}$$

$$C_i = \sum_c x_{ci}^{ctrl} \lambda_c \tag{4.2}$$

$$M_i = \sum_m pot_{m,a_i} \tanh\big(\frac{1}{2} spd_{m,a_i} x_{mi}^{exp}\big) \tag{4.3}$$

where α is the base term, C_i is the control term and M_i is the media term for respondent i.

In this formulation y_i corresponds to the KPI level of interest for respondent i. Moreover, x_{ci}^{ctrl} is a control variable c obtained from the survey data for respondent i and λ_c is the corresponding control parameter for control variable c. Next, x_{mi}^{exp} is the media exposure expressed in terms of estimated contacts for media channel m for respondent i. The media term is modelled following the tanh function to capture the law of diminishing returns. This means that beyond a certain amount of money spent on a campaign, the increase in returns, or in this case, the increase in KPI levels, decreases. This means that the slope of the media curve always decreases. The potential *pot* of the contact curve refers to the height that the contact curve can reach, while the speed *spd* refers to the speed with which the contact curve increases.

The media term is defined in this way to account for the trade-off between the potential and speed of the media curve. An example of the media curve and the trade-off between potential and speed is presented in Figure 4.1. In this figure, the purple curve has a high potential value and a slow speed. This indicates that the maximum media effect that can be achieved is high, but much more contacts are required to achieve this. Meanwhile, the green curve has a low potential value and a high speed. This indicates that the maximum media effect that can be achieved is lower, but less contacts are required to achieve this.

Figure 4.1: Example of the media curve.

In the relevant data set we have three different modeling audiences, namely, tech-savvy communicators, family networkers and the other audience group. The client is interested in media effects for all three audiences. It is certainly a possibility to run a separate model for each audience, but this would require the assumption that the audiences are independent from each other. In reality, it may well be the case that audiences are not completely independent from each other. In contrast to assuming complete independence between audiences, it is safer to make the assumption that respondents within an audience group are more similar to each other than to respondents from other audience groups. A hierarchical Bayesian model tries to capture this discrepancy while still assuming some similarities across audience groups. A hierarchical Bayesian model allows us to assume a common distribution with a shared parameter from which the audience-specific parameters arise from. In this model formulation we have the audience-specific parameters pot_{m,α_i} and spd_{m,α_i}, which are sampled as follows:

$$pot_{m,a_i} \sim Normal\left(pot_m^{loc}, pot_m^{scl}\right) \tag{4.4}$$

$$pot_m^{loc} \sim Normal\left(pot\_loc_m^{loc}, pot\_loc_m^{scl}\right) \tag{4.5}$$

$$spd_{m,a_i} \sim Lognormal\left(spd_m^{loc}, spd_m^{scl}\right) \tag{4.6}$$

$$spd_m^{loc} \sim Lognormal\left(spd\_loc_m^{loc}, spd\_loc_m^{scl}\right) \tag{4.7}$$

where $pot_m^{loc} = 0$, $pot_m^{scl} = 0.10$, $pot\_loc_m^{loc} = 0$, $pot\_loc_m^{scl} = 0.10$, $spd_m^{loc} = -0.50$, $spd_m^{scl} = 0.80$, $spd\_loc_m^{loc} = -0.5$ and $spd\_loc_m^{scl} = 0.15$

Moreover, the base term α, the control parameter λ and the scale parameter y^{scl} are sampled as follows:

$$\lambda \sim Normal\Big(0, 1\Big) \tag{4.8}$$

$$\alpha \sim Normal\Big(0, 1\Big) \tag{4.9}$$

$$y^{scl} \sim HalfCauchy\Big(0, 1\Big) \tag{4.10}$$

In these equations the potential and speed of the contact curve are given by pot_{m,α_i} and spd_{m,α_i} for media channel m for the audience group of respondent i.

The media term is modelled following the tanh function to capture the law of diminishing returns as mentioned previously. This means that the slope of the media curve always decreases. The potential *pot* of the contact curve refers to the height that the contact curve can reach, while the speed *spd* refers to the speed with which the contact curve increases.

For the potential, we want to use a prior that can take any value, since we want to have it as a theoretical option to find negative potentials. Simultaneously, we want to be able to pull the potential closer to certain values using the prior, so it needs to be a distribution that allows us to do so. The normal distribution has these properties while simultaneously being easy to use in practice. For the speed, we want to use a prior that is strictly positive, and not too close to 0, since in that case the model estimation runs into problems due to the variable being almost linear. Often the data gives a weak signal on what the speed should be. Because of that, we want to use a distribution that allows us to give good guidance towards the value of the speed for these situations. On the other hand, if there is a clear signal in the data for the value of the speed, that should take precedence over the prior. A lognormal distribution allows for all of this, while being easy to work with due to being a transformation of a normal distribution.

The control parameter λ_c and the base term α have a weakly uninformative prior, such that the prior would not affect the posterior much and the data is allowed to give good guidance for this.

For the scale parameter of y, y^{scl}, a Half Cauchy distribution is used, which is equivalent to the positive part of the standard t-distribution with 1 degree of freedom. Subsequently, this term is divided by the square root of the respondent's assigned weight to ensure that respondents with a higher weight are fitted more accurately.

Multivariate Probit Model

The standard way of modeling KPIs in the default TMR model used by Nielsen is a univariate respondent-level model, which models the KPIs separately. However, to exploit the correlations between KPIs in the marketing funnel, we can use a multivariate model to model the KPIs jointly. Given that the KPIs are modelled in a binary way, a multivariate Probit model is proposed (Chib & Greenberg, 1998). In previous literature this model has been utilized in many different contexts. Mittal and Mehar, 2016 investigate the socio-economic factors affecting adoption of modern information and communication technology by farmers in India using the multivariate Probit model. Lansink, Berg, and Huirne, 2003 analyze the strategic planning of Dutch pig farmers using a multivariate Probit model. Lesaffre and Molenberghs, 1991 perform a multivariate probit analysis in the field of medical problems, while Rao and Winter, 1978 investigate the application of the multivariate Probit model for product design.

Despite the many applications of the multivariate Probit model in different fields, there has not been much research done of its application in the field of media and advertising, specifically with regard to KPI modeling, Hence, it is of interest to investigate that in a Bayesian manner in this paper.

In order to model the funnel KPIs in a multivariate manner, we make use of the multivariate Probit model, in which the order of the funnel KPIs is imposed through the latent utilities. In this case we have four funnel KPIs in which we observe ordering according to the location in the funnel as specified in the Data section. Due to the utilization of top-boxes, a respondent i makes binary choices over $J = 4$ decisions. This corresponds to the four funnel KPI questions. The response $y_{ij} = 1$ indicates that respondent i chooses 1 on decision j, and $y_{ij} = 0$ indicates that individual i chooses 0 on decision j with $i = 1, ..., n$, $j = 1, .., J$, with $J = 4$ in this case. In order to model this effectively following Greenberg, 2012, we introduce a latent variable y^*_{ij}, such that

$$y^*_{ij} = x'_{ij}\beta_j + u_{ij} \tag{4.11}$$

where x_{ij} and β_j are K_j x 1, and let $K = \sum K_j$. The observed choices are the y_{ij}, which are related to the latent data through

$$y_{ij} = \begin{cases} 0 & \text{if} \quad y^*_{ij} \leq 0 \\ 1 & \text{otherwise} \end{cases} \tag{4.12}$$

We define $y_i^* = (y_{i1}^*, ..., y_{iJ}^*)'$, and $u_i = (u_{i1}, ..., u_{iJ})'$ and aggregate all x_{ij} for $j = 1, ..., J$ vectors into a single matrix X as follows:

$$X_i = \begin{pmatrix} x_{i1}^* & 0 & \cdots & 0 \\ 0 & x_{i2}^* & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & x_{iJ}^* \vdots \end{pmatrix} \tag{4.13}$$

Moreover, upon defining $\beta = (\beta_1^*, ..., \beta_J^*)'$ and $u_i = (u_{i1}, ..., u_{iJ})'$, we have

$$y_i^* = X_i\beta + u_i \tag{4.14}$$

We assume that $u_i \sim N_J(0, \Sigma)$. Hence, in this case for $J = 4$, we have the following covariance matrix:

$$\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} & \sigma_{34} \\ \sigma_{14} & \sigma_{24} & \sigma_{34} & \sigma_{44} \end{pmatrix} \tag{4.15}$$

This covariance matrix Σ is equal to $\frac{Y^{scl}}{\sqrt{W_i}}$, where Y^{scl} is the covariance between KPIs, which is scaled by the respondent weights W_i . We require the scale to be positive, since a negative scale would suggest that certain KPIs are negatively correlated to each other. This contradicts our assumption that brand health KPIs are solely positively correlated with each other. Brand health KPIs are assumed to be positively correlated with each other, because it is not logical for brand health KPIs to move in the opposite direction following a certain marketing campaign. The covariance matrix Y^{scl} can be defined as follows:

$$Y^{scl} = \begin{pmatrix} y_1^{scl} & 0 & 0 & 0 \\ 0 & y_2^{scl} & 0 & 0 \\ 0 & 0 & y_3^{scl} & 0 \\ 0 & 0 & 0 & y_4^{scl} \end{pmatrix} \tag{4.16}$$

where y_j^{scl} is the variance that corresponds to a certain KPI j. Although it is possible, we choose to not impose the ordering of the funnel KPIs through these covariances. The model may be too restrictive if ordering of the funnel KPIs is imposed through the correlation coefficients. Hence, we choose to impose the ordering in a less restrictive way through the base effect. For this reason, the covariances between KPIs are assumed to be zero. The variances y_j^{scl}, $j = 1, ..., J$

are sampled through the Half-Cauchy distribution as follows:

$$y_j^{scl} \sim HalfCauchy\Big(0,1\Big) \quad \text{for} \quad j = 1, ..., J \tag{4.17}$$

These individual variances are aggregated as the diagonal values in Y^{scl}

In the regular regression in Equation (4.16), $X_i\beta$ is divided into a base term α, a control term C_i and a media term M_i, for each KPI in the multivariate model. Hence, we have $X_i\beta = (\alpha + C_i + M_i)'$. which is a vector of length J which is defined as follows:

$$C_i = \sum_c x_{ci}^{ctrl} \lambda_c \tag{4.18}$$

$$M_i = \sum_m pot_{m,\alpha_i} \tanh\left(\frac{1}{2} spd_{m,\alpha_i} x_{mi}^{exp}\right) \tag{4.19}$$

where x_{ci}^{ctrl} is a control variable c obtained from the survey data for respondent i and is of length J and λ_c is the control parameter for control variable c and is of length J. Moreover, x_{mi}^{exp} is the media exposure expressed in terms of estimated contacts for media channel m for respondent i. The potential and speed of the contact curve are given by pot_{m,α_i} and spd_{m,α_i} for media channel m for the audience group of respondent i, and is of length J.

As mentioned previously we have three different modeling audiences in this data set. In this model formulation we have the audience-specific parameters pot_{m,α_i} and spd_{m,α_i}, which are sampled as follows:

$$pot_{m,a_i} \sim \text{Multivariate normal}\left(\begin{bmatrix} pot_{m,1}^{loc} \\ pot_{m,2}^{loc} \\ pot_{m,3}^{loc} \\ pot_{m,4}^{loc} \end{bmatrix}, \begin{bmatrix} pot_{m,1}^{scl} & 0 & 0 & 0 \\ 0 & pot_{m,2}^{scl} & 0 & 0 \\ 0 & 0 & pot_{m,3}^{scl} & 0 \\ 0 & 0 & 0 & pot_{m,4}^{scl} \end{bmatrix}\right) \tag{4.20}$$

$$spd_{m,a_i} \sim \text{Multivariate lognormal}\left(\begin{bmatrix} spd_{m,1}^{loc} \\ spd_{m,2}^{loc} \\ spd_{m,3}^{loc} \\ spd_{m,4}^{loc} \end{bmatrix}, \begin{bmatrix} spd_{m,1}^{scl} & 0 & 0 & 0 \\ 0 & spd_{m,2}^{scl} & 0 & 0 \\ 0 & 0 & spd_{m,3}^{scl} & 0 \\ 0 & 0 & 0 & spd_{m,4}^{scl} \end{bmatrix}\right) \tag{4.21}$$

where where $pot_{m,j}^{scl} = 0.10$ and $spd_{m,j}^{scl} = 0.10$ for $j = 1, ..., J$. The audience-specific parameters $pot_{m,j}^{loc}$and$spd_{m,j}^{loc}$ for $j = 1, ..., J$ can be aggregated into J-dimensional parameters pot_m^{loc}, spd_m^{loc}, which are sampled from a common distribution as follows:

$$pot_m^{loc} \sim \text{Multivariate normal}\left(\begin{bmatrix} pot\_loc_{m,1}^{loc} \\ pot\_loc_{m,2}^{loc} \\ pot\_loc_{m,3}^{loc} \\ pot\_loc_{m,4}^{loc} \end{bmatrix}, \begin{bmatrix} pot\_loc_{m,1}^{scl} & 0 & 0 & 0 \\ 0 & pot\_loc_{m,2}^{scl} & 0 & 0 \\ 0 & 0 & pot\_loc_{m,3}^{scl} & 0 \\ 0 & 0 & 0 & pot\_loc_{m,4}^{scl} \end{bmatrix}\right) \tag{4.22}$$

$$spd_m^{loc} \sim \text{Multivariate lognormal}\left(\begin{bmatrix} spd\_loc_{m,1}^{loc} \\ spd\_loc_{m,2}^{loc} \\ spd\_loc_{m,3}^{loc} \\ spd\_loc_{m,4}^{loc} \end{bmatrix}, \begin{bmatrix} spd\_loc_{m,1}^{scl} & 0 & 0 & 0 \\ 0 & spd\_loc_{m,2}^{scl} & 0 & 0 \\ 0 & 0 & spd\_loc_{m,3}^{scl} & 0 \\ 0 & 0 & 0 & spd\_loc_{m,4}^{scl} \end{bmatrix}\right) \tag{4.23}$$

$$\lambda \sim \text{Multivariate normal}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right) \tag{4.24}$$

$$\alpha \sim \text{Multivariate normal}\left(\begin{bmatrix} 0.04 \\ 0.03 \\ 0.02 \\ 0.01 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right) \tag{4.25}$$

where $pot\_loc_{m,j}^{loc} = 0$, $pot\_loc_{m,j}^{scl} = 0.10$, $spd\_loc_{m,j}^{loc} = -0.5$ and $spd\_loc_{m,j}^{scl} = 0.15$.

The ordering of the funnel KPIs is imposed through the prior distribution of the intercept term α. The location and variance vector and matrix parameters are arranged according to the funnel KPIs. Hence, the first element of the location vector $\begin{pmatrix} 0.04 \\ 0.03 \\ 0.02 \\ 0.01 \end{pmatrix}$ corresponds to the Awareness KPI, while the second, third and fourth elements correspond to Favorability, Consideration and Purchase Intent, respectively. The elements of the location vector display a decreasing pattern, because we expect the base effects of the multivariate model to display transitive behavior between the funnel KPIs. By restricting this location vector to have decreasing elements, we can impose transitivity in the base effects. Thus, the base effect belonging to Awareness should be higher than the base effect for Favorability, which should be higher than the base effect for Consideration, which should be higher than the base effect for Purchase Intent.

Hamiltonian Monte Carlo No-U-Turn Sampler

Hamiltonian Monte Carlo an a Markov Chain Monte Carlo (MCMC) algorithm that avoids the random walk behavior and sensitivity to correlated parameters by taking a series of steps influenced by the first-order gradient information (Hoffman, Gelman, et al., 2014). The HMC uses the derivative of the sampling density to generate efficient transitions, which is very efficient if the parameter settings fit. The HMC converges to high-dimensional target distributions much more quickly than Metropolis Hastings or Gibbs sampling. On top of that, HMC preserves the target distribution, making the chain drift towards the typical set of the target distribution regardless of the initial starting point. A burn-in period is required to account for the bias in the first draws of the parameter space due to the fact that the Markov chain must convert first. The advantage of HMC over other sampling methods such as the Metropolis-Hastings (MH) algorithm is that HMC is able to make relatively large leaps to unexplored spaces, hence, moving more efficiently across the parameter space (Betancourt, 2017). This results in higher acceptance rates and faster convergence. The power of HMC lies in in its Hamiltonian dynamics, which is a reformulation of Lagrangian dynamics.

We specify the Hamiltonian function, which is a representation of total energy at a point in space according to Betancourt, 2017 as follows:

$$H(\theta, r) \equiv -log\, \pi(\theta, r) = -log\, \pi(r|\theta) - log\, \pi(\theta) \tag{4.26}$$

$$\equiv K(r, \theta) + V(\theta) \tag{4.27}$$

The total energy can be decomposed into two parts, namely, the kinetic energy $K(r, \theta)$, which corresponds to the density over the supplementary momentum, and the potential energy $V(\theta)$, which corresponds to the density of the target distribution. HMC uses a numerical integrator called the leapfrog integrator which takes discrete steps of size ϵ. After L leapfrog updates of size ϵ, the current position and momentum can be accepted or rejected.

HMC has a disadvantage of being sensitive to certain parameters in its performance, thus it requires careful selection of the parameters ϵ and L in the algorithm, because the performance and duration of the algorithm is dependent on these variables. If the number of steps L is too large, the computation time of the algorithm increases. In contrast, if L is too small, we risk having a random walk sampling method.

HMC No-U-Turn Sampler (HMC-NUTS) in an extension of HMC in the sense that it eliminates the requirement of setting L and ϵ as user input(Hoffman, Gelman, et al., 2014). The notion of time reversibility of the chain is necessary to guarantee convergence. The regular

HMC does not guarantee time reversibility. NUTS tackles this matter by preserving time reversibility by allowing the Hamiltonian simulation to run both forward and backward in time, which eliminates the need to set L. Moreover, the need to set the step size ϵ by the user is also eradicated by the possibility to obtain a satisfactory value for the step-size during burn-in. The obtained step size is then kept fixed for the remaining draws.

4.2 Performance Evaluation

To compare the performance of the multivariate model with the univariate model, we compare the difference between the parameter coefficients using several metrics. The Area Under the ROC curve (AUC) is commonly used in binary classification problems (Huang & Ling, 2005). The univariate model can be considered as the baseline model, since it is established that this is a statistically sound and robust model delivered to the client. By comparing the AUC of the multivariate model with the AUC of the univariate model, we can obtain an indication of the robustness of the multivariate model.

Furthermore, we are also interested in the base effects of a campaign. It is then of interest to observe whether the multivariate model, which takes into account the ordering of funnel KPIs, also exhibits this ordering in the base effects across the funnel KPIs. We compare the decomposition of base effects of the multivariate model with those of the univariate model.
Moreover, the credible interval (CI) is an interval within which an unobserved parameter value falls with a particular probability. By reporting on the credible interval (CI) coverage, we can attain information on how often the CI covers the true parameter values in terms of percentage. percentage. The CI are obtained from the 5% and 95% quantiles of the sample of draws from the posterior distributions.

Chapter 5

Results

We present the results of the empirical and simulation study through a comparison of the multivariate model with the baseline, univariate model on several metrics for each of the funnel KPIs. Firstly, we will look at the differences in statistical inferences between the models by evaluating and comparing the AUC values for both models.

Next, we will evaluate the parameter comparison by assessing the decomposition of base and media effects for the univariate model and the multivariate model. This analysis will give us more insights into the comparison of the estimated base and media effects between the models. Moreover, it will give us insights into the occurrence of a transitive behavior between the funnel KPIs in the base effects in both models.

Lastly, we will evaluate and compare the estimated posterior densities through the CI coverage. This will also give us more insights regarding the Bayesian robustness of the multivariate model compared to the univariate model.

5.1 Empirical Study

5.1.1 Statistical Inference

The Area Under the ROC Curve (AUC) for the multivariate and univariate models for each KPI are given in Table 5.1. By comparing the AUC values for the multivariate model with those from the univariate model, we can evaluate whether the multivariate model, in which ordering of the funnel KPIs is present, produces comparable classification results as the baseline, univariate model. We see that the multivariate model does not differ much from the baseline, univariate model. For the Favorability and Purchase Intent KPI, we observe the identical AUC value as the univariate model. Meanwhile, for the Awareness KPI, we observe a slightly higher AUC, with an increase of 0.31%. On the other hand , for the Consideration KPI we observe a slightly lower AUC for the multivariate model, namely, a decrease of 3.27%. It is evident that it in inconclusive whether the multivariate model outperforms the univariate model in terms of AUC values, however, the multivariate model certainly performs closely to the univariate model in terms of the ability to classify KPIs.

Table 5.1: AUC values for the multivariate and univariate models for each KPI.

	Awareness	Favorability	Consideration	Purchase Intent
Univariate model	0.648	0.819	0.795	0.839
Multivariate model	0.650	0.819	0.769	0.839

5.1.2 Parameter Comparison

Besides the comparison of the robustness of both models, we are also interested in the comparison of parameters generated by both models. The ordering of the funnel KPI is implemented in the base effect of the equation which models campaign effectiveness. Hence, it is of interest to mainly compare the base effects of the funnel KPIs between the multivariate and univariate model.

The division between the base and media effect for the multivariate and univariate model for Awareness for the three different audiences is depicted in Figure 5.1. Firstly, it is evident that the multivariate model estimates base and media effects comparable to those estimated by the univariate model for all audiences. The relative differences in base effects between the multivariate and univariate model for the audiences Tech Connecters, Other and Family Networkers are -0.38%, 3.80% and 1.25%, respectively. Hence, there are no large differences in the estimated base effects through introduction of ordering in the funnel KPI in the multivariate model for Awareness.

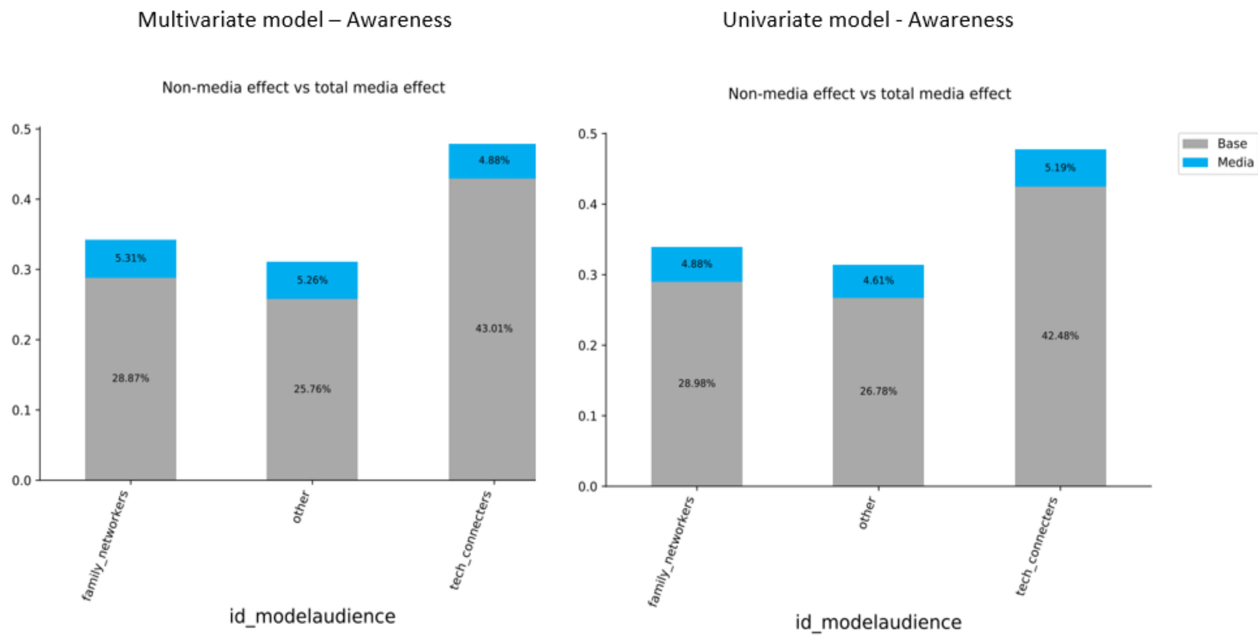


Figure 5.1: Comparison in base and media effects between the multivariate and univariate model for Awareness for the three audiences.

Similarly, the division between the base and media effect for the multivariate and univariate model for Favorability is depicted in Figure 5.2. In contrast to Awareness, there do appear to be some stark differences in the base effects between the multivariate and univariate model for the three audiences for Favorability. The relative differences in base effects between the multivariate and univariate models for the audiences Tech Connecters, Other and Family Networkers are -69.60%, -77.30% and -56.15%, respectively.

When comparing the base effects for Favorability in Figure 5.2 with the base effects for Awareness in Figure 5.1 for the baseline, univariate model, we observe that the base effects for Favorability are higher for all three audiences than the base effects for Awareness. Given the ordering of the funnel KPI in which Favorability follows after Awareness, we would expect Favorability to have lower base effects compared to Awareness, since theoretically only respondents who are aware of the product can have a favorable opinion of it. When comparing the base effects for Favorability with the base effects for Awareness for the multivariate model, in which the ordering of the funnel KPIs is present in the prior distribution specification for the base effect, we observe that the base effects for Favorability for the three audiences in Figure 5.2 are lower than the base effects for Awareness in Figure 5.1. This transitivity in the base effects between higher and lower funnel KPIs is anticipated. Hence, the multivariate model succeeds in incorporating this transitive behavior in base effects between Awareness and Favorability.

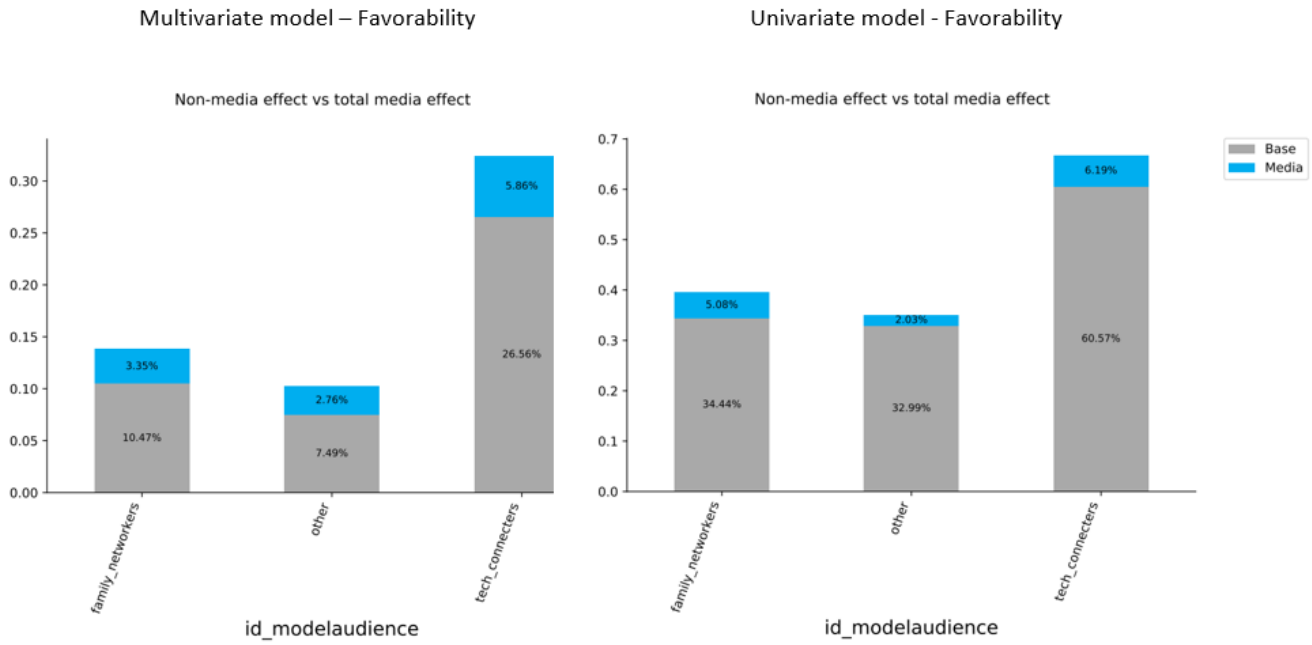


Figure 5.2: Comparison in base and media effects between the multivariate and univariate model for Favorability for the three audiences.

Moreover, the division between the base and media effects for the multivariate and univariate model for Consideration is depicted in Figure 5.3. Similar to Awareness, the multivariate model base and media effects estimates are similar to those estimated by the univariate model for all audiences. The relative differences in base effects between the multivariate and univariate model for the audiences Tech Connecters, Other and Family Networkers are 4.38%, -0.87% and 1.79%. Hence, there are no large differences in the estimated base effects through introduction of ordering in the funnel KPI in the multivariate model for Favorability.
Moreover, when comparing the base effects for Consideration in Figure 5.3 with the base effects for Favorability in Figure 5.2 for the multivariate model, we observe that the anticipated transitive property in the base effects between Favorability and Consideration is present, since Consideration has lower base effects across all three audiences compared to Favorability. Hence, the multivariate model succeeds in incorporating this transitive behavior in base effects between Favorability and Consideration.

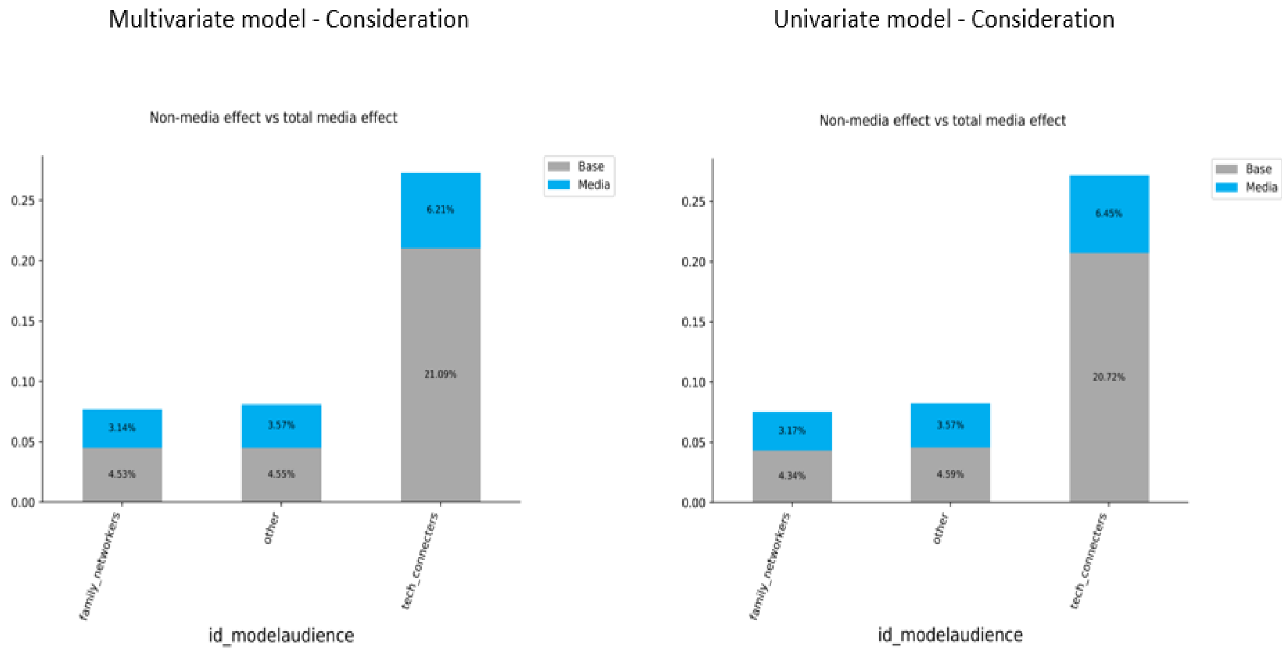


Figure 5.3: Comparison in base and media effects between the multivariate and univariate model for Consideration for the three audiences.

Moreover, the division between the base and media effects for the multivariate and univariate model for Purchase Intent is depicted in Figure 5.4. Similar to Awareness and Consideration, the multivariate model base and media effects estimates are similar to those estimated by the univariate model for all audiences. The relative differences in base effects between the multivariate and univariate model for the audiences Tech Connecters, Other and Family Networkers are -2.61%, -2.28% and 2.80%. Hence, there are no large differences in the estimated base effects through introduction of ordering in the funnel KPI in the multivariate model for Purchase Intent.

When comparing the base effects for Purchase Intent in Figure 5.4 with the base effects for Consideration in Figure 5.3 for the baseline, univariate model, we observe that the transitive property is present for the Family Networkers and Tech Connecters audiences. However, for the Other audience, the base effect for Purchase Intent is larger than the base effect for Consideration. Hence, transitivity in the base effect between Consideration and Purchase Intent is not present among all three audiences in the univariate model.

When comparing the base effects for Purchase Intent in Figure 5.4 with the base effects for Consideration in Figure 5.3 for the multivariate model, we observe the same trend as with the univariate model. Namely the transitive behavior in base effects is present for Family Networkers and Tech Connecters, but not for the Other audience group.

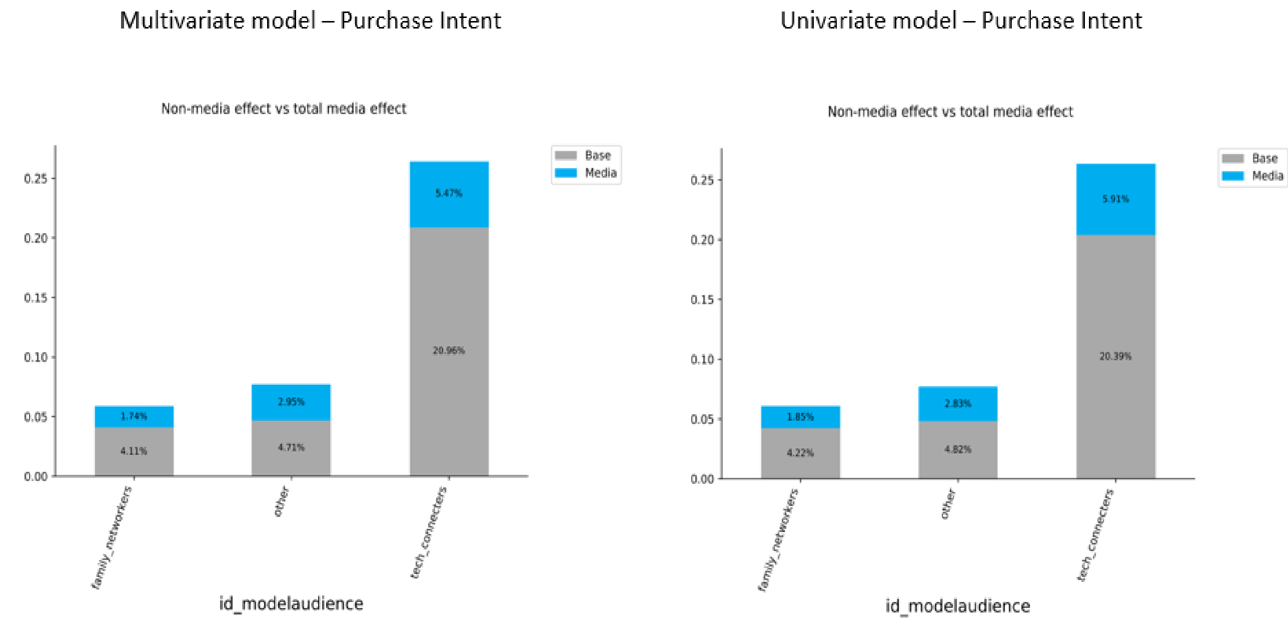


Figure 5.4: Comparison in base and media effects between the multivariate and univariate model for Purchase Intent for the three audiences.

From the aforementioned points we can conclude that the multivariate model succeeds in estimating base and media effects comparable to those estimated by the univariate model. Moreover, the univariate model unintentionally displays some of the transitive behavior in the funnel KPIs, namely, between Favorability and Consideration for all three audiences and between Consideration and Purchase Intent for Tech Connecters and Family Networkers. However, the univariate model fails to exhibit the transitive relation between Awareness and Favorability. Although the correlation between Favorability and Consideration and between Consideration and Purchase Intent is higher compared to the correlation between Awareness and Favorability, namely, 0.75 and 0.78 compared to 0.63, it is still desirable to see a transitive pattern in the base effect between Awareness and Favorability, since there is a correlation between them.
On the other hand, the multivariate model does succeed in incorporating the ordering of the funnel KPIs. It preserves the existing transitivity between Favorability and Consideration for all three audiences and between Consideration and Purchase Intent for Tech Connecters and Family Networkers. On top of that it manages to incorporate the transitive relation between Awareness and Favorability.

Additionally, the CI coverage averaged over all control and media variables for the multivariate model is presented in Table 5.2, where the true parameters are represented by the baseline, univariate model parameters estimates. It is evident that the multivariate model produces relatively high CI coverage among Awareness, Consideration and Purchase Intent. This is in line with the findings on the base effects in which we saw that the multivariate model produces similar base effects as the univariate model for Awareness, Consideration and Purchase Intent. On the other hand, the CI coverage for Favorability is relatively low, thus the CI from the multivariate model does not cover the true parameter values 26% of the time on average across all control and media variables. This is not surprising, since we observed that the multivariate model produces base effects that differ substantially from the univariate model. These results reinstate our hypothesis from earlier that the multivariate model performs comparably to the baseline, univariate model in terms of CI coverage for Awareness, Consideration and Purchase Intent. On the other hand, the multivariate model differs from the univariate model for Favorability, since this is the KPI for which transitivity is explicitly imposed through the multivariate model.

Table 5.2: Credible interval coverage for the multivariate model.

	Awareness	Favorability	Consideration	Purchase Intent
Multivariate model	0.98837	0.74419	0.98837	0.98837

5.2 Simulation Study

In this section we will evaluate the results from the simulation study. As part of the simulation study, we simulate the funnel KPIs, while keeping the media and control variable data unchanged. The KPIs are simulated in three different ways with varying levels of correlations between the KPIs. Due to the hypothetical transitive property of the funnel KPIs, each KPI is simulated sequentially based on the previous KPI.

Firstly, Awareness is generated based on a data generating process which uses the described prior specification as the true parameters. Subsequently, Favorability is generated through a binomial draw based on Awareness with a certain probability $p_{awareness-favorability}$. Next, Consideration is generated through a binomial draw based on Favorability with a certain probability $p_{favorability-consideration}$ and Purchase Intent is generated through a binomial draw based on Consideration with a certain probability $p_{consideration-purchase\,intent}$. We specify three different simulation instances with different probabilities. Firstly, we evaluate the situation in which there is a constant probability between the funnel KPIs. Namely, $p_{awareness-favorability} = p_{favorability-consideration} = p_{consideration-purchase\,intent} = 0.50$. Secondly, we will evaluate the case in which there is increasing probability between the funnel KPIs. Namely, $p_{awareness-favorability} = 0.60$, $p_{favorability-consideration} = 0.70$ and $p_{consideration-purchase\,intent} = 0.80$. Lastly, we will evaluate the case in which there is an decreasing probability between the funnel KPIs. Namely, $p_{awareness-favorability} = 0.80$, $p_{favorability-consideration} = 0.70$ and $p_{consideration-purchase\,intent} = 0.60$.

Since these simulated KPIs are different from the KPIs used in the empirical study the results from the simulation study cannot directly be compared to the baseline, univariate model utilized in the empirical study. Hence, we will mainly focus on comparing the three simulated data sets to each other and make the relevant inferences. First, we will report on the resulting correlations between the KPIs from the three different simulated data sets. It is then of interest to evaluate the different cases with varying levels of correlations between the funnel KPIs in order to gain insights into the impact of different correlation coefficients between funnel KPIs on the AUC values and the base effects.

5.2.1 Resulting Correlations

The correlation coefficients between the funnel KPIs resulting from the three different simulation strategies are provided in Table 5.3. It is of interest to evaluate the different levels of correlation and assess what impact they have on the results.

When observing the correlation coefficients between the funnel KPIs in Table 5.3, we observe that the multivariate model with constant Bernoulli probability between the funnel KPIs tends to underestimate the actual correlations in the empirical data, with the correlation between Consideration and Purchase Intent being underestimated by 12.82%.

Next, the multivariate model with decreasing Bernoulli probability between the funnel KPIs tends to overestimate the actual correlation in the empirical data for the correlation between Awareness and Favorability by 30.16%. In contrast, the correlation between Consideration and Purchase Intent is underestimated by 8.97%. Whereas, the correlation between Favorability and Consideration remains on par with the univariate model.

Lastly, the multivariate model with increasing Bernoulli probability between the funnel KPIs produces the most similar correlation structure between the funnel KPIs as in the univariate model. In this case the most dissimilar correlation coefficient is that of between Consideration and Purchase Intent, which is overestimated by 11.54%.

Table 5.3: Correlation coefficients between the funnel KPIs for the univariate and multivariate models with simulated KPIs.

	Aware - Favor	Favor - Consider	Consider - Purchase
Univariate model	0.63	0.75	0.78
Multivariate model - constant	0.57	0.66	0.68
Multivariate model - increasing	0.64	0.79	0.87
Multivariate model - decreasing	0.82	0.76	0.71

5.2.2 Statistical Inference

The AUC values for the univariate model and the three different simulated KPIs are given in Table 5.4. The AUC values for Awareness for the data sets with simulated KPIs is not given, because Awareness is simulated through a data generating process (DGP) which samples parameters from the prior distribution. Hence, the DGP produces estimates of Awareness that are close to the true value. The simulation is intended to explain more about the simulation of the transitive relation between KPIs and not so much the simulation of KPIs itself.

Table 5.4: AUC values for each KPI for the univariate and multivariate models with simulated KPIs.

	Awareness	Favorability	Consideration	Purchase Intent
Univariate model	0.648	0.819	0.795	0.839
Multivariate model - constant	-	0.825	0.782	0.766
Multivariate model - increasing	-	0.841	0.804	0.791
Multivariate model - decreasing	-	0.905	0.833	0.789

When observing the AUC values in Table 5.4, we see that for Favorability, all three simulated data sets produce higher AUC values. In fact, The multivariate model with decreasing Bernoulli probabilities for the funnel KPIs results in an AUC value above 0.90 for Favorability, which is an increase of 10.50% compared to the univariate model. The multivariate model with decreasing Bernoulli probabilities for the funnel KPIs also results in the highest AUC value among all models for Consideration, with an increase of 4.78% compared to the univariate model. However, this trend does not carry over to Purchase Intent, since the AUC value for Purchase Intent is highest for the univariate model. We observe no improvement in the AUC value for Purchase Intent among any of the multivariate models with differing simulation strategies. In fact, the AUC value for Purchase Intent for the multivariate model with constant Bernoulli probabilities reduces by 8.70% compared to the univariate model.

Hence, we can conclude that the multivariate model with decreasing Bernoulli probabilities for the funnel KPIs outperforms the univariate model in terms of AUC values for two KPIs, namely, Favorability and Consideration. Although the AUC value for Purchase Intent is lower in the multivariate model compared to the univariate model, it is not too far off being 5.96% lower. Meanwhile, the AUC values for Favorability and Consideration are 10.50% and 4.78% higher, respectively, for the multivariate model with decreasing Bernoulli probabilities compared to the univariate model.

5.2.3 Parameter Comparison

It is of interest to compare the parameters acquired by the simulations. In this case it is mainly of interest to compare the base effects of the funnel KPIs between the aforementioned models.

The decomposition between the base and media effects for the univariate and three different simulated multivariate models for Awareness is depicted in Figures 5.5 till 5.8. Upon initial sight, it is evident that the three simulated data sets all produce similar base and media effects to each other, and these effects are quite different from the base and media effects estimated by the univariate model. This is not surprising, since the simulated KPIs are different from the KPIs in the empirical data.

Firstly, we can observe that the base effects for Awareness for the Family Networkers audience and the Other audience are inflated by 90.80% and 122.97%, respectively, on average across the three simulated multivariate models. In contrast, the base effects for Awareness for Tech Connecters are deflated by 8.79% on average across the three multivariate models.

Moreover, the simulated data sets seem to severely underestimate the media effects, with most media effects being relatively close to zero, and in some cases there are negative media effects present.

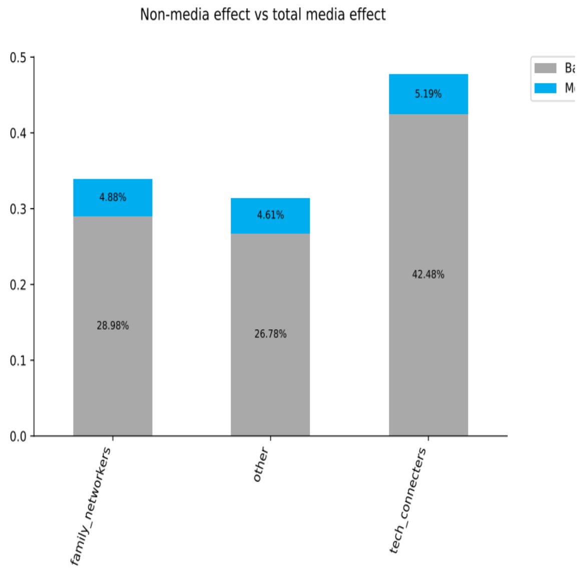


Figure 5.5: Base effect for the univariate model for Awareness for the three audiences.

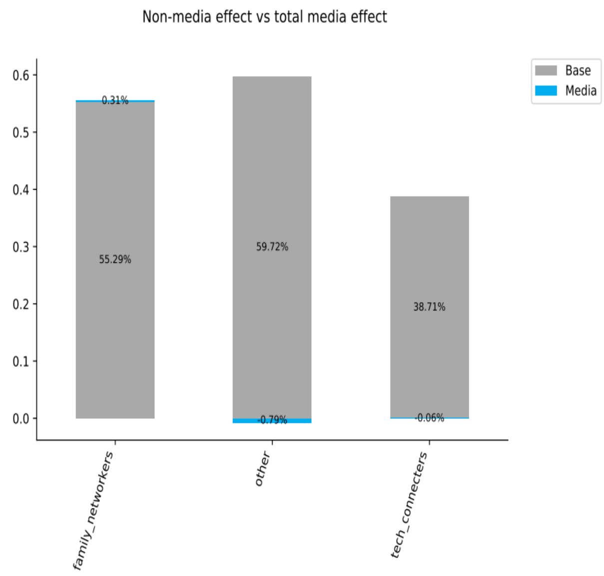


Figure 5.6: Base effect for the multivariate model with constant Bernoulli probabilities in simulation process for Awareness for the three audiences.

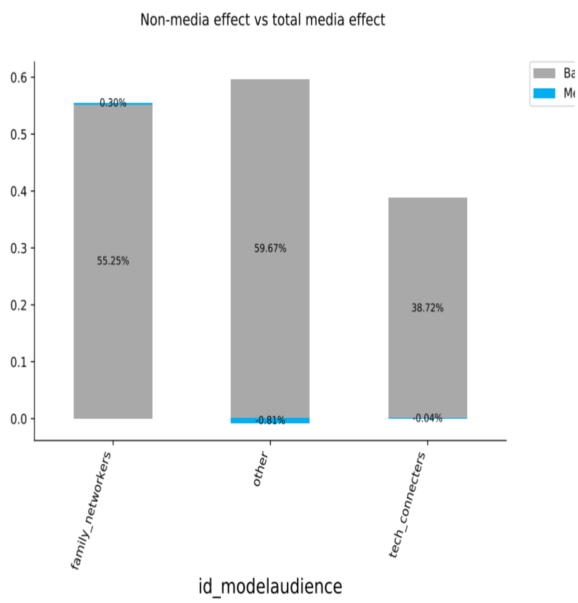


Figure 5.7: Base effect for the multivariate model with increasing Bernoulli probabilities in simulation process for Awareness for the three audiences.

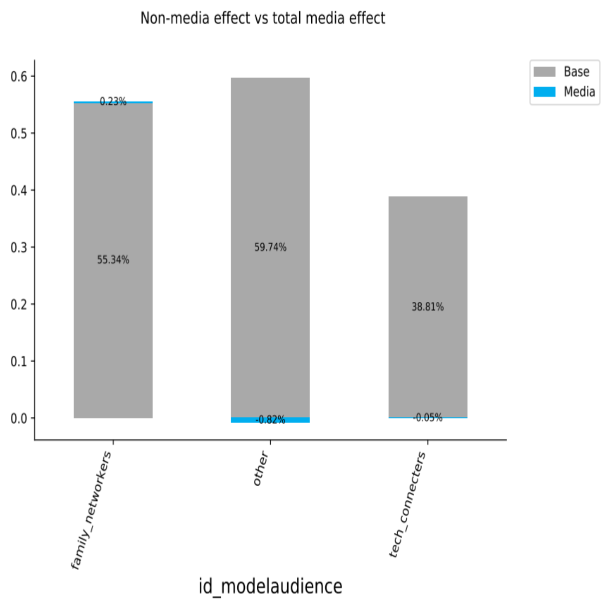


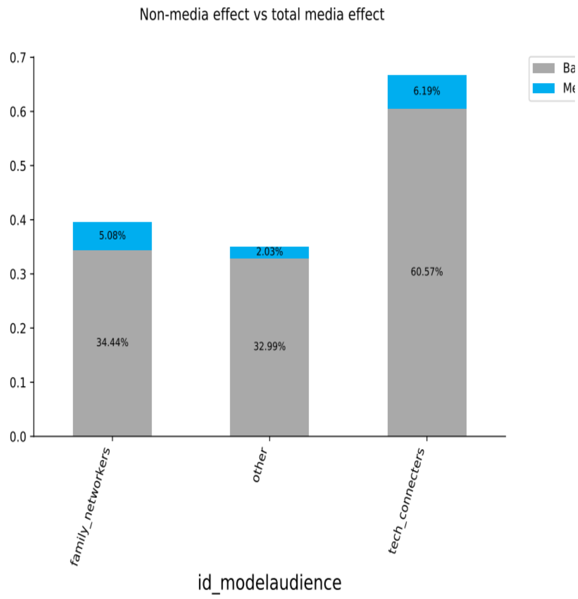
Figure 5.8: Base effect for the multivariate model with decreasing Bernoulli probabilities in simulation process for Awareness for the three audiences.

Similarly, the decomposition between the base and media effects for the univariate and the three different simulated multivariate models for Favorability is depicted in Figures 5.9 till 5.12. In contrast to Awareness, the base and media effects between the three simulated multivariate models are relatively different from one another, and each of them is relatively different from the univariate model.

The multivariate models tend to overestimate the base effect for the Family Networkers audience and the Other audience. On the other hand, there is no consistency in the over- or underestimation of the media effect for the Tech Connecters audience. This is because the base effects for the multivariate model with constant Bernoulli probabilities and the multivariate model with increasing Bernoulli probabilities underestimate the base effect, while the multivariate model with decreasing Bernoulli probabilities overestimates the base effect for the Other audience.

Moreover, similar to Awareness, the simulated data sets seem to underestimate the media effects, with most media effects being relatively close to zero, and in some cases there are negative media effects present.

Analogue to the empirical study, it is also of interest to compare the base effects within models, between consecutive funnel KPIs in order to assess whether transitivity is maintained between the KPIs. When comparing Figure 5.6 with Figure 5.10, Figure 5.7 with Figure 5.11 and Figure 5.8 with Figure 5.12, we observe that for all three multivariate models the base effects for Favorability are lower than those for Awareness for all three audiences. Hence, the anticipated transitive property in the base effects between Awareness and Favorability are present in the multivariate models. In this sense the simulated multivariate models outperform the univariate model, since the univariate model does not display this transitive property between Awareness and Favorability.



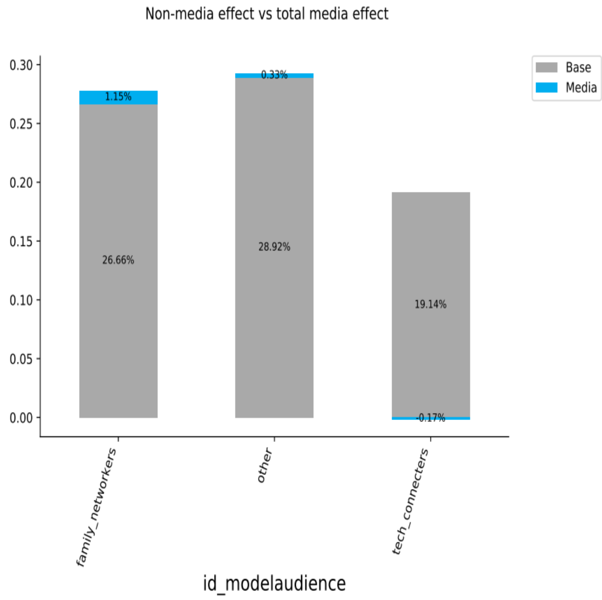
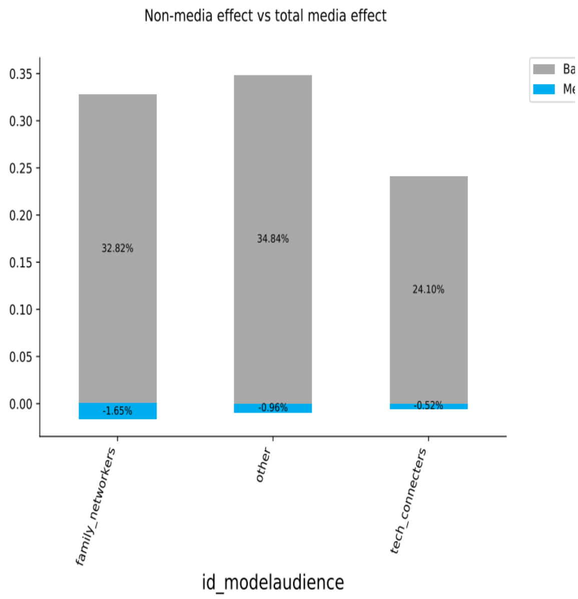


Figure 5.9: Base effect for the univariate model for Favorability for the three audiences.

Figure 5.10: Base effect for the multivariate model with constant Bernoulli probabilities in simulation process for Favorability for the three audiences.



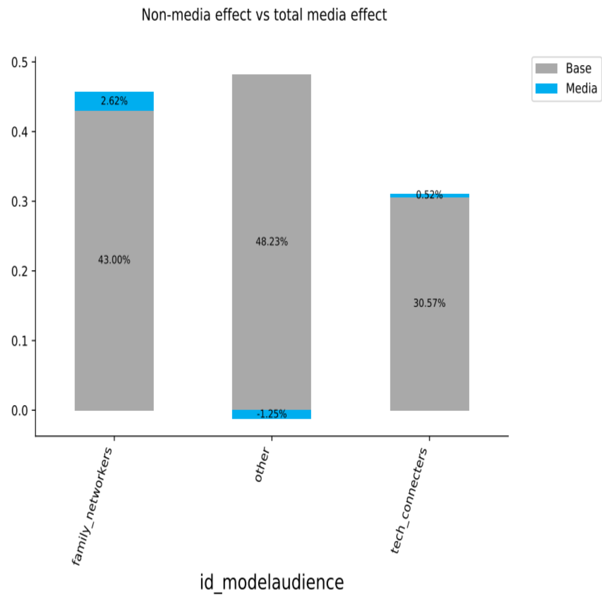


Figure 5.11: Base effect for the multivariate model with increasing Bernoulli probabilities in simulation process for Favorability for the three audiences.

Figure 5.12: Base effect for the multivariate model with decreasing Bernoulli probabilities in simulation process for Favorability for the three audiences.

Moreover, the decomposition between the base and media effects for the univariate and the three different simulated multivariate models for Consideration are depicted in Figures 5.13 till 5.16. Similar to Favorability, the base and media effects between the three simulated multivariate models are relatively different from one another, and each of them is relatively different from the univariate model.

The multivariate models tend to overestimate the base effect for the Family Networkers audience and the Other audience. For the Tech Connecters group we observe that for the multivariate model with constant Bernoulli probabilities and the multivariate model with increasing Bernoulli probabilities the base effect is underestimated. However, the multivariate model with decreasing Bernoulli probabilities estimates the base effect on par with the univariate model.

Moreover, similar to Awareness and Favorability, the simulated data sets underestimate the media effects.

When comparing the base effects within models, between Favorability and Consideration, so when comparing Figure 5.10 with Figure 5.14, Figure 5.11 with Figure 5.15 and Figure 5.12 with Figure 5.16, we observe that for all three multivariate models, the base effects for Consideration are lower than those for Favorability for all three audiences. Hence, the transitive property in the base effects between Favorability and Consideration are present.

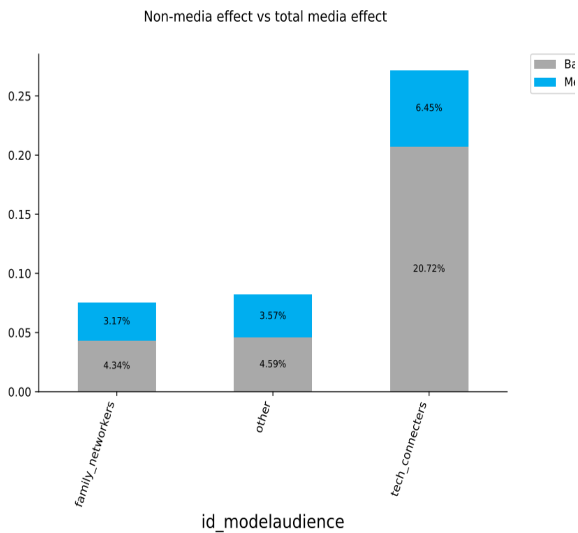


Figure 5.13: Base effect for the univariate model for Consideration for the three audiences.

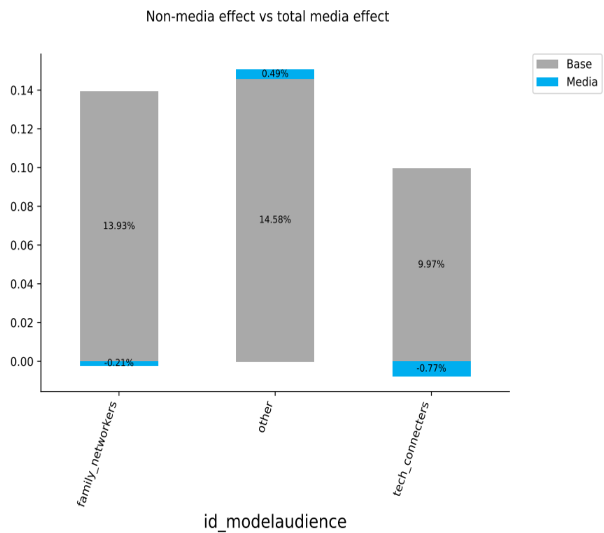


Figure 5.14: Base effect for the multivariate model with constant Bernoulli probabilities in simulation process for Consideration for the three audiences.

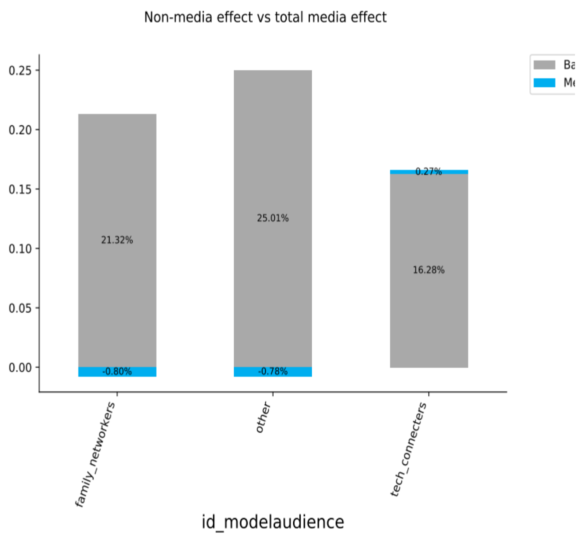


Figure 5.15: Base effect for the multivariate model with increasing Bernoulli probabilities in simulation process for Consideration for the three audiences.

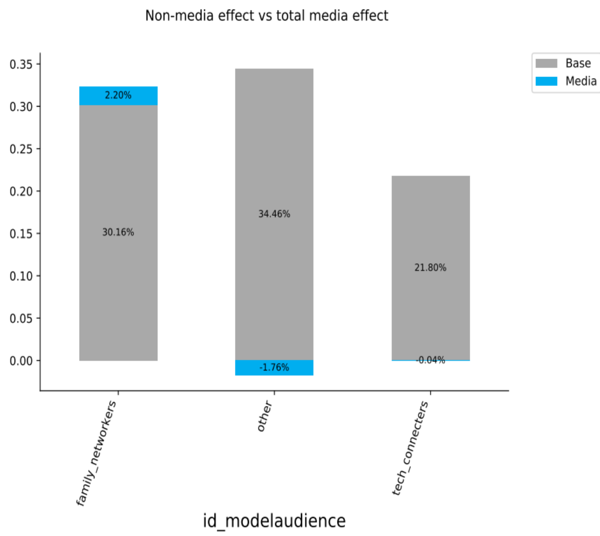


Figure 5.16: Base effect for the multivariate model with decreasing Bernoulli probabilities in simulation process for Consideration for the three audiences.

Lastly, the decomposition between the base and media effects for the univariate and the three different simulated multivariate models for Purchase Intent are depicted in Figures 5.17 till 5.20. The multivariate models with increasing and decreasing Bernoulli probabilities perform relatively similar to each other and the estimated base and media effects are relatively different compared to the univariate model. However, the multivariate model with constant Bernoulli probabilities yields quite different base and media effects compared to the other two simulated multivariate models.

It is noteworthy that similar to Awareness, the multivariate models tend to overestimate the base effect for the Family Networkers audience and the Other audience, while the base effects for the Tech Connecters audience are underestimated.

Moreover, similar to the other KPIs, the simulated multivariate models tend to produce very small and often negative media effects. A possible explanation for the estimation of small media effects could lie in the fact that the media effects estimated by the univariate model are not large to begin with, especially for the Family Networkers audience and the Other audience. Hence, estimation of the media effect can tend to be more uncertain.

When comparing the base effects within models, between Consideration and Purchase Intent, so when comparing Figure 5.14 with Figure 5.18, Figure 5.15 with Figure 5.19 and Figure 5.16 with Figure 5.20, we observe that for all three multivariate models, the base effects for Purchase Intent are lower than those for Consideration for all three audiences. Hence, the transitive property in the base effects between Consideration and Purchase Intent holds in this case as well.

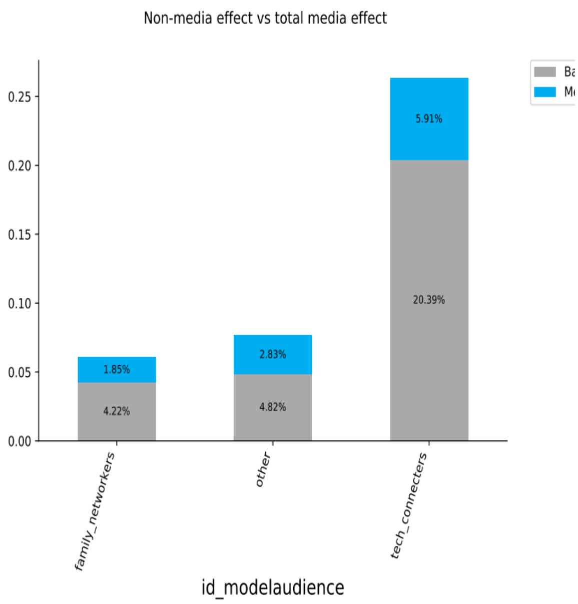


Figure 5.17: Base effect for the univariate model for Purchase Intent for the three audiences.

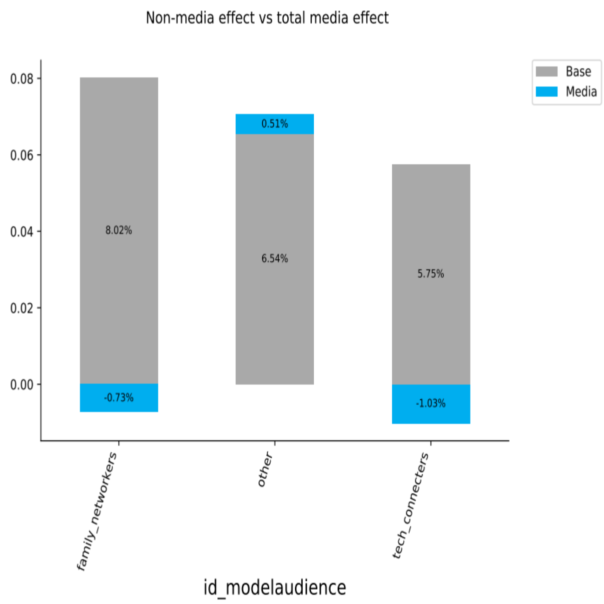


Figure 5.18: Base effect for the multivariate model with constant Bernoulli probabilities in simulation process for Purchase Intent for the three audiences.

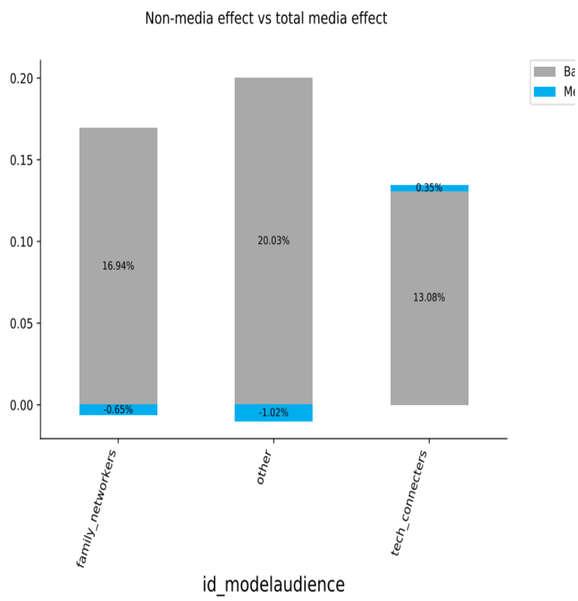


Figure 5.19: Base effect for the multivariate model with increasing Bernoulli probabilities in simulation process for Purchase Intent for the three audiences.

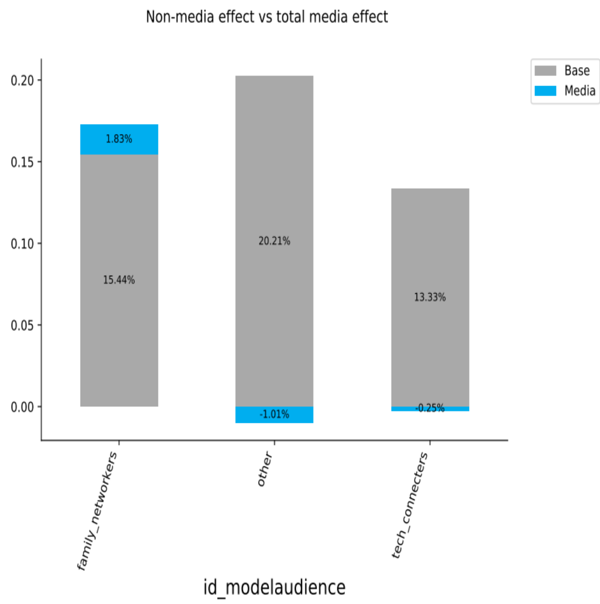


Figure 5.20: Base effect for the multivariate model with decreasing Bernoulli probabilities in simulation process for Purchase Intent for the three audiences.

From the aforementioned points we can conclude that regardless of the correlation structure imposed on the funnel KPIs, as long as there is an imposition of correlation the multivariate model guarantees transitivity in the base effects across the funnel KPIs for all three audiences. The present transitive behavior in the univariate model between Favorability and Consideration for all three audiences and between Consideration and Purchase Intent for Tech Connecters and Family Networkers is carried over to the three multivariate models with KPIs simulated using constant, increasing and decreasing Bernoulli probabilities. On top of that, the three multivariate models with KPIs simulated using constant, increasing and decreasing Bernoulli probabilities also exhibit transitive behavior in the base effects between Awareness and Favorability for all three audiences and between Consideration and Purchase Intent for the Other audience.

Chapter 6

Conclusion

This paper studies the implementation of a multivariate model, specifically the multivariate Probit model in the analysis of brand health metrics. In this study we consider KPIs in the marketing funnel, namely, Awareness, Favorability, Consideration and Purchase Intent. The data is obtained through Nielsen on the product called Portal from Facebook from the 2020 campaign in the United States. which ran from October 12$^{\text{th}}$ 2020 until January 4$^{\text{th}}$ 2021.

We examine our first research question: *How can KPI variables in the marketing funnel be modeled in a multivariate manner?* We conclude that a multivariate Probit model would be fitting for this task. The existing univariate hierarchical Bayesian model employed by Nielsen which measures the effectiveness of a marketing campaign can be extended upon. Firstly, by substituting the existing normal distribution of KPIs with a Probit model we take into consideration the binary recoding of the topboxes of the KPIs. Secondly, by extending this univariate model to a multivariate model we can incorporate correlations between the KPIs into the modeling. The correlation structure can be imposed in various ways, namely, through the intercept term, the control term based on the control variables or the media term based on the media variables. Since we are employing a Bayesian model, the correlation structure can be imposed through the prior distributions of any of the parameters belonging to the intercept term, control term or media term. Additionally, we find that the correlation structure for the funnel KPIs of interest in the utilized data set exhibits transitive property. It is of interest to not make the model too restrictive. Hence, the correlation structure is imposed in the prior distribution of the intercept term in a transitive manner. In this case instead of assuming an identical location parameter value for all KPIs, we impose transitivity in the location parameter value for the funnel KPIs.

Our second research question was the following: *How does a joint multivariate model for multiple KPI compare to a default univariate model for individual KPI?* We find that the multivariate model performs comparably to the univariate model in terms of AUC values, base effects and media effects decomposition and CI coverage. The multivariate model not only succeeds in exhibiting the existing transitive behavior between Favorability and Consideration for the Tech Connecters audience, the Family Networkers audience and the Other audience and between Consideration and Purchase Intent for the Tech Connecters and Family Networkers audience. On top of that the multivariate model also succeeds in imposing transitive behavior which is not present in the univariate model, thus between Awareness and Favorability for all three audiences and between Consideration and Purchase Intent for the Other audience. Moreover, we investigate the effect that the role of varying correlations between the funnel KPIs has on the appearance of transitivity in the base effects. Through simulation of KPIs based on constant, increasing and decreasing Bernoulli probabilities, we find that despite the magnitudes of correlation coefficients, the multivariate model succeeds in imposing transitive behavior in the base effects of the model.

Limitations and further research

This paper focuses on implementing the transitive behavior between funnel KPIs through the base term of the campaign effectiveness model by imposing ordering in the prior distribution of the intercept term. As further research it would be interesting to analyse the imposition of transitivity in other parts of this equation, such as through the media effects term.

Moreover, one could analyse a similar, less restrictive model in which the covariances between the funnel KPIs in the prior distribution are not assumed to be zero. In this case the ordering is also imposed in the covariance structure of the KPIs aside from (or on top of) the imposition of the ordering solely in the location parameter of the prior.

Due to time and computer memory constraints, the number of draws in the HMC-NUTS sampling is restricted to 100. While this number of draws does deliver sensible results, it would be of interest to increase the number of draws in the sampling algorithm to observe whether this causes large differences in the estimation results.

Moreover, the aim of this study is to facilitate the implementation of a multivariate model for joint modelling of funnel KPIs. As an extension, one can delve into multiple multivariate models aside from the multivariate Probit model such as a multivariate Logit model in order to compare their respective performances.

Bibliography

Ahmad, N. S., Musa, R., and Harun, M. H. M. (2016). "The impact of social media content marketing (SMCM) towards brand health". In: *Procedia Economics and Finance* 37, pp. 331–336.

Berg, J. D., Matthews, J. M., and O'Hare, C. M. (2007). "Measuring brand health to improve top-line growth". In: *MIT Sloan Management Review* 49.1, p. 61.

Betancourt, M. (2017). "A conceptual introduction to Hamiltonian Monte Carlo". In: *arXiv preprint arXiv:1701.02434*.

Chib, S. and Greenberg, E. (1998). "Analysis of multivariate probit models". In: *Biometrika* 85.2, pp. 347–361.

Colicev, A., Kumar, A., and O'Connor, P. (2019). "Modeling the relationship between firm and user generated content and the stages of the marketing funnel". In: *International Journal of Research in Marketing* 36.1, pp. 100–116.

Corsini, A et al. (2016). "Multivariate KPI for energy management of cooling systems in food industry". In: *Energy Procedia* 101, pp. 297–304.

Greenberg, E. (2012). *Introduction to Bayesian econometrics*. Cambridge University Press.

Hoffman, M. D., Gelman, A., et al. (2014). "The No-U-Turn sampler: adaptively setting path lengths in Hamiltonian Monte Carlo." In: *J. Mach. Learn. Res.* 15.1, pp. 1593–1623.

Huang, J. and Ling, C. X. (2005). "Using AUC and accuracy in evaluating learning algorithms". In: *IEEE Transactions on knowledge and Data Engineering* 17.3, pp. 299–310.

Johnson, G., Lewis, R. A., and Nubbemeyer, E. (2017). "The online display ad effectiveness funnel & carryover: Lessons from 432 field experiments". In: *Available at SSRN 2701578*.

Kornas, T. et al. (2019). "A multivariate KPI-based method for quality assurance in lithium-ion-battery production". In: *Procedia CIRP* 81, pp. 75–80.

Lansink, A. O., Berg, M. van den, and Huirne, R. (2003). "Analysis of strategic planning of Dutch pig farmers using a multivariate probit model". In: *Agricultural Systems* 78.1, pp. 73–84.

Lesaffre, E. and Molenberghs, G. (1991). “Multivariate probit analysis: a neglected procedure in medical statistics”. In: *Statistics in Medicine* 10.9, pp. 1391–1403.

Mezouar, H. et al. (2016). “Proposal of a modeling approach and a set of KPI to the drug supply chain within the hospital”. In: *2016 3rd International Conference on Logistics Operations Management (GOL)*. IEEE, pp. 1–6.

Mittal, S. and Mehar, M. (2016). “Socio-economic factors affecting adoption of modern information and communication technology by farmers in India: Analysis using multivariate probit model”. In: *The Journal of Agricultural Education and Extension* 22.2, pp. 199–212.

Oliveira, L. and Figueira, Á. (2017). “Improving the benchmarking of social media content strategies using clustering and KPI”. In: *Procedia Computer Science* 121, pp. 826–834.

Ouyang, Y. and Fallah, M. H. (2010). “A performance analysis for UMTS packet switched network based on multivariate KPIs”. In: *2010 Wireless Telecommunications Symposium (WTS)*. IEEE, pp. 1–10.

Rao, V. R. and Winter, F. W. (1978). “An application of the multivariate Probit model to market segmentation and product design”. In: *Journal of Marketing Research* 15.3, pp. 361–368.

Sawang, S. (2011). “Key performance indicators for innovation implementation: Perception vs. actual usage”. In: *Asia Pacific Management Review* 16.1, pp. 23–29.

Strelnik, E., Usanova, D., and Khairullin, I. (2015). “Key performance indicators in corporate finance”. In: *Asian Social Science* 11.11, p. 369.

White, G. P. (1996). “A survey and taxonomy of strategy-related performance measures for manufacturing”. In: *International Journal of Operations & Production Management*.