ERASMUS UNIVERSITY ROTTERDAM

Erasmus School of Economics

Bachelor Thesis IBEB

The Mobile OS Market: Switching Costs as a Strategic Variable

Name Student: Filippos Adam

Student ID number: 534285

Supervisor: Jurjen Kamphorst

Second Assessor: Josse Delfgaauw

Date Final Version: 17 August 2022

Abstract

The mobile OS market is a tightly concentrated duopoly with partially differentiated goods and a multitude of switching costs. In 2015, Apple, one of the two key players of the market launched “Move to iOS”, a tool that enabled users of its rival’s products to effortlessly switch brands. The exogenous switching cost model analyzed in this study, shows that the breakdown of lock-out barriers in a duopoly enables firms to capture a larger share of market demand. We further show that the presence of lock-out is equivalent to the presence of rival lock-in. Through an endogenous modeling of switching costs, we explain the emphasis placed by firms on actively abolishing lock-out barriers while simultaneously attaching users to their products through artificial constraints.

Contents

1 Introduction

The market for mobile operating systems (MOS) requires buyers to make numerous relationship-specific investments to make full use of the platforms. Learning to navigate a mobile interface constitutes a sunk cost for the user, and results in a skill which is only partly transferable to competing MOS. Similarly, in instances of external deviation, where consumers wish to switch MOS supplier, users were until recently required to manually migrate their data from one system to another. This manual migration constituted a *procedural switching cost* which is absent in cases of *intra-brand* switches. The relational specificity of assets in conjuncture to the MOS market translates to severe *switching costs* for any user looking to cross ecosystems. Extant literature outlines the strategic complement nature of switching costs as the imposition of endogenous lock-in triggers comparable actions by rival firms. Similar effects can be observed in instances of endogenous lock-out. Given the above relation, this paper uses a static Bertrand-type model to investigate firm's conduct of competition in duopolistic markets where *lock-in* and *lock-out* switching costs are a strategic variable.

Section 1.1 offers a detailed review of the relevant literature and positions the current investigation amongst the existing studies. Section 2.1 begins with a general account of the MOS market and provides a substantiated reasoning for the social relevance of the current investigation. Section 2.2 examines the conduct of competition and firms' strategic use of switching costs, the contents of this section help formulate accurate assumptions with regard to competitive behaviour in the MOS market. Section 3 presents the model, a static duopoly with partially differentiated products and switching costs as an exogenous strategic variable. Section 3.1 delves deeper into the system of demand used to model the market, and Section 3.2 examines the use of lock-in and lock-out switching costs. Section 3.3 considers an alternative model where switching costs are endogenous. In Section 4, the analysis is carried out and findings are discussed in Section 5.

1.1 Literature Review

Klemperer's (1987b, 1987a, 1987c, 1995) publications provide the theoretical foundation for the study of switching costs in consumer markets. In addition to characterizing the types of lock-in costs that can arise, Klemperer identified their implications on the distribution of market prices, quantities and profits. He noted that lock-in costs create brand loyalty and thereby market power for the producers, an inference further supplemented by Farrell and Shapiro (1988). In their own model, monopoly power emerges specifically over the *locked-in users* as firms are able to charge repeat purchasers in excess of competitor's price by an amount equal to and marginally smaller than the sum of switching costs. Given similar expectations by rival firms, the presence of lock-in can give rise to price increases above marginal costs and beyond the value of switching costs (Diamond, 1971).

Equivalent to the current investigation, Klemperer (1987c) provided an in-depth analysis of switching costs in price-setting oligopolies. Still, Klemperer considers a dynamic two-period model in which only exogenous lock-in costs are present. In accordance with that model, the presence of switching costs incentivizes competition in early periods as consumers buy into products for the first time and firms attempt to capture the largest possible share of demand. Consumer surplus is extracted in subsequent periods, the "mature" phase of the market, as buyers find it prohibitively expensive to switch suppliers.

In contrast to Klemperer's dynamic modeling of the market and exogenous application of switching costs, we consider a static one-period model and identify price, quantity and profit distributions under both exogenous and endogenous lock-in and lock-out costs. It is worth noting that the one-period games presented in Section 3 and analyzed in Section 4, focus on the "mature" phase of the market. As we show in Section 2.2, this choice stems from the fact that the MOS market, the practices of which form the primary motivation of this investigation, is *a priori* in a "mature" state.

The comprehensive theoretical intuition on lock-in switching costs, most clearly elaborated by Farrell and Klemperer (2007), has shown to be empirically observable (El-Manstrly, Paton, Veloutsou, & Moutinho, 2011). Still, certain caveats in relation to border cases where firms have failed to capture a sufficient share of total market demand, remain (Klemperer, 1987c). These considerations help identify parameters relevant to the efficacy of lock-in costs.

Klemperer (1987c) found that low market share firms may be incentivized to continue competing aggressively in subsequent periods. There are two key takeaways from Klemperer's bedrock model. First, and ex ante obvious, is the fact that switching costs are a relevant consideration for firms' strategic pricing decisions. Second, and maybe less discernible, is the extensive influence of the market demand composition on firm behavior in markets characterized by switching costs.

With the above theoretical intuition in mind, the efficacy of switching costs is dependent on two countervailing factors with respect to market demand. The frequency of *repeat orders*, that is, the rate at which ex ante committed buyers re-enter the market, and the frequency of *first-time orders*, that is, the rate at which ex ante uncommitted buyers enter the market. Repeat orders may be either *intra-* or *inter-brand*; intra-brand repeat orders are those originating from consumers that are a priori locked-in with the own firm, while inter-brand repeat orders originate from consumers that are a priori locked-in with the rival firm. The final consumer group is the first-time purchasers.

The sum of these three consumer categories constitutes the total market demand in the MOS market. Each subgroup's purchasing decision holds a different relationship with switching costs. As a result, the share of each category in relation to total market demand, dictates the optimal pricing strategy of the firm. Accounting for the artificiality of certain switching costs (Klemperer, 1987b), we further infer that the degree of lock-in and lock-out imposed by a firm through its product-design choices, becomes a strategic variable for all profit maximizing firms. The above form an integral element to the models developed in Section 3.

Research by Dubé, Hitsch and Rossi (2009) on the competitiveness of markets with switching costs, deviate from the theoretical and empirical consensus that switching costs soften price competition and lead to increased profits. Using consumer panel data, they demonstrate that as switching costs increase, equilibrium prices fall. A result intuitively based on the notion that market share investments override the "harvest" of consumer switching costs in empirical circumstances. They note that results similar to those proposed by Klemperer may only be observed in instances where switching costs are extremely high or infinite.

As discussed, Klemperer (1995) provides a detailed overview of switching costs and analyses a series of models all pointing to the same result. Research by Farrell and Shapiro (1988) and Padilla (1995) investigated switching costs under different entry circumstances, they also concluded that switching costs lead to increased profits. Both models employ the concept of a "fat-cat" to characterize firms with loyal consumer bases, market power and the ability to charge higher prices. Particularly, they find that "fat-cats" can invite entry and incentivize higher prices across the whole market due to positive externalities on the pricing decision of rivals (Farrell & Klemperer, 2007), a familiar notion in cases of competition in strategic substitutes.

Literature on the economic force behind lock-out switching costs, is severely limited. Hence, one of the primary objectives of this investigation is to help formalize the implications of lock-out on the distribution of prices, quantities and profits in oligopolistic industries. In that effort, literature on switching subsidies, a form of negative lock-out cost, can help formulate a baseline intuition. Chen (1997) analysed a two-period model in which firms can charge a separate price to locked-in and unaffiliated customers. He found that firms benefit from the ability to price discriminate between groups, while consumers are worse off. Given the positive profit differentials arising from the application of negative lock-out, we expect that firms in our model will also strive to reduce lock-out. This concept is further explored in the analysis of Section 4.

All relevant literature presented thus far, model switching costs as an exogenous parameter in the environment of firms. Still, the notion that firms can- and often do determine the degree of switching costs through "product-design", lends itself to an endogenous modeling of switching costs. A theoretical classification of endogenously and exogenously derived switching costs was made by Klemperer (1995). This classification aids in determining the environmental validity of the endogenous switching cost model variation developed in Section 3. Theoretical analysis of endogenous and exogenous switching costs by Shi (2013), proposes a strategic substitute relationship between the two. This is an intuitive notion considering their equivalent effect and the fact that endogenous switching costs require fixed investments by the firm. Similar to Caminal and Matutes (1990) we account for these costs through their inclusion in the profit function of the firm.

By considering both lock-in and lock-out switching costs, and examining industry distributions under both regimes, the current investigation aids in defining possible switching costs strategies and their use by firms. These results can be used to better regulate technology markets, characterized by their use of intricate switching cost. Moreover, the analysis of both endogenous and exogenous switching costs helps understand the empirically observable firm choices in the MOS market.

2 The MOS Market

2.1 Competition in the MOS Market

Information and communication technologies have had a significant impact in all aspects of human conduct. Even still, few products within that spectrum experienced the profound appeal and widespread adoption witnessed by mobile communication devices (Smith, Spence, & Rashid, 2011). The IBM Simon, a prototype of what would become the *modern smartphone* was first developed as far back as 1992 (Shen & Su, 2019). Nonetheless, it was not until the introduction of the first iPhone in 2007 that the idea of an all-in-one internet browser, media player, and mobile communication device permeated the mass market (Mickalowski, Mickelson, & Keltgen, 2008). In its wake, a plethora of hardware manufacturers began to produce their own devices with competing functionalities. Yet, as the hardware market broadened, the development of mobile OS software remained concentrated.

The concentration of resources and firms in MOS development is an inherent characteristic of the market. This is due to upstream production constraints and a desire for compatibility by downstream buyers. The development of MOS requires large sunk investments in R&D, specialized labor, and essential facilities. In addition, the product itself requires continual innovation and constant maintenance. The presence of scale economies, innate network effects, need for large fixed investments, and compatibility requirements, cause the concentration of market share by a handful of dominant firms.

During the market's infancy three tech giants competed in the market. Apple, with the introduction of iOS alongside the first iPhone. Google, with its early acquisition of Android, and Microsoft through the acquisition of Nokia's mobile division in 2013. Even still, Apple's and Google's *first-mover advantages* proved too large for Microsoft to succeed in the industry (Brahma, 2015). The ex-ante established networks of iOS and Android constricted Microsoft's residual demand and by 2017, Microsoft had already exited the market.

Following Microsoft's exit, Apple and Google further consolidated their positions, thereby instilling a clearly delimitated duopoly in the MOS market. Globally, there were 6.259 billion active smartphone users in 2021 (O'Dea, 2022), this comprises 83.72% of the world's population. As can be seen in Figure 1, from 2017 onward the near totality of the market is served solely by iOS and Android. Consequently, Apple and Google are in control of a large segment of consumer and social welfare, and as a result, their practices – be they anti-competitive or not – carry a large effect on social utility. Their global outreach amplifies the social externalities of their strategic actions, further motivating this theoretical research into the outcomes of duopolistic competition in markets with switching costs as a strategic variable.

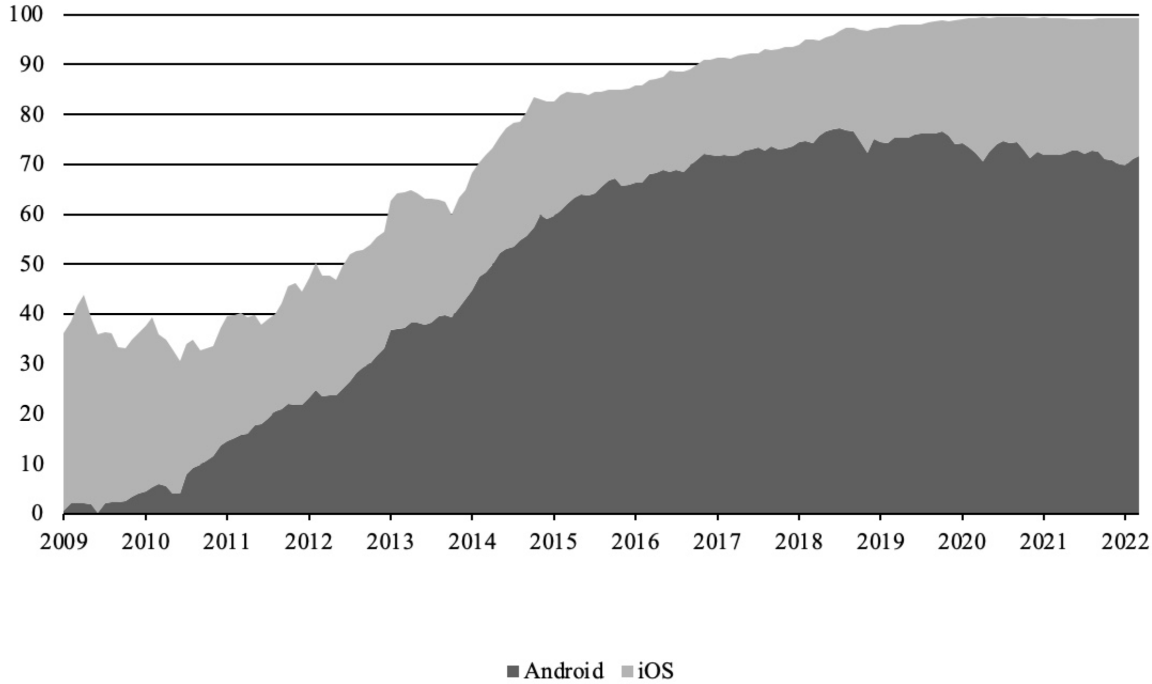

Figure 1: Global market share per mobile operating system, 2009-2021 (StatCounter, 2022).

2.2 Strategic Use of Switching Costs and Market Maturity

Various exogenous and endogenous switching costs are present in the MOS market. These include learning costs through consumer investments in MOS-specific navigation skills, contractual costs deriving MOS-specific service subscriptions (i.e., iCloud for iOS users, Google Drive for Android), and finally procedural switching costs (Burnham, Frels, & Mahajan, 2003) of deviation, those a customer must endure in order to switch platforms in the absence of automatic migration tools. These procedural switching costs, often in the form of manual and incomplete content migration when inter-brand switching, formed a large entry barrier for both Android and iOS users looking to cross over.

In September 2015, Apple took a substantial step towards broader MOS compatibility by launching "Move to iOS", a content migration tool that enabled Android users to seamlessly switch from one MOS to the other. Google followed suit a few years later with the introduction of its own content migration tool "Switch to Android". These actions help formulate a key assumption for the modeling of lock-out costs:

Assumption 1. *The procedural switching cost of deviating from one MOS to another is at the discretion of the receiving firm.*

The removal of endogenous procedural costs of deviation, coupled with the strategic compliment nature of the variable, signals a desire by firms to decrease lock-out barriers in the industry. It is this strategic choice by Apple and Google that motivates the endogenous modeling of switching costs in Section 3.2. The lack of sufficient literature on the industry effects of lock-out prompt the modeling of exogenous switching costs covered in Section 3.1.

3 Model

The main purpose of this paper is to examine the implications of switching costs for the competitiveness of markets. We do that by analyzing equilibrium outcomes in two static price-setting games, first, with exogenous switching costs, and second, with endogenous switching costs. Consider a duopoly where both firms simultaneously select prices. Let i, $i \in A, B$, be an arbitrary firm, and let p_i, $p_i \in \mathbb{R}_+$, denote its price. Firms choose price p_i to maximize profit π_i.

3.1 System of Demand

The above, outline a standard Bertrand-type model of simultaneous competition in strategic complements. In the duopoly setting, we can specify D_i if p_i and p_j are known. That is, a firm's resulting market demand can be specified given the prices of all firms in the relevant market. Therefore, for each firm i, there exists a demand function $D_i(p_i, p_j) : \mathbb{R}_+ \to \mathbb{R}$. The resulting demand function must satisfy the relevant natural properties of demand. Additionally, it must *in principle* be deducible from a representative agent's utility maximization problem.

Assumption 2 (Properties of Demand and Concavity)**.** *For all i, D_i is twice differentiable and strictly concave,*

$$D_i(0,0) > 0, \quad \text{and} \quad \frac{\partial^2 D_i}{\partial^2 p_i} < 0, \quad \text{and} \quad \frac{\partial D_i}{\partial p_i} < 0, \quad \text{for } i \neq j.$$

Assumption 3 (Gross Substitutability)**.** *Products in the relevant market are gross substitutes,*

$$\frac{\partial D_i}{\partial p_j} > 0, \quad \text{for } i \neq j.$$

Smoothness in the demand function is a stereotypical assumption for the sake of convenience. Naturally, when market prices are low, demand is positive. Moreover, firm demand is decreasing in firm's own price and increasing in rival firm's price. The assumption of gross substitutability between firm products in the relevant market is central in the study of competing MOS where consumers face the choice between iOS and Android. The goods need not be perfect substitutes, meaning one good need not directly replace the other, however, a drop in the price of one product still reduces the residual demand faced by the other firm. This is apparent in the MOS market with the existence of slight variations in functionality and aesthetic design between the competing products, leading to market power for both firms beyond that endowed through switching costs.

Assumption 4 (Symmetry)**.** *For fixed p_i, p_j, when $c_i = c_j$,*

$$D_i(p_i, p_j) = D_j(p_j, p_i).$$

Assumption 4 helps simplify the maximization problem of firms while maintaining the integrity of the model. Since marginal costs carry no relevant effect on the implications of switching costs for the competitiveness of markets, we further assume that $c_i = c_j = 0$. With the above in mind, we can formulate the first lemma with regard to the derivation of the demand system.

Lemma 1 (Price and Demand Order). *For fixed p_i, p_j where $p_i \leq p_j$. We have that:*

$$D_i(p_i, p_j) \geq D_j(p_j, p_i).$$

Proof. Starting from the symmetric environment of Assumption 4, we know that given $p_i = p_j \rightarrow D_i(p_i, p_j) = D_j(p_j, p_i)$. Consider a decrease in the price of Firm i, leading to $p_i \leq p_j$. Using Assumption 2, we see that the price decrease of firm i leads to a rise in quantity demanded for firm i. Conversely, consider an increase in the price of firm j, leading to $p_i \leq p_j$. Using Assumption 3, we see that the price increase by firm j leads to a rise in quantity demanded for firm i. This suffices to establish a trivial price and demand ordering relation. □

3.1.1 Linear Demand

We present the demand function employed in the static game of the following sections. Consider positive parameters A, B, C which will be justified in what follows. Given $B > C$, we define:

$$D_i(p_i, p_j) = A - Bp_i + Cp_j \quad \text{for } i \neq j. \tag{1}$$

In the proposed demand function $D_i = f(p_i, p_j)$ where $f(p_i, p_j) : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}$. The quantity demanded of firm i's product is smooth, increasing in p_i and decreasing in p_j. It is easy to see that system (1) satisfies Assumptions 2, 3 and 4. Moreover, we see that $D_i(0, 0) = A > 0$.

3.1.2 Derivation of Linear Demand

It is worthwhile showing that under certain conditions, there can exist preferences that rationalize the system of demand through consistent utility functions. We do this using Singh's and Vive's (1984) quadratic utility specification.

Naturally, $U(q_i, q_j) : \mathbb{R}^2_+ \rightarrow \mathbb{R}$, is the twice-differentiable and strictly concave utility function of a representative agent. Consumption of products i and j, increase utility, yet, require the incurment of costs equal to product price p_i and p_j. Given these conditions, an agent aims to solve the following utility maximization problem:

$$max_{q_i, q_j} \quad U(q_i, q_j) - (p_i q_i + p_j q_j) \tag{2}$$

The inverse demand can be obtained from the first order condition of the above maximization problem, the direct demand system D_i can be found by inverting this system for quantity as a function of price. The by-definition concavity of U implies that $\partial D_i / \partial p_i < 0$. Given Assumption 3, $\partial D_i / \partial p_j \geq 0$ is guaranteed at the level of demand, although not observable from the partial derivative of the utility function itself. The linear demand system introduced in Section 3.1.1 can be derived using the following utility specification (Singh & Vives, 1984; Sircar & Ledvina, 2010):

$$U(q_i, q_j) = \alpha \sum_{i=1}^{2} q_i - \frac{1}{2}(\beta \sum_{i=1}^{2} q_i^2 + \gamma \sum_{i \neq j} q_i q_j) \tag{3}$$

Proof. See Appendix A. □

It is worth noting that the above demand system is derived from the utility maximization problem of a representative market agent, rather than consumer. Although consumers purchase a single of the two products on offer, a representative agent must consider all consumer preferences when supplying the market. As a result, a representative agent aggregates the demand of all consumers in the market.

Objective Maximization

Having determined firm demand, we can now consider the firm's objective maximization function. In the absence of fixed and marginal costs, profit of firm i denoted by π_i is equal to total revenue. Expression (4) describes the profit function of firm i.

$$max_{p_i} \quad \pi_i(p_i, p_j) = p_i D_i \tag{4}$$

3.2 Exogenous Switching Costs and Changes in Effective Price

Switching costs are incorporated in the model either as lock-in or lock-out costs. Lock-in costs at the discretion of firm i are costs for current i consumers when switching to rival firm j. Conversely, lock-out costs at the discretion of firm i are costs for current j consumers when switching to firm i. In the exogenous variation, we consider changes in equilibrium outcomes based on the type of switching cost s_i^z, where $s_i^z \in \mathbb{R}_+$ and $z \in \{N, O\}$, where investor lock-in costs are denoted by N, and investor lock-out costs are denoted by O.

Consider the single-period duopoly presented above in which firms sell partially differentiated substitute goods. Further assume that it is a "mature-market", that is, fraction σ_i of the consumers purchased the product of firm i in the previous period and constitute its previous period's share of total market demand. Fraction σ_j purchased the product of firm j in the previous period. A fraction of consumers $\sigma_u = (1 - \sigma_i - \sigma_j)$, made no purchase in the previous period.

As we analyse the use of lock-in and lock-out switching costs, consumers of different brands will face different effective prices. Consumer segmentation through previous period market shares enables us to account for these differences. The sum of σ_i, σ_j, and σ_u, constitute total market demand and form demand function (5). In the absence of any variations in effective price, demand function (5) can be simplified to Expression (1).

$$\begin{aligned} q_i(p_i, p_j) = \sigma_i(A - Bp_i + Cp_j) \\ + \sigma_j(A - Bp_i + Cp_j) + \sigma_u(A - Bp_i + Cp_j) \Rightarrow \\ q_i(p_i, p_j) = A - Bp_i + Cp_j \end{aligned} \tag{5}$$

When lock-in costs are present in firm i's product, fraction σ_i of consumers will face a switching cost $s_i^N > 0$ of buying from firm j. Conversely, when lock-out costs are present in firm i's product, fraction σ_j of consumers will face a switching cost $s_i^O > 0$ of buying from firm i. In case of no switching costs by either firm, $s_i^z = 0 \vee s_j^z = 0$, consumer choices are solely determined by product preferences and the given market demand. The above cases can be summarised in the following demand function (6):

$$\begin{aligned} q_i(p_i, p_j) = \sigma_i(A - Bp_i + C(p_j + s_i^N + s_j^O)) + \sigma_j(A - B(p_i + s_i^O + s_j^N) + Cp_j) \\ + \sigma_u(A - Bp_i + Cp_j) \end{aligned} \tag{6}$$

The above demand assumes an independent realization of preferences in each period of sale. This further supposes that consumers preferences are not altered following the use of a product and that any heterogeneity in preferences is derived solely from changes in effective prices deriving from switching costs. In what follows, we consider the market demands that arise in the different producer environments with exogenous switching costs. These include one-sided lock-in, lock-out and their two-sided counterparts.

3.2.1 One-Sided

Switching costs of either type are considered to be one-sided when only one of the two firms in the relevant duopoly incorporate them in their product design. One-sided lock-in costs by firm i result in the following demand functions for firms i and j:

$$\begin{aligned} q_i = \sigma_i(A - Bp_i + C(p_j + s_i^N)) + \sigma_j(A - Bp_i + Cp_j) \\ + \sigma_u(A - Bp_i + Cp_j) \Rightarrow \\ q_i = A - Bp_i + Cp_j + \sigma_i C s_i^N \end{aligned} \tag{7}$$

$$\begin{aligned} q_j = \sigma_i(A - B(p_j + s_i^N) + Cp_i) + \sigma_j(A - Bp_j + Cp_i) \\ + \sigma_u(A - Bp_j + Cp_i) \Rightarrow \\ q_j = A - Bp_j + Cp_i - \sigma_i B s_i^N \end{aligned} \tag{8}$$

It is obvious that in this environment, the two firms no longer face a symmetric demand. As a result, we expect a non-symmetric equilibrium to arise in this market. The demand environments are reversed in case of lock-out. One-sided lock-out costs by firm i result in the following demand functions for firms i and j:

$$\begin{aligned} q_i = \sigma_i(A - Bp_i + Cp_j) + \sigma_j(A - B(p_i + s_i^O) + Cp_j) \\ + \sigma_u(A - Bp_i + Cp_j) \Rightarrow \\ q_i = A - Bp_i + Cp_j - \sigma_j B s_i^O \end{aligned} \tag{9}$$

$$\begin{aligned} q_j = \sigma_i(A - Bp_j + Cp_i) + \sigma_j(A - Bp_j + C(p_i + s_i^O)) \\ + \sigma_u(A - Bp_j + Cp_i) \Rightarrow \\ q_j = A - Bp_j + Cp_i + \sigma_j C s_i^O \end{aligned} \tag{10}$$

3.2.2 Two-Sided

Switching costs of either type are considered to be two-sided when both firms in the relevant duopoly incorporate them in their product design. Two sided lock-in costs result in the following demand functions for firms i and j:

$$
\begin{aligned}
q_i = \sigma_i(A - Bp_i + C(p_j + s_i^N)) + \sigma_j(A - B(p_i + s_j^N) + Cp_j) \\
+ \sigma_u(A - Bp_i + Cp_j) \Rightarrow \\
q_i = A - Bp_i + Cp_j + \sigma_i C s_i^N - \sigma_j B s_j^N
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
q_j = \sigma_i(A - B(p_j + s_i^N) + Cp_i) + \sigma_j(A - Bp_j + C(p_i + s_j^N)) \\
+ \sigma_u(A - Bp_j + Cp_i) \Rightarrow \\
q_j = A - Bp_j + Cp_i - \sigma_i B s_i^N + \sigma_j C s_j^N
\end{aligned} \tag{12}
$$

It is important to note that in this environment, the two firms face a symmetric demand. Since demand environments are reversed in the case of lock-out, a notion we further explore in Section 4, we expect profits under the two regimes to be trivially equivalent and symmetric. Two-sided lock-out costs result in the following demand functions for firms i and j:

$$
\begin{aligned}
q_i = \sigma_i(A - Bp_i + C(p_j + s_i^O)) + \sigma_j(A - B(p_i + s_j^O) + Cp_j) \\
+ \sigma_u(A - Bp_i + Cp_j) \Rightarrow \\
q_i = A - Bp_i + Cp_j + \sigma_i C s_i^O - \sigma_j B s_j^O
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
q_j = \sigma_i(A - B(p_j + s_i^O) + Cp_i) + \sigma_j(A - Bp_j + C(p_i + s_j^O)) \\
+ \sigma_u(A - Bp_j + Cp_i) \Rightarrow \\
q_j = A - Bp_j + Cp_i - \sigma_i B s_i^O + \sigma_j C s_j^O
\end{aligned} \tag{14}
$$

3.3 Endogenous Switching Costs as a Strategic Variable

In the model developed thus far, switching costs are treated as an exogenous variable beyond the control of the firm. This extension to the model endogenizes switching costs in an effort to observe firm choices in a setting where fixed investments by the firm can influence the level of lock-in and lock-out.

Assumption 5 (Switching Cost Artificiality)**.** *The level of switching costs present in the market can be artificially determined by firms through "product design".*

We consider a market where two-sided switching costs of both types are present, accordingly, firm i faces the following demand function:

$$
\begin{aligned}
q_i = \sigma_i(A - Bp_i + C(p_j + s_i^N + s_j^O)) + \sigma_j(A - B(p_i + s_j^N + s_i^O) + Cp_j) \\
+ \sigma_u(A - Bp_i + Cp_j) \Rightarrow \\
q_i = A - Bp_i + Cp_j + \sigma_i C s_i^N + \sigma_i C s_j^O - \sigma_j B s_j^N - \sigma_j B s_i^O
\end{aligned} \tag{15}
$$

In this variation, firms begin by simultaneously setting s_i^N, followed by s_i^O, where $s_i^z \in \mathbb{R}$. After firms observe each other's choices, prices are set and profits are realized.

Objective Maximization

In the endogenous extension, firms must account for fixed investments in the creation of switching costs, beyond those exogenously present in the market. Accordingly, and due to the presence of diminishing marginal returns, firms incorporate a convex cost function which is increasing in both directions of lock-in and lock-out. In the game with endogenous switching costs, Expression (5) describes the profit function of firm i.

$$max_{p_i} \quad \pi_i(p_i, p_j) = p_i q_i - k\left[(s_i^N)^2 + (s_i^O)^2\right] \tag{16}$$

Equilibrium, Timeline and Actions

The proposed models are static one-period price-setting duopolies with ex ante heterogeneous goods. We consider four separate settings with exogenous switching costs: 1) one-sided lock-in; 2) one-sided lock-out; 3) two-sided lock-in; 4) two-sided lock-out. In all four of these cases, the timeline and player actions are identical. Players set prices simultaneously according to the demand function their firm faces and in expectation of the rival firm's price. Once equilibrium prices and quantities are set, firms offer their products to the market. Finally, profits are realized. In the model extension, switching costs are an endogenous variable determined by the firms. Here, firms begin by setting s_i^O, followed by the choice of s_i^N. Firm choices are observed by the rival. After switching costs are determined, firms set equilibrium prices and quantities. Finally, profits are realized.

4 Analysis

We use a simple example to introduce the model. Consider the market described in Section 3.3, hereafter, the "baseline" model. Solving for the Nash equilibria of this model enables us to identify the distribution of prices, quantities, and firm profits in a differentiated market without switching costs.

Since consumers face no switching costs, any market power held by the firms derives from product heterogeneity as modelled through Assumption 3. This leads to a symmetric environment, wholly determined by the firms' demand function $D_i(p_i, p_j) = A - Bp_i + Cp_j$. We show in Appendix B, that given the above symmetric setting, and in accordance to the standard maximization procedure described in Section 3.3, firms will choose profit maximizing price, $p_i = A/(2B - C)$, and quantity, $q_i = AB/(2B - C)$, yielding firm profit, $\pi_i = (A^2B)/(2B - C)^2$, and industry profit, $\pi_i + \pi_j = 2\left[(A^2B)/(2B - C)^2\right]$.

Section 4.1 examines the distribution of profits in one-sided switching cost regimes. Section 4.2 examines two-sided switching cost regimes, Section 4.3 examines the endogenous switching cost variation of the model. Comparisons with the baseline equilibrium are made in order to examine the industry effects of switching costs and firm incentives for their imposition.

4.1 One-Sided Switching Costs

This section is devoted to the study of Nash equilibria in price games with one-sided switching costs. First, we characterize the distribution of prices and profits in a market with one-sided lock-in. Thereafter, we replicate this process for a market with one-sided lock-out. To do so, we build on the "baseline" model by incorporating switching costs into the environment of each player. The resulting "mature market" demand for firm i is described by Expression (17).

$$q_i = \sigma_i(A - Bp_i + C(p_j + s_i^N + s_j^O))+$$
$$+ \sigma_j(A - B(p_i + s_i^O + s_j^N) + Cp_j) + \sigma_u(A - Bp_i + Cp_j) \Rightarrow$$
$$q_i = (A - Bp_i + Cp_j) + \sigma_i C s_i^N + \sigma_i C s_j^O - \sigma_j B s_i^O - \sigma_j B s_j^N \quad (17)$$

Expression (17) provides a general overview of firm demand in a mature price-setting market with switching costs. Based on this, we can formulate the following proposition.

Proposition 1. *Lock-out costs $s_i^O > 0$, are equivalent to a rival implementation of lock-in costs $s_j^N > 0$.*

Proof. Consider demand for firm i under a single-sided lock-out regime $s_i^O > 0$ and demand for firm i under a single-sided lock-in regime $s_j^N > 0$. Under these circumstances, firm i faces the following demands:

$$q_i = (A - Bp_i + Cp_j) - \sigma_j B s_i^O \quad (18)$$

$$q_i = (A - Bp_i + Cp_j) - \sigma_j B s_j^N \quad (19)$$

Further assume that $s_i^O = s_j^N$. It is clear that if the above conditions are satisfied, equations (18) and (19) are equivalent. □

In what follows, we prove that if lock-in switching costs are present in firm i's product, the price and quantity demanded of firm i's product rises, while the price and quantity demanded of firm j's product falls. Consequently, the firm with lock-in gains while the rival firm loses. In consideration of Proposition 1, we further infer that market outcomes are reversed if lock-out switching costs are implemented by firm i.

4.1.1 Lock-in

Consider the situation in which only firm i's product has lock-in costs. Given the fact that substitute goods often carry equivalent product characteristics, it is rare for such a situation to arise in real markets. Even still, artificial switching costs such as capacity expansion discounts can be incorporated into MOS product-design, leading to periods of one-sided lock-in that increase the effective price of the rival's product for a fraction of consumers. Although these costs may be endogenously derived, examining them in an exogenous setting allows us to directly observe the implications of lock-in on market distributions. Expression (20) provides an overview of the switching costs present in the above market.

$$\text{One-sided Lock-in} \begin{cases} > 0, & \text{for } s_i^N \\ = 0, & \text{for } s_i^O \\ = 0, & \text{for } s_j^N \\ = 0, & \text{for } s_i^O \end{cases} \quad (20)$$

Demand for products i and j is determined by Expression (20) and described by the following demand functions:

$$q_i = \sigma_i(A - Bp_i + C(p_j + s_i^N)) + \sigma_j(A - Bp_i + Cp_j) + \sigma_u(A - Bp_i + Cp_j) \Rightarrow$$
$$q_i = A - Bp_i + Cp_j + \sigma_i C s_i^N \quad (21)$$

$$q_j = \sigma_i(A - B(p_j + s_i^N) + Cp_i) + \sigma_j(A - Bp_j + Cp_i) + \sigma_u(A - Bp_j + Cp_i) \Rightarrow$$
$$q_j = A - Bp_j + Cp_i - \sigma_i B s_i^N \quad (22)$$

It is clear that firms no longer face a symmetric environment. In particular, due to the presence of lock-in, the quantity demanded of product i increases on σ_i, multiplied by the size of the switching costs s_i^N and parameter C. Conversely, the quantity demanded of product j decreases on σ_i, multiplied by the size of the switching cost s_i^N and parameter B. We note that given $B > C$, overall market demand will decrease by $\Delta_{0\to1}Q = \sigma_i s_i^N(C - B)$, as compared to the "baseline" equilibrium.

Noncooperative Equilibrium

We use Expressions (21) and (22) to solve for the price-competition equilibrium. Firm i's first-order condition after substituting for q_i is:

$$\frac{\partial \pi_i}{\partial p_i} = A - 2Bp_i + Cp_j + \sigma_i C s_i^N = 0 \quad (23)$$

Where π_i is firm i's profits. Replicating the process for firm j and solving for price yields reaction functions:

$$p_i = \frac{A + Cp_j + \sigma_i C s_i^N}{2B} \quad (24)$$

$$p_j = \frac{A + Cp_i - \sigma_i B s_i^N}{2B} \quad (25)$$

Figure 2 illustrates why the lock-in cost disrupts the "baseline" symmetric equilibrium. Curves p_i^0 and p_j^0 describe the reaction function of firms i and j when no switching costs are present. The "baseline" equilibrium is reached at the intersection of the two curves, and described by price vector:

$$(p_i^0, p_j^0) = \left[\left(\frac{A}{2B - C}\right), \left(\frac{A}{2B - C}\right)\right] \quad (26)$$

When lock-in by firm i is accounted for, curve p_i^0 shifts outward to p_i^1. The shift is equal to $\sigma_i C s_i^N/2B$, equivalent to the change in firm i's reaction function. Similarly, curve p_j^0 shifts inward to p_j^1. The shift is equal to $\sigma_i B s_i^N/2B$, equivalent to the change in firm j's reaction function. Note that due to $B > C$, the absolute shift of firm j's reaction function is greater than that of firm i. It is now observable why the initial price pair no longer represents a Nash equilibrium in the new market structure. When accounting for lock-in in the product of firm i, the reaction functions of the two firms adjust in opposite directions and the following non-cooperative equilibrium prices arise:

$$(p_i^1, p_j^1) = \left[\left(\frac{A}{2B - C} + \frac{\sigma_i BC s_i^N}{(2B)^2 + C^2}\right), \left(\frac{A}{2B - C} - \frac{2\sigma_i B^2 s_i^N - \sigma_i C^2 s_i^N}{(2B)^2 + C^2}\right)\right] \quad (27)$$

Two conclusions immediately follow from this analysis.

Theorem 1. *A single-sided lock-in cost $s_i^N > 0$ leads to an increase in the price of firm i, equal to:*

$$\Delta_{0\to1}p_i = \frac{\sigma_i BC s_i^N}{(2B)^2 + C^2}$$

The rise in price is increasing on the size of the switching cost, previous period market share, parameter B and C. Additionally, it leads to a decrease in the price of firm j, equal to:

$$\Delta_{0\to 1}p_j = -\frac{2\sigma_i B^2 s_i^N - \sigma_i C^2 s_i^N}{(2B)^2 + C^2}$$

The drop in price is increasing on the size of the switching cost, previous period market share and parameter B. The drop is decreasing on C.

Proof. Decompose the price change into two stages. First, firm i raises its price by $\sigma_i C s_i^N$. By gross substitutability, this raises demand for rival firm j, and hence benefits them. Facing the new price of i, p_j^0 is no longer profit-maximizing, so firm j raises its price by Cp_i. Still, the presence of lock-in by firm i has a direct price decreasing effect on firm j, captured by $\left(\sigma_i B s_i^N\right)$ in its reaction function. For firm j to profit-maximize, it must now decrease its price by an equivalent amount, leading to equilibrium price $p_j^1 < p_j^0$. By gross substitutability, this decreases demand for firm j. Facing the new price p_j^1 firm i must now decrease its price to profit-maximize. As in the case of firm j, the price decreasing effect of gross substitutability is not sufficient to overcome the direct price increasing effects of the lock-in leading to equilibrium price $p_i^1 > p_i^0$. □

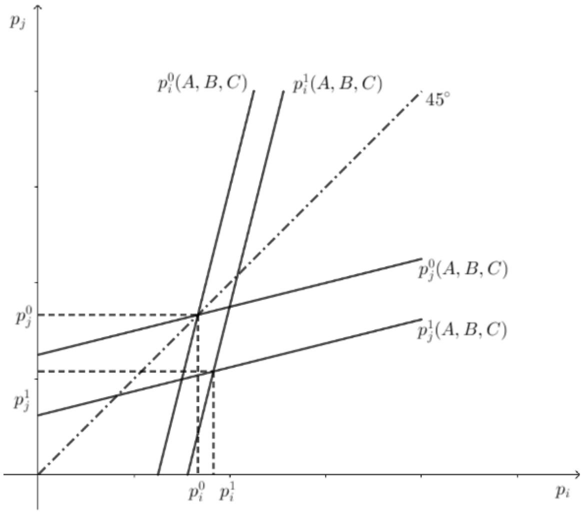

Figure 2: Firm i and j reaction functions and equilibrium prices under no switching costs, and single-sided lock-in by firm i.

Theorem 2. *A single-sided lock-in cost $s_i^N > 0$ leads to an increase in the profits of the imposing firm, i, as compared to the non-cooperative "baseline" distribution. The profit of rival firm j decreases, as do the overall industry profits.*

Proof. The proof can be obtained by evaluating profits under the new single-sided lock-in equilibrium. A detailed derivation of profits can be found in Appendix C, below, we present the change in firm and industry profits.

$$\Delta\pi_i^{0\to 1} = \left(\frac{\sigma_i BCs_i^N}{(2B)^2 + C^2}\right)^2 B$$

$$\Delta\pi_j^{0\to 1} = -\left(\frac{2\sigma_i B^2 s_i^N - \sigma_i C^2 s_i^N}{(2B)^2 + C^2}\right)^2 B$$

$$\Delta\pi^{0\to 1} = \Delta\pi_i^{0\to 1} + \Delta\pi_j^{0\to 1} = \left(\sigma_i s_i^N \frac{BC + C^2 - 2B^2}{(2B)^2 + C^2}\right)^2 B \quad (28)$$

$\Delta_{0\to 1}\pi$ is strictly smaller than zero, given $B > C$. □

4.1.2 Lock-out

In this subsection, we consider the distribution of prices and profits in a market with one-sided lock-out, a regime that naturally arose in the MOS market following the introduction of Apple's automatic migration tool. Given the inferences of Proposition 1, we show that firms acquire no real benefit from positive lock-out, even still, their removal may be a driver of firm profits. Consider the situation in which only firm i has lock-out costs. Expression (29) provides an overview of the switching costs present in the above market.

$$\text{One-sided Lock-out} \begin{cases} = 0, & \text{for } s_i^N \\ > 0, & \text{for } s_i^O \\ = 0, & \text{for } s_j^N \\ = 0, & \text{for } s_j^O \end{cases} \quad (29)$$

Demand for products i and j is determined by Expression (29) and described by the following demand functions:

$$q_i = \sigma_i(A - Bp_i + Cp_j) + \sigma_j(A - B(p_i + s_i^O) + Cp_j) + \sigma_u(A - Bp_i + Cp_j) \Rightarrow$$
$$q_i = A - Bp_i + Cp_j - \sigma_j Bs_i^O \quad (30)$$

$$q_j = \sigma_i(A - Bp_j + Cp_i) + \sigma_j(A - Bp_j + C(p_i + s_i^O) + \sigma_u(A - Bp_j + Cp_i) \Rightarrow$$
$$q_j = A - Bp_j + Cp_i + \sigma_j Cs_i^O \quad (31)$$

Since Proposition 1 established the trivial and reverse equivalence between one-sided lock-in and lock-out, we can extrapolate the results of Section 4.1.1 to the analysis of one-sided lock-out. We formulate the following proposition:

Proposition 2. *Given the reverse equivalence between one-sided lock-in and lock-out, postulated in Proposition 1, it follows that Theorem 1 and 2 of Section 4.1.1 are transferable to the analysis of one-sided lock-out.*

We derive that an imposition of one-sided lock-out by firm i will lead to a decrease in profit for firm i, and an increase in profit for firm j. Additionally, equilibrium price of firm i will be lower as compared to the "baseline" market; the equilibrium price of firm j will be higher.

4.2 Two-Sided Switching Costs

This section is devoted to the study of Nash equilibria in price games with two-sided switching costs. We simultaneously characterize the distribution of prices and profits under double-sided lock-in and lock-out costs as the two are wholly equivalent. Producer environment described by firm-specific demand is once again based on "mature market" consumer segmentation.

$$\begin{aligned} q_i = \sigma_i(A - Bp_i + C(p_j + s_i^N + s_j^O)) + \\ + \sigma_j(A - B(p_i + s_i^O + s_j^N) + Cp_j) + \sigma_u(A - Bp_i + Cp_j) \Rightarrow \\ q_i = (A - Bp_i + Cp_j) + \sigma_i C s_i^N + \sigma_i C s_j^O - \sigma_j B s_i^O - \sigma_j B s_j^N \end{aligned} \quad (32)$$

Proposition 3. *A two-sided application of lock-in is quantitatively equivalent to a two-sided application of lock-out.*

Proof. Assuming, $s_i^N = s_j^O = s_1 \vee s_j^N = s_i^O = s_2$, we find the following demand functions for firms i and j, respectively:

$$q_i = A - Bp_i + Cp_j + \sigma_i C s_1 - \sigma_j B s_2 \quad (33)$$

$$q_j = A - Bp_j + Cp_i - \sigma_i B s_1 + \sigma_j C s_2 \quad (34)$$

This proves that the demand function under two-sided lock-in and lock-out, are quantitatively equivalent. □

Given this equivalence, we focus our analysis on the case of two-sided lock-in. We prove that under certain conditions the presence of two-sided lock-in can be beneficial for firm i. The benefit is dependent on two parameters. The share of all potential customers that previously bought from firm i, and the level of s_i^N as compared to s_j^N. The larger the previous period demand held by firm i and the difference between s_i^N and s_j^O, the larger the benefit. Hence, firms benefit from lock-in costs that surpass those of the rival firm, and do so only in instances where a sufficient fraction of possible consumers previously bought from them. Two-sided lock-out results in the same market conditions, however, the fact that lock-in costs now depend on the rival firm, means firms benefit from lower lock-out costs. This conclusion supports the findings of the one-sided lock-in analysis.

4.2.1 Lock-in and Lock-out

Consider the situation in which both firms have implemented a lock-in cost. Expression (35) provides an overview of the switching costs present in the above market.

$$\text{Two-sided Lock-in} \begin{cases} > 0, & \text{for } s_i^N \\ = 0, & \text{for } s_i^O \\ > 0, & \text{for } s_j^N \\ = 0, & \text{for } s_i^O \end{cases} \quad (35)$$

Demand for products i and j is determined by Expression (35) and described by the following demand functions:

$$
\begin{aligned}
q_i = \sigma_i(A - Bp_i + C(p_j + s_i^N)) \\
+ \sigma_j(A - B(p_i + s_j^N) + Cp_j) + \sigma_u(A - Bp_i + Cp_j) \Rightarrow \\
q_i = A - Bp_i + Cp_j + \sigma_i C s_i^N - \sigma_j B s_j^N
\end{aligned} \tag{36}
$$

$$
\begin{aligned}
q_j = \sigma_i(A - B(p_j + s_i^N) + Cp_i) \\
+ \sigma_j(A - Bp_j + C(p_i + s_j^N)) + \sigma_u(A - Bp_j + Cp_i) \Rightarrow \\
q_j = A - Bp_j + Cp_i - \sigma_i B s_i^N + \sigma_j C s_j^N
\end{aligned} \tag{37}
$$

Firms face a symmetric environment as long $\sigma_i = \sigma_j$. Under this assumption, we find that in case of equal switching costs ($s_i^N = s_j^N$) firm and industry demand declines. The lower industry demand is a result of the higher effective prices due to the presence of switching costs; this effect can be clearly observed in Figure 3.

Theorem 3. *Firms benefit from highest possible lock-in. Simultaneously, they benefit from lowest possible lock-out.*

Proof. Below we present equilibrium profits for firm i (j). We find that firm profits are increasing on own previous period share of total market demand, level of lock-in s_i^N (s_j^N) and C. Conversely, firm profits are decreasing in rival previous period share of total market demand, level of lock-in s_j^N (s_i^N) and B. Detailed analysis of the maximization problem can be found in Appendix D.

$$
\pi_i^2 = \left(\frac{A + \sigma_i C s_i^N - \sigma_j B s_j^N}{2B - C} \right)^2 B \tag{38}
$$

$$
\pi_j^2 = \left(\frac{A + \sigma_j C s_j^N - \sigma_i B s_i^N}{2B - C} \right)^2 B \tag{39}
$$

□

Reflecting on the practices of the MOS market, Section 4.2.1 provides an intuitive rational as to why firms, namely Apple and Google, chose to introduce *automatic migration tools.* In essence, if the presence of lock-out barriers constitute a disadvantage for the imposing firm, their abolition does result in benefits. As shown in Figure 4, in the absence of lock-out barriers, firms are able to charge higher prices given the rise in residual demand. Moreover, since the absence of lock-out has beneficial effects even when pursued by only one of the two firms in the market, no excess inertia is observed. We infer that in endogenous switching cost circumstances, any delays in the removal of lock-out barriers by the rival firm are the result of costly investments or variables beyond those considered in this paper.

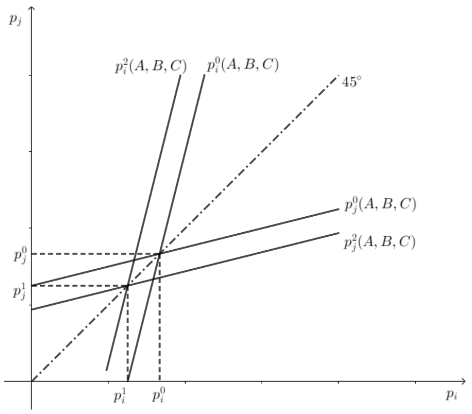

Figure 3: Double-sided lock-in.

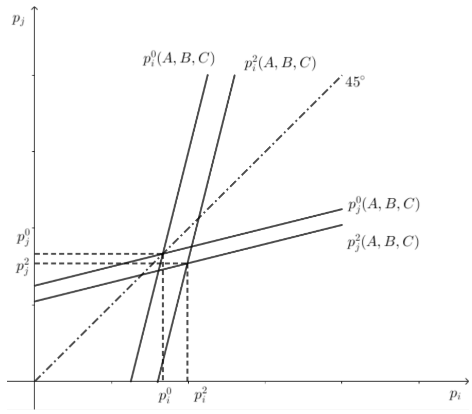

Figure 4: Removal of lock-out by firm i.

4.3 Endogenous Switching Costs

Consider the following profit maximization function, presented in the Section 3.2:

$$max_{p_i} \quad \pi_i(p_i, p_j) = p_i q_i - k\left[(s_i^N)^2 + (s_i^O)^2\right] \tag{40}$$

Assume now that switching costs are an endogenous strategic variable. Theorem 3 establishes a firm's incentive to maximize exogenous lock-in while simultaneously minimizing exogenous lock-out. Given that both endogenous and exogenous switching costs carry the same implications for market outcomes, we show that firms will choose negative lock-out, constituting investments in the abolition of lock-out barriers. Simultaneously, firms will endogenously increase lock-in costs in the positive direction. Expression (41) provides an overview of the switching costs present in the above market.

$$\text{Two-sided Endogenous Lock-out} \begin{cases} > 0, & \text{for } s_i^N \\ > 0, & \text{for } s_i^O \\ > 0, & \text{for } s_j^N \\ > 0, & \text{for } s_i^O \end{cases} \tag{41}$$

Demand for products i and j is determined by Expression (41) and described by the following demand functions:

$$q_i = A - Bp_i + Cp_j + \sigma_i C s_i^N + \sigma_i C s_j^O - \sigma_j B s_j^N - \sigma_j B s_i^O \tag{42}$$

$$q_i = A - Bp_i + Cp_j + \sigma_j C s_j^N + \sigma_j C s_i^O - \sigma_i B s_i^N - \sigma_i B s_j^O \tag{43}$$

Noncooperative Equilibrium

Following the standard maximization procedure, we begin by finding equilibium prices and quantities, followed by lock-out costs s_i^O and lock-in costs s_i^N. Below we present the relevant expressions for optimal lock-in and lock-out, detailed maximization of price and quantity can be found in Appendix E. Following price maximization, we substitute p_i^3 and q_i^3 into π_i, and solve for the first-order condition w.r.t s_i^O. Assuming symmetry and solving for $s_i^O = s_j^O = s^O$ yields optimal lock-out as a function of lock-in, Expression (44).

$$s^{O}=-\frac{B^{2}\sigma_{j}\left(A-Bs_{j}^{N}\sigma_{j}+Cs_{i}^{N}\sigma_{i}\right)}{-B^{3}\sigma_{j}^{2}+B^{2}C\sigma_{i}\sigma_{j}+k\left(2B-C\right)^{2}} \tag{44}$$

Having found optimal lock-out as an expression of s^N, we substitute s^O into π_i and solve for the first-order condition w.r.t s_i^N. Assuming symmetry and solving for $s_i^N = s_j^N = s^N$ results in Expression (45), equivalent to optimal firm lock-in:

$$\begin{aligned} s^{N*}=&\frac{ABC\sigma_{i}\left(-B^{3}\sigma_{j}^{2}+k(2B-C)^{2}\right)}{B^{6}\sigma_{j}^{4}-3B^{5}C\sigma_{i}\sigma_{j}^{3}-8B^{5}k\sigma_{j}^{2}} \\ &+2B^{4}C^{2}\sigma_{i}^{2}\sigma_{j}^{2}+12B^{4}Ck\sigma_{i}\sigma_{j}+8B^{4}Ck\sigma_{j}^{2}+16B^{4}k^{2}-4B^{3}C^{2}k\sigma_{i}^{2} \\ &-12B^{3}C^{2}k\sigma_{i}\sigma_{j}-2B^{3}C^{2}k\sigma_{j}^{2}-32B^{3}Ck^{2}+4B^{2}C^{3}k\sigma_{i}^{2}+3B^{2}C^{3}k\sigma_{i}\sigma_{j} \\ &+24B^{2}C^{2}k^{2}-BC^{4}k\sigma_{i}^{2}-8BC^{3}k^{2}+C^{4}k^{2} \end{aligned} \tag{45}$$

We can now derive optimal firm lock-out in equilibrium.

$$\begin{aligned} s^{O*}=&AB^{2}\sigma_{j}\frac{\left(B^{3}\sigma_{j}^{2}-B^{2}C\sigma_{i}\sigma_{j}-k(2B-C)^{2}\right)}{B^{6}\sigma_{j}^{4}-3B^{5}C\sigma_{i}\sigma_{j}^{3}-8B^{5}k\sigma_{j}^{2}} \\ &+2B^{4}C^{2}\sigma_{i}^{2}\sigma_{j}^{2}+12B^{4}Ck\sigma_{i}\sigma_{j}+8B^{4}Ck\sigma_{j}^{2}+16B^{4}k^{2}-4B^{3}C^{2}k\sigma_{i}^{2} \\ &-12B^{3}C^{2}k\sigma_{i}\sigma_{j}-2B^{3}C^{2}k\sigma_{j}^{2}-32B^{3}Ck^{2}+4B^{2}C^{3}k\sigma_{i}^{2}+3B^{2}C^{3}k\sigma_{i}\sigma_{j} \\ &+24B^{2}C^{2}k^{2}-BC^{4}k\sigma_{i}^{2}-8BC^{3}k^{2}+C^{4}k^{2} \end{aligned} \tag{46}$$

Finally, we derive equilibrium profits per firm:

$$\pi_{i}^{3}=B\left(p^{3}\right)^{2}-d\left(s^{N*}\right)^{2}-d\left(s^{O*}\right)^{2} \tag{47}$$

Theorem 4. *In the presence of endogenous switching costs, and within certain bounds, firms invest both in the removal of lock-out, and the creation of lock-in. Moreover, firms prioritize the removal of lock-out due to its direct effect on firm demand.*

Proof. We provide a partial proof to the above theorem through the use of comparative statistics, a method chosen due to the complexity of the equations. The comparative statistics adhere to all assumptions relevant to the model and the demand function. Bellow we present a summary of industry distributions in a symmetric setting, additional statistics can be found in Appendix F.

Assuming: $A = 10$, $B = 5$, $C = 4$, $k = 1$, and $\sigma_i = \sigma_j = 0.5$, we derive the following industry distributions,

$$\begin{aligned} s_i^N = 0.522660064644797, \qquad & s_i^O = -4.09187813767966 \\ q_i = 9.8205075304312, \qquad & p_i = 1.96410150608624 \\ & \pi_i = 2.27183339425586 \end{aligned}$$

□

Corollary 1. *The presence of endogenous switching costs leads to reduced or negative profits for the firms in the relevant market. This is due to large investments by both firms in the endogenous imposition of lock-in and abolition of lock-out.*

The choice of values was made in an effort to cover various instances of the market. Although the only provide a partial proof to the Theorems of Section 4.3 they help interpret the model and draw some conclusion with regard to firm incentives in a market with endogenous switching costs. It is also observed that within a certain range of values, the results of the endogenous model supplement those of the previous sections and Theorem 3.

5 Discussion

Inspired by the practices of the MOS market, this study set out to investigate firms' conduct of competition in duopolistic markets with differentiated goods where switching costs are a strategic variable. Switching cost is a broad term used to describe costs that bind customers to specific supplier of products. Although lock-in type costs are widely covered in the theoretical literature, consumer switching costs of the lock-out type have received little stand-alone attention; with the exception of Chen (1997) discussed in the literature review. The present paper analysed the effects of both exogenous and endogenous switching costs in a static game, on the distribution of prices, quantities, and profits in the industry.

The exogenous modeling of lock-out costs was motivated by the lack of literature on the direct effects their presence may have on market distributions. In order to evaluate the validity of the model, switching costs of the lock-in type were also considered. This enabled the comparison of our findings with those of the established literature, while supplementing the integrity of the model considering their presence alongside lock-out switching costs in the MOS market. We found that one-sided lock-in raises profits for the imposer above those observed in an equivalent market without switching costs, suggesting agreement with previous models (Farrell & Klemperer, 2007). Moreover, the model indicates that a firm faced by rival lock-in will be forced to lower prices, resulting in lower profits.

Having established the partial validity of the model through its cohesion with extant literature, we turned to the study of one-sided lock-out and two-sided switching costs of both types. Of interest, is the finding that lock-out switching costs are equivalent to a rival implementation of lock-in costs. Since one-sided lock-out hurts the firm facing them, firms will strive to reduce its presence. This finding is further clarified through the endogenous modeling of switching costs. Theorem 4, although only partially proven through comparative statistics, supplements the findings of the previous sections by showing that firms will prioritize the abolition of lock-out while simultaneously investing in the creation of higher lock-in barriers. While this is the most important finding of the current investigation, it is limited by the lack of a complete proof in determining the domain for which the proposed relation holds. We hope that future research will attempt solidify these findings through more concrete models.

It is noteworthy that the theoretical inferences of the model analysis are fully reflected by the empirical practices of firms in the MOS market. Specifically, we find that the presence of endogenous switching costs leads to lower prices and profit in the market, as firms "race" to attract new consumers by lower lock-out barriers and simultaneously entrench their current customers through higher lock-in costs. These finding are parallel to those by Dubé, Hitsch and Rossi (2009) who also found that the presence of switching costs fuel rather than soften price competition in the market. Furthermore, the proposed model substantiates Apple's strategic choice to invest in the abolition of endogenously present lock-out barriers, namely, the lack of *automatic migration tools.* Furthermore, Google's introduction of their own migration tool is in line with the notion that switching costs are a strategic complement. That is, if a firm abolishes its lock-out barriers, the rival firm will strive to abolish its own, and vice-versa in the case of lock-in.

The findings of the model help describe MOS market practices by uncovering the underlying economic forces diving Apple's and Google's strategic choices in relation to switching costs. The ability of these firms to discriminate amongst consumer groups enables them to capture an increasing share of social utility. Given the products' widespread appeal, antitrust authorities ought to consider new and existing theoretical inferences in an effort to protect market, consumers and society as a whole.

The current paper can aid in that effort, still, certain limitations are to be considered. The modelling of switching costs as homogeneous amongst all affected consumers, and the apparent redistribution of preferences at every period, form the two core limitations of the model. The constant distribution of preferences amongst consumers, fails to account for possible network effects and preference deviation following the use of a product. Although these are considerable oversights, we may assume that any change in preferences is captured by the effective price faced by each consumer group. Hence, unless a switching cost is present, there is no dis/incentive for the consumer to switch. As a result, the consumer faces the same choice every period. Modelling switching costs as homogeneous for all affected consumers, opposes Klemperer's time-tested model (1987c). The application of switching costs through a geometric function is arguably more representative of real market conditions. Still, the proposed assumption of homogeneous costs helps simplify the analysis while retaining all elements necessary to describe the relevant economic force.

To externally validate the model we limited our focus to specific subcategories of switching costs that homogeneously increase the effective price of products. Cloud capacity expansion has become a necessary after-market purchase for many smartphone users. As MOS suppliers offer discounts to existing users they create lock-in costs for their current customer base. These costs need not directly increase the revenue of suppliers, however, they do arise homogeneously within groups. Lock-out costs can also arise homogeneously. As discussed, in the absence of first-party migration tools, consumers need to expend effort in transferring their data. While search costs can be considered to arise heterogeneously, third-party paid migration tools are partial evidence for the existence of homogeneous lock-out costs. Although the above cannot fully justify this assumption of the model, it does provide sufficient reasoning to consider its findings.

Future research ought to take into account the above limitations when revisiting the topic of switching costs in markets with the ability to price discriminate among consumers. In addition to modeling switching costs heterogeneously within group, future research should focus on incorporating consumer preference stability. The presence of stable preferences may lead to more clearly defined consumer segments, thereby increasing the benefit of switching costs for firms. We anticipate future research to focus on developing more accurate models of endogenous switching costs that account for consumer preference instability, switching cost heterogeneity, and dynamic multi-period contact between sellers and buyers.

References

Brahma, M. (2015, 9). When a Giant Falls. *Vision*, *19*(3), 288–291. Retrieved from https://doi.org/10.1177/0972262915604915 doi: 10.1177/0972262915604915

Burnham, T. A., Frels, J. K., & Mahajan, V. (2003, 4). Consumer Switching Costs: A Typology, Antecedents, and Consequences. *Journal of the Academy of Marketing Science*, *31*(2), 109–126. Retrieved from https://journals.sagepub.com/doi/abs/10.1177/0092070302250897 doi: 10.1177/0092070302250897

Caminal, R., & Matutes, C. (1990). Endogenous switching costs in a duopoly model. *International Journal of Industrial Organization*, *8*(3), 353–373. Retrieved from https://www.sciencedirect.com/science/article/pii/016771879090002I doi: https://doi.org/10.1016/0167-7187(90)90002-I

Chen, Y. (1997, 12). Paying Customers to Switch. *Journal of Economics & Management Strategy*, *6*(4), 877–897. Retrieved from https://doi.org/10.1111/j.1430-9134.1997.00877.x doi: https://doi.org/10.1111/j.1430-9134.1997.00877.x

Diamond, P. A. (1971). A Model of Price Adjustment. *Journal of Economic Theory*, *3*(2), 156–178.

Dubé, J.-P., Hitsch, G. J., & Rossi, P. E. (2009, 8). Do Switching Costs Make Markets Less Competitive? *Journal of Marketing Research*, *46*(4), 435–445. Retrieved from https://doi.org/10.1509/jmkr.46.4.435 doi: 10.1509/jmkr.46.4.435

El-Manstrly, D., Paton, R., Veloutsou, C., & Moutinho, L. (2011). An empirical investigation of the relative effect of trust and switching costs on service loyalty in the UK retail banking industry. *Journal of Financial Services Marketing*, *16*(2), 101–110. Retrieved from https://doi.org/10.1057/fsm.2011.9 doi: 10.1057/fsm.2011.9

Farrell, J., & Klemperer, P. (2007, 1). Chapter 31 Coordination and Lock-In: Competition with Switching Costs and Network Effects. *Handbook of Industrial Organization*, *3*, 1967–2072. doi: 10.1016/S1573-448X(06)03031-7

Farrell, J., & Shapiro, C. (1988). Dynamic Competition with Switching Costs. *The RAND Journal of Economics*, *19*(1), 123–137. Retrieved from http://www.jstor.org/stable/2555402 doi: 10.2307/2555402

Klemperer, P. (1987a). The Competitiveness of Markets with Switching Costs. *The RAND Journal of Economics*, *18*(1), 138–150. Retrieved from http://www.jstor.org/stable/2555540 doi: 10.2307/2555540

Klemperer, P. (1987b). Entry Deterrence in Markets with Consumer Switching Costs. *The Economic Journal*, *97*, 99–117. Retrieved from http://www.jstor.org/stable/3038233 doi: 10.2307/3038233

Klemperer, P. (1987c, 5). Markets with Consumer Switching Costs*. *The Quarterly Journal of Economics*, *102*(2), 375–394. Retrieved from https://doi.org/10.2307/1885068 doi: 10.2307/1885068

Klemperer, P. (1995). Competition when consumers have switching costs: An overview with applications to industrial organization, macroeconomics, and international trade. *The review of economic studies*, *62*(4), 515–539.

Mickalowski, K., Mickelson, M., & Keltgen, J. (2008). Apple's iPhone launch: A case study in effective marketing. *The Business Review*, *9*(2), 283–288.

O'Dea, S. (2022). *Number of smartphone subscriptions worldwide from 2016 to 2027.*

Padilla, A. J. (1995, 12). Revisiting Dynamic Duopoly with Consumer Switching Costs. *Journal of Economic Theory*, *67*(2), 520–530. doi: 10.1006/JETH.1995.1083

Shen, L., & Su, A. (2019). Intervention of smartphone addiction. In *Multifaceted approach to digital addiction and its treatment* (pp. 207–228). IGI Global.

Shi, M. (2013). A theoretical analysis of endogenous and exogenous switching costs. *Quantitative Marketing and Economics*, *11*(2), 205–230. Retrieved from https://doi.org/10.1007/s11129-012-9129-4 doi: 10.1007/s11129-012-9129-4

Singh, N., & Vives, X. (1984). Price and Quantity Competition in a Differentiated Duopoly. *The RAND Journal of Economics*, *15*(4), 546–554. Retrieved from http://www.jstor.org/stable/2555525 doi: 10.2307/2555525

Sircar, R., & Ledvina, A. F. (2010). Dynamic Bertrand Oligopoly. *SSRN Electronic Journal*. doi: 10.2139/ssrn.1587347

Smith, M. L., Spence, R., & Rashid, A. T. (2011). Mobile phones and expanding human capabilities. *Information Technologies & International Development*, *7*(3), pp-77.

6 Appendix

6.1 Appendix A

We obtain the inverse demand from the first order condition of the maximization problem: max_q $U(q) - pq$. Using the below utility function we can derive linear demand system (1) from Section 3.1.1.

$$U(q_i, q_j) = \alpha \sum_{i=1}^{2} q_i - \frac{1}{2}(\beta \sum_{i=1}^{2} q_i^2 + \gamma \sum_{i \neq j} q_i q_j)$$

Setting up the maximization problem of the Representative consumer:

$$U(q_i, q_j) = \alpha (q_i + q_j) - \frac{1}{2}(\beta \left(q_i^2 + q_j^2\right) + \gamma (q_i q_j) \\ - (p_i q_i + p_j q_j)$$

Finding the first-order condition w.r.t q_i and solving for p_i:

$$\frac{\partial U}{\partial q_i} = \alpha - \beta q_i - \frac{1}{2}\gamma q_j - p_i = 0 \Rightarrow \\ p_i = \alpha - \beta q_i - \frac{1}{2}\gamma q_j$$

We obtain the inverse demand for product i and replicate the maximization problem for product j:

$$\frac{\partial U}{\partial q_j} = \alpha - \beta q_j - \frac{1}{2}\gamma q_i - p_j = 0 \Rightarrow \\ p_j = \alpha - \beta q_j - \frac{1}{2}\gamma q_i$$

By inverting the above system, we can write direct demand, $D_i(p_i, p_j)$, as:

$$D_i = q_i = A - Bp_i + Cp_j \\ D_j = q_j = A - Bp_j + Cp_i$$

6.2 Appendix B

Firm i's first-order condition is:

$$\frac{\partial \pi_i}{\partial p_i} = a - 2bp_i + cp_j = 0 \tag{48}$$

At a pure strategy symmetric equilibrium, $p_i = p_j = p$. This yields:

$$p^0 = p_i = p_j = \frac{a}{2b - c} \tag{49}$$

Substituting (38) into (4), yields quantity (39) and firm profit (40):

$$q^0 = q_i = q_j = \frac{ab}{2b - c} \tag{50}$$

$$\pi^0 = \pi_i = \pi_j = \frac{a^2 b}{(2b - c)^2} \tag{51}$$

6.3 Appendix C

The majority of the objective maximization process for the non-cooperative single-sided lock-in equilibrium can be found in Equations (20)–(25). This section of the Appendix derives equilibrium quantities and profits for the relevant market. We begin by substituting Equations (24) and (25) into Equations (18) and (19), respectively. This gives us the quantity sold by each firm.

$$q_i^1 = \frac{AB}{2B - C} + \frac{\sigma_i B^2 c C s_i^N}{(2B)^2 + C^2} \tag{52}$$

$$q_j^1 = \frac{AB}{2B - C} - \frac{2\sigma_i B^3 s_i^N - \sigma_i B C^2 s_i^N}{(2B)^2 + C^2} \tag{53}$$

We can now calculate firm and industry profits using objective maximization function (13).

$$\pi_i^1 = \left(\frac{A}{2B - C} + \frac{\sigma_i B C s_i^N}{(2B)^2 + C^2} \right)^2 B \tag{54}$$

$$\pi_j^1 = \left(\frac{A}{2B - C} - \frac{2\sigma_i B^2 s_i^N - \sigma_i C^2 s_i^N}{(2B)^2 + C^2} \right)^2 B \tag{55}$$

$$\pi^1 = \pi_i^1 + \pi_j^1 = \left(\frac{A}{2B - C} + \sigma_i s_i^N \frac{BC + C^2 - 2B^2}{(2B)^2 + C^2} \right)^2 B \tag{56}$$

6.4 Appendix D

This section of the Appendix derives equilibrium prices, quantities and profits for the non-cooperative two-sided lock-in version of the model. Using demand function (34), we attain the following profit function for firm i:

$$\pi_i = p_i (A - B p_i + C p_j + \sigma_i C s_i^N - \sigma_j B s_j^N) \tag{57}$$

The first order condition for firm i:

$$\frac{\partial \pi_i}{\partial p_i} = A - 2B p_i + C p_j + \sigma_i C s_j^N - \sigma_j B s_i^N = 0 \tag{58}$$

Solving for price through symmetry:

$$p_i^2 = \frac{A + \sigma_i C s_i^N - \sigma_j B s_j^N}{2B - C} \tag{59}$$

$$p_j^2 = \frac{A + \sigma_j C s_j^N - \sigma_i B s_i^N}{2B - C} \tag{60}$$

Finding q:

$$q_i^2 = \frac{AB + \sigma_i BCs_i^N - \sigma_j B^2 s_j^N}{2B - C} \tag{61}$$

$$q_j^2 = \frac{AB + \sigma_j BCs_j^N - \sigma_i B^2 s_i^N}{2B - C} \tag{62}$$

Finding π:

$$\pi_i^2 = \left(\frac{A + \sigma_i Cs_i^N - \sigma_j Bs_j^N}{2B - C}\right)^2 B \tag{63}$$

$$\pi_j^2 = \left(\frac{A + \sigma_j Cs_j^N - \sigma_i Bs_i^N}{2B - C}\right)^2 B \tag{64}$$

6.5 Appendix E

The Firm i's first-order condition w.r.t. p_i after substituting for q_i is:

$$\frac{\partial \pi_i}{\partial p_i} = A - 2Bp_i + Cp_j + \sigma_i Cs_i^N + \sigma_i Cs_j^O - \sigma_j Bs_j^N - \sigma_j Bs_i^O = 0 \tag{65}$$

Solving for price through symmetry and finding q_i, yields:

$$p_i^3 = \frac{A + \sigma_i Cs_i^N + \sigma_i Cs_j^O - \sigma_j Bs_j^N - \sigma_j Bs_i^O}{2B - C} \tag{66}$$

$$q_i^3 = \frac{AB + \sigma_i BCs_i^N + \sigma_i BCs_j^O - \sigma_j B^2 s_j^N - \sigma_j B^2 s_i^O}{2B - C} \tag{67}$$

This results in profit:

$$\pi_i^3 = \left(\frac{A + \sigma_i Cs_i^N + \sigma_i Cs_j^O - \sigma_j Bs_j^N - \sigma_j Bs_i^O}{2B - C}\right)^2 B - d(s_i^N)^2 - d(s_i^O)^2 \tag{68}$$

Substituting p_i^3 and q_i^3 into π_i and solving for the first-order condition w.r.t s_i^O yields the following:

$$\frac{\partial \pi_i}{\partial s_i^O} = -\frac{2\sigma_j B^2}{2B - C}\left(\frac{A + \sigma_i Cs_i^N + \sigma_i Cs_j^O - \sigma_j Bs_j^N - \sigma_j Bs_i^O}{2B - C}\right) - 2ds_i^O = 0 \tag{69}$$

Having found optimal lock-out as an expression of s^N (46), we substitute s^O into π_i. Solving for the first-order condition w.r.t s_i^N yields the following:

$$\begin{aligned}\frac{\partial \pi_i}{\partial s_i^N} = -2B^4C\sigma_i\sigma_j^2 k\left(A - Bs_j^N\sigma_j + Cs_i^N\sigma_i\right) + \\ 2C\sigma_i k\left(2B - C\right)\left(A - Bs_j^N\sigma_j + Cs_i^N\sigma_i\right) \\ \left(-B^3\sigma_j^2 + B^2C\sigma_i\sigma_j + B^2\sigma_j\left(B\sigma_j - C\sigma_i\right) - k\left(B - C\right)\left(2B - C\right) + k\left(2B - C\right)^2\right) - \\ 2s_i^N k\left(-B^3\sigma_j^2 + B^2C\sigma_i\sigma_j + k\left(2B - C\right)^2\right)^2 = 0\end{aligned} \tag{70}$$

We can now substitute equilibrium lock-in and lock-out into equilibrium price and quantity:

$$p^3 = \frac{Ak\left(-2B^4\sigma_j^2 + 2B^3C\sigma_i\sigma_j + B^3C\sigma_j^2 + 8B^3k - B^2C^2\sigma_i\sigma_j - 12B^2Ck + 6BC^2k - C^3k\right)}{\begin{aligned}&B^6\sigma_j^4 - 3B^5C\sigma_i\sigma_j^3 - 8B^5k\sigma_j^2 \\ &+ 2B^4C^2\sigma_i^2\sigma_j^2 + 12B^4Ck\sigma_i\sigma_j + 8B^4Ck\sigma_j^2 + 16B^4k^2 - 4B^3C^2k\sigma_i^2 \\ &- 12B^3C^2k\sigma_i\sigma_j - 2B^3C^2k\sigma_j^2 - 32B^3Ck^2 + 4B^2C^3k\sigma_i^2 + 3B^2C^3k\sigma_i\sigma_j \\ &+ 24B^2C^2k^2 - BC^4k\sigma_i^2 - 8BC^3k^2 + C^4k^2\end{aligned}} \tag{71}$$

$$q^3 = \frac{ABk\left(-2B^4\sigma_j^2 + 2B^3C\sigma_i\sigma_j + B^3C\sigma_j^2 + 8B^3k - B^2C^2\sigma_i\sigma_j - 12B^2Ck + 6BC^2k - C^3k\right)}{\begin{aligned}&B^6\sigma_j^4 - 3B^5C\sigma_i\sigma_j^3 - 8B^5k\sigma_j^2 \\ &+ 2B^4C^2\sigma_i^2\sigma_j^2 + 12B^4Ck\sigma_i\sigma_j + 8B^4Ck\sigma_j^2 + 16B^4k^2 - 4B^3C^2k\sigma_i^2 \\ &- 12B^3C^2k\sigma_i\sigma_j - 2B^3C^2k\sigma_j^2 - 32B^3Ck^2 + 4B^2C^3k\sigma_i^2 + 3B^2C^3k\sigma_i\sigma_j \\ &+ 24B^2C^2k^2 - BC^4k\sigma_i^2 - 8BC^3k^2 + C^4k^2\end{aligned}} \tag{72}$$

6.6 Appendix F

Below we present various comparative statistics for the endogenous switching cost model presented in Section 3.3 and analysed in Section 4.3.

Assuming: $A = 10$, $B = 5$, $C = 4$, $k = 1$, and $\sigma_i = \sigma_j = 0.5$, we derive the following industry distributions,

$$s_i^N = 0.522660064644797, \qquad s_i^O = -4.09187813767966$$
$$q_i = 9.8205075304312, \qquad p_i = 1.96410150608624$$
$$\pi_i = 2.27183339425586$$

Assuming: $A = 100$, $B = 90$, $C = 50$, $k = 1$, and $\sigma_i = \sigma_j = 0.5$, we derive the following industry distributions,

$$s_i^N = 11.1657923011819, \qquad s_i^O = -7.79140680804576$$
$$q_i = 22.5085085565766, \qquad p_i = 0.250094539517518$$
$$\pi_i = -179.751682678912$$

Assuming: $A = 100$, $B = 90$, $C = 50$, $k = 0$, and $\sigma_i = \sigma_j = 0.5$, we derive the following industry distributions,

$$s_i^N = 25, \qquad s_i^O = -20.0$$
$$q_i = 5.6843418860808 \times 10^{-14}, \qquad p_i = 0$$
$$\pi_i = 0$$

Assuming: $A = 100$, $B = 90$, $C = 50$, $k = 1$, and $\sigma_i = 0.75\sigma_j = 0.25$, we derive the following industry distributions,

$$s_i^N = -2.62444015885597, \qquad s_i^O = -2.59720000369645$$
$$q_i = 15.0060444658018, \qquad p_i = 0.166733827397797$$
$$\pi_i = -11.1311187787322$$