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# Optimal maintenance planning for offshore wind farms considering time-varying costs and limited manpower

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*Full name:* Roby Cremers

*Student number:* 537513

*Supervisor:* Dekker, R

*Second assessor:* Heuvel, W van den

## Abstract

With wind energy taking up a bigger share of the world wide electricity production each year and the desire to have switched to a fully sustainable global energy landscape by 2050, finding least-cost maintenance programs for wind turbine components becomes more and more important. In this thesis we analyse the problem of determining which maintenance activities should be carried out at times other than originally planned when encountering limited available manpower. We consider the period-dependent age replacement policy (p-ARP), block replacement policy (p-BRP) and modified block replacement policy (p-MBRP) to construct least-cost maintenance policies for a single component under time-varying costs. The first two form the groundwork for several algorithms that we propose in case of dealing with multiple components where only a limited number can be maintained at the same time. First, we propose the Dynamic Maintenance Delay (*DMD*) heuristic that deals with delaying preventive maintenance activities. Next, we include bringing forward maintenance by introducing the Dynamic Maintenance Replan (*DMR*) and Static Maintenance Replan (*SMR*) heuristics. We compare the performances means of simulation for 80 identical components. Moreover, we compare the performances with the optimal policy in case of two components.

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# 1 Introduction

The worldwide wind power capacity has been vastly increasing in the past couple of years. In 2020, wind was already accountable for 16% of the electric energy consumption in Europe (Wind Europe 2020) and Wind Europe (n.d.) expects wind to become the number one source of Europe’s electricity by 2027. With the number of wind turbines increasing, also the associated costs are growing. Operation and maintenance costs make up 20-25% of these costs and are significantly larger for offshore wind farms due to the harsh environment. To decrease maintenance costs it is important to perform the required operations at the right moments. This is because in Europe, wind energy generation is relatively low in summer while it peaks in winter months. Shutting down a wind turbine for maintenance purposes therefore results in a higher production loss during winter.

However, environmental conditions is not the only aspect that matters when scheduling maintenance. Another issue that plays a role is limited manpower. Because the wind industry has grown so rapidly, a skills gap has arisen. As a result, Europe is encountering a shortage of qualified personnel (European Wind Energy Technology Platform 2013). Planning all maintenance during the summer months therefore comes with the risk of not finding the required technicians. Furthermore, limited availability of personnel resources offshore makes it impossible maintain all components at the same time. In order to balance the workload, activities need to be performed at times other than those optimal from a maintenance perspective. This means they will either take place too early or too late. George et al. (2022) point out that this aspect of offshore maintenance, which has a great influence on the feasibility of a constructed schedule, has not been included in any recent studies.

Therefore, the main research question this paper tries to answer is: *How to optimally plan maintenance of an offshore wind turbine component, trying to minimize downtime costs and taking into account limited manpower?* First, this paper seeks to find the optimal maintenance planning for a single component trying to minimize lost production costs. After that, we analyse how to delay planned operations when there are not enough technicians available. Finally, we investigate the optimal policy for a wind farm, while trying to keep the fluctuation in required personnel considerably small.

In this paper, we use the period-dependent maintenance policies that were introduced by Schouten et al. (2022) as the fundament for a multi-component maintenance schedule, where we introduce a restriction on the number of permitted maintenance activities per month. The main contributions of this paper are three algorithms that can assist a maintenance manager in deciding which operations should be replanned when there are not enough service engineers available to carry out all activities.

The ideas presented in this paper are relevant to companies that own offshore wind farms as it can help them reduce costs. However, they can also be applied in other fields where time-varying downtime costs and limited manpower play a role, such as maintenance in aviation, repair of industrial machinery that manufacture seasonal products and restorations of tourist sites. Furthermore, although we focus on limited manpower in this paper, the results are also relevant when it is not desirable to plan all operations at the same time due to other causes, such as unavailability of spare parts, severe weather conditions or a minimum guaranteed production to be able to meet demand.

The remainder of this study is structured as follows. In Section 2 relevant literature and latest developments in the field are presented. This is followed by a problem description in Section 3 and an explanation of the methods used to answer the research question in Section 4. In Section 5, we present the results of this study. Finally, we end with some concluding remarks, limitations of our study and recommendations for further research in Section 6.

## 2 Literature review

There exist several studies investigating optimal maintenance planning. Schouten et al. (2022) consider different policies for a single component of an offshore wind turbine. They introduce period-dependent variants of the age, block and modified block replacement policies and compares these. The focus lies on preventive versus corrective maintenance. The former occurs when it was scheduled and the system is still operational until the actions start while the latter takes place after a breakdown. Schouten et al. (2022) include cost fluctuations due to varying wind speed and they model the life of a component using a discrete-time Markov decision process. An important result is that including cost fluctuations can save a significant percentage of costs. However, they do not take into account the fact that maintenance is not possible in some periods of the year due to severe weather.

A study that does incorporate this in their models is conducted by Zhong et al. (2019). What is more, next to minimizing costs they include reliability using multi-objective optimisation.

Do Van et al. (2013) propose to group the maintenance of different components. They conclude that when there is no negative economic dependence due to for example limited manpower, it is beneficial to combine maintenance for different components. Schouten et al. (2019) capture this same idea by including set-up costs that only need to be paid once when several maintenance activities are combined. They present modified mathematical formulations for the period ARP, BRP and MBRP models for one and two components and propose several heuristics that can be used in case of more components. Finally, Dekker et al. (1991) also define algorithms to combine execution of maintenance activities, using penalty costs for deviating from the original maintenance planning.

The recent literature on the topic of offshore maintenance has been reviewed by George et al. (2022). They found that there seem to be no studies that include constraints regarding the limited availability of personnel resources on offshore wind turbines and the impact of the time needed to carry out the maintenance. As the former restricts the number of employees that can be deployed, this can be seen as a limitation on the available staff.

This research first follows in the footsteps of Schouten et al. (2022) trying to replicate their results and in addition aims to include the limitation on manpower by restricting the number of maintenance actions per period when looking at an offshore wind farm. Similarly to Dekker et al. (1991) we implement this by defining penalty costs for deviating from the original execution dates.

## 3 Problem description

In this paper, we aim at finding an optimal maintenance policy, first for one wind turbine component and later we extend this to a multiple-component case. We assume that the distribution of the lifetime of a component is known and follows a Weibull distribution.<sup>1</sup> Once a component fails, immediate corrective maintenance (CM) is performed. To avoid this form of maintenance which entails relatively large downtime costs due to a lead time, one can decide to execute preventive maintenance (PM). In both cases, the component is replaced by an as-good-as-new component. In addition, we define for each component a maximum age and PM is performed whenever the component reaches this age.

An important assumption that we make is that the failure rate only depends on the age of the component, not on aspects as the age of the entire turbine, the state of other components or the time of the year. Something that does depend on the time of the year are the repair costs. As the average wind speed is much higher in the winter months, shutting down a turbine to conduct maintenance work is way more expensive in this period compared to the summer. We therefore propose time-varying cost functions with a yearly cycle. The time that it takes to perform

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<sup>1</sup>However, all results can also be applied to other distributions as long as the failure rate is increasing and the renewal density converges to one divided by the average lifetime fast enough.

the maintenance actions is presumed to be known and constant, just as the lead time for corrective maintenance. Furthermore, we do not take into account inflation, such that the costs do not change over the years.

When considering multiple components, we aim at finding those for which moving up or postponing PM is the least costly whenever there are not enough engineers available to perform all planned activities. For ease of implementation and because delaying CM is always more costly than delaying PM, we assume all CM can be executed immediately such that we only need to focus on replanning PM. We assume that we can also perform all PM at the maximum age, such that we regard this as CM in this sense. Finally, we disregard any positive economic, structural or stochastic dependence. This is not completely realistic, but in this paper we decide to solely focus on negative dependence.

## 4 Methodology

In accordance with the methodology introduced in the paper by Schouten et al. (2022), we first consider the optimal maintenance planning for a single component of an offshore wind turbine. The sets, parameters and variables they use that we adopt in this paper can be found in Appendix A. In Section 4.1, we will shortly explain all steps of the process but for further detail we refer to Schouten et al. (2022). After that, we will introduce a heuristic solution to deal with bringing forward and delaying PM activities in a multi-component case in Section 4.2.

### 4.1 Single-component maintenance

In the paper by Schouten et al. (2022), the life of a component is modeled by a discrete-time Markov decision process. Each state consists of the period of the year  $i_1 \in \mathcal{I}_1$  and the component age  $i_2 \in \mathcal{I}_2$ , where age zero indicates a failed component and  $M$  is the maximum component age. The possible actions given a state  $i = (i_1, i_2)$  are denoted by  $\mathcal{A}(i)$  and are either to maintain ( $a = 1$ ) or to do nothing ( $a = 0$ ), where the latter is only possible if  $i_2 \notin \{0, M\}$ .

As explained before, the maintenance costs depend on the period  $i_1 \in \mathcal{I}_1$ . They are denoted by  $c_p(i_1)$  for PM and  $c_f(i_1)$  for CM, and are a function of the yearly averages  $\bar{c}_p$  and  $\bar{c}_f$  respectively.

Given the Markov chain and cost functions, we can write down the mathematical model for different types of maintenance policies, where we aim at minimizing the long-term cost per period.

First, we consider a period age replacement policy (ARP), where each period  $i_1 \in \mathcal{I}_1 = \{1, 2, \dots, N\}$  of the year has associated a critical maintenance age  $t_{i_1}$  at or above which PM is performed. Schouten et al. (2022) show that there exists a p-ARP that is an optimal maintenance policy when considering time-varying costs and under mild conditions there exists a finite optimal p-ARP. They also introduce a linear programming (LP) formulation that should be solved in order to find this policy.

Second, Schouten et al. (2022) define a mixed integer LP (MILP) formulation for a period block replacement policy (p-BRP). Here, there is a finite cycle of  $m$  years after which the maintenance policy repeats itself. Within these  $m$  years, PM is performed at fixed times  $T_1, T_2, \dots, T_n \in \mathcal{I}_1 = \{1, 2, \dots, mN\}$ . Again, a few conditions need to be satisfied for the existence of an optimal p-BRP.

The third and final policy that we discuss is the period modified BRP (p-MBRP), which is a combination of the first two policies. It yields a set of periods  $T_1, T_2, \dots, T_n \in \mathcal{I}_1 = \{1, 2, \dots, mN\}$  and for each  $T_k, k = 1, 2, \dots, n$  we maintain if the component age has reached a critical maintenance age  $t_{T_k}$ . Again, Schouten et al. (2022) mention the conditions for the existence of an optimal p-MBRP. However, the MILP they present for finding the optimal p-MBRP is missing two constraints, namely

$$x_{i,0} + z_{i_1, i_2} \leq 1 \quad \forall i = (i_1, i_2) \in \mathcal{I} \quad (1a)$$

$$x_{i,1} - z_{i_1, i_2} \leq 0 \quad \forall i = (i_1, i_2) \in \mathcal{I}, i_2 \notin \{0, M\} \quad (1b)$$

Constraints (1a) make sure that we actually perform PM whenever we are in state  $i = (i_1, i_2) \in \mathcal{I}$  if we are supposed to maintain for age  $i_2 \in \mathcal{I}_2$  in period  $i_1 \in \mathcal{I}_1$ . To enforce  $x_{i,1}$  to become zero, such that we do not perform PM, whenever we are in state  $i = (i_1, i_2) \in \mathcal{I}$  if we are not supposed to maintain for age  $i_2 \in \mathcal{I}_2$  in period  $i_1 \in \mathcal{I}_1$ , we add restrictions (1b).

For each of these formulations, solving the (MI)LP gives us optimal values for the long-run probabilities that the system is in state  $i \in \mathcal{I}$  and decision  $a \in \mathcal{A}(i)$  is chosen,  $x_{i,a}^*$ . Given these values we can define the set  $\mathcal{I}_{LP} = \{i | x_{i,a}^* > 0\}$  and the optimal stationary policy  $R^*$ . For each state  $i \in \mathcal{I}$  we let  $R^*(i) = a$  if  $x_{i,a}^* > 0$ . For all other states  $i$ , we choose an action  $a$  such that  $\pi_{ij}(a) > 0$  for some  $j \in \mathcal{I}_{LP}$ , we let  $R^*(i) = a$  and add  $i$  to  $\mathcal{I}_{LP}$ . We do this iteratively until all states are in  $\mathcal{I}_{LP}$ .

## 4.2 Multi-component maintenance

For the case of multiple components, finding an optimal solution quickly becomes computationally expensive as the number of states grows exponentially with the number of components. Furthermore, when we add the restriction on the number of PM activities per period, the problem resembles the capacitated lot sizing problem, which has been proven to be NP-hard (Bitran et al. 1982). We therefore introduce heuristic solutions. Let us consider a set of components  $\mathcal{S}$  for which the lifetimes are independent and possibly follow different distributions. Our goal is to make a long-term average least-cost operational maintenance planning for all components, while taking into account limited manpower. As a first step, we come up with an algorithm to decide for which of the components to delay PM, for example when at the planned execution date the company encounters unavailability of service engineers or spare parts. After that, we consider moving up PM in addition to postponing and we will present heuristics to create an age and block maintenance planning while restricting the number of PM activities per period. We decided not to discuss p-MBRP in this paper, but a combination of the heuristics that we introduce for the other two policies can be used for this policy.

In the remainder of this section, we assume that we always have enough spare manpower and materials to perform all CM and all PM for components that have reached the maximum age  $M$ . Furthermore, we make use of the following lemma for which the proof can be found in [Appendix C](#):

**Lemma D1.** *When there is no economic, structural or stochastic dependence between different components, the policy that applies to each of the components the optimal replacement policy obtained when solving the model for that particular component, leaving aside all other components, is an optimal replacement policy for all components combined.*

### 4.2.1 Delaying maintenance

To determine for which components postponing preventive maintenance is the least costly, we first compute the optimal joint maintenance planning when disregarding any dependence. [Lemma D1](#) tells us that we can find this optimal policy by solving the p-ARP or p-BRP model for each of the components separately. Given this policy, we will later come up with an algorithm for delaying PM, but we first need to introduce additional notation and present some theoretical results.

For a component  $s \in \mathcal{S}$ , let  $g^s(R^s)$  denote the long-run average cost per period that follows from an optimal planning policy  $R^s$  obtained by solving one of the models from [Section 4.1](#). As the Markov decision chain for the life of this component is unichain, this average cost does not depend on the initial state. In the book written by Tijms (2003) it is explained that it can be proven that under mild conditions there exist bias values  $v_i^s(R^s), \forall i \in \mathcal{I}$ , which are defined in such a way that  $v_i^s(R^s) - v_j^s(R^s)$  measures the difference in the total costs for component  $s$  when starting in state  $i$  rather than starting in state  $j$ , if we follow policy  $R^s$ . We can find the relative value function  $v_i^s(R^s), \forall i \in \mathcal{I}$  and the long-run average costs  $g^s(R^s)$  under a policy  $R^s$  by solving the value-determination equations

$$v_i^s(R^s) = c_i^s(R_i^s) - g^s(R^s) + \sum_{j \in \mathcal{I}} \pi_{ij}^s(R_i^s) v_j^s(R^s), \quad \forall i \in \mathcal{I} \tag{2a}$$

$$v_q^s = 0, \tag{2b}$$

where  $c_i^s(a)$  is the immediate cost incurred when action  $a$  is chosen for component  $s$  in state  $i$  and  $\pi_{ij}^s(a)$  is the transition probability from state  $i$  to state  $j$  under action  $a$  associated with component  $s$ . In this system of equations,  $q \in \mathcal{I}$  is arbitrarily chosen.

It can be shown that the difference in the long-term total costs for component  $s$  when starting in state  $i$  and first taking action  $a$  after which using policy  $R^s$  rather than immediately using  $R^s$  is

$$\Delta^s(i, a, R^s) \approx c_i^s(a) + \sum_{j \in \mathcal{I}} \pi_{ij}^s(a) v_j^s(R^s) - g^s(R^s) - v_i^s(R^s). \quad (3)$$

Using the values for  $v_i^s(R^s)$ ,  $\forall i \in \mathcal{I}$  and  $g^s(R^s)$  obtained from solving (2)a-(2)b, we can compute the additional costs that are incurred when we decide not to perform maintenance for a certain component in the period for which it would be optimal. In particular, suppose we have  $R_i^s = 1$  for state  $i = (i_1, i_2)$ . If we decide not to perform PM in period  $i_1$ , an additional cost of  $\Delta^s(i, 0, R^s) = c_i^s(0) + \sum_{j \in \mathcal{I}} \pi_{ij}^s(0) v_j^s(R^s) - g^s(R^s) - v_i^s(R^s) = \sum_{j \in \mathcal{I}} \pi_{ij}^s(0) v_j^s(R^s) - g^s(R^s) - v_i^s(R^s)$  will be incurred. Note that this is the cost when we assume the shortage of manpower or material is a one-time thing, such that we only deviate from the policy  $R^s$  once.

We now know how to compute the additional cost of not performing PM at the planned execution date, but this is not the expected additional cost if we decide to delay the activity to a next period instead of not performing the PM at all. Let us denote the total penalty cost of postponing PM for component  $s \in \mathcal{S}$  in state  $i = (i_1, i_2) \in \mathcal{I}$  by  $\delta^s(i)$ . Before we can compute this penalty cost, we need to determine how to replan PM. Here, we distinguish between p-ARP and p-BRP.

For p-ARP, the optimal replacement policy yields for each period a threshold age above which it is optimal to perform PM. We can therefore simply apply the optimal policy  $R^s$  to determine when to perform the postponed PM. The expected cost of delay then becomes

$$\delta^s(i) = \Delta^s(i, 0, R^s). \quad (4)$$

For p-BRP, we perform PM in certain fixed periods. The value  $\Delta^s(i, 0, R^s)$  now shows the price of not maintaining at the planned execution date and waiting for the next block time for a new PM opportunity. However, to avoid CM with higher costs, we can decide to delay PM by  $x \in \{1, 2, \dots, \ell\}$  periods. We perform a so-called short-term shift, such that the next PM date remains the same and the interval between the two dates will be smaller. The expected cost of postponing by  $x$  periods is

$$\delta^s(i = (i_1, i_2), x) = \Delta^s(i, 0, R^s) - \Delta^s(i, 1, R^s) + \sum_{j \in \mathcal{I}} \pi_{ij}^s(x)(0) \Delta^s(j, 1, R^s) \quad \forall x \in \{1, 2, \dots, \ell\} \quad (5)$$

where  $\pi_{ij}^s(x)(0)$  is the  $x$ -step transition probability from state  $i$  to state  $j$  given that we do not perform PM in the current period. Note that the last part are the savings associated with preventively maintaining in  $x$  months from now, when we do not perform PM in the current period. The second term is needed in case the PM at period  $i_1$  was already a delayed PM, such that maintaining is not optimal and the costs are expected to decrease by the additional costs due to maintaining in this period. Bear in mind that this term is zero when the PM is optimal in this period. We determine the period to which we replan PM by choosing the value for  $x$  that minimizes the extra costs. The optimal period  $i_1^{s*}$  to perform the PM and the associated difference in costs  $\delta^s(i)$  are computed as

$$i_1^{s*} = i_1 + x^{s*} \pmod{N} = i_1 + \arg \min_{x \in \{1, \dots, \ell\}} \{\delta^s(i = (i_1, i_2), x)\} \pmod{N} \quad (6)$$

$$\delta^s(i = (i_1, i_2)) = \min_{x \in \{1, 2, \dots, \ell\}} \{\delta^s(i, x)\} = \delta^s(i, x^{s*}) \quad (7)$$

We will now define a heuristic to determine which activities should be delayed at the start of a period  $i_1 \in \mathcal{I}_1$ , given the component ages  $i_2^s \in \mathcal{I}_2, \forall s \in \mathcal{S}$ . The idea of the heuristic is that we first compute the cost of delay of PM for each component and then postpone PM for those components for which this additional cost is the lowest. This process can then be repeated for each period.

- Step 0. Let  $\mathcal{S}^P$  be the set of components for which PM is planned in period  $i_1$  and define the maximum number of PM activities in this period  $r$ , which could vary over different periods.<sup>2</sup> Initialise the set of components for which we delay PM,  $\mathcal{S}^D = \emptyset$ .
- Step 1. If  $|\mathcal{S}^P| \leq r$ , go to Step 3. Otherwise, for each  $s \in \mathcal{S}^P$  compute the cost of delaying the maintenance,  $\delta^s((i_1, i_2^s))$ , according to equation (4) or (7) respectively, by solving the system of equations (2)a-(2)b. Store the components  $s \in \mathcal{S}^P$  in non-decreasing order of the values  $\delta^s((i_1, i_2^s))$ .
- Step 2. Extract a component  $s$  from the top of list  $\mathcal{S}^P$ . Update  $\mathcal{S}^D = \mathcal{S}^D \cup \{s\}$  and  $\mathcal{S}^P = \mathcal{S}^P \setminus \{s\}$ . If  $|\mathcal{S}^P| = r$ , STOP, go to Step 3. Otherwise repeat Step 2.
- Step 3. We are done.  $\mathcal{S}^P$  is the set of components for which we perform PM in the current period and  $\mathcal{S}^D$  is the set of components for which we delay PM.
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The algorithm always terminates as the size of  $\mathcal{S}^P$  is finite and each time we visit Step 2 we either stop if the size of  $\mathcal{S}^P$  is not larger than  $r$  anymore or we decrease its size by one and go back to Step 2. Note that it is possible that we delay the same PM activity multiple times.

Finally, we can analyse the performance by means of simulation. Moreover, we will compare the costs when using the *DMD* heuristic and when using a naive heuristic that is described in [Appendix B](#).

#### 4.2.2 Bringing forward and delaying maintenance

So far we focused on how to postpone maintenance. However, when you know beforehand that you are dealing with a shortage of manpower, it could be beneficial to also consider bringing forward PM. We will therefore now focus on making a maintenance planning for multiple components with the constraint that there can be no more than a certain number  $r$  of PM activities per period. When this constraint is violated, we can either delay or move up PM for certain components. In this paper, we keep the value  $r$  fixed, but everything presented in this section can also be applied to a situation in which  $r$  varies over the year. For ease of notation the modulo operator (mod) is omitted everywhere in this section. This means that for example  $i_1 + v(+x)$  should actually be  $i_1 + v(+x) \pmod{N}$ .

In case of using p-ARP, we propose an ongoing process of replanning PM. This is necessary as the optimal number of PM activities in a certain period depends on the ages of the different components such that we do not know this number in advance. We will therefore present an algorithm that should be applied at the beginning of each period. We first determine the optimal maintenance planning for each component  $s \in \mathcal{S}$  separately. Denote this optimal policy by  $R^{s*}, \forall s \in \mathcal{S}$ . At the beginning of a period  $i_1 \in \mathcal{I}_1$ , we compute the expected number of PM activities for each of the next  $\ell$  periods. If this number exceeds  $r$  for any of these periods  $i_1 + v \in \mathcal{I}_1, v = 0, 1, \dots, \ell$ , we replan certain activities where we bring them forward or postpone them by a maximum of  $\ell$  months. We then only implement all changes that relate to the current period  $i_1$  and after that go to the next period.

Let  $\mathbb{1}_A$  be an indicator function that equals one if condition A is true. For a component  $s \in \mathcal{S}$  currently in state  $i = (i_1, i_2)$ , we compute the expected penalty cost of deviating from the original execution date  $i_1 + v$  by  $x \in \{-v, -v+1, \dots, -1, 1, \dots, \ell\}$  periods, such that we would maintain when the component is in state  $j = (i_1 + v + x, i_2 + v + x)$  instead of in  $z = (i_1 + v, i_2 + v)$  as

$$\begin{aligned} \delta^s(i, v, x) = & -\pi_{iz}^{s(v)} \Delta^s(z, 1, R^s) + \mathbb{1}_{x < 0} \cdot \pi_{ij}^{s(v+x)} \sum_{k=x+1}^0 \pi_{(i_1+v+x, 0), (i_1+v+k, k-x)}^s \Delta^s((i_1 + v + k, k - x), 0, R^s) \\ & + \mathbb{1}_{x > 0} \sum_{k=0}^{x-1} \pi_{i(i_1+v+k, i_2+v+k)}^s \Delta^s((i_1 + v + k, i_2 + v + k), 0, R^s) + \pi_{ij}^{s(v+x)} \Delta^s(j, 1, R^s) \end{aligned} \quad (8)$$

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<sup>2</sup>The value of  $r$  could be determined by the available manpower or spare parts or could be enforced by the management in order to keep the fluctuation of required personnel small over the year.

The first term is needed in case the PM in period  $i_1 + v$  is a replanned PM, such that it is not optimal to maintain in this period. We add for each period in between the original and new execution date (including the former) the probability that the component does not fail before we reach this period times the additional cost of not maintaining in this period. If we move up maintenance, this means that it should not fail before and not after the PM. Note that we restrict the action for the periods in between the original and new date to 0 (do nothing). We need the last part of the first line if  $\ell$  is at least the minimum critical maintenance age, as in this case PM in between the two dates could be optimal but we wish to prohibit this. The last term is the probability that the component does not fail before it reaches the period to which we replan times the additional cost of PM in this period.

Finally, given a set  $\mathcal{I}_1^A$  of possible periods to which we can replan the PM, we define the optimal period to perform the PM as the one that yields the lowest difference in costs. We can compute this optimal period and the corresponding expected additional costs respectively as

$$i_1^{s*} = i_1 + v + x^{s*} = (i_1 + v + \arg \min_{x : i_1 + v + x \in \mathcal{I}_1^A} \{\delta^s((i_1, i_2), v, x)\}) \quad (9)$$

$$\delta^s(i = (i_1, i_2), v) = \min_{x : i_1 + v + x \in \mathcal{I}_1^A} \{\delta^s((i, v, x)\} = \delta^s(i, v, x^{s*}) \quad (10)$$

Given the current period  $i_1 \in \mathcal{I}_1$  and the component ages  $i_2^s \in \mathcal{I}_2, \forall s \in \mathcal{S}$ , we introduce the following algorithm for replanning multi-component maintenance for p-ARP under restricted manpower:

---

*DMR (Dynamic Maintenance Replanning)*

---

Step 0. Let  $r$  be the maximum number of PM activities per period and  $Q$  the set of periods to which we are not allowed to replan PM. Initialize  $Q = \emptyset$ . Define  $\mathcal{S}_{i_1+v}^D, v = 0, 1, \dots, 2\ell - 1$  as the set of components for which we delayed PM to period  $i_1 + v$  and that have not failed in the meantime and  $\mathcal{S}_{i_1+v}^B, v = 0, 1, \dots, \ell - 1$  as the set of components for which we brought forward PM such that we already performed the PM and should not maintain in period  $i_1 + v$ .

Using these sets, we define  $\mathcal{S}_{i_1+v}^P, v = 0, 1, \dots, \ell$  as the sets of components for which we expect to perform PM in period  $i_1 + v$ . That is for  $v = 0, 1, \dots, \ell, \forall s \in \mathcal{S}$ , add  $s$  to  $\mathcal{S}_{i_1+v}^P$  if  $s \notin \mathcal{S}_{i_1+v}^B, i_2^s + v \leq \mathbb{E}(X|X \geq i_2^s), s \notin \mathcal{S}_{i_1+k}^P, k < v$  and either  $i_2^s + v \geq t_{i_1+v}$  or  $s \in \mathcal{S}_{i_1+v}^D$ . Note that we only look at the first planned PM for each component. Here, the expected failure time is

$$\mathbb{E}(X|X \geq i_2^s) = \sum_{j_2=0}^M j_2 P(X = j_2 + 1 | X \geq i_2^s) = \sum_{j_2=0}^{i_2^s-1} j_2 + \sum_{j_2=i_2^s}^{M-1} j_2 \frac{P(X \leq j_2+1) - P(X \leq j_2)}{1 - P(X \leq i_2^s)} + M \frac{1 - P(X \leq M)}{1 - P(X \leq i_2^s)}$$

Step 1. Find the smallest  $v = 0, 1, \dots, \ell$  for which  $|\mathcal{S}_{i_1+v}^P| > r$  and add period  $i_1 + v$  to  $Q$ . Define the set of available periods to which we can move PM activities  $\mathcal{I}_1^A = \{i_1, i_1 + 1, \dots, i_1 + v + \ell\} \setminus Q$ .

For each component  $s \in \mathcal{S}_{i_1+v}^P$ , compute the expected additional cost of replanning PM, that is  $\delta^s((i_1, i_2^s), v)$ , using (10). Sort the components in list  $\mathcal{S}_{i_1+v}^P$  in non-decreasing order of their values  $\delta^s((i_1, i_2^s), v)$  and go to Step 2. If such a  $v$  does not exist, go to Step 3.

Step 2. Subtract component  $s$  from the top of list  $\mathcal{S}_{i_1+v}^P$ . Update  $\mathcal{S}_{i_1+v}^P = \mathcal{S}_{i_1+v}^P \setminus \{s\}$  and  $\mathcal{S}_{i_1}^{s*} = \mathcal{S}_{i_1}^P \cup \{s\}$ , with  $i_1^{s*}$  from equation (9). Repeat this  $|\mathcal{S}_{i_1+v}^P| - r$  times. Go back to Step 1.

Step 3. STOP. There are no periods for which the number of planned PM activities exceeds the maximum number  $r$  anymore. In the current period  $i_1$ , we will maintain the components  $\mathcal{S}_{i_1}^P$ .

---

This heuristic is ought to be used at the start of each period. Note that the algorithm terminates after a finite number of steps. Each time we execute steps 1 and 2 we select a period that is not yet in the list  $Q$  and we add that period to  $Q$ . We ensure that the number of planned PM activities for that period does not exceed  $r$  anymore and once

a period is in  $Q$  we do not add any new PMs to that period, which means the algorithm does not cycle. Since the number of periods is finite, this procedure must terminate.

Something to bear in mind is the fact that the order in which we visit the different periods will most likely influence the obtained planning. In this paper, we stick to starting in the period closest to the current period for which the number of planned PM exceeds the threshold. The reason being that there exist  $\ell!$  possible orders in which to visit all periods which means it is very time-consuming to consider all possibilities. However, this might not be the optimal order.

In case of p-BRP and if we assume that the lifetimes of all components follow the same distribution, planning maintenance under restricted manpower is easier, as we know how many components we will maintain in each period. In case the distribution of the lifetime varies over different components, creating a planning following the heuristic described below is way more difficult, but in this case the *DMR* heuristic could be used. To determine the maintenance policy under limited manpower  $R^* = (R^{1*}, R^{2*}, \dots, R^{|S|^*})$  for a set  $S$  of identical components, we propose the following heuristic:

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*SMR-BRP (Static Maintenance Replanning - Block Replacement Policy)*

---

Step 0. Let  $S'$  be a copy of the component set,  $r$  the maximum number of PM per period and  $Q$  the periods for which the number of planned PM has reached the maximum  $r$ . Initialize  $Q = \emptyset$ .

Step 1. Solve the p-BRP model for one component. This gives the optimal policy  $R$ . Set  $R^{s*} = R, \forall s \in S$ .

Step 2. Look for all periods  $j_1 \in \mathcal{I}_1$  for which the number of planned PM activities  $\sum_{s \in S'} R^{s*}(j_1, 1)^3$  exceeds  $r$ . If no such period exists, go to Step 4. Otherwise, add all such  $j_1$  to the set  $Q$ . To obtain the suboptimal policy  $R'$  now solve the p-BRP model with the additional constraints

$$\text{Sort the components in } S' \text{ in non-increasing order of values } \begin{matrix} y_{i_1} = 0 \\ \forall i_1 \in Q \end{matrix} g^s(R') - g^s(R^{s*}). \quad (11)$$

Step 3. Randomly subtract  $r$  components from  $S'$ . These components will be maintained in period  $j_1$ . For all other components  $s \in S'$ , let  $R^{s*} = R'$ . Go back to Step 2.

Step 4. STOP. There are no periods for which the number of planned PM exceeds the maximum  $r$  anymore. The maintenance policy for the set of components  $S$  is  $R^* = (R^{1*}, R^{2*}, \dots, R^{|S|^*})$ . Note that because we try to minimize the long-term average costs per period, the time intervals between the period for which we start applying the policy and the first maintenance activities might not be optimal, but this is needed to ensure we have the optimal cycle in the long-run.

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We can once more evaluate the performances of our heuristics by means of simulation. We will compare the costs when using *DMR* to the costs when using the *DMD* heuristic presented in [Section 4.2.1](#). Furthermore, we analyse the difference in costs compared to the costs if we do not restrict the manpower in any period. In addition, we will once more compare the performances to a naive heuristic that is described in [Appendix B](#). Finally, for two components, we can examine the difference in cost in comparison with the optimal solution. The modified Markov chain and p-ARP and p-BRP models for two identical components can be found in [Appendix D](#).

## 5 Results

In this section, we first report on the replication of the results from Schouten et al. (2022). After that we show how our *DMD*, *DMR* and *SMR* heuristics for replanning PM perform using simulation. Note that in this section  $\Delta$  denotes how time-varying the costs are, which is different from the function for the additional costs of deviating from the optimal policy  $R^s$  by taking action  $a$ , which we denote by  $\Delta^s(i, a, R^s)$ . We used the solver CPLEX 20.1.0 in Java on an i7-8750H CPU 2.21 GHz 16GB RAM laptop with an Intel(R) UHD Graphics 630 graphics card for our computations.

<sup>3</sup>Note that p-BRP is defined in such a way that  $R^{s*}(j_1, 1) = R^{s*}(j_1, 2) = \dots = R^{s*}(j_1, M), \forall j_1 \in \mathcal{I}_1$ .

## 5.1 Single-component maintenance

For the single-component case, we replicated the results that were reported by Schouten et al. (2022). In general, the steps the authors took in writing down and implementing their models are clear and well-written. However, we did come across a few minor issues that we will now shortly discuss.

In Section 3.1 of the study by Schouten et al. (2022), it reads “where  $p_{i_2} = P(X = i_2 | X \geq i_2)$  indicates the failure probability at age  $i_2$ ”, but in order to obtain the same results we changed this to  $P(X = i_2 + 1 | X \geq i_2)$ . Alternatively, as we would propose, the transition probabilities could be altered so that  $p_{i_2}$  becomes  $p_{i_2+1} = P(X = i_2 + 1 | X \geq i_2)$ . Furthermore, as explained in Section 4.1, the MILP model for the p-MBRP missed the two sets of constraints (1)a and (1)b. Another small issue with regards to the models is that constraints (10d) from their research should only hold when the age of the component is not the maximum  $M$ , as PM should always be allowed in this case. Finally, it is not mentioned what values for  $m$  were used for the results in Table 4 of their article.

After these small adjustments, we were able to exactly reproduce Tables 2 and 3, for which our code runs in a little less than 5 minutes. For example when assuming a Weibull distribution with parameters  $\alpha = 1$  and  $\beta = 2$  for the lifetime of the component and using a constant-cost function ( $\Delta = 0\%$ ), we also get the optimal yearly costs 40 098, 41 501 and 40 311 for p-ARP, p-BRP and p-MBRP respectively, with a critical maintenance age of 6 months in each period for p-ARP, an optimal block of 6 months for p-BRP and p-MBRP and a corresponding critical age of 4 months for p-MBRP. We also obtain the same yearly costs of not maintaining at all, namely 53 855. Also for other values of  $\Delta$  and for  $\alpha = 3$  we obtain the same results as reported by Schouten et al. (2022). An exception is that for p-BRP, when  $\alpha = 3$ ,  $\beta = 2$  and  $\Delta = 10\%$ , we get an optimal solution in which we maintain in periods 9 and 30 instead of 6 and 21. However, note that period 9 is September and period 30 is June, while period 6 is June and period 21 is September. In the end, these are therefore two optimal solutions that come down to the exact same policy. The only real difference that we found concerns Figure 1 in which the critical maintenance ages for p-ARP are displayed when  $\alpha = 1$ . The critical maintenance age in November that we find to be optimal for  $\Delta = 40\%$  is 4 instead of  $M$ .

We did not replicate Table 4 from Schouten et al. (2022), but since these are obtained by only changing the parameter values, without changing anything about the model, we would also be able to replicate these results, as we found the first two tables of results to be correct.

Finally, Schouten et al. (2022) applied their models to a Gearbox example. Once more, this is no more than a change of parameter values. We therefore only discuss the calculation of the cost functions prior to the actual implementation of the models. The computations are pretty straight-forward. The costs of PM, which takes 10 days, in thousand euros is equal to the estimated manpower and material costs in thousand euros plus the function of the downtime costs in euros per day times 10 divided by 1000. The same holds for CM, but this takes 40 days such that everything is multiplied by 4. Nevertheless, we think there is a small mistake in formulas (15) and (16), as we feel that 12.9 should be 12.95 and 51.6 should be  $12.95 \cdot 4 = 51.8$ . We would like to make one final remark about the Gearbox example. The reported costs when PM can be planned on the days of the week with the lowest wind speed seem somewhat unrealistic. As maintenance takes 10 days, you always have the downtime costs of one full week and therefore never only the minimum over the week.

## 5.2 Multi-component maintenance

In the remainder this section, we will present all results for bringing forward and delaying PM in case of 80 identical components. Here, we assume that the lifetime of each component is independent and follows a Weibull distribution. We express the time in months, we always use a value for  $m$  that is equal to  $\alpha$  and for the maximum component age  $M$  we use the rounded up 99.9th percentile of the lifetime distribution. All heuristics are tested using a batch of 10 simulation runs, in which we apply the heuristic for 50.000 periods. We decided to simulate many periods as we are interested in the long-term behavior and we ran a batch to be able to make statements about the significance of

results. We measure this using t-tests, which we chose over sign tests as we wanted to incorporate the magnitude of differences rather than just the sign. Finally, we did not make use of a warm-up period to remove the effect of the initial conditions as the number of periods is quite large and we are not interested in the outputs for a single simulation run, but rather in how the outcomes differ amongst policies and heuristics. To make sure the initial conditions are always the same, we started in the first month of the year and let each component have age one month. In addition, we used the same random numbers for the same purposes: for each combination of parameter values and heuristic we drew the exact same sequence of failure times for each component for the same simulation run.

### 5.2.1 Multi-component maintenance for p-ARP

We start with our heuristics for the p-ARP model. All results for  $\alpha = 1$  year and  $\alpha = 5$  years can be found in [Appendix E.1](#) for delaying and in [Appendix E.3](#) for replanning. Here, one can also find more information on the average lifecycle of the components for different  $\alpha$ ,  $\Delta$  and  $r$ .

Applying the *DMD* heuristic described in [Section 4.2.1](#) for p-ARP for  $\alpha = 3$  years results in very high additional costs (see the results in [Appendix E.1](#) in [Table 21](#) and [Table 22](#)). After an analysis of the cause we realized this is because in the optimal solution for one component we maintain in only one month per year for  $\Delta \geq 30\%$ . This means that in the optimal solution without restricted manpower, we would like to maintain almost all 80 components (on average about 72) in July each year and delaying maintenance the way described in [Section 4.2.1](#) (not maintaining in the current period and after that follow the optimal policy) results in a delay of one year. This means we keep delaying and it is not possible to perform all PM activities. We therefore came up with a new procedure for when  $\alpha > 1$  that is very similar to the one lined out for p-BRP, but we now only perform the delayed PM if the component has not failed in the meantime. We perform maintenance in period  $i_1^{s*}$  that is computed conform formula (6), where  $\delta^s(i = (i_1, i_2)) = \min_{x \in \{1, 2, \dots, \ell\}} \{ \Delta^s(i, 0, R^s) - \Delta^s(i, 1, R^s) + \pi_{i(i_1+x \pmod N), i_2+x}^s(x) (0) \Delta^s((i_1+x \pmod N), i_2+x, 1, R^s) \}$ .

**Table 1** Results of delay of PM for p-ARP (*DMD*), with  $\alpha = 3$  years,  $\beta = 2$  and  $\ell = 1$ . Yearly costs, in 1000 €, difference in costs when including restrictions on manpower and number of times we delay PM for the set of 80 components and percentage of times this directly leads to a failure.

$\Delta$	costs				#PM	# delayed PM / % of times delay leads to CM					
	$r = 80$	$r = 60$	$r = 40$	$r = 20$		$r = 80$	$r = 60$	$r = 40$	$r = 20$		
0%	13.516	-0.00%	-0.00%	-0.00%	173 128	7	0%	18	0.6%	37	2.4%
10%	13.240	0.00%	0.00%	0.04%**	185 521	7	0%	21	0.5%	13 918	2.5%
20%	12.713	0.03%	0.05%	0.04%	296 039	17 236	1.9%	47 019	1.9%	78 207	2.0%
30%	11.784	0.17%**	0.46%**	4.03%**	297 566	48 327	1.8%	132 903	1.8%	194 598	3.0%
40%	10.852	0.26%**	0.69%**	8.37%**	301 088	51 725	1.8%	136 010	1.8%	222 541	3.1%
50%	9.905	0.38%**	1.01%**	11.80%**	303 654	54 167	1.7%	138 321	1.8%	224 492	3.1%

Note: the results are averages over 10 runs, each of 50 000 simulated periods, and  $r$  denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

**Table 2** Results of replanning PM for p-ARP (*DMR*) with  $\alpha = 3$  years and  $\beta = 2$ . Yearly costs, in 1000 €, difference in costs when including restrictions on manpower and the difference in comparison with *DMD*. The results are for  $\ell = 3$  but are exactly the same for  $\ell = 5, 7$ .

$\Delta$	Costs				difference compared to delaying maintenance (with $\ell = 1$ )					
	$r = 80$	additional costs			$r = 60$		$r = 40$		$r = 20$	
		$r = 60$	$r = 40$	$r = 20$						
0%	13.516	0.00%	0.00%	-0.00%	0.00%	(0.000)	0.00%	(0.000)	0.00%	(0.000)
10%	13.240	0.00%	0.00%	0.01%	-0.00%	(0.003)	-0.00%	(0.005)	-0.03% <sup>†</sup>	(0.035)
20%	12.713	0.46%**	1.37%**	0.40%**	0.43% <sup>§§</sup>	(0.082)	1.32% <sup>§§</sup>	(0.165)	0.36% <sup>§§</sup>	(0.131)
30%	11.784	0.83%**	2.29%**	3.04%**	0.66% <sup>§§</sup>	(0.028)	1.83% <sup>§§</sup>	(0.019)	-0.96% <sup>††</sup>	(0.073)
40%	10.852	1.28%**	3.39%**	4.56%**	1.02% <sup>§§</sup>	(0.030)	2.68% <sup>§§</sup>	(0.028)	-3.51% <sup>††</sup>	(0.061)
50%	9.905	1.90%**	4.75%**	6.42%**	1.51% <sup>§§</sup>	(0.038)	3.70% <sup>§§</sup>	(0.034)	-4.82% <sup>††</sup>	(0.103)

Note: the results are averages over 10 runs, each of 50 000 simulated periods, and  $r$  denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test,  $*p < 0.05$ ,  $**p < 0.01$ .  
<sup>†</sup> and <sup>††</sup> denote that the *DMR* heuristic performed significantly better than *DMD* heuristic and <sup>§</sup> and <sup>§§</sup> denote that the *DMR* heuristic performed significantly worse, based on a one-sided t-test, <sup>†, §, §§</sup>  $p < 0.05$ , <sup>††, §§</sup>  $p < 0.01$ .

The new results for *DMD* for  $\alpha = 3$  years and  $\ell = 1$  are summarized in [Table 1](#) and those for *DMR* for  $\alpha = 3$  years and  $\ell = 3$  (same as for  $\ell = 5, 7$ ) can be found in [Table 2](#). The additional costs are computed as the relative difference under a restricted number of permitted PM operations per period when applying the heuristic, compared to the costs without this constraint. For more digits, running times and standard deviations see [Table 13](#) in [Appendix E.1](#) and [Table 35](#) in [Appendix E.3](#) respectively. The results for *DMD* with  $\ell = 3, 5$  are similar and can be found in [Tables 14](#) and [15](#).

[Table 1](#) and [Table 2](#) show that the additional costs become larger when the constraint on available manpower is more restrictive and when there are larger cost fluctuations over the year (i.e. larger  $\Delta$ ). Furthermore, in [Table 1](#) it can be seen that the number of times we delay PM increases with  $\Delta$ . This is because all planned maintenance in the optimal policy is concentrated around one month a year when cost fluctuations are large. The number of delays are counted in such a way that each component can be delayed only once in a lifetime. This means that if we reach a period to which we delayed PM and again decide to delay for this component, this was counted as only one delay. Note that we never have significant cost savings, which makes sense as we compare to an optimal policy.

**Table 3** Results of delaying and replanning PM for p-ARP, with  $\alpha = 3$  years and  $\beta = 2$ . Percentage additional costs when including restrictions on manpower with respect to the optimal planning without this constraint, divided into a part due to more/less maintenance activities and a part due to maintenance in more/less expensive periods. For delay we used  $\ell = 1$  and for replan  $\ell = 3$  but for the latter results are exactly the same for  $\ell = 5, 7$ .

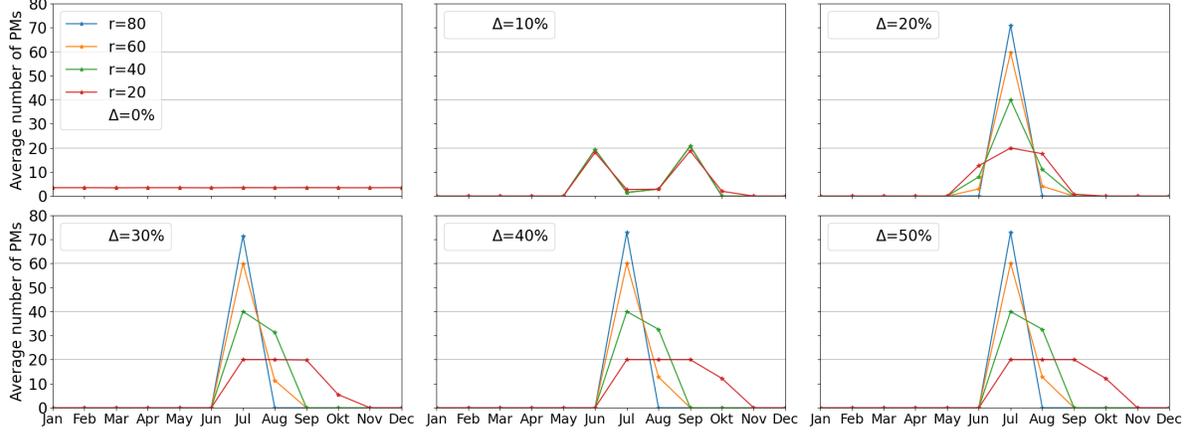
$\Delta$	more or less maintenance / more or less expensive periods					
	<i>DMD</i> (delay)			<i>DMR</i> (replan)		
	$r = 60$	$r = 40$	$r = 20$	$r = 60$	$r = 40$	$r = 20$
0%	-0.00% / 0.00%	-0.00% / 0.00%	-0.00% / 0.00%	0.00% / 0.00%	0.00% / 0.00%	-0.00% / 0.00%
10%	0.00% / 0.00%	0.00% / 0.00%	-0.02%* / 0.06%**	0.00% / -0.00%	0.00% / -0.00%	0.11%** / -0.09%**
20%	-1.06%** / 1.09%**	-2.93%** / 2.98%**	-4.92%** / 4.96%**	-0.10%** / 0.56%**	-0.07%** / 1.44%**	-4.08%** / 4.47%**
30%	-0.03%** / 0.19%**	-0.07%** / 0.53%**	-2.05%** / 6.08%**	-0.09%** / 0.91%**	-0.02%** / 2.32%**	-0.14%** / 3.18%**
40%	-0.07%** / 0.32%**	-0.16%** / 0.85%**	-0.41%** / 8.77%**	-0.04%** / 1.32%**	0.05%** / 3.34%**	-0.21%** / 4.77%**
50%	-0.09%** / 0.47%**	-0.21%** / 1.22%**	-0.57%** / 12.37%**	-0.02%** / 1.91%**	0.11%** / 4.64%**	-0.26%** / 6.68%**

Note: the results are averages over 10 runs, each of 50 000 simulated periods, and  $r$  denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

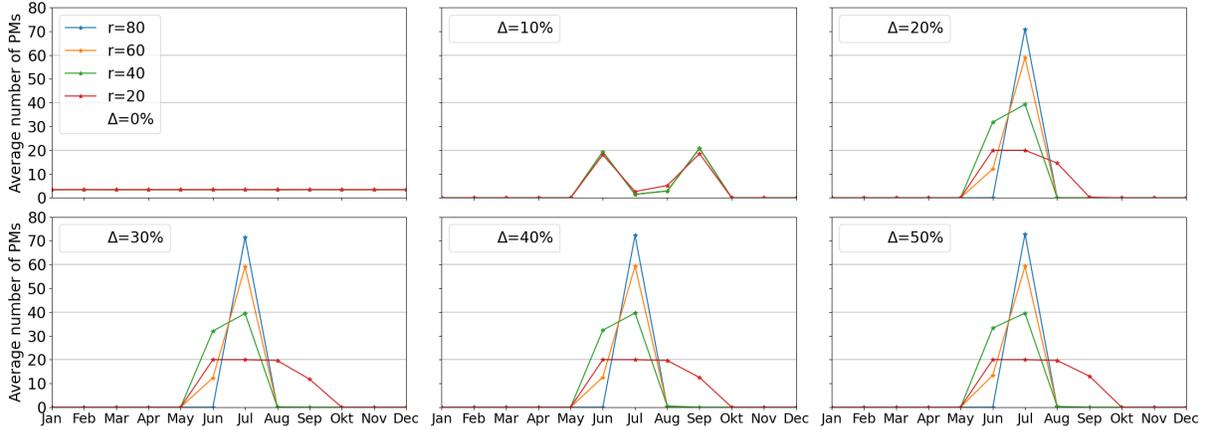
\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

In [Table 3](#) we made a distinction between the additional costs due to the fact that more or less maintenance is needed and the difference due to the fact that we maintain in more or less expensive periods. We measure the first part by taking the obtained maintenance plannings and calculating the difference in costs when using a constant cost function ( $\Delta = 0\%$ ). The second part is then the remainder of the difference. It turns out that on average we maintain less often but in more expensive periods when we delay PM. This result is in line with [Table 1](#) which shows that a component only fails in less than 3% of the times that we delay PM. Delaying thus caused limited additional CM while delaying without failure leads to less maintenance. Furthermore, the increasing extra costs when  $\Delta$  is large are mainly caused by the fact that we maintain in periods with high costs.

[Figure 1](#) and [Figure 2](#) present the average number of PM operations for each period for different combinations of  $\Delta$  and  $r$ , for *DMD* and *DMR* respectively. An important difference that can be derived from these graphs is that we perform maintenance in Oktober for  $r$  equal to 20 when delaying, but not in case of replanning PM. When we replan, we move up these activities such that we already perform them in June. As Oktober is more expensive than June this drives up the costs for *DMD*. This is in agreement with [Table 2](#) and [Table 3](#) where we see that the additional costs (due to PM in more expensive periods) are significantly lower for *DMR* when  $r = 20$  and  $\Delta$  large.



**Figure 1** Average number of PM activities for each period for the set of 80 components when delaying PM (*DMD*), for  $\alpha = 3$  years,  $\beta = 2$  and  $\ell = 1$ , for p-ARP with and without including constraints on the number of allowed PM activities per period  $r$  and for different  $\Delta$ .

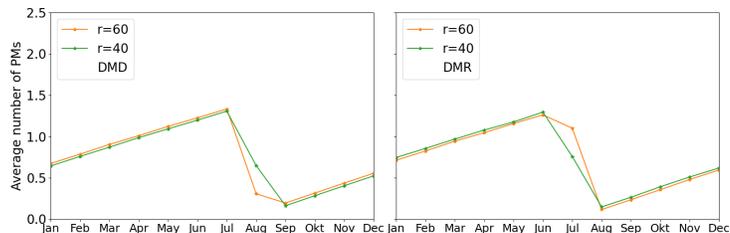


**Figure 2** Average number of PM activities for each period for the set of 80 components when replanning PM (*DMR*), for  $\alpha = 3$  years,  $\beta = 2$  and  $\ell = 3$ , for p-ARP with and without including constraints on the number of allowed PM activities per period  $r$  and for different  $\Delta$ . Results are exactly the same for  $\ell = 5, 7$ .

For  $\Delta = 20\%$  and  $r = 20$ , we see that the costs are lower for *DMD*. [Table 3](#) indicates this is because of relatively high cost savings from less maintenance. As presented by Schouten et al. (2022), the optimal policy for  $\Delta = 20\%$  is to maintain in June if the component is older than 21 months and in July if it is older than 11 months. This means that if we delay PM in July by one month, we skip maintenance in the next year, which results in less PM activities. [Figures 1 and 2](#) show that for  $\Delta = 20\%$  we delay more with *DMD* than when using *DMR*, such that the former results in more savings. Do note however that maintaining only after two years does cause more CM such that we more often maintain in expensive periods, as also evidenced by [Table 3](#).

Furthermore, we observe from [Table 2](#) that *DMD* results in lower costs than *DMR* when both the number of permitted activities per period and  $\Delta$  are large. This is partly caused by the fact that we on average need less PM activities when we delay PM, but mostly caused by the fact that we maintain in more expensive periods. [Figure 2](#) shows that the number of PM activities in the most optimal period (in which we maintain for  $r = 80$ ) is slightly lower than  $r$  when  $r$  is equal to 40 or 60, such that we maintain more in other - more expensive - months than necessary. The reason for this is because we replan PM for the optimal month July at the start of June based on the expected number of PM activities in July. However, the actual number when we reach July is sometimes lower due to failures.

The rest of the difference results from CM in more expensive periods, as manifested by [Figure 3](#), where we illustrate the average number of CM activities per period for  $\Delta = 50\%$  (for other values see [Tables 12](#) and [25](#)). When we are in June and replan for July, given that the component is still operative and we want to replan once, it makes sense to move up the PM to June rather than delay it to August. The reason being that the latter results in an increased risk of two failures before we reach next year July. However, when you repeatedly replan PM because of limited manpower, it turns out to be more optimal to maintain in August as this leads to CM in cheaper periods.



**Figure 3** Average number of CM activities for each period for the set of 80 components, for *DMD* and *DMR*, for p-ARP with  $\alpha = 3$  years,  $\beta = 2$  and  $\Delta = 50\%$ .

**Table 4** Comparison of the *DMD* and *DMR* heuristics to naive heuristics in which we choose randomly which PM activities to replan and where we move up or delay with equal probability when both are possible for replanning, for p-ARP with  $\alpha = 3$  years and  $\beta = 2$ . Percentage additional costs compared to the naive heuristics, for different restrictions on the maximum number of PM activities per period  $r$ . We used  $\ell = 1$  for *DMD* and  $\ell = 3$  for *DMR*.

$\Delta$	<i>DMD</i> (delay)						<i>DMR</i> (replan)					
	$r = 60$		$r = 40$		$r = 20$		$r = 60$		$r = 40$		$r = 20$	
0%	-0.000%	(0.001)	0.000%	(0.001)	0.000%	(0.001)	0.000%	(0.000)	-0.000%	(0.002)	0.000%	(0.001)
10%	-0.001%	(0.004)	0.001%	(0.008)	-0.015%	(0.037)	-0.001%	(0.002)	-0.001%	(0.007)	-0.034% <sup>†</sup>	(0.052)
20%	-0.013%	(0.116)	-0.043%	(0.123)	-0.192% <sup>††</sup>	(0.134)	0.247% <sup>§§</sup>	(0.068)	0.883% <sup>§§</sup>	(0.103)	-0.197% <sup>††</sup>	(0.144)
30%	-0.067% <sup>††</sup>	(0.045)	-0.115% <sup>††</sup>	(0.040)	-0.569% <sup>††</sup>	(0.164)	0.213% <sup>§§</sup>	(0.030)	0.706% <sup>§§</sup>	(0.040)	-0.454% <sup>††</sup>	(0.084)
40%	-0.059% <sup>††</sup>	(0.049)	-0.079% <sup>††</sup>	(0.058)	-0.430% <sup>††</sup>	(0.084)	0.375% <sup>§§</sup>	(0.037)	1.139% <sup>§§</sup>	(0.056)	-0.459% <sup>††</sup>	(0.048)
50%	-0.073% <sup>††</sup>	(0.037)	-0.105% <sup>††</sup>	(0.076)	-0.443% <sup>††</sup>	(0.109)	0.630% <sup>§§</sup>	(0.056)	1.607% <sup>§§</sup>	(0.081)	-0.415% <sup>††</sup>	(0.103)

Note: the results are averages over 10 runs, each of 50 000 simulated periods, and  $r$  denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

<sup>†</sup> and <sup>††</sup> denote that the *DMR* or *DMD* heuristic performed significantly better than the naive heuristics and <sup>§</sup> and <sup>§§</sup> denote that they performed significantly worse, based on a one-sided t-test, <sup>†,§</sup>  $p < 0.05$ , <sup>††,§§</sup>  $p < 0.01$ .

Finally, [Table 4](#) shows the results of a comparison with naive heuristics in which we choose randomly which activities to replan and where we move up or delay with equal probability for replanning. They are explained in more detail in [Appendix B](#). We see the same trends that we already discussed. *DMD* and *DMR* often perform better than the naive heuristics except for *DMR* when  $r$  is 40 or 60. This is again because we bring forward PM when using *DMR* but this leads to more expensive CM.

### 5.2.2 Multi-component maintenance for p-BRP

Using the *DMD* and *SMR* heuristics described in [Section 4.2](#) we can also make feasible maintenance plannings under restricted manpower for p-BRP. The results for  $\alpha = 3$  years and  $\beta = 2$  are presented in [Tables 5](#) and [6](#). More digits, running times and standard deviations are presented in [Tables 27](#) and [39](#). We find that the additional costs again increase with the value of  $\Delta$ . Results for  $\alpha = 1$  and  $\alpha = 5$  years can be found in [Appendix E.2](#) for delaying and in [Appendix E.4](#) for replanning. Here, one can also find more information on the average lifecycle of the components for several  $\alpha$ ,  $\Delta$  and  $r$ .

**Table 5** Results of delay of PM (*DMD*) for a set of 80 components for p-BRP, with  $\alpha = 3$  years,  $\beta = 2$  and  $\ell = 1$ . Yearly costs, in 1000 €, difference in costs when including restrictions on manpower and number of times we delay PM per simulation run and percentage of times this directly leads to a failure.

$\Delta$	costs				#PM	# delayed PM / % of times delay leads to CM						
	$r = 80$	additional costs				$r = 80$	$r = 60$		$r = 40$		$r = 20$	
		$r = 60$	$r = 40$	$r = 20$								
0%	14.158	-0.69%**	-0.76%**	-2.00%**	217 132	50 483	1.3%	106 126	2.0%	161 885	3.9%	
10%	13.807	-0.50%**	-0.49%**	-0.62%**	217 198	50 571	1.2%	106 205	2.0%	161 935	3.8%	
20%	13.133	-0.17%**	-0.10%**	3.09%**	327 846	77 909	1.4%	161 365	1.6%	245 132	3.0%	
30%	12.111	-0.03%**	0.18%**	5.94%**	327 846	77 909	1.4%	161 365	1.6%	245 132	3.0%	
40%	11.090	0.14%**	0.52%**	9.32%**	327 846	77 909	1.4%	161 365	1.6%	245 132	3.0%	
50%	10.068	0.34%**	0.93%**	13.39%**	327 846	77 909	1.4%	161 365	1.6%	245 132	3.0%	

Note: the results are averages over 10 runs, each of 50 000 simulated periods, and  $r$  denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test,  $*p < 0.05$ ,  $**p < 0.01$ .

A result that seems striking at first glance is the fact that we have significant savings for *DMD* when the costs are not largely time-varying. This however makes perfect sense when taking into account that an optimal p-BRP is not an overall optimal maintenance planning. p-BRP is defined such that we maintain each component in certain fixed periods, regardless of the age of the component. It is therefore reasonable that we can save costs by delaying maintenance for the components with the smallest ages. Using *DMD* we thus transform the policy into a mix of p-BRP and p-ARP.

Nevertheless, the additional costs when applying *DMD* turn out to be positive when the cost differences among periods are large. The extra costs of maintaining in more expensive periods exceed the savings of less maintenance in this case. This effect is bigger when  $r$  is small because in this case we have to delay PM for some components by more than one period, such that we end up in even more expensive periods, as can be seen in [Figure 4](#) which shows the number of PM in each period for *DMD*. Note that these explanations are also in line with [Table 7](#), which displays the distinction between differences in how often we maintain and differences in the cost when we maintain.

**Table 6** Results of replanning PM for p-BRP (*SMR*) with  $\alpha = 3$  years and  $\beta = 2$ . Yearly costs, in 1000 €, difference in costs when including restrictions on manpower and the difference in comparison with *DMD*.

$\Delta$	Costs				difference compared to delaying maintenance (with $\ell = 1$ )		
	$r = 80$	additional costs			$r = 60$	$r = 40$	$r = 20$
	$r = 60$	$r = 40$	$r = 20$				
0%	14.158	-0.00%	-0.01%	-0.01%	0.69% <sup>§§</sup> (0.030)	0.76% <sup>§§</sup> (0.039)	2.02% <sup>§§</sup> (0.067)
10%	13.807	0.00%	0.00%	0.10% <sup>**</sup>	0.51% <sup>§§</sup> (0.042)	0.49% <sup>§§</sup> (0.056)	0.72% <sup>§§</sup> (0.064)
20%	13.133	0.21% <sup>**</sup>	0.43% <sup>**</sup>	0.98% <sup>**</sup>	0.38% <sup>§§</sup> (0.023)	0.53% <sup>§§</sup> (0.022)	-2.04% <sup>††</sup> (0.146)
30%	12.111	0.34% <sup>**</sup>	0.69% <sup>**</sup>	3.97% <sup>**</sup>	0.37% <sup>§§</sup> (0.027)	0.51% <sup>§§</sup> (0.023)	-1.86% <sup>††</sup> (0.052)
40%	11.090	0.50% <sup>**</sup>	1.01% <sup>**</sup>	5.79% <sup>**</sup>	0.36% <sup>§§</sup> (0.032)	0.49% <sup>§§</sup> (0.026)	-3.23% <sup>††</sup> (0.060)
50%	10.068	0.69% <sup>**</sup>	1.39% <sup>**</sup>	7.97% <sup>**</sup>	0.35% <sup>§§</sup> (0.039)	0.46% <sup>§§</sup> (0.031)	-4.78% <sup>††</sup> (0.072)

Note: the results are averages over 10 runs, each of 50 000 simulated periods, and  $r$  denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test,  $*p < 0.05$ ,  $**p < 0.01$ .  
 $\dagger$  and  $\dagger\dagger$  denote that the *DMR* heuristic performed significantly better than *DMD* heuristic and  $\S$  and  $\S\S$  denote that the *DMR* heuristic performed significantly worse, based on a one-sided t-test,  $\dagger, \S p < 0.05$ ,  $\dagger\dagger, \S\S p < 0.01$ .

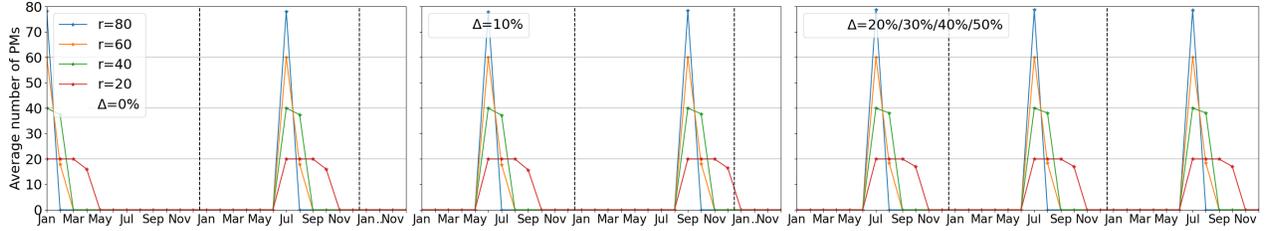
**Table 7** Results of delaying and replanning PM for p-BRP, with  $\alpha = 3$  years,  $\beta = 2$  and  $\ell = 1$  for delay. Percentage additional costs when including restrictions on manpower with respect to the optimal planning without this constraint, divided into a part due to more/less maintenance and a part due to maintenance in more/less expensive periods.

$\Delta$	more or less maintenance / more or less expensive periods					
	<i>DMD</i> (delay)			<i>SMR</i> (replan)		
	$r = 60$	$r = 40$	$r = 20$	$r = 60$	$r = 40$	$r = 20$
0%	-0.69% <sup>**</sup> / 0.00%	-0.76% <sup>**</sup> / 0.00%	-1.99% <sup>**</sup> / 0.00%	-0.00% / 0.00%	-0.01% / 0.00%	-0.01% / 0.00%
10%	-0.68% <sup>**</sup> / 0.18% <sup>**</sup>	-0.81% <sup>**</sup> / 0.32% <sup>**</sup>	-2.12% <sup>**</sup> / 1.50% <sup>**</sup>	0.00%* / 0.00%	0.00% / 0.00%	0.00% / 0.10% <sup>**</sup>
20%	-0.45% <sup>**</sup> / 0.28% <sup>**</sup>	-0.65% <sup>**</sup> / 0.54% <sup>**</sup>	-1.70% <sup>**</sup> / 4.78% <sup>**</sup>	0.00% / 0.21% <sup>**</sup>	0.00% / 0.43% <sup>**</sup>	-3.33% <sup>**</sup> / 4.32% <sup>**</sup>
30%	-0.49% <sup>**</sup> / 0.46% <sup>**</sup>	-0.70% <sup>**</sup> / 0.88% <sup>**</sup>	-1.84% <sup>**</sup> / 7.78% <sup>**</sup>	0.00% / 0.34% <sup>**</sup>	0.00% / 0.69% <sup>**</sup>	-0.00% <sup>**</sup> / 3.97% <sup>**</sup>
40%	-0.54% <sup>**</sup> / 0.67% <sup>**</sup>	-0.77% <sup>**</sup> / 1.29% <sup>**</sup>	-2.01% <sup>**</sup> / 11.33% <sup>**</sup>	0.00% / 0.50% <sup>**</sup>	0.00% / 1.01% <sup>**</sup>	-0.00% <sup>**</sup> / 5.79% <sup>**</sup>
50%	-0.59% <sup>**</sup> / 0.93% <sup>**</sup>	-0.84% <sup>**</sup> / 1.77% <sup>**</sup>	-2.22% <sup>**</sup> / 15.60% <sup>**</sup>	0.00% / 0.69% <sup>**</sup>	0.00% / 1.39% <sup>**</sup>	-0.00% <sup>**</sup> / 7.97% <sup>**</sup>

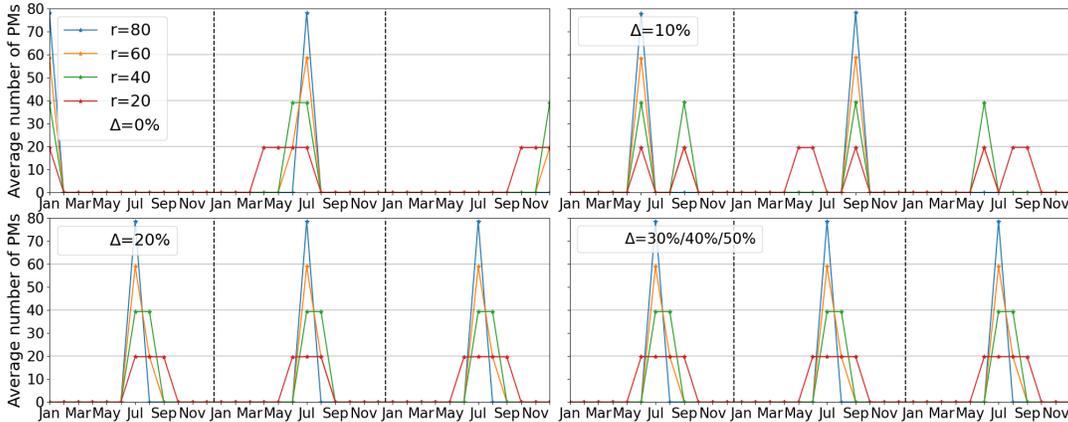
Note: the results are averages over 10 runs, each of 50 000 simulated periods, and  $r$  denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test,  $*p < 0.05$ ,  $**p < 0.01$ .

Logically, we do not see cost savings for replanning PM, as our *SMR* heuristic uses a combination of different p-BRP policies and we compare the corresponding costs to the optimal p-BRP policy. From there, it also follows that *DMD* yields significantly lower costs than *SMR* for small values of  $\Delta$  and large values of  $r$  (Table 6). Nonetheless, for large monthly deviations of the maintenance costs and very restrictive conditions on available manpower *SMR* performs better than *DMD*.



**Figure 4** Average number of PM activities for each period for the set of 80 components when delaying PM (*DMD*), for  $\alpha = 3$  years,  $\beta = 2$  and  $\ell = 1$ , for p-BRP with and without including constraints on the number of allowed PM activities per period  $r$  and for different  $\Delta$ . The vertical lines separate the years.



**Figure 5** Average number of PM activities for each period for the set of 80 components when replanning PM (*SMR*), for  $\alpha = 3$  years and  $\beta = 2$ , for p-BRP with and without including constraints on the number of allowed PM activities per period  $r$  and for different  $\Delta$ . The vertical lines separate the years.

When comparing [Figure 5](#) (PM per period for *SMR*) with [Figure 4](#) (for *DMD*) we see that the differences are the largest for  $\Delta = 10\%$ . When we compute an entire new planning for some components, we maintain several of them in the next year during the summer rather than during the last three months of the year. For other values of  $\Delta$  the policies resulting from *DMD* and *SMR* are quite similar, besides the fact that we move up PM activities for some components by a few months for *SMR* instead of delaying them for *DMD*.

**Table 8** Comparison of the *DMD* and *SMR* heuristics to naive heuristics in which we choose randomly which PM activities to replan, for p-BRP with  $\alpha = 3$  years and  $\beta = 2$ . Percentage additional costs compared to the naive heuristics, for different restrictions on the maximum number of PM activities per period  $r$ . We used  $\ell = 1$  for *DMD*.

$\Delta$	<i>DMD</i> (delay)			<i>SMR</i> (replan)		
	$r = 60$	$r = 40$	$r = 20$	$r = 60$	$r = 40$	$r = 20$
0%	-0.263% <sup>††</sup> (0.041)	-0.189% <sup>††</sup> (0.053)	-0.656% <sup>††</sup> (0.071)	-0.316% <sup>††</sup> (0.198)	-0.314% <sup>††</sup> (0.198)	-0.310% <sup>††</sup> (0.198)
10%	-0.239% <sup>††</sup> (0.039)	-0.186% <sup>††</sup> (0.061)	-0.656% <sup>††</sup> (0.087)	-0.462% <sup>††</sup> (0.229)	-0.601% <sup>††</sup> (0.251)	-0.554% <sup>††</sup> (0.252)
20%	-0.231% <sup>††</sup> (0.039)	-0.178% <sup>††</sup> (0.059)	-0.652% <sup>††</sup> (0.087)	-0.635% <sup>††</sup> (0.270)	-0.932% <sup>††</sup> (0.319)	-0.827% <sup>††</sup> (0.340)
30%	-0.222% <sup>††</sup> (0.039)	-0.170% <sup>††</sup> (0.058)	-0.648% <sup>††</sup> (0.090)	-0.836% <sup>††</sup> (0.321)	-1.314% <sup>††</sup> (0.401)	-1.135% <sup>††</sup> (0.454)
40%	-0.212% <sup>††</sup> (0.041)	-0.160% <sup>††</sup> (0.057)	-0.644% <sup>††</sup> (0.095)	-1.072% <sup>††</sup> (0.384)	-1.760% <sup>††</sup> (0.500)	-1.485% <sup>††</sup> (0.5918)
50%	-0.200% <sup>††</sup> (0.045)	-0.148% <sup>††</sup> (0.057)	-0.639% <sup>††</sup> (0.103)	-1.354% <sup>††</sup> (0.461)	-2.287% <sup>††</sup> (0.618)	-1.889% <sup>††</sup> (0.752)

Note: the results are averages over 10 runs, each of 50 000 simulated periods, and  $r$  denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

<sup>†</sup> and <sup>††</sup> denote that the *DMR* or *DMD* heuristic performed significantly better than the naive heuristics and <sup>§</sup>, based on a one-sided t-test, <sup>†</sup> $p < 0.05$ , <sup>††</sup> $p < 0.01$ .

Finally, let us analyse how the heuristics perform compared to the naive heuristics described in [Appendix B](#), where we randomly choose which PM to replan and we delay and move up with equal probability. The outcomes in [Table 8](#) show that *DMD* and *SMR* always perform significantly better.

### 5.2.3 Analysis performance heuristics for two components

As a final analysis of the performances of the heuristics that are presented in this paper, we compare the average monthly costs of the heuristics with the monthly costs for the optimal solution for two components. See the results in [Table 9](#). Note that the heuristics perform very well in case of p-ARP for small values of  $\Delta$  but entail significantly larger costs than optimal for large  $\Delta$ . For p-BRP, we see significantly lower costs for *DMD*, which is, as explained before, caused by the fact that we turn the policy into a combination of p-BRP and p-ARP which is more optimal. The *SMR*, which is a pure p-BRP, performs very well and never gives significantly larger costs.

**Table 9** Percentage difference in costs of delaying and replanning PM compared to the optimal solution for two components, for  $\alpha = 3$  years and  $\beta = 2$ . For *DMD* we used  $\ell = 1$  and for *DMR*  $\ell = 3$ .

$\Delta$	p-ARP				p-BRP			
	<i>DMD</i> (delay)		<i>DMR</i> (replan)		<i>DMD</i> (delay)		<i>SMR</i> (replan)	
0%	0.15%	(0.3434)	0.15%	(0.3673)	-0.655% <sup>††</sup>	(0.2618)	-0.000%	(0.0387)
10%	-0.13%	(0.6191)	-0.24%	(0.6755)	-0.417% <sup>††</sup>	(0.4286)	0.000%	(0.0000)
20%	-0.15%	(0.5717)	1.43% <sup>§§</sup>	(0.1982)	-0.557% <sup>††</sup>	(0.3075)	0.014%	(0.0690)
30%	0.24% <sup>§§</sup>	(0.2222)	2.18% <sup>§§</sup>	(0.1857)	-0.567% <sup>††</sup>	(0.2849)	0.000%	(0.0000)
40%	0.47% <sup>§§</sup>	(0.2104)	3.42% <sup>§§</sup>	(0.2198)	-0.532% <sup>††</sup>	(0.3599)	0.032%	(0.1621)
50%	1.06% <sup>§§</sup>	(0.2031)	4.88% <sup>§§</sup>	(0.2330)	-0.559% <sup>††</sup>	(0.3196)	0.000%	(0.0000)

Note: the results are averages over 10 runs, each of 50 000 simulated periods, brackets denote standard deviations. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

<sup>†</sup> and <sup>††</sup> denote that the average costs of the heuristic solutions are significantly smaller than the optimal costs and <sup>§</sup> and <sup>§§</sup> denote that the costs of the heuristics are significantly larger, based on a one-sided t-test, <sup>†, §</sup>  $p < 0.05$ , <sup>††, §§</sup>  $p < 0.01$ .

## 6 Discussion and conclusion

In this paper, we analysed the period-dependent versions of the ARP, BRP and MBRP maintenance policies for a single off-shore wind turbine component and we formulated and investigated three heuristic solutions to construct a least-cost maintenance program for a group of components under restricted manpower. We first presented the *DMD* (Dynamic Maintenance Delay) heuristic to delay preventive maintenance (PM) for p-ARP and p-BRP. The idea behind this is that we delay preventive maintenance (PM) for those components for which this is the least costly. After that, we formulated the *DMR* (Dynamic Maintenance Replan) heuristic for p-ARP, for which the idea is similar to *DMD* but is used to also move up maintenance in addition to delaying. Finally, we introduced *SMR* (Static Maintenance Replan) for p-BRP, where we use a combination of multiple suboptimal maintenance programs to create a feasible maintenance planning for multiple identical components. We tested these using simulation for 80 identical components.

Our main findings are that *DMR* in general performs better than *DMD* when the number of permitted PM operations per period is small. This is because anticipating on periods with a large number of planned maintenance results in PM in less expensive periods. When the restriction on manpower is weak however, we see that *DMR* does not work as well and *DMD* performs better. This may be caused by the fact that delaying PM instead of moving up by one period results in component failure in less expensive periods. Furthermore, when maintenance costs are not largely

time-varying, differences between the two heuristics are small. As the *DMR* is more heuristic, more complicated and more time-consuming, we would consider *DMD* to be the better option in this case.

For p-BRP we saw that applying *DMD* can result in cost savings, as this heuristic turns the policy into a combination of p-BRP and p-ARP. We delay PM for components that are not likely to fail yet and the number of maintenance operations therefore decreases. Nevertheless, the additional costs of maintaining in more expensive periods exceed these savings in case of large cost variations over the months and few available technicians. We can then use *SMR* for a better supoptimal planning.

Finally, to all heuristics applies that the costs in addition to the costs without the constraint on manpower clearly increase when the costs become more largely time-varying and when the restriction on manpower becomes more restrictive.

A limitation of this study lies in the measures on which we base our decisions of replanning maintenance. To evaluate the expected additional cost of PM in a different period, we compute the difference in the long-term total expected costs when diverging from the optimal policy once. However, we do not stick to the optimal policy after this deviation, such that this estimated cost is not the actual cost. The actual cost depends on future decisions and hence on the state of and actions performed on other components. The heuristics could therefore be further improved by finding a more accurate computation of the difference in costs of replanning maintenance, that incorporates future effects of decisions at the present time.

Moreover, we did not take into account the fact that there are periods of the year where it is impossible to perform maintenance on off-shore wind turbines due to severe weather conditions. The consequences of available weather windows could be accounted for by incorporating advance and delay of corrective maintenance in addition to preventive maintenance in the heuristics that we presented and allowing for the maximum number of activities per period to vary over the year.

Another interesting extension to our models would be to make the failure rates of the components dependent on the part of the year. Next to an effect on the maintenance costs, it is not unthinkable that the weather conditions also influence the probability of failure for an off-shore wind turbine.

What would also be a valuable follow-up study is to analyse the results of our heuristics for the multi-component case when different components follow different (types of) distributions. In addition, a modified *SMR-BRP* heuristic could be presented to be able to handle this.

Finally, we disregarded any positive economic, structural or stochastic dependence in this paper. This is not realistic in most cases. Dekker et al. (1991) for example have shown that combining maintenance can reduce costs. Ideally, these two forms of dependence that are both present in reality would be combined. For example, clustering of maintenance operations when systems are geographically close could be considered in deciding for which components to replan PM.

## References

- [1] Gabriel R. Bitran and Horacio H. Yanasse. “Computational Complexity of the Capacitated Lot Size Problem”. In: *Management Science* 28.10 (1982), pp. 1174–1186. URL: <http://www.jstor.org/stable/2630946>.

- [2] Rommert Dekker, Adriaan Smit, and Jeroen Losekoot. “Combining maintenance activities in an operational planning phase: a set-partitioning approach”. In: *IMA Journal of Management Mathematics* 3.4 (1991), pp. 315–331.
- [3] Phuc Do Van, Anne Barros, Christophe Bérenguer, Keomany Bouvard, and Florent Brissaud. “Dynamic grouping maintenance with time limited opportunities”. In: *Reliability Engineering System Safety* 120 (2013), pp. 51–59. URL: <https://www.sciencedirect.com/science/article/pii/S0951832013000926>.
- [4] European Wind Energy Technology Platform. *Workers wanted: The EU wind energy sector skills gap*. [https://www.windplatform.eu/fileadmin/ewetp\\_docs/Documents/reports/Workers\\_Wanted\\_TPwind.pdf](https://www.windplatform.eu/fileadmin/ewetp_docs/Documents/reports/Workers_Wanted_TPwind.pdf). 2013.
- [5] Biju George, Jonathan Loo, and Wei Jie. “Recent advances and future trends on maintenance strategies and optimisation solution techniques for offshore sector”. In: *Ocean Engineering* 250 (2022), p. 110986. URL: <https://www.sciencedirect.com/science/article/pii/S0029801822004127>.
- [6] Thijs Nicolaas Schouten, Rommert Dekker, and Ayse Sena Eruguz. “Optimal maintenance policies for wind turbines under time-varying costs”. In: (2019). URL: <https://thesis.eur.nl/pub/47321/>.
- [7] Thijs Nicolaas Schouten, Rommert Dekker, Mustafa Hekimoğlu, and Ayse Sena Eruguz. “Maintenance optimization for a single wind turbine component under time-varying costs”. In: *European Journal of Operational Research* 300.3 (2022), pp. 979–991. URL: <https://www.sciencedirect.com/science/article/pii/S0377221721007633>.
- [8] Henk C Tijms. *A first course in stochastic models*. John Wiley and sons, 2003.
- [9] Wind Europe. *Wind energy – powering the European Green Deal*. <https://windeurope.org/about-wind/campaigns/european-green-deal/>.
- [10] Wind Europe. *Wind energy in Europe in 2020 - Statistics and the outlook for 2021-2025*. <https://windeurope.org/intelligence-platform/product/wind-energy-in-europe-2020-statistics-and-the-outlook-for-2021-2025/>. 2020.
- [11] Shuya Zhong, Athanasios A. Pantelous, Mark Goh, and Jian Zhou. “A reliability-and-cost-based fuzzy approach to optimize preventive maintenance scheduling for offshore wind farms”. In: *Mechanical Systems and Signal Processing* 124 (2019), pp. 643–663. URL: <https://www.sciencedirect.com/science/article/pii/S0888327019300950>.

# Appendix

## A Notation

**Table 10** Sets, parameters and variables of the maintenance planning problem

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Sets	
$\mathbb{N}^+ = \{1, 2, \dots\}$	Set of positive integers
$\mathcal{I}_1$	Set of periods
$\mathcal{I}_2$	Set of component ages
$\mathcal{I} = \mathcal{I}_1 \times \mathcal{I}_2$	State space of the Markov decision process for one component
$\mathcal{A}(i_1, i_2)$	Set of possible actions in the Markov decision process in state $(i_1, i_2)$
Parameters	
$\alpha$	Scale parameter of the Weibull distribution of the components' lifetime
$\beta$	Shape parameter of the Weibull distribution of the components' lifetime
$c_p(i_1)$	PM cost in period $i_1 \in \mathcal{I}_1$
$c_f(i_1)$	CM cost in period $i_1 \in \mathcal{I}_1$
$\bar{c}_p$	Yearly average PM cost
$\bar{c}_f$	Yearly average CM cost
$N$	Number of periods in a year
$m$	Number of years in a p-BRP cycle
$M$	A large number representing the maximum age of a component
$r$	The maximum number of allowed PM activities per period
$\ell$	The maximum number of periods that we move up or delay a PM activity
(Functions of) random variables	
$X$	Random lifetime of a component
$\mathbb{E}(X)$	Average lifetime of a component
$p_x = P(X = x + 1   X \geq x)$ $= \frac{P(X \leq x+1) - P(X \leq x)}{1 - P(X \leq x)}$	Discretized failure rate of a component at age $x$ , i.e. the conditional probability that the component fails before it reaches age $x + 1$
Decision variables	
$x_{i,a} \leq 0$	Long-run probability that the system is in state $i = (i_1, i_2) \in \mathcal{I}$ at the beginning of a period and decision $a \in \mathcal{A}(i_1, i_2)$ is chosen
$y_{i_1} \in \{0, 1\}$	Whether we maintain preventively in period $i_1 \in \mathcal{I}_1$ , for p-(M)BRP
$z_{i_1, i_2} \in \{0, 1\}$	Whether we maintain for age $i_2 \in \mathcal{I}_2$ in period $i_1 \in \mathcal{I}_1$ , for p-MBRP
$t_{i_1} \in \mathbb{N}^+$	Threshold age for performing PM at period $i_1 \in \mathcal{I}_1$ , for p-ARP/p-MBRP

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## B Naive heuristics

To analyse the performance of the heuristics that we use in this paper, we compare them with naive heuristics. All these naive heuristics are implemented in such a way that we randomly choose for which components to replan PM when necessary.

### Comparison with *DMD*

For comparison with *DMD* we define a naive heuristic for which we always delay PM by one period when there are too many preventive maintenance activities planned in a certain period. Note that it is possible that preventive maintenance for a certain component is delayed multiple times. This means even though we always delay by one period, the PM is not always performed one period later.

### Comparison with *DMR*

For the naive heuristic for replanning in case of p-ARP, we let  $\ell = 1$ . This means that at the start of each period  $i_1$ , we replan for the current and next period. We start by looking at the current period  $i_1$  and when the number of permitted PM activities for this period is exceeded we randomly choose for which components we replan the PM to period  $i_1 + 1$ . If the number of planned PM for period  $i_1 + 1$  exceeds  $r$  after this, we choose the components for which we replan the PM to period  $i_1 + 2$ . If the number of preventive maintenance operations in period  $i_1$  is allowed but the expected number for period  $i_1 + 1$  is too high, we move up (to period  $i_1$ ) or delay (to period  $i_1 + 2$ ) PM with equal probability as long as the the number of PM for period  $i_1$  is less than or equal to  $r$ . Finally, we only implement all changes that apply to the current period  $i_1$ .

### Comparison with *SMR*

Finally, we have a naive heuristic for replanning in case of p-BRP. Let the optimal p-BRP for a single component be to preventively maintain at times  $T_1, \dots, T_n < mN$ . We now randomly select  $r$  components for which we apply this policy. To each of the remaining components (that do not have a policy yet) we assign a maintenance planning in which we perform PM at times  $T_1 + x, \dots, T_n + x$ , where  $x$  takes value  $+1$  or  $-1$  with equal probability. We draw  $x$  for each component separately. Finally, in case of  $r = 20$ , if there are too many components that have to be maintained at times  $T_1 + x, \dots, T_n + x$ , we replan PM for components to  $T_1 + x + x, \dots, T_n + x + x$ , until the constraint on available manpower is satisfied.

## C Proofs

**Lemma C2.** *When there is no economic, structural or stochastic dependence between different components, the policy that applies to each of the components the optimal replacement policy obtained when solving the model for that particular component, leaving aside all other components, is an optimal replacement policy for all components combined.*

*Proof.* As there is no dependence between the components, the cost of each action in each state for an arbitrary component is independent of the actions that are performed for the other components and the states that these components are in. It now follows that the optimal action does not depend on any other factor than the time period and the age of that particular component and can therefore be derived from solving the replacement policy model for that component. This leads to the result.  $\square$

## D Optimal policies for two components

### D.1 Markov process

For the situation with two components, we can expand the state space of the Markov process to include the state of the second component. The state space  $\mathcal{I}$  will then consist of states  $(i_1, i_2, i_3)$  with  $i_1$  the period of the year,  $i_2$  the age of the component of the first wind turbine and  $i_3$  the age of the component of the second turbine. The possible actions are now to do nothing ( $a = 0$ ), to replace only the first component ( $a = 1$ ), to replace only the second component ( $a = 2$ ) or to replace both ( $a = 3$ ). The state-dependent action space can then be written down as:

$$\mathcal{A}(i_1, i_2, i_3) = \begin{cases} \{3\} & \text{if } i_2 \in \{0, M\} \text{ and } i_3 \in \{0, M\}, \\ \{1, 3\} & \text{if } i_2 \in \{0, M\} \text{ and } i_3 \notin \{0, M\}, \\ \{2, 3\} & \text{if } i_2 \notin \{0, M\} \text{ and } i_3 \in \{0, M\}, \\ \{0, 1, 2, 3\} & \text{otherwise,} \end{cases} \quad (12)$$

where  $M$  represents the maximum age of the components. The transition probabilities  $\pi_{(i_1, i_2, i_3)(j_1, j_2, j_3)}(a)$  to go from state  $(i_1, i_2, i_3)$  to state  $(j_1, j_2, j_3)$  given the action  $a \in \mathcal{A}(i_1, i_2, i_3)$  are now

$$\pi_{(i_1, i_2, i_3)(j_1, j_2, j_3)}(0) = \begin{cases} (1 - p_{i_2})(1 - p_{i_3}) & \text{for } j_1 = i_1 + 1(\text{mod } N), j_2 = i_2 + 1, j_3 = i_3 + 1, i_2 \notin \{0, M\}, i_3 \notin \{0, M\} \\ (1 - p_{i_2})p_{i_3} & \text{for } j_1 = i_1 + 1(\text{mod } N), j_2 = i_2 + 1, j_3 = 0, i_2 \notin \{0, M\}, i_3 \notin \{0, M\} \\ p_{i_2}(1 - p_{i_3}) & \text{for } j_1 = i_1 + 1(\text{mod } N), j_2 = 0, j_3 = i_3 + 1, i_2 \notin \{0, M\}, i_3 \notin \{0, M\} \\ p_{i_2}p_{i_3} & \text{for } j_1 = i_1 + 1(\text{mod } N), j_2 = 0, j_3 = 0, i_2 \notin \{0, M\}, i_3 \notin \{0, M\} \\ 0 & \text{else,} \end{cases} \quad (13a)$$

$$\pi_{(i_1, i_2, i_3)(j_1, j_2, j_3)}(1) = \begin{cases} (1 - p_1)(1 - p_{i_3}) & \text{for } j_1 = i_1 + 1(\text{mod } N), j_2 = 1, j_3 = i_3 + 1, i_3 \notin \{0, M\} \\ (1 - p_1)p_{i_3} & \text{for } j_1 = i_1 + 1(\text{mod } N), j_2 = 1, j_3 = 0, i_3 \notin \{0, M\} \\ p_1(1 - p_{i_3}) & \text{for } j_1 = i_1 + 1(\text{mod } N), j_2 = 0, j_3 = i_3 + 1, i_3 \notin \{0, M\} \\ p_1p_{i_3} & \text{for } j_1 = i_1 + 1(\text{mod } N), j_2 = 0, j_3 = 0, i_3 \notin \{0, M\} \\ 0 & \text{else,} \end{cases} \quad (13b)$$

$$\pi_{(i_1, i_2, i_3)(j_1, j_2, j_3)}(2) = \begin{cases} (1 - p_{i_2})(1 - p_1) & \text{for } j_1 = i_1 + 1(\text{mod } N), j_2 = i_2 + 1, j_3 = 1, i_2 \notin \{0, M\} \\ (1 - p_{i_2})p_1 & \text{for } j_1 = i_1 + 1(\text{mod } N), j_2 = i_2 + 1, j_3 = 0, i_2 \notin \{0, M\} \\ p_{i_2}(1 - p_1) & \text{for } j_1 = i_1 + 1(\text{mod } N), j_2 = 0, j_3 = 1, i_2 \notin \{0, M\} \\ p_{i_2}p_1 & \text{for } j_1 = i_1 + 1(\text{mod } N), j_2 = 0, j_3 = 0, i_2 \notin \{0, M\} \\ 0 & \text{else,} \end{cases} \quad (13c)$$

$$\pi_{(i_1, i_2, i_3)(j_1, j_2, j_3)}(3) = \begin{cases} (1 - p_1)^2 & \text{for } j_1 = i_1 + 1(\bmod N), j_2 = 1, j_3 = 1 \\ (1 - p_1)p_1 & \text{for } j_1 = i_1 + 1(\bmod N), j_2 = 1, j_3 = 0 \\ p_1(1 - p_1) & \text{for } j_1 = i_1 + 1(\bmod N), j_2 = 0, j_3 = 1 \\ p_1^2 & \text{for } j_1 = i_1 + 1(\bmod N), j_2 = 0, j_3 = 0 \\ 0 & \text{else,} \end{cases} \quad (13)d$$

where  $p_{i_2} = \mathbb{P}(X = i_2 + 1 | X \geq i_2)$  and  $p_{i_3} = \mathbb{P}(X = i_3 + 1 | X \geq i_3)$  are the probabilities of failure after the components reach age  $i_2$  or  $i_3$  respectively but before they reach the next age and mod is the modulo operator.

The cost of taking action  $a$  in state  $(i_1, i_2, i_3) \in \mathcal{I}$  is now

$$c_{(i_1, i_2, i_3)}(a) = \begin{cases} 0 & \text{if } a = 0, \\ c_p(i_1) & \text{if } a = 1, i_2 \neq 0 \text{ or } a = 2, i_3 \neq 0, \\ c_f(i_1) & \text{if } a = 1, i_2 = 0 \text{ or } a = 2, i_3 = 0, \\ 2c_p(i_1) & \text{if } a = 3, i_2 \neq 0, i_3 \neq 0, \\ c_p(i_1) + c_f(i_1) & \text{if } a = 3, i_2 \neq 0, i_3 = 0 \text{ or } a = 3, i_2 = 0, i_3 \neq 0, \\ 2c_f(i_1) & \text{if } a = 3, i_2 = 0, i_3 = 0. \end{cases} \quad (14)$$

## D.2 period Age Replacement Policy

Let  $\mathcal{I}^{b_1} = \mathcal{I}_1 \times \{0\} \times \mathcal{I}_3$  denote the set of states where the first component has failed and  $\mathcal{I}^{b_2} = \mathcal{I}_1 \times \mathcal{I}_2 \times \{0\}$  the set of states where the second component has failed. Note that  $\mathcal{I}^{b_1} \cap \mathcal{I}^{b_2} = \mathcal{I} \times \{0\} \times \{0\}$  are all states where both components are in the failed state.

The LP for the p-ARP then becomes

$$\min \sum_{i=(i_1, i_2, i_3) \in \mathcal{I} \setminus \mathcal{I}^{b_1}} c_p(i_1)(x_{i,1} + x_{i,3}) + \sum_{i=(i_1, i_2, i_3) \in \mathcal{I} \setminus \mathcal{I}^{b_2}} c_p(i_1)(x_{i,2} + x_{i,3}) \quad (15)a$$

$$+ \sum_{i=(i_1, i_2, i_3) \in \mathcal{I}^{b_1}} c_f(i_1)(x_{i,1} + x_{i,3}) + \sum_{i=(i_1, i_2, i_3) \in \mathcal{I}^{b_2}} c_f(i_1)(x_{i,2} + x_{i,3})$$

$$\text{s.t.} \quad \sum_{a \in \mathcal{A}(i)} x_{i,a} - \sum_{j \in \mathcal{I}} \sum_{a \in \mathcal{A}_j} \pi_{ji}(a) x_{j,a} = 0, \quad \forall i = (i_1, i_2, i_3) \in \mathcal{I} \quad (15)b$$

$$\sum_{i_2 \in \mathcal{I}_2} \sum_{i_3 \in \mathcal{I}_3} \sum_{a \in \mathcal{A}(i_1, i_2, i_3)} x_{i_1, i_2, i_3, a} = \frac{1}{N}, \quad \forall i_1 \in \mathcal{I}_1 \quad (15)c$$

$$x_{i,a} \geq 0, \quad \forall i = (i_1, i_2, i_3) \in \mathcal{I}, \forall a \in \mathcal{A}(i) \quad (15)d$$

Note that the purposes of the objective and the constraints are the same as in the LP for the p-ARP described in Schouten et al. (2022).

To see what happens when there can be only one preventive action per period, we can restrict

$$x_{i,3} = 0, \forall i = (i_1, i_2, i_3) \in \mathcal{I}, i_1 \notin \{0, M\}, i_2 \notin \{0, M\}. \quad (15)e$$

### D.3 period Block Replacement Policy

For p-BRP we perform PM for the first component at times  $T_{1,1}, T_{1,2}, \dots, T_{1,n} < mN$  for some  $n \in \mathbb{N}^+$  and for the second component at times  $T_{2,1}, T_{2,2}, \dots, T_{2,l} < mN$ . Define the decision variables

$$y_{i_1,1} = \begin{cases} 1 & \text{if we maintain the first component preventively in period } i_1 \in \mathcal{I}_1, \forall i_1 \in \mathcal{I}_1 \text{ and} \\ 0 & \text{else} \end{cases} \quad (16)a$$

$$y_{i_1,2} = \begin{cases} 1 & \text{if we maintain the second component preventively in period } i_1 \in \mathcal{I}_1, \forall i_1 \in \mathcal{I}_1. \\ 0 & \text{else} \end{cases} \quad (16)b$$

In addition to (15)a-(15)d, we have the constraints

$$x_{i,0} + x_{i,2} + y_{i_1,1} \leq 1, \quad \forall i = (i_1, i_2, i_3) \in \mathcal{I} \quad (17)a$$

$$x_{i,0} + x_{i,1} + y_{i_1,2} \leq 1, \quad \forall i = (i_1, i_2, i_3) \in \mathcal{I} \quad (17)b$$

$$x_{i,1} + x_{i,3} - y_{i_1,1} \leq 0, \quad \forall i = (i_1, i_2, i_3) \in \mathcal{I}, i_1 \notin \{0, M\} \quad (17)c$$

$$x_{i,2} + x_{i,3} - y_{i_1,2} \leq 0, \quad \forall i = (i_1, i_2, i_3) \in \mathcal{I}, i_2 \notin \{0, M\} \quad (17)d$$

$$y_{i_1,1} + y_{i_1,2} \leq r, \quad \forall i_1 \in \mathcal{I}_1 \quad (17)e$$

$$y_{i_1,1} \in \{0, 1\}, \quad \forall i_1 \in \mathcal{I}_1 \quad (17)f$$

$$y_{i_1,2} \in \{0, 1\}, \quad \forall i_1 \in \mathcal{I}_1 \quad (17)g$$

and the right hand side of (15)c becomes  $\frac{1}{mN}$ . Restrictions (17)a and (17)c ensure that if we are supposed to preventively maintain the first component in a certain period  $i_1 \in \mathcal{I}_1$ , we take an action for which we maintain this component ( $a = 1$  or  $a = 3$ ) and the other way around. The constraints (17)b and (17)d have the same purpose but for the second component. Note that constraints (17)e limit the permitted number of preventive actions per period to  $r$ . In the situation of two components, this is only a real restriction when  $r = 1$ . To control the required manpower per period, we can also use constraints (15)e but we chose to use (17)e as it is necessary to restrict the values of  $y_{i_1,1}$  and  $y_{i_1,2}$  for constraints (17)a and (17)b to work. Namely, if we allow  $y_{i_1,1}$  and  $y_{i_1,2}$  to be both equal to one, then it could happen that for example  $x_{i,2} = 1$  even though  $y_{i_1,1} = 1$ .

After implementing this model, we discovered that it did not solve in a reasonable time when  $\alpha \geq 3$  years for all values of  $\Delta$ . In an attempt to make the process of solving these models faster, we made the state space smaller by removing actions 3 when both  $i_1 \notin \{0, M\}$  and  $i_2 \notin \{0, M\}$  and added some constraints. We know that for  $\alpha \geq 3$ , it is never optimal to maintain more than twice a year, so we added the restrictions

$$\sum_{i_1=n \cdot 12+1}^{(n+1) \cdot 12} y_{i_1,1} \leq 1, \quad n = 0, \dots, |\mathcal{I}_1|/12 - 1 \quad (18)\text{a}$$

$$\sum_{i_1=n \cdot 12+1}^{(n+1) \cdot 12} y_{i_1,2} \leq 1, \quad n = 0, \dots, |\mathcal{I}_1|/12 - 1 \quad (18)\text{b}$$

Furthermore, in case that  $\Delta \geq 20\%$ , the optimal policy for one component is to always maintain in July. As the costs are period-dependent and become larger when the month is further away from July, it is logical that it will not be optimal to maintain one of the two components in December, January, February or March. We therefore add the constraints

$$y_{n \cdot 12+k,1} = 0, \quad n = 0, \dots, |\mathcal{I}_1|/12 - 1, k = 1, 2, 3, 12 \quad (19)\text{a}$$

$$y_{n \cdot 12+k,2} \leq 1, \quad n = 0, \dots, |\mathcal{I}_1|/12 - 1, k = 1, 2, 3, 12 \quad (19)\text{b}$$

For  $\Delta \leq 10\%$  the model still did not solve in a reasonable time, even after implementing these changes. However, for these values of  $\Delta$  the optimal solution is quite straightforward, so we did not need to solve the model. For  $\Delta = 0\%$  the optimal solution for one component is any policy with a block time of 18 months. We can therefore construct our own policy for two components where both components have a block time of 18 months and we do not maintain them in the same month. The monthly cost is then exactly twice the optimal monthly cost for one component in the optimal solution. This solution we constructed must therefore be optimal. In case of  $\Delta = 10\%$ , the optimal p-BRP policy for a single component is to maintain in period 6 and 21, which is the month June and next year the month September after which we have a year with no preventive maintenance. As mentioned in [Section 5.1](#), this comes down to the same policy as maintaining in September in the first year, not preventively maintaining in the next year and maintain in June in the third year, which means maintaining in period 9 and 31. We can therefore assign one component the policy for which we maintain in periods 6 and 21 and we apply to the other component the policy in which we maintain in periods 9 and 31. The monthly cost is then exactly twice the optimal monthly cost for one component in the optimal solution. This solution we constructed must therefore be optimal.

## E Additional tables and figures

### E.1 Delaying for p-ARP

**Table 11** Results of delay of PM for a set of 80 components for p-ARP, with  $\alpha = 1$  year and  $\beta = 2$ . Yearly costs, in 1000 €, difference in costs when including restrictions on manpower and number of times we delay PM per simulation run and percentage of times this directly leads to a failure.

$\Delta$	costs	additional costs			#PM	# delayed PM / % of times delay leads to CM					
	$r = 80$	$r = 60$	$r = 40$	$r = 20$		$r = 80$	$r = 60$		$r = 40$		$r = 20$
0%	40.060	-0.00%	0.00%	0.00%	552 490	7	2.8%	21	6.3%	183	9.1%
10%	40.004	0.00%	-0.00%	0.00%	488 143	6	3.2%	17	4.6%	205	9.2%
20%	39.667	0.00%	0.03%**	0.37%**	525 516	32	8.1%	42 116	8.7%	124 991	12.3%
30%	39.192	-0.00%	0.44%**	2.13%**	524 014	44	26.6%	41 834	29.4%	169 210	28.5%
40%	38.439	0.00%	0.12%**	1.79%**	524 971	1 710	6.1%	77 325	8.6%	232 063	21.1%
50%	37.611	0.22%**	2.81%**	4.99%**	524 813	8 154	54.4%	109 010	45.0%	262 085	36.2%

Note: the results are averages over 10 runs, each of 50 000 simulated periods, and  $r$  denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test,  $*p < 0.05$ ,  $**p < 0.01$ .

**Table 12** Results of delay of PM for a set of 80 components for p-ARP, with  $\alpha = 1$  year and  $\beta = 2$ . Yearly costs, in thousands of €, and difference in costs when including different restrictions on manpower.

$\Delta$	costs	average costs and percentage additional costs with respect to $r=80$								
	$r = 80$	$r=60$		$r=40$		$r=20$				
0%	40.060	40.060	-0.0001%	(0.0002)	40.060	0.0001%	(0.0008)	40.060	0.0005%	(0.0018)
10%	40.004	40.004	0.0004%	(0.0021)	40.004	-0.0001%	(0.0032)	40.005	0.0018%	(0.0081)
20%	39.667	39.667	0.0000%	(0.0008)	39.681	0.0338%**	(0.0256)	39.813	0.3670%**	(0.0737)
30%	39.192	39.191	-0.0011%	(0.0038)	39.364	0.4415%**	(0.0702)	40.027	2.1328%**	(0.0878)
40%	38.439	38.441	0.0048%	(0.0095)	38.484	0.1191%**	(0.0420)	39.125	1.7854%**	(0.0792)
50%	37.611	37.695	0.2224%**	(0.0395)	38.668	2.8102%**	(0.1256)	39.487	4.9875%**	(0.1493)

Note: the results are averages over 10 runs, each of 50.000 simulated periods, brackets indicate standard deviations and  $r$  denotes the number of permitted PM operations per period.

We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test,  $*p < 0.05$ ,  $**p < 0.01$ .

Total runtime for the results in this table was 3.5 minutes ( $\approx 0.00004$  seconds per month).

**Table 13** Results of delay of PM for a set of 80 components for p-ARP, with  $\alpha = 3$  year and  $\beta = 2$ . Yearly costs, in thousands of €, and difference in costs when including different restrictions on manpower. The results are for  $\ell = 1$ .

$\Delta$	costs		average costs and percentage additional costs with respect to r=80							
	$r = 80$		r=60		r=40		r=20			
0%	13.516	13.516	-0.0002%	(0.0006)	13.516	0.0006%	(0.0008)	13.516	-0.0009%	(0.0024)
10%	13.240	13.240	0.0004%	(0.0032)	13.241	0.0019%	(0.0044)	13.246	0.0416%**	(0.0241)
20%	12.713	12.717	0.0318%	(0.0774)	12.719	0.0504%	(0.1526)	12.718	0.0411%	(0.0967)
30%	11.784	11.803	0.1660%**	(0.0149)	11.838	0.4568%**	(0.0216)	12.259	4.0302%**	(0.0643)
40%	10.852	10.879	0.2550%**	(0.0235)	10.927	0.6945%**	(0.0463)	11.759	8.3666%**	(0.1240)
50%	9.905	9.943	0.3805%**	(0.0236)	10.006	1.0148%**	(0.0418)	11.074	11.8034%**	(0.1818)

Note: the results are averages over 10 runs, each of 50.000 simulated periods, brackets indicate standard deviations and r denotes the number of permitted PM operations per period.

We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

Total runtime for the results in this table was 5.5 minutes ( $\approx 0.00007$  seconds per month).

**Table 14** Results of delay of PM for a set of 80 components for p-ARP, with  $\alpha = 3$  years and  $\beta = 2$ . Yearly costs, in 1000 €, difference in costs when including restrictions on manpower and number of times we delay PM for all components combined and percentage of times this directly leads to a failure. The results are for  $\ell = 3$ , but are exactly the same for  $\ell = 5$ .

$\Delta$	costs				#PM	# delayed PM / % of times delay leads to CM					
	$r = 80$	$r = 60$	$r = 40$	$r = 20$		$r = 80$	$r = 60$	$r = 40$	$r = 20$		
0%	13.516	-0.00%	-0.00%	-0.00%	173 128	7	0%	18	0.6%	37	2.4%
10%	13.240	0.00%	0.00%	0.04%**	185 521	7	0%	21	0.5%	13 918	2.5%
20%	12.713	0.03%	0.05%	0.04%	296 039	17 236	1.9%	47 019	1.9%	78 207	2.0%
30%	11.784	0.17%**	0.46%**	<b>4.23%**</b>	297 566	48 327	1.8%	132 903	1.8%	<b>174 453</b>	<b>3.2%</b>
40%	10.852	0.26%**	0.69%**	<b>8.95%**</b>	301 088	51 725	1.8%	136 010	1.8%	222 <b>675</b>	<b>3.2%</b>
50%	9.905	0.38%**	1.01%**	<b>13.96%**</b>	303 654	54 167	1.7%	138 321	1.8%	224 <b>774</b>	<b>3.3%</b>

Note: the results are averages over 10 runs, each of 50 000 simulated periods, differences compared to  $\ell = 1$  in bold, and r denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

**Table 15** Results of delay of PM for a set of 80 components for p-ARP, with  $\alpha = 3$  years and  $\beta = 2$ . Yearly costs, in thousands of €, and difference in costs when including different restrictions on manpower. The results are for  $\ell = 3$ , but are exactly the same for  $\ell = 5$ .

$\Delta$	costs		average costs and percentage additional costs with respect to r=80							
	$r = 80$		r=60		r=40		r=20			
0%	13.516	13.516	-0.0002%	(0.0006)	13.516	0.0006%	(0.0008)	13.516	-0.0009%	(0.0024)
10%	13.240	13.240	0.0004%	(0.0032)	13.241	0.0019%	(0.0044)	13.246	0.0416%**	(0.0241)
20%	12.713	12.717	0.0318%	(0.0774)	12.719	0.0504%	(0.1526)	12.718	0.0411%	(0.0967)
30%	11.784	11.803	0.1660%**	(0.0149)	11.838	0.4568%**	(0.0216)	12.259	<b>4.2260%**</b>	<b>(0.1591)</b>
40%	10.852	10.879	0.2550%**	(0.0235)	10.927	0.6945%**	(0.0463)	11.759	<b>8.9543%**</b>	<b>(0.1221)</b>
50%	9.905	9.943	0.3805%**	(0.0236)	10.006	1.0148%**	(0.0418)	11.074	<b>13.9647%**</b>	<b>(0.1883)</b>

Note: the results are averages over 10 runs, each of 50.000 simulated periods, differences compared to  $\ell = 1$  in bold, brackets indicate standard deviations and r denotes the number of permitted PM operations per period.

We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

Total runtime for the results in this table was 7 minutes ( $\approx 0.0008$  seconds per month).

**Table 16** Results of delay of PM for a set of 80 components for p-ARP, with  $\alpha = 5$  years and  $\beta = 2$ . Yearly costs, in 1000 €, difference in costs when including restrictions on manpower and number of times we delay PM for all components combined and percentage of times this directly leads to a failure. The results are for  $\ell = 1$ .

$\Delta$	costs				#PM	# delayed PM / % of times delay leads to CM					
	$r = 80$	$r = 60$	$r = 40$	$r = 20$		$r = 80$	$r = 60$	$r = 40$	$r = 20$		
0%	8.133	-0.00%	-0.00%	-0.00%	107 218	7	0%	19	1.1%	38	1.9%
10%	7.878	-0.00%	0.00%	0.00%	114 607	7	0%	19	1.0%	436	1.5%
20%	7.434	-0.00%	0.01%*	0.38%**	146 161	17	0%	1 457	1.4%	63 404	1.3%
30%	6.980	-0.00%	0.01%**	0.67%**	148 152	22	0.5%	1 790	1.3%	65 296	1.3%
40%	6.521	-0.00%	0.03%**	1.03%**	150 384	21	0%	2 085	1.2%	67 474	1.3%
50%	6.054	-0.00%	0.05%**	1.50%**	152 935	20	0%	2 742	1.1%	69 959	1.3%

Note: the results are averages over 10 runs, each of 50 000 simulated periods, and r denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

**Table 17** Results of delay of PM for a set of 80 components for p-ARP, with  $\alpha = 5$  years and  $\beta = 2$ . Yearly costs, in thousands of €, and difference in costs when including different restrictions on manpower. The results are for  $\ell = 1$ .

$\Delta$	costs		average costs and percentage additional costs with respect to r=80					
	$r = 80$		$r=60$		$r=40$		$r=20$	
0%	8.133	8.133	-0.0001% (0.0002)	8.133	-0.0000% (0.0012)	8.133	-0.0006% (0.0029)	
10%	7.878	7.878	-0.0010% (0.0026)	7.878	0.0002% (0.0056)	7.878	0.0016% (0.0168)	
20%	7.434	7.434	-0.0005% (0.0009)	7.435	0.0088%* (0.0090)	7.462	0.3758%** (0.0273)	
30%	6.980	6.980	0.0003% (0.0012)	6.981	0.0144%** (0.0112)	7.027	0.6684%** (0.0326)	
40%	6.521	6.521	-0.0005% (0.0011)	6.522	0.0260%** (0.0068)	6.588	1.0320%** (0.0425)	
50%	6.054	6.054	-0.0001% (0.0012)	6.057	0.0485%** (0.0106)	6.145	1.4970%** (0.0629)	

Note: the results are averages over 10 runs, each of 50.000 simulated periods, brackets indicate standard deviations and r denotes the number of permitted PM operations per period.

We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test,  $*p < 0.05$ ,  $**p < 0.01$ .

Total runtime for the results in this table was 9 minutes ( $\approx 0.001$  seconds per month).

**Table 18** Results of delay of PM for a set of 80 components for p-ARP, with  $\alpha = 5$  years and  $\beta = 2$ . Yearly costs, in 1000 €, difference in costs when including restrictions on manpower and number of times we delay PM for all components combined and percentage of times this directly leads to a failure. The results are for  $\ell = 3$ , but are exactly the same for  $\ell = 5$ .

$\Delta$	costs				#PM $r = 80$	# delayed PM / % of times delay leads to CM					
	$r = 80$	$r = 60$	$r = 40$	$r = 20$		$r = 60$	$r = 40$	$r = 20$			
0%	8.133	-0.00%	-0.00%	-0.00%	107 218	7	0%	19	1.1%	38	1.9%
10%	7.878	-0.00%	0.00%	0.00%	114 607	7	0%	19	1.0%	436	1.5%
20%	7.434	-0.00%	0.01%*	<b>0.45%**</b>	146 161	17	0%	<b>1 453</b>	1.4%	<b>63 412</b>	<b>1.4%</b>
30%	6.980	0.00%	<b>0.03%**</b>	<b>0.83%**</b>	148 152	22	<b>0.9%</b>	<b>1 697</b>	<b>1.5%</b>	<b>65 284</b>	1.4%
40%	6.521	-0.00%	<b>0.05%**</b>	<b>1.27%**</b>	150 384	21	0%	<b>2 153</b>	<b>1.4%</b>	<b>67 489</b>	1.3%
50%	6.054	0.00%	<b>0.09%**</b>	<b>1.85%**</b>	152 935	20	<b>0.5%</b>	<b>2 798</b>	<b>1.2%</b>	<b>69 962</b>	1.3%

Note: the results are averages over 10 runs, each of 50 000 simulated periods, differences compared to  $\ell = 1$  in bold, and r denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test,  $*p < 0.05$ ,  $**p < 0.01$ .

**Table 19** Results of delay of PM for a set of 80 components for p-ARP, with  $\alpha = 5$  years and  $\beta = 2$ . Yearly costs, in thousands of €, and difference in costs when including different restrictions on manpower. The results are for  $\ell = 3$ , but are exactly the same for  $\ell = 5$ .

$\Delta$	costs		average costs and percentage additional costs with respect to r=80					
	$r = 80$		r=60		r=40		r=20	
0%	8.133	8.133	-0.0001% (0.0002)	8.133	-0.0000% (0.0012)	8.133	-0.0006% (0.0029)	
10%	7.878	7.878	-0.0010% (0.0026)	7.878	0.0002% (0.0056)	7.878	0.0016% (0.0168)	
20%	7.434	7.434	-0.0005% (0.0009)	7.435	0.0099%* ( <b>0.0100</b> )	7.468	0.4472%** ( <b>0.0289</b> )	
30%	6.980	6.980	0.0000% ( <b>0.0011</b> )	6.982	0.0275%** ( <b>0.0104</b> )	7.038	0.8292%** ( <b>0.0397</b> )	
40%	6.521	6.521	-0.0004% (0.0011)	6.524	0.0465%** ( <b>0.0124</b> )	6.603	1.2662%** ( <b>0.0380</b> )	
50%	6.054	6.054	0.0000% ( <b>0.0017</b> )	6.060	0.0906%** ( <b>0.0126</b> )	6.166	1.8487%** ( <b>0.0496</b> )	

Note: the results are averages over 10 runs, each of 50.000 simulated periods, brackets indicate standard deviations, differences compared to  $\ell = 1$  in bold and r denotes the number of permitted PM operations per period.

We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

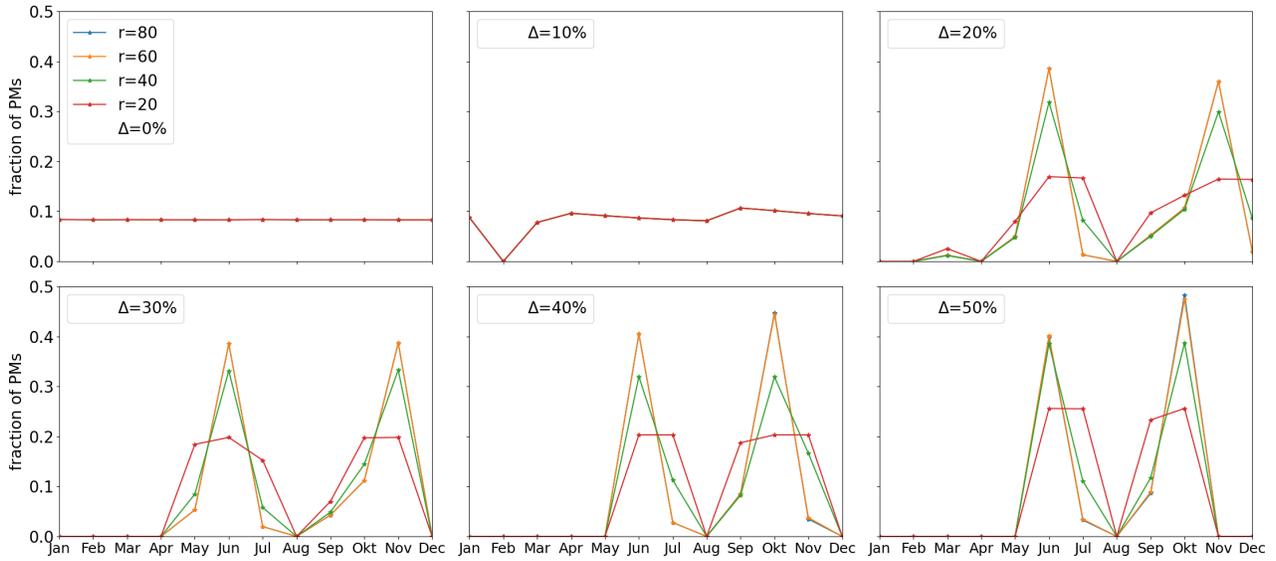
Total runtime for the results in this table was 9 minutes ( $\approx 0.001$  seconds per month).

**Table 20** Results of delay of PM for a set of 80 components for p-ARP. Percentage additional costs when including restrictions on manpower with respect to the optimal planning without this constraint, divided into a part due to more/less maintenance activities and a part due to maintenance in more/less expensive periods. The results for  $\alpha = 5$  years are for  $\ell = 1$ .

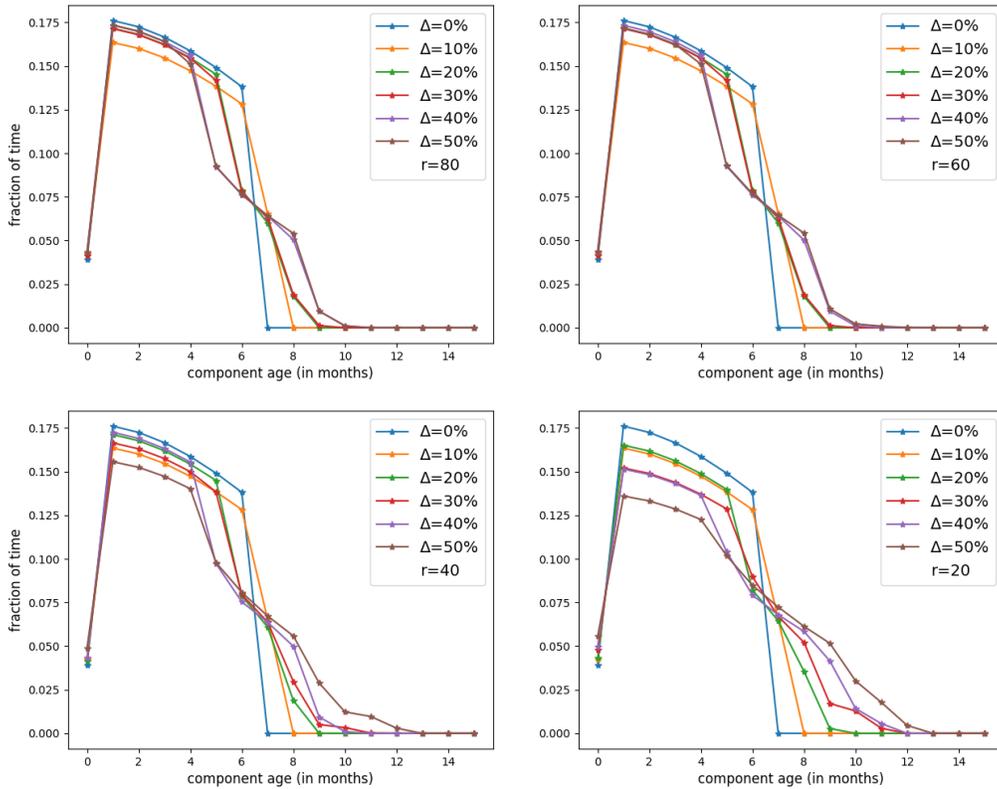
$\Delta$	more or less maintenance / more or less expensive periods					
	$\alpha = 1, \beta = 2$			$\alpha = 5, \beta = 2$		
	$r = 60$	$r = 40$	$r = 20$	$r = 60$	$r = 40$	$r = 20$
0%	-0.00% / 0.00%	0.00% / 0.00%	0.00% / 0.00%	-0.00%/0.00%	-0.00% /0.00%	-0.00% /0.00%
10%	0.00% / 0.00%	0.00% /-0.00%	0.00% / 0.00%	-0.00%/0.00%	-0.00% /0.00%	-0.00% /0.01%**
20%	0.00% /-0.00%	0.01% / 0.02%**	0.40%**/-0.03%**	-0.00%/0.00%	0.00% /0.01%**	-0.07%**/0.45%**
30%	-0.00% /-0.00%	0.49%**/-0.05%**	1.80%**/ 0.33%**	-0.00%/0.00%	-0.00% /0.02%**	-0.09%**/0.76%**
40%	-0.02%**/ 0.02%**	-0.18%**/ 0.30%**	1.44%**/ 0.35%**	-0.00%/0.00%	-0.01% /0.03%**	-0.14%**/1.17%**
50%	0.04%**/ 0.18%**	1.33%**/ 1.48%**	3.29%**/ 1.70%**	-0.00%/0.00%	-0.02%**/0.07%**	-0.19%**/1.69%**

Note: the results are averages over 10 runs, each of 50 000 simulated periods, and r denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

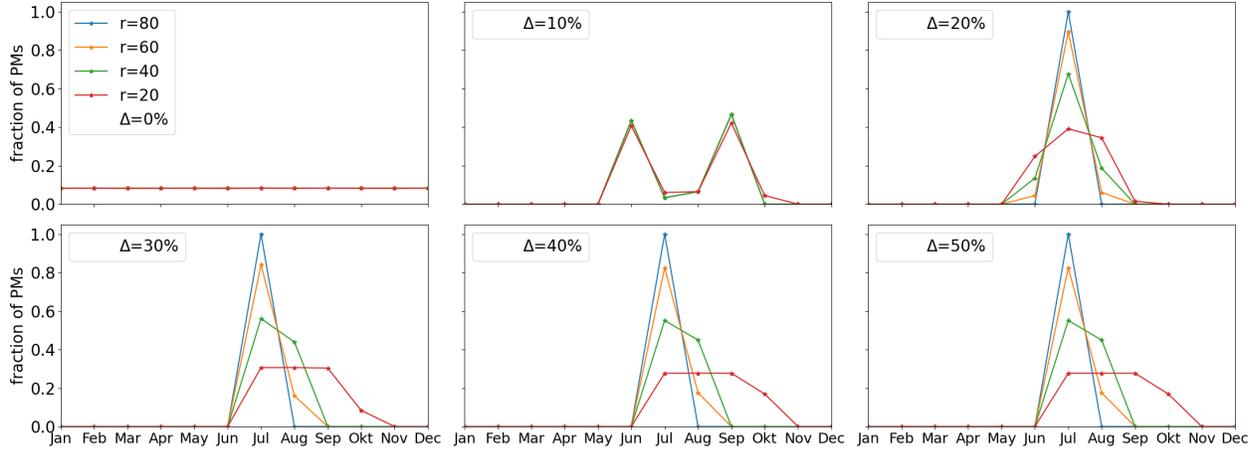
\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .



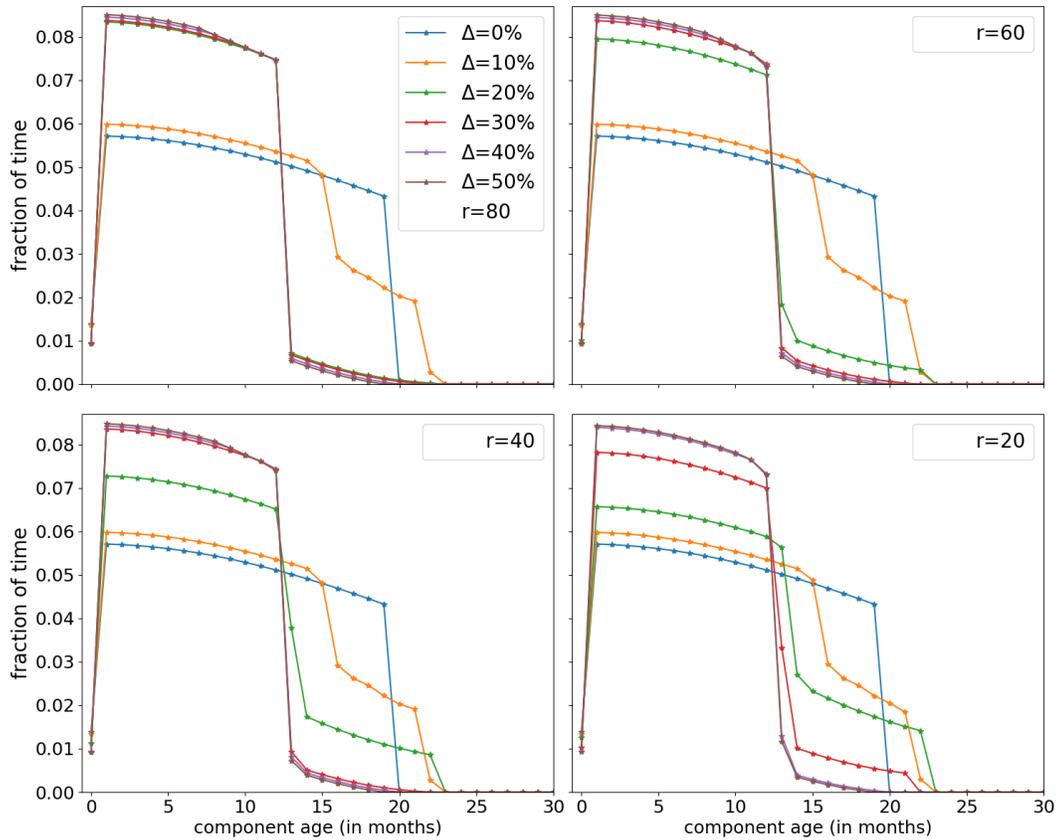
**Figure 6** Distribution of the number of PM activities per period for the set of 80 components when delaying PM, for  $\alpha = 1$  year and  $\beta = 2$ , for p-ARP with and without including constraints on the number of allowed PM activities per period  $r$  and for different  $\Delta$ . Note that the lines for  $r = 80$  and  $r = 60$  coincide.



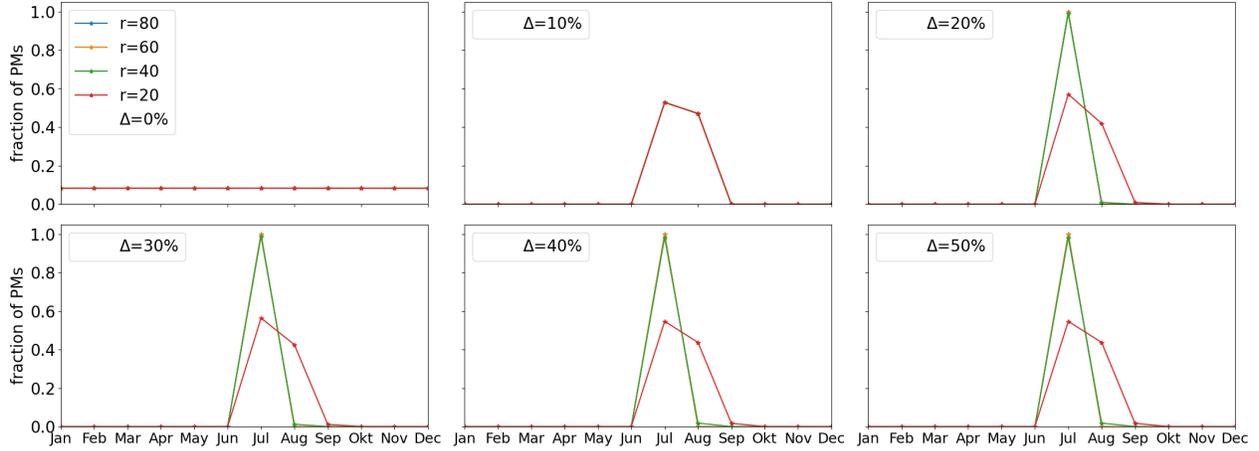
**Figure 7** Age distribution of the set of 80 components for  $\alpha = 1$  year and  $\beta = 2$ , for p-ARP when delaying PM, with and without including constraints on the number of allowed PM activities per period  $r$ .



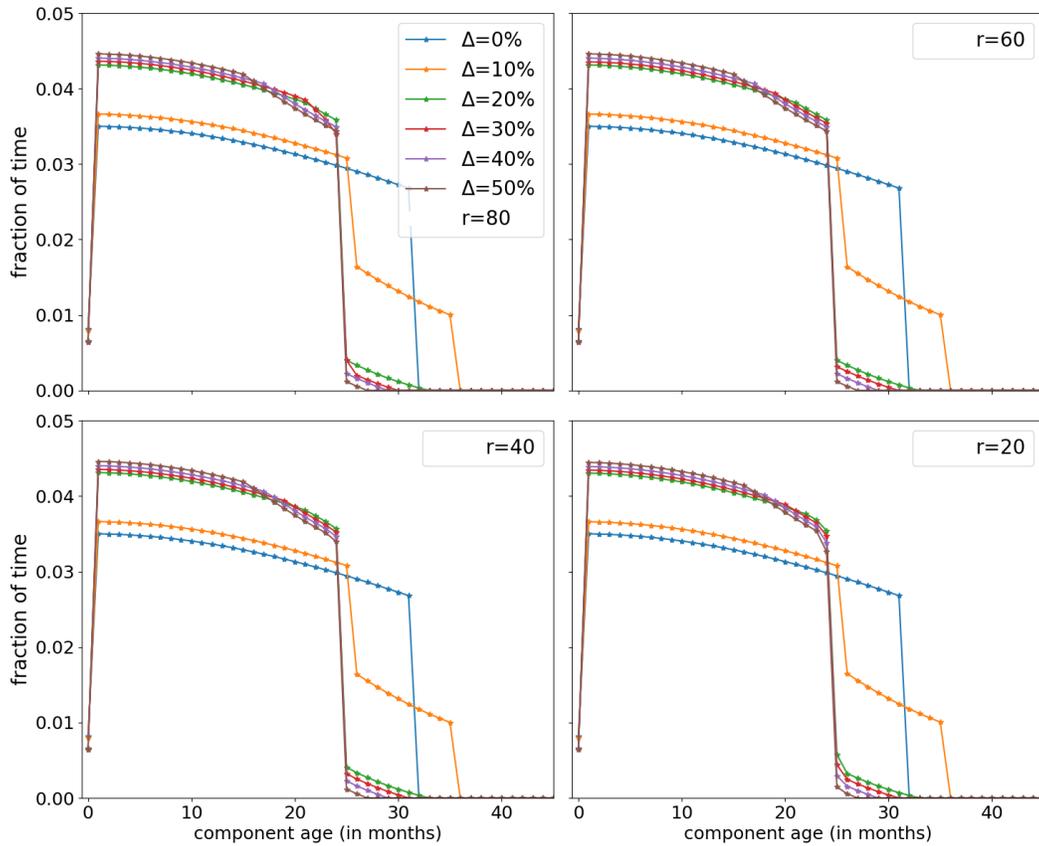
**Figure 8** Distribution of the number of PM activities per period for the set of 80 components when delaying PM, for  $\alpha = 3$  years,  $\beta = 2$  and  $\ell = 1$ , for p-ARP with and without including constraints on the number of allowed PM activities per period  $r$  and for different  $\Delta$ .



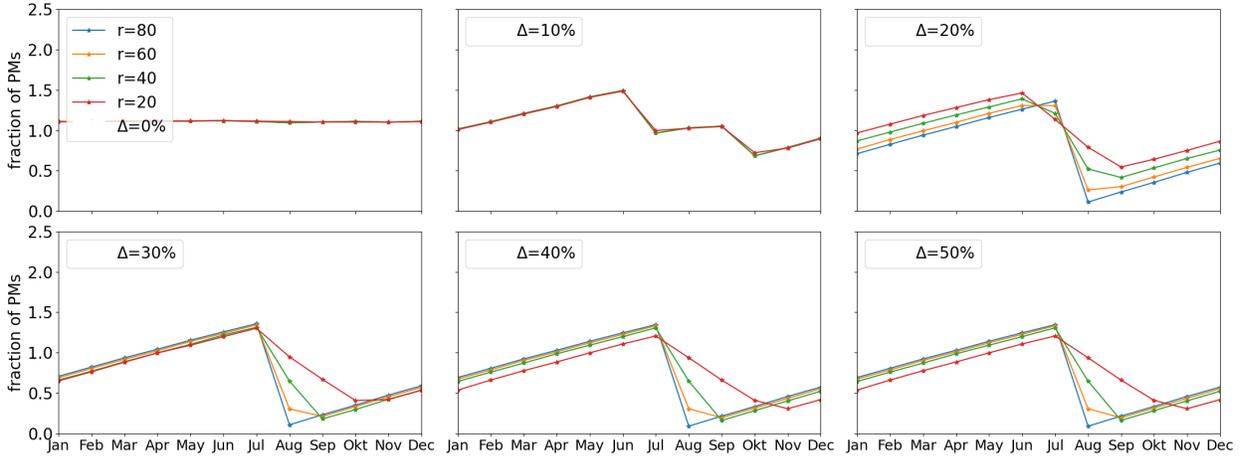
**Figure 9** Age distribution of the set of 80 components for  $\alpha = 3$  years,  $\beta = 2$  and  $\ell = 1$ , for p-ARP when delaying PM, with and without including constraints on the number of allowed PM activities per period  $r$ .



**Figure 10** Distribution of the number of PM activities per period for the set of 80 components when delaying PM, for  $\alpha = 5$  years,  $\beta = 2$  and  $\ell = 1$ , for p-ARP with and without including constraints on the number of allowed PM activities per period  $r$  and for different  $\Delta$ . Note that the lines for  $r = 80$  and  $r = 60$  coincide.



**Figure 11** Age distribution of the set of 80 components for  $\alpha = 5$  years,  $\beta = 2$  and  $\ell = 1$ , for p-ARP when delaying PM, with and without including constraints on the number of allowed PM activities per period  $r$ .



**Figure 12** Average number of CM activities per period for the set of 80 components when delaying PM, for  $\alpha = 3$  years and  $\beta = 2$ , for p-ARP with and without including constraints on the number of allowed PM activities per period  $r$  and for different  $\Delta$ . The results are for  $\ell = 3$  but are exactly the same for  $\ell = 5$  and  $\ell = 7$ .

**Table 21** Results of delay of PM for a set of 80 components for p-ARP, following Section 4.2.1, with  $\alpha = 3$  years and  $\beta = 2$ . Yearly costs, in 1000 €, difference in costs when including restrictions on manpower and number of times we delay PM for all components combined and percentage of times this directly leads to a failure.

$\Delta$	costs		additional costs			#PM $r = 80$	# delayed PM / % of times delay leads to CM				
	$r = 80$	$r = 60$	$r = 40$	$r = 20$	$r = 60$		$r = 40$	$r = 20$			
0%	13.516	-0.00%	-0.00%	-0.00%	173 128	7	0%	18	0.6%	37	2.4%
10%	13.240	0.00%	0.00%	0.07%*	185 521	7	1.4%	20	2.0%	14 281	15.7%
20%	12.713	0.31%**	0.85%**	1.97%**	296 039	24 034	25.6%	66 558	25.6%	113 866	26.0%
30%	11.784	1.81%**	5.06%**	18.39%**	297 566	41 321	28.1%	113 530	28.2%	142 169	41.4%
40%	10.852	3.31%**	9.20%**	26.07%**	301 088	46 018	27.7%	120 678	27.9%	144 819	42.5%
50%	9.905	5.29%**	14.38%**	35.41%**	303 654	49 066	27.3%	124 966	27.8%	146 200	43.0%

Note: the results are averages over 10 runs, each of 50 000 simulated periods, and  $r$  denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

Total runtime for the results in this table was 3 minutes.

**Table 22** Results of delay of PM for a set of 80 components for p-ARP, following Section 4.2.1, with  $\alpha = 5$  years and  $\beta = 2$ . Yearly costs, in 1000 €, difference in costs when including restrictions on manpower and number of times we delay PM for all components combined and percentage of times this directly leads to a failure.

$\Delta$	costs	additional costs			#PM	# delayed PM / % of times delay leads to CM						
	$r = 80$	$r = 60$	$r = 40$	$r = 20$		$r = 80$	$r = 60$	$r = 40$	$r = 20$	$r = 60$	$r = 40$	$r = 20$
0%	8.133	-0.00%	-0.00%	-0.00%	107 218	7	0%	19	1.1%	38	1.9%	
10%	7.878	-0.00%	0.00%	-0.00%	114 607	7	0%	19	1.0%	397	2.9%	
20%	7.434	-0.00%	0.01%	4.30%**	146 161	8	9.6%	545	17.2%	97 376	19.8%	
30%	6.980	-0.00%	0.01%	7.34%**	148 152	13	10.0%	725	16.5%	100 115	20.1%	
40%	6.521	-0.00%	0.04%**	10.87%**	150 384	12	9.8%	902	26.0%	102 450	20.4%	
50%	6.054	-0.00%	0.03%**	15.13%**	152 935	11	12.3%	1 192	13.9%	104 369	20.9%	

Note: the results are averages over 10 runs, each of 50 000 simulated periods, and  $r$  denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test,  $*p < 0.05$ ,  $**p < 0.01$ .

Total runtime for the results in this table was 3 minutes.

## E.2 Delaying for p-BRP

**Table 23** Results of delay of PM for a set of 80 components for p-BRP, with  $\alpha = 1$  year and  $\beta = 2$ . Yearly costs, in 1000 €, difference in costs when including restrictions on manpower and number of times we delay PM per simulation run and percentage of times this directly leads to a failure. The results are for  $\ell = 1$ .

$\Delta$	costs	additional costs			#PM	# delayed PM / % of times delay leads to CM						
	$r = 80$	$r = 60$	$r = 40$	$r = 20$		$r = 80$	$r = 60$	$r = 40$	$r = 20$	$r = 60$	$r = 40$	$r = 20$
0%	41.463	-1.27%**	-1.50%**	-2.85%**	623 010	124 266	4.5%	292 437	6.2%	464 647	10.8%	
10%	41.385	-1.22%**	-1.43%**	-2.56%**	623 494	124 726	4.3%	292 930	6.0%	465 101	10.5%	
20%	40.898	-1.13%**	-1.29%**	-2.06%**	623 494	124 726	4.3%	292 930	6.0%	465 101	10.5%	
30%	40.346	-1.05%**	-1.31%**	-1.47%**	624 895	126 103	3.9%	294 314	5.6%	466 344	9.9%	
40%	39.425	-0.92%**	-1.15%**	-0.72%**	624 895	126 103	3.9%	294 314	5.6%	466 344	9.9%	
50%	38.468	-0.33%**	0.09%**	1.58%**	627 322	128 405	3.3%	296 548	4.8%	466 375	9.5%	

Note: the results are averages over 10 runs, each of 50 000 simulated periods, and  $r$  denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test,  $*p < 0.05$ ,  $**p < 0.01$ .

**Table 24** Results of delay of PM for a set of 80 components for p-BRP, with  $\alpha = 1$  year and  $\beta = 2$ . Yearly costs, in thousands of €, and difference in costs when including different restrictions on manpower. The results are for  $\ell = 1$ .

$\Delta$	costs		average costs and percentage additional costs with respect to r=80					
	$r = 80$		$r=60$		$r=40$		$r=20$	
0%	41.463	40.937	-1.2676%** (0.0261)	40.842	-1.4976%** (0.0255)	40.283	-2.8456%** (0.0449)	
10%	41.385	40.879	-1.2206%** (0.0442)	40.795	-1.4252%** (0.0593)	40.325	-2.5608%** (0.0628)	
20%	40.898	40.437	-1.1270%** (0.0449)	40.372	-1.2856%** (0.0602)	40.054	-2.0628%** (0.0646)	
30%	40.346	39.924	-1.0470%** (0.0379)	39.818	-1.3080%** (0.0513)	39.751	-1.4748%** (0.0753)	
40%	39.425	39.063	-0.9189%** (0.0385)	38.972	-1.1488%** (0.0534)	39.142	-0.7179%** (0.0730)	
50%	38.468	38.340	-0.3312%** (0.0454)	38.503	0.0920%** (0.0754)	39.075	1.5780%** (0.1086)	

Note: the results are averages over 10 runs, each of 50.000 simulated periods, brackets indicate standard deviations and r denotes the number of permitted PM operations per period.

We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

Total runtime for the results in this table was 8 minutes ( $\approx 0.001$  seconds per month).

**Table 25** Results of delay of PM for a set of 80 components for p-BRP, with  $\alpha = 1$  year and  $\beta = 2$ . Yearly costs, in 1000 €, difference in costs when including restrictions on manpower and number of times we delay PM per simulation run and percentage of times this directly leads to a failure. The results are for  $\ell = 3$  but are exactly the same for  $\ell = 5$ .

$\Delta$	costs				#PM	# delayed PM / % of times delay leads to CM							
	$r = 80$	$r = 60$	$r = 40$	$r = 20$		$r = 80$	$r = 60$	$r = 40$	$r = 20$	$r = 80$	$r = 60$	$r = 40$	$r = 20$
0%	41.463	-1.74%**	-2.03%**	-2.46%**	623 010	124 955	6.2%	293 149	6.8%	470 269	11.4%		
10%	41.385	-1.59%**	-1.85%**	-2.20%**	623 494	125 511	6.3%	293 684	6.8%	471 029	11.1%		
20%	40.898	-1.48%**	-1.69%**	-1.69%**	623 494	124 726	4.3%	292 930	6.0%	465 101	10.5%		
30%	40.346	-1.16%**	-1.53%**	-0.59%**	624 895	125 321	5.9%	293 586	6.6%	468 210	11.1%		
40%	39.425	-0.85%**	-1.16%**	0.23%**	624 895	127 206	7.3%	295 533	7.0%	471 165	10.8%		
50%	38.468	0.26%**	1.026%**	2.78%**	627 322	129 602	6.1%	298 231	6.6%	463 555	10.7%		

Note: the results are averages over 10 runs, each of 50 000 simulated periods, and r denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

**Table 26** Results of delay of PM for a set of 80 components for p-BRP, with  $\alpha = 1$  year and  $\beta = 2$ . Yearly costs, in thousands of €, and difference in costs when including different restrictions on manpower. The results are for  $\ell = 3$  but are exactly the same for  $\ell = 5$ .

$\Delta$	costs		average costs and percentage additional costs with respect to r=80							
	r = 80		r=60		r=40		r=20			
0%	41.463	40.740	-1.7434%**	(0.0363)	40.622	-2.0280%**	(0.0329)	40.443	-2.4587%**	(0.0542)
10%	41.385	40.728	-1.5855%**	(0.0534)	40.621	-1.8454%**	(0.0698)	40.476	-2.1952%**	(0.0776)
20%	40.898	40.294	-1.4753%**	(0.0381)	40.208	-1.6942%**	(0.0649)	40.205	-1.6942%**	(0.0649)
30%	40.346	39.878	-1.1596%**	(0.0607)	39.728	-1.5311%**	(0.0694)	40.107	-0.5927%**	(0.0625)
40%	39.425	39.090	-0.8501%**	(0.0593)	38.967	-1.1612%**	(0.0709)	39.514	0.2250%**	(0.0861)
50%	38.468	38.340	0.2617%**	(0.0430)	38.503	1.0257%**	(0.0572)	39.075	2.7840%**	(0.1087)

Note: the results are averages over 10 runs, each of 50.000 simulated periods, brackets indicate standard deviations and r denotes the number of permitted PM operations per period.

We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

Total runtime for the results in this table was 9 minutes ( $\approx 0.001$  seconds per month).

**Table 27** Results of delay of PM for a set of 80 components for p-BRP, with  $\alpha = 3$  year and  $\beta = 2$ . Yearly costs, in thousands of €, and difference in costs when including different restrictions on manpower. The results are for  $\ell = 1$ .

$\Delta$	costs		average costs and percentage additional costs with respect to r=80							
	r = 80		r=60		r=40		r=20			
0%	14.158	14.061	-0.6875%**	(0.0234)	14.050	-0.7626%**	(0.0355)	13.877	-1.9896%**	(0.0619)
10%	13.807	13.738	-0.5016%**	(0.0392)	13.740	-0.4886%**	(0.0520)	13.721	-0.6232%**	(0.0559)
20%	13.133	13.110	-0.1689%**	(0.0208)	13.119	-0.1035%**	(0.0229)	13.538	3.0853%**	(0.0443)
30%	12.111	12.108	-0.0290%**	(0.0227)	12.133	0.1825%**	(0.0224)	12.831	5.9394%**	(0.0519)
40%	11.090	11.105	0.1366%**	(0.0261)	11.148	0.5213%**	(0.0226)	12.123	9.3194%**	(0.0649)
50%	10.068	10.102	0.3358%**	(0.0314)	10.162	0.9287%**	(0.0236)	11.416	13.3851%**	(0.0848)

Note: the results are averages over 10 runs, each of 50.000 simulated periods, brackets indicate standard deviations and r denotes the number of permitted PM operations per period.

We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

Total runtime for the results in this table was 11 minutes ( $\approx 0.001$  seconds per month).

**Table 28** Results of delay of PM for a set of 80 components for p-BRP, with  $\alpha = 3$  year and  $\beta = 2$ . Yearly costs, in 1000 €, difference in costs when including restrictions on manpower and number of times we delay PM per simulation run and percentage of times this directly leads to a failure. The results are for  $\ell = 3$  but are exactly the same for  $\ell = 5$ .

$\Delta$	costs				#PM	# delayed PM / % of times delay leads to CM							
	$r = 80$	additional costs				$r = 80$	$r = 60$		$r = 40$		$r = 20$		
0%	14.158	-1.14%**	-1.25%**	-2.30%**	217 132	50 522	1.9%	106 137	2.3%	162 180	4.0%		
10%	13.807	-0.77%**	-0.74%**	-0.62%**	217 198	50 610	1.7%	106 194	2.2%	162 168	3.9%		
20%	13.133	0.81%**	2.15%**	12.15%**	327 846	78 014	2.7%	161 568	3.1%	251 016	5.3%		
30%	12.111	0.37%**	0.56%**	10.06%**	327 846	77 948	1.6%	161 377	1.7%	246 952	3.6%		
40%	11.090	0.77%**	1.12%**	10.12%**	327 846	77 949	1.6%	161 374	1.7%	245 213	3.1%		
50%	10.068	1.19%**	1.74%**	16.39%**	327 846	77 935	1.5%	161 368	1.7%	245 283	3.1%		

Note: the results are averages over 10 runs, each of 50 000 simulated periods, and  $r$  denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

**Table 29** Results of delay of PM for a set of 80 components for p-BRP, with  $\alpha = 3$  year and  $\beta = 2$ . Yearly costs, in thousands of €, and difference in costs when including different restrictions on manpower. The results are for  $\ell = 3$  but are exactly the same for  $\ell = 5$ .

$\Delta$	costs		average costs and percentage additional costs with respect to $r=80$							
	$r = 80$		$r=60$		$r=40$		$r=20$			
0%	14.158	13.996	-1.1482%**	(0.0387)	13.982	-1.2485%**	(0.0587)	13.283	-2.2950%**	(0.0617)
10%	13.807	13.701	-0.7718%**	(0.0467)	13.705	-0.7399%**	(0.0467)	13.721	-0.6224%**	(0.0895)
20%	13.133	13.239	0.8131%**	(0.0347)	13.414	2.1468%**	(0.0447)	14.729	12.1529%**	(0.0863)
30%	12.111	12.156	0.3721%**	(0.0313)	12.179	0.5564%**	(0.0352)	13.330	10.0624%**	(0.1002)
40%	11.090	11.175	0.7704%**	(0.0480)	11.213	1.1151%**	(0.0365)	12.212	10.1207%**	(0.0638)
50%	10.068	10.188	1.1896%**	(0.0522)	10.243	1.7358%**	(0.0403)	11.719	16.3915%**	(0.0934)

Note: the results are averages over 10 runs, each of 50.000 simulated periods, brackets indicate standard deviations and  $r$  denotes the number of permitted PM operations per period.

We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

Total runtime for the results in this table was 9 minutes ( $\approx 0.001$  seconds per month).

**Table 30** Results of delay of PM for a set of 80 components for p-BRP, with  $\alpha = 5$  years and  $\beta = 2$ . Yearly costs, in 1000 €, difference in costs when including restrictions on manpower and number of times we delay PM per simulation run and percentage of times this directly leads to a failure. The results are for  $\ell = 1$ .

$\Delta$	costs				#PM	# delayed PM / % of times delay leads to CM						
	$r = 80$	additional costs				$r = 80$	$r = 60$			$r = 40$		$r = 20$
0%	8.542	-0.44%**	-0.51%**	-1.37%**	131 492	31 498	0.7%	64 857	1.2%	98 220	2.3%	
10%	8.232	-0.20%**	-0.04%**	0.84%**	131 511	31 498	0.7%	64 855	1.2%	98 230	2.3%	
20%	7.841	0.06%**	0.50%**	3.26%**	131 512	31 499	0.7%	64 851	1.2%	98 259	2.3%	
30%	7.436	-0.07%**	0.17%**	3.85%**	131 524	31 526	0.7%	64 874	1.2%	98 260	2.3%	
40%	6.936	0.47%**	1.17%**	10.80%**	198 033	48 051	0.7%	98 084	0.9%	148 148	1.8%	
50%	6.329	0.76%**	1.75%**	15.20%**	198 033	48 051	0.7%	98 084	0.9%	148 148	1.8%	

Note: the results are averages over 10 runs, each of 50 000 simulated periods, and  $r$  denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

**Table 31** Results of delay of PM for a set of 80 components for p-BRP, with  $\alpha = 5$  years and  $\beta = 2$ . Yearly costs, in thousands of €, and difference in costs when including different restrictions on manpower. The results are for  $\ell = 1$ .

$\Delta$	costs		average costs and percentage additional costs with respect to $r=80$							
	$r = 80$		$r=60$		$r=40$		$r=20$			
0%	8.542	8.504	-0.4438%**	(0.0199)	8.499	-0.5072%**	(0.0248)	8.425	-1.3748%**	(0.0623)
10%	8.232	8.215	-0.2018%**	(0.0494)	8.229	-0.0378%**	(0.0471)	8.301	0.8407%**	(0.1259)
20%	7.841	7.846	0.0611%**	(0.0443)	7.880	0.4993%**	(0.0511)	8.097	3.2609%**	(0.1233)
30%	7.436	7.431	-0.0746%**	(0.0670)	7.449	0.1724%**	(0.0774)	7.723	3.8505%**	(0.0561)
40%	6.936	6.969	0.4728%**	(0.0975)	7.017	1.1707%**	(0.1190)	7.685	10.8037%**	(0.2241)
50%	6.329	6.377	0.7636%**	(0.1124)	6.439	1.7522%**	(0.1410)	7.291	15.2017%**	(0.2758)

Note: the results are averages over 10 runs, each of 50.000 simulated periods, brackets indicate standard deviations and  $r$  denotes the number of permitted PM operations per period.

We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

Total runtime for the results in this table was 55 minutes ( $\approx 0.007$  seconds per month).

**Table 32** Results of delay of PM for a set of 80 components for p-BRP, with  $\alpha = 5$  years and  $\beta = 2$ . Yearly costs, in 1000 €, difference in costs when including restrictions on manpower and number of times we delay PM per simulation run and percentage of times this directly leads to a failure. The results are for  $\ell = 3$  but are exactly the same for  $\ell = 5$ .

$\Delta$	costs				additional costs				#PM	# delayed PM / % of times delay leads to CM			
	$r = 80$	$r = 60$	$r = 40$	$r = 20$	$r = 80$	$r = 60$	$r = 40$	$r = 20$		$r = 80$	$r = 60$	$r = 40$	$r = 20$
0%	8.542	-0.77%**	-0.84%**	-1.61%**	131 492	31 497	1.0%	64 854	1.3%	98 321	2.4%		
10%	8.232	-0.20%**	-0.02%**	0.96%**	131 511	31 513	1.0%	64 871	1.3%	98 331	2.4%		
20%	7.841	0.34%**	0.80%**	3.73%**	131 512	31 513	1.0%	64 871	1.3%	98 314	2.4%		
30%	7.436	0.31%**	0.57%**	4.61%**	131 524	31 545	0.9%	64 892	1.3%	98 311	2.3%		
40%	6.936	1.77%**	2.43%**	13.27%**	198 033	48 069	1.0%	98 092	1.0%	148 557	1.9%		
50%	6.329	1.96%**	2.91%**	20.10%**	198 033	48 045	0.9%	98 085	1.0%	148 587	2.0%		

Note: the results are averages over 10 runs, each of 50 000 simulated periods, and  $r$  denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

**Table 33** Results of delay of PM for a set of 80 components for p-BRP, with  $\alpha = 5$  years and  $\beta = 2$ . Yearly costs, in thousands of €, and difference in costs when including different restrictions on manpower. The results are for  $\ell = 3$  but are exactly the same for  $\ell = 5$ .

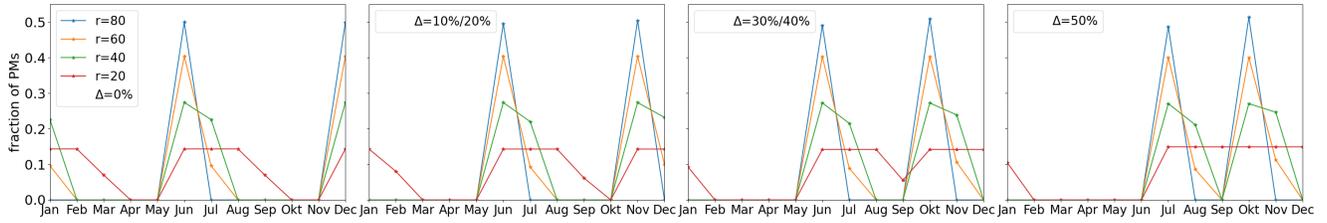
$\Delta$	costs		average costs and percentage additional costs with respect to $r=80$							
	$r = 80$		$r=60$		$r=40$		$r=20$			
0%	8.542	8.476	-0.7741%**	(0.0195)	8.470	-0.8436%**	(0.0451)	8.405	-1.6116%**	(0.0587)
10%	8.232	8.216	-0.1954%**	(0.0491)	8.230	-0.0194%**	(0.0495)	8.310	0.9553%**	(0.1214)
20%	7.841	7.868	0.3387%**	(0.0398)	7.904	0.7978%**	(0.0385)	8.133	3.7285%**	(0.1556)
30%	7.436	7.459	0.3082%**	(0.0542)	7.479	0.5705%**	(0.0870)	7.780	4.6132%**	(0.1051)
40%	6.936	7.059	1.7668%**	(0.1126)	7.105	2.4291%**	(0.1187)	7.857	13.2716%**	(0.2485)
50%	6.329	6.453	1.9638%**	(0.1298)	6.513	2.9115%**	(0.1348)	7.600	20.0951%**	(0.3275)

Note: the results are averages over 10 runs, each of 50.000 simulated periods, brackets indicate standard deviations and  $r$  denotes the number of permitted PM operations per period.

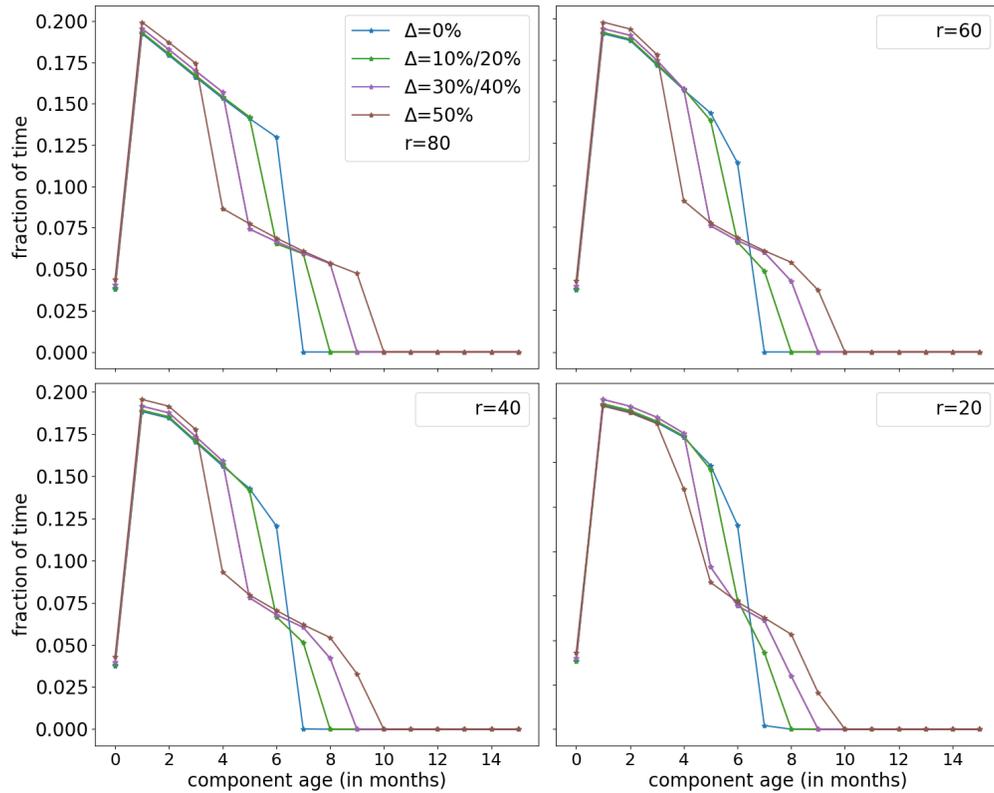
We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

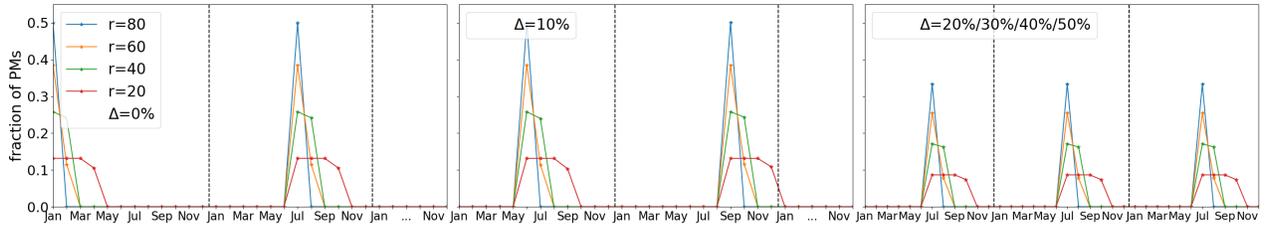
Total runtime for the results in this table was 60 minutes ( $\approx 0.007$  seconds per month).



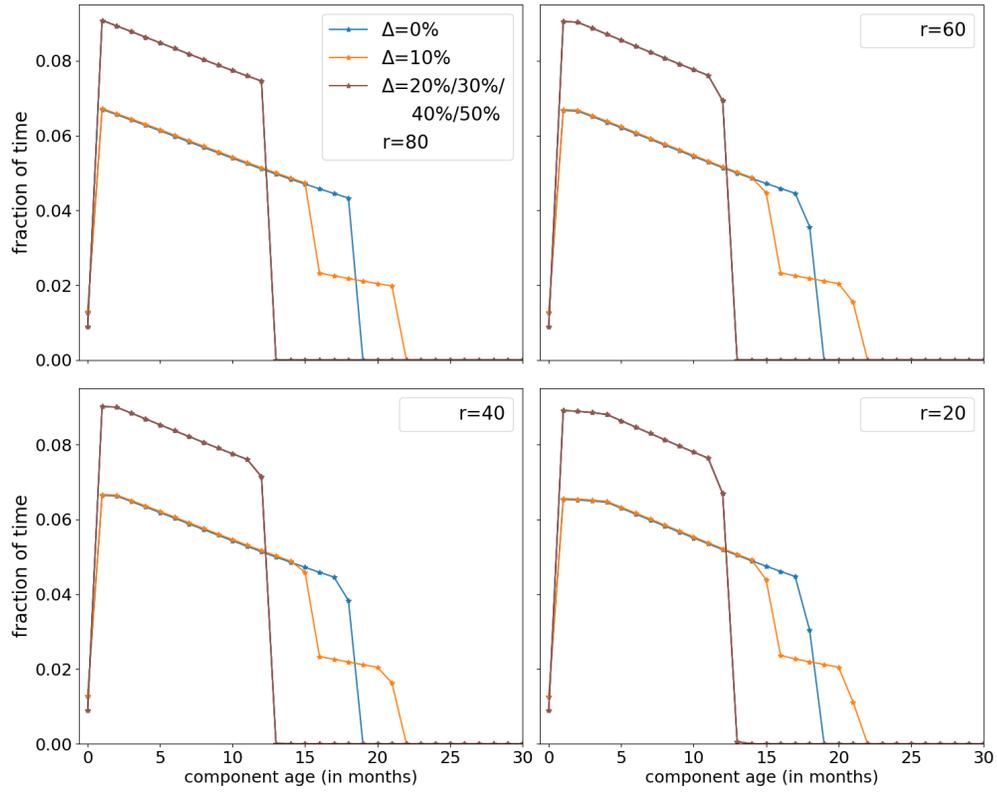
**Figure 13** Distribution of the number of PM activities per period for the set of 80 components when delaying PM, for  $\alpha = 1$  year and  $\beta = 2$ , for p-BRP with and without including constraints on the number of allowed PM activities per period  $r$  and for different  $\Delta$ . The results are for  $\ell = 1$ .



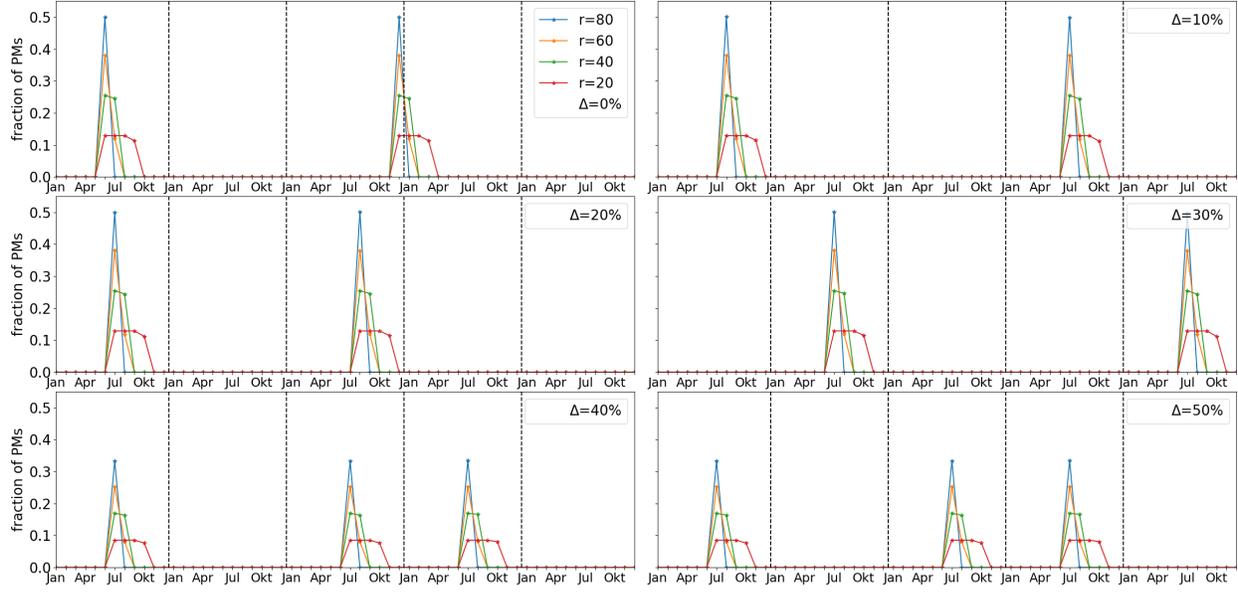
**Figure 14** Age distribution of the set of 80 components for  $\alpha = 1$  year and  $\beta = 2$ , for p-BRP when delaying PM, with and without including constraints on the number of allowed PM activities per period  $r$ . The results are for  $\ell = 1$ .



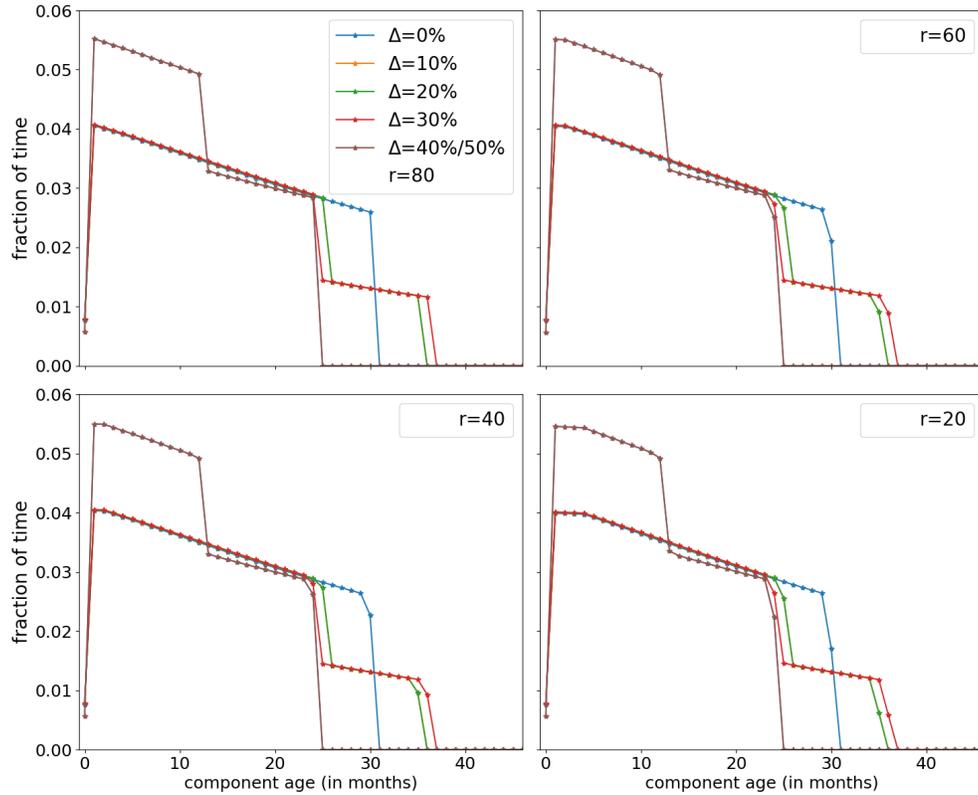
**Figure 15** Distribution of the number of PM activities per period for the set of 80 components when delaying PM, for  $\alpha = 3$  years and  $\beta = 2$ , for p-BRP with and without including constraints on the number of allowed PM activities per period  $r$  and for different  $\Delta$ . The results are for  $\ell = 1$ .



**Figure 16** Age distribution of the set of 80 components for  $\alpha = 3$  years and  $\beta = 2$ , for p-BRP when delaying PM, with and without including constraints on the number of allowed PM activities per period  $r$ . The results are for  $\ell = 1$ .



**Figure 17** Distribution of the number of PM activities per period for the set of 80 components when delaying PM, for  $\alpha = 5$  years and  $\beta = 2$ , for p-BRP with and without including constraints on the number of allowed PM activities per period  $r$  and for different  $\Delta$ . The results are for  $\ell = 1$ .



**Figure 18** Age distribution of the set of 80 components for  $\alpha = 5$  years and  $\beta = 2$ , for p-BRP when delaying PM, with and without including constraints on the number of allowed PM activities per period  $r$ . The results are for  $\ell = 1$ . The lines for  $\Delta = 10\%$  and  $\Delta = 20\%$  almost coincide.

### E.3 Replanning for p-ARP

**Table 34** Results of replanning PM for a set of 80 components for p-ARP, with  $\alpha = 1$  year and  $\beta = 2$ . Yearly costs, in thousands of €, and difference in costs when including different restrictions on manpower. The results are for  $\ell = 3$  but are exactly the same for  $\ell = 5$  and  $\ell = 7$ .

$\Delta$	costs		average costs and percentage additional costs with respect to r=80							
	r = 80		r=60		r=40		r=20			
0%	40.060	40.060	-0.0001%	(0.0002)	40.060	0.0001%	(0.0008)	40.060	0.0006%	(0.0023)
10%	40.004	40.004	-0.0012%	(0.0017)	40.004	-0.0014%	(0.0031)	40.002	-0.0051%	(0.0110)
20%	39.667	39.667	0.0005%	(0.0016)	39.683	0.0411%**	(0.0372)	39.733	0.1658%**	(0.0482)
30%	39.192	39.191	-0.0003%	(0.0024)	39.207	0.0385%*	(0.0465)	39.304	0.2860%**	(0.0848)
40%	38.439	38.442	0.0092%**	(0.0066)	38.487	0.1261%**	(0.0407)	38.701	0.6837%**	(0.0799)
50%	37.611	37.622	0.0281%*	(0.0297)	37.775	0.4359%**	(0.0699)	37.868	0.6812%**	(0.0615)

Note: the results are averages over 10 runs, each of 50.000 simulated periods, brackets indicate standard deviations and r denotes the number of permitted PM operations per period.

We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

The total runtime for the results in this table was 15 minutes ( $\approx 0.0018$  seconds per month).

**Table 35** Results of replanning PM for a set of 80 components for p-ARP, with  $\alpha = 3$  years and  $\beta = 2$ . Yearly costs, in thousands of €, and difference in costs when including different restrictions on manpower. The results are for  $\ell = 3$  but are exactly the same for  $\ell = 5$  and  $\ell = 7$ .

$\Delta$	costs		average costs and percentage additional costs with respect to r=80							
	r = 80		r=60		r=40		r=20			
0%	13.516	13.516	0.0000%	(0.0004)	13.516	0.0001%	(0.0014)	13.516	-0.0007%	(0.0012)
10%	13.240	13.240	0.0001%	(0.0008)	13.240	0.0004%	(0.0035)	13.242	0.0149%	(0.0306)
20%	12.713	12.771	0.4595%**	(0.0228)	12.887	1.3738%**	(0.0216)	12.763	0.3984%**	(0.1092)
30%	11.784	11.881	0.8251%**	(0.0200)	12.054	2.2930%**	(0.0209)	12.141	3.0354%**	(0.0602)
40%	10.852	10.990	1.2768%**	(0.0349)	11.219	3.3910%**	(0.0455)	11.347	4.5618%**	(0.0926)
50%	9.905	10.093	1.8964%**	(0.0286)	10.376	4.7534%**	(0.0613)	10.541	6.4168%**	(0.0855)

Note: the results are averages over 10 runs, each of 50.000 simulated periods, brackets indicate standard deviations and r denotes the number of permitted PM operations per period.

We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

The total runtime for the results in this table was 12.5 minutes ( $\approx 0.0015$  seconds per month).

**Table 36** Results of replanning PM for a set of 80 components for p-ARP, with  $\alpha = 5$  years and  $\beta = 2$ . Yearly costs, in thousands of €, and difference in costs when including different restrictions on manpower. The results are for  $\ell = 3$  but are exactly the same for  $\ell = 5$  and  $\ell = 7$ .

$\Delta$	costs		average costs and percentage additional costs with respect to r=80							
	$r = 80$		r=60		r=40		r=20			
0%	8.133	8.133	0.0003%	(0.0010)	8.133	0.0003%	(0.0013)	8.133	-0.0001%	(0.0023)
10%	7.878	7.878	-0.0004%	(0.0012)	7.878	-0.0003%	(0.0058)	7.877	-0.0086%	(0.0273)
20%	7.434	7.434	0.0001%	(0.0006)	7.436	0.0226%**	(0.0089)	7.436	1.0018%**	(0.0284)
30%	6.980	6.981	0.0003%	(0.0010)	6.984	0.0445%**	(0.0080)	7.097	1.6760%**	(0.0338)
40%	6.521	6.521	0.0003%	(0.0008)	6.526	0.0755%**	(0.0189)	6.680	2.4442%**	(0.0314)
50%	6.054	6.054	0.0015%*	(0.0019)	6.062	0.1310%**	(0.0232)	6.256	3.3346%**	(0.0550)

Note: the results are averages over 10 runs, each of 50.000 simulated periods, brackets indicate standard deviations and r denotes the number of permitted PM operations per period.

We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

The total runtime for the results in this table was 12.5 minutes ( $\approx 0.0015$  seconds per month).

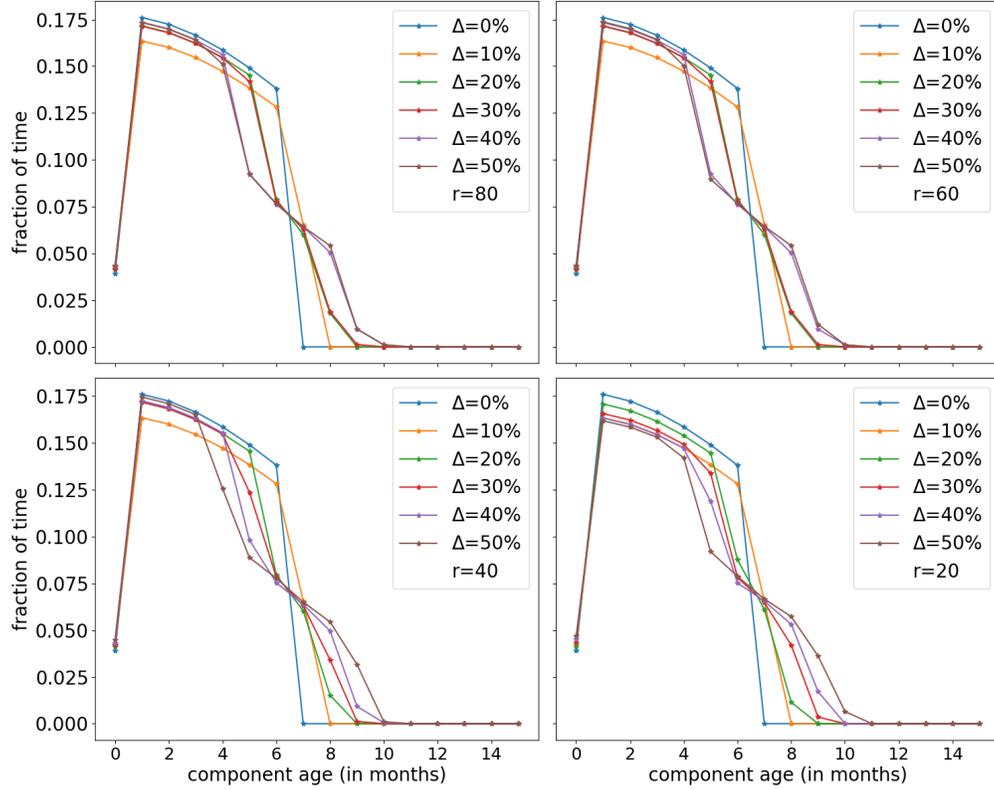
**Table 37** Results of replanning PM for p-ARP. Percentage additional costs when including restrictions on manpower with respect to the optimal planning without this constraint, divided into a part due to more/less maintenance activities and a part due to maintenance in more/less expensive periods.

$\Delta$	more or less maintenance / more or less expensive periods					
	$\alpha = 1, \beta = 2$			$\alpha = 5, \beta = 2$		
	$r = 60$	$r = 40$	$r = 20$	$r = 60$	$r = 40$	$r = 20$
0%						
10%	-0.00%* / -0.00%	0.00% / 0.00%	0.00% / 0.00%	0.00%/0.00%	0.00% / 0.00%	-0.00% / 0.00%
20%	0.00% / -0.00%**	-0.05%** / 0.09%**	-0.24%** / 0.41%**	-0.00% / 0.00%*	-0.00% / 0.03%**	-0.01% / 1.01%**
30%	0.00% / -0.00%**	0.86%** / -0.82%**	0.62%** / -0.34%**	-0.00% / 0.00%*	-0.01%* / 0.05%**	0.02% / 1.61%**
40%	-0.02%** / 0.02%**	-0.25%** / 0.37%**	-0.85%** / 1.53%**	-0.00% / 0.00%	-0.00% / 0.08%**	0.02%* / 2.42%**
50%	0.24%** / -0.21%**	2.33%** / -1.89%**	0.89%** / -0.21%**	0.00% / 0.00%*	-0.01% / 0.14%**	0.01% / 3.33%**

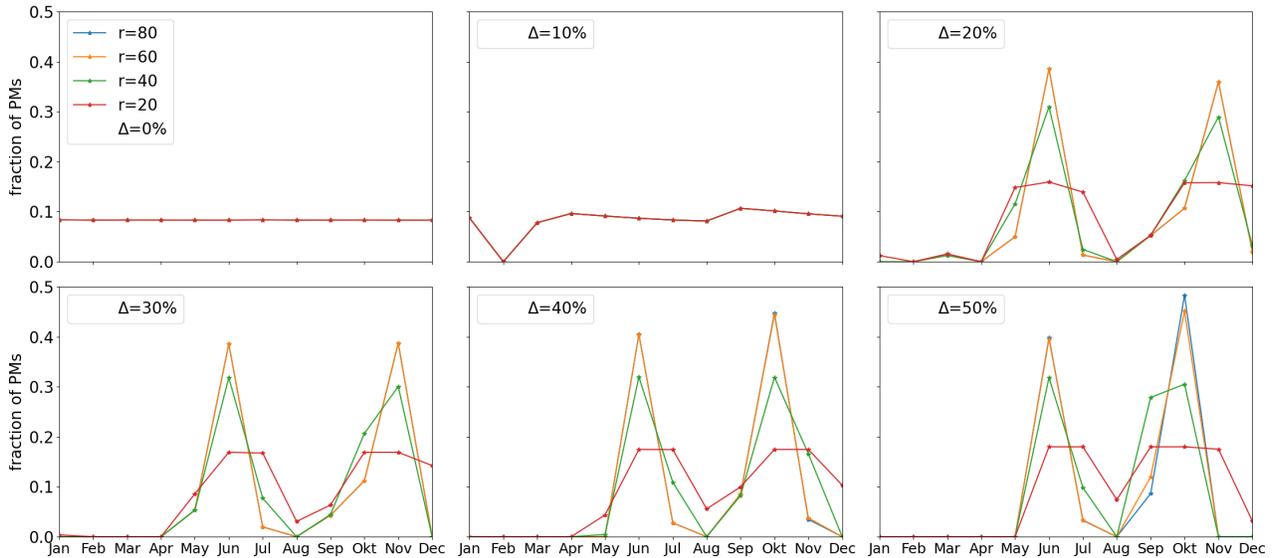
Note: the results are averages over 10 runs, each of 50 000 simulated periods, and r denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

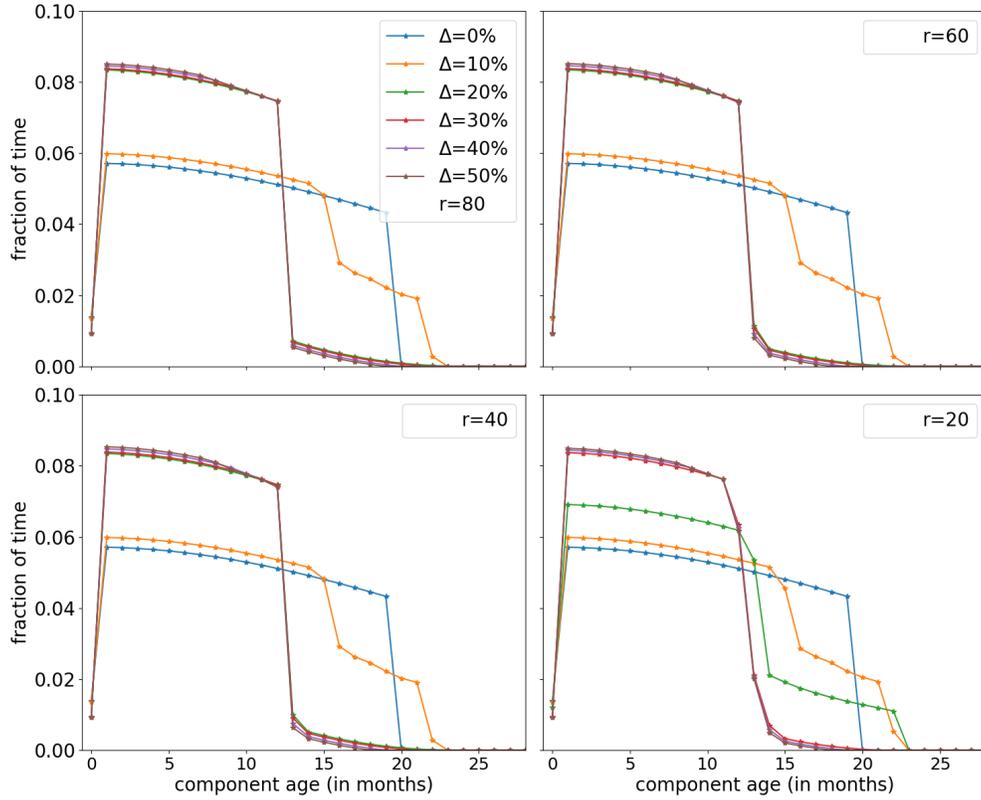
Results are for the maximum number of periods to move up/delay PM  $\ell = 3$  but are exactly the same for  $\ell = 5, 7$ .



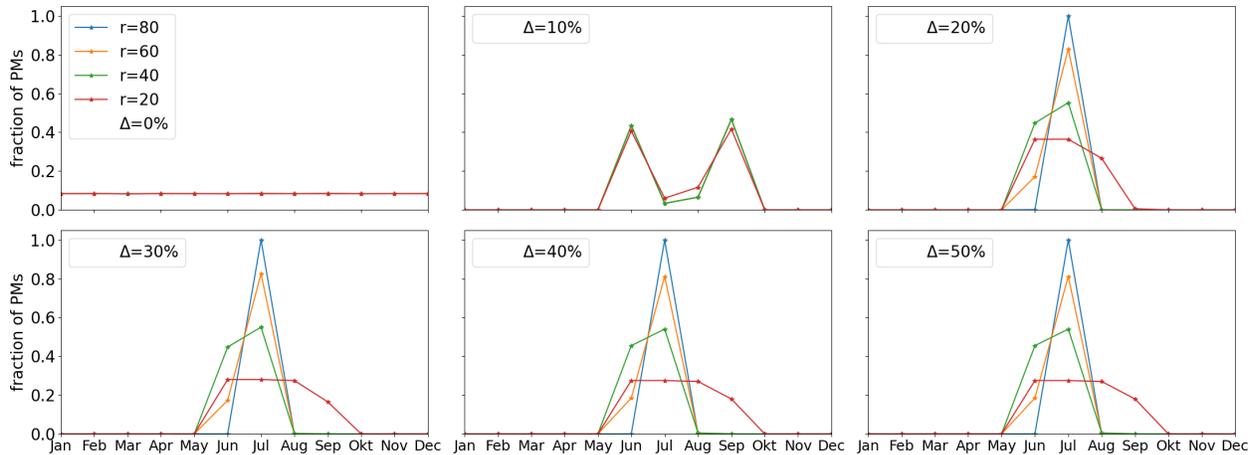
**Figure 19** Age distribution of the set of 80 components for  $\alpha = 1$  and  $\beta = 2$ , for p-ARP when replanning PM, with and without including constraints on the number of allowed PM activities per period  $r$ . The results are for  $\ell = 3$  but are exactly the same for  $\ell = 5$  and  $\ell = 7$ .



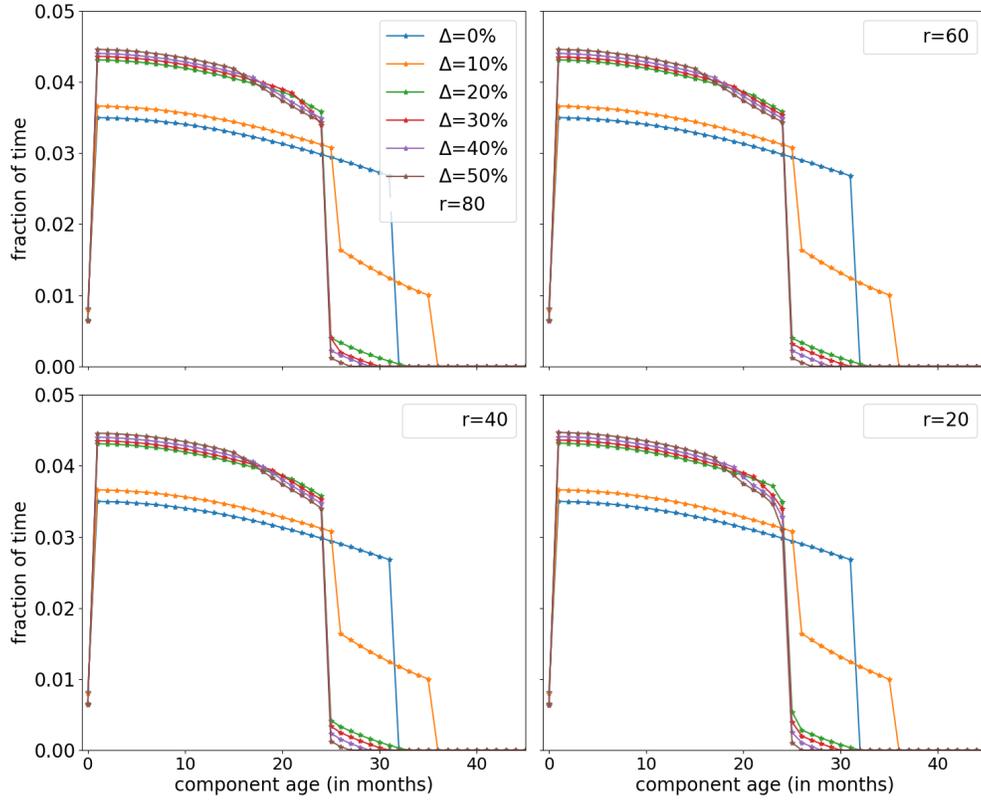
**Figure 20** Distribution of the number of PM activities per period for the set of 80 components when replanning PM, for  $\alpha = 1$ ,  $\beta = 2$ , for p-ARP with and without including constraints on the number of allowed PM activities per period  $r$  and for different  $\Delta$ . The results are for  $\ell = 3$  but are exactly the same for  $\ell = 5$  and  $\ell = 7$ . Note that the lines for  $r = 80$  and  $r = 60$  almost coincide.



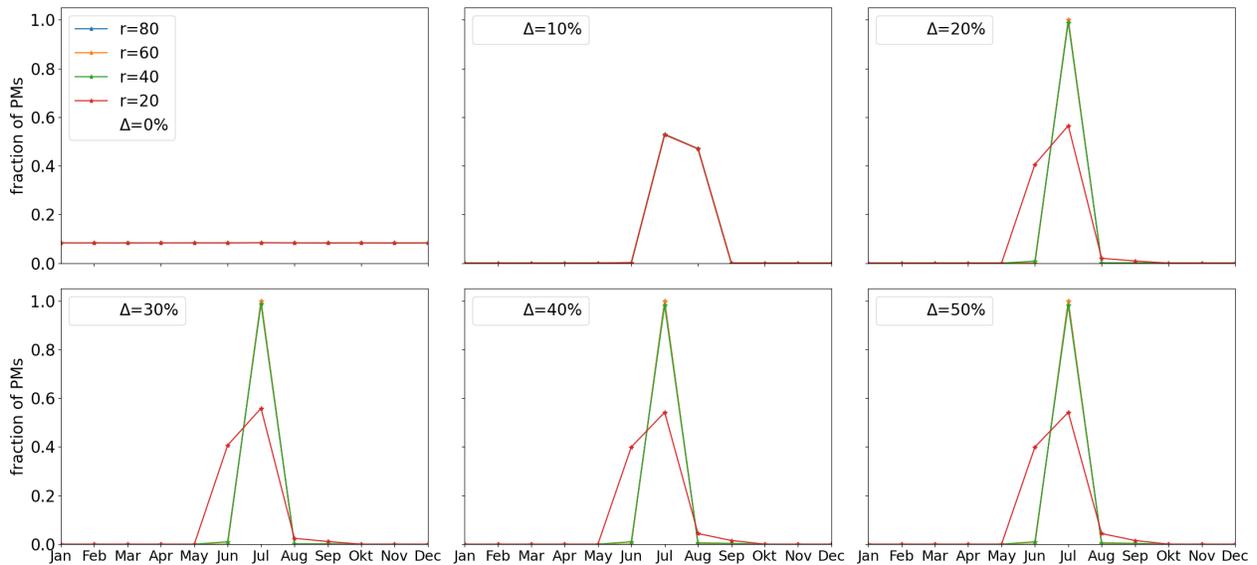
**Figure 21** Age distribution of the set of 80 components for  $\alpha = 3$  and  $\beta = 2$ , for p-ARP when replanning PM, with and without including constraints on the number of allowed PM activities per period  $r$ . The results are for  $\ell = 3$  but are exactly the same for  $\ell = 5$  and  $\ell = 7$ . Note that the lines for  $\Delta = 20\%$ ,  $\Delta = 30\%$ ,  $\Delta = 40\%$  and  $\Delta = 50\%$  are really close for  $r > 20$ .



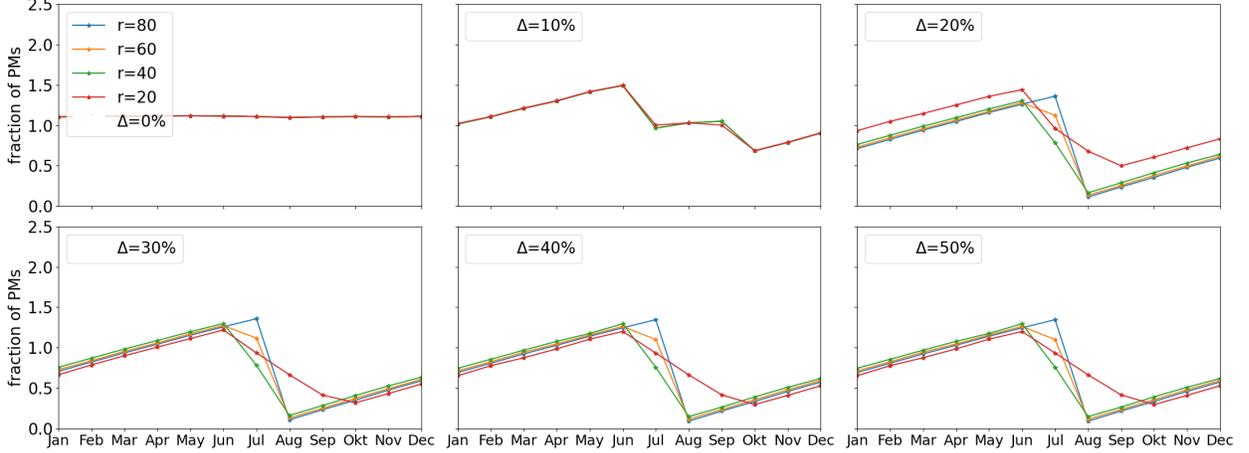
**Figure 22** Distribution of the number of PM activities per period for the set of 80 components when replanning PM, for  $\alpha = 3$ ,  $\beta = 2$ , for p-ARP with and without including constraints on the number of allowed PM activities per period  $r$  and for different  $\Delta$ . The results are for  $\ell = 3$  but are exactly the same for  $\ell = 5$  and  $\ell = 7$ .



**Figure 23** Age distribution of the set of 80 components for  $\alpha = 5$  and  $\beta = 2$ , for p-ARP when replanning PM, with and without including constraints on the number of allowed PM activities per period  $r$ . The results are for  $\ell = 3$  but are exactly the same for  $\ell = 5$  and  $\ell = 7$ .



**Figure 24** Distribution of the number of PM activities per period for the set of 80 components when replanning PM, for  $\alpha = 5$ ,  $\beta = 2$ , for p-ARP with and without including constraints on the number of allowed PM activities per period  $r$  and for different  $\Delta$ . The results are for  $\ell = 3$  but are exactly the same for  $\ell = 5$  and  $\ell = 7$ . Note that the lines for  $r = 80$  and  $r = 60$  coincide.



**Figure 25** Average number of CM activities per period for the set of 80 components when replanning PM, for  $\alpha = 3$  years and  $\beta = 2$ , for p-ARP with and without including constraints on the number of allowed PM activities per period  $r$  and for different  $\Delta$ . The results are for  $\ell = 3$  but are exactly the same for  $\ell = 5$  and  $\ell = 7$ .

#### E.4 Replanning for p-BRP

**Table 38** Results of replanning PM for a set of 80 components for p-BRP, with  $\alpha = 1$  year and  $\beta = 2$ . Yearly costs, in thousands of €, and difference in costs when including different restrictions on manpower.

$\Delta$	costs		average costs and percentage additional costs with respect to $r=80$							
	$r = 80$		$r=60$		$r=40$		$r=20$			
0%	41.463	41.463	-0.0001%	(0.0009)	41.463	-0.0002%	(0.0030)	41.463	0.0002%	(0.0048)
10%	41.385	41.399	0.0359%**	(0.0025)	41.415	0.0722%**	(0.0026)	41.442	0.1390%**	(0.0181)
20%	40.898	40.928	0.0731%**	(0.0040)	40.958	0.1466%**	(0.0045)	41.080	0.4446%**	(0.0114)
30%	40.346	40.362	0.0403%**	(0.0076)	40.380	0.0831%**	(0.0080)	40.735	0.9637%**	(0.0422)
40%	39.425	39.447	0.0547%**	(0.0106)	39.469	0.1130%**	(0.0107)	39.955	1.3434%**	(0.0192)
50%	38.468	38.618	0.3905%**	(0.0080)	38.768	0.7812%**	(0.0139)	40.371	4.9469%**	(0.0682)

Note: the results are averages over 10 runs, each of 50.000 simulated periods, brackets indicate standard deviations and  $r$  denotes the number of permitted PM operations per period.

We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test,  $*p < 0.05$ ,  $**p < 0.01$ .

The total runtime for the results in this table was 6.5 minutes.

**Table 39** Results of replanning PM for a set of 80 components for p-BRP, with  $\alpha = 3$  year and  $\beta = 2$ . Yearly costs, in thousands of €, and difference in costs when including different restrictions on manpower.

$\Delta$	costs		average costs and percentage additional costs with respect to r=80							
	r = 80		r=60		r=40		r=20			
0%	14.158	14.158	-0.0025%	(0.0073)	14.157	-0.0071%	(0.0133)	14.157	-0.0087%	(0.0171)
10%	13.807	13.808	0.0046%*	(0.0075)	13.808	0.0037%	(0.0102)	13.821	0.0968%**	(0.0121)
20%	13.133	13.160	0.2119%**	(0.0068)	13.189	0.4270%**	(0.0088)	13.261	0.9800%**	(0.1394)
30%	12.118	12.153	0.3444%**	(0.0109)	12.195	0.6941%**	(0.0142)	12.592	3.9721%**	(0.0235)
40%	11.090	11.145	0.5014%**	(0.0157)	11.202	1.0105%**	(0.0206)	11.731	5.7855%**	(0.0352)
50%	10.068	10.138	0.6901%**	(0.0215)	10.208	1.3911%**	(0.0283)	10.870	7.9667%**	(0.0502)

Note: the results are averages over 10 runs, each of 50.000 simulated periods, brackets indicate standard deviations and r denotes the number of permitted PM operations per period.

We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

The total runtime for the results in this table was 30 minutes.

**Table 40** Results of replanning PM for a set of 80 components for p-BRP, with  $\alpha = 5$  year and  $\beta = 2$ . Yearly costs, in thousands of €, and difference in costs when including different restrictions on manpower.

$\Delta$	costs		average costs and percentage additional costs with respect to r=80							
	r = 80		r=60		r=40		r=20			
0%	8.542	8.542	0.0002%	(0.0016)	8.542	0.0004%	(0.0017)	8.542	0.0004%	(0.0022)
10%	8.232	8.232	-0.0011%	(0.0143)	8.232	0.0014%	(0.0205)	8.232	0.0032%	(0.0180)
20%	7.841	7.841	0.0066%*	(0.0074)	7.841	0.0062%	(0.0128)	7.841	-0.0003%	(0.0217)
30%	7.436	7.436	-0.0092%	(0.0190)	7.436	-0.0080%	(0.0252)	7.461	0.3350%**	(0.0607)
40%	6.936	6.945	0.1286%**	(0.0819)	6.952	0.2238%**	(0.0903)	7.045	1.5621%**	(0.1609)
50%	6.329	6.343	0.2226%**	(0.0895)	6.354	0.4066%**	(0.1099)	6.505	2.7906%**	(0.0873)

Note: the results are averages over 10 runs, each of 50.000 simulated periods, brackets indicate standard deviations and r denotes the number of permitted PM operations per period.

We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .

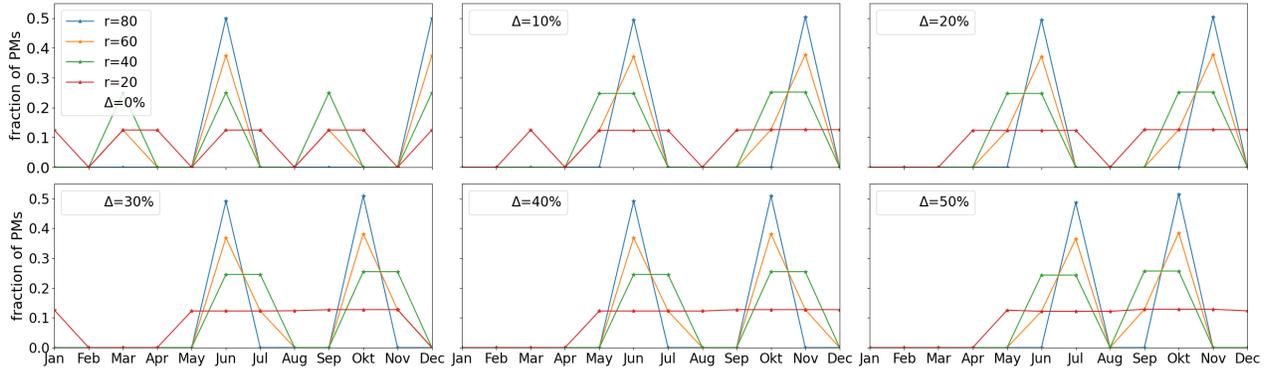
The total runtime for the results in this table was 90 minutes.

**Table 41** Results of replanning PM for p-BRP. Percentage additional costs when including restrictions on manpower with respect to the optimal planning without this constraint, divided into a part due to more/less maintenance activities and a part due to maintenance in more/less expensive periods.

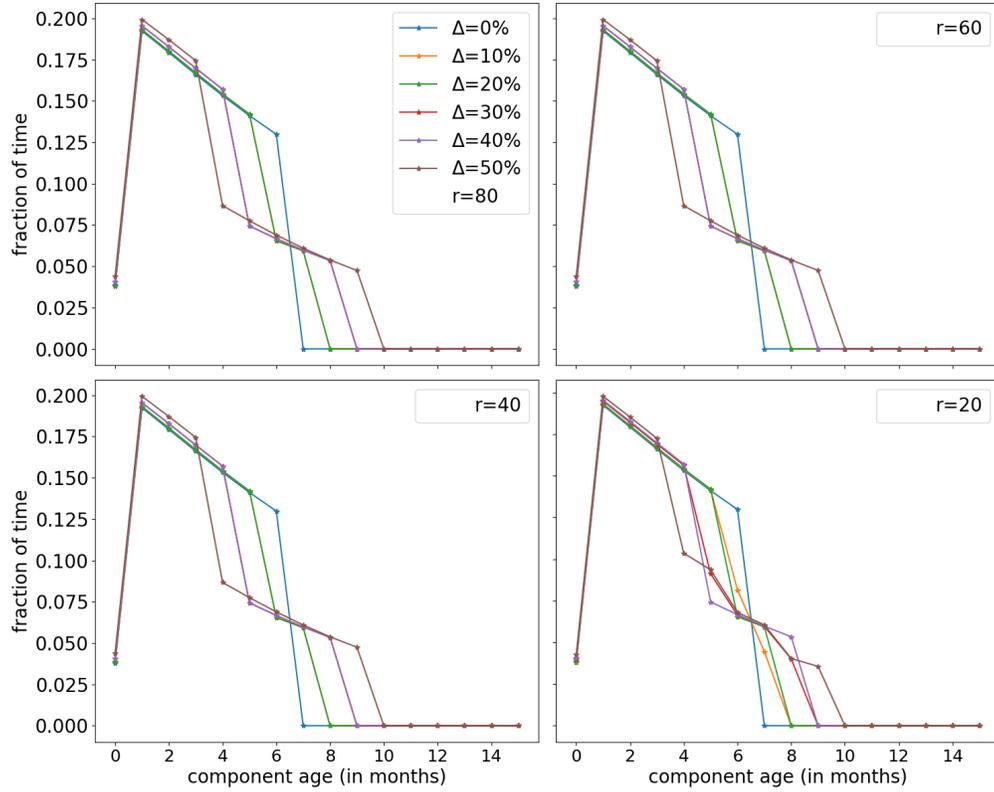
$\Delta$	more or less maintenance / more or less expensive periods					
	$\alpha = 1, \beta = 2$			$\alpha = 5, \beta = 2$		
	$r = 60$	$r = 40$	$r = 20$	$r = 60$	$r = 40$	$r = 20$
0%	-0.00%/0.00%	-0.00%/0.00%	0.00% /0.00%	0.00% / 0.00%	0.00% / 0.00%	0.00% / 0.00%
10%	-0.00%/0.04%**	-0.00%/0.07%**	-0.23%**/0.37%**	-0.00% /-0.00%	0.00% /-0.00%	0.00% /-0.00%**
20%	-0.00%/0.07%**	-0.00%/0.15%**	-0.00% /0.44%**	0.01% / 0.00%**	0.00% / 0.00%	-0.00% / 0.00%**
30%	0.00%/0.04%**	0.00%/0.08%**	-0.77%**/1.73%**	-0.01% /-0.00%*	-0.01% /-0.00%*	-0.12%**/ 0.45%**
40%	0.00%/0.05%**	0.00%/0.11%**	0.00%**/1.34%**	-0.17%**/ 0.30%**	-0.36%**/ 0.59%**	-0.84%**/ 2.41%**
50%	-0.00%/0.39%**	-0.00%/0.78%**	-2.13%**/7.07%**	-0.19%**/ 0.41%**	-0.40%**/ 0.81%**	-0.55%**/ 3.34%**

Note: the results are averages over 10 runs, each of 50 000 simulated periods, and  $r$  denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

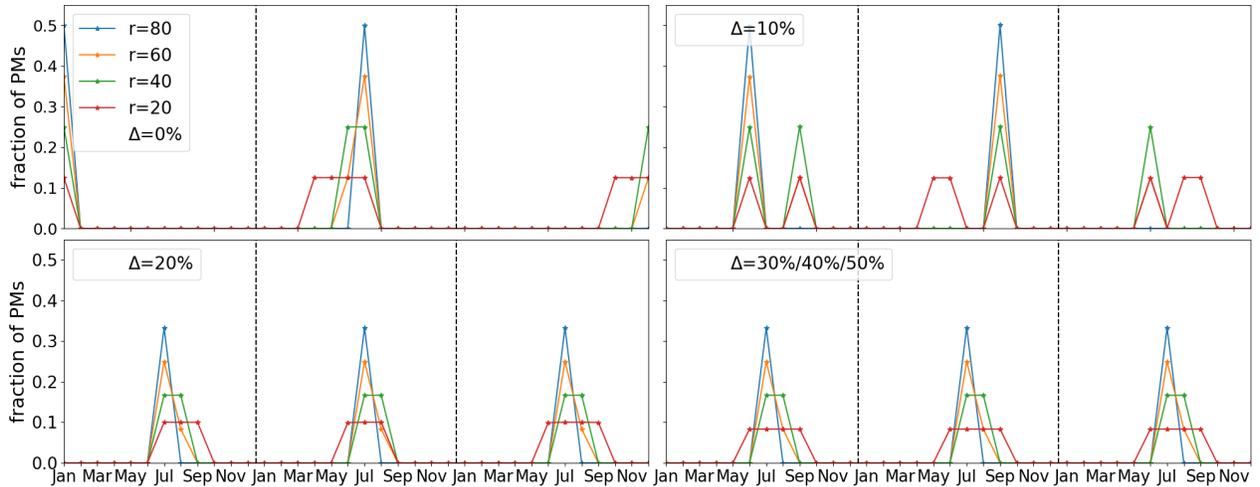
\* and \*\* denote the cost difference is significantly different from zero, based on a two-sided t-test, \* $p < 0.05$ , \*\* $p < 0.01$ .



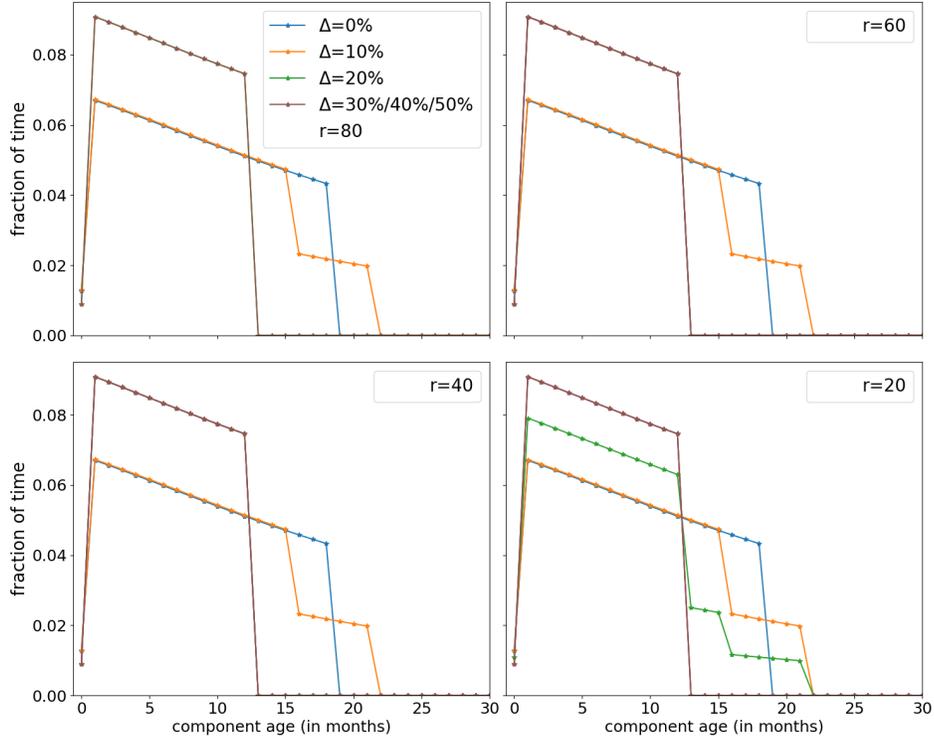
**Figure 26** Distribution of the number of PM activities per period for the set of 80 components when replanning PM, for  $\alpha = 1$  year and  $\beta = 2$ , for p-BRP with and without including constraints on the number of allowed PM activities per period  $r$  and for different  $\Delta$ . The results are for  $\ell = 1$ .



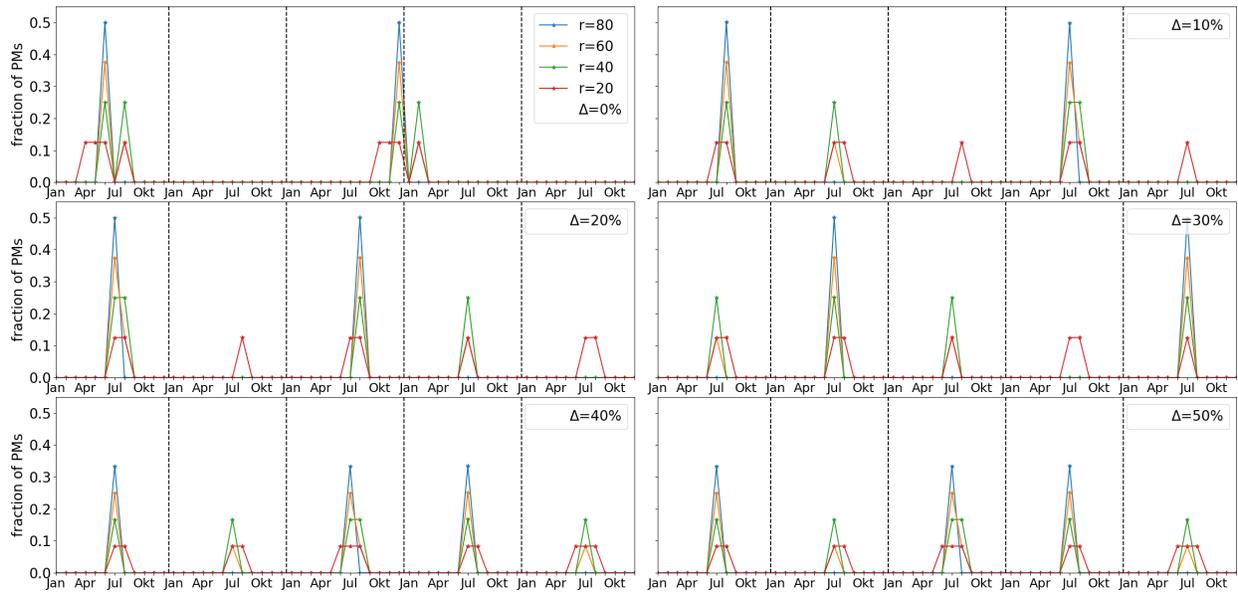
**Figure 27** Age distribution of the set of 80 components for  $\alpha = 1$  year and  $\beta = 2$ , for p-BRP when replanning PM, with and without including constraints on the number of allowed PM activities per period  $r$ . The results are for  $\ell = 1$ . Note that the lines for  $\Delta = 10\%$  and  $\Delta = 20\%$  coincide when  $r > 20$ , just as the lines for  $\Delta = 30\%$  and  $\Delta = 40\%$ .



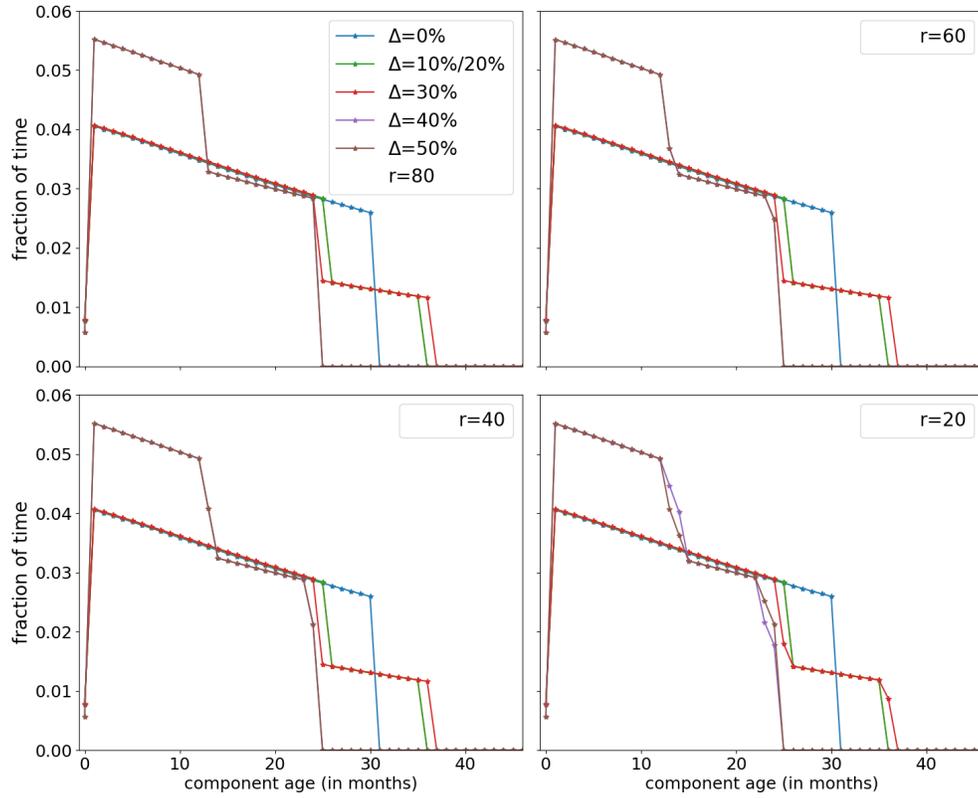
**Figure 28** Distribution of the number of PM activities per period for the set of 80 components when replanning PM, for  $\alpha = 3$  years and  $\beta = 2$ , for p-BRP with and without including constraints on the number of allowed PM activities per period  $r$  and for different  $\Delta$ . The results are for  $\ell = 1$ .



**Figure 29** Age distribution of the set of 80 components for  $\alpha = 3$  years and  $\beta = 2$ , for p-BRP when replanning PM, with and without including constraints on the number of allowed PM activities per period  $r$ . The results are for  $\ell = 1$ . Note that the line for  $\Delta = 20\%$  coincides with the line for  $\Delta = 30\%, 40\%, 50\%$  when  $r > 20$ .



**Figure 30** Distribution of the number of PM activities per period for the set of 80 components when replanning PM, for  $\alpha = 5$  years and  $\beta = 2$ , for p-BRP with and without including constraints on the number of allowed PM activities per period  $r$  and for different  $\Delta$ . The results are for  $\ell = 1$ .



**Figure 31** Age distribution of the set of 80 components for  $\alpha = 5$  years and  $\beta = 2$ , for p-BRP when replanning PM, with and without including constraints on the number of allowed PM activities per period  $r$ . The results are for  $\ell = 1$ . Note that the lines for  $\Delta = 40\%$  and  $\Delta = 50\%$  coincide when  $r > 20$ .

## E.5 Comparison heuristics with naive heuristics and optimal solutions

**Table 42** Comparison of the *DMD* and *DMR* heuristics to naive heuristics in which we choose randomly which PM activities to replan and where we move up or delay with equal probability when both are possible for replanning, for p-ARP with  $\alpha = 1$  year and  $\beta = 2$ . Percentage additional costs compared to the naive heuristics, for different restrictions on the maximum number of PM activities per period  $r$ . We used  $\ell = 1$  for *DMD* and  $\ell = 3$  for *DMR*.

$\Delta$	<i>DMD</i> (delay)						<i>DMR</i> (replan)					
	$r = 60$		$r = 40$		$r = 20$		$r = 60$		$r = 40$		$r = 20$	
0%	-0.000%	(0.000)	0.000%	(0.000)	-0.001%	(0.002)	-0.000%	(0.000)	0.000%	(0.001)	0.000%	(0.002)
10%	0.000%	(0.003)	0.000%	(0.003)	-0.001%	(0.008)	-0.001% <sup>†</sup>	(0.002)	-0.001%	(0.004)	-0.007%	(0.016)
20%	-0.000%	(0.002)	-0.099% <sup>††</sup>	(0.037)	-0.114% <sup>††</sup>	(0.057)	0.001%	(0.003)	-0.106% <sup>††</sup>	(0.031)	-0.400% <sup>††</sup>	(0.062)
30%	-0.000%	(0.005)	0.246% <sup>§§</sup>	(0.081)	1.479% <sup>§§</sup>	(0.088)	-0.002%	(0.006)	-0.097% <sup>††</sup>	(0.059)	-0.354% <sup>††</sup>	(0.070)
40%	-0.002%	(0.008)	-0.080% <sup>††</sup>	(0.073)	0.613% <sup>§§</sup>	(0.105)	-0.015% <sup>†</sup>	(0.017)	-0.408% <sup>††</sup>	(0.062)	-0.682% <sup>††</sup>	(0.100)
50%	0.180% <sup>§§</sup>	(0.044)	2.405% <sup>§§</sup>	(0.146)	3.456% <sup>§§</sup>	(0.173)	-0.028% <sup>†</sup>	(0.033)	-0.217% <sup>††</sup>	(0.063)	-0.988% <sup>††</sup>	(0.108)

Note: the results are averages over 10 runs, each of 50 000 simulated periods, and  $r$  denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

<sup>†</sup> and <sup>††</sup> denote that the *DMR* or *DMD* heuristic performed significantly better than the naive heuristics and <sup>§</sup> and <sup>§§</sup> denote that they performed significantly worse, based on a one-sided t-test, <sup>†,§</sup>  $p < 0.05$ , <sup>††,§§</sup>  $p < 0.01$ .

**Table 43** Comparison of the *DMD* and *DMR* heuristics to naive heuristics in which we choose randomly which PM activities to replan and where we move up or delay with equal probability when both are possible for replanning, for p-ARP with  $\alpha = 5$  years and  $\beta = 2$ . Percentage additional costs compared to the naive heuristics, for different restrictions on the maximum number of PM activities per period  $r$ . We used  $\ell = 1$  for *DMD* and  $\ell = 3$  for *DMR*.

$\Delta$	<i>DMD</i> (delay)						<i>DMR</i> (replan)					
	$r = 60$		$r = 40$		$r = 20$		$r = 60$		$r = 40$		$r = 20$	
0%	-0.000%	(0.000)	-0.000%	(0.001)	0.001%	(0.001)	0.000%	(0.001)	0.000%	(0.002)	0.000%	(0.002)
10%	-0.002%	(0.005)	-0.001%	(0.008)	-0.010%	(0.026)	-0.001%	(0.003)	-0.001%	(0.007)	-0.023% <sup>†</sup>	(0.037)
20%	-0.001%	(0.001)	0.001%	(0.012)	-0.051% <sup>†</sup>	(0.072)	-0.000%	(0.001)	0.006% <sup>§</sup>	(0.008)	0.237% <sup>§§</sup>	(0.042)
30%	-0.001% <sup>†</sup>	(0.002)	-0.007%	(0.015)	-0.073% <sup>††</sup>	(0.044)	0.000%	(0.001)	0.016% <sup>§§</sup>	(0.008)	0.437% <sup>§§</sup>	(0.008)
40%	-0.001% <sup>†</sup>	(0.001)	-0.000%	(0.015)	-0.051% <sup>††</sup>	(0.063)	0.000%	(0.001)	0.014%	(0.027)	0.673% <sup>§§</sup>	(0.053)
50%	-0.001% <sup>†</sup>	(0.001)	0.004%	(0.016)	-0.046% <sup>†</sup>	(0.057)	0.001% <sup>§</sup>	(0.001)	0.034% <sup>§§</sup>	(0.013)	0.863% <sup>§§</sup>	(0.076)

Note: the results are averages over 10 runs, each of 50 000 simulated periods, and  $r$  denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

<sup>†</sup> and <sup>††</sup> denote that the *DMR* or *DMD* heuristic performed significantly better than the naive heuristics and <sup>§</sup> and <sup>§§</sup> denote that they performed significantly worse, based on a one-sided t-test, <sup>†,§</sup>  $p < 0.05$ , <sup>††,§§</sup>  $p < 0.01$ .

**Table 44** Comparison of the *DMD* and *SMR* heuristics to naive heuristics in which we choose randomly which PM activities to replan and where we move up or delay with equal probability when both are possible for replanning, for p-BRP with  $\alpha = 1$  year and  $\beta = 2$ . Percentage additional costs compared to the naive heuristics, for different restrictions on the maximum number of PM activities per period  $r$ . We used  $\ell = 1$  for *DMD*.

$\Delta$	<i>DMD</i> (delay)			<i>SMR</i> (replan)		
	$r = 60$	$r = 40$	$r = 20$	$r = 60$	$r = 40$	$r = 20$
0%	-1.141% <sup>††</sup> (0.034)	-1.025% <sup>††</sup> (0.069)	-2.765% <sup>††</sup> (0.059)	-0.148% <sup>††</sup> (0.132)	-0.154% <sup>††</sup> (0.132)	-0.159% <sup>††</sup> (0.132)
10%	-1.136% <sup>††</sup> (0.051)	-1.074% <sup>††</sup> (0.063)	-2.823% <sup>††</sup> (0.080)	-0.159% <sup>††</sup> (0.144)	-0.160% <sup>††</sup> (0.144)	-0.268% <sup>††</sup> (0.145)
20%	-1.100% <sup>††</sup> (0.049)	-1.049% <sup>††</sup> (0.060)	-2.814% <sup>††</sup> (0.081)	-0.172% <sup>††</sup> (0.145)	-0.172% <sup>††</sup> (0.145)	-0.190% <sup>††</sup> (0.149)
30%	-1.000% <sup>††</sup> (0.063)	-1.047% <sup>††</sup> (0.068)	-2.812% <sup>††</sup> (0.068)	-0.223% <sup>††</sup> (0.147)	-0.414% <sup>††</sup> (0.159)	-0.384% <sup>††</sup> (0.203)
40%	-0.939% <sup>††</sup> (0.066)	-0.989% <sup>††</sup> (0.075)	-2.770% <sup>††</sup> (0.068)	-0.286% <sup>††</sup> (0.163)	-0.546% <sup>††</sup> (0.183)	-0.448% <sup>††</sup> (0.249)
50%	-0.896% <sup>††</sup> (0.063)	-1.110% <sup>††</sup> (0.053)	-3.911% <sup>††</sup> (0.044)	-0.315% <sup>††</sup> (0.133)	-0.520% <sup>††</sup> (0.142)	1.343% <sup>§§</sup> (0.309)

Note: the results are averages over 10 runs, each of 50 000 simulated periods, and  $r$  denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

<sup>†</sup> and <sup>††</sup> denote that the *SMR* or *DMD* heuristic performed significantly better than the naive heuristics and <sup>§</sup> and <sup>§§</sup> denote that they performed significantly worse, based on a one-sided t-test, <sup>†, §</sup>  $p < 0.05$ , <sup>††, §§</sup>  $p < 0.01$ .

**Table 45** Comparison of the *DMD* and *SMR* heuristics to naive heuristics in which we choose randomly which PM activities to replan and where we move up or delay with equal probability when both are possible for replanning, for p-BRP with  $\alpha = 5$  years and  $\beta = 2$ . Percentage additional costs compared to the naive heuristics, for different restrictions on the maximum number of PM activities per period  $r$ . We used  $\ell = 1$  for *DMD*.

$\Delta$	<i>DMD</i> (delay)			<i>SMR</i> (replan)		
	$r = 60$	$r = 40$	$r = 20$	$r = 60$	$r = 40$	$r = 20$
0%	0.000% (0.000)	0.000% (0.000)	0.000% (0.000)	-0.384% <sup>††</sup> (0.234)	-0.384% <sup>††</sup> (0.234)	-0.384% <sup>††</sup> (0.234)
10%	-0.066% <sup>††</sup> (0.029)	-0.078% <sup>††</sup> (0.038)	-0.285% <sup>††</sup> (0.058)	-0.293% <sup>††</sup> (0.170)	-0.361% <sup>††</sup> (0.178)	-0.632% <sup>††</sup> (0.164)
20%	-0.066% <sup>††</sup> (0.032)	-0.078% <sup>††</sup> (0.041)	-0.286% <sup>††</sup> (0.060)	-0.384% <sup>††</sup> (0.199)	-0.532% <sup>††</sup> (0.218)	-0.502% <sup>††</sup> (0.210)
30%	-0.065% <sup>††</sup> (0.037)	-0.079% <sup>††</sup> (0.045)	-0.287% <sup>††</sup> (0.063)	-0.494% <sup>††</sup> (0.234)	-0.739% <sup>††</sup> (0.267)	-0.681% <sup>††</sup> (0.282)
40%	-0.064% <sup>††</sup> (0.043)	-0.081% <sup>††</sup> (0.051)	-0.288% <sup>††</sup> (0.069)	-0.632% <sup>††</sup> (0.279)	-0.993% <sup>††</sup> (0.329)	-0.893% <sup>††</sup> (0.378)
50%	-0.063% <sup>††</sup> (0.051)	-0.083% <sup>††</sup> (0.059)	-0.289% <sup>††</sup> (0.076)	-0.807% <sup>††</sup> (0.336)	-1.314% <sup>††</sup> (0.407)	-1.150% <sup>††</sup> (0.499)

Note: the results are averages over 10 runs, each of 50 000 simulated periods, and  $r$  denotes the number of permitted PM operations per period. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

<sup>†</sup> and <sup>††</sup> denote that the *SMR* or *DMD* heuristic performed significantly better than the naive heuristics and <sup>§</sup> and <sup>§§</sup> denote that they performed significantly worse, based on a one-sided t-test, <sup>†, §</sup>  $p < 0.05$ , <sup>††, §§</sup>  $p < 0.01$ .

**Table 46** Percentage difference in costs of delaying and replanning PM compared to the optimal solution for two components, for  $\alpha = 1$  year and  $\beta = 2$ . For *DMD* we used  $\ell = 1$  and for *DMR*  $\ell = 3$ .

$\Delta$	p-ARP				p-BRP			
	<i>DMD</i> (delay)		<i>DMR</i> (replan)		<i>DMD</i> (delay)		<i>SMR</i> (replan)	
0%	0.02%	(0.0810)	0.02%	(0.0810)	-1.130% <sup>††</sup>	(0.2178)	0.002%	(0.0049)
10%	-0.11%	(0.3242)	-0.15%	(0.3383)	-1.256% <sup>††</sup>	(0.2901)	0.008%	(0.0268)
20%	0.16%	(0.3689)	0.07%	(0.4421)	-1.201% <sup>††</sup>	(0.2620)	0.016%	(0.0549)
30%	0.94% <sup>§§</sup>	(0.6273)	-0.16%	(0.3790)	-1.264% <sup>††</sup>	(0.3154)	0.016%	(0.0910)
40%	-0.14%	(0.4763)	-0.10%	(0.5997)	-1.149% <sup>††</sup>	(0.3204)	0.024%	(0.1245)
50%	3.08% <sup>§§</sup>	(0.9319)	0.37% <sup>§§</sup>	(0.3991)	0.247% <sup>§</sup>	(0.4043)	0.485% <sup>§§</sup>	(0.4458)

Note: the results are averages over 10 runs, each of 50 000 simulated periods, brackets denote standard deviations. We let  $\bar{c}_f = 50$ ,  $\bar{c}_p = 10$ ,  $\Delta_f = \Delta \cdot \bar{c}_f$  and  $\Delta_p = \Delta \cdot \bar{c}_p$  for different  $\Delta$ .

<sup>†</sup> and <sup>††</sup> denote that the average costs of the heuristic solutions are significantly smaller than the optimal costs (note that this only happens for delay of p-BRP, and that this is possible because the optimal p-BRP is not an overall optimal policy) and <sup>§</sup> and <sup>§§</sup> denote that the costs of the heuristics are significantly larger, based on a one-sided t-test, <sup>†, §</sup>  $p < 0.05$ , <sup>††, §§</sup>  $p < 0.01$ .

## F Explanation of programming code

In this appendix we will explain how all results presented in this paper were obtained. As mentioned in [Section 5](#), we programmed everything in Java, using the solver CPLEX 20.1.0. to obtain the optimal solutions of the different models that were presented.

### F.1 Replication part: single-component

For the replication part, we used only classes in the packages ‘markov’ and ‘oneTwoComponent’. Everything that concerns the Markov decision process and the cost functions can be found in the package ‘markov’ and the code for solving the p-BRP, p-ARP and p-MBRP models can be found in the eponymous classes in the package ‘oneTwoComponent’. When the class ‘MainOneComp’ in the package ‘oneTwoComponent’ is run, the results described in [Section 5.1](#) are obtained. To see only part of the results, parts of the parameter values in lines 29-51 can be commented. Furthermore, to obtain the average yearly costs of not maintaining at all, you should comment lines 55-59 and uncomment lines 66-96. We implemented this by adding the constraint that there is no preventive maintenance allowed to the model for p-BRP (restricting all y-variables to be zero). Note that the objective values that are reported by the models for the different policies must be multiplied by 12 to obtain the results that were reported in this paper and the paper by Schouten et al. ([2022](#)), as the objectives are the monthly costs while we report yearly costs.

### F.2 Extension: multi-component

For the multiple-component case, we added the ‘multiComponent’ package. Here, you can find a class to solve the value-determination equations, classes to model the set of components, an enumeration class ‘Policy’ that can take values p-ARP, p-BRP and p-MBRP and classes for the different types of heuristics and classes to perform the simulations to test the performances of these heuristics.

To obtain [Table 1](#), the class ‘SimulationDelay’ should be run. To obtain the results of delay of p-ARP for other combinations of values for  $\alpha$  and  $\ell$ , the values that are (un)commented should be changed. To obtain the same results for p-BRP line 26 should be commented and line 27 uncommented after which the class is run again.

For [Table 2](#), one needs to run the ‘simulationReplanARP’ class. Again, to find the results for other values of  $\alpha$  and  $\ell$  you need to comment and uncomment the corresponding lines between 36 and 58. Here, we get a heap space error for  $\ell = 7$  and  $\alpha \geq 3$  and  $\delta \geq 30\%$ , as we save the additional costs for too many combinations of state, v and x. To fix this, lines 375-382 of ‘ReplanHeuristicARP’ should be commented. As a result, the running time will increase, but the needed storage space decreases.

To find the corresponding results for p-BRP, the class ‘simulationReplanBRP’ was created, in which again the parameter values can be changed in the same way as for p-ARP.

Running these versions also automatically gives the results for [Table 3](#) and [Table 7](#) and the distribution of the PM activities per period. These can be transformed into the figures about the average number of PM activities by multiplying the numbers for each month by the total number of PM activities (which is also printed when running the classes) and dividing by the total number of years that fit in 50.000 periods. The same can be done for CM. Moreover, also the age distributions are printed, for which the graphs can be found in [Appendix E](#).

To create [Table 4](#), one should run the classes ‘SimulationDelayNaive’ for the left part and ‘SimulationReplanNaiveARP’ for the right part. The values of  $\alpha$  and  $\ell$  can once more be changed by means of commenting. For the left side of [Table 8](#) line 31 of ‘SimulationDelayNaive’ should be commented and line 32 should not be commented. The right part can be obtained by running the class ‘SimulationReplanNaiveBRP’. Again, the values for  $\alpha$  and  $\ell$  can be changed to obtain the results that are presented in [appendixFigures](#).

Finally, we can obtain [Table 9](#) about the comparison with the optimal solutions for two components by running the classes ‘SimulationDelay2Comp’, ‘SimulationReplanARP2Comp’ and ‘SimulationBRP2Comp’ from the package ‘oneTwoComponent’. To obtain [Table 46](#), once again change the parameter values. For this table, where  $\alpha = 1$ , the lines 175, 176 and 192 of the class ‘Transition’ from the package ‘markov’ should be commented. We added these lines to make the state space smaller, such that the models solve faster for  $\alpha = 3$ . However, the results that we present in [Table 46](#) were obtained before we added these lines. This could give a slightly different solution when there exist multiple optimal solutions. For  $\Delta = 0\%$  there are always multiple optimal solutions because each period has the same costs for PM such that only the interval between to block times matters and for other values we always have two optimal solutions as we can change which component gets assigned which of the two policies. As we draw different random numbers for the two components, this can give slightly different outcomes in a simulation.