Combined time- and condition-based maintenance for a wind turbine under time-varying costs

Bachelor Thesis

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Abstract

This paper proposes a combined time-based maintenance (TBM) and condition-based maintenance (CBM) strategy under time-varying costs, specifically targeted at offshore wind turbine maintenance. The main contribution lies in the ability of the model to maintain its performance under imperfect condition monitoring. The model is optimized by means of a discretization of time and a mixed integer linear programming formulation. We find that both in a constant cost setting and under time-varying costs, the combined TBM and CBM strategy notably outperforms the singular policies. With highly imperfect condition monitoring, a pure CBM policy breaks down while the combined model performs well. Additionally, we find that a TBM approach benefits more from the consideration of time-varying costs than a CBM approach. Last, a marginal cost analysis (MCA) is presented to provide more insights into the model. Since a MCA has a short horizon and does not take into account the cyclical nature of time-varying costs, the MCA only performs well under constant costs.

Keywords: Condition-based maintenance · Time-based maintenance · Marginal cost analysis

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1 Introduction

The European Union aims to be climate-neutral in 2050. One important step towards this goal is the construction of wind farms, as wind turbines are an efficient source of sustainable energy. To illustrate, the European Commission aims at generating 60GW of power in offshore wind parks by 2030, which is enough to power approximately 45 million homes (The European Commission, 2021). By 2050, this amount should be increased to 340 GW. Wind turbines at offshore wind farms are exposed to severe weather, including lightning, snow and extreme temperatures. Consequently, operation and maintenance costs amount to 15-30% of the cost of energy generated by offshore wind farms (Besnard et al., 2013). The optimization of these costs impacts the competitiveness of wind energy considerably.

Wind turbine maintenance can be classified into preventive maintenance (PM) and corrective maintenance (CM) (Dao et al., 2021). CM takes place when a system is not working properly and performs below the intended level. However, for offshore wind turbines this is not suitable, as failures often have serious consequences, such as severe damage to the gear box or rotor blades (Tchakoua et al., 2014). PM is done before a system fails and can be time-based or condition-based. Two common time-based maintenance (TBM) policies are the age-replacement policy (ARP) and the block-replacement policy (BRP) (de Jonge & Scarf, 2020). An ARP strategy performs maintenance at a certain critical age, whereas with a BRP approach one performs maintenance after a fixed time interval.

The ARP and BRP models assume costs to be constant over time, however, wind turbine maintenance costs are likely to vary over the year. To illustrate, the average wind speed in IJmuiden, a coastal city in the Netherlands, was measured to be 9.6 m/s in January and 7.4 m/s in July between 1971 and 2022 (Royal Netherlands Meteorological Institute, 2022). When maintenance is performed in winter, there is a higher loss of production due to the higher average wind speeds. Therefore, maintenance costs are likely to be higher in winter. Schouten et al. (2022) introduce the p-ARP and p-BRP policies, which improve upon the ARP and BRP strategies by considering time-varying costs. These policies reach savings by shifting more maintenance to summer.

Whereas TBM schedules maintenance by a time-dependent policy, condition-based maintenance (CBM) uses condition monitoring to schedule maintenance. Measuring instruments or visual inspections are used to monitor the state of a wind turbine component, after which a maintenance decision is made. For instance, the temperature and oil level can be measured. Over the past decades, CBM approaches have been gaining popularity due to the increase in possibilities of real-time monitoring and the analysis of the condition of a component without visual inspection (Tchakoua et al., 2014). One advantage of condition-based maintenance (CBM) is that it usually leads to a better timing of the maintenance, as one ideally conducts maintenance just before the performance of a component deteriorates. However, the effectiveness of CBM largely depends on the quality of the condition monitoring (Kim et al., 2016). Low quality data on the condition of a wind turbine component can lead to an unnoticed failure, which results in high CM costs. Additionally, malfunctions of measuring equipment may require costly visual inspections, due to labor and transportation costs.

On the one hand, we thus see that TBM is easy to plan, while CBM depends on the changing state of the wind turbine components. On the other hand, CBM results in lower costs due to a more effective maintenance planning when condition monitoring is accurate. This paper aims to overcome the disadvantages of the separate strategies by combining a TBM and a CBM approach. When using a combined technique, one uses TBM to spot failures that are missed by the condition monitoring. At the same time, the CBM component of the policy leads to an early identification of imminent failures, which results in a reduction of maintenance costs. Moreover, time-varying costs are considered to adjust for the difference in production loss over the year. This leads to the following research question:

What is the effect of jointly optimizing a combined TBM and CBM model for a single wind turbine under time-varying costs?

While there is a large body of literature on both TBM and CBM approaches, research combining the two approaches is sparse. Dao et al. (2021) take a first step in this direction by integrating an imperfect TBM approach with a CBM strategy under constant costs, which results in lower costs compared to the separate strategies. However, in this approach the TBM interval is fixed at one year, whereas different TBM approaches may yield better results. Furthermore, the option to coordinate maintenance with periods of low wind speeds is neglected.

This paper contributes to the existing literature in three ways. First, it extends the existing literature by optimizing a model that combines TBM and CBM, which is a novel method for handling imperfect condition monitoring. These findings have practical value, as an increasing number of wind turbines contain sensors that transmit real-time information regarding the condition of the components (Tchakoua et al., 2014). Second, it considers time-varying costs for a combined TBM and CBM model, whereas previously time-varying costs have only been analysed in separate models (Schouten et al., 2022). Maintenance costs are reduced by shifting maintenance to periods with lower average wind speeds. Last, both an approach based on a Markov decision process (MDP) and an approach based on a marginal cost analysis (MCA). While a MDP is suitable for finding the optimal maintenance policy, a MCA provides more insights into the analysis by using interpretable criteria.

The main findings of this paper show that both under constant costs and under time-varying costs a combined TBM and CBM model outperforms the separate strategies. We find that the accuracy of the condition monitoring largely determines the proportion of TBM and CBM in the optimal maintenance schedule. Additionally, we find that a TBM approach benefits more from incorporating time-varying costs than a CBM strategy. This illustrates that TBM can more easily be postponed than CBM, as the delay of CBM is more likely to result in the failure of a component.

The paper continues as follows. In Sect. 2, the relevant literature is summarized, after which the model is outlined in Sect. 3. Then, the different maintenance policies are described in more detail in Sect. 4. The corresponding numerical results are discussed in Sect. 5. Then, a marginal cost approach is presented in Sect. 6. Last, we conclude and provide recommendations for future research in Sect. 7.

2 Literature review

In this section, we will first give a brief overview of literature reviews regarding maintenance optimization related to wind energy. Afterwards, specific research on TBM and CBM models is described. Then, literature on deterioration modelling and imperfect condition monitoring is discussed. Subsequently, literature on the incorporation of time-varying costs into maintenance optimization is reviewed. Last, we summarize literature on the application of a marginal cost analysis in the analysis of maintenance policies.

2.1 Maintenance optimization in the context of wind energy

When optimizing all aspects of an offshore wind park, one encounters several challenges. To illustrate, there are many components that each deteriorate in a different way, the accessibility for maintenance is limited and weather conditions can impair the transportation of material and manpower. Research into maintenance optimization in this field has increased over the last decade and an extensive review of maintenance optimization models and strategies is given by Shafiee and Sørensen (2019). In this review, literature is classified based on five criteria: the type of system, the planning horizon, the failure modelling, the optimization model and the maintenance strategy. Two other reviews on maintenance optimization that are relevant for this paper are written by de Jonge and Scarf (2020) and Ding and Kamaruddin (2015). These reviews explicitly distinguish single-component and multi-component systems, alongside different condition monitoring methods.

2.2 TBM and CBM models

The two most common TBM strategies are the age-replacement policy (ARP) and the block-replacement policy (BRP). An ARP strategy schedules maintenance at a critical age, while under a BRP components are replaced at a given interval. When PM and CM costs are constant and the probability of failure is increasing, an ARP policy is optimal for minimizing average costs (Ross, 1970). The disadvantage of TBM policies is that maintenance may be carried out too early. Consequently, a component is sometimes replaced while it could have functioned well for a longer period.

A concise review on CBM strategies is given by Kang et al. (2019). CBM strategies aim at monitoring the state of wind turbine components and choosing the optimal maintenance activity accordingly. CBM consists of three primary phases: condition monitoring, fault diagnosis and prognosis (Kang et al., 2019). In the condition monitoring phase, one gathers information regarding the state of the wind turbine components. Several examples of condition monitoring are vibration analysis, oil monitoring and temperature measurement. In the fault diagnosis phase, the data from condition monitoring is used to identify potential system failures. Last, one must make a prognosis regarding the remaining lifetime of the wind turbine components and choose the optimal maintenance activity accordingly.

Although there is a large amount of research on TBM and CBM models, literature comparing and integrating the two types of strategies is sparse. Both de Jonge et al. (2017) and Kim et al.

(2016) conclude that the quality and costs of condition monitoring determine which strategy is dominant. To illustrate, when one can visually inspect a component at a low cost, CBM will result in better maintenance decisions. Dao et al. (2021) analyse a combined imperfect TBM and CBM approach by performing yearly PM alongside frequent inspections that may lead to additional maintenance. They find that, under constant costs, combining a TBM and CBM approach yields better results compared to applying the approaches separately. This paper builds upon this work by jointly optimizing a combined TBM and CBM model in a setting with time-varying costs.

2.3 Deterioration modelling

Since a CBM approach relies on the state of a component, modelling the degradation process is an essential part of finding the optimal policy. There is a large body of literature regarding deterioration modelling, which provides useful insights for this paper. One common approach is to base the model on the P-F curve, as depicted in Figure 1. The P-F curve shows how a component deteriorates over time. While operating, a component deteriorates until it cannot carry out its function anymore. This point of failure is called functional failure 'F'. The first point where an indication of the deterioration of the component may be detected is referred to as a potential failure 'P'. The deterioration process can be modelled by dividing the P-F curve into several regions. To illustrate, van Horenbeek et al. (2013) divide the curve into five regions: a working stage, three deteriorating stages and a failed stage. Tchakoua et al. (2014) use the P-F curve to classify different methods of condition monitoring. Methods such as vibration analysis and the detection of noise can be used to spot early signs of failure, whereas heat and smoke will only be observed when failure is imminent.

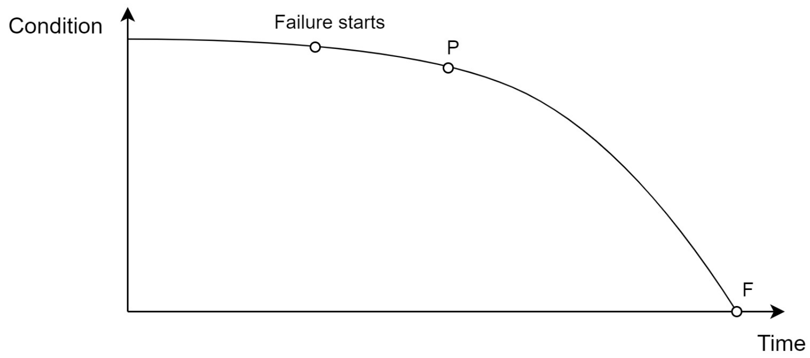


Fig. 1: P-F curve.

A common method to model the deterioration process is by means of a MDP. The system state is normally divided into three types: perfect functioning, deteriorating and functional failure. This three-state Markov model has been applied by McMillan and Ault (2008), who subsequently use a Monte Carlo simulation to evaluate the performance. When analysing a CBM approach, one usually divides the deteriorating stage into several sub-types, such as minor degradation and severe degradation. Two examples of such approaches are given by Besnard and Bertling (2010) and Ossai et al. (2016), who respectively use a five-state and six-state Markov model for wind turbine maintenance optimization. Besnard and Bertling (2010) show that for a high failure rate online inspections are more efficient than visual inspections, as the high costs of a visual inspection outweigh the loss of precision due to online monitoring. For a more extensive review of the application of Markov models we refer to Dawid et al. (2015), who consider different Markov models, such as a hidden Markov model and a partially observable MDP.

2.4 Imperfect condition monitoring

It is a common assumption to consider condition monitoring to be perfect. This is demonstrated by Raza and Ulansky (2019) in their categorization of CBM models according to underlying assumptions. However, Kang et al. (2019) emphasize that the accuracy of condition monitoring is largely responsible for the effectiveness of a CBM policy. When the data regarding the performance of a wind turbine component is noisy, it becomes difficult to make condition-based maintenance decisions. On the one hand, the data may indicate a failure while the component is in a good condition, resulting in unnecessary maintenance costs. On the other hand, it can be the case that the condition monitoring misses a fault, leading to costly CM. In fact, Schouten et al. (2022) motivate the use of TBM approaches over CBM approaches by the limited accuracy of condition monitoring.

There has been little research on incorporating imperfect condition monitoring into a maintenance optimization model. The most important works are written by Raza and Ulansky (2019) and van Horenbeek et al. (2013). In a case study, van Horenbeek et al. (2013) show that the effectiveness of a CBM policy highly depends on the quality of the condition monitoring. When the accuracy of the condition monitoring goes down, the performance of a CBM policy declines rapidly. Raza and Ulansky (2019) built a model for the optimization of a CBM approach with online condition monitoring. They make use of a confusion matrix to distinguish between false positives and false negatives that arise from the imperfect information from the condition monitoring system. Both works thus show that imperfect information leads to higher maintenance costs. This paper extends upon the existing literature by considering a combined TBM and CBM model to reduce the impact of imperfect information, in a setting with time-varying costs.

2.5 Incorporating time-varying costs

In maintenance optimization models it is commonly assumed that maintenance costs are constant over time (Ding & Kamaruddin, 2015). However, for wind turbine maintenance this assumption does not hold, as the costs of lost production mainly depend on the wind speed. Since the wind speed largely depends on the season, these costs vary in a cyclic way. Schouten et al. (2022) use this by extending the ARP, BRP and modified block replacement policy (MBRP) to a time-varying cost setting. This leads to the the p-ARP, p-BRP and p-MBRP models. For each of these models, Schouten et al. (2022) find that a higher degree of variation in costs over the months leads to larger savings compared to the constant cost setting. These savings are obtained by shifting more PM towards the summer months. While Schouten et al. (2022) only considers TBM models under time-varying costs, we will also analyse a CBM model and a combined TBM and CBM model under time-varying costs. Therefore, we will show under what circumstances it is beneficial to take into account time-varying costs.

2.6 Marginal cost analysis

A last stream of literature that is relevant for this paper is the application of a MCA in the analysis of maintenance policies. The main approach in the literature described in previous sections is based on a MDA. One advantage of such an approach is that it provides optimal policies for certain maintenance strategies, such as the BRP and ARP policies (Schouten et al., 2022). However, the optimization of a MDP does not give many insights into the analysis. In fact, Ding and Kamaruddin (2015) and de Jonge and Scarf (2020) indicate that the complexity of many maintenance optimization models contributes to the gap between academia and the industry. A MCA narrows this gap, since it is an approach at a micro-level that gives more insights into the analysis. Therefore, a MCA can be more easily applicable in the industry. However, due to the short-term perspective of the approach there may be losses compared to the optimal maintenance schedule (Dekker & Roelvink, 1995).

The MCA is based on the marginal cost function, which represents the costs of postponing PM for one period. The analysis makes use of a well-known economic principle: at the optimum, the marginal cost is equal to the average costs. When the average costs exceed the marginal costs, the long-run average costs will go down when postponing maintenance. Similarly, if the marginal cost is higher than the average costs, postponing maintenance will increase the long-run average costs. Berg (1980) first introduced MCA in the field of maintenance optimization and proves the aforementioned optimality condition for an ARP and BRP approach. A big advantage of this criterion is that it has a clear interpretation, which is important for usage in the industry. Additionally, with a MCA one may find maintenance policies for problem structures that are not easily solved with a MDP. To illustrate, Dekker and Roelvink (1995) use a MCA to find a maintenance policy for a group of wind turbine components. The results indicate that using the marginal cost function reduces the costs by 0% to 10% over the optimal BRP policy.

3 Model description

In this section, we will describe our model specifications and the cost structure. After giving an overview of the model, we introduce the state space in Sect. 3.1. Afterwards, the modelling of the deterioration process is specified in Sect. 3.2. Finally, the transition probabilities between states and the cost structure are presented in Sect. 3.3 and Sect. 3.4 respectively.

This paper extends the model of Schouten et al. (2022) and thus uses a similar notation. To give a brief overview of our model, we will explain it based on the four features that were introduced by Dekker (1996). Specifically, we describe the technical system, the deterioration process, the available information and the model objective:

1. The technical system that we will consider is a single wind turbine component. When it breaks down, it should be repaired immediately, as the wind turbine cannot function without it. When it is repaired, it is replaced by an as-good-as-new component.
2. The deterioration process consists of two stages and is described in more detail in Sect. 3.2. The first stage corresponds to the transition of a working component to showing the first signs of failure. The second stage reflects the transition from showing the first faults to a

functional failure, which happens through a maximum of m stages. The two stages follow independent stochastic deterioration processes.

3. The deterioration process in each stage is assumed to be known. Additionally, the PM and CM costs are time-varying over the seasons. We consider pure TBM policies, pure CBM policies and combined policies.
4. The aim of the model is to minimise the long-run average costs. The models are solved by means of time discretization and the formulation of a discrete-time MDP. The MDP is solved by means of a mixed integer linear programming (MILP) formulation. This is described more thoroughly in Sect. 4.3.

In the remainder of this section, the model will be described in more detail. First, the state space and the deterioration process are extended upon. Afterwards, the transition probabilities and the time-varying cost structure are outlined. A nomenclature is given in Table 1 below and a list of abbreviations can be found in Appendix A.1.

Table 1 Nomenclature

Symbol	Description
N	Number of periods in a year
$\mathbb{N}^+$	Set of positive integers, {1, 2, ...}
$\bar{\mathbb{N}}$	Set of extended natural numbers, {0, 1, 2, ..., ∞}
$\mathcal{I}_1$	Set of periods within a year
$\mathcal{I}_2$	Set of component condition states
$\mathcal{I}_{2,observed}$	Set of observed condition states
$\mathcal{I}_{2,unobserved}$	Set of unobserved condition states
$\mathcal{I}_3$	Set of component ages
$\mathcal{I}$	State space of Markov decision process
T	Random lifetime of a component
T_k	Random lifetime of a component in stage i (k = 1, 2)
m	Number of deteriorating stages
$c_p(i_1)$	Costs of PM in period i_1
$c_f(i_1)$	Costs of CM in period i_1
$\bar{c}_p$	Average PM costs
$\bar{c}_f$	Average CM costs
$p_{observed}$	The probability of observing the deterioration process of a component
F	The state representing the failure of a component
M	A large number representing the maximum age of a component
$\mathcal{A}(i_1, i_2, i_3)$	The set of possible actions in the Markov decision process in state (i_1, i_2, i_3)
$F_L(t)$	Distribution function for stage 2, i.e., probability of failure after t periods in stage 2
c_T	Coefficient of variation for T
p_{i_3}	The chance of a component leaving stage 1 at age i_3
p_{i_2,j_2}	The chance of a component going from condition i_2 to condition j_2 in stage 2

3.1 State space

The life of a component is represented by a discrete-time MDP. Let $\mathcal{I}_1 \subset \mathbb{N}^+$ be the periods in a year, $\mathcal{I}_2 \subset \bar{\mathbb{N}}$ the condition of a component and $\mathcal{I}_3 \subset \bar{\mathbb{N}}$ the age of a component. Note that

$\bar{\mathbb{N}}$ is the set of extended integers and $\mathcal{I}_1 = \{0, 1, ..., N\}$, where N is the number of periods in a year. Furthermore, $\mathcal{I}_2 = \{1, ..., F\}$, where a higher value for $i_2 \in \mathcal{I}_2$ reflects a worse state and state F represents a functional failure. We have $\mathcal{I}_3 = \{0, 1, ..., M\}$, where M is the maximum age, at which PM must be performed. The maximum age is solely considered to have a finite state space and should not influence the outcome. Therefore, we take M sufficiently large, such that it is never reached under the optimal policy. In each state we can take an action, we either replace a component ($a = 1$) or do nothing ($a = 0$). When a component has failed, CM is mandatory. Under a pure CBM approach, we do not maintain if there are no observed signs of failure. Therefore, the state-dependent action space for a CBM approach is:

$$\mathcal{A}_{CBM}(i_1, i_2, i_3) = \begin{cases} \{1\} & \text{if } i_2 = F \vee i_3 \geq M \\ \{0\} & \text{if } i_2 = 1 \\ \{0,1\} & \text{otherwise} \end{cases} \quad (1)$$

When there is a TBM component in the maintenance approach, one may also perform maintenance when there are no observed signs of failure. Therefore, the state-dependent action space is expanded to:

$$\mathcal{A}(i_1, i_2, i_3) = \begin{cases} \{1\} & \text{if } i_2 = F \vee i_3 \geq M \\ \{0,1\} & \text{otherwise} \end{cases} \quad (2)$$

Note that for combined approaches, the action space $\mathcal{A}$ is used, as $\mathcal{A}_{CBM} \subset \mathcal{A}$.

3.2 Deterioration process

We consider a wind turbine component in a discrete time setting. The deterioration process is modelled similar to van Horenbeek et al. (2013) and Tchakoua et al. (2014), by considering the P-F curve as introduced in Sect. 2.3. Specifically, we thus consider a two-stage approach. The first stage corresponds to the transition from a fully functional component to a potential failure P, which takes time T_1. From this point onward, the component enters a second stage in which it deteriorates through a maximum of m states. The second stage is either observed or unobserved. The time that passes from the potential failure P to the functional failure F is denoted by T_2. The model is depicted in Figure 2. We divide the condition state space into an observed and an unobserved part, thus $\mathcal{I}_2 = \mathcal{I}_{2,observed} \cup \mathcal{I}_{2,unobserved}$. Let $\mathcal{I}_{2,observed} = \{1, 2, ..., m+1, F\}$ and $\mathcal{I}_{2,unobserved} = \{m+2, ..., F-1\}$, where the as-good-as-new state corresponds to condition 1 and the state of failure corresponds to condition $F = 2m + 2$.

To further illustrate the deterioration process, we consider the deterioration of a wind turbine blade. One of the most severe failures for a wind turbine is the failure of a rotor blade, which results in approximately 11 days of downtime (Tchakoua et al., 2014). One common way of monitoring the condition of a rotor blade is by using acoustic emission sensors that detect high frequencies, often caused by cracks in the rotor blades. However, these sensors only detect failures that are nearby (Li et al., 2016). Consequently, only a section of the blades is monitored. When a crack forms, it occurs with a probability $p_{observed}$ in a section of the blade that is monitored. In this case, the whole deterioration process is observed. With a probability $1 - p_{observed}$, the crack is unobserved and the fault is only observed when there is a functional failure. Note that once the observed deterioration process is entered, there is no transition to the unobserved states, and vice versa.

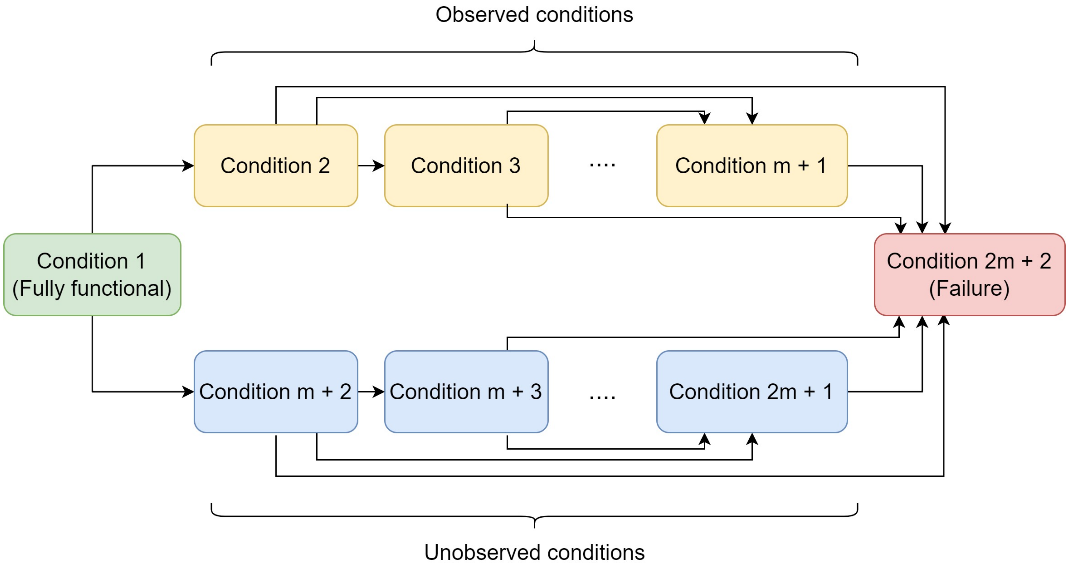


Fig. 2: Visual representation of the condition state space

3.3 Transition between states

The transition between states depends on whether maintenance is performed or not. If maintenance is performed, a component reaches an as-good-as-new status. Let $\pi_{(i_1,i_2,i_3),(j_1,j_2,j_3)}(a)$ be the transition probability from state (i_1, i_2, i_3) to state (j_1, j_2, j_3) under action a. For the first stage, the probability of failure only depends on the age of the component i_3. This gives

$$\pi_{(i_1,1,i_3),(j_1,j_2,j_3)}(0) = \begin{cases} (1 - p_{observed}) \cdot p_{i_3} & \text{for } j_1 = i_1 + 1 \pmod{\text{N}},\ j_2 = m + 2,\ j_3 = i_3 + 1 \\ p_{observed} \cdot p_{i_3} & \text{for } j_1 = i_1 + 1 \pmod{\text{N}},\ j_2 = 2,\ j_3 = i_3 + 1 \\ 1 - p_{i_3} & \text{for } j_1 = i_1 + 1 \text{ mod(N)},\ j_2 = 1,\ j_3 = i_3 + 1 \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

Where $p_{i_3} = \mathbb{P}(T_1 = i_3 \mid T_1 \geq i_3)$, which is the probability that the component shows the first signs of faults at age i_3, given that it has age i_3. When this has happened, the component moves into the second stage. Here, the transition probabilities depend both on the current condition i_2 and the new condition j_2. This gives for the conditions $i_2 > 1$:

$$\pi_{(i_1,i_2,i_3),(j_1,j_2,j_3)}(0) = \begin{cases} p_{i_2,j_2} & \text{for } j_1 = i_1 + 1 \pmod{\text{N}},\ j_3 = i_3 + 1,\ i_2 > 1 \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

where p_{i_2,j_2} represents the probability of going from condition i_2 to condition j_2. Whenever maintenance is performed ($a = 1$), the component returns to condition 1 with age 0. This gives:

$$\pi_{(i_1,i_2,i_3),(j_1,j_2,j_3)}(1) = \begin{cases} (1-p_{observed})\cdot p_1 & \text{for } j_1 = i_1+1 \pmod{\text{N}},\ j_2 = m+2,\ j_3 = 1 \\ p_{observed}\cdot p_1 & \text{for } j_1 = i_1+1 \pmod{\text{N}},\ j_2 = 2,\ j_3 = 1 \\ 1-p_1 & \text{for } j_1 = i_1+1 \pmod{\text{N}},\ j_2 = 1,\ j_3 = i_3+1 \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

3.4 Cost parameters

This paper follows the approach of Schouten et al. (2022) and considers time-dependent maintenance costs. It is assumed that the costs for PM are equal to $c_p(i_1)$ and the costs for CM are $c_f(i_1)$, with $i_1 \in \mathcal{I}_1$. The costs for CM are likely to be higher than the costs for PM, since complete failure of one component is often accompanied by damage to other components. The cost of taking action a in state $i = (i_1, i_2, i_3)$ is thus:

$$c_{(i_1,i_2)}(a) = \begin{cases} 0, & \text{if } a = 0 \\ c_p(i_1), & \text{if } a = 1,\ i_2 \neq F, \\ c_f(i_1), & \text{if } a = 1,\ i_2 = F. \end{cases} \tag{6}$$

4 Maintenance policies

In this section, various maintenance policies will be discussed. First, pure TBM and CBM maintenance strategies are described. Next, a combined approach is introduced. For each approach, we distinguish between a policy under constant costs and a policy under time-varying costs.

4.1 TBM and CBM approaches

Pure TBM and CBM policies will be considered as benchmark policies, to which the combined approach will be compared. The ARP and p-ARP policies as described in Schouten et al. (2022) will be considered as the benchmark TBM policies. In a p-ARP policy, each period $i_1 \in \mathcal{I}_1$ has a critical maintenance age $t(i_1) \in \bar{\mathbb{N}}\backslash\{0\}$ at or above which PM is performed. In an ARP policy, costs are constant over the periods and we have $t(i_1) = t$, $\forall i_1 \in \mathcal{I}_1$. The ARP and p-ARP policies are based on the age since the component was first used. Alongside the TBM benchmark policies, we will also consider two CBM benchmark maintenance policies. Let us first define a condition replacement policy (CRP), under constant costs.

Definition 1 A CRP policy is a policy with a threshold condition k, such that maintenance is performed when $i_2 \geq k$, $i_2, k \in \{2, ..., m+1\}$.

A CRP policy thus has a threshold structure that is similar to an ARP policy, however, the critical threshold is based on the condition instead of the age. Note that when we set $k = F$, we only perform CM. When the costs are time-varying, it may be optimal to do more maintenance in months with lower maintenance costs (Schouten et al., 2022). This leads to the introduction of a period-dependent condition replacement policy (p-CRP).

Definition 2 A p-CRP policy is a policy in which each period $i_1 \in \mathcal{I}_1$ has a threshold $k(i_1)$. If the wind turbine has condition $i_2 \geq k(i_1)$ in period i_1, PM is performed.

One may set $k(i_1) = F$ for $i_1 \in \mathcal{I}_1$, which means that in period i_1 no PM is performed, regardless of the condition of the component. In fact, when $k(i_1) = F$, $\forall i_1 \in \mathcal{I}_1$, the p-CRP policy reduces to a CM approach.

4.2 Combined TBM and CBM approach

When there is imperfect condition monitoring, it may be the case that there are unobserved signs of failure. If we use a pure CBM approach in this setting, there will be unexpected failures that lead to CM. On the other hand, if we use a pure TBM approach, we neglect the information that is provided by the condition monitoring. This paper extends upon the existing literature by combining a TBM and a CBM approach, in order to reach potential savings in case of imperfect condition monitoring. Specifically, an ARP and CRP approach are integrated, which leads to the policy as defined below.

Definition 3 A combined age and condition replacement policy (CACRP) is a policy in which there is a condition threshold k. If a component is in a condition k or worse, PM is performed. Additionally, PM is performed when a critical age threshold t is exceeded.

The CRP, ARP and CM policies are nested within the CACRP policy. Specifically, if we set $k = F$, CBM is never performed and we obtain an ARP policy. Similarly, if $t = M$, TBM never occurs and the policy reduces to a CRP approach. Last, if both $k = F$ and $t = M$, the CACRP approach reduces to a CM policy. As with the ARP and CRP strategies, we also introduce a time-dependent variant of the CACRP policy.

Definition 4 A period-dependent combined age and condition replacement policy (p-CACRP) is a policy in which there is a period-dependent condition threshold $k(i_1)$. If a component is in a condition state $k(i_1)$ or worse in period i_1, PM is performed. Additionally, for each period there is a critical age threshold $t(i_1)$. If in period i_1 the age $t(i_1)$ is exceeded, PM is performed.

The p-ARP and p-CRP approach are nested in the p-CACRP policy and can be obtained by the same restrictions as described before for the CACRP policy. Additionally, the p-CACRP strategy reduces to the CACRP approach by setting $k(i_1) = k$ and $t(i_1) = t$ for $\forall i_1 \in \mathcal{I}_1$.

4.3 LP formulations

In this section the linear programming (LP) formulations of the maintenance strategies are given, which are constructed by following the approach of Tijms (2003). For the p-CACRP policy, we introduce the decision variables y_{i_1,i_3}:

$$y_{i_1,i_3} = \begin{cases} 1, \text{ if we maintain at age } i_3 \text{ in period } i_1 \text{ with no observed signs of failure,} \\ 0, \text{ else.} \end{cases} \tag{7}$$

This following LP formulation leads to the optimal p-CACRP policy with respect to the long-run average cost.

$$
\begin{aligned}
\text{minimize} \quad & \sum_{i\in\mathcal{I}} c_i(1)x_{i,1} && && (8a)\\
\text{subject to} \quad & \sum_{i\in\mathcal{I}}\sum_{a\in\mathcal{A}(i)} \pi_{(i,j)}(a)x_{i,a} = \sum_{a\in\mathcal{A}(j)} x_{j,a} \quad \forall j\in\mathcal{I}, && && (8b)\\
& x_{i_1,i_2,i_3,0} + y_{i_1,i_3} \leq 1 && i_2 \in \{1\}\cup\mathcal{I}_{2,unobserved}, \forall (i_1,i_3)\in\mathcal{I}_1\times\mathcal{I}_3, && (8c)\\
& x_{i_1,i_2,i_3,1} - y_{i_1,i_3} \leq 0 && i_2 \in \{1\}\cup\mathcal{I}_{2,unobserved}, \forall (i_1,i_3)\in\mathcal{I}_1\times\mathcal{I}_3, && (8d)\\
& \sum_{i_2\in\mathcal{I}_2}\sum_{i_3\in\mathcal{I}_3}\sum_{a\in\mathcal{A}(i_1,i_2,i_3)} x_{i_1,i_2,i_3,a} = \frac{1}{N} && \forall i_1\in\mathcal{I}_1, && (8e)\\
& x_{i,a} \geq 0 && \forall i\in\mathcal{I}, a\in\mathcal{A}_{CBM}(i), && (8f)\\
& y_{i_1,i_3} \in \{0,1\} && \forall i_1\in\mathcal{I}_1, i_3\in\mathcal{I}_3. && (8g)
\end{aligned}
$$

The decision variables $x_{i,a}$ can be interpreted as the long-run probability of being in state $i = (i_1, i_2, i_3) \in \mathcal{I}$ and taking action $a \in \mathcal{A}$. As maintenance costs are only incurred when taking action $a = 1$, the objective function 8a minimizes the maintenance costs. Constraint 8b ensures that in the steady state the transitions into a state and out of a state are equal. The constraints 8c and 8d ensure that one cannot take a different action in an unobserved condition than in condition 1. To illustrate, if one would not perform maintenance at age 5 in condition 1, one must do the same in each unobserved condition. This must be the case, as in practice you cannot distinguish condition 1 from the unobserved conditions.

Since it is possible to move to the failed state ($i_2 = F$) from any other state, the LP formulation satisfies the weak unichain condition. Therefore, it will provide the optimal p-CACRP policy. Let $x^*_{i,a}$ be the optimal solution to the LP problem. Similar to Schouten et al. (2022), we define $\mathcal{I}_{LP} = \{i|\exists a \in \mathcal{A} \text{ with } x^*_{i,a} > 0\}$. Now we define the strategy R^* by $R^*(i) = a$ if $x^*_{i,a} > 0$. For each remaining state $i \in \mathcal{I}$, we choose an action $a \in \mathcal{A}$ such that $\pi_{ij}(a) > 0$ for some $j \in \mathcal{I}_{LP}$. In this way we add the states i recursively to $\mathcal{I}_{LP}$ until no state remains. With this algorithm, the average optimal strategy R^* is obtained.

The optimal maintenance strategies for the ARP, p-ARP, CRP and p-CRP approaches can be obtained with similar LP formulations. Specifically, one can obtain these formulations by enforcing the model restrictions as depicted in Table 2. Note that all models are nested within the p-CACRP model.

Table 2 Overview of maintenance policies and the restrictions on the LP as formulated in Sect. 4.3

Policy	Restriction
ARP	$p_{observed} = 0$ and $\Delta = 0$
p-ARP	$p_{observed} = 0$
CRP	State-dependent action space $\mathcal{A}_{CBM}$ and $\Delta = 0$
p-CRP	State-dependent action space $\mathcal{A}_{CBM}$
CACRP	$\Delta = 0$
p-CACRP	

5 Numerical results

In this section the numerical results will be presented. First, we replicate a section of Schouten et al. (2022), to obtain benchmark TBM models. Then, the two-stage deterioration process that will be used in the numerical analysis is described. Specifically, we discuss the parameters that can be altered and analyse the failure rate. Afterwards, the results in a constant cost setting and the results in a time-varying costs setting are discussed. Both the costs and the resulting maintenance policies are considered.

5.1 Replication of Schouten et al. (2022)

To obtain a benchmark for the combined models that are discussed in subsequent sections, we first replicate the models as described in Schouten et al. (2022). Specifically, we replicate Table 2 on page 985. Besides the p-ARP strategy, we also compute the cost for the p-BRP and the p-MBRP policies. For a thorough explanation of these policies, we refer to Schouten et al. (2022).

Table A1 in Appendix A.2 shows that the same results were obtained up to the third decimal. The only difference can be found in the computation time for the p-MBRP model. Whereas the original paper shows a computation time of 3 seconds, our computation time was approximately 1 second. This difference may be explained by the solver that was used. Our results were obtained by using Gurobi 9.5.1 in Python, whereas the results from Schouten et al. (2022) are obtained via CPLEX 12.8.0 in Java (Gurobi Optimization, LLC, 2022). Additionally, the results may differ due to the hardware that was used. Our results are obtained by means of an HP envy x360 convertible pc, which has an i7 core processor. The tolerance used in our optimization is 10^{-4}. Our code for the replication of the results is made publicly available on https://github.com/asternfeld/repl_schouten2022.

Note that in the following description, we use the notation of Schouten et al. (2022), which differs from the notation that is used in other parts of this paper. While replicating the results, we encountered two flaws in the original paper. First, in MILP formulation 12 in Section 4.3 of Schouten et al. (2022) there are no constraints that formulate the relationship between x and z. In our implementation, the following two constraints were added:

$$z_{i_1,i_2} + x_{i_1,i_2,0} \leq 1 \tag{9}$$

$$x_{i_1,i_2,1} - z_{i_1,i_2} \leq 0 \tag{10}$$

If $z_{i_1,i_2} = 1$, maintenance is performed in period i_1 when age i_2 is exceeded. Constraint (9) ensures that if $z_{i_1,i_2} = 1$, it must hold that $x_{i_1,i_2,0} = 0$. This ensures that the decision variables x and z do not contradict each other. Similarly, constraint (10) ensures that if there is no maintenance ($z_{i_1,i_2} = 0$), it must hold that $x_{i_1,i_2,1} = 0$. With these added constraints, the same results were obtained as in Schouten et al. (2022). Therefore, it is likely that the constraints were used in their computations but were omitted in the paper by mistake. A second minor flaw can be found in the cdf of the Weibull distribution in Section 5. The correct cdf is $F(x) = 1 - exp\left(-\left(\frac{x}{\alpha}\right)^{\beta}\right)$, whereas the minus sign inside the exponent is missing in Schouten et al. (2022).

5.2 Weibull and Gamma deterioration process

The first stage is assumed to follow a discretized Weibull distribution, which is in line with Schouten et al. (2022). From condition 1, it is only possible to go to the observed condition 2 or the unobserved condition m + 2. The probability of observing the transition to condition 2 is reflected by $p_{observed}$, where $p_{observed} \in [0,1]$. For the second stage we follow the approach of de Jonge (2019) and assume a gamma process, which is a flexible distribution and thus suitable for a wide variety of deterioration processes. This process is most appropriate for a monotonically degrading component (van Noortwijk, 2009). It is not possible to transition from an observed state to an unobserved state, nor is it possible to transition the other way around. Additionally, a wind turbine can only improve by means of maintenance. Let us denote the time spent in stage 1 by T_1 and the time spent in stage 2 by T_2.

Stage 1: Weibull distribution deterioration

The first stage is modelled by means of a discretized Weibull distribution with cdf $F(x) = 1 - exp\left(-\left(\frac{x}{\alpha}\right)^{\beta}\right)$ for $x \in \bar{\mathbb{N}}$, where $\alpha > 0$ is the scale parameter and $\beta > 0$ is the shape parameter. Therefore, the probability p_{i_3} of showing the first signs of failure at age i_3 can be calculated as shown in equation (11). One can then calculate the transition probabilities according to the specification in equation (3) (Schouten, 2019).

$$\begin{aligned} p_{i_3} &= \mathbb{P}(i_3 \leq T_1 \leq i_3 + 1) \\ &= 1 - exp\left\{-\left(\frac{i_3+1}{\alpha}\right)^{\beta} + \left(\frac{i_3}{\alpha}\right)^{\beta}\right\} \end{aligned} \tag{11}$$

For the analysis of maintenance policies, it is of interest to consider the expected time spent in stage 1 ($\mathbb{E}(T_1)$) and the variance of the time spent in stage 1 ($\mathbb{V}ar(T_1)$). The expected value can be calculated as shown below in Eq. (12) (Schouten, 2019). It follows from this expression that an increase in the scale parameter α leads to an increase in the average time spent in stage 1. On the other hand, a change in the shape parameter β does not have a large effect on the mean.

$$\mathbb{E}(T_1) = \sum_{k=0}^{n} exp\left\{-\left(\frac{k}{\alpha}\right)^{\beta}\right\} \quad \text{for large } n \tag{12}$$

The variance of the time spent in stage 1 can be calculated as shown below. An increase in the scale parameter α results in an increase of the variance, whereas increasing β leads to a decrease in the variance.

$$\begin{aligned} \mathbb{V}ar(T_1) &= \mathbb{E}(T^2) - (\mathbb{E}(T_1))^2 \\ &= \sum_{k=0}^{n}(2k+1)exp\left\{-\left(\frac{k}{\alpha}\right)^{\beta}\right\} - \left(\sum_{x=0}^{n} exp\left\{-\left(\frac{k}{\alpha}\right)^{\beta}\right\}\right)^2 \quad \text{for large n} \end{aligned} \tag{13}$$

Stage 2: Gamma deterioration process

Whereas in the first stage the deterioration depends on the age of the component, the deterioration in the second stage depends on the condition of the component. The additional deterioration per period is modelled by a discretized homogeneous gamma process. We use the following density function, with scale parameter $\delta > 0$ and shape parameter $\theta > 0$:

$$f_{\delta,\theta}(x) = \frac{1}{\Gamma(\theta)\delta^{\theta}} x^{\theta-1} e^{-\frac{x}{\delta}}, \tag{14}$$

where $\Gamma(\theta) = \int_0^\infty z^{\theta-1} e^{-z} dz$ is the gamma function. The corresponding cdf is denoted by $F_{\delta,\theta}$. As a closed-form expression does not exist, the cdf must be calculated numerically. We follow the discretization approach of de Jonge (2019), by considering the stochastic process $X(t)$, which reflects the amount of deterioration after spending t periods in stage 2. The process $\{X(t),\ t \geq 0\}$ has shape function $b(t)$ and scale a. Additionally, one must specify the period length Δt and the number of intervals of equal length that the condition state space is divided into, denoted by m. We then see that the independent increments satisfy $X(t + \Delta t) - X(t) \sim f_{a\Delta t,b}$ for $\Delta t > 0$. Therefore, $\mathbb{E}(X(t)) = abt$ and $\mathbb{V}ar(X(t)) = a^2bt$ reflect the expectation and variance of the deterioration level at time t respectively. When the variance is higher, there is a larger probability of observing a large jump in the deterioration level. To compute the transition probabilities, we first consider the probability h_i of moving from condition k to condition $k + i$:

$$h_i = \begin{cases} q_0, & \text{if } i = 0, \\ q_i - q_{i-1}, & \text{if } i = 1, 2, ..., \end{cases} \tag{15}$$

with $q_i = \frac{1}{\Delta x} \int_0^{\Delta x} F_{a\Delta t,b}((i+1)\Delta x - x)dx$. This results in the transition probabilities:

$$p_{i_2,j_2} = \begin{cases} h_{j_2 - i_2}, & \text{if } i_2 < j_2 \wedge (i_2, j_2 \in I_{2,unobserved} \vee i_2, j_2 \in I_{2,observed}), \\ 1 - \sum_{k=1}^{m+1-i} h_k, & \text{if } 1 < i_2 < F \wedge j_2 = F, \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

A functional failure F corresponds to the critical deterioration threshold L. Therefore, the probability of failure at time t can be represented by $P(X(t)) \geq L$. This probability can be computed by

$$F_L(t) = \mathbb{P}(X(t) > L) = \int_L^\infty f_{bt,a}(x)dx. \tag{17}$$

In the remainder of this paper, we rescale the deterioration levels, such that failure occurs when the deterioration level is higher than $L = 1$. As for stage 1, we will consider the mean and variance of the time spent in stage 2 for the analysis of maintenance policies. We therefore note that the probability of failure between t_1 and t_2 is $F_L(t_2; a, b) - F_L(t_1; a, b)$, which means that the expected time spent in stage 2 can be computed as

$$E(T_2) = \sum_{i=0}^{\infty} i\epsilon(F_L((i+1)\epsilon; a, b) - F_L(i\epsilon; a, b)), \tag{18}$$

for sufficiently small ϵ. Whereas de Jonge et al. (2017) and de Jonge (2019) use $\mathbb{V}ar(X(1)) = ba^2$ as a measure of the variation in the deterioration process, we are specifically interested in the variance of the time spent in stage 2. The variance can be computed as

$$\begin{aligned}\mathbb{V}\text{ar}(T_2) &= \sum_{i=0}^{\infty} \mathbb{P}(i\epsilon \leq T_2 \leq (i+1)\epsilon)(i\epsilon - \mathbb{E}(T_2))^2 \\ &= \sum_{i=0}^{\infty} (F_L((i+1)\epsilon; a, b) - F_L(i\epsilon; a, b)(i\epsilon - \mathbb{E}(T_2))^2.\end{aligned} \tag{19}$$

Parameter settings

Overall, the deterioration process is determined by six parameters. We can adjust the shape parameters α and a, the scale parameters β and b and the discretization parameters m and ΔT. For the remainder of this paper, we fix $m = 3$, as we focus our analysis on the effects of the shape and scale parameters and on the effects of considering time-varying costs . It is of particular interest to consider the mean time to failure (MTTF), where MTTF $= E(T)$, as this is useful for the interpretations of the results. Additionally, we consider the coefficient of variation $c_T = \frac{\sqrt{\mathbb{V}ar(T)}}{\mathbb{E}(T)}$, as this is a unitless measure of the variability with respect to the mean. When the coefficient of variation is increases, the distribution of the lifetime of a component becomes less peaked around the mean. Generally, this results in a worse performance of TBM strategies, as planning maintenance based on the lifetime of a component is harder. Since the deterioration processes of stage 1 and 2 are independent, we can calculate these measures as illustrated below.

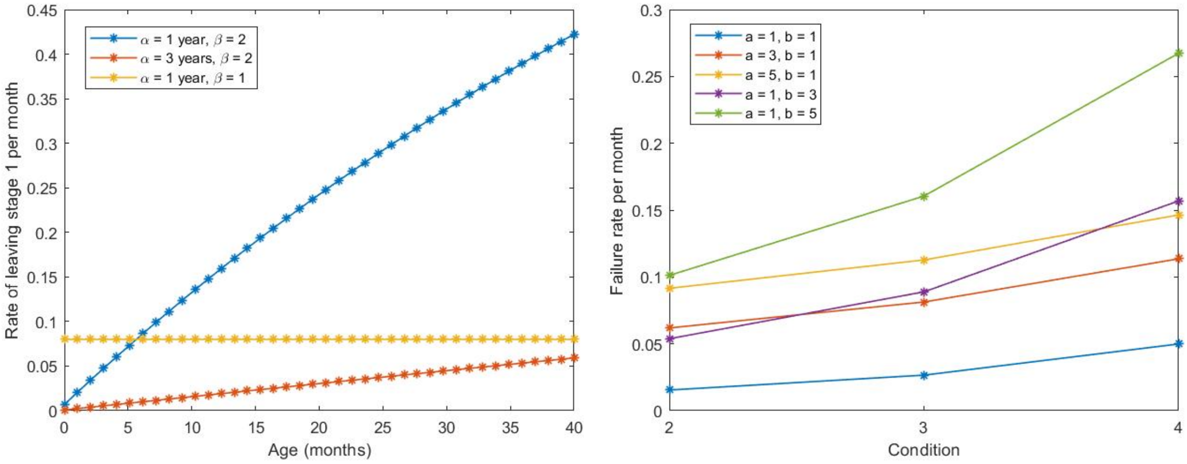
$$MTTF = E(T_1) + E(T_2), \tag{20}$$

$$c_T = \frac{\sqrt{\mathbb{V}ar(T)}}{\mathbb{E}(T)}, \quad \text{where } \mathbb{V}ar(T) = \mathbb{V}ar(T_1) + \mathbb{V}ar(T_2). \tag{21}$$

Failure rate

Based on the two stages as described before, we can now analyse the probability of failure under different parameter settings. Figure 3 shows the rate of transitioning from stage 1 to stage 2 and the probability of failure in stage 2 for varying values of α, β, a and b. The discrete analog of the failure rate as described in Dekker and Roelvink (1995) is used. The failure rates in stage 1 and stage 2 are thus $\mathbb{P}(T_1 = t \mid T_1 \geq t)$ and $\mathbb{P}(T_2 = t \mid T_2 \geq t)$, respectively. Note that the failure rate in stage 2 is independent of the time spent in stage 2 and only depends on the current condition. In fact, $\mathbb{P}(T_2 = t \mid T_2 \geq t) = p_{i_2,F}$.

Figure 3a demonstrates that an increase of the scale parameter α results in a steeper curve, which indicates that on average a component shows the first faults at a lower age. When $\beta = 1$, the curve is flat, which means that the probability of showing the first signs of failures is independent of the age. Specifically, when $\beta = 1$, it holds that $T_1 \sim exp(\alpha)$. Figure 3b illustrates that an increase of the scale parameter a results in a higher failure probability in each condition. An increase in the shape parameter b also leads to a higher failure probability in each condition, but also results in a steeper curve. This means that for higher values of b, a component in a worse state will deteriorate relatively faster. This is in line with the P-F curve as described in Sect. 3.1, therefore different values of b will be considered in the analysis.



(a) Rate of leaving stage 1 **(b)** Failure rate stage 2

Fig. 3: Rate of leaving stage 1 and failure rate in stage 2 for various parameter settings.

5.3 Constant costs setting

Let us start by considering a constant costs setting, thus $c_p(i_1) = \bar{c}_p, \forall i_1 \in \mathcal{I}_1$. We discretize time into months, hence $\varDelta T = \frac{1}{12}$. The LP given in Sect. 4.3 is then solved using Gurobi 9.5.1 in Python. Table 3 shows the results for various values of $p_{observed}$ and the stage 2 shape parameter b, with $\alpha = 1$ year. The differences are calculated with respect to the benchmark ARP model with the same parameters for the deterioration process.

The results show that for small values of $p_{observed}$, a CRP approach performs worse than the ARP approach. However, for larger values of $p_{observed}$ the CRP approach outperforms the benchmark ARP model. This illustrates that a CRP approach is highly dependent on the quality of the condition monitoring. When the condition monitoring is accurate, one may rely fully on the observations. If there are severe measurement errors, it is necessary to also use periodic maintenance. For larger values of the shape parameter b, CRP and CACRP approaches perform better. This can be explained due to the coefficient of variation being larger for higher values of b. Therefore, there is more variability in the time of failure, which means the failure times are less concentrated. Consequently, the ARP policy performs worse, as the failures are centered less around a critical age. In contrast, a CRP approach is affected less by a change in the shape parameter b. A CRP approach depends on the observations of faults and on the expected duration of the two stages but does not rely on the failure times being concentrated around one age. Furthermore, the results demonstrate that a CACRP approach reduces to an ARP approach for $p_{observed} = 0$ and reduces to a CRP approach for $p_{observed} = 1$. Since the ARP and CRP strategies are thus nested within the CACRP policy, the CACRP approach always performs best.

Table A2 in Appendix A.4 shows the results for $\alpha = 3$ years. Generally, these results reflect the same findings as described before. Note that a higher value of α results in a relatively higher duration of stage 1. Consequently, the MTTF is higher and the costs for each maintenance policy decrease. For higher values of $p_{observed}$, the CRP and CACRP policies outperform the ARP approach by relatively more compared to the setting with $\alpha = 1$. This is caused by the higher MTTF, as this increases the benefit of performing CBM. When early signs of failure are observed, it is more beneficial to perform PM, since a component is likely to remain working for a long time after maintenance. Additionally, one can see that the coefficient of variation is lower compared to the setting with $\alpha = 1$. This means that the failure times are more concentrated, which results in a better performance of the ARP policy. Consequently, the ARP approach outperforms the CRP by more for low values of $p_{observed}$.

Table 3 Yearly costs in thousands of euros and the corresponding differences with respect to the ARP benchmark model, under constant costs.

	b = 1				b = 3				b = 5			
	MTTF = 2.4 years				MTTF = 1.6 years				MTTF = 1.2 years			
	$c_T = 0.56$				$c_T = 0.69$				$c_T = 0.76$			
	Benchmark ARP costs 13.791				Benchmark ARP costs 20.782				Benchmark ARP costs 24.420			
	CRP		CACRP		CRP		CACRP		CRP		CACRP	
$p_{observed}$	Costs	Difference	Costs	Difference	Costs	Difference	Costs	Difference	Costs	Difference	Costs	Difference
0*	18.269	32.47%	13.791	0%	33.234	59.92%	20.782	0%	39.295	60.91%	24.420	0%
0.2	16.809	21.88%	13.463	-2.38%	29.996	44.34%	19.813	-4.67%	34.756	42.33%	23.004	-5.80%
0.4	15.918	15.42%	13.134	-4.76%	26.275	26.43%	18.650	-10.26%	29.731	21.75%	21.388	-12.42%
0.6	14.952	8.42%	12.767	-7.43%	21.954	5.64%	17.169	-17.39%	24.139	-1.15%	19.311	-20.92%
0.8	13.903	0.81%	12.138	-11.99%	16.874	-18.80%	15.052	-27.57%	17.877	-26.79%	16.340	-33.09%
1	10.817	-21.56%	10.817	-21.56%	10.817	-47.95%	10.817	-47.95%	10.817	-55.70%	10.817	-55.70%

Note. In each model, we have $\bar{c}_p = 10$ and $\bar{c}_f = 50$. Let $\alpha = 1$ year and $\beta = 2$, which gives $\mathbb{E}(T_1) = 0.9$ years. For stage 2 we set $a = 1$ and let b vary.
*A CRP model with $p_{observed} = 0$ is equivalent to a pure CM approach. The costs in this setting are thus the same as doing no PM.

We have thus seen that a CACRP approach leads to cost savings compared to the ARP and CRP policies, when there is imperfect condition monitoring. To further investigate this maintenance strategy, Table 4 displays the optimal CACRP strategies for different parameter settings. One can see that when $p_{observed}$ increases, the critical age for TBM increases. This indicates that when condition monitoring is more accurate, there is more reliance on CBM and thus less frequent TBM. The same conclusion can be drawn by the proportion of TBM relative to all PM. For higher values of $p_{observed}$, there is relatively less TBM and more CBM. On the other hand, when condition monitoring is inaccurate, there is relatively more TBM, as there is a higher chance of unobserved failures.

Furthermore, when the gamma parameter b is higher, a component deteriorates faster after the first faults are observed. Therefore, when considering a setting with $\alpha = 1$ year, the critical maintenance condition is lower for $b = 3$ compared to $b = 1$. When α increases, the expected time until the first faults occur rises, which leads to two changes. First, the critical age for TBM increases, which is caused by the slower deterioration process in stage 1. Second, the critical condition for performing CBM goes down. This effect demonstrates how the two stages of the deterioration process are interconnected. When the expected time a component stays in condition 1 increases, performing CBM will yield a higher benefit. This is the case because performing CBM will bring a component back to condition 1, in which it will now stay longer. Therefore, a higher value of α thus leads to a policy which performs CBM sooner.

Table 4 Maintenance strategies for a CACRP approach under differing parameters for the deterioration process.

	$\alpha = 1$ year, $\beta = 2$						$\alpha = 3$ years, $\beta = 2$					
	a = 1, b = 1			a = 1, b = 3			a = 1, b = 1			a = 1, b = 3		
	CACRP			CACRP			CACRP			CACRP		
$p_{observed}$	Age	Condition	%TBM	Age	Condition	%TBM	Age	Condition	%TBM	Age	Condition	%TBM
0	14	F	100%	9	F	100%	25	F	100%	20	F	100%
0.2	14	3	86.12%	9	2	86.90%	27	2	90.27%	22	2	92.27%
0.4	3	3	69.72%	10	2	74.76%	29	2	78.67%	24	2	81.24%
0.6	3	3	51.39%	11	2	53.97%	33	2	62.25%	29	2	65.79%
0.8	2	2	24.51%	14	2	32.13%	40	2	38.57%	39	2	38.63%
1	M	2	0%	M	2	0%	M	2	0%	M	2	0%

Note. The critical age in stage 1 and the critical condition in stage 2 are given, alongside the percentage TBM, with respect to all PM. In each model, it holds that $c_p = 10$ and $c_f = 50$.

5.4 Time-varying costs

In this section, the effects of time-varying costs are discussed. The model can be applied with time-varying costs over the weeks or months, but this requires a high level of computational power. Therefore, we consider time-varying costs over the seasons, hence $N = 4$. Specifically, periods 1, 2, 3 and 4 are winter, spring, summer and fall respectively. We follow the approach of Schouten et al. (2022) and set $c_p(i_1) = \bar{c}_p + \Delta \cdot \bar{c}_p \cdot \cos\left(\frac{2\pi i_1}{N} - \frac{2\pi}{N}\right)$ and $c_f(i_1) = \bar{c}_f + \Delta \cdot \bar{c}_f \cdot \cos\left(\frac{2\pi i_1}{N} - \frac{2\pi}{N}\right)$. In this specification, maintenance costs are highest in winter and lowest in summer. The cost structure is described more thoroughly in Appendix A.3. In this section, we first give the cost savings for various parameter settings and then analyse the resulting maintenance policies.

Cost savings

Table 5 shows the costs and savings for different values of Δ. These results demonstrate that taking into account time-varying costs results in cost savings for each maintenance approach. However, these savings are smaller for a p-CRP approach compared to the p-ARP policy. This illustrates that CBM is more urgent and harder to postpone. With TBM, one can more easily delay maintenance until summer, as there is no indication the wind turbine will fail imminently. This is harder for CBM, as there is a higher risk for a component to fail soon.

Table 5 Yearly costs and differences with respect to the constant cost model ($\Delta = 0$).

	p-ARP		p-CRP		p-CACRP	
Δ	Costs	Savings	Costs	Savings	Costs	Savings
0	12.694	0%	12.746	0%	11.768	0%
10%	11.795	7.08%	12.376	2.90%	11.494	5.33%
20%	10.673	15.92%	11.904	6.61%	10.473	11.00%
30%	9.552	24.75%	11.431	10.32%	9.397	20.15%
40%	8.424	33.64%	10.959	14.02%	8.312	29.37%
50%	7.296	42.52%	10.487	17.72%	7.226	38.60%

Note. Let $\bar{c}_p = 10$ and $\bar{c}_f = 50$. In each model, $\alpha = 1$ year, $\beta = 2$, a = 1, b = 1 and $p_{observed} = 0.6$.

In a constant cost setting, a p-CRP approach outperforms a p-ARP strategy for high values of $p_{observed}$. However, in a setting with time-varying costs, there are more savings for a p-ARP approach compared to the p-CRP policy. Figure 4 displays the long run average costs for a p-ARP and p-CRP strategy for different values of $\varDelta$ and $p_{observed}$. This figure shows that $\varDelta$ affects the costs for a p-CRP strategy less than the costs of a p-ARP approach, which confirms that p-ARP has more benefit from the consideration of time-varying costs. In this setting, p-ARP generally outperforms p-CRP. Only for a small value of $\varDelta$ and a high $p_{observed}$, the p-CRP approach obtains lower costs. However, these results depend on the parameter setting. Figure A3 in appendix A.5 shows that the performance of the p-ARP strategy deteriorates when $\beta = 1$. As described in Sect. 5.3, this is caused by a higher coefficient of variation. Now, it largely depends on $p_{observed}$ and $\varDelta$ whether p-ARP or p-CRP results in lower costs.

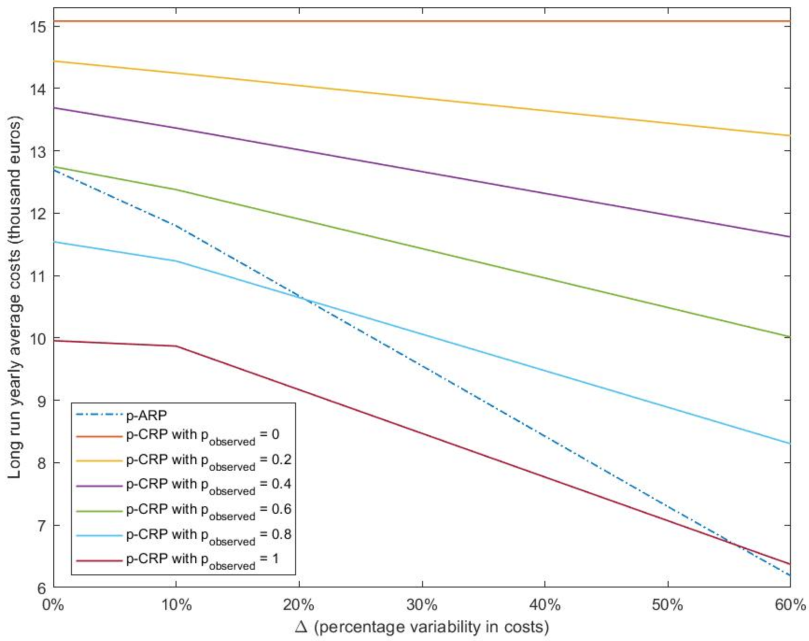


Fig. 4: Long run yearly average costs for p-ARP and p-CRP approaches, for varying levels of $p_{observed}$ and $\varDelta$. Let $\beta = 2$, $\alpha = 1$ year and $a = 1$.

The savings depend highly on the parameter setting in the deterioration process. Therefore, we consider $\alpha = 0.5, 1, 3$ years, $\beta = 1, 2$, $a = 1, 3, 5$ years and $b = 1, 2$ and compute the savings for each combination. Table 6 summarizes the results for the p-ARP, p-CRP and p-CACRP model for each parameter. The savings are averaged over each combination of the possible values of the other parameters.

Table 6 shows that a higher value of α leads to higher savings. The explanation is that for a higher value of α, the MTTF increases. Consequently, one can coordinate the maintenance with the seasons more easily, as the chance of failure before the optimal maintenance moment is smaller. Similarly, a higher value of the shape parameter β results in higher savings. This is due to the distribution becoming more peaked, which leads to less failures before the PM happens.

An increase in the gamma shape parameter b results in a significant decrease in savings, due to both the lower MTTF and the higher coefficient of variation. For $b = 5$, the savings of a p-CRP policy are close to 0, for all values of Δ. This indicates that when the rate of deterioration in stage 2 is high, it is not profitable to coordinate CBM with the season. The effect of the parameter a is similar, albeit smaller.

When comparing the savings of the three different maintenance policies, it becomes clear that the p-ARP policy results in the largest savings compared to its constant cost alternative. This illustrates that TBM is less urgent than CBM. When maintenance is based on the age of a component, one can postpone maintenance to a more favorable season with a relatively low risk of failure. In contrast, the delay of CBM has a higher risk of leading to failure of the component. After all, when observing early signs of failure, the component is likely to fail in the near future. This becomes even clearer when b increases, as this reduces the expected time spent in stage 2 ($\mathbb{E}(T_2)$). For $b = 3$ or $b = 5$, a p-CRP approach reaches almost no savings over the constant cost case. Whereas a p-CRP policy only benefits from incorporating time-varying costs for high levels of Δ and a small b, the p-CACRP approach also obtains savings for lower variations in costs.

Table A3 in appendix A.5 displays the long run yearly average costs for each parameter. The results are averaged over all possible combinations of the other parameters. These results show that the p-CACRP policy results in substantial savings over the p-ARP and p-CRP policies in most scenarios. Especially for larger values of b and for $\beta = 1$, p-CACRP outperforms p-ARP. This is caused by more variation in the deterioration process, which reduces the performance of the p-ARP strategy.

Table 6 Average percentage savings of the p-ARP, p-CRP and p-CACRP maintenance policies, compared to the constant cost ($\Delta = 0$) model.

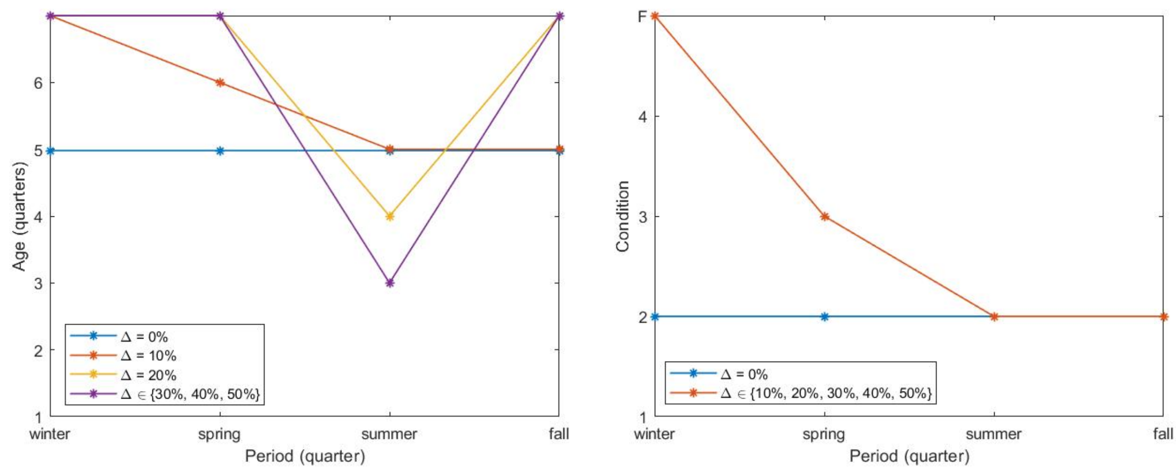
Δ	Method	$\alpha = 0.5$	$\alpha = 1$	$\alpha = 3$	$\beta = 1$	$\beta = 2$	a = 1	a = 2	b = 1	b = 3	b = 5	Average
	p-ARP	1.31%	3.01%	3.11%	2.01%	2.94%	2.74%	2.21%	5.17%	1.22%	1.05%	2.48%
10%	p-CRP	0.22%	0.34%	0.10%	0.18%	0.26%	0.33%	0.11%	0.65%	0.01%	<0.01%	0.24%
	p-CACRP	0.97%	2.28%	2.46%	1.22%	2.59%	2.10%	1.70%	2.28%	1.91%	1.52%	1.90%
	p-ARP	3.32%	7.93%	7.36%	5.14%	7.27%	6.89%	5.52%	11.28%	3.95%	3.38%	6.20%
20%	p-CRP	0.68%	0.80%	0.73%	0.61%	0.86%	1.07%	0.40%	2.18%	0.03%	<0.01%	0.82%
	p-CACRP	3.63%	6.23%	6.58%	3.89%	7.07%	6.07%	4.90%	7.42%	5.07%	3.95%	5.48%
	p-ARP	6.67%	13.59%	13.00%	9.39%	12.78%	12.07%	10.10%	17.69%	8.39%	7.18%	11.09%
30%	p-CRP	1.60%	1.52%	1.71%	1.34%	1.88%	2.09%	1.13%	4.60%	0.23%	<0.01%	1.79%
	p-CACRP	6.89%	11.09%	11.19%	7.26%	12.19%	10.71%	8.74%	13.60%	8.73%	6.85%	9.73%
	p-ARP	11.33%	19.87%	20.05%	14.52%	19.65%	18.52%	15.65%	24.51%	14.67%	12.07%	17.08%
40%	p-CRP	2.7%	2.44%	3.05%	2.30%	3.17%	3.44%	2.04%	7.28%	0.87%	0.05%	2.73%
	p-CACRP	10.68%	16.85%	16.08%	11.21%	17.86%	15.92%	13.16%	20.54%	12.82%	10.24%	14.54%
	p-ARP	17.40%	26.37%	28.07%	20.45%	27.43%	25.84%	22.04%	31.76%	21.75%	18.31%	23.94%
50%	p-CRP	4.34%	3.46%	4.63%	3.49%	4.80%	5.09%	3.87%	9.99%	2.00%	0.44%	4.21%
	p-CACRP	15.31%	23.20%	21.46%	15.68%	24.30%	21.75%	18.23%	27.91%	17.61%	14.46%	19.99%

Note. The average savings are given for each deterioration process parameter, averaged over the other parameters. For all models, $p_{observed} = 0.6$, $\bar{c}_p = 10$ and $\bar{c}_f = 50$.

Time-varying maintenance policies

Let us now analyse how the optimal maintenance policy changes for different levels of Δ. Figures 5 and 6 show how the critical age and critical condition for a p-CACRP approach vary over the periods in two different settings. Figure 5 corresponds to a setting in which $b = 1$, which means that the MTTF is 2.4 years. One can then see that if there are time-varying costs ($\Delta > 0$), the critical maintenance age in winter is M, which indicates that there will never be TBM in winter. For higher levels of Δ, the critical maintenance age in summer decreases. In fact, for $\Delta \geq 20\%$, TBM will only be performed in summer. This aligns with the results of Schouten et al. (2022). In the same setting, Figure 5b illustrates that when costs vary over time there will also be no CBM in winter. However, the CBM will not be purely centered in summer. Also in spring and fall, there will be CBM, although the critical maintenance condition is lowest in summer and fall. This again shows that CBM cannot be postponed easily, as the chance of failure in the near future is relatively high when observing early signs of failure.

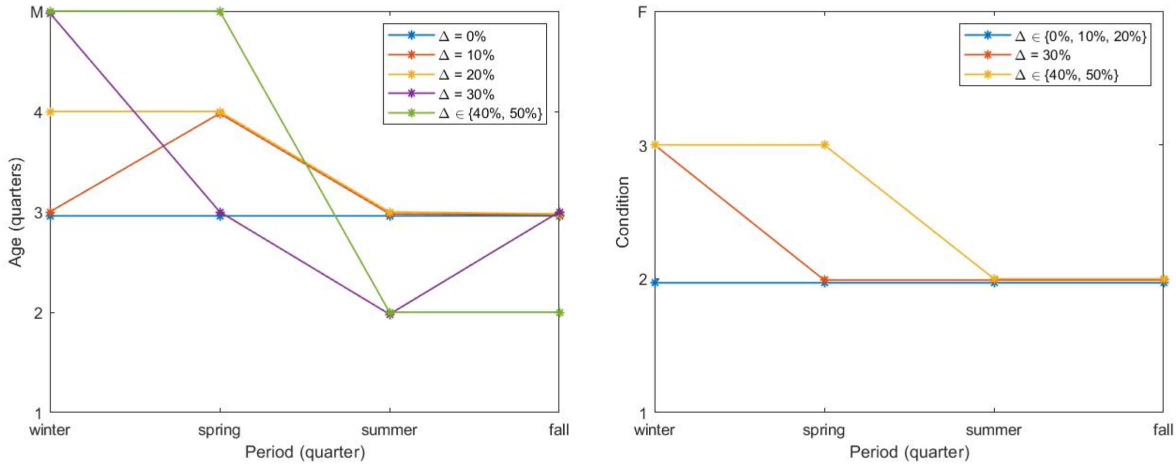
Figure 6 illustrates how the results change when considering a setting with $b = 5$, which leads to a MTTF of 1.2 years. One can now see that coordination with the seasons becomes harder, due to the faster deterioration process. In Figure 6a it is shown that for lower values of Δ, the main policy change is a higher critical maintenance age in spring. Intuitively, one waits more often for maintenance until summer, as costs are then lowest. For higher levels of Δ, maintenance is mainly done in summer and fall. Whereas in the setting with $b = 1$ maintenance was only done in summer, we now also have maintenance in fall with the aim of preventing failure in winter. Figure 6b shows that with a faster deterioration process, the critical maintenance condition only changes for relatively high cost variations over the seasons. In fact, even with $\Delta \in \{40\%, 50\%\}$, CBM may still be optimal in every period. As seen before, the critical condition is higher in winter and spring. These results illustrate that the effect of time-varying costs is dependent on the deterioration process. When a component deteriorates slowly, coordinating maintenance with the seasons is easier and will lead to higher benefits.



(a) Critical maintenance age

(b) Critical maintenance condition

Fig. 5: Critical maintenance age and condition for p-CACRP with $a = 1$, $b = 1$, $\alpha = 1$ year, $\beta = 2$, $\bar{c}_p = 10$, $\bar{c}_f = 50$ and $p_{observed} = 0.6$



(a) Critical maintenance age **(b)** Critical maintenance condition

Fig. 6: Critical maintenance age and condition for p-CACRP with $a = 1$, $b = 5$, $\alpha = 1$ year, $\beta = 2$, $\bar{c}_p = 10$, $\bar{c}_f = 50$ and $p_{observed} = 0.6$

6 Marginal cost analysis

One disadvantage of using a MDP to optimize the p-ARP, p-CRP and p-CACRP models is the lack of interpretability. In practical applications, it is a major advantage if there are clear and intuitive criteria on which the maintenance decision is based. Therefore, we will now discuss maintenance policies that are based on a MCA.

6.1 Formulation of the Marginal Cost Approach

For this analysis, we will consider the model with $m = 1$, as this eases the computation of failure rates. As the MCA takes a short-term perspective, we introduce the observed condition i_2^*. The new state space is thus $i = (i_1, i_2^*, i_3)$. We define i_2^* as:

$$i_2^* = \begin{cases} 1, \text{ if no early signs of failure have been observed,} \\ 2, \text{ if early signs of failure have been observed,} \\ 3, \text{ if there is a functional failure.} \end{cases} \tag{22}$$

Note that when no faults are observed, there may still be unobserved faults. In each scenario, we consider two choices:

1. Replace the component preventively,
2. Replace the component upon failure or maintain preventively in the next period.

We now consider the marginal costs of choosing option 2 over option 1. The first option results in the costs $c_p(i_1)$, as we do PM. Opting for the second choice means that we wait one more period before we perform PM. This results in the costs $(1 - r(i_2^*, i_3))c_p(i_1 + 1) + c_f(i_1 + 1)r(i_2^*, i_3)$, where $r(i_2^*, i_3)$ is the failure rate in condition i_2^* with age i_3. If there is no failure, we only have the PM costs in period $i_1 + 1$, whereas we incur the CM costs $c_f(i_1 + 1)$ if there is a failure. We now see that the marginal costs (MC) of waiting one more period before doing maintenance is

$$MC = c_p(i_1 + 1) - c_p(i_1) + (c_f(i_1 + 1) - c_p(i_1 + 1))r(i_2^*, i_3) \tag{23}$$

We can now form a decision criteria based on a well-known economic principle. When minimizing the average total costs, at the optimal point the marginal costs should equal the average costs. After all, when the marginal costs are lower than the average costs, one can benefit from waiting longer as this will decrease the overall average costs. On the other hand, when the marginal costs are higher than the average costs, maintenance takes place too late as the total average costs are now increasing. This leads to the following replacement criterium (RC):

$$RC = c_p(i_1 + 1) - c_p(i_1) + (c_f(i_1 + 1) - c_p(i_1 + 1))r(i_2^*, i_3) - g^* \tag{24}$$

Specifically, we will replace the component if $RC \geq 0$, as in this case the marginal costs are higher or equal than the average total costs. The total average costs will be estimated by considering the average costs from a CACRP strategy.

6.2 Computation of the failure rates

For the computation of the failure rates, the discrete analog of the failure rate as described by Dekker and Roelvink (1995) is used. The failure rate can be computed via

$$\begin{aligned} r(i_2^*, i_3) &= \frac{\mathbb{P}(T = t)}{\mathbb{P}(T \geq t)} \\ &= \mathbb{P}(T = t \mid T \geq t). \end{aligned} \tag{25}$$

Nest, two different scenarios should be considered. In the first case, there have been no observed signs of failure. The rate of failure now depends on both stages. There is only a chance of failure if there has already been an unobserved transition to stage 2. Additionally, there should be a transition from stage 2 into a state of functional failure over the next period. This gives:

$$r(1, i_3) = (1 - p_{observed})\mathbb{P}(T_1 \leq i_3 - 1)p_{3,4}. \tag{26}$$

Note that $\mathbb{P}(T_1 \leq i_3 - 1)$ can be calculated with the cdf of the Weibull distribution. In the second scenario, one has observed signs of failure. This means that one can be certain that the true condition is $i_2 = 2$. Therefore, the failure rate is

$$r(2, i_3) = p_{2,4}. \tag{27}$$

6.3 Constant cost setting

In this section, the results of the MCA under constant costs will be described. First, the marginal costs of postponing maintenance will be analysed. Afterwards, the long-run average costs of the MCA strategy will be compared to the costs of an ARP, CRP and CACRP approach.

Marginal costs without observation of faults

Let us first consider the marginal cost function, after which the long-run average costs of a MCA strategy are analysed. Figure 7 shows the marginal costs when having observed no signs of failure for different parameter settings. The markers on the curves indicate at what age the marginal costs equal the long run average costs based on a CACRP policy. At these points, TBM should be performed according to the replacement criterion. One can see that the marginal cost curves are increasing, which indicates that postponing maintenance becomes more costly when a component is older. This is caused by a larger rate of failure, due to a higher probability of having left stage 1 unobserved. Figure 7 shows that for larger values of b the curve becomes steeper, as $r(2, i_3)$ is larger for any value of $i_3 \in \mathcal{I}_3$. When $\alpha = 3$ years, the average time spent in stage 1 increases. Consequently, the marginal costs are lower for each age. When $\alpha = 1$ year and $b = 1$, the MCA indicates that TBM should never be done. The rate of failure in stage 2 is relatively low, therefore the marginal costs of postponing maintenance when no faults have been observed are lower than the long-run average costs.

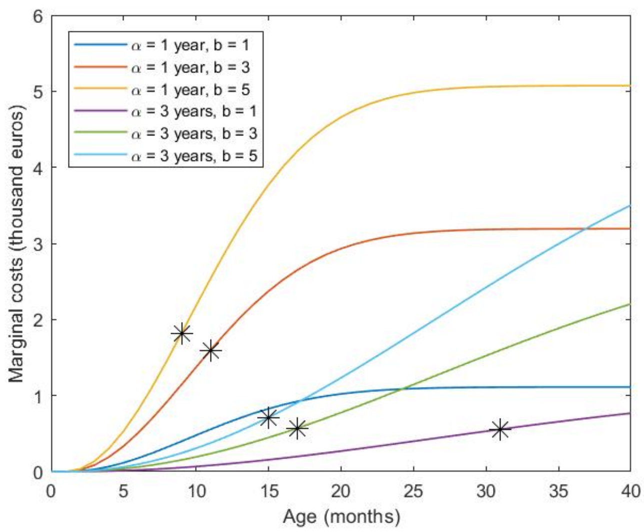


Fig. 7: Marginal costs of postponing maintenance one period when no signs of failure have been observed. The markers indicate the intersection with the long run average CACRP costs. Let $c_p = 10$, $c_m = 50$, $\beta = 2$ and $p_{observed} = 0.6$.

Marginal cost after observation of faults

When signs of failure have been observed, one may choose for CBM. Table 7 displays the MC and the RC for different parameter settings for stage 2. Note that in stage 2, the probability of failure is independent of the age of a component. Table 7 shows that in every parameter setting, the RC is positive. This indicates that CBM should be done whenever early signs of failure have been observed. For higher values of a and b, the RC increases, which indicates that postponing maintenance is more costly. This is caused by a faster deterioration process in stage 2, since an increase in a or b leads to a decrease in $\mathbb{E}(T_2)$.

Table 7 Marginal cost (MC) and replacement criterium when faults have been observed

	$a = 1$			$a = 3$		
	$b = 1$	$b = 3$	$b = 5$	$b = 1$	$b = 3$	$b = 5$
MC	2.780	9.979	12.681	5.283	14.108	20.931
RC	1.487	8.267	10.775	3.742	12.156	18.823

Note. Let $c_p = 10$, $c_m = 50$, $\beta = 2$ and p = 0.6. The marginal cost refers to the cost of postponing maintenance one period.

Long-run average costs

Let us then compare the long-run average costs of the MCA to the ARP, CRP and CACRP strategies. Table 8 shows the costs of each approach and the corresponding savings with respect to a CM approach, for different parameter settings. The MCA obtains significant savings on the CM costs, in each setting. When $\alpha = 1$ and b = 1, the MCA strategy is the same as a CRP approach, as TBM is never performed. For $\alpha = 1$ and b = 3 or b = 5, the MCA approach outperforms both the CRP and ARP strategies. For b = 3, the MCA approach is the same as a CACRP approach, whereas it is slightly worse for b = 5. Last, the MCA approach performs worse than the CRP approach for $\alpha = 3$ and b = 3 or b = 5. This is caused by a relatively low critical age when no signs of failure have been observed. On the other hand, it performs better than the ARP approach for these parameter settings.

Table 8 Long run yearly average costs for a CRP, ARP, CACRP and MC approach, for different parameter settings. Additionally, savings with respect to the CM policy are given.

	$\alpha = 1$						$\alpha = 3$					
	$b = 1$		$b = 3$		$b = 5$		$b = 1$		$b = 3$		$b = 5$	
CM costs	22.687		37.078		41.980		9.410		13.025		14.079	
	Costs	Savings	Costs	Savings	Costs	Savings	Costs	Savings	Costs	Savings	Costs	Savings
CRP	17.747	21.78%	23.670	36.16%	25.163	40.06%	6.536	30.54%	7.948	38.98%	8.318	40.92%
ARP	16.978	25.16%	23.489	36.65%	26.622	36.58%	8.176	13.11%	10.533	19.13%	11.408	18.97%
CACRP	14.527	35.97%	18.508	50.08%	20.322	51.59%	6.394	32.05%	7.658	41.21%	8.048	42.84%
MCA	17.747	21.78%	18.508	50.08%	20.488	51.20%	6.402	31.97%	8.368	35.75%	9.763	30.66%

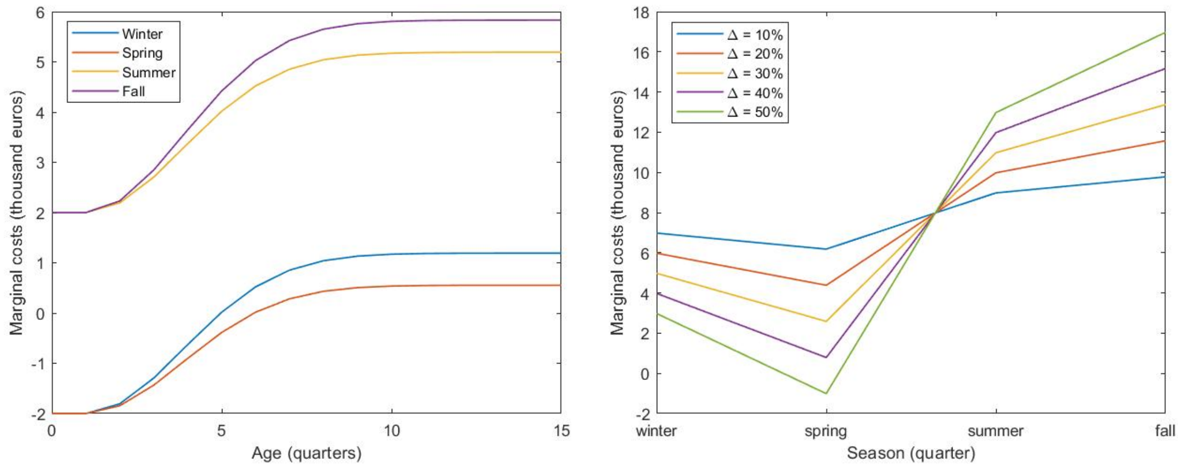
Note. Let $c_p = 10$, $c_m = 50$, $\alpha = 1$ year, $\beta = 2$ and p = 0.6.

6.4 Time-varying costs

Let us now consider a MCA in a setting with time-varying costs. As with the MDP, the time will be discretized into four seasons. We will first describe the marginal costs over the seasons, after which the long-run average costs are presented for different degrees of cost variation.

Marginal costs

Figure 8 shows the marginal costs of postponing maintenance one period in a setting with time-varying costs. Figure 8a gives the marginal cost when no signs of failure have been observed, for different ages of a component. The marginal cost in is highest in fall and lowest in spring. When postponing maintenance from fall to winter, the costs increase, as maintenance is most expensive in winter. On the other hand, the marginal costs are lowest when postponing maintenance in spring, as the costs in summer are lowest. Figure ?? in Appendix A.6 provides a further analysis of these marginal costs and shows the marginal costs of postponing maintenance when no signs of failure have been observed with $\Delta = 50\%$. In this setting, the differences between the marginal costs across seasons increase. Figure 8b shows the marginal costs of delaying maintenance after signs of failure have been observed, for several values of Δ. Again, the marginal costs are lowest in spring and highest in fall. When Δ is larger, the differences in marginal costs over the seasons get larger, as there is more cost variation.

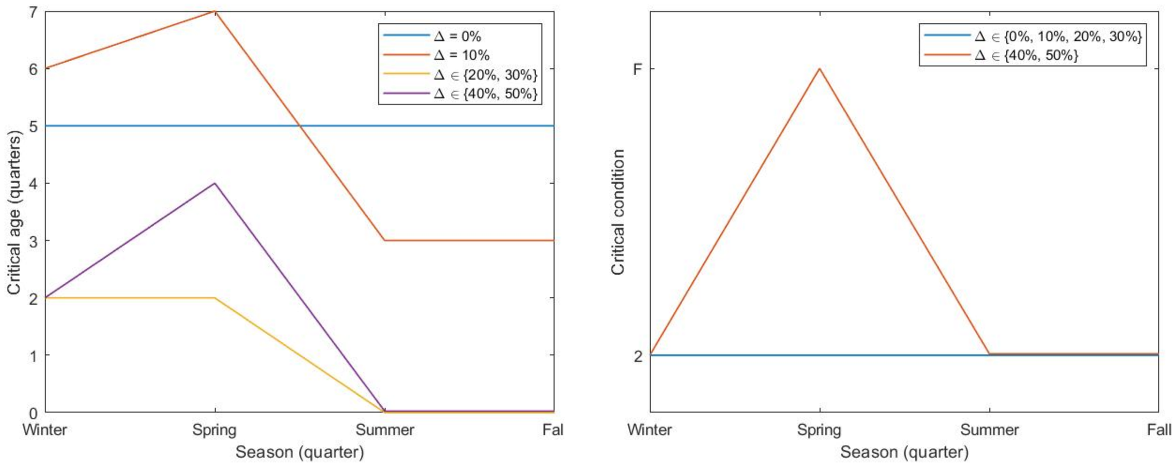


(a) No observation of signs of failure, $\Delta = 20\%$ **(b)** Observed signs of failure

Fig. 8: Marginal costs of postponing maintenance one period. We distinguish between a setting where signs of failure have not been observed and a setting in which they have been observed. Let $a = 1$, $b = 1$, $\alpha = 1$ year, $\beta = 2$, $\bar{c}_p = 10$, $\bar{c}_f = 50$ and $p_{observed} = 0.6$

Long-run average cost In order to analyse the performance of the MCA strategy under time-varying costs, we first consider the critical maintenance age and critical maintenance condition in each season. Figure 9 displays the critical age and critical condition in each season for Δ between 0% and 50%. Figure 9a shows that when Δ is 20% or higher, the critical maintenance age is 0 in fall and winter. This illustrates one of the disadvantages of a MCA approach. The RC compares the costs of doing maintenance immediately and the costs of doing maintenance one period later. It thus takes a short-term perspective and does not consider the option of doing maintenance in the further future. The difference between PM costs in summer and fall is sufficiently high for $\Delta > 20\%$, such that it is the optimal choice to do PM in summer at age 0 when taking a short-term point of view. Similarly, PM costs in winter are significantly higher than in fall, hence even at age 0 the MCA indicates to do PM in fall.

Figure 9b shows that for $\Delta \leq 30\%$, maintenance is always done when signs of failure are observed. For larger values of Δ, CBM is not done in spring. It is then postponed, due to the lower maintenance cost in summer. Note that there is CBM in winter, which indicates that the CM costs in spring are sufficiently high to warrant PM in winter.



(a) Critical maintenance age **(b)** Critical maintenance condition

Fig. 9: Critical maintenance age and critical maintenance condition in each season. Let $a = 1$, $b = 1$, $\alpha = 1$ year, $\beta = 2$, $\bar{c}_p = 10$, $\bar{c}_f = 50$ and $p_{observed} = 0.6$

Table 9 shows the long-run average costs of the CACRP and MCA strategies under time-varying costs. The results show that a MCA performs worse when incorporating time-varying costs. In fact, generally the costs increase for $\Delta \geq 20\%$, compared to the constant cost setting. As described before, this is caused by the short-term perspective of a MCA. Maintenance is performed even at age 0, which leads to higher costs. The losses due to incorporating time-varying costs are lower for larger values of b, since a component then deteriorates faster. When a component deteriorates faster, it is more reasonable to do maintenance at a low age. Whereas the MCA performed well under constant costs, one can see that the CACRP now outperforms the MCA by a large margin. In Sect. 5.4 we saw that the CACRP gained savings by incorporating time-varying costs in a model with m = 3. Table 9 shows that also for m = 1, the CACRP approach reaches savings for every level of Δ.

Table 9 Yearly costs in thousands of euros and the corresponding savings with respect to the constant cost model.

	b = 1				b = 3				b = 5			
	CACRP		MCA		CACRP		MCA		CACRP		MCA	
Δ	Costs	Diff.	Costs	Diff.	Costs	Diff.	Costs	Diff.	Costs	Diff.	Costs	Diff.
0%	13.284		13.284		16.582		16.582		18.074		19.094	
10%	12.893	-2.94%	13.084	-1.51%	15.992	-3.56%	16.323	-1.56%	17.450	-3.45%	17.450	-8.61%
20%	12.160	-8.46%	20.194	52.02%	15.173	-8.50%	21.303	28.48%	16.608	-8.11%	21.959	15.01%
30%	11.318	-14.80%	19.163	44.26%	14.301	-13.76%	20.182	21.71%	15.681	-13.24%	20.787	8.87%
40%	10.153	-23.57%	17.623	32.66%	13.393	-19.23%	19.290	16.33%	14.712	-18.60%	20.340	6.53%
50%	8.924	-32.82%	16.514	24.31%	12.369	-25.41%	17.966	8.35%	13.657	-24.44%	18.884	-1.10%

Note. In each model, $c_p = 10$ and $c_f = 50$. Let $\alpha = 1$ year, $\beta = 2$, a = 1, b = 1 and m = 1.

7 Conclusion

This paper has presented a combined TBM and CBM approach, which was analysed by using both a discrete-time MDP and a MCA. We find that under constant costs, the CACRP policy obtains savings over both the standard ARP and CRP approaches. When condition monitoring is inaccurate, the CACRP policy prevents unobserved failures by scheduling TBM at a relatively low critical age. On the other hand, if condition monitoring is accurate the CACRP strategy only chooses for TBM at a high critical age, since most imminent failures are spotted by condition monitoring. Under time-varying costs, a p-ARP policy reaches more savings than a p-CRP approach. In fact, when there is a high degree of variability in maintenance costs over the years, a p-ARP policy generally results in lower costs than a p-CRP policy. Under time-varying costs, the savings of the p-CACRP approach with respect to the p-ARP policy are smaller compared to the constant cost setting.

A MCA provides more insight in the model and performs reasonably well in a constant cost setting. However, in a setting with time-varying costs, a MCA is not suitable due to its short-term view. When maintenance costs in the next period are significantly higher than the current costs, a MCA will indicate that maintenance should be performed. However, this may not be optimal, since maintenance costs will decrease again in future periods. For a setting with time-varying costs, the MCA should be adapted such that it considers the cyclical cost pattern, which is left to future studies.

Let us conclude by discussing two limitations of this study that lead to opportunities for future research. First, this work considers one type of failure that is either observed or unobserved, while in practice there are multiple types of failures. Each type of failure may occur at a different frequency and may result in different maintenance costs, which is defined as a competing failure process. Future research can focus on developing a combined TBM and CBM approach for a competing failure process. In this setting, condition monitoring may only detect certain failures and a combined TBM and CBM policy may reach savings by preventing functional failures due to unobserved defects.

A second limitation of the present work lies in the modelling of condition monitoring. The results indicate that the accuracy of condition monitoring is largely responsible for the effectiveness of the CBM component of a policy. However, this work took a simplified approach by modelling the accuracy of condition monitoring as the probability of observing a fault. A more sophisticated approach is taken by Raza and Ulansky (2019), who model noise as a random process and specifically consider false positives and false negatives. Future work can further analyse the performance of a combined TBM and CBM approach, with a more detailed condition monitoring model. This will result in a more thorough understanding of the effect of condition monitoring on the optimal maintenance policy.

8 References

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A Appendix

A.1 Abbreviations

Wind turbine (WT)

Time-based maintenance (TBM)

Condition-based maintenance (CBM)

Age replacement policy (ARP)

Block replacement policy (BRP)

Condition replacement policy (CRP)

Combined age and condition replacement policy (CACRP)

Period-dependent age replacement policy (p-ARP)

Period-dependent block replacement policy (p-BRP)

Period-dependent modified block replacement policy (p-MBRP)

Period-dependent condition replacement policy (p-CRP)

Period-dependent combined age and condition replacement policy (p-CACRP)

Markov decision process (MDP)

Linear program (LP)

Marginal cost analysis (MCA)

Preventive maintenance (PM)

Corrective maintenance (CM)

Mean time to failure (MTTF)

A.2 Replication of Schouten et al. (2022)

In Table A1, the results of the replication of Table 2 on page 985 of Schouten et al. (2022) are given. Further explanation is provided in Sect. 5.1.

Table A1 Yearly costs in thousands of euros and the savings with respect to the constant cost case.

	p-ARP		p-BRP (m = 1)			p-MBRP (m = 1)			
Δ	Costs	Savings	Costs	Savings	Months	Costs	Savings	Months	Ages
0%	40.098		41.501			40.311			
10%	40.035	0.16%	41.420	0.20%	6, 11	40.263	0.12%	6, 11	4, 4
20%	39.701	0.99%	40.933	1.37%	6, 11	39.855	1.13%	6, 11	4, 4
30%	30.224	2.18%	40,316	2.75%	6, 10	39.338	2.41%	6, 10	5, 3
40%	38.461	4.08%	39.439	4.97%	6, 10	38.556	4.35%	6, 10	5, 3
50%	37.635	6.14%	38.446	7.31%	7, 10	37.773	6.30%	6, 10	5, 3
CPU	<1 s		<1 s			1 s			

Note. Maintenance months are given for p-BRP and p-MBRP, with the corresponding critical age for the p-MBRP model. In each setting we let $\bar{c}_p = 10$, $\bar{c}_f = 50$, $\alpha = 1$ year and $\beta = 2$.

A.3 Time-varying cost specification

In this appendix, we will provide a more detailed view on the time-varying cost setting. When we set the number of periods to 4 ($N = 4$), the costs are computed as follows:

$$c_p(i_1) = \bar{c}_p + \Delta\bar{c}_p \cos\left(\frac{1}{2}\pi i_1 - \frac{1}{2}\pi\right) \tag{A1}$$

$$c_f(i_1) = \bar{c}_f + \Delta\bar{c}_f \cos\left(\frac{1}{2}\pi i_1 - \frac{1}{2}\pi\right) \tag{A2}$$

Consequently, the highest costs are in period 1 (winter) and the lowest costs are in period 3 (summer). The costs in period 2 (spring) and period 4 (fall) are equal in this specification. To assess the accuracy of this specification, we consider wind speeds over the seasons in IJmuiden. We use daily data from (Royal Netherlands Meteorological Institute, 2022) to estimate the average wind speed per season. The data contains the daily average wind speeds in IJmuiden, a city at the coast in the Netherlands, from 1971 to 2022. Let us first consider the average wind speeds per month. Figure A1 shows the estimated averages alongside a cosine fit.

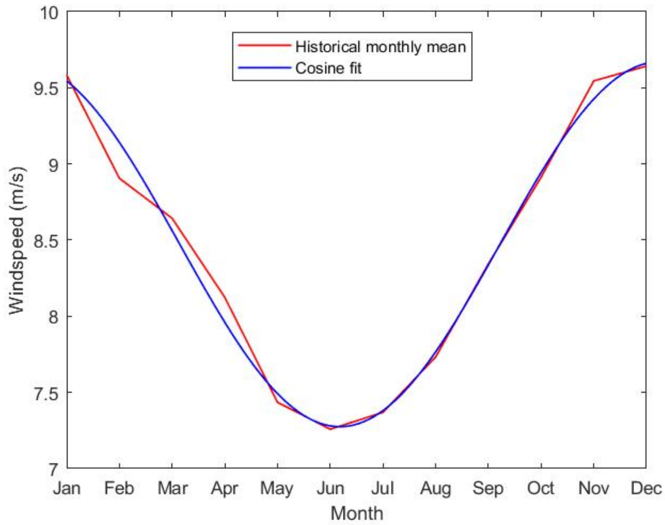

Fig. A1: Average wind speed in IJmuiden over the months, with cosine fit. Estimated by using daily data on wind speed between 1971 and 2022.

Let us now group the months by season, as this is applied in our analysis. We define the months as follows:

- Winter: December, January and February
- Spring: March, April and May
- Summer: June, July and August
- Fall: September, October and November

Figure A2 displays the estimated seasonal average wind speed with a cosine fit. One can see that compared to a monthly setting, some accuracy is lost. However, the cosine fit still seems to be a reasonable specification.

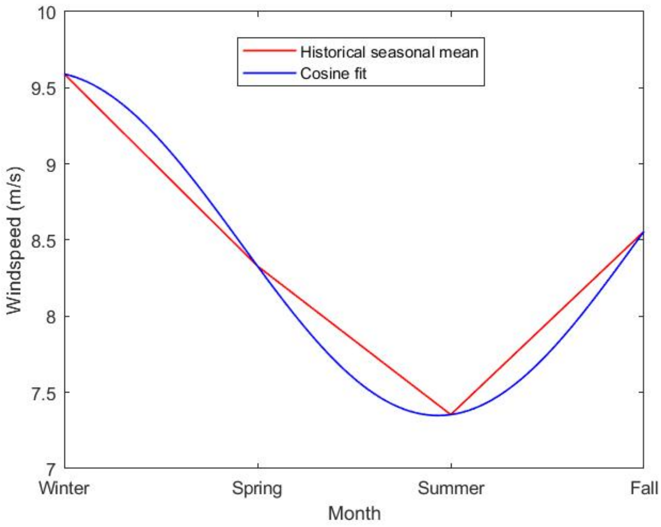

Fig. A2: Average wind speed in IJmuiden over the seasons, with cosine fit. Estimated by using daily data on wind speed between 1971 and 2022.

A.4 Results in a constant cost setting

Table A2 displays the long-run yearly average costs for the ARP, CRP and CACRP maintenance policies in a constant cost setting. Various values of b and $p_{observed}$ are considered. Whereas in Sect. 5.3 the results for $\alpha = 1$ year were displayed, Table A2 gives the results for $\alpha = 3$ years. A comparison of the results for $\alpha = 1$ year and $\alpha = 3$ years is given in Sect. 5.3.

Table A2 Yearly costs in thousands of euros and the corresponding differences with respect to the ARP benchmark model under constant costs.

	b = 1 MTTF = 4.2 years $c_T = 0.70$				b = 3 MTTF = 3.4 years $c_T = 0.85$				b = 5 MTTF = 3.0 years $c_T = 0.90$			
	Benchmark ARP costs 7.243				Benchmark ARP costs 10.004				Benchmark ARP costs 11.044			
	CRP		CACRP		CRP		CACRP		CRP		CACRP	
$p_{observed}$	Costs	Difference	Costs	Difference	Costs	Difference	Costs	Difference	Costs	Difference	Costs	Difference
0*	12.018	65.93%	7.243	0%	15.647	56.41%	10.004	0%	16.589	50.21%	11.044	0%
0.2	10.830	49.52%	6.897	-4.78%	13.559	35.54%	9.316	-6.88%	14.227	28.82%	10.183	-7.80%
0.4	9.459	30.60%	6.481	-10.52%	11.334	13.29%	8.493	-15.10%	11.764	6.52%	9.169	-16.98%
0.6	7.860	8.52%	5.949	-17.87%	8.961	-10.43%	7.46	-25.43%	9.195	-16.74%	7.918	-28.30%
0.8	5.971	-17.56%	5.194	-28.29%	6.424	-35.79%	6.036	-39.66%	6.511	-41.04%	6.239	-43.51%
1	3.716	-48.70%	3.716	-48.70%	3.705	-62.96%	3.705	-62.96%	3.716	-66.35%	3.716	-66.35%

Note. In each model, $\bar{c}_p = 10$ and $\bar{c}_f = 50$. Let $\alpha = 3$ years and $\beta = 2$, which gives $\mathbb{E}(T_1) = 2.7$ years. For stage 2 we set $a = 1$ and let b vary.
*A CRP model with $p_{observed} = 0$ is equivalent to a pure CM approach. The costs in this setting are thus the same as doing no PM.

A.5 Results in a time-varying cost setting

Figure A3 gives the long-run yearly average costs for a p-ARP and p-CRP approach, for varying values of $\varDelta$ and $p_{observed}$. In Figure 4 in Sect. 5.4 the costs are given with $\beta = 2$, whereas here we consider $\beta = 1$. When $\beta = 1$, the p-ARP approach performs worse. This is caused by a higher coefficient of variation, since the failure times are now less concentrated around one critical age.

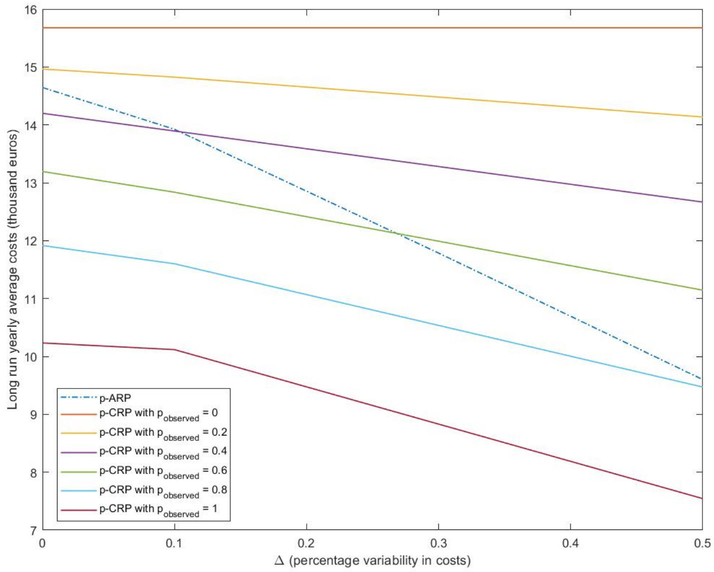

Fig. A3: Long run yearly average costs for p-ARP and p-CRP approaches, for varying levels of $p_{observed}$ and $\varDelta$. Let $\beta = 1$, $\alpha = 1$ year and $a = 1$.

Table A3 Yearly average costs of the p-ARP, p-CRP and p-CACRP policies.

$\varDelta$	Method	$\alpha = 0.5$	$\alpha = 1$	$\alpha = 3$	$\beta = 1$	$\beta = 2$	a = 1	a = 2	b = 1	b = 3	b = 5	Average
	p-ARP	28.602	21.835	11.592	23.298	18.060	19.193	22.164	14.936	21.976	25.124	20.678
0%	p-CRP	29.508	18.557	8.663	18.785	18.899	17.906	19.778	14.305	20.377	21.844	18.842
	p-CACRP	24.735	16.910	8.461	18.152	15.252	15.852	17.552	13.195	17.659	19.252	16.702
	p-ARP	28.360	21.283	11.246	22.915	17.677	18.799	21.793	14.219	21.757	24.913	20.296
10%	p-CRP	29.479	18.496	8.432	18.751	18.854	17.844	19.761	14.187	20.377	21.844	18.803
	p-CACRP	24.497	16.541	8.221	17.915	14.924	15.550	17.289	12.887	17.361	19.011	16.420
	p-ARP	27.912	20.320	10.745	22.237	17.082	18.138	21.180	13.334	21.244	24.400	19.659
20%	p-CRP	29.340	18.402	8.345	18.656	18.736	17.728	19.664	13.871	20.373	21.844	18.700
	p-CACRP	23.912	15.900	7.781	17.388	14.340	14.952	16.776	12.165	16.837	18.592	15.864
	p-ARP	27.028	19.179	10.108	21.230	16.313	17.266	20.277	12.393	20.349	23.573	18.772
30%	p-CRP	29.091	18.258	8.220	18.503	18.543	17.529	19.517	13.433	20.292	21.844	18.523
	p-CACRP	23.186	15.125	7.322	16.741	13.681	14.278	16.143	11.326	16.227	18.079	15.211
	p-ARP	25.756	17.881	9.320	20.050	15.255	16.129	19.176	11.403	19.105	22.449	17.652
40%	p-CRP	28.790	18.057	8.049	18.315	18.283	17.257	19.340	12.983	20.102	21.810	18.299
	p-CACRP	22.325	14.205	6.859	15.992	12.934	13.515	15.411	10.399	15.533	17.456	14.463
	p-ARP	24.049	16.531	8.431	18.746	13.928	14.832	17.842	10.365	17.680	20.967	16.337
50%	p-CRP	28.329	17.819	7.823	18.038	17.942	16.915	19.066	12.559	19.775	21.638	17.990
	p-CACRP	21.244	13.190	6.389	15.181	12.034	12.645	14.570	9.462	14.706	16.655	13.608

Note. The costs for each parameter are averaged over the other parameters. Let $p_{observed} = 0.6$, $\bar{c}_p = 10$ and $\bar{c}_f = 50$.

Table A3 shows the long-run yearly average costs of the p-ARP, p-CRP and p-CACRP approach, under various parameter settings. For each parameter setting, the costs are averaged over each combination of the possible values of the other parameters. The results shows that the gamma parameter b has the highest impact on the costs. For $b = 1$, the lowest costs are obtained, as this will give the slowest deterioration process. Furthermore, we see that the p-CACRP policy generally gives significant savings over the p-ARP and p-CRP strategies.

A.6 Marginal cost analyis

Figure A4 displays the marginal costs of postponing maintenance for one period, when no signs of failure have been observed. In Figure 8a in Sect. 6.4 these marginal costs are reflected for $\Delta = 20\%$, whereas we now consider $\Delta = 50\%$. One can see that the shape of the curve is similar for $\Delta = 20\%$ and $\Delta = 50\%$. However, for $\Delta = 50\%$ the differences between the seasons are larger. One should note that the marginal costs in fall are higher than in summer. To explain this, we consider the marginal cost criterion (as in (23)):

$$MC = c_p(i_1 + 1) - c_p(i_1) + (c_f(i_1 + 1) - c_p(i_1 + 1))r(i_2^*, i_3)$$

Both when postponing maintenance from summer to fall and from fall to winter, the PM costs and CM costs increase. However, in winter the difference between PM costs and CM costs is largest. Therefore, the marginal costs are largest in fall, as the term $c_f(i_1 + 1) - c_p(i_1 + 1)$ is largest. Intuitively, this illustrates that we want to avoid performing corrective maintenance in winter, as this is highly costly. A similar analysis holds when comparing the marginal costs in spring and winter.

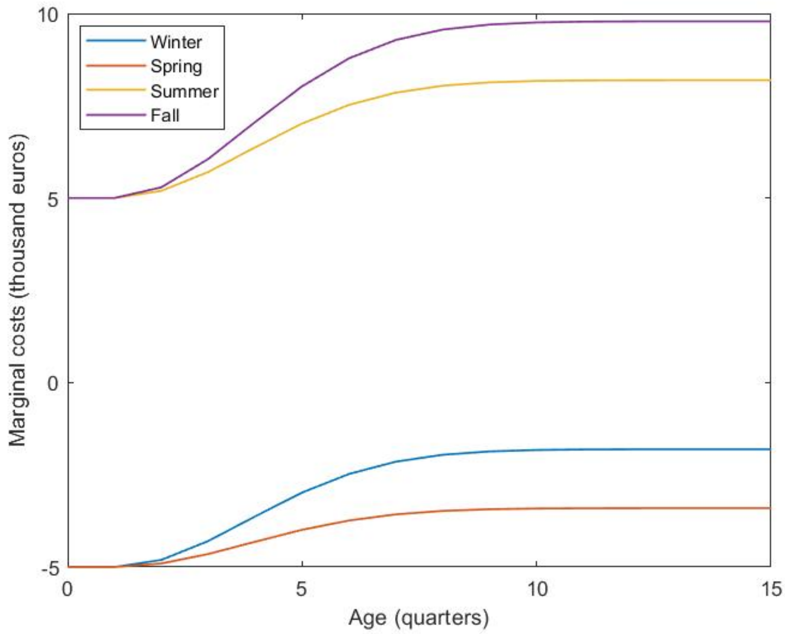

Fig. A4: Marginal costs of postponing maintenance one period, when signs of failure have not been observed. Let $\Delta = 50\%$, $a = 1$, $b = 1$, $\alpha = 1$ year, $\beta = 2$, $\bar{c}_p = 10$, $\bar{c}_f = 50$ and $p_{observed} = 0.6$