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Optimal maintenance policies for wind turbines under time-varying costs

Author:
Emily Fields

Student number:
505456

Supervisor:
Prof. dr. ir. Rommert Dekker

Second Assessor:
Dr. Wilco van den Heuvel

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Abstract

In this paper, two new, multi-component models for maintenance optimization under time-varying costs are presented, specifically targeted towards offshore wind turbine maintenance. The single-component modified block replacement policy for time-varying costs (p-MBRP) is extended to two components with economic dependence in the form of setup costs. A further adaptation to this model is also presented, in which corrective maintenance (CM) can be delayed to take advantage of shared setup costs with planned preventive maintenance (PM) of another component. Mixed integer linear programming formulations are presented to optimize the parameters of these policies. Numerical results show that savings of up to 19% can be achieved via combining the two extensions.

Keywords: Preventive maintenance (PM) · Corrective maintenance (CM) · p-MBRP

Contents

1 Introduction

As the world attempts to shift toward a more sustainable living pattern, it is becoming increasingly important to switch energy generation from exhaustible to renewable sources. One form of renewable energy that is particularly valuable in countries with a coastline is offshore wind energy: as of 2019, 11 European countries use energy from 4149 offshore wind turbines, generating approximately 16 gigawatts of energy (DeCastro et al., 2019). However, the costs of offshore wind energy remain prohibitive in some cases, not only in terms of capital but also in terms of maintenance (Blanco, 2009). Given the urgent need to transition to more sustainable energy sources, it is crucial that maintenance costs be optimized to allow offshore wind energy to be an economically feasible alternative to oil and gas. Indeed, results in Garcia-Teruel et al. (2022) highlight the importance of customizing maintenance policies to the offshore scenario in order to minimize the environmental impact of the renewable technology.

There exist two types of maintenance for offshore wind turbines: corrective maintenance (CM) involves repairing or replacing a component after it has broken down, while preventive maintenance (PM) is a form of maintenance in which components are preventively maintained before failure, with the goal of increasing their lifetime. In general, it is assumed that CM cannot be planned, as it must always be performed immediately to prevent downtime. Maintenance optimization models therefore focus on optimizing the scheduled PM times to minimize costs.

The seminal work of Barlow and Proschan (1965) introduces some of the first such maintenance optimization models. Three maintenance policies are presented, called the age replacement, block replacement, and modified block replacement policies (ARP, BRP, and MBRP, respectively). An ARP is a policy according to which a component is replaced preventively once it has reached a certain age. On the other hand, under a BRP, components are replaced at a given interval, regardless of their age. MBRP is a combination of the two, where components have scheduled maintenance times based on intervals, but maintenance is only performed at these times if the component has reached a certain age.

A feature specific to wind turbines is the variability in the costs of maintenance. Namely, when a wind turbine is maintained, it faces some downtime, when it cannot generate energy. As energy generation in this case depends greatly upon wind speeds, the costs of maintaining an offshore wind turbine are thus also dependent on the wind speeds. Fortunately, these wind speeds are somewhat predictable (on average) according to the seasons, so can be accounted for in policy-making to optimize maintenance times such that they can take advantage of periods with lower costs. Such model adjustment is presented in Schouten et al. (2022), where the three standard ARP, BRP, and MBRP models are extended to the case of time-varying maintenance costs, leading to three new periodic versions of the models, denoted p-ARP, p-BRP, and p-MBRP.

Of course, a wind park does not usually consist of just one turbine. It is thus crucial to consider maintenance decisions not only on a single-turbine basis, as there exist costs that can be shared when maintaining several turbines at the same time. An example of such a cost is the travel via a support vessel to the offshore wind farm. Schouten (2019) presents a multi-component version of the p-MBRP model in Schouten et al. (2022), in which there is a setup cost that can be shared when several components undergo maintenance at the same time.

In the model of Schouten (2019), several simplifying assumptions are made that do not necessarily accurately reflect reality. First, the setup cost is assumed to be constant over time. In reality, however, setup costs might be expected to change depending also on the time of year, for example if travel costs to the wind farm are higher in windier periods with bigger waves. The present work thus builds upon the multi-component model in Schouten (2019) to allow setup costs to vary with time in the same way as variable maintenance costs. Second, the model in Schouten (2019) does not allow the timing of CM to be altered; it is assumed that CM is always necessarily performed immediately upon failure. Yet, Nakagawa (1982) shows that there is potential in delaying CM in certain scenarios. Consider an example in which setup costs are very high, and a component fails only one period before the next scheduled PM activity for another component. In this case, it might be optimal to accept the lost production and delay the maintenance until further components are maintained, to save on setup costs. For this reason, the present work develops a new policy, denoted p-MBRP2+DCM, to allow for this possibility. As well as reducing costs directly via the sharing of setup costs and the delaying of CM to a cheaper period, such a policy also allows for increased predictability of maintenance times, which might lower costs even further in practice.

Briefly, the present work contributes to the literature by extending the existing p-MBRP model in Schouten et al. (2022) to the two-component case with economic dependence, adapting it to more realistic conditions by relaxing some assumptions. The two-component case is considered rather than the multi-component case for ease of exposition and computation. Specifically, the model will be extended, under its existing assumptions, to the two-component case with economic dependence in the form of time-varying setup costs. Then, the model will be further adapted to include the opportunity to delay CM. To the best of my knowledge, time-varying setup costs have not been considered in the literature, while delayed CM has been explored only in the constant cost case.

Hence, the aim of this report is to answer the following research question:

> **RQ.** *How can existing block-based maintenance policies for wind turbines with time-varying costs be adapted to a multi-component setting with shared setup costs?*

This research question is split into three subquestions that give direction to the research, as follows:

> **RQ.1.** *How does the single-component p-MBRP model in Schouten et al. (2022) perform in a multi-component setting with economic dependency in the form of setup costs?*

> **RQ.2.** *How can the single-component p-MBRP model in Schouten et al. (2022) be extended to a multi-component setting with economic dependence in the form of time-varying setup costs, assuming that CM occurs immediately?*

> **RQ.3.** *How can the single-component p-MBRP model in Schouten et al. (2022) be extended to a multi-component setting with economic dependence in the form of time-varying setup costs, assuming that CM can be delayed?*

Thus, two models for the multi-component case, one with and one without the possibility to delay CM, will be presented and compared to both one another and the single-component policy.

The paper continues as follows. First, the related works and their results are discussed in more detail in Sect. 2, after which the exact problem is described in Sect. 3. Subsequently, the models are presented in Sect. 4, in which the single-component model in Schouten et al. (2022) as well as the two new multi-component models are outlined, and the experimental setup is described. Thereafter, the results of each of the models are compared to one another in Sect. 5. Last, conclusions with the main findings and suggestions for future research are presented in Sect. 6.

2 Literature review

There exists a wide range of literature regarding maintenance policies, across many different industries and in both theoretical and technical fields of research. Moreover, there has recently been a lot of research into improving renewable energy sources in order to facilitate the energy transition. This section will begin by discussing the economic and environmental value of research regarding offshore wind turbines. Next, single-component maintenance optimization models will be discussed, with a brief description of each of the main three policy types outlined in Nakagawa (1982), as well as some of the competing models. Then, adaptations of single-component models to the case with time-varying costs will be described. Last, a short review of existing multi-component models will be provided.

2.1 Economic setting

As of 2022, offshore wind energy is an established technology, with wind turbines themselves considered reliable and developed. As such, there remain only limited possibilities to improve the technology itself. To maximize the potential of this renewable energy source, it is therefore crucial that to make the operation and maintenance of offshore wind turbines more efficient. In fact, Röckmann et al. (2017) identifies that operational and maintenance costs form 25-30% of the total lifecycle cost of an offshore wind turbine. On the other hand, production losses due to downtime have been estimated to be close to 12 million euros per year for a 500 megawatt offshore wind farm located 50 km offshore in the North Sea (Rinaldi et al., 2021). This is a significant loss, indicating a trade-off which is crucial to take into account when planning PM frequency. Namely, while scheduling PM too often can be expensive, scheduling it not often enough can lead to high downtime costs.

In order to minimize total lifecycle costs of an offshore wind turbine, a decision maker thus must take into account two key aspects: the likelihood that the turbine will fail given a certain maintenance schedule, and the cost of maintenance performed according to that maintenance schedule (Rinaldi et al., 2021). The former is based on the predicted reliability of each wind turbine component, that is, how long it will survive between maintenance actions. The latter can be called supportability, and relates to the allocation of resources to different maintenance actions. It can be that the cost of maintenance varies depending on how many maintenance actions are carried out at once, as certain costs (such as transportation costs to the offshore wind farm) can be shared under certain conditions.

Maintenance costs of an offshore wind turbine can be separated into two categories: preventive and corrective (PM and CM). The goal of PM is to replace components that are likely to fail soon. Such maintenance is scheduled in advance, and might include such tasks as inspection, blade cleaning, calibration, replacement of sensors, and oil and filter changes (Walford, 2006). Such costs can be estimated relatively accurately, although variations may occur due to fluctuations in labor costs or the cost of replacement components. On the other hand, CM is unscheduled maintenance, that must take place when a component fails (leading the whole turbine to fail). The costs of such maintenance can be separated into direct and indirect costs: direct costs are those of labor, equipment, and replacement parts, while indirect costs are the lost revenue due to downtime of the failed turbine. The costs due to lost revenue depend upon the total downtime (during repair) as well as on the wind energy that could have been produced in this time. It can be reported as lost kWh or in monetary terms.

2.2 Single-component models

The basic variations of the three ARP, BRP, and MBRP models defined in Barlow and Proschan (1965) are well adapted to the discrete time case in Nakagawa (1984). Studying these maintenance models in discrete time is relevant because it is often the case that the condition of a component, that is, whether it has failed or not, cannot be tracked continuously. Nakagawa (1984) presents more precise definitions of the ARP, BRP, and MBRP models. In an ARP, a component is replaced either upon failure or at some critical maintenance age t, whichever comes first. On the other hand, in a BRP, a component is replaced at a given cycle independent of its age, with CM carried out by replacing a failed unit by a new one between scheduled replacements. The precise definition of an MBRP model varies across the literature. Nakagawa (1984) defines it as a BRP policy in which failed units undergo minimal repair between scheduled replacements. On the other hand, Berg and Epstein (1976) and Schouten et al. (2022) define it as a BRP policy in which a component is only replaced at the scheduled time if it has reached a pre-specified critical maintenance age. In this work, the latter definition is applied throughout.

Of the three models, ARP leads to the lowest costs. BRP is more wasteful, as it can frequently occur that under block replacement a component that has recently undergone CM is preventively maintained, despite being as-good-as-new. On the other hand, BRP has the advantage that maintenance is more predictable, making such a policy more convenient in practice. This trade-off is the motivation for the development of the MBRP model of Berg and Epstein (1976), which successfully strikes a balance between practicality and efficiency.

Additionally, several adaptations of the standard ARP, BRP, and MBRP models have been developed in recent years, in which different replacement/repair policies are applied, several of which are outlined in Archibald and Dekker (1996). In particular, several models consider alternative approaches for CM of a broken component than to simply replace it with a good-as-new component, as is the case in the base model of Barlow and Proschan (1965). For example, Nakagawa (1983) considers minimal repair, in which case the lifetime distribution of a repaired component is the same as that of the same component if it had not failed, while Tango (1978) replaces broken components with used components that have been replaced preventively. Such adaptations are easily implementable in the standard formulation in Schouten (2019), and lead to cost savings under certain parameter settings.

On the other hand, an entirely different alternative to the three standard models (and their variants) exists in the form of condition-based maintenance policies. These take advantage of data from sensors and manual inspections to determine when PM is warranted. Such models are presented in Besnard and Bertling (2010), Lu et al. (2018), Maillart (2006), and Tian et al. (2011). The most significant drawback of such models is a lack of accurate data regarding the condition of wind turbines, meaning perfect prediction of failures is far from possible.

2.3 Time-varying costs

In the case of offshore wind turbines specifically, there tends to be a lot of variability in the costs of maintenance, as the lost energy generation from the downtime during maintenance is almost entirely dependent on the wind speed, which is variable. This somewhat complicates the existing general models, as they assume the cost of maintenance to be constant regardless of when maintenance is performed. Fortunately, wind speeds are somewhat predictable on a seasonal basis. For example, Ailliot et al. (2006) presents an autoregressive model to describe time-varying wind speeds, giving an indication of when costs are greatest, namely in the winter months. Such predictable trends in maintenance costs across the year can be incorporated into the policies described in Sect. 2.2 to take advantage of periods with lower wind speeds and thus lower maintenance costs.

Indeed, Schouten et al. (2022) extends the standard ARP, BRP, and MBRP models to the case of time-varying maintenance costs, leading to three new periodic versions of the models, denoted p-ARP, p-BRP, and p-MBRP. The p-ARP model is adapted from the standard ARP such that the critical maintenance age is defined per period, implying that the optimal critical maintenance age can be higher in months with higher wind speeds, which pushes maintenance activities to months with lower maintenance costs. For p-BRP, the adjustment is that maintenance activities must not always occur after the same interval, that is, the first PM might occur after 4 months, the second after 6, and so on, provided that there is a cycle that repeats over m years. Last, for p-MBRP, both adjustments for p-ARP and p-BRP are combined, namely period-dependent critical maintenance ages and varying intervals between PMs across a cycle.

It is proven in Schouten et al. (2022) that, of the p-ARP, p-BRP, and p-MBRP policies, p-ARP leads to the lowest costs, given standard assumptions. On the other hand, p-BRP leads to the highest costs, but is considered the most convenient, as maintenance can be planned far in advance, with no unpredictable changes. Fortunately, the p-MBRP model deals with this tradeoff, by sacrificing some of the predictability of the p-BRP model to reduce the wastefulness of repairing almost-new components and thus save costs.

2.4 Multi-component models

More recently, multi-component extensions of existing single-component models have been developed to account for interactions between components. Such interactions can be grouped into three categories: economic dependence, structural dependence, and stochastic dependence (Thomas, 1986). Incorporating all of these dependencies into one model leads to excessively complicated or computationally expensive models, so most multi-component maintenance models limit themselves to considering only one (Dekker et al., 1997). In the present work, I focus on economic dependence in the form of setup costs. Setup costs are costs that are incurred each time maintenance is carried out, but that are the same regardless of how many components are maintained. By planning overlapping maintenance times across components, total maintenance costs can be reduced by sharing setup costs.

One of the first extensions of the standard models in Barlow and Proschan (1965) to the multi-component case is in Archibald and Dekker (1996). In this work, the MBRP model is extended to a discrete time framework with economic dependence, but no stochastic or structural dependence, via extending the Markov decision space. They find in their numerical results that their multi-component MBRP model outperforms the standard single-component MBRP model quite significantly, but also almost reaches the performance of the standard ARP model, which has lower costs than MBRP in all cases. Schouten (2019) takes a similar approach to define time-varying policies for multiple components. Such models allow the policy to respond to economic dependence only by altering the scheduled PM periods to be more aligned across components. They do not take into account the possibility to share setup costs across CM and PM activities. Namely, they assume always that CM occurs immediately upon failure, and that PM times are not affected, other than via the critical maintenance age, by failures.

Opportunistic maintenance models, on the other hand, allow for the possibility to adapt the PM schedule when failures occur. Namely, they take advantage of CM activities in order to perform PM tasks at a lower cost, by bringing forward scheduled PM activities when a failure occurs, leading to savings in setup costs. Such models are presented in Besnard et al. (2009), Kang and Soares (2020), Li et al. (2020), and Zhou and Yin (2019). While such models lead to significant cost savings (Besnard et al., 2009), they are inconvenient in practice, as maintenance activities become extremely unpredictable, which can lead to further costs in practice. In contrast, a more predictable alternative to bringing forward PM is the option of delaying CM. In this way, PM times remain fixed once a policy is planned. Specifically, Nakagawa (1982) considers delaying CM of a broken component until the next scheduled PM of another component, leading to extended downtime but a more predictable maintenance schedule, leading to cost savings for some scenarios in the constant-cost case. The usefulness of such an approach naturally depends greatly upon the relative size of setup costs and the costs of downtime. It might be particularly valuable in a situation with time-varying costs, as delaying CM then not only allows setup costs to be shared but also allows CM to be performed in a cheaper period.

3 Problem description

3.1 Single-component models

For now, consider a single wind turbine component in a continuously operating offshore wind farm. I consider two types of maintenance: PM, in which a component is replaced preventively, and CM, in which a failed component is replaced. In the single-component framework, it is assumed that CM must be performed directly to prevent excessive production loss. Maintenance is carried out in the form of replacing a failed component by an as-good-as-new component.

Time is discretized into periods, in this case months. The component lifetime is denoted by X, with $\mathbb{P}(X = k) > 0$ for all $k \in \overline{\mathbb{N}} \setminus \{\infty\}$, where $\overline{\mathbb{N}} = \{0, 1, ..., \infty\}$ is the set of extended natural numbers. It is assumed that the component's time to failure X follows a discretized Weibull distribution as defined in Schouten (2019), with cdf $F(x) = 1 - \exp(-(\frac{x}{\alpha})^{\beta})$ and pmf $P(X = x) = F(x) - F(x-1)$, for $x \in \overline{\mathbb{N}}$, where $\alpha > 0$ is the scale parameter, measured using the same units of time as x (months), and $\beta > 0$ is the shape parameter. This differs from the distribution presented in Schouten et al. (2022), which incorrectly gave the cdf $F(x) = 1 - \exp((\frac{x}{\alpha})^{\beta})$. Failures are assumed to happen at the end of a period, meaning that $P(X = x)$ represents the probability that the component fails just before age x. Maintenance costs are assumed to vary according to a seasonal cycle, based on weather conditions. I consider a yearly cycle in which costs may be different in each month. The notations used are defined in Table 1.

Table 1. *Nomenclature for single-component models.*

Parameter	Meaning
X	Lifetime of component
α	Scale parameter of discrete Weibull distribution
β	Shape parameter of discrete Weibull distribution
N	Number of periods per year
m	Number of years in a cycle
$\mathbb{N}^+$	Set of positive integers, $\{1, 2, ...\}$
$\overline{\mathbb{N}}$	Set of extended natural numbers, $\{0, 1, 2, ..., \infty\}$
$\mathcal{I}_1$	Set of periods within a year
$\mathcal{I}_2$	Set of component ages
$\mathcal{I}$	State space of the MDP
M	Large number representing maximum age of component
$\mathcal{A}(i_1, i_2)$	Set of possible actions in MDP in state $(i_1, i_2) \in \mathcal{I}$
$\pi_{(i_1,i_2)(j_1,j_2)}(a)$	Transition probability from state $(i_1, i_2) \in \mathcal{I}$ to state $(j_1, j_2) \in \mathcal{I}$ under action $a \in \mathcal{A}(i_1, i_2)$
p_x	Probability that component fails in period directly before reaching age x
$c_f(i_1)$	CM cost in period $i_1 \in \mathcal{I}_1$
$c_p(i_1)$	PM cost in period $i_1 \in \mathcal{I}_1$
$\overline{c}_f$	Average CM cost
$\overline{c}_p$	Average PM cost
$c_{(i_1,i_2)}(a)$	Cost of taking action $a \in \mathcal{A}(\rangle_\infty, \rangle_\in)$ in state $(i_1, i_2) \in \mathcal{I}$

This subsection is structured as follows. First, the Markov decision process and cost structure in the single-component case are described. Then, the existing p-ARP and p-BRP models are briefly outlined, before the p-MBRP model upon which the multi-component models build is presented in detail.

3.1.1 Markov decision process (MDP) and cost parameters

The life of the component is modeled via a discrete-time MDP. This MDP has a partially ordered state space $\mathcal{I} = \mathcal{I}_1 \times \mathcal{I}_2$. $\mathcal{I}_1 = \{1,2,...,mN\} \subseteq \mathbb{N}^+$ is the set of periods considered, where N is the number of periods per year and m is the number of years in a PM cycle, while $\mathcal{I}_2 = \{0,1,2,...\} \subseteq \overline{\mathbb{N}}$ is the set of component ages. This follows the specification of Schouten (2019).

3.1.1.1 Action space

There are two possible actions, either to replace the component ($a = 1$) or to do nothing ($a = 0$). As it is assumed that CM must be performed directly, when the age of the component is zero, maintenance must be performed. In all other cases, maintenance can either be performed or not performed. Thus, the state-dependent action space is thus defined as

$$\mathcal{A}(i_1,i_2) = \begin{cases} \{1\} & \text{if } i_2 \in \{0,M\} \\ \{0,1\} & \text{otherwise,} \end{cases} \tag{1}$$

where $i_1 \in \mathcal{I}_1$ and $i_2 \in \mathcal{I}_2$, and M is the maximum age of the component, at which age a PM is necessarily performed. In case it is optimal never to perform a PM, it could be that $M = \infty$.

3.1.1.2 Transition probabilities

The transitions of the Markov chain depend on whether a PM is planned or a failure occurs. In this model, it is assumed that failures occur at the end of the period and maintenance is performed at the beginning of the period. In case a failure occurs, there is a jump to a state with a higher time period and age 0, as CM is necessarily performed. On the other hand, in case there is no failure, CM is not performed, and a choice must be made as to whether or not to perform PM. In case maintenance is performed, whether it be PM or CM, there is an instantaneous jump to age 0, beyond which point the component might either reach age 1 by the end of the period, or have a failure and end with age 0. Thus, after maintenance, a state with a higher time period and either age 0 or age 1 is reached.

It is assumed that while the probability of failure of course depends on the age of the component, it is not affected by the time of year, unlike the maintenance costs. $\pi_{(i_1,i_2)(j_1,j_2)}(a)$ denotes the transition probability from state (i_1,i_2) to state (j_1,j_2), $i_1, j_1 \in \mathcal{I}_1$, $i_2, j_2 \in \mathcal{I}_2$, under action $a \in \mathcal{A}(i_1,i_2)$. The following transition probabilities are based on Schouten et al. (2022), with the correction that the transition probabilities under action 0 depend on p_{i_2+1} rather than on p_{i_2}, as the probability of failure between i_2

and i_2+1 is required, not that between i_2-1 and i_2. This leads to

$$\pi_{(i_1,i_2)(j_1,j_2)}(0) = \begin{cases} 1-p_{i_2+1} & \text{for } j_1 = i_1+1 \pmod N,\ j_2 = i_2+1,\ i_2 \notin \{0,M\}, \\ p_{i_2+1} & \text{for } j_1 = i_1+1 \pmod N,\ j_2 = 0,\ i_2 \notin \{0,M\}, \\ 0 & \text{else,} \end{cases} \tag{2a}$$

$$\pi_{(i_1,i_2)(j_1,j_2)}(1) = \begin{cases} 1-p_1 & \text{for } j_1 = i_1+1 \pmod N,\ j_2 = 1, \\ p_1 & \text{for } j_1 = i_1+1 \pmod N,\ j_2 = 0, \\ 0 & \text{else,} \end{cases} \tag{2b}$$

where p_x gives the failure probability at age x and mod is the modulo operator.

To be more precise, p_x is defined as to represent the probability that the failure time of the component is equal to x given that it is at least x, that is, the probability that the component fails at the end of the period after which it would have otherwise reached age x. Thus, it is defined as

$$p_x = \mathbb{P}(X = x | X \geq x) = \frac{P(X = x)}{P(X \geq x)} = \frac{F(x) - F(x-1)}{1 - F(x-1)}. \tag{3}$$

3.1.2 Cost structure

The costs of maintenance, both CM and PM, are dependent only on the period of the year $i_1 \in \mathcal{I}_1$. They are denoted by $c_f(i_1)$ and $c_p(i_1)$ respectively. The cost of taking an action $a \in \mathcal{A}(i_1,i_2)$ in state $i = (i_1,i_2) \in \mathcal{I}$, denoted by $c_{(i_1,i_2)}(a)$, can thus be calculated as

$$c_{(i_1,i_2)}(a) = \begin{cases} 0 & \text{if } a = 0, \\ c_p(i_1) & \text{if } a = 1,\ i_2 \neq 0, \\ c_f(i_1) & \text{if } a = 1,\ i_2 = 0. \end{cases} \tag{4}$$

The yearly average PM and CM costs are denoted as $\overline{c}_p$ and $\overline{c}_f$, respectively. In line with Schouten (2019), the time-varying PM and CM costs are then defined as

$$c_p(i_1) = \overline{c}_p + \Delta_p \cos\left(\frac{2\pi i_1}{N} + \phi\right), \tag{5a}$$

$$c_f(i_1) = \overline{c}_f + \Delta_f \cos\left(\frac{2\pi i_1}{N} + \phi\right), \tag{5b}$$

where time i_1 is expressed in months, $\Delta_f = \Delta\overline{c}_f$, and $\Delta_p = \Delta\overline{c}_p$. Thus, $N = 12$ and $i_1 = 1$ for January. Then, $\phi = \frac{-2\pi}{N}$ is chosen such that the lowest costs are obtained in July and the highest in the winter months, mirroring Northern European wind patterns.

3.2 Multi-component models

Now, multiple wind turbine components are considered. These components have no stochastic dependence, that is, the failure of one component does not affect the failure of another component. Economic dependence, however, is present in the form of maintenance setup costs. Namely, for both CM and PM, there is a setup cost associated with carrying out maintenance. Such a setup cost can be considered to be the cost of sending a support vessel to the offshore wind park: once the support vessel is there, it is possible to carry out multiple PM or CM activities directly after one another, sharing the setup costs.

I consider two different assumptions on the timing of CM: (1) that CM must be performed directly (at the start of the period following the failure) to prevent any production loss, $r = A$, or (2) that CM can be delayed and performed at the same time as the next scheduled PM of another component, with a penalty assumed for lost production, $r = B$. In the latter case, I assume that if all components have failed, all components are immediately maintained. Maintenance is again carried out in the form of replacing a failed component by an as-good-as-new component.

Time is again discretized into months, with the lifetime of each component modeled by a discretized Weibull distribution as described in Sect. 3.1. As I consider only economic dependence, I assume that each component has a lifetime independent of the lifetime of the other components, denoted by X^k. For ease of exposition, I consider a two-component case, where the components are named component k for $k = 1,2$. Each component might have a different lifetime distribution, that is, the parameters of the Weibull distribution can vary across components. Maintenance costs are again assumed to vary by month. The notations used in these models are given in Table 2.

This subsection first defines the Markov decision process and cost structure in this multi-component case, based on the definitions in Sect. 3.1. Then, it presents the formulations of the two new multi-component models.

3.2.1 Markov decision process

Compared to the single-component model, the state space of the two-component model must be extended to include an extra dimension, namely the age of the second component. Thus, the MDP for the two-component case has state space $\mathcal{I} = \mathcal{I}_1 \times \mathcal{I}_2 \times \mathcal{I}_3$, where $\mathcal{I}_1 = \{1,2,...,mN\} \subseteq \mathbb{N}^+$ is the number of periods in a year, $\mathcal{I}_2 \subseteq \overline{\mathbb{N}}$ is the age of the component 1, and $\mathcal{I}_3 \subseteq \overline{\mathbb{N}}$ is the age of component 2.

3.2.1.1 Action space

The action space also becomes two-dimensional, with an action defined for each component. This leads to 4 possible actions: $\{0,0\}$, which means maintenance is performed for neither component, $\{1,0\}$, which means maintenance is performed only for component 1, $\{0,1\}$, which means maintenance is performed only for component 2, and $\{1,1\}$, which means maintenance is performed for both components. As described in Sect. 3.2, I consider two assumptions on the timing of CM, defining whether or not CM must be performed immediately. Thus, for the two-component case, I define a distinct action space for

Table 2. *Nomenclature for two-component models.*

Parameter	Meaning
X_k	Lifetime of component k
α_k	Scale parameter of discrete Weibull distribution of lifetime of component k
β_k	Shape parameter of discrete Weibull distribution of lifetime of component k
N	Number of periods per year
m	Number of years in a cycle
$\mathbb{N}^+$	Set of positive integers, $\{1,2,...\}$
$\overline{\mathbb{N}}$	Set of extended natural numbers, $\{0,1,2,...,\infty\}$
$\mathcal{I}_1$	Set of periods within a year
$\mathcal{I}_2$	Set of ages of component 1
$\mathcal{I}_3$	Set of ages of component 2
$\mathcal{I}$	State space of the MDP
M_k	Large number representing maximum age of component k
$\mathcal{A}^r(i_1,i_2,i_3)$	Set of possible actions in MDP in state $(i_1,i_2,i_3) \in \mathcal{I}$ under assumption r
$\pi^r_{(i_1,i_2,i_3)(j_1,j_2,j_3)}(a)$	Transition probability from state $(i_1,i_2,i_3) \in \mathcal{I}$ to state $(j_1,j_2,j_3) \in \mathcal{I}$ under action $a \in \mathcal{A}^r(i_1,i_2,i_3)$ under assumption r
p^k_x	Probability that component k fails in period directly before reaching age x
$c^k_f(i_1)$	CM cost for component k in period $i_1 \in \mathcal{I}_1$
$c^k_p(i_1)$	PM cost for component k in period $i_1 \in \mathcal{I}_1$
$c_s(i_1)$	Setup cost in period $i_1 \in \mathcal{I}_1$
$\overline{c}^k_f$	Average CM cost for component k
$\overline{c}^k_p$	Average PM cost for component k
$\overline{c}_s$	Average setup cost
$c^r_{(i_1,i_2,i_3)}(a)$	Cost of taking action $a \in \mathcal{A}^3(i_1,i_2,i_3)$ in state $(i_1,i_2,i_3) \in \mathcal{I}$ under assumption r
$l^k(i_1)$	Penalty for lost production when component k is broken but not repaired in period $i_1 \in \mathcal{I}_1$
$\overline{l}^k$	Average lost production cost for component k
$n(i_1)$	Next scheduled maintenance period after $i_1 \in \mathcal{I}_1$

each of these assumptions, denoted $\mathcal{A}^r(i_1,i_2,i_3)$.

When CM must be performed immediately, it means that a component with age zero must always be maintained. Thus, the action space under this assumption is defined as given in the Appendix in (A.17). On the other hand, if CM can be delayed, it is no longer a requirement that maintenance occurs when a component reaches age zero. Hence, the action space is adjusted, leading to (A.18).

3.2.1.2 Transition probabilities

Once the action space is defined, a similar logic as in the single-component case can be applied to define the transition probabilities for the two-component case. From the distinct action spaces follow of course two distinct sets of transition probabilities, depending on the assumption on CM timing. The transition probability from state $i = (i_1,i_2,i_3) \in \mathcal{I}$ to state $j = (j_1,j_2,j_3) \in \mathcal{I}$, $i_1, j_1 \in \mathcal{I}_1$, $i_2, j_2 \in \mathcal{I}_2$, $i_3, j_3 \in \mathcal{I}_3$, under action $a \in \mathcal{A}^r(i_1,i_2)$, is denoted by $\pi^r_{(i_1,i_2,i_3)(j_1,j_2,j_3)}(a)$, for $r = A,B$.

The transition probabilities under the assumption that CM must be performed immediately follow directly from those in the single-component case, given that components fail independently of one another and I apply the same assumptions as in Schouten et al. (2022). This leads to the transition probabilities defined in (A.19a)-(A.19d). On the other hand, if CM can be delayed, the transition probabilities are changed to allow for the possibility not to maintain a broken component. Specifically, for a single component, even when no maintenance takes place, a transition from one period to the next in which the age stays equal to zero is possible. This leads to the extended transition probabilities defined in (A.20a)-(A.20d).

3.2.2 Cost structure

It is again assumed that maintenance costs for both CM and PM depend only on the period of the year $i_1 \in \mathcal{I}_1$. However, components are non-identical, the CM and PM costs are assumed to be different per component, meaning CM costs are $c_f^1(i_1)$, $c_f^2(i_1)$, and PM costs are $c_p^1(i_1)$, $c_p^2(i_1)$, for components 1 and 2, respectively. Moreover, I introduce for each period $i_1 \in \mathcal{I}_1$ a maintenance setup cost $c_s(i_1)$ that is incurred whenever any maintenance action takes place, whether it be PM or CM. This cost is the same regardless of how many maintenance actions occur, meaning it can be shared between any combination of PM and CM actions. However, it varies with time. Intuitively, this cost might be considered to represent the cost of sending a team out to the wind farm, for example. Such a cost might depend on the level of wind on that day, as high waves might make travel by boat more difficult.

In case CM is delayed, there is no maintenance cost in this period, yet there is still a loss in production. I thus apply a penalty $l^k(i_1)$ for lost production in the periods in which a component k is broken, i.e., has age zero, but is not under maintenance. This penalty is assumed to be time-varying in the same way as maintenance costs, that is, according to (5a) and (5b), where the yearly average cost of lost production is defined as $\bar{l}^k$, $k = 1,2$. If CM is always performed immediately, there is no need to include such a penalty. Thus, under the assumption that CM must be performed immediately, costs are defined as in (A.21), whereas if CM can be delayed, there are defined as in (A.22).

The yearly average PM and CM costs for components 1 and 2, respectively, are denoted as $\bar{c}_p^1$, $\bar{c}_p^2$, and $\bar{c}_f^1$, $\bar{c}_p^2$, respectively. Based on these averages, the time-varying PM and CM costs are calculated as in (5a) and (5b). Moreover, a yearly average setup cost $\bar{c}_s$ is defined, upon which the setup cost for a given period depends in the same manner as for the component-specific maintenance costs. It is noted that the setup cost in the case of CM of both components is only incurred once, as both CM activities will be carried out at the beginning of the following period. This differs from Schouten (2019), who assumes that CM occurs immediately for each component.

4 Analysis

4.1 Single-component models

4.1.1 p-ARP

In a standard ARP, a component is replaced whenever it has reached a given age. This model is adapted to the time-varying case by defining a separate critical maintenance age for each time period, leading to the p-ARP model. The present work follows closely the formulation in Schouten et al. (2022), with two corrections. First, the transition probabilities are adapted as described in Sect. 3.2.1.2 to depend upon the correct failure probability. Second, the cumulative density function of the discretized Weibull distribution is corrected, as stated in Sect. 3.1. Both x and α are measured in months.

4.1.2 p-BRP

In a standard BRP, components are replaced preventively after a fixed amount of time since the previous PM, regardless of whether a CM was performed in that time. In the time-varying case, the intervals between PM times are allowed to vary, with some pattern that repeats every cycle of m years, for some $m \in \mathbb{N}^+$, leading to a p-BRP policy. In such a policy, maintenance is performed at periods $T_1, T_2, ..., T_n < mN$, where $T_k \in \mathbb{N}^+, \forall k = 1, 2, ..., n$, for $n \in \mathbb{N}^+$. Again, the model applied in this research follows closely that of Schouten et al. (2022), with the same adaptations as applied for the p-ARP.

4.1.3 p-MBRP

The disadvantage of the standard BRP model is that it may replace components at a very young age if CM has just occurred. Berg and Epstein (1976) introduce a critical maintenance age before which a component is not replaced preventively, leading to a modified BRP. This modification is extended to the time-varying cost case in Schouten et al. (2022) by allowing the critical maintenance age to vary by period, giving rise to the p-MBRP policy defined in Definition 1, adapted from Schouten et al. (2022). Moreover, in Schouten et al. (2022), sufficient conditions for the p-MBRP to be better than a policy with no PM are presented, for some finite m, T_k, and $t_{(k)}$, stated in Theorem 1.

Definition 1. Consider a finite cycle of $m \in \mathbb{N}^+$ years. A p-MBRP is a policy in which possible block times T_k for $k = 1, 2, ..., n$, with $n \in \mathbb{N}^+$, are $T_1, T_2, ..., T_n \leq mN$. PM is performed at occasion $k \in \{1, 2, ..., n\}$ only if a critical maintenance age $t_{(k)}$ has been reached, with this critical maintenance age defined as $t_{(k)} \leq T_k - T_{k-1}$ for $k = 2, 3, ..., n$ and $t_{(1)} \leq T_1 - T_n + mN$.

Theorem 1. If $\bar{c}_f > \bar{c}_p$ and $p_\infty = \lim_{k\to\infty} p_k = \frac{1}{\mathbb{E}(X)} \frac{\bar{c}_f}{\bar{c}_f - \bar{c}_p}$, then there exists an optimal p-MBRP with finite cycle m, finite block times $T_1, T_2, ..., T_N$, and finite critical ages $t_{(1)}, t_{(2)}, ..., t_{(n)}$.

The optimal p-MBRP policy can be found using a multiple integer linear programming (MILP) formulation. The decision variables $x_{i,a}$ represent the long-run probability that the system is in state $i = (i_1, i_2) \in \mathcal{I}$ at the beginning of the period and the decision $a \in \mathcal{A}(i)$ is chosen. I define a set $\mathcal{I}^b = \mathcal{I}_1 \times \{0\}$ to be the set of states where the component has failed. Furthermore, two additional decision variables are introduced. The variable y_{i_1} gives in which periods PM is performed, defined as

$$y_{i_1} = \begin{cases} 1 & \text{if PM occurs in period } i_1 \in \mathcal{I}_1 \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Similarly, the variable z_i represents for which period and age PM takes place, given by

$$z_i = z_{(i_1,i_2)} = \begin{cases} 1 & \text{if PM occurs for age } i_2 \in \mathcal{I}_2 \text{ in period } i_1 \in \mathcal{I}_1, \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

The following MILP formulation finds the p-MBRP policy that minimizes the long-run average cost.

$$\text{minimize} \quad \sum_{i=(i_1,i_2)\in\mathcal{I}\setminus\{\mathcal{I}_b\}} c_p(i_1)x_{i,1} + \sum_{i=(i_1,i_2)\in\mathcal{I}_b} c_f(i_1)x_{i,1} \tag{8a}$$

$$\text{subject to} \quad \sum_{a\in\mathcal{A}(i)} x_{i,a} - \sum_{j\in\mathcal{I}}\sum_{a\in\mathcal{A}(j)} \pi_{ji}(a)x_{j,a} = 0, \qquad \forall i = (i_1,i_2)\in\mathcal{I}, \tag{8b}$$

$$\sum_{i_2\in\mathcal{I}_2}\sum_{a\in\mathcal{A}(i)} x_{i,a} = \frac{1}{mN}, \qquad \forall i_1\in\mathcal{I}_1, \tag{8c}$$

$$z_{i_1,i_2} - y_{i_1} \le 0, \qquad \forall i_1\in\mathcal{I}_1, \forall i_2\in\mathcal{I}_2, \tag{8d}$$

$$z_{i_1,i_2} - z_{i_1,j_2} \le 0, \forall i = i_1,i_2\in\mathcal{I}, \qquad \forall j_2\in\mathcal{I}_2 : i_2 < j_2, \tag{8e}$$

$$t_{i_1} + j_1 y_{j_1} + mNy_{j_1} \le mN + i_1, \qquad \forall i_1, j_1\in\mathcal{I}_1 : j_1 < i_1, \tag{8f}$$

$$t_{i_1} + j_1 y_{j_1} \le mN + i_1, \qquad \forall i_1, j_1\in\mathcal{I}_1 : j_1 > i_1, \tag{8g}$$

$$My_{i_1} - Mz_{i_1,i_2} - t_{i_1} \le M - 1 - i_2, \qquad \forall i_1\in\mathcal{I}_1, \forall i_2\in\mathcal{I}_2, \tag{8h}$$

$$Mz_{i_1,i_2} + t_{i_1} \le M + i_2, \qquad \forall i_1\in\mathcal{I}_1, \forall i_2\in\mathcal{I}_2, \tag{8i}$$

$$x_{i,1} - z_i \le 0, \qquad \forall i = (i_1,i_2)\in\mathcal{I} : i_2 \ne 0, \tag{8j}$$

$$x_{i,0} + z_i \le 1, \qquad \forall i = (i_1,i_2)\in\mathcal{I} : i_2 \ne 0, \tag{8k}$$

$$z_i = 0, \qquad \forall i = (i_1,i_2)\in\mathcal{I} : i_2 = 0, \tag{8l}$$

$$x_{i,a} \ge 0, \qquad \forall i = (i_1,i_2)\in\mathcal{I}, \forall a\in\mathcal{A}, \tag{8m}$$

$$z_i \in \{0,1\}, \qquad \forall i = (i_1,i_2)\in\mathcal{I}, \tag{8n}$$

$$y_{i_1} \in \{0,1\}, \qquad \forall i_1\in\mathcal{I}_1, \tag{8o}$$

$$t_{i_1} \in \mathbb{N}^+, \qquad \forall i_1\in\mathcal{I}_1. \tag{8p}$$

The objective function (8a) represents the long-run average cost of the policy. The constraints are explained as follows. (8b) ensures that the inflows and outflows between states are equal. (8c) ensures that the long-run probabilities of being in each state sum to mN, since mN periods over a cycle of m years are considered. (8d) is trivial based on the definitions of z_i and y_{i_1}, while (8e) ensures that if PM occurs for age i_2 in period i_1, then PM must also occur for all ages $j_2 > i_2$ in this period. (8f) and (8g) ensure that if $y_{i_1} = 1$, t_{i_1} is less than or equal to the time since the previous PM, taking into account the repetition of the cycle. Moreover, (8h) and (8i) define the relationship between z_i and y_{i_1},

$$z_i = z_{(i_1,i_2)} = \begin{cases} y_{i_1} & \text{if } i_2 \ge t_{i_1}, \\ 0 & \text{otherwise,} \end{cases} \tag{9}$$

where the if statement is linearised using M. Finally, (8j)-(8l) are added compared to Schouten et al. (2022) to define the relationship between the decision variables $x_{i,a}$ and z_i, namely to ensure that $z_i = 1$ when $x_{i,1} > 0$ and $i_2 \neq 0$, $z_i = 0$ when $x_{i,0} > 0$ and $i_2 \neq 0$, and $z_i = z_{i_1,i_2} = 0$ when $i_2 = 0$, that is, when CM is performed. The remaining cases, for which $x_{i,1} = x_{i,0} = 0$, are defined by (8e).

4.2 Multi-component models

4.2.1 p-MBRP2+DCM

A two-step model is presented to incorporate the possibility to delay CM in the p-MBRP model, denoted p-MBRP2+DCM. First, PM times are optimized under the assumption that CM cannot be delayed. Then, the fixed PM times and critical ages resulting from that model are used as inputs into a new model that optimizes the conditions under which CM should be delayed (if at all). This second stage will necessarily lead to costs that are the same or lower than those arising from the first stage.

4.2.1.1 Stage 1: p-MBRP2

As in the single-component case, the optimal p-MBRP policy for two components is found with an MILP formulation. The definition and existence of such a policy follow closely from those presented in Sect. 4.1.3. However, as components are non-identical, I allow for different critical maintenance ages per component.

The critical maintenance age of component k in period $i_1 \in \mathcal{I}_1$ is denoted by $t^k_{i_1}$. Again, the decision variables $x_{i,a}$ represent the long-run probability of the system being in state $i = (i_1, i_2, i_3) \in \mathcal{I}$ at the beginning of the period and the decision $a \in \mathcal{A}^A(i)$ being chosen.

For ease of notation, the set $\mathcal{A}$ is defined as the set of all possible actions across all states, namely $\mathcal{A} = \{\{0,0\},\{1,0\},\{0,1\},\{1,1\}\}$. Then, $\mathcal{A}_k$ is defined as the set of actions in which component k is maintained, that is, $\mathcal{A}_1 = \{\{1,0\},\{1,1\}\}$ and $\mathcal{A}_2 = \{\{0,1\},\{1,1\}\}$, and $\mathcal{A}^C_k$ as the set of actions in which component k is not maintained, $\mathcal{A}^C_k = \mathcal{A} \setminus \mathcal{A}_k$.

Two additional decision variables are also included. The variable y_{i_1} is defined as in the single-component model in 6, but per component, denoted $y^k_{i_1}$ for $k = 1,2$. This leads to

$$y^k_{i_1} = \begin{cases} 1 & \text{if PM is performed for component } k \text{ in period } i_1 \in \mathcal{I}_1 \\ 0 & \text{otherwise.} \end{cases} \tag{10}$$

Additionally, the variable $z^k_{i_1,i_{k+1}}$ is introduced to represent for which period and age component k is maintained, where $k = 1,2$ in the two-component case. Hence, it is defined as

$$z^k_{i_1,i_{k+1}} = \begin{cases} 1 & \text{if component } k \text{ is maintained preventively in period } i_1 \in \mathcal{I}_1 \text{ at age } i_{k+1} \in \mathcal{I}_{k+1}, \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

Using these decision variables, a MILP formulation to find the optimal p-MBRP2 policy can be formulated. The formulation follows a very similar structure as the single-component p-MBRP formulation in (8). Namely, the objective function is close to identical, except that setup costs are included in the cost term. The constraints also remain almost identical, with the change that they must now hold for both components, $k = 1,2$, and for the now extended MDP, with different transition probabilities and action spaces.

Thus, the MILP formulation to find the p-MBRP2 policy that minimizes the long-run average cost of maintenance, across two components, is defined as follows.

$$\text{minimize} \quad \sum_{i=(i_1,i_2,i_3)} \sum_{a \in \mathcal{A}^A} c_i^A(a) x_{i,a} \tag{12a}$$

$$\text{subject to} \quad \sum_{a \in \mathcal{A}^A(i)} x_{i,a} - \sum_{j \in \mathcal{I}} \sum_{a \in \mathcal{A}^A(j)} \pi_{ji}^A(a) x_{j,a} = 0, \qquad \forall i = (i_1,i_2,i_3) \in \mathcal{I}, \tag{12b}$$

$$\sum_{i_2 \in \mathcal{I}_2} \sum_{i_3 \in \mathcal{I}_3} \sum_{a \in \mathcal{A}^A(i)} x_{i,a} = \frac{1}{mN}, \tag{12c}$$

$$z_{i_1,i_{k+1}}^k - y_{i_1}^k \leq 0, \qquad \forall i_1 \in \mathcal{I}_1, \forall i_{k+1} \in \mathcal{I}_{k+1}, k = 1,2, \tag{12d}$$

$$z_{i_1,i_2}^k - z_{i_1,j_2}^k \leq 0, \qquad \forall i = (i_1,i_2,i_3) \in \mathcal{I}, \forall j_{k+1} \in \mathcal{I}_{k+1} : i_{k+1} < j_{k+1}, \tag{12e}$$

$$t_{i_1}^k + j_1 y_{j_1}^k + mN y_{j_1}^k \leq mN + i_1, \qquad \forall i_1, j_1 \in \mathcal{I}_1 : j_1 < i_1, k = 1,2, \tag{12f}$$

$$t_{i_1}^k + j_1 y_{j_1}^k \leq mN + i_1, \qquad \forall i_1, j_1 \in \mathcal{I}_1 : j_1 > i_1, k = 1,2, \tag{12g}$$

$$M_k y_{i_1}^k - M_k z_{i_1,i_{k+1}}^k - t_{i_1} \leq M_k - 1 - i_{k+1}, \qquad \forall i_1 \in \mathcal{I}_1, \forall i_{k+1} \in \mathcal{I}_{k+1}, k = 1,2, \tag{12h}$$

$$M_k z_{i_1,i_{k+1}}^k + t_{i_1} \leq M_k + i_{k+1}, \qquad \forall i_1 \in \mathcal{I}_1, \forall i_{k+1} \in \mathcal{I}_{k+1}, k = 1,2, \tag{12i}$$

$$\sum_{a \in \mathcal{A}_k^C} x_{i,a} + z_{i_1,i_{k+1}}^k \leq 1, \qquad \forall i = (i_1,i_2,i_3) \in \mathcal{I} : i_{k+1} \neq 0, k = 1,2, \tag{12j}$$

$$\sum_{a \in \mathcal{A}_k} x_{i,a} - z_{i_1,i_{k+1}}^k \leq 0, \qquad \forall i = (i_1,i_2,i_3) \in \mathcal{I} : i_{k+1} \neq 0, k = 1,2, \tag{12k}$$

$$z_{i_1,i_{k+1}}^k = 0, \qquad \forall i = (i_1,i_2,i_3) \in \mathcal{I} : i_{k+1} = 0, \tag{12l}$$

$$x_{i,a} \geq 0, \qquad \forall i = (i_1,i_2,i_3) \in \mathcal{I}, \forall a \in \mathcal{A}^A, \tag{12m}$$

$$z_{i_1,i_{k+1}}^k \in \{0,1\}, \qquad \forall i_1 \in \mathcal{I}_1, \forall i_{k+1} \in \mathcal{I}_{k+1}, k = 1,2, \tag{12n}$$

$$y_{i_1}^k \in \{0,1\}, \qquad \forall i_1 \in \mathcal{I}_1, k = 1,2, \tag{12o}$$

$$t_{i_1}^k \in \mathbb{N}^+, \qquad \forall i_1 \in \mathcal{I}_1, k = 1,2. \tag{12p}$$

The constraints follow easily from those in the single-component model. Note, however, that in this model, setup costs are included in the cost term.

4.2.1.2 Stage 2: DCM

Inspired by Nakagawa (1982), the p-MBRP2 policy in 12 is altered to incorporate the relaxation of the assumption that CM must be immediate. Specifically, CM is delayed if a failure occurs within a certain number of periods before the next scheduled PM of the other component, given it is not itself failed. If a failure occurs more than this number of periods before the next scheduled PM of the other component, CM is immediate. Also, in case both components are failed in a period, CM is carried out immediately for both components. Moreover, a penalty is introduced to account for lost production in case CM is delayed. The policy resulting from this is called p-MBRP2+DCM.

Compared to the formulation in (12), several changes are made. First, the transition probabilities, action spaces, and cost parameters for assumption $r = B$ are used, rather than for $r = A$. Second, crucially, $y^k_{i_1}$, $t^k_{i_1}$, and $z^k_{i_1,i_{k+1}}$ are fixed, based on the results of step 1 of the sequential model. Third, a new function $n^k(i_1)$ is defined to denote the next scheduled maintenance period for component k after a given period $i_1 \in \mathcal{I}_1$, where $n^k(i_1) \in \mathcal{I}_1$, $k = 1,2$.

Moreover, additional decision variables are introduced. Namely, the binary decision variables $f^k_{i_1}$ are added to the model to track in which periods CM is performed for component k, for $k = 1,2$, given that both components are not broken. CM that is performed because both components are broken is considered a special case that cannot be accounted for in the policy, so is excluded from the definition of $f^k_{i_1}$. The decision variables $f^k_{i_1}$ are thus defined by

$$f^k_{i_1} = \begin{cases} 1 & \text{if CM is performed only for component } k \text{ in period } i_1 \in \mathcal{I}_1, \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

Additionally, decision variables $d^k_{i_1} \in \mathbb{N}$ are defined to denote the number of periods that CM can be delayed from period $i_1 \in \mathcal{I}_1$, for component k, $k = 1,2$, called the critical delay. Again, this definition does not refer to the case in which both components are broken. In case the number of periods until the next scheduled PM (for either component) is greater than $d^k_{i_1}$, or if both components are in the failed state, CM occurs immediately; otherwise, it is delayed to (at least) the following period.

Given that both components are not broken, CM should be performed for a given component in a period $i_1 \in \mathcal{I}_1$ only if the number of periods until the next scheduled PM of the other component is at least the critical delay $d^k_{i_1}$. Recall that $f^k_{i_1}$ does not refer to the state in which both components are failed. Then, $f^k_{i_1}$ is related to $d^k_{i_1}$ via

$$f^1_{i_1} = \begin{cases} 1 & \text{if } n^2(i_1) - i_1 \geq d^1_{i_1} +, n^2(i_1) \geq i_1, \\ 1 & \text{if } n^2(i_1) - i_1 + mN \geq d^1_{i_1}, n^2(i_1) < i_1, \\ 0 & \text{otherwise.} \end{cases} \tag{14a}$$

$$f^2_{i_1} = \begin{cases} 1 & \text{if } n^1(i_1) - i_1 \geq d^2_{i_1} +, n^1(i_1) \geq i_1, \\ 1 & \text{if } n^1(i_1) - i_1 + mN \geq d^2_{i_1}, n^1(i_1) < i_1, \\ 0 & \text{otherwise.} \end{cases} \tag{14b}$$

Now, the relation of these new decision variables with the existing decision variables is defined in (12), $x_{i,a}$. It must be that $f^1_{i_1} = 1$ if $x_{i,\{1,0\}} + x_{i,\{1,1\}} \neq 0$ for $i_2 = 0$, $i_3 \neq 0$, $\forall i_1 \in \mathcal{I}_1$, and $f^1_{i_1} = 0$ if $x_{i,\{0,1\}} + x_{i,\{0,0\}} \neq 0$ for $i_2 = 0$, $i_3 \neq 0$, $\forall i_1 \in \mathcal{I}_1$. Similarly, $f^2_{i_1} = 1$ if $x_{i,\{0,1\}} + x_{i,\{1,1\}} \neq 0$ for $i_3 = 0$, $i_2 \neq 0$, $\forall i_1 \in \mathcal{I}_1$, and $f^2_{i_1} = 0$ if $x_{i,\{1,0\}} + x_{i,\{0,0\}} \neq 0$ for $i_3 = 0$, $i_2 \neq 0$, $\forall i_1 \in \mathcal{I}_1$. Last, to enforce that CM is performed immediately for both components if both are failed, it is enforced that $x_{i,\{1,1\}} \geq \varepsilon$ if $i_2 = i_3 = 0, \forall i_1 \in \mathcal{I}_1, \forall i_2 \in \mathcal{I}_2, \forall i_3 \in \mathcal{I}_3$.

This leads to the following MILP formulation for the second stage of p-MBRP2+DCM, called DCM.

$$\begin{aligned}
\text{minimize} \quad & \sum_{i=(i_1,i_2,i_3)} \sum_{a \in \mathcal{A}^B} c^B_i(a) x_{i,a} && \text{(15a)}\\
\text{subject to} \quad & \sum_{a \in \mathcal{A}^B(i)} x_{i,a} - \sum_{j \in \mathcal{I}} \sum_{a \in \mathcal{A}^B(j)} \pi^B_{ji}(a) x_{j,a} = 0, \qquad \forall i = (i_1,i_2,i_3) \in \mathcal{I}, && \text{(15b)}\\
& \sum_{i_2 \in \mathcal{I}_2} \sum_{i_3 \in \mathcal{I}_3} \sum_{a \in \mathcal{A}^B(i)} x_{i,a} = \frac{1}{mN}, && \text{(15c)}\\
& x_{i,a} = 0, \qquad \forall a \in \mathcal{A}^C_k, \forall i = (i_1,i_2,i_3) \in \mathcal{I} : z^k_{i_1,i_{k+1}} = 1, k = 1,2, && \text{(15d)}\\
& x_{i,a} = 0, \qquad \forall a \in \mathcal{A}_k, \forall i = (i_1,i_2,i_3) \in \mathcal{I} : i_{k+1} \neq 0, z^k_{i_1,i_{k+1}} = 0, k = 1,2, && \text{(15e)}\\
& \sum_{a \in \mathcal{A}^C_1} x_{i,a} + f^1_{i_1} \leq 1, \qquad \forall i = (i_1,i_2,i_3) \in \mathcal{I} : i_2 = 0, i_3 \neq 0, && \text{(15f)}\\
& \sum_{a \in \mathcal{A}^C_2} x_{i,a} + f^2_{i_1} \leq 1, \qquad \forall i = (i_1,i_2,i_3) \in \mathcal{I} : i_3 = 0, i_2 \neq 0, && \text{(15g)}\\
& \sum_{a \in \mathcal{A}_1} x_{i,a} - f^1_{i_1} \leq 0, \qquad \forall i = (i_1,i_2,i_3) \in \mathcal{I} : i_2 = 0, i_3 \neq 0, && \text{(15h)}\\
& \sum_{a \in \mathcal{A}_2} x_{i,a} - f^2_{i_1} \leq 0, \qquad \forall i = (i_1,i_2,i_3) \in \mathcal{I} : i_3 = 0, i_2 \neq 0, && \text{(15i)}\\
& x_{i,\{1,1\}} > 0, \qquad \forall i = (i_1,i_2,i_3) \in \mathcal{I} : i_2 = 0, i_3 = 0, && \text{(15j)}\\
& M_1 - M_1 f^1_{i_1} + 1 + n^2(i_1) - i_1 - d^1_{i_1} \leq M_1, \qquad \forall i_1 \in \mathcal{I}_1 : n^2(i_1) \geq i_1, && \text{(15k)}\\
& M_1 f^k_{i_1} + d^1_{i_1} - n^2(i_1) + i_1 \leq M_1, \qquad \forall i_1 \in \mathcal{I}_1 : n^2(i_1) \geq i_1, && \text{(15l)}\\
& M_1 - M_1 f^1_{i_1} + 1 + n^2(i_1) - i_1 + mN - d^1_{i_1} \leq M_1, \qquad \forall i_1 \in \mathcal{I}_1 : n^2(i_1) < i_1, && \text{(15m)}\\
& M_1 f^1_{i_1} + d^1_{i_1} - n^2(i_1) + i_1 - mN \leq M_1, \qquad \forall i_1 \in \mathcal{I}_1 : n^2(i_1) < i_1, && \text{(15n)}\\
& M_2 - M_2 f^2_{i_1} + 1 + n^1(i_1) - i_1 - d^2_{i_1} \leq M_2, \qquad \forall i_1 \in \mathcal{I}_1 : n^1(i_1) \geq i_1, && \text{(15o)}\\
& M_2 f^2_{i_1} + d^2_{i_1} - n^1(i_1) + i_1 \leq M_2, \qquad \forall i_1 \in \mathcal{I}_1 : n^1(i_1) \geq i_1, && \text{(15p)}\\
& M_2 - M_2 f^2_{i_1} + 1 + n^1(i_1) - i_1 + mN - d^2_{i_1} \leq M_2, \qquad \forall i_1 \in \mathcal{I}_1 : n^1(i_1) < i_1, && \text{(15q)}\\
& M_2 f^2_{i_1} + d^2_{i_1} - n^1(i_1) + i_1 - mN \leq M_2, \qquad \forall i_1 \in \mathcal{I}_1 : n^1(i_1) < i_1, && \text{(15r)}\\
& x_{i,a} \geq 0, \qquad \forall i = (i_1,i_2,i_3) \in \mathcal{I}, \forall a \in \mathcal{A}^B, && \text{(15s)}\\
& f^k_{i_1} \in \{0,1\}, \qquad \forall i_1 \in \mathcal{I}_1, k = 1,2, && \text{(15t)}\\
& d^k_{i_1} \in \mathbb{N}, \qquad \forall i_1 \in \mathcal{I}_1, k = 1,2. && \text{(15u)}
\end{aligned}$$

The explanation of the constraints is as follows. (15d)-(15e) relate the decision variables $x_{i,a}$ to the predefined values $z^k_{i_1,i_{k+1}}$: (15d) ensures that maintenance occurs if a PM is scheduled, while (15e) ensures that maintenance does not occur if no PM is scheduled and the component is not broken. (15f)-(15i) are added to define the relation between $x_{i,a}$ and $f^k_{i_1}$, while (15j) ensures that CM is performed immediately if both components are failed. (15k)-(15r) define the relation between $d^k_{i_1}$ and $f^k_{i_1}$ via a linearization of (14a)-(14b) using the relatively big number M_k, $k = 1,2$.

4.3 Experimental setup

The models are optimized using the Gurobi optimizer in Python 3.9. The code is available at .... The long-run average costs per year of each of the policies are calculated as in Schouten (2019) and Schouten et al. (2022) as N times the optimal objective value. Further details regarding the programming files are given in Section 4.3.2.

The long-run annual costs of the policies resulting from the single-component models are compared with those in Schouten et al. (2022), to ensure the results can be replicated exactly. This comparison is performed for one set of parameters, namely $\bar{c}_f = 50$, $\bar{c}_p = 10$, $\alpha = 12$, and $\beta = 2$.

Next, the multi-component models are evaluated for various sets of parameter combinations, to identify which model performs best in which scenario. Namely, I consider two new multi-component policies. The first policy is p-MBRP2, in which it is assumed that CM occurs immediately. Such a policy is calculated by solving only stage 1 of p-MBRP2+DCM, and relates to RQ.2. The second policy is p-MBRP2+DCM, in which CM can be delayed under certain conditions. This policy is calculated by solving both stages of p-MBRP2+DCM, and relates to RQ.3.

The performances of these multi-component models are compared to the baseline model, the single-component p-MBRP. Given that this model does not account for shared setup costs in its calculation of long-run average costs, some additional steps are required to extrapolate the results of the single-component model to the two-component setting. Only after these steps are completed can the performance of the policy in the two-component setting be evaluated, as described in RQ.1. These steps are described in detail in Section 4.3.1.

4.3.1 Single-component p-MBRP policy costs in two-component setting

First, the optimal policy for each individual component is calculated using the p-MBRP model in formulation (8), with one slight adjustment. Namely, the objective function is changed to include also a setup cost, which is incurred in addition to PM and CM costs. Thus, rather than minimizing (8a), $\sum_{i=(i_1,i_2)\in\mathcal{I}\setminus\{\mathcal{I}_b\}}(c_p(i_1)+c_s(i_1))x_{i,1}+\sum_{i=(i_1,i_2)\in\mathcal{I}_b}(c_f(i_1)+c_s(i_1))x_{i,1}$ is minimized, where $c_s(i_1)$ is time-varying as defined as in the multi-component case. This approach is taken to calculate the optimal policy of each of the two components, independently of the maintenance policy of the other component given the single-component nature of the model in 8.

Then, the resulting policies are implemented in a two-component setting, with the same assumption that CM is necessarily performed immediately. That is, the policy parameters $y^k_{i_1}$, $t^k_{i_1}$, and $z^k_{i_1,i_{k+1}}$ are fixed, and an MILP formulation with decision variables $x_{i,a}$ is solved to determine the state action frequencies, from which the average costs can be determined. The MILP formulation uses the cost parameters, action space, and transition probabilities of formulation (12), with the relation between $x_{i,a}$ and $z^k_{i_1,i_{k+1}}$ as in formulation (15). Moreover, additional constraints are added to ensure that CM is performed at age zero or the maximum age, as this is the assumption applied here.

This leads to the following model, used to obtain the costs of the single-component p-MBRP model in a two-component setting with setup costs.

$$\text{minimize} \quad \sum_{i=(i_1,i_2)\in\mathcal{I}\setminus\{\mathcal{I}_b\}} (c_p(i_1)+c_s(i_1))x_{i,1} + \sum_{i=(i_1,i_2)\in\mathcal{I}_b} (c_f(i_1)+c_s(i_1))x_{i,1} \tag{16a}$$

$$\text{subject to} \quad \sum_{a\in\mathcal{A}^A(i)} x_{i,a} - \sum_{j\in\mathcal{I}} \sum_{a\in\mathcal{A}^A(j)} \pi^A_{ji}(a)x_{j,a} = 0, \qquad \forall i=(i_1,i_2,i_3)\in\mathcal{I}, \tag{16b}$$

$$\sum_{i_2\in\mathcal{I}_2} \sum_{i_3\in\mathcal{I}_3} \sum_{a\in\mathcal{A}^A(i)} x_{i,a} = \frac{1}{mN}, \tag{16c}$$

$$x_{i,a} = 0, \qquad \forall a\in\mathcal{A}^C_k, \forall i=(i_1,i_2,i_3)\in\mathcal{I} : z^k_{i_1,i_{k+1}} = 1, k=1,2, \tag{16d}$$

$$x_{i,a} = 0, \qquad \forall a\in\mathcal{A}_k, \forall i=(i_1,i_2,i_3)\in\mathcal{I} : i_{k+1}\neq 0, z^k_{i_1,i_{k+1}} = 0, k=1,2, \tag{16e}$$

$$x_{i,a} = 0, \qquad \forall a\in\mathcal{A}^C_k, \forall i=(i_1,i_2,i_3)\in\mathcal{I} : i_{k+1} = 0, k=1,2, \tag{16f}$$

$$x_{i,a} = 0, \qquad \forall a\in\mathcal{A}^C_k, \forall i=(i_1,i_2,i_3)\in\mathcal{I} : i_{k+1} = M_k, k=1,2, \tag{16g}$$

$$x_{i,a} \geq 0, \qquad \forall i=(i_1,i_2,i_3)\in\mathcal{I}, \forall a\in\mathcal{A}^A \tag{16h}$$

4.3.2 Programming files

To find the yearly long-run average costs of the standard single-component p-ARP, p-BRP, and p-MBRP models, the files titled 'pARP', 'pBRP', and 'pMBRP' can be used. The chosen parameters can be inputted at the beginning of the file. Based on these parameters, the relevant decision variables, constraints, and objective function are generated and added to the model, in correspondence with the formulations described in Sections 4.1.3, 4.1.4, and 4.1.5. The model is then solved via the Gurobi optimizer when the code is run. The values of each of the decision variables, along with the optimal objective value and long-run average annual costs, are printed.

For the two-component setting, obtaining the costs is slightly more complex. For p-MBRP, first the file 'pMBRP' is run to obtain the optimal policy for each component, based on formulation (8). Then, the resulting policies are inputted into the file titled 'pMBRP+', which then solves the model in (16) to calculate the long-run annual cost of the policy. For p-MBRP2, the file 'pMBRP2' is run with the desired parameters, giving as output directly the optimal policy and corresponding costs. Using this policy as a starting point, the optimal maintenance schedule and annual costs of p-MBRP2+DCM are found, by inputting the policy obtained for p-MBRP2 into the file 'pMBRP2+DCM' with the required parameters. This results in the optimal policy (with delays) for p-MBRP2+DCM with the corresponding costs.

5 Results

5.1 Single-component models

The single-component p-ARP, p-BRP, and p-MBRP models are optimized under the parameters $N = 12$, $m = 1$, $\bar{c}_f = 50$, $\bar{c}_p = 10$, $\alpha = 12$, and $\beta = 2$, for different Δ ranging from 0 to 0.5 in increments of 0.1. The long-run annual costs obtained are identical to those in Table 2 of Schouten et al. (2022). It is observed that p-ARP leads to the lowest costs, followed by p-MBRP and then p-BRP. p-ARP also has the fastest computation time, at just 0.01 seconds on average across all parameter combinations, followed by p-BRP with approximately 0.05 seconds, and p-MBRP with approximately 0.15 seconds.

5.2 Multi-component models

5.2.1 Identical components

First, the models are evaluated for two identical components, $\bar{c}_f = \bar{c}_f^1 = \bar{c}_f^2$, $\bar{c}_p = \bar{c}_p^1 = \bar{c}_p^2$, $\bar{l} = \bar{l}^1 = \bar{l}^2$, $\alpha = \alpha^1 = \alpha^2$, and $\beta = \beta^1 = \beta^2$.

5.2.1.1 Basic scenario

Table 3 presents the long-run average yearly costs for the p-MBRP, p-MBRP2, and p-MBRP2+DCM models for $\bar{c}_f = 5\bar{c}_p$, $\bar{c}_s = 0.5\bar{c}_p$, and $\bar{l} = \bar{c}_p$, with Δ varying from 0% to 50%, in 10% increments. Given that the two components are identical, it is optimal in each of the three policies to schedule PM at the same time for both components.

Table 3. *Yearly costs, in thousands of €, for p-MBRP, p-MBRP2, and p-MBRP2+DCM, with $N = 12$, $m = 1$, $\bar{c}_f = 50$, $\bar{c}_p = 10$, $\bar{c}_s = 5$, $\bar{l} = 10$, $\alpha = 12$, $\beta = 2$.*

	p-MBRP			p-MBRP2				p-MBRP2+DCM		
Δ	Costs	Months	Ages	Costs	Savings	Months	Ages	Costs	Savings	Delay
0%	94.196	4,10	6,6	93.892	0.32%	5,11	4,4	93.892	-	-
10%	93.951	5,10	7,5	93.690	0.28%	6,11	4,3	93.690	-	-
20%	93.172	6,10	8,4	92.700	0.51%	6,11	4,3	92.384	0.34%	4-5
30%	91.267	6,10	8,4	91.040	0.25%	6,10	5,3	89.623	1.56%	2-5
40%	89.823	6,9	9,3	89.097	0.81%	6,10	5,3	85.478	4.06%	2-5
50%	88.620	8	1	87.040	1.78%	7,10	5,3	80.934	7.02%	1-6

Note. Savings of p-MBRP2 are given relative to p-MBRP, savings of p-MBRP2+DCM relative to p-MBRP2.
Note. The delay column indicates in which period CM is not carried out.

It can be observed from Table 3 that p-MBRP2 schedules PM more frequently than p-MBRP, as the critical ages are lower. This implies that less failures occur under this policy. This is because the model aims to reduce the number of unplanned CM activities, as the setup costs of these can frequently not be shared. That is, when a failure occurs and there is no PM scheduled, a setup cost is incurred twice.

In preventing this failure, a lower setup cost per maintenance action could be achieved, which might reduce costs overall. In fact, Table 3 shows that p-MBRP2 outperforms p-MBRP for all Δ. Moreover, the cost savings of p-MBRP2 compared to p-MBRP are increasing in Δ, as scheduling maintenance more frequently is particularly attractive when the cost of unplanned CM can be very high in certain months.

Additionally, p-MBRP2+DCM in all cases performs at least as well as p-MBRP. It performs better whenever CM is delayed, which is when $\Delta \geq 20\%$. CM is never delayed for $\Delta < 20\%$, because the savings from delaying CM to a cheaper PM period are not high enough to compensate the penalty for lost production. The savings from delaying CM are increasing in Δ, as the greatest gains can be obtained from delaying CM when there are large cost variations between periods. While the penalty for delaying CM is also period-dependent, on average this is lower than the average CM cost. Given that the cost deviations are defined based on a factor of the average cost, this implies that deviations in CM cost outweigh deviations in the penalty for lost production when delaying CM.

It should be noted that the 'savings' from delaying CM include not only the immediate financial savings, such as a possibly lower CM cost or an avoided setup cost, but they also implicitly account for the benefit of having a newer component later. For example, for $\Delta = 20\%$ in Table 3, delaying CM from period 5 to 6 saves 4.5 in setup costs and 3.66 in CM costs, but costs 9 in lost production. Still, it is optimal because the benefit of having a newer component later compensates this immediate loss.

5.2.1.2 Scenario with zero setup costs

When setup costs are zero, different patterns in savings are observed. It is interesting to observe whether the newly developed p-MBRP2+DCM policy has benefits also in a scenario without setup costs. The results for this scenario are shown in Table 4, which shows the long-run average yearly costs for the p-MBRP, p-MBRP2, and p-MBRP2+DCM models when $\bar{c}_f = 5\bar{c}_p$ and $\bar{l} = \bar{c}_p$ as in Table 3, but $\bar{c}_s = 0$.

Table 4. *Yearly costs, in thousands of €, for p-MBRP, p-MBRP2, and p-MBRP2+DCM, with* $N = 12$, $m = 1$, $\bar{c}_f = 50$, $\bar{c}_p = 10$, $\bar{c}_s = 0$, $\bar{l} = 10$, $\alpha = 12$, $\beta = 2$.

	p-MBRP			p-MBRP2				p-MBRP2+DCM		
Δ	Costs	Months	Ages	Costs	Savings	Months	Ages	Costs	Savings	Delay
0%	80.622	1,7	4,4	80.622	-	1,7	4,4	80.622	-	-
10%	80.526	6,11	4,4	80.526	-	6,11	4,4	80.526	-	-
20%	79.710	6,11	4,4	79.710	-	6,11	4,4	79.710	-	-
30%	78.676	6,10	5,3	78.676	-	6,10	5,3	78.676	-	-
40%	77.112	6,10	5,3	77.112	-	6,10	5,3	75.934	1.53%	2-5
50%	75.547	6,10	5,3	75.547	-	6,10	5,3	72.137	4.51%	2-5

Note. Savings of p-MBRP2 are given relative to p-MBRP, savings of p-MBRP2+DCM relative to p-MBRP2.
Note. The delay column indicates in which period CM is not carried out.

In the scenario of Table 4, there is no advantage of p-MBRP2 over p-MBRP, as both models always lead to the same policy and costs. However, p-MBRP2+DCM may lead to lower costs that p-MBRP and p-MBRP in some cases, as CM can be pushed back to a cheaper period if the savings of doing so, both from cheaper CM and from having a newer component later, are greater than the penalty for the lost production. As shown in Table 4, the savings from delaying CM to a cheaper period are again greatest when Δ is highest. When $\Delta < 40\%$, the cost savings from delaying CM combined with the benefit of having a newer component later are not enough to compensate the penalty for lost production when delaying CM, hence the policy remains identical to p-MBRP and p-MBRP2, leading to the same costs.

5.2.1.3 Sensitivity analysis

Based on the above results, the factors that most influence the savings of delaying CM appear to be the setup costs, the penalties for lost production, and Δ. Table 5 shows the long-run average yearly savings of p-MBRP2+DCM compared to p-MBRP2 for various combinations of $\bar{c}_s$, $\bar{l}$, and Δ, with $\bar{c}_f = 5\bar{c}_p$.

Table 5. *Yearly costs savings for p-MBRP2+DCM compared to p-MBRP2, with $N = 12$, $m = 1$, $\bar{c}_f = 50$, $\bar{c}_p = 10$, $\alpha = 12$, $\beta = 2$.*

	$\Delta = 0$				$\Delta = 0.3$				$\Delta = 0.5$			
$\bar{l}$	$\bar{c}_s = 5$	$\bar{c}_s = 10$	$\bar{c}_s = 15$	$\bar{c}_s = 20$	$\bar{c}_s = 5$	$\bar{c}_s = 10$	$\bar{c}_s = 15$	$\bar{c}_s = 20$	$\bar{c}_s = 5$	$\bar{c}_s = 10$	$\bar{c}_s = 15$	$\bar{c}_s = 20$
5	1.65%	4.43%	6.66%	10.25%	10.07%	12.70%	22.27%	25.45%	19.53%	28.57%	31.91%	34.73%
10	-	0.65%	2.08%	3.90%	1.56%	3.64%	5.07%	8.25%	7.02%	9.26%	12.63%	16.37%
15	-	-	0.55%	1.29%	-	0.34%	0.81%	1.43%	0.43%	0.93%	2.33%	4.79%
20	-	-	-	0.47%	-	-	0.30%	0.74%	-	-	-	0.40%

Overall, it can be concluded from Table 5 that the greatest gains from delaying CM for identical components are obtained when setup costs are high, costs of lost production are low, and costs vary a lot with time. Each of these effects are as to be expected based on the policy structure. Namely, when setup costs are high, there is more to be gained by by performing several maintenance actions simultaneously. Thus, delaying CM can lead to greater cost savings. Similarly, when the costs of lost production are low, the penalty for delaying CM is lower, meaning it can lead to greater savings. Last, when costs vary a lot with time, there is even greater benefit to delaying CM to a period with cheaper CM costs, as the difference from one period to the next is larger. This again leads to cost savings.

Moreover, Table 3 shows that p-MBRP2 performs at least equally as good as, and often better than, p-MBRP, particularly when Δ is large. That p-MBRP2 performs better than p-MBRP is to be expected, as the two models are nested. On the other hand, the reason why p-MBRP2 outperforms p-MBRP particularly strongly for higher Δ is less clear, and can only be somewhat speculated based on observed maintenance policies. In Table 3, one can observe that under the p-MBRP2 policy, for high Δ, the first maintenance month is earlier in the year than under p-MBRP, while the second maintenance month is later. This means that the gap over the winter between PM actions is smaller, meaning there is a lower risk of failures in the winter months. When Δ is higher, CM in the winter months is particularly expensive. Thus, by reducing the number of CM activities that must be performed in the winter, p-MBRP2 leads to cost savings, which are naturally higher when the costs of CM in winter are high, that is, when Δ is large.

5.2.2 Non-identical components

5.2.2.1 Components with different CM costs

First, components that differ in terms of maintenance cost are considered. Table 6 shows the expected annual maintenance costs and policies for two components with identical lifetime distributions, penalties for lost production, and PM costs, but different CM costs. Namely, $\bar{c}_f^1 = 5\bar{c}_p$ and $\bar{c}_f^2 = 2.5\bar{c}_p$, meaning CM is cheaper for component 2 than for component 1, so PM is less frequent.

Table 6. *Yearly costs, in thousands of euros, for p-MBRP, p-MBRP2, and p-MBRP2+DCM, with* $N = 12$, $m = 1$, $\bar{c}_p = 10$, $\bar{c}_s = 5$, $\bar{l} = 10$, $\alpha = 12$, $\beta = 2$, $\bar{c}_f^1 = 50$, $\bar{c}_f^2 = 25$.

	p-MBRP					p-MBRP2						p-MBRP2+DCM			
		Months		Ages				Months		Ages				Delay	
Δ	Costs	1	2	1	2	Costs	Savings	1	2	1	2	Costs	Savings	1	2
0%	85.463	4,10	12	6,6	1	81.383	4.77%	1,7	1,7	4,4	6,6	81.383	-	-	-
10%	83.902	5,10	8	7,5	1	80.359	4.22%	4,9	9	7,3	1	80.006	0.44%	-	8
20%	81.981	6,10	8	8,4	1	78.748	3.94%	5,9	9	5,3	1	78.477	0.34%	-	8
30%	79.520	6,10	8	8,4	1	76.608	3.66%	8	8	1	1	76.154	0.59%	7	7
40%	76.927	6,9	8	9,3	1	73.593	4.33%	8	8	1	1	72.744	1.15%	2-5,7	7
50%	71.297	8	8	1	1	71.297	-	8	8	1	1	68.641	3.73%	1-5	7

Note. Savings of p-MBRP2 are given relative to p-MBRP, savings of p-MBRP2+DCM relative to p-MBRP2.
Note. The delay column indicates in which period CM is not carried out.

It can be observed that PM is scheduled less frequently for component 2, as the lower CM cost reduces the financial damage of unplanned failures. Most notably, under the standard p-MBRP, PM is only scheduled once per year for component 2, and the PM times rarely overlap across the components, leading to excess setup costs compared to the multi-component models. On the other hand, the new p-MBRP2 model always schedules the PM of the less frequently maintained component at one of the PM times of the more frequently maintained component, saving setup costs. Additionally, p-MBRP2 has a lower critical age than p-MBRP, leading to less unplanned failures and thus fewer missed opportunities to share setup costs. This, in combination with the more frequently overlapping PM times, leads to relatively significant savings of the p-MBRP2 model compared to p-MBRP, at close to 5% in most cases. It is also observed that p-MBRP and p-MBRP2 can spuriously lead to the same policy costs, as is observed in Table 6 for $\Delta = 50\%$.

Is is noted that the savings of p-MBRP2+DCM compared to p-MBRP2 are minor compared to in the case with identical components, only exceeding 1% for $\Delta =\geq 40\%$. This is likely because, when CM is cheap for a component, cost fluctuations also are smaller, meaning there is less advantage to be gained by delaying CM, while the cost of doing so remains the same.

5.2.2.2 Components with different lifetime distributions

Next, the performance of the models for two components with different lifetime distributions is considered. Table 7 shows the long-run average yearly maintenance costs for two components with identical maintenance costs and shape parameter β, but different scale parameter α. Specifically, $\alpha^1 = 12$ and $\alpha^2 = 6$. Then, component 2 is expected to have a shorter lifetime than component 1, meaning it will fail more often and PM should be scheduled more frequently.

Table 7. *Yearly costs, in thousands of €, for p-MBRP, p-MBRP2, and p-MBRP2+DCM, with* $N = 12$, $m = 1$, $\bar{c}_f = 50$, $\bar{c}_p = 10$, $\bar{c}_s = 5$, $\bar{l} = 10$, $\beta = 2$, $\alpha^1 = 12$, $\alpha^2 = 6$.

	p-MBRP					p-MBRP2						p-MBRP2+DCM			
		Months		Ages				Months		Ages				Delay	
Δ	Costs	1	2	1	2	Costs	Savings	1	2	1	2	Costs	Savings	1	2
0%	143.832	4,10	2,6,10	6,6	3,3,3	141.762	1.44%	5,10	2,5,10	4,4	3,2,2	140.989	0.55%	-	4,9
10%	144.109	5,10	1,5,9	7,5	3,3,3	141.011	2.15%	6,10	2,6,10	5,3	3,2,2	139.979	0.73%	-	3-5,9
20%	146.045	6,10	1,5,9	8,4	3,3,2	140.097	4.07%	6,10	2,6,10	5,3	3,2,2	137.444	1.89%	5	1-5
30%	142.881	6,10	1,6,9	8,4	3,3,2	139.182	2.59%	6,10	2,6,10	5,3	3,2,2	133.422	4.14%	3-5	1-5,12
40%	138.235	6,9	1,6,9	9,3	3,3,2	137.756	0.35%	7,10	2,7,10	5,3	3,2,2	128.190	6.94%	3-6	1-6,11-12
50%	140.384	8	6,9,12	1	3,2,3	135.937	3.17%	7,10	2,7,10	5,3	4,2,2	121.690	10.48%	1-6	1-6,12

Note. Savings of p-MBRP2 are given relative to p-MBRP, savings of p-MBRP2+DCM relative to p-MBRP2.
Note. The delay column indicates in which period CM is not carried out.

With different component lifetimes, p-MBRP2 leads in all cases to slightly better results than p-MBRP. This is again because PM is performed at a lower critical maintenance age, leading to fewer unplanned failures, and PM times overlap across components, meaning setup costs can be shared. Moreover, it is again observed that the benefits of delaying CM are increasing in Δ, as larger cost fluctuations make delaying CM to a cheaper period more beneficial. It is interesting to note that for $\Delta = 50\%$ PM of component 1 is scheduled only once per year under p-MBRP, compared to twice per year under p-MBRP2. This is a clear example of the mechanism by which p-MBRP2 outperforms p-MBRP for non-identical components: by scheduling PM more frequently, greater use can be made of the possibility to share setup costs, and unplanned failures (and their associated costs) are less likely.

5.2.2.3 Components with different costs of lost production

Now, two components with the same maintenance costs and lifetime distributions, but different costs of lost production for delayed CM, $\bar{l}^1 = 10$ and $\bar{l}^2 = 15$, are examined. The costs under the various policies are presented in Table 8. In view of the p-MBRP and p-MBRP2 models, the two components are identical, as $\bar{l}^k$ is not included in those models. Thus, the PM months and critical ages are not dependent on the component. Despite this fact, both cases are presented here for comparability with Table 7 and Table 8.

Table 8. *Yearly costs, in thousands of €, for p-MBRP, p-MBRP2, and p-MBRP2+DCM, with* $N = 12$*,* $m = 1$*,* $\bar{c}_f = 50$*,* $\bar{c}_p = 10$*,* $\bar{c}_s = 5$*,* $\beta = 2$*,* $\alpha = 12$*,* $\bar{l}^1 = 10$*,* $\bar{l}^2 = 15$*.*

	p-MBRP					p-MBRP2						p-MBRP2+DCM			
		Months		Ages				Months		Ages				Delay	
Δ	Costs	1	2	1	2	Costs	Savings	1	2	1	2	Costs	Savings	1	2
0%	94.196	4,10	4,10	6,6	6,6	93.892	0.32%	5,11	5,11	4,4	4,4	93.892	-	-	-
10%	93.951	5,10	5,10	7,5	7,5	93.690	0.28%	6,11	6,11	4,3	4,3	93.690	-	-	-
20%	93.172	6,10	6,10	8,4	8,4	92.700	0.51%	6,11	6,11	4,3	4,3	92.549	0.16%	4-5	-
30%	91.267	6,10	6,10	8,4	8,4	91.040	0.25%	6,10	6,10	5,3	5,3	90.334	0.78%	2-5	-
40%	89.823	6,9	6,9	9,3	9,3	89.097	0.81%	6,10	6,10	5,3	5,3	87.309	2.01%	2-5	5
50%	88.620	8	8	1	1	87.040	1.78%	7,10	7,10	5,3	5,3	83.811	3.71%	1-6	4-6

Note. Savings of p-MBRP2 are given relative to p-MBRP.
Note. Savings of p-MBRP2+DCM are given relative to p-MBRP2.
Note. The delay column indicates in which period CM is not carried out.

The behaviour of p-MBRP and p-MBRP2 under the conditions illustrated in Table 8 is identical to that in Table 3, as all parameters are identical with the exception of $\bar{l}^k$, $k = 1,2$. However, the p-MBRP2+DCM model leads to slightly higher costs than in Table 3 in all cases, as the penalty for lost production relating to component 2 is higher. It is also notable that despite the penalty for component 1 being the same, the periods in which CM is delayed differ in Table 8 compared to 3, with CM delayed less frequently for $\Delta = 20\%$ and $\Delta = 30\%$. This illustrates the dependence of the choice to delay CM of one component on the choice to delay CM of the other: while the scheduled PM times are the same in both scenarios, the increased penalty of delaying CM of component 2 not only affects the CM delay of component 2, but it also affects in which cases CM of component 1 is delayed, as it is no longer beneficial to delay the CM of both components to the same period. The dependence of CM choices between components serves as another example to show that it is essential to consider components simultaneously, unlike existing single-component models that are unable to respond to this dependence.

5.2.2.4 Sensitivity analysis

It is clear from the above results that p-MBRP2 outperforms p-MBRP in almost all cases as it has the advantage that it is able to plan overlapping maintenance times across components. The relationship between the costs in the two scenarios remains somewhat spurious, as depending on the exact parameter combinations it might occur that p-MBRP coincidentally gives the same policy as p-MBRP2. Comparing p-MBRP2+DCM to p-MBRP2, however, is much more interesting. The above analysis has shown that the following parameters influence the savings of p-MBRP2+DCM compared to p-MBRP2: (1) the PM/CM costs of each component, (2) the lifetime distribution of each component, and (3) the cost of lost production incurred when delaying CM of each component. Moreover, it is clear from the model formulation that altering $\bar{c}_s$ will also affect the resulting policy.

Table 9 gives the average savings of p-MBRP2+DCM over p-MBRP2 for each parameter, averaged over the other parameter settings, for $\Delta = 0\%, 30\%, 50\%$. For the purpose of clarity, I choose to vary only the parameters $\bar{c}_f^k$, α^k, $\bar{c}_s$, and $\bar{l}^k$. In this way, an excessive number of parameter combinations need not be considered, but changes in the three key parameters shown to influence the savings of p-MBRP2+DCM compared to p-MBRP2 in Tables 6, 7, and 8 can be observed. I also examine changes in the setup cost, as this can be expected to have an effect on the resulting policy.

Table 9. *Average savings of p-MBRP2+DCM relative to p-MBRP2 for given Δ for each parameter, averaged over the other parameter settings, for $N = 12$, $m = 1$, $\bar{c}_p^1 = \bar{c}_p^2 = 10$, $\beta^1 = \beta^2 = 2$.*

Δ	$\alpha^1 = 12$, $\alpha^2 = 6$	$\alpha^1 = 12$, $\alpha^2 = 24$	$\bar{c}_f^1 = 50$, $\bar{c}_f^2 = 25$	$\bar{c}_f^1 = 50$, $\bar{c}_f^2 = 100$	$\bar{l}^1 = 10$, $\bar{l}^2 = 15$	$\bar{l}^1 = 10$, $\bar{l}^2 = 5$	$\bar{c}_s = 5$	$\bar{c}_s = 10$	Average
0%	12.24%	0.49%	1.91%	10.82%	0.27%	12.46%	5.82%	6.91%	6.37%
10%	12.50%	1.11%	2.27%	11.34%	0.59%	13.03%	5.99%	7.62%	6.81%
20%	13.38%	1.37%	2.57%	12.18%	1.14%	13.61%	6.48%	8.27%	7.38%
30%	15.21%	2.33%	3.55%	13.99%	2.33%	15.21%	7.78%	9.76%	8.77%
40%	17.11%	4.83%	5.45%	16.49%	4.34%	17.60%	9.68%	12.25%	10.97%
50%	19.53%	8.70%	8.28%	19.96%	7.33%	20.91%	12.68%	15.55%	14.12%

It can be observed that the savings of delaying CM are increasing in Δ across all parameter combinations, as observed also in the case for identical components. The reasoning for this is the same. Similarly, the savings of p-MBRP2+DCM are decreasing in $\bar{l}^2$, the penalty for delaying CM for component 2, as is also the case with identical components. Moreover, the savings of p-MBRP2+DCM increase also with higher $\bar{c}_f^2$, in line with the analysis in Sect. 5.2.2.1. Savings are also decreasing in α_2, as identified in Sect. 5.2.2.2. Last, it is observed that savings are increasing in $\bar{c}_s$. This is to be expected, as one of the key benefits of delaying CM is that setup costs can be shared, and the cost savings of doing so naturally increase as $\bar{c}_s$ increases.

5.2.3 Synthesis

5.2.3.1 Policies overview

An overview of the features of each of the three models is presented in Table 10.

Table 10. *Overview of p-MBRP, p-MBRP2, and p-MBRP2+DCM policies.*

	p-MBRP	p-MBRP2	p-MBRP2+DCM
Accounts for setup costs	No	Yes	Yes
Possible to delay CM	No	No	Yes
PM frequency	Relatively infrequent	Relatively frequent	Relatively frequent
PM critical ages	Relatively high	Relatively low	Relatively low

Overall, it can be seen that p-MBRP2+DCM leads to equally good results as p-MBRP in all cases, and frequently outperforms it under the studied parameter combinations. The cost savings of p-MBRP2+DCM compared to p-MBRP arise from three main factors. The first is that, by scheduling PM more frequently, the extension of p-MBRP to p-MBRP2 reduces the occurrence of unplanned failures, which have higher costs and are relatively likely to need to be carried out in a period in which there is no PM scheduled. The second reason is that, by accounting for shared setup costs in the policy optimization, p-MBRP2 has the advantage over p-MBRP that it can schedule PM at overlapping times for each component, so setup costs are shared across components, leading to lower overall costs. Last, p-MBRP2+DCM further improves p-MBRP2 by delaying CM in two ways. First, delaying CM can lead to savings on variable maintenance costs, as CM can be delayed from a period with high maintenance costs (such as in winter) to a period with lower maintenance costs (such as summer). Second, CM can be pushed back to a period in which PM is already planned, allowing setup costs to be shared.

5.2.3.2 Identical versus non-identical components

Compared to the case with identical components, the benefits of p-MBRP2+DCM compared to p-MBRP2 are limited in the non-identical component case. Delaying CM is most attractive when PM is scheduled at the same times for both components, as this allows for the greatest sharing of setup costs. When one component has more or less frequent PM than the other, it becomes more difficult to delay CM maintenance to the same period for both components. Thus, the benefits of p-MBRP2+DCM are lower for non-identical components, although still important at close to 5% in some cases.

On the other hand, the cost savings of p-MBRP2 compared to p-MBRP are far larger for non-identical components than for identical components. This is because p-MBRP schedules PM for identical components at the same time anyway, so the benefit of p-MBRP2 is limited to reducing the risk of CMs that must take place outside of these PM times. On the other hand, for non-identical components, p-MBRP performs quite poorly, as it cannot take into account the PM times of the other component, meaning PM times only spuriously overlap. In this case, p-MBRP2 leads to high cost savings, by scheduling overlapping PM times as well as reducing the risk of unplanned CM as in the case with identical components.

For $m = 1$, computation times were observed to be very similar across all parameter combinations, for both identical and non-identical components. Namely, p-MBRP could be optimized in less than a second, whereas p-MBRP2 took around 45 seconds, and p-MBRP2+DCM around 90 seconds, plus the time taken to input the results of stage 1 into stage 2. However, when considering a higher m, the computation time increased significantly, making it infeasible to run many different settings for this longer time span. Notably, with identical components and $m = 2$, the p-MBRP2 had still not been solved after running it for over an hour. The computation time would only increase for larger m and for p-MBRP2+DCM, making the present formulation limited to rather simple cases. Heuristics could be used to resolve this for larger time frames.

6 Conclusion

In this paper, the single-component p-MBRP model for time-varying costs in Schouten et al. (2022) is refined and generalized to the two-component case with economic dependence. The resulting model is called p-MBRP2. A new model is also developed, which relaxes the existing assumptions of the economic setting to allow for CM to be delayed when it is profitable to do so, building upon the p-MBRP2 model to the p-MBRP2+DCM model. In relation to RQ.1, the numerical results indicate that the single-component p-MBRP model in Schouten et al. (2022) does not perform particularly well in a multi-component setting. On the other hand, the models developed in response to RQ.2 and RQ.3, p-MBRP2 and p-MBRP2+DCM, respectively, both outperform the single-component p-MBRP model according to the numerical results, for both identical and non-identical components, with p-MBRP2+DCM leading to further cost savings compared to p-MBRP2 in settings with low penalties for lost production and high setup costs.

An advantage of the present model beyond the cost savings identified by the numerical results relates to its practicality. More precisely, based on the model structure, p-MBRP2+DCM can lead to increased predictability of maintenance actions, particularly CM. Namely, after optimizing p-MBRP2+DCM, planners know in advance in which periods they will never need to perform maintenance. This might lead to further cost savings with regard to such costs as those of obtaining labor on short notice.

On the other hand, a limitation of the present model is that the two-stage MILP formulation can be excessively computationally expensive. As additional components are added, the state space of the MDP grows exponentially, meaning computation times will rise significantly. This makes solving the resulting integer programming problem computationally impossible for large numbers of components. To resolve this, Schouten (2019) applies various heuristics, which lead to suboptimal yet acceptable results. This is relevant also for future research related to the new models derived in the present work, such that they can be applied to larger wind farms.

Moreover, the present work considers only low α and m, leading to policies based on a yearly cycle. However, it might be expected in practice that offshore wind turbines have a lifetime of several years, so higher m and α should be considered. This is theoretically possible within the framework of the present models, but is computationally unrealistic, as the running times of the formulations become extremely long. Future research could apply heuristics to make the present policy computationally feasible for longer lifetimes and policy cycles, that more closely resemble the economic reality.

Also, it should be noted that I consider only small-scale maintenance actions which can be carried out using a support vessel, to which continuous access is available. I do not consider more complex maintenance that requires a jack-up vessel. Such a vessel must typically be transported from across the world, and is in high demand and short supply. This implies huge lags between breakdown and obtaining such a vessel, that cannot be easily predicted or scheduled, making such maintenance unrealistic to include in a PM and CM scheduling model. As of the present, it seems unlikely that such maintenance will be able to be planned in advance in the near future. However, further research might wish to consider such maintenance in developing PM and CM models for the more standard maintenance actions. It might be that performing standard maintenance actions more frequently could reduce the likelihood of needing such a jack-up vessel in the future, thus leading to cost savings.

Moreover, a further limitation of the present model is that in its decision to delay CM for one component, it does not account for the age of the other component. For this reason, it might be possible that the CM of one component is delayed to occur at the same time as the next scheduled PM of another component, but that the other component has in fact not reached the critical age by this time, and so PM is not carried out. In this case, the cost of lost production from delaying CM is incurred without any benefit of sharing setup costs. Future models might delay CM to the next period only if the age of the other component is high enough that it is possible that it will have reached the necessary critical age by the next PM period. In this way, such unnecessary delaying of CM would not occur.

Last, it is noted that, ideally, the PM and CM times should be optimized jointly in the p-MBRP2+DCM model, to obtain the greatest cost savings. However, in the present MILP formulation, this is difficult to implement due to the non-linear dependence of the new decision variables introduced in stage 2 of the model on the planned PM policy. This simplification is acceptable in a standard scenario because, when CM costs are relatively high compared to PM costs (as is generally the case), PM activities are likely to significantly outnumber CM activities, meaning there is a relatively low chance that implementing the possibility to delay CM would lead to different block times and critical ages in the standard p-MBRP model. In this sense, the choice of sequential optimization is justified. Still, future research might combine both stages into one model, for completeness.

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A Appendix

A.1 Multi-component model

A.1.1 Action space

Assuming that CM is performed immediately, $r = A$, the state-dependent action space is defined as

$$\mathcal{A}^A(i_1,i_2,i_3) = \begin{cases} \{\{1,1\}\} & \text{if } i_2 \in \{0,M_1\}, i_3 \in \{0,M_2\} \\ \{\{1,0\},\{1,1\}\} & \text{if } i_2 \in \{0,M_1\}, i_3 \notin \{0,M_2\}, \\ \{\{0,1\},\{1,1\}\} & \text{if } i_3 \in \{0,M_2\}, i_2 \notin \{0,M_1\}, \\ \{\{0,0\},\{0,1\},\{1,0\},\{1,1\}\} & \text{otherwise,} \end{cases} \tag{A.17}$$

where $i_1 \in \mathcal{I}_1$, $i_2 \in \mathcal{I}_2$, and $i_3 \in \mathcal{I}_3$, and M_1 and M_2 are the maximum ages of components 1 and 2, respectively, at which age a PM is necessarily performed.

Assuming that CM can be delayed, $r = B$, the state-dependent action space is defined as

$$\mathcal{A}^B(i_1,i_2,i_3) = \begin{cases} \{\{1,1\}\} & \text{if } i_2 = M_1, i_3 = M_2, \\ \{\{1,0\},\{1,1\}\} & \text{if } i_2 = M_1, i_3 \neq M_2 \\ \{\{0,1\},\{1,1\}\} & \text{if } i_3 = M_2, i_2 \neq M_1, \\ \{\{0,0\},\{0,1\},\{1,0\},\{1,1\}\} & \text{otherwise,} \end{cases} \tag{A.18}$$

where $i_1 \in \mathcal{I}_1$, $i_2 \in \mathcal{I}_2$, and $i_3 \in \mathcal{I}_3$, and M_1 and M_2 are as for $r = A$.

A.1.2 Transition probabilities

Assuming that CM is performed immediately, $r = A$, the transition probabilities are given by

$$\pi^A_{(i_1,i_2,i_3)(j_1,j_2,j_3)}(\{0,0\}) = \begin{cases} (1-p^1_{i_2+1})(1-p^2_{i_3+1}) & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = i_2+1,\ j_3 = i_3+1, \\ & i_2 \notin \{0,M_1\},\ i_3 \notin \{0,M_2\}, \\ (1-p^1_{i_2+1})p^2_{i_3+1} & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = i_2+1,\ j_3 = 0, \\ & i_2 \notin \{0,M_1\},\ i_3 \notin \{0,M_2\}, \\ p^1_{i_2+1}(1-p^2_{i_3+1}) & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = 0,\ j_3 = i_3+1, \\ & i_2 \notin \{0,M_1\},\ i_3 \notin \{0,M_2\}, \\ p^1_{i_2+1}p^2_{i_3+1} & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = 0,\ j_3 = 0, \\ & i_2 \notin \{0,M_1\},\ i_3 \notin \{0,M_2\}, \\ 0 & \text{else,} \end{cases} \tag{A.19a}$$

$$\pi^A_{(i_1,i_2,i_3)(j_1,j_2,j_3)}(\{1,0\}) = \begin{cases} (1-p^1_1)(1-p^2_{i_3+1}) & \text{for } j_1 = i_1+1 \pmod N,\ j_2 = 1, \\ & j_3 = i_3+1,\ i_3 \notin \{0, M_2\}, \\ (1-p^1_1)p^2_{i_3+1} & \text{for } j_1 = i_1+1 \pmod N,\ j_2 = 1, \\ & j_3 = 0,\ i_3 \notin \{0, M_2\}, \\ p^1_1(1-p^2_{i_3+1}) & \text{for } j_1 = i_1+1 \pmod N,\ j_2 = 0, \\ & j_3 = i_3+1,\ i_3 \notin \{0, M_2\}, \\ p^1_1 p^2_{i_3+1} & \text{for } j_1 = i_1+1 \pmod N,\ j_2 = 0, \\ & j_3 = 0,\ i_3 \notin \{0, M_2\}, \\ 0 & \text{else,} \end{cases} \tag{A.19b}$$

$$\pi^A_{(i_1,i_2,i_3)(j_1,j_2,j_3)}(\{0,1\}) = \begin{cases} (1-p^1_{i_2+1})(1-p^2_1) & \text{for } j_1 = i_1+1 \pmod N,\ j_2 = \\ & i_2+1, \\ & j_3 = 1,\ i_2 \notin \{0, M_1\}, \\ p^1_{i_2+1}(1-p^2_1) & \text{for } j_1 = i_1+1 \pmod N,\ j_2 = 0, \\ & j_3 = 1,\ i_2 \notin \{0, M_1\}, \\ (1-p^1_{i_2+1})p^2_1 & \text{for } j_1 = i_1+1 \pmod N,\ j_2 = \\ & i_2+1, \\ & j_3 = 0,\ i_2 \notin \{0, M_1\}, \\ p^1_{i_2+1}p^2_1 & \text{for } j_1 = i_1+1 \pmod N,\ j_2 = 0, \\ & j_3 = 0,\ i_2 \notin \{0, M_1\}, \\ 0 & \text{else,} \end{cases} \tag{A.19c}$$

$$\pi^A_{(i_1,i_2,i_3)(j_1,j_2,j_3)}(\{1,1\}) = \begin{cases} (1-p^1_1)(1-p^2_1) & \text{for } j_1 = i_1+1 \pmod N,\ j_2 = 1, \\ & j_3 = 1, \\ (1-p^1_1)p^2_1 & \text{for } j_1 = i_1+1 \pmod N,\ j_2 = 1, \\ & j_3 = 0, \\ p^1_1(1-p^2_1) & \text{for } j_1 = i_1+1 \pmod N,\ j_2 = 0, \\ & j_3 = 1, \\ p^1_1 p^2_1 & \text{for } j_1 = i_1+1 \pmod N,\ j_2 = 0, \\ & j_3 = 0, \\ 0 & \text{else,} \end{cases} \tag{A.19d}$$

where p^k_x gives the failure probability of component k at age x, $k = 1,2$, and mod is the modulo operator. The precise definition of p^k_x follows from the definition of p_x in (3).

Assuming that CM can be delayed, $r = B$, the transition probabilities are given by

$$\pi^B_{(i_1,i_2,i_3)(j_1,j_2,j_3)}(\{0,0\}) = \begin{cases} (1-p^1_{i_2+1})(1-p^2_{i_3+1}) & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = i_2+1,\ j_3 = i_3+1, \\ & i_2 \notin \{0,M_1\},\ i_3 \notin \{0,M_2\}, \\ (1-p^1_{i_2+1})p^2_{i_3+1} & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = i_2+1,\ j_3 = 0, \\ & i_2 \notin \{0,M_1\},\ i_3 \notin \{0,M_2\}, \\ (1-p^1_{i_2+1}) & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = i_2+1,\ j_3 = 0, \\ & i_2 \notin \{0,M_1\},\ i_3 = 0, \\ p^1_{i_2+1}(1-p^2_{i_3+1}) & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = 0,\ j_3 = i_3+1, \\ & i_2 \notin \{0,M_1\},\ i_3 \notin \{0,M_2\}, \\ (1-p^2_{i_3+1}) & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = 0,\ j_3 = i_3+1, \\ & i_2 = 0,\ i_3 \notin \{0,M_2\}, \\ p^1_{i_2+1}p^2_{i_3+1} & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = 0,\ j_3 = 0, \\ & i_2 \notin \{0,M_1\},\ i_3 \notin \{0,M_2\}, \\ p^1_{i_2+1} & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = 0,\ j_3 = 0, \\ & i_2 \notin \{0,M_1\},\ i_3 = 0, \\ p^2_{i_3+1} & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = 0,\ j_3 = 0, \\ & i_2 = 0,\ i_3 \notin \{0,M_2\}, \\ 1 & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = 0,\ j_3 = 0, \\ & i_2 = 0,\ i_3 = 0, \\ 0 & \text{else}, \end{cases} \tag{A.20a}$$

$$\pi^B_{(i_1,i_2,i_3)(j_1,j_2,j_3)}(\{1,0\}) = \begin{cases} (1-p^1_1)(1-p^2_{i_3+1}) & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = 1,\ j_3 = i_3+1,\ i_3 \notin \{0,M_2\}, \\ (1-p^1_1)p^2_{i_3+1} & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = 1,\ j_3 = 0,\ i_3 \notin \{0,M_2\}, \\ (1-p^1_1) & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = 1,\ j_3 = 0,\ i_3 = 0, \\ p^1_1(1-p^2_{i_3+1}) & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = 0,\ j_3 = i_3+1,\ i_3 \notin \{0,M_2\}, \\ p^1_1 p^2_{i_3+1} & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = 0,\ j_3 = 0,\ i_3 \notin \{0,M_2\}, \\ p^1_1 & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = 0,\ j_3 = 0,\ i_3 = 0, \\ 0 & \text{else}, \end{cases} \tag{A.20b}$$

$$\pi^B_{(i_1,i_2,i_3)(j_1,j_2,j_3)}(\{0,1\}) = \begin{cases} (1-p^1_{i_2+1})(1-p^2_1) & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = i_2+1,\ j_3 = 1,\ i_2 \notin \{0, M_1\}, \\ p^1_{i_2+1}(1-p^2_1) & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = 0,\ j_3 = 1,\ i_2 \notin \{0, M_1\}, \\ (1-p^2_1) & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = 0,\ j_3 = 1,\ i_2 = 0, \\ (1-p^1_{i_2+1})p^2_1 & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = i_2+1,\ j_3 = 0,\ i_2 \notin \{0, M_1\}, \\ p^1_{i_2+1}p^2_1 & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = 0,\ j_3 = 0,\ i_2 \notin \{0, M_1\}, \\ p^2_1 & \text{for } j_1 = i_1+1 \pmod N, \\ & j_2 = 0,\ j_3 = 0,\ i_2 = 0, \\ 0 & \text{else,} \end{cases} \tag{A.20c}$$

$$\pi^B_{(i_1,i_2,i_3)(j_1,j_2,j_3)}(\{1,1\}) = \begin{cases} (1-p^1_1)(1-p^2_1) & \text{for } j_1 = i_1+1 \pmod N,\ j_2 = 1, \\ & j_3 = 1, \\ (1-p^1_1)p^2_1 & \text{for } j_1 = i_1+1 \pmod N,\ j_2 = 1, \\ & j_3 = 0, \\ p^1_1(1-p^2_1) & \text{for } j_1 = i_1+1 \pmod N,\ j_2 = 0, \\ & j_3 = 1, \\ p^1_1 p^2_1 & \text{for } j_1 = i_1+1 \pmod N,\ j_2 = 0, \\ & j_3 = 0, \\ 0 & \text{else,} \end{cases} \tag{A.20d}$$

where p^k_x and mod are as for $r = A$.

A.1.3 Cost structure

Under the assumption that CM is immediate, $r = A$, the cost of taking an action $a \in \mathcal{A}^A(i_1,i_2,i_3)$ in state $i = (i_1,i_2,i_3) \in \mathcal{I}$ is calculated as

$$c^A_{(i_1,i_2,i_3)}(a) = \begin{cases} 0 & \text{if } a = \{0,0\}, \\ c_s(i_1) + c^1_p(i_1) & \text{if } a = \{1,0\},\ i_2 \neq 0, \\ c_s(i_1) + c^1_f(i_1) & \text{if } a = \{1,0\},\ i_2 = 0, \\ c_s(i_1) + c^2_p(i_1) & \text{if } a = \{0,1\},\ i_3 \neq 0, \\ c_s(i_1) + c^2_f(i_1) & \text{if } a = \{0,1\},\ i_3 = 0, \\ c_s(i_1) + c^1_p(i_1) + c^2_p(i_1) & \text{if } a = \{1,1\},\ i_2 \neq 0,\ i_3 \neq 0, \\ c_s(i_1) + c^1_p(i_1) + c^2_f(i_1) & \text{if } a = \{1,1\},\ i_2 \neq 0,\ i_3 = 0, \\ c_s(i_1) + c^1_f(i_1) + c^2_p(i_1) & \text{if } a = \{1,1\},\ i_2 = 0,\ i_3 \neq 0, \\ c_s(i_1) + c^1_f(i_1) + c^2_f(i_1) & \text{if } a = \{1,1\},\ i_2 = 0,\ i_3 = 0. \end{cases} \tag{A.21}$$

Under the assumption that CM can be delayed with a penalty, $r = B$, the cost of taking an action $a \in \mathcal{A}^B(i_1, i_2, i_3)$ in state $i = (i_1, i_2, i_3) \in \mathcal{I}$ is calculated as

$$c^B_{(i_1,i_2,i_3)}(a) = \begin{cases} 0 & \text{if } a = \{0,0\},\, i_2 \neq 0,\, i_3 \neq 0, \\ l^1(i_1) & \text{if } a = \{0,0\},\, i_2 = 0,\, i_3 \neq 0, \\ l^2(i_1) & \text{if } a = \{0,0\},\, i_2 \neq 0,\, i_3 = 0, \\ l^1(i_1) + l^2(i_2) & \text{if } a = \{0,0\},\, i_2 = 0,\, i_3 = 0, \\ c_s(i_1) + c^1_p(i_1) & \text{if } a = \{1,0\},\, i_2 \neq 0,\, i_3 \neq 0, \\ c_s(i_1) + c^1_p(i_1) + l^2(i_1) & \text{if } a = \{1,0\},\, i_2 \neq 0,\, i_3 = 0, \\ c_s(i_1) + c^1_f(i_1) & \text{if } a = \{1,0\},\, i_2 = 0,\, i_3 \neq 0, \\ c_s(i_1) + c^1_f(i_1) + l^2(i_1) & \text{if } a = \{1,0\},\, i_2 = 0,\, i_3 = 0, \\ c_s(i_1) + c^2_p(i_1) & \text{if } a = \{0,1\},\, i_2 \neq 0,\, i_3 \neq 0, \\ c_s(i_1) + c^2_p(i_1) + l^1(i_1) & \text{if } a = \{0,1\},\, i_2 = 0,\, i_3 \neq 0, \\ c_s(i_1) + c^2_f(i_1) & \text{if } a = \{0,1\},\, i_2 \neq 0,\, i_3 = 0, \\ c_s(i_1) + c^2_f(i_1) + l^1(i_1) & \text{if } a = \{0,1\},\, i_2 = 0,\, i_3 = 0, \\ c_s(i_1) + c^1_p(i_1) + c^2_p(i_1) & \text{if } a = \{1,1\},\, i_2 \neq 0,\, i_3 \neq 0, \\ c_s(i_1) + c^1_p(i_1) + c^2_f(i_1) & \text{if } a = \{1,1\},\, i_2 \neq 0,\, i_3 = 0, \\ c_s(i_1) + c^1_f(i_1) + c^2_p(i_1) & \text{if } a = \{1,1\},\, i_2 = 0,\, i_3 \neq 0, \\ c_s(i_1) + c^1_f(i_1) + c^2_f(i_1) & \text{if } a = \{1,1\},\, i_2 = 0,\, i_3 = 0. \end{cases} \tag{A.22}$$