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Detecting change-points in the U.S. and Chinese Stock Markets during COVID-19

Supervisor:

M.F.O. Welz

Author:

Vincent de Quelerij

464628

Second assessor:

A. Tetereva

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Abstract

The COVID-19 outbreak in early 2020 has had a considerable impact on financial markets worldwide. Historically, such global health crises tend to result in long-term structural changes in volatility and correlation between markets. To explore this phenomenon, this thesis investigates structural breaks for the world's two largest economies - the U.S. and China - during the COVID-19 pandemic. We use a newly proposed method by Zhao et al. (2021) that allows for temporal dependence and provides a versatile detection of structural breaks. We use daily log returns of the S&P500 index and Shanghai Stock Exchange Composite index ranging from 2016 to 2021. Main findings include simultaneous structural changes in volatility for both markets around April 1st 2020 and no structural changes in correlation between the two markets. Furthermore, we estimate a structural break in volatility in the Chinese stock market on July 30th 2020, possibly corresponding to the recovery from a second COVID-19 outbreak in China from June to July 2020. These findings find purpose in scenarios including risk management and modelling.

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1 Introduction

The introduction of new technology and rapid globalization has led to a fast and ever-changing world over the past decades, resulting in a higher interdependence of financial markets around the globe (Banerjee and Guhathakurta 2020). The interdependence of stock markets has been excessively researched and confirmed in literature. Many studies show that major global events affect virtually all stock markets (e.g. Bertero and Mayer, 1990; Luchtenberg and Vu, 2015). Correlation among stock markets may even structurally change after such occurrences (Baele, 2005). For example, higher correlations occur during bull periods (Ang and Chen, 2002) and during financial crises (Hartmann et al., 2004). Lee and Chou (2020) find that major macroeconomic or financial events permanently affect correlations between stock markets that do not revert to their initial levels after economic recovery. Such events that cause structural developments in time series include epidemics, economic crises, political changes and wars.

One event of such major economic impact is the COronaVIrus Disease-19 (COVID-19) outbreak that first appeared in 2019 in Wuhan City, China (Ruiz Estrada et al., 2020). On March 12th, 2020, the World Health Organization (WHO) declared COVID-19 as a pandemic that has been a prevalent topic in research ever since. Şenol and Zeren (2020) show a decline in financial markets in January 2020 that reverts fairly rapidly. The virus originates from January and February 2020 where Asian markets (especially China) show a decline. A further decline in all financial markets happened after the announcement by WHO in March 2020. One of the pioneering studies after the outbreak, conducted by Zeren and Hizarci (2020), confirms the existence of a relationship between COVID-19 cases and various stock markets. This thesis investigates the presence of structural changes in multiple aspects of the current two largest global stock markets, the U.S. and Chinese stock markets, during the COVID-19 pandemic. Such research is relevant since incorporating structural breaks in volatility estimation is always effective and leads to a more accurate capture of downside risk and asymmetry in volatility responses (Tsuji, 2022). Hence, capturing structural breaks is highly important in risk management. It may inform an allocation of assets to lower investment portfolio risk. Additionally, it enables identification of major events before and after structural breaks in time series (Lee and Chou, 2020).

Zhao et al. (2021) introduce a change-point estimation method that is applicable for time series segmentation, based on self-normalization, incorporating a nested local-window segmentation algorithm. The self-normalization change-point (SNCP) method is fully nonparametric and does not require temporal dependence of the time series, contrary to other proposed methods. The latter is a valuable characteristic, as most time series do include temporal dependence. Zhao et al. (2021) show the versatility of the SNCP method as it allows for the detection of structural breaks for a vast class of parameters, also in a unified manner. We use this convenient property to extensively research structural breaks of different (unified) parameters for the U.S. and Chinese stock markets. We also investigate the intercorrelation between the two markets. Specifically, the first research question of this thesis is as follows:

RQ1: *Do structural changes exist in the volatility of the U.S. and Chinese stock market returns during the COVID-19 pandemic?*

This thesis tries to answer this question using the SNCP method to examine change-points in variance and Value-at-Risk (VaR) for both stock markets. These are two commonly used measures for volatility in financial markets. While the variance is a symmetric measure, VaR captures the downside risk of returns below the expected amount. It is a high quantile of the return distribution (we use 90% and 95% quantiles). We estimate change-points for all parameters and their multi-parameter combinations.

One might argue that there is one obvious structural break around March 2020 and hence no change-point detection method is needed. However, it is important to confirm the presence of such a structural break with statistical evidence. Moreover, this thesis focuses on structural breaks during the pandemic that occur not only at the initial outbreak of COVID-19.

Secondly, we investigate the intercorrelation between the two stock markets. Therefore, we introduce the second research question:

RQ2: *Does the correlation between the U.S. and Chinese stock markets structurally change during the COVID-19 pandemic?*

We again use the SNCP method to attempt to answer this particular research question, as its described versatility allows us to estimate change-points for correlation between two time series.

This thesis differs from existing literature in multiple ways. First, we use a newly proposed change-point estimation method that allows for temporal dependence and versatility in (combined) parameters as opposed to other methods. Secondly, we use recent data and perform analysis on both large- and small-scale data sets, while all mentioned literature has a rather small data selection only up to June 2020.

We use log returns of the commonly used S&P500 index (SP500) as this index represents a considerable amount of roughly 80% of the total U.S. equity market with its data widely available. Therefore, the SP500 represents the U.S. stock market in this thesis. For China, we conduct research on log returns of the Shanghai Stock Exchange Composite Index (SSE), widely used as an indicator for the Chinese stock market. We use data ranging from Januray 2016 to December 2021. We also zoom in on this data by only looking at data ranging from December 2019 to December 2021 as this includes just the COVID-19 pandemic and may result into more subtle insights and change-point estimates due to the observation period being smaller.

Additionally, we try to replicate findings from Zhao et al. (2021). Specifically, their real-world data application on financial data (Section 5.2). We refer to our main research as the extension. Our main findings for the extension include simultaneous structural breaks in volatility for both markets around April 1st 2020. We estimate an additional change-point in volatility for the SSE on July 30th 2020. The latter change-point indicates a different recovery trajectory for the SSE after the initial COVID-19 outbreak, possibly due to a second outbreak in China from June 2020 to July 2020 (Wu et al., 2020). Furthermore, we detect no structural changes in the correlation between the two markets, which suggests that COVID-19 did not affect long-term intercorrelation between the U.S. and Chinese stock markets. It appears that COVID-19 has similar effects on both markets.

Finally, in the context of volatility, we analyse whether incorporating a structural break improves a widely used and researched symmetric Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) model, proposed by Bollerslev (1986). Specifically, we use a GARCH(1,1) model. This analysis is important as it might show and reinforce the importance of detecting structural breaks. The analysis gives an indication that incorporating structural breaks in forecasting models might improve forecast accuracy.

The remainder of this thesis is structured as follows. To start with, Section 2 gives an overview of the data that we use and how we retrieve and clean the data sets accordingly. Secondly, Section 3 gives the set-up of our methodology and dives into the use of the SNCP method. Section 4 then shows replication and extension results by applying our methods. Next, we thoroughly discuss the results and tackle limitations of our work in Section 5. Finally, Section 6 concludes.

2 Theory

Co-movement between markets is an excessively researched phenomenon in literature. King and Wadhwani (1990) establish interdependence between markets even in the stock market crash in October 1989. The crash resulted in a contemporaneous fall of all stock markets, despite different economic circumstances. They show that an occurrence of one mistake in an initial stock market leads to a contagion to other markets. Moreover, they find empirical evidence that the contagion effect's size increases with the initial market's volatility. Market co-movement is less apparent in Asian countries than in western countries, possibly due to vast geographical distances and more hindrance due to cultural and language diversity (Lee and Chou, 2020). Also, economic integration between Asian countries differs from that between western countries (Arshanapalli et al., 1995).

For RQ1, we hypothesize any structural breaks in China to occur in late February 2020 - late March 2020 (Zeren and Hizarci, 2020; Kusumahadi and Permana, 2021) and in the U.S. in late February 2020 - early April 2020 (Çütcü and Kilic, 2020; Yilmazkuday, 2021; Hong et al., 2021). Generally, we expect structural changes in the Chinese stock market to occur earlier as opposed to the U.S. stock market due to China being the origin of the COVID-19 outbreak (e.g. Gunay, 2020).

Correlation between the two stock markets has been excessively researched before the COVID-19 outbreak (e.g. Lee and Chou, 2020). Again, Hartmann et al. (2004) state that correlations are often higher during financial crises. Additionally, Just and Echaust (2020) find evidence of a higher correlation between financial markets during the COVID-19 pandemic, including the U.S. and Chinese market. This leads to the hypothesis of a possible structural break during the COVID-19 pandemic as correlations tend to get higher in such periods.

Another relevant and important topic is whether structural breaks contribute to forecasting models. Literature states that when structural breaks are unknown, some robust models show good forecasting performance even with the existence of such a structural break (e.g. Choi et al., 2010; Pesaran et al., 2013). For this thesis, it is more relevant whether incorporating known structural breaks improves forecasting models. Boot and Pick (2020) show that structural changes of small magnitude should not be incorporated into models. Moreover, they find that relevant structural changes for forecast accuracy happen less often than most existing tests suggest in the context of macroeconomic time series. Incorporating structural breaks into models situationally might or might not improve forecasting accuracy.

Finally, volatility has an unknown ground truth. Hence, choosing an adequate benchmark for volatility is important if we want to compare models with and without structural breaks. Andersen et al. (2006) state that the squared log returns, although naive, are a straightforward benchmark when evaluating the forecast accuracy of GARCH models. Let r_t be the daily log returns. Then, by definition, $VAR(r_t) = E(E(r_t^2) - E(r_t)^2)$. When working with daily (and not weekly or monthly) log returns, the naive assumption is that expected log returns to be approximately zero. Hence, the equation shrinks to $VAR(r_t) = E(r_t^2)$, the squared log returns. As their paper bundles a variety of literature, it deems appropriate to use squared log returns as a benchmark for volatility in this thesis. Some famously used prediction error metrics are the mean squared error (MSE), root mean squared error (RMSE) and the mean absolute error (MAE) (e.g. Molinaro et al., 2005; Hansen and Lunde, 2005), which we use in this thesis to evaluate forecasts.

3 Data

3.1 Replication

The data consists of daily log-returns of SP500 between June 2006 and December 2010. A total of $n = 2^{10} = 1024$ observations satisfies a required power of 2 that is needed and further explained in Section 4.2. We use data from Wall Street Journal (2022) for the (closed) SP500 for the dates May 30th 2006 to December 31th 2010 to capture all (negative) log returns. A major issue arises as this accounts for 1158 data points, 134 too many. When taking a closer look at the results of Zhao et al. (2021), we find observations missing. There seems to be no systematic or randomized way in which they removed the data points, hence requiring manual analysis of each individual data point. Appendix A thoroughly shows the manual cleaning of the data set, shrinking it down to the 1024 used data points. It is worth mentioning that the missing observations do not appear to be randomly selected by Zhao et al. (2021). Some meaningful (groups of) observations are missing, which may lead to biased outcomes. Although the data cleaning process is obscure and very susceptible to mistakes, the cleaned data set seems to lead to the pursued replication.

3.2 Extension

We use data between January 2016 and December 2021 of the SP500 and SSE acquired from Wall Street Journal (2022) and Yahoo!Finance (2022) respectively. This data set covers most of the pandemic and recovery (all though COVID-19 is still an ongoing phenomenon), while keeping the impact on the economy from the current Russian invasion in Ukraine (Ivana Kottasová and Regan, 2022) to a minimum, as we want to isolate the effect of COVID-19 as much as possible. In order to obtain data sets of equal size, necessary for investigating correlations, we exclude all closed market days during this period for both the SP500 and SSE. This results in a total of $n = 1411$ observations. Furthermore, we zoom in on the data set and use only the last $n_* = 491$ observations corresponding with data from December 2019 to December 2021. The (negative) daily log returns of the total data set show in Figure 1 for both indices.

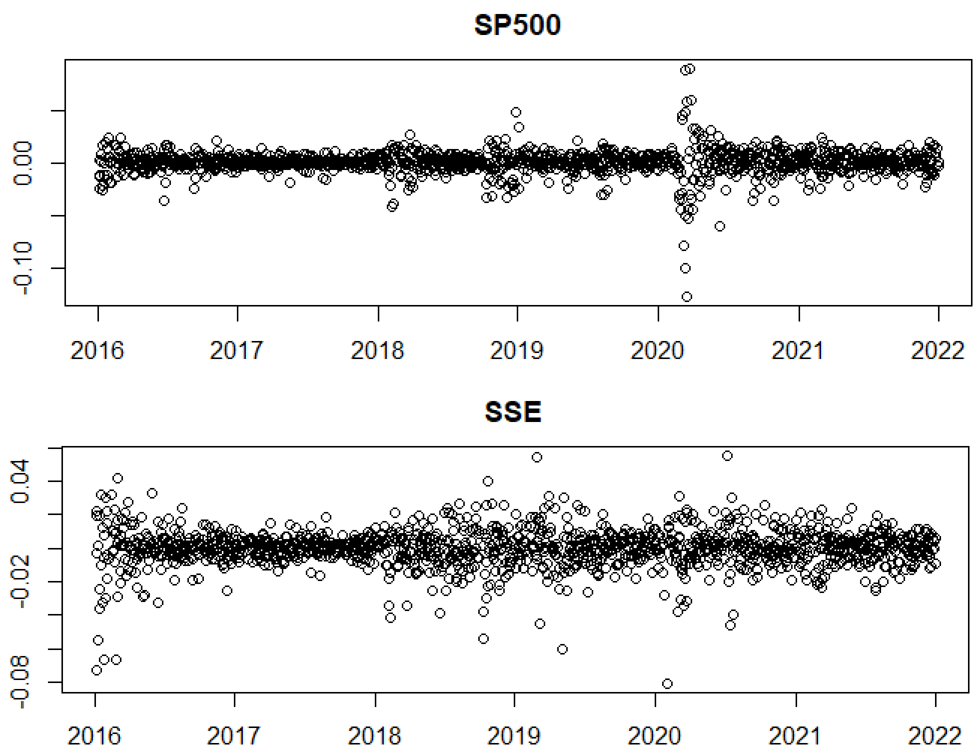

Figure 1: Daily (negative) log returns of the S&P 500 index and the Shanghai Stock Exchange Composite index from January 2016 - December 2021 with a total of n = 1411 observations for both indices.

The SSE seems to contain more volatility in general than the SP500, possibly due to the lower market volume. Moreover, both indices seem to increase in volatility rather simultaneously, which gives a visual indication of positive correlations between both markets.

4 Methodology

4.1 SNCP

In this thesis, we follow the notation used by Zhao et al. (2021). Let $\{Y_t\}_{t=1}^n$ be a piecewise stationary (multivariate) time series with a fixed dimension, $Y_t \in \mathbb{R}^p$, $p \geq 1$. Let $\{Y_t\}_{t=1}^n$ contain $m_o \geq 0$ unknown change-points $0 < k_1 < ... < k_{m_o} < n$. It follows that $k_0 = 0$, $k_{m_0+1} = n$ and that the time series is partitioned into segments, where segment i contains observations $\{Y_t\}_{t=k_{i-1}+1}^{k_i}$. To complete the notation for the time series, we define

$$Y_t = Y_t^{(i)} \ , \ k_{i-1}+1 \leq t \leq k_i, \ \text{ for } i = 1, \cdots, m_o + 1 \tag{1}$$

as the data generating process for $\{Y_t\}_{t=1}^n$, where every $\{Y_t^{(i)}\}_{t\in\mathbb{Z}}$ has cumulative distribution function $F^{(i)}$ for which it is required that for some vector-valued functional $\boldsymbol{\theta} \in \mathbb{R}^d$ with $d \geq 1$, $\boldsymbol{\theta}_i = \boldsymbol{\theta}(F^{(i)}) \neq \boldsymbol{\theta}_{i+1}$, for all $i = 1, \cdots, m_o$. For this thesis, we choose $\boldsymbol{\theta}$ to be the 90th and 95th quantile functionals, the variance functional and their multivariate combinations. Additionally, we use the correlation functional for bivariate time series.

To determine the unknown number of change-points and their locations, we define for $1 \leq t_1 < k < t_2 \leq n$ the following notation:

$$T_n^*(t_1, k, t_2) = D_n^*(t_1, k, t_2)^\top V_n^*(t_1, k, t_2)^{-1} D_n^*(t_1, k, t_2) \ \text{ for } 1 \leq t_1 < k < t_2 \leq n, \tag{2}$$

where

$$D_n^*(t_1, k, t_2) = \frac{(k - t_1 + 1)(t_2 - k)}{(t_2 - t_1 + 1)^{3/2}}(\widehat{\boldsymbol{\theta}}_{t_1,k} - \widehat{\boldsymbol{\theta}}_{k+1,t_2}), \quad V_n^*(t_1, k, t_2) = L_n^*(t_1, k, t_2) + R_n^*(t_1, k, t_2),$$

$$L_n^*(t_1, k, t_2) = \sum_{i=t_1}^{k} \frac{(i - t_1 + 1)^2(k - i)^2}{(t_2 - t_1 + 1)^2(k - t_1 + 1)^2}(\widehat{\boldsymbol{\theta}}_{t_1,i} - \widehat{\boldsymbol{\theta}}_{i+1,k})(\widehat{\boldsymbol{\theta}}_{t_1,i} - \widehat{\boldsymbol{\theta}}_{i+1,k})^\top,$$

$$R_n^*(t_1, k, t_2) = \sum_{i=k+1}^{t_2} \frac{(t_2 - i + 1)^2(i - 1 - k)^2}{(t_2 - t_1 + 1)^2(t_2 - k)^2}(\widehat{\boldsymbol{\theta}}_{i,t_2} - \widehat{\boldsymbol{\theta}}_{k+1,i-1})(\widehat{\boldsymbol{\theta}}_{i,t_2} - \widehat{\boldsymbol{\theta}}_{k+1,i-1})^\top,$$

with $\widehat{\boldsymbol{\theta}}_{a,b} = \boldsymbol{\theta}(\widehat{F}_{a,b})$, $\widehat{F}_{a,b}$ being the empirical distribution of the time series containing observations a to b with logically $1 \leq a < b \leq n$. Note that for $d = 1$, the test statistic shrinks down to a one dimensional non-vectorized case.

We then choose a collection of nested windows around k. Define $\epsilon \in (0, 1/2)$ and window size $h = \lfloor n\epsilon \rfloor$. We now construct the set of nested windows for all $k = h, \cdots, n-h$ as

$$H_{1:n}(k) = \left\{ (t_1, t_2) \middle| t_1 = k - j_1 h + 1, j_1 = 1, ..., \lfloor k/h \rfloor; t_2 = k + j_2 h, j_2 = 1, ..., \lfloor (n-k)/h \rfloor \right\}.$$

For each separate k, maximize the test statistic from Equation 2 such that

$$T^*_{1,n}(k) = \max_{(t_1,t_2) \in H_{1:n}(k)} T^*_n(t_1, k, t_2).$$

Lastly, denote $W_{s,e} = \{(t_1, t_2) | s \leq t_1 < t_2 \leq e\}$ and $H_{s:e}(k) = H_{1:n}(k) \bigcap W_{s,e}$, the nested window set on the subsample $\{Y_t\}_{t=s}^{e}$ and K_n a certain prespecified threshold. The set $H_{s:e}(k)$ allows us to determine the maximal test-statistic over a subsample, namely $T^*_{s,e}(k) = \max_{(t_1,t_2) \in H_{s:e}(k)} T^*_n(t_1, k, t_2)$. All relevant notation is now present to construct recursive the recursive procedure. Algorithm 1 specifies the procedure formally. Intuitively, we start with the complete time series. If $K_n \geq \max_{k=1,..,n} T^*_n(k)$, the algorithm estimates no change-point. Otherwise, the algorithm continues on the subset up to and from $\widehat{k}^*$ corresponding to the maximized test-statistic. The algorithm continues until it detects no (further) change-points.

Algorithm 1: multivariate SNCP pseudocode for multiple change-point estimation

Input: Time series $\{Y_t\}_{t=1}^{n}$, threshold K_n and window size $h = \lfloor n\epsilon \rfloor$

Output: Set containing $\widehat{m}$ estimated change-points $\widehat{\mathbf{k}} = \left(\widehat{k}_1, \cdots, \widehat{k}_{\widehat{m}}\right)$

Initialization: SNCP$(1, n, K_n, h)$

Procedure: SNCP(s, e, K_n, h)

if $e - s + 1 < 2h$ **then**
- Stop

else
- $\widehat{k}^* = \arg\max_{k=s,\ldots,e} T^*_{s,e}(k)$;
- **if** $T^*_{s,e}(k) \leq K_n$ **then**
  - Stop
- **else**
  - $\widehat{\mathbf{k}} = \widehat{\mathbf{k}} \cup \widehat{k}^*$;
  - SNCP$(s, \widehat{k}^*, K_n, h)$;
  - SNCP$(\widehat{k}^* + 1, e, K_n, h)$;
- **end**

end

One important note is that the choice of K_n controls the Type I error, as Zhao et al. (2021) show $T^*_{1,n}(k)$ to converge to some limiting distribution $G^*_{\epsilon,d}$. Setting K_n as the $(1-\alpha)\times 100\%$ quantile level of $G^*_{\epsilon,d}$ corresponds to Type I error control of α. Despite $G^*_{\epsilon,d}$ being unknown, Zhao et al. (2021) obtained limiting distribution of various choices of ϵ and d by simulation. We use their simulation results to select K_n. Table 1 shows critical values of $G^*_{0.05,d}$, corresponding to window size scale $\epsilon = 0.05$.

Table 1: Critical values for $G^*_{\epsilon,d}$, the limiting null distribution for $\epsilon = 0.05$.

$1-\alpha$ \ d	1	2	3
90%	141.9	208.2	275.0
95%	165.5	237.5	309.1
99%	224.2	309.4	386.4

Notes: Critical values generated by means of simulation by Zhao et al. (2021).

4.2 Replication

We try to replicate the results in Section 5.2 of Zhao et al. (2021) concerning the SP500 investigation. In particular, we investigate the volatility behaviour of the SP500 during the financial crisis of 2008. We apply SNCP to investigate structural breaks in the variance, Value-at-Risk (90% and 95% quantiles) and their multi-parameter combinations. We again follow the notation used by Zhao et al. (2021) to establish seven SNCP estimators: SNV, SNQ_{90}, SNQ_{95}, $\text{SNQ}_{90,95}$, SNQ_{90}V, SNQ_{95}V and $\text{SNQ}_{90,95}\text{V}$. V corresponds to the variance and 90 and 95 to the respective quantiles. We implement various methods in addition to the SNCP for comparison reasoning. For variance and autocorrelation changes, we compare with both Cho and Fryzlewicz (2012) (MSML) and Korkas and PryzlewiczV (2017) (KF). We compare with Matteson and James (2014) (ECP) for multi-parameter change. We will not go into detail on the comparison methods since the purpose is merely to replicate their findings. However, there are two important notes. First, the ECP method requires temporal independence, which is barely the case in time series. Secondly, MSML only handles sample sizes of n as a power of 2.

Lastly, Zhao et al. (2021) mention that they use window size scale $\epsilon = 0.05$ for all SNCP estimators. They additionally state that they use critical values at $\alpha = 0.1$ for all SNCP estimators which corresponds to a choice of K_n at the 90% quantile of $G^*_{\epsilon,d}$.

4.3 Extension

4.3.1 Change-point detection

To generate the results to answer the main research questions, we use previously established SNCP estimators along with one extra parameter, SNC, which estimates change-points in the correlation between two time series. For all performed estimations, we use $\epsilon = 0.05$ as a window size scale, in line with the original use for real-world applications by Zhao et al. (2021). They state that $\epsilon = 0.05$ might not perform the best for every window size, but it does have the best performance for high and low signal-to-noise ratio cases. Moreover, it guards best against faulty change-point dates that deviate from actual change-point dates. Lastly, we set the threshold K_n at 90, 95 and 99% quantiles of $G^*_{\epsilon,d}$. As previously stated, we use critical values provided by Zhao et al. (2021) for all SNCP estimators.

4.3.2 Modelling Structural Breaks

To conclude the methodology, we conduct a very basic analysis on forecast improvement when incorporating structural breaks. As previously stated, we use a GARCH(1,1) model as it is widely used in volatility modelling. The GARCH(1,1) model formulates as follows.

$$\begin{aligned} \mathrm{r}_t &= \mu + z_t \sigma_t, \\ \sigma^2_{t+1} &= \omega + \alpha z^2_t + \beta \sigma^2_t, \end{aligned} \tag{3}$$

with r_t being the daily log returns, σ^2_t the conditional variance at time t. Furthermore, the model assumes $z_t \sim N(0,1)$. $\omega > 0, \alpha > 0$ and $\beta \geq 0$ to guarantee $\sigma^2_t \geq 0 \ \forall t$. We estimate the GARCH(1,1) model using the first 80% of the observation period to forecast the remaining observations. We also estimate variances using a GARCH(1,1) model using the first 80% of the observation period, excluding data points before a certain structural break. There is no correct universal size for out-of-sample portions in literature. Our data sets are appropriately large for 80% of the data to be sufficient to estimate the parameters and compare models while ensuring that the out-of-sample period consists of enough observations to predict. Let $\hat{\sigma}^2_t$ be the one-step-ahead forecasts for conditional variance at time t by the GARCH(1,1) model. The MSE, RMSE and MAE are then formulated as follows.

$$
\begin{aligned}
MSE &= \sum_{t=1}^{n} \frac{\left(\hat{\sigma}_t^2 - r_t^2\right)^2}{n} \\
RMSE &= \sqrt{MSE} \\
MAE &= \sum_{t=1}^{n} \frac{\left|\hat{\sigma}_t^2 - r_t^2\right|}{n}
\end{aligned} \tag{4}
$$

Note that we keep this analysis reasonably simple as its purpose is merely to give some insights on the importance of change-point detection and not to discuss the GARCH model and its assumptions excessively. We choose the observation period for this analysis after the change-point estimation.

4.4 Code

This thesis uses RStudio 2022.02.0 for programming in R. Zhao et al. (2021) provide user-friendly R code that we use for change-point estimation for the replication and the extension part. Additionally, this thesis uses EViews 11 for GARCH(1,1) model estimation and forecast evaluation. The author of this thesis can supply all data and programming codes as a .zip file upon request. Of course, all rights for the R code go to Zhao et al. (2021).

5 Results

5.1 Replication

After requiring the appropriate size for the data set, we start the replication. Table 2 shows the replication results, with in red the diverging outcomes from the results of Zhao et al. (2021). MSML seemingly confirms the validity of the cleaned data set as it gives identical outcomes to that of Zhao et al. (2021) when rounding the SP500 log returns to three decimals (which we apply for all other methods as well). Next, R uses a random number generator to produce the KF outcomes. A commonly used seed is 101, and it produces almost the exact results. CP4 differs from the actual result by two trading days, suggesting that Zhao et al. (2021) uses another seed or just use the first random result they obtain. To conclude the comparison methods, ECP produces an exact replication of the desired outcomes when looking at a significance level $\alpha = 0.05$ (for $\epsilon = 0.05$).

Only the SNCP estimators remain. We find very similar results to the original outcomes. A possible explanation for diversions (e.g. missing change-points) is that the SNCP change-point estimates are not robust to changes in the window size. $\epsilon = 0.05$ results in a window size of $h = \lfloor n\epsilon \rfloor = 51$. Alternating between window sizes produce different (and additional) outcomes, often also extremely close to the original results. However, not one window size produces the exact outcomes. This may indicate that Zhao et al. (2021) use some sort of a weighted choice of change-point estimates of multiple window size choices around 51, which would contradict their statement on the used window size. Moreover, Zhao et al. (2021) were not clear about the data selection. Although perfect replication for MSML suggests that our data set is equal to that of Zhao et al. (2021), it is not guaranteed and could be an additional explanation for not replicating all results. Finally, adjusting K_n does not change the replication results meaningfully.

Table 2: Estimated change-points by MSML, KF, ECP and various SNCP estimators for log returns of the S&P 500 index between June 2006 and December 2010.

	Method	CP1	CP2	CP3	CP4	CP5	CP6
SP500	SNV			07/17/2007	09/16/2008	12/05/2008	XXX
	MSML	07/28/2006	02/23/2007	07/18/2007	09/02/2008	12/01/2008	04/20/2009
	KF	08/01/2006	01/23/2007	07/23/2007	08/18/2008		04/20/2009
	ECP			07/20/2007	09/17/2008		04/21/2009
	SNQ_{90}			XXX	08/04/2008		05/18/2009
	SNQ_{95}			XXX	08/04/2008		05/18/2009
	$SNQ_{90,95}$			07/09/2007	09/17/2008		XXX
	$SNQ_{90}V$			07/17/2007	09/16/2008	12/05/2008	04/20/2009
	$SNQ_{95}V$			07/09/2007	09/16/2008	12/05/2008	04/21/2009
	$SNQ_{90,95}V$			07/09/2007	09/16/2008	12/05/2008	04/20/2009

Notes: Dates present in the U.S. date format MM/DD/YYYY. In red are the (missing) change-points that are not exact replicates of the original results. XXX indicates a missing change-point in the replication. Specifically, this Table tries to replicate Table 7 from Zhao et al. (2021).

5.2 Extension

This subsection shows the results for answering the main research questions. We present outcomes for two different data sets and close out with a short exploration on change-point frequency in both data sets.

5.2.1 Change-point detection

We now present outcomes of the change-point analysis on the SP500 and SSE between 2016 - 2021. Table 3 shows change-point estimation dates. Results show no structural break for intercorrelation during the COVID-19 pandemic. One structural break is estimated in May 2018, during the trade war during the Trump Presidency. Similar change-points show for all other SNCP estimators, mostly during the start of 2018, the beginning of the trade war between China and the U.S (Itakura, 2020). Structural changes in variance and high quantiles are estimated in 2016 for the SSE, possibly due to the recovery after the 2015-2016 stock market turbulence in China (Han et al., 2019). The last change-point estimations range from December 2019 to January 2020. This period may indicate the start of the COVID-19 outbreak. The fast stabilization right after the COVID-19 (although its change in the total volume of the indices) is a possible explanation for no change-points in the variation for both stock markets. Although $\epsilon = 0.05$ is the recommended window size scale, the SNCP method seems to lack robustness for different window sizes. For example, Appendix B shows fairly different change-point estimates for $\epsilon = 0.04$.

Table 3: Estimated change-points by SNCP estimators for log returns of the S&P 500 index and the Shanghai Stock Exchange Composite index between January 2016 and December 2021.

	Method	CP1	CP2	CP3
SP500 & SSE	SNC		05/25/2018	
SP500	SNV		01/09/2018**	
	SNQ_{90}		01/10/2018*	
	SNQ_{95}		01/12/2018**	
	$SNQ_{90,95}$		01/16/2018**	12/02/2019**
	$SNQ_{90}V$		01/11/2018**	
	$SNQ_{95}V$		01/11/2018**	
	$SNQ_{90,95}V$		01/11/2018**	12/02/2019**
SSE	SNV	06/16/2016*	01/23/2018**	
	SNQ_{90}		06/28/2018*	01/23/2020*
	SNQ_{95}	11/10/2016	02/05/2018*	07/05/2019
	$SNQ_{90,95}$		06/28/2018	01/23/2020*
	$SNQ_{90}V$	06/16/2016	02/05/2018**	01/23/2020
	$SNQ_{95}V$	06/14/2016**	02/05/2018**	
	$SNQ_{90,95}V$	06/14/2016**	01/12/2018**	

Notes: Dates present in the U.S. date format MM/DD/YYYY.
Only a date indicates a change-point at the 90% threshold.
** indicates a change-point at the 95% threshold.*
*** indicates a change-point at the 99% threshold.*

We now perform the same analysis on a smaller data set only containing the COVID-19 pandemic. The results of the SNCP estimators for data ranging from December 2019 - December 2021 for a total of $n_* = 491$ observations show in Table 4. The SNC estimator supports our earlier findings of no structural break in the correlation between the SSE and SP500 during the COVID-19 pandemic. A similar impact on both markets of the COVID-19 outbreak is a possible explanation for no structural changes in correlation between the two markets. However, we find some additional change-points opposed to the outcomes of Table 3. Various estimators indicate a structural break in variance for the SP500 on April 1st 2020. A related result presents itself for the SSE as all variance-incorporated SNCP estimators estimate change-points on March 27th 2020. Such dates are in line with our hypothesis and existing literature (Zeren and Hizarci, 2020; Kusumahadi and Permana, 2021); Çütcü and Kilic, 2020; Yilmazkuday, 2021; Hong et al., 2021). For SSE, various SNCP estimators detect change-points later in 2020, possibly due to a decrease in volatility after the initial outbreak. One change-point for variance is measured on July 30th 2020 by SNV and SNQ_{95}V.

Figure 2 visualizes the results of SNV. Change-point estimations of SNV are marked with red vertical lines. There seems to be a spike in volatility right before the second change-point estimate of the SSE on July 30th 2020. Wu et al. (2020) show the presence of a second outbreak of COVID-19 in June 2020 to July 2020. This along with Figure 2 indicates that the change-point estimate at July 30th 2020 possibly corresponds to the recovery after the quick containment of the second COVID-19 outbreak in China.

Table 4: Change-point estimates by SNCP estimators for daily log returns of the S&P 500 index and the Shanghai Stock Exchange Composite index from December 2019 to December 2021.

	Method	CP1	CP2
SP500 & SSE	SNC		
SP500	SNV	04/01/2020*	
	SNQ_{90}		
	SNQ_{95}	02/28/20*	06/10/2020**
	$SNQ_{90,95}$	02/28/20	
	$SNQ_{90}V$	04/01/2020**	
	$SNQ_{95}V$	04/01/2020**	
	$SNQ_{90,95}V$	04/01/2020**	
SSE	SNV	03/27/2020**	07/30/2020**
	SNQ_{90}		11/09/2020
	SNQ_{95}		
	$SNQ_{90,95}$		11/10/2020
	$SNQ_{90}V$	03/27/2020*	12/04/2020*
	$SNQ_{95}V$	03/27/2020*	07/30/2020*
	$SNQ_{90,95}V$	03/27/2020	

Notes: Dates present in the U.S. date format MM/DD/YYYY.
Only a date indicates a change-point at the 90% threshold.
** indicates a change-point at the 95% threshold.*
*** indicates a change-point at the 99% threshold.*

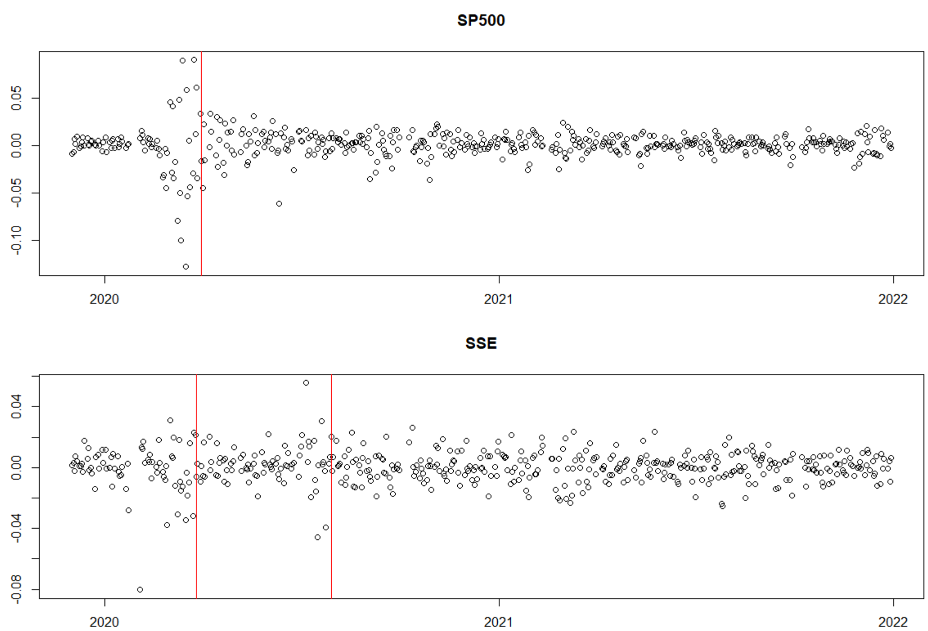

Figure 2: SNV change-point estimations (in red) of daily log returns of the S&P 500 index and the Shanghai Stock Exchange Composite index from December 2019 - December 2021.

5.2.2 Modelling Structural Breaks

Table 4 shows one change-point on April 4th 2020 for the variance for the SP500. We use this estimated structural break to see if it improves forecast accuracy of a simple GARCH(1,1) model. The observation period consists of $n_* = 491$ observations, for which we use the first 80% (393) observations for model estimation. GARCH$_{SP500F}$ and GARCH$_{SP500CP}$ are the GARCH(1,1) models that use the full in-sample data and the post change-point in-sample data respectively. Visual representation of the out-of-sample forecasts shows in Appendix C. The main results show in Table 5. All error measures turn out very low. Therefore, differences in model performance are remarkably small. Hence, we cannot make bold statements. However, all evaluation metrics show lower values for the model incorporating the structural break. We can carefully say that incorporating the structural break in this simple case improves forecast accuracy of the variance (and thus volatility), although barely. According to Boot and Pick (2020), this could suggest that this structural break is of a large enough magnitude to be considered relevant in forecasting models. The low metric values can be explained due to the daily log returns being relatively small towards the end of 2021, which is the out-of-sample period.

Table 5: Various evaluation metrics for out-of-sample forecasts of the variance of daily log returns of the S&P500 index using two GARCH(1,1) models. Squared daily log returns serve as a benchmark.

	MSE ($*10^{-9}$)	RMSE ($*10^{-5}$)	MAE ($*10^{-5}$)
GARCH$_{SP500F}$	9.98	9.99	6.73
GARCH$_{SP500CP}$	9.13	9.56	6.52

Notes: GARCH$_{SP500F}$ uses all in-sample data points.
GARCH$_{SP500CP}$ uses in-sample data points after the estimated change-point on April 1st 2020.

6 Discussion

This section discusses the results and tackles some limitations of this thesis.

Although mentioned before, the replication raises some questions on the data cleaning process of Zhao et al. (2021) for their real-world data application on financial data. They state the observation period of June 2006 to December 2010 and their total observations, but not how they exclude data points to obtain that exact number of 1024 observations. Appendix A provides in-depth visualization of the excluded points from the total observation period. The main finding is that the excluded points seem not to be randomly removed but instead removed in clusters (e.g. around Christmas). Sometimes, even extreme observations are excluded, which could substantially influence change-point detection. Therefore, we question if there is a bias in their choice of data and think that Zhao et al. (2021) should elaborate more on the choice of data in Section 5.2 in their paper.

We now discuss our primary research, the extension part of this thesis.

First, volatility is essentially an observable variable, and the ground truth is unknown. We examine structural changes from different angles using the SNCP method's versatility. One limitation persists, however, since daily data assumes consistent volatility during days and treats gaps of multiple days the same as consecutive days. These assumptions rarely match reality.

Next, the SNCP method is proven to be very robust for different thresholds (Zhao et al., 2021), but has varying results for different window sizes. $\epsilon = 0.05$ is recommended but does not lead to any structural breaks in the analysis of the SP500 from 2016 to 2021. Appendix B gives an example of a smaller window size that leads to some change-points and hence would lead to different conclusions. This makes it hard to make bold conclusions on the discrete estimated change-points. As stated earlier, the choice of $\epsilon = 0.05$ guards best for deviation of estimated change-point dates from actual change-point dates and hence seems the appropriate choice.

Additionally, Zhao et al. (2021) state that the SNCP method is not suitable for time series with frequent change-points. Although there seems to be no visual evidence of such scenarios, this rests as an exciting topic for further research. One may investigate whether the observation periods may be a frequent-change-point scenario and choose and compare with other estimations methods that account for this (e.g. Fryzlewicz, 2020).

Since $\{Y_t\}_{t=1}^{n}$ should be piecewise stationary, this may also be researched more thoroughly for the data set. The visualization of the data gives no indication of the data violating the piecewise stationarity condition.

Finally, the GARCH(1,1) models show small absolute differences in forecast performance for predicting the variance of the log returns of the SP500. The low daily log returns can partly explain this. Also, the choice of a naive benchmark might have influenced the results, and one might be interested in looking at realized variances and more excessive research on forecasting performance. We use 20% of the data as an out-of-sample period. This somewhat arbitrary choice can also be varied to get more in-depth results. Nonetheless, our findings give careful indication that incorporating a structural break for variance may increase out-of-sample forecast models of the variance and hence volatility.

7 Conclusion

This thesis investigates the presence of structural changes in and between the U.S. and Chinese stock markets during the COVID-19 pandemic. This is done by investigating a data set of daily log returns for the SP500 and SSE from January 2016 to December 2021. We also look only at the data covering the pandemic itself (December 2019 to December 2021) to get additional insights. We use a newly proposed change-point estimation method by Zhao et al. (2021), mainly because of its versatility and allowance for temporal dependence in the data. We examine structural changes in correlation between the two markets. We estimate change-points for variance, 90% and 95% quantiles and their multi-parameter combinations for both individual markets.

Our main findings include a simultaneous structural break in volatility for both the U.S. and Chinese markets around April 1st 2020. This date is in line with previous literature findings. For the Chinese market, a second structural break is estimated for July 30th 2020, possibly due to the second COVID-19 outbreak in China (Wu et al., 2020). Moreover, we find no evidence of a structural break in the correlation between the markets. This result does not match our hypotheses and literature, as the correlation between markets tends to increase during financial crises. However, this might indicate that COVID-19 has similar long-term effects on both markets and does not substantially differ their intercorrelation. Additionally, we show that incorporating structural breaks in forecasting models might result in better forecast accuracy. However, we cannot make any bold conclusions as we perform very basic analysis on this matter.

Since the COVID-19 pandemic is still ongoing, our findings need to be taken into consideration with care. Further future research on structural changes should be conducted after full world-wide recovery from the virus. Secondly, in order to get more insights on the micro-level, one could have closer looks at smaller window sizes to spot differences between the two major economies, that of the U.S. and China. Lastly, further research on the importance of the estimated change-points could be conducted. Possibly by showing the predictive ability of multiple models while including or excluding one or multiple change-points.

8 Acknowledgements

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Appendix A Cleaning Replication Data

As stated in Section 5.1, May 30th 2006 to December 31th 2010 accounts for 1158 data points, not in line with the required 1024 points. We compare a plot as R output of the negative log returns with figure S.4 of Zhao et al. (2021) and manually remove data points that are missing in their figure. Figure 3 shows the manual comparisons. The most important takeaway is the red dots, exactly 134 points that are excluded to get n = 1024 observations. After the manual cleaning, still no pattern seems present in which data points are used and which are not. The data selection appears to be nonrandom as there is sometimes a cluster of data points missing. The nonrandom selection invites some possible criticism on influence on the obtained results, as some extreme data points are left out (e.g. Figure 3 observation 600).

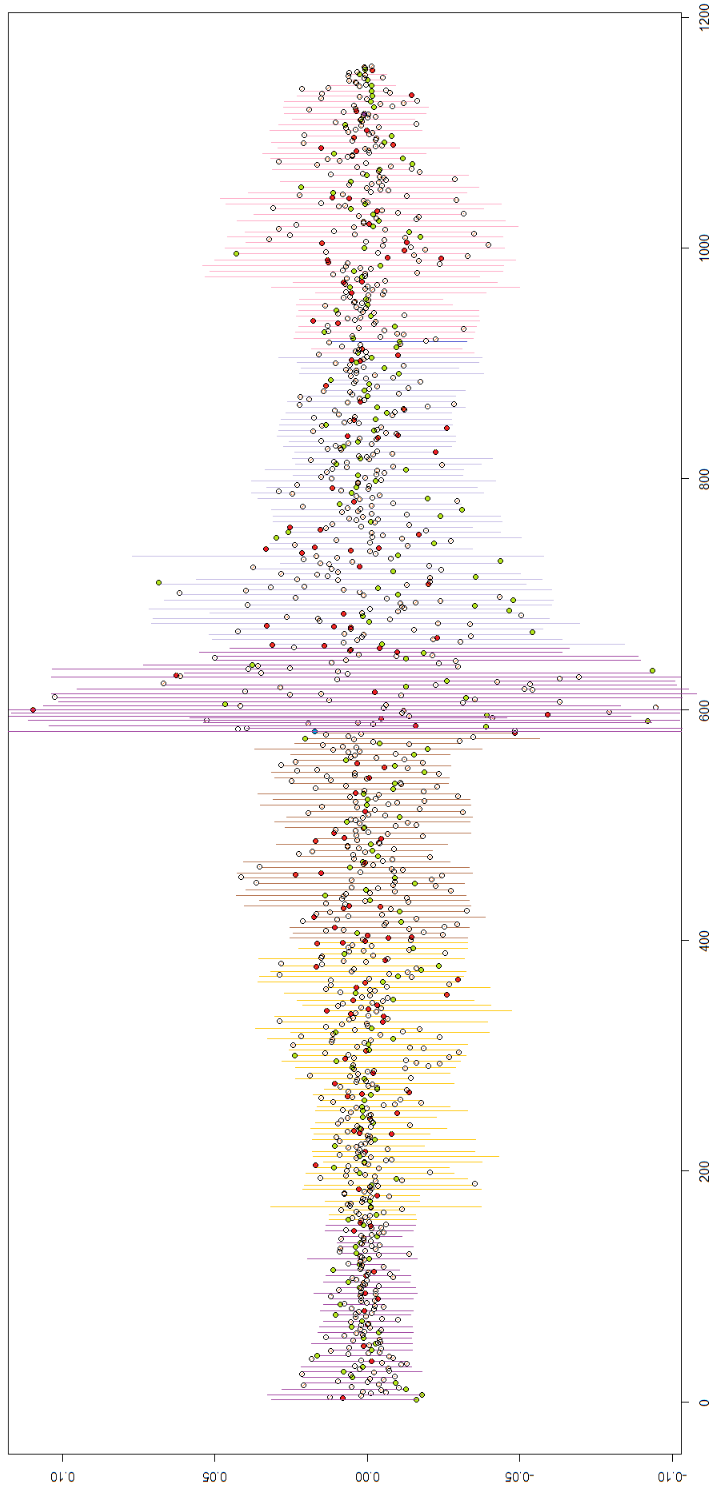

Figure 3: Visualization of the data cleaning process. The red dots are the excluded data points to obtain a total of n = 1024 observations.

Appendix B Alternate Window Size Results

Table 6 shows estimated change-point comparable to Table 3, but with window size scale $\epsilon = 0.04$. Note that K_n is unknown for this window size scale and hence we cannot draw any conclusions from this Table. We set thresholds K_n as the 90% thresholds for $\epsilon = 0.05$. Although this seems very obscure at first sight, the SNCP method shows robustness for different threshold values and hence Table 3 still is able to give some insights on different change-points estimates when adjusting the window size scale.

Table 6: Estimated change-points by SNCP estimators for log returns of the S&P 500 index and the Shanghai Stock Exchange Composite index from January 2016 to December 2021. Results show for window size scale $\epsilon = 0.04$

	Method	CP1	CP2	CP3	CP4
SP500 & SSE	SNC			07/03/2018	
SP500	SNV		01/17/2018	09/26/2018	
	SNQ_{90}			01/10/2018	02/10/2020
	SNQ_{95}	07/22/2016	01/25/2018		
	$SNQ_{90,95}$		01/16/2018	10/17/2019	04/08/2020
	$SNQ_{90}V$		01/16/2018	09/26/2018	
	$SNQ_{95}V$		01/17/2018	09/26/2018	
	$SNQ_{90,95}V$		01/16/2018	09/26/2018	
SSE	SNV		01/16/2018		
	SNQ_{90}		06/27/2018		
	SNQ_{95}			10/17/2019	04/08/2020
	$SNQ_{90,95}$		02/14/2018		02/03/2020
	$SNQ_{90}V$		01/16/2018		02/04/2020
	$SNQ_{95}V$	06/13/2016	01/16/2018	05/20/2019	
	$SNQ_{90,95}V$	06/16/2016	01/17/2018	05/20/2019	

Notes: No specified threshold as reliable critical values of the limiting distribution are unknown.

Appendix C Out-of-sample Forecasts

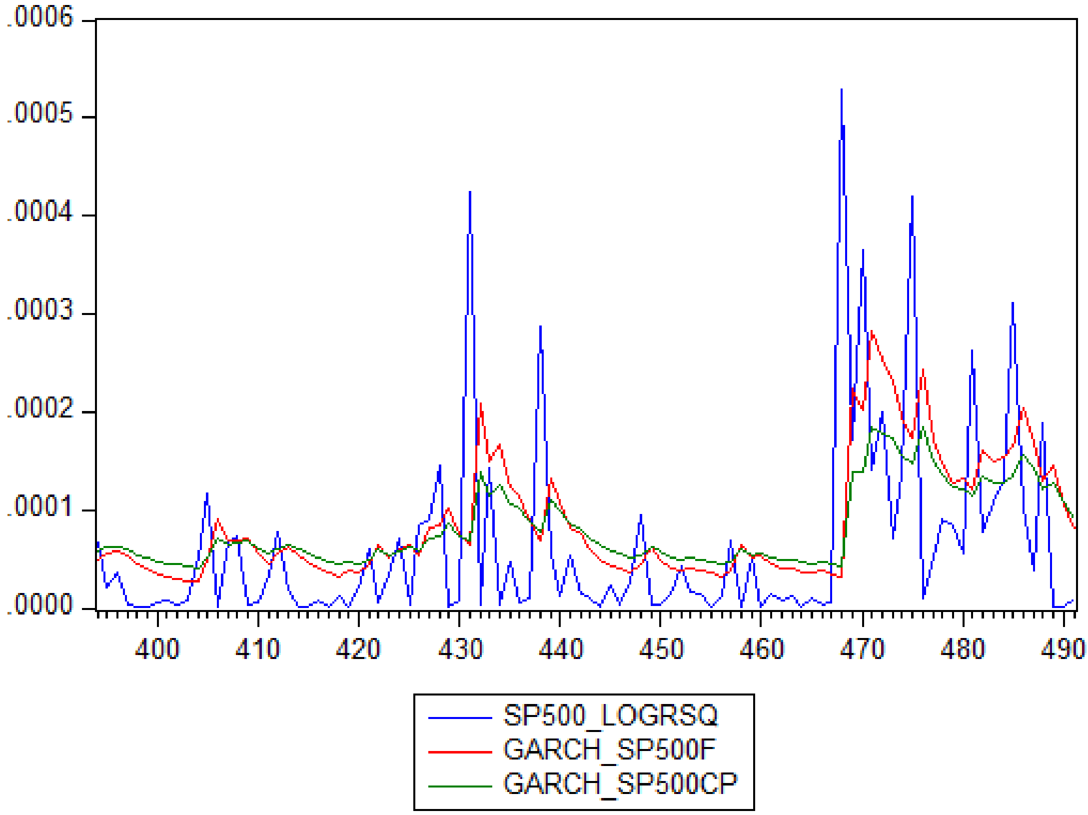

Figure 4: out-of-sample forecasts of the variance of daily log returns of the S&P 500 index from August 3 2021 to December 31 2021. The benchmark variances are squared daily log returns. We compare two GARCH(1,1) models estimated using the sample from December 2 2019 to August 3 2021. GARCH_{SP500F} uses the full in-sample data and $\text{GARCH}_{SP500CP}$ uses in-sample data after April 1st 2020 where a change-point is estimated. The y-axis shows the variance forecasts and the x-axis the observation numbers.

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