Detecting Breakpoints in Stock Market Volatility During the Financial Crisis of 2008

ERASMUS UNIVERSITY ROTTERDAM

Erasmus School of Economics

Guus Capel; 501390,

Supervisor: M.F.O Welz,

Second assessor: A. Tetereva

July 3 2022

Abstract

The stock market endures great periods of stress during a financial crisis. With that, the market becomes more volatile and the economy suffers from uncertainty and inefficiencies. In this paper we focus on stock market indexes (S&P500, S&P ASX, STOXX600 and AEX) to analyse breakpoints in the financial crisis of 2008. We implement a Self-Normalization approach using the (negative) log daily returns in the period June 2006 until December 2010. We find four common break dates for all markets, which occur in June 2007, September and December 2008, and around May 2009.

Keywords: Financial Crisis; Stock Market Index; Breakpoints; Segmentation; Self-Normalization

Contents

1 Introduction

The stock market contributes to a large part of the financial market, and with that, the stock market and the economy are tied together. Friend (1972) investigates the impact of the stock market on the economy, and he finds that there is an effect on the efficiency of the economy. The stock market affects the economy in multiple ways, an important one being the fact that the stock market functions as a mechanism to efficiently allocate investments (Fama, 1970). Antonios (2010) explains in an empirical analysis the causal relationship between the stock market development and economic growth for Germany. In his paper the long-run relationship between the development of the stock market and economic growth is found to be significant. Specifically, they state that a unidirectional causality with direction from the stock market to economic growth was found.

A financial crisis affects the functioning and the performance of different markets, including the stock markets. During a crisis, stock markets go through a period of high volatility due to large uncertainty and less trust from market participants (Engelhardt et al., 2021). As an example, the beginning of the Great Recession is believed to be in 2007 when the American housing market crashed. The next big hit occurred in 2008 when the investment bank Lehman Brothers did not receive a bailout from the United States government and went bankrupt. From that point onward uncertainty about the direction of the financial market increased. As explained before, this uncertainty creates a more volatile period. This volatility in the market again has an impact on the economy. Hasan (2017) observed that the volatility in the capital market significantly affects economic growth, especially in developing financial sectors. Similar results are found by Samuel et al. (2019).

Since the stock market and its volatility are integrated into the economy, it is of interest to detect changes in this volatility. These changes in volatility are reflected in economic and financial parameters such as return and variance, and therefore are meaningful to investigate. For analysis using past data it would be useful to know about these changes in parameters for model-building purposes. This is because when a structural break occurs in the data, then the model estimation should incorporate this change by allowing different parameter estimations before and after the break (Franses et al., 2014). Boot & Pick (2020) also researched this by evaluating the Mean Squared Prediction Error of models when accounting for structural changes. They found that many structural breaks do not have a significant effect on model accuracy. This shows that structural breaks should be identified and handled with care.

Because of this, it is of interest how certain parameters change during a crisis. Moreover, we will investigate whether there are structural changes in the volatility of the stock market during the financial crisis of 2008 (Great Recession). By doing so we obtain much information about when models should be adjusted due to changes in parameters. When these adjustments are correctly made models based on this data will perform significantly better.

This brings us to the main research question we will answer in this paper:
"Are there structural changes in stock market volatility during the financial crisis of 2008?"
For this purpose we analyse the volatility of the S&P500, S&P ASX, STOXX600 and AEX in the period June 2006 until December 2010. Here we measure the volatility with the variance, and the Value-at-Risk of the time series. We implement the non-parametric Self-Normalization algorithm first introduced by Shao (2010). This method does not rely on hefty assumptions of an underlying distribution of the data. Furthermore, it already has an implemented application on financial data (Zhao et al., 2021). To first validate the methods we replicate the financial application of Zhao et al. (2021). This forms the basis of our research, as they have implemented this algorithm for various parameter estimators.

Using this algorithm we find that there are four distinguished breaks in the specified time frame. These breaks occur in June 2007, September 2008, December 2008 and lastly between May and June 2009. We see that these breaks occur for all the analysed stock market indexes, however the breaks are detected on different dates.

With these results we get a better understanding of the behavior of stock markets during a crisis, and help recognize typical behavior in the next crisis. Moreover, these results can be used to validate earlier research done on patterns in the economy during stressful periods.

This paper proceeds in the following manner: Section 2 covers the theoretical background. Then Section 3 states what data is used. Subsequently, Section 4 explains the methods used. Section 5 discusses the main findings and lastly, Section 6 concludes the research.

2 Theory

In order to answer the main research question we investigate multiple sub-questions, which will be stated and motivated in the following section. The first sub-question is:
"*When are there structural changes in the volatility of the S&P500 during the 2008 financial crisis?*"

The Great Recession started after the American housing market crashed due to the large number of derivatives swilling around which were worth a lot less than the bankers had previously imagined. At this time it was uncertain how much exposure banks had and of what magnitude the losses were. Because of this, trust evaporated quickly and banks stopped cooperating with each other. After this there was a period of uncertainty and an increase in volatility in the economy. Schwert (2011) investigates this phenomenon and state that this increase in volatility was short lived in comparison to the Great Depression. We therefore state the hypothesis as: *There are multiple structural changes during the financial crisis at various points in time.* Lopes & Polson (2010) states that volatility increased at the beginning of the crisis for the S&P500 and in other market indexes. Schwert (2011) shows that stock market volatility has changed over time and that high volatility during the 2008 financial crisis was short-lived when compared to the Great Depression. This suggests the need to investigate the volatility of the Great Recession and the possible changes in the volatility.

To detect any change in volatility one needs a specific measurement to track these possible changes. The most obvious measurement is the variance, which is widely used in the assessment of volatility by for example Mei et al. (2018), Caporale et al. (2006) and Daly (2008). Besides the variance, another measurement used is the Value-at-Risk (shortly VaR) which is a statistic that quantifies the supremum of possible financial loss with a certain probability. Alternatively, instead of calculating this probability that corresponds to the financial loss, one can compute the losses that correspond to a certain threshold probability. Here one can notice that the VaR can then be written as the corresponding quantile of the financial loss. Koenker & Bassett Jr (1978) proposes a robust framework to use quantile regressions to estimate the VaR, which was later also incorporated in the study of Buchinsky (1998). Tan et al. (2019) has also performed similar research in the usage of quantiles to estimate the VaR and found it to be adequate in evaluating volatility. Therefore, the volatility of the S&P500 index can be estimated by using these two volatility measurements.

Our second sub-question will be a natural extension and will look at different markets that are affected by the same financial crisis. We define the second sub-question as:
"*Are there similarities in the number and location of breakpoints in the volatility across American, European and Australian stock market indexes?*"

Even though the financial crisis began in the United States it quickly escalated to a worldwide recession. After American banks went bankrupt, western governments were forced to fund their banks in order to keep them from collapsing. This governmental bailout kept banks from bankruptcy but was too late to save the global economy. Here it is known that European markets are closely linked to the American markets, especially compared to the Asian market. The link of the Australian market to the American and Asian market makes it more interesting to analyse than Asian countries. We hypothesize: *We can find similar, if not latent, effects in the European and Australian stock markets.* To answer the question we analyse comparable index funds in Europe and Australia. Due to the globalization of many stock markets there is an increase in the linkage between markets that originate from different countries (Slimane et al., 2013). Doman & Doman (2013) showed in their research that the dependencies of these markets change over time.

However, with their research they could not conclude with certainty that there is an increase in the dependency in times of a crisis. Another aspect of the globalization of stock markets is the correlation between international markets, which was also pointed out by Slimane et al. (2013). Longin & Solnik (2001) investigated this correlation between international equity markets, specifically whether there was an increase in correlation during volatile periods. In their paper they state that the correlation is mainly affected by the market trend and therefore it is not of interest to further investigate the correlation in this paper. Using this we will investigate whether or not different markets exhibit similar changes in volatility during a crisis.

3 Data

In order to investigate the financial crisis using the segmentation methods proposed before we will use the opening prices of stock market indexes. These time series are obtained from *investing.com (SP500)* (2022), *WSJ (STOXX600)* (2022), *investing.com (SP/ASX200)* (2022) and *investing.com (AEX)* (2022).

3.1 Description

The first data set is used to perform a replication study of the financial application of Zhao et al. (2021). They use the S&P500 to investigate the behavior of the financial crisis. Zhao et al. (2021) mentioned that they perform the analysis using daily return from June 2006 until December 2010. However, they did not precisely state the starting date, ending date and how they calculated the daily return. Therefore we attempt to precisely replicate their research. We will use the daily simple (negative) log returns, calculated from the opening prices of each day. Since we look at openings prices there are only observations during the trading week, resulting in five observations per week. We use the data from June 1^{st} 2006 until December 31^{st} 2010, resulting in 1145 observations. In this data set some observations are missing, which are most likely holidays. Since it is never more than one day we discard that day from the analysis and calculate the 2-day simple return which then is used as one observation.

After the replication study we extend the research to multiple different data sets, namely "STOXX Europe 600" for Europe, "The SP/ASX 200" for Australia and "AEX" for the Netherlands. Using these indexes we can cover different parts of the world economy and compare the segmentation results between the different markets. To help in this comparison we use the same set-up as before, calculating the daily simple (negative) log returns from the opening prices from consecutive observations. Again because of some missing observations we have a different number of observations but the data is obtained for the same time span June 1^{st} 2006 until December 31^{st} 2010. For S&P/ASX200, STOXX600 and AEX we have respectively 1159, 1153 and 1174 observations.

3.2 Characteristics

In Table 1 we provide the key characteristics of the (negative) daily log returns for the four indexes. Here we see that for the time period there was an overall negative mean, which shows that on average the returns were negative. We note that the mean of the S&P500 is the lowest at $3.5 \cdot 10^{-5}$ compared to the highest for the AEX at $1.9 \cdot 10^{-4}$ Furthermore, we observe a fairly high variance (and standard deviation) indicating that the crisis was indeed a volatile period. The different indexes show relatively equal variances. Here the AEX has the highest variance at $2.87 \cdot 10^{-4}$ and the S&P ASX the lowest with $1.97 \cdot 10^{-4}$

Table 1: Key characteristics for S&P500, S&P ASX, STOXX 600 and AEX for June 2006 to December 2010

	S&P 500	**S&P ASX**	**STOXX 600**	**AEX**
Mean	$3.50 \cdot 10^{-5}$	$1.11 \cdot 10^{-4}$	$1.25 \cdot 10^{-4}$	$1.869 \cdot 10^{-4}$
Var	$2.67 \cdot 10^{-4}$	$1.97 \cdot 10^{-4}$	$2.31 \cdot 10^{-4}$	$2.87 \cdot 10^{-4}$
St. Dev	$1.63 \cdot 10^{-2}$	$1.40 \cdot 10^{-2}$	$1.52 \cdot 10^{-2}$	$1.69 \cdot 10^{-2}$
Min	-0.11	-0.06	-0.09	-0.10
Max	0.09	0.08	0.08	0.10
n	1145	1159	1153	1174

Figure 1-4 presents a graphical illustration of the data.

Daily (negative) log returns for the indexes in the period Jun 2006 - Dec 2010

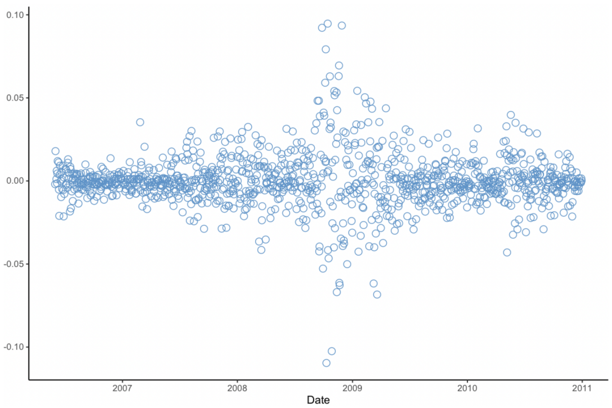

Figure 1: S&P500

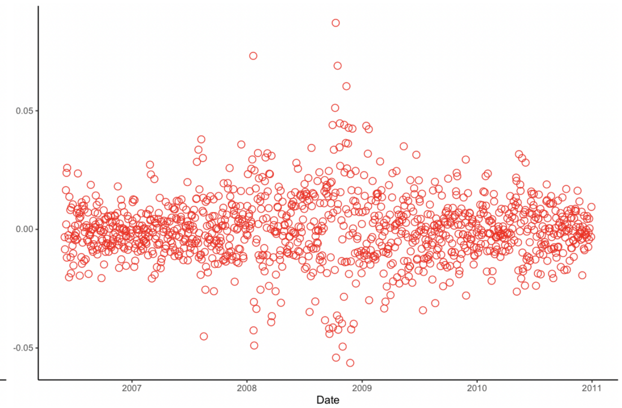

Figure 2: S&P ASX

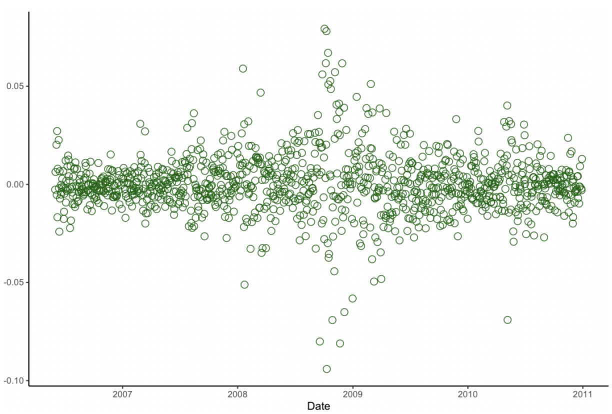

Figure 3: STOXX600

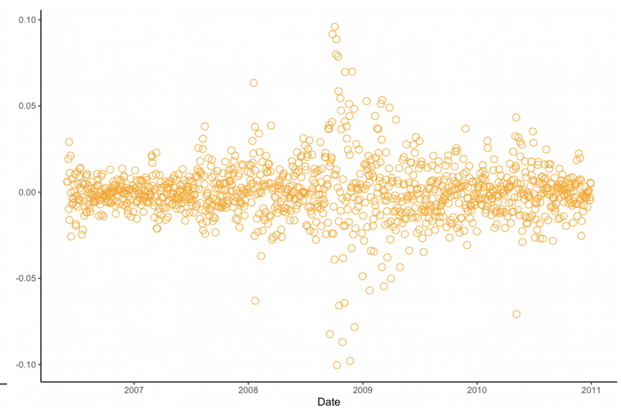

Figure 4: AEX

For the S&P500 in Figure 1 we can see that for most periods the observations are centered around the mean. A clear deviation from this is between 2008 and 2009, where observations lay further away from -0.10 to 0.10 in the fall and winter of 2008. Furthermore, at the end of 2007 it can be noticed that observations begin to spread out indicating a possible change in variance.

Next, one can see that the S&P ASX shows similar patterns, especially at the end of 2008. Generally, observations are less centered in comparison to the S&P500. We also observe that from the fall of 2007 until 2009 there are relatively more days with negative returns. Overall, we see less obvious possible breaks, but recognize similarities to the other indexes.

Then, we look at Figure 3 for the STOXX600 index. Here it can be noticed that the returns of the STOXX600 and the S&P500 are very alike, showing the same deviations around the end of 2008. It can be seen that after 2009 the observations are less clustered and spread out more evenly.

Lastly, the AEX again is centered strongly around the mean at the beginning of the sample and spreads out at the end. In between these periods we notice that the daily returns deviate from the general pattern. Especially we observe a break in 2008 with returns hitting both the minimum and the maximum within a short time frame.

Combining the graphs, the overall pattern shows that in the beginning the returns are relatively stable and close together. Halfway through 2008, the returns start to stretch out, which happened more rapidly for the S&P500 and more steadily for the S&P ASX and the AEX.

4 Methodology

To analyse the financial data of the various index funds we implement a segmentation algorithm first proposed by Shao (2010). This theoretical paper investigates a Self-Normalization (SN) method for segmenting time series, and therefore can be used for detecting structural changes. This method distinguishes itself because it works in a non-parametric way. This means that is does not rely on assumptions about the underlying distribution of the data. Furthermore, the SN method as proposed works by using test statistics fully based on the differences in the segmentation of the data meaning that little assumptions for parameters are needed. Zhao et al. (2021) tested and implemented the SN algorithm in various applications and for multiple parameters. They show how versatile the SN method is, and that it can be implemented for the detection of breakpoints for single or multiple parameters.

4.1 Vector based breakpoint detection

In this section we explain the vector-based detection method, which can be used for single parameter estimation by using dimension equal to one. By allowing a vector-based functional we can implement this method to detect changes in higher dimensional time series or in multiple different parameters.

As explained in Section 2 we intend to detect changes in the volatility of the daily (negative) log returns. Also explained is that we will measure the volatility of a given index fund by analysing the variance and the Value-At-Risk (VaR) of a time series. Here the 90% and 95% quantile are used to represent the VaR. Also stated by Zhao et al. (2021), since the central tendency of the daily log-returns of stock indexes is stable over time we can use high quantiles to measure the volatility of a distribution.

Since we only implement the algorithm for single or vector based parameters of and not for higher dimensional time series there are seven parameters of interest: SNV, SNQ_{90}, SNQ_{95}, $SNQ_{90,95}$, $SNQ_{90}V$, $SNQ_{95}V$, $SNQ_{90,95}V$.

4.1.1 The Self-Normalization method

As is standard for this literature, we assume that the time series $\{Y_t\}_{t=1}^{n}$ is piecewise stationary where n is the number of observations. This means that if the time series has a number of unknown breakpoints equal to m_0, we can partition the time series in $m_0 + 1$ segments (here $m_0 \geq 0$) . We can define, without loss of generality, the unknown breakpoints as $0 < k_0 < k_1 < ... < k_{m_0} < n$. If we define $k_0 = 0$ and $k_{m_0+1} = n$ then the i'th segment contains stationary observations $\{Y_t\}_{t=k_{i-1}+1}^{k_i}$ that share the same parameter of interest.

These parameters are represented by $\boldsymbol{\theta_i} = F_i(\boldsymbol{\theta})$ for $i = 1, ..., m_0 + 1$. Here F_i is the underlying distribution of segment i which may be different for each segment. In the following formulas we use $\hat{\boldsymbol{\theta}}_{\boldsymbol{i}}$ as the estimate of the parameter $\boldsymbol{\theta}_i$

Next we will define the test statistic and the self-normalizer. For this the computation are done for certain segments containing breakpoint k starting at t_1 and ends at t_2.

$$T_n(t_1,k,t_2) = D_n(t_1,k,t_2)^\top V_n(t_1,k,t_2)^{-1} D_n(t_1,k,t_2) \tag{1}$$

where $D_n(t_1,k,t_2) = \frac{(k-t_1+1)(t_2-k)}{(t_2-t_1+1)^{1.5}}(\hat{\boldsymbol{\theta}}_{t_1,k} - \hat{\boldsymbol{\theta}}_{k+1,t_2})$, $V_n(t_1,k,t_2) = L_n(t_1,k,t_2) + R_n(t_1,k,t_2)$, and

$$L_n(t_1,k,t_2) = \sum_{i=t_1}^{k} \frac{(i-t_1+1)^2(k-i)^2}{(t_2-t_1+1)^2(k-t_1+1)^2}(\hat{\boldsymbol{\theta}}_{t_1,i} - \hat{\boldsymbol{\theta}}_{i+1,k})^\top$$

$$R_n(t_1,k,t_2) = \sum_{i=k+1}^{t_2} \frac{(t_2-1+1)^2(k-i)^2}{(t_2-t_1+1)^2(t_2-k)^2}(\hat{\boldsymbol{\theta}}_{i,t_2} - \hat{\boldsymbol{\theta}}_{k+1,i-1})^\top$$

Here the test statistic test for a breakpoint at time $k(t_1 < k < t_2)$, such the parameter $\hat{\boldsymbol{\theta}}_{\boldsymbol{t_1,t_2}}$ experiences a structural change on the interval $(t_1;t_2)$. Furthermore, $D_n(t_1,k,t_2)$ is used such that the detected breakpoints are only identified because of the difference in the specific parameter. The self-normaliser, $V_n(t_1,k,t_2)$, is an inconsistent long term estimate of the variance but may suffer from inflation if $L_n(1,k,n)$ and $R_n(1,k,n)$ are inflated. This may happen when there are other breakpoints between t_1 and t_2, which results in a severe power loss of the test statistic T_n. To overcome this Zhao et al. (2021) proposes to incorporate a local nested window algorithm. By doing so, we compute a maximal SN test based on multiple nested local windows around breakpoint k and use this instead of the global test statistic. Specifically we define the nested window as:

$$H_{1:n}(k) = \{(t_1,t_2)|t_1 = k - j_1 h + 1, j_1 = 1, ...\lfloor\frac{k}{h}\rfloor; t_2 = k + j_2 h, j_2 = 1, ...\lfloor\frac{n-k}{h}\rfloor\} \tag{2}$$

Here n is the sample size and $h = \lfloor n\epsilon \rfloor$ is the window size, where ϵ is a fixed scalar such that $\epsilon \in (0, 0.5)$. We note that for $k < h$ and $k > n \min h$ the window is the empty set. Furthermore, for $k = 1, ..., n$ the maximal test statistic based on the corresponding nested window is defined as:

$$T_{1,n}(k) = \underset{(t_1,t_2)\in H_{1:n}(k)}{max} T_n(t_1,k,t_2) \tag{3}$$

4.1.2 Intuition of the algorithm

Now that the test statistics and the variables are defined and specified we will discuss the SN algorithm used to detect breakpoints. For this we need as input, the full time series $\{Y_t\}_{t=1}^n$, a threshold K_n which is determined a priori, and the window size $h = \lfloor n\epsilon \rfloor$.

With the threshold K_n we can control the Type I error by taking the $1 - \alpha$ quantile as value. Zhao et al. (2021) shows that the distribution of the test statistics of dimension d converges to a certain distribution $G_{d,\epsilon}$. In this paper we use a Type I control level of $\alpha = 0.1$, 0.05 and 0.01 which results in $K_n = 90$, 95 and 99% respectively.

In order to specify the window size we need to predetermine ϵ, this then represents the minimum distance between two consecutive breakpoints. We fix $\epsilon = 0.05$ such that the window size is 5% of the sample size. With this input the algorithm calculates the maximal test statistic $T_{1,n}(k)$ for $k = 1, ..., m$, where we say that the time series has no breakpoints if this test statistic is below the threshold. Otherwise, when $T_{1,n}(k) \leq K_n$ we specify the breakpoint as $\tilde{k} = \underset{k=1,...,N}{argmax} T_{1,N}(k)$ and partition the sample in the two segments $(t_1; \tilde{k})$ and $(\tilde{k} + 1; n)$. Hereafter the algorithm is implemented for the new segments until there are no more breakpoints or when all segments are smaller than the window size.

4.1.3 Assumptions

In order to use the SN method we should validate three assumptions, which are specified in Section 3 of Zhao et al. (2021). In this paper we only touch upon the working of the assumptions but will refer to Zhao et al. (2021) for the full details and the validations of the mathematical proof.

Assumption one is the "Invariance Principle" which is required for each stationary segment. This assumption basically is a functional central limit theorem that is frequently used in the SN framework and can easily be validated using for example weak dependence conditions.

In the second assumption it is stated that the remainder term is required to be asymptotically negligible. Lastly, the third assumption regulates the behavior of the parameter's functional. This is a relatively strong condition but trivially holds when the parameter of interest is the only parameter to change within a specific bound. This is a common assumption within the breakpoint detection literature (Korkas & PryzlewiczV, 2017; Wied et al., 2012).

The consistency of the SN method is established using the proposed assumptions and additional theorems, which are given in Section 3 of Zhao et al. (2021). The difference between the 1-dimensional and multi-dimensional estimation is that for the higher dimensional functionals an additional assumption on the linearity of parameters is needed. We note that the multi-valued method is validated using a different argument[1] which needs a computationally difficult matrix inverse of the self-normalizer.

5 Results

In this section we will discuss the results we obtained by implementing the SN method for the four aforementioned indexes. In order to best implement the SN algorithm we contacted the authors of Zhao et al. (2021) and they provided us with the code used in their research. In order for the code to work properly for our univariate time series we adjusted their code slightly.

As explained in their paper, they use a confidence interval of 90% to represent the threshold value K_n. We have extended this to also include the values for 95 and 99%. Because of this we obtain 3 sets of breakpoints per estimated parameter. Note that we differentiate between these sets by indicating the highest threshold for which the breakpoint is detected using asterisks.

5.1 Replication - S&P 500

As mentioned earlier we will split the results in two, where we begin with the replication of the financial data case of Zhao et al. (2021). Here we use the S&P 500 data to investigate possible breakpoints in the volatility of the index. The found breakpoints are represented in Table 2 below. For the sake of comparison we have provided the break dates found by Zhao et al. (2021) below the dates we find.

The first thing we note is that all SN methods have found two breaks in the first half of the data. The first being in June 2007, this is considered the start of the crisis. Most estimators found this break at the end of the month. However, the break in variance was

[1]Stated in Theorem 3.2 of Zhao et al. (2021)

Table 2: Estimated break dates for the S&P500 from June 2006 to December 2010 - compared to Zhao et al. (2021)

Method	*CP1*	*CP2*	*CP3*	*CP4*
SNV	2 Aug 2007** (17/07/2007)	18 Sept 2008*** (16/09/2008)	10 Dec 2008*** (05/12/2008)	27 May 2009* (27/05/2009)
SNQ_{90}	15 Jun 2007*** (12/06/2007)	4 Sept 2008*** (04/08/2008)	-	- (18/05/2009)
SNQ_{95}	18 Jun 2007*** (09/07/2007)	11 Sept 2008*** (17/09/2008)	-	- (30/04/2009)
$SNQ_{90,95}$	15 Jun 2007*** (09/07/2007)	4 Sept 2008*** (17/09/2008)	-	- (18/05/2009)
$SNVQ_{90}$	21 Jun 2007*** (17/07/2007)	18 Sept 2008*** (16/09/2008)	11 Dec 2008*** (05/12/2008)	2 Jun 2009*** (27/05/2009)
$SNVQ_{95}$	22 Jun 2007** (09/07/2007)	12 Sept 2008*** (16/09/2008)	11 Dec 2008** (08/12/2008)	28 May 2009** (21/04/2009)
$SNVQ_{90,95}$	18 Jun 2007*** (09/07/2007)	12 Sept 2008*** (16/09/2008)	8 Dec 2008*** (05/12/2008)	- (20/04/2009)

$* \rightarrow 90\%, ** \rightarrow 95\%, *** \rightarrow 99\%$

found in the month August. As indicated by the asterisks, most estimators found the breaks for the threshold of 99%, except the SNV and $SNVQ_{95}$ which found it for the 95% threshold.

A second break that was found by all estimators is in September 2008. On 15 September 2008 the Lehman Brothers bank filed for bankruptcy resulting in the acceleration of the crisis. It is known that after this point market behavior was unpredictable and volatile. That the algorithm detected this as a breakpoint is therefore expected and shows that the SN methods are adequate in detecting changes in volatility. Furthermore, we observe that four estimators found another break in 2008. SNV, $SNVQ_{90}$, $SNVQ_{95}$ and $SNVQ_{90,95}$ found a break in the first week of December 2008, with a threshold value of 99% or 95%. Lastly, the variance and variance/quantile multiple-parameter estimator found the end of the financial crisis in May 2009. The specific date differs across the different estimators.

If we compare our results to that of Zhao et al. (2021), we see that they are relatively similar. Note that we do not find a break in 2009 for the quantile estimators whereas Zhao et al. (2021) finds this breakpoint for all estimators. For the breaks that we do find the specific dates may differ because Zhao et al. (2021) does not clearly specify how to calculate the log returns. Overall enough similarities are found that we can correctly implement these methods for the other indexes.

5.2 Extension - Global stock market indexes

We will now proceed with the extension part of this research where we investigate three more index funds. The break dates we found for these indexes are presented in Tables 3, 4 and 5 and will therefore also be discussed separately.

5.2.1 S&P ASX

In this section we discuss the breaks we find for the S&P ASX during the financial crisis between 2006 and 2010. The results are presented in Table 3 where the highest threshold value for a certain break is indicated by the asterisk.

Table 3: Estimated break dates for the S&P ASX from June 2006 to December 2010

Method	*CP1*	*CP2*	*CP3*	*CP4*
SNV	-	11 Sept 2008**	2 Dec 2008**	-
SNQ_{90}	-	-	-	26 Jan 2009***
SNQ_{95}	25 Jul 2007***	-	-	30 Jan 2009*
$SNQ_{90,95}$	26 Jul 2007***	-	12 Dec 2008**	-
$SNVQ_{90}$	-	-	-	26 Jan 2009***
$SNVQ_{95}$	31 Jul 2007***	5 Sept 2008**	8 Dec 2008**	-
$SNVQ_{90,95}$	31 Jul 2007***	-	8 Dec 2008***	-

$* \rightarrow 90\%, ** \rightarrow 95\%, *** \rightarrow 99\%$

The first thing we observe is that the methods detect various different breakpoints. The algorithm detects two breaks for the variance in 2008, namely in September and December. Furthermore, we see that a breakpoint in July 2007 is found by SNQ_{95},$SNQ_{90,95}$,$SNVQ_{95}$ and $SNVQ_{90,95}$ at a threshold value of 99%. Again this is considered to be the beginning of the financial crisis and affected also the Australian markets.

The acceleration part in 2008 is, besides the variance, also detected for the quantile and the multi-parameter estimates of $SNQ_{90,95}$, $SNVQ_{95}$ and $SNVQ_{90,95}$. Noticeable is that this break is less detected than for the S&P500, which may be because of the fact that the crisis originated in the United States. The fourth break is found in January 2009, and is detected by SNQ_{90}, SNQ_{95} and $SNVQ_{90}$. Here we observe that this is the only breakpoint flagged by $SNVQ_{90}$, whereas this method detected similar breaks as other methods for the S&P500. This gives further evidence in the difference in breaks for geographical different indexes in this crisis.

With these breakpoints the most crucial parts of the crisis are detected. Namely, the beginning in 2007, the acceleration in 2008 and the end in 2009 are all recognized by the algorithms.

5.2.2 STOXX 600

Next, we analyse the STOXX600 to investigate the breaks found in the European market. The found breaks are displayed in a similar form as before in Table 4.

Table 4: Estimated break dates for the STOXX 600 from June 2006 to December 2010

Method	*CP1*	*CP2*	*CP3*	*CP4*
SNV	-	16 Sept 2008***	-	-
SNQ_{90}	4 Jul 2007*	-	-	-
SNQ_{95}	26 Jul 2007***	-	-	-
$SNQ_{90,95}$	23 May 2007**	-	-	-
$SNVQ_{90}$	12 Jul 2007*	17 Sept 2008***	-	-
$SNVQ_{95}$	17 Jan 2008**	16 Sept 2008***	8 Dec 2008***	-
$SNVQ_{90,95}$	-	16 Sept 2008***	-	-

$* \rightarrow 90\%, ** \rightarrow 95\%, *** \rightarrow 99\%$

The first interesting observation we make is that the break dates found by various algorithms are less consistent than for the previous indexes. Furthermore we see that the end of the crisis does not show a significantly high change in volatility to be flagged by any algorithm.

The break in 2007 is found by SNQ_{90}, SNQ_{95}, $SNQ_{90,95}$, and $SNVQ_{90}$ for different days but mostly in July. We also see that $SNVQ_{95}$ detects a different first break, namely in January 2008. Here we note that this method only detects breaks in 2008 in the month January, September and December.

The middle (or acceleration) part of the crisis in September 2008 is detected for SNV, $SNVQ_{90}$, $SNVQ_{95}$ and $SNVQ_{90,95}$. In comparison, we see that this break is detected more often than for the S&P ASX but a lot less than for the S&P500.

The last break that is detected appeared in December 2008, and is found for the 99% threshold by $SNVQ_{95}$. That there is no break found in 2009 may be caused by the fact that the STOXX600 is a large index which slowly grew back to normal levels after the crisis hit. If so, the algorithm does not detect a break as there is no sudden change in volatility.

5.2.3 AEX

Lastly, we investigate the AEX, where the breaks are presented in Table 5.

Table 5: Estimated break dates for the AEX from June 2006 to December 2010

Method	*CP1*	*CP2*	*CP3*	*CP4*
SNV	-	16 Sept 2008***	8 Dec 2008*	-
SNQ_{90}	20 Jul 2007*	-	-	25 Aug 2010**
SNQ_{95}	23 Jul 2007**	12 Sept 2008***	-	24 Apr 2009*
$SNQ_{90,95}$	-	24 Jul 2008***	-	-
$SNVQ_{90}$	-	16 Sept 2008***	-	-
$SNVQ_{95}$	-	16 Sept 2008***	8 Dec 2008**	-
$SNVQ_{90,95}$	-	16 Sept 2008***	-	-

$* \rightarrow 90\%, ** \rightarrow 95\%, *** \rightarrow 99\%$

Firstly, again we see that there are four obvious breaks during the crisis. The first being in July 2007, which is detected by both 90 and 95% quantile estimators at the 90 and 95% threshold value. In comparison to the other indexes, a break in the beginning was less present for the AEX as only two methods detected a change in volatility.

The second break is again in September 2008 and is found by all but one method. The precise date is fairly consistent among estimators and we note also that it only deviates a couple of days between different indexes. This shows just how severe the shock to the economy was when the volatility changed across the globe in such a short time frame.

Lastly, the final breaks are found in December 2008, April 2009 and August 2010. Of these breaks we suspect that only December 2008 and April 2009 are of interest since the last is only detected once across all indexes. These breaks are also only found for lower threshold values whereas the other dates have been detected for 99% with different methods.

Overall we find four common breaks in volatility in stock markets found by the various methods. These breaks are July 2007, September 2008, December 2008 and lastly between January and July 2009. As explained before these periods represent important stages of the financial crisis. These breaks are detected for all four indexes showing the similarities between international indexes. We note that the S&P500 endured great breaks in volatility in the beginning of the crisis as every method detects similar breaks in 2007 and 2008.

Since foreign banks are active participants on the United States markets they too experienced similar effects in the volatility. Especially European banks and investors are active such that the effects that spilled over to their financial systems are relatively significant.

According to The Reserve Bank of Australia their banks had little exposure in the housing crash, but the SN algorithm still detected a break in volatility. This in combination with the rest of our results we see that even though the crisis had similar effects on the different indexes, the magnitude may differ.

Of the methods used to detect these breaks we see that there are some inconsistencies between the different methods. Since there is no concrete test to determine which method is better, we will not go in detail to answer this. Seeing that all methods fail to detect breaks that were flagged by other methods we would suggest to always implement multiple estimators in analysing time series. To give a better overview of the used methods we provide tables of the results per method in the appendix.

6 Conclusion

In this paper we answer the main research question: *"Are there structural changes in stock market volatility during to the financial crisis of 2008?"* For this, we analyse four major stock market indexes across the globe. Namely, the S&P500, S&P ASX, STOXX600 and the AEX. Using the daily opening prices of an index we calculated the (negative) daily log returns to obtain four time series.

We measured the volatility by means of the variance, the 90% and the 95% quantile. For the breakpoint detection we implemented a non-parametric algorithm introduced by Shao (2010). Following the implementation of Zhao et al. (2021) we analysed seven vector-valued estimators for each time series.

To provide a clear and decisive answer we formulated two sub-questions, which together would form the answer for the main research question. This first sub-question is as follows: *"When are there structural changes in the volatility of the SP500 during the 2008 financial crisis? "*

With the previously mentioned methods we found that there are four breaks in the volatility between June 2006 and December 2010. These break occur in June 2007, September 2008, December 2008 and May 2009. From all estimators, the one for the variance, and the vector of variance and 90% or 95% quantile found all breaks. The other methods found two or three of the breaks, however June 2007 and September 2008 was found by all methods.

The second sub-question is: *"Are there similarities in the number and location of breakpoints in the volatility across American, European and Australian stock market indexes?"*

We see that this question builds upon the first sub-question and allows for a similar approach as before. We used the same methods as for the S&P500 to detect breaks in the time series for the other indexes. What we found by doing so is that overall the same break dates are found for the indexes. For the S&P ASX we found the breaks in July 2007, September 2008, December 2008 and January 2009. However, no single estimator found all breaks, and none of the breaks were found by all methods. The same holds for the AEX, where we even find a break in 2010. For the STOXX600 we do not find the same breaks as for the other indexes. The breaks that are found by most methods are in July 2007 and September 2008, which is in line with other findings. Nevertheless, the estimator for the vector of variance and 95% quantile finds breaks in January 2008 and December 2008. Lastly, we observe that no break in 2009 or 2010 is found.

So to provide a concrete answer, we can conclude that there are four breakpoints in the volatility during the Great Recession. These breaks are in the summer of 2007 and 2008, the winter of 2008 and the latest in the summer of 2009. As these breaks are found for a variety of indexes that capture the movements of different stock markets we have good evidence for the validity of these findings.

With these findings, we can segment the financial crisis into different parts based on their differences in volatility. Using these segments we can better account for these changes when this time frame is used in model building techniques. As stated before, not accounting for structural changes can highly impact parameter estimations. Furthermore, the breaks we find can be used in a theoretical manner by further investigating the movements of the economy during the different segments. This paper also shows that the used algorithms are very robust and easily implemented for other time series.

As a topic of further research we would suggest the following: one can use these results to investigate what precisely changes in the volatility by calculating these differences per segment. By doing so we have an idea about the magnitude of these changes and we can compare the breaks between time series based on the size of the structural change. Secondly, further research can be done on government influences on the economy and investigate if there is a change in the market as a response to new policies.

References

Antonios, A. (2010). Stock market and economic growth: an empirical analysis for germany. *Business and Economics Journal*, *2010*(1), 12.

Boot, T., & Pick, A. (2020). Does modeling a structural break improve forecast accuracy? *Journal of Econometrics*, *215*(1), 35-59. Retrieved from https://www.sciencedirect.com/science/article/pii/S0304407619301824 doi: https://doi.org/10.1016/j.jeconom.2019.07.007

Buchinsky, M. (1998). Recent advances in quantile regression models: a practical guideline for empirical research. *Journal of human resources*, 88–126.

Caporale, G. M., Pittis, N., & Spagnolo, N. (2006). Volatility transmission and financial crises. *Journal of economics and finance*, *30*(3), 376–390.

Daly, K. (2008). Financial volatility: Issues and measuring techniques. *Physica A: Statistical Mechanics and its Applications*, *387*(11), 2377-2393. Retrieved from https://www.sciencedirect.com/science/article/pii/S0378437108000216 doi: https://doi.org/10.1016/j.physa.2008.01.009

Doman, M., & Doman, R. (2013, 08). Dynamic linkages between stock markets: The effects of crises and globalization. *Portuguese Economic Journal*, *12*. doi: 10.1007/s10258-013-0091-1

Engelhardt, N., Krause, M., Neukirchen, D., & Posch, P. N. (2021). Trust and stock market volatility during the covid-19 crisis. *Finance Research Letters*, *38*, 101873. Retrieved from https://www.sciencedirect.com/science/article/pii/S1544612320316871 doi: https://doi.org/10.1016/j.frl.2020.101873

Fama, E. (1970). Efficient capital markets: A review of theory and empirical work. *Journal of Finance*, *25*, 383–417.

Franses, P. H., Dijk, D. v., & Opschoor, A. (2014). *Time series models for business and economic forecasting* (2nd ed.). Cambridge University Press. doi: 10.1017/CBO9781139049894

Friend, I. (1972). The economic consequence of the stock market. *American Economic Review*, *62*(2), 212. Retrieved from https://search-ebscohost-com.eur.idm.oclc.org/login.aspx?direct=true&db=buh&AN=4504252&site=ehost-live&scope=site

Hasan, M. Z. (2017). Transmission of international energy price shocks to australian stock market and its implications for portfolio formation. *Asian Economic and Financial Review*, *7*(4), 393-412. Retrieved from https://www.proquest.com/scholarly-journals/transmission-international-energy-price-shocks/docview/1896346833/se-2?accountid=13598 (Copyright - Copyright Asian Economic and Social Society 2017; Last updated - 2021-11-17; SubjectsTermNotLitGenreText - Australia)

investing.com (aex). (2022). Retrieved from https://www.investing.com/indices/netherlands-25

investing.com (sp500). (2022). Retrieved from https://nl.investing.com/indices/us-spx-500

investing.com (sp/asx200). (2022). Retrieved from https://www.investing.com/indices/aus-200

Koenker, R., & Bassett Jr, G. (1978). Regression quantiles. *Econometrica: journal of the Econometric Society*, 33–50.

Korkas, K. K., & PryzlewiczV, P. (2017). Multiple change-point detection for non-stationary time series using wild binary segmentation. *Statistica Sinica*, 287–311.

Longin, F., & Solnik, B. (2001). Extreme correlation of international equity markets. *The journal of finance*, *56*(2), 649–676.

Lopes, H. F., & Polson, N. G. (2010). Extracting sp500 and nasdaq volatility: The credit crisis of 2007-2008. *Handbook of Applied Bayesian Analysis*, 319–342.

Mei, D., Zeng, Q., Zhang, Y., & Hou, W. (2018). Does us economic policy uncertainty matter for european stock markets volatility? *Physica A: Statistical Mechanics and its Applications*, *512*, 215-221. Retrieved from https://www.sciencedirect.com/science/article/pii/S0378437118309658 doi: https://doi.org/10.1016/j.physa.2018.08.019

Samuel, U. E., Abner, I. P., Inim, V., & Ndubuaku, V. (2019). Monthly stock market volatility on economic growth in nigeria. *International Journal of Mechanical Engineering and Technology*, *10*(10), 131–144.

Schwert, G. W. (2011). Stock volatility during the recent financial crisis. *European Financial Management*, *17*(5), 789–805.

Shao, X. (2010, June). A self-normalized approach to confidence interval construction in time series. *Journal of the Royal Statistical Society. Series B: Statistical Methodology*, *72*(3), 343–366.

Slimane, F. B., Mehanaoui, M., & Kazi, I. A. (2013). How does the financial crisis affect volatility behavior and transmission among european stock markets? *International Journal of Financial Studies*, *1*(3), 81-101. Retrieved from https://www.proquest.com/scholarly-journals/how-does-financial-crisis-affect-volatility/docview/1537914378/se-2 (Copyright - Copyright MDPI AG 2013; Document feature - ; Tables; Graphs; Equations; Last updated - 2021-09-10; SubjectsTermNotLitGenreText - Europe)

Tan, S.-K., Ng, K.-H., Chan, J. S.-K., & Mohamed, I. (2019). Quantile range-based volatility measure for modelling and forecasting volatility using high frequency data. *The North American Journal of Economics and Finance*, *47*, 537-551. Retrieved from https://www.sciencedirect.com/science/article/pii/S1062940818300548 doi: https://doi.org/10.1016/j.najef.2018.06.010

Wied, D., Krämer, W., & Dehling, H. (2012). Testing for a change in correlation at an unknown point in time using an extended functional delta method. *Econometric Theory*, *28*(3), 570–589.

Wsj (stoxx600). (2022). Retrieved from https://www.wsj.com/market-data/quotes/index/XX/SXXP/historical-prices

Zhao, Z., Jiang, F., & Shao, X. (2021). Segmenting time series via self-normalization. *arXiv preprint arXiv:2112.05331*.

A Additional tables

Table 6: Estimated break dates by SNV from June 2006 to December 2010

SNV	*CP1*	*CP2*	*CP3*	*CP4*
S&P 500	2 Aug 2007**	18 Sept 2008***	10 Dec 2008***	27 May 2009*
S&P ASX	-	11 Sept 2008**	2 Dec 2008**	-
STOXX 600	-	16 Sept 2008***	-	-
AEX	-	16 Sept 2008***	8 Dec 2008*	-

$* \rightarrow 90\%, ** \rightarrow 95\%, *** \rightarrow 99\%$

Table 7: Estimated break dates by SNQ_{90} and SNQ_{95} from June 2006 to December 2010

SNQ_{90}	*CP1*	*CP2*	*CP3*	*CP4*
S&P 500	15 Jun 2007***	4 Sept 2008***	-	-
S&P ASX	-	-	-	26 Jan 2009***
STOXX 600	4 Jul 2007*	-	-	-
AEX	20 Jul 2007*	-	-	25 Aug 2010**
SNQ_{95}	*CP1*	*CP2*	*CP3*	
S&P500	18 Jun 2007***	11 Sept 2008***	-	-
S&P ASX	25 Jul 2007***	-	-	30 Jan 2009*
STOXX 600	26 Jul 2007***	-	-	-
AEX	23 Jul 2007**	12 Sept 2008***	-	24 Apr 2009*

$* \rightarrow 90\%, ** \rightarrow 95\%, *** \rightarrow 99\%$

Table 8: Estimated break dates by $SNVQ_{90}$, $SNVQ_{95}$ and $SNVQ_{90,95}$ from June 2006 to December 2010

$SNQ_{90,95}$	*CP1*	*CP2*	*CP3*	*CP4*
S&P500	15 Jun 2007***	4 Sept 2008***	-	-
S&P ASX	26 Jul 2007***	-	12 Dec 2008**	-
STOXX 600	23 May 2007**	-	-	-
AEX	-	24 Jul 2008***	-	-
$SNVQ_{90}$	*CP1*	*CP2*	*CP3*	*CP4*
S&P 500	21 Jun 2007***	18 Sept 2008***	11 Dec 2008***	2 Jun 2009***
S&P ASX	-	-	-	26 Jan 2009***
STOXX 600	12 Jul 2007*	17 Sept 2008***	-	-
AEX	-	16 Sept 2008***	-	-
$SNVQ_{95}$	*CP1*	*CP2*	*CP3*	*CP4*
S&P500	22 Jun 2007**	12 Sept 2008***	11 Dec 2008**	28 May 2009**
S&P ASX	31 Jul 2007***	5 Sept 2008**	8 Dec 2008**	-
STOXX 600	17 Jan 2008**	16 Sept 2008***	8 Dec 2008***	-
AEX	-	16 Sept 2008***	8 Dec 2008**	-
$SNVQ_{90,95}$	*CP1*	*CP2*	*CP3*	*CP4*
S&P500	18 Jun 2007***	12 Sept 2008***	8 Dec 2008	-
S&P ASX	31 Jul 2007***	-	8 Dec 2008***	-
STOXX 600	-	16 Sept 2008***	-	-
AEX	-	16 Sept 2008***	-	-

$* \rightarrow 90\%, ** \rightarrow 95\%, *** \rightarrow 99\%$