ERASMUS UNIVERSITY ROTTERDAM

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ESTIMATING STRUCTURAL CHANGES IN WEATHER USING SELF-NORMALIZATION

Bachelor Thesis Econometrics and Operations Research

Major in Econometrics

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Abstract

Researchers heavily discuss climate change and its consequences among which changes in the weather. Often a time span is mentioned over which the weather changed, "this decade" or "past century". This paper aims to establish significant evidence and exact dates of changes in the weather. In doing so, we propose a method to estimate change points in climatic variables and support future research in this field. We use data on temperature, precipitation, and humidity in the United Kingdom and data on wind speed of tropical cyclones between 1772 and 2021 to find signs of change in the weather. Using the non-parametric SNCP procedure and parametric Quandt-Andrews test we find structural changes among all variables mostly between 1985 and 2000. Most notably is an 1.13°C increase of UK mean temperature in 1989.

Keywords: Climate change; Change point estimation; weather; Self-Normalization; Quandt-Andrews test

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1 Introduction

According to the Intergovernmental Panel on Climate Change (IPCC), earth's climate is seriously changing (IPCC, 2014). It was only two decades ago that the article of Weart (2003) alerted about rapid climate change. At first sight changes in weather were seen as temporary incidents. However, literature shows that rising concentrations of greenhouse gasses in the atmosphere result in more frequent heatwaves, more rainfall, and more intense natural disasters (Houghton, 2005). This will result in a severe loss of different species in flora and fauna (Levinsky et al., 2007; Thuiller et al., 2005). Moreover, climate change does not only effect the biodiversity, but also worldwide economies in the coming century. Tol (2009) reviews that welfare loss can become a few percent per income, but an even larger loss is estimated for less developed economies. Summarizing, the consequences of climate change are very serious. This paper aims to establish significant evidence of structural changes in the climate and exactly pinpoint when these changes happened. In doing so we verify the findings of more extreme weather as mentioned by Houghton (2005). This way we contribute to raising awareness for the possible changes in climate. Furthermore, we propose a method to find structural changes climatic variables and encourage further research in this field. In this research we define weather according to Moore et al. (2010): "the state of the atmosphere at a specific place on a specific time". There are different atmospheric variables that describe that state.

In our research we introduce the commonly used atmospheric variables temperature, wind speed, precipitation and humidity to capture the weather (Hollinger & Angel, 2009; Kusriyanto & Putra, 2018; Fogno Fotso et al., 2022). By assessing a possible change in the mean of these variables over a long period, we may find evidence for a changing climate. Climate is namely referred to as a statistical representation of the weather averaged over a long period (Moore et al., 2010). Thus, if we estimate change points in weather using a representative number of observations, we establish evidence for climate change. Therefore we pose the central question is this research as:

"When did structural changes occur in the weather over the past century?".

To formulate an answer to this question we will use weather data of the atmospheric variables mentioned above. We use data from different sources for this purpose. Data from the temperature, precipitation and humidity is measured in the United Kingdom (UK) over a period from 1772 to 2021. Changes in wind speed are examined using observations of the maximum wind speed of tropical cyclones between 1981 and 2006. We use the recently proposed non-parametric Self-Normalization Change Point (SNCP) algorithm to estimate change points in mean and variance

(Zhao et al., 2021). When applicable, we will model the atmospheric variables using an autoregressive (AR) model. This allows us to verify the SNCP change points using the well-esthablished Quandt-Andrews test (Andrews, 1993). Using both the non-parametric Self-Normalization procedure and the parametric Quandt-Andrews test, we find change points in each atmospheric variable. Most of these happened in the period 1985-2000. Moreover, we found an 1.13°C increase in England temperature after 1989, an increase in wind speed, and change of total precipitation variability in 1993. By empirical application of the Self-Normalization procedure and support of the Quandt-Andrews test we show that these methods are capable of estimating changes in climatic data. Moreover, our findings support future simulations of climate change by precise estimation of change points in weather. By estimating changes in the weather this study expresses that the UK climate is changing and contributes to raising awareness for this topic.

This study proceeds in the following manner: in Section 2 we discuss the theoretical background and introduces the sub-questions. Section 3 and 4 concern data and describe the methods we apply respectively. In Section 5 we show our model decisions and findings. Lastly, Section 6 and 7 conclude the study.

2 Literature

In 1896 the Swedisch Svante Arrhenius first showed that changes in atmospheric carbon-dioxide levels could result in higher surface temperatures[1]. Recent evidence of climate change includes retracting glaciers and the degradation of polar ice (Thompson, 2010). Other literature shows changes in temperature or extreme weather (IPCC, 2014). Studies also reasoned that some effects may be attributed to natural climatic variability (Francis & Hengeveld, 1998). It is common in this literature that a time span over which change in weather took place, like “past decade” or “past century”. We try to find more precise dates of changes in long term averages of the weather. An earlier study showed that climate change is often perceived as something distant. Knowledge of the exact points in time may change the perceived proximity of climate change for the public (Brügger et al., 2015).

It is of interest for researchers to discover new procedures for estimating change points in climatic data. Pinpointing when these changes in the weather exactly occurred supports climatologist in their research to determine the effects and causes of changes in atmospheric variables. Reliable

[1] https://climate.nasa.gov/evidence/, accessed June 22 2022.

procedures for change point estimation support accurate simulations of climate change scenarios. Cubasch et al. (1995) show that the starting time of simulations can be crucial for its outcomes, as a "cold start" in simulations can underestimate global warming.

2.1 Offline change point estimation

Offline change point estimation, also known as time series segmentation, is a procedure used to split potential ordered non-homogeneous time series into homogeneous segments. That is, separating an ordered time series such a way that within a segment the data shows similar patterns (Zhao et al., 2021). In this study change point estimation is a powerful tool that allows us to track down potential changes in the weather.

Estimation of change points of the weather is previously done by Fealy & Sweeney (2005). The authors detected change in atmospheric variability in the North Atlantic during 1988-1990. They state: "The present analysis has identified 1988 – 1990 as a period that warrants further investigation because of its potential climatic significance.". Jaiswal et al. (2015) indicated change points in annual series of temperature, wind speed and sunshine hours in India over the period 1990-2000. There are also various studies concerned with trend analysis on climatic variables. Often, the non-parametric Mann-Kendall test is used to test for a significant positive or negative trend. Gocic & Trajkovic (2013) use this method to investigate meteorological variables in Serbia from 1980 - 2010; we will elaborate on their results next Section. Alyousifi et al. (2021) show that a significant trend is present in Malaysia's air pollution index. The downside of the Mann-Kendall test is that it is only able to accept or reject the null-hypothesis of a trend present in the data. In this fashion it is not possible to detect change in a specific parameter of interest, for example mean or variance. Zhao et al. (2021) also perform a climate studies regarding change points. They use their newly proposed Self-Normalization Change Point (SNCP) procedure on climate data to exemplify the effect of climate change. Specifically, they analyse mean annual temperature of central England and maximum wind speeds of tropical cyclones. Using SNCP they discovered mean shifts of the annual temperature in England namely, 1919 and 1988. Moreover, they find one change point in maximum windspeed of cyclones estimated in 1988. An advantage of the SNCP procedure is that it is capable of estimating multiple change points in a variety of parameters of interest, consider mean, variance, (auto-)correlation, and quantiles. Furthermore, it is also able to identify changes in multivariate time series.

2.2 Changes in weather

Zhao et al. (2021) already showed a brief application of the SNCP technique to climate data, but this study tries to extend the analysis to multiple atmospheric variables and parameters of interest. As mentioned in Section 1 we will use SNCP on temperature, wind speed, precipitation and humidity series. By investigating multiple series we further show the applicability of this method. We also estimate change points in the variables across multiple parameters of interest. This motivates the four guiding sub-questions for the central question in this research.

First, we are interested in differences in the level data of the atmospheric variables. We pose the following sub-question: *1. "Are there changes in mean of atmospheric variables in the past century?"*. To be able to flag changes we will use the Self-Normalization technique of Zhao et al. (2021). This method seems appropriate since it has previously been used by Zhao et al. (2021) on climatic data, and shows good theoretical properties. We will answer this question by performing SNCP on the separate time series of the variables. We hypothesize that a change point is found in the temperature, wind speed, and humidity series. This hypothesis is based on the similar research of Jaiswal et al. (2015) who show change points in these variables in India. Based on the work of Gocic & Trajkovic (2013), who found no significant trends in Serbia precipitation series, we hypothesize that the precipitation series will not show any change points.

Secondly, we want to use the Self-Normalization technique to explore the possibilities of change in uncertainty of these weather related variables, as measured by the variance. We formulate the second sub-question as: *2. "Are there changes in variance of atmospheric variables in the past century?"*. For this question we will again use SNCP, but now use the variance as parameter of interest. Earlier work of Stott (2016) reasoned that the weather will become more extreme. Periods of drought and extreme rainfall are mentioned. We hypothesize that there are changes in variance present in all atmospheric variables, and follow the literature that the weather has become increasingly more uncertain. Furthermore, we want to check whether simultaneous changes in mean and variance are present in the atmospheric variables. We pose the third sub-question as: *3. "Are there simultaneous changes in mean and variance of atmospheric variables in the past century?"*. We reason from the previous hypotheses that there are simultaneous change points are present in the variables. Lastly, we want to verify the results of the SNCP procedure. We will do so in a parametric fashion by using the Quandt-Andrews test (Andrews, 1993). This test is introduced for the estimation of single unknown break points in parameter estimates. When applicable, we will model the time series using an autoregressive (AR) model, and find changes in parameter estimates. Changing parameters in an AR model indicates that the relational structure between previous observations changed. This thus confirms a change in relation and therefore explains a structural change. We can then check whether there are matching change points with the SNCP procedure to verify the break dates. We form the fourth question as: *4. "Are the flagged change points of SNCP coinciding with the Quandt-Andrews change points?"*. Due to the promising simulation results of SNCP and the wide-applicability of the Quandt-Andrews test, we hypothesize that when an autoregressive model is in place the change points will coincide.

3 Data

This section discusses the data we use for the analysis of changes in the weather. The state of the atmosphere, weather, is measured by different variables, we will refer to them as atmospheric variables. We first specify the atmospheric variables we use and thereafter provide characteristics of the data.

3.1 Atmospheric variables

Commonly used variables that describe the weather are: temperature, wind speed, precipitation and humidity (Hollinger & Angel, 2009; Kusriyanto & Putra, 2018; Fogno Fotso et al., 2022). In this study we will mainly use observations of the variables in the UK. This is because of the availability of early weather measurements in the UK. We also use data of tropical cyclones to capture wind speed, in this fashion we capture changes in extreme weather. We use different time spans for each variable due to the availability of some measurements. All observations fall within the interval from 1772 to 2021. Next, we will discuss all variables in detail and how we prepared the data for analysis.

Temperature As measure for the temperature we use the Hadley Centre Central England Temperature (HadCET) dataset[2], described by Parker et al. (1992). The dataset is globally the longest instrumental record of temperature. It contains the mean, maximum, and minimum value of the temperature in degree Celsius. The temperature measures a roughly triangualar surface of the UK spanned by London, Lancashire, and Bristol, on a daily basis. The dataset is available on a

[2]https://www.metoffice.gov.uk/hadobs/hadcet/

daily, monthly, or seasonal frequency. Since we partially redo the study of Zhao et al. (2021), we use the daily mean temperature measures starting in 1772. This includes the daily mean temperature from 1772 to date, but we only use the data until 2019. The available data is corrected for urban warming: since 1974 all mean observations are corrected by -0.2°C. We prepared the data by adding 0.2°C for all observations after 1974, because we want to include the effect of urban warming. Thereafter we aggregated the data to obtain yearly mean values.

Wind speed The wind speed data[3] is obtained from Scheitlin et al. (2010). It contains the maximum lifetime wind speed ($WmaxST$ in dataset) of 2098 tropical cyclones. The data is satellite derived and contains observations from cyclones between 1981 and 2006. To analyse the wind speed we use the observations of the maximum wind speed in kilometer per hour.

Precipitation The data[4] on precipitation is obtained from the Met Office National Climate Information Centre. It contains montlhy, seasonal and annual total precipitation for the UK in millimeters. We make use of the year ordered statistics on rainfall in the UK. The yearly total precipitation ranges from 1836 to 2021.

Humidity As measure for the humidity we use the average annual vapour pressure in the UK in hectopascal (hPa). The yearly observations of the data[5] range from 1901 till 2019. For a detailed derivation of the data we refer to Harris et al. (2020). In the present analysis we only use the annual observations of vapour pressure.

3.2 Characteristics of atmospheric variables

In Table 1 the descriptive statistics of the atmospheric variables are shown. The mean, variance, and autocorrelation columns are based on the total number of observations noted in the last column. Temperature and humidity means are based on monthly averages over a year, whereas the precipitation mean is based on the yearly total rainfall amount. The average wind speed is 77.20 km/h, measured over 2098 tropical cyclones. Note that the standard deviation relative to the mean is highest for the variables wind speed and precipitation. We also denote the first-order autocorrelation in Table 1. This is because we want to model the atmospheric variables using an AR model. This model specification is justifiable when autocorrelation is present in the data. The bold values

[3]https://myweb.fsu.edu/jelsner/temp/Data.html
[4]https://www.metoffice.gov.uk/research/climate/maps-and-data/uk-and-regional-series
[5]https://data.ceda.ac.uk/badc/cru/data/cru˙cy/cru˙cy˙4.04/data

in the autocorrelation column denote a significant presence of autocorrelation at a 95% level. Only the variables temperature and humidity show significant autocorrelation. This is a first sign that an AR model is appropriate for these variables. However, wind speed and precipitation show no significant autocorrelation. The fact that wind speed does not show autocorrelation is probably due to the fact that the observations are independent cyclones. Since the first-order autocorrelation is not significant, an AR specification is not appropriate here. We will assess this further in Section 5.

Table 1: Descriptive statistics of atmospheric variables: mean, standard deviation, first-order autocorrelation, and number of observations. Significant first-order autocorrelations are noted in bold.

	Mean	St.dev.	Autocorr.	Obs.
Temperature ($^{\circ}C$)	9.41	0.69	**0.38**	247
Wind speed (km/h)	77.20	24.14	-0.03	2098
Precipitation (mm)	1072.11	119.35	0.11	186
Humidity (hPa)	9.92	0.29	**0.38**	119

4 Methodology

This Section covers the technical specification of the methods and procedures in this study. We will introduce Self-Normalization, the autoregressive model, and the Quandt-Andrews test. First, we discuss the general problem of estimating change points in time series. Let $\{Y_t\}_{t=1}^{n}$ be a piece-wise stationary time series with n observations and dimension d. Time series segmentation is concerned with the segmentation of this series into stationary segments. Following the notation of Zhao et al. (2021), we assume $m_0 \geq 0$ unknown change points $0 < k_1 < ... < k_{m_0} < n$. In this fashion we split the time series into $m_0 + 1$ segments. The aim is to estimate the unknown m_0 change points and their location.

4.1 Self-Normalization

Self-Normalization was first proposed by Shao (2010). The method is thereafter used to test time series for change points but not for estimation of change points. The difference is that the latter does not depend on an educated guess of the location of a change point. Zhao et al. (2021) explore the use of Self-Normalization for change point estimation. They propose the SNCP procedure, which is able to detect multiple change points in multivariate time series, $d > 1$, across different parameters of interest θ. A convenient advantage of this method is that it is fully non-parametric and thus does not require any distributional assumptions on the time series $\{Y_t\}_{t=1}^{n}$. Therefore this method is suitable to estimate change points for different atmospheric variables. We will only use SNCP on univariate time series, $d = 1$, since the number of observations differs across the variables. As parameter of interest we consider mean, variance, and common mean-variance (multi-parameter). When using the multi-parameter version of this method the parameter of interest, θ, becomes a vector containing the individual parameters of interest. In this section we only describe the Self-Normalization procedure for univariate series, multiple change points and single-parameter estimation. The vectorized procedure for multi-parameter change points is described by Zhao et al. (2021) in Section 3.3.

First we define change point $k_0 = 0$ and $k_{m_0+1} = n$. For the time series $\{Y_t\}_{t=1}^{n}$, we introduce segments $i = 1, ..., m_0 + 1$, where segment i contains observations $\{Y_t\}_{t=k_{i-1}+1}^{k_i}$. We further define $Y_t^{(i)}$ when observation Y_t is in segment i. In the data generation process observation $Y_t^{(i)}$ follows cumulative distribution function (CDF) $F^{(i)}$. Then the method is able to flag changes in any parameter θ_i which is a function of $F^{(i)}$, $\theta_i = \theta(F^{(i)})$. One could consider functionals as mean, variance, quantile or autocorrelation. Next, it is trivial that observations in segment i share the same parameter θ_i, for $i = 1, ..., m_0 + 1$. Note that a change point is identified by $\theta_i \neq \theta_{i+1}$, that is, when segment i and segment $i+1$ do not share the same parameter value. The SN test statistic $T_n(t_1, k, t_2)$ on subsample $\{Y_t\}_{t=t_1}^{t_2}$ is calculated as follows

$$T_n(t_1, k, t_2) = \frac{D_n(t_1, k, t_2)^2}{V_n(t_1, k, t_2)}, \quad \text{for } 1 \leq t_1 < k < t_2 \leq n, \tag{1}$$

where t_1 and t_2 are respectively the begin and end of a window of observations, k is tested as significant change point, $D_n(t_1, k, t_2) = \frac{(k-t_1+1)(t_2-k)}{(t_2-t_1+1)^{1.5}}(\hat{\theta}_{t_1,k} - \hat{\theta}_{k+1,t_2})$ measures the change of $\hat{\theta}_{t_1,k}$ and $\hat{\theta}_{k+1,t_2}$, the non-parametric estimators of θ on $\{Y_t\}_{t=t_1}^{k}$ and $\{Y_t\}_{t=k+1}^{t_2}$ respectively, and $V_n(t_1, k, t_2) = L_n(t_1, k, t_2) + R_n(t_1, k, t_2)$ is the self-normalizer, where

$$L_n(t_1,k,t_2) = \sum_{i=t_1}^{k} \frac{(i-t_1+1)^2(k-i)^2}{(t_2-t_1+1)^2(k-t_1+1)^2}(\hat{\theta}_{t_1,i} - \hat{\theta}_{i+1,k})^2,$$

$$R_n(t_1,k,t_2) = \sum_{i=k+1}^{t_2} \frac{(t_2-1+1)^2(k-i)^2}{(t_2-t_1+1)^2(t_2-k)^2}(\hat{\theta}_{i,t_2} - \hat{\theta}_{k+1,i-1})^2.$$

If we set $t_1 = 1$ and $t_2 = n$ the test statistic $T_n(t_1,k,t_2)$ becomes the global SN test statistic for single change point estimation. Under the multiple change points alternative this global test statistic experiences power loss due to inflation of the self-normalizer $V_n(t_1,k,t_2)$. When the subsample $\{Y_t\}_{t=t_1}^{t_2}$ contains observations of different segments, the parameter of interest is not properly estimated. This results in larger values of $V_n(t_1,k,t_2)$ and thus lower values of the test statistic $T_n(t_1,k,t_2)$.

To overcome this problem Zhao et al. (2021) propose a nested local-window segmentation algorithm. By doing so for each k a maximal SN test is obtained on different nested windows covering k. First, specify an $\epsilon \in (0,1/2)$ and define window size $h = \lfloor n\epsilon \rfloor$. Then for each $k = h,..,n-h$, a nested local window set $H_{1:n}(k)$ is defined as

$$H_{1:N}(k) = \Big\{(t_1,t_2)\Big| t_1 = k - j_1 h + 1, j_1 = 1,...\lfloor k/h \rfloor; t_2 = k + j_2 h, j_2 = 1,...\lfloor n - k/h \rfloor\Big\}.$$

Then for each $k = 1,...,n$, based on its nested window, a maximal SN the test statistic $T_{1,n}(k)$ is defined as

$$T_{1,n}(k) = \max_{(t_1,t_2)\in H_{1:n}(k)} T_n(t_1,k,t_2), \tag{2}$$

which is equal to 0 when $H_{1:n}(k)$ is the empty set. This procedure is designed to overcome the inflation of V_n under multiple change points.

4.1.1 SNCP procedure

The SNCP procedure is then as follows: first specify a threshold K_n that controls the Type-I error, α, of the test. We do so by setting K_n equal to the $(1-\alpha)$ quantile of $G^*_{\epsilon,d}$, the simulated distribution of $\max_{k=1,...,n} T_n(k)$ under the assumption of no change point with known ϵ and d.

Secondly $T_{1,n}(k)$ is calculated based on the full sample. If $\max_{k=1,...,n} T_n(k) \leq K_n$, no change point is declared. Otherwise if $\max_{k=1,...,n} T_n(k) > K_n$, SNCP declares a change point and set $\hat{k} = \arg\max_{k=1,...,n} T_{1,n}(k)$. SNCP is then recursively applied on subsamples $\{Y_t\}_{t=1}^{\hat{k}}$ and $\{Y_t\}_{t=\hat{k}}^{n}$ until no change point is found. In this study we set $\epsilon = 0.05$ and K_n according to the 90% quantile of $G^*_{\epsilon,d}$, to control for a Type-I error of $\alpha = 0.1$. We choose these values as they coincide with the values used in the climate studies of Zhao et al. (2021). For details on the simulation of the quantiles we refer to Zhao et al. (2021).

4.1.2 Assumptions

Assume that segmentation i occurs at proportion of the sample $k_i/n \to \tau_i \in (0,1)$ for $i = 1, ...m_0$ as $n \to \infty$. Then if we define τ_0 and τ_{m_0+1}, and assume $\min_{1\leq i\leq m_0+1}(\tau_i - \tau_{i-1}) = \epsilon_0 > \epsilon$, where ϵ is the SNCP window size. We observe that that ϵ imposes an upper bound to the number of segments m_0 in a sense that $m_0 \leq 1/\epsilon$. In other words ϵ defines the minimum spacing between two change points. Zhao et al. (2021) claim that SNCP is robust for choices of K_n and ϵ as long as $\epsilon_0 > \epsilon$. In practice ϵ is set rather small to overcome this problem but depends on the application. Other assumptions are present for this procedure, among which assumptions to ensure consistency and convergence, but are out of scope for this study. All assumptions are discussed in Zhao et al. (2021).

4.2 Autoregressive model

A common univariate time series model is the autoregressive (AR) model, see for example Franses et al. (2014). It is suitable when the observations of a univariate time series, assume again Y_t for $t = 1, ..., n$, depends on its p most recent lags. By reasoning we can write the conditional distribution of Y_t as $f(Y_t|Y_{t-1}, ..., Y_{t-p})$. If we assume that the dependence on its own lagged values is linear, we can describe Y_t as

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + ... + \phi_p Y_{t-p} + \varepsilon_t, \quad t = p+1, p+2, ..., n, \tag{3}$$

where $\phi_1, ..., \phi_p$ are unknown parameters, p is the number of lags included in the model, and ε_t is a white noise series. The white noise series is unobserved and has to be estimated from the data. A series is considered white noise when

I) $E(\varepsilon_t) = 0 \quad$ for all $t = 1, ..., n$,

II) $E(\varepsilon_t^2) = \sigma^2$ for all $t = 1, ..., n$,

III) $E(\varepsilon_s \varepsilon_t) = 0$ for all $s, t = 1, ..., n$ and $s \neq t$,

where σ^2 denotes the unconditional variance of Y_t. We can summarize the white noise assumptions as: I) zero mean, II) homoskedastic, and III) no serial correlation. I) only holds for model 3 if the data is centered around 0. From the descriptive characteristics of the data in Table 1, we observe that the mean of the variables is not equal to zero. To accompany for this in our model we include an intercept α in the model. The AR model then becomes $Y_t = \alpha + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + ... + \phi_p Y_{t-p} + \varepsilon_t$. The relation between the unconditional mean μ of Y_t and α becomes

$$\mu = \frac{\alpha}{1 - \phi_1 - ... - \phi_p}. \tag{4}$$

To identify an AR model Franses et al. (2014) suggest to use the autocorrelation function (ACF) and partial autocorrelation function (PACF). For an AR model of order p one expects a slowly declining ACF and p significantly different from zero PACF values. In this study we follow this reasoning to model the atmospheric variables using an AR model. We will discuss the ACF and PACF of the atmospheric variables in Section 5 and present final models there. Estimation of an AR model can easily be done by ordinairy least squares (OLS) (Franses et al., 2014). We also use this method for estimating the AR parameters.

For the AR model to be valid we need the white noise properties I-III to hold, and we add an additional assumption of normality for the Quandt-Andrews test to hold. These assumptions are verified in Section 5.

4.3 Quandt-Andrews test

In the Quandt-Andrews (QA) test for unknown break points in parameter estimates is based on the Chow break test (Andrews, 1993; Chow, 1960). This method is used in our research to verify the change points flagged by the SNCP procedure. Use of the Chow and QA test in AR models is described by Enders (2014). Muthuramu et al. (2019) further reviews the use of QA test in time series regression models, which includes the AR model. An advantage of this method is that we are able to plot the individual Chow statistics. This will provide us with a better insight on changing parameters in the model on different intervals.

The Quandt-Andrews test uses the maximum F-statistic of individual Chow break tests on trimmed data. First we explain the notation and use of the Chow break test and thereafter explain the Quandt-Andrews test. We build upon the same notation of previous Section about AR modelling.

4.3.1 Chow break test

As mentioned before the Chow break test is used to test for user specified change points in parameter estimates in linear regression. In the AR setting in Equation 3 this regards parameters $\phi_1, ..., \phi_p$. To perform this test the sample with n observation is split into two sub-samples; one sample of observations before the a priori specified change point consisting of n_1 observations, and one sample after the specified change point consisting of $n_2 = n - n_1$ observations. OLS is then applied to obtain two estimates $\hat{\boldsymbol{\phi}}_1, \hat{\boldsymbol{\phi}}_2$, which denote the vector of length p of estimated coefficients of $\phi_1, ..., \phi_p$ based on samples of length n_1 and n_2 respectively. The null-hypothesis is then $H_0 : \boldsymbol{\phi}_1 = \boldsymbol{\phi}_2$ tested against alternative $\boldsymbol{\phi}_1 \neq \boldsymbol{\phi}_2$. To test this we use the following F-statistic,

$$F = \frac{(S_0 - S_1 - S_2)/w}{(S_1 + S_2)/(n_1 + n_2 - 2w)} \sim F(w, n_1 + n_2 - 2w), \tag{5}$$

where S_0, S_1, S_2 are the sum of squared residuals of the samples based on n, n_1, n_2 observations respectively, and w is the number of explanatory variables estimated using OLS. Using the F-distribution we can calculate the p-value and draw a conclusion based on this test. The theoretical assumptions of this test are described by Heij et al. (2004). We use white noise assumptions I-III in Section 4.2. Moreover, we make use of the normality assumption of the residuals, such that quantiles of the F-distribution are correct. We will verify normality of the residuals by means of the Jarque-Bera test for normality, see Bera & Jarque (1982).

4.3.2 Maximum F-statistic

The QA test is a natural extension of the Chow test. For this test a series of k Chow tests are performed between observations γ_1 and γ_2. The Chow test is then performed between every two observations between (γ_1, γ_2) obtaining k Chow F-statistics. Trimming is done such that the theoretical properties of the Chow test remain valid. We specify a trimming parameter $\pi_0 = \gamma_1/n = \gamma_2/n$. For our application we set this equal to the minimum segment size used in SNCP, namely $\epsilon = \pi_0 = 0.05$. The Chow statistics are summarized into one statistic, the QLR statistic

$$QLR = \max_{\gamma_1 \leq \gamma \leq \gamma_2} (F(\gamma)), \tag{6}$$

where γ is one of every two observations a Chow test is performed in between observations γ_1 and γ_2, and $F(\gamma)$ denotes the individual chow statistic at γ. This QLR statistic is then used to test H_0 : no breakpoints between γ_1 and γ_2, against the alternative that there are breakpoints in between γ_1 and γ_2. The distribution of this test-statistic is not trivial, we will use the approximate asymptotic p-values of Hansen (1997).

5 Results

This section discusses the change points flagged by the SNCP for mean, variance and multi-parameter as well as the AR models and the QA test. The SNCP results are obtained using the software environment *R*, computation of the AR models and QA test are done in *Eviews*. For all SNCP procedures we set window size $\epsilon = 0.05$ and K_n such that the confidence level is equal to 90%. At the end of this Section we compare the change points as well as show a graphical illustration. Note that the first two paragraphs of Section 5.1 replicate the climate studies of Zhao et al. (2021).

5.1 Self-Normalization change in mean

To obtain the results for SNCP with parameter of interest mean, we run the procedure separately for each atmospheric variable. We use the number of observations reported in Table 1.

The SNCP procedure for mean flagged one change point in the mean annual temperature series of central England, namely in 1989. This suggests that the mean temperature on sample 1772-1989 is equal to 9.28°C, and on sample 1990-2018 is equal to 10.41°C. An increase of 1.13°C in mean annual temperature. Surprisingly, this change point does not coincide with the change points found by Zhao et al. (2021) using the same data and method. They found two change points in temperature: 1919 and 1988. We suggest that this difference is due to data adjustments. This step was not accurately covered in their studies. Note however that the change point 1989 we find in this study is close to the 1988 change point in the study of Zhao et al. (2021). Also interesting is that the temperature data is corrected for global warming from 1974 on wards by 0.2°C. After removing this correction we find a different change point, and find a larger change in temperature.

One change point was detected in the mean of maximum wind of tropical cyclones. This change was pinpointed in 1988, meaning that the average maximum wind speed from 1981-1988 equals 74.47 km/h, whereas from 1989-2006 this mean increases to 78.33 km/h. The mean maximum wind speed thus increased about 4 km/h. This change point does coincide with the results on wind speed of Zhao et al. (2021). We believe this is due to the fact that the same number of cyclones is used in both studies and no cleaning was necessary for this data.

As measure for humidity we use the mean annual vapour pressure from 1901-2019. The SNCP procedure detects one change point in mean, namely in 1996. The average vapour pressure form 1901-1996 is then 9.84 hPa, on the latter subsample the mean is equal to 10.22. This means that the water vapour pressure in the air increased in the last century in the UK and thus has become more humid. We also note that when temperature increases vapour pressure also increases.

To answer the first sub-question regarding the change in level data of the atmospheric variables, we note that there are mean shifts in the variables temperature, wind speed and humidity in the past century. There was no significant change in mean present in the precipitation series. This is in line with the findings of Gocic & Trajkovic (2013), who do not find a trend in Serbia's precipitation series. Therefore, the posed hypothesis of change points in all variables except precipitation was right.

5.2 Self-Normalization change in variance

For SNCP change in variance we use the same number of observations and run the procedure separately for each variable.

Using SNCP, we find one change point in variance of total yearly precipitation in the UK. This change point is 1993. We calculate the variance on both subsamples 1836-1993 and 1994-2021, being 12171.86 and 16445.32 respectively. In terms of standard deviation 110.33 and 128.24. In other words the spread of observations of precipitation has increased in the past century.

We also note one change point in variance of vapour pressure in 2008. On the regime before 2008 the variance is equal to 0.077 and after 2008 equal to 0.070. This thus indicates less spread in observations of humidity.

Revisiting our second sub-question, we find change in variance of two atmospheric variables: precipitation and humidity. We hypothesized wrongly that we would find change points in variance across all variables. We found no change point in variance of the mean annual temperature series of central England. Moreover, SNCP also detects no change point in variance of the wind speed of tropical cyclones.

5.3 Self-Normalization change in multi-parameter: mean and variance

Lastly, we use the SNCP procedure to esitmate simultaneous change points in both mean and variance. We thus use multi-parameter SNCP.

We estimated one change point for mean annual central England temperature, being 1988. Interestingly, this deviates one year from the single parameter mean SNCP. Also, there seems to be a common change point for mean and variance but not for variance separately. If we estimate the mean, μ_{temp} and variance, σ^2_{temp} for the sample before 1988 we obtain $(\mu_{temp}, \sigma^2_{temp}) = (9.27, 0.34)$ and after 1988 $(\mu_{temp}, \sigma^2_{temp}) = (10.42, 0.26)$. The change in mean around 1988 was also flagged by SNCP for mean.

Concerning humidity, as measured by the vapour pressure, two common change points in mean and variance were found: 1995 and 2009. The time series is thus split into three regimes 1901-1995, 1996-2009, and 2010-2019 with mean and variance, $(\mu_{humid}, \sigma^2_{humid}) = (9.85, 0.064)$, $(\mu_{humid}, \sigma^2_{humid}) = (10.2, 0.065)$, and $(\mu_{humid}, \sigma^2_{humid}) = (10.17, 0.076)$ respectively. Note that the last two regimes approximately share the same mean. This is also captured by SNCP for mean which indicated a change point in 1996 which is close to 1995 indicated by multi-parameter. Furthermore, we note that the variance is similar for the first and second regime and changed in the third regime starting in 2009. This point is similar to the one found by SNCP for variance in 2008.

For answering our third sub-question we investigate simultaneous changes in mean and variance of the weather in the past century. We hypothesized that we would find simultaneous change in all variables, which is not true. We only find synchronous change in temperature and humidity. For the atmospheric variables regarding the maximum wind speed of tropical cyclones and the yearly total precipitation in the UK no significant change points for common mean and variance were estimated.

5.4 Autoregressive modelling and Quandt-Andrews test

We will now introduce the AR models and perform the QA test to verify the change points estimated by SNCP.

We only find two 95% significant PACF values for the variables temperature and humidity. Therefore, we make use of an AR(2) model for the variables temperature and humidity. An AR model is not appropriate to model the variables wind speed and precipitation, due to no significant values of the PACF. Detailed analysis of the ACF and PACF is given in Appendix A.

5.4.1 Models

We found evidence that an AR(2) model seems appropriate for the variables temperature and humidity. We denote the variables at time t as $temp_t$ and $humid_t$ respectively, and denote the length of the series n_{temp} and n_{humid} respectively. Since both variables do not have zero mean we include intercepts, α and β in the models. The models then become

$$temp_t = \alpha + \phi_1 temp_{t-1} + \phi_2 temp_{t-2} + \varepsilon_t \quad \text{for} \quad t = 3, ..., n_{temp}, \tag{7}$$

$$humid_t = \beta + \lambda_1 humid_{t-1} + \lambda_2 humid_{t-2} + \upsilon_t \quad \text{for} \quad t = 3, ..., n_{humid}, \tag{8}$$

where ϕ_i and λ_i denote the autoregressive parameters for $i = 1, 2$ in Equation 7 and 8 respectively.

The estimated parameters of model 7 are shown in Equation 9, where $\hat{\varepsilon_t}$ denotes the estimated remainder term at time t with mean < 0.001, and standard errors are shown in parentheses. Note that all coefficients are significantly different from zero at a 95% level. We discuss the white noise assumptions I-III and the normality assumption in Appendix B.

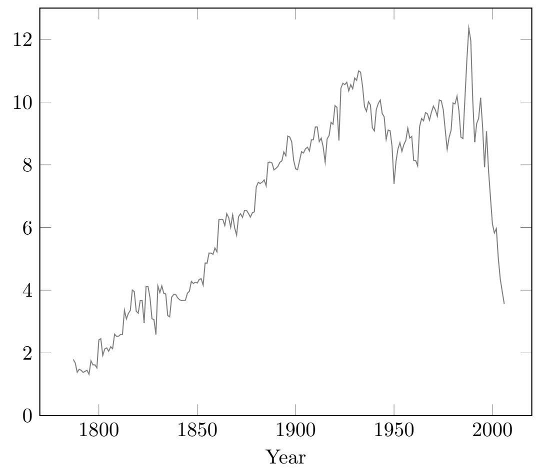
$$temp_t = \underset{(0.65)}{3.83} + \underset{(0.06)}{0.27}\, temp_{t-1} + \underset{(0.06)}{0.33}\, temp_{t-2} + \hat{\varepsilon_t} \quad \text{for} \quad t = 3, ..., n_{temp} \tag{9}$$

We now perform the QA test with trimming parameter $\pi_0 = 0.05$. The values of the individual Chow F-statistics are shown in Figure 1. We observe that the individual Chow statistics gradually increases, then drops down and then attains its maximum value. This indicates that the further in the sample the test is performed, the more evidence of changing parameters is present. The F-statistic spikes in 1988 with a p-value of < 0.001. We thus reject the null-hypothesis of no change points between 1787 and 2006. This provides evidence for the fact that there are change points within the sample.

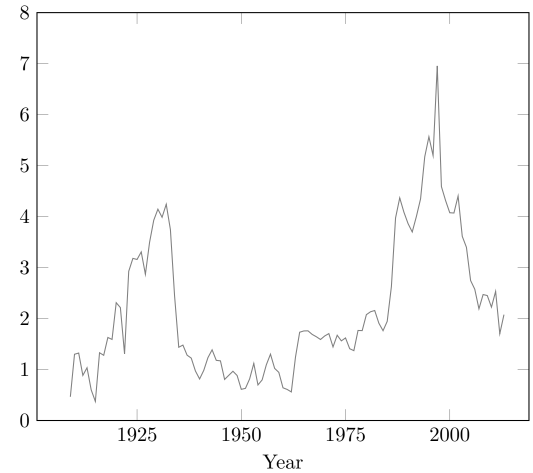
For the atmospheric variable humidity we found the following estimated parameters

$$humid_t = \underset{(1.00)}{4.83} + \underset{(0.09)}{0.28}\, humid_{t-1} + \underset{(0.09)}{0.23}\, humid_{t-2} + \hat{\upsilon}_t \quad \text{for} \quad t = 3, ..., n_{humid}, \tag{10}$$

where $\hat{\upsilon}_t$ is the estimated remainder term at time t with zero mean and standard errors are in parentheses. Again note that all parameters are significant at a 95% confidence level. The validity



(a) Individual Chow F-statistics temperature



(b) Individual Chow F-statistics humidity

Figure 1: Individual F-statistics between every two observation of Quandt-Andrews test for (a) Temperature and (b) Humidity.

of this model is discussed in Appendix C.

Next, we perform the QA test on the humidity data. Figure 1 shows two clear spikes in the F-statistics, with a low period in between. The null-hypothesis of no change points in the data between 1909 and 2013 is again rejected. The maximum F-statistic is attained in 1997 with a p-value of 0.01.

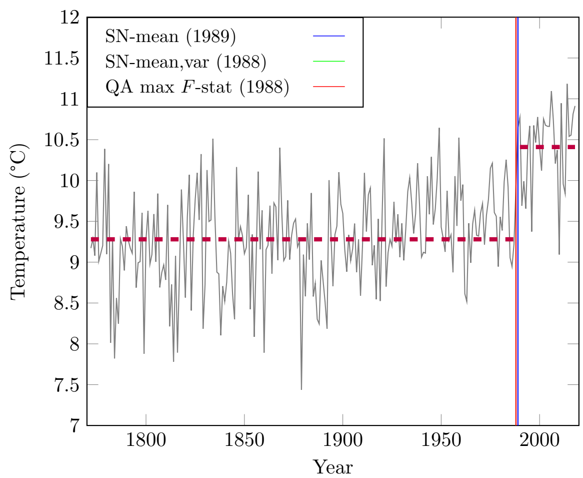
5.4.2 Verification of Self-Normalization change points using Quandt-Andrews

Lastly, we compare the change points found using Self-Normalization and the Quandt-Andrews test. Since we are only able to perform the QA test when a valid AR model is in place, comparison is only possible for the variables temperature and humidity. The QA test only tests the null-hypothesis of no change point within the trimmed data. This hypothesis is rejected for both temperature and humidity. The conclusions of SNCP and QA thus coincide, namely that change points are present in variables temperature and humidity. An advantage of the QA test is that we are able to pinpoint when in the sample the most significant change in parameters occurred based on the maximum F-statistic. Interestingly, this maximum statistic for temperature occurs at 1988, which closely coincides with the mean change point flagged by SNCP in 1989, and exactly coincides with the variance change point obtained from SNCP. Moreover, the maximum F-statistic of humdity appears in 1997, this date closely matches with the mean change point of 1996 estimated by SNCP.

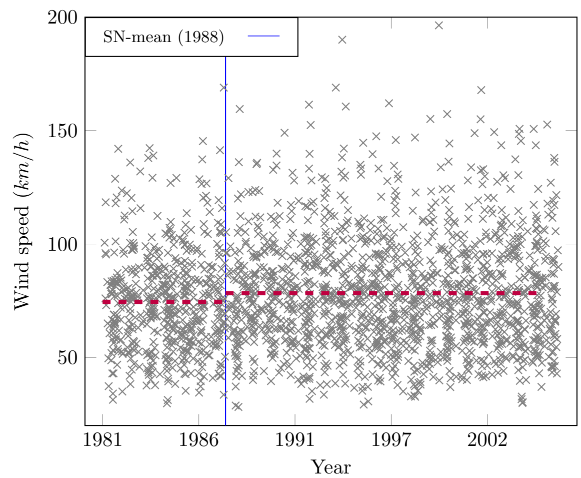
Our hypothesis regarding the fourth sub-question whether the change points flagged by SNCP coincide with those of the QA test is thus true. When a valid model is in place, the date at which the maximum F-statistic occurred closely matches the SNCP change points in mean.

5.5 Change points in atmospheric variables

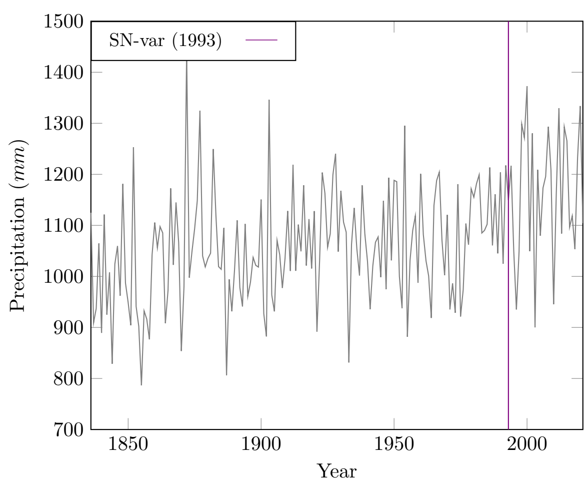
A graphical representation of all change points found in the atmospheric variables is shown in Figure 2. Interestingly, we observe that all change points happened between 1988 and 2009, which is in the latter stage of the previous century. Furthermore, we note that change points closely coincide when both SNCP and the QA test is performed on the variable.



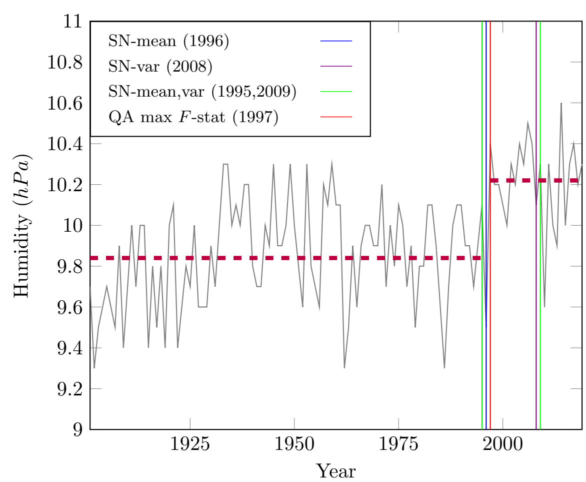
(a) Central England temperature



(b) Tropical cyclone maximum wind speed



(c) Yearly total UK precipitation



(d) Yearly average UK humidity

Figure 2: Change points estimated using Self-Normalization (SN) and Quandt-Andrews (QA) test for atmospheric variables: (a) Temperature, (b) Wind speed, (c) Precipitation, and (d) Humidity. Purple dashed horizontal lines show segment averages estimated by SN-mean.

6 Conclusion

In this study we aim to find significant evidence of changes in the weather. The central question in this research is:

"When did structural changes occur in the weather over the past century?".

To answer this question we introduce variables that describe the state of the atmosphere at a given place at a given time. We use measurements over the period 1772 until 2021 of temperature, precipitation, and humidity in the UK and maximum wind speeds of tropical cyclones to describe that state. To establish evidence of change in the weather we examine the mean and variance separate and simultaneous using the SNCP procedure, a newly proposed time series segmentation algorithm proposed by Zhao et al. (2021). To verify the SNCP change points that we utilize the well-established Quandt-Andrews test.

At a 90% confidence level, we found changes in all weather related variables across various parameters. The estimated change points mainly occurred at the end of past century. Moreover, all change points were estimated between 1988 and 2009. We found an interesting change in mean UK temperature increasing from 9.28°C before 1989 to 10.41°C thereafter. We also found an increase in the mean wind speed of tropical cyclones and increase in variance of total UK precipitation in 1993. An autoregressive model was used to model temperature and humidity, and then perform the Quandt-Andrews test. The change points found using this method closely coincide with the change points flagged by SNCP in these variables.

From the obtained results we conclude that significant change in weather has happened in the past century. We are able to attribute this change to specific change points in time across different variables and parameters. We also found a good agreement between the SNCP procedure and the Quandt-Andrews test. Indeed, a long-term average of the weather, climate, is changing as recorded by IPCC (2014).

Current findings suggest that the climate in the UK is changing over the studied period. We mention the proximity of climatic change in the previous century and contribute to raising awareness on this topic. Moreover, we propose how to pinpoint exact change dates of atmospheric variables. This will support scientist in future climate simulations by helping accurately model changes in weather. We also show the use of Self-Normalization and autoregressive modelling in climate science. This can be used in future research to investigate change in different atmospheric variables or weather changes in different countries.

Discussion and further research During the research on changes in weather we found some improvements we want to share. Firstly, due to the parametric nature of the Quandt-Andrews test, we were not able to use this method on the variables wind speed and precipitation. For further research on the verification of SNCP we consider different non-parametric methods to estimate change points. In this way, no distributional assumptions are required in the estimation. One could consider the binary segmentation algortihm proposed by Bai & Perron (2003). Another advantage of this method over the Quandt-Andrews test is that it is able to find multiple change points.

Secondly, the optimal number of lags for the autoregressive model was chosen based on the number of significant lags observed in the partial autocorrelation function. Liew (2004) shows that the Akaike information criterion (AIC), a measure for model fit, performs well to select the optimal lag length in an AR model. It would be of interest to determine the optimal lag length using the AIC.

Lastly, the models were verified using visual inspection on the residuals. In the future we could also make use of for example the Breusch Pagan test, or the Breusch-Godfrey test, to test for heteroskedasticity and serial correlation respectively (Breusch & Pagan, 1979; Godfrey, 1978). This would enable us to significantly test the model assumptions.

7 Limitations

One limitation of this study is that it only covers the variables temperature, precipitation, and humidity in the UK. To establish evidence of global climate change, we would need to investigate changes in the weather in different countries, and possibly gather data of more atmospheric variables over a longer time span. Wind speed is included in the data to cover the replication of the climate studies in Zhao et al. (2021). We can therefore draw no conclusions about the wind speed in the UK. Secondly, we want to mention the quality of the data. It was hard to find the overall accuracy and quality of the data. We believe that this is due to some historic data, of which is it hard to track the initial source and the lack of accurate measuring instruments in the past.

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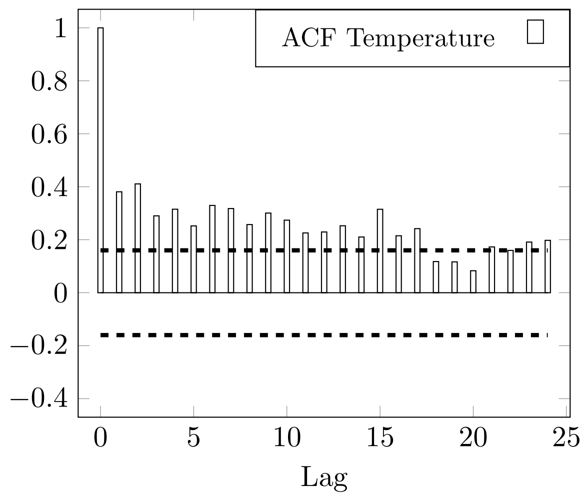
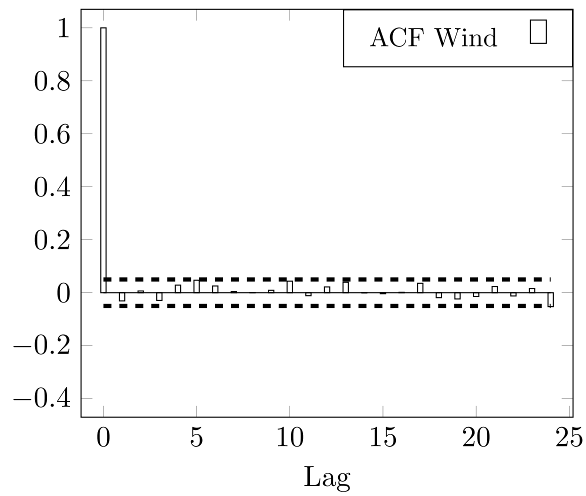
A Results: (Partial) Autocorrelation function

In addition to Section 5, we show the ACF and PACF of the atmospheric variables. We first plot the ACF and PACF for each atmospheric variable to investigate whether an AR model is justified for these variables.

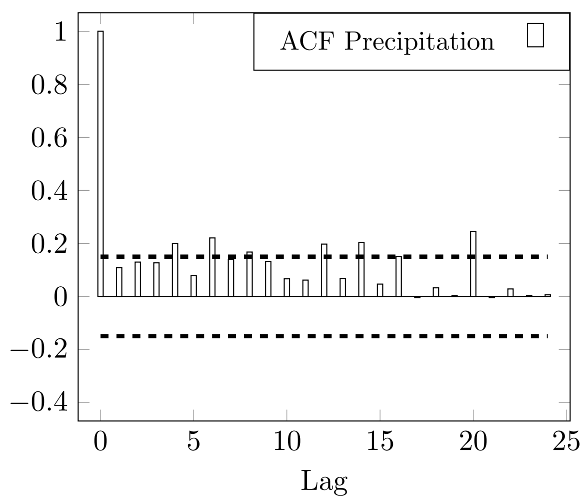
From Figure 3, we immediately observe no significant autocorrelation for the variables wind speed and precipitation, as the significance is denoted by dashed lines. However, we do observe a significant slowly declining autocorrelation for the variables temperature and humidity. For further investigation we plot the PACF in Figure 4. We observe that the partial autocorrelation is significant for the first two lags of the variables temperature and humidity. Observing two significant lags for these variables indicates the use of an AR(2) model for these variables. We do not observe significance for the first lags for the other variables.



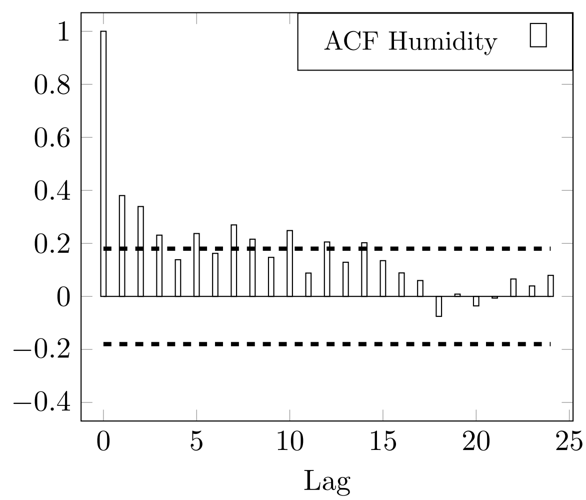
(a) Temperature



(b) Wind speed

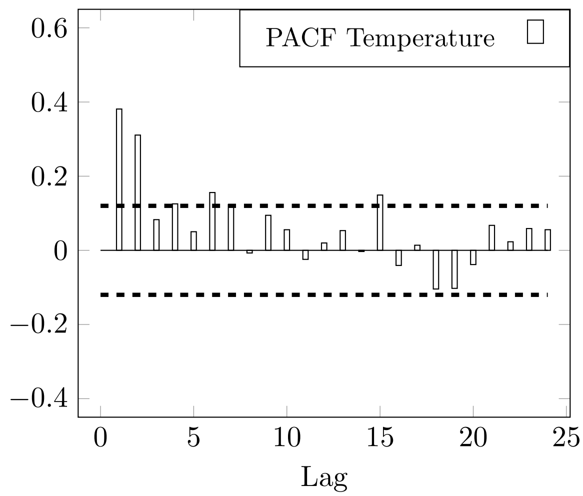


(c) Precipitation

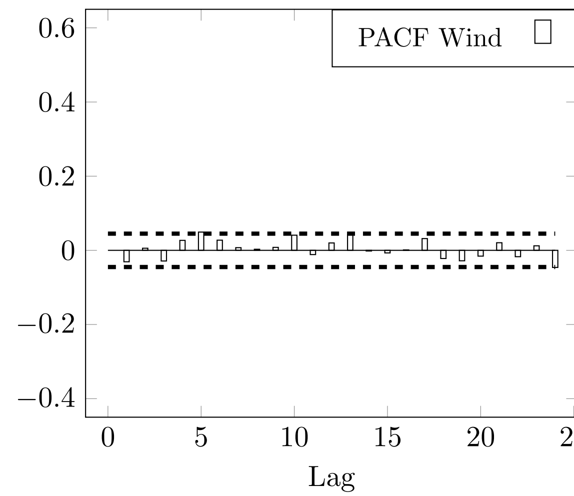


(d) Humidity

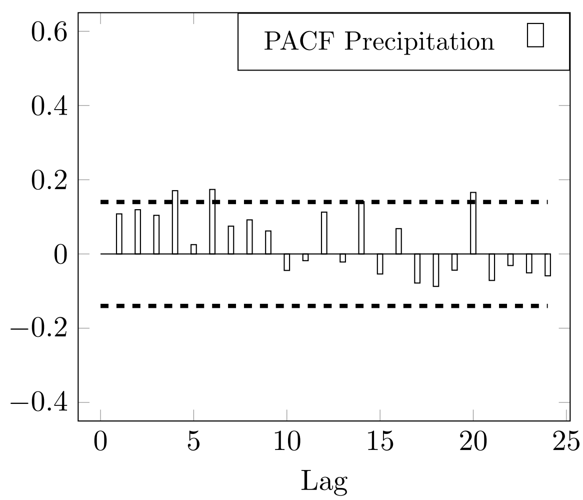
Figure 3: Autocorrelation function (ACF) for atmospheric variables: (a) Temperature, (b) Wind speed, (c) Precipitation, and (d) Humidity. Black dashed horizontal lines show 0.95 confidence level significance.



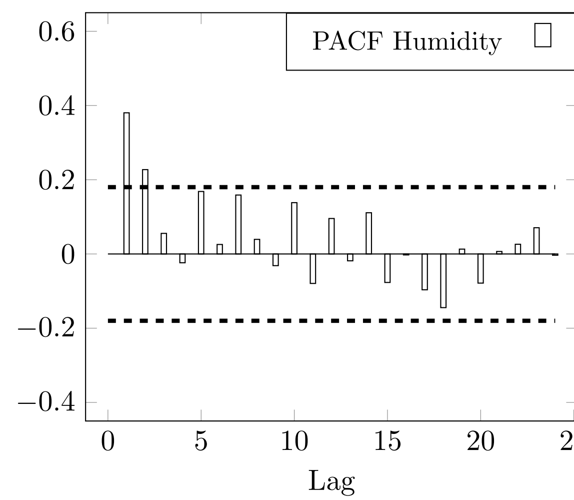
(a) Temperature



(b) Wind speed



(c) Precipitation



(d) Humidity

Figure 4: Partial autocorrelation function (PACF) for atmospheric variables: (a) Temperature, (b) Wind speed, (c) Precipitation, and (d) Humidity. Black dashed horizontal lines show 0.95 confidence level significance.

B Results: Regression outcomes temperature and model justification

This Section covers model 7. It is concerned with the estimates of the parameters and justification of the model. We will discuss the white noise properties and the normality assumption.

The OLS estimates of the coefficients are shown in Table 2. Note that all variables included in the regression are significant. To be certain of a valid model we test the residuals for the desired white noise properties I-III in Section 4, and normality. The mean of the residuals is equal to 0.00, which is as desired. We verify the assumptions of homoskedasticity and serial correlation by graphical inspection. The residuals are shown in Figure 5, we observe that the residuals are nicely centered around 0. We also note that the spread of the residuals is larger at the beginning of the sample and becomes less near the end of the sample indicating heteroskadasticity, not equal variance within the residual series. Moreover, near the end of the sample we observe that a slight upwards trend in the residuals, indicating serial correlation. These findings are evidence that the model parameters fail to describe the end of the sample. Despite these findings we continue the analysis. For normality, we use the Jarque-Bera test with null-hypothesis that the residuals are normally distributed. We obtain a p-value of 0.77 and conclude that the data is normally distributed.

Table 2: Regression results Equation 7

Dependent variable: temp		
Observations: 245		
Coefficient	Value	Std. error
α	3.83	0.65
ϕ_1	0.27	0.06
ϕ_2	0.33	0.06
R-squared	0.24	
Durbin-Watson	2.05	

C Results: Regression outcomes humidity and model justification

This Section covers model 8. It is concerned with the estimates of the parameters and justification of the model. We will discuss the white noise properties and the normality assumption.

Table 3 shows the OLS estimates of Equation 8. We note that all independent variables are significant and check for the white noise properties of the residuals. The mean of the residuals is equal to 0.00. Again we perform a graphical inspection to verify homoskedasticity and no serial correlation. Figure 5 shows the residuals of Equation 8. From the graph we observe that the residuals again show signs of higher and lower variance, and higher and lower values clustered together. This shows that the white noise properties do not exactly hold. We suggest that this is due to the presence of structural breaks in the data. We will again proceed our analysis. Lastly, we perform the Jarque-Bera test for normality. With a p-value of 0.61, we conclude that the residuals are normally distributed.

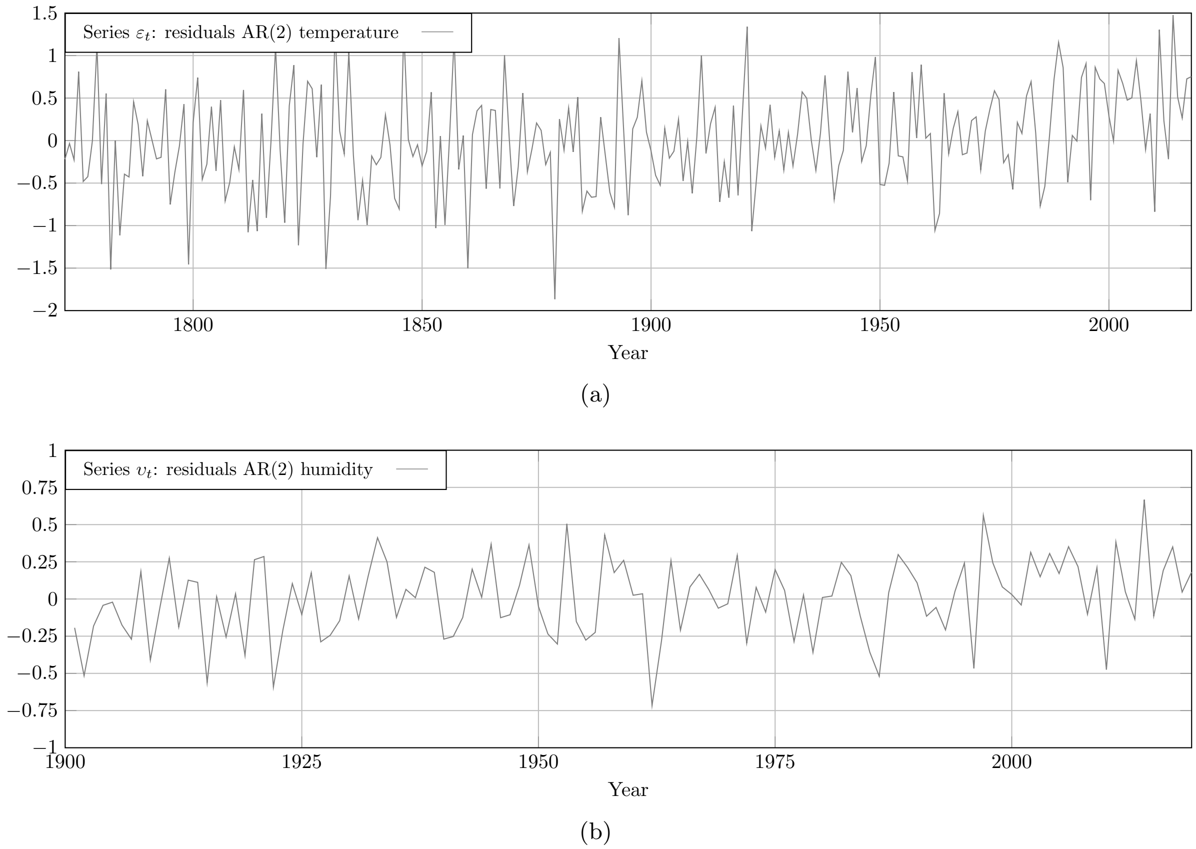


Figure 5: Residuals in AR models for (a) Temperature and (b) Humidity

Table 3: Regression results Equation 8

Dependent variable: humid		
Observations: 116		
Coefficient	Value	Std. error
β	4.83	1.00
λ_1	0.28	0.09
λ_2	0.23	0.09
R-squared	0.19	
Durbin-Watson	2.06	