Bachelor Thesis [International Bachelor Econometrics and Operations Research]

ERASMUS UNIVERSITY ROTTERDAM

Erasmus School of Economics

Bitcoin: An Analysis of Volatility, Returns, Trading Volume and Numbers of Trades

Author:

Dayao Jin

Student ID number:

496758

Supervisor:

(Mikhail) M Zhelonkin

Second Assessor:

(Dick) DJC van Dijk

Date Final Version: July 3, 2022

Acknowledgements

The views stated in this thesis are those of the author and not necessarily those of the supervisor, second assessor, Erasmus School of Economics or Erasmus University Rotterdam.

Abstract

With various statistical tools, a growing literature records important findings on Bitcoin's characteristics like volatility, trading volumes and prices. In this research, we apply the Heterogeneous AutoRegressive-Jumps (HARJ) model, the Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) models and the Generalized AutoRegressive Conditional Heteroskedasticity MIxed-DAta Sampling (GARCH-MIDAS) model with CBOE Nasdaq 100 Volatility (VXN) to study Bitcoin's volatility. In addition, we forecast daily Bitcoin returns with a MultiLayer Perceptron (MLP). Eventually, we conduct a Vector AutoRegression (VAR) model to study impulse responses of returns, number of trades and trading volume of Bitcoin and Ethereum on their own shocks. Our analysis shows that five days earlier realized volatility and jumps roughly capture present daily volatility of Bitcoin in the HARJ model. A MLP machine learning measure provides excessively extreme predictions for daily logarithm returns of Bitcoin and this measure is non-ideal during periods with extreme returns. The increase of weekly VXN suggests a small decrease in the Bitcoin volatility one week later. Lastly, we also see that both the numbers of trades and trading volume of Ethereum have positive responses to shocks of Ethereum returns and numbers of Bitcoin trades while returns of both coins do not react to shocks in other factors.

Contents

1 Introduction

Cryptocurrency, a highly volatile asset, is becoming popular among usual investors. Some representatives coins like Bitcoin and Ethereum, are attention-grabbing assets in the cryptocurrency market (Smales, 2022). Nevertheless, Gandal et al. (2018) suspect that whether cryptocurrency is a trustworthy asset as it may actually be manipulated under so-called "decentralization" and massive suspicious Bitcoin trades are linked to increases of exchange rate.

There is a growing amount of research regarding cryptocurrencies nowadays. In this research, we mainly focus on volatility and return prediction of Bitcoin. The first half of this research aims to replicate partial results by Pichl and Kaizoji (2017). They find that using a modified Heterogeneous AutoRegressive-Jumps (HARJ) model, initially introduced by Andersen et al. (2007), Bitcoin's realized volatility is well captured with the lagged realized volatility and lagged jumps from 1, 5 and 10 days earlier. Andersen et al. (2007) discover that the HARJ model is effective in capturing realized volatility on stock and foreign exchange markets. We follow the description of Pichl and Kaizoji (2017), with possibly different approaches to check whether we could obtain similar results. This forms our first research question:

To what extent can the Heterogeneous AutoRegressive-Jumps (HARJ) model capture dynamics of Bitcoin realized volatility?

On the other hand, as machine learning is increasingly popular among quantitative finance, we also use a supervised learning technique to predict daily logarithm returns of Bitcoin. The main idea also follows the work by Pichl and Kaizoji (2017). Our result provides an insight about whether this tool is effective in Bitcoin return prediction. This raises our second research question:

How does a MultiLayer Perceptron (MLP) perform in predicting daily logarithm returns of Bitcoin?

Conrad et al. (2018) suggest that using relevant variables in a lower frequency (rather than daily) could provide additional information on Bitcoin's long-term volatility. They apply the Generalized AutoRegressive Conditional Heteroskedasticity Mixed-DAta Sampling (GARCH-MIDAS) model with several monthly indexes to capture long-term (monthly) component of Bitcoin volatility. The GARCH-MIDAS model could inspect both the short-term and the long-term Bitcoin volatility. In this thesis, we select weekly CBOE Nasdaq 100 Volatility (VXN) to see how it additionally explains changes of the weekly Bitcoin volatility. We also analyse basic characteristics of Bitcoin volatility with the classical Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) and the Threshold Generalized AutoRegressive Conditional Heteroskedasticity (TGARCH) model. The core question of this part is:

To what extent does weekly CBOE Nasdaq 100 Volatility (VXN) suggest weekly volatility of Bitcoin?

We also interest whether a coin relationship exists between Bitcoin and Ethereum. Ethereum is the cryptocurrency with the second largest market cap (and Bitcoin is the first place) and possibly relates to Bitcoin. A final part of this research is to inspect responses of daily returns, numbers of trades and trading volume of Bitcoin and Ethereum to shocks from these variable. By doing so, we could deeper understand the relationship across these two coins and this may provide superior factors for constructing advanced forecast models in the future. In this part, we apply Vector AutoregRessive (VAR) model and conduct an impulse response analysis. This leads to the following question:

> *What interactions exist among returns, numbers of trades and trading volume of Bitcoin and Ethereum?*

This research provides the following findings. The square root version of the HARJ model moderately captures Bitcoin's realized volatility. The lagged realized volatility and lagged jumps five days ago have significant and opposite influence to the present realized volatility. With an input of a moving window of past ten daily logarithm returns, employing a MLP machine learning technique provides an over extreme prediction on the present daily logarithm return. As suggested by the GARCH (1,1) model and the TGARCH (1,1) model, Bitcoin's daily volatility are conditional stationary and insensitive to negative returns. The GARCH-MIDAS model shows that one standard deviation increase in weekly VXN suggests a 0.34% decrease of Bitcoin volatility in the next week. Lastly, trading volume of Bitcoin has a positive reaction to the number of trades of Bitcoin. Furthermore, both the number of trades and trading volume of Ethereum positively react to increases of Ethereum returns and Bitcoin trades. Additionally, Ethereum volume has a positive response to Bitcoin's trading volume. Returns of both coins show no significant reactions to shocks of the other factors.

This thesis proceeds with the following structure: Section 2 provides a detailed literature review, data description is in Section 3. We present our methodology in Section 4. Section 5 discusses our results and Section 6 concludes this research.

2 Literature Review

Cryptocurrency market draws public attention with a huge increase in its market cap. As a symbol of cryptocurrency, Bitcoin is not only the first cryptocurrency and but also the most expensive cryptocurrency. One reason behind is that Bitcoin provides people a decentralized anonymous payment, benefiting people on low trackable trades (Sasson et al., 2014). Nevertheless, Gervais et al. (2014) disagree with that and believe there are some parties have decisive influences on bitcoin prices by adjusting cryptocurrency service, mining and resolution processes. Anyhow, according to the ISO (International Organization for Standardization) 4217 standards, the abbreviation of Bitcoin changes from "BTC" to "XBT". Starting with "X" means that Bitcoin is not a currency subject to a particular country.

Given the popularity of cryptocurrency, governments plan to pose more restrictions on the cryptocurrency industry. Riley (2021) points out, the countries which are legalizing cryptocurrencies payments, devote their efforts on solving problems like tax avoidance and illegal financial financing activities. Next to financial security, mining, the key to secure transaction credibility of cryptocurrencies, consumes massive energy and causes vast CO2 emission. Badea and Mungiu-Pupzan (2021) suggest authorities should also legitimate ecological responsibility to reduce adverse environmental influences caused by mining.

In the view of investment, public has divergent valuations on cryptocurrencies. An earlier study by Kristoufek (2015) finds that Bitcoin provides properties of both traditional financial and speculative assets. During the Covid-19 pandemic, Bitcoin prices accelerate as many people believe that Bitcoin is a hedging or even a safe-haven asset during high inflation periods according to Choi and Shin (2022). Nevertheless, Chen et al. (2020) indicate that higher trading volumes of Bitcoin only results negative returns in general and Bitcoin behaves more like other traditional financial assets than safe-haven assets in the pandemic.

Cryptocurrency draws researchers' attention. With development of technologies, Bitcoin leaves massive relevant data to facilitate research. Many studies apply different statistical tools, which are used on other financial indexes, on cryptocurrencies. For instance, GARCH model, one famous model measuring volatility of returns, is widely applied in studies of Bitcoin. For example, Ardia et al. (2019) show that a Markov switching GARCH model has a superior performance in Value at Risk (VaR) forecast than GARCH models with a single regime.

In addition, some researchers focus on the relationship between Bitcoin's volatility and other financial indexes. For example, Conrad et al. (2018) find that the monthly realized volatility of S&P 500 index has a 17% positive effect on Bitcoin's monthly volatility. Estrada (2017) discovers a similar relationship between CBOE Volatility index (VIX) and realized volatility of Bitcoin. Conrad et al. (2018) state, understanding Bitcoin's volatility not only deepen insights on cryptocurrencies but also provides more insights on global economic activities. Pichl and Kaizoji (2017) apply the HARJ model, originally introduced by Andersen et al. (2007), to capture daily realized volatility of Bitcoin. Andersen et al. (2007) show this HARJ model is a well-founded tool on exchange spot markets, equity markets and US Treasure bonds. Results of Pichl and Kaizoji (2017) show previous realized volatility and jumps are significant to forecast present realized volatility of Bitcoin.

Price prediction is also a popular topic in quantitative finance. Some traditional econometric tools are powerful in predicting Bitcoin prices, Azari (2019) explores that the traditional AutoRegressive Integrative Moving Average (ARIMA) model is efficient in predicting Bitcoin prices, particularly in the short-term forecast. Nowadays, machine learning is widely used to predict prices for different assets, including cryptocurrencies. In some cases, compared to traditional time-series models, machine learning outperforms in predicting cryptocurrencies prices. For example, McNally et al. (2018) implement a Bayesian optimised Recurrent Neural Network (RNN) and a Long Short Term Memory (LSTM) method, and they conclude both methods offer superior Bitcoin price prediction than the ARIMA model. Pichl and Kaizoji (2017) implement a multilayer perceptron (MLP) and receive a relatively satisfying prediction. They also propose RNN and LSTM may further improve prediction performance and this is exactly verified by McNally et al. (2018).

In addition, some previous researches about cryptocurrencies implement the Vector AutoregRessive (VAR) model on studying relationship between cryptocurrencies and other assets. Giudici and Abu-Hashish (2019) use VAR correlation networks and confirm that Bitcoin prices usually are unrelated to classic asset prices. They also find that their model is capable to predict Bitcoin prices with an error of 11% average price. Some researchers also apply VAR model to learn relationship between coins. For instance, Beneki et al. (2019) investigate the impulse responses by a VAR model and spot a delayed positive response of Bitcoin volatility on a positive shock of Ethereum returns. Luu Duc Huynh (2019) uses the VAR model and find Ethereum prices are relatively independent to other coins and Bitcoin is a receiver of spillover effects from other coins.

The following three points are original contributions of this research. The first part of this research aims to replicate partial results of Pichl and Kaizoji (2017) about Bitcoin's realized volatility in the HARJ model and prediction of Bitcoin's returns based on a machine learning method. Their conclude that both methods provide reasonable predictions and their conclusion is not rigorous. They simply assess prediction accuracy via seeing the difference between predicted values and actual values on graph. Our research uses two prediction error measurements to examine the prediction accuracy of the methods they propose. Secondly, Conrad et al. (2018) link some stock market indexes, luxury goods indexes and metal indexes with the GARCH-MIDAS model to study monthly Bitcoin volatility. Previous works show VIX and realized volatility of S&P 500 are effective in capturing the long-term volatility of Bitcoin. In this research, we apply the GARCH-MIDAS model with an alternative index as a potential Bitcoin volatility driver: weekly CBOE Nasdaq 100 volatility (VXN). VXN is the volatility index for Nasdaq 100 and Nasdaq 100 skews heavily toward technology companies than other major market indexes. Considering the strong technology characteristic of Bitcoin and some cryptocurrencies related firms (for example, the manufacturers of graph cards, which are essential to mining processes in cryptocurrencies, Nvidia and AMD) are also listed in NASDAQ exchange, VXN may provide a fresh explanation on Bitcoin volatility in weekly basis. Eventually, this research includes the numbers of trades of Bitcoin and Ethereum in a VAR model. Adding this new factor may explore more dynamics of daily returns and trading volumes of cryptocurrencies.

3 Data

This section presents an overview of used variables in this research. To answer our research questions, we mainly use the OHLCVT (Open, High, Low, Close, Volume, Trades) data containing Bitcoin-USD and

Ethereum-USD relationships from Kraken's database. There are six different time frames for the data: 1, 5, 15, 60, 720 and 1440 minutes. We select Bitcoin's close prices in 5 minutes frequency to aggregate daily realized volatility of Bitcoin in the HARJ model. For the the remaining parts, we choose the 1440 minutes (daily) data for Bitcoin and Ethereum. In the first extension, we use another variable in weekly frequency, the CBOE Nasdaq 100 Market Volatility (VXN). Its data is downloaded from Yahoo finance.

3.1 Data Transformation: Close Prices to Logarithm Returns

Although close prices are straightforward to tell value changes of cryptocurrencies, compared to linear price scales, the logarithm returns are more stable under severe price changes, especially for prices of highly volatile assets like cryptocurrencies. The logarithm form offers a symmetric representation in both positive and negative price changes and a zero return represents constant price levels. The expression of logarithm returns (R_t) is in Equation 1.

$$R_t = \log\left(\frac{C_t}{C_{t-1}}\right), \tag{1}$$

where C_t is the close price of a coin (Bitcoin or Ethereum) at time t.

3.2 Data Description

Table 1 demonstrates the descriptive statistics (mean, median, maximum, minimum, standard deviation, Skewness and Kurtosis) of all used variables in this research. There are three panels in Table 1. Panel A includes the data used in the replication part and the first extension. Panel B and panel C correspond to data in the first and the second extension. The sample period of both Panel A and B covers 1 May 2014 - 8 April 2017 (1064 daily observations), CBOE Nasdaq 100 Market Volatility (VXN) is weekly data and hence with only 155 observations. Panel C concerns both Bitcoin and Ethereum. Because Kraken only records Ethereum information from 11 August 2015, the sample period of panel C is from 11 August 2015 to 12 April 2017 (608 observations).

Table 1: Descriptive statistics of used variables.

Variable	Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis
Panel A: Daily Bitcoin returns (2014M5-2017M4)							
Bitcoin Close Prices	489.752	427.485	1276.999	175	241.933	1.113351	3.802262
Bitcoin Log Returns	0.000386	0.000474	0.097405	-0.12661	0.015895	-0.852423	11.56911
Panel B: Weekly Volatility Index (2014M5-2017M4)							
CBOE Nasdaq Market Volatility Index (VXN)	17.1184	16.1	42.95	10.31	4.21209	1.696772	6.923727
Panel C: Daily Logarithm returns, volume and trades for Bitcoin and Ethereum (2015M8-2017M4)							
Bitcoin Log Returns	0.001058	0.001298	-0.072363	0.048939	0.013489	-0.974940	8.601841
Bitcoin Volume	1278.701	858.8206	9817.799	1.572498	1454.161	2.362505	10.12454
Bitcoin Trades	1473.104	936	18790	10	1831.767	3.450545	22.61887
Ethereum Log Returns	0.002612	9.89e-06	0.165349	-0.13689	0.034896	0.238577	6.561737
Ethereum Volume	32706.49	19091.7	522545	0.74443	48676.03	4.233461	30.30843
Ethereum Trades	906.7467	479	12955	1	1388.251	3.94222	25.51363

In panel A, we see the standard deviation suggests the Bitcoin market is quite volatile in this period. The Skewness and Kurtosis show Bitcoin prices are not normally distributed like most financial assets. After taking logarithm, we find Bitcoin provides a positive return over this period. Additionally, the logarithm returns switch to negative skewed compared to positive skewed close prices. To study the weekly volatility of Bitcoin, we select weekly VXN as a predictor and its descriptive statistics is in Panel B. We also interest cointegration between coins in terms of logarithm returns, trading volume and numbers of trades. Panel C shows different characteristics in volume and trades. Ethereum has a larger trading volume while Bitcoin has higher numbers of trades. This may suggest between 2015M8-2017M4, especially that is an emerging period for cryptocurrencies, Bitcoin attracts more individual investors than Ethereum. Both coins exhibit strong volatility according to their means and standard deviations.

Figure 1 presents close prices of Bitcoin and Ethereum. With a decrease from 600 USD to 200 USD at the end of 2014, Bitcoin prices fluctuate at the level of 300 USD until November 2015. Since then, Bitcoin prices climbed at most times and spiked to 1200 USD in March 2017. At this point, the price of Bitcoin is fourfold than that 16 months ago. For Ethereum, its value has much smaller scales than Bitcoin, only with a highest price 50 USD in this period. However, Ethereum experiences a first boost from February to June in 2016, and its price rises from a penny coin to more than 10 USD. Since February 2017, Ethereum becomes five-fold in one month. Ethereum prices show a rapid increase speed than Bitcoin.

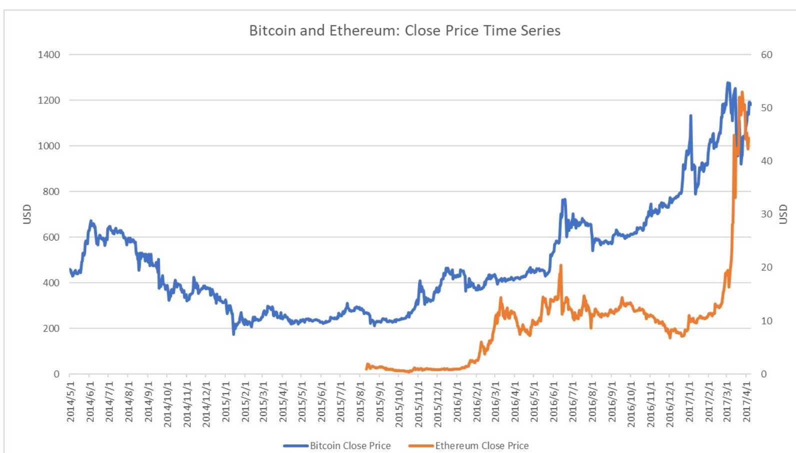


Figure 1: Close prices of Bitcoin and Ethereum in USD over May 2014 / August 2015 - April 2017.

4 Methodology

4.1 Bitcoin's Daily Realized Volatility

Pichl and Kaizoji (2017) define a HARJ model, modifying from the specification of Andersen et al. (2007), to investigate the relationship among Bitcoin's realized volatility, its historical values and historical jumps from 1, 5 and 10 days ago. The sample period concerns May 2014 - 8 April 2017 (1064 valid trading days). The programming language for regression replication is R and the package *highfrequency* by Boudt et al. (2017) is used for implementation.

To begin with, the definition of realized volatility (**RV**) is as follows:

$$\mathbf{RV}_{t+1}(\triangle) = \sum_{j=1}^{1/\triangle} r_{t+j\cdot\triangle}^2 \,, \tag{2}$$

where r^2 is the squared logarithm return, t represents time (days) and $\triangle$ is five minutes. On the left side, the realized volatility corresponds to a relatively lower frequency index (daily in this case). On the right side, squared logarithm return corresponds to relatively higher frequency index (five minutes in this case). Eventually, this expression aims to make one day realized volatility equalling to the sum of all squared logarithm returns every five minutes. Theoretically, as Bitcoin market is open 24/7, there should be $\frac{1}{\triangle}$ = 288 five minutes intervals per day. Nevertheless, Kraken data suggests not every trading day contains exactly 288 records. This is likely due to technical issues and inactive trading periods. Therefore, the sum calculation of realized volatility in one day is based on the actual number of records in one day, rather than 288.

To obtain a HARJ model specified by Pichl and Kaizoji (2017),we also define jumps (**J**) by providing its necessary component bi-power variation (**BV**) first.

$$\mathbf{BV}_{t+1}(\triangle) = \mu_1^{-2} \sum_{j=2}^{1/\triangle} |r_{t+j\cdot\triangle}||r_{t+(j-1)\cdot\triangle}| \,, \tag{3}$$

where $\mu_1 \equiv \sqrt{\frac{2}{\pi}} = \mathbb{E}(|Z|)$, the mean of absolute value of Z and Z is a standard normally distributed variable.

$$\mathbf{J}_{t+1}(\triangle) = max[\mathbf{RV}_{t+1}(\triangle) - \mathbf{BV}_{t+1}(\triangle), 0] \,. \tag{4}$$

After obtaining **BV**, jumps are calculated as Equation (4) and this ensures jumps have non-negative values. The detailed derivations and motivations can be found in the work by Andersen et al. (2007). Equation (5) provides the HARJ model defined by Pichl and Kaizoji (2017), and for simplicity, we omit $\triangle$.

$$\sqrt{\mathbf{RV}}_{t+1} = \beta_0 + \beta_1\sqrt{\mathbf{RV}}_t + \beta_2\sqrt{\mathbf{RV}}_{t-5} + \beta_3\sqrt{\mathbf{RV}}_{t-10} + \beta_4\sqrt{\mathbf{J}}_t + \beta_5\sqrt{\mathbf{J}}_{t-5} + \beta_6\sqrt{\mathbf{J}}_{t-10} \,. \tag{5}$$

Equation (5) considers square roots of realized volatility and square roots of jumps from 1, 5 and 10 days earlier. We should notice that their HARJ formula concerns the values from 1, 6 and 11 days ago. Even so, we carry on the setting of (1, 5, 10) days as the alternative setting is meaningless in their economic evaluation. The command code of the given HARJ model is as follows.

HARmodel(x, periods = c(1,5,10),transform="sqrt",periodsJ = c(1,5,10), type = "HARJ"), where x is a data frame containing date, **RV** and **J**. *periods* and *periodsJ* correspond to the lagged values for **RV** and **J** 1, 5 and 10 days earlier. *transform="sqrt"* takes square root for **RV** and **J**. *type="HARJ"* is the default setting for HARJ models in the package.

To evaluate the prediction accuracy of the HARJ model, we inspect the Mean Absolute Prediction Error ($MAPE$) of fitted realized volatility. $MAPE$ is a measurement of the average errors in this forecast and less unambiguous than other measurements (Chai & Draxler, 2014). We compare this $MAPE$ with the mean of the actual (square root of) realized volatility to see the range of prediction errors of this model. In addition, we calculate the Root Mean Squared Prediction Error ($RMSPE$). Because the $RMSPE$ weighs higher for large errors, we could tell whether obvious mangnitudes of variation occur in errors. The formulas for the $MAPE$ and $RMSPE$ are in Equation (6) and Equation (7).

$$MAPE = \frac{1}{N}\sum_{t=1}^{N} |\sqrt{\mathbf{RV}}_{t,actual} - \sqrt{\mathbf{RV}}_{t,fitted}| \,, \tag{6}$$

$$RMSPE = \frac{1}{N}\sum_{t=1}^{N} (\sqrt{\mathbf{RV}}_{t,actual} - \sqrt{\mathbf{RV}}_{t,fitted})^2 \,, \tag{7}$$

where $\mathbf{RV}_{t,actual}$ is the actual realized volatility at day t, $\mathbf{RV}_{t,fitted}$ is the fitted realized volatility in the HARJ model. N is the total number of trading days.

4.2 Predictions of Bitcoin's Daily Returns Based on Machine Learning

Machine learning is popular among financial predictions. In the paper by Pichl and Kaizoji (2017), they apply a MultiLayer perceptron (MLP) to forecast daily logarithm returns of Bitcoin. This research follows a similar neural network setting to predict daily logarithm returns of Bitcoin. Additionally, this thesis provides some missing details in their paper, which are essential to implement this neural network. The programming language for this implementation is R, using the package *neuralnet* by Gnther and Fritsch (2016).

The studied period for this MLP measure is also 1 May 2014 - 8 April 2017 (1064 valid trading days). For each trading day, with an input of daily logarithm returns of past ten days, a fully-connected network with two hidden layers provides a prediction output of daily return that day. The two hidden layers uses a (10, 5) configuration and the gradient vanish threshold is 0.005. Since this neural network needs ten earlier inputs for each trading day, we remove the data of first ten days. Before implementing the neural network, the daily logarithm returns are scaled as shown in Equation (8).

$$R_t = 0.08\left(\frac{R_t - \mu}{\sigma}\right) \,, \tag{8}$$

where R_t is the logarithm return on day t, the mean of logarithm returns over the sample period is $\mu = 0.000386$, and the standard deviation of logarithm returns over the sample period is $\sigma = 0.015895$.

Multiplying a coefficient of 0.08 ensures the modified logarithm returns have zero mean and a standard deviation of 0.08 as Pichl and Kaizoji (2017) describe.

To see the prediction performance of this neural network, Pichl and Kaizoji (2017) select the first two thirds data as the training set and the remaining one thirds part as the test set. However, they actually split the data from 2 May 2016 based on Figure 4(b) and Figure 5(a) in their paper. Therefore, we also separate the entire sample period into two subperiods from 2 May 2016. The first subperiod serves as the training set and the second subperiod serves as the test set.

Similar to the last subsection, we examine this neural network's prediction accuracy by inspecting the Mean Absolute Prediction Error ($MAPE$) and the Root Mean Absolute Prediction Error ($RMSPE$) between (scaled) actual logarithm returns and (scaled) predicted logarithm returns. Formulas of the two prediction error measures have the same specification in Equation (6) and Equation (7), but with actual and predicted (fitted) logarithm returns as variables this time.

4.3 Long-Term (Weekly) and Short-Term (Daily) Bitcoin Volatility

Some previous studies apply several variants of the GARCH model to study volatility of different financial assets/indexes. For example, Dyhrberg (2016) finds that the asymmetric GARCH model is useful in risk management for a cryptocurrency investment portfolio. Some scholars additionally introduce external regressors to strengthen the power of GARCH models. For instance, Conrad et al. (2018) combine MIxed DAta Sampling (MIDAS) and GARCH models to study Bitcoin volatility. Their model discovers the effects on the long term Bitcoin volatility from external monthly financial indexes like realized volatility of S&P 500, Nikkei 225, Gold and Copper indexes.

In this research, we first use the GARCH (1,1) model and the TGARCH (1,1) model to learn basic properties of Bitcoin volatility over 1 May 2014 - 8 April 2017. After that, we apply a GARCH-MIDAS model with weekly CBOE Nasdaq 100 Volatility (VXN) to study weekly Bitcoin volatility. The following GARCH-MIDAS setting are mainly based on the description in Conrad and Kleen (2020).

4.3.1 The GARCH (1,1) Model and the TGARCH (1,1) Model

Equation (9), Equation (10) and Equation (11) form the GARCH (1,1) model by Bollerslev (1986).

$$r_t = \mu + \epsilon_t \,, \tag{9}$$

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \,, \tag{10}$$

$$\epsilon_t = z_t\sigma_t \tag{11}$$

where r_t is the Bitcoin logarithm returns at time t, μ is the mean, ϵ_t is the return difference between the actual return at time t and the mean return. σ^2 is the conditional volatility at time t, $z_t \sim N(0,1)$. The following parameter restrictions guarantee only positive volatility is obtained: $\omega > 0$, $\alpha \geq 0$ and $\beta \geq 0$ for all t.

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \eta\epsilon_{t-1}^2 I[\epsilon_{t-1} < 0] + \beta\sigma_{t-1}^2 \,. \tag{12}$$

The TGARCH (1,1) model, introduced by Zakoian (1994), has identical specifications as the GARCH (1,1) model, except its conditional volatility additionally considers leverage effects η of negative returns. Equation (12) provides the conditional volatility form of TGARCH model while the other two formulas are the same as the GARCH (1,1) model. The indicator function $I[A] = 1$ if A happens, and 0 otherwise. The programming language is R with the package *rugarch* by Ghalanos (2020).

4.3.2 The GARCH-MIDAS Model

In this part, we analyse both the short-term and the long-term Bitcoin volatility via the GARCH-MIDAS model. With this model, we are able to explore potential drivers of the long-term Bitcoin volatility with varaiables in different frequency. The following equations show the construction of the GARCH-MIDAS model, and they are based on Conrad et al. (2018). In this research, we choose weekly CBOE Nasdaq 100 Market Volatility Index (VXN) as a potential explanatory variable of the long-term (weekly) Bitcoin volatility. Previous GARCH-MIDAS investigations (Kleen, 2018) usually apply other volatility indexes like realized volatility of S&P 500 and VIX, and these indexes are more focus on the volatility of general stock markets. Due to strong technology characteristic of Bitcoin, Nasdaq 100 volatility index may provide more accurate information on Bitcoin volatility as Nasdaq has a higher weight of technological stocks (Jeon & Jang, 2004). The estimation measure is quasi-maximum likelihood and the implementation is done via the package *mfGARCH* in R by Kleen (2018).

Equation (13) and Equation (14) provide the specification of Bitcoin returns in the GARCH-MIDAS model. Unlike the GARCH (1,1) model, this model considers both $h_{i,t}$ and τ_t, which are the short-term and the long-term components of conditional Bitcoin volatility. t represents week t and i represents day i within week t.

$$r_{i,t} = \mu + \epsilon_{i,t} \,, \tag{13}$$

$$\epsilon_{i,t} = \sqrt{h_{i,t}\tau_t} z_{i,t} \,. \tag{14}$$

The short-term component $h_{i,t}$ of Bitcoin volatility is in daily frequency and follows GARCH (1,1) process with restrictions: $\alpha \geq 0$, $\beta \geq 0$ and $\omega > 0$.

$$h_{i,t} = (1 - \alpha - \beta) + \alpha\frac{\epsilon_{i-1,t}^2}{\tau_t} + \beta h_{i-1,t} \,. \tag{15}$$

The long-term component of Bitcoin volatility τ_t is in weekly frequency. m is the mean, X is the explanatory variable (weekly VXN), π is the coefficient. k is the lag and we choose K=52 to all weeks within one year. The coefficient π is the multiplication of θ and $\phi_k(\omega_1, \omega_2)$, where θ decides the sign effect of X_t and $\phi_k(\omega_1, \omega_2)$ is a weight scheme.

$$\tau_t = m + \sum_{k=1}^{K} \pi_k X_{t-k} \,, \tag{16}$$

$$\pi_k = \theta \phi_k(\omega_1, \omega_2) \,. \tag{17}$$

The weight scheme is restricted to non-negative and sum to one. In addition, the used package restrict $\omega_1 = 1$ to ensure weights are monotonically declining. Equation (18) shows the Beta weight scheme for $\phi_k(\omega_1, \omega_2)$.

$$\phi_k(\omega_1, \omega_2) = \frac{(k/(K+1))^{\omega_1 - 1} \cdot (1 - k/(K+1))^{\omega_2 - 1}}{\sum_{j=1}^{K} (j/(K+1))^{\omega_1 - 1} \cdot (1 - j/(K+1))^{\omega_2 - 1}} \,. \tag{18}$$

4.4 Interactions among Returns, Trading Volume and Numbers of Trades in Bitcoin and Ethereum

To answer the second extension question, we need the response of individual variables to an shock in another variable. Kraken's OHLCVT (Open, High, Low, Close, Volume, Trades) data set provides the daily trading volume (Volume) and the daily numbers of individual trades (Trades). In this part, we apply the following Vector AutoRegressive (VAR) model by Sims (1980), and conduct an impulse response analysis to study the mutual effects of logarithm returns, volumes and trades of Bitcoin (XBT) and Ethereum (ETH). Unlike Bitcoin information, the records for Ethereum information is only available from 11 August 2015. Hence, we select the sample period from 11 August 2015 - 13 April 2017 (608 Observations).

$$\mathbf{Y}_t = \mu + \sum_{i=1}^{p} \mathbf{B}_i \mathbf{Y}_{t-i} + \boldsymbol{\varepsilon}_t \qquad \text{for} \quad 1 \le t \le T, \tag{19}$$

where

$$\mathbf{Y}_t = [\text{XBT Log Returns}, \text{XBT Volume}, \text{XBT Trades}, \text{ETH Log Returns}, \text{ETH Volume}, \text{ETH Trades}]' \tag{20}$$

In Equation (19), $\mathbf{Y}_t$ denotes a [6 × 1] vector with the above variables at time t. μ is a constant vector, $\mathbf{B}_i$ represents the coefficient matrix with lag i, $\boldsymbol{\varepsilon}_t$ is the vector of shocks and $\mathbb{E}[\varepsilon_t \varepsilon_t'] = \Sigma$. t represents day t, from 1 to T (T=608). p is a selected lag number.

Firstly, we select a suitable lag order for this VAR model. Relevant selection criteria are Akaike Information Criterion (AIC), Schwarz Information Criterion (SC), and Hannan-Quinn Information Criterion

(HQ). The corresponding part in the result explains the our motivation of choices. Additionally, it is important to check stability of this VAR model. If this VAR model is unstable, shock impacts explode and this leads to an invalid model for interpretation. We use the Augmented Dickey-Fuller (ADF) test to check the stationarity of each variable. To examine stability of this VAR model, we will check whether all eigenvalues of the coefficient matrix are within the unit cycle. If all eigenvalues are less than one, this VAR system is stable and we can draw useful conclusion from the impulse response analysis. The implementation is done via EViews.

5 Results and Discussions

5.1 Bitcoin's Daily Realized Volatility in the HARJ Model

In this part, Table 2 presents the regression results of the HARJ model and Figures 2 compares the actual realized volatility and forecasted realized volatility by the HARJ model.

As can be seen in Table 2, we obtain several significant estimates. With a significant constant of 0.006, the square root of realized volatility 5 days earlier has significant positive influences on the present (square root of) realized volatility. If releasing the significance level to 10%, then the square root of realized volatility a day earlier also shows a similar but slightly smaller effect. In terms of jumps, only the square root of jumps 5 days ago shows a significantly negative relationship with current realized volatility. For values from 10 days ago, both the square root of realized volatility and the square root of jumps display no significant influences on the current realized volatility.

For this regression, we have a R^2 of 0.3757, which may suggest this HARJ model cannot provide precise prediction on the daily realized volatility of Bitcoin. The $MAPE$ of this model is 0.00887. The square root of realized volatility has a mean of 0.021597. On average, this shows the prediction error range is between 0.012727 and 0.030647. The forecast's distance from the true value is about 41%, which means the prediction by this model is indeed not very precise. The $RMSPE$ is 0.01508 and there are some magnitude variations in prediction errors. In Figure 2, we see this model does not perform well in the time of extreme volatility. The time on the top right of Figure 2 suggests that this model considers lags up to ten days such that the starting date is 11 May 2014.

Table 2: Regression results of the Heterogeneous AutoRegressive Jumps (HARJ) model. The dependent variable is the daily (square root) realized volatility of Bitcoin in US dollars. The daily realized volatility is aggregated by realized volatility in a frequency of five minutes.

Coef.	**Estimate**	**Std.error**	**t-value**	**p-value**	**Signf.**
β_0	0.0060749	0.0009451	6.428	1.97e-10	***
β_1	0.3263158	0.1811037	1.802	0.0719	.
β_2	0.5629379	0.2403979	2.342	0.0194	*
β_3	-0.035053	0.2328086	-0.151	0.8803	
β_4	0.1347861	0.1334814	1.010	0.3128	
β_5	-0.505911	0.2268156	-2.230	0.0259	*
β_6	0.1910648	0.2711929	0.705	0.4813	

Note. The codes of **Signf.** represent different significance levels (in parenthesis): ***(99.9%), **(99%), *(95%), .(90%) and blank if less than 90%.

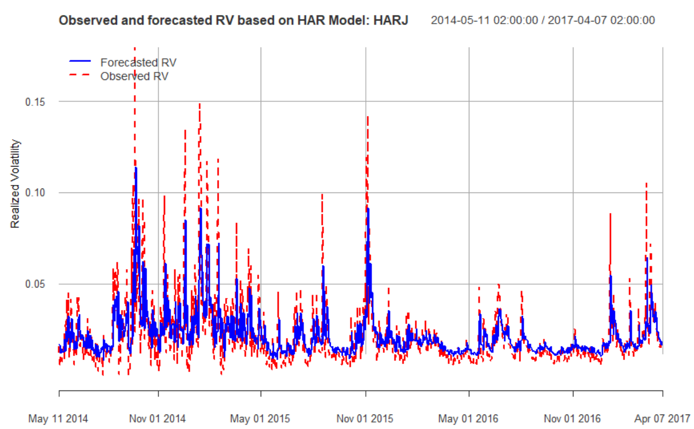


Figure 2: Actual realized volatility and predicted realized volatility of Bitcoin in US dollars by the Heterogeneous AutoRegressive Jumps (HARJ) model over 1 May 2014 - 8 April 2017.

Lastly, our results are somehow different from Pichl and Kaizoji (2017). For the regression results, their results additionally suggest the square root of realized volatility and the square root of jumps from 10 days ago are significant estimates. Furthermore, for the same significant regressors, they report much lower p-values. The signs of the estimates are mostly matched except β_4, but it is insignificant in both our and their results though.

One possible explanation could be that our data for calculating realized volatility is different from theirs. The actual values of realized volatility and bi-power variation are calculated from logarithm returns. Nevertheless, after the first step of replicating the paper by Pichl and Kaizoji (2017), we obtain different logarithm returns compared to theirs. They do not report any treatments on the raw data and the logarithm returns are straightforward to calculate, but we obtain different logarithm returns. Figure 2 also hints different input data. In Figure 2, the actual realized volatility only exceeds 0.15 once whereas their plot has seven pikes more than 0.15. The general trend in our plot and theirs is similar. However, for some periods with extreme realized volatility, like around January 2015 and May 2016, the shapes of observed realized volatility vary.

5.2 Predictions of Bitcoin's Daily Returns Based on Machine Learning

Following the description in Section4.2, the machine learning result for the test set (2 May 2016 - 8 April 2017) is present in Figure 3 and Figure 4. Figure 3 is the plot of original and predicted daily Bitcoin logarithm returns over the test sample period, and Figure 4 shows the density of original and predicted daily logarithm returns. These returns are scaled as in Equation (8).

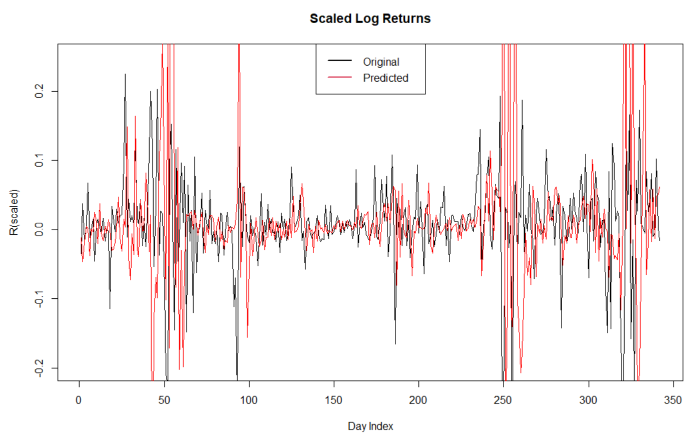


Figure 3: The original and the MultiLayer Perceptron (MLP) predicted daily logarithm returns of Bitcoin over the test set (2 May 2016 - 8 April 2017).

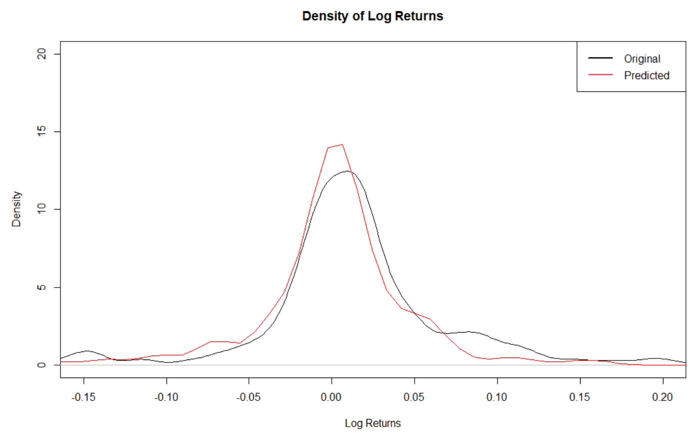


Figure 4: The density distribution of scaled logarithm returns for both the original and the MultiLayer Perceptron (MLP) predicted daily logarithm returns of Bitcoin.

For this machine learning result, the prediction measurement indicator $RMSPE$ is 0.262677 and the $MAPE$ is 0.087975. It is obvious some large prediction errors occur as the difference between two measurements is significantly large. Considering the mean of scaled logarithm returns is 0.008089, it seems the prediction error interval is over wide. In Figure 3, we see this MLP method roughly captures the trends of the original daily logarithm returns, the upward and downward trends are moderately matched. However, the predicted values exhibit greater magnitude changes than the actual values in cases of extreme returns switch like at day 50, 250 and 320. Additionally, this neural network sometimes predicts daily returns in the opposite direction compared to actual ones. For example, around day 100 and 260, this neural network predicts sharp decreases whereas the actual returns have upward trends. Figure 4 indicates that the density of predicted logarithm returns has a higher peak than actual returns and over predict (large) negative returns as shown in Figure 3 shows. This result leaves a future study path: Is it possible to improve the prediction performance with more advanced machine learning techniques.

This subsection also aims to replicate the results by Pichl and Kaizoji (2017). However, our result is different from theirs significantly. Firstly, like mentioned in Section 5.1, the different raw data are the main reason why magnitudes of the our result differ. In addition, Pichl and Kaizoji (2017) do not provide many details about scaling logarithm returns. It is also possible that the scaling methods used in this research differ from their practices and this would possibly widen the gaps between their results and ours. Lastly, Pichl and Kaizoji (2017) only report their original and predicted values for less than 300 days. However, their test set, one thirds of the complete sample period (May 2014 - 8 April 2017), should lead to around 350 days and absolutely more than 300 days.

5.3 Weekly and Daily Bitcoin Volatility in GARCH Models

This part presents the results of studying Bitcoin's volatility with the GARCH (1,1), TGARCH (1,1) and GARCH-MIDAS models. Table 3 presents the estimation results of the GARCH (1,1) and the TGARCH (1,1) model for Bitcoin logarithm returns.

Table 3: The estimation results for the daily Bitcoin volatility in the GARCH (1,1) model and the TGARCH (1,1) model over 1 May 2014 - 8 April 2017.

Parameter	μ	ω	α	β	η
The GARCH (1,1) model					
Estimate	0.000720	0.000007	0.156583	0.836544	-
Std. Error	0.000340	0.000002	0.021329	0.009476	-
p-value	0.034013	0.001176	0.000000	0.000000	-
The TGARCH (1,1) model					
Estimate	0.000816	0.000854	0.195794	0.810782	0.048539
Std. Error	0.000231	0.000254	0.029651	0.033646	0.074289
p-value	0.000412	0.000783	0.000000	0.000000	0.513506

In Table 3, we see that both the mean of returns and the constant ω are significant, even though the estimate of the constant is very close to zero. The sum of α and β equals to 0.99 and is smaller than 1. Therefore, this GARCH (1,1) model is covariance stationary but it shows strong persistence of variance. The TGARCH (1,1) model shows that the Bitcoin's daily volatility is insensitive to negative returns as η is an insignificant estimate. Bitcoin's volatility does not show asymmetric property on returns and this is consistent to the conclusion by Bouri et al. (2017).

Table 4: The estimation results for the weekly Bitcoin volatility in the GRACH-MIDAS model considering the weekly CBOE NASDAQ 100 Voltility (VXN) over 1 May 2014 - 8 April 2017.

Parameter	μ	α	β	m	θ	ω_2
Estimate	0.10447543	0.21291632	0.77768712	3.72554046	-0.16907115	1.00000206
Std. Error	0.09645122	0.07158138	0.05207983	1.48122044	0.07425561	0.66752809
p-value	2.156564e-03	4.710554e-04	4.440892e-16	2.855026e-01	3.878685e-01	3.758946e-01

Table 4 shows the parameter estimates for the GARCH-MIDAS model considering weekly VXN as the long-term (weekly) volatility component. The short-term (daily) component of Bitcoin volatility follows the GARCH(1,1) process and we still see that daily conditional variance stationary. For the long term Bitcoin volatility, VXN shows a significant and negative influence on it as θ is negative. $\phi_1(\omega_1, \omega_2)$, the weight scheme with one week lag (this weight scheme is not present in the table as it needs manual print), is 0.02. As the coefficient π_1 is the multiplication of θ and $\phi_1(\omega_1, \omega_2)$, this suggests that a one standard deviation increase in VXN in a week leads to about -0.338% Bitcoin's volatility in the next week.

5.4 Bitcoin and Ethereum: Returns, Trading Volume and Numbers of Trades in the VAR Model

This part first shows the lag selection and stability inspection of the VAR model, and then exhibits our impulse responses analysis. Table 5 presents suggested VAR lag orders based on AIC, SC and HQ considering maximum 10 lags. According to Heij et al. (2004), both SC and HQ offer consistent estimators of lags for VAR models. Because the goal of our VAR model is observing the authentic effects of impulse responses, we prioritize selecting correct lag orders rather than minimizing prediction errors. Therefore, we do not consider the suggestion by AIC as it emphasizes more on reducing forecast errors. With the consideration of parsimony modelling, we select 2 lags as SC suggests.

Table 5: Based on selection criteria, the suggested lag orders in the Vector AutoRegression (VAR) model.

AIC	SC	HQ
8	2	6

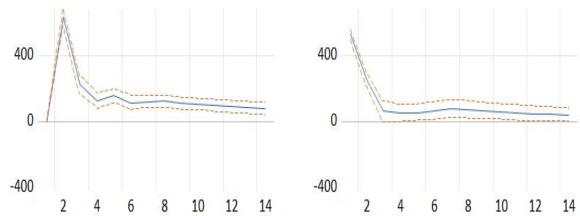
Note. The considered maximum lag order is 10.

As could be expected, taking logarithm returns indeed leads coins' returns to stationary processes. In Table 6, we see the results of the Augmented Dickey–Fuller (ADF) test show returns of both coins are stationary. In addition, apart from the Bitcoin trades, all other three variables are stationary under 95% confidence levels. If we loosen the confidence level to 90%, all six variables are stationary. Hence, we conclude the used these variables are suitable for constructing a VAR model. According to Eviews' results, no root of the coefficient matrix lies outside the unit circle. Therefore, this VAR model satisfies the stability condition and can be used to examine impulse responses.

Table 6: Results of the Augmented Dickey–Fuller (ADF) test for logarithm returns, volume and trades of Bitcoin and Ethereum over 11 August 2015 - 13 April 2017.

Variable	XBT Log Returns	XBT Volume	XBT Trades	ETH Log Returns	ETH Volume	ETH Trades
t-stats	-24.70828	-3.418578	-2.658772	-25.93423	-6.104142	-4.773666
Prob.	0.0000	0.0107	0.0820	0.0000	0.0000	0.0001

Due to a page limit, Figure 5, Figure 6 and Figure 7 only present valid impulse responses. The complete plot of all impulse responses is in the Appendix. In each plot, the blue line is the median impulse responses, the orange dashed lines are the boundaries of 95% confidence regions. The horizontal axis (in days) provides two weeks to investigate dynamics of concerned responses. The vertical axis are the scales of the affected variable. The unit of a shock is one standard deviation. If one plot has boundaries covering both positive and negative areas with median impulse response in the middle right after an impulse, no conclusion can be made. This is because, the response with that shape may have no reaction to shocks and this impulse responses become invalid to report.

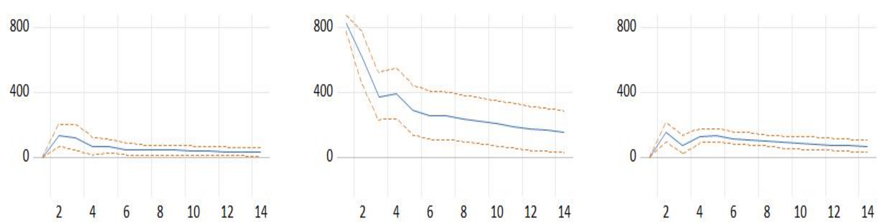


(a) Bitcoin trades (b) Bitcoin volume

Figure 5: Bitcoin volume's responses to a shock of

Note. The impulse refers to one standard deviation shock. The blue line is the median impulse response and the orange dashed lines are the boundaries of 95% confidence regions. The observation period is 14 days.

In Figure 5(a), for one standard deviation shock of Bitcoin trades (1831 trades), we see Bitcoin's trading volume jumps to 600 within three days. After that, Bitcoin's trading volume drops in two day and this response's effect dies out gradually. For Bitcoin trading volume itself (see Figure 5(b)), the response to one shock of its own diminishes in three days.

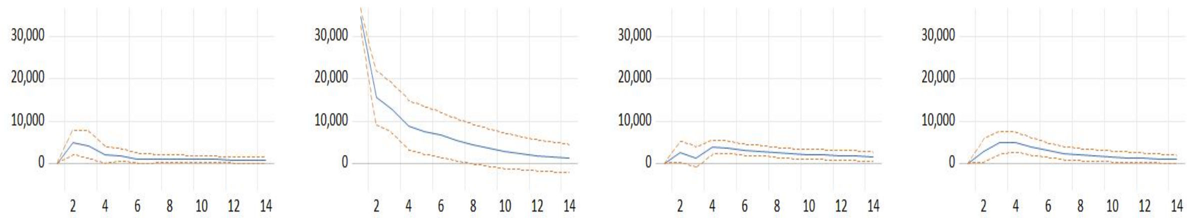


(a) Ethereum returns (b) Ethereum trades (c) Bitcoin trades

Figure 6: Ethereum trades' responses to a shock of

Note. See Figure 5

For one standard deviation shock of Ethereum returns, the numbers of Ethereum trades increases and this refers to more active trading in three days, causing about 150 more trades. Considering the median of Ethereum trades (479), this is response is influential. The vanishment of self response to Ethereum's numbers of trade takes nearly two weeks as shown in Figure 6(b). In Figure 6(c), we see an response between two coins, Ethereum trades positively respond to a positive increase of Bitcoin trades but this response is only significant in two days and dissipates afterwards.



(a) Ethereum returns (b) Ethereum volume (c) Bitcoin trades (d) Bitcoin volume

Figure 7: Ethereum volume's responses to a shock of

Note. See Figure 5

The trading volume of Ethereum shows a positive reaction to increases of Ethereum returns, Bitcoin trades and Bitcoin volume as shown in Figure 7. About a 5000 more trading Ethereum volume responds to one positive shock in Ethereum returns, this effect lasts roughly four days. Given that the mean of Ethereum's trading volume is 19091, this volume increase is crucial. Ethereum's trading volume also exhibits responses to Bitcoin's information. For positive shocks in the numbers of trades and the trading volume in Bitcoin, Ethereum's trading volume replies with positive responses in five days. Lastly, Ethereum's trading volume needs nearly 12 days to return to the original level after one own standard deviation shock.

For these two coins, their returns show no responses to shocks in any other variables. Since that trading volume and numbers of trades cannot affect returns of the two coins, this may show the idea of intrinsic value for Bitcoin and Ethereum, at least within this sample period. In addition, Bitcoin's number of trades is also insensitive to other factors. Lastly, most of those impulse responses ease up after three days. This may remind investors to pay extra attention to the cryptocurrency market in first three days after one shock mentioned above.

6 Conclusion

This research studies the daily Bitcoin volatility over May 2014 - April 2017 using Heterogeneous AutoRegressive Jumps (HARJ) model and the (Threshold) Generalized AutoRegressive Conditional Heteroskedasticity ((T)GARCH) models. Our findings show the HARJ model, modified by Pichl and Kaizoji (2017), moderately captures the dynamics of Bitcoin's daily realized volatility with earlier realized volatility and earlier jumps from five days ago, but this model does not provide very accurate predictions. The GARCH (1,1) and TGARCH (1,1) models demonstrate that Bitcoin's daily conditional volatility is stationary and symmetric to both positive and negative returns. For the same period, this study also uses MultiLayer Perceptron (MLP) technique to predict daily logarithm returns of Bitcoin. The prediction with this machine learning measure tends to over forecast the magnitudes of daily returns. We also apply the GARCH-MIDAS model with weekly CBOE Nasdaq 100 Volatility (VXN) as an explanatory variable to study weekly Bitcoin volatility. We find an increase of weekly VXN would slightly decrease Bitcoin volatility one week later. Lastly, this research implements a Vector AutoRegression (VAR) model to study the impulse responses of logarithm returns, numbers of trades and trading volume of Bitcoin and Ethereum on their own shocks over August 2015 - April 2017. In the VAR model, we see that the numbers of trades and trading volume of Ethereum positively react to increases of Ethereum returns and numbers of Bitcoin trades. Additionally, Bitcoin's trading volume has a positive reaction to its numbers of trades. Furthermore, returns of both coins are insensitive when encountering shocks in trading volume or numbers of trades.

The limitations in this research and recommendation for future work are as follows. For the GARCH-MIDAS model, our result finds that VXN somehow provides limited information about weekly Bitcoin volatility. There are two suggestions for this. Firstly, future study could attempt other weekly variables like VIX or commodity indexes to examine whether more prominent factors in weekly basis exist. Secondly, the frequency gap between weekly data and daily data is much smaller than that with monthly data. As previous studies focus on monthly variables, the variables in bi-weekly (or other intermediate frequencies) could be fresh sources to study volatility in Bitcoin. In the VAR model, we discover that in the residual covariance matrix, for Ethereum, its trading volume and numbers of trades are (significantly) positively correlated. This means their shocks are also positively correlated. Although the shocks of either of them receive no responses, we could consider some restrictions on these two variables beforehand. Further researchers could also attempt trading volume and/or numbers of trades on other assets when developing VAR models. In addition, as we find three days is a common timing pattern in the impulse responses, future study could attempt study Bitcoin and other cryptocurrencies in a basis of three days.

References

Andersen, T. G., Bollerslev, T., & Diebold, F. X. (2007). Roughing it up: Including jump components in the measurement, modeling, and forecasting of return volatility. *The Review of Economics and Statistics*, *89*(4), 701–720.

Ardia, D., Bluteau, K., & Rüede, M. (2019). Regime changes in Bitcoin GARCH volatility dynamics. *Finance Research Letters*, *29*, 266–271.

Azari, A. (2019). Bitcoin price prediction: An ARIMA approach. *ArXiv Preprint ArXiv:1904.05315*.

Badea, L., & Mungiu-Pupzan, M. C. (2021). The economic and environmental impact of Bitcoin. *IEEE Access*, *9*, 48091–48104.

Beneki, C., Koulis, A., Kyriazis, N. A., & Papadamou, S. (2019). Investigating volatility transmission and hedging properties between Bitcoin and Ethereum. *Research in International Business and Finance*, *48*, 219–227.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, *31*(3), 307–327.

Boudt, K., Cornelissen, J., Payseur, S., Nguyen, G., Kleen, O., & Sjoerup, E. (2017). Highfrequency: Tools for highfrequency data analysis. *R Package Version 0.5*.

Bouri, E., Azzi, G., & Dyhrberg, A. H. (2017). On the return-volatility relationship in the Bitcoin market around the price crash of 2013. *Economics*, *11*(1).

Chai, T., & Draxler, R. R. (2014). Root mean square error (RMSE) or mean absolute error (MAE)?–arguments against avoiding RMSE in the literature. *Geoscientific Model Development*, *7*(3), 1247–1250.

Chen, C., Liu, L., & Zhao, N. (2020). Fear sentiment, uncertainty, and Bitcoin price dynamics: The case of COVID-19. *Emerging Markets Finance and Trade*, *56*(10), 2298–2309.

Choi, S., & Shin, J. (2022). Bitcoin: An inflation hedge but not a safe haven. *Finance Research Letters*, *46*, 102379.

Conrad, C., Custovic, A., & Ghysels, E. (2018). Long-and short-term cryptocurrency volatility components: A GARCH-MIDAS analysis. *Journal of Risk and Financial Management*, *11*(2), 23.

Conrad, C., & Kleen, O. (2020). Two are better than one: Volatility forecasting using multiplicative component GARCH-MIDAS models. *Journal of Applied Econometrics*, *35*(1), 19–45.

Dyhrberg, A. H. (2016). Bitcoin, gold and the dollar–a GARCH volatility analysis. *Finance Research Letters*, *16*, 85–92.

Estrada, J. C. S. (2017). Analyzing Bitcoin price volatility. *Working Paper. University of California, Berkeley.*

Gandal, N., Hamrick, J., Moore, T., & Oberman, T. (2018). Price manipulation in the Bitcoin ecosystem. *Journal of Monetary Economics*, *95*, 86–96.

Gervais, A., Karame, G. O., Capkun, V., & Capkun, S. (2014). Is Bitcoin a decentralized currency? *IEEE Security & Privacy*, *12*(3), 54–60.

Ghalanos, A. (2020). Rugarch: Univariate GARCH models. *R Package Version 1.4-1*.

Giudici, P., & Abu-Hashish, I. (2019). What determines Bitcoin exchange prices? a network VAR approach. *Finance Research Letters*, *28*, 309–318.

Gnther, F., & Fritsch, S. (2016). Neuralnet: Training of neural networks. *R Package Version 1.33*.

Heij, C., de Boer, P., Franses, P. H., Kloek, T., & van Dijk, H. K. (2004). *Econometric Methods with Applications in Business and Economics*. Oxford University Press.

Jeon, B. N., & Jang, B.-S. (2004). The linkage between the US and Korean stock markets: The case of NASDAQ, KOSDAQ, and the semiconductor stocks. *Research in International Business and Finance*, *18*(3), 319–340.

Kleen, O. (2018). Mfgarch: Mixed-frequency GARCH models. *R Package Version 0.1*.

Krištoufek, L. (2015). What are the main drivers of the Bitcoin price? evidence from wavelet coherence analysis. *PLoS One*, *10*(4), 1–15.

Luu Duc Huynh, T. (2019). Spillover risks on cryptocurrency markets: A look from VAR-SVAR granger causality and student'st copulas. *Journal of Risk and Financial Management*, *12*(2), 52.

McNally, S., Roche, J., & Caton, S. (2018). Predicting the price of Bitcoin using machine learning. *2018 26th Euromicro International Conference on Parallel, Distributed and Network-Based Processing (PDP)*, 339–343.

Pichl, L., & Kaizoji, T. (2017). Volatility analysis of Bitcoin. *Quantitative Finance and Economics*, *1*(4), 474–485.

Riley, J. (2021). The current status of cryptocurrency regulation in China and its effect around the world. *China and WTO Review*, *7*(1), 135–152.

Sasson, E. B., Chiesa, A., Garman, C., Green, M., Miers, I., Tromer, E., & Virza, M. (2014). Zerocash: Decentralized anonymous payments from Bitcoin. *2014 IEEE Symposium on Security and Privacy*, 459–474.

Sims, C. A. (1980). Macroeconomics and reality. *Econometrica: Journal of the Econometric Society*, 1–48.

Smales, L. A. (2022). Investor attention in cryptocurrency markets. *International Review of Financial Analysis*, *79*, 101972.

Zakoian, J.-M. (1994). Threshold heteroskedastic models. *Journal of Economic Dynamics and Control*, *18*(5), 931–955.

Appendix

1. The Codes and Data Set

This research also provides the relevant codes and data sets corresponding to four different parts in our methodology. The codes are with necessary comments.

2. The Plot of Weekly CBOE Nasdaq 100 Market Volatility Index (VXN)

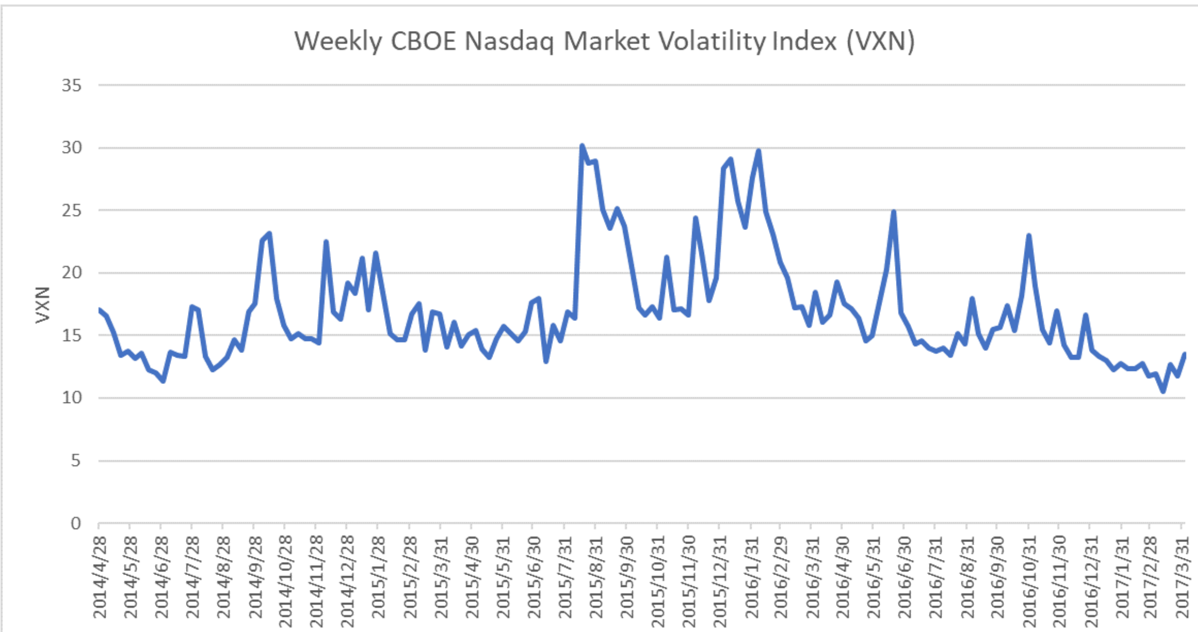
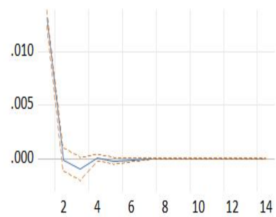
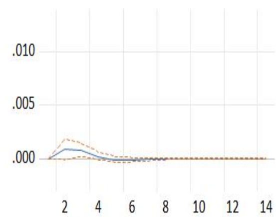


Figure 8: Weekly CBOE Nasdaq Market Volatility Index (VXN) over the study period (1 May 2014 - 8 April 2017) for the GARCH-MIDAS model. The series starts from 28 April 2014 (17th Friday of 2017) is because this index uses the close prices on every Friday.

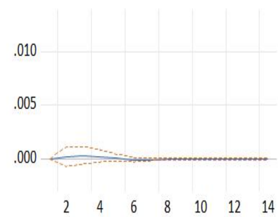
3. Complete Plots of All Impulse Responses



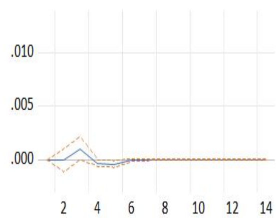
(a) Bitcoin returns



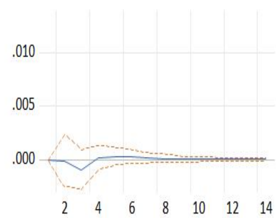
(b) Bitcoin volume



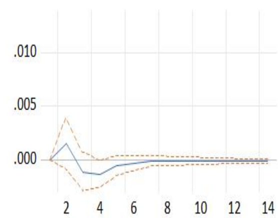
(c) Bitcoin trades



(d) Ethereum returns



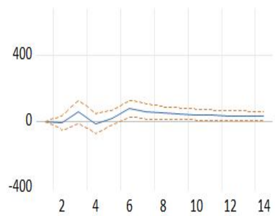
(e) Ethereum volume



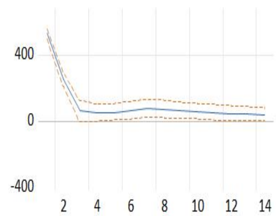
(f) Bitcoin trades

Figure 9: Bitcoin returns' responses to a shock of

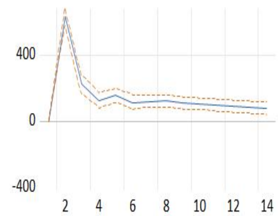
Note. The impulse refers to one standard deviation shock. The blue line is the median impulse response and the orange dashed lines are the boundaries of 95% confidence regions. The observation period is 14 days.



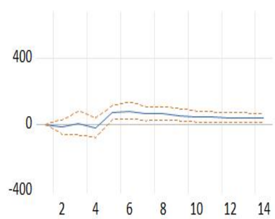
(a) Bitcoin returns



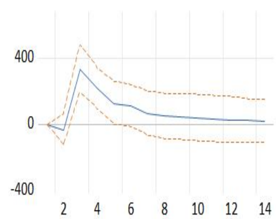
(b) Bitcoin volume



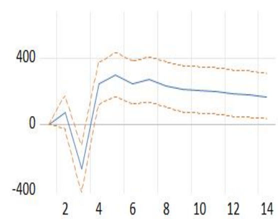
(c) Bitcoin trades



(d) Ethereum returns



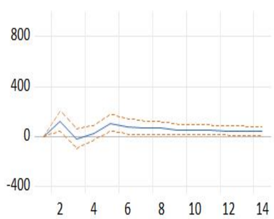
(e) Ethereum volume



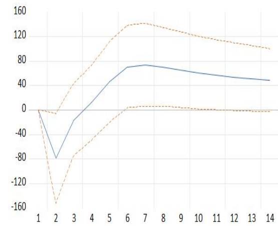
(f) Bitcoin trades

Figure 10: Bitcoin volume's responses to a shock of

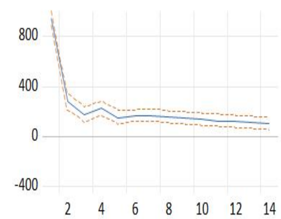
Note. The impulse refers to one standard deviation shock. The blue line is the median impulse response and the orange dashed lines are the boundaries of 95% confidence regions. The observation period is 14 days.



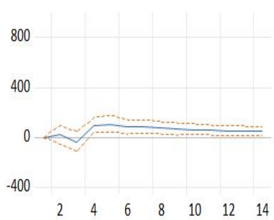
(a) Bitcoin returns



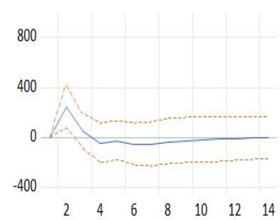
(b) Bitcoin volume



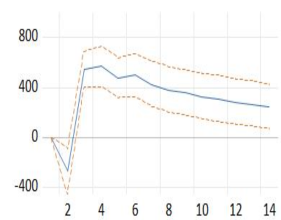
(c) Bitcoin trades



(d) Ethereum returns



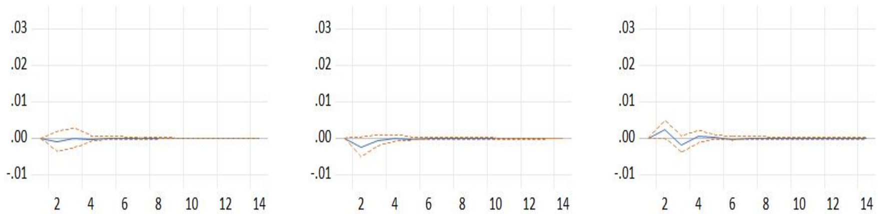
(e) Ethereum volume



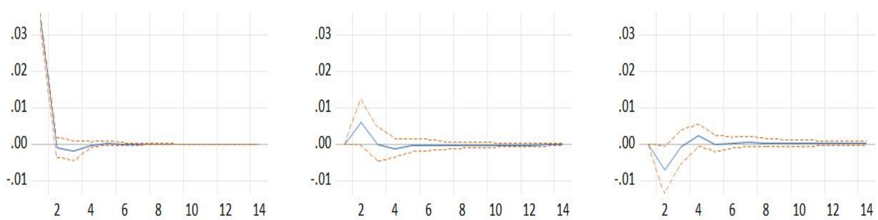
(f) Bitcoin trades

Figure 11: Bitcoin trades' responses to a shock of

Note. The impulse refers to one standard deviation shock. The blue line is the median impulse response and the orange dashed lines are the boundaries of 95% confidence regions. The observation period is 14 days.



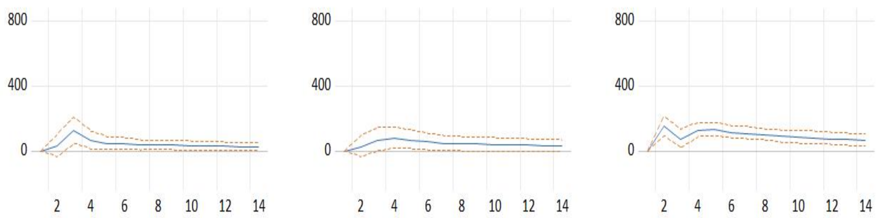
(a) Bitcoin returns (b) Bitcoin volume (c) Bitcoin trades



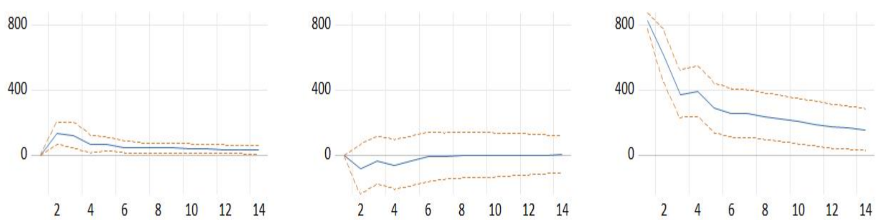
(d) Ethereum returns (e) Ethereum volume (f) Bitcoin trades

Figure 12: Ethereum returns' responses to a shock of

Note. The impulse refers to one standard deviation shock. The blue line is the median impulse response and the orange dashed lines are the boundaries of 95% confidence regions. The observation period is 14 days.



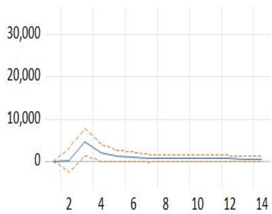
(a) Bitcoin returns (b) Bitcoin volume (c) Bitcoin trades



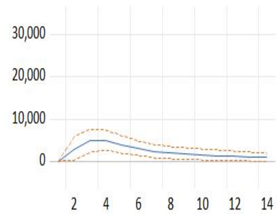
(d) Ethereum returns (e) Ethereum volume (f) Bitcoin trades

Figure 13: Ethereum volume's responses to a shock of

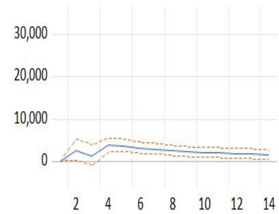
Note. The impulse refers to one standard deviation shock. The blue line is the median impulse response and the orange dashed lines are the boundaries of 95% confidence regions. The observation period is 14 days.



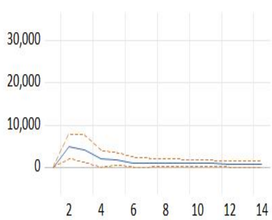
(a) Bitcoin returns



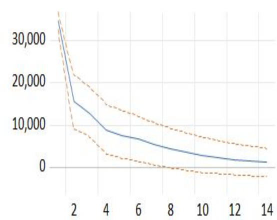
(b) Bitcoin volume



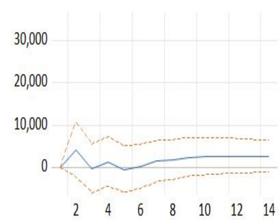
(c) Bitcoin trades



(d) Ethereum returns



(e) Ethereum volume



(f) Bitcoin trades

Figure 14: Ethereum trades' responses to a shock of

Note. The impulse refers to one standard deviation shock. The blue line is the median impulse response and the orange dashed lines are the boundaries of 95% confidence regions. The observation period is 14 days.