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How to find the best fit: Fama-French three-factor model with macroeconomic risk factors added.

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Abstract

We test the new Principal Component Analysis methodology of Lettau & Pelger (2020a) that is used for estimating latent asset pricing factors that fit the time series and cross-sectional returns. The estimator is named the Risk-Premium PCA (RP-PCA) and adds an extra penalty term on the cross-sectional pricing errors of the returns. We extend the empirical literature of Lettau & Pelger by comparing RP-PCA and PCA to various portfolios adjusted for Fama-Frech and macroeconomic factor risk. We find results that show that, on average, RP-PCA outperforms PCA in having higher Sharpe ratios and lower cross-sectional pricing errors. This outperformance becomes more evident when the effect of strong systematic risk factors are removed from the portfolios beforehand. In addition, the results showed that adding macroeconomic factors to the Fama-French three factor model led to higher Sharpe ratios and lower cross-sectional pricing errors. However, it also became clear that the original Fama-French factors are more important collectively than the added macroeconomic factors for gold, oil, GDP growth and USD.

Contents

1 Introduction

Financial research indicates that the cross-sectional variance of financial assets can be explained through a variety of systematic risk factors, such as the well-known beta of the Capital Asset Pricing Model (CAPM; Treynor, 1962). The search for these risk factors has provided us with over 300 different indicators that explain the cross section of expected stock returns (Harvey et al., 2016). Navigating through this "factor zoo" raises the question as to which factors are most critical in explaining the expected returns. So what kind of factors determine the asset prices and could play a role in building a model that can help explain returns in the best and robust manner?

In 1952, Markowitz kicked off the theoretical discussions through his work on Modrern Portfolio Theroy. Sharpe (1964), Lintner (1965) and Mossin (1966) then moved on to create - on the foundation laid out by Markowitz - the first and most fundamental version of the general equilibrium model, the CAPM. The CAPM is valid under certain assumptions and uses the market beta, also called the market risk premium, as its key and sole risk factor. Although the CAPM is still widely used as an asset pricing model, researchers have found empirical results that seemed to indicate that the anomalies found were too frequent and persistent to confirm that CAPM was indeed the whole story, notwithstanding the fact that any empirical test would always bump into the issues related to the way in which the market portfolio is measured/defined (Gibbons, 1982; Coggin & Hunter, 1985). Therefore, people started searching for improvements and/or alternatives. In 1976, Ross developed the Arbitrage Pricing Theory (APT) which predicts a relation between the cross-sectional expected returns and the exposure to *a set* of systematic risk factors. Unlike the CAPM, the APT is not an equilibrium model and therefore does not identify systematic factors, nor does it assume that there should be only one of them. Thus, to implement APT, it was expected and shown that multiple risk factors would play a structural role in explaining asset returns.

In 1993, Eugene Fama and Kenneth French came up with two new factors next to the CAPM beta, SMB and HML. Stateted differently, other than Ross with his APT they did not totally distance themselves from CAPM, but instead focused on creating an improved version of it that would better stand the empirical tests. By combining the SMB and HML factors with the beta of the original CAPM, the famous Fama-French three-factor model was constructed. In recent decades, the three-factor model has been considered the most important model for explaining stock returns. Unfortunately, the three-factor model is not able to explain all of the cross-sectional variation of the returns (Griffin, 2000) and therefore the search for other systematic factors and/or improved model structures has continued.

To identify these systematic factors in large cross-sectional datasets, Lettau & Pelger (2020a) came up with a new method to identify these risk factors by using a generalization of the well-known Principal Component Analysis (PCA), called Risk-Premium PCA (RP-PCA). RP-PCA works in such a way that it resolves cross-sectional pricing errors by adding an additional penalty term to the standard objective function of PCA. This additional penalty term ensures that the PCA now takes into account information of the first and second moments of the data. Another paper of Lettau & Pelger (2020b) found that RP-PCA is superior to traditional PCA in several respects and is asymptotically more efficient. In particular, it became clear that RP-PCA outperforms PCA in the case of having 'weak factors' in the data, factors that only affect a subset of the total assets. Our work will be closely related to the paper of Lettau & Pelger (2020b) in that we will not only look for new variabes over-and-above what is included in Fama-French, but we will also check for model improvements by going from PCA to RP-PCA.

Since the method used in Letta & Pelger (2020a, 2020b) is relatively new, we extend their empirical applications by testing whether RP-PCA still outperforms PCA in a different setting with the same portfolios of stocks. Namely, Lettau & Pelger's articles showed that RP-PCA performs especially well when the latent factors in the portfolio returns are weak. To verify this relationship, we empirically tested this on a dataset, where the effects of 'strong' factors, such as the Fama-French factors, were removed from the returns in advance. This was done by regressing the portfolio returns on a number of 'proven' and 'unproven' systematic risk factors. By doing so, we removed the systematic risk embodied in these factors from the returns, and therefore should obtain returns that are composed of less cross-sectional variation.

The 'proven' systematic risk factors in this paper are the Fama-French factors, whose influence on the explanation of stock returns has been demonstrated time and time again. For the 'unproven' systematic factors, we wanted to add to the existing literature of the 'factor zoo' (Harvey et al., 2016) by examining whether macroeconomic factors could also play a role in explaining returns over-and-above the role these factors may have played already indirectly via the Fama-French factors. It is well known that macroeconomic variables can have a substantial impact on the stock market (Humpe & Macmillan, 2009; Sirucek, 2012). If only already, because the daily financial news, and the buying and selling behavior of market participants are to a large extent based on their evaluation of the market climate. With these market climate evaluations often being based to quite some extent on top-down considerations in which macroeconomic news plays a key role. It would therefore not be illogical when we end up seeing this effect during more extreme periods of climate change or shocks. A great example of this is the oil crisis of November 1973, which had a very negative influence on the stock returns. Therefore, the objective is to create a more sophisticated model that better predicts the performance of the stock market by adding macroeconomic factors to the original Fama-French three-factor model. The macroeconomic factors that have been created are an Oil factor, a Gold factor, a GDP factor and an USD factor. They have been constructed/proxied on the basis of ETFs, such that the factors are tradeable and more easily accessible to all investors. With these macroeconomic and Fama-French factors, different portfolios have been constructed and adjusted for the systematic risk associated with these factors.

The objective of this paper was therefore twofold, (i) to verify the dominance of RP-PCA over PCA using a transformed dataset and (ii) to investigate whether the inclusion of macroeconomic factors, next to Fama-Frech factors, can better explain the cross-sectional variation in returns. Applying RP-PCA to the portfolios, the 'replication' portfolios and our constructed adjusted portfolios, yielded results consistent with the conclusions made by Lettau & Pelger (2020b). For the replication portfolios, we observed similar results, showing that RP-PCA consistently outperforms PCA based on higher Sharpe Ratios and lower cross-sectional pricing errors, while the unexplained idiosyncratic variation is of similar magnitude. Also for our adjusted portfolios, we found that, on average, RP-PCA outperforms PCA, both in-sample and out-of-sample. Moreover, we observed that removing the systematic risk factors from the portfolios in advance, led to even larger difference between the RP-PCA and PCA methods. This confirms the hypothesis that the outperformance of RP-PCA becomes especially evident when we remove the obvious, stronger factors from the data, leaving only the 'weaker' factors behind. Moreover, the results showed that the addition of macroeconomic factors to the three-factor model led, on average, to higher maximal Sharpe Ratios and lower idiosyncratic pricing errors for both in-sample and out-of-sample.

The rest of the paper is structured as follows. In Section 2 we provide a literature review on RP-PCA and the relationship between the factors and stock returns. Section 3 shows us the data that is used for the empirical applications. Section 4 elaborates on the methodology of the RP-PCA and the construction of the macroeconomic factors. Section 5 shows us the empirical results and Section 6 concludes.

2 Literature review

2.1 Background on RP-PCA

In recent years, there has been extensive research on approximate factor models in the field of finance (e.g. Bai & Ng, 2008; Ludvigson and Ng, 2009). Typically, Principal Component Analysis (PCA) finds hidden factors in the data that capture most of the time-series variation in the data. What often happens with PCA is that it performs poorly in the case of weak factors (Onatski, 2012). Factors can be weak when they have relatively little explanatory power over the idiosyncratic noise. In some cases, economic theory can impose a structure on the first moments of the dataset. Ensuring that this information is taken into account can significantly improve the estimation of the factors, especially when these factors have little explanatory power for the variance.

Therefore, Lettau and Pelger (2020a) came up with a new improved version of the traditional Principal Component Analysis that also takes into account information in the first moments. They did this by using the arbitrage pricing theory (APT) of Ross (1976) which states that exposure to systematic risk factors should explain cross-sectional expected returns. Traditionally, PCA constructs factors that are unable to account for the cross-sectional explanatory power of *average returns*, but is instead only able to capture the time-series *covariation*. For that reason, Lettau and Pelger (2020a) came up with the new estimator Risk-Premium Principal Component Analysis (RP-PCA) that adds an extra penalty component that accounts for pricing errors in the mean returns. And as has been proven by Lettau & Pelger (2020b), adding this additional information to the object of interest helps to significantly improve the estimation of the factors. The outperformance of RP-PCA over PCA is most evident when the methods need to identify weak factors. Strong factors are those that affect all underlying assets, such as the well-known Fama-French factors. Weak factors are more difficult to find because they affect only a subset of the assets. Anomaly-based factors or asset-pricing factors often fall into this category. RP-PCA is able to detect these weak factors with associated high Sharpe Ratios, while PCA fails to do so.

This paper adds to the existing econometric literature on estimating factors from large cross-sectional data sets. For example, Bai (2003) treated a large static dimensional factor model and Forni et al. (2000) created a new dynamic PCA method. These papers all created methods under the assumption of a strong factor structure. Onatski (2012) showed that using PCA for large factor models can produce rather cumbersome results when weak factors are involved.

2.2 Fama-french factors

Many researchers have spent time on the traditional Capital Asset Pricing Model (CAPM) to help improve the literature on explaining cross-sectional stock returns. The CAPM is the traditional asset pricing model and uses only one factor, the market risk premium, to help explain stock returns. Although the original CAPM is an exceptional model, any empirical test will one or the other way have to be transposed by dropping the expectations (E) sign and going instead for a test in which realized empirical variables are used as the basis. The risk premium can be defined as the difference between the expected market return and the risk-free rate.

$$R_{it} - R_{ft} = \alpha_{it} + \beta_1 \left(R_{Mt} - R_{ft}\right)$$

Papers such as Basu (1977) and Banz (1981) found empirical return patterns that were not consistent with the returns patterns expected by the CAPM. In 1992 and 1993, Fama and French came up with two groundbreaking papers that showed that size and value, measured by market capitalization and book-to-market ratio respectively, were two very important strong factors that helped explain the cross-sectional returns of stocks much better than the standard 1-factor CAPM. With that, they extended the traditional CAPM by adding size risk and value risk factors to the model. This model is the well-known three factor model that explains stock returns by the market risk premium, the small minus big (SMB) factor and the high minus low (HML) factor.

$$R_{it} - R_{ft} = \alpha_{it} + \beta_1 \left(R_{Mt} - R_{ft}\right) + \beta_2 SMB_t + \beta_3 HML_t + \epsilon_{it}$$

Here the R_{it}, R_{ft} and R_{Mt} represent the returns of portfolio i, the returns of the riskfree rate and the returns of the total market portfolio m, respectively.

The SMB factor is a variable for the tendency of small-cap companies to outperform large-cap companies in the long run. The HML factor consists of firms with the 'highest' book-to-market ratio minus those with the 'lowest' book-to-market ratio. Generally, stocks with a high book-to-market ratio are considered *value stocks*, while stocks with low book-to-market ratio stocks are considered *growth stocks.* The HML represents a factor that reflects the trend of value stocks to outperform growth stocks over a longer time period. The factors are considered to be strong factors due to them playing a large role in affecting the US stock markets. (Fama & French, 1995; Lambert & Hübner, 2014). The α in the CAPM and the three-factor model represents the excess or abnormal return. If the model were fully capable of explaining the stock returns, the estimated alpha should be significantly close to zero. Hence, the α, combined with the ϵ, is the part of the model that is not explained by the incorporated risk factors.

2.3 Oil

As Adelman (1993) has stated, the effects of fluctuations in the oil prices on OECD countries should not be downplayed. These fluctuations in oil prices are so significant that economic growth forecasts are often made under the assumption that there are no oil shocks in the world. For this reason, the relationship between crude oil prices and stock returns has been extensively studied in the past (e.g. Apergis & Miller, 2009; Narayan & Sharma, 2011; Scholtens & Yurtsever, 2012)

Although it is widely believed that changes in the price of crude oil have an effect on the stock market, there is still no consensus on what the relationship between oil prices and stock returns is. And that is not per se a big surprise, knowing that some countries are net oil importers and other net exporters. With oil companies normally being relatively big firms on their local stock markets, this could boil down to a situation in which oil price increases are good news for oil exporting countries and their stock markets and bad news for the net importers, and vice versa. But the return effect is of course not the full story. It is also perfectly possible that the nervousness in the oil markets translates into volatile pricing that - via the infusion of this volatility and nervousness into stock markets - does also have an important indirect effect irrespective of the direction of expected oil price changes.

Kling (1985) found evidence that increases in the price of crude oil would lead to declines in the stock markets, while Chen et al. (1986) later argued that fluctuations in the oil prices do not significantly affect asset prices at all. A study by Huang et al. (1996) focused on the relationships between changes in future oil prices and stock returns. Using a vector autoregression model, they concluded that oil returns of futures only affected the returns of certain individual oil companies in a negative way, while this effect did not occur for broader indexes of stocks such as the S&P 500. On the other hand, Jones and Kaul (1996) used quality data to examine the effect of oil shocks on the international stock markets. Unlike Huang et al., they did find a significant negative relationship between fluctuations in the oil prices and the aggregate stock markets of Canada and the US, and less so for Japan and the UK.

Sadorsky (1999) also investigated the relationship between crude oil returns and real stock returns (i.e. with the incorporation of inflation-adjustment), specifically for the US. He concluded that both changes in the oil price and oil price volatility played a role in affecting stock returns. In addition to establishing this relationship, he found evidence that the dynamics of crude oil prices changed over time. Killian & Park (2009) found that *demand* shocks to oil prices led to increases in stock returns, while oil *supply* shocks had no significant effect.

Hence, it seems that oil prices have an effect on stock returns, but an unambiguous relationship has yet to be found. And it might be the case that such a strong relationship can never be expected without knowing if a country of analysis is a net producer or consumer of oil. In our study we use a US data set, with the US being a net exporter of fossil fuels.

2.4 Gold

Gold is a versatile metal because it can be used as a medium of exchange, a unit of value and also act like a source of wealth. (Goodman, 1956). Traditionally, gold was often seen as a good indicator of future inflation and is therefore often used as a hedge against inflation and as a safe alternative to the stock market. Baur & Lucey (2010) argued that an asset can be regarded as a hedge when there is no correlation or a negative correlation with another asset in times of turbulent markets. They found that gold can be used as a hedge for the stock market and can be a save haven in times of large fluctuations and market nervousness. In addition, there have been several other researchers who have tried to investigate whether gold can be used as a hedge for the stock market. (e.g. Baur & McDernott, 2010; Chkili et al., 2014).

There have also been studies that attempted to infer diversification opportunities between the gold and stock markets. Another study by Kumar (2014) examined the return and volatility spillovers between gold and the Indian stock market. He was able to conclude that adding gold in a portfolio led to a better diversified portfolio with higher risk-adjusted returns. And Arouri et al. (2015) found results between gold prices and the Chinese stock market that show significant returns and volatility spillovers between the two. Moreover, they argue that the risk of portfolios composed solely of stocks is reduced by the addition of gold due to the hedging against 'equity market risk' in bearish periods.

Therefore, the general belief is that the gold price and stock returns have an inverse relationship which becomes particularly evident in the event of a severe decline in the stock market.

2.5 GDP

Often the stock market is seen as a sentiment indicator that can influence the GDP. When equity markets move down or up, often the sentiment in the economy does too. And the opposite is also seemingly logical: when the economy is growing, business and consumers normally spend more. And spending more does also translate into buying more products and services provided by the stock-market-listed companies. Therefore, people often think that GDP growth is also good for the stock markets and its stock holders. However, a study by Ritter (2005) showed that during the period 1900-2002, there was actually a negative correlation between per capita GDP growth and real stock returns. Another study by by Dimson et al. also showed the relation between GDP per capita and real stock returns for 16 different countries. Over a period from 1900-2000, he found a negative and seemingly unrelated relationship between GDP growth and real stock returns.

Another study, conducted by Siegel, (1998) also asserted the existence of a negative correlation between per capita GDP growth and real stock returns. This relationship between GDP growth and stock returns, according to Siegel, is probably due to the fact that high growth had already been incorporated into the stock prices. Investors all the the time analyze trends in the past and translate that into expectations for the future. So whenever the expectation is that the economy is going to improve, more buyers than sellers will enter the market and prices tend to go up. The opposite happens when market participants believe that GDP is expected to go down.

Nonetheless, economic theory and standard economic growth models such as Solow's (1956) growth model predict a positive relationship between economic growth and stock returns in the long run. Unfortunately, there is inconsistency between the theory and the empirical results that have been obtained. Although there have been papers that have also found a positive correlation between GDP and and stock returns (e.g. Fama, 1990;, Lovatt & Parkih, 2000; GallegatI, 2008), these relate to a much shorter sample period than in the aforementioned cases.

According to Madsen et al. (2013), this ambiguous relationship between GDP growth and stock returns is due to underlying output volatility caused by shocks in the productivity. In other words, Madsen et al. argue that it has to do with the predictability of GDP growth and thus largely depends on the persistence of these shocks. Using a dataset of 20 OECD countries, they found that with persistent output volatility, the relationship between GDP growth and stock returns is positive, thus supporting the theory.

2.6 USD

The value of the US dollar (USD) has tended to go up an down a lot in recent decades. Stock returns could be positively or negatively affected by these fluctuations in the value of the USD. Because on the one hand, a depreciation of the USD leads to an increase in exports and an improved competitive position in the world. This while one could also argue that the domestic price levels will go up and the costs of imports will increase due to the depreciation. The first line of thought is expected to lead to an increase in stock returns, while the second line of thought should lead to a decrease in stock returns. Therefore, it is necessary to test empirically how changes in the value of the USD affect the stock market.

A study by Solnik (1987) shows that there is a weak positive correlation between changes in the exchange rates and stock returns. This is while Soenen & Hennigar (1988) showed that there was a significant negative correlation between fluctuations in the value of the USD and the US stock market. Goodwin et al. (1992) confirmed this relation by showing that equity markets are negatively affected by changes in the value of the dollar. In contrast, Aggarwal (1981) found exactly the opposite of Goodwin et al., namely that the USD has a significant positive effect on the returns of US stocks.

On the other hand, studies have also been published showing that stock prices adjust quickly to changes in the USD (Franck & Young, 1972). Similarly, a study by Bahmani-Oskooee & Sohrabian (1992) found no long-run relationship between the effective exchange rate of the USD and the composite stock price of the S&P 500.

3 Data

For the empirical application three different cases will be done. First, we will make use of double-sorted portfolios from the website of Professor Kenneth French. The double-sorted set of 5x5 quintile portfolios are categorized based on size and book-to-market, accruals, investment, profitability, momentum, short-term reversal, volatility and idiosyncratic volatility. For the second case we will be making use of a large cross-section of single sorted decile portfolios. These portfolios are based on anomaly characteristics and can be obtained from the website of Serhiy Kozak. The single-sorted decile portfolios we use are constructed on the basis of 37 characteristics that were selected in the paper of Kozak, Nagel and Santosh (2020). For both cases we will be making use of a monthly dataset covering the period from November 1963 till December 2017, which provides us with 650 observations.

For the third case we will be constructing our own dataset by transforming the double-sorted portfolios of Kenneth French. This time we make use of daily data, which is available for the 5x5 portfolios based on size and book-to-market, investment, operating profitability, short-term reversal and momentum. To transform this dataset we make use of the three factors from the Fama-French model (available on the website of Kenneth French). As indicated earlier, for the macroeconomic factors we will make use of tradeable securities and have therefore opted to go with relevant ETFs. For the oil and gold factors we will be making use of daily data from the United States Oil Fund, LP (USO) ETF and the SPDR Gold Shares (GLD) ETF. For the USD exchange rate we make use of the Invesco DB US Dollar Index Bullish Fund (UUP) ETF. An appropriate ETF for the US GDP is unfortunately not readily available. We will therefore use as a proxy an ETF focused on the index of industrial production (Fulop & Gyomai, 2012), where we will use the Industrial Select Sector SPDR Fund (XLI) ETF. The data range for these daily datasets is from 22 February 2007 till 29 April 2022.

4 Methodology

4.1 PCA

In our research, we assume that we are working with portfolios consisting of excess returns that follow an approximate factor model. The model for the returns of N test assets over a time series of T observations, looks as follows:

$$X_{nt} = \mathrm{F}_t \Lambda_n^\top + e_{nt} \quad n = 1, \dots, N, t = 1, \dots, T \tag{1}$$

$$\iff \underbrace{\mathrm{X}}_{T\times N} = \underbrace{\mathrm{F}}_{T\times K} \underbrace{\Lambda^\top}_{K\times N} + \underbrace{\mathrm{e}}_{T\times N}, \tag{2}$$

In this model, the X_{nt} represent the excess returns of the assets n. The rationale is that the K factors represent the systematic component of the excess returns, $\mathrm{F}_t \Lambda_n^\top$, and a nonsystematic component e_{nt}. The F_t represent the latent factors and the $\Lambda_n^\top$ represent the betas whose values are unknown and thus need to be estimated. We assume that the we have uncorrelated factors and residuals, yielding the following covariance matrix for the excess returns:

$$\boldsymbol{\Sigma}_X = \mathrm{Var}(\mathrm{X}) = \Lambda\,\mathrm{Var}(\mathrm{F})\Lambda^\top + \mathrm{Var}(\mathrm{e}) \tag{3}$$

To estimate the unknown factors F and their betas λ, we use PCA. Here we should note that information from the second moment is used by the estimated PCA factors, while information from the first moments is ignored. The idea of PCA is to identify several hidden patterns, the principal components, that seek to maximize the variability in the data while remaining uncorrelated. The standard way to perform factor analysis is to use PCA on the sample covariance matrix of X:

$$\frac{1}{T}\mathrm{X}^\top\mathrm{X} - \overline{\mathrm{X}}\overline{\mathrm{X}}^\top \tag{4}$$

PCA then proceeds to acquire N orthogonal factors by utilizing the eigendecomposition of $\boldsymbol{\Sigma}_X$. These orthogonal factors are then ranked by their eigenvalues, which means that they are ranked on the basis of their variance. Next, the K factors with the largest corresponding eigenvalues will be selected as the 'true' factors in equation 2. The loadings $\hat{\Lambda}_{PCA}$ can then be estimated from a K x N eigenvector matrix. The factors $\mathbf{F}$ can then be obtained by performing a regression on equation 2 using the estimated Λ_{PCA}. This yields the following estimator for the orthogonal factors $\mathbf{F}$

$$\widehat{\mathrm{F}}_{\mathrm{PCA}} = \mathrm{X}\widehat{\Lambda}_{\mathrm{PCA}}\left(\widehat{\Lambda}_{\mathrm{PCA}}^\top\widehat{\Lambda}_{\mathrm{PCA}}\right)^{-1} \tag{5}$$

The above procedure can be described as Stock and Watson (2002) have done, who minimize the following function:

$$\widehat{\mathrm{F}}_{\mathrm{PCA}}, \widehat{\Lambda}_{\mathrm{PCA}} = \underset{\Lambda,\mathrm{F}}{\operatorname{argmin}} \frac{1}{NT}\sum_{n=1}^{N}\sum_{t=1}^{T}\left(\left(X_{nt} - \overline{\mathrm{X}}_n\right) - \left(\mathrm{F}_t - \overline{\mathrm{F}}\right)\Lambda_n^\top\right)^2 \tag{6}$$

Through this fucntion, one can derive that the means of the assets depend on the loadings and factors. The goal of Lettau & Pelger (2020a) was to modify this function in such a way that it does take information of the first moments of the test assets X into account.

4.2 Arbitrage Pricing Theory

In 1976, Stephen Ross came up with the arbitrage pricing theory (APT) which tells us that the expected cross-section of excess returns can be explained by a number of systematic risk factors times a corresponding risk premium of these factors. As mentioned earlier, in this study the test assets X and the factors F consist of excess returns for which the APT assumes the following relationship:

$$\mathrm{E}\left[\mathrm{X}_n\right] = \Lambda_n\mathrm{E}[\mathrm{F}] \tag{7}$$

To ensure that this implied relationship of APT holds, it involves minimizing the cross-sectional pricing error between the excess returns and the systematic factors and loadings.

$$\min_{\Lambda,\mathrm{F}} \frac{1}{N} \sum_{n=1}^{N} \left(\overline{\mathrm{X}}_n - \overline{\mathrm{F}} \Lambda_n^\top \right)^2 \tag{8}$$

Lettau and Pelger (2020a) incorporated this APT function by including an extra penalty function, which mimimizes the cross-sectional pricing errors in the original PCA function in equation 6. This gave us the following updated objective function for PCA:

$$\widehat{\mathrm{F}}_{\mathrm{RP}}, \widehat{\Lambda}_{\mathrm{RP}} = \underset{\Lambda,\mathrm{F}}{\operatorname{argmin}} \underbrace{\frac{1}{NT} \sum_{n=1}^{N} \sum_{t=1}^{T} \left(X_{nt} - \mathrm{F}_t \Lambda_n^\top \right)^2}_{\text{unexplained TS variation}} + \gamma \underbrace{\frac{1}{N} \sum_{n=1}^{N} \left(\overline{\mathrm{X}}_n - \overline{\mathrm{F}} \Lambda_n^\top \right)^2}_{\text{XS pricing error}}, \tag{9}$$

Here the γ indicates the weight of this APT mean restriction. Called the risk-premium PCA (RP-PCA), this new method minimizes the weighted average of the unexplained time series variation while simultaneously minimizing the cross-sectional pricing errors. Intuitively, one can see that the RP-PCA is a special case of PCA where one applies PCA to a different covariance matrix as in equation 4, but rather one that overweights the means:

$$\mathbf{\Sigma}_{\mathrm{RP}} = \frac{1}{T} \mathrm{X}^\top \mathrm{X} + \gamma \overline{\mathrm{X}} \overline{\mathrm{X}}^\top \tag{10}$$

Here it is important to note that if one sets $\gamma =$ -1, one ends up with the covariance matrix of equation 4 which again leads to the standard PCA method. Similar to PCA, the eigenvectors of the K greatest eigenvalues are proportional to the factor loadings of RP-PCA. The difference is that the eigenvalues of RP-PCA are more akin to a general concept of 'signal strength' of a given factor, while the eigenvalues of PCA are identical to the factor variances.

4.3 RP-PCA as OLS estimator

The RP-PCA loadings $\hat{\Lambda}$ relate to the betas when one uses ordinary least squares (OLS). Using the RP-PCA loadings, the factors $\hat{F}$ can be estimated by performing a time-series regression of the excess returns on the factors:

$$X_{nt} = \widehat{\mathrm{F}}_t \ \mathrm{B}_n^\top + e_{nt} \tag{11}$$

Equation 2 does not include an intercept and therefore the error term will not necessarily have a mean equal to zero. To remedy this, one can include an intercept and assess the time-series regression by the size of the pricing errors α_n:

$$X_{nt} = \alpha_n + \widehat{\mathrm{F}}_t \ \mathrm{B}_n^\top + e_{nt} \tag{12}$$

Equation 11 is the same as the first term in the objective function of equation 9, while the second term describes the difference between $\bar{X}$ and $E[\hat{F}_t]\hat{B}_n^\top$. If $\gamma = 0$, it is obvious that the RP-PCA estimator $\hat{\Lambda}_n$ is equal to the OLS estimator of $\hat{B}_n$. We can rewrite the time-series regression of equation 11 as follows such that, independent of γ, we have that equation 9 is equal to the OLS objective function and thus $\hat{\Lambda} = \hat{B}$:

$$X_{nt} = \widehat{\mathrm{F}}_t \ \mathrm{B}_n^\top + e_{nt} \tag{13}$$

Where $\tilde{\gamma} = \sqrt{\gamma + 1} - 1$ and $\tilde{\mathrm{F}}_{nt} = \widehat{\mathrm{F}}_t + \tilde{\gamma}\overline{\mathrm{F}}_t$, $\tilde{X}_{nt} = X_{nt} + \tilde{\gamma}\bar{X}_{nt}$. We will use equation 12 so that we can also grasp the implications of the pricing of a factor model.

In a nutshell, RP-PCA is performed by applying PCA to equation 10 to obtain the loadings $\hat{\Lambda}$. With the loadings, the factors are constructed: $\widehat{\mathrm{F}} = \mathrm{X}\widehat{\Lambda}\left(\widehat{\Lambda}^\top\widehat{\Lambda}\right)^{-1}$. With the constructed factors $\hat{F}$, equation 12 is estimated by OLS to obtain $\hat{\alpha}$, $\hat{B}$ and $\hat{e}$. After the regression, the performance of the model will be evaluated using three different criteria, that will be calculated for both in-sample and out-of-sample. For the in-sample case we have the maximal Sharpe Ratio that can be calculated by the tangency portfolio of the mean-variance frontier that is spanned by the factors, $\hat{b}_{MV} = \Sigma_F^{-1}\mu_F$. If Σ_F^{-1} is a diagonal matrix, the Sharpe Ratio can be calculated as follows

$$\hat{b}_{MV} = \frac{E[b^\top \hat{F}_t]}{\sigma(b^\top \hat{F}_t)} \tag{14}$$

Which implies the following SDF

$$M_t = 1 - \widehat{\mathrm{b}}_{\mathrm{MV}}^\top \left(\widehat{\mathrm{F}}_t - \mathrm{E}\left[\widehat{\mathrm{F}}_t\right]\right) \tag{15}$$

In addition to the Sharpe Ratio, we evaluate the model by calculating the root-mean-squared pricing error $RMS_\alpha = \sqrt{\hat{\boldsymbol{\alpha}}^\top \hat{\boldsymbol{\alpha}}/N}$ and the unexplained idiosyncratic time variance $\bar{\sigma}_e^2 = \frac{1}{N}\sum_{n=1}^N \mathrm{Var}\left(\hat{\mathbf{e}}_n\right) / \frac{1}{N}\sum_{n=1}^N \mathrm{Var}\left(\mathbf{X}_n\right)$ by making use of the $\hat{\alpha}$ and $\hat{\epsilon}$ obtained by OLS on equation 12.

The calculation of the three different criteria for the out-of-sample case works somewhat differently. First, the loadings are estimated by using pre-specified 'rolling windows'. Then the factors $\hat{F}_{t+1}$ are predicted using the excess returns at t+1 and the information from the estimated loadings up to time t. The b will again be computed via $\hat{b}_{MV} = \Sigma_F^{-1}\mu_F$ for each estimation window, after which the out-of-sample return $b^\top \hat{F}_{t+1}$ will be calculated. After covering the entire sample, the OOS Sharpe Ratio can be calculated via $\hat{b}_{MV} = \frac{E[b^\top \hat{F}_{t+1}]}{\sigma(b^\top \hat{F}_{t+1})}$. Moreover, the B_n can be computed for each estimation window, such that we can calculate the OOS pricing errors $\hat{\alpha}_{n,t+1} = X_{n,t+1} - \mathbf{F}_{t+1}\mathbf{B}_n^\top$. Then the OOS pricing errors can be calculated via $RMS_\alpha = \sqrt{\bar{\boldsymbol{\alpha}}^\top \bar{\boldsymbol{\alpha}}/N}$ where $\bar{\alpha}_n = \frac{1}{T}\sum_{t=1}^T \hat{\alpha}_{n,t+1}$. The OOS calculation of the idiosyncratic variance will be done as follows $\bar{\sigma}_e^2 = \frac{1}{N}\sum_{n=1}^N \mathrm{Var}\left(\hat{\alpha}_n\right) / \frac{1}{N}\sum_{n=1}^N \mathrm{Var}\left(\mathbf{X}_n\right)$.

4.4 Transforming the datasets

The macroeconomic factors that we use are Oil, Gold, US GDP and the USD exchange rate. To construct these macroeconomic factors, we have selected appropriate ETFs for all the corresponding macroeconomic variables. The data consists of daily closing prices and therefore the daily prices are transformed into log returns:

$$r_{macro_j,t} = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{16}$$

The portfolios that will be transformed are the double-sorted portfolios. Of all these portfolios, the excess returns are calculated by subtracting the risk-free rate from the original returns. Then the following three regressions are performed on all 5x5 portfolios i using the Fama-French and macroeconomic factors:

$$(R_{it} - R_{ft}) = \alpha_{1it} + \beta_{11}(R_{mt} - R_{ft}) + \beta_{12}SMB_t + \beta_{13}HML_t + \epsilon_{1it} \tag{17}$$

$$(R_{it} - R_{ft}) = \alpha_{2it} + \beta_{21}OIL + \beta_{22}GLD + \beta_{23}USD + \beta_{24}GDP + \epsilon_{2it} \tag{18}$$

$$(R_{it} - R_{ft}) = \alpha_{3it} + \beta_{31}(R_{mt} - R_{ft}) + \beta_{32}SMB_t + \beta_{33}HML_t + \beta_{34}OIL + \beta_{35}GLD + \beta_{36}USD + \beta_{37}GDP + \epsilon_{3it} \tag{19}$$

Using these regressions, we adjust the portfolio returns for the risk of the Fama-French factors and the macroeconomic risk indicators by moving the estimated variables to the left side of the equation. Therefore, the only things in the portfolios that will remain unexplained are the abnormal returns $\hat{\alpha}$ and the residual returns $\hat{\epsilon}$. Thus the updated portfolios will contain the excess returns adjusted for the factors:

$$(R_{it} - R_{ft}) - \hat{\beta}_{11}(R_{mt} - R_{ft}) - \hat{\beta}_{12}SMB_t - \hat{\beta}_{13}HML_t = \hat{\alpha}_{1it} + \hat{\epsilon}_{1it} \tag{20}$$

$$(R_{it} - R_{ft}) - \hat{\beta}_{21}OIL - \hat{\beta}_{22}GLD - \hat{\beta}_{23}USD - \hat{\beta}_{24}GDP = \hat{\alpha}_{2it} + \hat{\epsilon}_{2it} \tag{21}$$

$$(R_{it} - R_{ft}) - \hat{\beta}_{31}(R_{mt} - R_{ft}) - \hat{\beta}_{32}SMB_t - \hat{\beta}_{33}HML_t - \hat{\beta}_{34}OIL - \hat{\beta}_{35}GLD - \hat{\beta}_{36}USD - \hat{\beta}_{37}GDP = \hat{\alpha}_{3it} + \hat{\epsilon}_{3it} \tag{22}$$

Hence, equation 20 and equation 21 show the double-sorted portfolios adjusted for Fama-French risk and macroeconomic risk, respectively. This while equation 22 takes into account both the Fama-French and macroeconomic factors. This will be done for all five double-sorted portfolios, providing us with 5 x 3 new datasets of excess returns. RP-PCA and PCA will be performed on these adjusted portfolios to examine whether RP-PCA still outperforms PCA when significant factors are removed from the dataset in advance.

5 Empirical results

In this section, the RP-PCA methodology is tested in practice to see if it performs significantly better than the traditional PCA methodology. This will be tested empirically in three ways. First, French's double-sorted portfolios will be used, after which the methods will be applied to the large cross-sectional anomaly portfolios of Kozak, Nagel and Santosh (2020). Both these two empirical applications have been done before in the research of Lettau & Pelger (2020b). We add to the existing empirical literature of RP-PCA by applying the methodology and comparing it with PCA to the adjusted double-sorted portfolios as described in Section 4.

All models will be compared based on the three different measures: (a) the maximum Sharpe Ratio, (b) the root-mean-squared pricing errors (RMS_α) and (c) the average idiosyncratic variance ($\bar{\sigma}^2_\epsilon$). For the calculation of the out-of-sample measures for the 'normal' double-sorted and the single-sorted portfolios using monthly data, a 20-year rolling window (T*=240) is used to make predictions of the out-of-sample returns and pricing errors at $t + 1$. For the adjusted portfolios containing daily returns a rolling window of 1 year (T*=250) will be used.

5.1 Double-sorted portfolios

The first analysis compares the RP-PCA and PCA methodology based on the criteria for the 5x5 double-sorted portfolios. In addition, these methods are compared to the well-known Fama-French three-factor model (FF) that uses the market risk premium, SMB (sorted on size) and HML (sorted on book-to-market) factors as explained in Section 2. We slightly differ here from the empirical application as has been done by Lettau & Pelger (2020b), who made use of an HML factor sorted on the second characteristic of the portfolios, due to uncertainty about how these HML factors were constructed. The choice of the 'original' HML factor still yields the same conclusions as in Lettau & Pelger's paper. In the application of the double-sorted portfolios, the $\gamma = 20$, although the results are robust to this setting.

The results for the out-of-sample (OOS) values of the RP-PCA, PCA and FF methods can be seen in Table 1. In the table, it is immediately noticeable that for seven out of the eight cases, the maximal OOS Sharpe Ratio of the RP-PCA is higher than that of the PCA and FF models. Moreover, RP-PCA dominates PCA and FF in terms of smaller cross-sectional pricing errors. For the idiosyncratic variance, we observe that PCA and RP-PCA outperform the FF model because the PCA methodology, by construction, minimizes the idiosyncratic variance by trying to capture as much covariation as possible. Concluding from the table, we see that in five of the eight cases PCA actually gives us higher idiosyncratic variation $\bar{\sigma}^2_\epsilon$ than RP-PCA.

An explanation on why RP-PCA performs better than classical PCA can be derived via three different dimensions. The compositions of the factors and/or the order of the factors may differ between PCA and RP-PCA. And in addition, RP-PCA can sometimes discover latent factors that PCA cannot detect. An example of these three different ways will be shown for the 'size/accruals and the 'size/short-term reversal portfolios.

In Figure A.1 in the appendix, we see the values of the maximal Sharpe Ratios, root-mean-squared pricing

Table 1: Out-of-sample values of RP-PCA, PCA and FF models for double-sorted portfolios

	SR			RMS_α			$\overline{\sigma}_e^2$		
	RP-PCA	PCA	FF	RP-PCA	PCA	FF	RP-PCA	PCA	FF
SIZE&BM	**0.21**	0.18	0.17	**0.17**	0.17	0.18	7.97%	**7.91%**	7.97%
SIZE&ACC	**0.21**	0.12	0.15	**0.09**	0.11	0.10	6.74%	**6.44%**	7.17%
SIZE&INV	**0.26**	0.23	0.23	**0.13**	0.15	0.13	**6.95%**	7.00%	7.07%
SIZE&OP	0.13	0.14	**0.15**	**0.09**	0.09	0.11	**6.93%**	7.08%	8.54%
SIZE&ST-REV	**0.16**	0.11	0.10	**0.18**	0.19	0.20	7.89%	**7.86%**	10.87%
SIZE&MOM	**0.21**	0.18	0.01	**0.20**	0.21	0.30	**8.30%**	8.40%	13.76%
SIZE&IVOL	**0.29**	0.23	0.23	**0.16**	0.17	0.22	**6.22%**	6.24%	7.11%
SIZE&VOL	**0.27**	0.21	0.21	**0.18**	0.19	0.23	**6.27%**	6.30%	7.04%

Note: *Out-of-sample maximal Sharpe Ratios, root-mean-squared pricing errors and unexplained idiosyncratic variance for different 5x5 double-sorted portfolios.* ***Bold*** *numbers indicate the best model for the specific performance measure.*

errors and the unexplained idiosyncratic variance for different numbers of factors as a function of γ. It turns out that for the unexplained time-series variation, the value is independent of the choice of γ. For the size/accrual portfolios, it can be seen that the models with one or two factors also have Sharpe Ratios and pricing errors α that are unaffected by the choice of γ. This implies that for these models, RP-PCA and PCA perform similarly. In contrast, the addition of a third factor (yellow line) changes the constant level of the Sharpe Ratios and the pricing errors. These SR and RMS_α increase and decrease, respectively, as γ increases from -1 to 4, until they become constant again.

For the size/short-term reversal portfolio, the right panel of Figure A.1 shows us that for the one-factor model, PCA and RP-PCA are equivalent because the out-of-sample SR and RMSα are constant over γ. However, things change for models with a higher order of factors such as the two-factor model (orange line). For the two-factor model, the Sharpe Ratio first falls until $\gamma = 4$ and then rises again till $\gamma = 20$ before remaining at a constant level. This while the three-factor model only rises moderately for higher values of γ.

Figure A.2 in the appendix shows the heatmaps of the first three statistical factors of the size/accrual and the size/short-term reversal portfolios to explain the intuition behind the aforementioned phenomena. In panel (a) and (b) the first two heatmaps of the RP-PCA and PCA factors look similar. The first two factors represent the market and SMB factor of the Fama-French model. This can be observed because the first heatmap has positive weights for all portfolios with a slight preference for the smaller sized portfolios. The second heatmap shows us the SMB factor as we go short in the large-stock portfolios and long in the small-stock portfolios due to the negative and positive loadings, respectively. The third factor differs for RP-PCA and PCA in that it gives us little information and no clear distinguishable pattern that can be derived from the heatmap of PCA. This while for RP-PCA we can observe negative weights for high-accrual portfolios and positive weights for low-accrual portfolios, which is similar to a HML (based on accruals) factor. The RP-PCA is thus able to detect a 'latent' factor in the size/accrual portfolio which implies the increase and decrease in the SR and RMS_α for changes in γ in Figure A.1.

As with the size/accrual portfolios, the first factor of the size/short-term reversal portfolios looks similar.

The first factor again represents the market factor of the Fama-French model. This time, however, the difference between RP-PCA and PCA is not due to PCA's inability to detect certain factors, but rather the order of the factors being different. In the case of RP-PCA in panel (c), the second factor has negative weights for high-reversal portfolios and positive weights for the small-reversal portfolios, while this is the third factor for PCA and represents a 'long-minus-short factor'. The third factor for RP-PCA is the SMB factor, which in turn is the second factor for PCA but then with a change of sign (Lettau & Pelger (2020b) do not find this change of sign in their heatmap, which is most likely due to a difference in scaling). The intuition behind this interchange is that RP-PCA prefers the 'long-minus-short' reversal factor because of its greater contribution to the cross-sectional dimension, while the SMB factor is more responsible for the common time-series variation preferred by PCA. This stems from the fact that the return dispersion is much smaller along the size dimension than along the reversal dimension, which leads RP-PCA to assign extra weight to the high SR factors.

5.2 Single-sorted portfolios

PCA and RP-PCA are designed to handle much larger cross-sectional datasets and therefore we will use a dataset with large, single-sorted decile portfolios as described in Section 3. The dataset is based on different anomaly characteristics, whose portfolios are more likely to contain 'weak' latent factors. The original data sets consists of 10 decile portfolios for all 37 anomaly characteristics, which gives a total of 370 portfolio. Just like Letta & Pelger, we will mostly use only the first and last deciles of the portfolios ($N = 74$) because these deciles contain most of the relevant information, as will be shown later.

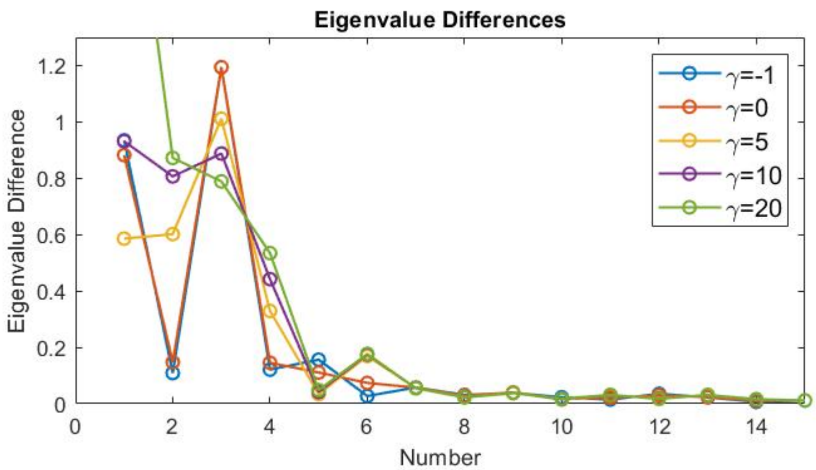
5.2.1 The choice of γ and the optimal number of systematic factors.

The analysis begins by determining how many 'systematic' factors and 'idiosyncratic factors' are in the data and how different values of γ affect this relationship. We determine the number of factors by looking at the pattern of successive eigenvalue differences of the form $\frac{1}{T}\mathrm{X}^\top\mathrm{X} + \gamma\overline{\mathrm{X}}\overline{\mathrm{X}}^\top$. Figure 1 shows us these patterns of eigenvalue differences for different values of γ. In the upper panel the first and last deciles of the sample are used and in the lower panel the full sample is used. Using the Onatski criterion, we conclude that the fifth eigenvalue difference for $\gamma < 10$ and the sixth eigenvalue difference for $\gamma \geq 10$ are lower than the critical value. This illustrates that the fifth systematic factor is weak and can only be detected if one chooses γ high enough. For the full sample in panel (b), the optimal number of factor does not differ with the N=74 portfolios, indicating that most of the relevant information for determining the number of systematic factors is present in the first and last deciles of the portfolios.

5.2.2 Estimation: RP-PCA vs PCA

For figure 2, we compare the in-sample and out-of-sample SR, RMSα and $\overline{\sigma}_e^2$ of the 74 first-decile and last-decile portfolios. We set $\gamma = 10$ and calculate the three criteria for the different models based on their

$N = 74$



$N = 370$

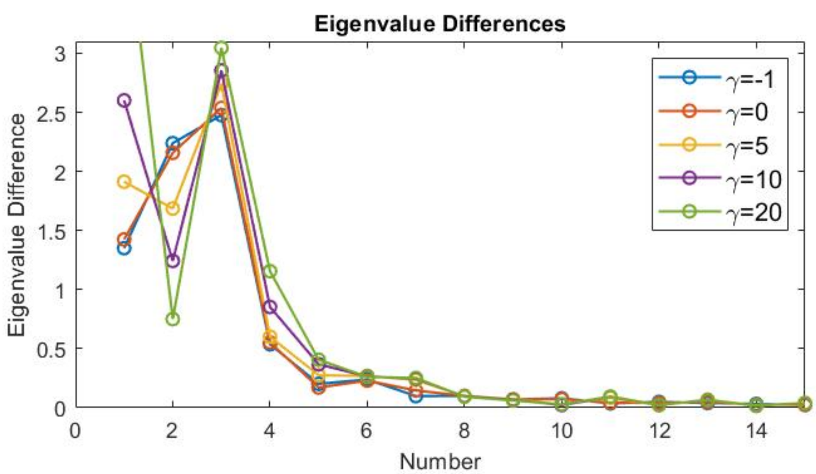


Figure 1: Eigenvalue differences for Single-sorted portfolios

Note: *The differences of successive eigenvalues of the covariance matrix* $\mathbf{\Sigma}_{\mathrm{RP}} = \frac{1}{T}\mathbf{X}^{\top}\mathrm{X} + \gamma\overline{\mathrm{X}}\overline{\mathrm{X}}^{\top}$ *for different weights* γ*. The* $N = 74$ *consists of the first and last deciles of the 37 single-sorted decile portfolios.*

different number of systematic factors. In the first row, we find that adding factors to the IS increases the SR, but the size of the increase varies per factor. In addition, it can be obtained that the SRs are significantly lower for PCA than for RP-PCA. It also stands out that for the RP-PCA case, especially the second and fifth factors lead to a sudden increase in the SR. The patterns of the OOS SRs are similar to those of the IS, so there is no problem of excessive overfitting. And adding more factors than five gives only slight improvements in the SR for RP-PCA, confirming the presence of the five systematic factors for RP-PCA.

For the RMSα, it is important to note that in all cases the pricing errors are larger for PCA than for RP-PCA. For the IS pricing error, especially adding the second factor and fifth factor leads to a rapid drop in the α for RP-PCA. And for IS and for OOS, we see that adding additional factors after adding the fifth factor leads to only small reductions in the pricing errors. Furthermore, the pricing errors in the five-factor

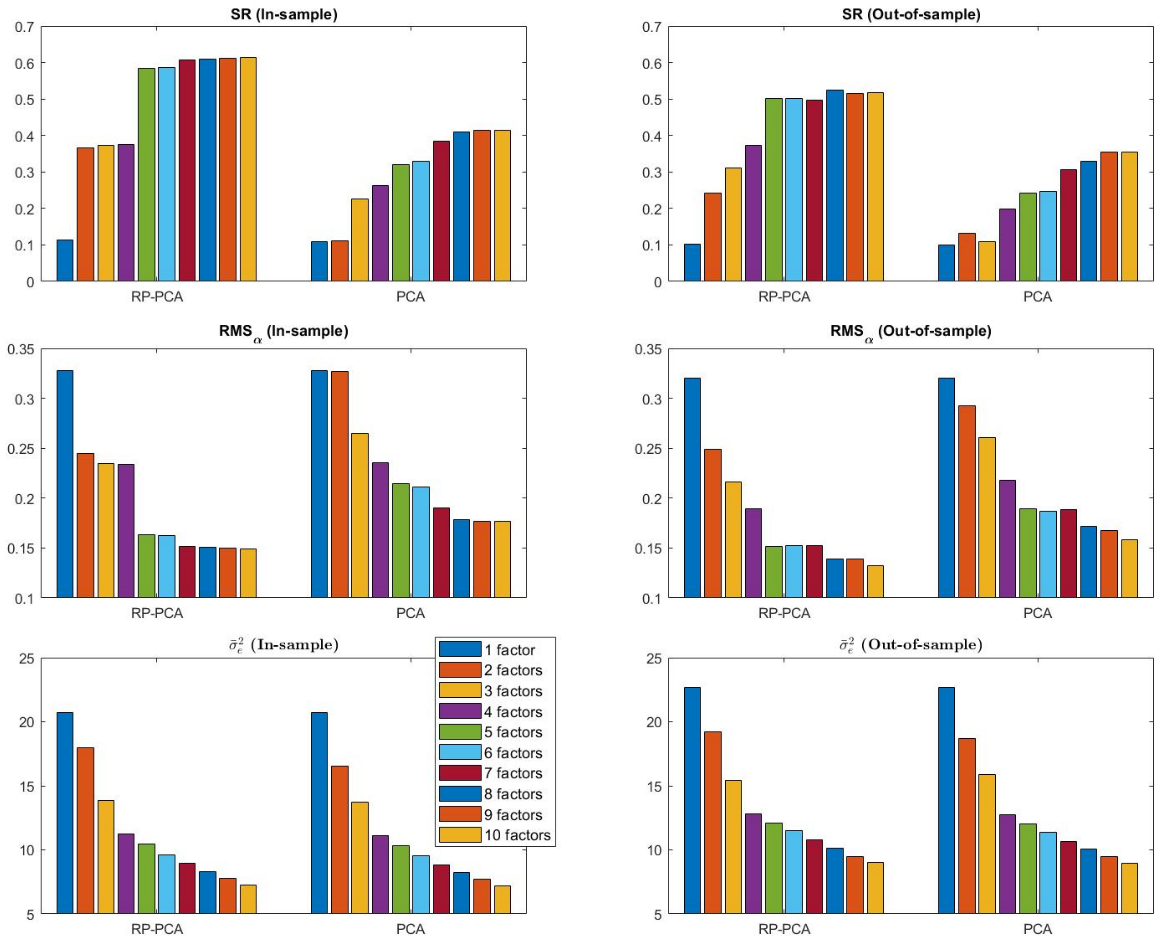


Figure 2: Performance measures of fit for 74 decile-1 and decile-10 portfolios
Note: *The maximal Sharpe Ratios, root-mean-squared pricing errors and unexplained idiosyncratic variance for various numbers of factors, both for in-sample and out-of-sample. The weight γ is set to 10.*

model of the RP-PCA are almost half the size of the pricing errors of PCA, both for IS and OOS.

The last row shows the idiosyncratic variance whose figures seem similar for the RP-PCA and PCA models. Remember that PCA by construction minimizes the unexplained variation $\bar{\sigma}^2_\epsilon$ in-sample, but not necessarily out-of-sample. Here the differences between the IS and OOS case are negligible. RP-PCA thus leads to higher Sharpe Ratios and lower pricing errors than PCA, while still capturing similar amounts of time-series covariation.

In Table 2 we compare the three different criteria for the RP-PCA and PCA methods using different numbers of factors and different samples. For comparison, we also use the three- and five-factor Fama-French model. Panel (a) shows that RP-PCA dominates PCA and Fama-French in terms of the maximal Sharpe Ratios and the pricing errors. This while the idiosyncratic variation of RP-PCA takes on similar values to PCA. As a robustness check, the results of the full sample and the shorter sample (T=530) of 98 portfolios are shown in panel (b) and (c). From these results we can conclude that a RP-PCA model with five factors is the preferred model to use.

Table 2: The values of the fit of RP-PCA, PCA and Fama-French models

	In-sample			Out-of-sample		
Model (K)	SR	RMS_α	$\overline{\sigma}_e^2$	SR	RMS_α	$\overline{\sigma}_e^2$
Panel A: 74 portfolios						
RP-PCA (3)	**0.37**	**0.23**	13.88%	**0.31**	**0.22**	**15.42%**
PCA (3)	0.23	0.27	**13.74%**	0.11	0.26	15.88%
Fama-French (3)	0.21	0.31	17.49%	0.14	0.25	16.54%
RP-PCA (5)	**0.59**	**0.16**	10.43%	**0.50**	**0.15**	12.11%
PCA (5)	0.32	0.21	**10.30%**	0.24	0.19	**12.04%**
Fama-French (5)	0.32	0.26	16.05%	0.24	0.19	13.91%
Panel B: 370 portfolios						
RP-PCA (3)	**0.24**	0.17	12.79%	**0.20**	**0.15**	**14.39%**
PCA (3)	0.17	**0.17**	**12.70%**	0.13	0.15	14.74%
Fama-French (3)	0.21	0.18	14.61%	0.12	0.16	14.89%
RP-PCA (5)	**0.59**	**0.13**	10.82%	**0.47**	**0.12**	12.70%
PCA (5)	0.25	0.14	**10.69%**	0.18	0.14	**12.57%**
Fama-French (5)	0.32	0.16	13.60%	0.21	0.13	13.74%
Panel C: 98 portfolios						
RP-PCA (3)	**0.44**	**0.31**	13.67%	**0.27**	**0.25**	16.42%
PCA (3)	0.19	0.32	**13.34%**	0.11	0.26	**16.41%**
Fama-French (3)	0.21	0.39	17.02%	0.12	0.29	17.52%
RP-PCA (5)	**0.73**	**0.23**	10.36%	**0.48**	**0.17**	12.84%
PCA (5)	0.37	0.25	**10.22%**	0.23	0.21	**12.74%**
Fama-French (5)	0.34	0.31	15.25%	0.22	0.22	14.74%

Note: *The numbers in* ***bold*** *are the best-performing models for given K in terms of maximal Sharpe Ratios, root-mean-squared pricing errors and unexplained idiosyncratic variation. In panel A an B are 74 and 370 decile-1 and decile-10 portfolios, respectively. Panel C shows 98 decile-1 and decile-1- for a time period November 1973 - December 2017 (T=530). Panel A and B are from November 1963 till December 2017 (T=650).*

Figure A.3 in the appendix shows the individual cross-sectional pricing errors based on a five-factor model and ranks the anomalies according to their Sharpe Ratio. For PCA, it appears that for both in-sample and out-of-sample, the α's become highest for the high-SR anomalies. For RP-PCA, the pricing errors are significantly lower than PCA for most anomalies, especially for the high-SR anomalies. In addition, the differences in pricing errors between RP-PCA and PCA for short-term reversal (*strev*) and momentum (*mom*) become much smaller OOS. Therefore, we conclude that RP-PCA gives lower pricing errors, especially OOS, compared to PCA and in particular does a better job for the portfolios that are most mispriced by PCA, i.e. the high Sharpe Ratio portfolios.

5.2.3 Cross-sectional factors vs time-series

Table 3 shows the mean, variance and Sharpe Ratios of individual RP-PCA and PCA factors. The factors have been normalized as in Lettau & Pelger (2020b) so that comparisons can be made between the factors. As is often the case in factor modelling, the first factor has much higher variance than the rest of the factors.

As expected, the PCA has ranked its factors according to their variance. For example, the second factor has a variance of 102.55, but a mean of 0.18, which is lower than almost all factors. This indicates that this

Table 3: Characteristics of individual factors

	RP-PCA				PCA			
Factor	Mean	Variance	SR	Mean rank	Mean	Variance	SR	Mean rank
1	5.02*	1935.06	0.11	**1**	4.83*	1944.86	0.11	**1**
2	2.32*	66.32	0.35	**2**	0.18	102.55	0.02	9
3	0.30	100.99	0.07	**4**	1.65*	69.13	0.2	**2**
4	0.10	65.45	0.03	6	1.05*	64.16	0.13	**3**
5	0.73*	26.34	0.45	**3**	0.83	20.21	0.18	**4**
6	0.03	19.55	0.04	9	0.34*	19.42	0.08	7
7	0.14*	17.96	0.16	**5**	0.79*	16.17	0.20	**5**
8	0.05	15.42	0.06	7	0.57*	15.10	0.15	6
9	0.04	13.55	0.06	8	0.20.	13.48	0.05	8
10	0.03	11.97	0.04	10	0.04	11.96	0.01	10

Note: *In the table the mean, variance and Sharpe Ratio of the incremental uncorrelated factor component are shown. The* ***bold*** *numbers indicate the top five factors with the highest means while the (*) indicate the significant means.*

factor is probably not priced, but does capture a lot of co-movement. The ranking of the RP-PCA factors depends not only on the variances but also on the means. So has the second factor of RP-PCA a lower variance than the third factor, but it does have a higher and significant mean. Thus, this second factor is priced but does capture less time-series variation than the third factor. Hence, for RP-PCA, the third and fourth factors capture more co-movement while the second and fifth factor have higher means, yielding higher Sharpe Ratios and thus a higher probability of being priced, which in turn is important for capturing more of the cross-sectional returns.

Table 4: The in-sample and out-of sample fit for RP-PCA for a subset of factors

	In-sample			Out-of-sample		
Factors	SR	RMS_α	$\overline{\sigma}_e^2$	SR	RMS_α	$\overline{\sigma}_e^2$
[1,2,3,4,5]	0.59	0.16	10.43%	0.50	0.15	11.84%
[1]	0.11	0.33	20.75%	0.10	0.32	22.65%
[1,2,5]	0.57	0.23	17.07%	0.50	0.18	18.11%
[2,5]	0.41	1.69	74.72%	0.38	0.34	21.97%
[1,3,4]	0.12	0.33	13.93%	0.02	0.31	15.89%
[3,4]	0.03	0.66	93.06%	-0.08	0.52	55.88%

Note: *In-sample and out-of-sample Sharpe Ratios, root-mean squared pricing errors and unexplained idiosyncratic variation for subset of factors. The weight γ = 10.*

To check whether the aforementioned claims are correct, the RP-PCA factors are divided into several subsets for which the three criteria are calculated. The first subset of [1] is very similar to the CAPM in that the first factor is highly correlated with the market return. It captures almost 80% of the total time-series variation.

For the subset [1,2,5] we see that the Sharpe Ratio is almost identical to the model that includes all five factors. The pricing errors are slightly higher and the idiosyncratic variation is about one and a half times as large. Omitting factor one from this subset still yields a reasonably high SR of about 0.34, but the removal of the first factor drastically increases the RMSα and the $\bar{\sigma}_\epsilon^2$ for the IS fit and to a lesser extent of the OOS fit. This difference is due to the assumption of orthogonality of the factors for the IS fit, while this is not necessarily the case out-of-sample. For the OOS factors, the second and fifth factor are able to capture some of the variation caused by the first factor. It can thus be concluded that a factor model with the first, second and fifth factor captures slightly less time-series variation, but does provide us with similar SRs and α's.

For the subsets of factor 3 and 4, it works the other way around. The SRs and RMSα are similar to a model with only the first factor. Therefore, the factors do not significantly affect the SRs and the RMSα's. However, the idiosyncratic variation of subset [1,3,4] is only a little higher than the full specification.

Hence, the first RP-PCA factor is the most essential factor and is relevant to both the cross-sectional and time-series fit. To capture the time-series variation, it is best to use a three-factor model with factors [1,3,4], while to capture most of the cross-sectional returns, a model with subset [1,2,5] should be used.

5.2.4 Portfolio weights in RP-PCA and PCA SDFs

The SDF is a linear combinations of the factors and is therefore also a linear combination of the test assets. Figure A.4 in the appendix shows the implied SDF by the RP-PCA and PCA factors for the single-sorted anomaly portfolios. Again, the anomaly portfolios are ranked according to their Sharpe Ratios. In Figure A.4 it can be seen that the SDFs are composed of long-short portfolios because the weights for almost all decile-1 and decile-10 portfolios are negative and positive, respectively. For the RP-PCA SDF, we see that the weights are highest for the high-SR portfolios, while this effect does not occur for the PCA loadings. This implies that the RP-PCA SDF is mainly composed of high-SR portfolios which explains the higher Sharpe Ratios of the RP-PCA factors compared to the PCA factors.

5.2.5 Portfolio weights and returns

To learn more about the differences between RP-PCA and PCA, we look at the relation between portfolio weights and returns. In figure A.5 of the appendix, we show the average returns of the 74 anomaly portfolios on the horizontal-axis and the SDF loadings on the vertical-axis. The composition of the SDF has a strong connection with the returns of the portfolios for the RP- PCA case as shown in the figure for RP-PCA. We see in the figure that portfolios with negative weights also have the lowest returns, while the portfolios with positive loadings yield higher returns.

For PCA, we see, as with RP-PCA, a positive correlation between the returns and the portfolio weights, but this time the relationship is weaker. We now have portfolios with negative weights but with fairly high returns. And indeed, the correlation between the returns and loadings for PCA is 0.56, compared to the correlation of 0.87 for RP-PCA. We have previously established that the SDF loadings for RP-PCA have a stronger correlation with the returns than PCA and it is therefore to be expected that the implied SR of

Table 2 is considerably higher for RP-PCA than PCA, namely 0.50 versus 0.24.

5.3 Adjusted portfolios

This section compares the adjusted portfolios based on size and book-to-market, investment, operating profitability, short-term reversal and momentum. For a valid comparison, the original portfolios that have not been adjusted for Fama-French and macroeconomic risk are used as benchmark portfolios. Since we are working with daily data and a different sample period than was used with the double-sorted portfolios of Section 5.1, it does not necessarily make sense to use the same assumptions on the number of factors K and especially the rolling window $T*$. Therefore, in Figure 3 the cumulative variance for the number of factors is shown for the non-adjusted size/book-to-market and the size/momentum portfolio. The figure shows the amount of variance explained for the first 10 factors of RP-PCA. It can be seen that the first factor of RP-PCA already explains more than than 85% of the total variance for both portfolios. Based on the figure we can conclude that selecting three principal components makes sure that 95% of the total variance will be explained. Therefore, for our further analysis we have chosen to set $K = 3$ factors.

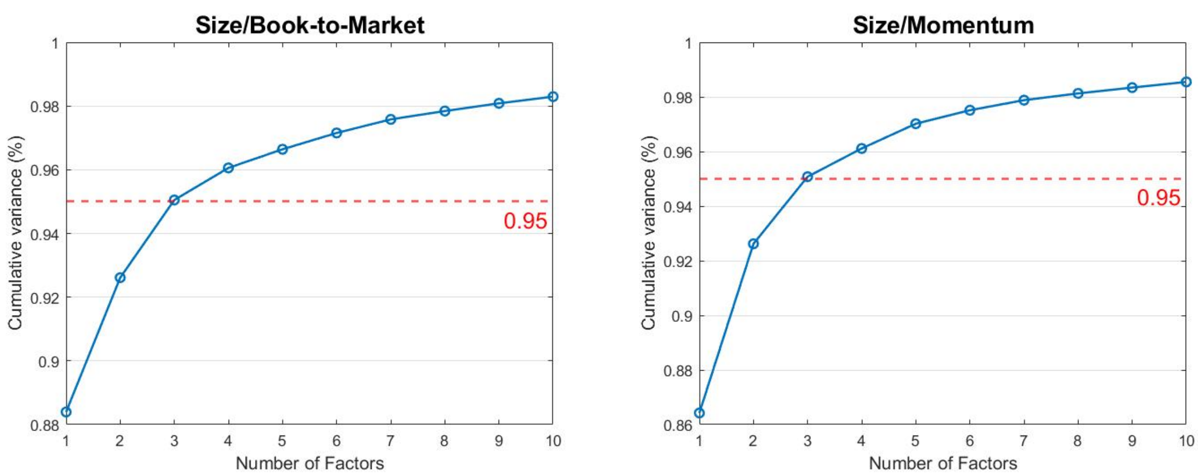


Figure 3: Cumulative variance explained by factors
Note: *The total variance explained by number of factors for the size/book-to-market and size/momentum portfolios. Red line indicates a threshof of 95% variance explained.*

Not only the choice of K is important for our analysis, but also the choice of the rolling window for the out-of-sample analysis. Since we have moved from monthly to daily observations, we should be able to obtain more precise out-of-sample results if the sample period would remain the same. Unfortunately, the data of the macroeconomic factors were only available from February 2007, so we have a much shorter sample range at our disposal. We have decided to go with a rolling window of 1 year (T*=250) for our out-of-sample analysis. The reason for the choice of a rolling window of 1 year is that it is often used in practice in analysing returns (e.g. Wang et al., 2011; Aloui et al., 2016) and also reduces the computational cost of RP-PCA. A rolling window of 1 year gives us the benefit that it takes into account anomalies such as the momentum cycle and the January effect. On the other hand, it is also not too long such that the out-of-sample results are unable to capture business cycles.

Nevertheless, as a robustness check to see if the choice of rolling window $T*$ has a large impact on the out-of-sample results, we compute the OOS Sharpe Ratios for different rolling windows and portfolios. For this computation we set $\gamma = 20$ as Lettau & Pelger (2020b) showed that the results were not sensitive to this choice. Table 5 gives us the results, showing that the out-of-sample results are very sensitive to the choice of the rolling window. Moreover, there is no clear window that 'outperforms' the others in terms of having stable OOS Sharpe Ratios, when compared to the IS Sharpe Ratios. Ranking all the rolling windows for each portfolio from best to worst in terms of stability relative to the IS Sharpe Ratios, shows us that the 1 year rolling window slightly dominates the other rolling windows. Therefore, for the reasons stated above, we decided to choose a rolling window of 1 year (T*=250), but keep in mind that the out-of-sample results are sensitive to this choice.

Table 5: Sharpe Ratios for different rolling windows

	In-Sample	Month	1/2 Year	1 Year	3 Years	5 Years
SIZE&BM	0.0347	0.0571	0.0443	0.0425	0.016	**0.033**
SIZE&INV	0.0365	0.0573	**0.0363**	0.0263	0.0172	0.0251
SIZE&OP	0.0365	0.0394	0.0274	**0.0366**	0.0182	0.0152
SIZE&ST-REV	0.0374	**0.0486**	0.0184	0.0145	0.0021	0.0021
SIZE&MOM	0.0352	0.0465	**0.0244**	0.0134	0.0044	0.0274

Note: *The table shows the in-sample and out-of-sample maximal Sharpe Ratios for RP-PCA using different rolling windows. The rolling windows T* are set to 21, 125, 250, 750 and 1250, respectively. Factors $K = 3$ and $\gamma = 20$.*

5.3.1 Influence of macroeconomic variables

The Fama-French factors are known to play a role in describing the returns of stock portfolios, as described in Section 2. The market risk premium provides the investor with excess return to compensate for the additional risk of investing in that particular portfolio. The SMB shows the long-run tendency of small-cap companies outperforming large-cap companies and the HML shows the long-term trend of value companies outperforming growth companies.

Regarding the macroeconomic factors, as shown in Section 2, the empirical influence on stocks is not so clear and often even ambiguous. Therefore, it is critical to show how the macroeconomic factors affect our portfolios.

Panel (a) in Figure 4 shows us the heatmaps for each macroeconomic factor, regressed on the non-adjusted size/book-to-market portfolio, to examine the individual effect of the macroeconomic variables on the excess returns.

For the oil factor, we find that all coefficients are positive. This means that an increase in oil prices has a positive impact on stock returns. Since the US is an oil-producing country and the portfolios are built upon the NYSE, NASDAQ and the Amex, we are not surprised to find this correlation. Furthermore, it appears that large-cap companies in particular benefit from a rise in the oil price, which is probably due to the fact that most oil companies are large multinationals. This is because oil firms have to incur high start-up costs to buy all the machinery and oil fields.

The second heatmap from panel (a) shows us that the gold price in general has a positive impact on the portfolio returns. This seems counter-intuitive to the relationships found in previous research, described in Section 2. Possible reasons for this could be that we did not take inflation into account or that we had to work with a short sample size. What is striking, however, is that the effect is strongly negative for large-cap and high book-to-market firms, the so-called 'large value' companies. One possible reason for this is that gold can be seen as an alternative to these stocks. When the stock market is falling, large value companies are often considered as more reliable than small/growth stocks and can therefore, just like gold, be used as a 'hedge' for the stock market.

For the GDP factors, we observe very large positive coefficients. Theoretically, this makes sense, since an increase in the US GDP should lead to higher profits for firms, less unemployment and more confidence in the economy as a whole, which should have a positive effect on stock returns. It seems large capitalizations in particular benefit from GDP growth and that companies with low book-to-market ratios are also slightly more affected by a rise in GDP.

The final heatmap in panel (a) for the USD factors shows us large negative signs for all portfolios. This implies that a depreciation of the USD has led to higher stock returns in the period from 2007 to 2022. As mentioned in Section 2, this implies that empirically, the positive effects of the depreciation, higher exports and an improved international competitive position, outweigh the negative effects of having higher costs of imports and having an increased domestic price level. The USD exchange rate appears to play a lesser role for smaller firms.

In panel (b) we see the influence of each macroeconomic variable when all factors are taken together, the Fama-French factors and the macroeconomic factors. The heatmaps show us the coefficients for the macroeconomic factors from the regression of equation 19. The coefficients of the Fama-French factors on the excess returns are not shown because their influence has been examined extensively in the past (see Section 2). The first thing to note is that the coefficients of the macroeconomic factors are quite small because the Fama-French factors play a large role in explaining the excess returns of the portfolios. Nonetheless, most of the coefficients are statistically significant, implying that the macroeconomic variables do influence the excess returns.

What is interesting about the oil factor in panel (b) is that it primarily plays a role for large-cap companies. This makes sense, because as mentioned earlier, oil companies - the ones most directly affected - tend to have a large market capitalization and the US stock market incorporates quite a large number of companies in this industry.

For the coefficients of the gold factor in the multiple regression, it is difficult to distinguish a clear pattern. It is notable, however, that the gold price has little effect on the low book-to-market firms, the growth companies. The gold price has more of an effect on the excess returns of value companies, especially for the large caps, yielding a similar relationship as seen earlier in the heatmap of panel (a).

(a) **Single regression** (b) **Multiple regression**

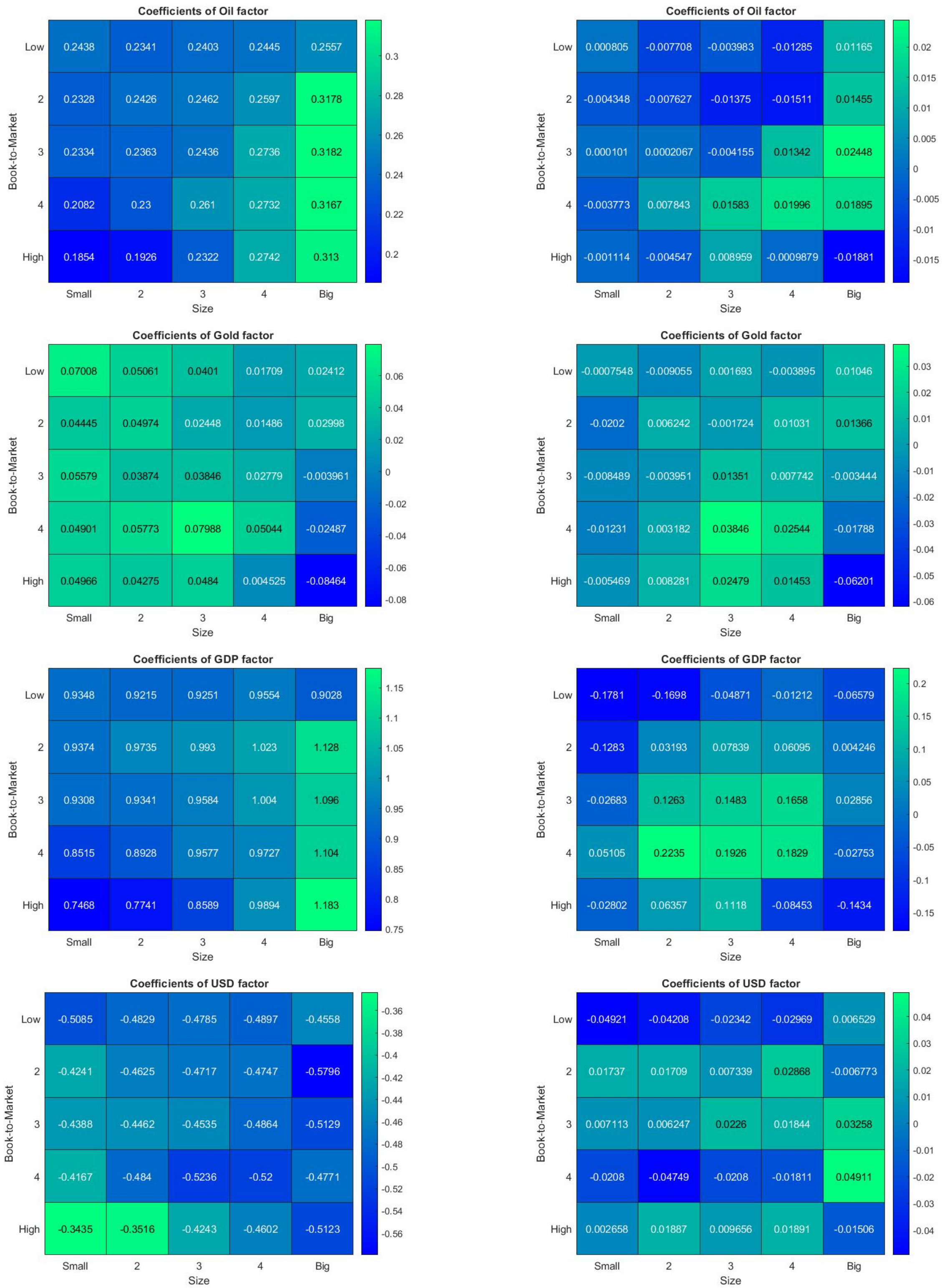


Figure 4: Heatmaps of macroeconomic coefficients for a single and multiple regression
Note: *Heatmaps showing the coefficients of the macroeconomic factors regressed on the size/book-to-market portfolio. The single regression uses only the macroeconomic factor as explanatory variable. The multiple regression regresses all macroeconomic and Fama-French factors on the portfolio as in equation 19.*

The coefficients of the GDP factor in panel (b) show the largest coefficients on the diagonal of the heatmap. Moreover, it seems that especially in the middle zone of the heatmaps, i.e., not in the outer portfolios, the returns are positively affected by an increase in GDP, while the large value companies and the small growth companies are negatively affected by GDP. One possible explanation for this observed pattern is that much emphasis is often placed on the differences between small vs large-cap and value vs growth companies. As a result, mid-cap companies and 'neutral' companies (in terms of book-to-market) can sometimes be overlooked when the economy is booming.

Finally, it is also difficult to discern a clear pattern for the USD coefficients in panel (b). The only thing that stands out is that growth companies are more negatively affected than value companies. This could be because growth stocks - often in the large cap-market segments synonymous with glamour stocks - tend to get more attention from foreign investors, who are more susceptible to changes in the USD exchange rate.

Hence, Fama-French factors have been proven to help explain the stocks returns and judging from the heatmaps it seems that macroeconomic factors also play a role in explaining the excess returns.

5.3.2 Estimation results for adjusted portfolios: RP-PCA vs PCA

Table 6 shows the in-sample maximal Sharpe Ratio, the root-mean-squared pricing error and the unexplained idiosyncratic variance for the adjusted portfolios. The Sharpe Ratios and the pricing errors have been converted to monthly values rather than daily values to facilitate interpretation.

What is immediately striking about the results of the in-sample analysis is that RP-PCA consistently dominates PCA in terms of maximal Sharpe Ratios and cross-sectional pricing errors. It seems that RP-PCA still outperforms PCA in finding (additional) latent factors, when one removes the very prominent Fama-French and macroeconomic risk from the dataset in advance. Although the PCA performs better in terms of idiosyncratic variance, recall that PCA by construction minimizes the unexplained idiosyncratic variation and that the differences are quite small for all portfolios.

Comparing panel A with panel D, it becomes clear that adjusting the portfolios for the risk of the Fama-French factor improves the Sharpe Ratios for four out of the five cases and the pricing errors for three out of the five cases. Especially for the size/short-term-reversal portfolio, there is a huge increase in the Sharpe Ratio. However, the momentum portfolio does not seem to benefit from the inclusion of the Fama-French factors.

Looking at panel B, we find that again in four out of five cases that the portfolios adjusted for macroeconomic risk have higher Sharpe Ratios. Adjusting for the macroeconomic factors does not seem to produce better results in explaining excess returns when looking at the pricing errors, for all the RMSα the benchmark portfolios have smaller cross-sectional pricing errors.

Lastly, panel C adjusts the portfolios for Fama-French *and* macroeconomic risk and provides us with four out of five times higher Sharpe Ratios and in three out of five cases lower pricing errors compared to the non-adjusted portfolios. In addition, it has, on average, the highest Sharpe Ratios and second-smallest pricing errors of all portfolios.

Comparing all panels with each other reveals that the difference between RP-PCA and PCA is more prominent in the portfolios adjusted for Fama-French risk than for macroeconomic risk. Table 7 displays the differences between the Sharpe Ratios and pricing errors between RP-PCA and PCA. What is particularly noteworthy is that removing the Fama-French factors and macroeconomic factors from the excess returns yields even larger in-sample differences between RP-PCA and PCA, compared to the other panels. This suggest that when these 'evident' factors are removed from the portfolios, RP-PCA is better in finding additional latent factors than PCA. This follows from the research of Lettau & Pelger (2020b) who state that RP-PCA is better in detecting weak factors than PCA.

Table 6: In-sample fit of RP-PCA and PCA for (non)-adjusted portfolios

	In-sample					
	SR		RMS_α		$\overline{\sigma}_e^2$	
	RP-PCA	PCA	RP-PCA	PCA	RP-PCA	PCA
Panel A: Adjusted for FF-risk						
SIZE&BM	**0.184**	0.170	**0.125**	0.128	55.14%	**55.13%**
SIZE&INV	**0.174**	0.164	**0.109**	0.111	57.42%	**57.41%**
SIZE&OP	**0.206**	0.203	**0.101**	0.101	41.11%	**41.11%**
SIZE&ST-REV	**0.263**	0.198	**0.316**	0.323	34.25%	**34.20%**
SIZE&MOM	**0.084**	0.077	**0.156**	0.157	33.44%	**33.44%**
Panel B: Adjusted for Macro-risk						
SIZE&BM	**0.181**	0.177	**0.194**	0.196	17.78%	**17.78%**
SIZE&INV	**0.258**	0.254	**0.109**	0.110	17.64%	**17.64%**
SIZE&OP	**0.145**	0.144	**0.230**	0.231	19.66%	**19.65%**
SIZE&ST-REV	**0.201**	0.193	**0.359**	0.360	16.40%	**16.40%**
SIZE&MOM	**0.195**	0.193	**0.159**	0.160	16.28%	**16.28%**
Panel C: Adjusted for FF+Macro-risk						
SIZE&BM	**0.210**	0.193	**0.132**	0.135	56.11%	**56.10%**
SIZE&INV	**0.214**	0.196	**0.115**	0.119	58.92%	**58.91%**
SIZE&OP	**0.257**	0.253	**0.095**	0.096	42.30%	**42.29%**
SIZE&ST-REV	**0.286**	0.197	**0.311**	0.325	34.18%	**34.09%**
SIZE&MOM	**0.097**	0.086	**0.170**	0.172	33.54%	**33.53%**
Panel D: Non-adjusted						
SIZE&BM	**0.159**	0.156	**0.172**	0.173	4.96%	**4.96%**
SIZE&INV	**0.167**	0.166	**0.102**	0.102	4.42%	**4.41%**
SIZE&OP	**0.167**	0.161	**0.164**	0.169	5.76%	**5.76%**
SIZE&ST-REV	**0.171**	0.167	**0.352**	0.353	4.86%	**4.86%**
SIZE&MOM	**0.161**	0.160	**0.126**	0.126	4.97%	**4.97%**

Note: *The table is showing the in-sample maximal Sharpe Ratios, root-mean-squared pricing errors and the unexplained idiosyncratic variation for the (non)-adjusted portfolios. The values for the SR and RMSα are monthly Sharpe Ratios and pricing errors. Portfolios adjusted for FF-risk and macro-risk are portfolios with excess returns minus the impact of the Fama-French and macroeconomic factors. Numbers in* ***bold*** *indicate which method performed best for the particular portfolio on the respective criteria. Factors is $K = 3$ with weight $\gamma = 20$*

In Table 8 we see the out-of-sample results reported for the three different criteria. What stands out

Table 7: Differences in fit for RP-PCA and PCA

Portfolios	In-Sample		Out-of-sample	
	SR	RMS_α	SR	RMS_α
(A) Adjusted for FF-risk	0.097	0.013	0.499	0.004
(B) Adjusted for macro-risk	0.019	0.005	0.167	**0.028**
(C) Adjusted for FF + macro-risk	**0.136**	**0.023**	**0.586**	0.025
(D) Non-adjusted	0.013	0.005	0.093	0.004

Note: *Absolute differences in Sharpe Ratio and root-mean-squared pricing errors between RP-PCA and PCA for the (non-)adjusted portfolios, both in-sample and out-of-sample.*

is that RP-PCA outperforms PCA in terms of higher Sharpe Ratios, but this time not in terms of having smaller pricing errors. This while for the idiosyncratic variance, PCA no longer strictly dominates RP-PCA, although the differences between the two methods are still quite small.

Comparing panels A, B and C, we see that C produces the highest Sharpe Ratios and the lowest cross-sectional pricing errors for three of the five portfolios. Comparing the Sharpe Ratios of panel C to the portfolios of panel D, we observe that the adjusted portfolios outperform the non-adjusted portfolios 60% of the time, especially for the size/short-term reversal portfolio of RP-PCA (0.387 to 0.066). Furthermore, for all the adjusted portfolios of panel C, the cross-sectional pricing errors are lower than for the non-adjusted portfolios of panel D. Hence, in OOS, the portfolios that take into account the Fama-French *and* macroeconomic factors clearly do a better job of explaining the excess returns. However, it is more difficult to capture the time-series variation when compared to panel D.

Looking at the differences between RP-PCA and PCA for the Sharpe Ratios in Table 7, we see that the differences are way larger out-of-sample than in-sample. Furthermore, the Sharpe Ratio difference is again largest for the portfolios adjusted for Fama-French and macroeconomic risk, with a difference of 0.586. This proves once again that RP-PCA outperforms PCA when 'strong' factors are removed, and other latent factors have to be found.

Regarding the difference in pricing errors for the RP-PCA compared to the PCA, we note that the differences are also way larger out-of-sample. The difference is the largest for panel B, closely followed by panel C with differences of 0.028 and 0.025 respectively. This is a lot higher than the difference of 0.004 of panel D and shows us two things: (1) the dominance of RP-PCA over PCA and (2) the even higher dominance of RP-PCA when only weak factors are present in the data.

Table 8: Out-of-sample fit of RP-PCA and PCA for (non)-adjusted portfolios

	Out-of-sample					
	SR		RMS_α		$\overline{\sigma}_e^2$	
	RP-PCA	PCA	RP-PCA	PCA	RP-PCA	PCA
Panel A: Adjusted for FF-risk						
SIZE&BM	**0.113**	0.069	0.115	**0.108**	**51.29%**	51.36%
SIZE&INV	**0.168**	0.099	0.101	**0.098**	**55.19%**	55.42%
SIZE&OP	**0.089**	0.081	0.134	**0.117**	40.63%	**40.56%**
SIZE&ST-REV	**0.368**	0.074	**0.261**	0.292	33.78%	**33.71%**
SIZE&MOM	**0.129**	0.043	**0.112**	0.111	**32.41%**	32.49%
Panel B: Adjusted for Macro-risk						
SIZE&BM	**0.116**	0.067	**0.172**	0.183	16.81%	**16.80%**
SIZE&INV	**0.113**	0.103	**0.123**	0.133	**16.16%**	16.23%
SIZE&OP	**0.162**	0.118	**0.166**	0.169	**17.23%**	17.29%
SIZE&ST-REV	**0.019**	-0.018	0.330	**0.33**	**16.46%**	16.47%
SIZE&MOM	**0.147**	0.120	**0.132**	0.143	15.83%	**15.82%**
Panel C: Adjusted for FF + Macro-risk						
SIZE&BM	**0.154**	0.109	0.117	**0.111**	**52.41%**	52.48%
SIZE&INV	**0.189**	0.120	**0.111**	0.114	**57.06%**	57.12%
SIZE&OP	**0.138**	0.108	0.122	**0.111**	**41.90%**	41.9%
SIZE&ST-REV	**0.387**	0.092	**0.250**	0.285	**33.70%**	33.71%
SIZE&MOM	**0.172**	0.025	**0.109**	0.114	**32.64%**	32.77%
Panel D: Non-adjusted						
SIZE&BM	**0.194**	0.170	**0.152**	0.156	**4.49%**	4.50%
SIZE&INV	**0.120**	0.106	0.119	**0.116**	4.05%	**4.04%**
SIZE&OP	**0.167**	0.137	**0.166**	0.167	**4.80%**	4.81%
SIZE&ST-REV	**0.066**	0.040	**0.324**	0.324	4.84%	**4.83%**
SIZE&MOM	0.062	**0.063**	**0.120**	0.122	4.87%	**4.85%**

Note: *The table is showing the out-of-sample maximal Sharpe Ratios, root-mean-squared pricing errors and the unexplained idiosyncratic variation for the (non-)adjusted portfolios. The values for the SR and RMSαA are monthly Sharpe Ratios and pricing errors. Numbers in* ***bold*** *indicate which method performed best for the particular portfolio on the respective criteria. The rolling window is set to T*=250, for $K = 3$ factors and weight $\gamma = 20$.*

6 Conclusion

In this paper, we test a brand-new estimator for latent asset pricing components that makes use of information in the first and seconds moments of the returns; the Risk-Premium Principal Component Analysis (RP-PCA). The goal of this new method is to extract factors that capture time-series variation on the one hand and provide us with smaller cross-sectional pricing errors on the other. The RP-PCA can be seen as a transformation of the original PCA combined with Ross's theory of the APT, resulting in an additional penalty term on the pricing errors.

The key findings can be described as follows. For the double-sorted portfolios we have seen that RP-PCA outperforms PCA and FF in most cases, especially in terms of higher maximal Sharpe Ratios and lower cross-sectional pricing errors. Furthermore, we have seen that the main difference between RP-PCA and PCA lies in the detection, compositions and order of the factors.

For the 370 single-sorted anomaly portfolios, only five RP-PCA factors are needed to explain most of the time-series variation and cross-sectional moments. For these factors, the RP-PCA method gave us higher maximal Sharpe Ratios and smaller cross-sectional pricing errors than PCA, both in-sample and out-of-sample. This is while the captured time-series variation remained very similar for both methods.

The first factor of the single-sorted portfolios that is selected according to RP-PCA is a long-only factor and has a high mean and high variance. And it is closely related to the overall market performance. The second and fifth factors have low variance but yield high Sharpe Ratios due to the weights of the factors being significantly correlated with the high means of the portfolio returns. Building a model with factors 1,2 and 5 gives us a model that is able to capture similar amounts of cross-sectional returns differences as a model that makes use of all five factors. The third and fourth factor on the other hand help capture the time-series co-movement, but have low means and provide us with low Sharpe Ratios. Thus, a model consisting of factors 1, 3 and 4 does not help us explain the cross-sectional return differences, but does capture almost all of the time-series variation.

When comparing the RP-PCA with the PCA factors, the PCA factors have significantly lower Sharpe Ratios than the RP-PCA factors. This is because as a result of the additional penalty term on the cross-sectional pricing errors, the portfolio loadings of the SDF are tilted towards the characteristic-portfolios that have higher return premia. For different specifications and samples these empirical findings are robust.

For portfolios consisting of daily returns we also found evidence that RP-PCA dominates PCA for the in-sample analysis, in terms of higher Sharpe Ratios and lower cross-sectional pricing errors, while yielding similar idiosyncratic variance. For the out-of-sample case, RP-PCA strictly dominates PCA in terms of higher Sharpe Ratios. The dominance is less strong for cross-sectional pricing errors, but overall RP-PCA also outperforms PCA here. The difference between RP-PCA and PCA was more noticeable for the adjusted portfolios. The more systematic risk we removed from the excess returns, the greater the differences in performance between RP-PCA and PCA became. This verifies Lettau & Pelger's assertion that RP-PCA performs especially well if the factors are weak.

In addition, we have proven that regardless of the method you use, removing Fama-French risk from the returns yields, on average, higher Sharpe Ratios and lower cross-sectional pricing errors for both in-sample and out-of-sample cases. This proves once again that Fama-French factors are helpful in describing stock returns. For macroeconomic factors, we obtain similar conclusions in-sample, but not out-of-sample. However, removing both macroeconomic risk and Fama-French risk from the portfolios gave us the highest Sharpe Ratios and lowest cross-sectional pricing errors, both in-sample and out-of-sample. This shows that there may be room for improvement on the Fama-French three-factor model through additional addition of macroeconomic components. As indicated earlier, one of the problems with the incorporation of macroeconomic variables is always that market participants do incorporate expectations about such variables in their analysis. Stated differently, at any moment in time, they do not only look at the value of these macro variables at that moment, but also the expected trends in these variables. This can translate into situations where the negative effect of, for example, a downfall in the level of a specific variable is already incorporated in the prices before the actual deterioration happens. And if it later turns out that the deterioration was less big than expected, it may actually at that point in time lead to a positive rebound. This introduces potential ambiguity. However, our results do indicate that the incorporation of four important macroeconomic indicators (oil price, gold price, GDP growth and USD exchange rate) can improve the results of the standard Fama-French three-factor model. Albeit that the latter do continue to explain the bulk of cross-sectional variation in stock returns. A detailed examination of the implications of adding macroeconomic factors to the three-factor model is beyond the scope of this paper, but could nonetheless be a very interesting topic for further research.

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A Appendix

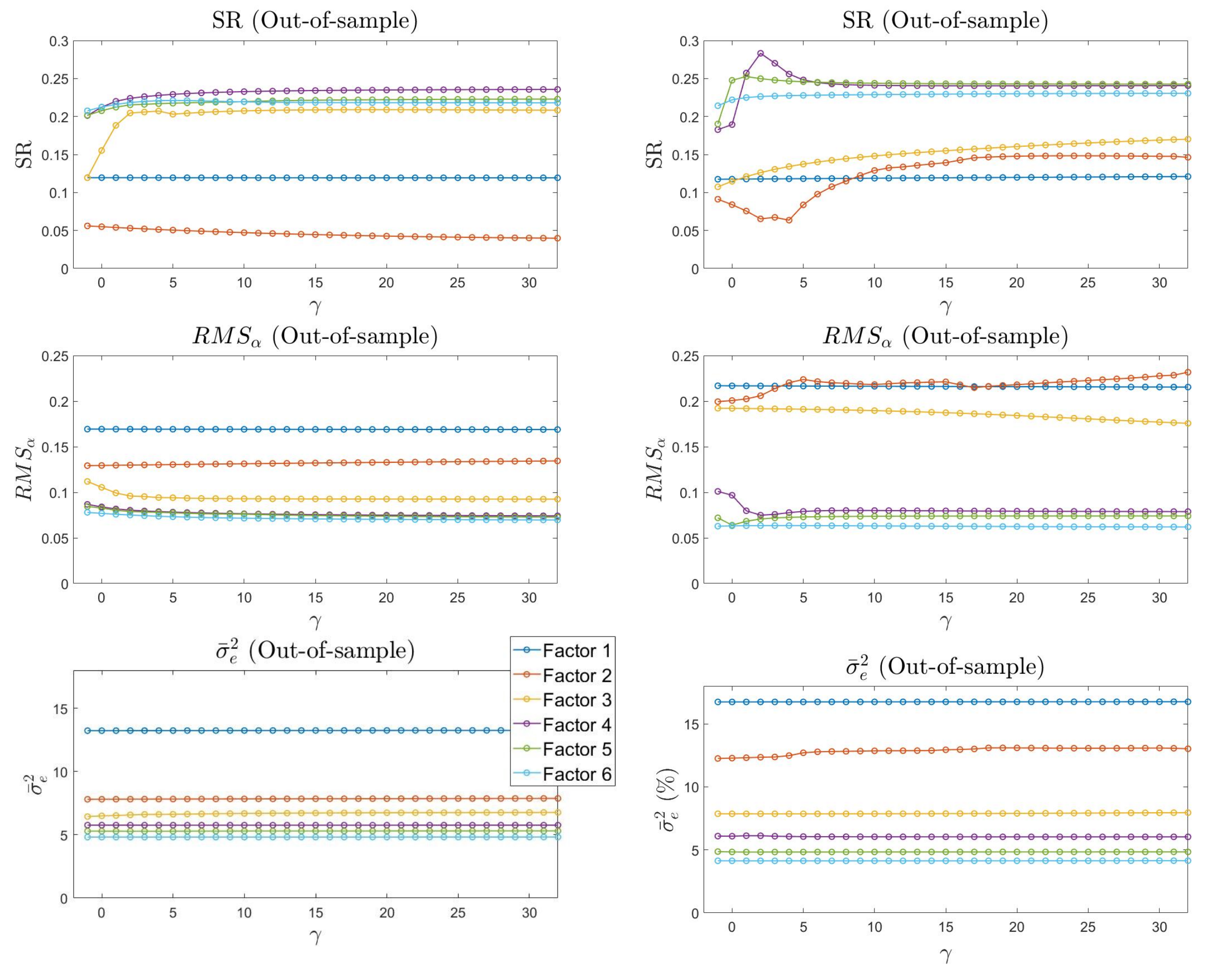
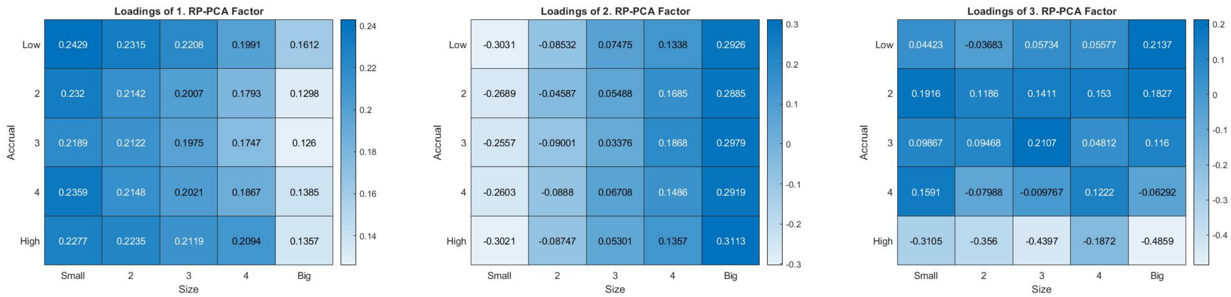


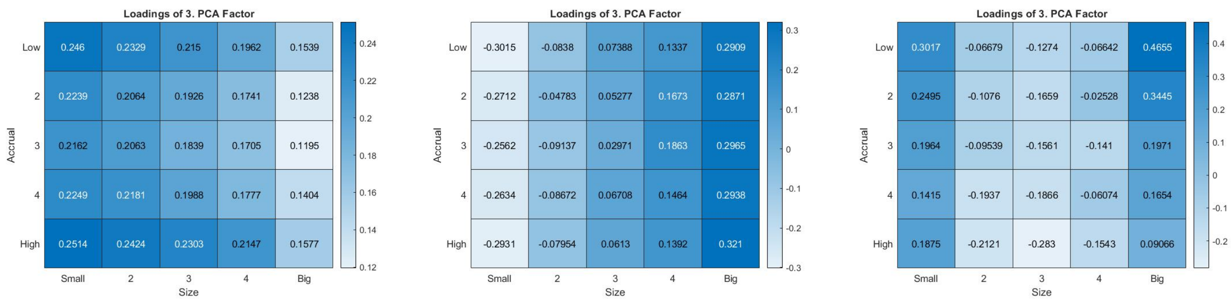
Figure A.1: Out-of-sample values as a function of γ

Note: *The out-of-sample maximal Sharpe Ratios, root-mean-squared pricing errors, and unexplained idiosyncratic variance for size/accrual portfolios (Left) and size/Short-term reversal portfolios (Right) as a function of γ for different number of factors.*

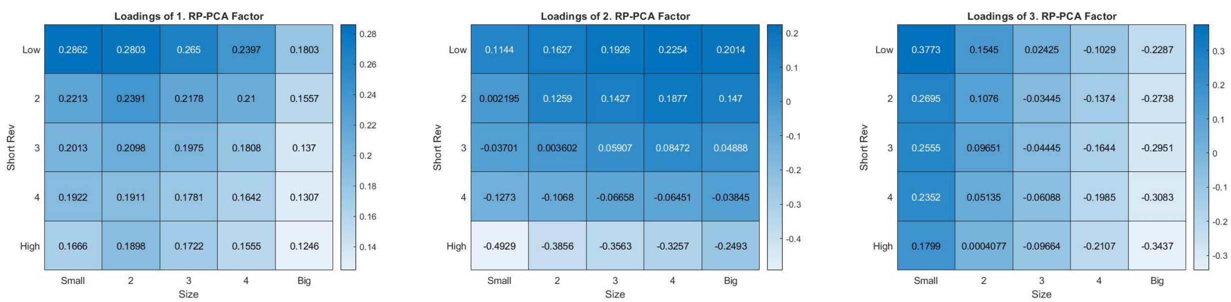
(a) Size/Accrual: RP-PCA



(b) Size/Accrual: PCA



(c) Size/Short-Term Reversal: RP-PCA



(d) Size/Short-Term Reversal: PCA

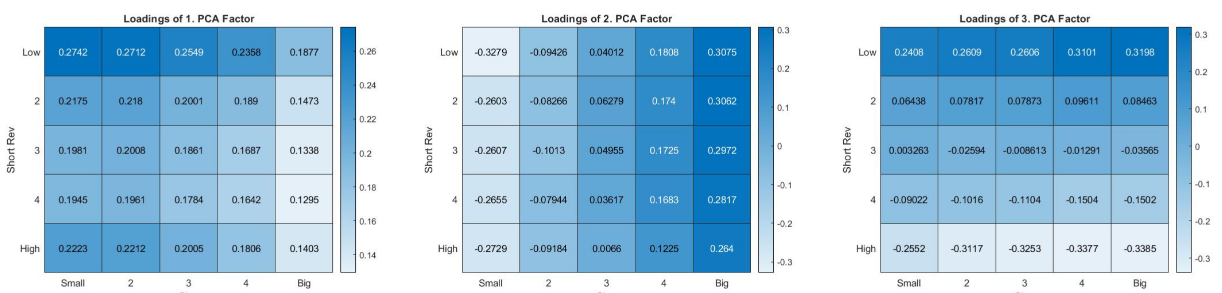


Figure A.2: Factors for Size/Accrual and Size/Short-Term Reversal portfolios
Note: *Heat map of portfolio loadings of $K = 3$ factors. RP-PCA weight is set to $\gamma = 20$*

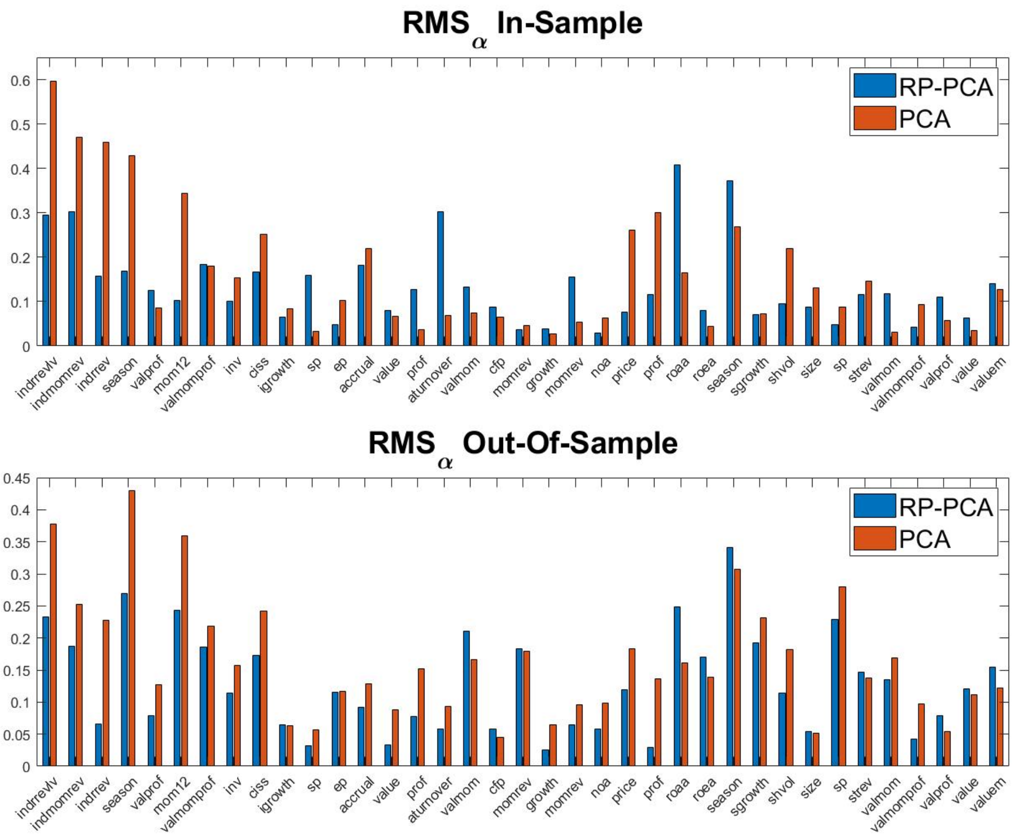


Figure A.3: Root-mean-squaed pricing errors α per characteristic
Note: *In-sample and out-of-sample, root-mean squared pricing errors for the 37 single-sorted portfolios (N=74) using $K = 5$ factors.*

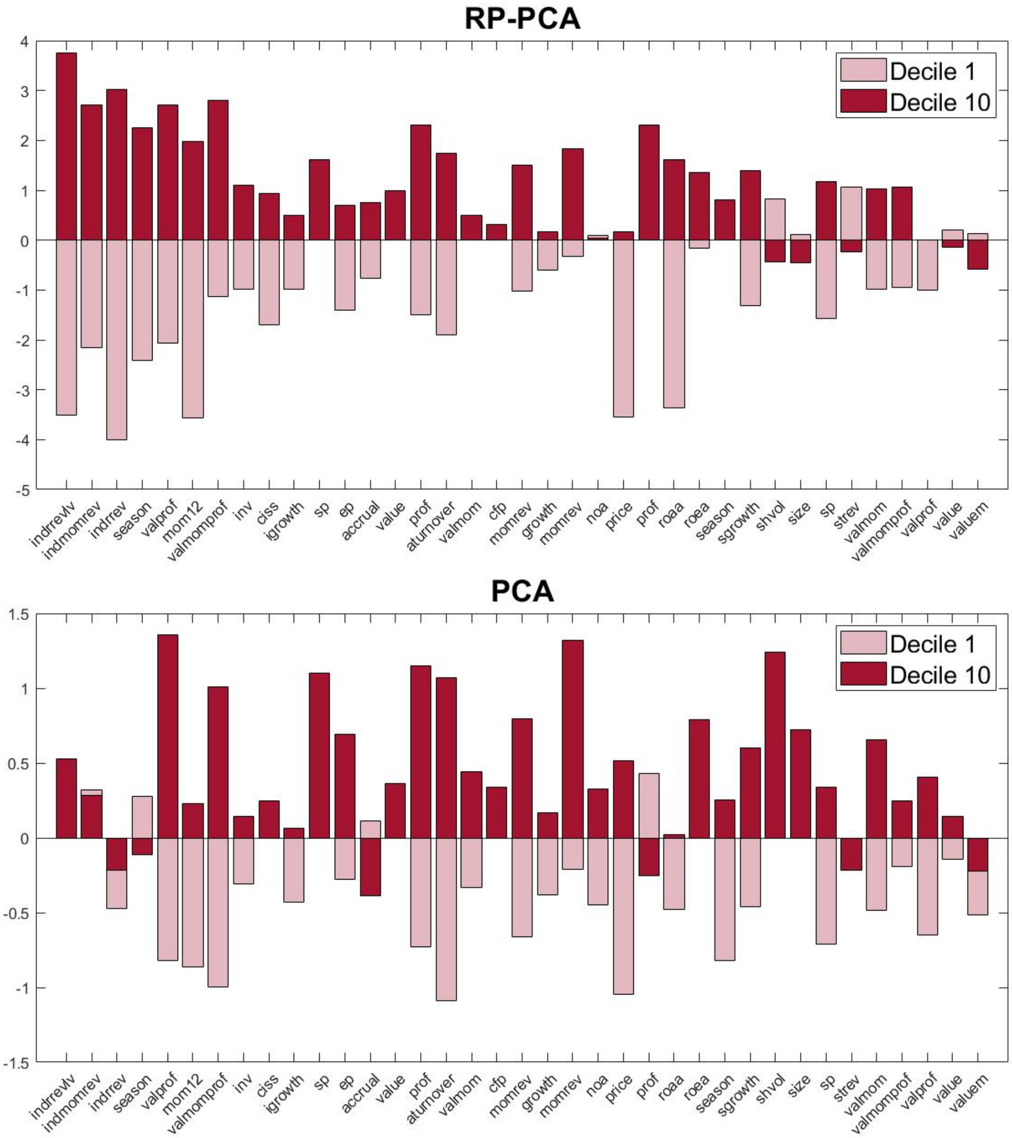


Figure A.4: Portfolio loadings for SDFs of RP-PCA and PCA
Note: *The RP-PCA and PCA portfolio weights of SDF for the single-sorted anomaly portfolios. The characteristics on the x-axis are sorted by their Sharpe Ratio.*

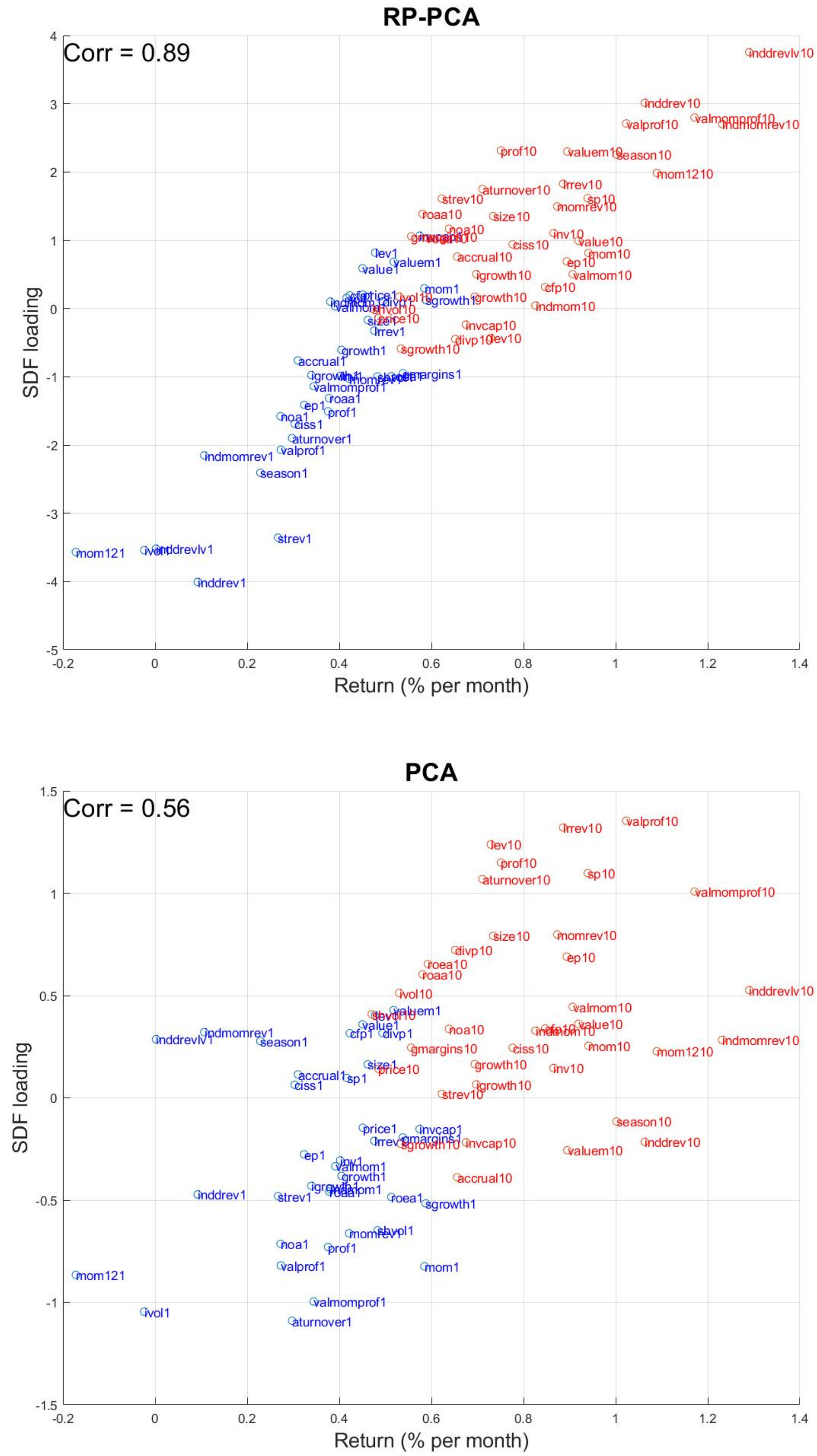


Figure A.5: Portfolio weights of the RP-PCA and PCA SDFs
Note: *Scatter plots of the mean returns of the portfolios on the horizontal axis and the SDF loadings on the vertical axis for decile-1 and decile-10 of the 37 single-sorted portfolios.*