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Bachelor Thesis

Quantitative Finance

Cryptocurrency factor models involving sector-specific risks

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Abstract

In this paper, we have used return information of ETFs (Exhange-Traded Funds) to make 3 additions to the cryptocurrency 3-Factor Model (Shen et al., 2020). For this we have used weekly data of a representative subset of 49 cryptocurrencies along with the returns of a financial, an environmental and a cybersecurity ETF in the period July 2018 to December 2021. First, we have added the average weekly return. Then, we have added the weekly biggest return, keeping the sign. Finally, we have introduced a regime-switching component. Each week's regime is mainly determined by Fuzzy C-Means. The OLS estimation results and the rolling adjusted R-squareds indicate that the 3-Factor Model is here too the most preferable. The average and biggest magnitude factor yield marginal and inconsistent improvements. The regime-switching model is more promising, but it does not seem to outperform the 3-Factor Model either.

Keywords: cryptocurrency, factor models, ETFs, Fuzzy C-Means

Contents

1 Introduction

Numerous studies have researched cryptocurrencies in recent years. These studies mainly seek relationships between cryptocurrencies and generic data such as the S&P500, The EPU index (Economic Policy Uncertainty index), the VIX or social network usage (Georgoula et al. (2015), Bouri et al. (2017), Conrad et al. (2018), Demir et al. (2018), Wang et al. (2020), López-Cabarcos et al. (2021), among others). These generic variables allow us to relate cryptocurrency behaviour with general economic developments.

The aforementioned variables are very broad since they include almost every sector. It is likely that cryptocurrencies are more sensitive to developments in certain specific sectors. For example: about 40% of the assets included in the S&P500 are active in the technology sector, 14% of the S&P500 are active in the healthcare sector and 11% in the financial sector[1]. If we relate the S&P500 with the behaviour of cryptocurrencies, then one could argue that the healthcare sector and the financial sector are over- and underrepresented, respectively. So, being able to incorporate only relevant sectors into a model may provide improvements. Including sector-specific information in a model can be realized by using efficient ETFs (Exchange-Traded Funds) (Tse and Martinez, 2007). We thus assume that ETF prices reflect all available news events. Cryptocurrencies are also relatively sensitive to those specific news events, so, their prices will also adjust accordingly. This means that you may be able to explain some coins' behaviour by the way the ETFs react to the same news events.

In this paper, we investigate how extending the cryptocurrency 3-Factor Model (Shen et al., 2020) with return information of crypto related sector ETFs affect the explanatory performance in the period 2018-2021. We do this with weekly returns of 49 representative cryptocurrencies and 3 ETFs that track the financial, environmental and cybersecurity sector. The benchmark is the C-CAPM (cryptocurrency-CAPM) and the three additions are: the average ETF returns, the biggest return in magnitude (keeping the sign) and a regime-switching component mainly determined by Fuzzy C-Means (Dunn (1973), Bezdek (1981)). We evaluate the models with rolling adjusted R-squareds

We find that the 3-Factor Model is still the best performing model. Because despite the relatively high adjusted R-squareds due to the small dataset, we find that the improvements of it obtained by the extension with the average and the biggest magnitude factor are small and inconsistent. The regime-switching factor does provide more consistent improvements, but not enough to conclude that it outperforms the 3-Factor Model. The latter is promising and demonstrates the potential of machine learning techniques in finance.

Section 2 represents the literature. Next, in Section 3 we discuss the data. The methods are described in Section 4, while the results thereof are reported in Section 5. The conclusions are given in Section 6. We refer to the study of Shen et al. (2020) as SUW (Shen, Urquhart and Wang)

[1] https://www.spglobal.com/spdji/en/indices/equity/sp-500/overview

2 Literature

In this section, we discuss the literature. First, we briefly review the Efficient Market Hypothesis (EMH) (Fama, 1970) and the efficiency of developed asset markets. After that, we look at the inefficiency of cryptocurrencies.

The EMH states that the prices in security markets incorporate all available relevant information. This makes consistently beating the market very challenging. Although, there are many studies in favour of the EMH in the early days surrounding the publication of this hypothesis, over time a sentiment arose that claims that markets can be predicted to a certain extent (Rosenberg et al. (1985), among others). However, in more recent years, others studies have found EMH supportive results (Welch and Goyal (2008), among others). Malkiel (2003) also shows support for the EMH in an extensive literature review. In his review, he refutes the three main schools of thought that claim predictability. He states that momentum strategies do not lead to consistent outperformance over time. Furthermore, he contradicts arguments from behavioural finance by claiming that underreaction is as common as overreaction. This again implies a random pattern of financial behaviour. Finally, he claims that fundamental factors with predictive power lose this power after when they are discovered, the latter is again in line with the EMH. Therefore, we reasonably assume that the EMH applies to developed asset markets and that deviation from it are only short-lived and unpredictable.

However, the assumption of efficiency is less valid for other asset classes. Bekaert and Harvey (2002) state that returns of emerging equity markets have higher serial correlation and react less towards company-specific news than developed markets. Urquhart (2016) has shown the inefficiency of Bitcoin and compares it with an emerging market asset which is becoming more efficient over time. The maturity process of Bitcoin is found by other studies too (Bouoiyour et al. (2016), Bariviera et al. (2017), among others). Since cryptocurrencies are less efficient, it seems reasonable to model their behaviour. Here too, numerous research has been done to find adequate explanatory variables. Finding factors is challenging because cryptocurrencies lack fundamental statistics. On the other hand, cryptocurrencies do have very sector-specific risks. Two of those are monetary policy/banking sector and environmental impact (Badea and Mungiu-Pupzan, 2021). Another obvious risk factor is cybersecurity (Zamani et al., 2020). One way to track these specific risks is by using efficient and sector-specific ETFs[2] (Tse and Martinez, 2007). With these ETFs, we may be able to explain some of the cryptocurrencies' behaviour.

3 Data

We continue with the required data for the models in this section. We denote everything in US Dollars (USD) and we collect weekly data on each week's Friday (this in order to capture all price developments within a trading week). The sample period is from 27 July 2018 to 31 December 2021 (180 weekly observa-

[2] https://www.investopedia.com/terms/e/etf.asp

tions in total). The 4-week secondary market US T-Bill rate is used as the risk-free rate[3].

The cryptocurrency dataset is not as extensive as in SUW. Instead, a representative subset is used by selecting only the data of the top 49 capitalized coins at the beginning of 2018[4]. We do not include *Bitconnect* (BCC), *Experience Points* (XP), *Voyager Token* (VGX), *Tether* (USDT), *Kin* (KIN) and *Veritasium* (VERI) because not enough relevant data of these coins are available. The chosen coins are representative since they represent all market capitalizations throughout the sample period. Coins with a capitalization more than 10 billion USD are classified as large capitalization coins. Coins with a capitalization between 1 billion and 10 billion USD are considered middle capitalization coins, while small capitalization coins are the coins with a capitalization less than 1 billion USD[5]. The capitalizations of the top 49 at the beginning of each year is shown in Table 1.

Table 1: Number of coins in each capitalization category at the beginning of each year

	2018	**2019**	**2020**	**2021**	**2022**	**average**
large capitalization	5	3	3	4	6	4.2
middle capitalization	25	6	10	13	13	13.4
small capitalization	19	40	36	32	30	31.4

From Table 1 it follows that the market capitalization composition of the top 49 is quite stable over time. Roughly 10% are large, 30% are middle and the remaining 60% are small. Changes over time are allowed and even desired, but this does not lead to underrepresentation of a particular category during the sample period.

The *iShares Global Financials ETF* (IXG)[6] is chosen to represent the banking sector risk. This is because it mainly consists of traditional financial institutions, which can be seen as counterparts of cryptocurrencies. The environmental risk is tracked with the *iShares Global Clean Energy ETF* (ICLN)[7]. This ETF tracks companies who develop and exploit sustainable energy technology. Finally, the ETF which tracks the cybersecurity risk is the *ETFMG Prime Cyber Security ETF* (HACK)[8]. This ETF is the oldest cybersecurity ETF and contains companies that are active in providing cybersecurity technology and services.

Missing values in our dataset are estimated by means of inverse distance weighted interpolation. This allows us to estimate (small) intervals with observations that are most related to the missing ones.

[3]https://fred.stlouisfed.org/series/WTB4WK
[4]https://coinmarketcap.com/historical/20180107/
[5]https://www.coinbase.com/learn/crypto-basics/what-is-market-cap
[6]https://www.ishares.com/us/products/239742/ishares-global-financials-etf
[7]https://www.ishares.com/us/products/239738/ishares-global-clean-energy-etf
[8]https://etfmg.com/funds/hack/

4 Methodology

With the data, we start with the construction of the J-K portfolios. After that, we discuss the dependent variables, the 3 factors of SUW and our ETF factors. Then, we will give an overview of the different models. The robustness check of those models is described in the end.

4.1 J-K Portfolios

We follow the idea of SUW by constructing weekly J-K portfolios (Jegadeesh and Titman, 1993). The construction of these portfolios happens with a J and K equal to 1,2,3 or 4 weeks (in total 16 strategies). In every week t we construct the following equally weighted portfolios:

- **Sell portfolio**: the portfolio containing the 5 cryptocurrencies with the lowest returns over the past J weeks.
- **Buy portfolio**: the portfolio containing the 5 cryptocurrencies with the highest returns over the past J weeks.
- **Buy-Sell portfolio**: the portfolio in which we buy the 5 best performing cryptocurrencies and sell the 5 worst performing cryptocurrencies over the past J weeks.

After K weeks we liquidate our positions and we calculate the holding period return. With these returns, we perform a t-test. Based on the test results, we determine how we construct our (in)dependent variables. The mean returns and the corresponding t-statistic of the 16 J-K portfolios are given in Table 2.

Table 2: Mean returns and t-statistics of the J-K portfolios

	J	K = 1	K = 2	K = 3	K = 4
1	**Buy**	1.660	3.329*	4.596**	6.261**
		(1.456)	(1.860)	(2.053)	(2.322)
1	**Sell**	1.760	2.661	3.822*	4.864*
		(1.544)	(1.572)	(1.810)	(1.969)
1	**Buy-Sell**	-0.100	0.668	0.774	1.397
		(-0.103)	(0.461)	(0.419)	(0.680)
2	**Buy**	1.809	3.055*	5.455**	6.863**
		(1.491)	(1.744)	(2.189)	(2.283)
2	**Sell**	1.500	2.437	3.795*	5.573**
		(1.291)	(1.497)	(1.779)	(2.128)
2	**Buy-Sell**	0.310	0.618	1.660	1.290
		(0.321)	(0.477)	(0.987)	(0.683)
3	**Buy**	2.077	4.242**	5.898**	8.058**
		(1.618)	(2.169)	(2.274)	(2.579)
3	**Sell**	1.711	3.333*	4.915**	6.791**
		(1.485)	(1.882)	(2.175)	(2.407)
3	**Buy-Sell**	0.366	0.909	0.983	1.267
		(0.329)	(0.576)	(0.527)	(0.583)
4	**Buy**	1.702	2.981*	5.485**	7.955**
		(1.320)	(1.656)	(2.305)	(2.564)
4	**Sell**	1.649	3.589**	4.880**	6.492**
		(1.423)	(2.017)	(2.103)	(2.520)
4	**Buy-Sell**	0.053	-0.608	0.605	1.463
		(0.053)	(-0.444)	(0.326)	(0.666)

Note: The t-statistics are given in brackets. 10%, 5% and 1% significant results are indicated with a *, ** and ***, respectively.

Noticeable is the fact that given a J, if we increase the K, the return of both the Buy and Sell portfolio rises. Along with this, the significance also increases. This means that the longer you hold your positions in the sample period of this paper, the higher the average return of your investment will be. This result corresponds to the growth of the cryptocurrency market from 2020 onward (Appendix A)

Contrary to SUW, we do not find a comprehensive presence of significant negative returns across the Buy-Sell portfolios. Instead, the returns are mainly positive and no longer significant. The latter implies that the most significant result at $J = 1$ and $K = 1$ and the momentum effect at $J = 4$ and $K = 1$ no longer appears in this paper. A possible explanation for this is the increasing efficiency of cryptocurrencies. According to the aforementioned literature, strategies that deliver significant returns disappear after being published. Since we need to know which lagged returns to sort by while constructing the (in)dependent variables and we do not have any significant Buy-Sell results in this paper to determine that, we have decided to use the same method as in SUW. Our paper approximates their research and therefore a reasonable guess is to follow their 1-1 approach.

4.2 Dependent Variables

The construction of the dependent variables is not the same as the 5×5 approach in Fama and French (2012). The latter is due to the fact that we work with fewer coins than in SUW. Dividing our 49 coins in 5×5 portfolios will result in very small portfolios, which in turn negatively influences its representativeness. So instead, we work with 3×3 portfolios with the breakpoints of both the weekly market capitalization (Large-cap, Mid-cap and Small-cap) and the prior return (Low, Mediocre and High) approximating the 33^{rd} and the 67^{th} percentiles. Because 49 is not divisible by 3, we have decided that the Small-cap set consists of a bit more coins than the other two (19 instead of 15). And since 19 is not divisible by 3 either, we have decided that its Mediocre set should contain 9 coins instead of the usual 5 in order to keep the methods symmetrical. The skewed allocation of the coins based on market capitalization corresponds to the skewed results in Table 1. The results of sorting this way are 3×3 portfolios of which we use the weekly returns as dependent variable in our regressions. Table 3 provides an overview of the number of coins per size category, prior return category and portfolio, while Table 4 presents the mean excess return and its t-statistic of each dependent variable

Table 3: Number of coins in each of the 3×3 portfolios

	Low	**Mediocre**	**High**	total
Large-cap	5	5	5	15
Mid-cap	5	5	5	15
Small-cap	5	9	5	19
total	15	19	15	49

Table 4: Mean excess returns and t-statistics of the 3×3 portfolios

	Low	**Mediocre**	**High**	Low-High
Large-cap	0.726	1.117	1.958*	-1.232
	(0.607)	(1.198)	(1.920)	(-1.548)
Mid-cap	1.026	1.306	1.930	-0.905
	(0.847)	(1.185)	(1.618)	(-0.956)
Small-cap	0.726	0.501	-0.498	1.224
	(0.634)	(0.468)	(-0.347)	(1.108)
Small-cap - Large-cap	0.000	-0.616	-2.455**	
	(0.000)	(-0.883)	(-2.005)	

Note: The t-statistics are given in brackets. 10%, 5% and 1% significant results are indicated with a *, ** and ***, respectively.

From Table 4 we notice that only the returns of Large-cap coins with High prior returns are significant. Furthermore, we do not find that the average returns of small coins are greater than those of the large coins. Here the difference between Small- and Large-cap in the Low and Mediocre category is insignificant, while in the High category we see that the Large-caps significantly outperforms the smaller coins. A possible explanation for this is that larger coins are more likely to grow consistently over time, while smaller coins are more driven by temporary hypes.

4.3 3 Factors

we regress the dependent variables on both the factors of SUW and on the new ETF factors. The market factor thereof is:

$$MKT_t = \sum_{i=1}^{49} ret_{i,t} \times \frac{Cap_{i,t}}{TotalCap_t} \tag{1}$$

where MKT_t is the return of the capitalization weighted market portfolio in week t. $ret_{i,t}$ and $Cap_{i,t}$ are the return and market capitalization of the i^{th} cryptocurrency in week t respectively. $TotalCap_t$ is the sum of the 49 market capitalizations in week t.

The size and reversal factors are created with the weekly returns of the 2×3 capitalization weighted portfolios as in SUW. However, the breakpoints deviate from those in that paper. Instead of classifying the coins with the bottom 10% market capitalization as small (S) and the top 90% as big (B), we classify the bottom 90% as small coins and the top 10% as big coins. The breakpoints of the prior returns are set to be the 33^{rd} percentile and the 67^{th} percentile. The prior return percentiles are classified as high prior returns (U), mediocre prior returns (M) or low prior returns (D). By taking the intersection of size and prior returns we end with 2×3 weekly portfolios: BD_t, BM_t, BU_t, SD_t, SM_t and SU_t. The number of coins per portfolio is shown in Table 5.

Table 5: Number of coins in each of the 2×3 portfolios

	D	**M**	**U**	total
B	2	2	2	6
S	14	15	14	43
total	16	17	16	49

The approach above deviates slightly from the one in SUW, but they are reasonable because: **[1]** it approximates the original approach **[2]** SUW has shown its robustness **[3]** it is consistent with Table 1. Finally, we construct the weekly size (SMB_t) and reversal (DMU_t) factor with the returns of the 2×3 portfolios:

$$SMB_t = \frac{1}{3}(SU_t + SM_t + SD_t) - \frac{1}{3}(BU_t + BM_t + BD_t) \tag{2}$$

$$DMU_t = \frac{1}{2}(BD_t + SD_t) - \frac{1}{2}(BU_t + SU_t) \tag{3}$$

Some summary statistics of the 3 factors of SUW are given in the left part of Table 6. The right part contains their correlations.

Table 6: Descriptive statistics of the 3 factors

	Summary statistics				**Correlation**		
	Mean	**Std.Dev.**	**Skewness**	**Kurtosis**	$\boldsymbol{RMRF_t}$	$\boldsymbol{SMB_t}$	$\boldsymbol{DMU_t}$
$\boldsymbol{RMRF_t}$	0.939	11.002	0.082	5.277	1.000		
$\boldsymbol{SMB_t}$	-0.219	5.770	0.437	6.046	0.056	1.000	
$\boldsymbol{DMU_t}$	-0.628	8.319	0.847	8.705	0.077	0.138	1.000

From the left part we see that the returns of both SMB_t (-0.219) and DMU_t (-0.628) are negative, while that of the $RMRF_t$ is positive (0.939). Furthermore, all three factors are leptokurtic and positively skewed (although, the skewness at $RMRF_t$ is almost zero). The highest correlation displayed on the right part equals 0.138 (between SMB_t and DMU_t). Because this value is relatively low, we conclude that multicollinearity has little influence on the estimates of the parameters of the 3-Factor Model.

4.4 ETF Factors

In addition to the 3 factors of SUW, we now introduce the ETF factors. These are made with the returns of the 3 ETFs and a data mining technique called Fuzzy C-Means (Dunn (1973), Bezdek (1981)). In this paper, we introduce 3 ETF factors:

1. **Average**: the average performance of the 3 ETFs in week t by using an equally weighted average of the returns of the 3 ETFs.

2. **Biggest Magnitude**: the most aberrant return in week t by using only the return which is the biggest in magnitude, keeping the sign.

3. **Regime**: 3 dummy variables which capture the different states of the stock/cryptocurrency market by means of clustering the 3 ETF returns in week t.

In the remaining part of this subsection, we explain how the 3 Regime dummy variables are made. The construction of the Regime dummies is mainly determined by the Fuzzy C-Means algorithm. Fuzzy C-Means divides a set of unlabelled observations (i.e., the 180 weekly 3×1 vectors with ETF returns) into "fuzzy" clusters. The fuzziness is due to the fact that observations do not have to belong 100% to one cluster. Instead, each observation belongs to each cluster with a weight varying from 0 to 1, depending on the amount of similarity with that cluster (these weights add up 1). The reason we choose Fuzzy C-Means is the fact that the resulting clusters are "fuzzy". This fuzziness usually fits better with the mostly elliptical distributions of returns, than clustering approaches that cut right through these subtle structures.

In this paper, we make 4 clusters with a seed equal to 0. We use 4 clusters in order to divide the weeks into 3 different regime categories. The Fuzzy C-Means clustering algorithm is applied in its most basic form, which means that the fuzziness exponent is 2, the distance metric is the L2-norm (for both vectors and matrices), the convergence value is 1×10^{-9} and the random cluster initialization is done with the same method as in K-means++ (Arthur and Vassilvitskii, 2007). We choose K-Means++ because this ensures that the clustering results depend much less on the random initialization and more on the actual underlying cluster structures. The procedure is described below. First, we introduce some general notation, then we give the algorithm.

seed	the seed for the random initialization
C	the number of clusters (indexed by j)
m	the fuzziness exponent
$\|\|*\|\|$	the vector and matrix norm
δ	the convergence value
N	the number of vectors (indexed by i)
x_i	the i^{th} vector (observation)
$u_{i,j}$	the weight of vector i for cluster j
c_j	the representative center of cluster j
$U^{(k)}$	the $N \times C$ weight matrix containing the weights of each vector for each cluster in the k^{th} step of Fuzzy C-Means

Algorithm 1 Fuzzy C-Means clustering with K-Means++ initialization procedure

Select C random c_j's with the initialization procedure of K-Means++:
  Use the seed to randomly select the first c_j from the x_i's
  Repeat
    Compute the distance metric of each vector x_i to its closest c_j, denoted as d_i
    Assign to each vector x_i a probability p_i which equals $\frac{(d_i)^2}{\sum_{q=1}^{N}(d_q)^2}$
    Vector x_i becomes the next initial cluster representative with a probability p_i
  Until We have found C initial c_j's

Calculate the entries of $U^{(0)}$ as follows: $u_{i,j} = \frac{1}{\sum_{q=1}^{C}(\frac{||x_i - c_j||}{||x_i - c_q||})^{\frac{2}{m-1}}}$

Update the c_j's as follows: $c_j = \sum_{i=1}^{N} \frac{(u_{i,j})^m}{\sum_{i=1}^{N}(u_{i,j})^m} \times x_i$

Repeat
  Calculate a new weight matrix with $u_{i,j} = \frac{1}{\sum_{q=1}^{C}(\frac{||x_i - c_j||}{||x_i - c_q||})^{\frac{2}{m-1}}}$
  Update the c_j's as follows: $c_j = \sum_{i=1}^{N} \frac{(u_{i,j})^m}{\sum_{i=1}^{N}(u_{i,j})^m} \times x_i$
Until we meet the convergence criterion: $||U^{(k+1)} - U^{(k)}|| < \delta$

The end result is a (local) minimization of the following objective function: $\sum_{i=1}^{N}\sum_{j=1}^{C}(u_{i,j})^m||x_i - c_j||^2$.

The 4 resulting clusters are then ranked in ascending return order. So, the first cluster contains the most bearish weeks, while the fourth cluster contains the most bullish weeks. For each week (observation), we multiply the weight of that observation for a cluster by the number of that cluster (i.e., 1,2,3 or 4). This results in a weekly sentiment time series that varies between 1 and 4 throughout the sample period. A value close to 1 implies a bearish sentiment, while a value close to 4 implies the opposite. For each week, we then look at the value of that sentiment. If a week's sentiment is in [1,2), then we classify this week as "bearish", if it is in [2,3), then we classify it as "neutral" and if it is in [3,4], then we classify it as "bullish". Based on this classification, we determine which dummy variable should be set to 1 and which should remain 0 in that week. If we take neutral market sentiment as the reference sentiment, then we can measure the effect on the intercept of a bear/bull market sentiment by adding the bear and bull dummy variable to the 3-Factor model.

In the remainder of this paper, we denote the weekly Average factor as ETF_t^A. The weekly Biggest Magnitude factor is gives as ETF_t^B and the 3 Regime dummies are denoted as ETF^i with i equal to 1 (bearish), 2 (neutral) or 3 (bullish).

4.5 Factor Models

With the (in)dependent variables stated above, we construct the factor models. This subsection gives a brief overview of the 5 factor models in this paper. The C-CAPM and the 3-Factor Model are as follows:

$$r_{i,t} - Rf_t = \alpha_i + \beta_{i,1} RMRF_t + \epsilon_{i,t} \tag{4}$$

$$r_{i,t} - Rf_t = \alpha_i + \beta_{i,1} RMRF_t + \beta_{i,2} SMB_t + \beta_{i,3} DMU_t + \epsilon_{i,t} \tag{5}$$

with $r_{i,t}$ the weekly return of one of the 3×3 portfolios. Rf_t is the risk-free rate and $RMRF_t$ is the excess market return ($MKT_t - Rf_t$). SMB_t and DMU_t are the weekly size and reversal factor, respectively.

Besides (4) and (5), we construct 3 other models (Average, Biggest Magnitude and Regime) with factors based on the returns of the ETFs:

$$r_{i,t} - Rf_t = \alpha_i + \beta_{i,1} RMRF_t + \beta_{i,2} SMB_t + \beta_{i,3} DMU_t + \beta_{i,4} ETF_t^A + \epsilon_{i,t} \tag{6}$$

$$r_{i,t} - Rf_t = \alpha_i + \beta_{i,1} RMRF_t + \beta_{i,2} SMB_t + \beta_{i,3} DMU_t + \beta_{i,4} ETF_t^B + \epsilon_{i,t} \tag{7}$$

$$\begin{aligned} r_{i,t} - Rf_t = \alpha_i &+ \beta_{i,1} RMRF_t + \beta_{i,2} SMB_t + \beta_{i,3} DMU_t \\ &+ \beta_{i,bear} ETF_t^1 + \beta_{i,bull} ETF_t^3 + \epsilon_{i,t} \end{aligned} \tag{8}$$

with ETF_t^A, ETF_t^B and ETF^i ($i = 1, 2, 3$) representing the ETF factors. Note the α_i that in the regime-switching model (8) represents the reference sentiment, while the $\beta_{i,bear}$ and $\beta_{i,bull}$ represents the change of α_i during a bear or bull market, respectively.

We use Ordinary Least Squares to estimate the parameters in the models described above. Unlike in SUW, we do not use the R-squared as evaluation criteria to compare the models. This is because the 3-Factor Model itself is already an expansion of C-CAPM. When extra regressors are added, the R-squared will always increase or stay the same in value, regardless of the added value of the extra variables. Therefore, to determine whether the extra ETF factors improves the model in a more proper way, we use the adjusted R-squared when comparing the models.

4.6 Robustness Check

The robustness check is performed with rolling regressions. The length of the window is 52 weeks and the step length is 1 week. The rolling adjusted R-squared for each model and each independent variable are plotted in a graph afterwards. This allows us to visually compare the results. We also calculate for each model how often this model has the highest adjusted R-squared compared to the other models for the same dependent variable. This numeric result is given as a percentage of the time interval of the rolling regression.

5 Results

We first discuss the ETF factors. After that, the model estimation results are reported and we end this section with the robustness checks. All results are rounded to 3 decimal places.

5.1 ETF Factors

The mean Average factor equals 0.354 and the mean Biggest Magnitude factor is 0.642. The highest correlation between the Average factor and the 3 factors of SUW equals 0.233 (between ETF_t^A and $RMRF_t$). For the Biggest Magnitude factor, the highest correlation is 0.212 (between ETF_t^B and $RMRF_t$). The highest correlation among the explanatory variables in the regime-switching model equals -0.265 (between ETF_t^1 and ETF_t^3) Due to these relative low correlations, we also assume here that multicollinearity is not a substantial problem when estimating the parameters of (6), (7) and (8).

The clustering is performed with the *"ppclust"* package in R[9]. A 2D graph with the clustering results is shown in Appendix B and Table 7 gives the representatives of each cluster. From Figure 1 below, we infer that the market is bullish in about 23.3% of the sample period. It is neutral in about 58.3% of the sample period and it is bearish in the remaining 18.3% of the time. It is also noticeable that sentiment fluctuates more frequently and more extremely from 2020 onward (77^{th} observation). This can be seen from the higher volatility in Figure 1a and the higher weights for clusters 1 and 4 in Figure 1b. The increase in market uncertainty and the appearance of more extreme ETF returns corresponds with the crypto/stock boom period after 2020 (Appendix A).

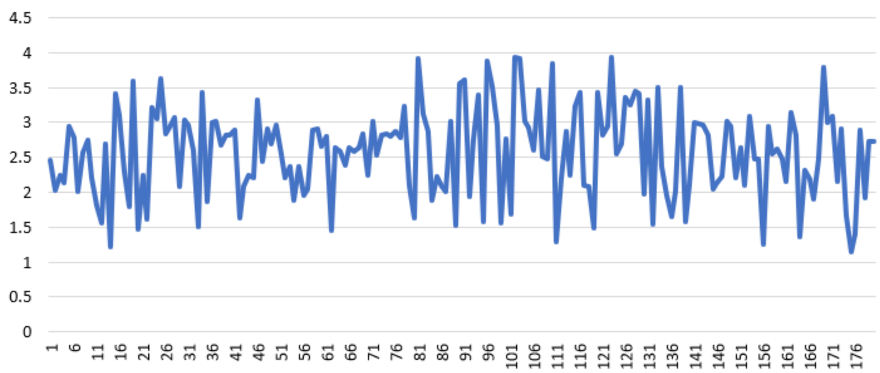
Table 7: Representatives of each cluster

	HACK	ICLN	IXG
Cluster 1	-3.989	-4.667	-3.136
Cluster 2	-0.367	-1.044	-0.670
Cluster 3	1.130	1.393	0.811
Cluster 4	3.610	6.217	2.985

Note: HACK, ICLN and IXG represents the cybersecurity, environmental and banking sector ETF, respectively.

[9]https://cran.r-project.org/web/packages/ppclust/index.html

(a) Cluster weights



(b) Sentiment

Figure 1: Cluster weights and sentiment over time

5.2 Factor Models

With the stated (in)dependent variables, we have performed 5 regressions for each of the 3×3 dependent variables. The intercept estimates and its t-statistic of the regression of each regression are given in Appendix C. In this subsection, we discuss the summary statistics of these regressions. These statistics are: mean absolute intercept, mean adjusted R-squared and mean standard deviation of the intercept. Table 8 gives an overview of these statistics per model.

Table 8: Regression summary statistics of the 5 models

	$\lvert\alpha\rvert$	adj. R^2	$s(\alpha)$
C-CAPM	0.569	0.633	0.698
3-Factor Model	0.530	0.714	0.616
Average	0.527	0.714	0.617
Biggest Magnitude	0.530	0.715	0.617
Regime	0.171	0.712	0.812

From Table 8 it follows that the mean intercept of the C-CAPM is always higher compared to the other models (excluding Regime, because the mean intercepts of this model is can not be compared). In addition, the average adjusted R-squared is very high for each model. This is due to the small number of coins in the dataset. Because we work with few coins, the independent and dependent variables may resemble each other. Despite this, we still see that adding additional factors gives extra explanatory power because the mean adjusted R-squared of C-CAPM is always lower than that of the other models.

So, despite the high adjusted R-squareds caused by using a small dataset, we are still able to find similar results as in SUW. This supports the representativeness of our dataset and it also shows that adding additional factors (especially the size and reversal factor) consistently improves the C-CAPM. However, adding the ETF factors does not lead to major ameliorations of the 3-Factor Model. Instead, the improvements made by including the ETF factors are less consistent as will be shown next in the rolling regression analysis. The regressions also show that the intercept of Large-caps with a High prior return are always positive (and sometimes significant) while those of the Low-caps with a High prior return are always negative. This shows that large cap coins grow more consistently than small cap coins in our sample period.

5.3 Robustness Check

For each of the 3×3 dependent variables we have performed a rolling regression on each model. The plots of the rolling adjusted R-squareds are given in Appendix D. The first aspect to notice is that the decreases of the rolling adjusted R-squared in the period after 2020 is ubiquitous, which means that these models have less explanatory power during the more volatile boom period. These plots also show that the C-CAPM is consistently worse compared to the other 4 models. However, the difference between the other 4 models is visually less clear. That is why we calculate for each dependent variable what percentage of time a certain model has the highest adjusted R-squared. The detailed results can be found in Appendix E. Here we discuss only the average percentage of time of each model.

We find that on average roughly (ignoring C-CAPM) 33% of the highest adjusted R-squareds are achieved with the 3-Factor Model. The Average and Biggest Magnitude model together account for another 33% of the highest adjusted R-squareds and the Regime model is responsible for the remaining one-third of the highest adjusted R-squareds. This shows that, on average, the two best models (3-Factor Model and Regime) are responsible for about two-thirds of the highest adjusted R-squareds.

To make a distinction between the two best models, we compare their percentages without the presence of Average and Biggest Magnitude. Then we find that the 3-Factor Model is responsible for roughly 60% of all highest adjusted R-squareds over time and the remaining 40% is achieved by the Regime model. From this we conclude that even after removing the Average and Biggest Magnitude factor, the 3-Factor Model is still, on average, the better performing one. A possible explanation for this is that the extra Average and Biggest Magnitude factors do not provide much improvement compared to the already well-performing 3-Factor Model. The Regime model is promising, but it does not seem to outperform the 3-Factor Model.

6 Conclusions

In this paper, we have investigated the effect of extending the 3-Factor Model of Shen et al. (2020) with return information of crypto related sector ETFs on the explanatory performance in the period 2018-2021. The criteria with which we compare the explanatory power is the adjusted R-squared. Despite the relatively high adjusted R-squared of each model due to the small dataset, we find that here too the 3-Factor Model is consistently better than the C-CAPM, because the rolling adjusted R-squared of the former is almost always higher than the latter. Adding the Average and Biggest Magnitude factor to the 3-Factor Model produces models that differ little in performance from the 3-Factor Model. However, making the 3-Factor Model to have a regime-switching intercept with dummy variables based on ETF sentiment does result in relatively more improvements. In addition, we find that large capitalized coins continue to grow more consistently than small capitalized coins and that the explanatory power of almost every model decreases during the more volatile period after 2020.

Yet, the major limitation of this study is the rather small dataset of 49 coins. As mentioned before, this probably has caused the high adjusted R-squareds. Which in turn makes differences in model performance less clear. Other improvements could be to make the Fuzzy C-Means clustering iterative, which makes the algorithm even less sensitive to the random initialization or to use other criteria/statistical tests when evaluating the models .

Taking everything together, we conclude that here too the 3-Factor Model is still the most preferred model. This is mainly due to its simplicity and its rather robust performance. The regime-switching model comes in second place. The latter is promising and may be extended by for example making the slope coefficient regime-switching, using other ETFs or more general variables, or changing the way how we have determined the regimes. In short, further work is required to establish the viability of this model and its possible elaborations.

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7 Appendix A

Figure 2: Total cryptocurrency market capitalization during the sample period

Image obtained through Coinmarketcap.com[10].

[10]https://coinmarketcap.com/charts/

8 Appendix B

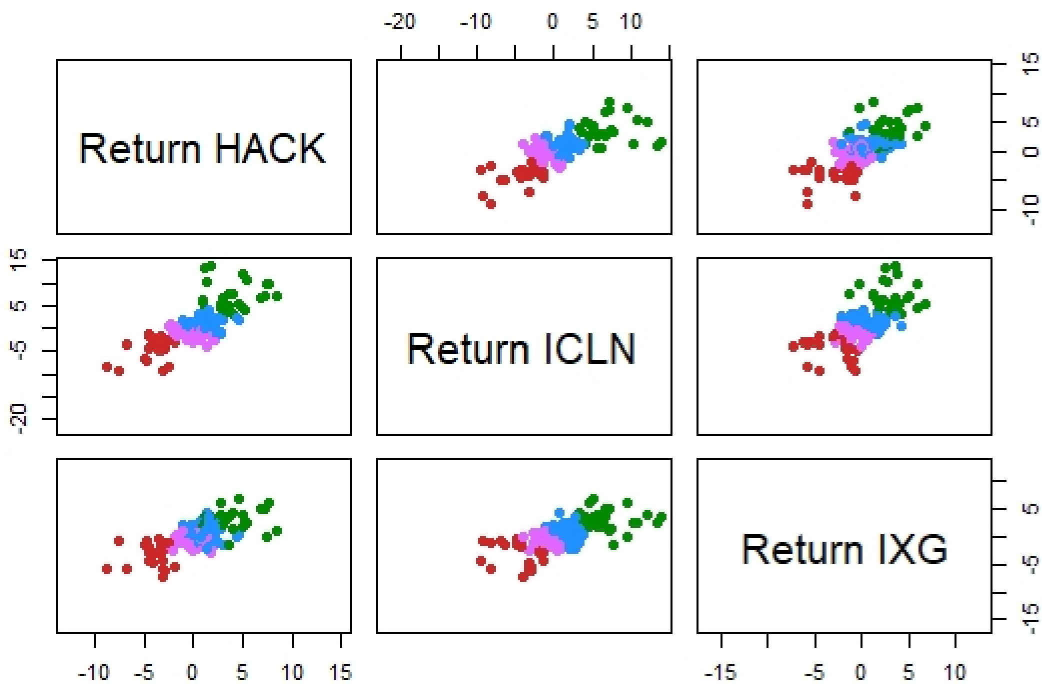


Figure 3: 2D Fuzzy C-Means clustering results

The observations are coloured according to the cluster they are most related to (i.e., according to the highest weight of the observation).

9 Appendix C

The intercept estimates and its t-statistics of the 5 models are given below. In each table, the left panel presents the intercept estimates, while the right panel presents the corresponding t-statistic. 10%, 5% and 1% significant results are indicated with a *, ** and ***, respectively.

Table 9: Intercept estimates and its t-statistic of the C-CAPM

	α			t(α)		
	Low	**Mediocre**	**High**	**Low**	**Mediocre**	**High**
Large-cap	-0.387	0.187	0.910**	-0.558	0.412	2.058
Mid-cap	-0.125	0.281	0.970	-0.186	0.439	1.141
Small-cap	-0.255	-0.472	-1.536	-0.336	-0.726	-1.375

Table 10: Intercept estimates and its t-statistic of the 3-Factor Model

	α			t(α)		
	Low	**Mediocre**	**High**	**Low**	**Mediocre**	**High**
Large-cap	0.084	0.235	0.754*	0.147	0.515	1.866
Mid-cap	0.293	0.432	0.980	0.543	0.806	1.475
Small-cap	-0.046	-0.387	-1.559	-0.067	-0.660	-1.448

Table 11: Intercept estimates and its t-statistic of the Average Model

	α			t(α)		
	Low	**Mediocre**	**High**	**Low**	**Mediocre**	**High**
Large-cap	0.154	0.218	0.756*	0.270	0.476	1.859
Mid-cap	0.291	0.467	1.028	0.537	0.868	1.542
Small-cap	-0.020	-0.398	-1.411	-0.029	-0.657	-1.315

Table 12: Intercept estimates and its t-statistic of the Biggest Magnitude Model

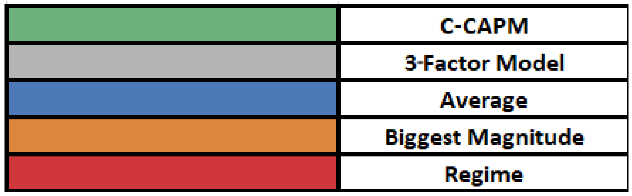
	α			t(α)		
	Low	**Mediocre**	**High**	**Low**	**Mediocre**	**High**
Large-cap	0.189	0.216	0.753*	0.333	0.471	1.850
Mid-cap	0.307	0.496	1.027	0.566	0.924	1.538
Small-cap	0.010	-0.376	-1.343	0.015	-0.620	-1.255

Table 13: Intercept estimates and its t-statistic of the Regime Model

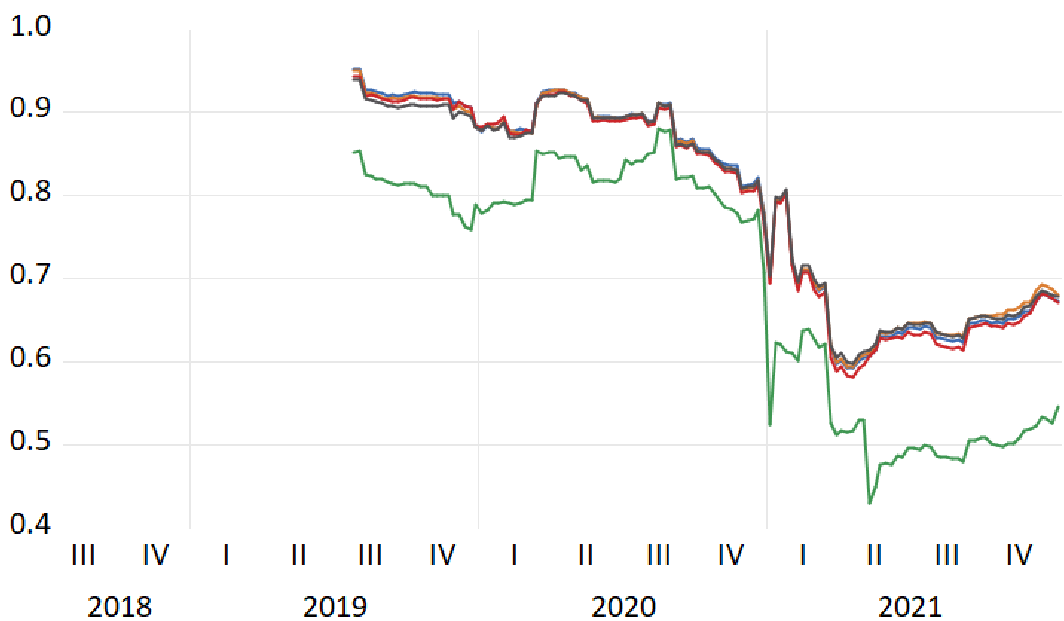
	α			t(α)		
	Low	**Mediocre**	**High**	**Low**	**Mediocre**	**High**
Large-cap	0.019	0.766	0.205	0.025	1.281	0.387
Mid-cap	0.527	0.457	1.419	0.740	0.644	1.617
Small-cap	0.063	-1.079	-0.841	0.069	-1.360	-0.592
	$\beta_{i,bear}$			t($\beta_{i,bear}$)		
	Low	**Mediocre**	**High**	**Low**	**Mediocre**	**High**
Large-cap	0.717	-2.152*	0.944	0.464	-1.757	0.869
Mid-cap	-0.985	-0.038	-0.669	-0.675	-0.026	-0.372
Small-cap	-0.454	1.388	-0.880	-0.242	0.854	-0.302
	$\beta_{i,bull}$			t($\beta_{i,bull}$)		
	Low	**Mediocre**	**High**	**Low**	**Mediocre**	**High**
Large-cap	0.327	-0.485	1.594	-0.234	-0.438	1.624
Mid-cap	-0.183	-0.076	-1.346	-0.159	-0.058	-0.829
Small-cap	-0.089	1.794	-2.385	-0.053	1.222	-0.907

10 Appendix D

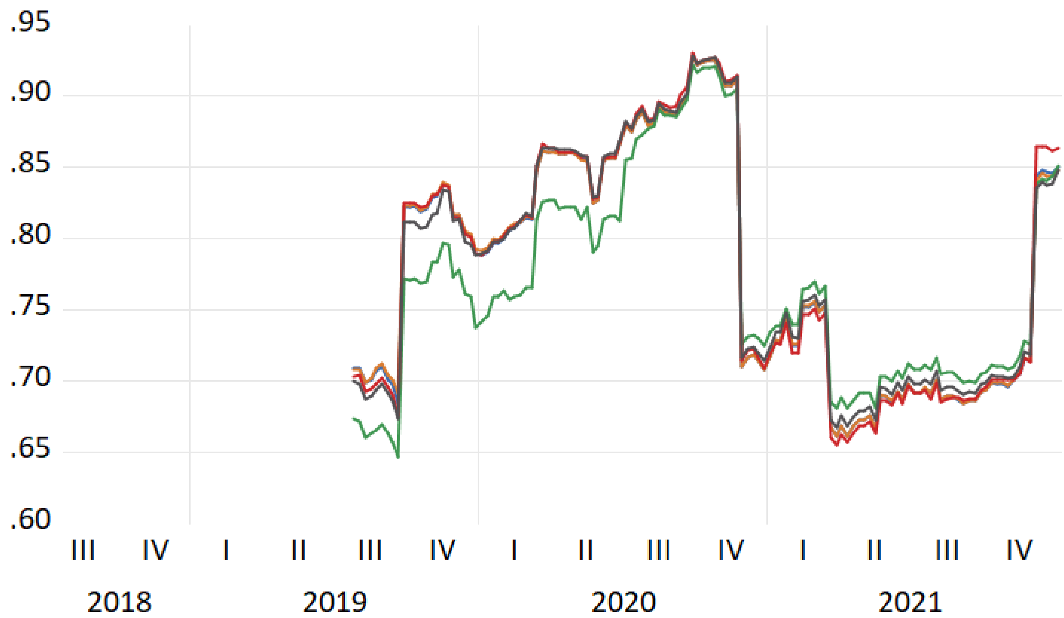
Figure 4 contains the rolling adjusted R-squared plots of each model belonging to each of the 3×3 dependent variables. The first subfigure is a colour legend. In the other subfigures we indicate in the caption below which of the 3×3 portfolios is used as the dependent variable.



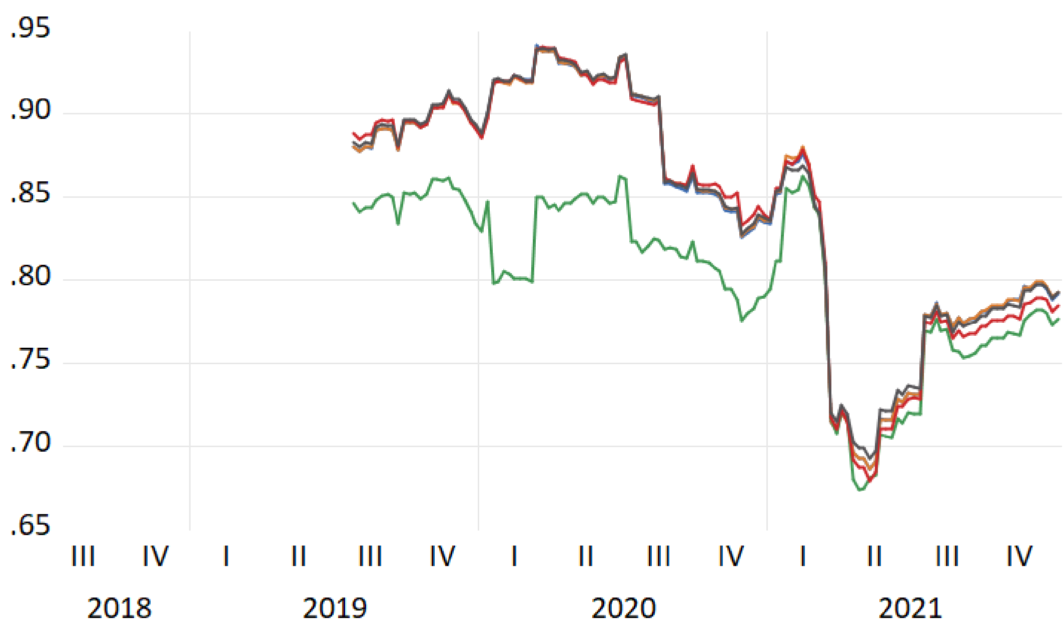
(a) Colour legend



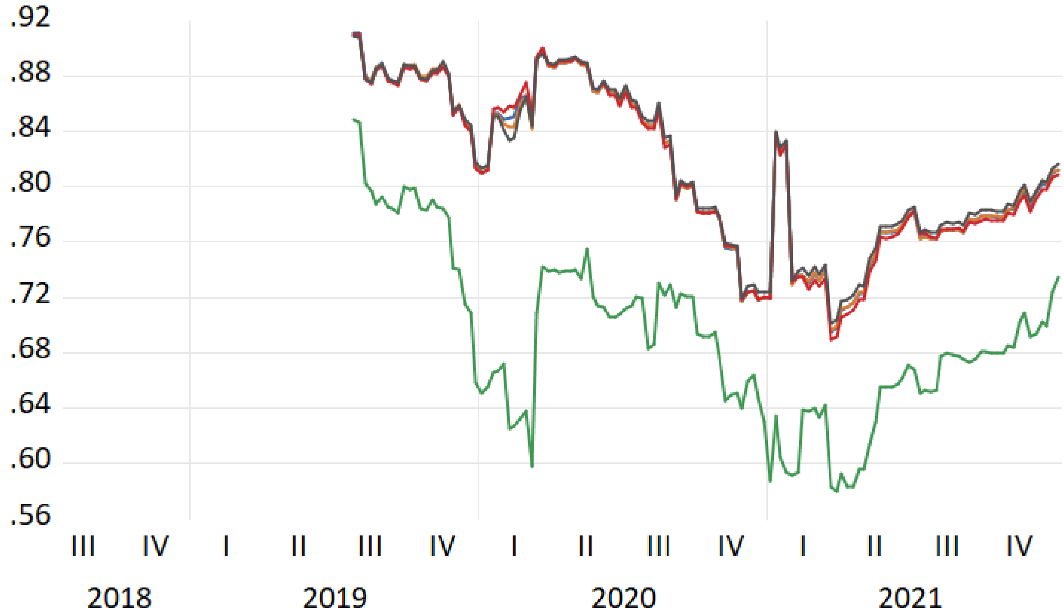
(b) Rolling adjusted R-squareds of Large-cap, Low prior return



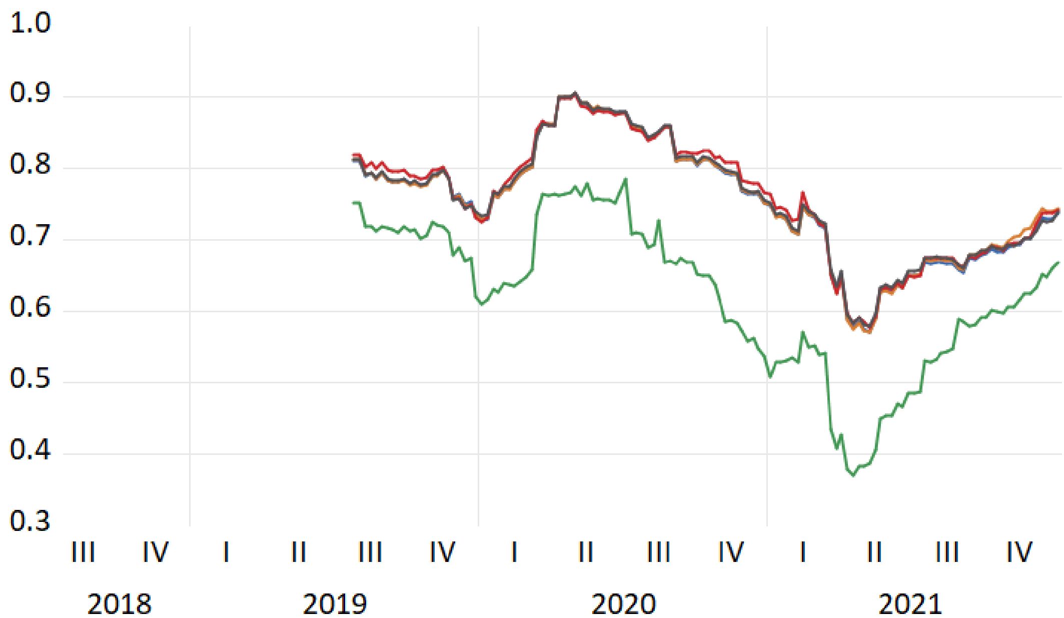
(c) Rolling adjusted R-squareds of Large-cap, Mediocre prior return



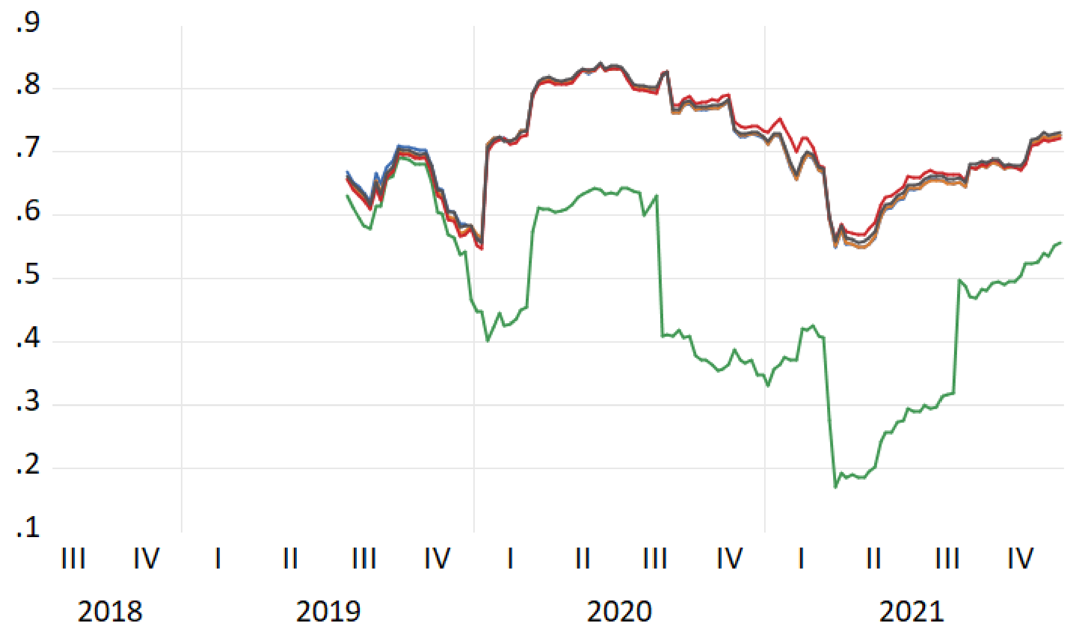
(d) Rolling adjusted R-squareds of Large-cap, High prior return



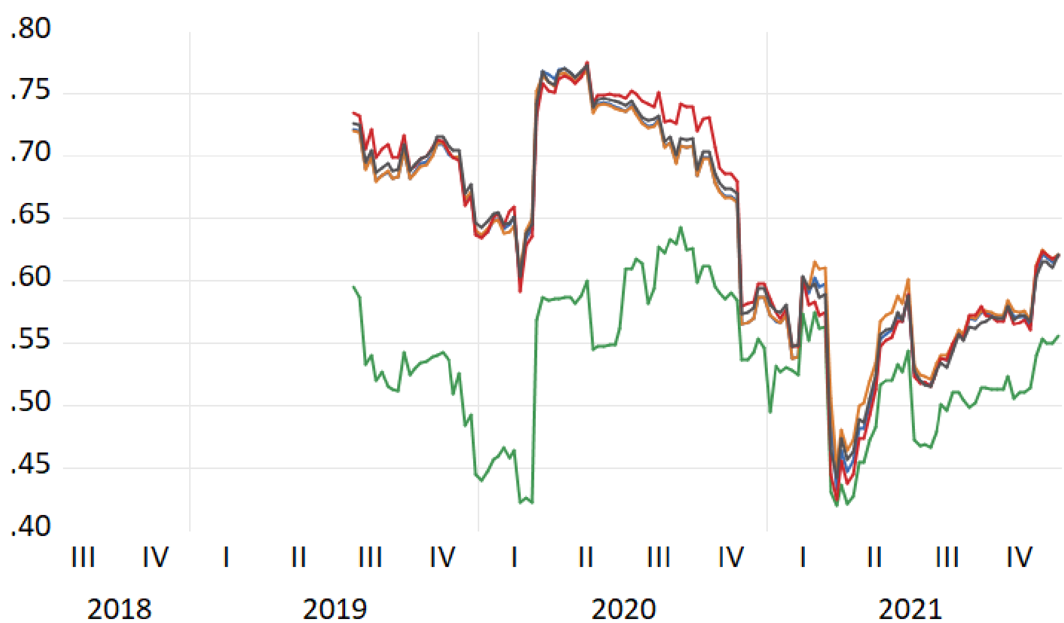
(e) Rolling adjusted R-squareds of Mid-cap, Low prior return



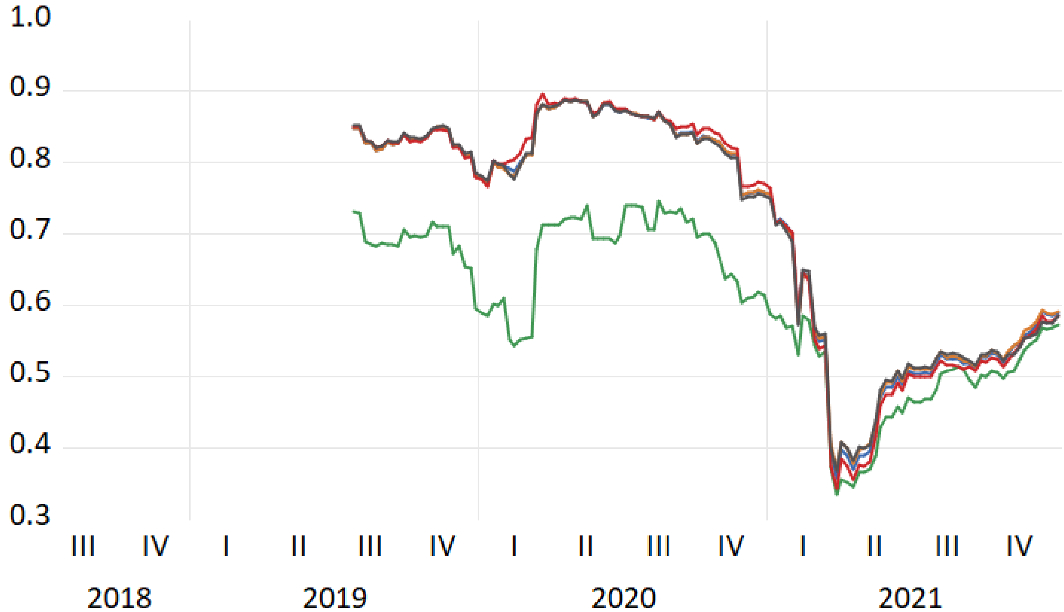
(f) Rolling adjusted R-squareds of Mid-cap, Mediocre prior return



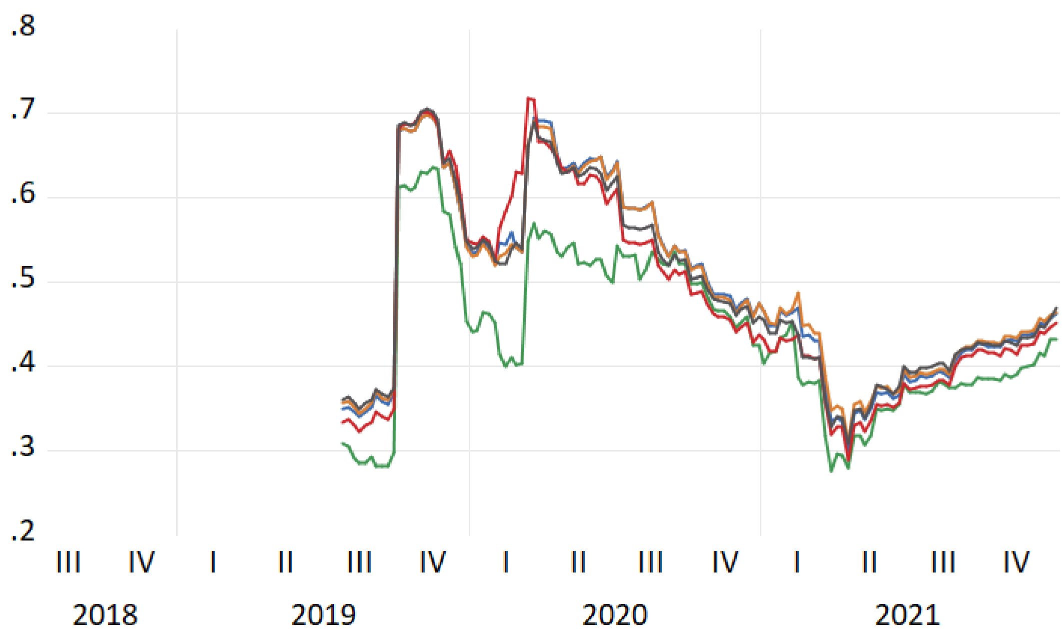
(g) Rolling adjusted R-squareds of Mid-cap, High prior return



(h) Rolling adjusted R-squareds of Small-cap, Low prior return



(i) Rolling adjusted R-squareds of Small-cap, Mediocre prior return



(j) Rolling adjusted R-squareds of Small-cap, High prior return

Figure 4: Rolling adjusted R-squared of each dependent variable regressed on each model.

11 Appendix E

Table 14: Percentage of time a model has the highest adjusted R-squared

		Average	Biggest Magnitude	C-CAPM	Regime	3-Factor Model
Large-cap	**Low**	51.563	22.656	0	7.031	18.750
	Mediocre	1.563	17.969	41.406	22.656	16.406
	High	7.031	20.313	0	29.688	42.969
Mid-cap	**Low**	4.688	6.250	0	8.594	80.469
	Mediocre	3.125	18.750	0	48.438	29.688
	High	18.750	3.906	0	43.750	33.594
Small-cap	**Low**	3.125	35.938	0	42.188	18.750
	Mediocre	2.344	16.406	0	35.153	46.094
	High	30.469	30.460	0	13.281	25.781
Mean		13.629	19.183	4.601	27.864	34.733

Table 15: Percentage of time a model has the highest adjusted R-squared, excluding Average and Biggest Magnitude

		C-CAPM	Regime	3-Factor Model
Large-cap	**Low**	0	30.469	69.531
	Mediocre	41.406	39.844	18.750
	High	0	36.719	63.281
Mid-cap	**Low**	0	10.156	89.844
	Mediocre	0	61.719	38.281
	High	0	43.750	56.250
Small-cap	**Low**	0	50.781	49.219
	Mediocre	0	45.313	54.688
	High	1.563	16.406	82.031
Mean		4.774	37.240	57.978