Erasmus School of Economics

Bachelor Thesis Econometrics and Operational Research

Forecasting Global Government Bond Yields using Shifting Endpoints and Global Factors[1]

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Abstract. This paper focuses on forecasting global government bond yields using the combination of a global method with the shifting endpoints method, or time dependant means. It is investigated if this method performs better than others. Making accurate forecasts, helps investor deciding if a bond has an acceptable price. Furthermore, on a the global level, it helps policymakers on examining the effect of other countries' yield on their own yield. The researched yields are from Euro area countries and the US used on several dynamic Nelson-Siegel methods. It is found that the application of the shifting endpoints method on the global one, increases it forecasting accuracy. However, the shifting endpoints method on the dynamic Nelson-Siegel method generally performs better.

[1] The views stated in this thesis are those of the author and not necessarily those of the supervisor, second assessor, Erasmus School of Economics or Erasmus University Rotterdam.

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1 Introduction

The bond market is an important part of the total financial market. It accounts for roughly 40% of the total market of the United States with an estimated size of 46 trillion US dollars. Worldwide, this is estimated to be more than 119 trillion US dollars, according to the Securities Industry and Financial Markets Association. The sheer size of the bond market makes it an interesting study with a lot of research already done. The government bond market in particular is popular for its relatively stable returns, which in turn makes it popular for estimating and forecasting the yields. One of the most prevalent method used for estimating the bond yields is the Nelson-Siegel method from Nelson and Siegel (1987) with various extensions as this is a parsimonious method. This is of high importance for, among others, investors for determining whether a given bond has an appropriate yield. A more recently popular method is the dynamic extension of the Nelson-Siegel method from Diebold and Li (2006), who model the forward rate and yield curve. More of most importantly, this method also forecast these curves. Another interesting and relatively recent method is from the research done by Van Dijk et al. (2014), who use the dynamic Nelson-Siegel method with shifting endpoints, or time dependant means. They argue that some of these shifting endpoints have an improved forecasting ability when compared to the method from Diebold and Li (2006). Furthermore, in a later study, Diebold, Li, and Yue (2008) present a global dynamic Nelson-Siegel method using global factors and conclude that these global effects have a significant effect on government bonds from Germany, Japan, the UK and the US.

When the two aforementioned methods are combined, a new question is created: How does the global dynamic Nelson-Siegel method with shifting endpoints perform on forecasting the global government bond yields? As both methods perform well in the cited literature, one might assume that applying the shifting endpoints method on the global method will create better results than applying either methods separately. Here, the global government bond market is defined to be the two largest bond markets in the world: the European Union and the United States. This differs form previous literature, as most is done on only US or country specific European bond yields. Moreover, most methods are only applied for estimating the yield curve, in this paper it is also used to forecast. Due to the high importance of the bond market, it is therefore interesting for investors to use this method for forecasting the global bond yields and for researchers by suggesting this new method for other specifications of the Nelson-Siegel methods.

In this paper, government bond yield and macro-economic data of the United States, Germany, Spain, France, Italy and the Netherlands are used. The methods used are mainly the dynamic Nelson-Siegel model from Diebold and Li (2006), the shifting endpoints method from Van Dijk et al. (2014) and the global dynamic method from Diebold, Li, and Yue (2008). Additionally, some benchmark methods are included. These are random walk methods and the four factor dynamic Nelson-Siegel model from Björk and Christensen (1999). This four factor method is applied with and without the application of the shifting endpoints method. Lastly, I compare the forecasting abilities of the dynamic Nelson-Siegel, shifting endpoints and global methods using the bootstrapping approach from Van Dijk et al. (2014), which accordingly specifies statistical significance whether the forecasting accuracy of one method is better or worse than that of some selected methods.

In this research it is found that the application of the shifting endpoints method improves the global method. However, the shifting endpoints method perform better when applied to the dynamic Nelson-Siegel method. Nonetheless, the global method has a generally higher forecasting accuracy than the four factor method of Björk and Christensen (1999).

This can be interesting for investors for attaining higher returns. It is also of interest for regional and country specific (monetary) policymakers, such as the European Central Bank (ECB), for forecasting short- and long-term interest rates. Additionally, it can also be useful for investigating the effect of other countries on one country's government bond yield. Scientifically, it is interesting by comparing several methods that are not often used against one another.

This research paper is structured as follows. In Section 2, I review earlier research done on this topic and introduce some sub-questions. In Section 3, I introduce the data that I use and how it is obtained. Following, in Section 4, I present the methods, for which I give the results of these in Section 5. Lastly, I conclude the research in Section 6.

2 Literature Review

The root of the mentioned methods, the Nelson-Siegel method by Nelson and Siegel (1987), was primarily for focusing on the fit of the model. A popular extension of this method is the one by Diebold and Li (2006) who apply it for forecasting the forward rate and yield curves. They found that their extension performs well compared to the regular Nelson-Siegel method. Using this method, Van Dijk et al. (2014) include shifting endpoints, where the mean in the AR(1) processes are now time dependant with several specification methods. They reason that the assumption that specifies stationary and mean-reverting processes with constant unconditional mean can not be made easily. For forecasting accuracy they additionally present a bootstrapping method for statistical significance, which is needed instead of the usual Diebold-Mariano test.

They conclude that the shifting endpoints, specifically the use of surveys and realized measures, improves the forecasts of the yields significantly.

Another adaptation of the Nelson-Siegel method is the four-factor method from Björk and Christensen (1999), which extends the Nelson-Siegel method by adding an additional factor. De Pooter (2007) uses this method and concludes that this extended method has an increased in-sample fit and a better performance in forecasting for all maturities, but mainly for long maturities. De Pooter also adds that this method is easy to estimate, has no potential multicollinearity issues and performs similar to the method of Svensson (1994), which is a more excessive four-factor method. The method from Björk and Christensen (1999) is therefore a preferred method. The method is then made dynamic by adding AR(1) processes with shifting endpoints to the factors similar to Van Dijk et al. (2014). As this extension performs well on the Nelson-Siegel method, one might suspect this to also perform well on the four factor method. These specifications are used as benchmark methods.

To answer the main research question, it is useful to add a global method, as the mentioned models are all on the country level. This is the method from Diebold, Li, and Yue (2008) who extend the dynamic method from Diebold and Li (2006) to be able to forecast multiple countries at once. Here, they use a set of zero-coupon bond yields of the US, Germany, Japan and the UK where the factors now depend on global and country specific factors. They find that the global yield factors are significant in explaining the country yield curve dynamics. This method is one of the main focus in my research. Where the other main focus is on the shifting endpoints method from Van Dijk et al. (2014). It is therefore also interesting to first inspect the performance of both methods separately on the data. As both methods perform well in the mentioned literature, it is suspected that these also perform well on the data in this research. It is then intriguing to compare these two.

The global dynamic extension contains another interesting aspect, considering the use of the bond yields and general economies of the US and the EU in this research. The US and the EU have several found spillover effects between and within the regions. Therefore, one might suspect that the use of the global method improves the forecasting accuracy compared to the country level methods. Curcuru et al. (2018) find significant spillover effects between the US and Germany in government bond yields when the FOMC or ECB make monetary policy announcements. Moreover, Antonakakis and Vergos (2013) find that there exists a great spillover effect in bond yields from the periphery Euro area countries to the core countries. Additionally, they find that pairs of countries from the core of the EU or the periphery have a relative high return spillover effect from sovereign bond yields. Likewise, Schwendner et al. (2015) conclude that there is significant evidence for correlation in government bond yields within the core or periphery of the Euro area. However, they also find that the correlation between the core and periphery has decreased since 2011, suggesting that the use of the global factors might not be as desirable. Lastly, Perego and Vermeulen (2016) find that macro-economic variables, such as inflation, debt and economic growth, contribute to a high correlation between stocks and government bonds in the Euro area. It is therefore interesting to investigate the countries of the United States, Germany, Spain, France, Italy and the Netherlands. Since there exists some spillover effect between and within the core and periphery of the Euro area and between the EU and the US. Thus, this includes countries such as the US and Germany, which already have been proven to be correlated, but also countries from both the core and periphery of the Euro area, which is also shown to have some spillover effect. Concluding, von Hagen et al. (2011) states that Germany has become a safe-haven for investment opportunities similar to the US, after the economic crisis of 2008. Combining these findings, I therefore primarily focus on the US and on Germany.

3 Data

For my research, I use several government bond yields from two sources. The first source is for obtaining the yields of the United States Treasury. The second one is used for obtaining the yields for Euro area countries, where there are also other variables given.

3.1 US Treasury bond yields

For the United States, I use data from Liu and Wu (2021) who report monthly and yearly government bond yields from the US Treasury from June 1961 to December 2021. These yields are annualized continuously-compounded zero-coupon yields in percentage points. Liu and Wu give these yields for all bonds with a constant-maturity in each month with maturities from 1 up to 360 months. Following with the research done by Van Dijk et al. (2014), I focus on the bond yields with maturities $m = 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108$ and 120 months. The descriptive statistics of these yields can be found in Table 1, where I provide the mean, standard deviation, minimum and maximum of the yields. Additionally, I present the empirical autocorrelations of orders 1, 12 and 30 of the yields ($\hat{\rho}(1)$, $\hat{\rho}(12)$ and $\hat{\rho}(30)$ respectively). These statistics are made using the yields from January 1995 to August 2021, to correspond to the same range of data of the used European yields seen in Section 3.2. Thus, there are a total of 320 yield observations. The statistics differ slightly from the statistics obtained by Van Dijk et al. (2014) as I use a different dataset and range for the data. However, this difference becomes obsolete when the same data range is used.

Table 1: Descriptive statistics of the US Treasury bond yields with several maturities from January 1995 to August 2021.

Maturity	Mean	SD	Min	Max	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
3	2.217	2.127	0.009	6.280	0.989	0.773	0.390
6	2.307	2.144	0.028	6.275	0.989	0.778	0.393
9	2.383	2.153	0.045	6.518	0.989	0.782	0.400
12	2.444	2.157	0.064	6.729	0.988	0.786	0.410
15	2.494	2.154	0.086	6.882	0.987	0.791	0.422
18	2.540	2.145	0.106	6.989	0.986	0.794	0.437
21	2.585	2.130	0.115	7.066	0.986	0.797	0.451
24	2.631	2.113	0.123	7.129	0.985	0.799	0.465
30	2.729	2.080	0.114	7.231	0.984	0.803	0.488
36	2.835	2.041	0.123	7.303	0.983	0.806	0.507
48	3.038	1.961	0.172	7.402	0.981	0.806	0.534
60	3.214	1.884	0.231	7.439	0.980	0.803	0.554
72	3.378	1.811	0.313	7.445	0.979	0.802	0.568
84	3.523	1.763	0.382	7.461	0.978	0.801	0.577
96	3.649	1.725	0.445	7.510	0.978	0.800	0.585
108	3.759	1.692	0.488	7.529	0.977	0.799	0.591
120 (level)	3.870	1.672	0.530	7.560	0.977	0.800	0.598
Slope	1.653	1.176	-0.605	4.358	0.970	0.531	-0.108
Curvature	-0.825	0.889	-2.695	1.305	0.955	0.642	0.157

Note: For each maturity, I present the mean, standard deviation, minimum value, maximum value and the first, twelfth and thirtieth autocorrelation. Additionally given are the empirical values for the level, slope and curvature factors of the yield curve. These are computed as the 120-month yield, as the 120-month minus the 3-month yield and as twice the 24-months minus the 3- and 120-month yield, respectively.

Additionally reported are the empirical estimations of the level, slope and curvature of the yield curves. These are defined as the longest maturity yield of 120 months for the level, the longest minus the shortest maturity yield of 3 months for the slope, and two times the 24-month maturity yield minus the sum of the shortest and longest maturity yield for the curvature, following Van Dijk et al. (2014). Furthermore, it holds that the yields follow the stylized facts of the yield curve. Firstly, the yield curve is upward sloping and concave. Secondly, the yields are persistent with larger autocorrelations for longer maturities. And lastly, the volatility, or the standard deviation, of the yields decrease as the maturity length increases.

Next, as I additional use macro-economic variables for each country in my research, I obtain the Consumer Price Index (CPI) and the Total Industry Production Index (IPT) from the Federal Reserve Bank of Philadelphia. Both variables are monthly observed with monthly vintages.

3.2 Euro Area bond yields

For the Euro area countries, I use data from Van der Wel and Zhang (2021)[2] who also report monthly and yearly government bond yields, but for a large set of countries, e.g. the UK, Japan and Canada. Additionally, they provide macro-economic variables, such as an inflation index and an industrial production growth index. The yields are given for bonds with maturities $m = 3$, 6, 12, 24, 36, 48, 60, 72, 84, 96, 108 and 120 months. Using this dataset, I apply the yields, the industrial production growth and the inflation of Germany (DE), Spain (ES), France (FR), Italy (IT) and the Netherlands (NL) for my research on Euro area countries. All observed variables from these countries are given from January 1995 up to August 2021 with a total of 320 observations for each country. Similar to the US bond yields, I give the descriptive statistics of these countries. For Germany, this can be found in Table 2. For the other countries, these can be found in the Appendix.

Table 2: Descriptive statistics of the German bond yields with several maturities from January 1995 to August 2021.

Maturity	Mean	SD	Min	Max	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
3	1.628	1.888	-1.003	5.140	0.989	0.822	0.628
6	1.663	1.896	-0.906	5.201	0.989	0.819	0.625
12	1.738	1.924	-0.916	5.683	0.987	0.818	0.626
36	2.049	2.079	-0.963	6.828	0.984	0.837	0.678
48	2.239	2.133	-0.967	7.072	0.984	0.842	0.681
60	2.406	2.149	-0.934	7.147	0.985	0.845	0.682
72	2.579	2.208	-0.916	7.425	0.985	0.845	0.673
84	2.731	2.234	-0.888	7.542	0.985	0.846	0.668
96	2.864	2.241	-0.831	7.587	0.986	0.846	0.663
108	2.967	2.219	-0.768	7.595	0.986	0.842	0.653
120 (level)	3.047	2.180	-0.707	7.521	0.986	0.843	0.649
Slope	1.419	0.896	-0.117	3.673	0.962	0.461	-0.081
Curvature	-0.913	0.589	-2.291	0.419	0.906	0.341	-0.149

Note: For each maturity, I present the mean, standard deviation, minimum value, maximum value and the first, twelfth and thirtieth autocorrelation. Additionally given are the empirical values for the level, slope and curvature factors of the yield curve. These are computed as the 120-month yield, as the 120-month minus the 3-month yield and as twice the 24-months minus the 3- and 120-month yield, respectively.

Again, I additionally present the empirical estimations of the level, slope and curvature of the yield curve. These are made using the same empirical estimation approach as for the US yields. Moreover, the yields for the Euro area countries also mostly follow the stylized facts of the yield curve.

[2] I am thankful for Michel van der Wel and Yaoyuan Zhang for sharing the data with me and allowing me to use it for my research.

4 Methodology

In this section I present the specifications of each method used for my research. There is a main focus on Nelson-Siegel models, specifically the dynamic extension from Diebold and Li (2006), given in Section 4.1, the extension from Van Dijk et al. (2014), given in Section 4.3, and the global dynamic extension from Diebold, Li, and Yue (2008), given in Section 4.4. The specification of Björk and Christensen (1999) is shortly mentioned in Section 4.2. Lastly, in Section 4.5, I present the model comparison method from Van Dijk et al. (2014).

4.1 Dynamic Nelson-Siegel Model

Firstly, I present the dynamic Nelson-Siegel model from Diebold and Li (2006), which is an extension of the original Nelson-Siegel model from Nelson and Siegel (1987). The yields, denoted as $y_t(\tau)$ for month $t = 1, ..., T$ and with M maturities of $\tau = \tau_1, \ldots, \tau_M$ months, is here defined as

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right) + \varepsilon_t(\tau), \tag{1}$$

where β_{1t}, β_{2t} and β_{3t} can be viewed as latent factors as explained by Diebold and Li (2006). The factor β_{1t} has a constant loading, thus is viewed as a level factor with a long-term component. The factor β_{2t} has a loading which starts at 1 for $\tau = 0$ and decreases to 0 when τ increases, thus is viewed as a slope factor with a short-term component. Lastly, the factor β_{3t} has a loading of 0 when $\tau = 0$, then first increases and then decreases to 0 again, when τ increases. This factor is therefore seen as a curvature factor with a medium-term component. The decay parameter λ_t determines how fast the level factor decreases to zero and what the maturity is when the curvature factor is at its maximum value. Diebold and Li (2006) use a constant $\lambda_t = 0.0609$ for all months t such that β_{3t} achieves its maximum value at a maturity of approximately $\tau = 30$ months. The low λ is used to produce better fits for long maturities, while a large λ is better for short maturities. Setting this parameter as a constant, eases the difficulty of estimating the other parameters with no large loss of generality. Given that λ is constant, ordinary least squares is then used to consistently estimate the three factors.

Diebold and Li (2006) then add separate AR(1) processes for forecasting the factors, given as

$$\beta_{j,t+1} = \mu_j + \phi_j\left(\beta_{jt} - \mu_j\right) + \eta_{j,t+1} \tag{2}$$

for $j = 1, 2, 3$, where μ_j and ϕ_j are the parameters to be estimated. The error term $\eta_{j,t+1}$ is assumed to have mean 0 and variance σ_j^2, where the errors are mutually and serially independent for all factors j and months t. The forecast of the yields are then obtained via the forecasts of the factors from (2) with consistent estimations using ordinary least squares. I abbreviate this method as DL. Additionally, I add a benchmark method, which treats the factors of (2) as a random walk. Therefore, this coincides with $\mu_j = 0$ and $\phi_j = 1$ for all factors j. I label this method as RW.

4.2 Björk and Christensen Model

Additionally, is the four factor method from Björk and Christensen (1999) and applied in the research of De Pooter (2007). The yields $y_t(\tau)$ are here defined as

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right) + \beta_{4t}\left(\frac{1-e^{-2\lambda_t\tau}}{2\lambda_t\tau}\right) + \varepsilon_t(\tau), \qquad (3)$$

where β_{4t} is the added fourth factor that is another short-term loading. This loading decays faster to zero compared to the one of β_{2t}. Without a large loss of generality, it is then again used that $\lambda_t = 0.0609$ for all t, such that the four factors can be consistently estimated using ordinary least squares. The factors, and thus yields via (3), can then be forecasted using (2), where now $j = 1, 2, 3, 4$. I abbreviate this method as BC. I again add a benchmark method that treats the factors as a random walk. I label this as BCRW.

4.3 Shifting Endpoints

Considering the method of Diebold and Li (2006), Van Dijk et al. (2014) argue that the assumption of a stationary process in (2) is not appropriate. They suggest the allowance of time-varying unconditional mean. The factors are then defined as

$$\beta_{j,t+1} = \mu_{j,t+1} + \phi_j\left(\beta_{jt} - \mu_{jt}\right) + \eta_{j,t+1}. \qquad (4)$$

It follows to specify the time-varying unconditional means. Van Dijk et al. (2014) suggest three specification methods. I use two of these, as one of the methods involves Blue Chip long-range surveys, which I have no access to. However, they do not conclude that this specific specification performs significant better than some other ones.

Exponential Smoothing The first specification uses exponential smoothing for the parameter μ_{jt}, such that

$$\mu_{j,t+1} = \alpha\beta_{jt} + (1-\alpha)\,\mu_{jt}, \quad \text{for } t = 1, 2, \ldots \qquad (5)$$

$$= \alpha\sum_{l=0}^{t-2}(1-\alpha)^l\,\beta_{j,t-l} + (1-\alpha)^{t-1}\,\beta_{j1}, \qquad (6)$$

with $0 < \alpha < 1$ as the smoothing parameter and $\mu_{j1} = \beta_{j1}$. Van Dijk et al. (2014) suggest using a relative small value for α, such that it gives a smoother forecasting pattern. They find that using $\alpha = 0.1$ works well for the monthly data. The factors are then given as

$$\beta_{j,t+1} = \omega_j \beta_{jt} + (1 - \omega_j)\, \mu_{jt} + \eta_{j,t+1}, \tag{7}$$

where $\omega_j = \phi_j + \alpha$. By iterating (5) and (7), one gets the h-step ahead forecasts of β_{jt} as

$$\hat{\beta}_{j,t+h|t} = \omega_j \hat{\beta}_{j,t+h-1|t} + (1 - \omega)\, \hat{\mu}_{j,t+h-1|t}, \quad \text{for } h = 1, 2, ... \tag{8}$$

where

$$\hat{\mu}_{j,t+h-1|t} = \alpha \hat{\beta}_{j,t+h-2|t} + (1 - \alpha)\, \hat{\mu}_{j,t+h-2|t}. \tag{9}$$

Using this method, Van Dijk et al. (2014) apply it first for forecasting the level factor β_{1t} alone, while using (2) for the other factors, after which they apply the method for all factors. I label these methods ESL and ESLSC respectively, in line with Van Dijk et al. (2014). Similarly, I use this method with the BC model. I first apply the exponential smoothing for β_{1t}, and AR(1) representations for β_{2t}, β_{3t} and β_{4t}. Secondly, I use this method for all four factors. I label these methods BCESL and BCESLSC respectively.

Realized Measures The other specification considered for the shifting endpoints from Van Dijk et al. (2014) is the use of exponential smooth realized macro-economic variables. These are inflation and industrial production growth data. The real-time exponential smoothing of these macro-economic series is done with a smoothing parameter of $\alpha_{RM} = 0.01$, as these variables are known for being rather noisy. The two variables are then used to estimate the regressions

$$\beta_{1t} = \theta_{01} + \theta_{11}\pi_t^{ES} + \zeta_{1t}, \tag{10}$$

$$\beta_{2t} = \theta_{02} + \theta_{12}\gamma_t^{ES} + \zeta_{2t}, \tag{11}$$

where π_t^{ES} and γ_t^{ES} are the real-time exponential smoothed realized inflation and industrial production growth respectively. For forecasting the level factor β_{1t}, it is then used that $\mu_{1t} = \theta_{01} + \theta_{11}\pi_t^{ES}$. For the slope factor β_{2t}, it is used that $\mu_{2t} = \theta_{02} + \theta_{12}\gamma_t^{ES}$ in (4). This follows from Van Dijk et al. (2014). Furthermore, it is assumed that the out-of-sample value of μ_{jt} remains the same as the last known observation, for $j = 1, 2$ in (4). For the curvature factor β_{3t}, the constant mean AR(1) representation in (2) is used for forecasting. Van Dijk et al. (2014) then differentiate between two methods. One where they only use this technique for the level factor, with the constant mean AR(1) for the other factors. And one where they use this for the level factor with inflation and the slope factor with industrial production growth, with AR(1) for the curvature factor. I abbreviate these methods as RZI and RZIG respectively, in line with Van Dijk et al. (2014). This method is similarly applied for the BC method. Since the fourth added factor is also depicted as a slope factor, thus reasoned to be in line with economic growth, I then consider the additional regression

$$\beta_{4t} = \theta_{03} + \theta_{13}\delta_t^{ES} + \zeta_{3t}, \tag{12}$$

where the exponentially smoothed industrial production is used as δ_t^{ES}. Two separate methods are then defined. One where only the realized measures method for the level factor is used, using a constant mean AR(1) for the other three factors. And one where this method is used for the level and both slope factors. I give these methods the labels BCRZI and BCRZIG respectively.

4.4 Global Dynamic Nelson-Siegel Model

The final method in this research is the one from Diebold, Li, and Yue (2008) where they use country and global specific factors and macro-economic variables for estimating and forecasting the yield curves. This method is summarized by using a dynamic Nelson-Siegel model for each country individually, similar to Diebold and Li (2006), where the country specific loadings β_{ijt} are additionally loaded on global factors B_{jt} and country idiosyncratic factors, for each country $i \in \{\text{DE}, \text{ES}, \text{FR}, \text{IT}, \text{NL}, \text{US}\}$. Additionally, Diebold, Li, and Yue (2008) simplify the Nelson-Siegel model by removing the curvature factor, where they argue that this factor is often estimated with a low precision for most countries for short and large maturities. However, I keep this curvature factor and loading as De Pooter (2007) mentions that the three factor NS model performs significantly better than the two factor model. The global yield $Y_t(\tau)$ is then defined as

$$Y_t(\tau) = B_{1t} + B_{2t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + B_{3t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right) + E_t(\tau), \tag{13}$$

where B_{jt} are the global latent dynamic factors for $j = 1, 2, 3$ and where λ_t is again made constant at 0.0609 for all months t. Furthermore, $E_t(\tau)$ is denoted as the error term at month t with maturity τ. The global factors are further specified with a VAR(1) representation as

$$\begin{pmatrix} B_{1t} \\ B_{2t} \\ B_{3t} \end{pmatrix} = \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{pmatrix} \begin{pmatrix} B_{1,t-1} \\ B_{2,t-1} \\ B_{3,t-1} \end{pmatrix} + \begin{pmatrix} U_{1t} \\ U_{2t} \\ U_{3t} \end{pmatrix}, \tag{14}$$

where it is assumed that the error terms U_{jt} have zero mean and are homoscedastic with no serial correlation and variance $\tilde{\sigma}_j^2$, for $j = 1, 2, 3$. Furthermore, it is assumed that $\tilde{\sigma}_k^2 \tilde{\sigma}_l^2 = 0$ for all $k \neq l$, thus no correlation between the errors. Diebold, Li, and Yue (2008) then allow the country specific factors, which are separately specified via (1) for each country specific yield $y_{it}(\tau)$ with country specific error $\varepsilon_{it}(\tau)$, to load on the global factors as

$$\beta_{ijt} = \alpha_{ij} + \rho_{ij} B_{jt} + \nu_{ijt}, \tag{15}$$

where β_{ijt} are the j-th factor of country i at month t, for $j = 1, 2, 3$. Furthermore, α_{ij} are the constant terms, ρ_{ij} the loadings on the global factors B_{jt} and ν_{ijt} the country idiosyncratic factors with assumed zero mean. The country idiosyncratic factors are then, similar to the global factors, allowed to follow a VAR(1) representation. These are specified as

$$\begin{pmatrix} \nu_{i1t} \\ \nu_{i2t} \\ \nu_{i3t} \end{pmatrix} = \begin{pmatrix} \varphi_{i11} & \varphi_{i12} & \varphi_{i13} \\ \varphi_{i21} & \varphi_{i22} & \varphi_{i23} \\ \varphi_{i31} & \varphi_{i32} & \varphi_{i33} \end{pmatrix} \begin{pmatrix} \nu_{i1,t-1} \\ \nu_{i2,t-1} \\ \nu_{i3,t-1} \end{pmatrix} + \begin{pmatrix} u_{i1t} \\ u_{i2t} \\ u_{i3t} \end{pmatrix}, \tag{16}$$

where u_{ijt} are the error terms with assumed (co)variance of $E[u_{ijt} u^{kls}] = \sigma_{k,j}^2$ for $i = k$, $j = l$ and $t = s$, and zero otherwise. Lastly, it is given that the error terms U_{jt} and u_{ijt} are orthogonal.

State Space Representation Combining the mentioned equations, one obtains the state space representation, useful for estimation. For simplicity, I rewrite the total representation from Diebold, Li, and Yue (2008) in country specific state space representations using blocks from the total one. The equations then combine, for an individual country i, into

$$\begin{pmatrix} y_{it}(\tau_1) \\ \vdots \\ y_{it}(\tau_M) \end{pmatrix} = A \begin{pmatrix} \alpha_{i1} \\ \alpha_{i2} \\ \alpha_{i3} \end{pmatrix} + C \begin{pmatrix} B_{1t} \\ B_{2t} \\ B_{3t} \end{pmatrix} + A \begin{pmatrix} \nu_{i1t} \\ \nu_{i2t} \\ \nu_{i3t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{it}(\tau_1) \\ \vdots \\ \varepsilon_{it}(\tau_M) \end{pmatrix}, \tag{17}$$

where

$$A = \begin{pmatrix} 1 & \left(\frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1}\right) & \left(\frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1}\right) \\ \vdots & \vdots & \vdots \\ 1 & \left(\frac{1-e^{-\lambda\tau_M}}{\lambda\tau_M}\right) & \left(\frac{1-e^{-\lambda\tau_M}}{\lambda\tau_M} - e^{-\lambda\tau_M}\right) \end{pmatrix}, \text{ and} \tag{18}$$

$$C = \begin{pmatrix} \rho_{i1} & \rho_{i2}\left(\frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1}\right) & \rho_{i3}\left(\frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1}\right) \\ \vdots & \vdots & \vdots \\ \rho_{i1} & \rho_{i2}\left(\frac{1-e^{-\lambda\tau_M}}{\lambda\tau_M}\right) & \rho_{i3}\left(\frac{1-e^{-\lambda\tau_M}}{\lambda\tau_M} - e^{-\lambda\tau_M}\right) \end{pmatrix}, \tag{19}$$

where $y_{it}(\tau)$ is the yield of country i at month t with maturity τ. The largest opportunity of using this state space is that the global yields are not required to be observed. Only the global factors B_{jt} are needed, which can be obtained via Principle Components Analysis (PCA) on all the country specific factors β_{ijt}, $j = 1, 2, 3$.

Estimation Estimation of this method is then as done as follows. First, estimate the individual country specific Nelson-Siegel factors using (1). Then, use PCA on all the level, slope and curvature factors separately for extracting the global factors. With the global factors, estimate equations (14) to (16), such that iterated forecasts can be made with the estimated equations and the state space representation of (17).

For simpler estimation, Diebold, Li, and Yue (2008) assume that the VAR(1) representations in (14) and (16) have diagonal coefficient matrices. This is done with the reasoning that the factors have almost no dynamic interaction across each other in their research. This implies that the global factors and the country idiosyncratic factors follow separate and uncorrelated AR(1) models. I label this method as GDNS.

Shifting Endpoints on Global Factors To answer the main research question, I apply the methods of Van Dijk et al. (2014) on the method of Diebold, Li, and Yue (2008). Specifically, it questions whether shifting endpoints on the global factors improve the forecasting ability of the global method of Diebold, Li, and Yue (2008), and possibly other methods. The method that I use on the global factors is the exponentially smoothing methods, therefore exclude the realized measure methods. Finding appropriate realized measures that captures the fluctuations of global (specifically US and EU) economies has been difficult. Therefore, I use exponentially smoothing on (1) the global level factor, which is the first factor found in the PCA on the level factors, and (2) all found global factors. I label these methods as GESL and GESLSC respectively.

4.5 Forecasting Accuracy

When comparing the forecasting accuracies, it is not as direct as applying the Diebold-Mariano (DM) test. The statistical significance of this test is not as reliable when the models that are being compared are nested, which is the case for many of the methods in this research. Van Dijk et al. (2014) give an appropriate adjustment to regulate for the statistical significance of the test statistic. This is done using two bootstrap methods. The first one uses a nulhypothesis where the DL method is the proper data generated process (DGP). The bootstrapping is then done using the following steps:

1. Estimate the factors $\{\beta_{jt}\}$ for $j = 1, 2, 3$ for the three factor models and for $j = 1, 2, 3, 4$ for the four factor models using (1), to simultaneously obtain errors $\varepsilon_t(\tau)$. These factors are then fitted on AR(1) models to obtain the errors and the fitted values, which I denote here as η_{jt} and $\hat{\beta}_{jt}$ respectively.
2. Create a bootstrap sample for these $\hat{\beta}_{jt}$ with errors η_{jR}, where R is a randomly chosen point in time with replacement. Therefore, $\hat{\beta}_{jt} = \beta_{jt} + \eta_{jR}$. Additionally, I create a bootstrap sample for the yields using the randomly chosen errors $\varepsilon_R(\tau)$ with replacement with the same randomly chosen point in time R, and using the bootstrapped samples from the factors. This is similarly done for the macro-economic variables. These are assumed to follow a random walk, thus the errors here are the first difference. The bootstrap samples for these variables are thus created using the previous observation and errors with replacement at time T.
3. Using these bootstrap samples, new RMSPE are computed such that I compute the ratio of the RMSPE of all methods relative to that of the DL method.

The second bootstrap then uses a nulhypothesis where the RW method is the true DGP, such that in the first step the three of four factors now follow a random walk process. Each bootstrap is then done 300 times to replicate the null distribution of the test statistic, thus obtaining an empirical distribution. The p-values can be then obtained using a one-sided test, with null of better forecasting accuracy of DL (RW) than the other methods and alternative of worse accuracy of DL (RW). Thus this implies a test statistic of at least 1.

4.6 Additional Remarks

Lastly, I include a few benchmark models (with label). The first one treats the yield levels as a random walk (RWY), the second one is an AR(1) process on the yields levels (AR) and the third is a VAR(1) process on the yield levels using all given yields with different maturities for estimation (VAR).

Concluding, considering the shifting endpoint methods from Van Dijk et al. (2014), I conserve some space by excluding (including) the main results of the methods ESL (ESLSC), RZI (RZIG), BCESL (BCESLSC), BCRZI (BCRZIG) and GESL (GESLSC). This is done with the reasoning that Van Dijk et al. (2014) find similar results to each pair of methods, for example the ESL and ESLSC pair, where the more involved model performs better, may it be small. The results for all methods can be found in the Appendix.

Table 3: All mentioned econometric methods with their corresponding label.

Label	Model Description
DL	The Dynamic Nelson-Siegel method from Diebold and Li (2006).
RW	Random walk process for the factors from DL.
RWY	Random walk process for the yield levels.
AR	Autoregressive model of order 1 on the yield levels.
VAR	Vector Autoregressive model of order 1 on the yields, using all maturities for estimation.
ESL	Shifting endpoints method using exponential smoothing on the level factor from DL.
ESLSC	Shifting endpoints method using exponential smoothing on all three factors from DL.
RZI	Shifting endpoints method using realized measures on the level factor from DL.
RZIG	Shifting endpoints method using realized measures on the level and slope factor from DL.
BC	The Dynamic Nelson-Siegel method using the Björk and Christensen model.
BCRW	Random walk process for the factors from BC.
BCESL	Shifting endpoints method using exponential smoothing on the level factor from BC.
BCESLSC	Shifting endpoints method using exponential smoothing on all four factors from BC.
BCRZI	Shifting endpoints method using realized measures on the level factor from BC.
BCRZIG	Shifting endpoints method using realized measures on the level and both slope factors from BC.
GDNS	The dynamic Nelson-Siegel model with global factors from Diebold, Li, and Yue (2008).
GESL	GDNS where shifting endpoints using exponential smoothing is applied on the global level factor.
GESLSC	GDNS where shifting endpoints using exponential smoothing is applied on all global factors.

5 Results

In this section, I present the found results using the given methods. In Section 5.1 contains the main findings of the methods. Afterwards in Section 5.2, I test the statistical significance of the improvements of the methods against the DL and RW models.

5.1 Main Findings

For my research, I use several methods for computing iterated forecasts, see Table 3 for a summary with their corresponding labels. The forecasts are made similarly to Van Dijk et al. (2014). I use out-of-sample forecasts with an expanding window, with the period from January 1995 for the initial estimation and with January 2005 as the first forecast, ending at August 2021. Here, forecast horizons of $h = 6, 12, 24$ months ahead are used for a few selected maturities of $m = 3, 12, 36, 60, 120$ months for each country. When estimating the methods, I use the maturities given in Section 3.1 for the US and the ones given in Section 3.2 for the EU area countries, as the data differs in this aspect. The first half of Table 4 shows the RMSPE's for the selected forecasts on the US Treasury bond yields. The second half of Table 4 shows this for the German government bond yields. For conserving space, I present all results of all used methods in the Appendix in Tables 10 - 15. The results differ from those of Van Dijk et al. (2014) as the data obtained is from a different source applied on a different time period for estimation and forecasting[3].

For the global dynamic Nelson-Siegel (GDNS) method, I first obtained three factors from each set of level, slope and curvature factors of all individual countries in the Nelson-Siegel model. The global level factor explains more than 92.2% of the total variance in the level factors. For this factor, it is found that most country level factors contribute (in percentage) relatively the same. However, it is notable that Germany (17.5%), France (17.9%) and the Netherlands (17.7%) have a slightly larger contribution than the other countries, where both groups of countries in this split have approximately the same contribution. The first factor of the PCA on the slope factors explains approximately 72.7% of the total variance in the slope factors. The countries France (20.6%) and the Netherlands (19.2%) have a relative higher contribution for this factor compared to the other countries, where the US (11.4%) has a marginally smaller contribution. The last factor, the global curvature factor, explains 65.8% of the total variance in curvature factors. This last factor has the most contribution from the countries Germany (20.1%), France (22.4%) and the Netherlands (22.9%), with the marginally lowest from the US (8.6%). Concluding, I obtain three global factors for the GDNS model. Here it is found that France and the Netherlands have a relative high contribution to the factors, especially for the global slope and curvature factors. However, all countries do still have a relatively high and equal contribution to all factors.

Table 4 shows that the VAR method almost always performs worse than the benchmark methods RW, RWY, BCRW and AR, even by 246.8% compared to RW for a horizon of 24 months and a maturity of 120 months for the US. This is also seen in Tables 10 - 15. I therefore exclude this method from further comparison.

The first half of Table 4 shows the RMSPE's for the US, where the GDNS method performs well compared to the BC and BCRZIG methods, as it improves the RMSPE's by $4.4 - 40.4\%$. However, the BCESLSC method performs better when considering long maturities of $m = 36, 60$ and 120 months, where the GDNS method worsens by $1.9 - 24.0\%$. Furthermore, the GESLSC method also performs better than the GDNS for these long maturities by $0.6 - 10.6\%$. However, the BCESLSC method again performs better in long maturities compared to GESLSC and for all forecasts made with $h = 24$, where it improves by $2.0 - 16.7\%$. Comparing the benchmark methods RW, RWY, BCRW and AR to the GDNS and GESLSC methods, it is notable that the RW, RWY and BCRW generally perform better than these methods and the other methods. Only for $h = 24$ and short maturities of $m = 3, 12$ months does the GDNS method improve the RMSPE of RW and RWY by $7.6 - 8.6\%$. However, the methods do generally improve the AR

[3] The results will be the very similar when using the same data range of Van Dijk et al. (2014) for estimating and forecasting. These results can be given on request.

method for the long maturity of 120 months by $5.1 - 17.6\%$. Additionally, the methods perform better for $h = 24$ and the short maturities of $m = 3, 12$ months compared to BCRW and AR, where these improve by $3.4 - 18.3\%$. Lastly, comparing GDNS and GESLSC to DL, ESLSC and RZIG, it can be seen that the global methods generally perform better than the DL and RZIG methods by $0.3 - 14.5\%$. Compared to the ESLSC method, it is seen that the GDNS method only has a lower RMSPE for again $h = 24$ and $m = 3, 12$ with improvements of 13.6% and 11.9% for the maturities respectively. GESLSC however, almost always worsens when compared to ESLSC.

Next, in the second half of Table 4 I show the same methods using the German government bond yields. It occurs that GDNS and GESLSC mostly decrease the RMSPE relative to BC between $1.6 - 43.2\%$. However for $h = 24$, GDNS worsens for all maturities except for 12 months. The GESLSC only worsens here for the 3 and 12 month maturities. The same holds when comparing to BCRZIG, where the two methods mostly perform well with the exception of different maturities for $h = 24$. With BCESLSC, I note that GDNS mostly performs worse, except for the short maturities of $m = 3, 12$ months where it improves by $2.2 - 12.2\%$. Similarly, GESLSC performs well for short maturities of $m = 3, 12, 36$ for $h = 6, 12$ and improves by $6.3 - 22.6\%$ here. Comparing GDNS and GESLSC, it appears that GESLSC almost always performs better. Only for the horizon of $h = 24$ months and the short maturities $h = 3, 12$ does the GDNS perform better, thus the exponential smoothing extension in general improves the standard method here. Compared to the benchmark methods RW, RWY, BCRW and AR, I note that the GDNS method always performs worse, by $2.4 - 42.5\%$, and that the GESLSC method mostly perform worse, such that there is no pattern to be found where it does do well. This also holds for DL, where both methods generally perform worse. However, GESLSC still improves DL by $2.6 - 7.2\%$ for $h = 6, 12$. Comparing the two methods to ESLSC, it is seen that GDNS and GESLSC worsen for almost every horizon and maturity. For RZIG, GDNS worsens again, however GESLSC mostly improves the RMSPE by $4.7 - 12.0\%$, where RZIG only performs better for $h = 6$ and $m = 120$ and for $h = 24$ and $m = 3, 12$.

For the other EU countries, some conflicting but generally similar results are found. Here, the benchmark methods RW, RWY, BCRW and AR generally perform better than any other method and the GESLSC method generally improves GDNS and the BC, BCESLSC and BCRZIG methods.

Table 4: Root Mean Square Prediction Error for several forecasts and methods on the US and DE bond yields.

Forecast horizon:	$h = 6$ months					$h = 12$ months					$h = 24$ months				
Maturity (months):	3	12	36	60	120	3	12	36	60	120	3	12	36	60	120
US Treasury bond yields															
DL	0.81	0.71	0.76	0.77	0.62	1.35	1.21	1.17	1.13	0.92	2.07	1.93	1.78	1.65	1.32
RW	0.70	0.64	0.63	0.64	0.59	1.21	1.09	0.92	0.83	0.74	1.98	1.85	1.50	1.28	0.97
RWY	0.67	0.65	0.62	0.63	0.59	1.18	1.10	0.90	0.82	0.74	1.96	1.86	1.49	1.26	0.99
AR	0.67	0.65	0.63	0.64	0.63	1.21	1.10	0.91	0.87	0.88	2.21	2.00	1.46	1.31	1.26
VAR	0.78	0.83	0.95	1.12	1.11	1.36	1.38	1.45	1.52	1.47	2.36	2.40	2.55	2.75	3.36
ESLSC	0.72	0.65	0.64	0.64	0.56	1.24	1.11	0.95	0.86	0.70	2.09	1.92	1.52	1.25	0.89
RZIG	0.84	0.77	0.77	0.78	0.60	1.36	1.26	1.15	1.07	0.80	1.98	1.88	1.64	1.44	1.03
BC	1.29	1.16	1.01	0.93	0.76	1.80	1.69	1.51	1.38	1.14	2.19	2.19	2.07	1.90	1.59
BCRW	0.67	0.65	0.62	0.62	0.59	1.19	1.10	0.90	0.82	0.74	1.96	1.86	1.49	1.26	0.99
BCESLSC	0.99	0.90	0.74	0.68	0.58	1.44	1.33	1.05	0.91	0.74	1.98	1.89	1.50	1.24	0.93
BCRZIG	1.18	1.00	0.88	0.84	0.68	1.60	1.41	1.28	1.17	0.90	1.89	1.83	1.74	1.55	1.15
GDNS	0.79	0.69	0.74	0.74	0.59	1.25	1.13	1.10	1.05	0.83	1.81	1.69	1.59	1.47	1.16
GESLSC	0.81	0.72	0.73	0.73	0.58	1.35	1.22	1.11	1.03	0.78	2.07	1.93	1.66	1.45	1.04
German government bond yields															
DL	0.60	0.61	0.61	0.58	0.50	0.93	0.91	0.85	0.81	0.69	1.35	1.28	1.20	1.16	1.00
RW	0.58	0.58	0.57	0.54	0.47	0.90	0.85	0.75	0.70	0.61	1.34	1.28	1.12	1.01	0.84
RWY	0.56	0.59	0.56	0.52	0.48	0.87	0.87	0.74	0.68	0.64	1.31	1.32	1.10	0.98	0.89
AR	0.55	0.59	0.58	0.55	0.50	0.84	0.84	0.81	0.78	0.71	1.21	1.17	1.21	1.17	1.05
VAR	0.61	0.64	0.61	0.58	0.53	0.91	0.92	0.86	0.80	0.72	1.31	1.30	1.23	1.13	0.99
ESLSC	0.58	0.57	0.56	0.54	0.47	0.91	0.84	0.75	0.71	0.61	1.43	1.33	1.15	1.03	0.85
RZIG	0.63	0.62	0.62	0.61	0.50	0.97	0.93	0.89	0.87	0.74	1.37	1.30	1.28	1.27	1.13
BC	1.02	0.95	0.81	0.71	0.58	1.28	1.23	1.08	0.97	0.82	1.39	1.37	1.32	1.28	1.15
BCRW	0.56	0.59	0.56	0.52	0.48	0.87	0.87	0.74	0.68	0.63	1.32	1.31	1.11	0.98	0.87
BCESLSC	0.74	0.72	0.63	0.57	0.50	1.04	1.00	0.84	0.74	0.66	1.38	1.37	1.17	1.04	0.92
BCRZIG	1.00	0.92	0.78	0.71	0.59	1.24	1.18	1.05	0.99	0.87	1.32	1.33	1.35	1.36	1.28
GDNS	0.66	0.63	0.65	0.64	0.52	1.01	0.95	0.95	0.94	0.80	1.42	1.34	1.37	1.39	1.26
GESLSC	0.58	0.58	0.58	0.57	0.51	0.91	0.86	0.79	0.77	0.69	1.47	1.39	1.22	1.14	1.01

Note: I use iterative out-of-sample forecasts that are made for the period January 2005 - August 2021, with estimation starting from January 1995 and using an expanding window. I report out-of-sample root mean square prediction errors for these forecast with several methods for given forecast horizons and maturities. Labels of the methods can be found in Table 3.

5.2 Statistical Improvement Tests

I continue with the statistical significance of the improvements for some methods, where I exclude the benchmark methods for focusing on the main research and for conserving space. The first half of Table 5 shows the first bootstrap that tests the null of better performance of the DL method than the selected methods for the US and Germany[4]. At a significance level of 5% one can see that, for the US and Germany, the improved forecasting accuracy of ESLSC compared to DL is statistically significant for all forecasts. This contradicts that Van Dijk et al. (2014) find that this method only is significantly better for predicting only one forecast. For Germany, it is also found that RZIG has a significant better accuracy for $h = 24$ and $m = 36, 60, 120$ months and GDNS for $h = 6$ and $m = 3, 36, 60$, for $h = 12$ and $m = 3, 60$ and for $h = 24$ and $m = 60, 120$ months.

The second half of Table 5 shows the results for the second bootstrap that tests the null of better performance of the RW method than the selected methods. Shown is that ESLSC has some significant improved forecasting accuracy compared to RW. For the US, this holds for $h = 6, 12$ with $m = 120$ months and for the entire forecast horizon of 24 months. For Germany, this holds for $h = 6$ with $m = 120$ months and for $h = 24$ and $m = 3, 12, 36$ months. Additionally, GESLSC has some significant improvements for $h = 24$ with $m = 3$ months for the US and for $h = 24$ and $m = 3, 12$ months for Germany.

6 Conclusions

In this paper, it is researched how the global method from Diebold, Li, and Yue (2008) and the shifting endpoints extension of Van Dijk et al. (2014) perform on global government bond yields. These methods are extensions of the dynamic Nelson-Siegel method. The methods are then used combined with each other and compared to each other. There exist some conflicting results in the findings. Explicitly, it appears that the use of shifting endpoints on the global method generally improves the original global method. However, the exponentially smoothed specification of the shifting endpoints method mostly outperforms all specifications of the global method. Moreover, the realized measures specification of Van Dijk et al. (2014) often performs worse than most researched methods. This contrast with the findings of Van Dijk et al. (2014), since they find that the realized measures specification performs better than the exponentially smoothed one.

The global method is also compared to country level methods. The global method performs well compared to country level specifications, where the use of shifting endpoints improves it

[4] The results for the other countries and methods can be given on request, for both bootstraps.

Table 5: The bootstrap p-values with null of better forecasting ability of the (1) DL method and (2) RW method, then the compared and selected models for US and DE yields.

Forecast horizon:	$h = 6$ months					$h = 12$ months					$h = 24$ months				
Maturity (months):	3	12	36	60	120	3	12	36	60	120	3	12	36	60	120
Null of better accuracy of DL on US Treasury bond yields															
ESLSC	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
RZIG	0.19	0.11	0.10	0.10	0.12	0.16	0.10	0.09	0.09	0.15	0.25	0.16	0.13	0.17	0.22
GDNS	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
GESLSC	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Null of better accuracy of DL on German government bond yields															
ESLSC	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
RZIG	0.92	0.98	0.95	0.96	1.00	0.73	0.92	0.68	0.41	0.62	0.19	0.21	0.01	0.00	0.00
GDNS	0.00	0.08	0.04	0.00	1.00	0.01	0.90	0.12	0.00	0.57	1.00	1.00	1.00	0.03	0.00
GESLSC	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Null of better accuracy of RW on US Treasury bond yields															
DL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
ESLSC	1.00	1.00	1.00	0.99	0.04	1.00	1.00	1.00	1.00	0.02	0.00	0.00	0.00	0.00	0.00
RZIG	1.00	1.00	1.00	1.00	1.00	0.91	0.94	0.97	0.96	0.94	0.58	0.62	0.81	0.86	0.86
GDNS	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
GESLSC	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.52	1.00	1.00	0.98
Null of better accuracy of RW on German government bond yields															
DL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
ESLSC	1.00	0.99	0.78	0.84	0.00	1.00	1.00	1.00	1.00	0.10	0.00	0.00	0.00	0.97	0.85
RZIG	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.89	0.97	1.00	1.00	1.00
GDNS	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	1.00	1.00	1.00	1.00
GESLSC	1.00	1.00	0.99	1.00	0.77	0.97	1.00	1.00	1.00	1.00	0.00	0.00	0.42	1.00	1.00

Note: I use bootstrap p-values where I do a one-sided test where the null is that the Diebold-Li (DL) method and random walk (RW) method have better forecasting accuracy, for the first and second half of the table respectively, with the alternative that the other compared method has better accuracy.

further. However, the dynamic Nelson-Siegel method and the random walk methods often perform better than any specification of the global method.

Concluding, the shifting endpoints method on the global methods generally performs well. However, the exponentially smoothing specification on the dynamic Nelson-Siegel method or the random walk methods have a higher out-of-sample forecasting accuracy. These findings imply that the method of Van Dijk et al. (2014) has an excellent performance on forecasting when applied to existing methods. Furthermore, it improves the forecasting accuracy of other methods when applied. Therefore, by applying this method on already used ones, it can help investors and (monetary) policymakers with increasing their forecasting accuracy.

Lastly, I give some suggestions for further research. First, one could find and apply useful survey forecasts. This proved to be one of the best performed specification of Van Dijk et al. (2014). Furthermore, it is interesting to find suitable global macro-economic variables to be used in the global method with shifting endpoints. This specification can be more desirable than the exponential smoothing one, as this is argued to perform better by Van Dijk et al. (2014) when applied to the dynamic Nelson-Siegel method.

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A Appendix

A.1 Code Description

The results in this paper are made using the programme *RStudio* with version *2022.02.0+443*, and *R* version 4.1.3 (R Core Team 2022). The code used is given in the file "ThesisCode.R". Here, I present in blocks (in order) the data collection, some used functions, the data descriptive statistics, the forecast function of all methods (with PCA and the global methods in separate blocks) and the model comparison methods. The comparison is first made with solely the RMSPE, then with the bootstrapping method of Van Dijk et al. (2014). Additionally used are the files "DE_yields_monthly.csv", "ES_yields_monthly.csv", "FR_yields_monthly.csv", "IT_yields_monthly.csv", "NL_yields_monthly.csv", "US_yields_monthly.csv", "US_IP_monthly_index.csv" and "US_CPI_index.csv" containing the used raw data.

Before running the code, please check if the required packages are installed. Check this by running the library functions in lines *8* to *11*. If you do not have certain packages, please install these using the install functions in lines *2* to *7*.

A.2 Descriptive Statistics for ES, FR, IT and NL

Table 6: Descriptive statistics of the Spanish bond yields with several maturities from January 1995 to August 2021.

Maturity	Mean	SD	Min	Max	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
3	2.218	2.456	-0.772	9.400	0.981	0.719	0.415
6	2.309	2.445	-0.688	9.730	0.979	0.700	0.387
12	2.445	2.467	-0.644	10.271	0.976	0.676	0.354
24	2.773	2.552	-0.595	11.463	0.972	0.673	0.355
36	3.021	2.580	-0.530	11.844	0.972	0.673	0.353
48	3.232	2.592	-0.484	12.127	0.972	0.675	0.352
60	3.439	2.579	-0.404	12.237	0.973	0.679	0.347
72	3.628	2.567	-0.321	12.364	0.974	0.683	0.344
84	3.804	2.545	-0.223	12.486	0.974	0.685	0.338
96	3.960	2.520	-0.125	12.485	0.974	0.686	0.331
108	4.089	2.493	-0.020	12.424	0.975	0.690	0.324
120 (level)	4.199	2.466	0.077	12.364	0.975	0.691	0.318
Slope	1.981	1.195	0.209	6.128	0.959	0.611	0.164
Curvature	-0.872	0.725	-2.115	2.278	0.778	0.190	-0.152

Note: For each maturity, I present the mean, standard deviation, minimum value, maximum value and the first, twelfth and thirtieth autocorrelation. Additionally given are the empirical values for the level, slope and curvature factors of the yield curve. These are computed as the 120-month yield, as the 120-month minus the 3-month yield and as twice the 24-months minus the 3- and 120-month yield, respectively.

Table 7: Descriptive statistics of the French bond yields with several maturities from January 1995 to August 2021.

Maturity	Mean	SD	Min	Max	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
3	1.752	1.998	-0.904	7.930	0.986	0.741	0.554
6	1.782	2.007	-0.849	7.420	0.986	0.751	0.559
12	1.850	2.013	-0.766	7.041	0.986	0.765	0.573
24	2.033	2.066	-0.859	7.126	0.984	0.790	0.614
36	2.217	2.102	-0.863	7.339	0.984	0.803	0.634
48	2.419	2.120	-0.811	7.534	0.984	0.811	0.640
60	2.598	2.118	-0.743	7.662	0.983	0.813	0.638
72	2.746	2.120	-0.681	7.768	0.983	0.815	0.632
84	2.908	2.125	-0.609	7.828	0.984	0.817	0.625
96	3.066	2.125	-0.535	7.890	0.984	0.818	0.619
108	3.204	2.107	-0.461	7.962	0.984	0.817	0.613
120 (level)	3.321	2.085	-0.379	8.012	0.984	0.816	0.605
Slope	1.570	0.868	-0.132	3.328	0.954	0.362	-0.029
Curvature	-1.007	0.615	-2.135	0.604	0.913	0.378	-0.097

Note: For each maturity, I present the mean, standard deviation, minimum value, maximum value and the first, twelfth and thirtieth autocorrelation. Additionally given are the empirical values for the level, slope and curvature factors of the yield curve. These are computed as the 120-month yield, as the 120-month minus the 3-month yield and as twice the 24-months minus the 3- and 120-month yield, respectively.

Table 8: Descriptive statistics of the Italian bond yields with several maturities from January 1995 to August 2021.

Maturity	Mean	SD	Min	Max	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
3	2.398	2.737	-0.676	11.000	0.982	0.720	0.419
6	2.523	2.707	-0.541	11.287	0.980	0.702	0.387
12	2.659	2.718	-0.526	11.744	0.979	0.684	0.363
24	3.041	2.776	-0.424	13.266	0.973	0.654	0.323
36	3.313	2.758	-0.293	13.775	0.971	0.639	0.306
48	3.548	2.725	-0.196	13.930	0.971	0.637	0.299
60	3.759	2.694	-0.020	14.009	0.971	0.634	0.283
72	3.947	2.667	0.079	13.899	0.971	0.642	0.279
84	4.122	2.623	0.240	13.793	0.972	0.642	0.273
96	4.275	2.586	0.347	13.715	0.972	0.645	0.275
108	4.417	2.561	0.438	13.952	0.973	0.641	0.261
120 (level)	4.533	2.532	0.537	14.138	0.973	0.632	0.245
Slope	2.135	1.176	-0.442	5.397	0.948	0.557	0.112
Curvature	-0.849	0.756	-2.299	2.033	0.823	0.177	-0.220

Note: For each maturity, I present the mean, standard deviation, minimum value, maximum value and the first, twelfth and thirtieth autocorrelation. Additionally given are the empirical values for the level, slope and curvature factors of the yield curve. These are computed as the 120-month yield, as the 120-month minus the 3-month yield and as twice the 24-months minus the 3- and 120-month yield, respectively.

Table 9: Descriptive statistics of the Dutch bond yields with several maturities from January 1995 to August 2021.

Maturity	Mean	SD	Min	Max	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
3	1.628	1.872	-1.037	5.140	0.988	0.815	0.620
6	1.665	1.899	-0.898	5.328	0.988	0.814	0.624
12	1.761	1.936	-0.889	5.819	0.987	0.817	0.634
24	1.949	2.011	-0.905	6.498	0.985	0.831	0.663
36	2.153	2.077	-0.921	6.932	0.984	0.837	0.675
48	2.349	2.115	-0.881	7.211	0.984	0.841	0.679
60	2.529	2.128	-0.818	7.329	0.984	0.843	0.677
72	2.692	2.146	-0.756	7.459	0.984	0.843	0.672
84	2.850	2.160	-0.712	7.528	0.985	0.844	0.665
96	2.991	2.163	-0.652	7.591	0.985	0.843	0.658
108	3.109	2.156	-0.608	7.663	0.985	0.840	0.649
120 (level)	3.205	2.140	-0.548	7.734	0.985	0.836	0.641
Slope	1.577	0.917	0.096	3.998	0.966	0.453	-0.056
Curvature	-0.935	0.588	-1.985	0.392	0.909	0.362	-0.134

Note: For each maturity, I present the mean, standard deviation, minimum value, maximum value and the first, twelfth and thirtieth autocorrelation. Additionally given are the empirical values for the level, slope and curvature factors of the yield curve. These are computed as the 120-month yield, as the 120-month minus the 3-month yield and as twice the 24-months minus the 3- and 120-month yield, respectively.

A.3 Complete RMSPE Tables

Table 10: Root Mean Square Prediction Error for several forecasts and methods on the US Treasury bond yields.

Forecast horizon:	$h = 6$ months					$h = 12$ months					$h = 24$ months				
Maturity (months):	3	12	36	60	120	3	12	36	60	120	3	12	36	60	120
DL	0.81	0.71	0.76	0.77	0.62	1.35	1.21	1.17	1.13	0.92	2.07	1.93	1.78	1.65	1.32
RW	0.70	0.64	0.63	0.64	0.59	1.21	1.09	0.92	0.83	0.74	1.98	1.85	1.50	1.28	0.97
RWY	0.67	0.65	0.62	0.63	0.59	1.18	1.10	0.90	0.82	0.74	1.96	1.86	1.49	1.26	0.99
AR	0.67	0.65	0.63	0.64	0.63	1.21	1.10	0.91	0.87	0.88	2.21	2.00	1.46	1.31	1.26
VAR	0.78	0.83	0.95	1.12	1.11	1.36	1.38	1.45	1.52	1.47	2.36	2.40	2.55	2.75	3.36
ESL	0.74	0.67	0.68	0.67	0.57	1.22	1.11	1.00	0.92	0.74	1.84	1.72	1.47	1.29	0.95
ESLSC	0.72	0.65	0.64	0.64	0.56	1.24	1.11	0.95	0.86	0.70	2.09	1.92	1.52	1.25	0.89
RZI	0.87	0.78	0.79	0.79	0.61	1.43	1.31	1.20	1.10	0.81	2.15	2.01	1.74	1.51	1.06
RZIG	0.84	0.77	0.77	0.78	0.60	1.36	1.26	1.15	1.07	0.80	1.98	1.88	1.64	1.44	1.03
BC	1.29	1.16	1.01	0.93	0.76	1.80	1.69	1.51	1.38	1.14	2.19	2.19	2.07	1.90	1.59
BCRW	0.67	0.65	0.62	0.62	0.59	1.19	1.10	0.90	0.82	0.74	1.96	1.86	1.49	1.26	0.99
BCESL	1.21	1.08	0.92	0.83	0.66	1.66	1.53	1.32	1.17	0.90	1.94	1.92	1.75	1.54	1.16
BCESLSC	0.99	0.90	0.74	0.68	0.58	1.44	1.33	1.05	0.91	0.74	1.98	1.89	1.50	1.24	0.93
BCRZI	1.24	1.11	0.95	0.88	0.69	1.71	1.59	1.39	1.24	0.93	2.08	2.07	1.88	1.64	1.20
BCRZIG	1.18	1.00	0.88	0.84	0.68	1.60	1.41	1.28	1.17	0.90	1.89	1.83	1.74	1.55	1.15
GDNS	0.79	0.69	0.74	0.74	0.59	1.25	1.13	1.10	1.05	0.83	1.81	1.69	1.59	1.47	1.16
GESL	0.79	0.71	0.74	0.74	0.59	1.26	1.15	1.10	1.04	0.81	1.82	1.72	1.59	1.45	1.10
GESLSC	0.81	0.72	0.73	0.73	0.58	1.35	1.22	1.11	1.03	0.78	2.07	1.93	1.66	1.45	1.04

Note: I use iterative out-of-sample forecasts that are made for the period January 2005 - August 2021, with estimation starting from January 1995 and using an expanding window. I report out-of-sample root mean square prediction errors for these forecast with several methods for given forecast horizons and maturities. Labels of the methods can be found in Table 3.

Table 11: Root Mean Square Prediction Error for several forecasts and methods on the German bond yields.

Forecast horizon:	$h = 6$ months					$h = 12$ months					$h = 24$ months				
Maturity (months):	3	12	36	60	120	3	12	36	60	120	3	12	36	60	120
DL	0.60	0.61	0.61	0.58	0.50	0.93	0.91	0.85	0.81	0.69	1.35	1.28	1.20	1.16	1.00
RW	0.58	0.58	0.57	0.54	0.47	0.90	0.85	0.75	0.70	0.61	1.34	1.28	1.12	1.01	0.84
RWY	0.56	0.59	0.56	0.52	0.48	0.87	0.87	0.74	0.68	0.64	1.31	1.32	1.10	0.98	0.89
AR	0.55	0.59	0.58	0.55	0.50	0.84	0.84	0.81	0.78	0.71	1.21	1.17	1.21	1.17	1.05
VAR	0.61	0.64	0.61	0.58	0.53	0.91	0.92	0.86	0.80	0.72	1.31	1.30	1.23	1.13	0.99
ESL	0.59	0.62	0.60	0.57	0.50	0.92	0.91	0.83	0.78	0.67	1.33	1.28	1.16	1.08	0.92
ESLSC	0.58	0.57	0.56	0.54	0.47	0.91	0.84	0.75	0.71	0.61	1.43	1.33	1.15	1.03	0.85
RZI	0.66	0.63	0.63	0.63	0.51	1.03	0.95	0.92	0.90	0.75	1.52	1.40	1.36	1.33	1.16
RZIG	0.63	0.62	0.62	0.61	0.50	0.97	0.93	0.89	0.87	0.74	1.37	1.30	1.28	1.27	1.13
BC	1.02	0.95	0.81	0.71	0.58	1.28	1.23	1.08	0.97	0.82	1.39	1.37	1.32	1.28	1.15
BCRW	0.56	0.59	0.56	0.52	0.48	0.87	0.87	0.74	0.68	0.63	1.32	1.31	1.11	0.98	0.87
BCESL	1.01	0.95	0.81	0.70	0.57	1.27	1.22	1.06	0.94	0.77	1.35	1.34	1.25	1.18	1.04
BCESLSC	0.74	0.72	0.63	0.57	0.50	1.04	1.00	0.84	0.74	0.66	1.38	1.37	1.17	1.04	0.92
BCRZI	1.04	0.96	0.82	0.73	0.60	1.31	1.25	1.10	1.03	0.89	1.47	1.45	1.42	1.41	1.30
BCRZIG	1.00	0.92	0.78	0.71	0.59	1.24	1.18	1.05	0.99	0.87	1.32	1.33	1.35	1.36	1.28
GDNS	0.66	0.63	0.65	0.64	0.52	1.01	0.95	0.95	0.94	0.80	1.42	1.34	1.37	1.39	1.26
GESL	0.66	0.68	0.67	0.64	0.54	1.02	1.01	0.97	0.92	0.79	1.40	1.36	1.33	1.30	1.14
GESLSC	0.58	0.58	0.58	0.57	0.51	0.91	0.86	0.79	0.77	0.69	1.47	1.39	1.22	1.14	1.01

Note: I use iterative out-of-sample forecasts that are made for the period January 2005 - August 2021, with estimation starting from January 1995 and using an expanding window. I report out-of-sample root mean square prediction errors for these forecast with several methods for given forecast horizons and maturities. Labels of the methods can be found in Table 3.

Table 12: Root Mean Square Prediction Error for several forecasts and methods on the Spanish bond yields.

Forecast horizon:	$h = 6$ months					$h = 12$ months					$h = 24$ months				
Maturity (months):	3	12	36	60	120	3	12	36	60	120	3	12	36	60	120
DL	0.85	0.86	0.88	0.89	0.72	1.18	1.22	1.24	1.25	1.07	1.55	1.56	1.66	1.72	1.57
RW	0.75	0.76	0.73	0.73	0.63	1.04	1.06	1.00	1.00	0.91	1.48	1.55	1.52	1.54	1.43
RWY	0.72	0.80	0.73	0.70	0.64	1.01	1.13	1.00	0.97	0.94	1.45	1.60	1.51	1.50	1.49
AR	0.72	0.80	0.77	0.74	0.67	1.02	1.12	1.09	1.06	1.00	1.45	1.56	1.61	1.58	1.52
VAR	0.99	1.06	1.01	0.97	0.83	1.27	1.29	1.31	1.32	1.23	1.76	1.76	1.83	1.87	1.81
ESL	0.89	0.90	0.88	0.88	0.72	1.29	1.31	1.26	1.24	1.06	1.79	1.77	1.77	1.78	1.61
ESLSC	0.79	0.80	0.78	0.78	0.67	1.12	1.16	1.10	1.09	0.98	1.66	1.72	1.68	1.66	1.53
RZI	0.77	0.81	0.81	0.82	0.70	1.02	1.10	1.12	1.12	1.02	1.24	1.35	1.43	1.48	1.43
RZIG	0.76	0.81	0.79	0.79	0.70	1.03	1.10	1.08	1.08	1.01	1.24	1.31	1.35	1.42	1.41
BC	1.44	1.28	1.07	1.02	0.81	1.71	1.62	1.47	1.42	1.20	1.90	1.87	1.87	1.88	1.72
BCRW	0.72	0.79	0.73	0.71	0.64	1.01	1.10	0.99	0.97	0.93	1.45	1.59	1.50	1.50	1.47
BCESL	1.60	1.41	1.16	1.07	0.84	1.98	1.84	1.60	1.50	1.24	2.29	2.20	2.10	2.05	1.84
BCESLSC	1.00	0.98	0.87	0.82	0.72	1.31	1.35	1.20	1.14	1.04	1.71	1.81	1.72	1.69	1.61
BCRZI	1.37	1.21	1.00	0.94	0.77	1.55	1.48	1.35	1.30	1.13	1.58	1.61	1.64	1.67	1.59
BCRZIG	1.27	1.14	1.02	0.97	0.78	1.37	1.37	1.36	1.33	1.15	1.34	1.49	1.64	1.69	1.59
GDNS	0.95	0.95	0.97	0.99	0.81	1.42	1.44	1.47	1.47	1.29	2.06	2.13	2.25	2.30	2.14
GESL	0.91	0.91	0.91	0.91	0.77	1.35	1.37	1.34	1.32	1.17	1.94	1.97	2.02	2.04	1.91
GESLSC	0.85	0.84	0.85	0.85	0.74	1.25	1.26	1.24	1.23	1.12	1.89	1.95	1.95	1.95	1.84

Note: I use iterative out-of-sample forecasts that are made for the period January 2005 - August 2021, with estimation starting from January 1995 and using an expanding window. I report out-of-sample root mean square prediction errors for these forecast with several methods for given forecast horizons and maturities. Labels of the methods can be found in Table 3.

Table 13: Root Mean Square Prediction Error for several forecasts and methods on the French bond yields.

Forecast horizon:	$h = 6$ months					$h = 12$ months					$h = 24$ months				
Maturity (months):	3	12	36	60	120	3	12	36	60	120	3	12	36	60	120
DL	0.64	0.66	0.63	0.62	0.50	1.00	0.99	0.93	0.89	0.72	1.41	1.37	1.34	1.31	1.11
RW	0.57	0.59	0.52	0.50	0.45	0.90	0.86	0.72	0.67	0.57	1.34	1.28	1.08	0.97	0.80
RWY	0.55	0.59	0.52	0.49	0.47	0.88	0.87	0.71	0.64	0.62	1.31	1.31	1.07	0.94	0.87
AR	0.55	0.58	0.53	0.52	0.50	0.85	0.84	0.77	0.74	0.70	1.20	1.19	1.14	1.13	1.08
VAR	0.63	0.67	0.61	0.56	0.50	0.94	0.99	0.91	0.83	0.72	1.36	1.39	1.33	1.24	1.10
ESL	0.64	0.68	0.63	0.60	0.51	1.00	1.01	0.92	0.85	0.68	1.39	1.36	1.27	1.20	0.99
ESLSC	0.58	0.59	0.53	0.52	0.46	0.92	0.88	0.74	0.69	0.59	1.41	1.35	1.13	1.02	0.85
RZI	0.69	0.68	0.67	0.66	0.53	1.09	1.06	1.03	1.00	0.82	1.61	1.55	1.55	1.53	1.33
RZIG	0.69	0.70	0.67	0.66	0.53	1.11	1.07	1.03	1.00	0.82	1.63	1.56	1.55	1.53	1.32
BC	1.04	1.02	0.88	0.78	0.62	1.36	1.33	1.18	1.07	0.87	1.56	1.56	1.50	1.45	1.28
BCRW	0.55	0.59	0.51	0.49	0.46	0.88	0.88	0.71	0.64	0.59	1.31	1.31	1.07	0.94	0.84
BCESL	1.04	1.02	0.88	0.77	0.60	1.36	1.33	1.16	1.03	0.81	1.54	1.53	1.42	1.33	1.13
BCESLSC	0.78	0.79	0.66	0.59	0.51	1.08	1.07	0.88	0.77	0.65	1.43	1.42	1.19	1.06	0.93
BCRZI	1.06	1.04	0.90	0.81	0.64	1.41	1.39	1.25	1.15	0.96	1.73	1.73	1.68	1.64	1.47
BCRZIG	1.09	1.05	0.91	0.81	0.65	1.46	1.42	1.27	1.17	0.97	1.80	1.80	1.74	1.69	1.50
GDNS	0.63	0.63	0.65	0.67	0.53	0.97	0.95	0.98	0.99	0.82	1.40	1.36	1.42	1.45	1.28
GESL	0.62	0.65	0.65	0.64	0.52	0.95	0.97	0.94	0.91	0.74	1.34	1.32	1.31	1.29	1.09
GESLSC	0.54	0.55	0.54	0.55	0.47	0.85	0.83	0.75	0.74	0.64	1.38	1.32	1.17	1.11	0.95

Note: I use iterative out-of-sample forecasts that are made for the period January 2005 - August 2021, with estimation starting from January 1995 and using an expanding window. I report out-of-sample root mean square prediction errors for these forecast with several methods for given forecast horizons and maturities. Labels of the methods can be found in Table 3.

Table 14: Root Mean Square Prediction Error for several forecasts and methods on the Italian bond yields.

Forecast horizon:	$h = 6$ months					$h = 12$ months					$h = 24$ months				
Maturity (months):	3	12	36	60	120	3	12	36	60	120	3	12	36	60	120
DL	0.86	0.89	0.94	0.92	0.75	1.23	1.24	1.30	1.28	1.08	1.63	1.59	1.68	1.67	1.50
RW	0.73	0.83	0.84	0.81	0.69	1.04	1.15	1.18	1.14	1.00	1.46	1.58	1.64	1.61	1.45
RWY	0.72	0.82	0.84	0.81	0.69	1.03	1.14	1.19	1.15	1.02	1.44	1.58	1.63	1.61	1.48
AR	0.72	0.82	0.85	0.82	0.71	1.04	1.13	1.20	1.17	1.04	1.44	1.57	1.62	1.58	1.45
VAR	1.05	1.07	1.05	0.97	0.82	1.43	1.46	1.47	1.39	1.23	1.77	1.81	1.85	1.80	1.64
ESL	0.92	0.95	0.96	0.92	0.76	1.38	1.38	1.37	1.32	1.13	1.90	1.85	1.87	1.85	1.67
ESLSC	0.77	0.88	0.89	0.85	0.74	1.15	1.27	1.29	1.24	1.09	1.69	1.81	1.84	1.79	1.62
RZI	0.78	0.85	0.90	0.88	0.76	1.06	1.14	1.22	1.20	1.08	1.26	1.31	1.47	1.49	1.42
RZIG	0.80	0.87	0.90	0.86	0.76	1.09	1.16	1.21	1.18	1.08	1.23	1.30	1.45	1.47	1.42
BC	1.66	1.49	1.25	1.13	0.86	1.80	1.69	1.53	1.43	1.18	1.92	1.88	1.84	1.79	1.59
BCRW	0.73	0.83	0.84	0.80	0.69	1.04	1.15	1.18	1.14	1.00	1.45	1.59	1.63	1.60	1.46
BCESL	1.76	1.58	1.30	1.16	0.87	1.97	1.83	1.60	1.47	1.19	2.20	2.11	2.01	1.93	1.71
BCESLSC	1.09	1.11	1.03	0.94	0.77	1.37	1.43	1.36	1.27	1.09	1.71	1.79	1.80	1.75	1.59
BCRZI	1.55	1.40	1.18	1.06	0.82	1.60	1.52	1.40	1.31	1.11	1.54	1.55	1.60	1.58	1.46
BCRZIG	1.35	1.29	1.17	1.06	0.82	1.37	1.40	1.39	1.31	1.10	1.30	1.43	1.58	1.57	1.45
GDNS	0.88	0.92	0.97	0.94	0.79	1.29	1.34	1.40	1.36	1.18	1.72	1.75	1.85	1.83	1.67
GESL	0.83	0.89	0.91	0.88	0.77	1.22	1.27	1.29	1.24	1.10	1.59	1.60	1.65	1.62	1.50
GESLSC	0.78	0.83	0.86	0.83	0.74	1.12	1.18	1.20	1.16	1.06	1.54	1.59	1.57	1.53	1.43

Note: I use iterative out-of-sample forecasts that are made for the period January 2005 - August 2021, with estimation starting from January 1995 and using an expanding window. I report out-of-sample root mean square prediction errors for these forecast with several methods for given forecast horizons and maturities. Labels of the methods can be found in Table 3.

Table 15: Root Mean Square Prediction Error for several forecasts and methods on the Dutch bond yields.

Forecast horizon:	$h = 6$ months					$h = 12$ months					$h = 24$ months				
Maturity (months):	3	12	36	60	120	3	12	36	60	120	3	12	36	60	120
DL	0.62	0.62	0.60	0.59	0.52	0.97	0.95	0.89	0.87	0.76	1.42	1.36	1.31	1.29	1.15
RW	0.58	0.58	0.51	0.50	0.46	0.91	0.85	0.71	0.67	0.62	1.35	1.28	1.08	0.98	0.84
RWY	0.56	0.58	0.52	0.48	0.47	0.89	0.86	0.71	0.65	0.65	1.33	1.30	1.07	0.94	0.90
AR	0.56	0.57	0.55	0.52	0.51	0.85	0.82	0.79	0.77	0.74	1.20	1.16	1.18	1.17	1.10
VAR	0.61	0.65	0.59	0.55	0.52	0.94	0.97	0.88	0.81	0.73	1.43	1.44	1.32	1.20	1.04
ESL	0.61	0.64	0.59	0.57	0.51	0.96	0.95	0.86	0.82	0.71	1.39	1.34	1.23	1.16	1.00
ESLSC	0.59	0.58	0.51	0.51	0.46	0.94	0.86	0.71	0.68	0.62	1.48	1.37	1.13	1.02	0.88
RZI	0.61	0.60	0.58	0.59	0.51	0.96	0.91	0.86	0.87	0.76	1.43	1.35	1.31	1.31	1.16
RZIG	0.63	0.61	0.59	0.60	0.51	0.98	0.92	0.88	0.88	0.76	1.45	1.36	1.33	1.33	1.19
BC	1.05	1.02	0.88	0.79	0.64	1.35	1.33	1.18	1.08	0.91	1.51	1.52	1.47	1.43	1.30
BCRW	0.57	0.59	0.51	0.49	0.47	0.89	0.87	0.70	0.66	0.63	1.33	1.30	1.06	0.95	0.88
BCESL	1.03	1.00	0.86	0.76	0.61	1.32	1.30	1.14	1.02	0.84	1.47	1.47	1.36	1.29	1.14
BCESLSC	0.76	0.75	0.63	0.56	0.51	1.07	1.05	0.85	0.76	0.68	1.43	1.41	1.19	1.06	0.96
BCRZI	1.02	0.98	0.85	0.77	0.62	1.29	1.26	1.13	1.05	0.90	1.46	1.47	1.43	1.42	1.31
BCRZIG	1.09	1.08	0.96	0.86	0.67	1.40	1.44	1.32	1.21	0.99	1.70	1.78	1.73	1.65	1.44
GDNS	0.63	0.62	0.63	0.64	0.54	0.97	0.94	0.94	0.95	0.84	1.38	1.34	1.39	1.42	1.30
GESL	0.63	0.66	0.63	0.62	0.54	0.98	0.98	0.93	0.90	0.78	1.35	1.32	1.30	1.29	1.14
GESLSC	0.55	0.56	0.52	0.52	0.49	0.89	0.83	0.73	0.72	0.67	1.45	1.36	1.16	1.10	0.98

Note: I use iterative out-of-sample forecasts that are made for the period January 2005 - August 2021, with estimation starting from January 1995 and using an expanding window. I report out-of-sample root mean square prediction errors for these forecast with several methods for given forecast horizons and maturities. Labels of the methods can be found in Table 3.