Assessing the importance of incorporating climate risk in investment decisions with factor models.

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Abstract

The advantages of the Risk Premium Principal Component Analysis (RP-PCA) model of Lettau and Pelger (2020) are analysed for standard test assets in the literature. Then, the importance of climate risk in financial markets within this framework is evaluated using factor models. The performance, in and out of sample, of the climate factor created by Gimeno Nogués and González Martınez (2022) is analysed, and using their stocks rankings according to emissions, a climate anomaly is added to the analysis of Lettau and Pelger (2020). The results show that climate risk is an essential component of systematic risk in financial markets. Comparing model performance across a variety of factor models we find that including climate risk information is more efficiently done through RP-PCA, and it helps improve performance both in and out of sample. Moreover, the addition of a climate-related anomaly has become more important after the Paris Agreement of 2015, and the structural change throughout the period indicates that more investment should be directed towards "greener" assets.

Contents

1 Introduction

Factor models can explain large cross-sections of information with only the estimation of a few factors. Nowadays, these models have become central in asset pricing theory (Connor & Korajczyk, 2010). They avoid investors having to analyse big dimensions of individual assets and trading strategies, and reduce co-movements and asset risk metrics into a reduced number of factors (Zaffaroni, 2019). Asset pricing theory states that co-movements between a cross-section of assets are driven by so called systematic risk factors (Lettau & Pelger, 2020), which constitute the portion of total risk that affects the whole market and is usually firm independent. When applying a factor model to a big and varied cross-section of assets or portfolios, the set of factors captured provides a good proxy of the systematic risk prevalent in the market. Capturing this risk efficiently can be essential to obtaining a broad picture of how the market is behaving, and deriving optimal investment strategies.

One of the first research papers where factor models are applied to the field of asset returns constitutes the 3 and 5 factor models derived by Fama and French (1993) and Fama and French (2015). The 5 factors constructed by Fama and French (2015) focus on capturing size, value, profitability, and investment patterns of stock returns to best explain their joint behaviour. In their research, they find that these 5 factors can explain between 71% and 94% of the cross-sectional variance of the portfolios examined (Fama & French, 2015). These models however have some limitations, which lie especially in their inability to capture correctly the low average returns of small stocks whose returns, regardless of their low profitability, are behaving like firms who invest aggressively (Fama & French, 2017). Additionally, many papers have tested the validity of the Fama and French factors by applying them in markets like Turkey (Zeren et al., 2019), Italy (Silvestri & Veltri, 2011), and Australia (Faff, 2004), amongst many others, were the factors proved insufficient. For these reasons, many researchers have made their contributions to improving upon the 5 factor model of Fama and French (2015), in attempts to find robust factors able to explain the common risk underlying different cross-sections of returns. Following all their findings, over 300-factor candidates have been found to have predictive power on stocks (Harvey et al., 2016). But the question remains on how to find the best small set of factors that can most accurately predict returns of portfolios, and capture efficiently systematic risk in a market.

A big input towards solving this central question is made by statistical factor models, and more specifically those using principal component analysis (PCA). These models, first introduced by Connor and Korajczyk (1988), use the second moment of portfolio returns to extract the co-movements and dynamics of large cross-sections of returns in a restricted amount of factors. In these PCA models, both the factors and factor loadings are assumed to be unknown, and they are estimated using eigenvalues decomposition of the returns. The estimation procedure assumes that the initial pool of observations is driven by a set of "inter-correlated quantitative dependant variables" (Abdi & Williams, 2010), and through the factor estimation, the essential information to describe these observations is extracted. Its application in the field of asset and portfolio returns is essential since it provides some clarity on which, out of all the potential factor candidates, are the most efficient and necessary to describe the cross-section of returns in a simplified model. The main critique of these models lies in the fact that only the second moment information is incorporated (Lettau & Pelger, 2020), and also the fact that high dimensionality issues arise when panel dimensions are big (Kelly et al., 2020). The first critique is of particular concern when it comes to findings the most accurate factors. This is because, according to the Arbitrage pricing theory (APT) of Ross (Roll & Ross, 1980), the means of excess returns carry valuable information about the factors, and its incorporation in the model should yield more efficient estimates.[1] Focusing on incorporating the insights from Ross' Arbitrage pricing theory into PCA models, Lettau and Pelger (2020) propose a generalization of the PCA model, called Risk Premium-PCA (RP-PCA). In RP-PCA, both the information of the first and second moment of returns are considered, to provide more accurate and efficient factor estimates.

In the past years a new risk, defined as climate risk, has entered the markets, and it is increasingly gaining importance when it comes to portfolio creation and asset pricing. Climate risk is described as the threat that rising global warming and worsening environmental conditions, can "damage the performance of an economy and investment returns" (Sturkenboom, 2020). Stocks highly exposed to climate risk, meaning their returns are likely to drop when climate risk rises, either as a result of explicit physical impact or through a change in regulation policies, should receive a risk premia for their exposure. Much of the existing literature in this field is focused on understanding the role climate risk plays in financial markets. In this line, many papers have focused on analysing whether firms with different environmental metrics exhibit significant differences in returns which should be considered when making investment choices. Additionally, there is a focus on understanding whether climate risk is currently being priced correctly. For example, Chen and Silva Gao (2012) evaluate whether corporate climate risk is incorporated in capital markets, and find that firms with higher climate risk have a higher cost of equity and debt. Chava (2014) confirm these findings by showing that investors require higher returns on stocks excluded from environmental screens. Additionally, Campiglio et al. (2019) contribute to showing the existence and increasing the effect of climate risk in financial markets, and further show that it is still not correctly priced in markets due to the lack of awareness by investors. Monasterolo and De Angelis (2020) also show that, even though investors are increasingly considering low-carbon assets as an attractive addition to their portfolios, the risk high-carbon assets carry is still not correctly priced in markets. Pricing climate risk correctly is not only essential for investors to transition into green investment, but also to incentivise firms to transform into cleaner and more sustainable business models.

[1]PCA models uniquely incorporate information of the second moment when constructing the eigenvalue decomposition that determines the factors. According to APT, the incorporation of the first moment should yield more efficient outcomes, and hence an improvement model which considers both the second and first moment can be constructed.

Regarding the previous discussion about findings on the underlying factor driving systematic risks in markets, it is not yet clear whether including climate risk in factor analysis can significantly impact the performance of models and forecasting results. The paper by Gimeno Nogués and González Martınez (2022) provides valuable research in this domain. Motivated by the growing importance of climate risk in the financial market, they develop a green factor, called GMP, which proves a relevant addition to the Fama and French framework and can help investors take climate-related characteristics into their investment decisions. After their research, the question remains on whether there are other ways to incorporate climate risk information into factor models, which we aim to answer in this research paper. Statistical factor models, such as PCA and RP-PCA, can be potential candidates to alternatively incorporate climate risk information in factor models. Additionally, with their factor estimation, climate risk importance in financial markets can be evaluated, helping share insights on how it can be best priced in markets. Climate-related characteristics of firms tend to affect the mean of returns more intensely than their variance (Górka & Kuziak, 2022), and motivated by the idea that RP-PCA can detect weaker factors that PCA cannot (Lettau & Pelger, 2020), analysing these models performance in capturing excess returns from climate sorted portfolios [2] can hence also help give additional insights on the performance of RP-PCA.

Following our main aim of understanding the extent to which climate risk is currently affecting markets, we divide the paper into two sections. The first focus lies in replicating the main findings of Lettau and Pelger (2020), which show that RP-PCA can outperform PCA models, especially in finding weak factors with high Sharpe Ratios. The replication section helps understand the differences between these two models and gives further clarity on which factors are needed to capture cross-sectional and time series movements of returns. These are essential steps to tackle the climate risk evaluation since it lays out the foundations of the models, and the estimations of market risk factors that will be taken as a benchmark later on. The first research question of this paper is hence:

What are the gains of RP-PCA models in capturing the time series and cross-section of stock returns?

Such analysis will be separated into 3 sub-questions. The first sub-question aims to answer; How does RP-PCA perform with respect to other models in a small cross-section of double sorted portfolio monthly excess returns? By answering this we will get better insights into the methodology of RP-PCA, in understanding its differences compared to the other models widely used in the literature. The second sub-question answers; How does RP-PCA perform with respect to PCA and Fama-French factor models in a large cross-section of single sorted portfolio excess returns?[3] The answer to this question gives more clear evidence of which model

[2] A climate sorted portfolio refers to a set of portfolios constructed by ranking firms according to their emission levels, following Gimeno Nogués and González Martınez (2022) GMP factor construction.

[3] The distinction between double and single sorted portfolios, is based on how many characteristics are assets inside the portfolios created ranked. In the double sorted portfolios, assets are allocated to different portfolios based on 2 characteristics. In the single sorted portfolios, this is done only based on one characteristic.

performs best and why and helps answer the next sub-question. Our final sub-question aims to reduce the large pool of factors in the literature by answering; What are the factors that can explain the cross-section and time series of characteristics sorted portfolio returns? This section will closely follow the analysis done by Lettau and Pelger (2020).

Moving on to tackling the gaps in the literature regarding climate risk, we apply the models discussed in Lettau and Pelger (2020) with the climate factor insights of Gimeno Nogués and González Martınez (2022). By doing so, we aim to answer our second research question:

Is climate risk an important component of systematic risk?

The 3 sub-questions for this research question go as follows. First, we answer; How do factors formed by RP-PCA and PCA perform against the green factor model of Gimeno Nogués and González Martınez (2022)? By answering this question we can better understand the added value of taking climate risk directly into account, and to what extent the GMP information is included in the RP-PCA and PCA factors. Moving on we re-do the analysis done in the replication section, but now with the addition of "green" or climate sorted portfolios. This is done to evaluate performance across models, and also to understand whether the addition of a green anomaly significantly alters the previous factor structure. Finally, as climate risk is a recent phenomenon, its importance is expected to have increased during the sample analysed and hence we evaluate; How has the importance of considering climate risk in investment decisions changed through the past years?

Following our analysis, the results from the replication section are in line with Lettau and Pelger (2020). RP-PCA model outperforms PCA both in and out of sample. The main driver of RP-PCA's dominance lies in its ability to detect weaker factors, and assign higher SDF weights to portfolios with higher Sharpe Ratios and mean returns. In terms of climate risk, results show that it is an important factor in financial markets, and its importance has increased over time. Comparing the model of Gimeno Nogués and González Martınez (2022), we observe that it outperforms models like RP-PCA and PCA in sample, but it lacks out of sample. On the other hand, when including a climate-related anomaly in the set of sorted portfolios, we observe the performance of RP-PCA and PCA is improved both in and out of sample. Overall, this improvement is bigger for RP-PCA, which leads to the conclusion that climate risk information is better incorporated in RP-PCA models. Finally, when analysing the difference in climate risk importance through time we observe that after the Paris Agreement, the inclusion of climate risk becomes more important.

The paper is structured as follows. Firstly, we introduce all the data employed to answer our research questions. Secondly, the methodology of the factor models and performance metrics are explained in addition to the steps of the analysis. Thirdly, the results of both research questions are presented and discussed. Finally, we end with some concluding remarks on our results, including limitations and lines for further research.

2 Data

To perform the necessary analysis to answer the two main research questions, a set of data needs to be obtained. In this section, the data used is explained, including the source, frequency, main characteristics and essential preliminary transformation.

2.1 Data used in the replication of Lettau and Pelger (2020)

For the first research question, it should be noted that the methodology is replicated from Lettau and Pelger (2020), hence the data obtained closely follows the one used in such paper.

In this research question, the overall aim is to analyse the advantages and disadvantages of different factor models. To do so, we first need to establish a benchmark comparison for our models, which we take as the Fama and French 3 and 5 factors. For this section, the data for these factors are extracted from November 1963 until December 2017 on a monthly frequency from the Kenneth French data library. Additionally, we will be working with excess returns, which are equivalent to the portfolio's returns minus the risk free rate. This risk free rate is also extracted from the Kenneth French data library on a monthly frequency, and it is expressed in percentage terms, hence we transform it by dividing each observation by 100.

To answer the first sub-question, which analyses the performance of the models in a small cross-section, monthly returns of 8 different sets of 25 double sorted portfolios are used. The returns are extracted from November 1963 until December 2017, following the frequency used in Lettau and Pelger (2020). For each of the 8 sets, the portfolios are created by sorting according to size and one of the following characteristics; book-to-market; accruals; investment; profitability; momentum; short-term reversal; volatility; and idiosyncratic volatility. Each portfolio is composed of assets of the NYSE, AMEX and NASDAQ indexes, and the assets included are re-calibrated every year in June. This data is obtained from the Kenneth French data library. The returns extracted from this library are expressed in percentage, and hence each observation is divided by 100. For each of the 8 sets of double sorted portfolios, we subtract the risk free rate from each portfolio to obtain excess returns.

For the analysis of our second sub-question, which employs a larger cross-section, monthly returns of 37 sets of single sorted portfolios are considered from Haddad et al. (2020). The anomaly selection to sort the 37 different sets of portfolios follows from Lettau and Pelger (2020).[4] Each of the 37 sets consists of 10 portfolios corresponding to the deciles ranking according to the anomaly. The monthly returns of each decile are extracted from November 1963 until December 2017. The creation of the different portfolios follows the model created by Giglio et al. (2021), which generates term structures off any set of stocks. The composition and size of each decile portfolio are re-calibrated each year. As before, the returns from each portfolio are transformed into excess returns by subtracting the risk free rate at each observation.

[4] As an extension the 37 anomalies are expanded to 49, following to the selection of Lettau and Pelger (2020).

The summary statistics of the decile 1 and 10 monthly excess returns of the 37 single sorted portfolios are shown in table A1 of the appendix. Here we can observe that on average, decile 10 excess returns have larger means and similar standard deviations. Additionally, their Sharpe Ratio (SR) are also on average larger than those of the corresponding decile 1, showing they are more attractive investment options. In terms of decile 1 portfolios, "size", "valuem" and "gmargins" score the highest Sharpe Ratios, whereas "Ivol", "Mom12" and "Indrrevlv" score the lowest. In decile 10 portfolios, "Indmomrev", "Indrrevlv" and "Valmomprof" score the highest Sharpe Ratios, whereas "Strev", "Sgrowth", and "Accruals" score the lowest.[5]

2.2 Data used in climate risk analysis

To answer our second research question, additional data needs to be collected. For the first sub-question of this section, we require the data from the factor built by Gimeno Nogués and González Martınez (2022) (GMP). We select to work with the US-specific GMP factor, on a daily frequency and from the 1st of January 2002 until the 31st of December 2019.[6] The US green GMP factor of Gimeno Nogués and González Martınez (2022) is constructed by ranking carbon emission of assets from the S&P500 index and going long for low polluters and short for high polluters.[7] They also construct two other GMP factors by balancing by industry and size. Further details on this can be found in section 3. The GMP factor is added to the 5 Fama and French factors (Fama & French, 2015), to form the model employed in Gimeno Nogués and González Martınez (2022). Note that the 3 and 5 Fama and French factors and the risk free rate, are now extracted on a daily frequency from the 1st of January 2002 until December 2019. The single sorted portfolio data in excess returns is re-extracted following the new time frame and daily frequency, also available from Haddad et al. (2020). [8]

Unfortunately, our previous data sources do not count with single sorted portfolios based on climate metrics. However, in the research by Gimeno Nogués and González Martınez (2022), they derive a ranking of firms according to emissions levels, which fit well into this framework. To answer our second sub-question we hence consider the ranking of firms from this paper to create a 1 and 10 decile portfolios based on a new carbon emission anomaly which we name the "green" anomaly.[9] The monthly excess returns of these portfolios will be used from January 2002 until December 2019, and the risk free rate will be subtracted to obtain the excess return.

[5]Sharpe Ratios are a reward-to-variability ratio, which help investors analyse the returns of a certain asset with respect to their risk (Sharpe, 1998). The higher this value is the better, since for a certain levels of risk you can receive higher returns.

[6]The end date is restricted to 2019 in order to match the data availability of the single sorted portfolios.

[7]408 companies are ranked, and the 50 top and bottom are taken to construct the factor.

[8]As a robustness check for this specific sub-question, the analysis is repeated in monthly frequency from January 2002 until December 2019. This data is obtained equivalently as the previously stated. The GMP factor is aggregated from daily to monthly.

[9]It is important to note that, as opposed to the single sorted portfolios considered for the first research question, these new anomalies portfolios won't be re-calibrated per year. Following Gimeno Nogués and González Martınez (2022) findings emissions don't vary significantly per period, and taking the emissions of 2021 to rank for the first time suffices to create an efficient factor and ranking.

Table A2 of the Appendix shows the summary statistics of decile 1 and 10 excess returns of 38 anomalies single sorted portfolios. From this table, we can observe that the decile 1 excess returns have, on average, a smaller mean, higher standard deviation, and lower Sharpe Ratios. This also holds for the green anomaly, where we see that the decile 10 portfolio, which is constructed using the top 50 firms' returns from the ranking of Gimeno Nogués and González Martınez (2022) with the lowest emissions scores, has a 21% higher mean and a 9% higher SR than decile 1, signalling to the potential benefit of considering climate risk in portfolio creation. Additionally, we also observe that for this green anomaly, the standard deviation of the excess returns in decile 10 is 10% higher than in decile 1, further motivating the use of RP-PCA models since only considering the second moment might leave out essential information. It is also worthy of note to compare these statistics with one of the monthly excess returns displayed in Table A1 of the appendix. It stands out that the means and standard deviation of the monthly data are of a larger magnitude than such of the daily data. This might lead to differences in performance across the models when using the different data types.

3 Methodology

In this section, the different methods that will be employed are introduced, and the steps of analysis are explained.

Factor models are extensively used in the financial world. More specifically, what we are most interested in is, applying a set of factor models to excess returns of market portfolios to capture the risk inherent in the whole market, otherwise known as systematic risk. The main idea behind these factor models lies in describing a cross-section of excess returns with K common factors, where K is a relatively small number simplifying the computation and analysis. The factors should capture the common variations across all cross-sections of returns, which in other words can be identified as the systematic risk affecting all portfolios. This together with the factor loadings, which determine the exposure of these excess returns to the systematic risk factors, helps capture the dynamics of excess returns with the following equation, using Lettau and Pelger (2020) notation. Note that N is the total number of portfolios considered, and T is the time period length studied. [10] [11]

$$X_{n,t} = F_t \lambda_n^\top + e_{n,t}, \quad for\ n = 1, 2, ..., N,\ t = 1, 2, ..., T \tag{1}$$

$$\iff \underbrace{\mathrm{X}}_{T\times N} = \underbrace{\mathrm{F}}_{T\times K}\underbrace{\Lambda^\top}_{K\times N} + \underbrace{\mathrm{e}}_{T\times N}. \tag{2}$$

In equation (1), $X_{n,t}$ represents the excess return of a portfolio "n" at time "t", F_t shows the

[10]The first equation shows the equation for each portfolio, and the second equation represent the matrix form.
[11]Note that in this model we represent time varying factors and static factor loadings.

systematic factors at time “t”, λ_n corresponds to the factor loadings for portfolio “n”, and $e_{n,t}$ displays the idiosyncratic component. Equation 2 shows the matrix where the excess returns of each portfolio n for all time periods are added as rows into the matrix X. In equation (2), F contains K columns, each representing a specific factor which varies over time t, and $\Lambda^\top$ contains the factor loadings which represent the weight of impact of a factor k to portfolio n. As can be seen from the equations above, in these models we consider time-varying factors and fixed factor loadings, like in Lettau and Pelger (2020).

Risk of excess returns formula, assuming that factors are uncorrelated from the residuals in equation 2, can be determined by equation 3 below.

$$Var(X) = \Lambda Var(F) \Lambda^\top + Var(e). \tag{3}$$

Here $Var(F)$ equals σ_F, the variance-covariance matrix of factors, $Var(e)$ equals σ_e, the variance-covariance of the error term e in equation 2, and $Var(X)$ equals σ_X the variance-covariance of our portfolio’s excess return data. It is important to observe how the total risk, Var(X), can be defined in terms of systematic risk, $\Lambda Var(F) \Lambda^\top$, and idiosyncratic risk, Var(e).

It is important to distinguish between the three types of factor models; macro-economic factor models; characteristic-based factor models; and statistical factor models (Connor & Korajczyk, 2010). In the macro-economic model, the factors are pre-determined by using observable financial or macro-economic variables, and the factor loadings are estimated via a standard regression. An example of a macro-economic model is the 3 or 5 Fama and French model, which we will use as a benchmark through the analysis. In the characteristic-based models, factor loadings are taken as observed characteristics of the portfolios or assets, such as book-to-market, and factors are estimated in a cross-sectional regression. Finally, in statistical models, both the factors and factor loadings are taken as unobserved variables to be estimated.

3.1 Principal Component Analysis

In an attempt to restrict the number of factors to the most essential ones, the statistical factor models can prove effective. In these models, we aim to find the factors that best describe the common dynamics and variations of returns. An efficient way to find these factors is to apply Principal Component Analysis (PCA) to the variance-covariance matrix. This technique provides essential advantages with respect to macro-economic factor models like the ones proposed by Fama and French (2015). As explained by Zaffaroni (2019) relying uniquely on observed factors can lead to omitted variable bias or bias due to mismeasured factors. Additionally, estimating the risk factors also avoids problems arising when too many factors with low estimation power are added (Gagliardini et al., 2019). PCA is described by (Abdi & Williams, 2010) as “a multivariate technique that analyzes a data table in which observations are described by several inter-correlated quantitative dependent variables”. When applying PCA to our N portfolio excess returns data, we obtain a set of orthogonal variables, called the principal components, which can best describe the co-movements and similarities of the portfolio's excess returns.[12] For our analysis, PCA is applied to the covariance-variance matrix of portfolio excess returns, to describe the systematic risk component of equation 3. This covariance-variance matrix is described in the equation below:[13]

$$Var(X) = \frac{1}{T} X^{\top} X - \bar{X}\bar{X}^{\top}. \tag{4}$$

Applying PCA to the equation above creates N orthogonal factors from the eigendecomposition of Var(X) (Lettau & Pelger, 2020). Each of the N factors has a variance, which is called the eigenvalue. These N factors are ranked according to their eigenvalues, from biggest to smallest. The eigenvectors corresponding to the top K eigenvalues are selected to estimate the true factors of equation 2. The corresponding KxN matrix of eigenvectors is defined as $\hat{\Lambda_{PCA}}$, representing the factor loadings. The factors F_{PCA} are estimated using the equation below:

$$\hat{F_{PCA}} = X * \hat{\Lambda_{PCA}}(\hat{\Lambda_{PCA}^{\top}} * \hat{\Lambda_{PCA}})^{-1} \tag{5}$$

To remain consistent with the replication of Lettau and Pelger (2020) we perform standardization of the eigenvalue loadings such that they sum up to 1. The objective function of this model can be found in equation A1 of the Appendix.

It is important to note that in performing PCA, only the second moment of returns is considered, the first moment information is disregarded. In much of the existing literature, the matrix of excess returns "X" is first demeaned before PCA is applied since it is assumed that the mean of X is insignificant and close to zero. This can be seen, as an example, in the research done by Stock and Watson (2002) where they apply PCA for forecasting and assume E(X)=0.

3.2 Risk Premium - PCA

The restriction in PCA of setting the means of excess returns to zero, or not considering them, takes away a lot of valuable information. This is indeed shown by the Ross' Arbitrage Pricing Theory (APT) (Roll & Ross, 1980), which states that "expected excess returns are explained by the exposure to the risk factors times the risk premium of the factors" (Lettau & Pelger, 2020), or in other words:

$$E(X_n) = \Lambda_n E(F). \tag{6}$$

Following the first sub-question of this paper, which focuses on replicating the findings of Lettau and Pelger (2020), we use the Risk Premium - PCA (RP-PCA) to incorporate the mean of excess returns into the model. As explained in their paper, this model is equivalent to applying

[12]Refer to Abdi and Williams (2010) for an extensive methodology on how principal component works in terms of the formation of the orthogonal variables.

[13]Following the analysis we aim to replicate of Lettau and Pelger (2020) the number of cross-section N and time series T are both large, such that N/T converges to finite limit.

PCA to the matrix in the equation below instead of Var(X) in equation 4. The objective function of this model can be found in equation A2 of the Appendix.

$$\Sigma_{RP} = \frac{1}{T} X^\top X + \gamma \bar{X} \bar{X}^\top. \tag{7}$$

As explained by Lettau and Pelger (2020), γ represents the weight given to "the average cross-sectional pricing error relative to the times-series error in standard PCA" (Lettau & Pelger, 2020). Note that when γ is set to -1, the RP-PCA model is equivalent to the PCA model.

The procedure of factor and factor loadings estimation is the same as the one described in the PCA section. The main difference in this model is that now the first and second moments are both considered, as a result of allowing the mean of X to be unrestricted. This model can be viewed as a generalisation of PCA, which "includes a penalty on the pricing error in expected returns"(Lettau & Pelger, 2020).

The properties of RP-PCA, for $\gamma \geq -1$, make it a more attractive statistical factor model than PCA. This idea mainly lies behind the ability of RP-PCA to detect weaker factors which provide valuable information.[14] In general, whether a certain factor is detected or not depends on whether the factor strength at explaining a certain part of the systematic variance, with respect to the noise variance is above a specific threshold. According to Lettau and Pelger (2020) an adequate selection of $\gamma \geq -1$, is capable of increasing the strength of factors and making them more detectable and more precise estimation.

The main challenge of RP-PCA lies in determining the optimal value for γ. The approach is two-fold, especially in the presence of weak factors. On the one hand, a very large γ helps weaker factors be detected more easily. On the other hand, if γ is chosen too large it can negatively impact the correlation of the estimated factors with true factors and deteriorate out of sample performance Lettau and Pelger, 2020. For this reason, we perform a few sensitivity analysis where we first start with γ=-1 and increase by one unit until no further significant improvement is observed in the performance metrics of the models.

3.3 Performance metrics

To answer both our research questions it is important to be able to interpret and evaluate the factor models used. To do so we will use a set of performance metrics which will allow us to judge the performance of the different factor models both in sample and out of sample. Note these performance metrics follow from Lettau and Pelger (2020).

To start, an important metric we compute is the Maximal Sharpe Ratio (SR) spanned by

[14] The main idea is that when applying RP-PCA or PCA, the first factor estimated is usually very correlated with the market. If we want to obtain additional information to the first factor which tends to affect all portfolios simultaniously, we must find weak factors which only affect a subset of these portfolios but can provide further insights. With RP-PCA these weak factors are more easily detectable since the mean information is also taken into consideration.

the estimated factors. The formula for this is shown below, where $\widehat{\mathrm{b}}_{\mathrm{MV}}$ represents the SR:[15]

$$\widehat{\mathrm{b}}_{\mathrm{MV}} = \Sigma_F^{-1}\mu_F. \tag{8}$$

Using the SR and the estimated factors we can then calculate the implied Stochastic Discount Factor (SDF). The SDF methodology was introduced by Cochrane (1996). The SDF gives a weight to each portfolio, which represents the position an investor should take in each portfolio to correct for their risk, such that all portfolios give the risk free rate (Zaffaroni, 2019). The formula for the SDF weights is shown below:

$$M_t = 1 - \widehat{\mathrm{b}}_{\mathrm{MV}}^\top \left(\widehat{\mathrm{F}}_t - \mathrm{E}\left[\widehat{\mathrm{F}}_t\right]\right) \tag{9}$$

As explained by Lettau and Pelger (2020), the highest the SR, the closer the model is to the true SDF. Hence when comparing across models, those yielding higher SR are preferred.

Using the estimated factors we can also create an OLS regression to describe the matrix of portfolio returns, by the following equation:

$$X_{nt} = \alpha_n + \widehat{F}_t B_n^\top + e_{nt}, \tag{10}$$

where $\widehat{\alpha}$, $\widehat{B}$, $\widehat{e}$ are estimated for each asset "n". [16] In equation 10, α_n represents the pricing errors of using the estimated factors. From this estimation we can then calculate the Root Mean Squared pricing error (RMS) and the Idiosyncratic Variance ($\bar{\sigma_e^2}$):

$$RMS_\alpha = \sqrt{\frac{\widehat{\alpha}^\top\widehat{\alpha}}{N}}. \tag{11}$$

$$\bar{\sigma_e^2} = \frac{1}{N}\sum_{n=1}^{N}\frac{Var(\widehat{e_n})}{Var(X_n)}. \tag{12}$$

The in sample performance of models is hence evaluated using the SR, RMS, and $\bar{\sigma_e^2}$. Additionally, the SDF results are used to gain further insights into the model. The calculation of the out of sample performance metrics requires first performing a rolling window, where F_{t+1} is estimated using information up to time t. For more detail on the procedure refer to the Appendix. For our first research question, following Lettau and Pelger (2020), we take 20 years, i.e 240 months, rolling window estimation. When answering the second research question which uses daily data, a 1-year rolling window is used.

[15]Here Σ_F is the variance-covariance matrix of the estimated factors $\widehat{F}$ and μ_F is the mean of the estimated factors $\widehat{F}$.

[16]In the RP-PCA model, when γ equals -1 or 0, the estimated $\widehat{}$ coefficient is equivalent to the factor loadings of the RP-PCA model.

3.4 Steps in the replication of Lettau and Pelger (2020)

Now that the foundations of the models and performance metrics have been laid out, it is also important to understand the different steps in the analysis of such models. Firstly, the steps taken to answer the first research question are outlined below.

To start, it is important to note that all model evaluations will be done with respect to the 3 and 5 Fama and French factor models as a benchmark. That is, we will fit these two models to the data and calculate the respective in and out of sample performance metrics.

In terms of the first sub-question, we first fit 3 factors PCA and RP-PCA models to a smaller cross-section of returns, namely the set of double sorted portfolios. We calculate the SR, RMS and $\bar{\sigma_e^2}$. To study the difference in performance across the models we performed a more detailed analysis for the double sorted portfolios based on Size/Accruals and Size/Short terms reversals. For both, we first calculate how the performance metrics of the RP-PCA model change as γ increases, to understand what would be an optimal value of such. This is done for a different amount of factors to see if there are major discrepancies in the sensitivity of γ depending on the number of factors chosen. Moving on, we then perform a detailed analysis of the difference in factors estimated by PCA and RP-PCA for these two sets of data.

Once the specific advantages and disadvantages of each model are better understood, we apply the models to a large cross-section of returns, able to represent systematic risk in a market. To do so we use the data of single sorted portfolios presented in section 2. First, the most optimal γ is selected, by analysing the eigenvalues of different γ values. Then the in sample and out of sample performance of 3 and 5 factor Fama and French, PCA and RP-PCA are compared. Moving on, we look more into the interpretation of each factor, to understand which factors explain the cross-section of returns and which the time series. To do so we create sub-models with specific factors and calculate the performance metrics on each combination. Finally, the SDF weights are calculated for further insights into the reasoning between the differences in performance. The SDF projection also allows to understand the structure of systematic risk in the market, and helps give insights into optimal trading strategies.

3.5 Steps to asses the importance of climate risk in markets.

Once the performance of PCA and RP-PCA has been evaluated and a set of optimal factors to capture systematic risk have been derived we move on to answering the second research question. This research question is focused on understanding to what extent climate risk is driving these factors and present in markets. To start this analysis we will assess the performance comparison of 5 factors Fama and French, PCA and RP-PCA, against 6 factors Fama and French with the addition of GMP from Gimeno Nogués and González Martınez (2022), PCA and RP-PCA.[17]Additionally, we will analyse the correlation of factors created with PCA and RP-PCA with the GMP factor to understand whether climate risk is already captured by the factors in

[17]The same performance metrics as in part 1 will be used to stay consistent.

those models. To do so we will perform a regression using the GMP factor as the dependent variable and the 6 factors from PCA, and RP-PCA separately in two regressions as explanatory variables.

Moving on, we add a green single sorted portfolio to the previous data set and perform RP-PCA and PCA. This green single sorted portfolio is constructed using the ranking of firms by Gimeno Nogués and González Martınez (2022). We take the top 50 firms with the highest emission and take an average of their stock returns to create the decile 1 portfolio, and vice versa with the top 50 less emitting firms for decile 10. To be consistent with the rest of the single sorted portfolios, these returns are then transformed into excess returns by subtracting the risk free rate. With this data, we re-analyse the optimal value for γ and the optimal number of factors to be used. To see whether adding this anomaly to the model significantly improved model performance, we will compare the model results in and out of sample to the case where the "green" anomaly is excluded. An analysis of the factor correlations across the two estimations, with and without the "green" anomaly can also help us see where this information has been included. This will help answer the question of whether climate change considerations significantly impact the model results, and how is this anomaly information incorporated.

Finally, we will evaluate how this performance metric changed over time, to see if the importance of including climate change risk has been increasing in the past years. Specially we study whether the signing of the Paris Agreement in 2015 has lead to a rise in the importance of including the green anomaly.

The GMP factor and the firms ranking according to emissions by Gimeno Nogués and González Martınez (2022) are essential data for the analysis in this section. To further understand the methodology behind such derivations please refer to the Appendix for a review of the paper and its methodology.

4 Results and Discussion

In this section, the results from the analysis previously explained are shown and discussed. Referring back to the two main research questions, this section is divided respectively to tackle each one. The first section is dedicated to presenting the results from the replication of Lettau and Pelger (2020), with the main aim of understanding the difference across models, and obtaining a set of factors that can capture systematic risk. In the second section, the results regarding the analysis of whether climate risk is prevalent in the market are presented.

4.1 Results of Lettau and Pelger (2020) replication

As previously explained, theoretically, combining the ideas of PCA and APT to create the RP-PCA model can yield more efficient and accurate estimates. This should hold, especially in the presence of weak factor models. If this holds we would expect to see higher Sharpe Ratios, lower

RMS, and lower $\bar{\sigma_e^2}$ in the applications of RP-PCA with respect to PCA. Following Lettau and Pelger (2020) we check whether this holds, firstly by applying to a small cross-section of double sorted portfolios excess returns, and secondly to a larger cross-section of single sorted portfolio excess returns. The second application also allows to gain insights on the factors needed to describe the complete set of risk factors able to capture the true SDF, and hence the systematic risk of a market.

4.1.1 Double sorted portfolios

For the double sorted portfolios, we estimate a 3 factor model for PCA and RP-PCA. For RP-PCA, we set γ equal to 20, to compare against the results of Lettau and Pelger (2020). The out of sample performance metrics is calculated for both models, in addition to the 3 Fama and French (FF) factor models which are used as a benchmark. The results are shown in the table below:

Table 1: Out of sample performance metrics for data set of double sorted portfolios

	SR			RMS_α			$\bar{\sigma_e^2}$		
	RP-PCA	PCA	FF	RP-PCA	PCA	FF	RP-PCA	PCA	FF
SIZE&BTM	**0.20**	0.18	0.17	**0.17**	0.17	0.18	7.97%	**7.91%**	7.97%
SIZE&ACC	**0.21**	0.12	0.15	**0.09**	0.11	0.10	6.74%	**6.44%**	7.17%
SIZE&INV	**0.26**	0.23	0.23	**0.13**	0.15	0.13	**6.95%**	7.00%	7.06%
SIZE&OP	0.13	0.14	**0.15**	**0.09**	0.10	0.11	**6.94%**	7.08%	8.54%
SIZE&STR	0.16	0.11	**0.20**	0.18	0.19	**0.11**	7.89%	**7.86%**	10.88%
SIZE&MOM	**0.21**	0.18	0.01	**0.20**	0.21	0.30	**8.30%**	8.40%	13.76%
SIZE&IVOL	**0.29**	0.23	0.23	**0.16**	0.17	0.22	**6.22%**	6.24%	7.11%
SIZE&VOL	**0.27**	0.21	0.21	**0.18**	0.19	0.23	**6.27%**	6.30%	7.04%

The Maximal Sharpe Ratio(SR); Root mean squared pricing error (RMS_α); and Idiosyncratic variance ($\bar{\sigma_e^2}$) are calculated for the 8 sets of double sorted portfolios, presented in the first column. For each of these datasets 3 factor models are performed; RP-PCA; PCA; and Fama and French (FF). For each of these models the amount of factor is set to 3, and for RP-PCA γ is equal to 20. The bold numbers show the best-performing model.

From table 1 different insights can be drawn. Firstly, in terms of the Sharpe Ratio(SR), the RP-PCA model is the best performing in 6 out of the 8 cases studied. Especially for the Size/Idiosyncratic volatility, Size/Volatility, and Size/Accruals, the SR of RP-PCA is much larger than the rest. In the remaining two cases the Fama and French model attain the highest Sharpe Ratio. It is interesting to note that PCA, in terms of the SR, is outperformed or equivalent to the Fama and French model in 6 out of 8 cases, showing the poor performance of this model even though factors are latent. Looking at the RMS we observe that RP-PCA is preferred in 7 out of the 8 cases, with only the Size/Short term reversal yielding better RMS in the Fama and French model. We also observe that in terms of this metric, PCA is preferred to Fama and French in 5 of the 8 cases, so the model is not as bad performing and usually not far off from RP-PCA results. The out of sample $\bar{\sigma_e^2}$ is lower in 5 out of 8 cases for RP-PCA, and 3 out of 8 cases for PCA. Additionally, RP-PCA and PCA outperform Fama and French in all cases for this metric. Consistently, for the 3 metrics, Size/Short term reversal sorted portfolio doesn't rank RP-PCA as the best-performing model, which is an interesting insight to look further into. Overall, we can clearly state that RP-PCA is the best performing model. In the broader picture, it also seems that PCA is preferred to Fama and French, but the SR performance of PCA is lacking. These results are mainly in accordance with Lettau and Pelger (2020), with the main difference that in their results PCA outperforms Fama and French.

Essentially, as explained by Lettau and Pelger (2020) the difference in performance between RP-PCA and PCA can be explained by the differences in "the detection of factors, the factor compositions, or the order of factors". To understand this further, we focus on two specific cases, the Size/Accruals and Size/Short term reversal portfolios. In figure A1 of the appendix, the out of sample SR, RMS, and $\bar{\sigma_e^2}$ have been plotted as a function of γ, starting with γ equal to 0 up to 30. This has been done for the RP-PCA model with 1, 2, 3, 4, 5, and 6 factors. For the Size/Accruals portfolios it can be observed that γ has little effect on all the metrics, except for SR and RMS in the 3 factor model, where the performance with γ equal to 0 is significantly worse than for γ from 5 onward. In the Size/Short term reversal portfolios the influence of γ is high in terms of the SR. For the 2 and 3 factor models, the SR is an increasing function of γ. From both portfolios, it can be observed that sensitivity to the choice of γ tends to arise mostly when a third factor is added, which shows that for 1 and 2 factor models RP-PCA is essentially very similar to PCA, but when a third factor is added discrepancies start to arise. Additionally, it is interesting to see that when a third factor is added, the performance metrics for the Size/Accruals portfolio are improved more intensively than in other cases. This shows that a potential 3 factor model might be optimal for this data set and that the performance gains of RP-PCA are inherent to this factor.

The third factor is hence causing the major differences between the models' performance. To analyse this difference, Figure 1 below shows the heat map of the factor structure for both portfolios for a 3 factor RP-PCA (with γ equal to 20) and PCA estimation. Figure 1 shows how for both double sorted portfolios, the first factor constitutes approximately a weighted average of all the 25 set portfolios within. For the Size/Accruals data, the second factor is also the same for RP-PCA and PCA, which can be interpreted as a small minus big portfolio structure. The difference between RP-PCA and PCA appears for the third factor, where for RP-PCA it is approximately a low minus high accruals, and for PCA no clear interpretation can be found. For the Size/Short term reversal, the second and third factors are both different. In RP-PCA, the second factor is a low reversal minus high reversal and the third factor is a small minus big. In the PCA model, the second factor is a big minus small and the third is a low minus high reversal. These results confirm the idea that, in terms of strong factors, both models perform quite similarly, but when it comes to identifying weaker factors they differ, and RP-PCA tends to be more efficient. These findings are in line with such presented in Lettau and Pelger (2020).

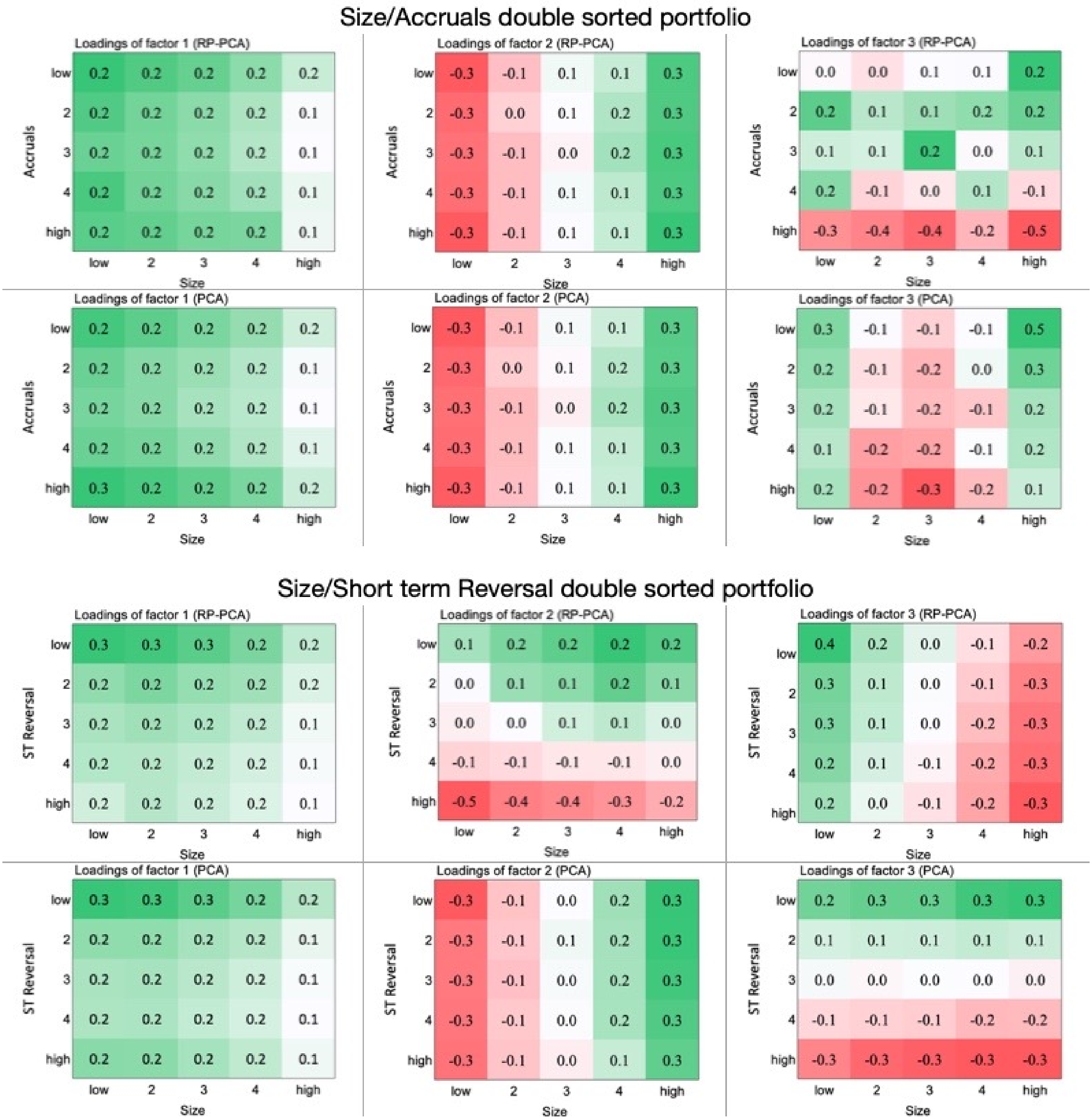

Figure 1. Heat maps for the factor loadings of size/accruals and size/short-term reversal

3 factor models are estimated on the double sorted portfolios of size/accruals, on the top panel, and size/short-term reversal, on the bottom panel, using RP-PCA, on top row, and PCA, on bottom row. In RP-PCA γ equals 20. Each row portrays a heat map of the factor loadings estimated within each model. The greener the loading on each of the portfolios, the more positive it is compared to the average in the graph. White is a neutral or average colour, and red shows negative/lower numbers.

4.1.2 Single sorted portfolios

As a second step to answering the first research question, we now analyse a large cross-section of returns, intending to select an optimal factor structure to capture systematic risk. To do this the single sorted portfolio data is employed, and the results are displayed below.

Firstly, we start by selecting an optimal γ value for the RP-PCA model, by calculating the sensitivity of percentage difference across the ordered eigenvalues. Such a figure can be found in Figure A2 of the appendix. As previously explained, the optimal choice of γ must be large enough such that weak factors are detectable, but not too large to maintain efficiency. In both of the eigenvalue graphs, the choice of γ equal to 5 or 10 looks most appropriate. For these two values, the first 10 factors become more detectable compared to the case where γ equals 0. We select γ equal to 10, to match Lettau and Pelger (2020). Additionally, we can observe that regardless of the choice of γ, after the 5th factor, the eigenvalue percentage differences stabilise at low levels. This hints toward a 5 factor model being the most appropriate since from the 6th factor onward, the variance that these factors can capture are significantly lower. This is again confirmed in Figure 2 below, where the in and out of sample performance metrics of RP-PCA, and PCA are plotted for a different number of factors. From Figure 2 we can observe that once the 5th-factor model is added, the SR, RMS and $\bar{\sigma_e^2}$ don't experience significant improvements, both in and out of sample, relative to the additions of factors before and including the 5th.

Figure 2 shows other important insights about the performance of the models. Firstly, it is clear that in terms of SR, and RMS, RP-PCA models are preferred in sample and out of sample. For the Idiosyncratic variance, the performance of PCA is slightly superior to RP-PCA, but results are very close together. This is in line with the fact that PCA maximises uniquely the amount of variance captured, and disregards other moments. For both models, the SR in and out of sample are relatively similar, which shows that over-fitting is highly unlikely, as expected from using statistical factor models. Additionally, the RMS and $\bar{\sigma_e^2}$ show a decreasing trend both in sample and out of sample, which supports this claim, and shows that the " factor structure is stable" (Lettau & Pelger, 2020). Table A3 of the Appendix also compares the performance of the models, including Fama and French, in and out of sample for 3 and 5 factor models. This table again confirms that RP-PCA is superior by large amounts in terms of SR and RMS, in and out of sample. When it comes to the idiosyncratic variance, it is the PCA model which performs best but is not far off from RP-PCA. This is indeed not surprising, since PCA minimises the unexplained variance directly. However, what is surprising is that RP-PCA, which considers the first and second moment, doesn't perform too far off from PCA in terms of this metric. Fama and French performance comes close to PCA in terms of SR, and RMS but is far off in terms of Idiosyncratic variance. Overall, we can conclude that RP-PCA dominates other models since even-though it performs slightly worse at minimising unexplained variance compared to PCA, it can largely outperform in terms of SR and RMS. Additionally, we find that there a 5 factors model is needed to describe the true SDF and systematic risk in the market.

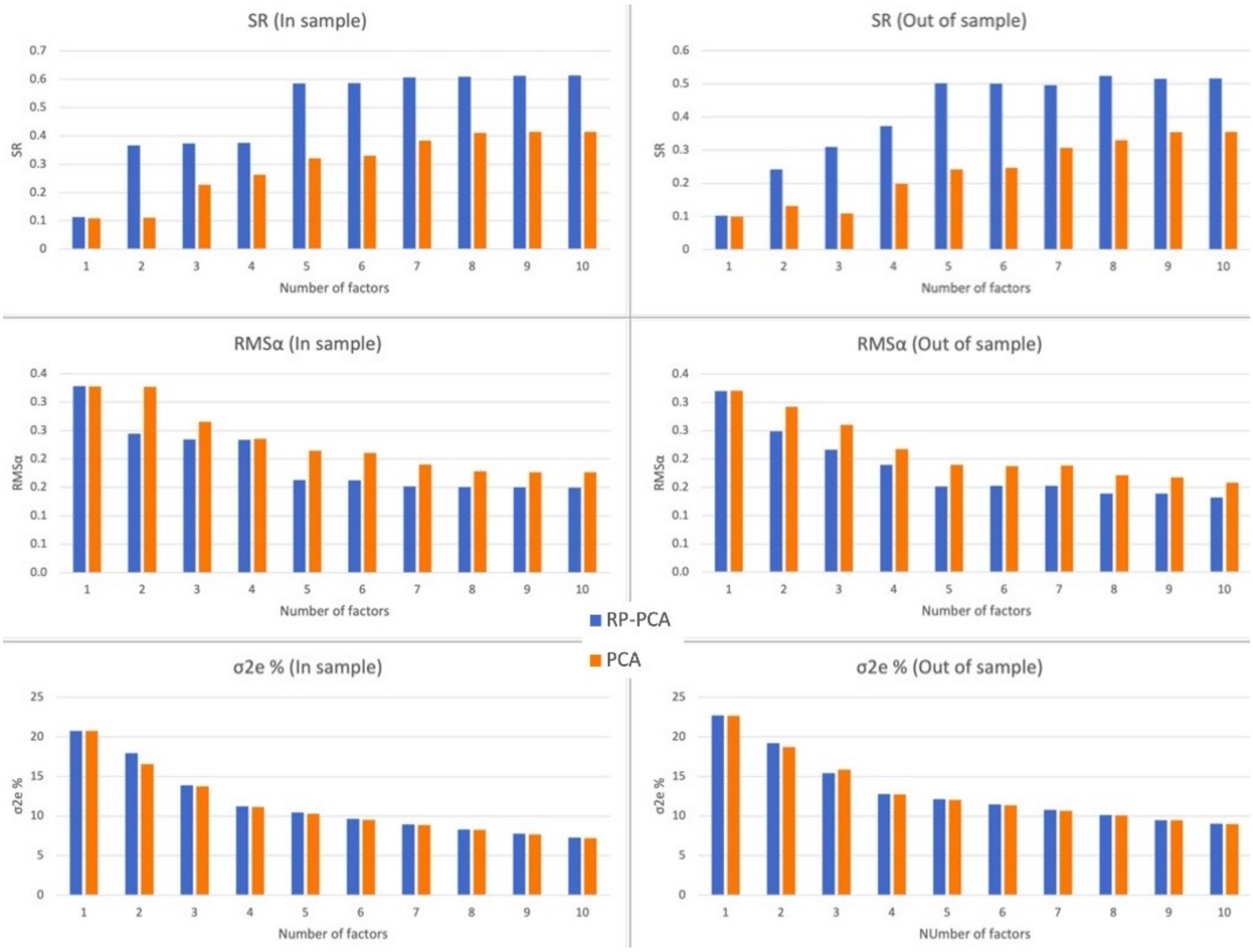

Figure 2. In and out of sample performance metrics for models with a different number of factors.
Using the N=74 single sorted portfolio data set, which only considers deciles 1 and 10 of each anomaly, RP-PCA, with γ equal to 10, and PCA models are estimated. This is repeated, and the number of factors estimated is altered each time, as shown on the x-axis. The performance metrics of each are calculated in sample and out of sample.

To look further into the difference across the models, Figure A3 of the Appendix plots the in and out of sample RMS, per anomaly for a 5 factor model. In these graphs, the anomalies are ranked by SR, from largest to smallest. Again, in general, we can observe how for most anomalies the RMS are lower for the RP-PCA model. More interestingly, it seems that RP-PCA performs much better with respect to PCA in terms of the anomalies with the highest SR, which tend to be the worst-performing for PCA. It is also interesting to see, that RP-PCA yields the highest RMS for anomalies with the lowest SR. These findings hold both in sample and out of sample. Overall, we can conclude that RP-PCA has clear advantages over PCA, especially in capturing the dynamics of the high SR portfolios, which are the most mispriced by PCA.

Next, we focus on understanding the differences in the factor estimation across the two models. In Table A4 of the Appendix, 10 factor models for PCA and RP-PCA are estimated, and the characteristics of the factors are displayed. Firstly, it is clear from this table that PCA ranks factors uniquely based on the variance. As a result, the second factor has the 9th smallest mean. Additionally, the factors with the largest SR aren't ranked first. With RP-PCA the situation is different, mainly due to the model dual optimisation of mean and variance. As can be seen in the RP-PCA factors, the variance and the mean are not monotonically decreasing, but still, the top 5 factors include the ones with the largest variance, SR, and means, except the 4th factor which has the 6th largest SR and mean. This could help explain why, RP-PCA performs so closely to PCA in terms of idiosyncratic variance, because even though this variable is not minimised directly, the factors with the largest explained variance are still selected. To investigate each of the top RP-PCA 5 factors more in detail, we follow the procedure by Lettau and Pelger (2020) where sub-models with specific factors are fitted to the data. The results can be found below:

Table 2: Subset of factors fit for RP-PCA

	In sample			Out of sample			
Factors	SR	RMS_α	$\bar{\sigma_e^2}$	SR	RMS_α	$\bar{\sigma_e^2}$	
[1,2,3,4,5]	0.59	0.16	10.43%	0.44	0.15	11.84	%
[1]	0.11	0.33	20.75%	0.10	0.32	22.64	%
[1,2,5]	0.57	0.23	17.07%	0.37	0.18	18.11	%
[2,5]	0.41	1.69	74.72%	−0.06	0.34	21.97	%
[1,3,4]	0.12	0.33	13.93%	0.04	0.31	15.89	%
[3,4]	0.03	0.66	93.06%	−0.05	0.52	55.88	%

The N=74 single sorted portfolio data is used. γ equals 10. A 5 factor RP-PCA model is performed. From this estimation, we take subsets of factors, depicted in the first column "Factors", and calculate both the IS and OOS performance metrics of the models using exclusively these subset of factors.

Table 2 reflects the usefulness of including the different 5 factors. Firstly, it is clear from the estimation with only 1 factor that this factor contributes both to the cross-sectional, and time series fit. This is mainly because fitting the model with only such factor yields adequate SR, RMS and $\bar{\sigma_e^2}$ results. Adding factors 2 and 5 to factor 1 helps largely to increase the SR, by around 420%, and lower the RMS, by 30%, but it only decreases $\bar{\sigma_e^2}$ by around 18%. In accordance with Lettau and Pelger (2020), this shows that factors 2 and 5 contribute to explaining the cross-section of returns but not the time series. On the other hand, adding factors 3 and 4 to factor 1 decreases the $\bar{\sigma_e^2}$ significantly by 33%, but has a negligible effect on the SR and RMS. Hence, it can be said that factors 3 and 4 uniquely contribute to explaining the time series.

Finally, we analyse the implied SDF spanned by the factors.[18] Figure A4 in the appendix,

[18]Recall that the SDF is calculated based on the Maximal Sharpe Ratio the factors can achieve. The SDF is also "perfectly negatively correlated with the tangency portfolio created by the factors" (Lettau & Pelger, 2020)

shows the SDF weights for each anomaly for RP-PCA and PCA, ranked according to SR. These weights are then plotted against their average monthly returns in Figure A5 of the appendix. From these figures, we can conclude that for both models, the decile 10 portfolios tend to take a positive position, while the decile 1 portfolios are usually negative, hinting toward a long-short structure. In Figure A4 we see how for RP-PCA the weights for the anomalies with higher SR are on average much higher than for low SR anomalies. On the other hand, this doesn't hold for PCA, where there is no clear relation between the weights of the SDF and the SR of the portfolios. This structure is also visible if Figure A5, where the weights of the SDF in RP-PCA are shown to be much more correlated to the average returns of the anomalies than in PCA. Overall, this shows that the advantage gains of RP-PCA in terms of SDF and RMS are occurring as a result of the model dynamically assigning the highest SDF weights to anomalies which have higher mean returns and SR, which is not done in the PCA model.

In Conclusion, it has been shown that the RP-PCA model outperforms PCA. RP-PCA yields better SR and RMS results than other models, both in and out of sample. Additionally, even though RP-PCA doesn't directly maximise the variance explained, like PCA, it is still able to yield very close idiosyncratic results to PCA. Fama and French are outperformed by PCA models, as opposed to the double sorted portfolios where the conclusion is unclear. RP-PCA generates fewer mispricing errors for anomalies with high SR, which drives its efficient performance. Also, in terms of SDF, RP-PCA gives larger weights to the anomalies with high SR, and the SDF generated is more correlated with mean returns than in PCA, also explaining the difference in performance. The optimal structure for the single sorted portfolios is to assign γ equal to 10 and estimate a 5 factor model. These 5 factors are enough to capture systematic risk. Factors 1,2, and 5 can be used to describe the cross-section of returns where as factors 1,3, and 4 can explain the time series of returns. It should be noted that these results are in accordance with the analysis done by Lettau and Pelger (2020), confirming the reliability of their results and reinforcing the importance of RP-PCA in the factor models' application to financial markets.

4.2 Analysing the existence of climate risk in markets

4.2.1 Comparing against a green factor model

We now perform a comparison across models with the addition of the 5 Fama and French factors plus the GMP factor of Gimeno Nogués and González Martınez (2022). This helps understand the importance of a climate risk factor, and to what extent such risk is captured by the estimated factors in PCA and RP-PCA.

As explained in the data section, we now shift towards daily data from the 1st of January 2002 until the 31st of December 2019. We also initially work on the same set of single sorted portfolios as before, following Lettau and Pelger (2020), but now in daily frequency. To perform such analysis, we first fit a 5 factor model using RP-PCA, PCA, and the original 5 Fama and French factors, which are all used as a benchmark for comparison. Secondly, we estimate 6 factor models with RP-PCA, PCA, and the 5 Fama and french factors plus GMP. This is done for the set of 370 and 74 portfolios. The results can be found in the table below:[19]

Table 3: Daily single sorted portfolios models fit

Model	In sample			Out of sample		
	SR	RMS_α	$\bar{\sigma_e^2}$	SR	RMS_α	$\bar{\sigma_e^2}$
Panel A: 74 portfolios						
RP-PCA(5)	0.045	0.695	8.729 %	0.037	0.807	7.621%
PCA(5)	0.045	0.697	8.728 %	0.035	0.820	7.620%
Fama-French(5)	0.065	**0.679**	13.498 %	0.025	**0.665**	8.683%
RP-PCA(6)	0.045	0.694	7.883 %	**0.038**	0.759	6.837%
PCA(6)	0.045	0.696	**7.883%**	0.037	0.765	**6.828%**
Fama-French and GMP(6)	**0.078**	0.683	13.494 %	−0.003	0.670	8.180%
Panel B: 370 portfolios						
RP-PCA(5)	0.042	0.561	7.7783%	0.027	0.562	7.022%
PCA(5)	0.041	0.561	7.783 %	**0.029**	0.560	7.022%
Fama-French(5)	0.065	0.555	9.740 %	0.025	**0.540**	7.474%
RP-PCA(6)	0.043	**0.554**	7.397 %	0.027	0.561	6.612%
PCA(6)	0.043	0.555	**7.397%**	0.026	0.557	**6.609%**
Fama-French and GMP(6)	**0.078**	0.555	9.737 %	0.003	0.544	7.252%

The same single sorted portfolios of 37 anomalies are considered, but now in a daily frequency from the 1st of January 2002 until the 31st of December 2019. In panel A only the decile 1 and 10 of each portfolio are considered, while in panel B all deciles are taken into the analysis. 3 sets of 5 factor models are considered, PCA, RP-PCA with γ equal to 10, and Fama and French. 3 sets of 6 factor models are considered, PCA, RP-PCA with γ equal to 10, and Fama and French with the GMP factor. The in and out of sample performance metrics are displayed, and the bold numbers represent the best performing model with a given number of factors K. RMS has been multiplied by 100 to see differences more clearly across models.

The first remark from table 3, is that RP-PCA and PCA performance is quite similar. It still holds that RP-PCA outperforms PCA in terms of SR and RMS, and PCA outperforms in terms of idiosyncratic variance, but the difference is now less obvious. This is mainly explained by the fact that daily data is now used, which makes the statistics smaller in general, and hence the difference becomes less obvious to 3 decimal places. What has significantly changed, is the performance of the Fama and French model in sample, which now outperforms both PCA and RP-PCA in terms of SR and RMS.

Focusing on the analysis of the model with GMP, for both data sets, it outperforms all other models in sample in terms of SR. It is also interesting to note, that such a model can achieve a 20% higher SR than the original 5 factors Fama and French model. In terms of RMS, the model also performs quite strongly, yielding better results in and out of sample compared to RP-PCA and PCA. On the other hand, it is also visible that the out of sample SR performance of the model with GMP is far behind the RP-PCA and PCA models. This could be a potential sign

[19]Note that here the out-of-sample rolling window is taken to be a year.

of overfitting. Additionally, as expected, the idiosyncratic variance both in and out of sample is far off from the RP-PCA and PCA results. Overall, it can be concluded that the model which includes a climate factor explicitly has some advantages since it can capture well the cross-section of returns in sample and produces an improvement of the RMS metric out of sample. However, out of sample, RP-PCA and PCA models are overall better performing, and for this reason, they could provide a good alternative way to incorporate climate risk information. This overall signals that in the current market structure, climate risk does take on an important role and should be considered in modelling systematic risk.

As a robustness check to these results, the same model analysis is re-performed but using monthly data, shown in table A5 of the appendix. The conclusions remain the same with monthly data, except that now RP-PCA more clearly outperforms PCA.

It remains ambiguous the extent to which information captured by the GMP factor is included in the estimated factors of RP-PCA and PCA. To give some clarity into this, two regressions are performed using the 6 estimated factors as explanatory variables regressed against GMP. The regression results can be found in table A6 of the Appendix. Overall, for both regressions, we observe extremely small R^2 coefficients of 0.0029. Additionally, only the coefficients for the 3rd and 5th factors are significant. This clearly shows that the information in the GMP factor is not well explained by the factors estimated by RP-PCA and PCA.

The GMP factor, which can be used as a proxy for climate risk factor, improves model performance predominantly in-sample but lacks out of sample. Considering climate risk might hence allow for more accurate SDF and systematic risk estimations, but a more efficient model to improve out of sample performance is needed. RP-PCA and PCA estimated factors using the original set of single sorted portfolios only capture an insignificant amount of climate risk. As shown in the previous section, RP-PCA models have a significant advantage over other models, and could hence prove an efficient way to incorporate climate risk information.

4.2.2 Single sorted portfolio analysis with a green anomaly

Now the "green anomaly", consisting of a decile 1 portfolio of high emitting firms, and a decile 10 portfolio of low emitting firms are incorporated. This helps analyse the extent to which the addition of this green anomaly alters the previous factor structure, which can be an indication of to what extent is climate risk important in the estimated factors. Additionally, by looking at the spanned SDF loadings of the green anomaly we can understand the characteristics of green investments compared to other anomalies.

To perform this analysis, we first need to determine for the new set of data the optimal γ and number of factors. First, the eigenvalue percentage differences for several γ values are plotted in Figure A8 of the appendix. From this graph, we see that the difference in eigenvalues is very similar across all the γ values. When zooming in to the graph, we observe that γ equal to 20 or 10 seems reasonable since they improve the detection of weak factors for the first 10 factors. To maintain consistency and facilitate comparison with previous results, we maintain γ equal to 10. Moving on, we plot the performance metrics in and out of sample for RP-PCA and PCA models with a varying number of factors, in Figure A9 of the Appendix. In sample the results are clear, the more factors that are added the better all performance metrics are. In terms of RMS and $\bar{\sigma_e^2}$, once the 5th factor is added, the metrics only decrease by small amounts compared to the previous changes. In terms of SR, after the 5th factor is added the increase in SR stabilises, but when the 9th factor is added there is a surprisingly big increase. When evaluating the out of sample performance it is more unclear. In terms of SR, the values increase until the 5th factor is added, then decrease until the 8th factor which creates an increase, and then decreases again when the 9th factor is added. Overall, the 5th factor yields the best SR for PCA, and the third-highest by RP-PCA, and hence seems an adequate choice. In terms of the RMS, the series decreases until the 4th factor and then stabilises after experiencing a small increase. Finally, the out of sample $\bar{\sigma_e^2}$ shows a decrease every time a factor is added, but after the 5th factor, this decrease is less prevalent. Overall, we can hence conclude that a 5 factor model seems the most accurate. It should, however, be noted, that the selection for this 5 factor model is less clear than in the previous section since the SR and RMS out of sample could also suggest a 4 factor model.

Having stabilised the adequate parameters of the model, we can now focus on understanding how the addition of a green anomaly has altered the factor structure. 5 factor models with RP-PCA (γ equal to 10) and PCA are performed with and without the green anomaly, and the in and out of sample performance metrics are recorded in Table 4 below.

Table 4: Performance metric of models on N=74 and N=76.

	In sample			Out of sample		
Model	SR	RMS_α	$\bar{\sigma_e^2}$	SR	RMS_α	$\bar{\sigma_e^2}$
RP-PCA(N=74)	**0.0452**	**0.695**	8.7286%	0.0375	0.807	7.6209 %
RP-PCA(N=76)	0.0441	0.698	8.6750%	**0.0767**	**0.717**	**7.4748%**
PCA(N=74)	0.0446	0.697	8.7285%	0.0349	0.820	7.6201 %
PCA(N=76)	0.0436	0.700	**8.6749%**	0.0295	0.748	7.5100 %

Here we consider the set of single sorted portfolio deciles 1 and 10 on the 37 anomalies, denoted as (N=74), and on the 38 anomalies, denoted as (N=76). For each, the RP-PCA, with γ equal to 10, and PCA models with 5 factors are estimated and the IS and OOS performance metrics are calculated. The bold numbers represent the best-performing model in terms of each metric. RMS is multiplied by 100 to see the difference across models.

Table 4 shows promising signs that the addition of this green anomaly has helped improve the model, and allowed a more accurate estimation of the true SDF, and systematic risk. In terms of RP-PCA, the model without the green anomaly only performs better in terms of the in-sample SR and RMS, but this is only 2% and 0.4% higher respectively. For the rest of the performance metrics, the model with the green anomaly performs best, especially for the out of sample results, where the SR is 105% higher and the RMS is 12% lower. In terms of PCA, the performance of the model with the green anomaly doesn't perform as well compared to RP-PCA. The in-sample performance is dominant for the model without the anomaly, except for the $\bar{\sigma_e^2}$, but the difference is small. In terms of out of sample performance, the model with the green anomaly obtains better RMS and $\bar{\sigma_e^2}$ but is lacking in terms of SR. Overall, it is shown that incorporating the green anomaly, is more efficiently done through RP-PCA. This is in line with our previous hypothesis, which stated that climate risk mainly affects the first moment of returns and hence will be best captured by RP-PCA. The improvements out of sample that arise when the green anomaly is added to RP-PCA, are a clear indication that the green characteristics play an important role in capturing the true SDF, and hence market risk.

In terms of the factor structure, the green anomaly information is mainly incorporated through factors 3 and 4 in both RP-PCA and PCA. This can be seen in table 5, where the correlation of the 6 factors estimated previously without the anomaly, with the 5 factors estimated with the anomaly, are shown. For factors 1, 2, and 5, the correlation with the previous factor is very close to 1. For the 3rd, and 4th factors the correlation has decreased to around 0.86 for RP-PCA and 0.88 for PCA, showing that these are the most significantly altered factors after the green anomaly introduction. Additionally, it is worth noting that the correlation decrease of the 3rd and 4th factors, is bigger for the RP-PCA model. This again confirms the previous statement, that the climate risk information is most efficiently incorporated through RP-PCA.

Adding a green anomaly yields improvements in the model and signals that climate risk is an important variable to consider but it must be noted that a complete "green" factor is not found. The highest correlation with GMP is achieved by the third factor and yields a value of 0.0233. When performing again the regression using the 5 estimated factors as explanatory variables on the GMP factor, we again obtain very low R^2 results, and significance only at the 5th factor. The results for such regression can be found in Table A7 of the Appendix. All in all, this shows that it is important to consider climate risk information, but with the current data, the addition of a complete green factor is not necessary.

Now that we have established that climate risk is a significant component of systematic risk in the market, it is necessary to understand the prevalent risk this "green anomaly" exerts with respect to other anomalies. To do so, the implied SDF weights by the 5 estimated factors are shown in Figure 3.

From Figure 3 we can observe that decile 1 portfolios mainly acquire negative SDF weights, and decile 10 portfolios mainly acquire positive SDF weights. This hints toward a long-short structure to hedge for the risk in the market. There are a few anomalies, however, which present positive weights for the decile 1 portfolios, and some, even exceed decile 10 weights. This holds for the "green" anomaly in both RP-PCA and PCA models. According to these SDF results, investors shall continue to take long positions in both high emission and low emission assets. Several reasons could explain these findings. First of all, much of the previous literature has shown that climate risk is still not priced correctly in markets, and hence this might be driving the positive SDF weight for the decile 1 portfolio of the green anomaly. Additionally, it could

Table 5: Correlation between the 6 factors model on N=74, with 5 factor model on N=76, and on the GMP factor.

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	GMP Factor
			RP-PCA factors				
Factor 1(N=76)	1.0000	-0.0061	-0.0020	-0.0039	-0.0042	-0.0010	0.0114
Factor 2(N=76)	-0.0050	1.0000	-0.0040	-0.0067	-0.0034	-0.0005	0.0087
Factor 3(N=76)	-0.0091	-0.0017	0.8620	0.5031	0.0128	-0.0030	0.0233
Factor 4(N=76)	-0.0055	-0.0015	-0.5090	0.8618	0.0136	-0.0018	-0.0196
Factor 5(N=76)	-0.0055	-0.0034	-0.0084	-0.0267	0.9992	-0.0024	-0.0337
GMP Factor	0.0113	0.0088	0.0297	-0.0052	-0.0336	0.0267	1.0000
			PCA factors				
Factor 1(N=76)	1.0000	-0.0005	0.0013	0.0029	0.0009	-0.0001	0.0114
Factor 2(N=76)	0.0006	1.0000	-0.0014	-0.0023	0.0001	0.0001	0.0088
Factor 3(N=76)	-0.0026	0.0024	0.8817	0.4707	0.0156	-0.0020	0.0261
Factor 4(N=76)	-0.0021	0.0015	-0.4716	0.8813	0.0193	-0.0016	-0.0159
Factor 5(N=76)	-0.0009	-0.0001	-0.0048	-0.0250	0.9991	-0.0016	-0.0335
GMP Factor	0.0113	0.0089	0.0302	-0.0018	-0.0334	0.0266	1.0000

The top row represents the 6 factors estimated on the set of N=74 daily single sorted portfolios. The first column represents the 5 factors estimated on the N=76 daily single sorted portfolios. The first panel shows the correlations of the RP-PCA factors with γ equal to 10, and the second panel shows the correlations of PCA factors.

also be that an alternative characteristic is correlated to this green anomaly, and is already capturing a long/short structure. For example, as Alessi et al. (2021) find, "more transparent and greener firms tend to be less correlated to the market, and more correlated to bigger firms, as opposed to opaque and brown firms". A third potential reason lies behind the idea that this climate risk is very recent, and has slowly been gaining attention in the past years. This could mean that even though the risk for "brown" portfolios has increased, it is still not negative enough to drive investment towards green portfolios. This is investigated further in section 4.2.3.

Finally, the SDF weights of each anomaly are plotted against the average mean returns of the portfolios, shown in Figure A11 of the Appendix. Here we can observe that both decile 1 and 10 mean returns, obtain much higher SDF weights compared to other anomalies. In particular, it is interesting to note that the SDF weight of decile 1 is much higher than decile 10, and that decile 1 obtains lower mean returns. In theory, investors should be willing to buy green assets for lower returns, since these assets carry less climate risk and can be used to hedge against it (Alessi et al., 2021). However, here the returns for the "green" portfolio are higher than the returns for the "brown" portfolio. Several explanations can help give insights into such results. Firstly, as Alessi et al. (2021) mention in the research, the "green" anomaly, should not only be driven by emissions but also by the transparency of the firms. Not explicitly including transparency as an anomaly to sort the portfolios might be distorting the results and driving the mean returns of "green" firms upwards. On this line, the fact that climate risk is still not priced correctly in markets could also be affecting these results.

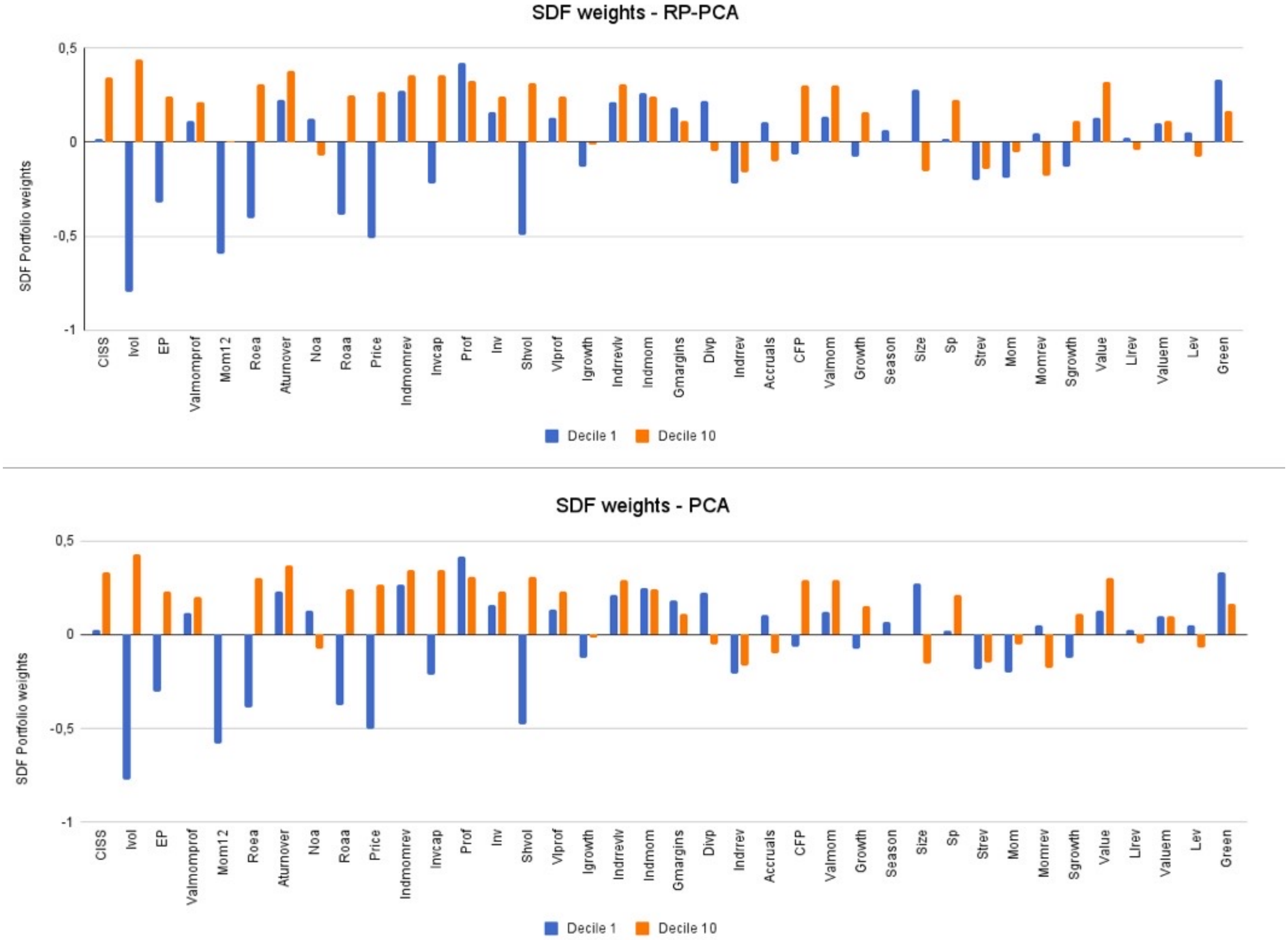

Figure 3. SDF Portfolio weights for each anomaly for the daily single sorted portfolios with N=76

Using the N=76 daily data, we estimate a 5 factor PCA and RP-PCA, with γ equal to 10 models. The SDF weights for each decile portfolio are calculated.

Thirdly, an alternative explanation lies behind the ideas exposed by Pastor et al. (2021), where they also find unexpected high returns for "green" assets. Pastor et al. (2021) findings show that "green stocks typically outperform brown stocks when climate concerns increase", and explain that high realized returns don't imply high expected returns but are rather portraying news about environmental issues. This, however, does not contradict the idea that stocks with higher risk should obtain higher returns in compensation, since additional to that, it should also be considered that pricing based on a climate-related characteristic is very forwards looking and dependent on future expectations. This implies that the high returns of "green" portfolio could be linked to investors' positive expectations of the behaviour of these assets. Overall, the fact that the decile 10 portfolio of the "green" anomaly obtains higher SDF weight compared to other portfolios with the same mean returns shows that it carries a strong weight in correcting for risk in the market, and should hence be considered for portfolio creations and investors decisions.

In summary, the addition of a "green" anomaly into the set of the single sorted portfolio has allowed an improvement of the models' performance mainly, out of sample. Additionally, the inclusion of climate risk is most effectively done through RP-PCA, since climate information is usually incorporated through the first moment. This is shown by the fact that model improvements, after the inclusion of the "green" anomaly, are larger for RP-PCA than PCA. The improvement in the model signals that climate risk information is an important component when measuring market risk, and should be taken into account by investors. Additionally, the SDF weight of the decile 10 portfolio from the "green" anomaly shows positive weights, showing that a positive position should be taken when investing in green assets and portfolios. Surprisingly, when comparing mean returns it is observed that decile 10 yields higher values. Overall, several explanations are found for this phenomenon and the overall conclusion signals that climate risk premium is subject to shocks and dependent on future expectations.

4.2.3 Evolution of climate risk importance over time

Climate risk has increased rapidly in past years. It hence remains important to assess whether the model improvements and the importance of climate risk, are different across the samples studied. This helps understand whether patterns such as the ones presented in the SDF results of Figure 3 are likely to reverse in the coming years and whether climate risk is expected to gain more importance in markets. To perform this analysis estimation of the RP-PCA and PCA 5 factor models has been done for 3 different time periods. The first time period is the full sample, the second time period takes observations before the Paris Agreement, which was signed on the 12th of December of 2015, and the last period takes on observations post the Paris Agreement. The performance metrics specific to the "green" anomaly are displayed in Table 6. [20]

There are several important insights reflected in Table 6. We focus throughout on the results of the RP-PCA model, which closely resemble PCA results. First, the mean return of the decile 10 portfolio has increased by 75%, compared to an increase in decile 1 of just 32%. This is in line with the findings of Pastor et al. (2021), since after the Paris Agreement, climate concerns rose, leading "green" assets to outperform "brown" assets. Again, this contradicts the idea that stocks with higher risk should obtain higher returns since "green" assets are less exposed to climate risk, but following the previous explanations, we can conclude this is a sign of a shock in concerns and change in expectations. The SDF weights reflect performance improvement since the decile 10 weight increases by 337% after the Paris Agreement. Following these results, investors should have directed more investment towards more sustainable firms after the Paris Agreement, confirming that climate risk has become an important variable in the market.

[20]Here we calculate the RMS specific to the "green" anomaly, the SDF, Sharpe Ratio and mean returns of the decile 1 and decile 10 of the "green anomaly", and the total amount of observations considered in each period.

Table 6: Evaluating the chance metrics of the green factor before and after the Paris Agreement signed on the 12th of December 2015

	Full sample	Pre-Paris Agreement	Post-Paris Agreement
Metrics with RP-PCA			
RMS_α in sample	0.7245	0.6674	0.9313
RMS_α out of sample	0.9599	0.8921	1.1838
SDF weight-Decile 1	0.3347	0.3342	0.7744
SDF weight-Decile 10	0.1656	0.1125	0.4915
Sharpe Ratio	0.0021	-0.0034	1.4650
Mean return(%)-Decile 1	0.0280	0.0261	0.0345
Mean return(%)-Decile 10	0.0337	0.0288	0.0505
Observations	4531	3512	1019
Metrics with PCA			
RMS_α in sample	0.7127	0.6433	0.8455
RMS_α out of sample	0.9618	0.8940	1.1710
SDF weight-Decile 1	0.3323	0.3212	0.7172
SDF weight-Decile 10	0.1649	0.1122	0.4783
Sharpe Ratio	0.0021	-0.0034	1.4650
Mean return(%)-Decile 1	0.0280	0.0261	0.0345
Mean return(%)-Decile 10	0.0337	0.0288	0.0505
Observations	4531	3512	1019

5 factor PCA and RP-PCA models, with γ equal to 10 models, are performed and the performance metrics specific to the green portfolios is calculated. The data of the green anomaly is taken from the 1st of January 2002 until the 31st of December 2019, denoted as the full sample, from the 1st of January 2002 until the 12th of December 2015, denoted as the Pre-Paris agreement, and from the 13th of December 2015 until the 31st of December 2019, denoted as Post-Paris Agreement.

On the other hand, it should be noted that the SDF weights of the decile 1 portfolio also increase after the Paris Agreement by 132%. This is smaller amount than the increase experienced by the decile 10 portfolio, but it is still surprising and worth looking into further for future research. In terms of the performance of the model, it is clear that the importance of incorporating climate risk into the model increases from one period to another, shown by the drastic rise in SR. In conclusion, Figure 2 shows that climate risk inclusion in the set of single sorted portfolios to capture the true SDF has become more and more important over the past years. This highlights that not only is climate risk an important factor in financial markets but also that this importance is expected to rise in the coming years. Additionally, the performance of assets coming from low emitting firms has improved, making them a more attractive investment option. Following the SDF results, directing more money towards "greener" investment options has the potential for a profitable investment strategy. Additionally, as can be seen, by the mean returns results, investors should also consider the investors' concerns on climate change and expectations as variables which drive the climate risk premium. Potential variables to do so can be found in the research by (Ardia et al., 2020) or Choi et al. (2020).

5 Conclusion

In accordance with Lettau and Pelger (2020), this paper shows that RP-PCA is a more efficient model for capturing more accurate factor structures. The performance of RP-PCA models, both for the small and large cross-section of returns, is dominant in terms of Sharpe Ratio and Root Mean Squared metrics in and out of sample. On the other hand, even though PCA tends to achieve better idiosyncratic variance results, RP-PCA is not far behind. This is surprising since PCA directly minimised idiosyncratic variance, whereas the RP-PCA optimisation is dual, with the addition of first moment information. RP-PCA's ability to closely match PCA's idiosyncratic variance results reinforces the idea that this model is superior. The 3 and 5 factors Fama and French model, closely matched PCA results in small cross-sections of returns, but are less optimal when studying larger cross-sections. The performance gain of RP-PCA can be explained by a few elements. Firstly, RP-PCA can better capture weaker factors, which are capable of explaining additional variance components to the market factor. Also, RP-PCA gives stronger SDF weights to anomalies with high Sharpe Ratios and high mean returns compared to PCA. This allowed RP-PCA to attain a factor structure closer to the true stochastic discount factor, and yield more accurate risk estimations. When looking at the large cross-section of returns, the results showed that a 5 factor model was necessary to capture systematic risk. Within these 5 factors, factors 1, 2, and 5 explain the cross-section of returns, whereas factors 1, 3, and 4 explain the time series component of the returns. Overall, we conclude that RP-PCA has evident performance gains, both in and out of sample, which arise due to the incorporation of first moment information.

Using these factor models, the importance of climate risk is evaluated. The GMP factor model of Gimeno Nogués and González Martınez (2022), which incorporates a climate factor, improves model performance upon RP-PCA and PCA in-sample but lacks out of sample. When adding a climate-related anomaly directly into the set of returns analysed, the performance in and out of sample of both RP-PCA and PCA improves. All this signals that climate risk is a determinant factor in predicting risk in the market and should therefore be incorporated into investors' decision-making. The performance gains experienced by the RP-PCA, are significantly larger than for the PCA model. It can hence be concluded that RP-PCA models more optimally incorporate this information into factor models. This agrees with the idea that climate information is mainly incorporated through the first moment, and hence RP-PCA is best able to incorporate it. Moving on, when analysing the SDF weights of the climate-related anomaly, positive weights are found for both high and low polluting firms. Considering that the importance of this factor has been increasing during the last year, particularly after the 2015 Paris Agreement, the time variation of these results is also analysed. Here we find that the SDF weights of the "green" portfolios have grown over time by 337%, compared to an increase of 132% for "brown" portfolios. This shows that, even though investment in less climate-friendly assets is still profitable as of today, investment profitability with respect to its risk, into green assets has grown significantly more. Also, these results are in line with the ideas presented in much of the previous literature, where they explain that climate risk is still not priced correctly in markets, and this limits the shift towards greener investment. Additionally, from the divided sample analysis, we observe that climate risk has become a more important factor in measuring market risk after the Paris Agreement. Based on the stochastic discount factor and mean returns results, more investment should be directed towards greener assets since their profitability has increased. Overall, this shows that climate risk is an important factor to measure market risk, and its importance is expected to rise in the coming years.

A limitation of these findings lies behind the creation of the "green" anomaly introduced into the set of single sorted portfolios. Its creation follows from the ranking of stocks based on emissions, done by Gimeno Nogués and González Martınez (2022). This ranking is done based on 2019 data, and hence our anomaly is not re-calibrated each year, nor does it change size and stocks considered. On the other hand, the rest of the anomalies are re-calibrated each year, which might lead to inconsistencies. For future research, more focus should be directed towards creating a "green anomaly", which is re-calibrated in time and can be added to the analysis. Additionally, it would also be useful to obtain this anomaly based on the full 10 deciles, since more insights could be drawn. Also, in terms of factor models, Miles et al. (2016) prove that time in-variant factor loadings are very restrictive in factor models' performance. In this sense, it would be interesting for future research to compare the performance across models with time-variant factor loadings, and see if the conclusions based on climate risks' importance are altered. Finally, given that we have found some evidence that climate risk is an essential component of systematic risk and financial markets, it is clear that more research needs to be directed towards understanding how best to price this risk. This is an essential component to ensure all parties are incentives to move toward a more sustainable economy.

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6 Appendix

6.1 Literature review of Gimeno Nogués and González Martınez (2022)

The extension of this paper is focused on linking the findings of Lettau and Pelger (2020) to the GMP factor and green portfolio ranking executed in the paper of Gimeno Nogués and González Martınez (2022). For this reason, it is important to understand the methodology and main findings of Gimeno Nogués and González Martınez (2022), to understand well the opportunities and limitations of this extension.

Gimeno Nogués and González Martınez (2022) focus on deriving "an indirect measure of the shade of green of a financial asset, even if in the absence of firms disclosure". This is an essential tool since the environmental data of firms is very limited and scarce. To do so, they take advantage of the growing interest in climate-related stock characteristics, and pricing of climate risk in the market. They create a green factor based on firms' CO2 emissions that in essence can be used as a proxy for firms' environmental scores.

This factor, called GMP, is constructed for the US and EU individually. The US factor, which is what we are interested in, data from all the SP 500 stocks are collected from 2002 until 2020. Financial companies are excluded, since their emissions are relatively small, and the 408 remaining are sorted from highest to lowest in terms of emissions. After this, they pick the highest 50, and lowest 50, and construct the portfolio by going long in the lowest and short in the highest polluters. They also re-do this portfolio with the highest and lowest 10% per industry, and also further re-do the portfolio by ensuring within each industry both small and big firms are included. The factor is essentially taken as the excess returns of the constructed portfolio. It should be noted that this portfolio ranking is only done once, with the most updated data, so the included stocks are not re-calibrated per year. This is because they conclude that firms' emission levels don't vary substantially to affect the rankings across the years.

Gimeno Nogués and González Martınez (2022) conclude that the GMP factor is relevant in the market, has predictive power and captures variation which was not yet captured by the Fama and French 5 factors. As an extension, they look at how different political events affect the contribution of the green factor and find that indeed through time some political events have affected its relevance. They also find that the information from the factor is an efficient proxy for green score of firms with no disclosure, and hence can help guide investors' decisions.

From a scientific perspective, Gimeno Nogués and González Martınez (2022) prove that including a climate factor can lead to efficiency gains in the Fama and French model. Since Lettau and Pelger (2020) show that RP-PCA generally outperforms both PCA and Fama and French models, it is interesting to look at whether including a climate-related anomaly in RP-PCA can generate an even better model result performance. From a theoretical perspective, it is very necessary to understand how climate risk is being incorporated into financial markets and how to best price it. By finding an efficient model able to extract climate risk's contribution to systematic risk, we can gain insights to better answer such questions.

6.2 Objective functions of the models

$$\text{PCA:} \quad \widehat{\mathrm{F}}_{\mathrm{PCA}}, \widehat{\Lambda}_{\mathrm{PCA}} = \underset{\Lambda,\mathrm{F}}{\mathrm{argmin}} \frac{1}{NT} \sum_{n=1}^{N} \sum_{t=1}^{T} \left(\left(X_{nt} - \overline{\mathrm{X}}_n \right) - \left(\mathrm{F}_t - \overline{\mathrm{F}} \right) \Lambda_n^{\top} \right)^2. \tag{A.1}$$

$$\mathrm{RP-PCA:} \quad \widehat{\mathrm{F}}_{\mathrm{RP}}, \widehat{\Lambda}_{\mathrm{RP}} = \underset{\Lambda,\mathrm{F}}{\mathrm{argmin}} \underbrace{\frac{1}{NT} \sum_{n=1}^{N} \sum_{t=1}^{T} \left(X_{nt} - \mathrm{F}_t \Lambda_n^{\top} \right)^2}_{\text{unexplained TS variation}} + \gamma \underbrace{\frac{1}{N} \sum_{n=1}^{N} \left(\overline{\mathrm{X}}_n - \overline{\mathrm{F}} \Lambda_n^{\top} \right)^2}_{\text{xS pricing error}}. \tag{A.2}$$

6.3 Performance metric out of sample

In the analysis throughout the paper, the 3 performance metrics, SR, RMS, and idiosyncratic variance, are computed both in and out of sample. In Section 3, the explanations of how these metrics are computed in sample are displayed. For out of sample, the computations change slightly and in this section, we explain how they are computed.

First, it should be noted that for the replication section of Lettau and Pelger (2020), the out of sample computations are done using a rolling window of 20 years is used, equivalent to 240 observations. On the other hand, for the analysis of climate risk, a rolling window of 1 year is used, equivalent to 252 observations. The difference between these rolling window sizes is driven by the difference in frequency and time frame considered in each analysis.

The out of sample computations first requires a rolling estimation of the factors. In order to predict F_{t+1} we employ the factor loadings resulting from "t-the rolling" length until t and excess returns at time t+1. Following equation 1, we are then left with one unknown which is the factor at t+1 which is estimated. The window is then moved until all the sample is covered, and the estimated factors are known.

Inside the estimation window, a few calculations are performed. Firstly, for the SR calculation, every time F_{t+1} is estimated we store the following values;

$$b_t = \Sigma_F^{-1} \mu_F \tag{A.3}$$

$$OOSreturns = b_t^{\top} \hat{F}_{t+1} \tag{A.4}$$

Moving on, inside the estimation window we also calculate the out of sample pricing error for each F_{t+1} estimation. To do so we use the equation below, where $B_n^{\top}$ is estimated like in equation 10.

$$\hat{\alpha}_{n,t+1} = X_{n,t+1} - \hat{F}_{t+1} B_n^{\top} \tag{A.5}$$

Finally, once all the estimations are obtained we can proceed to the final performance metrics calculations. The SR is calculated as the equation below, where $\hat{F}_{t+1}$ is a vector containing all the estimated factors at each time point and $b_t^\top$ is their respective value obtained from equation A.3.

$$SR = \frac{E(b_t^\top \hat{F}_{t+1}}{\sigma(b_t^\top \hat{F}_{t+1})} \tag{A.6}$$

To calculate the out of sample RMS and idiosyncratic variance we first calculate $\bar{\alpha}_n$ with the following equation, where $\hat{\alpha}_{n,t+1}$ follows from equation A.5.

$$\bar{\alpha}_n = \frac{1}{N} \sum_{n=1}^{N} \hat{\alpha}_{n,t+1} \tag{A.7}$$

With this value we can now calculate RMS and the idiosyncratic variance following equations 11 and 12 respectively, replacing $\hat{e}_n$ with $\bar{\alpha}_n$ in equation 12.

6.4 Code

The basis of the code is taken from Markus Pelger's website, more specifically in the section dedicated to the research done by Lettau and Pelger (2020). The "RPPCA" and "RPPCAOOS" Matlab codes are used. In order to arrive at the results, specific codes are written to obtain the necessary data for each figure and table. These codes are all slight adjustments from one another which build on the code of Markus Pelger.

6.5 Tables and Figures

6.5.1 Results from replication Lettau and Pelger (2020)

Table A1: Statistics of the decile 1 and decile 10 monthly excess returns of the 37 anomaly single sorted portfolios.

	Decile 1			Decile 10		
Anomaly	Mean	Standard Deviation	Sharpe Ratio	Mean	Standard Deviation	Sharpe Ratio
Accruals	0.003 10	0.061 17	0.050 71	0.006 56	0.058 70	0.111 75
Aturnover	0.002 97	0.045 18	0.065 82	0.007 11	0.050 28	0.141 38
CFP	0.004 23	0.054 35	0.077 79	0.008 47	0.055 24	0.153 31
CISS	0.003 03	0.056 43	0.053 71	0.007 76	0.041 82	0.185 60
Divp	0.004 94	0.055 48	0.089 01	0.006 52	0.049 07	0.132 85
EP	0.003 24	0.067 07	0.048 27	0.008 94	0.052 27	0.171 01
Gmargins	0.005 38	0.054 34	0.098 96	0.005 56	0.046 08	0.120 66
Growth	0.004 05	0.060 35	0.067 06	0.006 94	0.053 57	0.129 55
Igrowth	0.003 39	0.061 72	0.054 97	0.006 97	0.058 39	0.119 45
Indmom	0.003 80	0.065 42	0.058 15	0.008 26	0.054 96	0.150 26
Indmomrev	0.001 07	0.046 90	0.022 75	0.012 31	0.047 62	0.258 56
Indrrev	0.000 93	0.056 72	0.016 32	0.010 63	0.071 98	0.147 71
Indrrevlv	0.000 02	0.045 09	0.000 41	0.012 90	0.053 23	0.242 35
Inv	0.004 02	0.058 32	0.068 89	0.008 65	0.052 76	0.164 00
Invcap	0.005 74	0.065 97	0.087 05	0.006 75	0.045 56	0.148 12
Ivol	−0.000 24	0.085 17	−0.002 77	0.005 30	0.034 89	0.151 85
Lev	0.004 78	0.050 33	0.094 91	0.007 29	0.061 20	0.119 09
Lrrev	0.004 76	0.058 22	0.081 70	0.008 86	0.066 38	0.133 49
Mom	0.005 85	0.072 71	0.080 40	0.009 41	0.058 95	0.159 64
Mom12	−0.001 72	0.080 40	−0.021 39	0.010 90	0.060 43	0.180 29
Momrev	0.004 21	0.059 94	0.070 31	0.008 73	0.065 10	0.134 12
Noa	0.002 72	0.054 71	0.049 78	0.006 38	0.055 41	0.115 15
Price	0.004 51	0.085 01	0.053 06	0.004 85	0.042 42	0.114 26
Prof	0.003 76	0.044 10	0.085 15	0.007 51	0.046 67	0.160 89
Roaa	0.003 78	0.066 95	0.056 43	0.005 81	0.046 03	0.126 17
Roea	0.005 13	0.069 27	0.074 05	0.005 92	0.046 70	0.126 78
Season	0.002 29	0.055 85	0.040 94	0.010 02	0.059 03	0.169 74
Sgrowth	0.005 88	0.060 20	0.097 60	0.005 34	0.052 73	0.101 24
Shvol	0.004 83	0.075 10	0.064 30	0.004 70	0.035 90	0.131 06
Size	0.004 62	0.041 90	0.110 24	0.007 34	0.063 13	0.116 29
Sp	0.004 16	0.049 59	0.083 82	0.009 39	0.059 13	0.158 87
Strev	0.002 67	0.054 08	0.049 36	0.006 23	0.072 08	0.086 37
Valmom	0.003 91	0.058 98	0.066 32	0.009 07	0.052 52	0.172 64
Valmomprof	0.003 44	0.060 00	0.057 40	0.011 71	0.055 09	0.212 61
Valprof	0.002 73	0.053 18	0.051 33	0.010 23	0.054 24	0.188 60
Value	0.004 50	0.050 67	0.088 89	0.009 19	0.055 28	0.166 30
Valuem	0.005 18	0.050 56	0.102 41	0.008 95	0.069 90	0.128 04

The statistics of the decile 1 and 10 excess returns from the single sorted portfolios according to 37 anomalies are shown. The data of the excess returns follow a monthly frequency and is taken from January 1963 until December 2019.

Table A2: Statistics of the decile 1 and decile 10 daily excess returns of the 38 anomaly single sorted portfolios.

	Decile 1			Decile 10		
Anomaly	Mean	Standard Deviation	Sharpe Ratio	Mean	Standard Deviation	Sharpe Ratio
Accruals	0.000 37	0.013 38	0.027 34	0.000 45	0.014 57	0.030 65
Aturnover	0.000 20	0.013 08	0.015 30	0.000 34	0.011 04	0.030 91
CFP	0.000 37	0.013 57	0.027 55	0.000 44	0.014 63	0.030 13
CISS	0.000 24	0.014 42	0.016 97	0.000 46	0.011 18	0.041 41
Divp	0.000 23	0.013 92	0.016 78	0.000 38	0.016 72	0.022 71
EP	0.000 22	0.016 38	0.013 46	0.000 49	0.014 83	0.032 95
Gmargins	0.000 35	0.013 02	0.026 55	0.000 38	0.011 62	0.032 85
Green	0.000 28	0.012 10	0.023 13	0.000 34	0.013 35	0.025 22
Growth	0.000 35	0.013 35	0.026 47	0.000 38	0.013 21	0.028 81
Igrowth	0.000 31	0.014 45	0.021 42	0.000 37	0.013 20	0.028 39
Indmom	0.000 38	0.018 16	0.021 03	0.000 36	0.013 01	0.027 82
Indmomrev	0.000 32	0.013 33	0.024 38	0.000 43	0.011 37	0.038 05
Indrrev	0.000 30	0.014 56	0.020 49	0.000 43	0.017 34	0.024 52
Indrrevlv	0.000 29	0.011 87	0.024 16	0.000 42	0.013 65	0.031 06
Inv	0.000 35	0.015 34	0.022 51	0.000 41	0.012 59	0.032 96
Invcap	0.000 41	0.014 74	0.027 80	0.000 50	0.012 60	0.040 01
Ivol	0.000 26	0.019 56	0.013 13	0.000 35	0.009 71	0.035 76
Lev	0.000 37	0.011 85	0.031 17	0.000 31	0.020 18	0.015 20
Lrrev	0.000 41	0.014 59	0.028 26	0.000 33	0.016 57	0.019 66
Mom	0.000 58	0.018 79	0.031 05	0.000 44	0.014 46	0.030 32
Mom12	0.000 27	0.021 75	0.012 63	0.000 45	0.014 42	0.030 96
Momrev	0.000 36	0.014 44	0.024 66	0.000 35	0.016 12	0.021 78
Noa	0.000 29	0.013 45	0.021 77	0.000 50	0.013 67	0.036 83
Price	0.000 30	0.017 57	0.016 88	0.000 34	0.010 91	0.031 33
Prof	0.000 28	0.011 61	0.024 20	0.000 39	0.011 12	0.035 34
Roaa	0.000 30	0.016 25	0.018 56	0.000 37	0.011 31	0.033 14
Roea	0.000 28	0.016 40	0.017 12	0.000 36	0.011 02	0.033 06
Season	0.000 31	0.014 31	0.021 77	0.000 34	0.014 37	0.023 64
Sgrowth	0.000 38	0.014 06	0.026 93	0.000 32	0.013 93	0.023 25
Shvol	0.000 37	0.017 83	0.020 69	0.000 30	0.009 79	0.030 74
Size	0.000 31	0.011 39	0.027 33	0.000 33	0.011 38	0.028 97
Sp	0.000 34	0.012 55	0.026 89	0.000 40	0.014 08	0.028 48
Strev	0.000 24	0.014 45	0.016 39	0.000 35	0.019 60	0.017 73
Valmom	0.000 42	0.013 34	0.031 82	0.000 46	0.013 32	0.034 20
Valmomprof	0.000 30	0.014 18	0.020 96	0.000 50	0.012 62	0.039 86
Valprof	0.000 32	0.013 26	0.023 81	0.000 43	0.013 23	0.032 75
Value	0.000 37	0.011 52	0.032 54	0.000 40	0.014 64	0.027 48
Valuem	0.000 37	0.011 38	0.032 18	0.000 35	0.017 61	0.019 99

The statistics of the decile 1 and 10 excess returns from the single sorted portfolios according to 38 anomalies are shown. The data of the excess returns follow a daily frequency and is taken from the 1st of January 2022 until the 31st of December 2019. The green anomaly portfolios are constructed following the firm ranking in terms of firm emission by Gimeno Nogués and González Martınez (2022), where the top 50 ranked firms' returns are added to construct decile 10, and the bottom 50 ranked firm returns are added to construct decile 1.

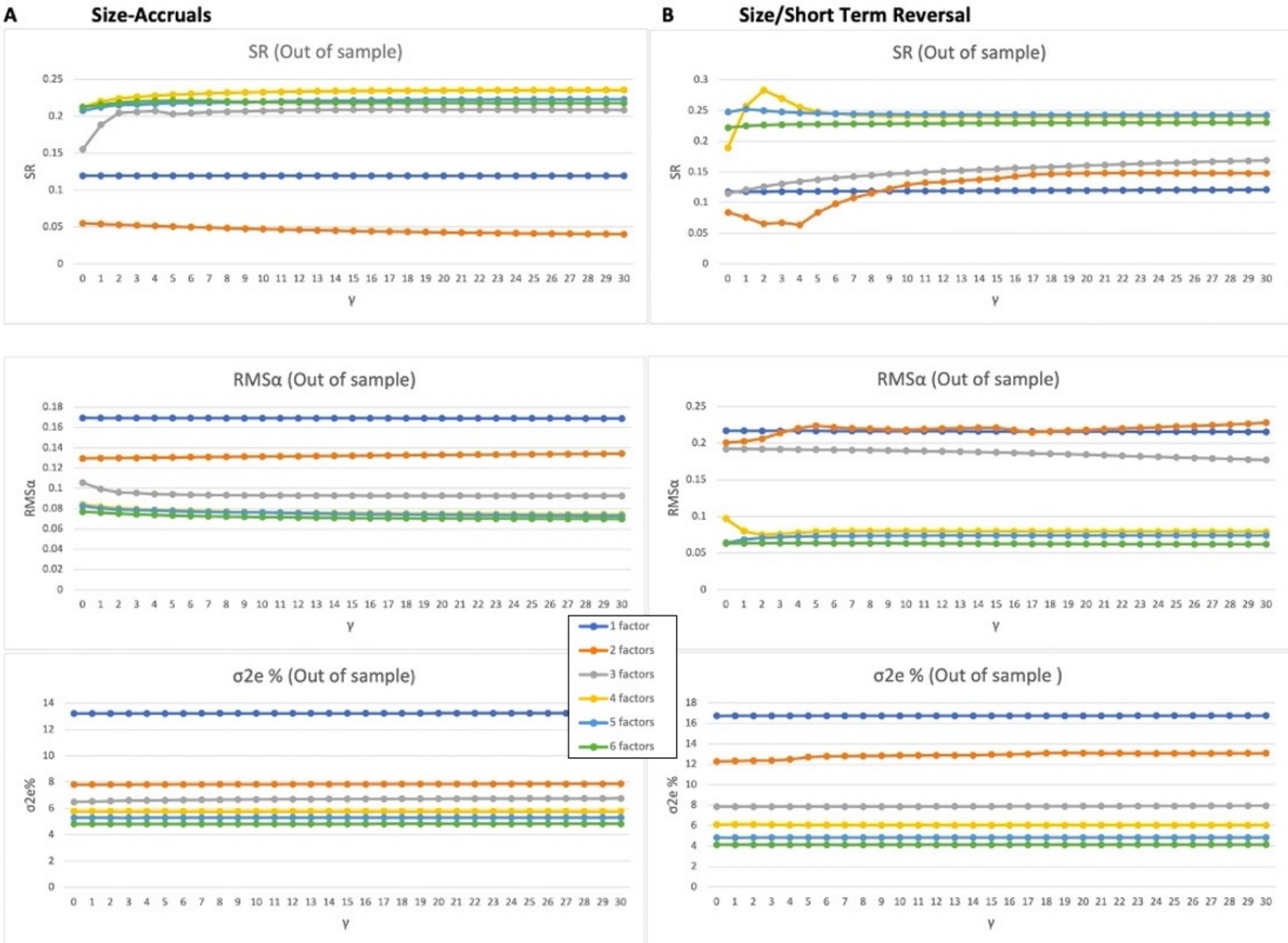

Figure A1. RP-PCA out of sample performance metrics as a function of γ

We consider two data sets of the double sorted portfolios, on the left the Size/Accruals, and on the right the Size/Short Term Reversal. The graphs show how the OOS performance metrics change as the choice of γ in the RP-PCA model varies (X-axis). This analysis is done for RP-PCA models with 1 up to 6 factors, each denoted in a different colour.

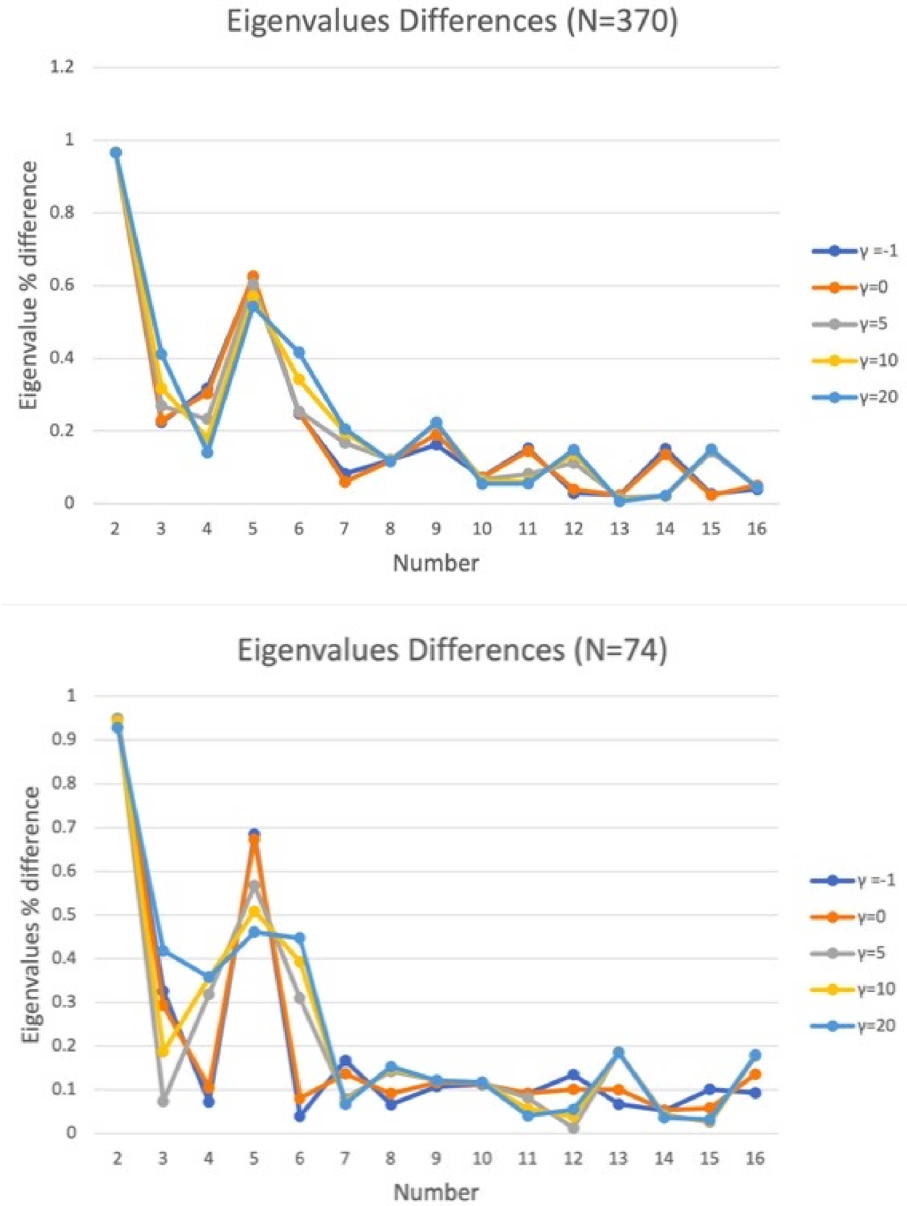

Figure A2. Eigenvalue percentage differences for the single sorted portfolio on different γ values
The eigenvalue percentage difference from the first until the 16th eigenvalue are calculated using the RP-PCA formula. This is done for a set of γ values, which each take a different colour. The single sorted portfolio data with 37 anomalies are considered. The top graph uses all deciles, while the bottom graph uses exclusively deciles 1 and 10 of each anomaly.

Table A3: Single sorted portfolios data set model fit

Model	In sample			Out of sample		
	SR	RMS_{α}	$\bar{\sigma_e^2}$	SR	RMS_{α}	$\bar{\sigma_e^2}$
Panel A: 74 portfolios						
RP-PCA (3)	**0.37**	**0.23**	13.88%	**0.31**	**0.22**	**15.42%**
PCA (3)	0.23	0.27	**13.74%**	0.11	0.26	15.88%
Fama-French (3)	0.21	0.31	17.49%	0.14	0.25	16.54%
RP-PCA (5)	**0.59**	**0.16**	10.43%	**0.50**	**0.15**	12.11%
PCA (5)	0.32	0.21	**10.30%**	0.24	0.19	**12.04%**
Fama-French (5)	0.32	0.26	16.05%	0.24	0.19	13.91%
Panel B: 370 portfolios						
RP-PCA (3)	**0.24**	**0.17**	12.77%	**0.20**	**0.15**	**14.41%**
PCA (3)	0.17	0.17	**12.68%**	0.13	0.15	14.49%
Fama-French (3)	0.21	0.18	14.61%	0.12	0.16	14.89%
RP-PCA (5)	**0.56**	**0.13**	10.80%	**0.47**	**0.12**	12.72%
PCA (5)	0.25	0.14	**10.67%**	0.18	0.14	**12.59%**
Fama-French (5)	0.32	0.16	13.60%	0.21	0.13	13.74%
Panel C: 98 portfolios						
RP-PCA (3)	**0.35**	**0.22**	13.48%	**0.17**	**0.22**	**16.29%**
PCA (3)	0.24	0.24	**13.36%**	0.09	0.23	16.54%
Fama-French (3)	0.21	0.34	17.26%	0.10	0.25	17.41%
RP-PCA (5)	**0.54**	**0.16**	10.31%	**0.39**	**0.15**	12.86%
PCA (5)	0.33	0.19	**10.21%**	0.19	0.18	**12.77%**
Fama-French (5)	0.34	0.24	15.48%	0.16	0.19	14.70%

The IS and OOS performance metrics are calculated for 3 different sets of data each on 6 models. Panel A contains the single sorted 37 anomalies portfolio, exclusively with decile 1 and 10 monthly excess returns, while Panel B contains the entire 10 deciles. In Panel C, a total of 49 anomalies are considered, of which monthly excess returns on deciles 1 and 10 are taken. In Panel A and B, the sample period goes from November 1963 until December 2017, whereas in Panel C the start of the sample is November 1973. In each of these panels RP-PCA, PCA and Fama and French models are performed both on 3 and 5 factors. In RP-PCA γ is set equal to 10. The bold numbers represent the best-performing model with the same number of factors K.

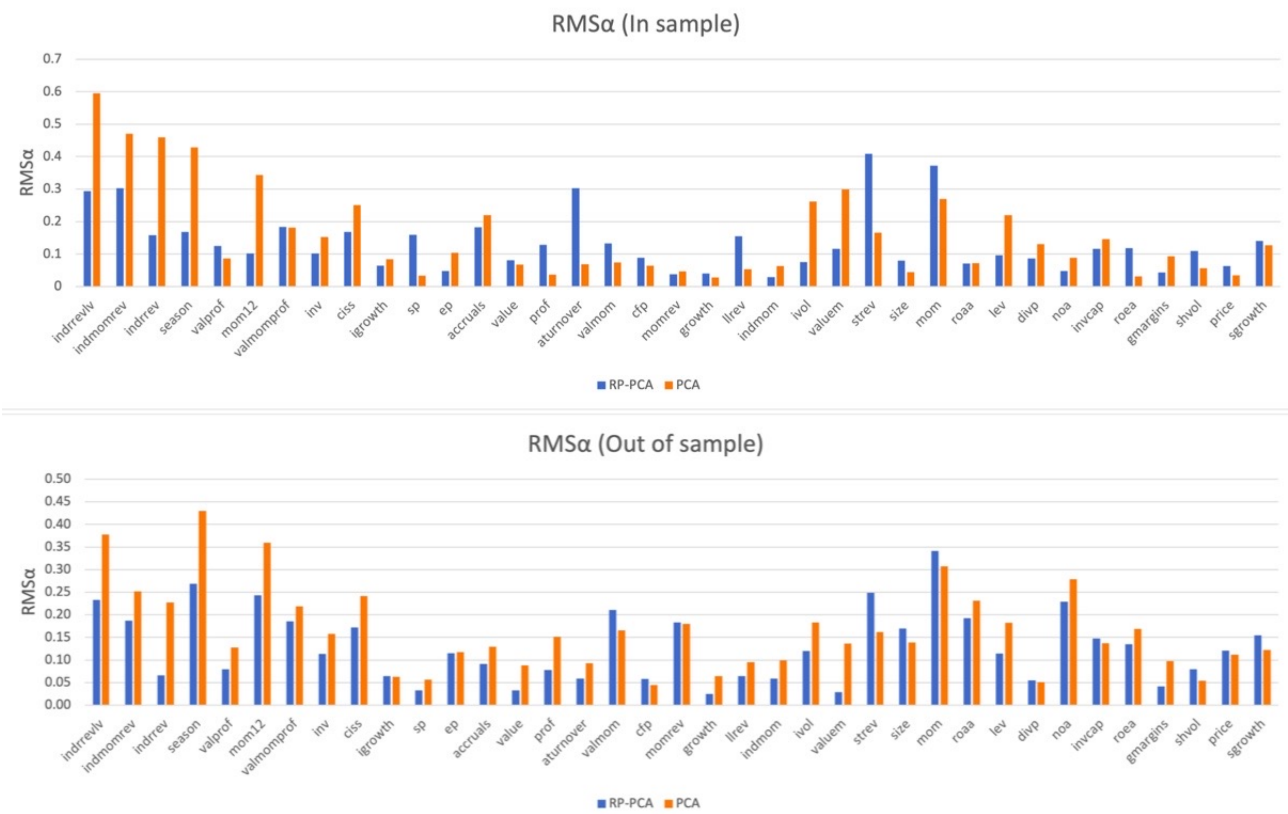

Figure A3. RMS_{α} in and out of sample by anomaly

Using the N=74 single sorted portfolios, we estimate a 5 factor PCA and RP-PCA model with γ equal to 10. The anomaly-specific RMS_{α} in and out of sample is then calculated for both models. The anomalies are ranked by anomaly specific Sharpe Ratio, where "Indrrevlv" attains the highest Sharpe ratio across all anomalies.

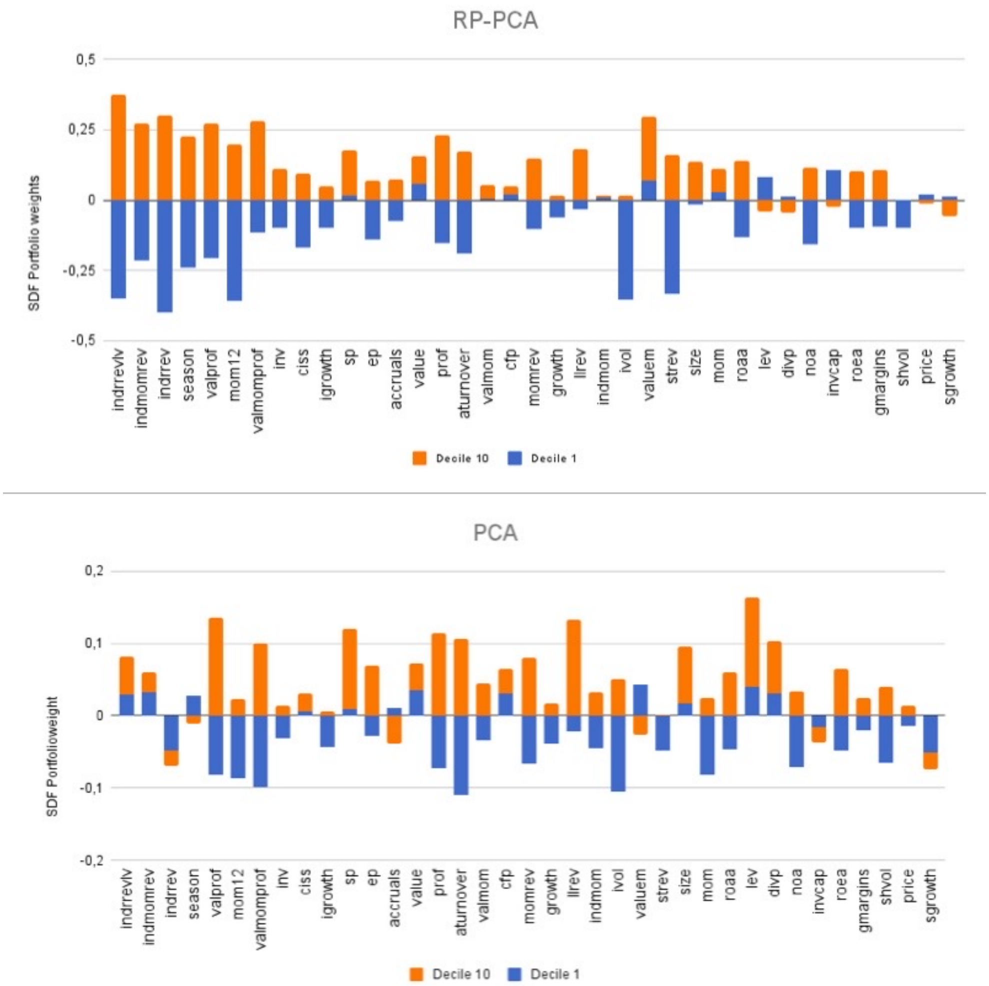

Figure A4. SDF portfolio weights in RP-PCA and PCA
Using the N=74 single sorted portfolio data, and performing a 5 factor PCA and RP-PCA, with γ equal to 10, models, the SDF weights per portfolio are calculated. The top graph shows the results for RP-PCA and the bottom for PCA. Again the anomalies are ranked by Sharpe ratio, from highest to lowest.

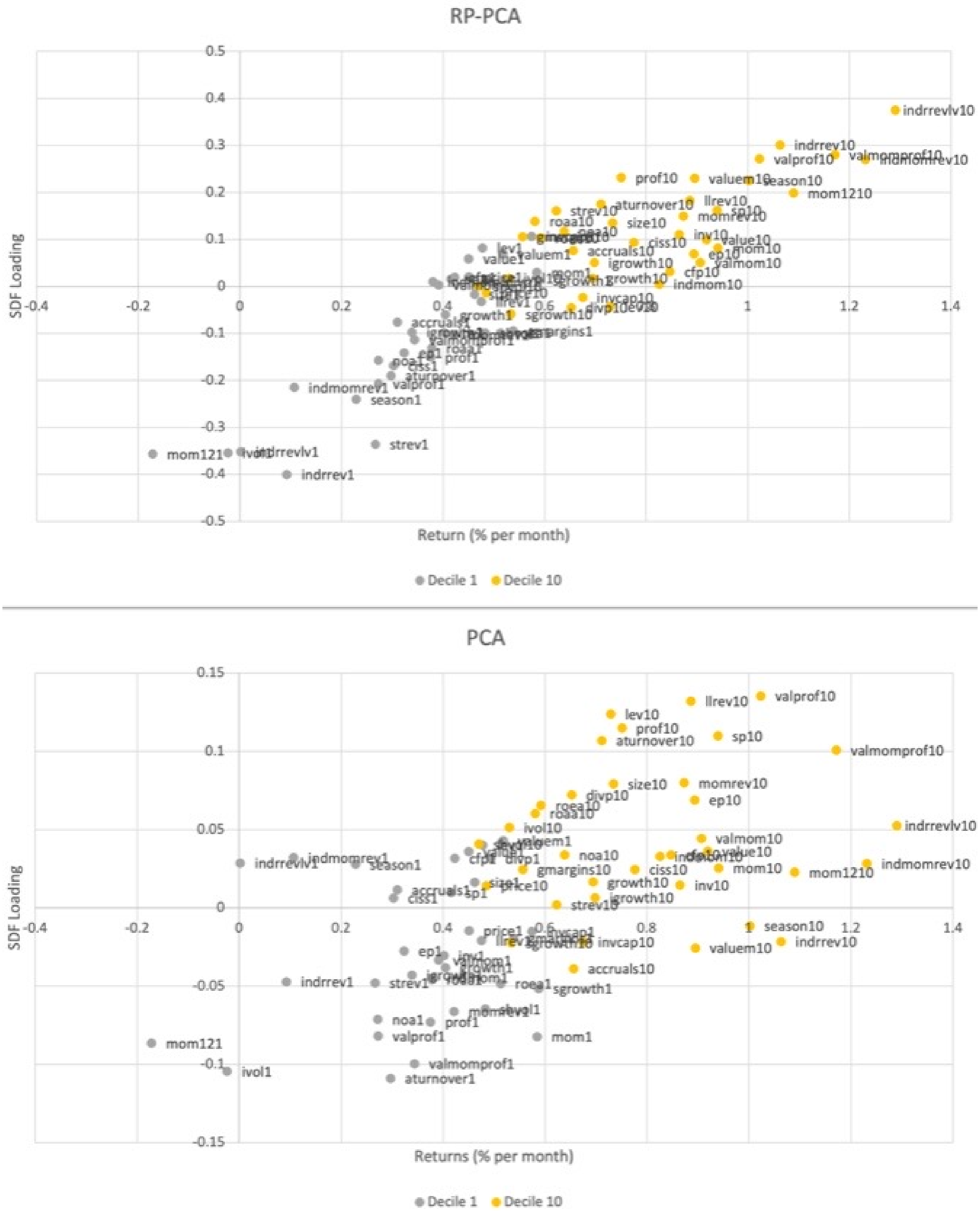

Figure A5. Portfolio returns and loadings of SDFs
Using the N=74 single sorted portfolio data, and performing a 5 factor PCA and RP-PCA, with γ equal to 10, models, the SDF weights per portfolio are calculated. They are plotted against the mean returns in percentage for each portfolio. The grey dots represent decile 1 portfolios, and the yellow dots represent decile 10 portfolios.

Table A4: Individual factor analysis

	RP-PCA				PCA			
Factor	Mean	Variance	SR	Mean rank	Mean	Variance	SR	Mean rank
1	5.02	1935.06	0.11	**1**	4.83	1944.86	0.11	**1**
2	2.32	66.32	0.28	**2**	0.18	102.55	0.02	9
3	0.30	100.99	0.03	**4**	1.65	69.13	0.20	**2**
4	0.10	65.45	0.01	6	1.05	64.16	0.13	**3**
5	0.73	26.34	0.14	**3**	0.83	20.21	0.18	**4**
6	0.03	19.55	0.01	9	0.34	19.42	0.08	7
7	0.14	17.96	0.03	**5**	0.79	16.17	0.20	**5**
8	0.05	15.42	0.01	7	0.57	15.10	0.15	6
9	0.04	13.55	0.01	8	0.20	13.48	0.05	8
10	0.03	11.97	0.01	10	0.04	11.96	0.01	10

For both RP-PCA and PCA, a 10 factor model is estimated and the statistics of each of the factors are shown above. The data set used is N=74 single sorted portfolios and γ is set to 10. The SR is the maximal Sharpe ratio of the individual factor, and the Mean rank ranks factors from highest mean (1) to lowest (10).

6.5.2 Results assessing the importance of climate risk in markets

Table A5: Monthly single sorted portfolios models fit

	In sample			Out of sample		
Model	SR	RMS_α	$\bar{\sigma_e^2}$	SR	RMS_α	$\bar{\sigma_e^2}$
Panel A: 74 portfolios						
RP-PCA(5)	0.311	0.142	10.179%	**0.226**	0.174	14.142%
PCA(5)	0.245	0.148	10.135%	0.159	0.177	14.220%
Fama-French(5)	0.340	**0.133**	16.869%	0.177	**0.168**	14.951%
RP-PCA(6)	0.325	0.139	9.278%	0.185	0.169	13.583%
PCA(6)	0.247	0.147	**9.221%**	0.178	0.171	13.723%
Fama-French and GMP(6)	**0.342**	0.135	16.755%	0.105	0.171	14.116%
Panel B: 370 portfolios						
RP-PCA(5)	0.299	0.123	10.065%	0.213	**0.138**	13.460%
PCA(5)	0.245	0.121	10.048%	0.170	0.141	13.481%
Fama-French(5)	0.340	**0.115**	13.210	0.183	0.141	13.850%
RP-PCA(6)	0.509	0.118	9.678%	**0.230**	0.138	13.212%
PCA(6)	0.245	0.121	**9.525%**	0.172	0.141	**13.206%**
Fama-French and GMP(6)	**0.342**	0.116	13.128%	0.074	0.142	13.509%

Single sorted portfolios of 37 anomalies are considered, on a monthly frequency from the 1st of January 2002 until the 31st of December 2019. In panel A only the decile 1 and 10 of each portfolio are considered, while in panel B all deciles are taken into the analysis. 3 sets of 5 factor models are considered, PCA, RP-PCA with γ equal to 10, and Fama and French. 3 sets of 6 factor models are considered, PCA, RP-PCA with γ equal to 10, and Fama and French with the GMP factor. The in and out of sample performance metrics are displayed, and the bold numbers represent the best performing model with a given number of factors K.

Table A6: Correlation between the green factor, GMP, and factors formed by RP-PCA and PCA

	Coefficient	Standard Error	P-values
Regression with RP-PCA factors			
Intercept	-0.0050^{***}	0.0017	0.0037
Factor 1	0.0116	0.0152	0.4483
Factor 2	0.0439	0.0737	0.5513
Factor 3	0.2272^{**}	0.1137	0.0457
Factor 4	-0.0401	0.1153	0.7283
Factor 5	-0.3143^{**}	0.1395	0.0243
Factor 6	0.2725^{*}	0.1517	0.0725
R^2	0.0029	-	-
Adjusted R^2	0.0016	-	-
Regression with PCA factors			
Intercept	-0.0050^{***}	0.0017	0.0037
Factor 1	0.0116	0.0152	0.4472
Factor 2	0.0442	0.0737	0.5484
Factor 3	0.2314^{**}	0.1136	0.0418
Factor 4	-0.0139	0.1153	0.9041
Factor 5	-0.3141^{**}	0.1395	0.0244
Factor 6	0.2716^{*}	0.1517	0.0735
R^2	0.0029	-	-
Adjusted R^2	0.0016	-	-

6 Factor models are estimated using RP-PCA, with γ equal to 10, and PCA on the daily frequency of N=74 single sorted portfolios. For each model, the 6 estimated factors are regressed against the GMP factor with a constant. The regression results are presented above, and the significance of the coefficient is presented using *.

Figure A6. Models performance metrics for the 370 daily portfolios
For the N=370 daily frequency single sorted portfolios, the in and out of sample performance metrics are displayed for RP-PCA, with γ equal to 10, and PCA 6 factor models.

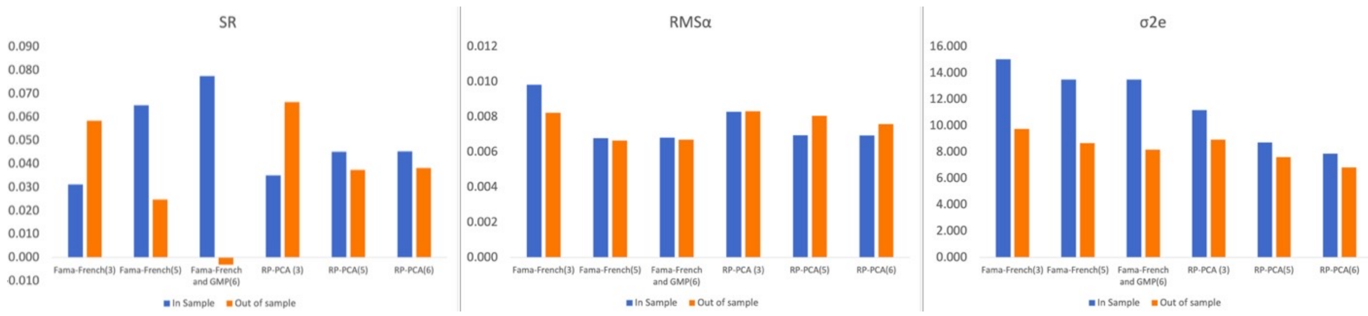

Figure A7. Models performance metrics for the 74 daily portfolios
For the N=74 daily frequency single sorted portfolios, the in and out of sample performance metrics are displayed for RP-PCA, with γ equal to 10, and PCA 6 factor models.

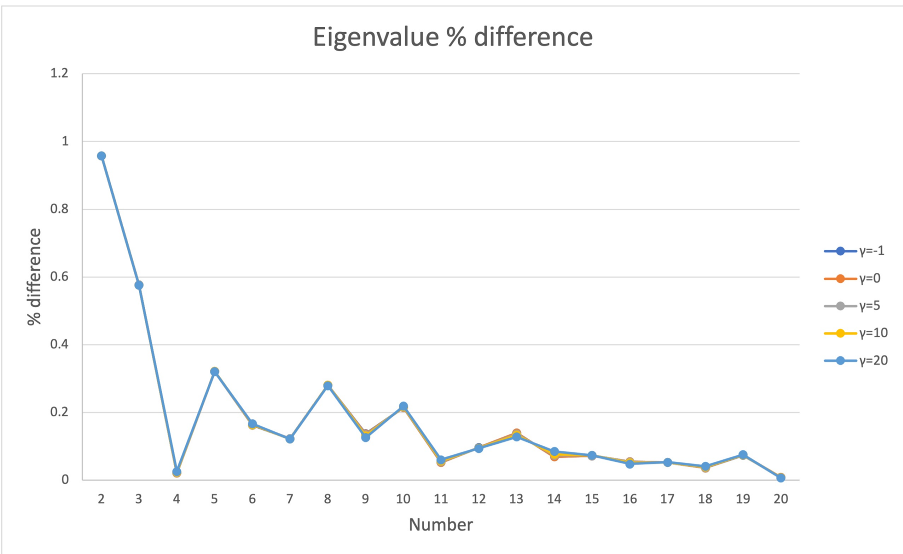

Figure A8. Eigenvalue percentage difference for N=76
The 37 anomaly daily frequency single sorted portfolios, with the addition of a green anomaly, are employed. The eigenvalues percentage difference from the first until the 16th eigenvalue are calculated using the RP-PCA formula. This is done for a set of γ values, which each take a different colour.

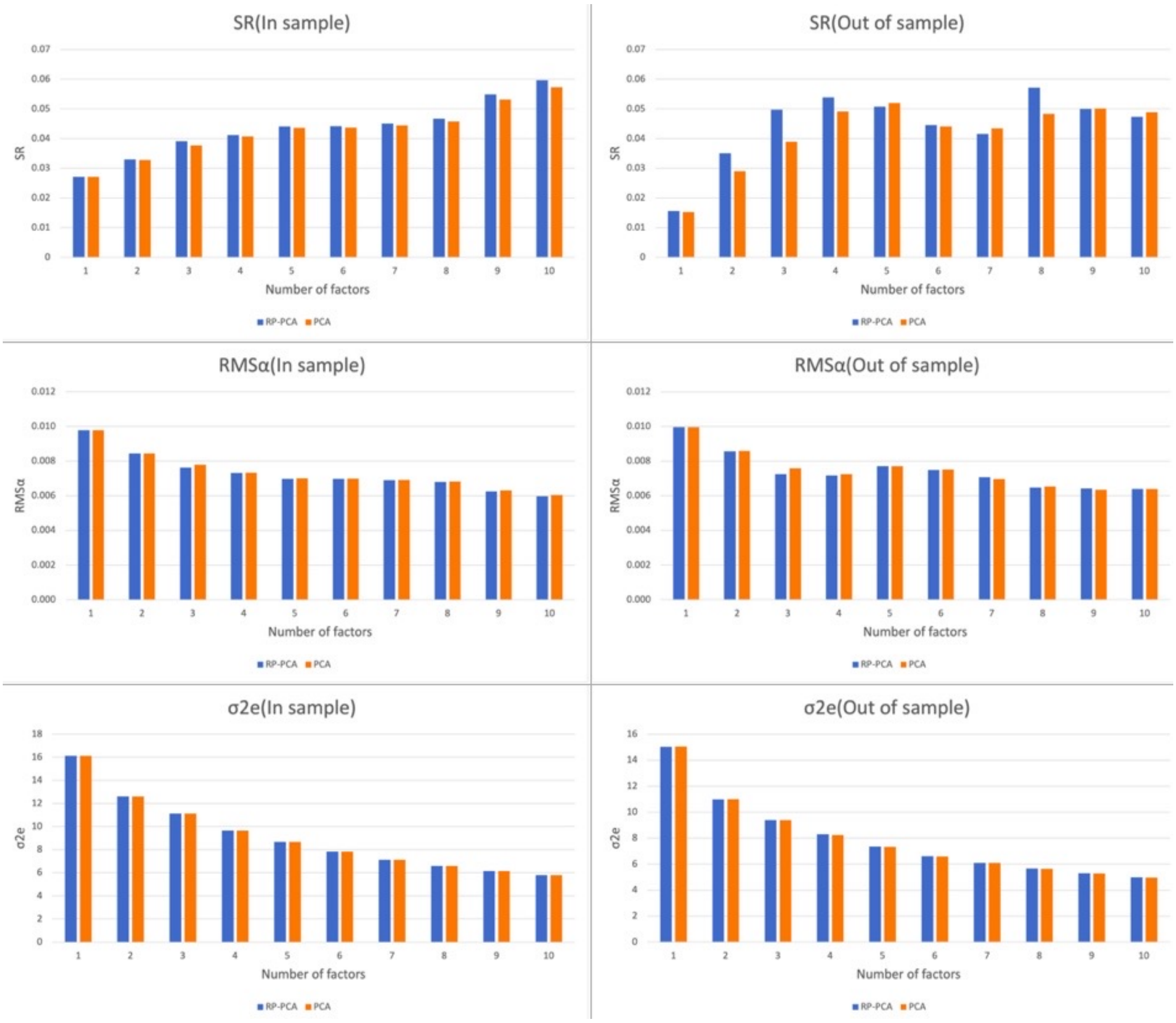

Figure A9. Performance metrics for different number of factors on N=76
The 37 anomaly daily frequency single sorted portfolios, with the addition of a green anomaly, are considered. Both RP-PCA, with γ equal to 10, and PCA models are estimated to calculate the IS and OOS performance metrics. This is done for a varying number of factors as expressed on the x-axis.

Table A7: Correlation between the green factor, GMP, and factors formed by RP-PCA and PCA with N=76

	Coefficient	Standard Error	P-values
Regression with RP-PCA factors			
Intercept	-0.0050^{***}	0.0017	0.0037
Factor 1	0.0115	0.0151	0.4435
Factor 2	0.0432	0.0737	0.5579
Factor 3	0.1774	0.1132	0.1173
Factor 4	-0.1508	0.1143	0.1872
Factor 5	-0.3139^{**}	0.1388	0.0238
R^2	0.0023	-	-
Adjusted R^2	0.0012	-	-
Regression with PCA factors			
Intercept	-0.0050^{***}	0.0017	0.0037
Factor 1	0.0116	0.0151	0.4421
Factor 2	0.0435	0.0737	0.5548
Factor 3	0.1992^{*}	0.1132	0.0785
Factor 4	-0.1225	0.1144	0.2840
Factor 5	-0.3135^{**}	0.1388	0.0240
R^2	0.0023	-	-
Adjusted R^2	0.0012	-	-

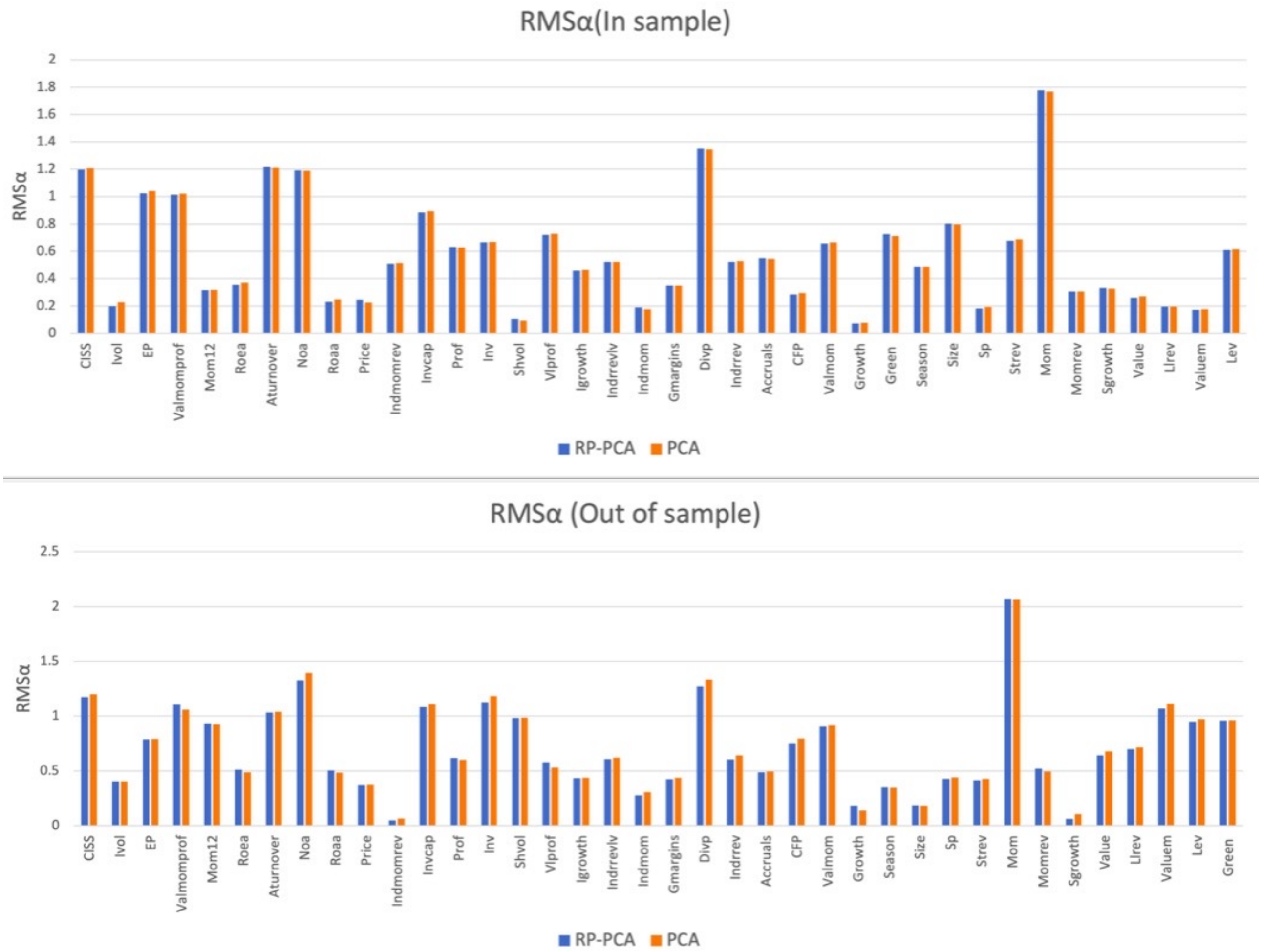

Figure A10. RMS per anomaly for the daily single sorted portfolios with N=76
Using the N=76 daily data, we estimate a 5 factor PCA and RP-PCA, with γ equal to 10 models. The anomaly specific RMS_α are calculated IS and OOS. The anomalies are ranked by anomaly specific SR, from highest to lowest.

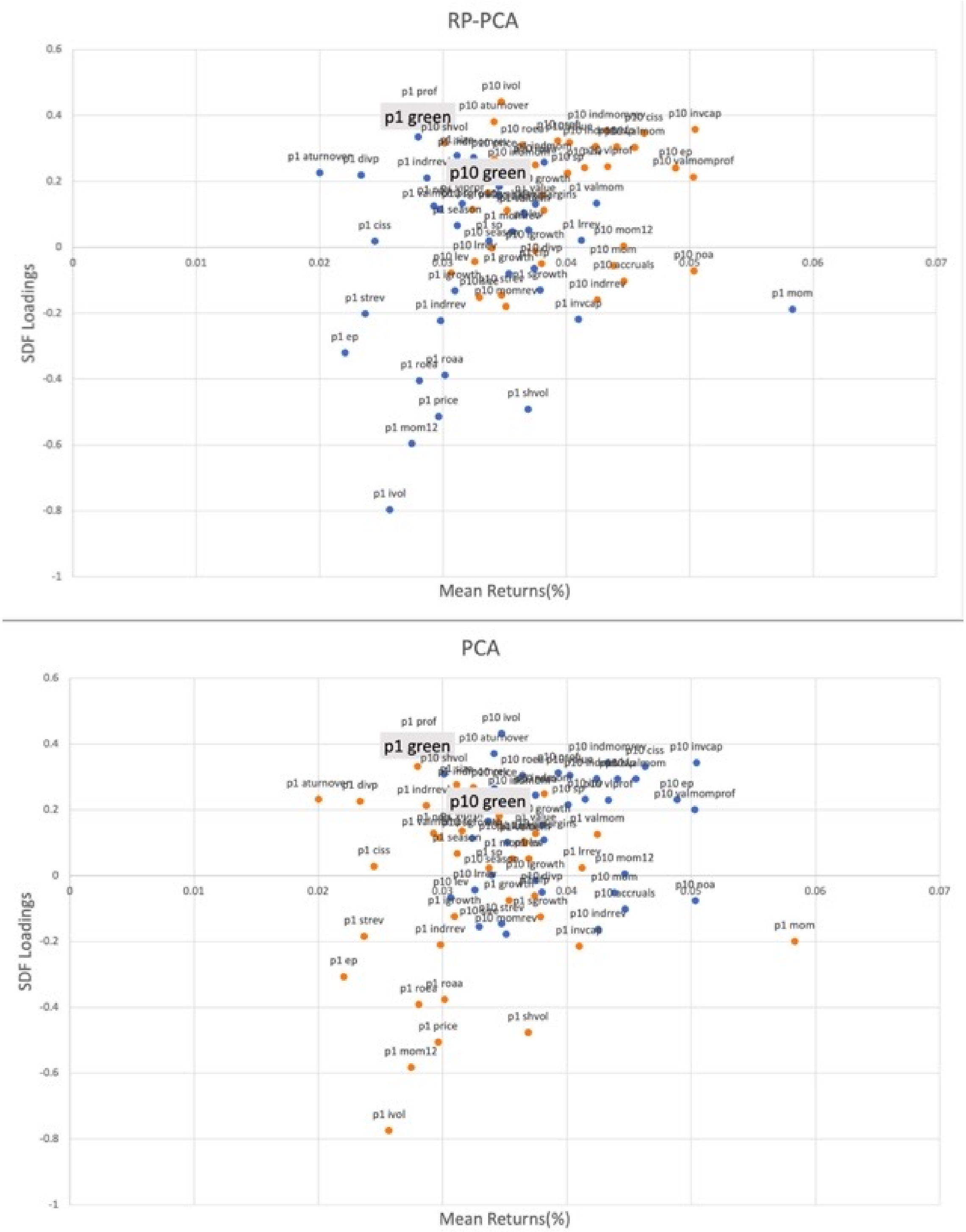

Figure A11. SDF Portfolio weights on mean returns per anomaly for the single sorted portfolios with N=76

Using the N=76 daily data, we estimate a 5 factor PCA and RP-PCA, with γ equal to 10 models. The SDF weights for each decile portfolio are calculated and plotted against the mean returns in percentage for each portfolio. For the top panel, the blue dots represent the 1 decile portfolios, and the yellow dots represent the 10 decile portfolios, this is reversed for the bottom panel.