ERASMUS UNIVERSITY ROTTERDAM ERASMUS SCHOOL OF ECONOMICS MSc Economics & Business Master Specialisation Financial Economics

# Option value of switchable tariff.

The effect of discontinuous times models on option value of the option, applied to a wind-park valuation.

A commence on 'Valuation of switchable tafriff for wind energy.' By Yu et al. (2005)

Author:M.J.P. BosStudent number:321998Thesis supervisor:R. HuismanFinish date:February 2010

ii

## **CHAPTER 1 INTRODUCTION**

Since the 90s a trend of deregulation in electricity markets has occurred. This fundamental change from a government controlled market to a privatized competitive market had consequences for electricity price behaviour. Weron (2007) noticed a new challenge for power market participants on their path to market liberalization due to an introduction of competition. In addition, Higgs and Worthington (2008) write about a significant influence on wholesale electricity price behaviour.

As a result of the increased dynamics in the electricity market, traditional modelling tools to approach financial valuations do not apply for electricity price forecasting anymore. Stochastic modelling, a method that models price evolution through time, has gotten lots of attention in respect to the outspoken features of electricity prices. These electricity prices exhibit for example mean-reversion, time-varying volatility, volatility clustering and seasonality. Also, the non-storable character of electricity makes a straightforward application like arbitrage price theory, a commonly used process in the bonds and stock market, not suitable. (Hull, 2008)

Another implication of the deregulation is the increased market risk. In comparison with stock and bond markets, energy markets show occasional price spikes. Nowadays, these spikes are a main source of risk in electricity markets. Market participants are exposed to increasing risk and have to adjust their risk management strategy in order to prevent unexpected losses. Important, when striving for an accurate risk analysis, is the price evolution of the underlying commodity. This is further expressed by Ethier and Mount (1998): *contingent claims valuation of physical assets and financial derivatives depend critically on the specification and estimation of the stochastic process that describes the price path*'. This is exactly one of the new challenges Weron (2007) stated. There is an urgent market need for such accurate modelling processes that capture the characteristics of the market in order to optimize risk management models and valuation analysis (Andriosopoulos and Nomikos, 2008).

A more realistic simulation of the price path automatically results in a more realistic volatility path. Volatility, an unobservable parameter, is one of the most important factors in determining the value of energy derivatives and real option analysis (Blanco et al 2001a). An accurate stochastic process influences the delta's, the price sensitivity, of exotic options like spread options used for real option analysis of generating utilities (Blanco et al 2001a). Furthermore volatility is a main value driver of options as well (De Jong and Huisman, 2002). Making a valuation with incorrect volatility parameters leads to an under or overestimation of the value, which is in the end an unreliable outcome (Weron, 2005). The main contribution of this research is to provide more clarity about the effect of volatility parameters – resulting from different stochastic processes – and their implications on financial value.

To examine the effect of this volatility values on financial value, two stochastic processes are selected and applied to a case study based on a research done by Yu et al. (2005). This case study describes the valuation of a wind-park in Spain by real option analysis. The value is determined by the flexibility offered by the government to a wind generator to choose between different tariffs as compensation for the generated electricity by their wind park. This 'flexibility' value is measured as a call option on windspread. The wind-spread measures the difference between the revenues for selling the wind power at market day-a-head spot price or fixed-price, minus the cost of wind generation. Important implications for the value of the wind-park is the forecast of the underlying, namely the hourly day-ahead spot price, by Yu et al. (2005) approached by a continuous time model. Classical stochastic modelling processes concentrated only on continuous time components, but more recent literature call for a model design including discontinuous components to mimic electricity prices (Ethier and Mount, 1998). As a consequence of their continuous/discontinuous character, these two different types of models result in different quantitative measures of volatility and other parameters of spot prices. The contribution of this research is summarized by the question; what is the effect a discontinuous models and their generated corresponding quantitative measures on the call option value calculated by a real option approach?

This study imitates the valuation of the wind park in same way as Yu et al. (2005) did, but compares two different stochastic processes: a Geometric Brownian motion with mean reversion (1) and an regime switching model (2) to forecast the electricity price. This research chooses a day-ahead spot prices case study, because price spikes are the most noticeable in comparison with monthly or yearly prices. With a Log likelihood estimation the belonging parameters, including volatility, of each process are determined. These serve as input for an Monte Carlo simulation to calculate the option value of the wind park using both stochastic processes. The option on switchable tariff is composed as a vanilla call option on market prices with a strike price of the fixed energy prices. In the end the result of the selected stochastic processes for their obtained option value is discussed. Special attention is given to the difference of both processes in determining the option value. In addition, after a general statement, the conclusions will also be related to the Spanish case specifically.

This research is structured as follows. First, the features of electricity markets are described shortly. Then recent literature about the most accurate approach of electricity prices is reviewed. There have been a number of studies, published after deregulation of the market, investigating several stochastic models for commodity prices. The focus of this study is on econometric models ranging from continuous time models to including discontinuous components application. A short outline of the wind park valuation case done by Yu et al can be read in chapter 4. After describing our

methodology and the data the real option value of both processes is calculated. Finally, the result of the processes and their obtained option values is discussed. The last chapter concludes.

## **CHAPTER 2 INTRODUCTION ELECTRCITY MARKETS**

In this part the most relevant features of the electricity market are outlined. The goal is to inform the reader about the most important characteristics of the electricity market. This knowledge can be used in later chapters to put forward an adequate model that pinpoints these unique characteristics. This paragraph illustrates only the universal characteristics that are shared by a majority of electricity markets. Escribano (2002) proves some regional difference among electricity markets due to regulation difference, but for the sake of simplicity the universal stylized facts about electricity spot prices are listed. Though, idiosyncratic features always have to be taken into account applying a general model to a specific market. Idiosyncratic features can for example be caused by the way capacity is generated: conventional thermal fuelled, hydro fuelled or otherwise, like wind or solar. (Huurman, 2008). After a short explanation of the electricity spot market itself and here inherent nature, some stylized facts are listed.

#### 2.1 Electricity spot market

Electricity prices that come about in a real-time commodity market are called spot prices. This is in contrast with future markets. Here, prices of non-yet-produced electricity is purchased and sold. In the spot market, electricity producers and buyers are brought together and submit their bids and offers in prices and quantities. Electricity can be sold on a time scale of minutes, days and up to a week. Energy producers in the day-a-head market offer energy based on their ability to produce energy for the following day (following 24 hours). They negotiate with buyers about the commodity prices using the same auction bidding systems by stock and commodities markets.<sup>1</sup>

Equilibrium of the market comes from the intersection between the electricity supply and demand (Fusai and Roncoroni, 2008). However, frequently a short disequilibrium between supply and demand is noticeable in the spot market. This disequilibrium is caused by different reasons – explained in the next paragraphs – but most of the time it can be ascribed by shocks of supply and demand side. Also empirical evidence proves electricity has a strong weather dependency, influencing both supply and demand. (Weron, 2008)

In general the nature of electricity is that, once generated, it cannot be stored or inventoried. This, non-storability, has major implications on the demand or supply side in the market. Electricity delivered at a specific hour cannot be substituted for electricity available shortly after or before. (Weron, 2005). This lack of transporting the commodity in different time frames nearly eliminates

http://www.energyvortex.com/energydictionary.html

arbitrage. (De Jong and Huisman, 2002) The demand curve of electricity is quite inelastic at least in short horizons (Fusai and Roncoroni, 2008) and an unexpected outage of for example a large coal generator can force the price up immediately because of the lack of inventories and no ability to smooth prices. Together, the features can be summarized in the most important stylized facts of the electricity market as listed and discussed below.

#### 2.1. Stylized fact: Mean reversion

Both Pilopovic (1997) and Baker et al (1998) found evidence for mean reversion in the energy and agricultural commodities. Pindyck states in his study (1999) on stochastic dynamics on price evolution, that prices tend to move around a certain trend and that exclusion of this trend in an attempt to forecast commodity prices will influence accuracy. In particular, commodity prices tend to hover around and drift over time to values determined by the cost of production and often experience large changes in price due to shock event. (Blanco et al, 2001a) This tendency, called mean reversion, means that events tend to be correlated. (Pilipovic, 1997). This high correlation between the future price and previous price(s) means that electricity prices do not follow a random path. (Huisman, 2009) The rate at which the market reverts back to its long run equilibrium differs in each market and is time-varying. (Weron, 2007).

#### 2.2 Stylized fact: Seasonality

Seasonality is the phenomenon that prices show different values according to a particular time frame. Both supply and demand side show seasonal fluctuations (Weron, 2005). This tendency occurs at all levels; annual, weekly and even daily fluctuations are visible in data. This seasonality is mainly a consequence of to the limited storable nature of electricity and the fluctuations in demand and supply over the time scales. These fluctuations in demand and supply, caused by several factors as temperature, are reflected in the price. The weather dependency nature of electricity is again noticeable. For example in summer the European Energy Exchange (EEX) experiences higher price levels in comparison with winters. On a daily scale, peak and load prices are visible. The weekly seasonal behaviour is generally due to variable intensity of business activities throughout the week. Annual fluctuations are mainly due to seasonal weather conditions. Thus, fluctuations of these kinds have to be taken into account when analyzing electricity data on each level.

#### 2.3. Stylized fact: Volatility and Spiky behaviour.

It is well known that volatility in the electricity market is the most severe one of all commodities. Numeral studies have devoted time to analyze the stylized facts concerning the volatility of electricity prices. (Andriosopoulos and Nomikos, 2008) Several studies, i.e. the Federal Energy Regulatory Commission, prove that historical volatility of electricity prices easily outrage these of other commodities (Higgs and Worthington, 2008). This extreme volatility is especially true for spot

prices, where volatility can be as high as 50% on the daily scale. In comparison with stock indexes with moderate volatility 1-1,5%, electricity spot market participants face higher market risks. (Weron, 2006). Andriosopoulos and Nomikos (2008) identify asymmetric tendencies in the volatility of energy markets. According to Knittel and Roberts (2005) upward spikes are more volatile than downward spikes. In addition, volatility is clustered, and as a consequence time varying. Empirical evidence suggests that electricity prices exhibit heteroscedasticity (Karakatsani and Bunn, 2004). At last, it is important to mention that data shows that spot prices are more volatile than future prices. (Baker et al 1998 pp 124-127) This observation is in line with the aforementioned stylized fact mean reversion, because in the long run the price is expected to revert to the marginal production cost and therefore there is less uncertainty and thus less volatile.

One of the pronounced features of electricity prices is that they frequently tend to have upward movements. These movements are significantly higher than the previous price level. This phenomenon is called spike. In econometric models these can also be referred to as jumps. An observation Weron (2005) makes is that spikes tend to be more severe during high price periods and can therefore considered to be seasonal. Price spikes typically do not last more than a day and are so called short-lived.

#### 2.4. Stylized fact 4: Heavy tails.

Important for stochastic modelling is examining how a price path is distributed. In financial data many empirical studies found significant variations from the normal distribution assumption. Empirical studies on electricity prices also reject the normality assumption. In addition, Andriosopoulos and Nomikos (2008) found, in line with other studies, evidence for positive skewness in energy prices, including electricity prices. This finding means that there is a higher probability on a high value than on a lower value occurring. The results on log-prices (also used to measure returns) are not that clear but show mixed results. Weron (2007), on the other hand, finds in his study on medium-term modelling of spot prices that the distribution of electricity returns are heavy–tailed, caused by the aforementioned spiky behaviour of electricity and the extreme volatility. In both distributions, log transformed or not, Gaussian and hyperbolic laws underestimate the tails of the distribution.

In the research done by Higgs and Worthington (2008) all spot markets show skewness and all have a kurtosis level higher then 3. Last finding could be an indication of a leptokurtic distribution. In an earlier study to the non linear behaviour of electricity prices by Knittel and Roberts (2005) they found indeed evidence for kurtosis. Their conclusion was that large deviations should not be considered as an exceptional event given the high kurtosis levels. Furthermore, positive swekness was found in the distribution of electricity prices. Huisman and Mahieu (2003) discovered that these fat tailed distributions fit the features of electricity as well; the upward spikes are more intense than the downward turnings.

Concluding, the evolution process of energy prices reveals some unusual characteristics. Knittel and Roberts (2005) states that there is a non linear mechanism that influences electricity prices, which increases modelling complexity. The asymmetric behaviour of volatility is an important fact this research. To achieve adequate modelling, aforementioned stylized facts have to be present in the chosen stochastic model.

# **CHAPTER 3 LITERATURE REVIEW**

This section reviews literature written about stochastic modelling of energy prices in search for a model that is adequate for application on asset valuation and financial derivatives. Because the research concentrates on the difference of quantitative measures of volatility as a consequence of the choice of the stochastic process and their influence on call option value in a real option approach, this section restricts the review on short time forecasting. Since countries deregulated their electricity markets, many models have been introduced that describe the behaviour of day-a-head prices in power markets (Huisman 2008). All researchers aim to improve the goodness-of-fit of their models on electricity prices. The model that has the highest goodness-of-fit generates the most accurate parameters. Where explorative work concentrates mainly on the more traditional continuous time models, more recent literature experiences a shift to discontinuous time models. Build on the latter even external factors are incorporated. Throughout time, based on each model several specifications, extensions or variations are made. Therefore, the aim of this section is to summarize the most relevant models briefly. Throughout the literature the aim of science is to include aforementioned stylized facts of electricity in order to make the most realistic forecast of competitive electricity spot prices.

In recent literature roughly two types of models on electricity price movements are distinguished, fundamental and econometric or stochastic process.

#### 3.1. Fundamental models

Fundamental models build the price processes based on equilibrium models for the electricity market.<sup>2</sup> An example of fundamental approach used in competitive markets is the Locational Marginal Pricing (LMP). Fundamental approach requires tremendous input data to perform an accurate forecast and approach, which is time consuming (Yu et al, 2005). The input of the data can be related to transmission network, fuel prices, parameters of all generators units and the market strategies of all participants (Yu et al, 2005). Relating this to the stylized fact of mean reversion, the fact that in the long run electricity prices convert back to their long run cost price, it seems a reasonable approach. While in theory LMP concept is useful, in practice there are some comments on how it should be estimated. Debate is about the availability information, namely the need for external data, and how data are taken into account. Exclusion of one factor might influence the accuracy of the whole estimation.

In response to this fundamental approach, econometric processes gained ground in energy literature. Stochastic processes describe the probabilistic evolution of the value of a variable through time (Hull, 2008) and therefore they can be used to describe dynamics of price changes over time (Huisman,

<sup>&</sup>lt;sup>2</sup> powerpoint presentation Rafal Weron, (2004): Pricing derivatives in electricity markets.

2009). The advantage of this approach is that only historical data of energy prices are needed. Because of the fact that more weight is put to these models in literature, some of these time models are explained. Worth mentioning, recent literature presents approaches that combine fundamental models with econometric processes. At the end of this review there will be a short note on these mixed models.

#### 3.2. Econometric models

#### 3.2.1 Continuous time series models

Continuous time series models are wildly used in financial analyses. Over time various variations of these models have come into existence. A short discussion follows below on the Brownian Motion Models and Mean Reverting Models.

#### Brownian motion

In many fields the Brownian motion is used for analysis. This continuous time model is also called Wiener process. A Wiener process is a process describing the evolution of a normally distributed variable with mean of zero and a standard deviation of 1 (Hull, 2008). It is denoted as dz. A Brownian motion is an example of a random process; the prices in the future are independent of the earlier price path. The path at time T is normally distributed with mean x and standard deviation of square root T. When time increases, the uncertainty, expressed in volatility, increases with the square root of time. (Hull, 2008).

The most commonly used model is the Geometric Brownian Motion (GBM) (1)<sup>3</sup>. This is a continuous time process in which the logarithm of the randomly varying quantity follows a Brownian motion:

$$dV = \mu V dt + \sigma V dz \tag{1}$$

Where  $\mu$  is the expected price, also mean price and  $\sigma$  the volatility of the dependent variable V (Hull, 2008). GBM is consistent with a weak form of efficient market hypothesis. The Markov process makes only the present value of the variable relevant for the predicting the future (Hull, 2008). Price paths are seen as random processes. In contrast with other forms of Brownian Motions, GBM implies that returns follow a lognormal distribution. This is a probability distribution of a random variable whose logarithm is normally distributed. In other words; logarithmic returns follow a normal distribution (Blanco, 2001a).

In energy price modelling log prices are widely applied and is preferred for the following reasons. First of all it is in line with the assumption that restricts prices to fall below zero when excluding

<sup>3</sup> http://www.puc-rio.br/marco.ind/stochast.html

imbalancing markets; the only market where negative prices exists. A feature of log prices is, indeed, that when raising the number *e* to power of any number (Ethier negative, or positive), the outcome can never become a negative number, or price in this case. A second advantage working with log prices is that the differences between two log prices are approximately the percentage change. (Huisman, 2009)

Despite GBM being a commonly used method for price evolution modelling, more and more researchers reject GBM as a suitable price process for various reasons. (Ethier and Mount, 1998). To begin with, assuming prices to follow a random walk is not in line with aforementioned stylized fact; electricity prices show a mean reverting pattern. Future prices seem to depend on previous prices instead of present prices only. Furthermore, variance of GBM increases proportional with the square root of time. Again, this is not in line with the time dependent volatility, volatility clustering and the knowledge that future prices seem less uncertain. The log normality assumption does not capture the skewness and kurtosis observations and as a consequence do not exhibit the dramatic price spikes frequently occurring as well.

#### Mean reversion models.

Another typical continuous time characterization is a mean reversion model. According to Pilipovic (1997) adding mean reversion to the model improves accuracy. The mean-reversion models bring more economic logic to the model in comparison to the Geometric Brownian Motion model. The main difference between the two processes is that in the mean-reversion model a component is added that gravitates the price when the current price is above the long run equilibrium price, but forces it back up when the current price is under this long run price, in line with observations in the market.

The format for a mean reverting process is given by Dixit & Pindyck (1994) and is called Geometric Ornstein-Uhlenbeck mode (2).

$$dV = \eta \left( V^* - V \right) V dt + \sigma V dz \tag{2}$$

Where  $\eta$  is the mean reverting component,  $V^*$  the long run equilibrium,  $\sigma$  the volatility and dz the Wiener process. Alike the GBM there exists a mean reverting model that considers x as log prices, called the Arithmetic Ornstein-Uhlenbeck process. Unlike the GBM process this process does not have a variance that is increasing in magnitude in time, but is growing to a steady state.<sup>4</sup>

Despite the improvement, the mean reversion models bring along some pitfalls that still have to be taken into account. Firstly, the mean reversion rates generated are constant, but empirical evidence shows that mean reversion rates fluctuate over time. Actually, they can differ hourly and for each month. (Blanco

<sup>4</sup> http://www.puc-rio.br/marco.ind/stochast.html

et al 2001b). Huisman (2008) brings up the fact that in models that contain only mean reversion components these mean reverting components reflect both the amount of mean-reversion in normal markets as well as the mean-reversion after a spiky event. Simply said, the mean reversion component might contain some uncertainty not captured by the volatility component.

Research on electricity price forecasting that are extending the mean reversion models also devote attention to the seasonality aspect and how this is captured best in the models. Because the time of the year, the day of the week and the hour of the day influence price patterns, it is important to incorporate this into the models (Weron 2008) An suggested approach used by Weron was to use a sinusoid function with a linear trend in order to mimic the fluctuations during the quarters of the year (Weron, 2005). Alternatively, according to Weron (2008) hourly and weekly fluctuations can be approached by an autoregressive structure of the model, or an introduction of a dummy variable into the analysis.

#### 3.2.2 Discontinuous time series

In later literature, the importance of discontinuous series gains more support as it is thought to increase the accuracy of the outcome. This is mainly due to the incorporation of spikes into the model. The literature distinguishes two ways to include these spikes; jump diffusion models and regime switching models

#### Jump diffusion models.

So far, literature discussed only linear models, while mentioned discussing the stylized facts non linear behaviour of electricity prices was noticed. Early work writes about adding a jump component to a basic GBM model. Important flaw of this work was that after a jump the price would stay on this new high level. (Huisman, 2009) In reaction on this Ethier (1997) suggested combining a jump process with mean reversion and capture the salient features of daily electricity spot prices. These models refer to as, mean reverting jump diffusion models (MRJD). An example of a MRJD model is given by Weron (2008):

$$dp_t = (\alpha - \beta p_t) dt + \sigma dW_t + J dq_t$$
(3)

The new formula is based on a GBM with mean reversion (slightly rewritten in comparison with (1)); the only part added is the jump component  $Jdq_t$  responsible for the spiky events (Weron, 2008). Here,  $W_t$  is the Wiener Process and several processes can drive the component that triggers the jump. Literature describes the Poisson process as the one that captures the characteristics of a jump the best (Escribano et al 2002).

It certainly proved to have value to concentrate on spikes in spot market prices forecasting. Their popularity can be explained from the fact that incorporating these jumps results in a more accurate forecast and as a consequence it can have major implications for asset valuation and financial derivatives. (Ethier and Mount, 1998). Excluding jumps would be inappropriate, because they are a drive of risk, but therefore a main value driver of an option. (De Jong and Huisman, 2002)

The main criticism on MRJD is the assumption that all the shocks die out at the same rate. (Escribano et al, 2002). But looking at the data, spikes tend to come back faster to an equilibrium than a normal movement. In 2007 Huisman, Huurman and Mahieu found that hourly electricity prices in dayahead markets mean revert around an hourly specific mean price level and that the speed of meanreversion is different over the hour.

In their search for ways to mimic these spikes and their gravitating behaviour again several methods are published. Researchers succeeded this by returning spikes to their equilibrium by a strong mean reversion rate, whereas other like Weron et al. (2004) did this by adding downward jumps. The latter has the advantage that on short time forecasting spikes tend to go back to their original level most of the time, within one day. Intuitively, introducing such a downward jump with a same magnitude feels realistic.

The disadvantages of these methods lie in the fact that jumps occurring in these jump diffusion models are not serially correlated or independent from the mean reverting process. Also, Escribano et al (2002) mentions the fact that a small excess capacity in the market influences the chance of the appearance of a shock in electricity prices. Furthermore, the question what the correct speed of mean reversion after a jump is, still remains. Both arithmetic as geometric mean-reverting jump diffusion models struggle with this question.

In response to this a more sophisticated models was introduced; regime switching models.

#### Regime switching models

Regime switching models were first introduced by Hamilton in 1989 in his explorative analysis of non-stationary time series and business cycles in the field of macroeconomics. Recently Ethier and Mount (1999) introduce regime switching models, followed by Huisman and Mahieu (2003).

The initiative of this new sophisticated model was the perspective that an electricity path could be divided in two states of the world. (Mount 2006). The first state of the world represents the relative wavy fluctuation of the price path. The second state, also called regime, fits the frequently high spiking events that often are the result of a specific market situation. The existence of these two regimes should then be translated to the underlying parameters that drive the model, and therefore a model should have two sets of parameters. The price model consists of a deterministic and a stochastic component, such that  $s_t = d_t + d_t$ 

 $x_i$ . Were the MRJD model suffers from problems with the speed to original price state, the regime switching model brings the solution with parameters related to each event and the possibility of change between these regimes for each period.

An example of a regime switching model is given in Huisman (2008). The model consists of a deterministic component with seasonal component  $\beta_l$  (3). (Huisman, 2008)

$$dt = \mu + \beta_1 W_t \tag{3}$$

The price dt is build op from a mean and a seasonal component, captured by a dummy  $W_t$ . The stochastic model is either a mean reverting simple autoregressive process (4) or a model with a new set of parameters reflecting the new mean price during a spike and the corresponding volatility (5), respectively formulated as:

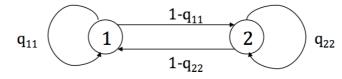
$$x_t = (1 - \alpha) x_{t-1} + \sigma \varepsilon_{2,t} \mid S_{t=1}$$
(4)

$$x_t = \mu_2 + \sigma \varepsilon_{2,t} | S_{t=2}$$
<sup>(5)</sup>

Adding  $x_{t-1}$  to the formula (4) means that the new price is dependent on the previous price time-serie, expressing the autocorrelation observed in electricity prices.

The switching mechanism is governed by the aforementioned Markov probability.<sup>5</sup> As explained before, the change of being in regime 1 or 2 in the future only depends on the present value. (Hull, 2008). The changes of being in one of the regimes is summarized in a transition probability as follows by Weron:

A two-state regime model:  $X_t = \{1,2\}$ 



Transition probabilities:

$$\mathbf{Q} = (q_{ij}) = \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix} = \begin{pmatrix} q_{11} & 1 - q_{11} \\ 1 - q_{22} & q_{22} \end{pmatrix}$$

The chance to be in a base regime (4) is higher than the chance of being in a spiking regime (5) (Weron, 2005). The outcome of this simple structured model seems to work well. Later, literature evolves to a more sophisticated way of how the chances to switch between regimes are set. Diebold at al (1994) have argued that treating transition probabilities as fixed parameters is restrictive. Mount et al (2006) introduces time-varying parameters, where the mean prices in the two regimes and the two transition

<sup>5</sup> powerpoint presentation Rafal Weron, (2004): Pricing derivatives in electricity markets.

probabilities are function of the load and/or the implicit reserve margin. They conclude that regimeswitching model can forecast spikes better and represent the volatile behavior of electricity prices set in a competitive environment associated with these price spikes effectively. Because of the lack on numbers on reserve margin, De Jong and Huisman (2002) introduce an alternative to measure time-varying parameters, namely temperature. Data about temperature are publicly available and decreases the time to get data.

Some disadvantages of this new method can be observed: the first comment is related to a practical matter, namely the calibration of this complex method and therefore time consuming. With respect to the content; the restrictive feature is that the variance for each regime is constant. Mount et al (2006) proves that this is unrealistic and argues that the probability of spike occurrence is time-varying. Debate is still going on about the best approach of this time-varying probability with the aim to mimic volatility as most accurate. Bierbrauer et al. (2004) state a regime switching model based on a Markov probability results in an unrealistic amount of spiking events in the electricity price path. Lastly, sometimes is seems difficult to combine regime-switching with seasonality's. ( spot market model for derivative pricing in energy markets)

#### 3.3 Contemporary literature

Nowadays, literature devotes time to the heteroskedasticity specifications of the volatility found by Karakatsani and Bunn (2004). Escribano et al (2002), noticed that adding a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) component to volatility, increases the log-likelihood value of the models. The models with GARCH and jump components outperformed in most electricity markets. Also Andopolous (2008) concludes; 'In general we find that the addition of the GARCH and EGARCH specifications for the volatility process improves the fit for most energy markets, and we can get to the conclusion that the models are generally consistent with each other for all the energy markets.' In addition, Andriosopoulos (2008) finds that introducing an EGARCH specification to the model mimics the volatility asymmetry quite well.

Capturing the time lacking component for the volatility of the underlying asset of an option also has valuation consequences. Still, there is discussion about the accuracy of this GARCH component, as Knittel and Roberts (2005) conclude statements opposite to those of Andriosopoulos & Nomikos (2008) and Escibano et al (2002). Huurman et al (2008) combine in their study fundamental information as temperature and wind predictions in order to increase the goodness-of-fit for electricity price forecasting. Their model is an example of the aforementioned combination of a fundamental with econometric approach. Using a combination of a GARCH specification and a weather component improves the accuracy of the electricity price forecasts.

Concluding, because electricity prices seem to be non-linear, accuracy is improved when jumps are incorporated into the model. All models without jumps can considered to be less accurate. Regime switching models are cheered because of their high accuracy. Contemporary literature devotes more time to the time-varying parameters, relating them to external variables as base load or temperature. More research is needed on the heteroskedastic character of volatility of electricity prices.

## **CHAPTER 4 CASE STUDY**

In this section our research question is applied to a case study. The case study is a valuation of a windpark. The value is determined by the flexibility of a switchable tariff offered by the government by a real option analysis written by Yu et al.(2005). The underlying of this switchable tariff, expressed in a vanilla call option, is the hourly price path of electricity. This paragraph elaborates on the way Yu et al. (2005) build their real option analysis and concentrates on the choice of the stochastic process of the underlying. The stochastic process chosen by the writers is aligned with the aforementioned literature about stochastic processes and criticized later because the chosen stochastic process does not align with contemporary perception.

#### 4.1 Introduction Case study "Valuation of switchable tariff for wind energy"

Due to increasing volatility the market risk has risen. This increased market risk forms an obstacle for the introduction of renewable energy into the electricity market. A trend of governments trying to integrate wind energy into the market has started. This supporting attitude of the government is necessary because dispatching wind electricity on the grid brings along many complications as a consequence of the characteristics of this commodity. The fact is that prediction of the generated load is hindered by the intermittent nature of wind, which puts some technological challenges to the system.

System Operators, responsible for the balancing on the grid, generally have rules for over or under dispatching the expected value on the grid, resulting sometimes in great imbalancing costs. Despite the challenges, governments try to increase the penetration levels of wind energy on the grid. Denmark is leading with a wind energy dispatch of 20% on the grid nowadays. Followed by countries as Germany and Spain as leading countries in dispatching wind energy on the grid. (Rivier Abbad, 2009)

In 2002, the Spanish government introduced a remuneration system for wind generators, called Royal Degree 2002, with the aim to stimulate wind energy on the Spanish grid. Wind generators were put to the choice between a fixed remuneration for their generated electricity selling/bidding into the daily spot market or they received the market price plus a bonus. Instead of a fixed regulated price, the time-varying and location dependent value of renewable energies now were taken into account. This remuneration system is also called switching tariff. Because wind generators have the capability to produce power in spiking situations, this power is expected to be worthy more in this new remuneration scheme (Yu et al, 2005) and influencing the overall

value of the wind park. Yu et al (2005) capture the choice between the two remuneration schemes – or: the value of this switching tariff – into a real option analysis and calculate this inherit flexibility for the wind generators given by the Spanish government. Where traditional valuation models ignore marginal flexibilities, real option analysis lends itself perfectly for this (Goulding, 2001).

#### 4.2. Switching tariff modelled by Yu et al (2005).

The switching tariff gives exogenous operating flexibility and is modelled as a real compound option on wind spread. A compound option gives the holder the right but not the obligation to choose between two scenarios. Wind spread is defined as: the price of electricity set in the daily market minus the cost of the generated wind. The price in the new remuneration scheme can either be the fixed tariff, or the market price plus a bonus of 50% of the market price itself, resulting in a wind spread on fixed prices and a wind spread on market prices.

Spread options are widely used in valuations of generation assets for example by Gourding 2001. Option spreads derive their value from the difference between the prices of two or more assets. The actual payoff results from the process of the two or sometimes more underlings minus the strike price. In the wind park case, drivers of the switchable tariff option are wind and electricity. For our analysis the latter is important and therefore wind scenarios are left out of consideration. The stochastic process of electricity has influences the value of the switchable tariff. The cost of generating wind are assumed to be small and for the sake of simplicity negligible assumed.

In this case, a wind generator has two possibilities: 1. Choosing the fixed tariff or don't generate any electricity, 2. Choosing the market tariff including a bonus of 50% or don't generate any electricity. The compound option is simplified by Yu et al (2005) by assuming that a wind park is always operating, because of the cost of generating wind is assumed to be lower that the electricity price in both remunerations. Adding this information results in capturing the switchable tariff together in a vanilla call option on the market electricity prices including the bonus with as strike price the amount of fixed tariff. This fixed tariff is 90% of the average price in that year.

#### 4.3 Stochastic model to approach the Spanish spot market.

Yu et al (2005) work with hourly spot market prices of the Spanish market of 2003 for a period of one year. They use a Geometric Brownian motion with mean reversion and seasonality to mimic the behaviour of Spanish hourly electricity prices *(6)*:

$$d(\ln P_t^E) = m^E (\ln P_t^E - m_t^E) dt + s^E dW_t^E$$
(6)

Here  $m^E$  is the mean-reverting component, seasonality is corrected by  $m_t^E$  and  $s^E dW^E$ stands for the volatility and Wiener process, respectively. With this model they predict hourly electricity prices and by a Monte Carlo Simulation. How the parameters of abovementioned formula are estimated is not mentioned in the article. Their work aims at valuing yearly and monthly switching tariff as compound real options, and concludes that monthly switching tariff is of more value to wind generators for its great flexibilities and accuracy of short term-forecast.

#### 4.4 Comments on the approach of Yu et al.

To relate this to our research question: does the choice of how the evolution of this price is modelled have influence on the value of the call option, in this case the value of the switchable tariff? Previous chapter concludes that continuous processes excluding jumps into the analysis results in a less accurate evolution of the electricity price and therefore the estimation of the financial derivative. This inaccuracy is mostly caused by the approximation of the volatility. A higher volatility of the underlying is being translated into higher (simple) option value (Smit and Trigeorigs, 2004). When working with a GBM with mean reversion, volatility parameters are usually higher on average than when a volatility is split up in two parts, one related to the normal uncertainty and one reflecting spiking events. These spiking events have a high volatility and it can be expected that taking these volatilities into account, approaches the high volatility of the electricity spot price better. By dividing up the calculation process in two components the effect on call option value is estimated. Derived from literature the hypothesis is tested:

Hypothesis I: the use of a discontinuous time process incorporates the spikes' high volatility better and thus drive up the value of the call option.

In the next chapters the influence of jumps on the call option value will be examined, by comparing price patch calibrated by a Monte Carlo simulation with different stochastic processes, one with and one without jump components. The accuracy of each stochastic model is estimated. Later, the vanilla call option of both situations is calculated and compared.

## **CHAPTER 5 METHODOLOGY**

#### 5.1 Continuous time model - GBM with mean reversion

The GBM Yu et al (2005) worked with (6) is reconstructed in this research with an adjusted equivalent (7). Prices are estimated by the formula  $s_i = d_i + x_i$ , which contains a deterministic- $(d_i)$  and a stochastic component  $(x_i)$  corrected by mean reversion:

$$dt = \mu + \beta_1 W_t$$
  

$$x_t = (1 - \alpha) x_{t-1} + \sigma \varepsilon_{2,t} | S_{t-1}$$
(7)

Abovementioned equation is normally distributed and the Wiener process makes future prices independent from past prices. The variance of the equation is held constant during the estimation. Like in the case study of Yu et al (2005) this study makes use of the transformed time series (natural logarithm). The equation also captures the autoregressive nature by electricity prices identified in the market by adding the last observed price into the equation  $(x_{t-1})$ . Making future log prices only dependent on log prices of the previous day seems to predict prices well according to Huisman (2009). Seasonality is captured by  $\beta_1 W_6$ , a weekend dummy only. Annual seasonal fluctuations are already captured by the autoregressive component (Weron, 2008). This model serves as the continuous time model, which is compared with the discontinuous model, introduced in the next paragraph.

#### 5.2 Discontinuous time model - Regime Switching Model

In literature jumps are incorporated in the evolution of price processes in different ways, for example using a MRJD or a regime-switching process. Both correct for the asymmetric type of volatility (Mount et al, 2005). This research prefers to work with a regime switching model with a Markov probability determining the switch between regimes. Specifying the models with such a probability allows the high-price regime to be more persistent than is the case in a MRJD model (Mount et al, 2005). This research on purpose chooses a pure price model without exogenous variable in order to compare both methods by isolating the jumps. Alike the mean reversion model, the Markov probability makes the change of being in a spiking regime dependent on a random factor. Also, the process is following normality. Both equations will be expressed in natural logarithm, so the distributions are lognormally distributed. As already described in the literature review, the regime switching model of Huisman (2008) lends itself perfectly for the restrictions made in this paragraph (8). For clarity reasons once repeated:

 $dt = \mu + \beta_1 W_t$ 

 $x_t = (1 - \alpha) x_{t-1} + \sigma \varepsilon_{2,t} \mid S_{t=1}$ 

$$x_t = \mu_2 + \sigma \varepsilon_{2,t} \mid S_{t=2}$$

Although the regime switching model devotes to each regime a set of parameters, these will be constant over time. Despite the fact that literature sees constant volatility sometimes as a restrictive factor and introduce GARCH models, there are some underlying reasons for specifying constant variances in each regime. (Mount et al, 2006). First of all the explanatory factor of these GARCH models is not convincing yet. (Knittel and Roberts, 2005). Secondly, when pricing derivatives it is important to know the overall variance of the spot price, and this is achieved with constant volatility as well. (Mount et al, 2006)

#### 5.3. Parameters estimation and goodness-of-fit

The parameters that fit the model relatively best of each process are calibrated by using a Log likelihood method. This is a derived from the Maximum Likelihood method, used quite often. Parameters are optimized in Scilab such that the error terms of the equation are at most likely or random as possible. The chosen distribution of the error term is normality for both processes with a mean of zero and standard deviation of one (Huiman 2009). The log-likelihood method maximizes the natural logarithm of the likelihood with respect to the parameters. This approach is assumed to lead to better optimizations, because instead of taking the product of log likelihoods the sum of the log-likelihoods are taken.

To judge which model with the generated parameters forecast electricity prices the best some studies suggest the generation of a LogLik value. A high LogLik value is considered to be a good indicator for the goodness-of-fit. Sadly, this approach is not appropriate for this research. Mainly because the models compared are not very alike (Huisman 2009). Therefore, this study assumes, based on contemporary literature, that incorporation of discontinuous models improve accuracy of the model (Ethier and Mount, 1998).

#### 5.4. Calculating the option value

Once the parameters are generated for both models, the Monte Carlo Simulation in Excel is used to forecast the price paths of electricity for each model. The Monte Carlo Simulation, a procedure of sampling random outcomes, works well in modelling payoffs that depend on the path followed by an underlying variable. (Hull, 2008) Especially with a set of several different, complex and non linear parameters, Monte Carlo proves excellence.

After applying a price path, the vanilla call option is reconstructed and the payoff is measured. The payoff is expressed in P max (0;S-K). Where S is the market price in the remuneration scheme, gained by the simulated electricity price times 50%, minus the strike price, the average electricity price from the remuneration scheme. The payoff is measured in months as Yu et al. (2005) showed that monthly switching tariff are of more value than yearly payoff. As explained by Yu et al. (2005) a generator is allowed to switch between remuneration schemes monthly. This payoff is run 1000 times by the Monte

Carlo simulation. The result is an average payoff; the flexible value of the monthly remuneration scheme expresses in euros per MWh. In a real option analysis the average payoffs are according to a risk-neutral environment, so they must be corrected for risk because this research works with real world data.<sup>6</sup> Goulding (2001) applied a 5% discount factor in a real option valuation of generation assets, which is adopted in this research as well.

<sup>6</sup> powerpoint presentation Rafal Weron, (2004): *Pricing derivatives in electricity markets.* 

# **CHAPTER 6 DATA AND DESCRIPTIVE ANALYSIS**

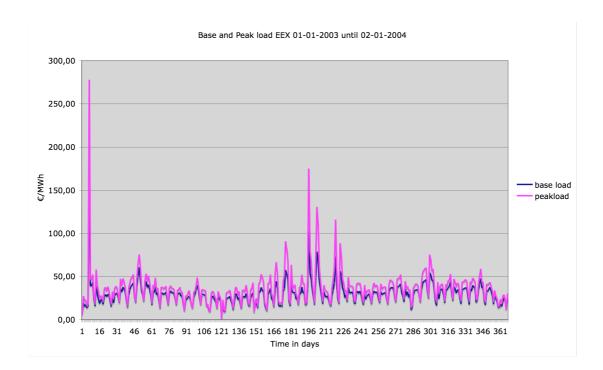
In our search to answer the question in what way the stochastic process influences the value of the call option in a real option analysis we use historical data of the European Energy Exchange (EEX). This section describes the data used for analysis. Characteristics of the data are discussed in the light of the stylized facts that are typical for the electricity spot price market. Furthermore, the alignment between models and the data is discussed.

In 2002 the EEX was established as a result of a merger between two German power exchanges and became the largest power exchange in Europe (Kosater 2006).<sup>7</sup> Germany allows just like the Spanish electricity market a relatively high amount of wind energy on the grid. Instead of hourly prices of electricity per kWh, this study prefers day-ahead spot prices of base and peak load per MWh of the dayahead-spot market. The data ranges over a period of 01-01-2003 until 02-01-2004. The data of 2004 contain the Thursday and Friday that complete week 1 of the year 2004. In total summed up to 367 observations.

Figure 1 plots the time series of base of peak load EEX over the year 2003. It shows that on average base prices are below peak prices. From the figure one exception is remarkable. Around the 122<sup>nd</sup> day in 2003, peak prices spike downwards to 0,80 €/MWh. Log prices even become negative at that specific moment, as shown in figure 2. Average prices of electricity of the EEX are for base load 29,43 €/MWh, and for peak load 36,91 per €/MWh.

Figure 1:

<sup>7 &</sup>lt;u>http://www.eex.com/en/EEX</u>

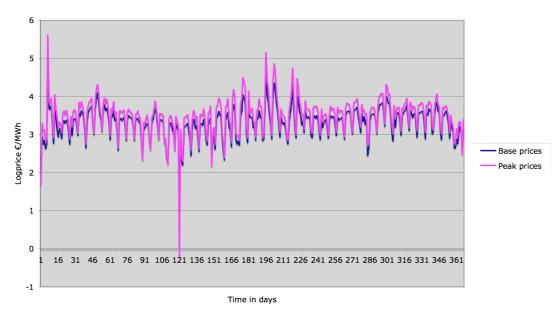


The mean reverting behaviour of the EEX is visible in the data. In comparison with other spot day-ahead-markets the mean reverting behaviour is not that severe. Mean reversion components and the autoregressive character of the equations should accommodate this mean reversion. Also, the EEX seems not to have much influence of annual seasonality. Similar conclusion was drawn by Weron (2007) who didn't observe a clear annual seasonality in the German exchange market looking at data ranging from 2001 until 2003. The motivation to run an analysis on EEX data instead of Spanish data is that results are not influenced by an annual seasonality. Nevertheless, weekly prices seem higher than weekend prices, which is captured in the equations by adding a weekend dummy and a seasonal parameter coefficient. Weekdays are marked with a 1 and weekends with a 0. A dummy lowers the deterministic components of both stochastic models with the amount of the parameter when a weekend day is observed. Another phenomenon observed from the data is that spiking behaviour in the EEX does seem seasonal. Only in summer and winter spikes occur. Again, Weron (2007) concludes similar. Europe experienced in the summer of 2003 a heat wave (Huurman et al, 2007).

Figure 2 shows log transformations of base and peak of EEX 2003. The graph shows a steady behaviour of the electricity price instead of an upward trend.

#### Figure 2:





Interestingly to show is the empirical distribution of the EEX in 2003. Results are graphed in Figure 3, found in appendix A.

The empirical distributions illustrated, show a highly non-normal character and exhibit the features of a leptokurtic distribution. Non-normality occurs when a price path experiences large price moments (Andriosopoulos and Nomikos, 2008). Thus, higher values have a higher chance of occurring in the EEX data. Extreme or rare event occur more often influenced by the volatility. Furthermore, the so-called fat tail is noticeable which explain the high skewness level generated from data. Table 1 shows the skewness and kurtosis level of both prices and log prices of base and peak load.

	Base Prices	Peak Prices	Log Base	Log Peak
Skewness	3,767549568	5,204972841	0,510209742	-1,14038926
Kurtosis	32,50391071	48,66159039	3,190071018	9,581857661

Both base and peak prices experience high positive skewness and excess kurtosis. Remarkable is the negative skewness in log peak prices. Huurman et al. (2007) gained negative skewness in hourly day ahead EEX data of 2004. The high kurtosis levels in the absolute prices of the EEX are are noticed by other researchers before. Escribano et al. (2002) described six various energy markets and the markets of Argentina, Australia (Vicotria), and the U.S (PJM) all show a similar shaped empirical distribution with similar corresponding skewness and kurtosis levels. Secondly we can conclude that price returns in the EEX market exhibit quite some volatility (Figure 5 and 6).

Essential for reliable research, the stylized facts described up till now in this chapter must be captured by the stochastic models chosen for our methodology. Mean reversion, seasonality and spiking events are captured. Though, at first glance the normality assumption seems violated, taken the fact that absolute prices seemed to follow a leptokurtic distribution. However, this possible flaw is solved by the expression of the equations in natural logarithms. So, the empirical distributions have more a normality shaped appearances, although base prices are clearer than peak prices. (Figure 7 and 8, Appendix A) This means that spiking behaviour is reduced by natural logarithms compared to absolute values (Huurman et al, 2007). Nevertheless, the reader must not ignore the negative skewness log peak prices exhibit.

The aim of this section was to gain more insight in the data and make a relation between the methodology and the data. Next section applies the methodology of the study on the EEX data.

# **CHAPTER 7: EMPIRICAL RESULTS**

In this section the empirical results of the switchable option value are presented and interpreted. For both methods, continuous and discontinuous, a distinction is made between base and peak prices. After a presentation of the parameters and a short descriptive analysis, price paths of base and peak prices are replicated by a Monte Carlo simulation and the payoff of the simple call option with P max (0; S-K), where S are market prices including a bonus of 50% and K is the strike price as being the fixed tariff, is calculated. In the last paragraph, time is devoted for understanding the obtained outcomes of the options. The hypothesis from Chapter 4 is recalled and conclusions are drawn, generally as well of this case study specifically. Finally the relevance of the conclusions is emphasized.

#### 7.1 Parameter estimates of GBM mean reversion- and Regime switching model.

As described in the methodology, the parameters are calibrated using a Log Likelihood method in Scilab. The estimates of the GBM mean reversion- and regime switching model are presented in table 2.

Parameter	GBM	Standard	GBM mean	Standard	Regime	Standard	Regime	Standard
	mean	error	reversion -	error	switching	error	switching	error
	reversion-		Peak prices		model –		model –	
	Base				Base Prices		Peak prices	
	prices							
μ	3.44	(0.030)	3.66	(0.034)	3.46	(0.022)	3.64	(0.033)
$\mu_2$	-		-		1.986D-12	(1831)	1.75	(0.011)
<b>β</b> 1	-0.46	(0.035)	-0.56	(0.045)	- 0.47	(0.025)	-0.55	(0.041)
α	0.52	(0.046)	0.61	(0.048)	0.51	(0.050)	0.59	(0.045)
<b>σ</b> <sub>1</sub>	0.29	(0.011)	0.37	( 0.014)	0.18	(0.011)	0.35	(0.013)
$\sigma_2$	-		-		0.77	(0.108)	0.24	(0.140)
$\lambda_1$	-		-		2.89	(0.409)	5.19	(0.728)
$\lambda_2$	-		-		0.10	(0.569)	- 22.7	(1448)
$P_{i,t}=Pr(S_{t=i}   S_{t-i})$	-		-		0.95		0.99	
1= <b>i</b> )								
$P_{i,t}=Pr(S_{2=i}   S_{t-}$	-		-		0,52		1.307D-10	
2=i)								

First of all, focus on the GBM mean reversion results. As expected, log mean price\_peak  $\mu$  is higher than log mean price\_base  $\mu$ . In Chapter 6, the peak price in general moved above the base price. The outcome is in line with observations before. In weekends EEX experiences lower prices and indeed,  $\beta_1$  shows for both models negative outcomes. The peak price is corrected in weekends more in comparison with base price. This is explained by less demand for peak load in weekends compared with peak prices throughout the week. In addition, the mean reverting component  $\alpha$  of peak price is higher than base. Peak price moves in general further above the equilibrium and therefore has to adjust more strongly. The quantitative measure of volatility  $\sigma_1$  of peak price is higher than for base prices, again in line with observations made in Chapter 6, where log returns on peak prices seem more volatile. All outcomes are significantly different from zero.

Now concentrate on the regime switching results. It can be seen that for the log mean prices  $\mu$  in do not substantially differ. Remarkable is the difference between the mean prices of the non-nomal state  $\mu_2$ . A recall of the formulas of the regime switching model (8) can be find in Chapter 5. For the base price,  $\mu_2$ approaches almost zero, on the other hand peak prices exhibit a noticeable higher value of 1, 75. Clearly, the regime switching model for peak prices gives a noticeable higher value in a non-normal state in comparison with base prices. Consequently, a deviation from the long run equilibrium peak price is immediately a substantial one in comparison with base prices. Instead, regime switching model with peak prices has a lower volatility in non-normal state( $\sigma_2$  0.24) than in the normal regime ( $\sigma_2$  0.35). Moreover, base prices in non-normal regime is the spiky state characterized a high volatility  $\sigma_2$  value of 0,77 and no substantial deviation of the long run average. Interestingly, is to interpreted the transition probabilities. The transition probabilities  $\lambda_1$  and  $\lambda_2$  are converted to conditional probabilities  $P_{i,t}=Pr(S_{t-1}|S_{t-1-1})$  and  $P_{i,t}=Pr(S_{2-i}|S_{t-2-i})$ , respectively the probability of staying in normal state and the probability of staying in a nonnormal state. The following can be concluded; the chance of switching between a normal and to a nonnormal regime considering peak prices is small, but if this occurs the effect of a spike is substantial. Analyzing base prices, the chance of getting in a non-normal regime is remarkably higher, but the size of the spike itself has not such an impact, given  $\mu_2$  value of almost zero, but a clearly substantially volatility.

The parameter estimates generated seems reasonable and in line with aforementioned characteristics of the EEX market and general electricity price behaviour and in line with previous studies. The difference in volatility values between the GBM mean reversion model and the regime switching model is well described by Weron (2008). In general the volatility of the mean reversion model is higher in the compared with regime switching models in normal state as an outlier, in this context a spike, influences the mean reversion coefficients. Instead, the volatility accompanied with spiky events are captured in the regime switching model by the non-normal state. What the effect is on call option value is examined in the next paragraph.

#### 7.2. Call option value

With the parameter estimates generated in the previous paragraph day ahead electricity price paths of both base and peak a Monte Carlo Simulation for each model simulates prices in Excel.

Recalls that the option is a vanilla call option calculated as P max (0; S-K). Strike price value K is 90% of the average price and in this case set on  $26,487 \notin$ /MWh and  $33,219 \notin$ /MWh. Yu et al. (2005) examined the flexible value of the remuneration scheme on monthly values. 1000 Simulations resulted in the following flexible monthly values, listed in table 3:

Table 3:

	GBM mean	GBM mean	Regime	Regime
	reversion - base	reversion - peak	switching - base	switching - peak
Average Payoff in €/MWh per	505	920	588	828
month				

It can be seen that the GBM mean reversion model, underestimates the option value on base prices and overestimated the option value on peak prices. How is this related to the hypothesis build in Chapter 4:

Hypothesis I: the use of a discontinuous time process incorporates the spikes' high volatility better and thus drive up the value of the call option.

The hypothesis is valid for base prices, but remarkable does not apply for peak prices. A possible explanation for the overestimation of option values by a continuous time model is, already found by Weron (2008), that one outlier influences, in this context a spike, upwards the coefficients of the whole estimation and therefore results in an overestimation of the option value. The data of a simulated peak price by GBM mean reversion is shown in Figure 10. Clearly, the price hovers with great amplitude around the long run equilibrium. In contrast with the regime switching model for peak prices where the model mimics a realistic amount of spikes a year and only occasional in a month (Figure 12). From the data of the EEX 01-01-2003 until 02-01-2004 around 4 substantial spikes were visible. When the probability of getting into a non-normal regime regarding peak prices is recalculated, the same amount op substantial spikes is gained.

The underestimation of the value by a continuous time model is explained by the fact that is does not mimic enough the volatility and therefore underestimates.

#### 7.3. Conclusion

In general this research concludes that using Geometric Brownian Motion with mean reversion in a real option analysis, underestimates the call option value applied to base prices but tends to overestimates the call option applied to peak prices. To apply this conclusion to the case study specifically; the flexible tariff was introduced by the Spanish government to be a incentive for generators on the grid. Such a decision from wind generators perspective must be based on a realistic future option value. This is not the case as concluded before. A wind generator must take the over and underestimation into account when taking a decision based on a simulation with a continuous time model like the GBM with mean reversion.

Nevertheless, some remarks must be taken into account. For a decision on base prices only the consideration of accepting the flexible remuneration scheme can be justified by the fact that the underestimation is not that substantial and computational time with a continuous time model is slightly shorter.

Moreover, the Spanish market suffers from some idiosyncratic features (Escribano et al, 2002). The conclusion of this research is based on EEX data only. Escribano (2002) noticed in his study that adding spikes and GARCH components into the analysis did not improve accuracy for the Spanish market. He explained this by the way the Spanish market is structured.

## REFERENCES

Andriosopoulos, K., Nomikos, N., (2008). *Modelling energy spot prices: "Empirical evidence from NYMEX*. Cass Business School, Londen ECIY 8TZ, UK.

Baker, M., Scott, M., Parsons, J., (1998). Alternative Models of Uncertain Commodity Prices for Use with Modern Asset Pricing Methods. The Energy Journal 19 (1): 115-148.

Bierbrauer, M., Trück, S., Weron, R., (2004). *Modeling electricity prices with regime switching models*. Lecture Notes in Computer Science, 3039, 859-867.

Blanco, C., Soronow, D., (2001a). Energy Price Processes: Used for Derivatives Pricing & Risk Management. Commodities Now, March 2001, Financial Engineering Associates.

Blanco, C., Soronow, D., (2001b). Mean Reverting Processes: Energy Price Processes used for Derivatives Pricing & Risk Management. Commodities Now, June 2001, Financial Engineering Associates.

Blanco, C., Soronow, D., (2001c). Jump Diffusion Processes: Energy Price Processes used for Derivatives Pricing & Risk Management. Commodities Now, September 2001, Financial Engineering Associates.

Bunn, D., Karakatsani, N., (2003). Forecasting Electricity Prices. London Business School, EMG Working Paper.

Goulding, A., (2001). What is it worth? Application of real options theory to the valuation of generation assets. London Economic International LLC, Working Paper.

Dixit, A., Pindyck, R., (1994). Investment Under Uncertainty. Princeton, NJ: Princeton University Press.

Diebold, F., Lee, J., Weinbach, G., (1994). Regime-switching with time-varying transition probabilities. Federal Reserve Bank of Philadelphia, No. 93-12 in Working Paper.

Ethier, R., (1997). *Competitive Electricity Markets and Long Term Investment*. Departmental Seminar, Dept. of Agricultural, Resource and Managerial Economics, Cornell University. Ithaca, NY.

Ethier, R., Mount, T., (1998). Estimating the volatility of spot prices in restructured electricity markets and the implications for option values. Cornell University, Working Paper.

Escribano, A., Pena, J., Villaplana P., (2002). *Modeling electricity prices: International evidence*. Discussion paper Universidad Carlos III de Madrid.

Fusia, G., Roncoroni, A., (2008). Implementing Models in Quantitative Finance: Methods and Cases. Springer.

Hamilton, J., (1994). Time Series Analysis. Princeton University Press.

Hamilton, J., (1989). A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. Econometrica 57 (2): 357-384.

Higgs, H., Worthington, A., (2008). Stochastic price modeling of high volatility, mean-reverting, spike-prone commodities: The Australian wholesale spot electricity market. Energy Economics 30,

Huisman, R., de Jong, C., (2003). *Option pricing for power prices with spikes*. Energy Power Risk Management 7.11, 12–16.

Huisman, R., Mahieu, R., (2003). Regime jumps in electricity prices. Energy Economics 25, 425-434.

Huisman, R., Huurman, C., Mahieu, R., (2007). *Hourly Electricity Prices in Day-Ahead markets*. ERIM Report Series Research In Management, ERS-2007-002-F&A.

Huisman, R., (2008). The influence of temperature on spike probability in day-ahead power prices. Energy Economics 30, 2697-2704.

Huisman, R. (2009). An Introduction to Models for the Energy Markets: The Thinking behind Econometric Techniques and Their Application. London: RISK Books.

Hull, J., (2008). Option Futures and other and derivatives. 17th edition. Prentice Hall.

Huurman, C., Ravazzolo, F., Zhou, C., (2007). The power of weather: Some empirical evidence on predicting dayahead power prices through weather forecasts. TI 2007-036/4. Tinbergen Institute Discussion Paper

Jong de, C., Huisman, R., (2002). Option Formulas for Mean-Reverting Power Prices with Spikes. Energy Global.

Karakatsani, N., Bunn, D., (2004). *Modeling the Volatility of Spot Electricity Prices*. London Business School, EMG Working Paper.

Knittel, C., Roberts M., (2005). An empirical examination of restructured electricity prices, Energy Economics 27(5), 791-817.

Kosater, P., (2006). On the Impact of Weather on German Hourly Power Prices. University of Cologne. Discussion Paper 1/06.

Mount, T., Ning, Y., Cai, X., (2006). Predicting price spikes in electricity markets using a regime-switching model with time-varying parameters. Energy Economics 28, 62–80.

Pindyck, D., (1999). The long-run evolution of energy prices. The Energy Journal 20, 1-27.

Smit, H., Trigeorgis, L., (2004). Strategic investment: real options and games. Princeton University Press.

Weron, R., Misiorek, A., (2007). Heavy tails and electricity prices: Do time series models with non-Gaussian noise forecast better than their Gaussian counterparts? Prace Naukowe Akademii Ekonomicznej we Wrocławiu Nr 1076, 472-480.

Weron, R., Misorek, A., (2008). Forecasting spot electricity prices: A comparison of parametric and semiparametric time series models. Munich Personal RePEe Archive.

Weron, R., Simonsen, I., Wilman, P., (2004). *Modeling highly volatile and seasonal markets: evidence from the Nord Pool electricity market.* The application of Econophysics, H. Takayasu (ed.), Springer, Tokyo, 182-191.

Weron, R., (2005). Market price of risk implied by Asian-style electricity options. Wrochlaw University of Technology, Poland.

Weron, R., (2006). Modeling and Forecasting Electricity Prices with Time Series Models. John Wiley & Sons.

Weron, R., (2008). Forecasting wholesale electricity prices: A review of time series models, in Financial Markets: Pricinples of Modelling, Forecasting and Decision-Making. eds W. Milo P. Wdowinski, FindEcon Monograph Series, WUL Lodz.

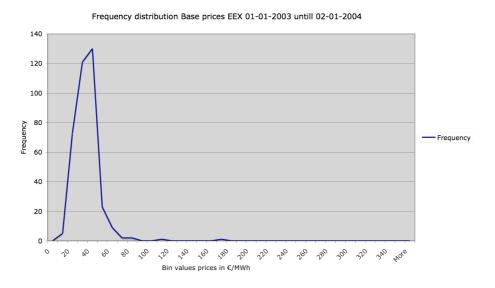
Pilipovic, D., (1997). Energy Risk: Valuing and Managing Energy Derivatives. New York, NY: McGraw-Hill.

Rivier Abbad, J., (2009). *Electricity market participation of wind farms: the success story of the Spanish pragmatism*. Energy Policy, doi:10.1016/j.enpol.2009.07.032

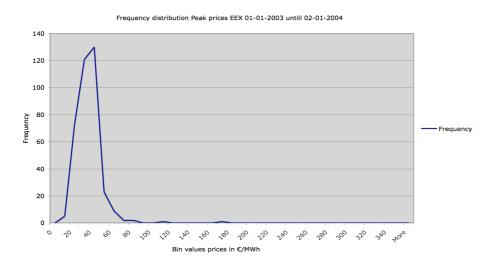
Yu, W., Sheble, G., Pecas Lopes, J., Matos, M., (2005). *Valuation of switchable tariff for wind energy*. Electric power systems research, Elsevier.

# APPENDIX A [Appendix title]

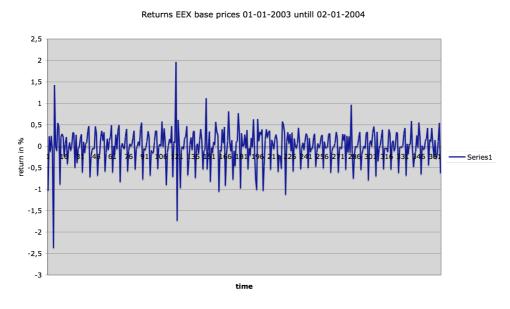
# Figure 3:





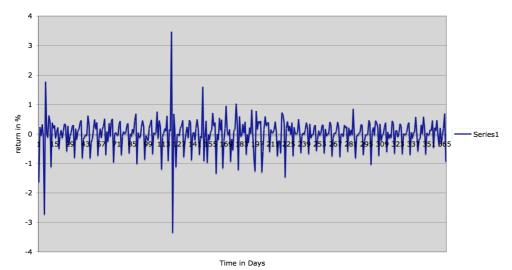








Returns EEX peak prices 01-01-2003 untill 02-01-2004



25



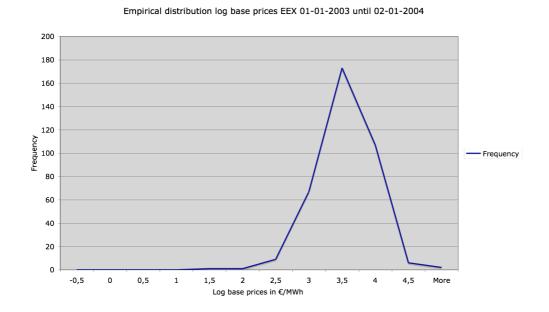
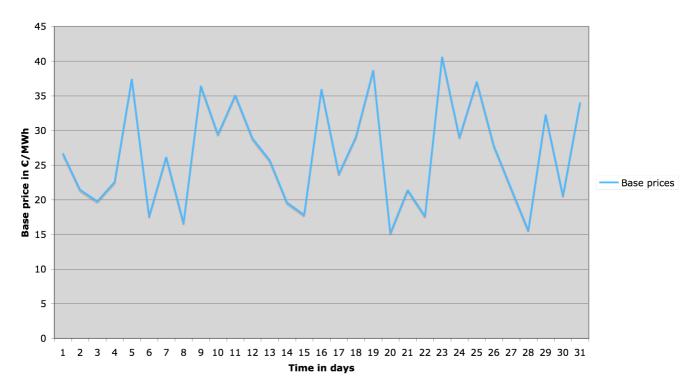


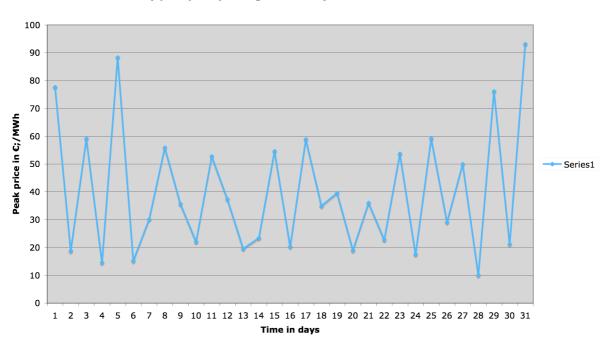
Figure 8:





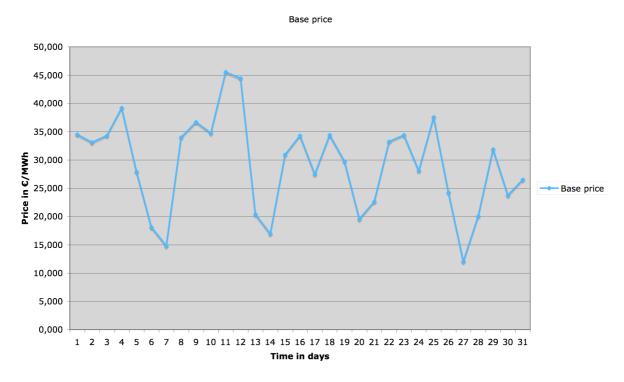
#### Simulation Base prices GBM mean reversion

Figure 10:



Montly path peak prices generated by GBM mean reversion model





Simulation Peak price by regime switching model.

