Realized Semibetas and Downside Risk

Master Thesis Financial Economics

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Abstract

Conversely to the Capital Asset Pricing Model and its downside adaptation, the newly proposed four-way decomposition of the traditional market beta into semibetas based on semicovariances is better able to capture the non-linear asymmetric dependencies in stock returns. As illustrated in this research, the superior forecasting ability embedded in the model translates to statistically and economically significant risk premiums for the semibetas associated with negative returns on the asset, thereby resolving the downside risk puzzle. The results are consistent with a model of ambiguity averse agents. The concordant negative semibeta is the main driver of the results, and consequently, this beta on "steroids" raises the bar in forecasting systematic risk and downside risk.

Keywords: Downside risk; realized (semi)(co)variance; semibetas; return predictability; cross-sectional return variation.

JEL: C58, G10, G11, G12, G17.

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Contents

"*Decisions taken in practice are less concerned with whether a little more of this or of that will yield the largest net increase in satisfaction than with avoiding known rocks of uncertain position or with deploying forces so that, if there is an ambush round the next corner, total disaster is avoided.*" – A. D. Roy (1952).

1. Introduction

Although the traditional Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965) prevails acknowledged as the foremost used asset pricing model among practitioners, ensuing evidence shows that the linear relationship of the expected excess return on an asset and its beta with the aggregate market portfolio is not sufficient to explain the cross-section of stock returns (Black et al., 1972). The conventional mean-variance analysis relies either on the assumption that returns are jointly normally distributed, or one assumes that investors manifest quadratic utility. The distribution of asset returns notwithstanding, shows signs of skewness and kurtosis, and in parallel, an individual that possesses quadratic preferences will disregard extreme adverse events in the left tail of the distribution of stock returns. Thus, in its essence, the traditional CAPM weights gains and losses uniformly, while it is recognized since Roy (1952) that investors are more susceptible to downside risk than to upside potential. Hence, the *semi*variance of negative stock returns is a more satisfactory measure of risk than the variance as it considers the asymmetric distribution of stock returns (Markowitz, 1959). The traditional beta is therefore an inadequate hedging measure during periods it is desired the most, specifically during times of market distress. Based on these insights, many theoretical models have been proposed such as the mean-semivariance CAPM (MS-CAPM) of Hogan and Warren (1972, 1974), the lower partial moment (LPM) framework of Bawa and Lindenberg (1977) and the generalization of Fishburn (1977) hereof, that controls for skewness and kurtosis in the distribution of asset returns.

The empirical literature largely corroborated the evidence that risk measures based on the concept of semivariances are more aligned with the actions of practitioners (Mao, 1970). Similar inferences hold in the behavioral framework of prospect theory, developed by Kahneman and Tversky (1979), where investors exhibit behavior of loss-aversion. Altogether, this indeed implies that investors place a larger emphasis on the left tail of asset returns and that they should be compensated for holding assets that covary strongly with the market when the aggregate market portfolio diminishes in value. However, empirical research on the pricing of downside risk in the cross-section of returns has been scarce until Ang et al. (2006) decomposed the regular beta into an asymmetric downside and upside beta, that are conditional to respectively market declines and upturns, to generate the downside version of the CAPM (D-CAPM).[1] The disintegration of the betas reveals that there is a premium present for the bearing of downside risk and therefore the D-CAPM offers a more conforming description of the cross-section of stock returns. Post and van Vliet (2004) support these findings as the MS-CAPM outperforms the traditional CAPM in tests conditional on market performance. The risk-return relation is especially discernible during bad states of the world, thereby advocating for the breakdown of risk into downside risk and upside potential. Building on this evidence, Bali et al. (2009) use Value at Risk (VaR), Expected Shortfall (ES), and tail risk as proxies for downside risk and as well find a positive relationship between downside risk and expected returns. Additionally, Lettau et al. (2014) expand the analysis to other asset classes like equity index options, sovereign bonds, commodities, and currencies. Correspondingly, the relationship between downside risk and expected stock returns is also evident. To recapitulate the above-mentioned empirical analyses, the CAPM conditional on downside risk, generally, provides a more thorough description of the cross-section of stock returns than the traditional unconditional CAPM.

In contrast to these findings, recent developments in the literature illustrate that the downside beta is not a persistent factor and that downside betas are inadequate to explain the cross-section of stock returns. Alongside these findings, Atilgan et al. (2018) and Atilgan et al. (2020b) fail to find a positive relationship between downside betas of international stocks and their expected returns in the cross-section. Atilgan et al. (2019) extend the analysis to other definitions of downside risk, such as tail betas, LPM, VaR, and ES measures, but also observe a negative relation between downside risk and expected returns. Atilgan et al. (2020a) give a more in-depth explanation for the left-tail momentum effect. Whereas in the process of under-diversification to higher-order moments one would expect lower prices to offset the increased probability and magnitude of large losses, it appears to be the case that investors underreact to information. Anchored beliefs of

[1] Empirical tests have been presented earlier on in the literature, albeit that these tests suffered from low statistical power (see for example the tests of Jahankhani (1976) on the MS-CAPM and that of Harlow and Rao (1989) on the LPM framework).

individuals accompanied by the gradual diffuse of information, assures that this effect is more conspicuous during events where bad news becomes available (i.e., during extreme adverse market events and financial crises). In conjunction, Levi and Welch (2020) state that downside betas are unable to outperform the predictions of traditional CAPM betas. Unexpectedly, the prevailing traditional beta is a more meaningful predictor for crashes than downside betas. This view is affirmed by the fact that the traditional beta is a better predictor for the downside beta in the following period than the current downside beta itself is. Barahona et al. (2021) provide a model of ambiguity averse agents and explain the connection between beta predictability and the pricing of risk. The model implies that lower (higher) predictability in betas results in declining (increasing) hedging demands and thus decreasing (increasing) risk premiums. Downside betas and VIX betas are not subject to a significant risk premium when ex-ante measures are used in the construction of portfolios.

Building on these inferences, one could deduce that the downside version of the CAPM does not provide an improvement over the conventional CAPM and practical applications for the D-CAPM cease to exist. Phrased in a different manner, the two-way decomposition is not a thorough measure of downside risk to satisfactorily explain the cross-section of stock returns. However, the two-way decomposition has the attractive feature that it values downside losses and upside gains in an asymmetric manner because stocks that perform poorly when the market drops should hold a risk premium. Nonetheless, stocks that perform well when the market drops should hold a negative risk premium as these stocks reduce total portfolio risk and thus can be applied as hedging instruments. Therefore, the two-way decomposition fails to fully derive the information that is captured in the asymmetric distribution of stock returns. The four-way decomposition as proposed in Bollerslev et al. (2022) alleviates this concern as it distinguishes *semi*betas by their signed covariation between the return of the individual asset and the aggregate market portfolio. The semibetas are constructed by dividing the *semi*covariance of the return on the individual asset with the return on the aggregate market portfolio by the variance of the return on the aggregate market portfolio. The novel CAPM that is comprised of semibetas leads to conclusions that harmonize with above mentioned theoretical predictions. There is a significant relationship between the semibetas associated with negative market returns and expected stock returns in the cross-section, but positive variation in market returns is not priced. An in-depth view of the semibetas reveals a non-linear asymmetric dependence structure, i.e., individual stocks that covary differently with the market will be priced in a separate manner. The results remain robust when accounting for other famously studied effects in the asset pricing literature and additional downside risk measures. The semibeta CAPM therefore provides a superior description of the cross-section of expected stock returns compared to other models, and consequently can be viewed as the best in its class.[2]

Although the breakthrough of this advanced model has relighted research into the downside risk literature, Bollerslev et al. (2022) refrain from a more rigorous analysis of the performance and forecasting abilities of the individual semibetas. Bollerslev et al. (2020a) manifest the superior performance of variance forecasts that exploit the information in the asymmetric distribution of stock returns by using realized semicovariances. Therefore, it is intriguing to investigate to what extent the four individual semibetas hold additional information compared to other beta measures and what the outperformance in forecasting drives. In doing so, the dynamics between the four semibetas and other beta measures are analyzed.

Constructively, this paper revisits the definition of traditional systematic risk. In other words, the traditional CAPM beta could effectively be a measure that is proxied better by one or potentially some combination of its underlying semicovariance-based components. Furthermore, it is interesting to apply these results in an asset pricing framework. Thus, to assess which semibetas earn a risk premium and to examine the relationships with downside risk. Hence, this research is pertinent for risk management purposes. The core inquiry of this research is therefore to evaluate the superior forecasting capacity of a four-way decomposition of the market beta into semibetas and its ability to resolve the downside risk puzzle. This central question is answered by providing three main contributions to the literature.

[2]Note that this supposition must be interpreted cautiously as there is an abundance of studies in the literature that analyze the dependencies of stock returns and downside risk. However, the majority of these studies rest on the incorporation of non-linear methods and the use of option data, while the concept of semibetas preserves linearity. The other methods comprise of far less intuitive measures and results and suggest misspecification of the models, e.g., different classes of GARCH models and volatility smiles and smirks in the options framework. These methods can therefore be deemed to be a class of their own.

First of all, using similar techniques as in Levi and Welch (2020), who refute the two-way decomposition, it is clear that the four-way decomposition of the market beta into semibetas yields superior forecasts. Upside and downside betas do not provide additional predictive value in estimating future betas. In line with previous research, the traditional beta is an even better predictor of the future downside beta than the current value of the downside beta itself is. On the contrary, a four-way decomposition provides more forecasting power and signals that there are non-linear asymmetric dependencies present among the semibetas. Much of the higher predictive performance can be attributed to the concordant negative semibeta, which is a more stable and better predictor of plain market betas in the period ahead than the current market beta itself is. Similarly, the concordant negative semibeta is a better predictor of downside risk as well.

Secondly, with the knowledge of this superior foreseeing behavior, the pricing implications of a fragmentation into semibetas is tested in the cross-section. Incorporating a more coarse return horizon than Bollerslev et al. (2022) with monthly betas constructed of daily returns, it is exhibited that semibetas concerned with negative variation in the return of the asset are subject to a statistically significant risk premium. These premiums also translate into economic significant gains. After controlling for well-established factor exposures in the literature, the negative concordant semibeta yields a significant annualized alpha of 5.13 percent. The GRS test statistic further underscores the outperformance in Sharpe ratios compared to the five-factor model of Fama and French (2015). These results are in line with the economic rationale found in Barahona et al. (2021), where ambiguity averse agents value predictability in the beta, thereby increasing hedging demand and risk premiums. Hence, the findings are able to resolve the downside risk puzzle.

Finally, the results are robust to different estimation windows and techniques. Nonetheless, inspired by the heightened caution stressed in Harvey et al. (2016), this research explores whether these empirical observations also spill over to non-classical financial research techniques. The random forests machine learning technique is able to capture non-linearities among the explanatory variables. The results show that a random forests classification problem is able to accurately forecast one-day-ahead outperformance of stocks. As indicated by the predictions, the semibetas encapsulate substantial information on top of that obtained by the market beta. Moreover, the semibetas are able to capture additional risk premiums and models including semibetas attain higher Sharpe ratios. The findings of the non-classical asset pricing techniques therefore resonate well with the results of the traditional methodology.

All in all, the four-way decomposition of the market beta into semibetas provides superior forecasting capabilities and holds valuable information that is left uncharted by the traditional beta. This additional information embedded in the semibetas with negative asset return variation is priced in the cross-section, and the results pertaining these semibetas subsequently resolve the downside risk puzzle.

There is a wide existing literature on the mean-semivariance framework and the decomposition of the traditional beta into downside and upside betas as outlined in the aforementioned studies. Because the four-way decomposition of the traditional CAPM beta into semibetas is a relatively new concept, research on the topic is scarce. This paper connects several strands in the literature that are related to the concept of semibetas. These include, to mention a few, the literature on tail and bear betas, good and bad volatility, and crash risk (Kelly and Jiang (2014), Van Oordt and Zhou (2016), Moreira and Muir (2017), Chabi-Yo et al. (2018), Lu and Murray (2019), Kapadia et al. (2019), Bollerslev et al. (2020b), Chabi-Yo et al. (2021), Baruník and Nevrla (2021), and Wang (2022), among others).

The remainder of this paper is structured as follows. Section 2 formalizes the concept of realized semicovariances and the ensuing four-way decomposition of semibetas. Section 3 presents the data that is employed in the empirical analysis. Section 4 discusses the enhanced forecasting ability of the semibetas. Section 5 considers the pricing of the semibetas in the cross-section. Section 6 reports results for non-classical asset pricing techniques. Section 7 concludes.

2. Realized Semicovariances and Semibetas

This section provides a concise overview of the underlying forces of stock price movements and econometric interpretation of the CAPM and the decompositions thereof. The importance of realized semi(co)variances and resulting semibetas lies central in the framework pioneered by respectively Andersen et al. (2001) and Barndorff-Nielsen and Shephard (2002). Andersen et al. (2006) shaped the empirical literature by applying these statistical inferences to construct realized betas. Analyzing this framework to a further extent highlights the main motivation for the empirical investigations.

2.1. Econometric Model of Stock Price Processes

To build the general framework of the development of asset prices over time, the model of Back (1991) is considered. Let $X_t = \{X_{1,t}, X_{2,t}, ..., X_{K,t}, t \geq 0\}$ denote a K-dimensional continuous-time continuous-state stochastic process of log-prices of the financial assets, indexed $k = 1, 2, ..., K$ at time t on the filtered probability space $(\Omega, \mathcal{F}, \mathbb{P})$. In this setting, the probability measure $\mathbb{P}$ defines the probabilities of reaching the possible states of the world denoted in Ω. The information up and until time t is described in the information filtration $\mathcal{F}_t$, where $\mathcal{F}_t \equiv \sigma(X_s | s \leq t)$.[3] The σ-field $\sigma(X_s|s \leq t)$ resembles the covariance matrix Σ for the subsets of X_0 until X_t. Explained differently, the filtration space defines the complete history of the asset prices, where $\mathcal{F}_s \subseteq \mathcal{F}_t$ for $0 \leq s \leq t \leq 1$, which represent the time period for which trade prices are available.

Further, a martingale can be defined as a stochastic process such that $\mathbb{E}_{\mathbb{P}}(X_t|\mathcal{F}_s) = X_s$ for all $s \leq t$. Therefore, conditional on the information available at time s, the expected value of the price of a financial security is its current price. Then, in an arbitrage-free market the log-price process X_t is a special semi-martingale, where the process can be fragmented into the sum of a local martingale and a finite path of variation:

$$X_t = X_0 + A_t + M_t \tag{1}$$

where M_t represents the unpredictable development of the local martingale around the deterministic drift of the locally finite variation process A_t. These processes are defined on the same dimensions as X_t and as such it follows mathematically that at time $t = 0$, $A_0 = M_0 = 0$. The unique canonical decomposition of the semi-martingale is special, because the finite variation process is assumed to be predictable and thus known at time t. Equation (1) defines the general setting, which include Itô processes, that are defined by the following stochastic differential equation (SDE) through the martingale representation theorem (Andersen et al., 2003):

$$X_t = X_0 + \int_0^t a_s \, ds + \int_0^t \sigma_s \, dW_s \tag{2}$$

where a denotes the locally bounded dimensional drift process, which together with the starting value at time $t = 0$, X_0, forms the predictable component. Furthermore, σ

[3]The filtration is augmented and therefore said to satisfy the usual conditions of right-continuity and completeness.

denotes a càdlàg (right-continuous process with limit on the left) $K \times K$ volatility process and W is a Wiener process with K dimensions (commonly referred to as the standard Brownian motion with $\sigma = 1$). The process W is obtained by the limiting process of a re-scaled sequence of independent Rademacher variables. The probability distribution function for Rademacher variables specifies an equal probability of going up or down, thereby bearing resemblance to the more economically familiar term of the random walk hypothesis of stock prices. Thus, in a setting where asset prices develop continuously, the K-dimensional process of stock returns R_t of the time span $[0, t]$ can be denoted as follows:

$$R_t = X_t - X_0 = \int_0^t a_s \, ds + \int_0^t \sigma_s \, dW_s \tag{3}$$

Note that this model does allow for leverage effects, referring to the tendency of the volatility of a stock to be negatively correlated with its return. Nelson (1991) observes the asymmetric behavior of this effect, where declines in prices are associated with greater increases in volatility than perceived for positive price movements. Equation (2) formalizes the no-jump setting, where asset prices develop continuously. However, stock prices show jumps as a reaction to sudden public news announcements Engle and Ng (1993). Allowing for these instantaneous jumps to occur, the semi-martingale can therefore be modified to

$$X_t = X_0 + \int_0^t a_s \, ds + \int_0^t \sigma_s \, dW_s + J_t \tag{4}$$

where J_t represents a pure jump process with jumps in X formulated as $\Delta X_t = X_t - X_{t-}$. X_{t-} is characterized by a càglàd process (left continuous process with limit on the right) such that $X_{t-} = \lim_{s \to t, s \leq t} X_s$.

2.2. Estimating Realized (Semi)(co)variances

The framework outlined above describes the stochastic process of log-prices and their returns. However, the SDEs do not convey how to estimate the variance of the stochastic prices of the assets. Building on the implications of Barndorff-Nielsen and Shephard (2002), let the realized variance (RV) be an estimator of the ex-post variation in the log-price process X_t:

$$RV \equiv \sum_{i=1}^{[T/\Delta_n]} (\Delta_i^n X)^\top (\Delta_i^n X) \tag{5}$$

where the time grid is partitioned into n sampled observations of realized stock prices on any time horizon $T > 0$, which is normalized to one. The time grid is then defined by

$\{i\Delta_n : 0 \leq i \leq [T/\Delta_n]\}$ with the ith return of X defined as $\Delta_i^n X \equiv X_{i\Delta_n} - X_{(i-1)\Delta_n}$. Further, it is demonstrated that this estimator converges to the quadratic variation (QV) at time one once more data becomes available[4]:

$$RV \xrightarrow{p} [X]_1 \tag{6}$$

where $[X]_1$ denotes the QV, which is a measure of the variance of a stochastic process such as those defined in the SDEs. As a matter of fact, the limiting operation of the QV process contains all information on ex-post variation in X_t for the Brownian semi-martingale defined in equation (2):

$$[X]_t = \int_0^t \sigma_s^2 \, ds, \tag{7}$$

$$d[X]_t = \sigma_t^2 \, dt \tag{8}$$

The RV statistic is therefore compelling and of great importance in describing fluctuations in asset prices. Consequently, in a no-jump setting, the realized variance by itself is sufficient to capture all the variation in asset prices. However, as the statistic is evaluated by squared returns, the sign of the return is inconsequential. Nevertheless, as previously argued, stock prices do not entirely develop continuously and exhibit additional shocks throughout the process. The QV of the process outlined in the jump-setting summarized in equation (4) at time t then transforms to:

$$[X]_t = \int_0^t \sigma_s^2 \, ds + \sum_{s \leq t} (\Delta X_s)^2 \tag{9}$$

The QV process now depends on an additional source of risk, which does not differentiate between positive and negative price jumps and therefore does not capture the asymmetric behavior of asset prices.[5] Thus, the realized variance does not extract information from the sign of the return on the aggregate market portfolio. As underlined in the introduction, the semivariance measure accounts for asymmetry in the distribution of stock returns and is therefore a gauge of downside risk. Following Barndorff-Nielsen et al. (2008), the realized variance can be disintegrated into the downside realized semivariance (RSV^-)

[4]This asymptotic inference is customarily specified as “in-fill asymptotics”.

[5]Withal, employing the bipower variation measure of Barndorff-Nielsen and Shephard (2004b) allows for an estimation robustly to jumps. However, this still does not extract information pertaining the asymmetric behavior of stock prices.

and upside realized semivariance (RSV^+) as follows:

$$RSV^- \equiv \sum_{i=1}^{[T/\Delta_n]} n(\Delta_i^n X)^\top n(\Delta_i^n X) \xrightarrow{p} \frac{1}{2}\int_0^t \sigma_s^2 \, ds + \sum_{s\leq 1} (\Delta X_s)^2 I_{\{\Delta X_s \leq 0\}} \tag{10}$$

$$RSV^+ \equiv \sum_{i=1}^{[T/\Delta_n]} p(\Delta_i^n X)^\top p(\Delta_i^n X) \xrightarrow{p} \frac{1}{2}\int_0^t \sigma_s^2 \, ds + \sum_{s\leq 1} (\Delta X_s)^2 I_{\{\Delta X_s \geq 0\}} \tag{11}$$

where $n(x) \equiv min\{x, 0\}$ and $p(x) \equiv max\{x, 0\}$ respectively define the vectors of negative and positive elements of the realized stock prices x. In accordance, the RSV^- and RSV^+ statistics converge to their bisected counterpart of the QV of the log-price process defined in equation (4). Note that I represents an indicator function that equals one if the statement in the brackets is true and zero otherwise. Therefore, an asymptotic analysis of the realized semivariances is able to differentiate between the sign of price jumps, while still maintaining the composite relationship $RV = RSV^- + RSV^+$. The signed jump variation can be defined as:

$$\Delta J_t \equiv RSV^+ - RSV^- \xrightarrow{p} \sum_{s\leq 1} (\Delta X_s)^2 I_{\{\Delta X_s \geq 0\}} - \sum_{s\leq 1} (\Delta X_s)^2 I_{\{\Delta X_s \leq 0\}} \tag{12}$$

Conforming the hypothesis of so-called co-jumps in prices and volatility, Patton and Sheppard (2015) find that negative jumps attribute to higher volatility levels in the future as opposed to positive jumps. Therefore, the preceding analysis shows that semivariances are able to capture the asymmetric behavior of price movements of stocks. Nevertheless, semivariances neglect certain information that is related to co-drifting, which entail steady price movements that occur for multiple stocks simultaneously. These movements are generally associated with news that is more difficult to interpret and correlate with the sign of the market.[6] Analogous to regular price jumps for an individual asset or co-jumps of price and volatility for an asset, stock prices can have simultaneous jumps, so-called co-jumps. Hence, semivariances fail to fully derive the movements of stock prices as the signed covariation of the asset with the market portfolio and with one another is not considered. To this extent, Bollerslev et al. (2020a) introduce the concept of *semi*covariances, which fragments the realized covariance matrix $\mathcal{C}$ as follows:

$$\mathcal{C} = \mathcal{N} + \mathcal{P} + \mathcal{M}^+ + \mathcal{M}^- \tag{13}$$

[6]Patton and Verardo (2012) find that idiosyncratic information affects the systematic risk of stocks and this information therefore leads to market-wide comovement in asset returns.

where $\mathcal{N}$ and $\mathcal{P}$ correspond to the concordant negative and positive realized semicovariance matrices and in parallel $\mathcal{M}^+$ and $\mathcal{M}^-$ are defined as the discordant mixed realized semicovariance matrices:

$$\begin{aligned}
\mathcal{N} &\equiv \sum_{i=1}^{[T/\Delta_n]} n(\Delta_i^n X) n(\Delta_i^n X)^\top, & \mathcal{P} &\equiv \sum_{i=1}^{[T/\Delta_n]} p(\Delta_i^n X) p(\Delta_i^n X)^\top, \\
\mathcal{M}^+ &\equiv \sum_{i=1}^{[T/\Delta_n]} n(\Delta_i^n X) p(\Delta_i^n X)^\top, & \mathcal{M}^- &\equiv \sum_{i=1}^{[T/\Delta_n]} p(\Delta_i^n X) n(\Delta_i^n X)^\top.
\end{aligned} \tag{14}$$

As shown in Bollerslev et al. (2020a), these realized semicovariance matrices converge in the probability limit to their latent counterparts. Realized semicovariances are able to capture additional information to that of realized semivariances in the form of co-drifting and co-jumps. Specifically, the $\widehat{\mathcal{P}} - \widehat{\mathcal{N}}$ spread is an estimator of signed co-jumps, and therefore intrinsically an estimator of deviation from Normality. Therefore, realized semicovariances are superior in describing the asymmetric fluctuations in asset prices.

2.3. CAPM (Semi)betas

Based on the theoretical framework established in previous subsections, the general playing field is established, which is concatenated to the empirical investigations in this paper as follows. In the financial econometrics literature, the Capital Asset Pricing Model (CAPM) as in Sharpe (1964) and Lintner (1965) is defined as:

$$\mathbb{E}[R] - \iota R_f = \frac{Cov(R, R_m)}{Var(R_m)} (\mathbb{E}[R_m] - R_f) \tag{15}$$

Where R is a column vector of size K that contains all the stochastic stock returns and ι denotes a K-dimensional column vector with ones. R_f and R_m respectively present the return on the risk-free asset and the aggregate market portfolio. The excess return on the assets $\mathbb{E}[R] - \iota R_f$ corresponds proportionally to the ratio of the covariances of the returns on the assets with the return on the aggregate market portfolio and the variance of the return on the aggregate market portfolio. This ratio is commonly referred to as the beta of a stock:

$$\beta \equiv \frac{Cov(R, R_m)}{Var(R_m)} \tag{16}$$

The beta of a stock is the most important measure of systematic risk and therefore an accurate estimation of its components is pivotal for portfolio management decisions (Li, 2015). However, due to the stochastic nature of stock returns and their (co)variances, true betas are unknown. Building on the implications of Barndorff-Nielsen and Shephard (2002, 2004a) on realized variances (RV) and realized covariances ($RCov$), Andersen et al. (2006) construct realized betas that are consistent estimators of true latent ex-post betas:

$$\widehat{\beta}_{k,t} \equiv \frac{RCov_{k,t}}{RV_t} = \frac{\sum_{i=1}^{[T/\Delta_n]} r_{k,t,i} r_{m,t,i}}{\sum_{i=1}^{[T/\Delta_n]} r_{m,t,i}^2} \xrightarrow{p} \beta_{k,t} \tag{17}$$

where the estimated univariate beta $\widehat{\beta}_{k,t}$ of stock k over the time period t is thus determined by respectively the return on stock k, $r_{k,t,i}$, and the return on the aggregate market portfolio m, $r_{m,t,i}$, over the time period t with $[T/\Delta_n]$ intraperiod intervals. However, tracing back to the work of Markowitz (1959), semivariances consider the asymmetric nature of asset returns. Applying these into the realized beta framework as elaborated in Ang et al. (2006), yields two separate betas:

$$\widehat{\beta}_{k,t}^{-} \equiv \frac{\sum_{i=1}^{[T/\Delta_n]} r_{k,t,i} r_{m,t,i}^{-}}{\sum_{i=1}^{[T/\Delta_n]} (r_{m,t,i}^{-})^2}, \qquad \widehat{\beta}_{k,t}^{+} \equiv \frac{\sum_{i=1}^{[T/\Delta_n]} r_{k,t,i} r_{m,t,i}^{+}}{\sum_{i=1}^{[T/\Delta_n]} (r_{m,t,i}^{+})^2}. \tag{18}$$

where the estimated downside (upside) beta $\widehat{\beta}_{k,t}^{-}$ ($\widehat{\beta}_{k,t}^{+}$) is only calculated on days for which the return on the market portfolio is negative (positive), with $r_{m,t,i}^{-} \equiv min(r_{m,t,i}, 0)$ ($r_{m,t,i}^{+} \equiv max(r_{m,t,i}, 0)$). Thus, the betas account for the asymmetric distribution of stock returns by conditioning on the sign of the return on the aggregate market portfolio.[7] As elaborated extensively, semivariances fail to fully derive the asymmetric behavior of stock prices as it does not account for co-drifting and co-jumps. Stocks tend to covary with the aggregate market portfolio and Bollerslev et al. (2022) show that the traditional market beta can be disaggregated into four components as follows:

$$\beta \equiv \frac{Cov(R, R_m)}{Var(R_m)} = \frac{\mathcal{N} + \mathcal{P} + \mathcal{M}^{+} + \mathcal{M}^{-}}{Var(R_m)} \equiv \beta^{\mathcal{N}} + \beta^{\mathcal{P}} - \beta^{\mathcal{M}^{+}} - \beta^{\mathcal{M}^{-}} \tag{19}$$

where $\mathcal{N}$, $\mathcal{P}$, $\mathcal{M}^{+}$, and $\mathcal{M}^{-}$ represent the semicovariance part of the total covariance of the asset with the market for the respective negative, positive, and mixed-sign states of the

[7] Ang et al. (2006) initially use the realized average return on the aggregate market portfolio as the target. However, implementing other cutoff points like the risk-free rate of return or the zero rate of return does not alter the results: "*the finding of a downside risk premium is being driven by emphasizing losses versus gains, rather than by using a particular cutoff point for the benchmark.*" In the same light, Bollerslev et al. (2021) investigate the concept of partial (co)variances and find with machine learning techniques that, when using a single threshold, the zero cutoff point remains superior.

world as in equation (14). The positive and negative state of the world are respectively defined by positive and negative returns on both the asset and the aggregate market portfolio. The mixed-sign $\mathcal{M}^+$ ($\mathcal{M}^-$) component is characterized by negative (positive) return on the asset, while the aggregate market portfolio exhibited a positive (negative) return. $\beta^{\mathcal{N}}$, $\beta^{\mathcal{P}}$, $\beta^{\mathcal{M}^+}$, and $\beta^{\mathcal{M}^-}$ betoken the semibetas measures by their corresponding semicovariance:

$$\beta_{k,t}^{\mathcal{N}} \equiv \frac{\mathcal{N}_{k,t}}{RV_t}, \qquad \beta_{k,t}^{\mathcal{P}} \equiv \frac{\mathcal{P}_{k,t}}{RV_t}, \qquad \beta_{k,t}^{\mathcal{M}^+} \equiv \frac{-\mathcal{M}_{k,t}^+}{RV_t}, \qquad \beta_{k,t}^{\mathcal{M}^-} \equiv \frac{-\mathcal{M}_{k,t}^-}{RV_t} \tag{20}$$

The mixed-sign semicovariances are negative by construction and therefore the corresponding semibetas are designed to be positive by adding a minus sign.[8] However, these semibetas stand for true betas and have not manifested yet. True latent betas can be estimated by their realized counterparts as shown in Barndorff-Nielsen and Shephard (2004a) and Bollerslev et al. (2020a):

$$\begin{aligned}
\widehat{\beta}_{k,t}^{\mathcal{N}} &\equiv \frac{\sum_{i=1}^{[T/\Delta_n]} r_{k,t,i}^- r_{m,t,i}^-}{\sum_{i=1}^{[T/\Delta_n]} r_{m,t,i}^2} \xrightarrow{p} \beta_{k,t}^{\mathcal{N}}, \qquad \widehat{\beta}_{k,t}^{\mathcal{M}^+} \equiv \frac{-\sum_{i=1}^{[T/\Delta_n]} r_{k,t,i}^- r_{m,t,i}^+}{\sum_{i=1}^{[T/\Delta_n]} r_{m,t,i}^2} \xrightarrow{p} \beta_{k,t}^{\mathcal{M}^+}, \\
\widehat{\beta}_{k,t}^{\mathcal{P}} &\equiv \frac{\sum_{i=1}^{[T/\Delta_n]} r_{k,t,i}^+ r_{m,t,i}^+}{\sum_{i=1}^{[T/\Delta_n]} r_{m,t,i}^2} \xrightarrow{p} \beta_{k,t}^{\mathcal{P}}, \qquad \widehat{\beta}_{k,t}^{\mathcal{M}^-} \equiv \frac{-\sum_{i=1}^{[T/\Delta_n]} r_{k,t,i}^+ r_{m,t,i}^-}{\sum_{i=1}^{[T/\Delta_n]} r_{m,t,i}^2} \xrightarrow{p} \beta_{k,t}^{\mathcal{M}^-}.
\end{aligned} \tag{21}$$

The construction of the semibetas is closely related to that of the downside and upside beta, where the following relationship holds:

$$\beta_{k,t}^+ = (\beta_{k,t}^{\mathcal{P}} - \beta_{k,t}^{\mathcal{M}^+}) \frac{\sum_{i=1}^{[T/\Delta_n]} r_{m,t,i}^2}{\sum_{i=1}^{[T/\Delta_n]} (r_{m,t,i}^+)^2} \tag{22}$$

$$\beta_{k,t}^- = (\beta_{k,t}^{\mathcal{N}} - \beta_{k,t}^{\mathcal{M}^-}) \frac{\sum_{i=1}^{[T/\Delta_n]} r_{m,t,i}^2}{\sum_{i=1}^{[T/\Delta_n]} (r_{m,t,i}^-)^2} \tag{23}$$

Therefore, if in each period the risk premiums of the betas with positive market returns (and likewise for the components of negative market returns) are equal, the semibeta-CAPM collapses to the D-CAPM. To put the fragmentation of semibetas into the traditional risk perspective, two thoughts should be contemplated. First of all, if stock returns and its aggregate market portfolio do follow a multivariate jointly Normal distribution, the concordant and discordant betas are similar and thus do not hold additional information over the traditional beta such that $\beta^{\mathcal{N}} = \beta^{\mathcal{P}}$ and $\beta^{\mathcal{M}^+} = \beta^{\mathcal{M}^-}$. It is broadly

[8]The concordant semicovariances are defined as the sum of outer-products of the same vector, which by default results in a positive semidefinite matrix. However, the sum of the vectors for the mixed semicovariance produce a hollow matrix, making it indefinite by construction.

accepted by the finance literature that this condition does not hold in practice thereby advocating for the decomposition into downside risk and upside potential formulated into semibeta terms. Secondly, acknowledging that the Normal distribution does not hold for stock returns, in a financial market free of frictions, the premia of $\beta^{\mathcal{P}}$ and $\beta^{\mathcal{M}^+}$, and equivalently $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$, must equal as any long-short position is able to arbitrage away the discrepancies. Nevertheless, Shleifer and Vishny (1997) perceive that short-selling constraints and arbitrage risk induce differing risk premiums for both the components associated with either negative or positive market returns. Limits-to-arbitrage, and thus the spread between $\beta^{\mathcal{N}} - \beta^{\mathcal{M}^-}$ and $\beta^{\mathcal{P}} - \beta^{\mathcal{M}^+}$ may further amplify during times of market turmoil and flights to liquidity (Brunnermeier and Pedersen, 2009). The semibetas thus have a time-varying nature. Hence, as all semibetas vary in magnitude over time, it connotes that each semibeta contains supplementary knowledge on top of that obtained by the others.

2.4. Coskewness and Cokurtosis

Besides the four-way decomposition of the traditional CAPM beta into four semibeta components, other downside risk measures have been advocated by the literature. These include as argued before, the betas of the D-CAPM, but also variates that capture higher-order moments with non-linearities such as coskewness (CSK) of Kraus and Litzenberger (1976) and cokurtosis (CKT) of Dittmar (2002). These gauges of non-Normality are calculated as follows:

$$CSK_{k,t} = \frac{\frac{1}{[T/\Delta_n]} \sum_{i=1}^{[T/\Delta_n]} (r_{k,t,i} - \overline{r}_{k,t})(r_{m,t,i} - \overline{r}_{m,t})^2}{\sqrt{\frac{1}{[T/\Delta_n]} \sum_{i=1}^{[T/\Delta_n]} (r_{k,t,i} - \overline{r}_{k,t})^2} \frac{1}{[T/\Delta_n]} \sum_{i=1}^{[T/\Delta_n]} (r_{m,t,i} - \overline{r}_{m,t})^2} \tag{24}$$

$$CKT_{k,t} = \frac{\frac{1}{[T/\Delta_n]} \sum_{i=1}^{[T/\Delta_n]} (r_{k,t,i} - \overline{r}_{k,t})(r_{m,t,i} - \overline{r}_{m,t})^3}{\sqrt{\frac{1}{[T/\Delta_n]} \sum_{i=1}^{[T/\Delta_n]} (r_{k,t,i} - \overline{r}_{k,t})^2} (\frac{1}{[T/\Delta_n]} \sum_{i=1}^{[T/\Delta_n]} (r_{m,t,i} - \overline{r}_{m,t})^2)^{3/2}} \tag{25}$$

where $\overline{r}_{k,t}$ and $\overline{r}_{m,t}$ represent the mean return on the asset and aggregate market portfolio over the specified time interval T. These measures place more emphasis on the tails of the probability distribution function of stock returns. Consequently, coskewness and cokurtosis serve as proxies for tail risk, and may therefore provide supplemental information over that disclosed by the semibetas.

3. Data

This section provides the sample construction and descriptive statistics of the data that is employed in the empirical analysis. Thereafter, the development of the semibetas over time is analyzed.

3.1. Data Sample and Descriptive Statistics

The data sample constitutes of daily data of stock returns for all contemporaneous stocks during the time span of July 1963 to December 2021 acquired from the Center for Research in Security Prices (CRSP) database. The aggregate value-weighted market portfolio rate of return and the risk-free rate of return (one-month Treasury Bill from Ibbotson and Associates, Inc.) are obtained from Kenneth French's library.[9]

Recession periods are defined according to the National Bureau of Economic Research (NBER). Following the basic premise in the literature, the sample is constricted to the following criteria. First of all, only stocks with share codes 10 and 11 are included in the sample. Furthermore, penny stocks, those with a price that falls below the threshold of five dollars, are excluded from the sample. Including these stocks would generate illiquidity concerns as these stocks suffer from wide bid-ask spreads. Monthly realized betas are established with daily returns, where a stock had to have at least 15 valid observations in a month to be included. The final data sample contains 299.287 ex-ante forecastable firm-month observations of the semibetas. This robust data sample provides a great balance between different samples in the literature as it takes into account the critique on the downside risk study of Ang et al. (2006) while adhering to the definitions in the original paper of Bollerslev et al. (2022).[10]

Table 1 lists the descriptive statistics for the sample. The estimates of the realized betas match the aggregate findings of Bollerslev et al. (2022), but nevertheless show an

[9] `https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`

[10] Ang et al. (2006) assert that "*one month of daily data provides too short a window for obtaining reliable estimates of downside variation*". On the contrary, testing different possibilities of overlapping and non-overlapping windows in the estimation of both monthly and yearly realized betas constructed with daily returns results in qualitatively similar distributions. Further, demeaning the returns is inconsequential, corroborating the evidence of French et al. (1987). Lastly, winsorizing returns at the 1 and 99 percent level or according to the specific winsorization as in Levi and Welch (2020) does not qualitatively change the distributional properties of the semibetas.

Table 1: **Descriptive Statistics.** Panel A displays the time series averages for the cross-sectional statistics. Monthly realized betas, coskewness and cokurtosis measures are established with daily returns of non-overlapping windows for all common, non-penny, CRSP stocks during the time span of July 1963 to December 2021. Panel B presents the time series averages of the Pearson's correlation coefficients of the estimated betas in the cross-section.

Panel A: Summary Statistics									
	β	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	β^+	β^-	CSK	CKT
Mean	0.85	0.55	0.70	0.23	0.17	0.86	0.81	-0.03	1.12
Median	0.78	0.48	0.61	0.16	0.11	0.77	0.75	-0.03	1.19
St.Dev.	0.77	0.35	0.45	0.23	0.20	0.98	1.09	0.30	0.89
Panel B: Correlation Matrix									
	β	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	β^+	β^-	CSK	CKT
β	1.00	0.71	0.77	-0.33	-0.32	0.83	0.76	0.02	0.69
$\beta^{\mathcal{N}}$		1.00	0.47	0.10	-0.06	0.36	0.86	-0.27	0.44
$\beta^{\mathcal{P}}$			1.00	-0.02	0.09	0.89	0.35	0.27	0.41
$\beta^{\mathcal{M}^+}$				1.00	0.31	-0.46	-0.07	-0.14	-0.44
$\beta^{\mathcal{M}^-}$					1.00	-0.07	-0.53	0.15	-0.44
β^+						1.00	0.34	0.29	0.56
β^-							1.00	-0.30	0.59
CSK								1.00	0.00
CKT									1.00

estimate below unity for the traditional beta. Levi and Welch (2020) argue that these differences could either arise from portfolio weighting or the non-monotonous relationship between the market capitalization of an asset and the market beta. Financial securities with a smaller market capitalization and higher expected betas are more likely to be excluded from the analysis, thereby suppressing the mean of the traditional beta. Therefore, a market beta below unity is not a result of price non-synchronicity, which is a proxy for firm-specific variation in stock returns.[11] As previously established, if stock returns were to follow a joint Normal distribution, the relation $\beta^{\mathcal{N}} = \beta^{\mathcal{P}}$ and $\beta^{\mathcal{M}^+} = \beta^{\mathcal{M}^-}$ would hold.

[11]The use of the aggregate equal-weighted market portfolio rate of return obtained from the CRSP database reinstates the relationship as the time series average of the cross-sectional mean for the traditional beta then equals unity again.

However, preliminary data analysis conveys this is not the case, as can be inferred from the top panel of Table 1.[12] Similarly as in Bollerslev et al. (2022), the correlations of the semibetas and the traditional beta are far from perfectly collinear, insinuating that the semibetas hold valuable information that is left uncharted by the traditional beta. Striking to the conclusions of Ang, Chen, and Xing (2006), the average β^{+} and β^{-} do not differ much from the traditional market beta. Secondly, the two-way decomposition correlates more with the traditional beta than the four-way decomposition does, with simultaneous non-perfect collinearity between the betas. Altogether, this preliminary analysis suggests that the semibetas emanate additional information to that appropriated by the traditional beta and that of its downside modification.

Figure 1 shows the time-series development of the monthly semibetas over the time span of July 1963 to December 2021. Both of the concordant semibetas and the discordant semibetas seem to move in a similar direction, in line with the positive correlations listed in Table 1. The concordant negative semibeta is the most stable over time. Interestingly,

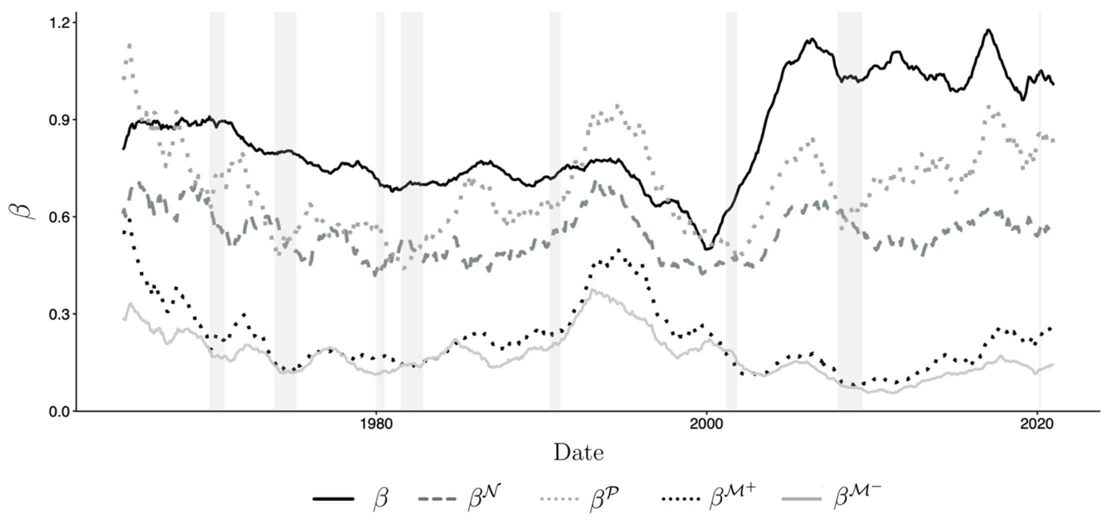

Figure 1: **Time-Series Analysis of Monthly Semibetas from July 1963 to December 2021.** The figure shows the time-series development of the monthly traditional beta and semibetas over the period of July 1963 to December 2021. The series are smoothed using a month $t-12$ to month $t+12$ moving average. The shaded regions represent recession periods as defined by the NBER.

[12]Auxiliary tests, such as QQ-plots and the Jarque-Bera goodness-of-fit test for Normality, reach similar conclusions. The Jarque-Bera test jointly tests for skewness and kurtosis and indicates that the distribution of returns is characterized by fatter tails, possibly caused by excessive returns during intervals of panic and euphoria.

the $\beta^{\mathcal{N}} - \beta^{\mathcal{P}}$ spread is much more precarious than the spreads of the negative semibeta with the discordant components. This should come as no surprise, as the $\mathcal{P} - \mathcal{N}$ spread is an estimator of signed co-jumps, and unexpected news is likely to cause fluctuation in this spread over time.

4. Predictability of Semibetas in the Cross-Section

The empirical analysis commences with the examination of the predictive performance of the semibetas relative to the forecasting power of the traditional CAPM beta and its downside adaptations. Inherently, this section is of great importance as further analysis relies on the incorporation of ex-ante semibetas in forecasting, hedging, and risk premium estimation. It is therefore a prerequisite that prevailing betas are able to accurately predict future betas.

4.1. Predictive Performance of Semibetas

The empirical analysis is initiated by showing that the preceding concordant semibeta of negative asset returns is more decisive in determining the traditional market beta in the next period than the market beta itself. Following the approach of Levi and Welch (2020), and thereby avoiding the critique on incorporating ex-post metrics, predictive regressions of various lagged betas on the beta in the current period are estimated. Throughout this analysis, it is important to maintain a focus on the relative predictive power of the betas. Tabel 2 reports the results of non-overlapping Pooled Ordinary Least Squares (OLS) regressions with clustered standard errors for monthly betas. Intercepts are included in each of the regressions, but not reported. Panel A reports the results for the traditional market beta and different specifications of lagged betas along with the t-statistics. The autoregressive (AR) model with a single lag shows that the traditional beta has a significant coefficient of 0.29, thus meaning that 29 percent of the historical beta translates into the future beta.[13]

[13] The first-order autocorrelations are downward biased due to measurement errors in the realized betas. Hansen and Lunde (2014) incorporate lags 4 through 10 as instrumental variables to estimate less noisy autocorrelations. These adjusted betas exhibit a higher degree of persistence than the autocorrelations that originate from a traditional OLS approach. Although this poses as additional evidence for the strong persistence for all realized betas, unadjusted measures henceforth used in the analysis suffice to show the superior relative predictive capability of the semibetas over other proposed beta measures.

Table 2: **Cross-Sectional Predictions of Monthly Betas with Lagged Betas with Pooled OLS.** Panel A and B respectively show the predictive regressions with β_t and $\beta_t^{\mathcal{N}}$ as dependent variables. The panels report coefficients and underneath t-statistics are calculated with clustered standard errors. Constants are included into the regression but not reported. Estimates are determined by all contemporaneous common, non-penny, CRSP stocks spanning from July 1963 to December 2021.

Panel A: Dependent Variable β_t												
β_{t-1}	0.29		0.29		0.21		0.32		0.30		0.30	
	5.88		*5.37*		*4.22*		*30.17*		*5.35*		*5.32*	
β_{t-1}^{+}		0.05	-0.01									
		2.60	*-1.01*									
β_{t-1}^{-}		0.05	0.00									
		3.37	*0.34*									
$\beta_{t-1}^{\mathcal{N}}$				0.54	0.30							0.51
				48.23	*5.14*							*32.31*
$\beta_{t-1}^{\mathcal{P}}$						0.24	-0.06					0.23
						3.46	*-0.94*					*3.35*
$\beta_{t-1}^{\mathcal{M}^+}$								-0.08	0.14			-0.17
								-5.90	*2.68*			*-6.77*
$\beta_{t-1}^{\mathcal{M}^-}$										-0.15	0.14	-0.20
										-7.42	*2.15*	*-8.98*
R^2	0.11	0.04	0.11	0.08	0.13	0.05	0.11	0.00	0.11	0.00	0.11	0.13
Panel B: Dependent Variable $\beta_t^{\mathcal{N}}$												
β_{t-1}	0.12				0.08							
	6.16				*4.51*							
β_{t-1}^{+}		0.02	0.02									
		2.55	*2.60*									
β_{t-1}^{-}		0.02	0.00									
		3.29	*0.83*									
$\beta_{t-1}^{\mathcal{N}}$			0.23	0.25	0.16		0.22		0.24		0.24	0.21
			34.48	*46.05*	*7.71*		*23.83*		*46.60*		*46.38*	*25.59*
$\beta_{t-1}^{\mathcal{P}}$						0.16	0.14					0.13
						3.64	*3.58*					*3.40*
$\beta_{t-1}^{\mathcal{M}^+}$								0.18	0.17			0.10
								26.46	*28.09*			*7.16*
$\beta_{t-1}^{\mathcal{M}^-}$										0.14	0.12	0.03
										9.94	*9.95*	*3.66*
R^2	0.07	0.02	0.07	0.06	0.09	0.08	0.13	0.02	0.08	0.01	0.07	0.13

Corroborating the evidence of Levi and Welch (2020), ex-ante historical upside and downside betas do not provide much explanatory power for the beta in the next month. Their predictive power almost completely vanishes and turns insignificant when the lagged value of the market beta is added to the model. The usefulness of the upside and downside betas in forecasting the subsequent traditional beta is therefore limited.

Contrary to the previous results, the negative concordant semibeta has a significant impact on the traditional beta of the next month. Interestingly, this predictive power does not disappear when the lagged value of the traditional beta is added to the model. The negative concordant semibeta therefore does possess marginal explanatory power over and above the CAPM beta, as well highlighted by the increased R^2. On top of that, the model illuminates that lagged values of the negative concordant semibeta even convey more relative forecasting capability than the traditional beta in the previous period itself has. The models containing the positive concordant semibeta reveal that this beta does have a significant influence on the traditional beta in the next period. However, this marginal explanatory power turns insignificant and is subsumed by the market beta when the latter is added to the model. Hence, the semibeta associated with positive concordant returns does not provide significant auxiliary forecasting power.[14] Nonetheless, both the discordant semibetas show their foreseeing aptness as the coefficients remain significant after incorporating the traditional beta into the model. Predicting the traditional beta with the full set of semibetas confirms that the negative semibeta is dominant over the other measures. This is verified by the magnitude of its coefficient, 0.51, which is more than twice the size of the coefficients of the other semibetas. The increased R^2 between the AR(1) model and the full semibeta specification further underlines the additional forecasting capability of the semibetas, with all highly significant coefficients. Hence, there is information embedded in the lagged asymmetric components above that of the traditional lagged beta. Controlling for the traditional beta in the full model causes perfect multicollinearity as all four components sum up to the traditional beta.[15] However, in unreported regressions where each of the components is successively replaced by the

[14]These results are further exacerbated by the fact that the current concordant negative semibeta is also a better predictor for the concordant positive future semibeta than the concordant positive beta itself is.

[15]Although the semibetas add up to the traditional beta, multicollinearity is not an issue for the variables of interest in the above estimated regressions as variance inflation factors remain far below 5.

conventional beta, the negative semibeta still enforces the largest influence and its predictive lead therefore does not collapse. These results are strengthened by the finding that besides the concordant negative semibeta, the other semibetas turn insignificant when the traditional beta is controlled for in the full specification. This is true even when the lagged upside and downside betas are added to the full specification. To conclude, the traditional beta seems inferior in forecasting its own value in the upcoming month, as the results are more strongly induced by the concordant negative semibeta.

The analysis outlined above suggests that the negative semibeta has a disproportionate influence on the estimation of the beta in the next period. To assess whether similar conjectures hold for the negative component itself, similar specifications of lagged betas are regressed on the negative semibeta in the period ahead. The results are exhibited in Panel B of Table 2 and indicate that the best relative predictor for the negative semibeta in the following period is its value in the current period. Adding historical values for the other semibetas does not really deteriorate the predictive ability of the autocoefficient that results from a simple AR(1) model. Nonetheless, the results show that the other semibetas exert some influence in predicting the future concordant negative semibeta, thereby disclosing that there are interdependencies between the semibetas.

4.2. Yearly Betas, Downside Betas, and Robustness Checks

To comply with a large literature that employs yearly realized betas based on daily data, the regressions are also estimated with yearly non-overlapping betas. Table A.1 in the Appendix lists the results. Not surprisingly, the larger estimation window for yearly betas results in larger first-order autocoefficients. Conforming the aforementioned findings, the coefficients of the upside and downside betas are insignificant after controlling for the traditional market beta. This implies that one can disregard the two-way decomposition and its accompanying betas in forecasting the market beta in the period ahead over longer time horizons as well. Notwithstanding, the negative concordant semibeta still possesses an equivalent power in predicting the future market beta as its own lagged value. This does not hold for the other semibetas, whose forecasting ability is subsumed by the traditional beta, emphasized by the insignificant coefficients. Ultimately, when forecasting the traditional beta, all semibetas exert information supplementary to that of the others as inferred by the significant coefficients in the full specification. The superiority of the four-way decomposition is again underscored by the higher R^2 compared to that of the CAPM model. Fundamentally, the forecasting ability of the negative concordant component does not meaningfully decrease in the full semibeta specification, which highlights its main dominance in driving the results. The results of Panel B in Table 2 remain robust when progressing from a monthly horizon to a yearly horizon for predicting negative semibetas. Panel B in Table A.1 shows that historical negative semibetas continue to be the best predictors of future negative semibetas. Altogether, the estimations with yearly betas yield qualitatively similar results as that for betas estimated on a monthly basis.

To further intuit the results, control regressions with the downside beta as the dependent variable are estimated. The outcomes of these regressions are listed in Table A.2 in the Appendix, where Panel A reports the results of non-overlapping monthly regressions and Panel B those based on a yearly frequency. Conforming the findings of Levi and Welch (2020), historical downside betas are poor predictors of future downside betas. Nevertheless, the traditional market beta is a much better predictor for the future downside beta than the downside beta itself is. Again, the semibeta decomposition conveys information above that of the market beta, as underlined by the increase in R^2. Withal, the negative concordant semibeta is a more meaningful measure than all the other betas, as its forecasting capacity does not collapse after controlling for the downside beta itself and the other semibetas. Thus, relative to other beta measures, the concordant negative semibeta is not only a better predictor of the future market beta, but also of its proposed downside modification. One can therefore assert that the concordant negative semibeta is superior in forecasting either systematic risk or downside risk.

One disadvantage of the Pooled OLS estimates is that a larger number of stocks in later years might tilt the estimation towards more recent periods. To mitigate this bias, Fama and MacBeth (1973) type regressions are estimated with both monthly and yearly betas to obtain an equal weighting for each time period.[16] Table A.3 and A.4 in the Appendix show the results. The results validate the evidence from the Pooled OLS regressions and concur with the conclusions outlined above. Therefore, the general findings are robust to estimates based on different estimation windows and to a time-varying number of stocks in the sample. The concordant negative semibeta is therefore superior in forecasting

[16] In line with Levi and Welch (2020), these regressions are not for serially correlated concerns, but merely to obtain time-series averages of cross-sectional coefficients.

systematic risk and downside risk as measured by the traditional CAPM beta and its downside modification.

4.3. On the Predictability of Spreads

In their analysis of the two-way decomposition, Levi and Welch (2020) argue that there is no stability in the spread between the upside and downside beta. The prediction difficulties in estimating downside betas are attributed to both measurement errors and a time-varying spread between the upside and downside beta. To examine what drives prediction errors in the negative concordant semibeta, several autoregressive models of the spread of the negative semibeta with the other semibetas are estimated. Table 3 lists the results. To assess whether the decay is caused by measurement errors or due to mean reversion in the spread, one-month and two-month (and yearly) lagged spreads are considered. In case there is no time-variation in the spread, the coefficients of both the one-period-lagged and two-periods-lagged spread should be equal. This is not the case for the $\beta_t^{\mathcal{N}} - \beta_t^{\mathcal{P}}$ spread for both a monthly and yearly horizon. The autocoefficients of these spreads are very small and differ in magnitude, suggesting that there is time-variation in the $\beta_t^{\mathcal{N}} - \beta_t^{\mathcal{P}}$ spread. Panel A and C show that the monthly spreads between the concordant negative beta and the discordant betas are quite persistent and a larger proportion of the historical spread is translated into the future spread. When moving to a yearly horizon, the coefficients for the one-year lagged spread and the two-years lagged spread differ more, but nevertheless still enforce a much larger influence on the current spread than the lagged values of the $\beta_t^{\mathcal{N}} - \beta_t^{\mathcal{P}}$ spread impose on its current value. Errors-in-variables concerns could partly be resolved by incorporating the instrumental variable estimation approach of Hansen and Lunde (2014). Underlying mean reversion makes it harder to predict the value of the concordant negative semibeta in the future. The yearly estimation horizon shows that there is some time-variability in the $\beta_t^{\mathcal{N}} - \beta_t^{\mathcal{M}^-}$ spread, corroborating the hypothesis of Shleifer and Vishny (1997) and Brunnermeier and Pedersen (2009) that limits-to-arbitrage and possibly liquidity spirals can lead to diverging spreads over time.

As follows from Table 3, the lagged spreads of the mixed components have much higher coefficients than that of the $\beta_t^{\mathcal{N}} - \beta_t^{\mathcal{P}}$ spread. This matches the image sketched by Figure 1. One could therefore assert that historical co-jumps in the asset do not

Table 3: **Autoregressive Models of the Semibeta Spreads.** Panel A and B show predictive Pooled OLS regressions of the lagged values of the spreads on the current spread while using respectively a monthly and yearly horizon. The panels report coefficients and underneath t-statistics are calculated with clustered standard errors. Panel C (D) shows the monthly (yearly) results for the Fama-Macbeth type regressions with Newey-West robust t-statistics using a lag length of 12 (1). Constants are included in all regressions but not reported. Estimates are determined by all contemporaneous common, non-penny, CRSP stocks spanning from July 1963 to December 2021.

Panel A: Monthly Betas Pooled OLS									
	$\beta_t^{\mathcal{N}} - \beta_t^{\mathcal{P}}$			$\beta_t^{\mathcal{N}} - \beta_t^{\mathcal{M}^+}$			$\beta_t^{\mathcal{N}} - \beta_t^{\mathcal{M}^-}$		
$SPREAD_{t-1}$	-0.03		-0.04	0.15		0.14	0.17		0.16
	-4.14		*-15.00*	*31.31*		*38.74*	*26.86*		*43.76*
$SPREAD_{t-2}$		0.00	0.00		0.16	0.14		0.19	0.16
		1.38	*0.91*		*36.85*	*37.82*		*29.27*	*29.39*
R^2	0.00	0.00	0.00	0.02	0.03	0.05	0.03	0.04	0.06
Panel B: Yearly Betas Pooled OLS									
	$\beta_t^{\mathcal{N}} - \beta_t^{\mathcal{P}}$			$\beta_t^{\mathcal{N}} - \beta_t^{\mathcal{M}^+}$			$\beta_t^{\mathcal{N}} - \beta_t^{\mathcal{M}^-}$		
$SPREAD_{t-1}$	-0.07		-0.11	0.50		0.49	0.54		0.50
	7.72		*-14.14*	*28.79*		*46.56*	*41.53*		*56.22*
$SPREAD_{t-2}$		-0.03	-0.03		0.45	0.20		0.48	0.21
		-3.31	*-3.88*		*25.28*	*15.72*		*34.31*	*56.22*
R^2	0.01	0.00	0.01	0.31	0.25	0.42	0.34	0.27	0.43
Panel C: Monthly Betas Fama-Macbeth Regressions									
	$\beta_t^{\mathcal{N}} - \beta_t^{\mathcal{P}}$			$\beta_t^{\mathcal{N}} - \beta_t^{\mathcal{M}^+}$			$\beta_t^{\mathcal{N}} - \beta_t^{\mathcal{M}^-}$		
$SPREAD_{t-1}$	-0.03		-0.03	0.28		0.22	0.27		0.21
	-4.10		*-4.27*	*18.74*		*22.59*	*17.61*		*19.54*
$SPREAD_{t-2}$		-0.00	-0.00		0.26	0.19		0.25	0.19
		-0.51	*-0.58*		*18.24*	*21.92*		*17.46*	*20.41*
R^2	0.48	0.48	0.49	0.36	0.35	0.39	0.23	0.23	0.27
Panel D: Yearly Betas Fama-Macbeth Regressions									
	$\beta_t^{\mathcal{N}} - \beta_t^{\mathcal{P}}$			$\beta_t^{\mathcal{N}} - \beta_t^{\mathcal{M}^+}$			$\beta_t^{\mathcal{N}} - \beta_t^{\mathcal{M}^-}$		
$SPREAD_{t-1}$	0.03		0.04	0.59		0.57	0.62		0.55
	1.12		*1.40*	*29.77*		*30.47*	*27.89*		*25.40*
$SPREAD_{t-1}$		0.03	0.02		0.52	0.19		0.55	0.21
		1.73	*1.35*		*20.34*	*10.42*		*17.23*	*9.57*
R^2	0.54	0.54	0.55	0.50	0.44	0.60	0.47	0.41	0.57

lead to improved forecasting for future co-jumps in the semibetas, as estimated by the $\beta_t^{\mathcal{N}} - \beta_t^{\mathcal{P}}$ spread. This further underscores the unexpected behavior of these co-jumps in prices. Altogether, compared to the analysis of Levi and Welch (2020) for the two-way decomposition, historical spreads of the negative concordant semibeta provide much more explanation of the future spread as highlighted by the coefficients and R^2. This means there is less uncertainty in estimating the negative semibeta as opposed to the downside beta.

5. Cross-Sectional Pricing of Semibetas

Building on the evidence of the previous section, the natural question arises whether the superior forecasting ability of the semibetas is priced in the cross-section of asset returns. To this end, this section employs Fama and MacBeth (1973) predictive regressions to estimate risk premiums of the individual semibetas and other measures of downside risk. Then, stocks are sorted into quintile portfolios and benchmarked against well-established asset pricing models, to analyze whether the statistical risk premiums of the semibetas translate into economically significant gains. Bollerslev et al. (2022) document the out-performance of trading strategies conditional on semibetas with negative market returns using high-frequency returns. This section investigates whether these pricing inclinations of the semibetas also impart in more coarse monthly measures based on daily returns.

5.1. Cross-Sectional Fama-MacBeth Predictive Regressions

To identify whether a certain sensitivity towards a particular semibeta proxies for risk exposure, predictive Fama-MacBeth regressions are estimated. The realized betas of each asset in the previous period are regressed on the returns of the asset in the current period. In this way, all information up and until time $t-1$ is available to estimate the returns at time t. This avoids the concerns expressed in Levi and Welch (2020), where market betas should not be employed to explain contemporaneous returns, as is done by Ang et al. (2006). Fama and MacBeth (1973) apply the following two-step procedure. First, the estimates of the betas at time $t-1$ and possibly a set of control variables are regressed on the returns, $r_{k,t}$, at the time period t for each particular month $t = 2, 3, ..., T$ for each stock k that is trading at that particular time. To illustrate, the simultaneous estimation

Table 4: **Fama-MacBeth Predictive Cross-Sectional Regressions.** This table shows the results of the time-series average of the risk premiums resulting from non-overlapping monthly Fama-MacBeth predictive cross-sectional regressions. The monthly betas, coskewness, and cokurtosis are measured with information available up and until time $t-1$ to estimate the monthly returns in the period ahead. The table reports annualized coefficients of the risk premiums and underneath Newey-West robust t-statistics are calculated with 12 lags. Constants are included into the regression but not reported. Estimates are determined by all contemporaneous common, non-penny, CRSP stocks spanning from July 1963 to December 2021.

β_{t-1}	$\beta^{\mathcal{N}}_{t-1}$	$\beta^{\mathcal{P}}_{t-1}$	$\beta^{\mathcal{M}^+}_{t-1}$	$\beta^{\mathcal{M}^-}_{t-1}$	β^{+}_{t-1}	β^{-}_{t-1}	CSK	CKT	R^2
1.46									0.22
1.50									
	15.65	-5.05	18.71	-1.94					0.25
	6.57	*-2.73*	*5.47*	*-0.61*					
					-3.52	5.42			0.23
					-5.11	*6.62*			
							-0.95	-0.30	0.21
							-0.65	*-0.40*	
	24.49	-9.03	21.19	-7.07			12.92	-1.76	0.26
	7.57	*-3.70*	*5.90*	*-1.98*			*5.61*	*-2.09*	

of the distinct risk premiums, λ^j_t, for the semibetas is developed as follows:

$$r_{k,t} = \lambda_{0,t} + \sum_j \lambda^j_t \beta^j_{k,t-1} + \epsilon_{k,t} \tag{26}$$

where $j = \mathcal{N}, \mathcal{P}, \mathcal{M}^+, \mathcal{M}^-$. This leaves $T-1$ estimates of coefficients for each risk premium in the cross-section, of which the time-series average is calculated:

$$\hat{\lambda}^j = \frac{1}{T-1} \sum_{t=2}^{T} \hat{\lambda}^j_t \tag{27}$$

Table 4 presents the annualized time-series average of the risk premiums of the cross-sectional regressions with Newey-West robust t-statistics calculated with 12 lags reported below the coefficients. First of all, a regression based on the plain market beta results in a risk premium of 1.46 percent per year with an insignificant t-statistic of 1.50. Alternatively, a full specification comprised of semibetas shows that betas affiliated with negative asset returns have a significant risk premium, with t-statistics convincingly above the 3.0

hurdle encouraged by Harvey et al. (2016). On the other hand, the betas associated with positive return variation in the asset fail to pass this barrier. The additional information that is captured in the semibeta-CAPM is once again underscored by the higher R^2 compared to the traditional CAPM. The two-way decomposition into upside and downside betas show significant coefficients but are nonetheless of a smaller magnitude. Further, it appears that other gauges of non-Normality such as coskewness and cokurtosis do not bear a significant risk premium. As coskewness and cokurtosis are associated with the pricing of tail risk, it is interesting to explore the dynamics between these forms of non-Normality and the semibetas. The final row in Table 4 shows that accounting for both betas and higher-order moments simultaneously yields significant risk premiums at a 5 percent significance level for all variates. The strongest conclusions again arise from the semibetas with negative asset returns.[17] These results are perfectly aligned with the model of ambiguity averse agents of Barahona et al. (2021). As previously demonstrated, the concordant negative semibeta is subject to higher predictability than the other betas. Hence, this beta experiences higher hedging demands than the others, and therefore earns a larger risk premium.

5.2. Univariate Portfolio Sorts

To begin with the pricing implications, several univariate portfolio sorts based on the different betas are analyzed. Stocks are sorted based on their prevailing beta into value-weighted quintile portfolios, of which the excess return in the next month is evaluated. The results are presented in Table 5. The table reports the average monthly excess returns and the standard deviation for the portfolios of each quintile ranging from 1 (Low) to 5 (High). The penultimate row in each panel shows the ex-ante mean beta of the respective beta measure and the bottom row displays the mean of the ex-post ranked betas. Fama and French (1992) illustrate that post-ranked betas of the portfolios should also show a monotonous relationship, because otherwise the prevailing betas would hold no predictive ability in forecasting the future beta.

[17] The conclusions of the regressions do not alter when changing the estimation window for the betas to a year or, as traditional in Fama-MacBeth type regressions, to five years. The betas associated with negative variation in returns on the asset bear the largest risk premium. Using overlapping monthly regressions also does not qualitatively alter the outcomes.

Table 5: **Univariate Portfolio Sorts of Monthly Betas.** All panels report monthly average excess returns and standard deviations for the portfolios and their ex-ante and ex-post beta loadings. The Newey-West robust t-statistic for the 5-1 (1-5) portfolios is listed along with the values for the MR^{all} test for monotonicity and its corresponding studentized Up and Down-test. Estimates are determined by all contemporaneous common, non-penny, CRSP stocks spanning from July 1963 to December 2021.

Panel A: β_{t-1}

	1	2	3	4	5	5-1	t-stat	MR^{all}	Up
Mean	0.60	0.68	0.78	0.77	0.77	0.17	*1.05*	0.04	0.36
St.Dev.	4.12	3.92	4.32	4.90	5.92	4.85			
β_{t-1}	-0.11	0.44	0.78	1.16	1.95	2.06			
β_t	0.49	0.65	0.80	0.98	1.30	0.81		0.00	

Panel B: $\beta_{t-1}^{\mathcal{N}}$

	1	2	3	4	5	5-1	t-stat	MR^{all}	Up
Mean	0.52	0.63	0.71	0.89	0.88	0.36	*1.98*	0.04	0.02
St.Dev.	3.70	3.93	4.30	5.13	6.16	5.03			
$\beta_{t-1}^{\mathcal{N}}$	0.16	0.34	0.49	0.67	1.09	0.93			
$\beta_t^{\mathcal{N}}$	0.39	0.47	0.53	0.60	0.76	0.37		0.00	

Panel C: $\beta_{t-1}^{\mathcal{P}}$

	1	2	3	4	5	1-5	t-stat	MR^{all}	Down
Mean	0.82	0.73	0.74	0.80	0.66	0.16	*1.05*	0.47	0.31
St.Dev.	4.00	4.22	4.32	4.92	5.98	4.88			
$\beta_{t-1}^{\mathcal{P}}$	0.22	0.43	0.61	0.84	1.39	-1.17			
$\beta_t^{\mathcal{P}}$	0.50	0.60	0.67	0.76	0.96	-0.46		0.00	

Panel D: $\beta_{t-1}^{\mathcal{M}^+}$

	1	2	3	4	5	5-1	t-stat	MR^{all}	Up
Mean	0.54	0.77	0.71	0.97	0.96	0.42	*3.05*	0.21	0.00
St.Dev.	4.59	4.54	4.50	4.79	5.34	3.88			
$\beta_{t-1}^{\mathcal{M}^+}$	0.04	0.10	0.16	0.26	0.58	0.54			
$\beta_t^{\mathcal{M}^+}$	0.16	0.18	0.21	0.25	0.34	0.18		0.00	

Panel E: $\beta_{t-1}^{\mathcal{M}^-}$

	1	2	3	4	5	1-5	t-stat	MR^{all}	Down
Mean	0.79	0.81	0.66	0.58	0.76	0.04	*0.28*	0.75	0.22
St.Dev.	4.72	4.50	4.50	4.46	5.14	3.67			
$\beta_{t-1}^{\mathcal{M}^-}$	0.02	0.06	0.11	0.20	0.47	-0.46			
$\beta_t^{\mathcal{M}^-}$	0.13	0.14	0.16	0.18	0.25	0.11		0.00	

Further, the long-short quintile 5-1 (1-5) portfolio is displayed for the betas associated with negative (positive) returns on the asset. Common practice in the finance literature is to perform a *t*-test on the mean return spread between the High-minus-Low portfolio. Despite its usefulness, the test has limited scope in evaluating a monotonous relationship between the prevailing betas and expected average returns over the full portfolio sorts. Therefore, Patton and Timmermann (2010) propose a test that considers the mean returns across all quantiles simultaneously. The Monotonic Relation (MR) test is a non-parametric test that does not entail assumptions on the functional form of the underlying data generating process and can therefore handle non-linear mappings between the betas. The resulting probability value of the MR test is listed under MR^{all}, and based on all possible comparisons across the quintile portfolios. Rejection of the null hypothesis implies a monotonous relationship. Following Patton and Timmermann (2010), bootstrap replications are set to one thousand and the mean block length is 10 months. As portfolios with low betas have smaller standard deviations than those with higher betas, studentized versions of the test are also considered. Probability values of these Up and Down tests (based on the expected direction of the relationship) are listed in the final column of Table 5. These tests are robust to heteroskedasticity that is possibly present in the stock returns.

The probability value of the MR^{all} test in Panel A shows that there is a monotonous relationship present in the portfolios sorted on the traditional beta. However, the studentized Up-test fails to reject the null of no increasing pattern. This corroborates the findings in Patton and Timmermann (2010) and dates back to Black (1972) who finds that the security market line is too flat, relative to the expectations of the CAPM. Nevertheless, post-ranked betas show a monotonous relation as illustrated by the probability value of 0.00. The High-minus-Low portfolio on the other hand only earns 0.17 percent per month on average and has an insignificant *t*-statistic. Then, moving over to Panel B of Table 5, it is shown that the negative concordant semibeta exhibits a monotonic relationship, highlighted both by the probability value of the MR^{all} test and its studentized counterpart for increasing returns. Ex-post betas show a monotonically increasing pattern as well. Finally, the 5-1 portfolio earns a significant monthly average excess return of 0.36 percent. This monotonic pattern is not present in the average returns of the portfolios sorted on the ex-ante positive semibeta as shown in Panel C. Even though, ex-post betas display a monotonic increasing relationship, a Low-minus-High portfolio fails to deliver significant excess returns. The strongest results are obtained by the discordant beta associated with positive market returns. However, the MR^{all} test fails to reject the null hypothesis of no monotonicity. Nonetheless, the studentized test for increasing returns has a corresponding probability value of 0.00. One can therefore conclude that there is an increasing pattern present in the average returns in the portfolios sorted on ex-ante $\beta^{\mathcal{M}^+}$. This pattern is also visible in ex-post betas. The High-minus-Low portfolio yields a significant excess return of 0.42 per month. Lastly, the $\beta_{t-1}^{\mathcal{M}^-}$-sorted portfolios mirror the results of the positive concordant beta sorted portfolios with a non-monotonous pattern in average returns and insignificant returns on the Low-minus-High portfolio. Pricing results are therefore the strongest for the semibetas associated with negative returns on the asset.[18] This corroborates the results of the Fama-MacBeth predictive regressions and the model of ambiguity averse agents as outlined earlier.

5.3. Semibeta Trading Strategies

The statistical significance of the pricing of semibetas in the cross-section has been affirmed by previous subsections. Moreover, there is also an economic rationale behind the risk premiums of the semibetas with negative asset returns. Investors want to be compensated for negative return variation and dislike ambiguity. Thus, one can assess the economic significance by exploring whether various zero-investment trading strategies earn positive excess returns. Following Novy-Marx and Velikov (2022), value-weighted long-short quintile portfolios are constructed where a long (short) position is taken into the upper (lower) quintile to realize a zero-investment portfolio that is monthly rebalanced.[19] The upper panel in Table 6 lists the annualized mean return, standard deviation, and Sharpe ratio for the semibeta trading strategies. In order to calculate risk-adjusted performance, each of the trading strategies is benchmarked against the three-factor model of Fama and French (1993) (FF3), the FF3 model with the momentum factor of Jegadeesh and Titman (1993) and the short-term reversal factor of Jegadeesh (1990) (FF3+2), and

[18] Applying sorts based on yearly betas and subsequent future monthly returns leads to qualitatively similar conclusions.

[19] Effects are more easily discernible with equal weighted portfolios. These portfolios are less representative as they contain a larger position in smaller and more illiquid stocks. Therefore, value-weighted portfolios are preferred in this exercise.

Table 6: **Semibeta Strategies and Benchmark Models with Monthly Betas.** The upper panel lists the annualized mean return of the long-short strategy and its corresponding standard deviation and Sharpe ratio. Strategies of $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^+}$ bet on the betas, while $\beta^{\mathcal{P}}$ and $\beta^{\mathcal{M}^-}$ bet against the beta. Zero-investment portfolios are constructed with value-weighted long-short positions, rebalanced on a monthly basis. The bottom-most panel displays estimates of the time-series regression on the FF3, FF3+2, and FF5 factor models with Newey-West robust t-statistics and annualized alphas. Estimates are determined by all contemporaneous common, non-penny, CRSP stocks spanning from July 1963 to December 2021.

	$\beta^{\mathcal{N}}$			$\beta^{\mathcal{P}}$			$\beta^{\mathcal{M}^+}$			$\beta^{\mathcal{M}^-}$		
Mean		4.37			1.94			5.12			0.43	
St.Dev.		17.41			16.89			13.46			12.73	
Sharpe		0.25			0.11			0.38			0.03	
α	1.32	-0.97	5.13	4.33	3.39	1.57	3.47	3.41	3.16	1.14	1.32	1.55
	0.60	*-0.47*	*2.39*	*2.37*	*1.72*	*0.84*	*2.21*	*2.24*	*1.82*	*0.78*	*0.83*	*0.87*
β^{MKT}	0.51	0.45	0.43	-0.43	-0.49	-0.36	0.02	-0.06	0.01	0.02	-0.02	0.02
	9.71	*7.52*	*9.93*	*-7.11*	*-8.06*	*-6.77*	*0.31*	*-1.44*	*0.21*	*0.60*	*-0.48*	*0.57*
β^{SMB}	0.21	0.16	0.07	-0.09	-0.12	-0.00	0.38	0.35	0.42	-0.25	-0.27	-0.30
	2.51	*2.07*	*0.96*	*-1.23*	*-1.84*	*-0.06*	*3.33*	*3.30*	*5.22*	*-3.31*	*-3.67*	*-4.56*
β^{HML}	-0.35	-0.36	-0.11	0.29	0.26	0.07	0.17	0.11	0.12	-0.08	-0.12	-0.02
	-3.41	*-3.67*	*-0.99*	*2.84*	*2.72*	*0.66*	*2.15*	*1.27*	*1.14*	*-0.88*	*-1.30*	*-0.24*
β^{MOM}		0.06			-0.03			-0.13			-0.10	
		0.86			*-0.34*			*-1.93*			*-1.54*	
β^{REV}		0.42			0.30			0.37			0.22	
		4.24			*3.02*			*4.60*			*2.87*	
β^{RMW}			-0.51			0.32			0.10			-0.09
			-6.16			*2.94*			*0.72*			*-0.64*
β^{CMA}			-0.58			0.51			-0.02			-0.03
			-4.32			*3.35*			*-0.17*			*-0.23*
R^2	0.34	0.38	0.39	0.24	0.26	0.27	0.10	0.18	0.11	0.04	0.08	0.05

the five-factor model of Fama and French (2015) (FF5). The bottom panel in Table 6 shows the results. It is clear that long-short trading strategies pertaining betas with downside variation in the returns on the asset outperform their positive counterparts.[20] $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^+}$ trading strategies attain a Sharpe ratio more than double that of the $\beta^{\mathcal{P}}$ and $\beta^{\mathcal{M}^-}$ strategies. Moving over to the risk-adjusted performance, the $\beta^{\mathcal{P}}$ and $\beta^{\mathcal{M}^-}$ portfolios do not earn a significant alpha after correcting for the risk exposures of the FF3+2 and FF5 factor models. Consequently, the $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^+}$ strategies are more relevant for the analysis. After calibrating against the FF3+2 risk exposures, the $\beta^{\mathcal{M}^+}$ still earns a significant annualized alpha of 3.41 percent per year. However, when benchmarking the

[20] For comparison, a long-short strategy of value-weighted quintile portfolios based on the traditional CAPM beta earns an insignificant alpha with an annualized return of 1.99 percent, standard deviation of 16.84, and a Sharpe ratio of 0.12.

portfolio against the well-established FF5 factor model, the yearly alpha of 3.16 percent turns insignificant at the 5 percent level. The $\beta^{\mathcal{N}}$ portfolio is the only portfolio that generates a significant alpha after accounting for the risk exposures of the FF5 factor model. The pricing implications of the concordant negative semibeta portfolios against this model are reinforced by the fact that this portfolio earns the highest observed significant annualized alpha of 5.13 percent. Taking a closer look at the factor loadings, the $\beta^{\mathcal{N}}$ portfolio has a larger tilt to the market than the $\beta^{\mathcal{M}^+}$ portfolio, whereas the latter is close to market neutral. When benchmarked against the FF5 factor model, the $\beta^{\mathcal{M}^+}$ strategy takes a much larger loading on the size factor than the $\beta^{\mathcal{N}}$ strategy.[21] Lastly, the $\beta^{\mathcal{N}}$ portfolio takes a larger negative position in the profitability and investment factors than the $\beta^{\mathcal{M}^+}$ strategy does. Summarizing the findings, one should bet on the betas associated with negative return variation on the asset.[22]

The aforementioned results imply mispricing in the market. However, in assessing market efficiency, pricing errors should be jointly tested as error terms of individual stocks might correlate. The Gibbons, Ross, and Shanken (GRS) test takes these concerns into account and applies the technique of a Seemingly Unrelated Regression (Gibbons et al., 1989). The GRS-test statistic intuitively explains how much the Sharpe ratio of the benchmark model can be improved based on the portfolio of test assets. Rejection of the null hypothesis that all alphas are zero suggests that the assets in the portfolios are mispriced and that the underlying benchmark model is not sufficient in explaining the cross-section of stock returns. Table 7 shows the results of the GRS-test. The $\beta^{\mathcal{N}}$ portfolio is the only portfolio of test assets that significantly improves the FF5 factor model as a probability value of 0.01 leads to the rejection of the null hypothesis. All other portfolios do not improve the FF5 factor model. Nonetheless, the test assets of the $\mathcal{M}^+$ portfolio show a higher Sharpe ratio compared to the semibeta portfolios pertaining positive returns

[21] The performance of the $\beta^{\mathcal{N}}$ strategy is therefore not likely to be dependent on small illiquid stocks such as the betting-against-beta strategy of Frazzini and Pedersen (2014).

[22] Throughout this research, the main goal is to establish the relative performance of $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^+}$ versus that of $\beta^{\mathcal{P}}$ and $\beta^{\mathcal{M}^-}$ and their implications for the risk-return relationship in the cross-section. Therefore, this research refrains from analyzing possible semi-β strategies involving combinations of semibetas or the impact of transaction costs as in Bollerslev et al. (2022). The results are also robust to different time horizons, as a yearly estimation window of betas with monthly rebalancing results in even larger annualized returns.

Table 7: **GRS-test Statistics and Probability Values.** This table reports the GRS-test statistics for the $\beta^{\mathcal{N}}$, $\beta^{\mathcal{M}^+}$, $\beta^{\mathcal{P}}$, $\beta^{\mathcal{M}^-}$ portfolios benchmarked against the FF3, FF3+2, and FF5 factor models. Strategies of $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^+}$ bet on the betas, while $\beta^{\mathcal{P}}$ and $\beta^{\mathcal{M}^-}$ bet against the beta. Corresponding probability values are listed below the values of the test statistic. Estimates are determined by all contemporaneous common, non-penny, CRSP stocks spanning from July 1963 to December 2021.

	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$
CAPM	0.00	7.35	6.50	0.10
	0.99	*0.01*	*0.01*	*0.75*
FF3	0.48	4.64	4.01	0.46
	0.49	*0.03*	*0.05*	*0.50*
FF3+2	0.25	2.66	3.82	0.58
	0.61	*0.10*	*0.05*	*0.45*
FF5	7.21	0.62	3.17	0.82
	0.01	*0.43*	*0.08*	*0.37*

on the assets. This is highlighted by the higher value of the GRS-test statistic in the final row of the table. These results corroborate the earlier findings discovered in the Fama-MacBeth regressions and the univariate portfolio sorts and are consistent with the model of an ambiguity averse agent of Barahona et al. (2021). To combine the results so far, one can conclude that the semibetas, and in particular the concordant negative semibeta, lead to increased forecastability. This enhanced predictive power is rewarded with a risk premium in the cross-section of asset returns and therefore shows a positive relationship between downside risk and expected excess returns. Thus, meaning that the downside risk puzzle is resolved when a four-way decomposition is taken into account.

6. Machine Learning Techniques and Semibetas

Hitherto, the superior forecasting ability of the semibetas compared to the conventional market beta has been affirmed by techniques arising from traditional asset pricing methodologies. Not only do the semibetas hold significant additional predictive power, but those associated with negative asset variation earn a positive risk-adjusted return as well. Therefore, it seems reasonable to conclude that a four-way decomposition of semibetas provides a more fitting delineation of the cross-section of asset returns. Most importantly, the downside risk puzzle has been resolved by showing a positive relationship between downside risk and expected asset returns in the cross-section. Nonetheless, Harvey et al. (2016) identify over three hundred factors within the so-called "zoo" of factors. In similar vein, Green et al. (2013) identify over three hundred return predictive signals (RPS). However, after orthogonalizing these against other factors, their explanatory power is largely reduced. Therefore, the semibeta pricing results should be interpreted with some caution.[23] Insofar the previous sections constitute as convincing evidence, it is important to ascertain whether these relationships readily follow from a non-classical, yet more statistically profound, approach as well.

This section extends the traditional asset pricing methodology and takes a more extensive perspective into the non-linear dependencies of the semibetas and its consequences in forecasting and asset pricing. In recent years, asset pricing analysis shifted towards more advanced statistical methods, where machine learning techniques gained significant ground. Gu et al. (2020) perform a comparative survey of the most common machine learning techniques and their ability to measure asset risk premiums. From this analysis, it can be concluded that machine learning forecasts result in significant economic gains and surpass the regression-based trading strategies with great length. The advantage of incorporating machine learning techniques over the use of traditional methods is that machine learning accounts for non-linearities among the explanatory variables and the functional form of the data generating process is correctly specified. Notably, random forests perform best and outperform deep neural networks and gradient-boosted trees in obtaining accurate daily one-day-ahead trading signals (Krauss et al., 2017). In forecasting stock market prices, Leung et al. (2000) show that classification problems are superior to level estimation models. To this end, the random forests classification technique is employed in this research to investigate whether prevailing semibetas hold significant forecasting ability of future stock returns and whether risk premiums are allocated to the semibetas.

[23] This is succinctly highlighted by the following two quotes that express concerns in respectively portfolio sorting and Fama-MacBeth regressions: "Sorts are awkward for drawing inference about which anomaly variables have unique information about average returns (Fama and French, 2008).", and "It is infeasible to examine non-linearities in RPS-returns relations in the manner undertaken in Fama and French (2008) (Green et al., 2014).".

6.1. Random Forests

The traditional asset pricing analysis is bounded by constraints to the number of predictors that can be incorporated without generating overfitting issues. On the other hand, a random forest is a non-parametric method that handles non-linear dependencies (Breiman, 2001). The forest is an ensemble technique and relies on bootstrap aggregation, so-called 'bagging'. From the training data, bootstrapped samples with replacement are generated to construct a decision tree, which aims to classify observations into groups that act comparably. In each individual tree, classification is ensured through sequential branching where a test in each of the internal nodes splits the observations into baskets based on whether the condition is satisfied or not. Predictions for the observations in each group equal the majority vote for the outcome variable of the observations in each partition. Ultimately, the results of the individual decision trees are aggregated and compose the forest. The random forest is thus constructed by combining multiple decision trees in order to generate aggregate predictions instead of relying on an individual tree. The randomness in the forest offers two advantages. First of all, random subsampling guarantees that bias is mitigated. Secondly, random feature selection at each node yields decorrelated trees. Fundamentally, overfitting is not a problem by virtue of the Strong Law of Large Numbers.

To be more specific, as explained in Friedberg et al. (2020), the predictions of a random forests model are mathematically derived by weighting the response variable Y_i for observation i as follows:

$$\hat{\mu}_{rf}(x_0) = \sum_{i=1}^{n} \alpha_i(x_0) Y_i \tag{28}$$

The prediction for observation x_0 is therefore defined by a linear combination of the n observations of the response variable. The forest weights $\alpha_i(x_0)$ are defined as:

$$\alpha_i(x_0) = \frac{1}{B} \sum_{b=1}^{B} \frac{I_{\{X_i \in L_b(x_0)\}}}{|L_b(x_0)|} \tag{29}$$

where B denotes the number of trees and I is an indicator function that equals one if the statement in the brackets is satisfied and zero otherwise. $L_b(x_0)$ represents the leaf of tree b and the indicator function therefore assesses whether observation i is located in the same leaf as observation x_0. $|L_b(x_0)|$ defines the cardinality of the leaf, i.e. the number of observations that are present in the leaf. The fraction within the sum therefore enumerates what proportion of the time observation i falls in the same leaf as x_0. Averaging this sum across all trees gives the weight for each observation of the response variable. In order to select the best features to use in the decision nodes, one can check the homogeneity of the samples after the split. The Gini impurity is such a measure for homogeneity and is specified as[24]:

$$GI = 1 - \sum_{i=1}^{N} p_i^2 \tag{30}$$

where p_i represents the probability that class i falls into the subset after the split for the N classes. The random forests algorithm tries every potential feature and split value in order to attain the largest decrease in Gini impurity. To accommodate random forests for its incorporation into the asset pricing spectrum, the methodology of Krauss et al. (2017) is considered. It is shown that random forests outperform other machine learning algorithms such as deep neural networks and gradient-boosted-trees in forecasting the probability that a particular stock beats the market. Further, random forests is the least subjected to downside risk as measured by Value-at-Risk and maximum drawdown. Hence, the random forests method is perfectly tailored to examine whether risk premiums are designated to the information captured in the semibetas.

6.2. Methodology

In order to execute the above mentioned exploration, the methodology is closely related to the four-phase procedure of Krauss et al. (2017) and Fischer and Krauss (2018). First, the data sample is split into non-overlapping training and testing sets. This separation is essential for in-sample training of the model and successive out-of-sample testing. Each batch comprises five years of data, with four years allocated to the training set and the latter year to the test set. In total, applying a sliding-window approach based on a yearly frequency, 55 non-overlapping test samples recursively loop over the full data sample. A few comments are at place for the motivation of the length of the training sets. The four year trading sample is chosen such that there are three full years of feature data available after calculating features that incorporate lagged data of the preceding year. The first year of the training sample is then removed, to obtain a three year training set of full data that is consistent with Krauss et al. (2017). On top of that, only stocks with full

[24]Other measures include for example information gain based on Entropy, but these do not yield significant advantages over the Gini impurity that has been widely used in the field of economics (Daniya et al., 2020).

data present in the final three years of the training window qualify for selection, whereas predictions in the trading window extend to the point that the stock is present in the respective year.

In the second step the feature space, i.e. the inputs for the model, is generated. Besides lagged return features, merely lagged beta features are considered as explanatory variables. In this setting, it is possible to derive what extra information is encapsulated in the semibetas. In this research, the returns $r_{t-1,h}$ are considered, defined as the return over the h-day period prior to and including time $t-1$. Similarly, betas $\beta_{t-1,n}$ represent the calculated β up and until time $t-1$ over the sampled frequency n. The following numbers are considered for $h \in \{\{1, 2, ..., 20\} \cup \{40, 60, ..., 240\}\}$ and n is chosen based on a monthly, quarterly, semi-annually, and yearly time horizons. The total feature space consists of 51 features, of which 31 are return inputs, and four inputs for the traditional beta and each semibeta. In accordance with Fischer and Krauss (2018), all inputs are standardized with their respective mean and standard deviation of the training sample. Then, these inputs are used on the right-hand side to explain the binary response variable Y^k_{t+1} for the stocks k. Y^k_{t+1} is in class one if the one-period return $r_{t,1}$ of security k exceeds the cross-sectional median calculated over all stocks in the given day, and belongs to class zero otherwise.

The third step is dedicated to the training of the model. The model uses the parameters $B = 100$ for the number of trees with maximum depth $J = 20$, where $m = \sqrt{p}$ features are randomly selected from the $p = 51$ possible features in every split.[25] The final step forecasts the probability $P^k_{t+1|t}$ that stock k beats the cross-sectional median based on the predictions of the specified model in step three.

6.3. Results

To emulate the results of Krauss et al. (2017), results for the full 1967-2021 period are presented in Panel A of Table 8 for a 10-10 portfolio, i.e. one that goes long (short) in the ten stocks with the highest (lowest) forecasted probability to outperform the cross-sectional median. Note that the less certain middle part of the probability ranking is

[25] Although Krauss et al. (2017) incorporate $B = 1000$ trees, this study considers $B = 100$ trees due to the computational aptitude that is required for the large data set. Nonetheless, this should be sufficient to assess the relative performance of the models in capturing a risk premium.

Table 8: **Random Forest Results for a 10-10 Portfolio during Different Time Horizons.** Descriptive statistics of average annualized returns, standard deviations, and Sharpe ratios for all portfolios of the listed models that go long and short in respectively the stocks with the ten highest and lowest probabilities. Portfolios are rebalanced on a daily basis. Model R solely contains lagged return features. Model R+B and R+S respectively add the traditional beta and semibeta features to the baseline model. Model R+ALL includes all features. Results for the full model are based on a three-year training period containing full data for all features, such that 55 non-overlapping test samples are obtained over the time span of 1967 until 2021. Similarly, results for the 2002-2021 subperiod are based on 20 of these non-overlapping test samples. Estimates are determined by all contemporaneous common, non-penny, CRSP stocks.

Panel A: Full Period 1967-2021				
	R	R+B	R+S	R+ALL
Mean	130.37	218.63	279.40	259.65
St.Dev.	31.28	34.15	32.74	32.61
Sharpe	4.17	6.40	8.53	7.96
Panel B: Prior Transaction Costs 2002-2021				
	R	R+B	R+S	R+ALL
Mean	51.85	68.49	77.07	71.38
St.Dev.	34.85	39.47	36.21	35.92
Sharpe	1.49	1.74	2.13	1.99
Panel C: Post-Transaction Costs 2002-2021				
	R	R+B	R+S	R+ALL
Mean	-8.23	1.84	7.04	3.59
St.Dev.	34.85	39.47	36.21	35.92
Sharpe	-0.24	0.05	0.19	0.10

thus censored. The first model (R) only includes the lagged return measures, thereby replicating the model used in the baseline paper of Krauss et al. (2017). The second model (R+B) adds the lagged traditional beta features measured on a monthly, quarterly, semi-annually, and yearly horizon. Both models serve as an important benchmark. The first one is to consider whether adding betas provides additional information to that of historical returns. The second one establishes the CAPM benchmark and responds to the question whether semibetas hold additional forecasting ability on top of the conventional market beta. The third model (R+S) contains all return features plus the four-by-four semibeta features. The last model (R+ALL) serves as a control and is a combination of the measures in the second and third model, thereby incorporating all return and available beta features.

Over the full sample period, the first benchmark model with only return features has an average annualized return of 130.37 percent with a Sharpe ratio surpassing 4.0. These returns seems extraordinary high, however, Krauss et al. (2017) even note a higher return of 176.27 percent with a Sharpe ratio of 5.11. Secondly, adding the lagged market beta measures leads to almost double the average returns and increases the Sharpe ratio. More importantly for this research, performance is further enhanced by adding only lagged semibetas and lagged returns to the model. This models performs best as highlighted by the higher return and lower standard deviation than the R+B model. Consequently, this leads to a higher Sharpe ratio of 8.53. Controlling for the market beta features in the R+ALL model actually leads to a slightly deteriorating performance, which might be an indication that the market beta blurs some of the information that the semibetas hold. Figure A.1 in the Appendix shows the cumulative performance over time. There are two main concerns that need to be addressed. First of all, the returns are not adjusted for transaction costs and daily rebalancing would partly mitigate the outperformance due to high turnover. Secondly, large returns prior to the start of the 2000s may be attributable to the fact that machine learning techniques were not manifested yet or applicable due to low computing power.[26] Krauss et al. (2017) explicitly mark 2001 as a turning point, which (maybe not so) coincidentally is the year Breiman (2001) published the seminal random forests paper. Therefore, the 2002-2021 subperiod is a natural period to consider in order to obtain a fair comparison between the results of non-classical techniques and the traditional methodology.

Panel B in Table 8 lists the results prior to transaction costs and corroborate the findings over the full sample period. To cast more light on a trading strategy that could be implemented empirically, Panel C subtracts the transaction costs of the returns. Following Avellaneda and Lee (2010), transaction costs are estimated to be 0.05 per share for each half-turn. As expected, performance largely deteriorates and strategies are arbitraged

[26]Nonetheless, these returns have been realized in the real world by for example the quantitative-driven hedge fund Rennaissance Technologies (Rubin and Collins, 2015).

away during the last twenty years. Incorporating only lagged returns would result in an average yearly loss of over 8 percent. Conversely, all models with betas yield an average positive return. However, solely incorporating semibetas yields a much higher return compared to models with the market beta, as underlined by the average annual returns of 7.04 percent versus 1.84 and 3.59 percent. The semibetas, and its additional embedded information, are therefore able to capture an additional risk premium over that of the traditional beta. Table A.5 in the Appendix reports the risk-adjusted performance against the FF3, FF3+2, and FF5 factor models. Succinctly stated, none of the models yield a significant alpha as expected.[27]

Figure 2 plots the cumulative performance over time based on the 2002-2021 subperiod. Here, the diverging performance of the different models becomes much more clear. The semibeta model is the only that permits a positive return over the 2002-2021 period. The other strategies, most notably the R and R+B strategies perform much worse. Semibeta strategies also exhibit a much more stable pattern, which is exemplified by its great performance at times of market turmoil during The Great Recession.

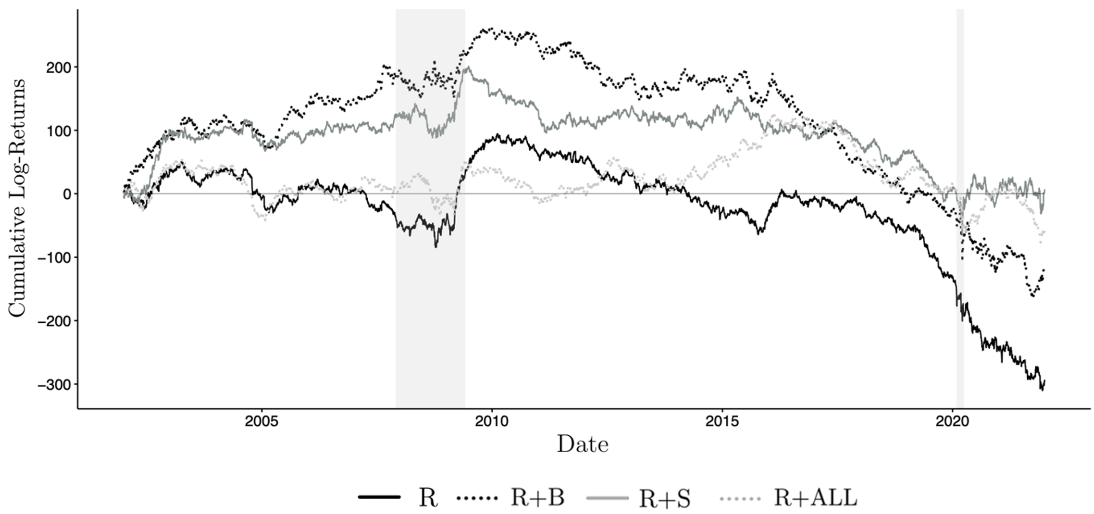

Figure 2: **Cumulative Performance of the 10-10 Portfolios from 2002 until 2021.** This figure shows the cumulative performance of the 10-10 Portfolio over time based on different prediction models classified by the random forests algorithm. Model R solely contains lagged return features. Model R+B and R+S respectively add the traditional beta and semibeta features to the baseline model. Model R+ALL includes all features. The shaded regions represent recession periods as defined by the NBER. Estimates are based on the subperiod of January 2002 until December 2021.

[27]Once again, the performed exercises must be interpreted on a relative basis. Here the semibeta models outperform the others and are able to capture a larger risk premium.

Table 9: **Random Forest Results for a High-minus-Low Portfolio during 2002-2021.** Descriptive statistics of average annualized return, standard deviation and Sharpe ratio for the portfolio that goes long and short in the quintiles with the stocks with the highest and lowest probabilities to outperform the cross-sectional median in the next day. Portfolios are value-weighted and therefore ensure a net investment of zero. Portfolios are rebalanced on a daily basis. Model R solely contains lagged return features. Model R+B and R+S respectively add the traditional beta and semibeta features to the baseline model. Model R+ALL includes all features. Results are based on 20 non-overlapping test samples from January 2002 to December 2021. Estimates are determined by all contemporaneous common, non-penny, CRSP stocks.

	R	R+B	R+S	R+ALL
Mean	10.08	14.68	14.72	16.67
St.Dev.	11.57	12.46	11.43	12.06
Sharpe	0.87	1.18	1.29	1.38

Moreover, as a control, the traditional finance approach of value-weighted long-short quintile portfolios is incorporated as in Novy-Marx and Velikov (2022). Altogether, this allows for a more fair comparison against the traditional semibeta trading strategies exhibited in Bollerslev et al. (2022). Results for the twenty-year subperiod are displayed in Table 9. These results point in the same direction as those for the 10-10 portfolio. Adding a beta measure to the basic return model yields an additional return of at least 4 percent on an annual basis. The difference between annual returns of the R+B and R+S models is negligible, but the R+S model has a lower standard deviation, leading to an increased Sharpe ratio by 11 basis points. The control model (R+ALL) outperforms the others by a slight margin. The evidence indicates that semibetas therefore add explanatory power and are useful in forecasting the performance of stocks when considering outperformance and underperformance compared to the cross-sectional median. Altogether, these results illustrate that adding betas to the model provides much more forecasting power. Semibetas hold supplemental information to that of the CAPM beta and seem to capture an additional risk premium accordingly. Thus, the conclusions outlined earlier do resonate well with the findings of non-classical asset pricing techniques.

7. Conclusion

Since its inception, the CAPM and its downside modifications into upside and downside betas have been subject to much debate for failing to find a positive downside risk-return relationship in the cross-section of stock returns. This research examined whether the recently proposed four-way decomposition of the market beta into semibetas provided superior forecasting capacity over the traditional market beta and assessed its ability to resolve the downside risk puzzle. The results show that the avant-garde semibeta-CAPM is the pre-eminent model in its class through three empirical contributions to the asset pricing literature.

First, while investigating the predictability of semibetas, it is highlighted that the upside and downside betas provide no additional forecasting value in estimating future betas. Furthermore, the traditional market beta itself is inferior to the concordant negative semibeta, as lagged realized measures of the latter are better predictors of future CAPM betas than lagged realizations of the former. This is also true in the prediction exercise of downside betas. The full specification of semibetas outperforms the predictions of other proposed models and measures in the literature, and suggests that there are non-linear asymmetric dependencies among the semibetas. The negative concordant semibeta asserts its dominance as the main driver of the results.

Secondly, with the knowledge of these augmented foreseeing abilities, the implications into the financial asset pricing methodology are examined. Consistent with a model of ambiguity averse agents, the semibetas associated with negative variation in the returns on the asset offer a higher predictability, experience increased hedging demand and subsequently earn a larger risk premium. The semibetas related to positive return variation of the security are not priced in the cross-section. This is demonstrated by several traditional asset pricing methods, such as predictive Fama-MacBeth regressions, portfolio sorts, and a time-series analysis of risk-adjusted performance of semibeta trading strategies. Altogether, the fragmentation of the traditional market beta into four semibetas is able to resolve the downside risk puzzle by finding a positive relationship between downside risk and expected stock returns in the cross-section.

The above mentioned results are robust to a variety of estimation horizons and techniques, and avoid critiques of using equal-weighted portfolios and ex-post measures. Nonetheless, inspired by the recently increased caution in the process of accepting new risk factors, machine learning techniques are employed to ascertain whether the findings readily follow from a non-classical approach as well. Simple trading strategies proxying outperformance show that the information encapsulated in the semibetas captures an additional risk premium and semibeta models therefore attain higher Sharpe ratios. The findings of non-classical techniques resonate well with those obtained by the traditional techniques.

All in all, the four-way decomposition of the market beta into semibetas provides superior forecasting capabilities and holds valuable information that is left uncharted by the traditional beta. An important limitation of this research is that the analysis does not incorporate high-frequency returns. Although monthly betas based on daily returns are more demanded by the literature for asset pricing purposes, its use blurs some of the effects that are discernible with high-frequency return horizons.[28] Hence, there is an important trade-off to consider as forecasts with high-frequency intraday data are better able to capture co-jumps and other deviations from Normality. Further, machine learning techniques are not transparent and provide so-called 'black box' predictions. Lastly, several other proposed measures in the literature incorporate more statistically advanced methods and use option data, that is forward-looking, into the analysis. Nevertheless, while acknowledging these limitations, the relative propensity of the concordant negative semibeta to drive the forecasting outperformance is so severe that it raises the question whether one should substitute the traditional market beta.

This brings the financial literature to an important crossing point and opens up various potential avenues for future research. For example, taking an even deeper look inside the covariance matrix and considering betas based on partial covariances could unveil more about the development of asset prices. Furthermore, the concept of semicovariances could also be implemented in good versus bad volatility measures. Most importantly, the relighted view on downside risk could be incorporated into macroeconomic outcomes and resulting policies, as prompted by Adrian et al. (2019) and Adams et al. (2021), and the forecasting of crashes, as elicited by Bollerslev et al. (2020a). All suggestions emphasize that the concordant negative semibeta, or beta on "steroids", raises the bar in forecasting systemic and downside risk.

[28] This is succinctly described by Bollerslev et al. (2022): "*Temporal aggregation generally tends to mute non-linear dependencies in returns, and as such the daily semibetas may better reveal the inherent asymmetric dependencies than the monthly beta measures constructed from coarser daily returns.*".

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A. Appendix

Table A.1: **Cross-Sectional Predictions of Yearly Betas with Lagged Betas with Pooled OLS.** Panel A and B respectively show the predictive regressions with β_t and $\beta_t^{\mathcal{N}}$ as dependent variables. The panels report coefficients and underneath t-statistics are calculated with clustered standard errors. Constants are included into the regression but not reported. Estimates are determined by all contemporaneous common, non-penny, CRSP stocks spanning from July 1963 to December 2021.

Panel A: Dependent Variable β_t												
β_{t-1}	0.52		0.49		0.38		0.59		0.52		0.52	
	14.43		*8.55*		*6.21*		*10.14*		*15.44*		*14.18*	
β_{t-1}^{+}		0.44	0.03									
		11.12	*0.62*									
β_{t-1}^{-}		0.03	-0.00									
		1.14	*-0.45*									
$\beta_{t-1}^{\mathcal{N}}$				1.03	0.38							0.87
				19.16	*3.44*							*13.27*
$\beta_{t-1}^{\mathcal{P}}$						0.52	-0.13					0.29
						8.12	*-1.60*					*4.67*
$\beta_{t-1}^{\mathcal{M}^+}$								-0.22	0.06			-0.34
								-3.64	*0.48*			*-1.97*
$\beta_{t-1}^{\mathcal{M}^-}$										-0.45	-0.03	-0.64
										-7.61	*-0.36*	*-4.62*
R^2	0.37	0.34	0.37	0.31	0.39	0.17	0.38	0.01	0.37	0.02	0.37	0.40
Panel B: Dependent Variable $\beta_t^{\mathcal{N}}$												
β_{t-1}	0.22				0.07							
	14.13				*3.97*							
β_{t-1}^{+}		0.19	0.08									
		11.11	*5.68*									
β_{t-1}^{-}		0.01	-0.01									
		1.09	*-3.59*									
$\beta_{t-1}^{\mathcal{N}}$			0.44	0.54	0.42		0.41		0.53		0.53	0.41
			12.25	*20.10*	*9.79*		*11.41*		*20.36*		*20.31*	*11.18*
$\beta_{t-1}^{\mathcal{P}}$						0.31	0.13					0.14
						8.53	*4.63*					*4.35*
$\beta_{t-1}^{\mathcal{M}^+}$								0.18	0.04			-0.04
								8.36	*1.95*			*-0.76*
$\beta_{t-1}^{\mathcal{M}^-}$										0.24	0.07	0.02
										10.07	*2.60*	*0.36*
R^2	0.28	0.26	0.38	0.35	0.36	0.25	0.38	0.02	0.35	0.02	0.35	0.38

Table A.2: **Cross-Sectional Predictions of Downside Betas with Lagged Betas with Pooled OLS.** Panel A and B respectively show the predictive regressions with the monthly and yearly β_t^- as the dependent variable. The panel reports coefficients and underneath t-statistics are calculated with clustered standard errors. Constants are included into the regression but not reported. Estimates are determined by all contemporaneous common, non-penny, CRSP stocks spanning from July 1963 to December 2021.

Panel A: Dependent Variable Monthly β_t^-												
β_{t-1}	0.31		0.31									
	6.24		*5.60*									
β_{t-1}^+			-0.00									
			-0.28									
β_{t-1}^-		0.05	0.00		0.02		0.04		0.05		0.05	
		3.52	*0.09*		*2.55*		*2.66*		*3.51*		*3.18*	
$\beta_{t-1}^{\mathcal{N}}$				0.57	0.54							0.53
				44.90	*31.86*							*31.04*
$\beta_{t-1}^{\mathcal{P}}$						0.26	0.24					0.25
						3.72	*3.65*					*3.62*
$\beta_{t-1}^{\mathcal{M}^+}$								-0.13	-0.12			-0.24
								-7.18	*-6.24*			*-8.46*
$\beta_{t-1}^{\mathcal{M}^-}$										-0.15	-0.02	-0.19
										-6.07	*-0.36*	*-7.11*
R^2	0.05	0.01	0.05	0.04	0.04	0.02	0.03	0.00	0.01	0.00	0.01	0.06
Panel B: Dependent Variable Yearly β_t^-												
β_{t-1}	0.52		0.49									
	14.15		*8.56*									
β_{t-1}^+			0.04									
			0.65									
β_{t-1}^-		0.09	-0.00		-0.00		0.05		0.09		0.08	
		1.28	*-0.42*		*-0.19*		*1.09*		*1.31*		*1.24*	
$\beta_{t-1}^{\mathcal{N}}$				1.02	1.03							0.86
				19.42	*16.49*							*13.11*
$\beta_{t-1}^{\mathcal{P}}$						0.52	0.47					0.30
						8.07	*6.40*					*4.67*
$\beta_{t-1}^{\mathcal{M}^+}$								-0.22	-0.23			-0.33
								-3.23	*-2.82*			*-1.79*
$\beta_{t-1}^{\mathcal{M}^-}$										-0.45	-0.33	-0.66
										-7.16	*-2.99*	*-4.58*
R^2	0.34	0.05	0.34	0.28	0.28	0.16	0.17	0.01	0.06	0.02	0.06	0.36

Table A.3: **Predictions of Monthly Betas with Lagged Betas in the Cross-Section with Fama-MacBeth Type Regressions.** Panel A and B respectively show the predictive regressions with β_t and $\beta_t^{\mathcal{N}}$ as dependent variables. The panels report coefficients and underneath Newey-West robust t-statistics are calculated with 12 lags. Constants are included into the regression but not reported. Estimates are determined by all contemporaneous common, non-penny, CRSP stocks spanning from July 1963 to December 2021.

Panel A: Dependent Variable β_t										
β_{t-1}	0.40		0.28		0.33		0.43		0.43	
	26.78		*21.33*		*19.44*		*28.21*		*27.89*	
$\beta_{t-1}^{\mathcal{N}}$		0.81	0.33							0.53
		21.94	*12.51*							*21.70*
$\beta_{t-1}^{\mathcal{P}}$				0.66	0.15					0.40
				22.07	*7.15*					*24.70*
$\beta_{t-1}^{\mathcal{M}+}$						-0.14	0.46			-0.07
						-2.84	*12.06*			*-2.64*
$\beta_{t-1}^{\mathcal{M}-}$								-0.25	0.36	-0.15
								-5.01	*10.97*	*-5.31*
R^2	0.21	0.19	0.23	0.18	0.23	0.08	0.23	0.08	0.23	0.25
Panel B: Dependent Variable $\beta_t^{\mathcal{N}}$										
β_{t-1}	0.17		0.07							
	23.26		*12.03*							
$\beta_{t-1}^{\mathcal{N}}$		0.42	0.29		0.27		0.41		0.43	0.27
		32.94	*29.34*		*29.34*		*31.22*		*33.20*	*29.41*
$\beta_{t-1}^{\mathcal{P}}$				0.34	0.22					0.22
				28.28	*34.87*					*33.27*
$\beta_{t-1}^{\mathcal{M}+}$						0.17	0.13			0.14
						8.98	*10.86*			*13.42*
$\beta_{t-1}^{\mathcal{M}-}$								0.14	0.21	0.11
								6.93	*16.35*	*10.83*
R^2	0.34	0.35	0.37	0.36	0.41	0.25	0.37	0.25	0.37	0.43

Table A.4: **Predictions of Yearly Betas with Lagged Betas in the Cross-Section with Fama-MacBeth Type Regressions.** Panel A and B respectively show the predictive regressions with β_t and $\beta_t^{\mathcal{N}}$ as dependent variables. The panels report coefficients and underneath Newey-West robust t-statistics are calculated with 1 lag. Constants are included into the regression but not reported. Estimates are determined by all contemporaneous common, non-penny, CRSP stocks spanning from July 1963 to December 2021.

Panel A: Dependent Variable β_t										
β_{t-1}	0.68		0.55		0.55		0.70		0.70	
	39.82		*14.74*		*13.15*		*39.39*		*42.63*	
$\beta_{t-1}^{\mathcal{N}}$		1.30	0.32							0.76
		26.84	*5.09*							*15.98*
$\beta_{t-1}^{\mathcal{P}}$				1.15	0.31					0.70
				20.69	*3.89*					*15.66*
$\beta_{t-1}^{\mathcal{M}^+}$						-0.22	0.33			-0.59
						-0.85	*4.58*			*-8.23*
$\beta_{t-1}^{\mathcal{M}^-}$								-0.27	0.46	-0.38
								-1.06	*4.88*	*-4.04*
R^2	0.55	0.49	0.57	0.48	0.57	0.16	0.57	0.16	0.58	0.59
Panel B: Dependent Variable $\beta_t^{\mathcal{N}}$										
β_{t-1}	0.30		0.09							
	32.46		*4.54*							
$\beta_{t-1}^{\mathcal{N}}$		0.66	0.50		0.37		0.65		0.66	0.37
		34.46	*13.70*		*22.58*		*34.39*		*37.79*	*19.58*
$\beta_{t-1}^{\mathcal{P}}$				0.59	0.32					0.32
				22.79	*14.17*					*15.20*
$\beta_{t-1}^{\mathcal{M}^+}$						0.36	0.12			0.01
						3.44	*2.67*			*0.25*
$\beta_{t-1}^{\mathcal{M}^-}$								0.42	0.21	0.14
								3.67	*4.65*	*3.65*
R^2	0.45	0.50	0.53	0.50	0.55	0.16	0.52	0.15	0.53	0.58

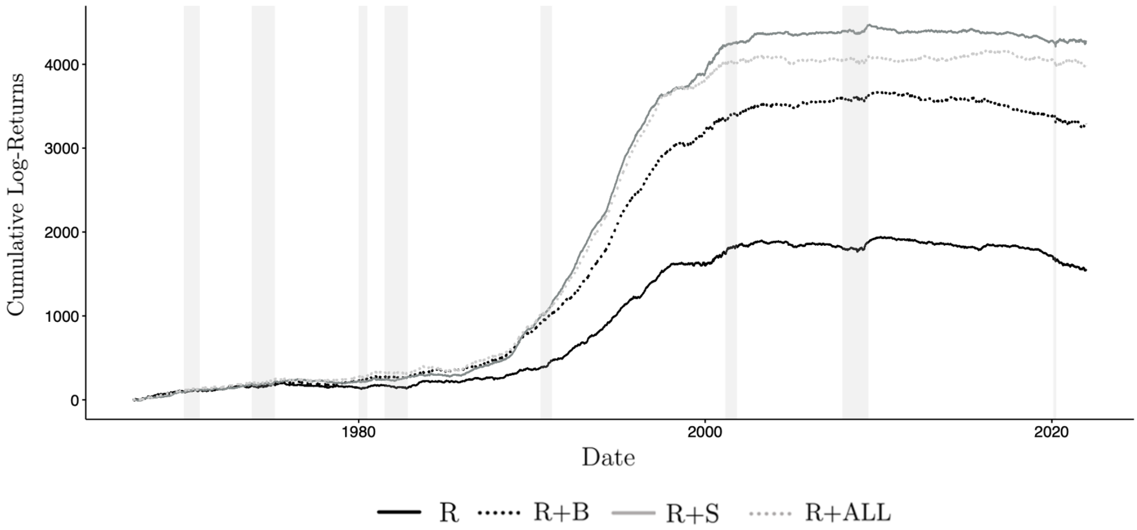

Figure A.1: **Cumulative Performance of 10-10 Portfolios from 1967 until 2021.** This figure shows the cumulative performance of the 10-10 Portfolio over time based on different prediction models classified by the random forests algorithm. Model R solely contains lagged return features. Model R+B and R+S respectively add the traditional beta and semibeta features to the baseline model. Model R+ALL includes all features. The shaded regions represent recession periods as defined by the NBER. Estimates are based on the period from January 1967 to December 2021.

Table A.5: **Risk-Adjusted Performance of Random Forests Trading Strategies with Returns and (Semi)Betas over 2002-2021.** The upper panel lists the annualized mean return of the long-short strategy and its corresponding standard deviation and Sharpe ratio. The strategies are self-financing and go long and short in the stocks with respectively the ten highest and lowest probabilities of outperforming the cross-sectional median on the next day. Portfolios are rebalanced on a daily basis. R includes only the lagged return features, R+B (R+S) the lagged return features and the traditional beta (semi beta) features. R+ALL includes all lagged return and beta features. The bottom-most panel displays estimates of the time-series regression on the FF3, FF3+2, and FF5 factor models with Newey-West robust t-statistics and annualized alphas.

		R			R+B			R+S			R+ALL	
Mean		-8.23			1.84			7.04			3.59	
St.Dev.		34.85			39.47			36.21			35.92	
Sharpe		-0.24			0.05			0.19			0.10	
α	-9.87	-13.02	-9.34	0.26	-4.55	0.27	5.09	-0.35	4.97	2.34	-1.94	2.00
	-1.36	*-1.87*	*-1.28*	*0.03*	*-0.55*	*0.03*	*0.62*	*-0.05*	*0.60*	*0.29*	*-0.25*	*0.25*
β^{MKT}	0.18	0.05	0.16	0.18	0.02	0.17	0.16	-0.03	0.15	0.14	-0.02	0.14
	4.11	*1.08*	*3.42*	*3.26*	*0.31*	*2.84*	*3.41*	*-0.59*	*2.96*	*3.17*	*-0.49*	*3.02*
β^{SMB}	0.01	0.01	0.03	-0.12	-0.12	-0.03	0.11	0.12	0.18	-0.10	-0.09	-0.02
	0.07	*0.21*	*0.38*	*-1.21*	*-1.45*	*-0.31*	*0.96*	*1.30*	*1.53*	*-1.13*	*-1.30*	*-0.26*
β^{HML}	0.09	0.10	0.17	0.20	0.25	0.33	0.03	0.06	0.09	0.13	0.13	0.23
	1.13	*1.28*	*1.88*	*2.03*	*2.84*	*3.01*	*0.37*	*0.93*	*0.99*	*1.58*	*1.71*	*2.55*
β^{MOM}		0.03			0.10			0.09			0.03	
		0.65			*1.87*			*1.88*			*0.63*	
β^{REV}		0.55			0.73			0.81			0.67	
		7.72			*8.68*			*8.71*			*8.74*	
β^{RMW}			0.05			0.23			0.19			0.24
			0.45			*1.80*			*1.80*			*2.23*
β^{CMA}			-0.45			-0.67			-0.50			-0.51
			-2.87			*-3.72*			*-2.62*			*-3.29*
R^2	0.01	0.06	0.02	0.01	0.08	0.02	0.01	0.10	0.02	0.01	0.08	0.02