QUANTITY PRECOMMITMENT AND PRICE COMPETITION

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1. Introduction

Imagine two clothing retailers both selling the same brand and located close to each other so that location does not matter in the eyes of consumers. They must both decide on how many clothes to buy from the upstream brand for the coming summer collection. They can only buy clothes once and after that they cannot buy any more. Or think about any market where production takes a lot of time and firms need to decide beforehand on how many stock they want to keep. If in the example of the two clothing retailers there is a 'Cournot market' both retailers would decide on how much stock (quantity) to buy and bring this quantity to the market. Then an auctioneer would match supply and demand and decide upon prices. But if this is a 'Bertrand market' firms would set prices and see what demand is. They would then buy exactly the amount of clothes demanded by consumers at their given prices. If we look at reality we know that neither of the two markets is true on its own. Retailers decide themselves on how much stock to buy and on what prices they ask. A more realistic market would thus be a market where firms choose their quantities as well as their prices. In this paper I study this situation.

In the existing literature this is an often-discussed topic. It is important to model how firms choose their quantities and prices to get a better understanding of the outside world. There is quite some discussion on, when firms choose their quantities and prices, and on what quantities and prices are optimal. Below I show some of the most important and well-known literature discussing this topic and we see that there is a lot of disagreement.

Kreps and Scheinkman (1983) look at this problem and model an oligopoly game where firms first choose their capacities simultaneously and independently and after production levels are made public compete in price competition. They find that there is a unique equilibrium level which coincides with the Cournot outcome. The key difference with the model of Kreps and Scheinkman (1983) and the one in this paper has to do with the cost structure. In this model we assume constant marginal cost while Kreps and Scheinkman (1983) make use of a convex cost curve.

Following the paper of Kreps and Scheinkman (1983), Yin and Ng (1997) model a simple duopoly game within the framework of Kreps and Scheinkman (1983), and find that the results found by Kreps and Scheinkman (1983) also hold when products are heterogeneous instead of homogeneous.

Like Kreps and Scheinkman (1983), Davidson and Deneckere (1986) look at a duopoly model where firms first choose their capacities before engaging in price competition. However, unlike Kreps and Scheinkman (1983) they show that the Cournot equilibrium is unlikely to emerge and that the equilibrium outcome tends to be more competitive than the Cournot equilibrium. Their results show a tendency towards asymmetric capacities and price dispersion.

Dudey (1992) investigates a somewhat similar problem. He also looks at a combination of price and quantity competition. But does that for a multiperiod model where consumers come to the market at different times and where firms can change their price at any time. Dudey (1992) concludes that if at least one seller out of two (duopoly) cannot supply total market demand, both firms will earn positive profits.

Furthermore Levitan and Shubik (1972) model a simple duopoly game where firms compete in prices and are capacity constrained. They assume that firms have equal capacities and that, when prices are equal, demand is split equally between the two firms. Although I make some different assumptions about market split and I do not assume that firms have equal capacities, I find just like Levitan and Shubik (1972) a subgame perfect Nash equilibrium in mixed strategies.

Dasgupta and Maskin (1986) study the existence of mixed-strategy equilibria for different games with discontinuous payoff functions. One of them is the Bertrand-Edgeworth game where two firms with capacity constraints choose their prices. In comparison with my assumption about market split and that of Levitan and Shubik (1972), Dasgupta and Maskin (1986) make yet another assumption. Namely that in the case when prices are equal, demand is split in proportion to the firms' capacities, as long as their capacities are not met.

In reality there is almost always some kind of product differentiation. Do not only think about the product itself but also about the location of the store, how the store looks, how it is equipped and even the people who work in the store can make a difference for some consumers. Bos and Vermeulen (2021a) solve the Bertrand/Cournot problem for any degree of product differentiation except for the case where the product are perfect substitutes (homogenous). In this paper we will make use of homogeneity and check whether there is some kind of equilibrium. Bos and Vermeulen (2021a) find in their paper that there exist pure-strategy equilibria. One of the critical assumptions they use in their model is that of limited spillover, this means that, if $p_1 < p_2$ and firm one cannot meet demand at the given price, not all consumers who were not supplied at firm one will go to firm two. Some consumers may go home, even though they would be willing to pay the higher price at firm two. Unlike Bos and Vermeulen (2021a), I do solve for complete spillover. In other words, if $p_1 < p_2$ and firm one cannot meet demand at the given price, all consumers who were not supplied at firm one but are willing to pay the higher price at retailer two, will go and buy from retailer two (of course if retailer two has sufficient capacity). Bos and Vermeulen (2021a) show that there is no pure-strategy equilibrium when there is complete spillover. In contrast, I show that if quantities are chosen before prices, a pure-strategy Nash equilibrium does exist.

Bos and Vermeulen (2021b) investigates price-quantity competition but this time with homogeneous goods. They find a pure-strategy equilibrium. What makes this paper different from mine has to do with two things, first of all that firm one and two choose a price-quantity pair in the same period whilst in my model the firms choose quantity in period zero and set prices in period one. The second difference has once again to do with spillover demand. Bos and Vermeulen (2021b) state that they allow for any degree of spillover demand but in a footnote they say that when there is complete spillover the Nash-equilibrium does not hold. As said before, I consider the case where quantity is chosen before prices, and where there is complete spillover.

Section 2 gives a general outline of the model. The main analyses and results are presented in section 3. Section 4 concludes the paper.

2. Model

Consider a market with two firms and a continuum of consumers. The firms sell a homogeneous good. The model consists of two periods. In period zero, both firms simultaneously and independently choose their quantities and then in period one, after quantities are made public, they compete in prices. The demand function is perfectly known to both firms. Demand is linear, in particular $D(p) = a - p$ where p is the price. This demand function implicitly assumes that consumers have different reservation prices. This is in contrast with the assumption made by Dudey (1992). He assumes that all consumers have the same reservation value for the good. Both firms face the same unit cost, c. The quantities firms choose are denoted by V_i and V_j . After period zero both firms know each other's quantities. We will use backward induction to solve this model and hence because both firms choose their quantities in period zero we do not need to worry about any costs in period one. Before we go into the analyses of this model I will make some assumptions.

A1 Efficient rationing rule

I use the same rationing rule as Kreps and Scheinkman (1983), namely 'efficient rationing'. This means that if $p_2 < p_1$ residual demand for firm one will be max $\{0, a - D(p_2) - p_1, a - V_2 - p_1\}$. This is in contrast with the rationing rules used by Davidson and Deneckere (1986). They make use of 'randomized rationing'. This means that if $p_2 < p_1$ residual demand for firm one will be $D(p_1)(\frac{D(p_2)-V_2}{D(p_2)})$.

A2 If prices are equal, the firm with the smallest quantity will sell first.

I make this assumption to avoid some difficult complications, and therefore makes the model better to interpret. This assumption is different form the assumption Levitan and Shubik (1972) make when prices are equal. Levitan and Shubik (1972) also look at capacity constraints and price competition within a duopoly setting. In their model if prices are equal demand is split equally between firms. They can easily make this assumption because they also assume that both firms have identical capacities. Because of this they do not encounter the complications we would if we do not assume **A2**. Furthermore, Dasgupta and Maskin (1986), make yet another assumption about how the market is split when prices are equal. The assume that in the case where prices are equal, demand is split in proportion to their capacities, as long as their capacities are not met. Moreover, in the case where prices and quantities are equal, market demand is split.

3. Analysis

I will know get into the analysis of the model and the main findings. Since we look at a two-period model I make use of backward induction. Because of that I will also start by showing the solution to period one and after that the solution to period zero.

Lemma. V_i and V_j will never be bigger than $a - c$.

Proof.
The maximum quantity a firm might sell is a, when firm i for instance chooses a higher quantity than a, firm j will act more competitive and lower its price. Thereby lowering the expected profits for firm i.

3.1 Period one

Recall that the demand function is $D(p) = a - p$. I found that under the assumptions made there are two different 'cases'. One case where $V_i + V_j \leq \frac{1}{2}a$ and one case where $V_i + V_j > \frac{1}{2}a$. In both cases I find different equilibrium strategies.

Without loss of generality we can assume that $V_j \leq V_i$. For any combination of V_i and V_j .

Proposition 1. Under A1 and A2, the equilibrium price is the market clearing price, when $V_i + V_j \leq \frac{1}{2}a$.

Proof.
Let $V_i = \frac{1}{2}a - x$ and $V_j = \frac{1}{4}a - y$, where $0 \leq y < \frac{1}{4}a$ and $V_j \leq x < \frac{1}{2}a$. The market clearing price is: $\frac{1}{2}a - x + \frac{1}{4}a - y = a - p$, solving gives $p^* = \frac{1}{4}a + x + y$. In case both firms set prices equal to p^* their profits are:

$$\Pi_i = \left(\frac{1}{2}a - x\right) \cdot \left(\frac{1}{4}a + x + y\right) = \frac{1}{8}a^2 + \frac{1}{4}ax + \frac{1}{2}ay - xy - x^2$$

$$\Pi_j = \left(\frac{1}{4}a - y\right) \cdot \left(\frac{1}{4}a + x + y\right) = \frac{1}{8}a^2 + \frac{1}{4}ax - xy - y^2$$

It can easily be seen pricing below p^* is never profitable as you will sell the same quantity against a lower price. Hence profits go down.

Pricing above p^* is also not profitable:
What happens to the profits of firm i if it sets its price equal to $p = \frac{1}{4}a + x + y + \varepsilon$, where ε is a small positive number. Because $p_j < p_i$ firm j will sell first. Residual demand for firm i is then: $D(p) = a - p - \left(\frac{1}{4}a - y\right) = \frac{3}{4}a + y - p$, substitute $p = \frac{1}{4}a + x + y + \varepsilon$ and we find that demand for firm i = $D(p) = \frac{3}{4}a + y - \left(\frac{1}{4}a + x + y + \varepsilon\right) = \frac{1}{2}a - x - \varepsilon$. This leads to a profit for firm i of:

$$\Pi_i = \left(\frac{1}{2}a - x - \varepsilon\right) \cdot \left(\frac{1}{4}a + x + y + \varepsilon\right)$$
$$= \frac{1}{8}a^2 + \frac{1}{4}ax + \frac{1}{2}ay - xy - x^2 + \frac{1}{4}a\varepsilon - y\varepsilon - 2x\varepsilon - \varepsilon^2$$

When $\frac{1}{4}a\varepsilon - y\varepsilon - 2x\varepsilon - \varepsilon^2 \leq 0$ holds, increasing your price does not benefit your profits. Rewriting gives: $\frac{1}{4}a \leq y + 2x + \varepsilon$, by the properties of x and y we can prove this.

$$\min y = 0 \; then \min x = \frac{1}{4}a$$
$$\frac{1}{4}a \leq 0 + 2 \cdot \frac{1}{4}a + \varepsilon$$
$$\frac{1}{4}a \leq \frac{1}{2}a + \varepsilon$$

And,

$$\max y \approx \frac{1}{4}a \; then \min x \approx 0$$
$$\frac{1}{4}a \leq \frac{1}{4}a + 2 \cdot 0 + \varepsilon$$
$$\frac{1}{4}a \leq \frac{1}{4}a + \varepsilon$$

Since both inequalities hold, we proved that increasing your price does not benefit your profits.

Next, I look what happens to the profits of firm j is it increases its price. What if firm j sets its price equal to $p = \frac{1}{4}a + x + y\ + \varepsilon$, where ε is a small positive number. Because $p_i < p_j$ firm i will sell first. Residual demand for firm j is then: $D(p) = a - p - \left(\frac{1}{2}a - x\right) = \frac{1}{2}a + x - p$, substitute $p = \frac{1}{4}a + x + y\ + \varepsilon$ and we find that demand for firm j = $D(p) = \frac{1}{2}a + x - \left(\frac{1}{4}a + x + y\ + \varepsilon\right) = \frac{1}{4}a - y - \varepsilon$. This leads to a profit for firm j of:

$$\Pi_j = \left(\frac{1}{4}a - y - \varepsilon\right) \cdot \left(\frac{1}{4}a + x + y\ + \varepsilon\ \right)$$
$$= \frac{1}{16}a^2 + \frac{1}{4}ax - xy - y^2 - 2y\varepsilon - x\varepsilon - \varepsilon^2$$

Here we can easily see that increasing your price does not benefit your profits.

Now consider the second case, so what happens if $V_i + V_j > \frac{1}{2}a$.
For some values of V_i and V_j this case contains pure-strategy equilibria. For other values of V_i and V_j this case does not contain any pure-strategy equilibrium. That's why for these values of V_i and V_j I make use of mixed-strategy equilibria.

Let's start with the easier one, namely the one where there are pure-strategy equilibria. Without loss of generality we can assume that $V_j \leq V_i$.

Proposition 2. When $V_i + V_j > \frac{1}{2}a\ and\ V_i \leq \frac{a - V_j}{2}$ there exists a pure-strategy equilibrium. The price we find in equilibrium is the market clearing price.

Proof.
Once again, without loss of generality suppose that $V_j \leq V_i$. Because of $V_i \leq \frac{a - V_j}{2}$ and $V_j \leq V_i$ it follows that $V_j \leq \frac{1}{3}a$. Let $V_i = \frac{1}{3}a + x$ and $V_j = \frac{1}{3}a - y$, where $0 \leq x < \frac{1}{6}a$ and $0 \leq y < \frac{1}{3}a$ and $x \leq \frac{1}{2}y$.

The market clearing price is: $\frac{1}{3}a + x + \frac{1}{3}a - y = a - p$, solving gives $p^* = \frac{1}{3}a - x + y$. In case both firms set their prices equal to p^* their profits are:

$$\Pi_i = \left(\frac{1}{3}a + x\right) \cdot \left(\frac{1}{3}a - x + y\right) = \frac{1}{9}a^2 + \frac{1}{3}ay + xy - x^2$$
$$\Pi_j = \left(\frac{1}{3}a - y\right) \cdot \left(\frac{1}{3}a - x + y\right) = \frac{1}{9}a^2 - \frac{1}{3}ax + xy - y^2$$

Like in the first case it can easily be seen pricing below p^* is never profitable as you will sell the same quantity against a lower price. Hence profits go down.

Pricing above p^* is also not profitable:
What happens to the profits of firm i if it sets its price equal to $p = \frac{1}{3}a - x + y\ + \varepsilon$, where ε is a small positive number. Because $p_j < p_i$ firm j will sell first. Residual demand for firm i is then: $D(p) = a - p - \left(\frac{1}{3}a - y\right) = \frac{2}{3}a + y - p$, substitute $p = \frac{1}{3}a - x + y\ + \varepsilon$ and we find

that demand for firm i = $D(p) = \frac{2}{3}a + y - \left(\frac{1}{3}a - x + y\ + \varepsilon\right) = \frac{1}{3}a + x - \varepsilon$. This leads to a profit for firm i of:

$$\Pi_i = \left(\frac{1}{3}a + x - \varepsilon\ \right) \cdot \left(\frac{1}{3}a - x + y\ + \varepsilon\ \right)$$
$$= \frac{1}{9}a^2 + \frac{1}{3}ay + xy - x^2 + 2x\varepsilon - y\varepsilon - \varepsilon^2$$

And hence, because $x \leq \frac{1}{2}y$ we see that increasing your price does not increase your profits.

As in the first case I will now look what happens to the profits of firm j is it increases its price. What if firm j sets its price equal to $p = \frac{1}{3}a - x + y\ + \varepsilon$, where ε is a small positive number. Because $p_i < p_j$ firm i will sell first. Residual demand for firm j is then: $D(p) = a - p - (\frac{1}{3}a + x) = \frac{2}{3}a - x - p$, substitute $p = \frac{1}{3}a - x + y\ + \varepsilon$ and we find that demand for firm j = $D(p) = \frac{2}{3}a - x - \left(\frac{1}{3}a - x + y\ + \varepsilon\right) = \frac{1}{3}a - y - \varepsilon$. This leads to a profit for firm j of:

$$\Pi_j = \left(\frac{1}{3}a - y - \varepsilon\right) \cdot \left(\frac{1}{3}a - x + y\ + \varepsilon\ \right)$$
$$= \frac{1}{9}a^2 - \frac{1}{3}ax + xy - y^2 - 2y\varepsilon + x\varepsilon - \varepsilon^2$$

Once again, because $x \leq \frac{1}{2}y$ we can easily see that increasing your price does not benefit your profits.

The higher quantity firm is more likely to differ from p^* and ask a higher price than the lower quantity firm. This may seem odd at first. However, remember that when prices are equal, the firm with the lower quantity will sell first. As a result there is only a one-sided effect when the higher quantity firm sets a higher price. Namely, the loss of its own consumers due to the higher price. This is called the income effect. But the higher quantity firm does not lose consumers because they switch to the lower priced firm. This is called the substitution effect. Vice versa, because the lower quantity firm would suffer from both the income as well as the substitution effect, it is less likely to differ from p^* and ask a higher price.

Finally we will look at the case when $V_i + V_j > \frac{1}{2}a\ and\ V_i > \frac{a - V_j}{2}$. In this case there exist no pure-strategy equilibrium. Therefore I make use of mixed-strategy equilibria.
Let us start by looking at what the possible payoffs are for firm i, if $V_j < V_i$ for different combinations of p_i and p_j :

$$\Pi_i(p_i, p_j) = \begin{cases} \min\{p_i V_i, p_i D(p_i)\} & if\ p_i < p_j \\ \max\{0, (a - V_j - p_i)p_i, (a - D(p_j) - p_i)p_i\} & if\ p_i \geq p_j \end{cases}$$

Hence, there is no difference between $p_i = p_j$ and $p_i > p_j$, this is due to **A2**.

These payoffs may seem fine, but they are not. If, for instance, firm j sets a lower price than firm i, firm j must sell all its quantity otherwise p_j is not a best reply, and vice versa. Therefore the actual payoffs for firm i look like:

$$\Pi_i(p_i, p_j) = \begin{cases} p_i V_i & if\ p_i < p_j \\ \max\{0, (a - V_j - p_i)p_i\} & if\ p_i \geq p_j \end{cases}$$

Now we know the possible payoffs, we can go and find the cumulative distribution function (CDF) for firm i and firm j. Define F_i as the CDF for i and F_j as the CDF for j.

CDF, F: [0,1], $x \in [0,1]$, $F(x)$ is the probability that the firm sets a price $p_i < x$. F is continuous. Firm i's payoff from setting price p_i is:

$$V_i p_i(1 - F(p_i)) + (a - V_j - p_i)p_i F(p_i)$$

Where $1 - F(p_i)$ is the probability that $p_j > p_i$ and $F(p_i)$ is the probability that $p_j \leq p_i$.

To get F_i we need to find the upper and lower bound of p_i , so what is the maximum and minimum price firm i is willing to choose. Let's call the upper bound $\overline{p}$ and the lower bound $\underline{p}$. The $\overline{p}$ can be found by taking the FOC of $(a - V_j - p_i)p_i$. We get that $\overline{p} = \frac{a-V_j}{2}$. Since $F(\overline{p}) = 1$ the payoff of setting $p_i = \frac{a-V_j}{2}$ is:

$$\left(a - V_j - \left(\frac{a - V_j}{2}\right)\right)\left(\frac{a - V_j}{2}\right) \cdot 1 = \left(\frac{a - V_j}{2}\right)^2$$

This means that any price firm i sets with positive probability has to yield a payoff of $\left(\frac{a-V_j}{2}\right)^2$.
$\forall\, x \in \left[\underline{p}, \frac{a-V_j}{2}\right] \rightarrow V_i x(1 - F(x)) + (a - V_j - x)xF(x) = \left(\frac{a-V_j}{2}\right)^2$

Solving for $F(x)$ gives:

$$V_i x - V_i xF(x) + axF(x) - V_j xF(x) - x^2 F(x) = \left(\frac{a - V_j}{2}\right)^2$$

$$F(x)(ax - V_i x - V_j x - x^2) = \left(\frac{a - V_j}{2}\right)^2 - V_i x$$

$$F_i(x) = \frac{\left(\frac{a - V_j}{2}\right)^2 - V_i x}{(a - V_i - V_j)x - x^2}$$

Since $F\left(\underline{p}\right) = 0$

$$\frac{\left(\frac{a - V_j}{2}\right)^2 - V_i \underline{p}}{(a - V_i - V_j)\underline{p} - \underline{p}^2} = 0$$

Solving for $\underline{p}$ gives:

$$V_i \underline{p} = \left(\frac{a - V_j}{2}\right)^2$$

$$\underline{p} = \frac{\left(\frac{a - V_j}{2}\right)^2}{V_i}$$

Thus firm i randomizes its price over $[\frac{\left(\frac{a-V_j}{2}\right)^2}{V_i}, \left(\frac{a-V_j}{2}\right)]$ according to $F_i(x) = \frac{\left(\frac{a-V_j}{2}\right)^2 - V_i x}{(a-V_i-V_j)x - x^2}$. Since firm j has no incentive to price lower then $\underline{p}$, hence firm j would sell the same quantity against a lower price, the expected profits for firm j, $\Pi_j = \underline{p}V_j$. This means that any price firm j sets with positive probability has to yield a payoff of $\frac{\left(\frac{a-V_j}{2}\right)^2}{V_i} V_j$.

$$\forall\, x \in \left[\frac{\left(\frac{a - V_j}{2}\right)^2}{V_i}, \overline{p}\right] \rightarrow V_j x\big(1 - F(x)\big) + (a - V_i - x)xF(x) = \frac{\left(\frac{a - V_j}{2}\right)^2}{V_i} V_j$$

Solving for $F(x)$ gives:

$$V_j x - V_j x F(x) + axF(x) - V_i x F(x) - x^2 F(x) = \frac{\left(\frac{a - V_j}{2}\right)^2}{V_i} V_j$$

$$F(x)\big(ax - V_j x - V_i x - x^2\big) = \frac{\left(\frac{a - V_j}{2}\right)^2}{V_i} V_j - V_j x$$

$$F_j(x) = \frac{\frac{\left(\frac{a - V_j}{2}\right)^2}{V_i} V_j - V_j x}{(a - V_j - V_i)x - x^2}$$

In the case before we saw that when $V_i \leq \frac{a-V_j}{2}$ there is no incentive for firm j to price higher than the market clearing price. When $V_i = \frac{a-V_j}{2}$ the market clearing price equals our $\overline{p}$ so when $V_i > \frac{a-V_j}{2}$ there is clearly never an incentive for firm j to ask a higher price as firm j will face even less residual demand. That's why firm j has the same $\overline{p}$ as firm i.

Thus firm j randomizes its price over $[\frac{\left(\frac{a-V_j}{2}\right)^2}{V_i}, \left(\frac{a-V_j}{2}\right)]$ according to $F_j(x) = \frac{\frac{\left(\frac{a-V_j}{2}\right)^2}{V_i} V_j - V_j x}{(a-V_j-V_i)x - x^2}$.

3.2 Period zero

Now that we have solved period one, and know each firm's pricing strategies for different cases, we can move on to period zero. In period zero both firms will independently and simultaneously choose their quantities. From the mixed strategies we learned what the expected profits are for firm i and j when $V_i > V_j$ holds. These are:

$$E\Pi_i = \left(\frac{a - V_j}{2}\right)^2 - cV_i$$

$$E\Pi_j = \frac{\left(\frac{a - V_j}{2}\right)^2}{V_i} V_j - cV_j$$

Where c is a constant unit cost.

Before we continue let us rewrite $E\Pi_j$ to $E\Pi_j = \left(\frac{a-V_j}{2}\right)^2 \cdot \frac{V_j}{V_i} - cV_j$. This notation will make it easier to solve for the equilibrium quantity.

One possible equilibrium is the Cournot equilibrium. In our model that means that $V_i = \frac{a-c}{3}$ and that $V_j = \frac{a-c}{3}$. If we substitute these values into the expected profit functions we immediately see that this is an equilibrium:

$$\left(\frac{a - (\frac{1}{3}a - \frac{1}{3}c)}{2}\right)^2 - \left(\frac{1}{3}a - \frac{1}{3}c\right)c = \left(\frac{a - (\frac{1}{3}a - \frac{1}{3}c)}{2}\right)^2 \cdot \frac{(\frac{1}{3}a - \frac{1}{3}c)}{(\frac{1}{3}a - \frac{1}{3}c)} - \left(\frac{1}{3}a - \frac{1}{3}c\right)c$$

$$\frac{1}{9}a^2 - \frac{2}{9}ac + \frac{13}{36}c^2 = \frac{1}{9}a^2 - \frac{2}{9}ac + \frac{13}{36}c^2$$

$$E\Pi_i = E\Pi_j$$

This is the only equilibrium that holds because the best reply for firm i to V_j , $Br_i(V_j)$, is always $\left(\frac{a-V_j}{2}\right)$. Firm i will never want to choose a higher quantity as he will not sell the extra quantity above $\left(\frac{a-V_j}{2}\right)$, hence this is the 'monopoly' quantity over the residual demand. Choosing a higher quantity thus only leads to higher costs.

Is there an incentive for firm j to choose a lower quantity? If firm j does choose a lower quantity we move from the mixed-strategy equilibrium to the pure-strategy equilibrium because we switch to the case where $V_i + V_j > \frac{1}{2}a \text{ and } V_i \leq \frac{a-V_j}{2}$. The equilibrium price in this case equals the market clearing price. So if firm j instead of choosing the Cournot quantity chooses $V_j = \frac{a-c}{3} - \varepsilon$, its profits will be:

$$\left(a - \left(\frac{1}{3}a - \frac{1}{3}c\right) - \left(\frac{1}{3}a - \frac{1}{3}c - \varepsilon\right)\right) \cdot \left(\frac{1}{3}a - \frac{1}{3}c - \varepsilon\right) - \left(\frac{1}{3}a - \frac{1}{3}c - \varepsilon\right)c$$

$$= \left(\frac{1}{3}a + \frac{2}{3}c + \varepsilon\right) \cdot \left(\frac{1}{3}a - \frac{1}{3}c - \varepsilon\right) - \left(\frac{1}{3}a - \frac{1}{3}c - \varepsilon\right)c$$

$$= \frac{1}{9}a^2 - \frac{2}{9}ac + \frac{1}{9}c^2 - \varepsilon^2$$

If we then compare the profit for firm j when it chooses the Cournot quantity and when it chooses a lower quantity respectively, we get:

$$\frac{1}{9}a^2 - \frac{2}{9}ac + \frac{13}{36}c^2 = \frac{1}{9}a^2 - \frac{2}{9}ac + \frac{1}{9}c^2 - \varepsilon^2$$

$$\frac{13}{36}c^2 = \frac{1}{9}c^2 - \varepsilon^2$$

The left-hand side of this equation is always bigger than the right-hand sight. In other words, the expected profits when choosing the Cournot quantity are always higher than the expected profits when choosing a quantity lower than the Cournot quantity. So, there is no incentive for firm j to choose a lower quantity than the Cournot quantity. To conclude, there is no incentive for firm i to choose a quantity other than $\frac{a-c}{3}$ and consequently there is neither an incentive for firm j to choose a quantity other than $\frac{a-c}{3}$. Therefore the subgame perfect Nash equilibrium is equivalent to the Cournot equilibrium. As a result, in the two-period game, there is a unique equilibrium outcome, namely the Cournot outcome where $V_i = V_j = \frac{a-c}{3}$ and where $p_i = p_j = \frac{a+2c}{3}$.

4. Concluding remarks

In this paper I showed what the optimal strategies are when two firms compete in prices but beforehand need to choose their quantities. Even though Kreps and Scheinkman (1983) made use of a convex cost curve, and I of a simple constant unit cost. The found equilibrium outcome is the same, namely the Cournot outcome. I showed that Bertrand and Cournot competition should not be seen as two complete opposites of each other. Both are correct in their own way. The Cournot quantity will arise in equilibrium but prices are not set by some auctioneer as is implicitly assumed by Cournot. Firms are the ones who set prices, just like in Bertrand. Combining Bertrand and Cournot as done in this model yields a far more realistic outcome than the two models on their own. Namely that firms choose their own quantities and their own prices and still make some profit.

When I first started this paper it was meant to model a three-period game. Where period zero would have been the same as in this paper, choosing quantities. But period one and two would both have been sales periods. I thought of this as a firm choosing quantities, then having a normal sales period and thereafter a sales period with discounts. I still think this is very meaningful research to do as it represents reality even better than my two-period model. A nice extension to the model in this paper would be to see if there exists asymmetric equilibrium due to the unit costs. Further extensions could be made by increasing the number of firms or replacing **A2** with equal demand split when prices are equal.

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